

4

Random Walks and Mobile Entities

In this chapter we consider a fundamental stochastic model, the *random walk*, which is the basis for a wide range of simulation models. Random walks, whether actually spatial or more metaphorically so—such as when stock price fluctuations are represented as random walks—are central to modelling change of any kind. The stochastic models considered in Section 1.4 focus on the expected distribution of aggregate outcomes of simple additive or multiplicative processes. Random walk models focus not on the aggregate outcomes of such processes, but on how individual members of the population arrive at the locations that combine to give their aggregate distribution. The key idea behind random walk models is that if we know something about the increments (the ‘steps’) in the walk, then we can say something useful about how the sequence of locations visited by the walk will develop over time. While none of this presumes that a walk plays out in space, random walk models are directly and naturally applicable to spatial cases, when they become a null model for processes involving the movement of entities in space. The interest of ecologists, geographers, anthropologists and social scientists more generally in spatial random walk models derives from this wide-ranging applicability.

4.1 Background and motivating examples

Examples of the sorts of phenomena where the concepts explored in this chapter may be relevant are illustrated in Figure 4.1. The movement of pedestrians is a critical driver of social systems, even in the contemporary heavily networked city. Pedestrian flows affect the success of retail businesses

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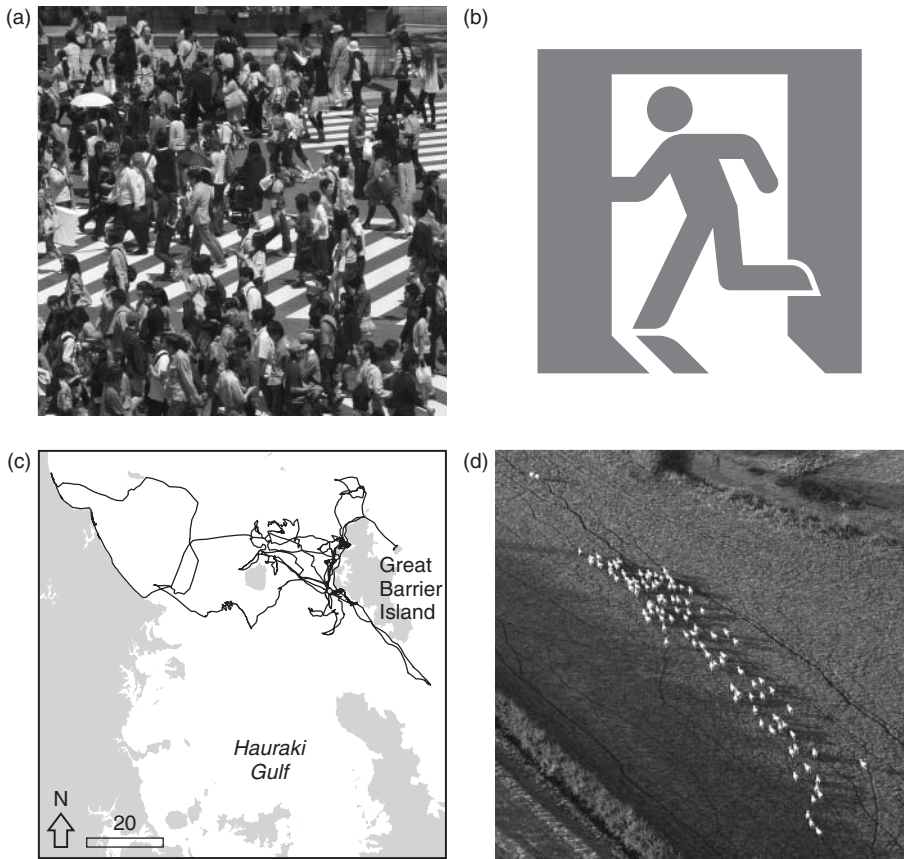


Figure 4.1 (a) Pedestrians on a crowded crossing in Shibuya, Tokyo. (b) Understanding movement in emergency situation is critical. (c) GPS tracks of nine Australasian gannets (*Morus serrator*) in the Hauraki Gulf, Auckland, New Zealand. (d) A flock of sheep in Warwickshire, England. *Sources:* (a) authors' collection, (b) © Foundation for Promoting Personal Mobility and Ecological Transportation, (c) image courtesy of Dr Todd Dennis, School of Biological Sciences, University of Auckland, and (d) photograph by Paul Englefield, 27 August 2006, http://commons.wikimedia.org/wiki/File:Sheep_herd.jpg.

and have an immediate effect on how we experience cities as places to live and work, as in Shibuya, Tokyo, in Figure 4.1(a). Understanding how such flows behave in emergency situations is particularly critical, when such behaviour can be a matter of life and death (Figure 4.1(b)). In ecology and the social sciences, movement data are now routinely collected and used to aid in understanding how species and individuals interact with one another and their environment. Recent developments in the global positioning system (GPS) and other tracking technologies allied with geographical information systems (GIS) have seen an explosion in the availability of highly

detailed empirical data describing the movement of people, animals and other entities. For example, the full extent of the long-range migrations of many animals, particularly birds, are now being explored using tracking, as shown in Figure 4.1(c). The complexities of herding and flocking behaviour are slowly being unravelled (Figure 4.1(d)) using both models and tracking technologies.

Random walks are a natural starting point for understanding both how to analyse such data and for simulating the processes that produce them. We develop a little of the mathematics behind simple random walks where individual steps in the walk are strictly independent of one another (Sections 4.2.1 and 4.2.2) before considering elaborations of this model where later steps in the walk are affected by earlier ones (Section 4.2.3) or by the environment (Section 4.2.4). An important class of walks undergoes super-diffusion as a result of non-finite variance of its step lengths, making their behaviour quite different, and we consider these cases in Section 4.2.5 (see also Chapter 5). In all these cases, the walk's trajectory is governed entirely by random effects and by its own previous evolution. In most real applications this is not a plausible representation, and so in later sections we consider how the evolution of a walk might be affected by the environment in which it unfolds (Section 4.3). We also look at simple models where the walk alters the environment in which it is evolving (Section 4.4), and at models of flocking where the most salient feature of the environment is the presence of other moving individuals (Section 4.5). The chapter closes with an overview of some of the ways in which random walk models have been applied in a range of disciplines (Section 4.6).

4.2 The random walk

4.2.1 Simple random walks

Although we can be fairly certain that few real mobile entities move entirely randomly, it is useful to start with a null model that incorporates no assumptions about underlying processes and to build from there. In physics, where particles often do move randomly or at least without conscious intention, and in related fields, there is an extensive literature on various types of random walk (see, for example, Berg, 1993, Spitzer, 2001, Rudnick and Gaspari, 2004, Révész, 2005, Lawler and Limic, 2010). Rudnick and Gaspari's book is recommended and Codling et al. (2008) review applications in theoretical ecology and biology, two fields which have contributed significantly to and drawn from the theory of random walks. These literatures deal with highly simplified and abstract models of random walk processes and form a natural starting point for our own exploration of these ideas.

Consider the two-dimensional integer coordinate plane \mathbb{Z}^2 and a walk beginning at the origin $(0, 0)$. Now, assume that a walk consists of a series of

‘steps’ of unit length in one of the four cardinal directions, north, south, east or west. Such a walk can only land at integer coordinate locations (x, y) , so that it is moving on a *lattice*. The choice of movement direction each time step is completely random, with no bias in any particular direction and no relationship between the direction of the previous step and the current one.



What sort of outcomes do we expect from such a model? How far will a walk(er) have moved from its starting point after a given number of steps? Will it ever return to the origin? Before considering some analytical results, it is helpful to show an example. Building a model to produce this type of walk is simple, requiring only the ability to randomly select which of the four neighbouring lattice locations to move to at each step. An example walk of 1000 steps is shown in Figure 4.2. Some features are immediately apparent, particularly the fact that the walk retraces its steps often. At any moment there is a one in four chance that the next step will reverse the previous one, so this is unsurprising. The overall effect is that the walk does not get as far as we might expect. This walk of 1000 unit-length steps only gets to $(19, -25)$, a journey that could have been made more directly in a mere 44 steps.

Such tortuous progress is typical. Figure 4.3 shows the points reached after 1000 time steps of 500 such random walks. It is clear from the density plot in (b) that many walks end near the origin, even after many steps. The mean distance of these 500 walks from the origin is 27.9 units, although the average *displacement* is 0 because there is no directional bias in step direction.

To derive an expectation of the distance from the origin of a lattice walk of n steps is relatively straightforward and is a result with many connections to fundamental results in statistics and mathematics (recall Galton’s board on

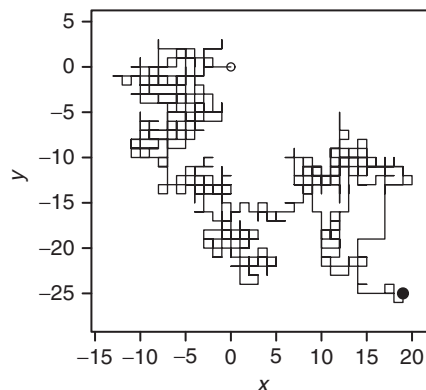


Figure 4.2 A simple 1000-step random walk on a lattice. The open circle indicates the start location at the origin $(0, 0)$, while the black circle shows where the walk has reached after 1000 steps.

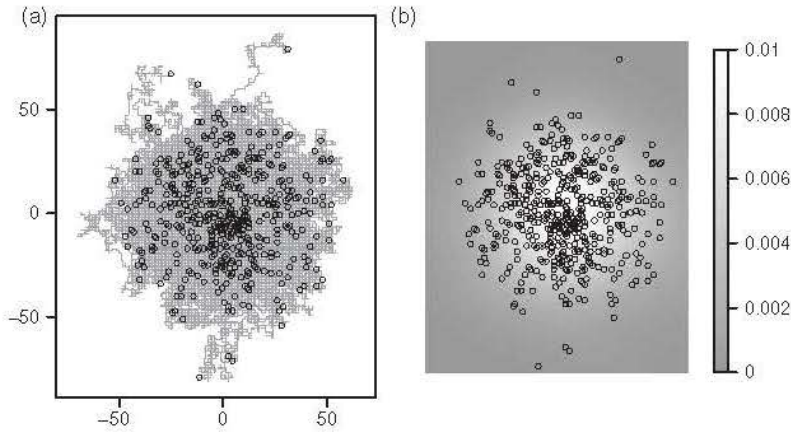


Figure 4.3 The end points of 500 random walks of 1000 steps (a) and their density (b).

pages 23ff.). The simplest approach is to consider a one-dimensional walk, where each step displaces the walker either to the right or the left along the number line. If the displacement at each step is a random variable S such that $s(t) = \pm D$, so that the walk moves either D units to the right or the left, then we can construct a *difference equation* for the position $X(t)$ after t steps, relative to its position at the previous time step:

$$X(t) = X(t-1) + s(t) \quad (4.1)$$

We can immediately deduce from this that the expected displacement at any time t is $X(t) = 0$, since the initial displacement is zero and each step is equally likely to take us in either direction. A more useful result avoids negative quantities by considering the *square root of the mean squared distance*, $\sqrt{\langle X^2(t) \rangle}$. Squaring each side of Equation 4.1 we obtain:

$$X^2(t) = X^2(t-1) + 2s(t)X(t-1) + s^2(t) \quad (4.2)$$

If we have N such walks, then the mean squared displacement is:

$$\langle X^2(t) \rangle = \frac{1}{N} \sum_{i=1}^N [X_i^2(t-1) + 2s_i(t)X_i(t-1) + s_i^2(t)] \quad (4.3)$$

The second term in this summation drops out because positive and negative values are equally probable, giving a difference equation for the mean squared displacement after t steps in terms of the mean squared displacement after $t-1$ steps:

$$\langle X^2(t) \rangle = \langle X^2(t-1) \rangle + s^2(t) \quad (4.4)$$

This tells us that the expected mean squared displacement increases by s^2 at each step, so that by deduction we have:

$$\langle X^2(t) \rangle = s^2 t \quad (4.5)$$

and, since $s = \pm D$, we have:

$$\begin{aligned} \langle X^2(t) \rangle &= D^2 t \\ \sqrt{\langle X^2(t) \rangle} &= D\sqrt{t} \end{aligned} \quad (4.6)$$

This means that the distance we can expect a random walk to progress from its starting point scales with the square root of the duration of the walk.

While this result has been derived in one dimension, it also applies in higher dimensional spaces. To see this, consider the four directions available to a lattice walk in two dimensions. Since each direction is independent of the other, the overall progress parallel to each coordinate axis is modelled by a one-dimensional random walk of half the duration because each step makes progress parallel to only one of the axes and no progress in the other direction. Thus we have:

$$\langle X^2(t) \rangle = \langle Y^2(t) \rangle = D^2 \left(\frac{t}{2} \right) \quad (4.7)$$

where $X(t)$ and $Y(t)$ denote the expected displacements parallel to the x and y axes. The root mean square distance covered $\sqrt{\langle R^2 \rangle}$ is given by Pythagoras's theorem and

$$\begin{aligned} \langle R^2 \rangle &= \langle X^2 \rangle + \langle Y^2 \rangle \\ &= D^2 \left(\frac{t}{2} + \frac{t}{2} \right) \\ \sqrt{\langle R^2 \rangle} &= D\sqrt{t} \end{aligned} \quad (4.8)$$

Similar reasoning means that Equation 4.8 holds for a lattice walk in any number of dimensions.



4.1

An obvious generalisation of the lattice random walk is a *simple random walk*. Here, each unit-length step may occur in any direction, randomly distributed on the interval $[0, 2\pi)$ radians. The result of Equation 4.8 again holds. In fact, this was the original model of a random or *drunkard's walk*, discussed by Pearson (1905a,b) and Lord Rayleigh (1905) in a quick-fire correspondence in *Nature*.

4.2.2 Random walks with variable step lengths



4.1

So far we have considered walks with a fixed step size. An obvious generalisation is to allow steps of varying lengths. Surprisingly, although Equation 4.8

no longer holds exactly, it turns out that the more general proportionality result:

$$\sqrt{\langle R^2 \rangle} \propto \sqrt{t} \quad (4.9)$$

applies for many types of walk. This result is related to the classical *central limit theorem* of statistics. The very general proportionality result of Equation 4.9 holds if we allow steps of any length drawn from a probability distribution *provided it has finite variance*. Thus if we draw walk step lengths from (say) an exponential distribution with mean $\lambda = 1$, although $\sqrt{\langle R^2 \rangle}$ is greater than before it still grows proportionate with the square root of the walk duration t . Examples of a simple random walk and an exponential step-length random walk are shown in Figure 4.4. The rate of increase in the root mean square distance covered by 100 exponential random walkers is shown in Figure 4.5, where the square root shape is evident, although the absolute value of the distance is greater than that of a simple walk with the same mean step size.

4.2.3 Correlated walks

Ecologists and biologists have used simple random walks such as those described in the previous section as models of animal movement and search strategies over several decades (see, for example, Pearson, 1906, Wilkinson, 1952, Morrison, 1978, Hoffmann, 1983). It is apparent, however, that animals, or for that matter people, rarely wander completely aimlessly in the manner of such walks—not even Pearson’s drunkard! We can make our random walks more ‘purposeful’ or *persistent* by biasing the walk direction from step to step so that the direction of the next step is chosen by selecting a random *turn angle* relative to the previous step (Marsh and Jones, 1988). If we permit only



4.1

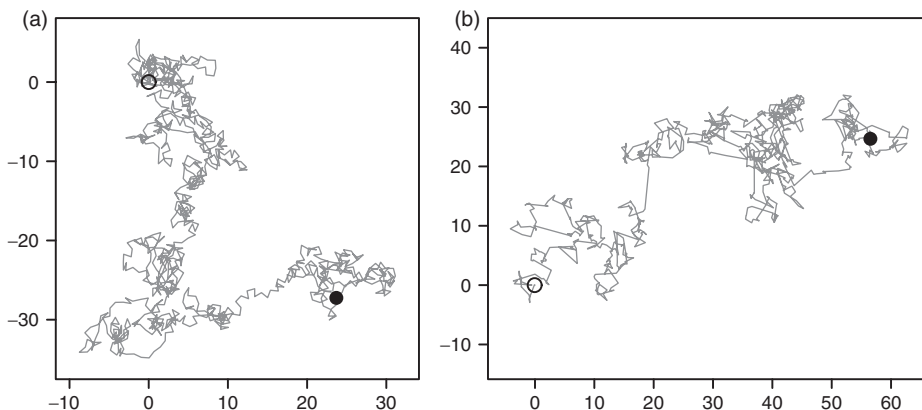


Figure 4.4 (a) A simple random walk and (b) a random walk with exponentially distributed ($\lambda = 1$) step lengths.

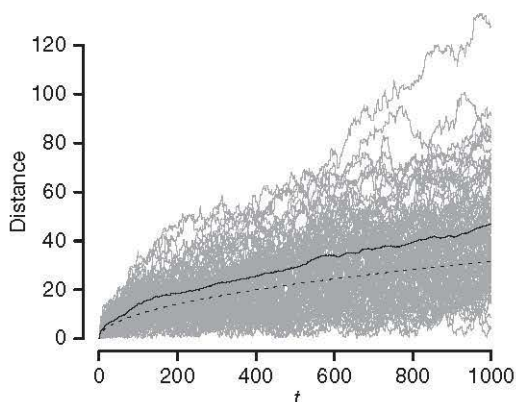


Figure 4.5 The root-mean-square distance traversed by 100 exponential step length random walks. The distances of all walks are shown in grey, and the square root of the number of steps (dashed) and root mean square distance of all walks (solid) are also shown.

small turn angles between steps, then walks move in near straight lines, and as we increase the probable turn angle walks become more tortuous. This effect is shown in Figure 4.6, where turn angles are chosen from a wrapped normal distribution with a standard deviation of, respectively, 10° , 30° and 50° . The more rapid progress of the first walk with its smaller turn angles (10°) is clear, keeping in mind the different scales on the axes of each plot.

Even so, with finite step lengths the square root proportionality relation *still* applies. The reason for this is that the directional correlation between steps is purely local so that over sufficiently long sequences of consecutive steps the correlation disappears. Thus successive steps are highly likely to be in the same or similar directions, but steps ten apart will be less strongly aligned with one another and, depending on the turn angle distribution, at some sufficiently long separation in time, steps become uncorrelated (Figure 4.7).

In the context of the analysis of polymer chains by random walk methods, the distance over which directional correlations remain has been labelled the *persistence length* of the walk (Patlak, 1953). Over successive steps up to the persistence length, a walk makes rapid, near-linear (or *ballistic*) progress due to the directional correlation. However, over sequences longer than this, the walk effectively becomes a simple random walk, albeit one at a different scale. Roughly speaking, if we consider a correlated random walk that exhibits persistence over L steps each of step length D , then over t steps, it behaves like a walk with step length DL and duration t/L . Substituting this into Equation 4.8 we have

$$\begin{aligned}\sqrt{\langle R^2 \rangle} &= DL\sqrt{t/L} \\ &= D\sqrt{tL}\end{aligned}\tag{4.10}$$



4.2

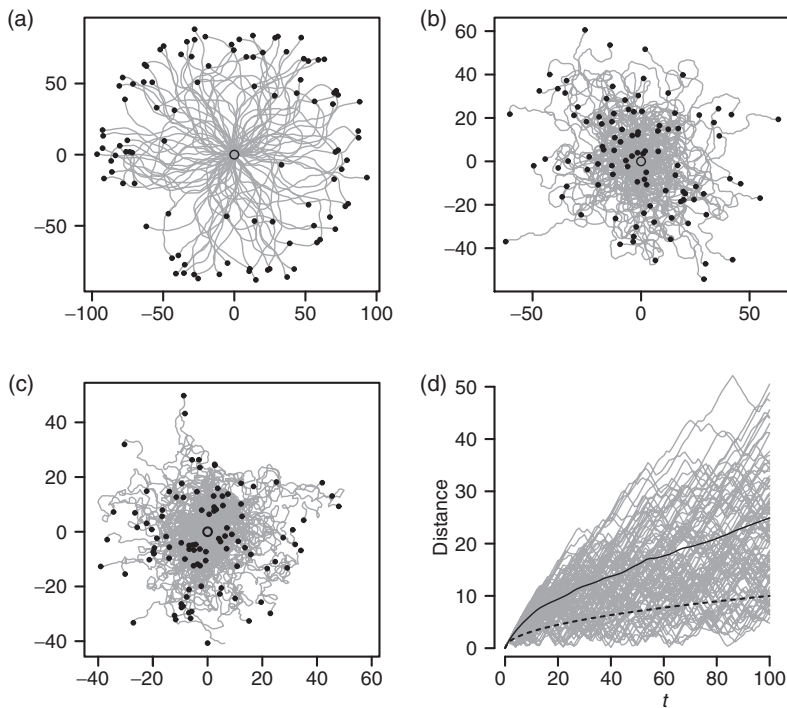


Figure 4.6 (a)–(c) 100-step random walks with turn angles chosen from a normal distribution, $\mu = 0^\circ$ and $\sigma = 10^\circ$, 30° and 50° , respectively. Note the different scale of each plot. (d) The distance from the origin of such walks still has the characteristic \sqrt{t} growth rate, shown here for the $\sigma = 50^\circ$ case. The dashed line shows the expected (slower) rate of progress for a simple random walk.

The net result is that the expected distance associated with the walk still scales with the square root of its duration t . Since the \sqrt{L} factor is a constant dependent on the degree of directional correlation, highly correlated walks will have longer persistence lengths and correspondingly larger scaling factors relative to the simple case. This point is clearly visible in Figure 4.6(d), and is even clearer in Figure 4.8, where the upper and lower straight lines show linear and square root proportionality, respectively. In each case, correlated walks show an initial linear trend but after some time this ‘rolls off’ to the slower square root relationship, with the persistence length decreasing progressively as the variability in the turn angle distribution increases (see Bartumeus et al., 2005).

A complete analysis of the expected distances for such walks is more complex. As Bovet and Benhamou (1988) explain, drawing on previous work by Tchen (1952) and Mardia (1972) (see also Batschelet, 1981, Mardia and

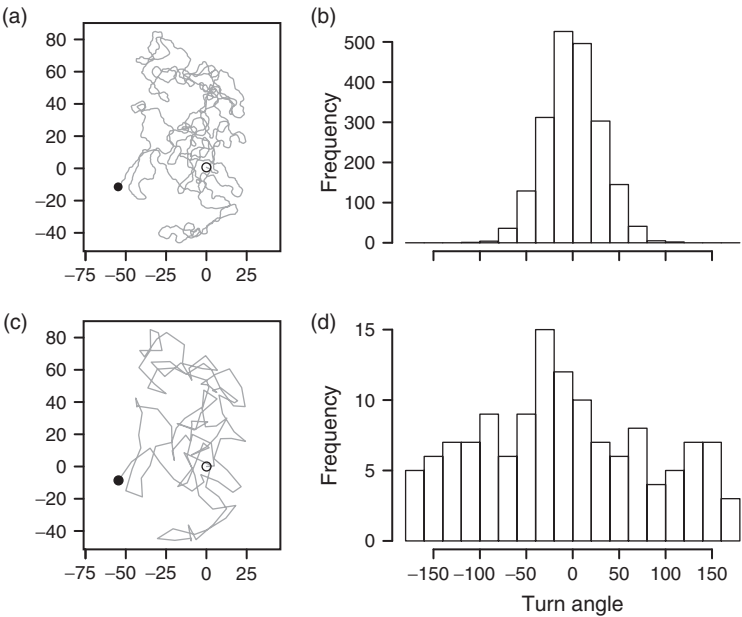


Figure 4.7 The turn angle distribution of a correlated angle walk is altered by aggregation. (a) and (b) A correlated random walk with turns drawn from a wrapped normal distribution $\sigma = 30^\circ$ and the distribution of the turn angles. (c) and (d) The walk and distribution of turn angles when the walk is aggregated by taking successive sets of 15 steps (effectively changing the grain of the walk). The distribution of turn angles is much more uniform and similar to what we would expect of a simple random walk. Aggregation over more steps will increase this effect.

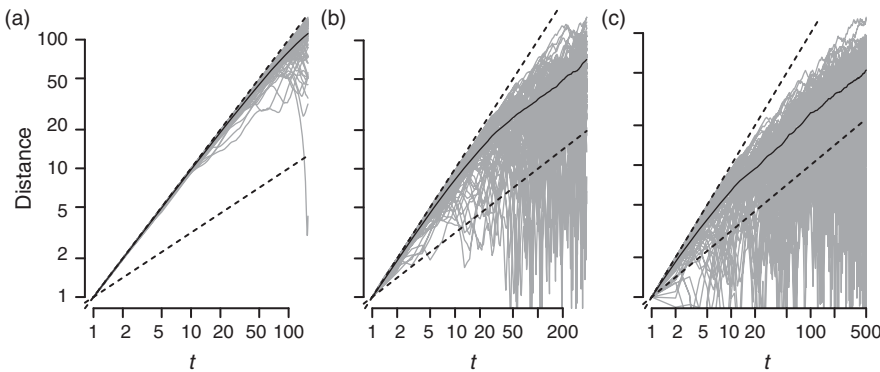


Figure 4.8 Expected distance of correlated random walks with turn angles chosen from a normal distribution, $\mu = 0^\circ$, and (a) $\sigma = 10^\circ$, (b) 30° and (c) 50° . Note the logarithmic time and distance scales.

Table 4.1 Scale factors for correlated random walks

Turn angle, σ ($^\circ$)	r	Scale factor $\sqrt{\frac{1+r}{1-r}}$
10	0.985	11.5
20	0.941	5.7
30	0.872	3.8
40	0.784	2.9
50	0.683	2.3
60	0.578	1.9
70	0.474	1.7
80	0.377	1.5
90	0.291	1.4

Jupp, 1999), the scaling factor for the expected distance relative to a simple random walk can be approximated by:

$$\sqrt{\langle R^2 \rangle} = D\sqrt{t(1+r)/(1-r)} \quad (4.11)$$

where r is a measure of the correlation between the directions of successive steps of the walk. For a wrapped normal distribution, as in the cases shown, $r = \exp -\sigma^2/2$ where σ is the standard deviation of the normal distribution measured in radians. Table 4.1 shows the resulting scale factors for a range of increasingly wide turn-angle distributions.

The rapid acceleration that results from a very strongly directed walk is apparent and, of course, in the limiting case with $\sigma = 0^\circ$, the root mean squared distance varies linearly with the duration of the walk, and it becomes ballistic point-to-point movement. Although subsequent work has provided more accurate estimates of the increased rate of progress expected due to directional correlation in successive steps (see Wu et al., 2000), the values in Table 4.1 demonstrate the qualitative impact of this refinement of the model and are reliable except at higher values of r , say when $r \gtrsim 0.9$ (Benhamou, 2004).

Directionally correlated walks are an example of a broad class of walks where each step is dependent on one or more of the previous steps. For example, steps might be similar to previous steps in length *and* direction. Even something as simple as a rule preventing an immediate return to the previous location makes steps non-independent of one another. Mathematicians have explored a variety of this broader class of correlated random walks.

Lazy walks are a modification of the simple random walk where with some probability in each time period, rather than take another step the walk remains in its current location. The most likely application of this sort of walk in a spatial context would be where the walker was engaged in some

other activity, such as shopping or feeding, although the mechanism driving movement in such a model would most likely relate to the requirements of that other activity (see Section 4.3). Lazy random walks are closely related to the ant in the labyrinth model discussed in Section 5.2.3, where pauses are enforced because not all movement directions are available at each step.

Reinforced walks are walks where the probability of making a particular step is greater if the link between two lattice sites has been crossed previously (edge-reinforced, see Davis, 1990a) or if the site has been visited previously (vertex-reinforced, see Pemantle, 1992). Pemantle (2007) provides an overview of both types of walk in the context of stochastic processes with ‘memory’ effects more generally, which are closely related to sampling processes without replacement. Such walks can exhibit surprisingly complicated behaviour, and they are relevant to understanding how terrain is explored and a territory, activity space or a path network is established by humans or other animals (see Boyer et al., 2012).

Self-avoiding walks are forbidden from returning to any previously visited location, or in more complex cases are excluded from approaching within some specified distance of any previously visited location (Madras and Slade, 1996). Such walks are commonly used as models of long-chain polymer molecules. They might also be considered a possible model for animal or human exploration in a new environment, since they force the walker to visit previously undiscovered sites.

In general, it is difficult to derive estimates of the effect of such varied inter-step dependencies on the expected distances traversed by a walk, although provided the step length distribution has finite variance, a reasonable null expectation is that the root mean squared distance traversed by a walk will be proportional to the square root of the walk duration.

4.2.4 Bias and drift in random walks

While correlation between the steps can produce the appearance of purposeful movement in a walk, another possibility is a persistent bias in the preferred movement direction. Such bias is easily incorporated into the models we have been considering by preferentially choosing the direction of each step with respect to some fixed direction. This approach can be applied equally well to lattice walks or walks in continuous space. The overall effect is to shift the peak of the probability surface associated with a walk away from the origin in the direction of the movement bias. Patlak (1953) provides a detailed, but mathematically demanding, account of this type of walk, while Turchin (1991) provides a more accessible introduction. Our interest in such walks is more practical than theoretical, and arises in situations where mobile

entities or ‘walkers’ interact with an environment, in particular when they respond to environmental gradients, with movement biased in the direction of increasing or decreasing availability of some resource. This kind of behaviour might be considered as foraging or search, and we examine the models that arise when walks interact with an environment in more detail in Section 4.3. For now we merely note that in practice it can be difficult to distinguish the effects of between-step directional correlation and directional bias in movement paths (Cai et al., 2006), especially over short time periods, yet another example of equifinality. Over longer time periods, if bias persists it will usually be obvious, since walks with correlated step directions ultimately show no directional bias.

4.2.5 Lévy flights: walks with non-finite step length variance

As has been noted, the additive central limit theorem governed distance–time relationship of Equation 4.8 holds for walks with step-length distributions of finite variance. When the step-length distribution does not meet this criterion, the outcome is quite different. Specifically, when step lengths are distributed according to a heavy-tailed distribution (e.g. a Pareto or a Cauchy distribution), then the walk is referred to as a *Lévy flight* (Viswanathan et al., 2011). An example is shown in Figure 4.9, with step lengths drawn from a Cauchy distribution. This walk is characterised by periods of short localised steps interspersed with long-range ‘jumps’. A distinction is sometimes (not always consistently) made between Lévy flights when steps in the walk are ‘instantaneous’ and Lévy walks where successive steps occur at constant velocity over time periods whose durations have a probability distribution with



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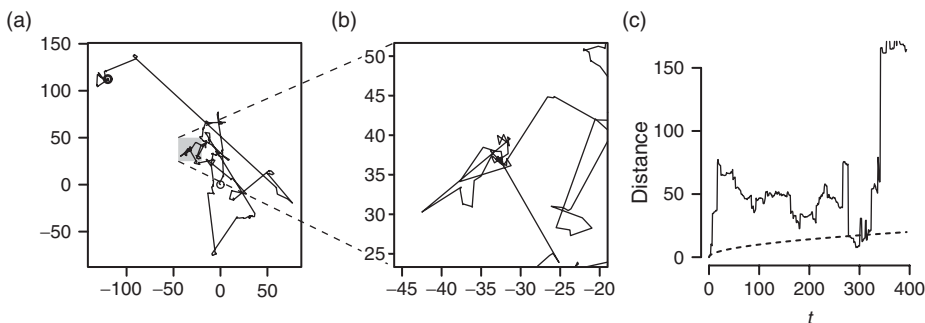


Figure 4.9 A walk of 281 steps, where each step is in a direction drawn from a uniform random distribution and step lengths are drawn from the Cauchy distribution with location parameter $x_0 = 0$ and scale $\lambda = 1.0$. (a) The full walk, (b) a zoomed-in region (grey area in (a)) and (c) the distance from the origin of the walk compared to a \sqrt{t} expectation.

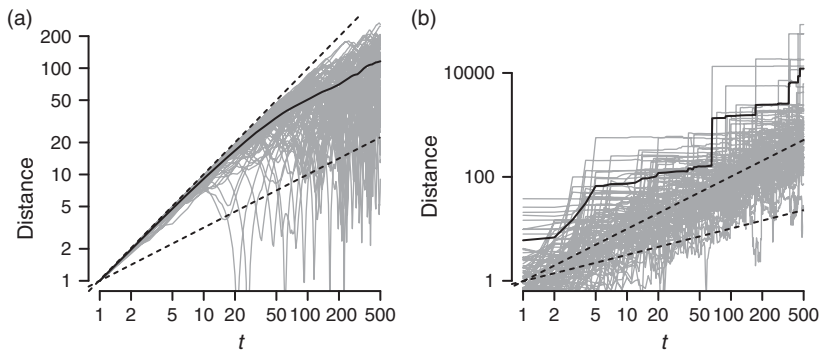


Figure 4.10 Comparison of the progress made by (a) 100 correlated random walks with exponential step lengths and (b) 100 Lévy flights with Cauchy distributed step lengths. Note the change in scales on the vertical axis and that both axes are logarithmic. The roll-off to diffusive behaviour already noted (see Figure 4.8) is clear for the correlated walks but does not occur for Lévy flights.

non-finite variance or alternatively where step length is constant but velocity varies (ben-Avraham and Havlin, 2000, Shlesinger and Klafter, 1986). For present purposes, the difference between flights and walks is unimportant, relative to a feature they share, which is that the overall progress made by the walk is dominated by the long-range jumps. As a result, such walks cover more ground more quickly than the other kinds of walk considered so far, as is clear in Figure 4.10. This characteristic of Lévy flights is termed *super-diffusive* behaviour (see also Section 5.2.3).

Super-diffusive walks, particularly Lévy flights with step lengths D , distributed according to a power law $P(D) \propto D^{-\mu}$ where $1 < \mu \leq 3$, have attracted considerable attention, at least in part because of the recent fascination with power-law distributions (Klafter and Sokolov, 2005). For example, the movement of dollar bills has been shown to be super-diffusive (Brockmann et al., 2006), while cell phone tracks also suggest such movement in the everyday lives of people (González et al., 2008). However, it is important to be cautious about such claims, particularly when they depend on potentially unreliable fitting to heavy-tailed probability distributions. Having sounded a note of caution, it is important also to acknowledge that many movement processes in social systems and in nature are super-diffusive. For example, it is apparent from data on rates of post-glacial recolonisation by trees that plant dispersal processes can be super-diffusive, implying that seed dispersal kernels are heavy-tailed (Clark, 1988, Clark et al., 1998).

However, the existence of super-diffusive movement in populations does not imply that the underlying processes are Lévy flights. Take, for example, human movement. Even cursory consideration of how and why people move

suggests that no single process accounts for the combination of habitual everyday movement (the daily commute), movement arising from regular but less frequent activities (such as weekend sports), movement over longer distance (such as annual vacations) and infrequent global-scale travel (perhaps due to migration or ‘once in a lifetime’ vacations). Thus an obvious alternative movement model to Lévy flights might incorporate multiple processes at a variety of scales, with each process modelled as a different kind of random walk. It is well understood that heavy-tailed probability distributions can result from composites of exponential distributions (see Petrovskii and Morozov, 2009), which further suggests that we might expect to find step lengths in empirical movement data consistent with Lévy flights if the generating mechanism were a combination of ordinary diffusive movement processes at a variety of scales (see, for example, Fritz et al., 2003).

In any case, all of the random walks that we have considered so far are limited as representations of any kind of intentional movement, in that they lack context or purpose, even when the resulting traces *look like* they might represent purposive movement. The mechanisms that drive these walks are entirely probabilistic and internal to the walks themselves, so that as models they are purely phenomenological: they produce effects that appear similar to observed patterns but are clearly not driven by the sorts of mechanisms that drive real entities in the real world, unless those entities are unthinking particles in random motion. In the next section we consider more purposeful models of movement.

4.3 Walking for a reason: foraging and search

The Nobel prize-winning economist and early originator of artificial intelligence Herbert Simon, considering the path of an ant, presents a simple parable:

We watch an ant make his laborious way across a wind- and wave-moulded beach. [...] I sketch the path on a piece of paper. It is a sequence of irregular, angular segments—not quite a random walk, for it has an underlying sense of direction, of aiming toward a goal. [...] Viewed as a geometric figure, the ant’s path is irregular, complex, hard to describe. But its complexity is really a complexity on the surface of the beach, not a complexity in the ant. (Simon, 1996, page 51)

Simon’s analogy suggests that the explanation for observed movement patterns, whether in terms of step lengths, turning angles, diffusivity or otherwise, may lie as much in the structure of the environment in which movement occurs as in the internal logic of the moving entities.

In earlier work, Simon (1956) explores the question of how simple the search behaviour of an organism can be, yet still allow it to survive in an environment with randomly distributed resources. The context for this paper is the debate around the limits to (human) rationality in decision-making (see also Simon, 1955, Conlisk, 1996). In economics, many models posit perfectly rational, *optimising* decision-makers able to make best choices among options whose relative merits are difficult to assess (the so-called *Homo economicus*). Simon suggests that limits on the availability and accuracy of information, and on the calculative abilities of organisms, argue for *bounded rationality* rather than perfect rationality. Under bounded rationality decisions are made using ‘good enough’ reasoning, as in, ‘this may not be the best choice, but it is good enough’, an approach Simon dubbed *satisficing*. Satisficing behaviour is a feature of many individual- and agent-based models of decision-making, and Simon’s (1956) paper provides a good starting point for exploring how such decision-making in search processes can account for observed movement patterns.



Simon’s (1956) model is not explicitly spatial, but is easily implemented in a spatial model. Consider a ‘walker’ searching for a target location on a two-dimensional lattice. At each step it may move in one of four available directions. The walker has a detection radius, or ‘vision’, v on the lattice, so that when it comes within v lattice sites of the target it switches ‘mode’ to move directly to the target by the shortest available path. Once at the target, the walker collects the resource, a new target location is randomly created and the walker begins searching again. Each movement step costs a single unit of energy and the walker starts with an energy level E . When the walker arrives at a target, its energy level is recharged to E , but if at any time the energy level falls to zero the walker dies.

Simon sets out a simple analysis of this model (see Simon, 1956, page 131ff), which we rehearse here, adjusted for our two-dimensional lattice setting. Critical variables are the proportion of lattice sites that provide resources, p_R , the maximum energy level, E , and the range of vision v . At each step, if the walker is moving forward into a previously unvisited part of the lattice, then $2v + 1$ new lattice locations come into view inside the range of vision. If the walker is turning to left or right, however, only $v + 1$ new sites come into view (see Figure 4.11). If the walker returns to a previously occupied site then no new sites come into view. As a result, the number of newly visible sites each step depends on the nature of the walk and the range of vision. For now, in order to develop the argument, we will assume that the mean number of newly visible sites, $\langle v \rangle \approx v$, meaning that v new lattice sites become visible each step, although we revisit this assumption below. The probability ρ that none of the newly visible sites will be a target location is:

$$\rho = (1 - p_R)^{(v)} \quad (4.12)$$

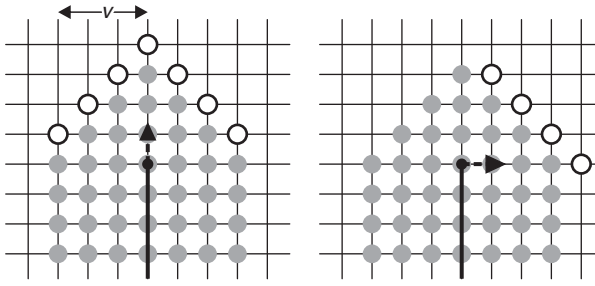


Figure 4.11 The number of newly visible sites on a lattice where the range of vision v is $2v + 1$ when the lattice walk moves forward into an unexplored region, but only $v + 1$ when the walk takes a turn to right or left. The latest movement step is shown as a dashed line. All grey spots are sites that have already been seen; black-outlined white circles are those newly visible once the step is taken.

The probability $P(k)$ that no target will be found in $k - 1$ moves, only for a target to be found on the k th move, is given by

$$P(k) = (1 - \rho)\rho^{k-1} \quad (4.13)$$

and the expected number of moves to find a target, M , is given by

$$\begin{aligned} M &= \sum_{k=1}^{\infty} k(1 - \rho)\rho^{(k-1)} \\ &= \frac{1 - \rho}{(1 - \rho)^2} = \frac{1}{1 - \rho} \end{aligned} \quad (4.14)$$

The probability of death P_{death} is equal to the probability that the number of moves to find a target will exceed $E - v$ and is given by

$$P_{\text{death}} = P(M \geq E - v) = \rho^{E-v} \quad (4.15)$$

Equations 4.13, 4.14 and 4.15 are standard results for the *geometric distribution*. The expected number of searches for which a walker will survive is similarly given by $1/P_{\text{death}}$, which we may consider the ‘life expectancy’ of the walker.

To get a feel for the numbers, if we consider a case where $p_R = 0.001$ and the range of vision is ten, then ρ from Equation 4.12 is 0.990 and the expected number of moves to spot a target, from Equation 4.14, is around 100. If the maximum energy level E is also 100 then the probability of death during a single search from Equation 4.15 is given by $0.990^{90} \approx 0.405$, giving a short life expectancy of only $2.46 = 1/0.405$ searches. If, on the other hand,



the maximum energy level is a lot higher than the expected search time at (say) $E = 1000$, then the probability of death during a single search is only 4.99×10^{-5} , giving a life expectancy of over 20 000 searches. Simon (1956) notes, although his exact numbers are different, that an energy level sufficient to sustain searches up to around ten times the expected search time seems reasonable for real organisms.

For now, we are more interested in how the characteristics of the walk executed during search affect survival chances. The critical issue is how the probability of not finding a target at each step changes with the characteristics of the walk. As described above, Simon's forager executes a simple random walk, and so is as likely to return to a lattice site just vacated as it is to move to one of the three other adjacent sites. This fact dramatically affects the exponent in Equation 4.12. We can demonstrate this experimentally by measuring the total area searched for random walks with differing degrees of directional persistence and fixed vision.

Results are shown in Figure 4.12 based on running a number of random walks with varying probability of turning, from zero (ballistic movement in a straight line) to one (a simple random walk). As the probability of turning increases, the walk's directional persistence decreases, and it becomes more tortuous and more likely to revisit previously visited sites. The number of sites seen within a range v , fixed at ten lattice units, falls so that a non-turning

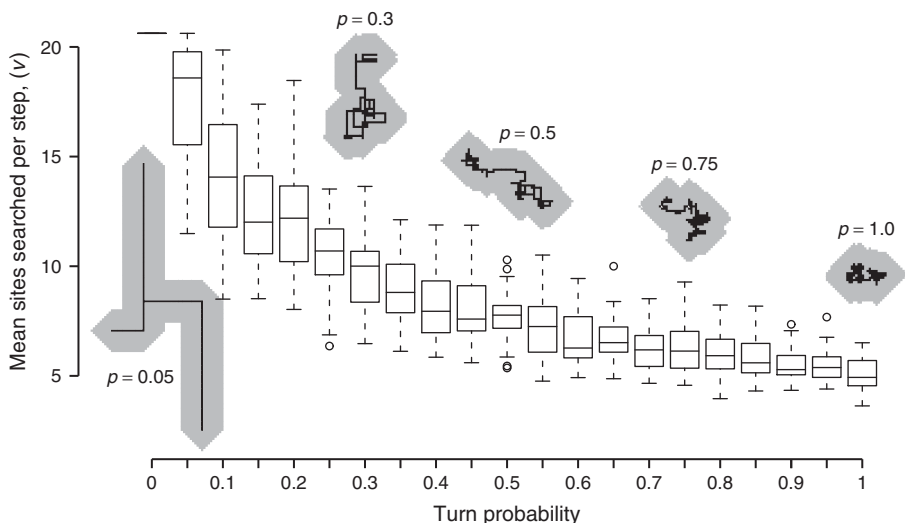


Figure 4.12 Variation in the region searched on the lattice with vision $v = 10$ for random walks as the probability of turning (p) at each step changes from zero to one. Examples of some walks and the associated 'sausage' or region explored are shown. Note that the lack of variation in the $p = 0$ case is because such walks are ballistic in a straight line with no change from run to run.

walk sees around 20 new lattice sites each step (i.e. $2v$) while the random walk sees only around five. These experimental findings are supported by work on *Wiener sausages*, the name given to the region within a specified distance of a random walk in various dimensions (see, for example, van den Berg et al., 2001). From this literature, the growth in area in two dimensions is not expected to increase linearly with time t , but to slow gradually, proportional to $t/\log t$. For now, we can treat the deceleration as a second-order effect and focus on the large differences in $\langle v \rangle$ that different walk geometries produce. Here, the change from ballistic motion to random walk behaviour sees the effective vision in terms of the exponential in Equation 4.12 fall from approximately $2v$ to $v/2$. For the numerical example given above, this changes the probability of not finding a target each time step from around 0.98 to 0.995. This may not seem like much, but the overall effect is to dramatically reduce the life expectancy when E is (say) 500 from 18 122 to just 11.6 searches. To get comparable life expectancy with the less efficient random walk movement pattern requires an approximately four-fold increase in the organism's storage capacity E . Thus, as we might expect, there are substantial benefits to be gained by searching more efficiently.

Although it is rarely recognised, probably due to the separation of the relevant literatures across economics, ecology and anthropology, much of the subsequent work on models of spatial search or foraging behaviour is effectively an exploration of elaborations of Simon's model. Below we briefly consider some of the many possible refinements of the model.

4.3.1 Using clues: localised search

If resources are not distributed purely at random, but instead tend to aggregate or cluster, then it may be profitable for a forager to adopt search behaviour that takes advantage of this. The most obvious change in behaviour is to assume that where resources have already been found, more resources are likely to be found (following Tobler's first law from Chapter 3). This entails behaviour where the region around a known resource is thoroughly explored, and also that between finds any available clues to the probable presence of resources are followed. A simple example of a clue to resource presence is to associate a resource density with each lattice location, using a kernel density method (see O'Sullivan and Unwin, 2010). This might represent how the environment provides cues suggesting that a particularly area is likely to be rich in the resource. Presented with a resource density signal an obvious search strategy is *hill-climbing*, where the forager moves to the adjacent lattice site with the highest resource density signal.

While this strategy is effective when it produces a clear preference for the next step in the walk, it can be problematic if, given no cues, the search reverts to a simple random walk. If instead of simple random behaviour between

resource concentrations the forager pursues less convoluted walk behaviour, then more ground is covered and the chance of encountering a new region of higher density is increased. Two other adjustments to search behaviour can also help with more effective searching.

Path memory involves the forager avoiding locations visited previously and found to have no resources. This requires the forager to have a memory of where it has previously been so that it can avoid those locations, when the walk is a form of self-avoiding walk, or revisit them, if they have been good sources in the past, when it is a reinforced walk. This approach can lead to some problems, particularly if self-avoidance behaviour is rigidly applied, when a walk may easily ‘trap’ itself because all surrounding available lattice sites have already been visited. Nevertheless, early work by Benhamou (1994) suggests that memory, as we might expect, improves the likelihood of successful foraging.

Mental (cognitive) maps build on memory-based approaches and involve the foraging organism constructing a mental map of the search region. A simple mental map might record locations which have been rich in resources on the previous visit, along with the time of that visit. A more detailed map might also include pathways to known resources and some concept of any time constraints on resource availability, such as seasonality of fruiting or opening hours of facilities.

The model we present in Chapter 8 uses mechanisms similar to these, and clearly departs some way from a simple random walk. Given how complicated the memory and mental maps of foraging organisms are (whether real or in models!) it is difficult to generalise about their effects on the resulting movement patterns.

4.3.2 The effect of the distribution of resources

As Simon’s parable of the ant suggests, the explanation of empirical movement data may lie in the landscapes where the movement occurs and the behaviour which the landscape causes. A more realistic example than those based on Simon’s model is provided by a model of spider monkeys foraging fruit-bearing trees (Boyer et al., 2006), which produces movement patterns similar to those observed empirically (Ramos-Fernández et al., 2004). It is worth nothing that very similar approaches to that described by Ramos-Fernández et al. (2004) have been used to explore the frugivorous foraging behaviours of birds by Morales and Carlo (2008) and, albeit in a more empirically and mechanistically grounded way, by Uriarte et al. (2011).

This example provides us with the basis for a plausible model of foraging or, more generally, resource search behaviour that is easily implemented.



Consider a landscape consisting of fruit-bearing trees distributed according to a homogeneous Poisson point process (see Section 2.2.2). Each tree is assigned a total quantity of resources (say fruit or seeds) z_i drawn at random from some distribution, say the exponential. Now, a single individual, starting from a tree near the centre of the study area, first exploits the resource, reducing its remaining resource to zero. In subsequent time steps the individual moves to the tree j that maximizes the quantity z_j/d_{ij} , where z_j is the resource available and d_{ij} is the distance to it. Having reached a new resource, the resource available is again reduced to zero. A typical walk produced by this process is shown in Figure 4.13, along with the distributions of the site resources and walk step lengths. Notably, the two distributions appear very similar. In Boyer et al.'s paper (2006) power-law distributed resource availabilities produce similarly distributed step lengths in the resulting walks. As a further example, Figure 4.14 shows how the walk and walk step-length distribution are changed when the resource distribution is log normal. Although little detailed work has been done on this question, it is plausible that different resource distributions will produce different walk step-length distributions.

More elaborate versions of this simple strategy produce surprisingly little variation in the resulting walk step-length distributions; for example, the search strategy may be unreliable in identifying the best next site and instead pick one of the n best sites at random. Another variation is to prevent the searcher from detecting available resources beyond some maximum distance from the current location. Except in cases where such a restriction severely affects the number of possible next sites, resulting movement patterns are

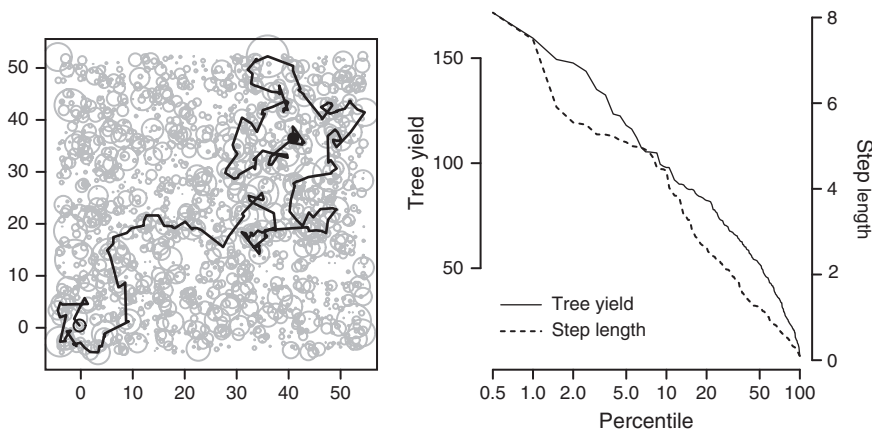


Figure 4.13 A typical walk produced by the process described in the text, when each step of the walk maximises the yield per distance moved z/d and the walk does not return to a previously visited resource. The right-hand panel shows the distribution of resources available at sites and the step lengths in the walk.

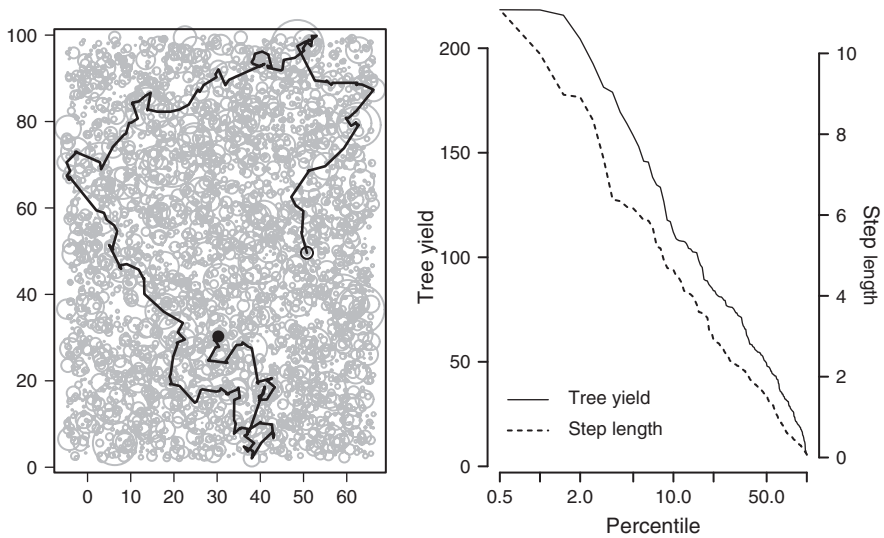


Figure 4.14 A typical walk produced when the movement dynamic is as in Figure 4.13, but resources are lognormally distributed.

surprisingly stable for a given resource distribution. This remains an active research area, and while it is likely that it will remain difficult to disentangle the effects of landscape pattern and search strategies, models will be an important research tool in this field (Mueller et al., 2010).

This example provides a simple framework for movement arising from search strategies in a landscape that provides some distribution of resources to be exploited. At the heart of any such strategy is a fundamental decision: whether to remain in the current location or move elsewhere to a potentially better opportunity. We might call this the ‘Should I stay or should I go?’ model, which is a very loose statement of the central problem of *optimal foraging theory* in ecology (see MacArthur and Pianka, 1966, Pyke et al., 1977). If a decision is made to go, a further decision, determining which among some set of possible targets is the most promising, is required. As in the example above, this decision might be based on an immediate evaluation of the available resources—the benefit available—relative to the cost of accessing them—the distance to them (and in classical optimal foraging theory the effort required to handle and process them). Alternatively, the choice of the next location might be less deterministic. For example, a series of possible target locations may be considered until the first ‘good enough’ choice is found, another example of Simon’s satisficing concept. The search process might also incorporate memory effects, so that an individual has prior knowledge of the available choices, based on previous experience, and newly discovered options are considered relative to one another and to locations

in memory. As an additional complication, memory may not be completely reliable, either in retaining details of the resources available at remembered locations or even in retaining them at all.

4.3.3 Foraging and random walks revisited

Recent research on animal foraging behaviour includes an extensive literature emphasising the importance of Lévy flights as search strategies (see Viswanathan et al., 2011, and the references therein). Much of this work followed assertions of Lévy flights in albatross movement (Viswanathan et al., 1996), although this claim was subsequently found to be erroneous (Edwards et al., 2007). Similar claims made for the movement of other species have also been found to be questionable (Edwards, 2011), the problem being both the technical difficulty of fitting empirical data to heavy-tailed distributions and over-optimistic claims based on relatively small variations in the scale of movements—say two or three orders of magnitude, when more are required to properly sustain claims for heavy-tailed distributions. This debate has been further energised by work showing that under conditions where resources are sparse and randomly distributed, Lévy flight search patterns are an efficient search strategy, perhaps even an optimal one (Viswanathan et al., 1999). However, it remains unclear how relevant such conditions are in practice (Benhamou, 2007) and recent work suggests that the specific model used by Viswanathan et al. (1999) has rather narrow applicability (James et al., 2011).

From the foregoing discussion, it should be clear that it is unsurprising that Lévy flight-like movement would be more efficient than simple random walks or even correlated walks as a search pattern *in some environments*. Super-diffusive Lévy walks cover ground much more quickly, and consequently have much larger associated search regions (see Figure 4.12, page 114). However, as is also clear from the foregoing, an organism responding to its environment may exhibit movement patterns that depend strongly on the structure of the environment, which may even mimic the heavy-tailed distributions of walk step lengths characteristic of Lévy flights. This is a good example where disentangling the mutually reinforcing effects of process and pattern is extremely difficult—a difficulty made even more challenging when movement processes also alter the environment in which they occur.

4.4 Moving entities and landscape interaction

The model we will examine here is similar to the previous one, but unfolds on a 60×60 (nontoroidal) lattice, which for convenience we say has N sites—obviously N is 3600. Sites are assigned a sequence number, representing the order in which they become productive, that is, when resources will be



4.6

available at the site. A patchy pattern is enforced by running 30 iterations of a 12-state voter model (see Section 3.4.4) and then assigning a sequence number between 0 and $N - 1$ to each site in the lattice ordered by the end states of the voter model. This produces a roughly clustered sequence in which sites yield resources. We will refer to the sites as s_i , where i is their sequence number. All sites from s_{N-n} to s_{N-1} , where n is a number of recently visited sites that we want to observe, have their initial yield z set to one, and all others are set to zero. As the model runs, at time step t , the site s_i , where $i = t \bmod N$, whose turn it is to become available, has its yield set to one, and the clock t is advanced by one. Thus we have a patchy environment in which resources become available in a ‘seasonal’ manner. An individual forager is then placed in this environment, at a randomly selected site with nonzero yield.

Each model time step, the forager first takes the resources at its current location by setting z to zero. It then moves by picking the closest site which currently has resources available. Once the forager has moved to the chosen site, the model proceeds to the next step. While one new site comes available each model time step and one is removed by foraging, sites will persist on average for n time steps. Although it is unsurprising, the interesting feature of this model is that it produces repeatable ‘seasonal’ behaviour over time, as shown in Figure 4.15, where at the same point in the model’s N step cycle, in successive ‘years’, the forager is exploring similar regions of the map and following similar paths. This example is deliberately designed to reinforce the point that the external drivers of movement in terms of the resources (or activities or, generally, the attractions) available across the landscape are as important to movement patterns as any internal logic of the movement itself.

This model was suggested in part by Boyer and Walsh (2010), who present a much more complicated model, with seasonally predictable fruiting of trees, distributed across a landscape. Monkeys foraging in this landscape with memory capability and a cognitive map of the environment end up following regularly repeating paths for a wide range of search behaviour parameter settings. Given the addition to this model of considerably more complexity in terms of both the resource landscape and the foraging behaviour, the mutual interaction of landscape and movement behaviour is intriguing. One finding is that it is important for the overall foraging success of the monkeys that they do not adhere too rigidly to a deterministic decision-making method for identifying their next target tree. Rather, they do better if they retain some randomness in their search behaviour. This prevents them from getting stuck in a ‘loop’ where they repeatedly re-exploit the same already known resources, rather than exploring and identifying potential new sites. This paper suggests that while, as Benhamou’s (1994) early work demonstrated, memory supports successful foraging, it is also important that an exploratory urge be maintained for long-term success. These issues are relevant to the model

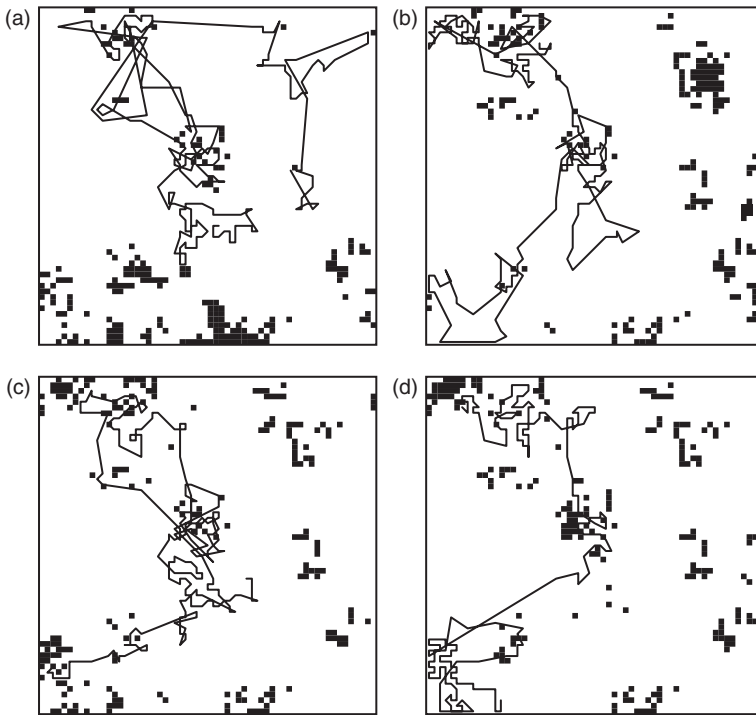


Figure 4.15 Seasonal patterns in the foraging model described in the text. These four snapshots show the currently live sites (black squares) in the model space and the most recent 200 steps of the forager's movement for four snapshots, 3600 model steps apart, that is, separated by a full model 'year'. Broadly the same regions in the space are active each time, and in all but one of the snapshots (a) the forager has recently been exploring similar areas of the map.

we present in Chapter 8. It is also noteworthy that the general framework of entities moving in and changing an environment as they move is closely related to the particle-based reaction–diffusion models briefly discussed in Section 3.7.1.

4.5 Flocking: entity–entity interaction

Models of *flocking* behaviour explore the conditions of imitation, attraction and repulsion between individual entities, under which spontaneous concerted movement of large groups—in other words, flocking—can emerge and be sustained. The most frequently cited original source for simulation models of this type is a conference paper by Craig Reynolds (1987), which presents the basic features of a model that allows the graphical simulation of flocking behaviour. Reynolds's model was not the first to consider flocking; Okubo

(1986) presents a thorough overview of theoretical attempts to understand collective behaviour and animal grouping up to that point in time.

Individuals in Reynolds's flock observe three basic rules. First, they avoid colliding with other individuals, essentially by not getting too close to one another, a *repulsion* effect. Second, they try to match their speed and direction of movement to their near neighbours, an imitation or *alignment* effect. Finally, individuals aim to remain in the flock by staying within range of near neighbours, an *attraction* effect. The motivation behind Reynolds's model was the simulation of flocking behaviour for special effects in films and computer games, and his model has formed the basis for developments in that field over recent decades.



4.7

A more formally presented academic model, even simpler than Reynolds's, is described by Vicsek et al. (1995). This model assumes that all individuals are moving at the same speed v and that they adjust their direction of movement or *heading* θ to the mean heading of nearby individuals within some range r , with the addition of a noise factor:

$$\theta(t+1) = \langle \theta(t) \rangle_r + \Delta\theta \quad (4.16)$$

where $\Delta\theta$ is a uniformly distributed random angle in some range $[-\eta, +\eta]$. All individuals adjust their heading according to Equation 4.16 each time step before moving a distance v in this new direction. If the speed v is considered a fraction of the neighbourhood range r and we set $r = 1$, then this model has only three free parameters: v , η and the density ρ of individuals per unit area. Figure 4.16 shows a typical sequence, starting from randomly located individuals with random headings. The rapid development of 'flocks' is apparent. Of course, this is unrealistic for many applications, where we might expect individuals to be moving at a range of different speeds and also to actively avoid one another, rather than stay apart solely due to the random term. Even so, this simple model demonstrates that flocking behaviour is not hard to generate and that the relationship between the search radius for alignment and the speed of movement is a critical parameter.

Elaborating this model rapidly becomes rather complex and demands some care in how the neighbouring individuals (or flock-mates), which affect the alignment and relative movement of individuals are determined. Figure 4.17(a) shows that the simplest approach using a square grid in which all individuals sharing the current grid cell are considered flock-mates may result in many more flock-mates than a more natural range-limited circular neighbourhood. Such a circular neighbourhood can be further elaborated by restricting it to an angular range, usually centred facing forward with respect to the current heading. One individual is excluded from the set of neighbours by this method in Figure 4.17(b). This diagram also shows how a preferred separation distance criterion (see the next paragraph for details) can be used to

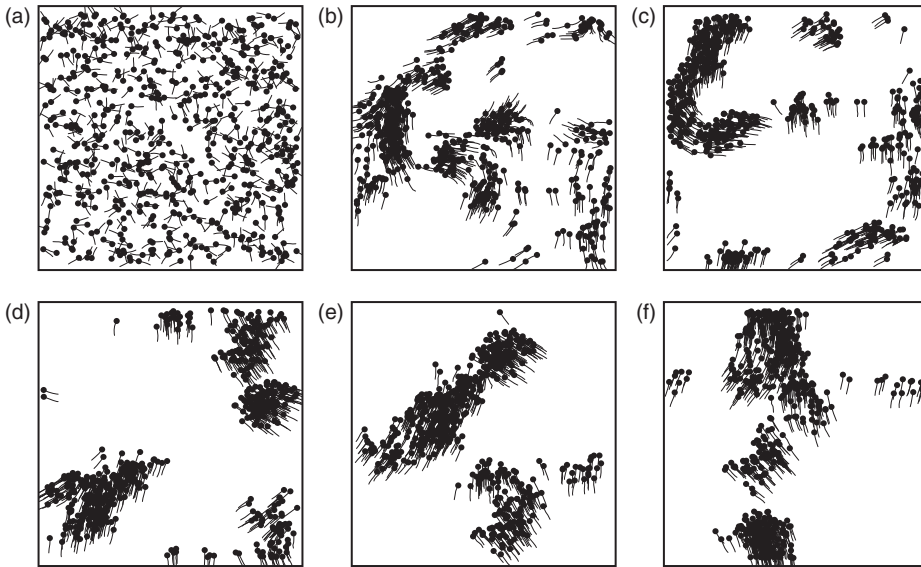


Figure 4.16 Development of flocks in Vicsek et al.'s (1995) model. Panels from (a) to (f) show system state at 30 time-step intervals, with $v = 0.3$, $\rho = 1$ and $\eta = 10^\circ$ in a 25×25 toroidally wrapped space. This example uses a simple grid-based method to select flock-mates. Tails on the individuals show movement over the previous five time steps.

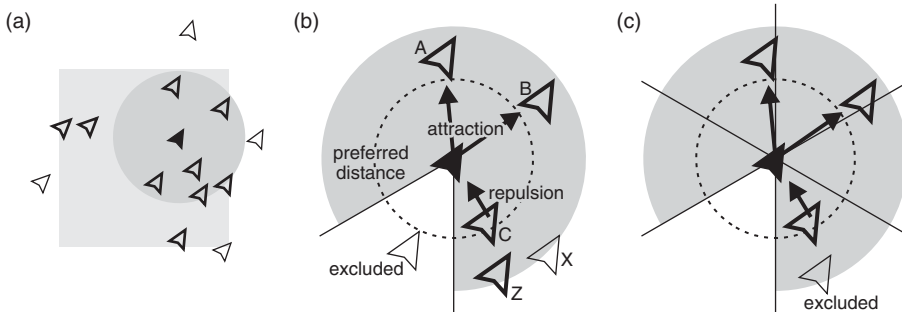


Figure 4.17 Details of the selection of flock-mates and the body-force. (a) Alternative definitions of flock-mates, one based on square cells in a lattice, which may include many more mates (heavy outlines) than those included by a circular radius. (b) Exclusion of some flock-mates by a cone angle restriction, and the attraction and repulsion effects associated with a preferred distance setting. (c) Exclusion of a second order mate by a 'pie slice' criterion. See text for details.

introduce forces of attraction and repulsion between individuals. For coherent behaviour to emerge, individuals accelerate towards, that is, are attracted by, nearby individuals more remote than the preferred distance (cases A and B) and are repelled by closer individuals (case C). This introduces a difficulty of how to handle individuals within the range of interest, but on the far side

of a closer flock-mate. While case X in Figure 4.17(b) is excluded as outside the radius of interest, case Z is in range. Forces of attraction towards Z and repulsion away from C have the potential to squeeze the flock together, even where individuals are trying to maintain a preferred distance. Figure 4.17(c) shows one way that this can be handled by introducing sectors (or ‘pie-slices’) around the central individual and only including the nearest individual in each sector as a flock-mate. This approach excludes Z from the flock-mates in the case shown. Another approach is to use a Voronoi tessellation (see pages 87ff.) of the space around individuals and for interactions only to occur between neighbours in the tessellation.

These details affect the sorts of flocks that emerge in models. Following Reynolds (1987), Grégoire et al. (2003) (see also Czirók and Vicsek, 2000) introduced a force acting between individuals so that they are attracted to one another when they are further apart than a preferred distance and repelled from one another when they are closer than this distance. Grégoire et al. term this effect a *body-force* and show that different combinations of the overall density of individuals, their speed of movement and the relative strength of the alignment and body forces produce different movement regimes, such that the behaviour resembles that of molecules in gaseous, liquid or solid states.



4.7

In Figure 4.18, a number of flocks produced by a model loosely based on this work are shown. In case (a) the ‘squeezing’ problem noted above is evident, when a simple definition of flock-mates is used—here the individuals that share each grid cell. Although the preferred distance is only 0.5 more remote individuals are considered flock-mates and attracted towards one another. Intervening nearer individuals, although they do not want to remain so close, are ‘trapped’, and highly compressed flocks result. In Figure 4.18(b) the preferred distance can be achieved by the flock because a six-sector nearest-neighbour ‘pie slice’ approach has been used. Figures 4.18(c) and (d) use the same neighbourhood approach and an increased preferred separation distance $r = 1.0$. The difference between these two cases is that the speed of individuals is no longer held constant in flock (d) and the greater freedom of manoeuvre available to individuals results in a more evenly spaced flock without the bunching evident in (c). Flocks (e) and (f) increase the preferred separation distance to $r = 2$. This distance is greater than is possible given the overall density of individuals. Even so, in case (e) a stable well-spaced flock still forms, albeit with some zig-zagging in the movement, as individuals attempt to space themselves more widely than the available space permits. The difference in Figure 4.18(f) arises from the relative strength of the body-force being increased four-fold. This increases how quickly individuals attempt to adjust their speed and direction of movement to remain well-spaced, and makes the system unstable because individuals overcompensate for the presence of others nearby and overshoot. The result is that a coherent flock is unable to form.

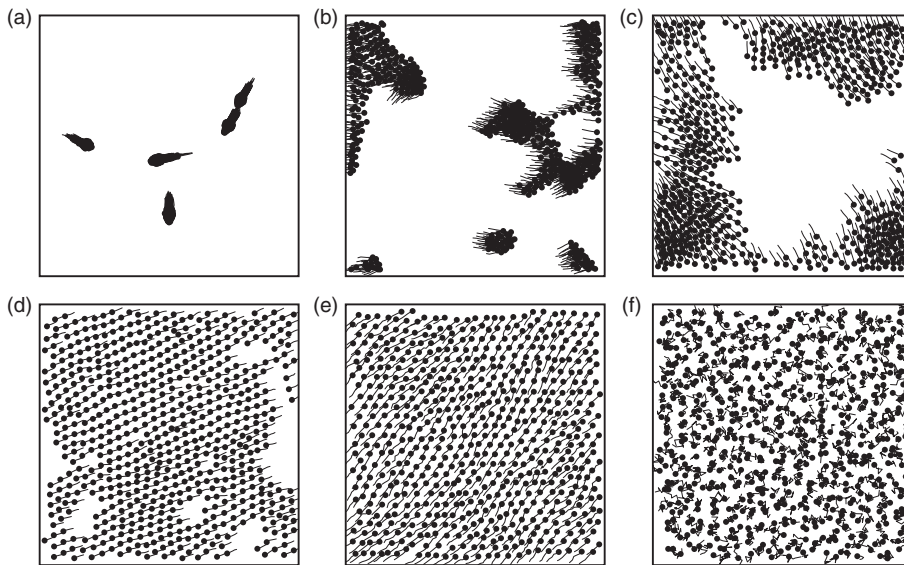


Figure 4.18 Differing outcomes for a more complex model loosely based on Grégoire et al.'s (2003). Each panel shows the model state after 150 iterations starting from a random initial state. In all cases the model space is a 25×25 toroidally wrapped grid and the density of individuals is $\rho = 1$. Other parameters as follows: (a) body-force with preferred distance $r = 0.5$, fixed speed $v = 0.3$, flock-mates based on shared grid cells; (b) body-force $r = 0.5$, $v = 0.3$, $k =$ six-sector nearest neighbours; (c) $r = 1$, $v = 0.3$, $k = 6$; (d) $r = 1$, $k = 6$, speed allowed to vary in the range $0.2 < v < 0.6$; (e) $r = 2$, $k = 6$, $0.2 < v < 0.6$; (f) $r = 2$, $k = 6$, $0.2 < v < 0.6$, but with the body-force increased four-fold.

Clearly, a wide variety of collective movement behaviours can be generated using an overall attraction–repulsion–alignment framework. As a result models with broadly this structure have been widely used to explore the behaviour of flocking animals. Schellinck and White (2011) provide a useful overview and cite many examples, particularly models of schooling fish (see, for example, Hemelrijk and Hildenbrandt, 2008, Huth and Wissel, 1992), but also of herds of land animals (Gueron and Levin, 1993, Turner et al., 1993). Perhaps more surprising is that similar models can be used to investigate the behaviour of human crowds (see Torrens, 2012, for an extensive review), particularly in emergency evacuation situations when the options available to people are limited (Helbing et al., 2000).

4.6 Applying the framework

Movement is a fundamental driver (and outcome) of many processes in social and environmental systems. Movement ecology has become an important focus of that discipline in recent years (Nathan et al., 2008, Sugden and

Pennisi, 2006). Similarly, in the social sciences there has been increased interest in human movement patterns (González et al., 2008), although this work is at a less advanced stage of development at the time of writing, in large measure due to privacy concerns around collecting tracking data for people as opposed to animals. Even so, the rapid development of what comes close to whole-population tracking via near-ubiquitous mobile phones and internet services is rapidly closing the gap, ushering in an era of computational social science (Lazer et al., 2009). In this section we briefly review examples from a wide range of literature to give a sense of the possibilities and also of the issues encountered when working with these models.

4.6.1 Animal foraging

As is clear from the many examples already discussed, models along the lines considered in this chapter find widespread application in attempts to understand the movement of animals in their environments. As we have noted, although pure random walks of one kind or another, with suitable tuning of turn-angle distributions and step-length distributions, can be used as phenomenological models, it is widely recognised that these are not satisfactory representations of most real movement behaviour, although they have a role as null models in the study of empirical data. Thus, for example, the considerable body of work in geographical information science on the analysis of moving point objects can utilise random walks as a basis for assessing real movement data (see, for example, Laube and Purves, 2006).

We have already seen work focused on the movement of animals driven by an environment in which resources are non-randomly distributed. Because they provide a plausible reason (or motivation) for the movement behaviour at different scales, such models are more convincing than purely internally driven random walks. Generally, movement related to exploitation of a resource is highly localised, while search behaviour is more wide ranging. Where resources are plentiful, localised movement from one site to the next is a viable strategy, but as resources are depleted more long-range and rapid search movement is called for (Sims et al., 2012). An interesting idea in the foraging theory literature is that most search behaviour is *saltatory*, meaning that it is ‘stop–go’ movement to a greater or lesser extent. A key consideration is then the length of moves during which no search happens and the duration of stops when search takes place, and the considerations discussed with respect to Simon’s model and Figure 4.11 (page 113) concerning the overlap between previously searched space and the newly accessible space become relevant (O’Brien et al., 1990).

More complicated mechanisms such as memory and path reinforcement of movement have also been developed. The paper by Boyer and Walsh (2010) is a case in point, deserving more detailed consideration. Their model

features a high-resolution landscape (a 200×200 lattice at 1 m resolution, with model ticks every 0.5 min) populated by fruiting trees, which bear power-law distributed resource loads, so that resources are concentrated in a small proportion of all trees. Trees fruit for a 30-day period starting at uniform randomly distributed times. Monkey foragers in this model can remember the location, resource yield and fruiting state of a tree when they last visited it, although this memory is lost if the tree fruits again when they are not present. Based on this information, the monkeys can predict when a tree will be in fruit and how much fruit it is likely to have available, at least for a limited period after they last encountered it. Because the model is intended to explore the relative benefits of memory and exploration, the monkeys' movement involves either moving to a known tree in memory or with probability p taking a random walk step. If the decision is to move to a known tree, then the choice of which tree is based on a cost-benefit comparison similar to that in an earlier model (Boyer et al., 2006) but with the expected resource availability based on information in memory (see Boyer and Walsh, 2010, page 5649 for details). Additionally, the probability p of a random step is linked to the expected yields of known trees, so that when no 'good' trees are known the probability of random exploratory movement increases.

Although this sounds complicated, it is a highly abstract representation of the reasoning processes of an intelligent animal and does not require that we assume an implausible level of knowledge and reasoning skill—in this sense it meets Simon's criteria for bounded rationality. Findings from this model point to an optimal balance between memory and random search. Memory improves the success of foraging but only up to a point because over-reliance on memory leads to non-discovery of important resources. In an interesting spatially-informed analysis, the authors show that habitual movement paths emerge, that is, sequences of visited trees that are repeated through a model run. Paths in the model also have heavy-tailed step-length distributions. Overall, the model confirms earlier findings that the distribution of resources is an important control on likely movement patterns. It also points to the importance of moving on from sterile debates about the optimal efficiency of single mechanism null models of movement, which are unlikely to apply to many real foragers. A general framework for the effects of memory on models of movement is suggested by Gautestad and Mysterud (2010), who coin the term 'memory dependent kinetics' and provide pointers to much of the relevant literature.

Another interesting variant in the foraging model literature looks at how flocks or herds forage. In these cases a tension between safety in numbers (staying close to the group) and the competition for food resources and finding new resources (pushing individuals away from the group) is central, and has been reported in models (Getz and Saltz, 2008, Bonnell et al., 2010) that directly consider the urge to find new resources relative to the need to stay

within a safe distance of the group. More general questions as to how animal groups make collective decisions revolve around resolving a similar tension between the interests of individuals and groups, and are reviewed by Conradt (2012). Work on very large herds has adopted a continuous space differential equation-based approach (Okubo, 1986, Gueron and Levin, 1993, Illius and O'Connor, 2000) rather than the flocking model methods discussed here, perhaps reflecting the difficulties of scale in such models (see the concluding remarks of this chapter).

4.6.2 Human 'hunter-gatherers'

There is no difference in principle between animal foraging behaviour and human hunter-gatherer behaviour, although it is likely that the decision-making behaviours of human hunter-gatherers and of animals differ, and will be represented differently in models. The cognitive capacities of human hunter-gatherers will most likely be more complicated to represent than animals in models, particularly when cultural aspects are included, but even this will depend on the model scale and the goals behind building it. Boyer et al. (2012) are careful to note that their conclusions regarding the effects of memory on foraging success and on spatial outcomes are equally applicable to primates and humans. As they note, 'How scaling laws emerge from the interplay between memory and landscape features remains elusive' (page 846). It is important to recognise that they make this point not with respect to human hunter-gatherers but in relation to models of contemporary human movement patterns, which typically occur in dense urban networks with numerous 'resources' of interest, such as places of employment, schools, shops, leisure activities and so on.

With regard specifically to early human hunter-gatherer activity, humans appear to have exhibited complex and varied relationships with environments in terms of the choices they made about how to survive in a particular landscape, and many of these choices related quite specifically to movement (Kelly, 1983). This is a context where the use of simulation models appears attractive because data are often limited and because of the impossibility of experiments. Exploration of how field data on artifact distributions might be explained by different movement strategies is an idea that has been explored in models of early human hunter-gatherer societies (Lake, 2000, 2001). Interestingly, the issues associated with how memory affects outcomes were identified early in the development of this field (Costopoulos, 2001). Movement at migration scales has also been a focus to a greater extent than in animal movement models (Mithen and Reed, 2002, Nikitas and Nikita, 2005), these models being at large scale (the whole Eurasian–African land mass) and more closely related to random walk models or even the contact process discussed in Section 3.4.1.

Models of human movement in the contemporary period are most prevalent in the transport literature, where origins and destinations distributed over networks are the dominant mode of representation and the methods discussed in this chapter are less relevant. More recent interest in random walk based models has derived from attempts to understand general patterns of human movement (Brockmann et al., 2006, González et al., 2008) and here the debate about Lévy movement has again appeared. While humans tend to range more widely than most animals, the argument that different scales of movement arise from different processes appears even stronger. Pedestrian models of human movement (Section 4.6.4) are more closely related to the models discussed in this chapter.

4.6.3 The development of home ranges and path networks

The repeated revisiting of pathways in Boyer and Walsh's (2010) model is immediately suggestive of a link between movement and the development of home ranges or activity spaces. Generally speaking, home range and movement approaches have tended to be treated separately (Börger et al., 2008), with the former analysed statically (perhaps using spatial partitioning models, see Section 3.6) and the latter more amenable to dynamic approaches. When repeatedly reused spaces emerge from a movement model, the potential arises to link these important perspectives on how space is used. Various types of reinforced random walks (Davis, 1990a, Pemantle, 1992) are one possible basis for such links, although as we have just seen, there is no necessity for purely phenomenological approaches to be adopted. Instead, memory effects, either in searching individuals or in the landscape itself (or both), might produce the outcomes. The latter case, where the movement has effects on the landscape and these subsequently strengthen the likelihood of repeated traversals of the same paths, has been explored in the context of the formation of informal pathways in an urban park (Helbing et al., 1997). Essentially the same 'active walker' model can be applied to the development of networks connecting settlement systems (see Batty, 2005) and to paths formed by animals or human foragers.

4.6.4 Constrained environments: pedestrians and evacuations

In the previous section, moving individuals might contribute to the formation over time of a network of paths across a landscape, which they are more likely to use than not. Pedestrian movement, on the other hand, almost always occurs in confined environments where, in addition to other pedestrians,

individuals in motion must steer a path through many and varied obstacles. This is not in principle a difficult additional feature for movement models to deal with. Some locations can be excluded from consideration for movement. Alternatively, the repulsion effects applied in flocking models can be applied to street furniture and other obstacles to enable pedestrians to avoid them. Early models of pedestrian movement used some combination of these features (see, for example, Haklay et al., 2001).

Importantly, it is immediately apparent in representing this kind of activity that several levels of movement behaviour are involved. Pedestrians are generally in a particular urban location for a reason, and are likely to be going somewhere, with some kind of plan. In pursuit of that plan, they may have a more immediate or intermediate goal in mind, such as the next street intersection. More immediately, they will be steering a path through any surrounding crowd and avoiding obstacles in the form of other pedestrians, street furniture and so on. While models based on the simple attraction–repulsion mechanisms of the flocking models in Section 4.5 are reasonably useful for the lowest level of behaviour in this scheme, they perform poorly at the higher levels. Torrens (2012) reviews models in this general area, although he is critical of most, and goes on to describe an intricate model that addresses the perceived failings by incorporating multiple levels of spatial decision-making and behaviour. This multi-level structure is not so different from that of some earlier models, although the end result is considerably more sophisticated.

Where such highly detailed models are most likely to prove their worth is in the critical context of planning for evacuation from complex buildings in emergency situations. It is here, where individual options are most limited, that the details of body size and shape, and of specific cognitive reactions are most likely to make a difference to overall outcomes. Additionally, there are rarely many data on which to base an assessment of the accuracy of models, and experiments are not an option. In this data-poor environment, a conservative approach using complex, detailed models may be appropriate. More to the point, highly simplified physically-based models, while they capture general patterns, are a risky choice on which to make potentially life and death decisions about maximum rates of egress from a building in an emergency. Notwithstanding the likely need for richer models, many simple models of emergency evacuation have been developed (see, for example, Helbing et al., 2000, Klüpfel et al., 2005, Pan et al., 2007, Moussaïd et al., 2011). These models all use mechanisms similar to those in the flocking models discussed in this chapter, but the constrained situation leads to emergent dynamics with wave effects, delays and other phenomena, the safe management of which is important in emergency situations. Insights from models can inform the detailed design of spaces to enhance flows, but testing such models remains a problem—Shiwakoti et al. (2009) even suggest using panicking ants as

a ‘hardware model’ (see Section 1.1.2) with which to evaluate simulation models, or perhaps even as a model for panicking humans!

4.6.5 Concluding remarks

A key challenge in all movement models is to retain a clear idea of the model scale and (especially) grain, and of the model scope, both spatially and temporally, in the context of the questions that are being addressed. Combining minute-by-minute routine foraging behaviour with seasonal and continental-scale migrations is never going to be easy, and it is important to consider if it is even necessary. If the focus of attention is on detailed movement at small scales over relatively short time periods, then longer range reorientations of movement may not even occur inside the model’s scope, but may instead be better represented by the disappearance of individuals that have finished feeding and the arrival into the model of new individuals. This allows the model to focus not on search, but on foraging itself (see, for example, Mercader et al., 2011). On the other hand, where longer range, seasonal or migratory movements are of interest, then localised, search-related movement may become irrelevant to the model structure given the disparate scales involved, and feeding need only be represented (if at all) by resource depletion effects ‘in place’ at a particular location. Indeed it may not even be necessary to represent individual entities and their behaviours, and in some cases the frameworks presented in Chapters 3 and 5 may be more appropriate. We revisit some of these issues of model scale and representation in a more general sense in Chapter 6.

