Chapter 1

Overview of uncertainties in modern power systems: uncertainty models and methods

Mohamed Ebeed[1], **Shady H. E. Abdel Aleem**[2]
[1]*Department of Electrical Engineering, Faculty of Engineering, Sohag University, Sohag, Egypt;*
[2]*Department of Mathematical, Physical and Engineering Science, 15th of May Higher Institute of Engineering, Helwan, Cairo, Egypt*

Chapter outline

1. Introduction

Nowadays, the development of power systems and the appearance of advanced and new energy concepts such as smart grids, renewable energy resources with energy storage systems, microgrids and nanogrids have introduced many operation and control opportunities but also caused many challenges in planning,

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investment, scheduling, and operation of modern power system networks. In this regard, uncertainty in energy systems is an ongoing issue, particularly in modern renewable energy systems, where it is difficult to accurately describe deterministic states of parameters in power networks because of the increase in the uncertainty associated with high penetration of renewables, or others, that could expose the systems to potential risks. Hence, from operation and management perspectives, the need for accurate decisions is necessary to decide between alternatives, calculate costs, expect revenue, and mitigate possible risks safely and reliably. The simple way is to have enough data available to help take the right decision in a time-effective framework; unfortunately, uncertainty means the absence of the confirmed data. Therefore, to manage the uncertainties, decision-making techniques have been investigated under uncertainties in many works to examine possible (and sometimes impossible) risks and consequences of all scenarios to accurately quantify the importance of uncertainty sources in practical terms to help understand how uncertainties will impact the performance of power systems [1,2].

Generally speaking, the uncertainty of parameters can be categorized into two main categories, namely uncertainty of technical and economic parameters. The technical parameters set describes parameters from two perspectives: topology and operation. The topology group indicates failure or outage of any element (generator, line or others) in power networks, while the operational group indicates uncertainty models of load demand alteration, load growth, renewables output (wind, PV, etc.), fluctuation, and uncertain penetration of plug-in electric vehicles (PEVs). The economic parameters set describes variations in electricity market price, gross domestic product, employment and unemployment rates, and economic growth. Recently, a third category of uncertainty has arisen in 2020, which is epidemics, pandemics, and disasters that affect all the energy sector from both techno-socio-economic perspectives, as over the past few months in 2020, the Covid-19 pandemic has caused an unprecedented global economic and social crisis that has significantly affected all aspects of life across the globe, including the energy sector [3]. Although electrical network operators were well prepared to cope with the immediate effects of the Covid-19 pandemic, it remains difficult to deal with and predict the wide range of consequences for the energy sector.

In this chapter, models of the uncertainty of parameters in modern power systems such as uncertainty models of load demand, wind energy, PV energy, PEVs, electricity price, and load growth are summarized to bring together several aspects of uncertainty management in power systems in planning and operation stages within an integrated framework. Then, uncertainty modeling methods such as probabilistic (numerical and analytical methods), possibilistic (α-cut and defuzzification), hybrid probabilistic-possibilistic (fuzzy scenario and fuzzy Monte Carlo), information gap decision theory, robust optimization, and interval analysis are presented in detail. Finally, the related future trends are explored.

2. Uncertainty models of parameters in power systems

2.1 Load demand uncertainty model

Generally, the uncertainty of load demand can be modeled using the normal or the Gaussian probability density functions (PDFs). Thus, the PDF of load demand can be expressed as follows [4,5]:

$$PDF_L(S_L) = \frac{1}{\sqrt{2\pi\sigma_L}} exp\left[-\frac{(S_L\ -\ \mu_L)2}{2\sigma_L^2}\right] \tag{1.1}$$

where PDF_L is the probability density function of the load demand. S_L is the apparent power of the load demand. $\sigma_L$and μ_L are the standard deviation and the mean values of the load demand, respectively.

2.2 Wind energy uncertainty model

The Weibull PDF is usually applied to define wind speed distribution [6]. It can be described as follows:

$$PDF_v(V) = \left(\frac{\beta}{\alpha}\right)\left(\frac{V}{\alpha}\right)^{(\beta-1)} \exp\left[-\left(\frac{V}{\alpha}\right)^{\beta}\right] \quad 0 \le V < \infty \tag{1.2}$$

where PDF_v(V) is the probability density of the wind speed (V). α denotes the scale parameter and β denotes the shape parameter for the PDF of the Weibull function.

Besides, the Rayleigh probability density function is also applied for modeling the variation of the wind speed, V, in which the Rayleigh probability density function is a special type of the Weibull PDF where $\beta\ = 2$. Hence, the Rayleigh PDF is modeled as follows [5,7–12]:

$$PDF_v(V) = \left(\frac{2V}{\alpha^2}\right) \exp\left[-\left(\frac{V^2}{\alpha^2}\right)\right] \tag{1.3}$$

Further, the output power from a wind power unit is expressed using the following models.

- *Model #1*

$$P_w(V) = \begin{cases} 0 & \text{for} \quad V < V_i \ \text{ and } \ V > V_o \\ P_r\left(\dfrac{V - V_i}{V_r - V_i}\right) & \text{for} \quad (V_i \le V \le V_r) \\ P_r & \text{for} \quad (V_r < V \le V_o) \end{cases} \tag{1.4}$$

where P_r is the rated output power of the wind turbine. Cut-in rated and cut-out wind speeds of the turbine are denoted as V_i, V_r, and V_o, respectively [11,12].

- *Model #2*

$$P_w(V) = \frac{1}{2}\rho A V^3 C_p \tag{1.5}$$

where ρ is the air density, A is the swept area of the rotor, V is the wind velocity (m/s), and C_p is the coefficient of performance [13].

- *Model #3* [13,14]

$$P_w(V) = \begin{cases} 0 & \text{for} \quad V < V_i \quad \text{and} \quad V > V_o \\ P_r\left(\dfrac{V^3 - V_i^3}{V_r^3 - V_i^3}\right) & \text{for} \quad (V_i \le V \le V_r) \\ P_r & \text{for} \quad (V_r < V \le V_o) \end{cases} \tag{1.6}$$

- *Model #4* [15]

$$P_w(V) = \begin{cases} 0 & \text{for} \quad V < V_i \quad \text{and} \quad V > V_o \\ P_r\left(\dfrac{V^2 - V_i^2}{V_r^2 - V_i^2}\right) & \text{for} \quad (V_i \le V \le V_r) \\ P_r & \text{for} \quad (V_r < V \le V_o) \end{cases} \tag{1.7}$$

2.3 PV energy uncertainty model

The lognormal probability density is used to express the solar irradiance [16–18]. It can be described as follows:

$$PDF_s(G_s) = \frac{1}{G_s \sigma_s \sqrt{2\pi}} exp\left[-\frac{(\ln(G_s) - \mu s)^2}{2\sigma_s^2}\right] \quad Gs > 0 \tag{1.8}$$

where PDF_s is the probability density of solar irradiance (G_s), and μs denotes the mean deviation and σ_s denotes the standard deviation.

The probability of solar irradiance can be also represented using the Beta distribution function as follows [4,19]:

$$PDF_T(T) = \begin{cases} \dfrac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)+\Gamma(\beta)} \times G_s^{\propto -1} \times (1 - G_s)^{\beta-1} & \text{If } 0 \le G_s \le 1,\ 0 \le \alpha, \beta \\ 0 & \text{otherwise} \end{cases} \tag{1.9}$$

where α, β are parameters of the beta probability function. The parameter of the Beta *PDF* can be evaluated using the mean and standard deviation of the random variable as follows:

$$\beta = (1-\mu_s) \times \left(\frac{\mu_s \times (1+\mu_s)}{\sigma_s^2}\right) - 1 \tag{1.10}$$

$$\sigma_s = (1-\mu_s) \times \left(\frac{\mu_s \times \beta}{(1-\mu_s)}\right) - 1 \tag{1.11}$$

Further, the output power of PV units can be calculated as a function of solar irradiance as expressed in the following models.

- *Model #1*

$$P_{PV,\ out} = \begin{cases} P_{sr}\left(\dfrac{G_s^2}{G_{std} \times X_c}\right) & \text{for } 0 < G_s \le X_c \\ P_{sr}\left(\dfrac{G_s}{G_{std}}\right) & \text{for } G_s \ge X_c \end{cases} \tag{1.12}$$

where P_{sr} denotes the rated power of the *PV* system. G_s is the solar irradiance at the location under consideration. G_{std} denotes the solar irradiance under standard environment conditions (about 1000 W/m^2). X_c represents a specific irradiance point [20,21].

- *Model #2* [22]

$$P_{PV,out} = \begin{cases} P_{sr}\left(\dfrac{G_s}{G_{std}}\right) & \text{for } 0 < G_s \le G_{std} \\ P_{sr} & \text{for } G_s \ge G_{std} \end{cases} \tag{1.13}$$

- *Model #3*

$$P_a(G_s) = \int P(G_s)\cdot f(G_s)\cdot dG_s \tag{1.14}$$

where P_a denotes the average output power from the PV module, $P(G_s)$ is the module output power, and $f(G_s)$ denotes the PDF [23].

- *Model #4*

$$P_{PV,\ out} = \zeta_{PV} \times A_{PV} \times G_s \tag{1.15}$$

where ζ_{PV} is the efficiency and A_{PV} is the area of the *PV* module (m^2), respectively [24].

- *Model #5*

$$P_{PV,\,out} = N_s \times N_p \times FF \times V_{oc} \times I_{sc} \tag{1.16}$$

$$V_{oc} = \frac{V_{Noc}}{1 + c_2 \ln\left(\frac{G_N}{G_a}\right)} \left(\frac{T_N}{T_a}\right)^{c1} \tag{1.17}$$

$$I_{sc} = I_{Nsc}\left(\frac{G_N}{G_a}\right)^{c3} \tag{1.18}$$

$$FF = \left(1 - \frac{G_N}{\left(\frac{V_{oc}}{I_{sc}}\right)}\right)\left(\frac{\frac{V_{oc}}{nKT/q} - ln\left(\frac{V_{oc}}{nKT/q} + 0.72\right)}{1 + \frac{V_{oc}}{nKT/q}}\right) \tag{1.19}$$

where N_s and N_p represents the number of series and parallel PV module, respectively. I_{sc}, V_{oc}, V_{Noc}, I_{Nsc}, and FF are the short-circuit current, open-circuit voltage, nominal open-circuit voltage, nominal short-circuit current, and the fill factor of the PV module, respectively. G_N and T_N are the nominal irradiance and temperature of the module, while G_a and T_a are the actual irradiance and temperature. c_1, c_2, and c_3 are constants. T denotes the module temperature. n denotes the density factor. K is the Boltzmann constant, and q is the charge of the electron [25].

- *Model #6*

$$P_{PV,\,out} = N_s \times N_p \times FF \times V \times I \tag{1.20}$$

$$I = S_a[I_{sc} + K_i(T_c - 25)] \tag{1.21}$$

$$V = V_{oc} - K_v \times T_c \tag{1.22}$$

$$FF = \frac{V_{mpp} \times I_{mpp}}{V_{oc} \times I_{oc}} \tag{1.23}$$

where K_i is the current temperature, T_c is the cell temperature coefficient, K_v is the voltage temperature coefficient, S_a is the average solar irradiance, I_{mpp} is the current at maximum power point (MPP), and V_{mpp} is the voltage at MPP [26,27].

2.4 PEVs uncertainty model

The PEVs also have a random nature to be considered. Three random variables are associated with the PEVs as the daily arrival time (initial parking time), the initial state of charge (SOC) of the EV battery, and vehicle travel concerning the distance, which can be represented using Eqs. (1.24)−(1.26), respectively [28,29]. PDF_T is the PDF of the daily arrival time, σ_T is the standard deviation of the daily arrival time, and μ_T is the mean of the daily arrival time. PDF_{SOC} is the PDF of the initial battery SOC, σ_{SOC} is the standard deviation of the initial battery SOC, and μ_{SOC} represents the mean value. In addition, PDF_d is the PDF of the traveling distance, σ_d and μ_d are the standard deviation and mean value of the traveling distance, respectively.

$$PDF_T(T) = \frac{1}{\sigma_T\sqrt{2\pi}} exp\left[-\frac{(T - \mu_T)^2}{2(\sigma_T)^2}\right] \tag{1.24}$$

$$PDF_{SOC}(SOC) = \frac{1}{\sigma_{SOC}\sqrt{2\pi}} exp\left[-\frac{(SOC - \mu_{SOC})^2}{2(\sigma_{SOC})^2}\right] \tag{1.25}$$

$$PDF_d(d) = \frac{1}{d\sqrt{2\pi}} exp\left[-\frac{(\ln(d) - \mu_d)^2}{2(\sigma_d)^2}\right] \tag{1.26}$$

2.5 Electricity price uncertainty model

The electricity price that is purchased from the grid is also one of the vital random parameters in power systems. The PDF of the electric price is given using Eq. (1.27) as follows [30,31], in which PDF_{EP} is the PDF of the electricity price, σ_{EP} and μ_{EP} are the standard deviation and mean value of the electricity price.

$$PDF_{EP}(EP) = \frac{1}{\sigma_{EP}\sqrt{2\pi}} exp\left[-\frac{(EP - \mu_{EP})^2}{2(\sigma_{EP})^2}\right] \tag{1.27}$$

2.6 Load growth uncertainty model

The load growth is very important to be handled in power system research and it is also considered a random parameter, in which $P_L(0)$ represents the initial power at the base year at an ith bus while $\Delta P_L(t)$ is the incremental load growth at year t. Hence, the load at year t of bus i is $P_L(t) = P_L(0) + \Delta P_L(t)$. Its PDF can be represented by Eq. (1.28) [32,33], where $PDF_{\Delta P_L}$ is the PDF, $\sigma_{\Delta P_L}$ is the standard deviation, and $\mu_{\Delta P_L}$ is the mean value of the load growth.

$$PDF_{\Delta P_L}(\Delta P_L) = \frac{1}{\sigma_{\Delta P_L}\sqrt{2\pi}} exp\left[-\frac{(\Delta P_L - \mu_{\Delta P_L})^2}{2(\sigma_{\Delta P_L})^2}\right] \tag{1.28}$$

3. Uncertainty modeling methods

Various uncertainty modeling methods are presented in the literature, and they are categorized into six main categories:

- *Probabilistic approaches*: The input parameters of a problem are random with a PDF identified for them. The commonly known probabilistic approaches or uncertainty modeling methods are categorized into numerical and analytical methods, such as Monte Carlo simulation (MCS), sequential MCS, Markov Chain MCS, pseudosequential MCS, and nonsequential MCS methods for the numerical-based methods and linearization, scenario-based, and PDF approximation methods for the analytical-based methods.
- *Possibilistic approach*: It rests on the fuzzy sets where the input parameters are represented using a membership function. The common methods in this group are the α-cut and defuzzification methods.
- *Hybrid possibilistic-probabilistic approaches*: The input parameters are mixed between both the possibilistic and probabilistic approaches. The common methods in this group are the Fuzzy-scenario and fuzzy-MCS methods.
- *Information gap decision theory*: Information gap decision theory (IGDT) approach specifies as to what extent the uncertain parameter can change while assuring the minimum income for the decision maker, in which robustness and opportuneness are the two basic features of IGDT.
- *Robust optimization*: The achieved decisions are taken based on solving a problem considering the worst-case scenario of a given uncertain dataset.
- *Interval analysis*: The input parameters are assumed to be taken from a known interval.

The listed uncertainty modeling methods help assist the decision maker in evaluating the consequences of different aspects of the problem under attention in the presence of uncertain parameters. Fig. 1.1 shows the well-known methods that have been applied for modeling uncertainty in electric power systems.

3.1 Probabilistic methods

3.1.1 Numerical methods

3.1.1.1 Monte Carlo simulation methods

The MCS method is a numerical method that uses random numbers associated with their PDFs to solve problems [31,34–37]. It is widely applied for modeling the uncertainty in power systems. In this method, a multivariate function exists, and it represents the output of the power system, for example, net energy saving, power loss, voltage profile, voltage stability, or a reliability metric, etc, so that $Y = f(X, Z)$, X is a vector that denotes the input parameter of the system including several uncertain parameters, that is,

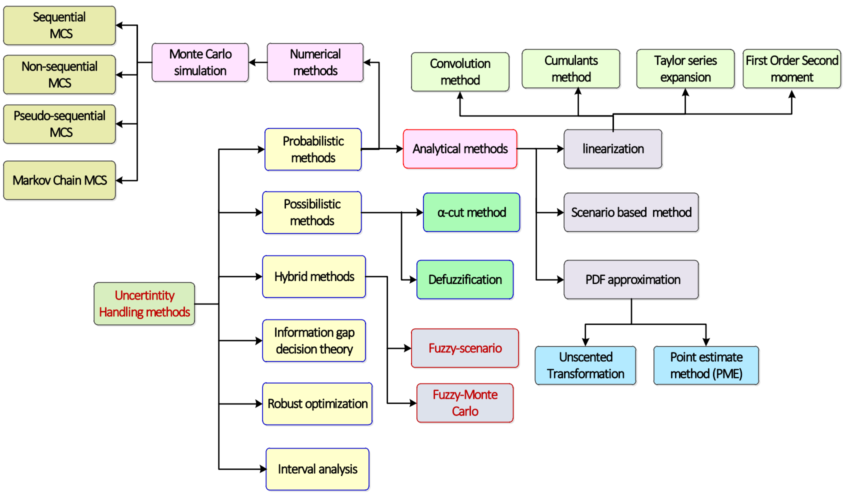

FIGURE 1.1 Methods used in the literature for modeling uncertainty in electric power systems.

$X = \{x_1, x_2, x_3, \ldots, x_n\}$. In a power system, X could be the load demand, wind turbine output power, PV output power, EVs' power profile, decisions from DGs owners, as well as the electricity price, and others. Z represents the power system decision variables including the sizing or locations of DGs, energy storage system, FACTS, and charging stations of EVs, etc.

MCS is an iterative process, in which the model is iteratively evaluated over a maximum iteration number (*Max.Iteration*), which usually ranges from 8000 to 10,000 iterations. In each iteration, a sample vector of the uncertain input variable is produced based on PDFs $\{x_1^l, x_2^l, x_3^l, \ldots, x_N^l\}$, then the output function is calculated for the produced vector.

$$Y = f\left(x_1^l, x_2^l, x_3^l, \ldots, x_N^l\right) \tag{1.29}$$

After performing the simulation up to the maximum iteration number, the approximate distribution of the output can be formulated in which it includes the mean and standard deviation of the output vector.

$$\mu_Y = \text{Mean}(Y_1, Y_2, Y_3, \ldots, Y_{Max.\ Iteration}) \tag{1.30}$$

$$\sigma_Y = \text{STD}(Y_1, Y_2, Y_3, \ldots, Y_{Max.\ Iteration}) \tag{1.31}$$

The steps procedure of the MCS method is illustrated in Fig. 1.2. The merits and demerits of MCS are summarized as follows [38,39]:

The main merits of MCS are as follows:

- It is a simple and flexible method to apply.
- It can be easily applied for nonconvex or nondifferentiable and complex problems.
- It can be applied to systems that have many uncertain variables.
- It can deal with all PDF types.

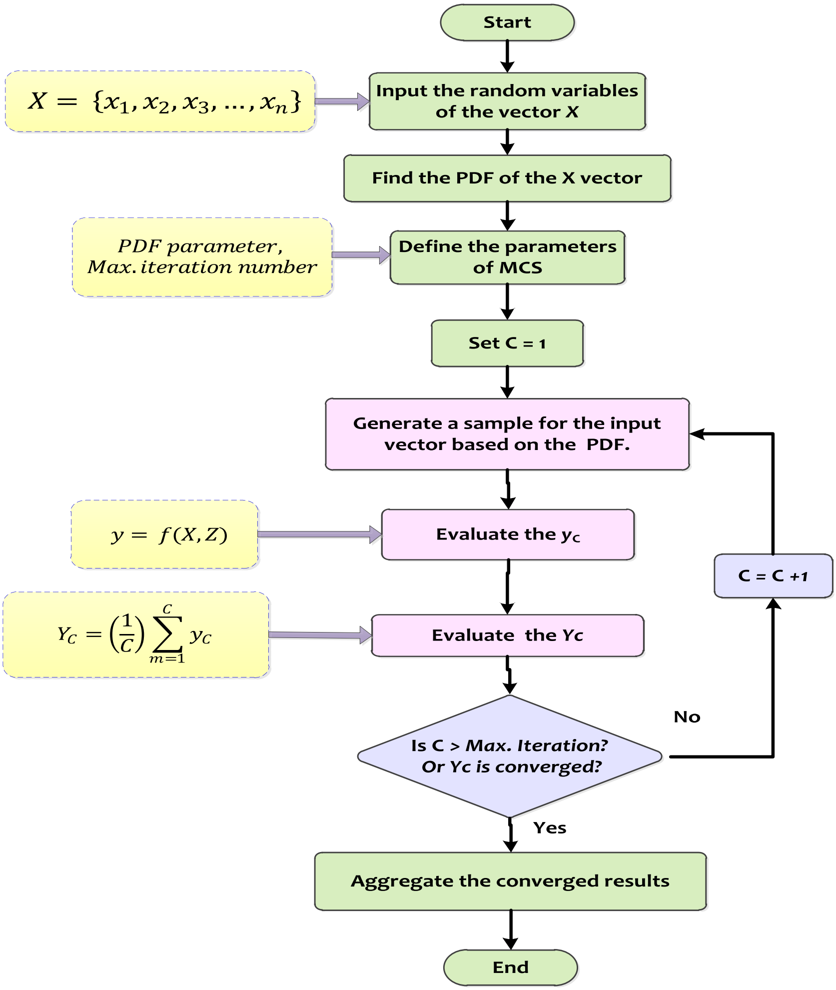

FIGURE 1.2 Flowchart of MCS.

On the other side, the demerits of MCS are as follows:

- It is an approximation method.
- This method is applied only when the PDF is defined.
- This method is iterative and repetitive, which indicates that the main feature of MCS is its high computation burden. Thus, several methods have been proposed in the literature for lessening the computation burden.

3.1.1.2 Sequential MCS method

Sequential MCS (SMCS) method or particle method is a numerical set of Monte Carlo methods that can provide an attractive and convenient method for computing posterior distributions [40,41]. Besides, they have considered sampling and resampling techniques designed to be derived from a sequence of PDFs and are applied for simulating the chronological behavior of a system. The main feature of this method is its flexibility, easiness, excellent computational power, and generality that help support its application in a wide range of applications with general settings.

In [42], the reliability indices with the presence of DG have been assessed considering the random interruption duration at the load point that has been modeled using the SMCS. Arabali et al. [43] presented a stochastic framework to assess the optimal integration of hybrid systems including wind, PV, storage system, and HVAC loads while SMCS was employed to obtain the sequential samples of system states for the chronological progress. Li et al. [44] presented probabilistic wind storm models for risk analysis of the distribution systems and SMCS was applied to simulate the impacts of wind storms. In Refs. [45], Lopes et al. assessed the integration of wind and hydropower units in the transmission system, and the SMCS was employed to obtain the time series of the wind and river inflows. In Ref. [46], SMCS was applied to assess the system reliability considering the effects of faults on load in distribution systems. In Ref. [47], SMCS has been utilized for evaluating the reliability of a smart grid considering real-time thermal rating electrical conductors under different weather conditions. In Ref. [48], SMCS was utilized for the reliability assessment of an electrical transmission system combining with a probabilistic prediction interval with the integration of wind power system. In Ref. [49], a well-designed method for the maximum power point tracking was proposed for a PV system considering the partial shading. SMCS has been applied to undertake the nonlinearity of time-varying at the adjustment of the step size voltage among the incremental conductance approach.

3.1.1.3 Pseudosequential MCS

The pseudosequential MCS (PSMCS) has been presented by Leite da Silva in 1994 [50]. This method is based on nonsequential sampling and chronological simulation of the subsequences associated with failed states only. The step procedure of this method is depicted in Fig. 1.3. In Ref. [51], PSMCS has been applied to evaluate customers' nodal reliability under PV power fluctuations. Celli et al. [52] presented an approach for assessing the smart distribution network reliability based on the adoption of the PSMCS. In addition, the authors in Ref. [53] presented an approach based on PSMCS-chronological simulation to evaluate the loss of load indices, with particular prominence with time-varying loads.

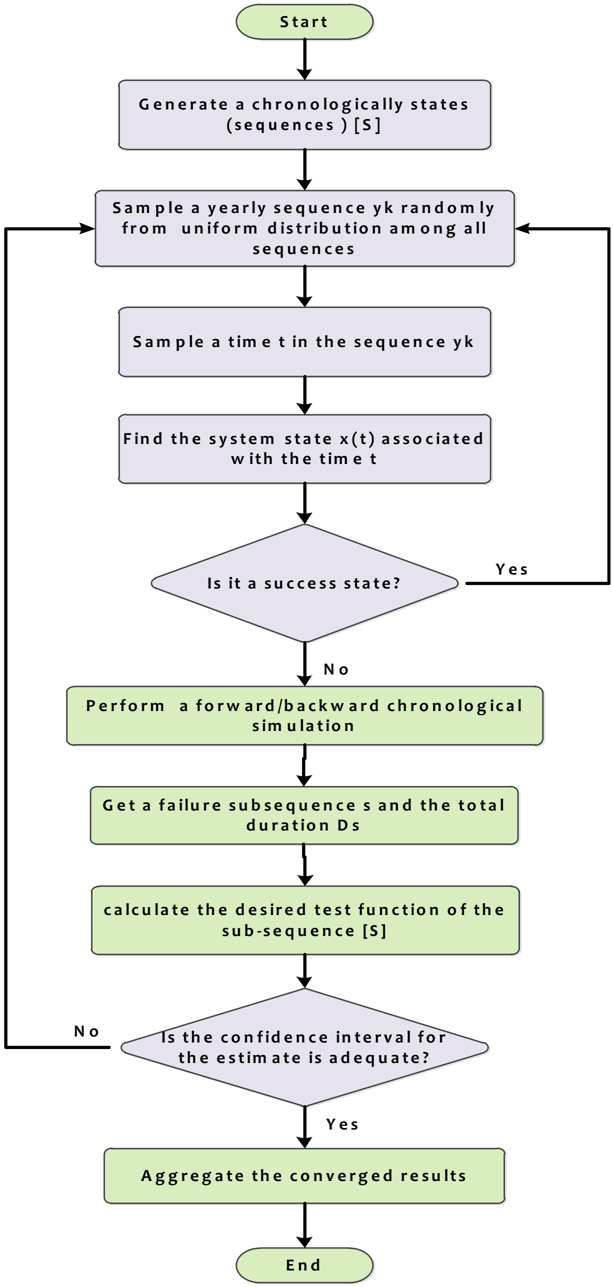

FIGURE 1.3 Flowchart of the PSMCS method.

3.1.1.4 Markov chain MCS

Markov chain MCS (MCMCS) is a dynamic MCS method and is usually utilized to handle the uncertainty of parameters of the system. In MCMCS, a Markov chain with MCS is used to create the samples depending on the probability distribution, in which the probability of producing a novel state in the chain is based on the current state only [54,55].

In the implementation of MCMCS, the probability of transition is defined in terms of the Metropolis method where Metropolis et al. proposed the transition probability from state m to $\overline{m}$, $q(m, \overline{m})$ while the portability of the accepted state is $\alpha(m, \overline{m})$. The step procedure of MCMCS based on the Metropolis–Hastings approach is depicted in Fig. 1.4 [54,56,57].

In [58], MCMCS was applied to generate a synthetic time series of wind power output. In Refs. [59], MCMCS was applied for forecasting the generated power of the PV system using statistics of the historical data. In Ref. [60], the reliability indices were captured by MCMCS to obtain the wind power time series for the evaluation of the generating capacity adequacy.

3.1.1.5 Nonsequential MCS

In the nonsequential MCS, the states are obtained randomly sampling regardless of the chronology of the events, unlike the sequential MCS that is based on the time-dependent states. The steps of this method are illustrated in Fig. 1.5.

In [61], the nonsequential MCS method was applied to minimize the harmful emissions of thermal generation units with wind power generation unit's behavior of hourly loads and wind generation. Amaral et al. [62], presented an efficient method based on the nonsequential MCS for the evaluation of a composite system. In addition, the authors in Ref. [63] have applied the nonsequential MCS method for evaluating well-being indices for composite generation and transmission systems. In Ref. [64], the reliability evaluation of power systems has assigned based on the sequential nonsequential MCS method and particle swarm optimization.

3.1.2 Analytical methods

The analytical methods are based on performing arithmetic with PDF of the uncertain input parameters. These methods are categorized as follows:

3.1.2.1 Scenario-based method

The scenario-based method is an efficient and simple method to model probabilistic uncertainties where the continuous space of uncertain function is transformed into discrete scenarios with corresponding probabilities, where the PDF curve is split into subregions [65–71]. Each region represents a scenario that has its probability. Suppose that the split regions $k = 1, 2, 3\ldots., N$

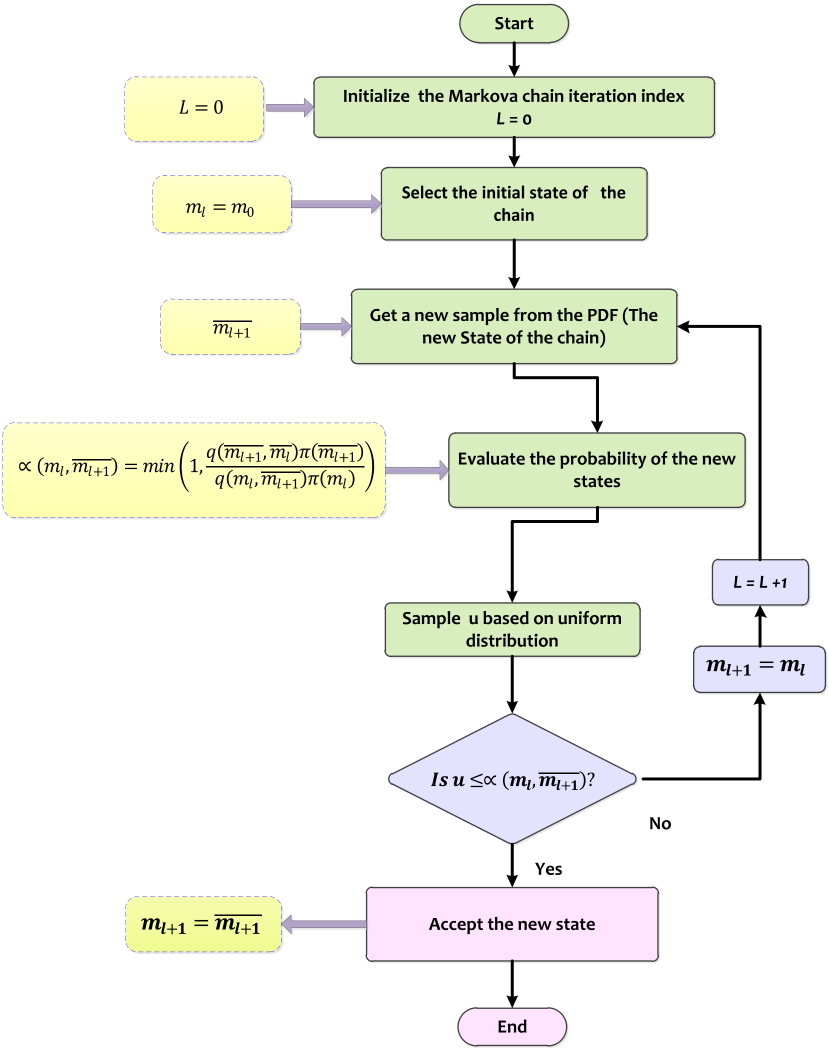

FIGURE 1.4 MCMCS based on the Metropolis–Hastings method.

and their corresponding probabilities are π_1, π_2, $\pi_3 \ldots$, π_N. Thus, the expected output value, $E(y)$ can be given as follows:

$$E(y) = \sum_{k=1}^{N} \pi_k \times f(x) \tag{1.32}$$

The scenario-based method is simple and efficient while the shortages of these methods are that this method is approximate and gives the expected

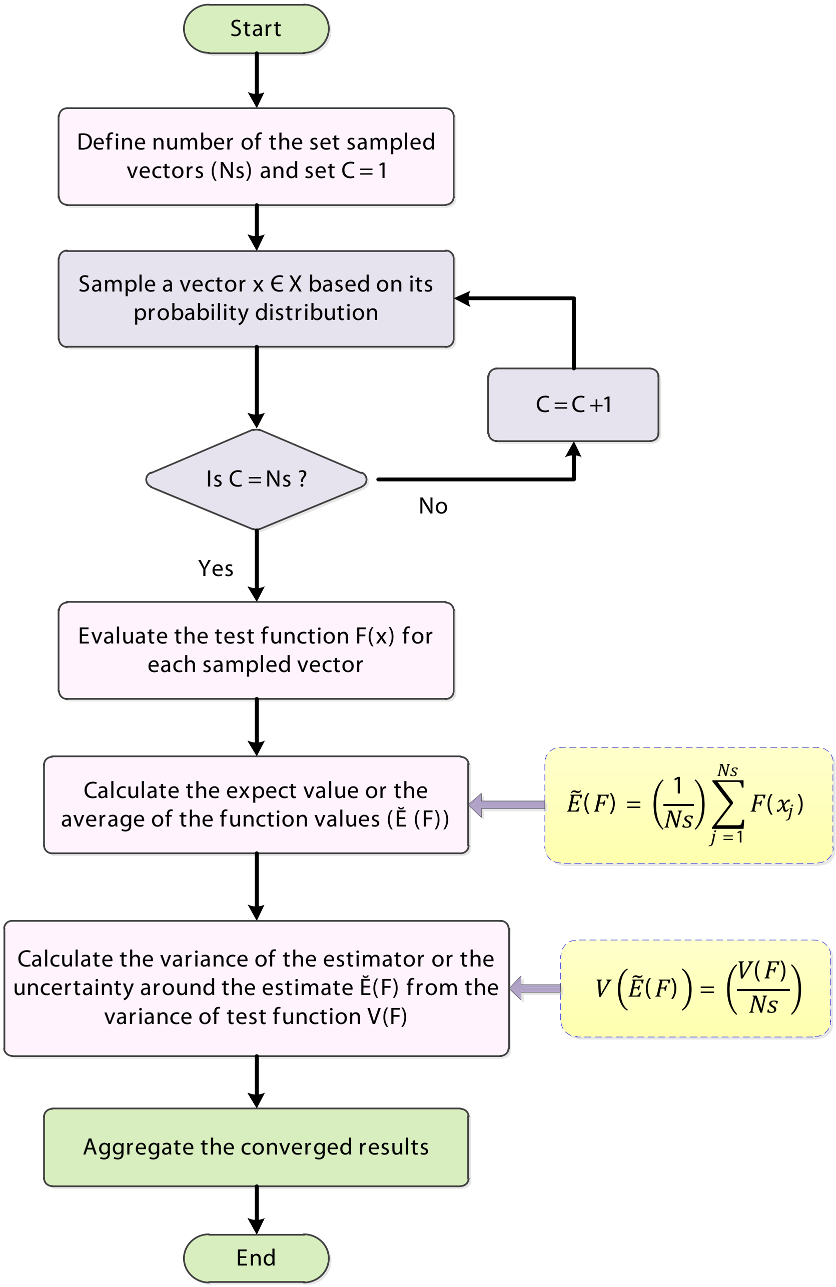

FIGURE 1.5 Flowchart of the nonsequential MCS method.

values of the output functions. In the literature, the scenario-based method was applied to handle the uncertainties of topological and loads for the optimal choice of pilot points for zonal voltage control [65]. In addition, it was applied to handle the uncertainty of wind, solar, and load uncertainties for optimal reactive power solutions. The authors in Refs. [68] have presented a combined generation−load model using the scenario-based method for optimal allocation of distributed generators. The authors in Refs. [69,70] applied the scenario-based method for considering the uncertainties of the output power of solar and wind units to minimize power loss. The authors in Ref. [71] applied the scenario-based method to solve the deterministic problem of maximization of the benefits of battery storage systems, minimization of the investment and replacement cost of the storage systems, and the operation and maintenance cost of the distributed generators with market price uncertainty.

3.1.2.2 PDF approximation methods

3.1.2.2.1 Unscented transformation method Unscented transformation (UT) method is an efficient method that has been applied for problems that use the linearization process [72,73]. Consider a function $Y = f(X)$ with uncertain input variables where X represents the vector of these variables with n-dimension and the mean and covariance values of this vector are $\overline{X} = m$ and P_X, respectively. The main job of the UT method is assigning the mean and covariance ($\overline{Y}$ and P_Y, respectively) of the output function. The steps of the UT method are shown in Fig. 1.6.

3.1.2.2.2 Point estimation method The point estimation method (PEM) is an efficient method for solving the probabilistic problem using its PDF. For example, if there are n uncertain parameters in a vector X, then PEM performs $2n$ calculations to obtain the expected values [74−78]. The PEM replaces the probabilistic distribution with a limited number of discrete points that match the distribution up to the third statistical moment. The PEM concentrates the statistical information provided by the first few central moments of a problem input random variable on n points for each variable, named concentrations. By using these points and the function Y (which relates the input and output variables), the information of the uncertainty associated with output random variables of the problem can be obtained. A step procedure of the PEM is illustrated in Fig. 1.7.

The merits of the PEM are summarized as follows [74−78]:

- The PEM is simple and robust.
- It needs less computational time compared with MCS.
- Using PEM, the convergence problem is avoided.
- The PEM is capable to solve problems with multiple uncertain parameters efficiently.

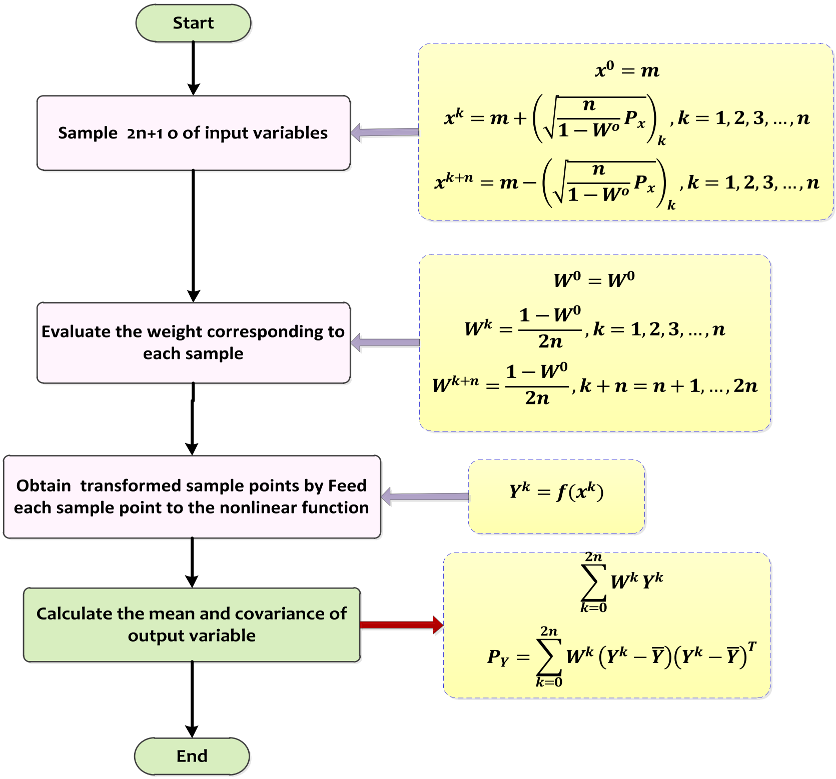

FIGURE 1.6 Flowchart of the UT method.

In addition, the demerits of the PEM are summarized as follows [78]:

- It provides the standard deviation and the mean values of the uncertain output only.
- The need for a description of the PDF of the problem.
- The obtained result of the PEM is a linear approximation to the real solutions.

3.1.2.3 Linearization methods

The linearization methods can be categorized into convolution, cumulants, Taylor series expansion, and first-order second-moment methods. They are presented as follows:

3.1.2.3.1 Convolution method The convolution method has been applied by Allan et al. [79,80] to solve the probabilistic AC load flow problem, where the method support determines the sum of any number of independent random variables.

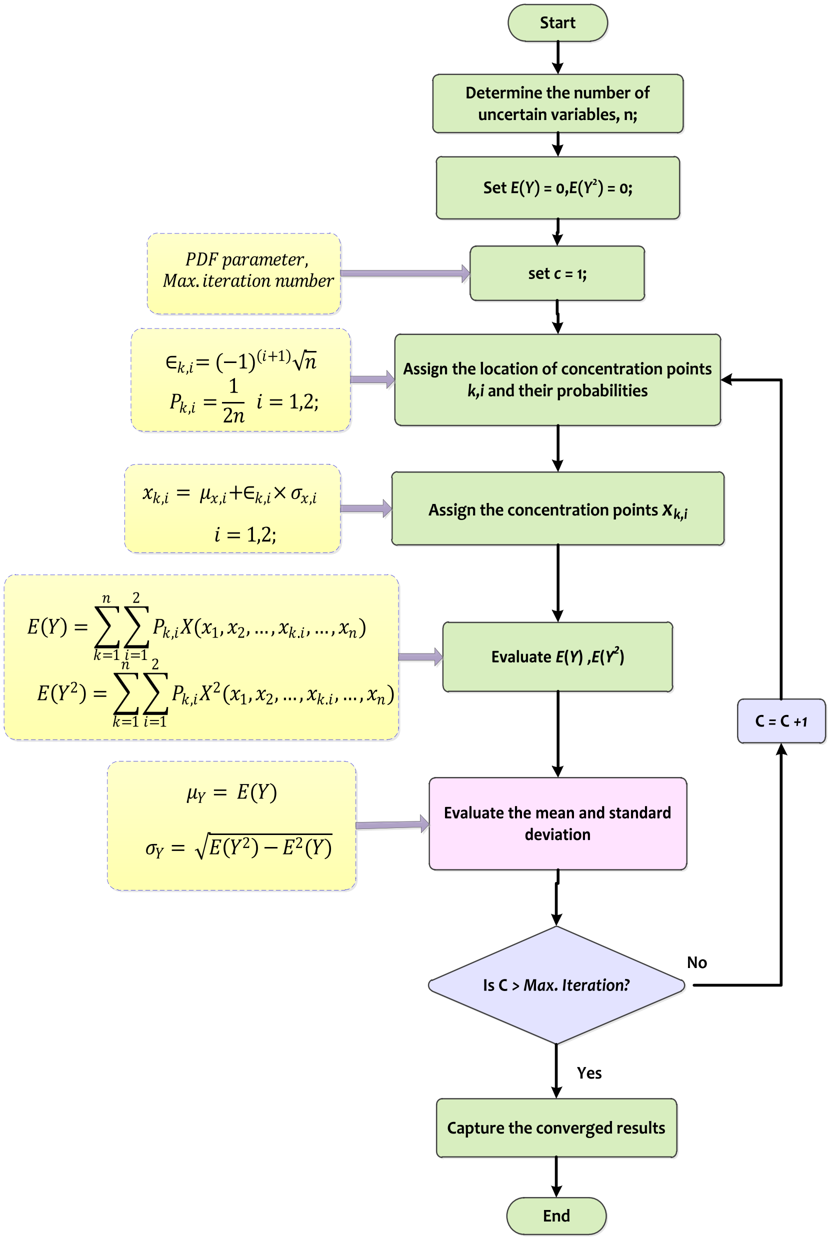

FIGURE 1.7 Flowchart of the two-point estimation method.

For instance, if X and Y are a set of probabilistic variables and their probability density functions are $f_X(x)$ and $f_Y(y)$, respectively, and Z is the output due to X and Y; thus:

$$Z = X + Y \tag{1.33}$$

Then

$$f_Z(z) = \int_{-\infty}^{\infty} f_X(x) f_Y(z-x) dx \tag{1.34}$$

Eq. (1.33) describes the convolution method that is applied for the summing of any number of independent random variables. However, this method suffers from the computational burden, particularly in large-scale problems (this can be solved by using the discrete Fourier transform [81].

3.1.2.3.2 Cumulants method The cumulant method is an efficient method that is employed to assign the PDF of random parameters when they are combined in a linear model [82–89]. The main advantage of this method is that the computational burden of this method is less than the convolution method.

If Z is a random variable derived from a linear combination of n several random variables $(x_{1,}x_{2,}\ldots,x_{n,})$, thus it can be calculated as follows:

$$Z = a_1x_1 + a_2x_2\ldots + a_nx_n \tag{1.35}$$

where x_i represents the ith coefficient of the linear combination model. The cumulant nth-order of Z can be found as follows [85]:

$$K_{n,z} = a_1^n K_{n,x_1} + a_2^n K_{n,x_2}\ldots + a_3^n K_{n,x_n} \tag{1.36}$$

where $K_{n,z}$ represents the nth order of the obtained variable (Z). K_{n,x_1} denotes the nth-order cumulant of the ith component random variable.

Besides, the cumulants method has been applied for probabilistic power flow [82,83]. Schellenberg et al. [84] applied the cumulants method in stochastic optimal reactive power planning to consider the stochastic nature of load demand and output power of wind generators. In addition, the method has been implemented to consider the uncertainty in the power system for probabilistic optimal power flow in Ref. [85,86]. In Ref. [87], the cumulant method has been utilized in assessing generation capacity reliability. The authors in Ref. [88] presented cumulants and the adaptive dynamic clone selection algorithm to assess passive filter planning. In Ref. [89], a comparison has been presented between three different methods including cumulant method, conventional MCS, and point estimation methods for probabilistic small-disturbance stability studies of large power systems with uncertainty considered.

3.1.2.3.3 Taylor series expansion Mathematically, Taylor series (TS) is a series expansion of a function into an infinite sum of terms, in which it can be applied to approximate a complex function while providing quantitative estimates on the error in this approximation. In Ref. [90], the TS expansion method was applied to the linearization of the cartesian coordinate formulation of nodal load flow equations for power system state estimation. The authors in Ref. [91] proposed TS expansion of the Markov chain stationary distribution to reproduce parametric uncertainty to reliability and performability indices in Markov reliability. Zhao et al. proposed the TS approximation and MCS combined with nonparametric probability density to consider parameter uncertainties [92]. Generally, in case of a function that has several random variables, $Y = f\left(x_{1,}x_{2,}\ldots,x_{n,}\right)$. By application, TS can be applied to expand the function around the value $x_{o_i}(i = 1, 2, \ldots, n)$ as follows:

$$Y = f(x_{o_1}, x_{o_2}\ldots, x_{o_n}) + \sum_{i=1}^{n}(x_i - x_{o_i})\left.\frac{\partial f}{\partial x_i}\right|_{x_o} + \frac{1}{2}\sum_{i=1}^{n}\left((x_1 - x_{o_1})\frac{\partial}{\partial x_1} + (x_2 - x_{o_2})\frac{\partial}{\partial x_2} + \ldots + (x_n - x_{\mathrm{n}})\frac{\partial}{\partial x_n}\right)^2 f \tag{1.37}$$

3.1.2.3.4 First-order second-moment method The first-order second-moment (FOSM) method is an efficient, fast, and high accuracy method used as an approximation method based on the first-order terms of the TS expansion. In Refs. [93], the FOSM method was applied for probabilistic load flow to consider the uncertainties of fluctuation of load demands and unit outages. In Ref. [94], it was employed to handle the uncertainty of the power transfer capability for evaluating the power system reliability. Li et al. [95] applied the FOSM method for probabilistic optimal power flow to consider the load fluctuations.

In general, the FOSM method is applied to assign the expected value and standard deviation of any random variable, in which the mean and variance of Y function of stochastic variable x with probabilistic density function $p(x)$ are given as follows:

$$E(Y) = \int_{-\infty}^{\infty} f(x)p(px)dx \tag{1.38}$$

$$var(Y) = \int_{-\infty}^{\infty} [f(x) - E(Y)]^{2}p(x)dx \tag{1.39}$$

In case of using TS to expand the function Y and by neglecting the orders higher than two from Eq. (1.37),

$$Y = f(x_{o_1}, x_{o_2}\ldots, x_{o_n}) + \sum_{i=1}^{n}(x_i - x_{o_i})\left.\frac{\partial f}{\partial x_i}\right|_{x_o} \tag{1.40}$$

If this function is expanded about the mean values $\mu_{o_1}, \mu_{o_2}\ldots, \mu_{o_n}$; then, it can be expressed as follows:

$$Y = f\left(\mu_{o_1}, \mu_{o_2}\ldots, \mu_{o_n}\right) + \sum_{i=1}^{n}\left(x_i - \mu_{o_i}\right)\left.\frac{\partial f}{\partial x_i}\right|_{\mu_X} \tag{1.41}$$

The expectation of Y can be expressed as follows:

$$E(Y) = f\left(\mu_{o_1}, \mu_{o_2}\ldots, \mu_{o_n}\right) \tag{1.42}$$

The variance of Y can be assigned by variances of input variables and the covariance of input variables by pair; thus,

$$\begin{aligned} var(Y) &= E\left[\left(\sum_{i=1}^{n}\left(x_i - \mu_{o_i}\right)\left.\frac{\partial f}{\partial x_i}\right|_{\mu_X}\right)^2\right] \\ &= \sum_{i=1}^{n}\left(\left.\frac{\partial f}{\partial x_i}\right|\right)^2 var(x_i) + 2\sum_{i=1}^{n}\sum_{j\neq i}^{n}\left(\left.\frac{\partial f}{\partial x_i}\right|\right)\left(\left.\frac{\partial f}{\partial x_j}\right|\right)cov(x_i, x_j) \end{aligned} \tag{1.43}$$

where $cov(x_i, x_j)$ represents the covariance between variables x_i and x_j, and it can be given as follows:

$$cov(x_i, x_j) = E\left[\left(x_i - \mu_{x_i}\right)\left(x_j - \mu_{x_j}\right)\right] \tag{1.44}$$

The second term of Eq. (1.39) represents the correlation between the input variables. This term equals zero if the input variables are independent. Therefore, Eq. (1.39) can be expressed as follows:

$$var(Y) = \sum_{i=1}^{n}\left(\left.\frac{\partial f}{\partial x_i}\right|\right)^2 var(x_i) \tag{1.45}$$

The standard deviation is given as follows:

$$\sigma_Y = \sqrt{\sum_{i=1}^{n}\left(\left.\frac{\partial f}{\partial x_i}\right|\right)^2 \sigma_{x_i}^2} \tag{1.46}$$

3.2 Possibilistic methods

The application of possibilistic methods for some cases in the power system is more powerful compared with the traditional probability theory-based methods in case of the presence of imprecise or inadequate data [96–98]. The possibilistic methods are categorized as follows:

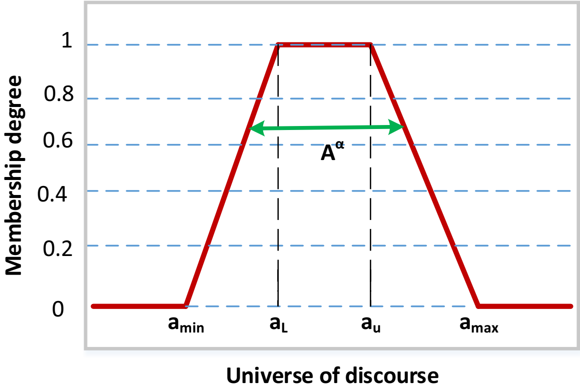

FIGURE 1.8 Fuzzy trapezoidal number illustration.

3.2.1 *α-Cut method*

In the possibilistic method, the possibility distribution P_X is used to model uncertainty of an uncertain value X. P_X is a membership function that defines how much each element x of the universe of discourse U related to X, where $X = (x_{min}, x_L, x_U, x_{min})$. Fig. 1.8 shows an illustration of the fuzzy trapezoidal numbers. The α-cut method is applied to find the possibility of the distribution corresponding to X as follows:

$$A^{\propto} = x \in U | P_X(x) \geq \alpha, \ 0 \leq \alpha \leq 1 \tag{1.47}$$

$$A^{\propto} = \left[\underline{A^{\propto}} \ \overline{A^{\propto}} \right] \tag{1.48}$$

where U is the universe of discourse of X that represents the range of its value. According to Fig. 1.8, the α-cut of the output variable Y is calculated as follows:

$$Y^{\propto} = \left[\underline{Y^{\propto}} \ \overline{Y^{\propto}} \right] \tag{1.49}$$

3.2.2 *Defuzzification*

The main goal of the defuzzification process is transforming a fuzzy number into a crisp one [97–99]. This process can be accomplished using the well-known centroid method. The defuzzified quantity X can be calculated as follows:

$$X = \frac{\int P_X(x) x \, dx}{\int P_X(x) \, dx} \tag{1.50}$$

3.3 Hybrid methods

3.3.1 Fuzzy scenario

This method combines the possibilistic and probabilistic approaches where some parameters are handled by the possibilistic approach using fuzzy arithmetic (refers to a model developed by Zadeh in 1965 [100]). Although the other parameters are handled by the possibilistic approach using the scenario-based approach [101], where $Y = f(X,Z)$ is a multivariable function, and X and Z are vectors of possibilistic and probabilistic uncertain parameters, respectively. In the fuzzy-scenario method, a set of scenarios are formed to describe Z's behavior. Then, the minimum and maximum values of the α-cut outputs are represented as follows:

$$\underline{Y^{\propto}} = \min \sum_{k=1}^{N} \pi_k \times f\left(X^{\propto},\ Z^k\right) \tag{1.51}$$

$$\overline{Y^{\propto}} = \max \sum_{k=1}^{N} \pi_k \times f\left(X^{\propto},\ Z^k\right) \tag{1.52}$$

where π_k represents the probability of the kth state.

Then, the defuzzified value of Y can be calculated as follows:

$$Y^* = \frac{\int P_Y(Y)dY}{\int P_Y(Y)\ dY} \tag{1.53}$$

3.3.2 Fuzzy MCS

This method is applied in case the system includes uncertainties with both probabilistic and possibilistic nature, where $f(X,\ \mathrm{Z})$ is a multivariable function, X and Z are vectors of possibilistic and probabilistic uncertain parameters, respectively. In this method, a combination between fuzzy and MCS methods is presented to solve the problems with both probabilistic and possibilistic nature, in which the variables are categorized into two groups to be separately solved based on the following loops [96,98,102,103]:

(1) First loop: The outer loop in which the sampling process of the probabilistic variable is made based on its PDF, that is, $\mathrm{Z_i^e}$ using MCS.
(2) Second loop: The inner loop that is based on evaluating the fuzzy α-cut method analysis to carry out the uncertainty diagnosis of the possibilistic variables. In this loop, the minimum and maximum values of α-cut outputs are calculated as follows:

$$\underline{Y^{\propto}} = \min f(X^{\propto},\ Z^e) \tag{1.54}$$

$$\overline{Y^{\propto}} = \max f(X^{\propto},\ Z^e) \tag{1.55}$$

3.4 Information gap decision theory

This method was proposed by Yakov Ben-Haim in 1980 [104]. It is wildly applied for severe uncertainty cases where the PDF is not used, which indicates its simplicity. It depends on measuring the differences between parameters and their estimations. Thus, the evaluation burden of this is significantly small. IGDT specifies as to what extent the uncertain parameter can change while assuring the minimum income for the decision maker. Robustness and opportuneness are the two basic branches for understanding IGDT as illustrated in Fig. 1.9. The robustness evaluates the resiliency to failure and opportuneness search for the chance of a windfall. In Fig. 1.9(A), the designer or the decision maker decreases the minimum profit to handle a more part of the uncertain space. However, in Fig. 1.9(B), the decision maker is optimistic, and the more the uncertain variables stray from the estimated amount, the more profit is gained [71,105].

The optimization problem in this situation is formulated as follows [105]:

$$\text{Minmization } J(x, \zeta) \tag{1.56}$$

subject to

$$g_j(x, \zeta) = 0 \quad j = 1, 2, \ldots, m \tag{1.57}$$

$$h_i(x, \zeta) \leq 0 \quad i = 1, 2, \ldots, k \tag{1.58}$$

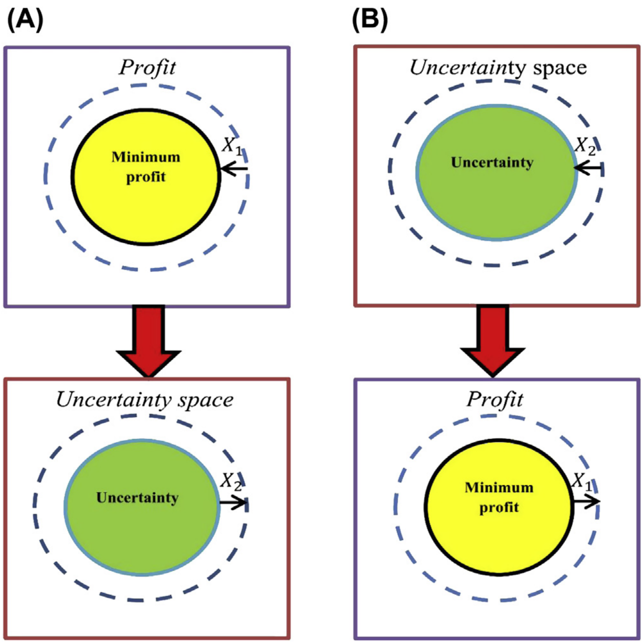

FIGURE 1.9 IGDT (A) robustness and (B) opportuneness.

$$\varphi \in \Gamma$$

where J is the considered the objective function. φ represents the uncertain input values vector. Γ represents a set of uncertainty that describes the actions of uncertain input values. g_j and h_j represent the equality and inequality system constraints, respectively. x represents the sets of control variables in the system, while ζ represents the dependent variables.

$$\forall\ \widetilde{\varphi} \in \Gamma(\widetilde{\varphi}, \alpha) = \left\{ \varphi : \left| \frac{\varphi - \widetilde{\varphi}}{\widetilde{\varphi}} \right| \leq \alpha \right\} \tag{1.59}$$

where $\widetilde{\varphi}$ represents the forecasted value of the uncertain parameter. α represents the maximum deviation of the uncertain parameter from the forecasted value.

In the literature, Saman et al. [106] proposed the risk-averse energy management strategy by optimal reconfiguration of the distribution networks in the presence of wind energy sources via the IGDT method. In Ref. [107], the IGDT has been applied to manage energy resources for supplying a certain load by distributed generations, pool market, DGs, and the bilateral contracts where the uncertainties of these resources are considered by the IGDT method. Besides, the IGDT was applied in Ref. [108] for managing the revenue risk of EV aggregator caused by the information gap between the forecasted and actual electricity prices. Soroudi et al. [109] presented a good model for solving unit commitment problems while incorporating wind energy units in the system and handing the uncertainty of wind power variations using the IGDT. In Ref. [110], IGDT has been applied for handling the uncertainty of the wind power generation system for solving the security-constrained unit commitment. Dehghan et al. [111] solved the transmission expansion planning by considering the uncertain capital costs and uncertain electricity demands using the IGDT.

3.5 Robust optimization

The robust optimization (RO) methods are widely used to handle uncertainties in power systems. The main principle behind the RO methods is to solve the optimization problem with the worst scenario of the uncertain parameters realized [112,113]. The RO method was proposed by Soyster et al. in Ref. [114] to solve problems in which it is hard to get the PDF of the parameters or in cases of unavailability of sufficient data. In Ref. [115], the RO method has been employed to handle the uncertainties of battery demand and electricity prices. In Ref. [116–118], RO methods have been applied to solve the economic dispatch problem with uncertainties considered. In addition, the RO method has been employed to solve the unit commitment problem with uncertainties of the system considered [119–124]. In Ref. [125–129], the RO method has been used to handle the uncertainties in energy management in microgrids to schedule the generation of the units. Besides, the RO method has been implemented to handle the uncertainties related to the electricity markets in Ref. [130–134].

3.6 Interval analysis

The interval analysis (IA) method was early applied for handling the uncertainty in 1966 [135]. In IA, input parameters are taken from a known interval, that is, the probability of the system can be evaluated in terms of the upper and the lower limits of the uncertain parameters that are predetermined values from its interval.

For a multivariate function, $f(x)$ and $\underline{x} \leq x \leq \overline{x}$ where $\underline{x}$ and $\overline{x}$ are the lower and upper limits of the uncertain parameters, the IA detects the probability of the multivariate function. For instance, Eq. (1.60) describes a Gaussian distribution of the parameter x. Thus,

$$\alpha(k) = f(x) = \frac{1}{\sigma\sqrt{2\pi}} exp\left[-\frac{(x-\ \mu)^2}{2(\sigma)^{\ 2}} \right] \tag{1.60}$$

where $\alpha(k)$ represents degree of belongingness (membership) and k is number of the degrees of belongingness. μ equals 1 when the degree of belongingness is 1. So, at this value, one can formulate it as follows:

$$x - 1 = \pm \left[\frac{-ln(\alpha(k))}{\pi} \right]^{\frac{1}{2}} \tag{1.61}$$

$$x = \pm\ \delta_k + 1 \tag{1.62}$$

where $\delta_k = \left[\frac{-ln(\alpha(k))}{\pi}\right]^{\frac{1}{2}}$ and $k = 1, 2, 3, \ldots N$. N is the number of point of linearization of the curve. In case of capturing three distinct intervals (regions); then, the certain values of these inbounds can be represented as follows:

$$\begin{aligned} &D_1 \leftrightarrow \{x\}, \text{i.e., point interval for } k = 1 \\ &D_2 \leftrightarrow \{x[1-\delta_2], x[1+\delta_2]\}, \text{for } k = 2 \\ &D_3 \leftrightarrow \{x[1-\delta_3], x[1+\delta_3]\}, \text{for } k = 3 \end{aligned} \tag{1.63}$$

In the literature, the IA method was used in many research works, For example, Zhang et al. [136] presented a novel reliability reconfiguration technique to enhance system reliability as well as the energy efficiency based on IA to enclose data uncertainties. The authors in Ref. [137] introduced a new method for solving the directional overcurrent relays coordination problem based on the IA to handle the uncertainties of the network topology. Das [138] solved the power flow problem in radial distribution networks and the IA has been applied to deal with input uncertainty of the system. The authors in Ref. [139] solved the distribution load flow by considering the uncertainties of load demand using the IA method.

4. Future trends

It is believed that future grids will face challenging developments such as a smart grid, microgrid and nanogrid, distributed energy resources, and large penetration of renewable energy sources. These innovative developments will pose different kinds of problems in the present systems. For the fruitful design of the energy and power systems, cooperation is needed between different perspectives of stakeholders when many scenarios based on different inconsistent criteria are intended.

Beyond optimization and uncertainty of parameters, decision-making (DM) techniques should be included in the problem formulation to get a solution for a complex problem. DM, in general, involves the allocation of a large number of resources of the economy that significantly influences all of the stakeholders. Opportunely, with the accelerated evolution of friendly computer-based and internet of things technologies, which are capable of performing complex tasks with a large number of decision variables with high computational speeds; DM models can be easily employed for global multi-criteria power system problems with uncertain parameters.

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