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A comprehensive review on uncertainty modeling techniques in power system studies

Morteza Aien [a,b,*], Ali Hajebrahimi [c], Mahmud Fotuhi-Firuzabad (Fellow member, IEEE)[b]

[a] Energy Department, Graduate University of Advanced Technology, Kerman, Iran
[b] Center of Excellence in Power System Management & Control (CEPSMC), Department of Electrical Engineering, Sharif University of Technology, Tehran, Iran
[c] Department of Mathematics and Statics, Laval University, Quebec, Canada

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A B S T R A C T

As a direct consequence of power systems restructuring on one hand and unprecedented renewable energy utilization on the other, the uncertainties of power systems are getting more and more attention. This fact intensifies the difficulty of decision making in the power system context; therefore, the uncertainty analysis of the system performance seems necessary. Generally, uncertainties in any engineering system study can be represented probabilistically or possibilistically. When sufficient historical data of the system variables is not available, a probability density function (PDF) might not be defined, they must be represented in another manner i.e. using possibilistic theory. When some of the system uncertain variables are probabilistic and some are possibilistic, neither the conventional pure probabilistic nor pure possibilistic methods can be implemented. Hence, a combined solution is needed. This paper gives a complete review on uncertainty modeling approaches for power system studies making sense about the strengths and weakness of these methods. This work may be used in order to select the most appropriate method for each application.

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* Corresponding author at: Sharif University of Technology, Azadi Avenue, Tehran, Iran. Tel.: +98 21 66165985.
E-mail address: morteza_aien@yahoo.com (M. Aien).

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1. Introduction

As well as increasing the share of renewable energy resources such as wind and solar generation and restructuring of power systems, new uncertainties are introduced in operation and investment decision making process. Therefore, one of the characteristic features of power system operation and planning is that the decision making problem is faced with significant level of uncertain information. Generally, the uncertain parameters of a given power system can be classified into two categories as:

- Uncertainty in a mathematical sense, which means difference between measured, estimated values and true values including errors in observation or calculation [1].
- Sources of uncertainty, including transmission capacity, generation availability, load requirements, unplanned outages, market rules, fuel price, energy price, market forces, weather and other interruptions, and so on [1].

The traditional and deterministic approaches in power system studies will be unavailing to deal with the aforementioned uncertainties associated in power system studies. Hence, different uncertainty modeling techniques for power markets under uncertainty have been developed so far. The existing uncertainty modeling techniques cover a wide range such as probabilistic approaches, possibilistic approaches, hybrid possibilistic–probabilistic approaches, information gap decision theory (IGDT), and robust optimization as shown in Fig. 1.

The main purpose of these methods is to measure the impact of uncertain input parameters on the system output parameters. However, the main difference between these methods is to utilize different approaches implemented for describing the uncertainty of input parameters. For instance, the probability density function (PDF) is used in probabilistic methods while in the fuzzy approach, a membership function (MF) is assigned for modeling uncertain parameters. A brief explanation about the uncertainty modeling by means of aforementioned methods is provided bellow:

- *Probabilistic approach:* In this approach, it is assumed that the PDF of input variables are known. One of the earliest works in stochastic programming was done by Dantzig in 1955 [2].
- *Possibilistic approach*: An MF is assigned for modeling of input parameters in this approach. The fuzzy arithmetic was introduced by Zadeh [3].
- *Hybrid possibilistic–probabilistic approaches*: Both random and possibilistic parameters are utilized to handle the uncertain input parameters in this approach [4,5].
- *Information gap decision theory*: In contrast to probabilistic and possibilistic decision theory, info-gap analysis does not use PDF or MF: it measures the deviation of errors (differences between parameters and their estimation), but not the probability of outcomes. It was first proposed by Yakov Ben-Haim [6].
- *Robust optimization*: The uncertainty sets are used for describing the uncertainty of input parameters. Using this technique, the obtained decisions remain optimal for the worst-case realization of the uncertain parameter within a given set. It was first proposed by Soyster [7].
- *Interval analysis*: It is assumed that the uncertain parameters are taking value from a known interval. It is somewhat similar to the probabilistic modeling with a uniform PDF. This method finds the bounds of output variables. It was introduced by Moore [8].

The rest of the paper is organized as the following. Section 2 discusses about probabilistic uncertainty modeling and introduces some of its approaches. In Section 3, the possibilistic uncertainty modeling is given. Section 4, presents the evidence theory and the proposed algorithm for joint propagation of probabilistic and possibilistic uncertainties. Information gap decision theory and its formulation is explained in Section 5. Robust optimization and interval optimization are briefly presented in Sections 6 and 7, respectively. Finally, Section 8 closes the paper providing some concluding remarks.

2. Probabilistic methods

Some of the uncertain input parameters follow a probability distribution function (PDF), such as wind speed pattern which follows a Wiebull PDF [4] as (1).

$$f(x) = \frac{\beta}{\alpha^{\beta}} x^{\beta-1} \exp\left[-\left(\frac{x}{\alpha}\right)^{\beta}\right] \tag{1}$$

And, the load uncertainty can be modeled with normal PDF as:

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{\frac{-(x-\mu)}{2\sigma^2}} \tag{2}$$

In this approach, consider $y = f(x)$ as a multivariate function and $x = \{x_1, x_2, x_3, ..., x_n\}$ is the vector of uncertain input random

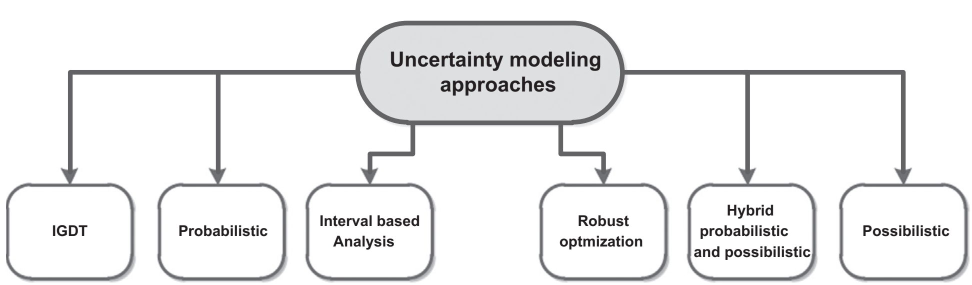

Fig. 1. The uncertainty modeling approaches.

variables. It is assumed that the PDF of input variables are known. The goal is to obtain the PDF of y or the output variable. In order to obtain the PDF of the output variable, several methods have been introduced in the literature for probabilistic analysis. Fig. 2 depicts a general classification of probabilistic methods based on the principle of the used methods.

As shown in Fig. 2, probabilistic methods can be categorized into two groups: numerical and analytical techniques.

2.1. Numerical approaches

One of the most common and accurate stochastic methods is Monte Carlo Simulation (MCS). MCS is recognized to be a system-size independent approach and is used when the system is highly nonlinear, complicated or has many uncertain variables [4]. The MCS is an iterative method which has the following steps:

Step 1. Set MCS counter $C=1$.

Step 2. Randomly generate a sample for the vector X using the PDF of each component x_i.

Step 3. Calculate y_c assuming that $X=X_c$ as $y_c=f(X_c)$.

Step 4. Calculate the expected value of y as $E(y)=\frac{\sum_c y_c}{c}$

Step 5. Calculate the variance of y, $\sigma(y)=E(Y^2)-E^2(Y)$.

Step 6. Stopping criteria is met? End, Else set counter $C=C+1$ and go to Step 2.

Step 7. End.

The authors in [9] address the problem of DG penetration with a Monte Carlo technique that accounts for the intrinsic variability of electric power consumption. In [10], two methods namely Taylor series Approximation and Monte Carlo simulation combined with nonparametric probability density estimation are proposed. The authors in [11] deploy Monte Carlo simulation techniques to systematically sample these random processes and emulate the side-by-side power system and transmission-constrained day-ahead market operations. The seismic probabilistic risk assessment is carried out by hierarchical modeling and Monte Carlo simulation in [12]. The reliability network equivalent techniques have been implemented in the MCS procedure to reduce the computational burden of the analysis in [13].

As shown in Fig. 2, sequential MCS [9–12,14–24], pseudo-sequential MCS [13,14,25,26], and non-sequential MCS [27–31] are three different types of MCS techniques utilized for probabilistic uncertainty analysis of engineering systems.

2.1.1. Sequential Monte Carlo simulation

Sequential Monte Carlo (SMC) methods are set of simulation-based methods which provide a convenient and attractive approach to compute the posterior distribution. Sequential Monte Carlo methods are very flexible, easy to implement, parallelizable and applicable in very general setting. Therefore, SMC method has been implemented in the previous studies. In order to preserve the characteristics of the time series of the variable energy resources (wind and river inflows) and the variable load, the SMC is employed in [15]. In [16], the impacts of wind on power distribution are accurately evaluated through a SMC enhanced by a temporal wind storm sampling strategy. In order to assess distribution system reliability in smart grids, SMC is applied in [17]. A pattern search-based optimization method is used in conjunction with a SMC in [18] to minimize the system cost and satisfy the reliability requirements. State sampling and SMC are used in [19] to estimate the reliability of the RBTS network. An SMC algorithm that can simultaneously assess composite system adequacy and detect wind power curtailment events is employed in [20]. In order to evaluate the adequacy of power systems with wind farms, an SMC simulation technique is developed in [21]. In [22], two main sampling techniques namely random sampling and Latin hypercube sampling are applied to the SMC technique for reliability evaluation of composite generation and transmission systems. A cross-entropy-based three-stage sequential importance sampling method is proposed in [23] to solve the low efficiency problem resulted from the low rate of component state transition during a fixed lead time. Sequential Monte Carlo simulation is utilized in [25,26] for composite power system reliability evaluation.

2.1.2. Non-sequential Monte Carlo simulation

Non-sequential Monte Carlo simulation method is sometimes called the state sampling approach. The concept is based on the fact that a system state is a combination of all component states and each component state can be determined by sampling the probability of the component appearing in that state. It is widely used in power system risk evaluation. For instance, reference [32]

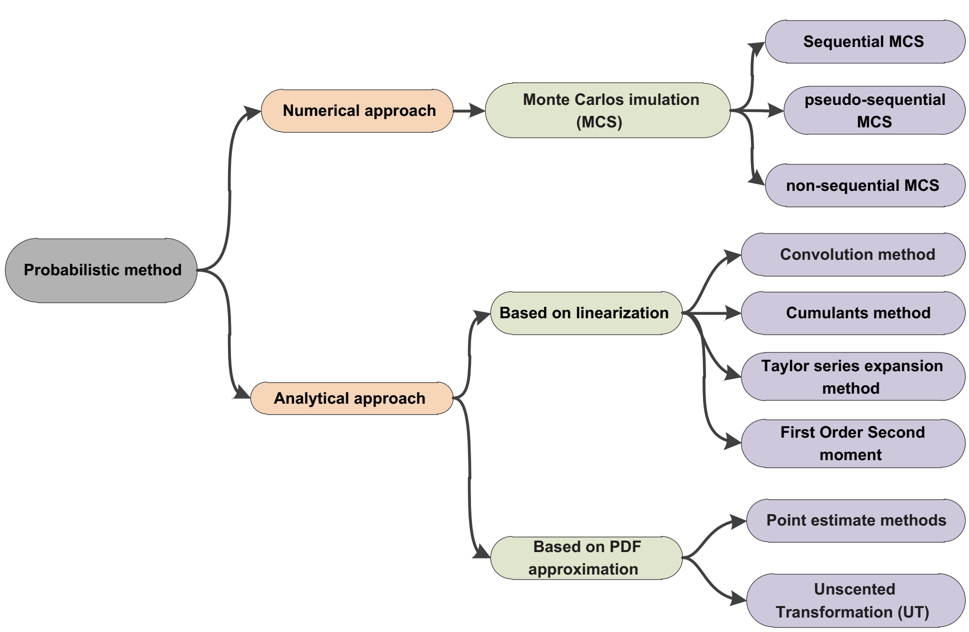

Fig. 2. Classification of probabilistic approaches.

utilizes non-sequential state transition sampling which can be used to estimate the actual frequency index without requiring an additional enumeration procedure for composite reliability assessment. This method is applied in [33] for power system adequacy assessment in optimal wind clustering study. An original non-sequential Monte Carlo simulation tool is developed in [27] to compute the optimal dispatch of classical (coal, oil, etc.) thermal generation in order to minimize polluting (NOx, CO_2, etc.) emissions in presence of wind power. Reference [28] presents an efficient method for composite system well-being evaluation based on non-sequential Monte Carlo simulation. A calculation method of wind farms' capacity credit based on non-sequential Monte-Carlo simulation is performed in [29]. A new methodology to evaluate well-being indices for a composite generation and transmission systems, based on non-sequential Monte Carlo simulation and pattern recognition techniques is presented in [31].

2.1.3. Pseudo-sequential Monte Carlo simulation

Generally, Monte Carlo methods require large amounts of random numbers and it therefore necessities the development of *Pseudo-sequential Monte Carlo simulation*, which is far quicker than previous Monte Carlo methods [26]. A method based on the pseudo-sequential Monte Carlo simulation technique has been proposed in [14] to evaluate the reserve deployment and customers' nodal reliability with high PV power penetration.

- The strength of MCS method can be listed as follows:
  – It is not needed to exactly know transfer function f for calculating y. The problem can be solved as a black box which receives samples and gives the related output.
  – It also works well in non-differentiable and non-convex problems (complex systems) as well as behavior problems.
  – It supports all PDF types.
  – It is intuitive and relatively easy to implement [34].

- The weaknesses of MCS method can be listed as follows:

– The computation burden is usually high since it is iterative and needs several evaluations of function f.

– The number of simulations needed increases as the degrees of freedom of the solution space increases. Therefore, in order to obtain accurate results, thousands of simulations (or many more) are usually required.

2.2. Analytical approaches

The analytical approaches analyze the system and its inputs using mathematical expressions, e.g. PDFs and obtain results also in terms of mathematical expressions. The basic idea of the analytical approach is to do arithmetic with PDFs of stochastic inputs variables. The analytical methods can themselves be categorized in two distinct groups.

2.2.1. The first group of analytical methods:

The first group methods are based on linearization such as:

- Convolution method

In various probabilistic power system studies, the convolution technique is implemented to determine the resulting PDFs [35,36]. Let $X_1, X_2, ..., X_n$ be n independent random variables with known PDFs $f_{X1}(X_1), f_{X2}(X_2), ..., f_{Xn}(X_n)$. Consider $Z = a_1X_1 + a_2X_2 + ... + a_nX_n$, $a_1, a_2, ..., a_n$ are coefficients for Z. Let $Y_i = a_iX_i$. Thus, the PDF of Z is given by

$$f_Z(Z) = \frac{1}{|a_1|} f_{X1}\left(\frac{Y_1}{a_1}\right) \otimes \frac{1}{|a_2|} f_{X2}\left(\frac{Y_2}{a_2}\right) \otimes \cdots \otimes \frac{1}{|a_n|} f_{Xn}\left(\frac{Y_n}{a_n}\right) \tag{3}$$

The main problem associated with this approach is that the technique demands a large amount of storage and computation time in large systems. Ref. [37] notes this problem and has applied the discrete Fourier transform (DFT) to reduce the computational burden. However, the computation may still be extensive. To prevent the convolution operation that appears in the calculation of the PDF of a linear combination of several random variables, the concept of cumulants is introduced.

- Cumulant method

Utilizing cumulants, it is possible to determine the PDF of a linear combination of several random variables by a simple arithmetic process instead of convolution. Moments and Cumulants method serves a feature extraction from a probability distribution and can avoid complicated convolution computation. The γth moment corresponding to a continuous random variable x is defined as follows:

$$\alpha_\gamma = \int_{-\infty}^{+\infty} x^\gamma dF(x) \tag{4}$$

where γ is the order of the moment and $F(x)$ is the cumulative probability density function of x. The moments about the mean value μ of x are called the central moments

$$\beta_\gamma = E[(x-\mu)^\gamma] = \int_{-\infty}^{+\infty} (x-\mu)^\gamma dF(x) \tag{5}$$

If X is a discrete random variable and there is a probability p_c for a corresponding component x_c of X, then the γth moment of X is defined as follows:

$$\alpha_\gamma = \sum_{c=1}^{\infty} p_c x_c^\gamma \tag{6}$$

On the other hand, cumulants are important features for random variables. The cumulants k_γ can be derived from the moments using recursion in closed forms as follows:

$$k_1 = \alpha_1 \tag{7}$$

$$k_2 = \alpha_2 - \alpha_1^2 \tag{8}$$

$$k_3 = \alpha_3 - 3\alpha_2\alpha_1 + 2\alpha_1^3 \tag{9}$$

and so on. The formulation to obtain the other cumulants can be found in [8]. It is obvious that k_1 and k_2 are equivalent to the expected value and the variance of the random variable, respectively. The cumulant method for the probabilistic optimal power flow problem is introduced in [38] and a cumulant based stochastic optimal reactive power planning method for distribution systems with high penetration of wind power generation is proposed in [39]. The authors of [40] introduced two extensions to the cumulant method for probabilistic optimal power flow (POPF) studies; the first is an enhancement to provide improved handling of limits within the P-OPF problem and the second is a way to include correlated variables. A cumulant method-based solution to solve a maximum loading problem incorporating a constraint on the maximum variance of the loading parameter is studied in [41]. The cumulant method and other estimation methods are investigated in [42] for small-disturbance stability assessment of uncertain power systems. A new method for studying passive filter planning using cumulants and the adaptive dynamic clone selection algorithm is introduced in [43]. For generation capacity reliability evaluation, a generalized cumulant model has been developed in [44]. The disadvantage of the cumulant- based algorithm is the loss of accuracy associated with truncation of the order of the cumulants used. When the moments or cumulants of the variables are known, the next step is to obtain the PDF. There are different approaches by using various types of series expansions to approximate the true function. The coefficients in the expansions can be computed from the moments or cumulants of the distribution. The three different approximation expansions are introduced in the following.

- Gram–Charlier a series

Allows many PDFs to be expressed as a series composed of a standard normal distribution and its derivatives. The cumulative distribution function (CDF) and the PDF of x can be written as

$$F(x) = \sum_{i=0}^{n} \frac{c_i}{i!} \phi^{(i)}(x) \tag{10}$$

$$f(x) = \sum_{i=0}^{n} \frac{c_i}{i!} \varphi^{(i)}(x) \tag{11}$$

where $\phi(x)$and $\varphi(x)$ represent the CDF and PDF of the standard normal distribution with $\mu = 0$ and$\sigma = 1$.

$$\varphi(x) = \frac{1}{\sqrt{2\pi}} e^{-x^2/2} \tag{12}$$

The coefficient c_i is a constant coefficient of the Gram–Charlier expansion

$$c_i = (-1)^i \int_{-\infty}^{+\infty} f(x) H_i(x) dx \tag{13}$$

where $H_i(x)$ is the Hermite polynomial. In this case, the orthogonal series expansion is in terms of the classical orthogonal functions. The Hermite polynomials is defined as

$$\varphi^{(i)}(x) = (-1)^i H_i(x) \varphi(x) \tag{14}$$

Then, the relationship between coefficients and central moments would be as

$$\begin{aligned} &c_0 = 1 \quad c_1 = c_2 = 0 \\ &c_3 = -\frac{\beta_3}{\sigma^3} \quad c_4 = \frac{\beta_3}{\sigma^3} - 3 \cdots \end{aligned} \tag{15}$$

Additional information about this method can be found in [45]. In line, some literatures which have utilized Gram Charlier method for power system uncertainty analysis are reviewed here. In order to compute the production cost of a power system in one iteration, an analytical method considering several terms of Gram–Charlier series is presented in [46]. A method using the Gram–Charlier series type A to forecast the PDF of power system loads is proposed in [47]. Literature [48], investigates the sensitivity of the bivariate Gram–Charlier series in the evaluation of reliability for several types of interconnected systems. The PDF of the voltage fluctuation is expressed by the Gram–Charlier series expansion in [49] to regulate the voltage profile for operation planning in distribution systems. The multivariate Gram–Charlier series as means of modelling the PDF which characterizes the uncertain parameters is employed in [50] for stochastic optimal energy dispatch. An efficient calculation method which is consistent with modern probabilistic production costing methods (e.g. Booth/Baleriaux or Gram–Charlier series) is proposed in [51] for evaluating the marginal cost meeting the hourly load. Preventive maintenance scheduling in power generation systems using a quantitative risk criterion is studied in [52] based on the Gram Charlier series of Edge worth Form. Literature [53] presents a novel fast probabilistic power-flow method based on the Gram–Charlier series expansion to deal with uncertainties associated with wind and solar generation. Some studies have combined both concept of cumulant method and Gram Charlier method in power system uncertainty analysis. A method using the Gram–Charlier series and a modification of the cumulants summation formula which accounts for statistical correlation is applied in [54] for production costing which treats power sources with statistically correlated outputs. Cumulants method is employed in [38–44,55,56] and both Cumulants and Gram-Charlier methods are applied in [48,57–59]. The concept of Cumulants and Gram–Charlier expansion theory are combined in [57] to obtain the PDF and CDF of locational marginal prices (LMPs). A method that combines the concept of cumulant and Gram-Charlier expansion theory to calculate probabilistic load flow is introduced in [58,59].

- Edgeworth expansion

If a random variable is normalized, the Edgeworth series expansion may be of value and this expansion can be written as

$$f(x) = \varphi(x) \left\{ 1 + \sum_{n=1}^{\infty} \sum_{\{k_m\}} H_{n+2r}(x) \prod_{m=1}^{n} \frac{1}{k_m!} \left(\frac{k_{m+2}}{(m+2)!} \right)^{k_m} \right\} \tag{16}$$

Where, $\{k_m\}$ is the cumulant m and in the inner summation consists of all non-negative integer solutions of the equation $k_1 + 2k_2 + \cdots + nk_n = n$, and $r = k_1 + k_2 + \cdots + k_n$. In reference [38], the concept of cumulant method and Edgeworth expansion are combined for probabilistic power flow. The cumulant method based on the well-known Edgeworth expansion is studied and compared with the recursive method in [60].

- Cornish–Fisher expansion

The Cornish–Fisher expansion is used to approximate the variable's quantile, which is the inverse function of the CDF. The quantile x of the probability q is the root of the cumulative distribution function $F(x) = q$. The method is based on the cumulants of the variable and the quantiles of the standard normal probability distribution. If the variable is normalized, considering the first five orders of cumulants, the expansion is given by

$$\begin{aligned} x(q) = \xi(q) &+ \frac{\xi^2(q) - 1}{6} k_3 + \frac{\xi^3(q) - 3\xi(q)}{24} k_4 \\ &- \frac{2\xi^3(q) - 5\xi(q)}{36} k_3^2 + \frac{\xi^4(q) - 6\xi^2(q) + 3}{120} k_5 \\ &- \frac{\xi^4(q) - 5\xi^2(q) + 2}{24} k_3 k_4 \\ &+ \frac{12\xi^4(q) - 53\xi^2(q) + 17}{324} k_3^3 \end{aligned} \tag{17}$$

where q is the probability, $x(q)$ is the quantile of the variable, $\xi(q)$ is the quantile of the standard normal distribution, $x(q) = F^{-1}(q), \xi(q) = \phi^{-1}(q)$. However, more theoretical derivative can be found in [61].

- Taylor series

  It is a common practice to approximate a function using a finite number of terms of its *Taylor series*. Taylor's theorem gives quantitative estimates on the error in this approximation. Any finite number of initial terms of the Taylor series of a function is called a Taylor polynomial. The Taylor series of a function is the limit of that function's Taylor polynomials, provided that the limit exists. A function may not be equal to its Taylor series, even if its Taylor series converges at every point. Taylor series expansion is utilized in [10,62,63] for power system reliability assessment and state estimation. The *Taylor series* of a real or complex-valued function $f(x)$ that is infinitely differentiable at areal or complex number a is

$$f(a) = f(a) + \frac{f'(a)}{1!}(x-a) + \frac{f''(a)}{2!}(x-a)^2 + \frac{f'''(a)}{3!}(x-a)^3 + \cdots. \tag{18}$$

- First-order second-method

First-order second-method derives its name from the fact that it is based on a first order Taylor series approximation of linearized function. It is evaluated at the mean values of the random variables, and only uses means and covariance of the random variables. One of the advantages of this method is that it allows the estimation of uncertainty in the output variable without knowing the shapes of PDFs of input variables in detail. Indeed, the mean value and the standard deviations of the input variables suffice to compute the mean value and standard deviation of the output. First-order second-method is implemented in [64–67] in order to deal with the uncertainties. A new probabilistic load flow method based on the first-order second-moment method is developed in [64]. Based on a first-order second-moment method (FOSMM), the POPF model which represents the probabilistic distributions of solution is determined in [65,67].

Linearization is often the common point among the aforementioned methods and it brings about the following shortcoming:

- Linearization transformation would be reliable if the error propagation can be well approximated by a linear function.

This difficulty caused to develop new methods do not need the linearization process that are placed in the second group of analytical methods.

2.2.2. The second group of analytical approaches

The second group of analytical methods are based on the PDF approximation. They benefit from the fact that it is easier to approximate a PDF than to approximate a nonlinear transformation function. The heart of these methods lie in how to generate appropriate samples of input variables that can maintain sufficient information about the input variable's PDF. The point estimation method (PEM) and Unscented Transformation (UT) method are examples of such methods that are discussed in the following.

- point estimation method

The point estimation method concentrates the statistical information provided by the first few central moments of a random variable on *i* points for each variable, named concentration. The two-point estimate method (2PEM) is one of the derivations of point estimation method. The mathematical procedure of this method is formulated as follows:

$$E(Y)^{(1)} = 0; E(Y^2)^{(1)} = 0. \tag{19}$$

In the first step, the initial values are assigned

$$\xi_{k,1} = \frac{\lambda_{k,3}}{2} + \sqrt{n + \left(\frac{\lambda_{k,3}}{2}\right)^2} \quad k = 1, \ldots, n$$
$$\xi_{k,2} = \frac{\lambda_{k,3}}{2} - \sqrt{n + \left(\frac{\lambda_{k,3}}{2}\right)^2} \quad k = 1, \ldots, n \tag{20a}$$

$$P_{k,1} = \frac{-\xi_{k,2}}{2n.\sqrt{n + \left(\frac{\lambda_{k,3}}{2}\right)^2}} \quad k = 1, \ldots, n$$

$$P_{k,2} = \frac{-\xi_{k,1}}{2n.\sqrt{n + \left(\frac{\lambda_{k,3}}{2}\right)^2}} \quad k = 1, \ldots, n. \tag{20b}$$

In the second step, the location and the probability of two concentrations are obtained using (20a) and (20b), respectively.

- The concentrations $x_{k,1}, x_{k,2}$ can be calculated as:

$$x_{k,1} = \mu_{X,k} + \xi_{k,1}.\sigma_{X,k}$$
$$x_{k,2} = \mu_{X,k} + \xi_{k,2}.\sigma_{X,k} \tag{21}$$

where $\mu_{X,k}, \sigma_{X,k}$, are the mean and variance of random variable k, respectively.

- Calculating output variable with regards to the vector X.

$$Y = f(X) \tag{22}$$

$$\boldsymbol{X} = [\mu_{k,1}, \ldots, x_{k,i}, \ldots, \mu_{k,n}] \ \forall i \in \{1, 2\} \tag{22a}$$

- Calculate (23) and (23a) for all random variables

$$E(Y)^{K+1} \cong E(Y)^K + \sum_{i=1}^{2} P_{k,i}.h(X) \tag{23}$$

$$E(Y^2)^{K+1} \cong E(Y^2)^K + \sum_{i=1}^{2} P_{k,i}.h^2(X). \tag{23a}$$

- The expected value and standard deviation of Y can be determined as

$$\Psi_Y = E(Y) \tag{24}$$

$$\sigma_Y = \sqrt{E(Y^2) - \psi_Y^2}. \tag{24a}$$

The strength of PEM method can be listed as follows:

- It is a non-iterative, computationally efficient technique.
- It is simple and easy to implement.
- There is no convergence problem.

The shortcomings of PEM method can be listed as follows:

- It only gives the mean and standard deviation of the uncertain output.
- No information about the shape of the PDF of the output is provided.
- It gives more reliable answers for non-skewed PDFs.
- It is only applicable in problems which are described using a PDF.
- The accuracy would be low when the number of random variables is large [68].
- The original method of 2PEM is cannot consider the correlation between random variables. However, the modified version of 2PEM is applicable to problems with spatial correlation among multiple uncertain input parameters.

This method is applied in various applications such as: the combination of Monte Carlo simulation and Point Estimation Method (PEM) is implemented to investigate the effects of network uncertainties in [69] for transmission expansion planning problem. A new probabilistic load flow solution algorithm based on an efficient point estimate method is proposed in [70]. The authors in [68] present an application of the 2PEM to account for uncertainties in the OPF problem in the context of competitive electricity markets. A probabilistic power flow based on the point estimate method is employed to include uncertainty in the wind power generation output and load demand in [71] for stochastic feeder reconfiguration. In order to minimize cost and increase efficiency, the 2PEM is used for energy management in [72]. The point estimate method has been improved to facilitate stochastic modelling of the aggregated power output of wind turbine generators in different locations in [73]. The major problem associated with aforementioned analytical methods is that these approaches do not consider the correlation among uncertain input variables. Although, the authors in [74,75] modified the method to handle the correlated input variables. The probabilistic methods that can inherently handle correlated variables are of significant interest since the correlation between uncertain input variables is an important issue in modern power system studies.

- Unscented transformation
 The Unscented Transformation (UT) method was developed to overcome the deficiencies associated with traditional probabilistic methods especially those use the linearization process. Analytical methods are established based on some mathematical assumptions and complex algorithms. The UT method is recognized as a powerful approach in assessing stochastic problems with/without correlated uncertain variables. It is a reliable method for calculating the statistics of output random variables undergoing a set of nonlinear transformations. Suppose that X is a vector of n-dimensional probabilistic uncertain variables which has mean$\overline{X}$, and covariance P_{XX}. Consider another uncertain variable Y relating to X through a nonlinear function such as (22). Where, f can be a set of nonlinear functions. The UT method can be used to obtain the mean and covariance of the output variable, Y and P_{YY}, through the following simple steps [76]:
- Step 1: Obtain $2\,n+1$ samples or sigma points of input variables through (25)–(27).

$$X^0 = m \tag{25}$$

$$X^k = m + \left(\sqrt{\frac{n}{1-W^0}P_{XX}}\right)_k, \quad k = 1, 2, ..., n \tag{26}$$

$$X^{k+n} = m - \left(\sqrt{\frac{n}{1-W^0}P_{XX}}\right)_k, \quad k = 1, 2, ..., n \tag{27}$$

- Step 2: Calculate the weight associated with each sample using (28)–(30).

$$W^0 = W^0 \tag{28}$$

$$W^k = \frac{1-W^0}{2n}, \quad k = 1, 2, \ldots, n \tag{29}$$

$$W^{k+n} = \frac{1-W^0}{2n}, \quad k+n = n+1, \ldots, 2n \tag{30}$$

 Note that the associated weights must meet the condition that

$$\sum_{k=0}^{2n} W^k = 1 \tag{31}$$

 In (26) and (27), $\left(\sqrt{\frac{n}{1-W^0}P_{XX}}\right)_k$ is the kth row or column of matrix square root of $\left(\frac{n}{1-W^0}P_{XX}\right)$. The matrix square root of positive definite matrix P means that there is a matrix $A = \sqrt{P}$ such that $P = AA^T$. Here, W^0 is the assigned weight to the point $\overline{X} = m$, named as the zeroth point. It controls the location of other points around the mean value of X.
- Step 3: Feed each sample point to the nonlinear function to yield a set of transformed sample points as

$$y^k = f(x^k) \tag{32}$$

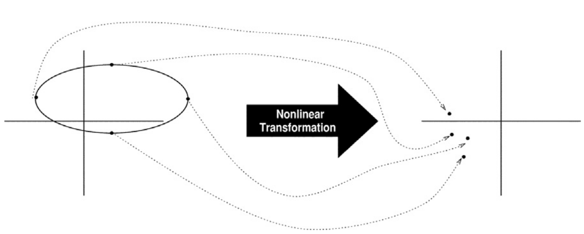

Fig. 3. The procedure of UT method.

It must be emphasized that in the UT method, the nonlinear function is considered as a black box; hence, no simplification or linearization is required. The procedure of UT method is depicted in Fig. 3.

- Step 4: Calculate the mean and covariance of output variable Y using (33) and (34), respectively

$$\overline{Y} = \sum_{k=0}^{2n} W^k Y^k \tag{33}$$

$$P_{YY} = \sum_{k=0}^{2n} W^k\left(Y^k - \overline{Y}\right)\left(Y^k - \overline{Y}\right)^T \tag{34}$$

It is clear that the basic UT algorithm is essentially very simple and easy to apply. For more details and extensions of the UT method, interested readers are referred to [76]. The UT method has been implemented for power system uncertainty assessment in several technical works. A new approach for probabilistic load flow evaluation using the unscented transformation method is proposed in [76]. The application of UT method in power system state estimation problem considering the correlations among instrument transformer signals is investigated in [77]. By embedding the unscented transformation into the Kalman filter process, a method is developed in [78] for power system dynamic state estimation.

The strengths of UT method can be listed as follows:

- It is time efficient.
- The accuracy would not decrease when the number of random variables is large
- It is applicable to problems with correlation among multiple uncertain input parameters.
- It is easy to implement.

 The deficiency of UT method can be listed as follows:
- Its running time depends on number of uncertain variables.
- It is only applicable in problems which the input variables are described using their PDF.

- Scenario-based decision making

Usually the uncertain parameters have infinite uncountable realizations. However, it is impossible to consider all these realizations. Instead, the realization space is divided into countable finite sections (scenarios) with a specific weight (probability). To do this, a list of scenarios is generated using the PDF of each uncertain parameter, X. The expected value of output variable, y, $E(y)$ is calculated as follows:

$$E(y) = \sum_s \pi_s \times f(X_s) \tag{35}$$

where $\sum_s \pi_s = 1$ and π_s is the probability of scenario s.

The strength of scenario-based decision making method can be listed as follows:

- It is computationally efficient and simple to implement.
- It converts the continuous space of uncertain environment into discrete finite scenarios with a given probability.

 The shortcomings of scenario-based decision making method can be listed as follows:
- It only gives the expected values of the uncertain output variables.
- The method is approximate.
- The model size tends to increase drastically with the number of considered scenarios.

– It requires probabilistic inputs and does not support uncertain parameters which are not described probabilistically.

This method is applied in various applications such as: Probabilistic determination of pilot points for zonal voltage control [78], stochastic scheduling for simulation of wind-integrated power systems [79,80] and wind power impact assessment [81].

The use of classical probability theory in the power systems is faced with two problems including: firstly, the power systems are not closed but open systems meaning that any exterior parameter may influence the system parameters. Secondly, selecting appropriate PDF for uncertain variables is not such an easy task, especially when the available data is inadequate or imprecise [4]. In this condition, the possibility theory may be an encouraging alternative.

3. Possibilistic method

The concept of possibilistic uncertainty modeling was first introduced by Zadeh [82]. In this method, the uncertain parameter is described using linguistic categories which have fuzzy boundaries [83]. Suppose the function *f* described in Eq. (22), *X* is a vector of input uncertain parameters to the system and *y* is the output variable. Various types of membership functions can be used to describe the membership degrees of possibilistic uncertain variables. Regardless of the membership function shape, the question is "how to determine the MF of output variable if MFs of input variables are known?".

3.1. α-cut method

In engineering systems, the possibilistic output variable $\tilde{Y}$ of a model of epistemic uncertain variables $\tilde{X}$ is usually represented in the form of a multivariate function $\tilde{Y} = h(\tilde{X}_1, \tilde{X}_2, ..., \tilde{X}_N)$. If the possibility distributions of the uncertain input variable $\tilde{X}$ are known, the possibility distribution of $\tilde{Y}$ can be obtained by means of α-cut method. For a given input variable, the α-cut of $\tilde{X}$ is defined as:

$$A^\alpha = \{x \in U \,|\, \pi_{\tilde{X}}(x) \geq \alpha, 0 \leq \alpha \leq 1\} \tag{36}$$

$$A^\alpha = \left[\underline{A}^\alpha, \overline{A}^\alpha\right] \tag{37}$$

Note that U is the universe of discourse of $\tilde{X}$ (i.e. the range of its possible values), $\underline{A}^\alpha$ and $\overline{A}^\alpha$ are the lower and upper limits of the A^α, respectively. A typical α-cut of a trapezoidal membership function is depicted in Fig. 4. Having the α-cut of each uncertain input variable, the α-cut of output variable $\tilde{Y}$ is calculated as

$$Y^\alpha = \left[\underline{Y}^\alpha, \overline{Y}^\alpha\right] \tag{38}$$

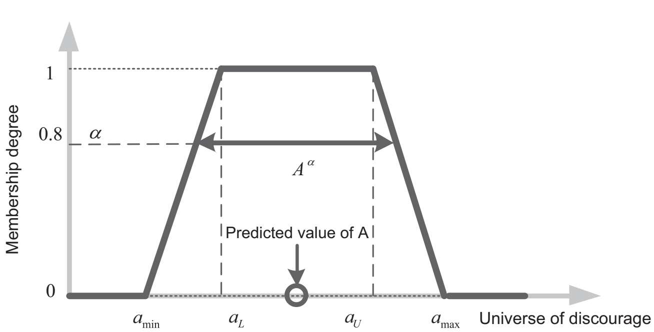

Fig. 4. A fuzzy trapezoidal membership function.

$$\underline{Y}^\alpha = \inf\left[h\left(F^\alpha_{\tilde{X}_1}, F^\alpha_{\tilde{X}_2}, \ldots, F^\alpha_{\tilde{X}_N}\right)\right] \tag{39}$$

$$\overline{Y}^\alpha = \sup\left[h\left(F^\alpha_{\tilde{X}_1}, F^\alpha_{\tilde{X}_2}, \ldots, F^\alpha_{\tilde{X}_N}\right)\right] \tag{40}$$

where $F^\alpha_{\tilde{X}_N}$ stands for the α-cut of the *N*th possibilistic input variable.

3.2. Defuzzification

The defuzzification converts a fuzzy number into a crisp one [17]. This process can be done using centroid method. The defuzzied value of a given fuzzy quantity i.e. $\tilde{X}$ is calculated as

$$X^* = \frac{\int \pi_{\tilde{X}}(x)x\,dx}{\int \pi_{\tilde{X}}(x)\,dx} \tag{41}$$

Ref. [84] presents a new possibilistic model for the multi objective planning of power distribution networks which finds out the optimal solutions corresponding to the simultaneous optimization of the fuzzy economic cost, level of fuzzy reliability, and exposure of network. A fuzzy evaluation tool for analysing the effect of investment and operation of DG units on active losses and the ability of distribution network in load supply at presence of uncertainties is proposed in [85]. In order to manage the uncertainties in the reliability input data of real power systems, a conceptual possibilistic approach using fuzzy set theory is presented in [86]. A new method based on fuzzy extension principle which can represent and propagate the possibilistic uncertainties associated with wind power in power system adequacy evaluation is proposed in [87]. The authors in [88], developed a harmonic load flow for harmonic analysis incorporating wind farm generation. A conceptual possibilistic approach using fuzzy set theory to manage the uncertainties in the reliability input data such as failure rate, repair time and operation of protective devices is developed in [89]. Literature [90] presents an improved approach which overcomes possibility of interaction between input parameters in possibilistic harmonic load flow. A methodology to aggregate uncertainly known bus loads, whose magnitudes and composition are described through fuzzy parameters, into a fuzzy admittance and injected harmonic currents of a possibilistic harmonic load-flow calculation is introduced in [91].

In real world, the uncertainty does exist in all parameters of a given system. The system parameters are influenced by this uncertainty, so it is not an eligible issue and must be taken into account in system performance studies. Performance assessment of an uncertain system is more onerous than a deterministic one. This difficulty escalates when there is incomplete data about some variables i.e. some uncertain variables are probabilistic and some are possibilistic. Sometimes, the decision makers are faced with a multivariate objective function, *y* =*f(X, Z)*, where both possibilistic uncertain parameters (*X*) and probabilistic uncertain ones (*Z*) exist. In other word, the function *f* describes the system model, *Z* is a vector of input uncertain parameters to the system described by PDF, *X* is a vector of input uncertain parameters to the system described by MF and *y* is the output variable. To deal with such cases, some methods are developed which are described next.

4. Combined possibilistic and probabilistic method

This tool would be useful to deal with the uncertainties which some of them can be modelled probabilistically and some of them are described possibilistically [81]. These methods combine possibilistic and probabilistic method through the following two major loops:

Outer loop: Probabilistic method.

Inner loop: Possibilistic method.

A brief explanation of these approaches is described as follows:

4.1. Possibilistic-Monte Carlo approach

In this method, the outer loop is Monte Carlo simulation and the inner loop is α-cut method. The following steps describe the mixed possibilistic-Monte Carlo approach

- Step 1: For each $z_i \in Z$, generate a value using its PDF, z_i^e
- Step 2: Calculate $\underline{Y}^\alpha$ and $\overline{Y}^\alpha$ as follows

$$\underline{Y}^\alpha = \min f(Z^e, X^\alpha) \tag{42}$$

$$\overline{Y}^\alpha = \max f(Z^e, X^\alpha) \tag{43}$$

$$X^\alpha = \left(\underline{X}^\alpha, \overline{X}^\alpha\right) \tag{44}$$

These steps are repeated to obtain the statistical data of the parameters of the output's MF such as PDF or expected values. The main problem with this method is that it is time consuming.

4.2. Possibilistic-scenario based approach

The following steps describe this approach [81]

- Step 1: Generate the scenario set describing the behavior of Z, Ω_j
- Step 2: Reduce the original scenario set to a small set, Ω_s.
- Calculate $\underline{Y}^\alpha$ and $\overline{Y}^\alpha$ as follows

$$\underline{Y}^\alpha = \min \sum_{s \in \Omega_s} \pi_s \times f(Z_s, X^\alpha) \tag{45}$$

$$\overline{Y}^\alpha = \max \sum_{s \in \Omega_s} \pi_s \times f(Z_s, X^\alpha) \tag{46}$$

$$X^\alpha = \left(\underline{X}^\alpha, \overline{X}^\alpha\right) \tag{47}$$

- Step 4: Deffuzzify the y.

In this method, the outer loop is scenario based and the inner loop is α-cut method. The strength of this method is that it is better than previous method from view point of computational burden; however, this method only calculates the mean of output variable and its accuracy is low.

4.3. Combined UT with fuzzy set theory analysis

The authors in Ref. [4] addresses a joint possibilistic and probabilistic method through the following two major loops:

Outer loop: Repeating UT sampling to perform the uncertainty analysis of probabilistic variables.

Inner loop: Executing fuzzy α-cuts analysis to perform the uncertainty assessment of possibilistic variables.

The combined possibilistic and probabilistic method is applied in various applications as the following. A hybrid possibilistic–probabilistic DG impact assessment tool which takes into account the uncertainties associated with investment and operation of renewable and conventional DG units on distribution networks is developed in [81]. The authors in [5] propose a hybrid possibilistic–probabilistic evaluation tool for analysing the effect of uncertain power production of distributed generations (DGs) on active losses of distribution networks.

5. Information Gap Decision Theory

The Information Gap Decision Theory (IGDT) is used to describe the uncertainties which cannot be modeled using PDF or MF due to the deficiency of available historical information. In this situation, the general optimization problem is stated as the following [92]:

$$\min_X f(X, \gamma) \tag{48}$$

$$\mathbf{H}_i(X, \gamma) \le 0, \mathrm{i} \in \Psi_{eq} \tag{49}$$

$$\mathbf{G}_j(X, \gamma) = 0, j \in \Psi_{ineq} \tag{50}$$

$$\gamma \in \Gamma \tag{51}$$

where γ is the vector of uncertain input parameters. Γ is the uncertainty set which describes the behavior of uncertain input parameters. The set of decision variables is represented by X. Mathematically, the uncertainty set is written as:

$$\forall \gamma \in \Gamma(\overline{\gamma}, \zeta) = \left\{ \gamma : \left| \frac{\gamma - \overline{\gamma}}{\overline{\gamma}} \right| \le \zeta \right\} \tag{52}$$

$\overline{\gamma}$ is the forecasted value of the uncertain parameter. ζ is the maximum possible deviation of actual value of uncertain parameter from its forecasted value. It is also called "the radius of uncertainty" which usually is uncertain for the decision maker. It could be assumed that in (48)–(51), the uncertain parameter would not violate from its predicted value as the following:

$$f_b = \min_X f(X, \overline{\gamma}) \tag{53a}$$

$$\mathbf{H}_i(X, \gamma) \le 0, \mathrm{i} \in \Psi_{eq} \tag{53b}$$

$$\mathbf{G}_j(X, \gamma) = 0, j \in \Psi_{ineq} \tag{53d}$$

Let name the result of (53) the objective function value (f_b). Now, the question is that what will happen if the realized uncertain parameter is different from our estimation. In order to cope with such an uncertain problem, two multifarious strategies may be used by the decision maker.

5.1. Risk averse

Is it possible to set the decision variables in order to hedge his decisions against the risks?

In this strategy, he attempts to make robust decisions against the possible errors stems from predicting the uncertain input parameters. It is clear that the most robust decision is reached when the objective function is protected against the maximum radius of uncertainty. This is mathematically formulated as follows:

$$\max_X \hat{\zeta} \tag{54}$$

$$\mathbf{H}_i(X, \gamma) \le 0, \mathrm{i} \in \Psi_{eq} \tag{55}$$

$$\mathbf{G}_j(X, \gamma) = 0, j \in \Psi_{ineq} \tag{56}$$

$$\left\{ \begin{array}{c} \zeta = \max_\zeta \zeta \\ f(X, \gamma) \le \Lambda_c \\ \Lambda_c = f_b(X, \gamma) + \varsigma_c |f_b(X, \gamma)|, \gamma \in \Gamma \end{array} \right\} \tag{57}$$

Λ_c is the critical value that the objective function should be immunized against surpassing it. ς_c is a positive parameter set by the decision maker. It specifies the degree of acceptable tolerance on increasing the value of base objective function (f_b) due to the possible undesired uncertainties.

5.2. Risk seeker

Is it possible to set the decision variables in order to take advantage from the existing uncertainties using the lack of information?

Herein, the decision maker tries to make the robust decision against the possible errors in prediction of the uncertain input parameters. In the risk seeker approach, the decision variables are set in a way that this can happen even with slight error (minimum radius of uncertainty) in prediction of uncertain parameters. The behavior of a risk seeker decision maker can be formulated as follows:

$$\min_X \hat{\zeta} \tag{58}$$

$$\mathbf{H}_i(X,\gamma) \le 0, \mathrm{i} \in \Psi_{eq} \tag{59}$$

$$\mathbf{G}_j(X,\gamma) = 0, j \in \Psi_{ineq} \tag{60}$$

$$\left\{ \begin{array}{c} \hat{\zeta} = \min_{\zeta}\zeta \\ f(X,\gamma) \le \Lambda_0 \\ \Lambda_0 = f_b(X,\gamma) - \varsigma_0 |f_b(X,\gamma)|, \gamma \in \Gamma \end{array} \right\} \tag{61}$$

Λ_0 is the opportunity value that the objective function should be less than it. ζ_0 is a positive parameter set by the decision maker. It specifies the degree of greediness on further decreasing the value of base objective function (f_b) due to the possible uncertainties.

Information gap decision theory (IGDT) is applied to handle the uncertainties associated with the volatility of wind power generation in [92]. A robust restoration decision-making model based on IGDT, which takes into account the uncertainty in the load and output of the DGs is reported in [93]. A non-probabilistic information-gap model is proposed in [94] to model the uncertainties in short-term scheduling of a generation company (GENCO). The reference [95] presents the application of IGDT to help the distribution network operators (DNOs) in choosing the supplying resources for meeting the demand of their customers. Day-ahead market price uncertainty was modelled in [96] using non-probabilistic information gap decision theory.

6. Robust optimization

It is a new approach for solving optimization problems affected by uncertainty specially in case of lack of full information on the nature of uncertainty [97]. It is described as follows:

Consider a function like $z=f(X, y)$ which is linear in X and nonlinear in y. The values of X are subject to uncertainty while the values of y are known. In robust optimization, it is assumed that no specified PDF is in hand for describing the uncertain parameter X. The uncertainty of X is modeled with an uncertainty set $X \in U(X)$; where, $U(X)$ is a set that parameter X can take value from it. The maximization of $z=f(X, y)$ can be formulated as (62)–(63).

$$\max_y z = f(X,y) \tag{62}$$

$X \in U(X)$

Since the value of z is linear with respect to X, it can be reformulated as:

$$\max_y z \tag{63}$$

$$s.t \left\{ \begin{array}{c} z \le f(\tilde{X},y) \\ h(\tilde{X},y) = A(y) * \tilde{X} + g(y) \\ \tilde{X} \in U(X) = \left\{ X \,|\, |X - \overline{X}| \le \hat{X} \right\} \end{array} \right\} \tag{64}$$

where, $\tilde{X}, \overline{X}, \hat{X}$ are the uncertain value, predicted value and maximum possible deviation of variable X from $\overline{X}$, respectively. The robust optimization seeks a solution which not only maximizes the objective function z but also insures the decision maker that if there exist some prediction error about the values of X, the z remains optimum with high probability. To this end, a robust counterpart version of the problem is constructed and solved. The robust counterpart of (62) is defined as follows:

$$\max_y z \tag{65}$$

$$z \le f(X,y) \tag{66}$$

$$\left\{ \begin{array}{c} \sum_i w_i \le \Gamma \\ 0 \le w_i \le 1 \\ f(X,y) = A(y) * \overline{X} + g(y) - \max_{w_i} \sum_i a_i(y) * \hat{X}_i * w_i \end{array} \right\} \tag{67}$$

Based on (65), two nested optimization problems are to be solved. Eq. (67) is linear with respect to w_i and has a dual form as follows:

$$\min \left[\Gamma\beta + \sum_i \xi_i \right] \tag{68}$$

$\beta + \xi_i \ge a_i(y) * \hat{x}_i$

Inserting the (68) into (65) gives:

$$\max_{y,\xi_i,\beta} z \tag{69}$$

$$z \le f(X,y) \tag{70}$$

$$\left\{ \begin{array}{c} f(X,y) = A(y) * \overline{X} + g(y) - \Gamma\beta - \sum_i \xi_i \\ \beta + \xi_i \ge A(y_i) * \hat{X}_i \end{array} \right\} \tag{71}$$

This method has been applied in various applications that a brief review is done in the following. An adaptive robust optimization model for multi-period economic dispatch is proposed in [98]. The robust optimization approach to perform an endogenous stress test for the spot prices as a function of the buy-and-sell portfolio of contracts and renewable energy generation scenarios is applied in [99]. Battery demand uncertainty is modelled in [100] using inventory robust optimization, while multi-band robust optimization is employed to model electricity price uncertainty. A robust optimization approach is adopted in [101] for considering forecast errors in load, variable renewable generation, and market prices for micro grid planning under uncertainty.

7. Interval analysis

In this method, the upper and lower bounds for each uncertain input parameter are defined that can be represented by an interval. Suppose a multivariate function of the form $f=(x_1, \ldots, x_n)$ and $lb_i \le x_i \le ub_i$ where lb_i, ubi are the lower and upper bounds of uncertain parameter x_i. The purpose is to find the lower and upper bounds of objective function f.

$$Prob = \int_a^d A_1 \frac{1}{\sigma\sqrt{2\pi}} e^{\frac{-(x-\mu)^2}{2\sigma^2}} = \frac{1}{\sigma\sqrt{2\pi}} \left[\int_a^b \frac{x-a}{b-a} e^{\frac{-(x-\mu)^2}{2\sigma^2}} + \int_b^c e^{\frac{-(x-\mu)^2}{2\sigma^2}} + \int_c^d \frac{x-d}{c-d} e^{\frac{-(x-\mu)^2}{2\sigma^2}} \right] \tag{72}$$

$$G(prob) = \mu_{B_2}(Prob) \tag{73}$$

The probabilistic distribution-based interval arithmetic approach is proposed in [102] to consider the uncertainty in load demand. In [103], in order to solve the directional overcurrent relays coordination

Table 1
Summaries of uncertainty modeling attributes.

Group	Main idea	Examples	Advantages	disadvantages
Probabilistic (simulation based)	Simulation of real state	Monte Carlo	Accurate, Simulation of the real world, Good for big and complex problems	Time consuming, High computational burden
Probabilistic (scenario based)	Scenario making	Scenario based methods	Accuracy depends on the number of selected scenarios, Faster than MCS	Can give only the mean values of output variables
Probabilistic (Analytical)	Linearization based	Gram Charlier, Cumulants, …	Fast	Cannot obtain high order moments accurately
Probabilistic (Analytical)	PDF approximation	2PEM UT	Fast, Accurate, Correlation modelling	Execution time depends on the number of uncertain variables
Possibilistic	Using fuzzy membership function	α-cut method	Can obtain the membership function of output variable	Time consuming, Cannot model correlation
Joint possibilistic–probabilistic	Modeling of both probabilistic and possibilistic uncertainties	Monte Carlo-fuzzy, Scenario based- fuzzy, UT- fuzzy	Can model the real world conditions, Both probabilistic and possibilistic uncertainties can be modelled	Time consuming
Information gap theory	Using forecasted values	Information gap decision theory	Useful for decision making in severe uncertainties	Complexity
Robust optimization	Using intervals	Robust optimization method	Useful when just an interval exists	Difficult to use in nonlinear problems
Interval analysis	Using intervals	Interval analysis method	Useful when just an interval exists	Cannot model the correlation between intervals

problem considering uncertainty in the network topology, a new approach based on the interval analysis has been proposed. Table 1 summarizes the uncertainty modelling attributes.

8. Conclusion

In this work, a comprehensive review on uncertainty modelling in power systems has been conducted. The classification of the methods has been done into different categories included are: probabilistic, possobilistic, joint possibilistic— probabilistic, information gap decision theory (IGDT), robust optimization, and interval analysis. It is attempted to provide the latest modifications done on each method. Then the comparison between these methods has been derived providing the strengths and weakness of the studied methods. This comparison helps the decision maker to choose the best method for the encountered uncertain problem to robust his decision against risks stem from the uncertain factors.

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References

[1] Zhu J. Optimization of power system operation. In: Piscataway NJ, editor. 1 ed. Chichester: Wiley-IEEE Press; 2009.
[2] Dantzig GB. Linear programming under uncertainty. Manag Sci 1955;1:197–206.
[3] Zadeh LA. Fuzzy sets. Inf Control 1965;8:338–53.
[4] Aien M, Rashidinejad M, Fotuhi-Firuzabad M. On possibilistic and probabilistic uncertainty assessment of power flow problem: a review and a new approach. Renew Sustain Energy Rev 2014;37:883–95.
[5] Soroudi A, Ehsan M. A possibilistic–probabilistic tool for evaluating the impact of stochastic renewable and controllable power generation on energy losses in distribution networks—a case study. Renew Sustain Energy Rev 2011;15:794–800.
[6] Ben-Haim Y. Info-gap decision theory: decisions under severe uncertainty. 2nd ed.. Oxford: Academic Press; 2006.
[7] Soyster AL. Technical note—convex programming with set-inclusive constraints and applications to inexact linear programming. Oper Res 1973;21:1154–7.
[8] Moore RE, Kearfott RB, Cloud MJ. Introduction to Interval Analysis. 1st ed. Philadelphia, PA: Society for Industrial and Applied Mathematics; 2009.
[9] Zio E, Delfanti M, Giorgi L, Olivieri V, Sansavini G. Monte Carlo simulation-based probabilistic assessment of DG penetration in medium voltage distribution networks. Int J Electr Power Energy Syst 2015;64:852–60.
[10] Zhao Y, Fan F, Wang J, Xie K. Uncertainty analysis for bulk power systems reliability evaluation using Taylor series and nonparametric probability density estimation. Int J Electr Power Energy Syst 2015;64:804–14.
[11] Degeilh Y, Gross G. Stochastic simulation of power systems with integrated intermittent renewable resources. Int J Electr Power Energy Syst 2015;64:542–50.
[12] Ferrario E, Zio E. Assessing nuclear power plant safety and recovery from earthquakes using a system-of-systems approach. Reliab Eng Syst Saf 2014;125:103–16.
[13] Ding Y, Wang P, Goel L, Billinton R, Karki R. Reliability assessment of restructured power systems using reliability network equivalent and pseudo-sequential simulation techniques. Electr Power Syst Res 2007;77:1665–71.
[14] Zhao Q, Wang P, Goel L, Ding Y. Evaluation of nodal reliability risk in a deregulated power system with photovoltaic power penetration. IET Gener Transm Distrib 2014;8:421–30.
[15] Lopes V, Borges C. Impact of the combined integration of wind generation and small hydropower plants on the system reliability. IEEE Trans Sustain Energy 2014;6:1169–77.
[16] Li G, Zhang P, Luh PB, Li W, Bie Z, Serna C, et al. Risk analysis for distribution systems in the Northeast U.S. under wind storms. IEEE Trans Power Syst 2014;29:889–98.
[17] Conti S, Rizzo S. Monte carlo simulation by using a systematic approach to assess distribution system reliability considering intentional islanding. IEEE Trans Power Deliv 2014;30:64–73.
[18] Arabali A, Ghofrani M, Etezadi-Amoli M, Fadali MS. Stochastic performance assessment and sizing for a hybrid power system of solar/wind/energy storage. IEEE Trans Sustain Energy 2014;5:363–71.
[19] Greenwood DM, Taylor PC. Investigating the impact of real-time thermal ratings on power network reliability. IEEE Trans Power Syst 2014;29:2460–8.
[20] de Magalhaes Carvalho L, da Rosa M, Martins Leite da Silva A, Miranda V. Probabilistic analysis for maximizing the grid integration of wind power generation. IEEE Trans Power Syst 2012;27:2323–31.
[21] Han X, Qu Y, Wang P, Yang J. Four-dimensional wind speed model for adequacy assessment of power systems with wind farms. IEEE Trans Power Syst 2013;28:2978–85.
[22] Shu Z, Jirutitijaroen P, da Silva AL, Singh C. Accelerated state evaluation and latin hypercube sequential sampling for composite system reliability assessment. IEEE Trans Power Syst 2014;29:1692–700.
[23] Wang Y, Guo C, Wu QH. A cross-entropy-based three-stage sequential importance sampling for composite power system short-term reliability evaluation. IEEE Trans Power Syst 2013;28:4254–63.
[24] Arya R. Estimation of distribution system reliability indices neglecting random interruption duration incorporating effect of distribution generation in standby mode. Int J Electr Power Energy Syst 2014;63:270–5.

[25] Manso LAF, Leite da Silva AM, Mello JCO. Comparison of alternative methods for evaluating loss of load costs in generation and transmission systems. Electr Power Syst Res 1999;50:107–14.
[26] JCO M, Pereira MVF, Leite da Silva A. Evaluation of reliability worth in composite systems based on pseudo-sequential Monte Carlo simulation. IEEE Trans Power Syst 1994;9:1318–26.
[27] Vallée F, Versèle C, Lobry J, Moiny F. Non-sequential Monte Carlo simulation tool in order to minimize gaseous pollutants emissions in presence of fluctuating wind power. Renew Energy 2013;50:317–24.
[28] Amaral TS, Borges CLT, Rei AM. Composite system well-being evaluation based on non-sequential Monte Carlo simulation. Electr Power Syst Res 2010;80:37–45.
[29] Zhang S, Li G, Zhou M. Calculation and analysis of capacity credit of wind farms based on Monte-Carlo simulation. IEEE power and energy society general meeting; 2010. p. 1–6.
[30] Yu X, Singh C. Expected power loss calculation including protection failures using importance sampling and SOM. IEEE power and energy society general meeting. Vol.1; 2004. p. 206–11.
[31] Leite da Silva A, de Resende LC, da Fonseca Manso L, Miranda V. Well-being analysis for composite generation and transmission systems based on pattern recognition techniques. IET Gener Transm Distrib 2008;2:202–8.
[32] Bakkiyaraj RA, Kumarappan N. Evaluation of composite reliability indices based on non-sequential Monte Carlo simulation and particle swarm optimization. IEEE congress on evolutionary computation, CEC; 2010. p. 1–5.
[33] Vallee F, Brunieau G, Pirlot M, Deblecker O, Lobry J. Optimal wind clustering methodology for electrical network adequacy studies using non sequential Monte Carlo simulation. In: Proceedings of the International conference on clean electrical power, ICCEP; 2011. p. 778–85.
[34] Smid JH, Verloo D, Barker GC, Havelaar AH. Strengths and weaknesses of Monte Carlo simulation models and Bayesian belief networks in microbial risk assessment. Int J Food Microbiol 2010;139:S57–63.
[35] Allan RN, Borkowska B, Grigg CH. Probabilistic analysis of power flows. Proc Inst Electr Eng 1974;121:1551–6.
[36] Allan RN, MRG Al-Shakarchi. Probabilistic a.c. load flow. Proc Inst Electr Eng 1976;123:531–6.
[37] Allan RN, Leite da Silva AM, Burchett RC. Evaluation methods and accuracy in probabilistic load flow solutions. IEEE Trans Power Appar Syst 1981:2539–46.
[38] Schellenberg A, Rosehart W, Aguado J. Cumulant-based probabilistic optimal power flow (P-OPF) with Gaussian and gamma distributions. IEEE Trans Power Syst 2005;20:773–81.
[39] Dadkhah M, Venkatesh B. Cumulant based stochastic reactive power planning method for distribution systems with wind generators. IEEE Trans Power Syst 2012;27:2351–9.
[40] Tamtum A, Schellenberg A, Rosehart WD. Enhancements to the cumulant method for probabilistic optimal power flow studies. IEEE Trans Power Syst 2009;24:1739–46.
[41] Schellenberg A, Rosehart W, Aguado JA. Cumulant-based stochastic nonlinear programming for variance constrained voltage stability analysis of power systems. IEEE Trans Power Syst 2006;21:579–85.
[42] Preece R, Huang K, Milanovic JV. Probabilistic small-disturbance stability assessment of uncertain power systems using efficient estimation methods. IEEE Trans Power Syst 2014;29:2509–17.
[43] Hong Y-Y, Liao W-J. Optimal passive filter planning considering probabilistic parameters using cumulant and adaptive dynamic clone selection algorithm. Int J Electr Power Energy Syst 2013;45:159–66.
[44] Alavi-Serashki MM, Singh C. A generalized cumulant method for generation capacity reliability evaluation. Electr Power Syst Res 1992;23:1–4.
[45] Coroiu F, Bucatariu I, Baloi A. Using the Gram–Charlier expansion in power systems reliability. In: Proceedings of 7th IEEE international symposium on applied computational intelligence and advanced informatics SACI; 2012. p. 59–62.
[46] Bloom JA. Representing the production cost curve of a power system using the method of moments. IEEE Trans Power Syst 1992;7:1370–7.
[47] Heydt G, Khotanzad A, Farahbakhshian N. A Method for the forecasting of the probability density function of power system loads. IEEE Trans Power Appar Syst 1981;PAS-100:5002–10.
[48] Ahsan Q, Schenk KF. Sensitivity study of the cumulant method for evaluating reliability measures of two interconnected systems. IEEE Trans Reliab 1987; R-36:429–32.
[49] Hong Y-Y, Luo Y-F. Optimal VAR control considering wind farms using probabilistic load-flow and gray-based genetic algorithms. IEEE Trans Power Deliv 2009;24:1441–9.
[50] Viviani GL, Heydt GT. Stochastic optimal energy dispatch. IEEE Trans Power Appar Syst 1981;PAS-100:3221–8.
[51] Bloom JA. Generation cost curves including energy storage. IEEE Trans Power Appar Syst 1984;PAS-103:1725–31.
[52] Billinton R, El-Sheikhi FA. Preventive maintenance scheduling in power generation systems using a quantitative risk criterion. Electr Eng J Can 1983;8:28–39.
[53] Hong Y-Y, Lin F-J, Lin Y-C, Hsu F-Y. Chaotic PSO-based VAR control considering renewables using fast probabilistic power flow. IEEE Trans Power Deliv 2014;29:1666–74.
[54] Bloom JA. Probabilistic production costing with dependent generating sources. IEEE Trans Power Appar Syst 1985:2064–71.
[55] Schellenberg A, Rosehart W, Aguado J. Introduction to cumulant-based probabilistic optimal power flow (P-OPF). IEEE Trans Power Syst 2005;20:1184–6.
[56] Li YZ, Yan MS, Fang RS. Two-area power system costing by the cumulant method using available capacity distribution. Electr Power Syst Res 1997;42:55–61.
[57] Nezhad AR, Mokhtari G, Davari M, Araghi AR, Hosseinian SH, Gharehpetian GB. A new high accuracy method for calculation of LMP as a random variable. In: Proceedings of the international conference on electric power and energy conversion systems, EPECS 09; 2009. p.1–5.
[58] Dong L, Li S, Yang Y, Bao H. The calculation of transfer reliability margin based on the probabilistic load flow. Power and energy engineering conference, APPEEC. Asia-Pacific; 2010. p. 1–4.
[59] Villanueva D, Feijóo AE, Pazos JL. An analytical method to solve the probabilistic load flow considering load demand correlation using the DC load flow. Electr Power Syst Res 2014;110:1–8.
[60] Morrow D, Gan L. Comparison of methods for building a capacity model in generation capacity adequacy studies. Communications, computers and power in the modern. environment conference proceedings. IEEE WESCANEX 93; 1993. p. 143–49.
[61] Stuart A, Ord K. Kendall's advanced theory of statistics, distribution theory. 1st ed. Chichester, West Sussex: Wiley; 2010.
[62] El-Ela AA. Fast and accurate technique for power system state estimation. Gener Transm Distrib IEE Proc C 1992;139:7–12.
[63] Dhople SV, Dominguez-Garcia AD. A parametric uncertainty analysis method for markov reliability and reward models. IEEE Trans Reliab 2012;61:634–48.
[64] Wan C, Xu Z, Dong ZY, Wong KP. Probabilistic load flow computation using first-order second-moment method. IEEE power & energy society general meeting; 2012. p. 1–6.
[65] Li X, Li Y, Zhang S. Analysis of probabilistic optimal power flow taking account of the variation of load power. IEEE Trans Power Syst 2008;23:992–9.
[66] Su C-L. Transfer capability uncertainty computation. In: Proceedings of the power systems technology (Powercon). vol. 1; 2004. p. 586–91.
[67] Madrigal M, Ponnambalam K, Quintana VH. Probabilistic optimal power flow. In: Proceedings of IEEE Canadian conference on electrical and computer engineering. vol.1; 1998. p. 385–88.
[68] Verbic G, Canizares CA. Probabilistic optimal power flow in electricity markets based on a two-point estimate method. IEEE Trans Power Syst 2006;21:1883–93.
[69] Moeini-Aghtaie M, Abbaspour A, Fotuhi-Firuzabad M. Incorporating large-scale distant wind farms in probabilistic transmission expansion planning -2014; Part I: theory and algorithm. IEEE Trans Power Syst 2012;27:1585–93.
[70] Su C-L. Probabilistic load-flow computation using point estimate method. IEEE Trans Power Syst 2005;20:1843–51.
[71] Malekpour AR, Niknam T, Pahwa A, Fard AK. Multi-objective stochastic distribution feeder reconfiguration in systems with wind power generators and fuel cells using the point estimate method. IEEE Trans Power Syst 2013;28:1483–92.
[72] Arabali A, Ghofrani M, Etezadi-Amoli M, Fadali MS, Baghzouz Y. Genetic-algorithm-based optimization approach for energy management. IEEE Trans Power Deliv 2013;28:162–70.
[73] Qiao S, Wang P, Tao T, Shrestha GB. Maximizing profit of a wind genco considering geographical diversity of wind farms. IEEE Trans Power Syst 2014;30:2207–15.
[74] Aien M, Fotuhi-Firuzabad M, Rashidinejad M. Probabilistic optimal power flow in correlated hybrid wind photovoltaic power systems. IEEE Trans Smart Grid 2014;5:130–8.
[75] Aien M, Khajeh MG, Rashidinejad M, Fotuhi-Firuzabad M. Probabilistic power flow of correlated hybrid wind-photovoltaic power systems. IET Renew Power Gener 2014;8:649–58.
[76] Aien M, Fotuhi-Firuzabad M, Aminifar F. Probabilistic load flow in correlated uncertain environment using unscented transformation. IEEE Trans Power Syst 2012;27:2233–41.
[77] Caro E, Valverde G. Impact of transformer correlations in state estimation using the unscented transformation. IEEE Trans Power Syst 2014;29:368–76.
[78] Wang S, Gao W, Meliopoulos APS. An alternative method for power system dynamic state estimation based on unscented transform. IEEE Trans Power Syst 2012;27:942–50.
[79] Amraee T, Soroudi A, Ranjbar AM. Probabilistic determination of pilot points for zonal voltage control. IET Gener Transm Distrib 2012;6:1–10.
[80] Sturt A, Strbac G. Efficient stochastic scheduling for simulation of wind-integrated power systems. IEEE Trans Power Syst 2012;27:323–34.
[81] Soroudi A. Possibilistic-scenario model for DG impact assessment on distribution networks in an uncertain environment. IEEE Trans Power Syst 2012;27:1283–93.
[82] Zadeh LA. Fuzzy sets as a basis for a theory of possibility. Fuzzy Sets Syst 1999;100:9–34.
[83] Yager RR. Determining equivalent values for possibilistic variables. IEEE Trans Syst Man Cybern B Cybern 2001;31:19–31.
[84] Ramirez-Rosado IJ, Dominguez-Navarro JA. Possibilistic model based on fuzzy sets for the multiobjective optimal planning of electric power distribution networks. IEEE Trans Power Syst 2004;19:1801–10.
[85] Soroudi A, Ehsan M, Caire R, Hadjsaid N. Possibilistic evaluation of distributed generations impacts on distribution networks. IEEE Trans Power Syst 2011;26:2293–301.

[86] Kim J-O, Singh C. Including uncertainty in LOLE calculation using fuzzy set theory. IEEE Trans Power Syst 2002;17:19–25.
[87] Bie Z, Sun C, Ning G, Gao Y. Adequacy evaluation of generating system recognizing random fuzzy wind speed. Power and energy engineering conference, APPEEC. IEEE PES Asia-Pacific; 2013. p. 1–5.
[88] Romero AA, Samudio C, Zini H, Ratta G. Modeling uncertainties in the harmonic distortion calculation in power systems, due to wind farms. In: Proceedings of IEEE international symposium on alternative energies and energy quality, SIFAE; 2012. p. 1–6.
[89] Hatami AR, Fotuhi-Firuzabad M. Evaluation of substation reliability indices and outage cost based on fuzzy arithmetic. IEEE-GCC conference GCC; 2006. p. 1–6.
[90] Romero AA, Zini HC, Ratta G. Modelling input parameter interactions in the possibilistic harmonic load flow. IET Gener Transm Distrib 2012;6:528–38.
[91] Romero AA, Zini HC, Ratta G, Dib R. A novel fuzzy number based method to model aggregate loads for harmonic load-flow calculation. Transmission and distribution conference and exposition Latin America, IEEEPES; 2008.
[92] Rabiee A, Soroudi A, Keane A. Information gap decision theory based OPF with HVDC connected wind farms. IEEE Trans Power Syst 2014;30:3396–406.
[93] Chen K, Wu W, Zhang B, Sun H. Robust restoration decision-making model for distribution Networks based on information gap decision theory. IEEE Trans Smart Grid 2014;6:587–97.
[94] Mohammadi-Ivatloo B, Zareipour H, Amjady N, Ehsan M. Application of information-gap decision theory to risk-constrained self-scheduling of GenCos. IEEE Trans Power Syst 2013;28:1093–102.
[95] Soroudi A, Ehsan M. IGDT based robust decision making tool for DNOs in load procurement under severe uncertainty. IEEE Trans Smart Grid 2013;4:886–95.
[96] Kazemi M, Mohammadi-Ivatloo B, Ehsan M. Risk-constrained strategic bidding of gencos considering demand response. IEEE Trans Power Syst 2015;30:376–84.
[97] Ben-Tal A, Ghaoui LE, Nemirovski A. Robust optimization. Princeton: Princeton University Press; 2009.
[98] Lorca A, Sun XA. Adaptive robust optimization with dynamic uncertainty sets for multi-period economic dispatch under significant wind. IEEE Trans Power Syst 2014;30:1702–13.
[99] Fanzeres B, Street A, Barroso LA. Contracting strategies for renewable generators: a hybrid stochastic and robust optimization approach. IEEE Trans Power Syst 2014;30:1825–37.
[100] Sarker MR, Pandzic H, Ortega-Vazquez MA. Optimal operation and services scheduling for an electric vehicle battery swapping station. IEEE Trans Power Syst 2014;30:901–10.
[101] Khodaei A, Bahramirad S, Shahidehpour M. Microgrid planning under uncertainty. IEEE Trans Power Syst 2014;30:2417–25.
[102] Chaturvedi A, Prasad K, Ranjan R. Use of interval arithmetic to incorporate the uncertainty of load demand for radial distribution system analysis. IEEE Trans Power Deliv 2006;21:1019–21.
[103] Noghabi AS, Mashhadi HR, Sadeh J. Optimal coordination of directional overcurrent relays considering different network topologies using interval linear programming. IEEE Trans Power Deliv 2010;25:1348–54.