Where do you want to start?

Mild 🌶

1. Francine has 26 letter tiles, one for each letter of the alphabet. How many ways could she arrange all of the letters in a circle?

$(26-1)! = 25!$

2. Brandon has scrabble tiles that spell M,A,T,H,E,M,A,T,I,C,S. He arranges them in order to make "words", how many words can he make if he uses…
   a. All the tiles?
   b. An arrangement of four tiles using only unique letters?
   c. Four of the tiles using unique letters, but the first letter must be an M?

11 tiles, 8 unique letters · 2 M's · 2 A's · 2 T's

a) $\frac{11!}{2!2!2!}$ b) $8 \cdot 7 \cdot 6 \cdot 5 = {}_8P_4$ c) $1 \cdot 7 \cdot 6 \cdot 5 = {}_7P_3$

3. Simplify the following:
   a. $\frac{(n+2)!}{n!}$ $\frac{(n+2)(n+1)(n!)}{n!}$

$(n+2)(n+1)$

$n^2 + 3n + 2$

   b. $\frac{(n+4)!}{(n-1)!} = \frac{(n+4)(n+3)(n+2)(n+1)(n)(n-1)!}{(n-1)!}$

$(n+4)(n+3)(n+2)(n+1)(n)$

Medium 🌶🌶

4. Simplify the following:
   a. $\frac{{}_nP_5}{{}_nC_5}$

$\frac{\frac{n!}{(n-5)!}}{\frac{n!}{5!(n-5)!}}$

$= 5!$

   b. $\frac{{}_nC_1}{{}_nC_{n-1}} = \frac{n!/(n-1)!(1!)}{n!/(n-(n-1)]!(n-1)!}$

$= \frac{n!/(n-1)!}{n!/(n-1)!}$

$= 1$ this makes sense b/c these are the same!

5. Let's go back to Brandon in Question 1 and his nerdy scrabble tiles. How many four-letter words can he make if he…
   a. Only has vowels? ${}_5P_4 = 120$ (sometimes ${}_6P_4$ for y)
   b. Has an S in any location?

$(4)\ ({}_{25}P_3)$

this is where we choose a spot for S — assign the other 3 letters to their places

Spicy 🌶🌶🌶

6. How many outfits can you make from 4 shorts, 3 shirts, 2 hats and 3 nose rings. Assume you must wear a shirt and shorts, but everything is optional.

$(4\cdot3\cdot2\cdot3) + (4\cdot3\cdot2) + (4\cdot3\cdot3) + (4\cdot3)$

all — incl. hat — incl. nose — just S or S

or… $(4\cdot3)(6+2+3+1)$ or $(4\cdot3)(4\cdot3)$

4 nose options, 3 hat options

7. Why is ${}_{100}C_{10} = {}_{100}C_{90}$? Explain in English, without just plugging the numbers into the formulas.

Choosing 10 items to include is just like choosing 10 to exclude, which you're doing when you include 90.

Review

8. In a state lottery, players choose 6 numbers between 1-49. The winning numbers are reported in increasing order.
   a. How many total choices are there?
   b. If [redacted] wants two of his numbers to be [redacted] birthday ([redacted]), then how many choices does [redacted] have?

a) ${}_{49}C_6$ (the #s will be reported in order regardless of how they're picked, so order doesn't matter here)

b) ${}_{47}C_4$