AN EQUATION OF THE HYPERBOLA

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ABSTRACT. We derive an equation, $g(x)$, of the hyperbola through three given points (α_i, β_i) $i = 1, 2, 3$ with asymptotes having slopes m_1 and m_2:

$$g(x) = (\frac{m_1 + m_2}{2})k(x) \pm (\frac{m_1 - m_2}{2})\sqrt{k^2(x) + 4r} - m_1 s - m_2 t,$$

where $k(x) = x + s + t$ and r, s and t are connected to the given points by the equations:

$$r + (s + a_i)(t - b_i) = 0, \qquad i = 1, 2, 3.$$

1. Introduction

Using elementary methods, we begin by solving the system:

$$r + (s + a_i)(t - b_i) = 0, \qquad i = 1, 2, 3, \tag{1}$$

for r, s, and t in terms of the (a_i, b_i), giving:

$$\begin{aligned} r &= a_3\hat{P}_{12} + a_1\hat{P}_{23} + a_2\hat{P}_{31}, \\ -s &= a_3\hat{Q}_{12} + a_1\hat{Q}_{23} + a_2\hat{Q}_{31}, \\ t &= b_3\hat{R}_{12} + b_1\hat{R}_{23} + b_2\hat{R}_{31}, \end{aligned}$$

where, for $Q_{ij} = \frac{(a_i - a_j)^2}{b_i - b_j}$ and $R_{ij} = a_k(b_i - b_j)$, $\quad \{i, j, k\} = \{1, 2, 3\}$

$$\begin{aligned} \hat{P}_{ij} &= \frac{a_i^2 - a_j^2}{Q_{12} + Q_{23} + Q_{31}}, \\ \hat{Q}_{ij} &= \frac{Q_{ij}}{Q_{12} + Q_{23} + Q_{31}}, \\ \hat{R}_{ij} &= \frac{R_{ij}}{Q_{12} + Q_{23} + Q_{31}}. \end{aligned}$$

For given point coordinates (α_i, β_i), $\quad i = 1, 2, 3$, define $\begin{pmatrix} a_i \\ b_i \end{pmatrix} = C \begin{pmatrix} \alpha_i \\ \beta_i \end{pmatrix}$, where

$$C = \begin{pmatrix} 1 & -1 \\ m_1 & m_2 \end{pmatrix}.$$

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2. Geometric Interpretation

Denote by Ω the graph of (1), regarding a_i as an independent variable, x, and b_i a dependent variable, $y = f(x)$:

$$\Omega = \{(x, y) \mid y = f(x)\},$$

and define $g(x)$ by the graph Γ satisfying $\Gamma = C\,\Omega$, i.e.

$$\{\begin{pmatrix} x \\ g(x) \end{pmatrix}\} = \{C \begin{pmatrix} x \\ f(x) \end{pmatrix}\}.$$

Letting $h(x) = x - f(x)$, evidently

$$g(x) = g \circ h \circ h^{-1}(x) = m_1 h^{-1}(x) - m_2 f \circ h^{-1}(x),$$

which, upon substitution for $h^{-1}(x)$, yeilds:

$$g(x) = (\frac{m_1 + m_2}{2})k(x) \pm (\frac{m_1 - m_2}{2})\sqrt{k^2(x) + 4r} - m_1 s - m_2 t,$$

where $k(x) = x + s + t$.

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