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Review of probabilistic load flow approaches for power distribution systems with photovoltaic generation and electric vehicle charging

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ABSTRACT

The currently increasing penetration of photovoltaic (PV) generation and electric vehicle (EV) charging in electricity distribution grids leads to higher system uncertainties. This makes it vital for load flow analyses to use probabilistic methods that take into account the uncertainty in both load and generation. Such probabilistic load flow (PLF) approaches typically involve three main components: (1) probability distribution models, (2) correlation models, and (3) PLF computations. In this review, state-of-the-art approaches to each of these components are discussed comprehensively, including suggestions of preferred modelling methods specifically for distribution systems with PV generation and EV charging. Research gaps that need to be explored are also identified. For further development of PLF analysis, improving input distribution modelling to be more physically realistic for load, PV generation, and EV charging is vital. Further correlation modelling efforts should focus on developing an effective spatio-temporal correlation model that is able to cope with high-dimensional inputs. The computational speed of PLF analysis needs to be improved to accommodate more complex distribution system models, and time-series approaches should be developed to meet operational needs. Furthermore, collection of higher-quality data is crucial for PLF studies, especially for improving the accuracy in the input variables.

1. Introduction

Load flow analysis is vital in solving problems related to power system planning and operation, as it provides information on the power flow at steady-state conditions [1]. Load flow analysis allows insight into power system performance by estimating line currents and bus voltages based on a known grid structure as well as injected and consumed power [2]. Traditionally, due to their simplicity, deterministic approaches have been used, in which worst-case scenarios are assumed, such as maximum consumption combined with minimum generation, and vice versa [3]. Hence, deterministic approaches disregard the uncertainties in the input variables arising from the stochastic nature of some loads and generators [4].

However, recently, such uncertainties from both generators and loads have increased due to the growing penetration of variable renewable energy (VRE) and the uptake of electric vehicles (EVs) in the power system [3]. The impact of VRE and EVs is significant not only on the transmission side, but also on the distribution side of the power system, as both VRE and EVs are widely dispersed in distribution networks [3]. Installed capacity of photovoltaic (PV) systems, as the main distributed VRE source connected in distribution systems, continues to grow and is predicted to claim a vital share of the future energy mix [5]. PV power generation has high uncertainties caused by atmospheric conditions, in particular cloud movements, that make the solar irradiance highly variable [6]. Integration of PV into the grid can lead to several adverse impacts including voltage rise due to excess generation, a decrease in power quality, equipment damage, reliability issues, and longer restoration time from outage [7].

EVs as a clean alternative for transportation has gained popularity around the world, with over one million electric cars sold in 2017 [8]. Charging of EVs also raises the uncertainty in the distribution system due to variability in charging behaviour, which depends on several parameters such as the number of the charged vehicles, battery capacity and status, and charging time duration [9]. EV charging integration into the distribution grid can increase the peak load demand, cause high voltage drops, and increase the power losses [10,11].

Analysing the impact of distributed VRE and EV charging on the distribution system requires a modelling approach that considers the

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uncertainty from existing loads, EVs and VRE generation. Hence, probabilistic methods that include input probability density functions (PDFs) are often more appropriate [12]. Such load flow analyses are often referred to as probabilistic load flow (PLF) [13]. These methods require the PDFs for all generators and loads in the system [14] and solve the power flow equations either numerically [12,15,16], analytically [17–19], or using an approximation methods [20–22]. When performing the PLF calculations, a correlation model is needed if dependencies between the input variables are to be taken into account [12]. The output from the PLF also comes in the form of PDFs, representing the uncertainties in the calculated system parameters.

1.1. Overview of past reviews on probabilistic power flow

Several previous reviews of PLF exist. One of the initial works [23] provided an extensive bibliography on this topic, covering studies published until 1988. A more recent study [13] reviewed PLF considering distributed generation; however, it discussed only basic computational techniques for PLF and only briefly covered uncertainty modelling. Uncertainty modelling techniques were extensively discussed in Refs. [24] and [25], but these reviews made broad overviews of the field, with probabilistic methods as one among several approaches, not treated in detail. Ref. [4] did a critical review with an in-depth coverage of PLF computation methods, including numerical, analytical, and approximation approaches, but input distribution modelling was not reviewed in detail. While all of the aforementioned papers discussed uncertainty in power flow studies, reviews covering PDF models used, or potentially used, for the input variables are relatively rare. Some recent studies have included reviews of PDF modelling [26,27], but these papers focused on energy system design and power system stability analysis, respectively, and not on PLF.

Previous reviews also rarely discussed the inclusion of uncertainty from EV charging in PLF. One study reviewed the general modelling process for EVs and PV on city-scale [28], but not in relation to PLF. It is important to note that awareness of the importance of the correlation between input variables is increasing in recent reviews [4,27]; however, correlation modelling has so far not been discussed in depth. Previous reviews also discussed PLF in general, without any specific focus on the distribution system.

This paper, for the first time, reviews PLF approaches, treating in detail the three components outlined above – distribution modelling of the inputs, correlation modelling, and the computational method – with a special focus on distribution networks with distributed PV generation and EV charging. With equal emphasis on input distribution and correlation modelling, and a comparison of the computational methods, it provides researchers in this field with an overview of state-of-the-art methods available for all the necessary components in PLF studies. The contributions of this paper are summarized as follows:

- Provide a review and highlight the state-of-the-art of the three main components in PLF analyses: (1) input uncertainty modelling, (2) correlation modelling, and (3) computational methods.
- Show how PLF analyses have been applied in studies of electricity distribution systems with PV generation and EV charging.
- Identify future research directions required to advance the work in this field and to improve the current state of the art.

1.2. Research questions

In this paper, the following research questions have guided the work:

- Which methods have been used, or could potentially be used, for modelling the uncertainty in existing load, PV power generation, and EV charging load, and the correlations among these?
- How can state-of-the-art methods among the aforementioned uncertainty and correlation modelling methods be combined to make more accurate PLF analyses?
- What are the advantages and disadvantages of the currently available computational methods for PLF analyses?
- How have PLF analyses on distribution networks with PV generation and EV charging been performed in terms of the aforementioned techniques?
- Which are the main research gaps regarding PLF modelling with PV power generation and EV charging?

1.3. Outline of the paper

The structure of the paper is as follows: Section 2 presents an introduction to the PLF process/method and its steps. Section 3 presents existing probability distribution models and highlights current state of the art. Similarly, Section 4 reviews existing methods used to represent correlations among the variables. Section 5 describes computational methods employed to solve the PLF problem including their advantages and disadvantages. The application of PLF calculation specifically for distribution networks with PV generation and EV charging is discussed in Section 6. Section 7 provides a concluding discussion on the findings and suggestions for further research are discussed as a concluding discussion.

2. Probabilistic load flow process

The purpose of the PLF analysis is to study the uncertainties of the inputs in power flow calculations [29]. Hence, the inputs are PDFs, and the outputs also are represented by PDFs. Fig. 1 shows an illustration outlining the PLF process for a power distribution system. In general, the process can be divided into three main stages: input uncertainty modelling, PLF computation and analysis of the PLF output.

The main objective of the input uncertainty modelling is to effectively represent the uncertainty of all load and generation in the network in terms of PDFs. In this study, the load is separated into existing load and EV charging. The power generation considered in this study is from PV only. These input PDFs are then used as input to the PLF computations. Current studies have shown the importance of including the dependencies, i.e., the correlations, between the input variables at this stage [12]. The accuracy of the uncertainty and correlation modelling could affect the quality of the results of PLF calculation. The correlation can be modelled as intra-variable (load-load, PV-PV, EV-EV), including the spatial correlation, or between variables (load-PV, load-EV, PV-EV).

The PLF computation stage solves the load flow problem using the input PDFs and correlation models. The load flow problem can be solved using a numerical method by repeatedly drawing samples from the input distributions and running a deterministic load flow solution for each set of samples, e.g., using a Monte Carlo method. It can also be performed analytically, usually based on cumulants, or based on an approximation using the point-estimation method (PEM).

The outputs of the PLF are presented as PDFs to represent the uncertainties. The exact method of PDF generation depends on the computation method. The numerical method produces a large number of output values that can be interpreted statistically as PDFs. The analytical and approximation methods, however, require expansion techniques to generate the output PDFs. When the study considers the time-domain, the output PDFs are presented for every considered time step.

3. Probabilistic distribution modelling

There are several approaches to include the effects of input parameter uncertainty on power system performance, as outlined in Fig. 2. A complete review of all of these methods, including the exact formulas involved, is given in [24]. To sum up, the main difference between these approaches is in the way the uncertainty is represented. The

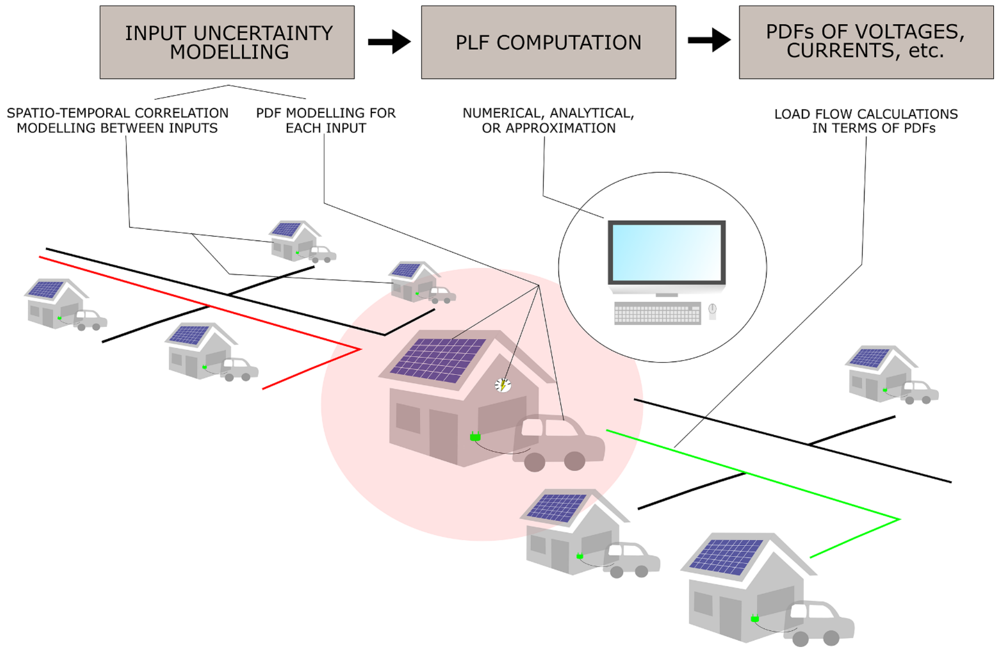

Fig. 1. Illustration of the PLF process for an example distribution network.

probabilistic method, which is used for PLF, represents the uncertainty in terms of PDFs. When a PDF cannot be defined, alternative options include fuzzy membership functions (possibilistic), deviation of errors (information gap decision theory), uncertainty sets (robust optimization), and a known interval (interval analysis) [30–33].

This section provides an overview of the various PDFs utilized in previous studies to describe the uncertainties associated with the input variables. The uncertainties considered are those related to the three input variables load, PV generation, and EV charging. Studies characterizing these input distributions did not come solely from the fields of PLF or even general power system studies [34]. These studies, however, are important to consider in further PLF studies to improve the current state of the art.

3.1. Load probability distributions

The electricity consumption in a distribution grid, in general, is complex to quantify and highly stochastic both spatially between load sources, and temporally over time scales from seconds to seasons [35]. These uncertainties arise from customer behaviours, variations in electrical equipment, and climatic conditions [4]. To represent these load uncertainties, several different PDFs have been utilized in previous studies. Distribution modelling of loads can be classified with regards to load type and aggregation. Most of the studies considered the whole aggregated system load, while others modelled a more specific type of power system load, such as a group of residential loads. The importance of residential load distribution modelling is increasing due to developments within this customer group, including time-dependent tariffs and distributed power generation [36].

In the studies that considered the aggregated power system load, the uncertainties were often modelled using a normal distribution [17–22,37–44]. However, several different methods were used to construct these normal PDFs. For example, Ref. [21] used a fully synthetic normal load in its base case. Ref. [17] used deterministic data for the mean value, and a fixed 7% of the mean value for the standard deviation, see Fig. 3(a). Ref. [18] obtained normal PDFs by fitting the distribution to historical data. Other studies also utilized historical data, but in the form of empirical PDFs instead of the fitting process [45–47]. Besides the normal distribution and empirical PDFs, a wide range of PDFs have been used, including lognormal [12,34], polynomial [15,48], beta [49], joint normal [50,51], Gumbel [52,53], and synthetic PDFs generated from a Brownian motion process [54].

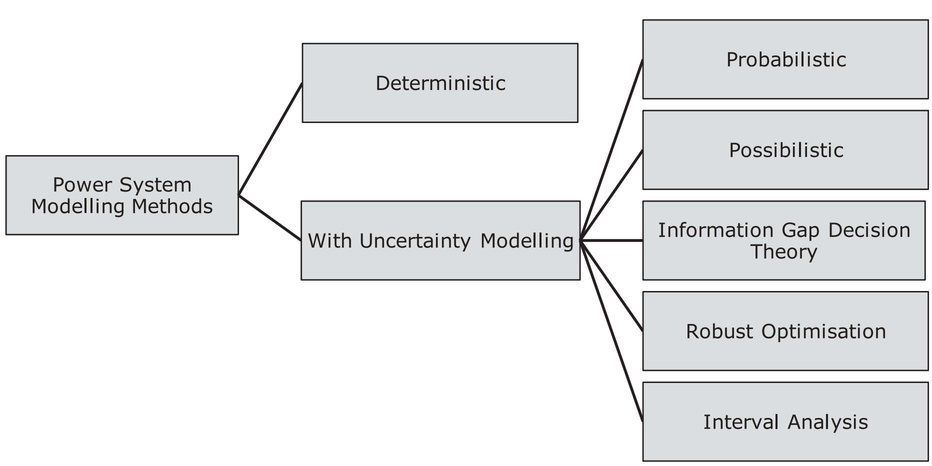

Fig. 2. Classification of power system modelling approaches [24].

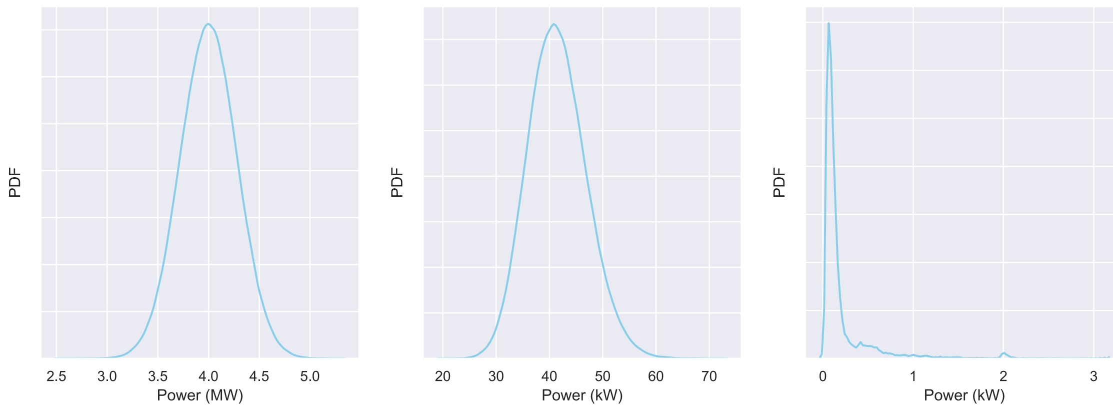

Fig. 3. (a) Example of normal distribution for aggregated load modelling using a deterministic mean value of 4 MW and a fixed standard deviation of 7% of the mean value [17]. (b) Example of gamma distribution for residential load modelling at 10.00 p.m of a winter weekday for 50 households [55]. (c) Example of an empirical distribution for one residential household in Australia [61].

Table 1
Overview of probability distributions for load used in uncertainty modelling.

Variable	Probability distribution
Aggregated power system load	Normal [17–22,37–44]
	Empirical PDF [45–47]
	Lognormal [12,34]
	Polynomial [15]
	Binomial [48]
	Beta [49]
	Joint normal [50,51]
	Gumbel [52,53]
	Brownian motion [54]
Aggregated residential-only load	Gamma [36,55]
	Beta [56,57]
	Exponential [58]
	Weibull [59,66]
	Lognormal [66]

In contrast, for residential loads, the normal PDF is rarely deployed. As seen in Table 1, none of the referred studies used the normal distribution. Instead, they used gamma [36,55], beta [56,57], exponential [58], Weibull [59], and polynomial [60] distributions. An example of the gamma PDF, used for load modelling, is shown in Fig. 3(b), based on the parameters in Ref. [55]. One of the main benefits of not using a normal distribution is avoiding negative power consumption [36]. Ref. [36] tested nine different distributions to model the load distribution of 100 residential customers in extra-urban areas. The result showed a skewed distribution of the residential loads, justifying not using a normal distribution for aggregated residential load, even though the smoothing effect of aggregated values does moderate the skewness. The result also showed that exponential, Gumbel, normal, and Rayleigh distributions did not accurately represent the load distribution, leaving gamma as the most suitable distribution. However, several other distributions also gave acceptable results: beta, inverse normal, lognormal, and Weibull distributions. Fig. 3(c) shows an example of highly skewed distribution for single residential load in Australia based on Ausgrid data explained in [61].

Some studies have proposed stochastic residential load model [62,63], but they have rarely been used in PLF studies. This model could be used for load modelling for systems dominated by household loads. Ref. [64] also proposed a stochastic load model with correlation between nodes based on historical smart meter data. Current research on probabilistic residential load modelling increasingly considers the PDF of residential net load, which is the load minus on-site PV power generation. Ref. [65] combined PV power production, household electricity consumption, and EV charging demand in the same probability distribution model for both individual and multiple households. The study illustrated the significant difference between the PDF of the net load and the conventional load, and also the difference between including or excluding EV charging.

3.2. PV probability distributions

A PV system consists of a number of PV modules that convert the energy from the sun to DC power, which depends directly on the irradiance [67]. The amount of total solar energy at the top of the atmosphere is relatively constant, represented by the solar constant [6,68]. However, the solar radiation on the earth's surface is highly variable due to several factors: the earth's orbit, geographic location, as well as absorption and scattering of solar radiation in the atmosphere, in particular from clouds [68]. Hence, PV uncertainty modelling can be approached directly as a PDF of the PV power yield or indirectly as a PDF related to the solar irradiance, subsequently fed into a PV system model. For the indirect approach, the PV power uncertainty can also be represented by the clear-sky index, a measure defined as the ratio between global solar irradiance and the corresponding theoretical clear-sky irradiance at the same time and location.

For solar irradiance, the PDF has been represented by normal [69–73] and beta [17–21,37,44,74] distributions. Besides the beta distribution, Ref. [21] also used an empirical PDF of the data in one out of four case studies. Another distribution used in Ref. [75] is the uniform distribution, which was utilized for the uncertainties of all inputs.

For the clear-sky index based approach, a wider range of PDFs has been used, typically mixtures of distributions such as for example normal, lognormal, or polynomial [76,77]. Some studies utilised a single peak distributions and other used bimodal distributions to represent clear and cloudy conditions. A study using irradiance data from Australia introduced a triple normal distribution [77], further elaborated upon in Ref. [78]. This approach represents the distribution for different cloud conditions as shown in Fig. 4. Some studies combined the PDF of irradiance with a PDF of PV temperature forecast error with the argument that the operating temperature of the solar cell considerably affects the power output [17–19].

The direct approach models the uncertainty of PV power production directly as a PDF. One of the advantages of this approach is that a PV output simulation from the solar radiation distribution is avoided. The uncertainty, using this approach, has been represented by exponential [79], Weibull [80,81], beta [82,83], normal [22], and empirical [47] PDFs.

Table 2 summarizes the PDFs used for both the direct and indirect approaches. It is important to note that the distribution modelling of the PV generation, both for the direct and indirect approaches, is often useful for only day-time or for specific sun angles to eliminate the stable and completely certain night data.

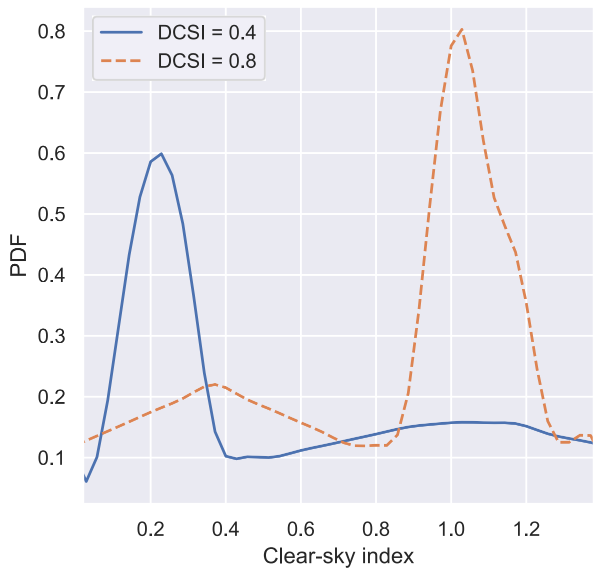

Fig. 4. Outline of the CSI probability distribution for two sky conditions. The solid line example (shifted to the left) represents a low daily clear-sky index (DCSI) of 0.4 and the dashed line example (shifted to higher values) represents a higher DCSI, of 0.8 [76,77].

Table 2
Overview of probability distributions for solar PV generation used in uncertainty modelling.

Indirect	Probability distribution
Solar irradiance	Normal [69–73] Beta [17–21,37,44,74] Empirical PDF [21] Uniform [75]
Clear-sky index	Single gamma [38,45,84–86] Single Boltzmann [87] Bi-exponential [88] Double normal [76,89,90] Double beta [91] Double Boltzmann [92] Double lognormal [76] Logistic combined with Weibull [93] Triple normal [77] N normal distributions [94] Normal-lognormal [76] Normal-polynomial [76] Lognormal-polynomial [76] Piece-wise uniform [95]
Sunny clear-sky index	Normal [78]
Cloudy clear-sky index	Double Normal [78]
Diffuse fraction	GEV [38,45,86]
Direct	**Probability distribution**
PV Power	Exponential [79] Weibull [80,81]Beta [82,83] Normal [22] Empirical PDF [47]
Complement variable	**Probability distribution**
Temperature forecast error	Normal [17–19]

No studies were found that included uncertainty from PV system components such as solar panels and inverters in the indirect approach. Nonetheless, the indirect approach based on the clear-sky index, that takes into account the condition of the sky, is preferred as it more accurately represents the environmental conditions. Furthermore, the effort to split the distribution into two or more distinct conditions provides a promising tool to model the system more realistically.

Table 3
Overview of probability distributions for EV charging used in uncertainty modelling.

Single EV	Probability distribution
Mileage	Lognormal [9,17,21,44,45,64,85], Normal [97], GEV [98], Empirical PDF [99,100]
Battery capacity	Normal [9,17,101]
Travel time	Lognormal [85]
Operating status for PHEV	Normal [9,17,101]
EV Charging Station	**Probability distribution**
Service time	Exponential [9,17,21,97]
Inter-arrival time	Exponential [9,17]
Customer arrival pattern	Poisson [9,20,85,97] Normal [45] Markov chain [103]
Overall charging demand	Weibull [9] Rayleigh [21] Normal [44,64,97]
Residential Charging	**Probability distribution**
Arrival/charging pattern	Poisson [9] Bernoulli [105] Normal [101] Markov chain [103]
Departure time	Weibull [98] Empirical PDF [99,100]
Arrival time	GEV [98] Empirical PDF [99,100]
Service time	Exponential [9,97]
Number of customer per house	Discrete [9]
Overall charging demand	Normal [9,105]

3.3. EV probability distributions

The uncertainties of EV charging demand arise from, e.g., human behaviour, EV operational parameters, and charging infrastructure. To properly capture these uncertainties of EV charging, the distribution modelling of EV requires utilizing multiple marginal distributions, as shown in Table 3. These distributions, however, considered EVs solely as loads. The application of vehicle-to-grid scheme as discussed in [96] were rarely considered.

To model the uncertainties of EV charging at charging stations and residential locations, usually a bottom-up model is first utilized to model several individual EVs. The main variable for modelling a single EV is the mileage or distance covered by the EV. The most popular distribution to use for the mileage is the lognormal, which is mostly based on fitted statistical data for vehicle driving data in cities [9,17,21,44,45,64,85]. Other studies have used the normal distribution [97], general expected value (GEV) [98], and empirical PDF [99,100].

The PDF depends on the type of EV, whether it is a fully electric vehicle or a plug-in hybrid (PHEV). To model PHEVs, the uncertainties in the operating status need to be taken into account. Other variables that are probabilistically modelled for single EVs are the battery capacity and the travel time. For both the battery capacity and operating status, the normal distribution has been used [9,17,101]. Ref. [85] modelled the distribution of single EV travel time using the lognormal PDF.

To model multiple EVs charging, assumptions regarding the EV charging infrastructure need to be made. Some models assumed both EV charging stations and residential charging locations are accessible by EV owners, while others just limited the accessibility to one of them. Several variables considered for EV charging stations are service durations, inter-arrival times, and customer arrival patterns. For the service durations and inter-arrival times, the exponential distribution is usually used [9,17,21,97]. For customer arrival patterns, according to

Table 3, a Poisson process is the most popular method. EV charging demand has also been modelled using Markov-chains model [102,103]. EV charging model should also consider the coincidence factor that represents how much the EVs are charged at the same time [102,104]. Some studies performed distribution fitting to the overall EV charging station demand. Although these studies used similar distributions, the final fitted distribution could be different [9,21,44,97].

For modelling arrival uncertainties for residential charging, some approaches exist. Some studies modelled the arrival or charging pattern similar to the EV charging station using Poisson [9], Bernoulli [105], and normal [101] distributions. Another study forecasted the departure and arrival times of the EV [98]. Other studies used copulas to generate a joint distribution from a marginal distribution of trip start time, end time, and distance [99,100]. Similar to the EV charging station models, several previous works also performed distribution fitting to the total residential charging demand [9].

The lognormal distribution seems to be suitable for modelling trip or daily mileage. Long trips are often rare (see, e.g., Refs. [99,106]) and the distribution of trip lengths becomes heavily tailed. This might also be represented by a Weibull distribution. Choosing between the two distributions can be based on the maximum likelihood estimate, as proposed in Ref. [107]. The number of EVs could also determine the distribution of the charging demand. Fig. 5 illustrates the comparison between the distributions of EV charging demand model for different number of EVs [108]. Fewer number of EVs produce a more discrete distribution.

Choosing the normal distribution for the battery capacity might be suboptimal. Firstly, similar to ICEV engine sizes, many EVs have similarly sized batteries, which makes certain battery capacities more likely than others. For example, Renault Zoe has a 41 kWh battery [109], Nissan Leaf has either 40 kWh or 62 kWh batteries [110], Kia Niro has a 64 kWh battery [111], Chevrolet bolt has a 60 kWh battery [112], BMW i3 18.8 kWh, Chevrolet Volt (PHEV) 18.4 kWh, and Tesla model S has a 100 kWh battery. Secondly, the distribution of battery sizes depends on the market share of the various EVs from various manufacturers.

A few papers, e.g., [9,17], rely on an M/M/c queuing process. This process is defined by a Poisson arrival rate (i.e., exponentially distributed inter-arrival time), exponentially distributed service time, and c is the number of parallel chargers [113].

4. Correlation modelling

Power system input variables are to some extent dependent on each other. Correlations between residential loads, presumably, occur due to human routines [45]. PV generation in nearby locations is also correlated due to similar weather patterns [114]. However, it is important to note that due to the dispersion-smoothing effect, there is substantial decorrelation of solar irradiance with distance [12]. Hence, a spatial correlation model is vital and has been proven to accurately reproduce and not overestimate the aggregated PV [12]. Similarly, even though not studied to the same extent as correlations between loads and between PV systems, correlations between EV charging patterns at dispersed locations have also been observed [115,116]. The state-of-charge of the EV, for example, is positively correlated with daily travelled distances [115]. The EV charging also spatially correlated due to spatial problems impact of EV mobility [116].

There are also correlations between different variables, such as between load and PV, load and EV, and between PV and EV. Taking these into consideration could impact the overall system uncertainties; in Ref. [18] the standard deviation of the voltage was significantly higher in a correlated case compared to an uncorrelated one. Recent studies show a growing interest in such correlations between input variables [27].

Generally, the joint variability of two random variables can be measured by the covariance [117]. A positive covariance means that higher values of one variable coincide with higher values of the other variable, and vice versa [117]. Hence, the covariance measures the strength of a linear relationship between two variables. For two random variables X and Y, both with a sample size of N, the covariance of X and Y is:

$$\mathrm{Cov}(X,\, Y) = \sum_{i=1}^{N} \frac{(x_i - \bar{x})(y_i - \bar{y})}{N}, \tag{1}$$

where $\bar{x}$ and $\bar{y}$ are the mean of X and Y respectively. This value, however, is often difficult to interpret due to its dependence on the magnitudes of the variations in X and Y. Therefore, the correlation coefficient (CC), the normalized covariance, is often used:

$$\mathrm{Corr}(X,\, Y) = \frac{\mathrm{Cov}(X,\, Y)}{\sigma_x.\ \sigma_y}, \tag{2}$$

where σ_x and σ_y are the standard deviations of X and Y respectively. The CC, however, assumes that a change in one variable is corresponding to a proportional change in another variable. In some cases, two variables are better represented as having a monotonic relationship (either increasing or decreasing), but not necessarily at a constant rate. In this case, the numerical values could be transformed to their rank on the sorted data, called ranking in statistics. Then a more suitable correlation measure is the rank CC, which measures the degree of similarity for two rankings. Some examples of rank CCs are the Spearman, Kendall, Goodman and Kruskal, and Somers rank CCs. A comprehensive overview of CCs is available in Ref. [118].

The interpretation of the CC, however, depends on the applied computational methods. On significant early work that included the CC for PLF was Ref. [119], where the authors developed a linear dependence framework to include the CC for analytical-based PLF. It was subsequently used in further studies [17,19]. This framework, however, will give only approximations of the output cumulants for non-linear dependence cases. Another analytical CC framework was shown in Ref.

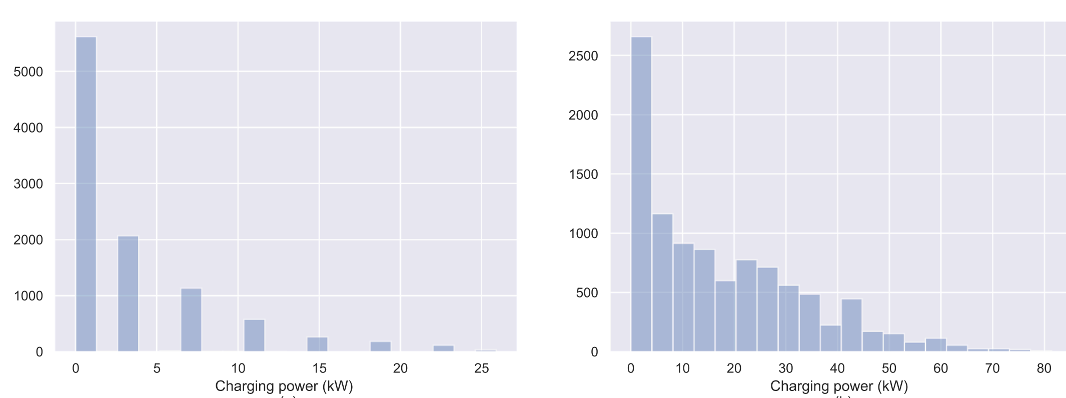

Fig. 5. Histogram of EV charging demand for (a) 20 EVs and (b) 100 EVs with charging power 3.7 kW [108].

[18] using a joint-cumulant method that generates a joint-moment and joint-cumulant based on the joint distribution of the variables and the given CC. The problem with this approach occurs when the number of variables is high, as it dramatically increases the number of calculated cumulants.

Even though these methods are popular due to their simplicity, they are often unable to define the dependency structure for more complex systems, when the distribution is not following the multivariate normal distribution. Therefore, for non-normal continuously distributed variables, copulas are often used to represent the dependency between the variables. The copula can be defined as a function that joins marginal distributions into a multivariate distribution function [117].

Copulas have been used for magnitude correlations between load, PVs, and EVs [45,86,120] and spatial correlations of PVs and EVs [12,100]. Copulas can also be used for both analytical and numerical PLF methods. Refs. [38,45] are examples of analytical approaches using the Gaussian copula. Another example of analytical-based PLF using a copula is Ref. [79], which utilized a fuzzy copula model for the correlation modelling. Several studies utilized copulas to model the correlation in numerical PLF [12,78,100,121]. The copula is convenient in numerical PLF, as it provides a multivariate distribution function from which correlated random samples can be drawn. However, for high-dimensional distributions, copulas are difficult to implement and are restricted to Gaussian copulas only. For this reason, pair copulas have been introduced, providing a bivariate copula function from any variants of copula families, which adds flexibility to the dependency structures [122].

Copulas have also been used in correlation modelling of load, PV, and EV for other applications than PLF. One study [123] used the Gaussian copula for modelling load-PV correlations for demand response, and another [120] used the Gaussian copula for modelling load-PV correlations for optimal scheduling of a solar-wind-storage hybrid generation system. load-PV correlations modelled with a copula were also used for studying low-voltage ride-through [124]. Another study performed student's *t* copula for modelling EV load demand only [115].

Another way to use the CC is to combine the CC with a transformation and Cholesky decomposition. The Cholesky decomposition is able to provide correlated random samples for numerical methods [15,125]. Even though the approximation method by default is not suitable for correlated variables, a combined modified Harr *2n* PE and Gauss-quadrature method showed its ability to cope with correlated variables using the decomposition method in PLF simulations [21,22]. A transformation method also generated promising results for another approximation-based PLF in Ref. [20].

The review of the literature showed that the choice of correlation method was largely independent of the input variables. This means that all of the correlation methods can be applied to any combination of input variables, as shown in Table 4. The choice of correlation method is more dependent on the PLF computational method and input dimensions. For future PLF analyses, considering multiple variables such as load, PV, and EV, an advanced pair copula could improve the performance. However, the main drawback of this method is the computational time, which requires a massive simplification on its application to find the best-fitting of full Pair Copula under given limited computational time [122].

5. PLF computational methods

The main objective of the PLF computation is to characterize the probability distribution of the output variables, e.g., the bus voltages and line currents, using the given statistical information on the input variables and the correlations between the variables as the input. In the literature, there are several methods that have been used for PLF computations. As seen in Fig. 6, probabilistic methods can be classified into three main groups: numerical, analytical, and approximation methods [4]. The procedures, characteristics, and advantages of each computational method are summarized in the following sections.

Table 4
Overview of correlation modelling methods used for the input correlation modelling for PLF.

	Computational methods	Correlation methods
PV-PV	Analytical	Linear Dependence [17,19]
		Joint Cumulant [18]
		Gaussian Copula [45,86]Fuzzy copula [79]
	Numerical	Gaussian Copula [12,78]
	Approximation	Pair copula [122]
Load-load	Analytical	Linear Dependence [17]
	Numerical	Gaussian copula [121]
		Cholesky decomposition [125]
	Approximation	Cholesky decomposition [21,22]
		Unscented transformation [20]
EV-EV	Analytical	Gaussian copula [45,86]
		Linear Dependence [17]
	Numerical	Gaussian Copula [100]
		Correlation methods
Load-PV	Analytical	Gaussian copula [45,86]
		Linear Dependence [17]
		Two-step correlated random sample [126]
	Numerical	Cholesky decomposition [15]
		Gaussian copula [120]
	Approximation	Unscented transformation [20]
		Cholesky decomposition [22]
Load-EV	Analytical	Gaussian copula [45,86]
		Two-step correlated random sample [126]
EV-PV	Analytical	Gaussian copula [45,86]
		Two-step correlated random sample [126]

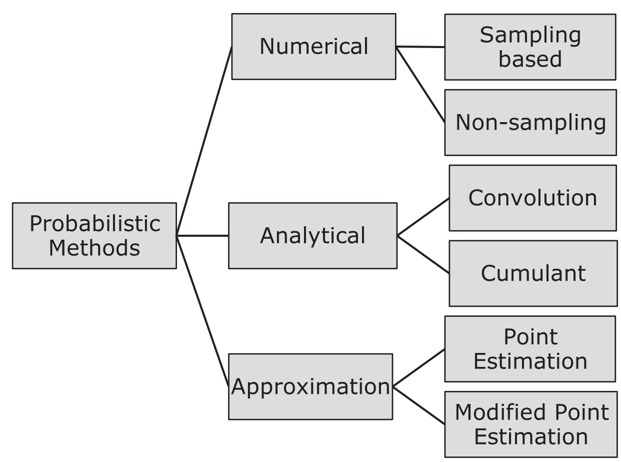

Fig. 6. Classification of PLF computational methods.

Most of the probabilistic methods still need to perform a deterministic load flow solution as one of the steps. The numerical methods repeatedly perform a deterministic power flow calculation for each set of new samples, while the analytical methods often perform it to find sensitivity matrices, one of the steps in cumulant-based analytical method that represent the relationship between input variables and output [27]. For the approximation methods, deterministic power flow solutions are often designed to calculate the output from every sample point to produce the output's raw moments. Deterministic load flow can be solved using one of several available load flow iteration methods. One of the most popular methods is Newton's method that is often used to solve load flow in transmission systems [127]. However, distribution systems have different characteristics compared with transmission system, and hence Newton's method is often difficult to use for distribution systems with low reactance-to-resistance (X/R) ratio [128]. Ref. [129] compared the performance of the forward-backward sweep (FBS) method and Newton's method in a distribution system and found that the FBS method is preferable for distribution systems. Even though the authors of PLF studies rarely inform about exact the load flow method, Ref. [17] stated that they utilized different approaches for PLF in transmission and distribution systems. For transmission systems, Newton's method was utilized, while for distribution systems, FBS method was used.

The need for faster computational methods increases as PLF is used for increasingly complex task, such as smart grid related scheduling and time-series operations [22,44,120]. Several studies propose fast PLF methods, referred to as simplified PLF methods, which have high computational efficiency [130,131]. These methods are based on a modification of the Jacobian matrix calculation. In Ref. [130], a modified system admittance matrix was used instead of the Jacobian matrix, while Ref. [131] employed linearization of the Jacobian matrix calculation. Another study utilized dynamic PLF based on analytical methods to consider time-series PLF with distributed power generation and EV charging [44].

5.1. Numerical

Numerical methods use numerical estimations to solve mathematical problems. In PLF computation, the numerical method called Monte Carlo Simulation (MCS) is often used, with general steps as given in Fig. 7. MCS estimates the output probability distributions by repeatedly drawing samples from the input probability distributions and solving the power flow problem for these samples. One of the most important parameters in MCS is the stopping criterion, which can be defined as a fixed number of simulations or as a certain limiting coefficient of variation as a measure of successive improvement.

MCS methods can be classified by their sampling method. The most basic method is a simple random sampling MCS, utilized, e.g., in Ref. [12]. However, this method needs a very large number of simulations (100,000 in the aforementioned study), which makes it infeasible in most situations. Several sampling methods have been introduced to increase the computational speed, such as the Latin hypercube (LH) [125,132] and uniform design (UD) [133] sampling. The LH removes an entire interval once the sample has been taken from that interval, in order to reduce the sampling time and ensure good coverage. The UD, on the other hand, ensures samples are scattered evenly over the domain.

Other studies have tried to improve the PLF simulation time using another type of MCS called quasi-MCS [97,134]. Quasi-MCS performs the computation using low-discrepancy (more uniformly distributed) sequences instead of sequences of pseudorandom numbers [97,134]. Ref. [135] compared the effectiveness of Sobol's quasi-random numbers to LH and normal random sampling and found that the former were more efficient. Sampling can also be done through Markov-chain Monte Carlo (MCMC), in which samples are drawn using Markov chains that have limiting distributions equal to the distributions of the input variables, see, e.g., Ref. [136]. Another effort to increase the efficiency of MCS-based PLF is by using nonparametric density estimators to decrease the trial numbers of MCSs [137,138].

Regular MCS is unable to capture time dependencies. One way to include time series in MCS is by performning a MCS-based PLF for every time step using forecast data [139]. Another available method is based on a special MCS that considers time sequences, called sequential MCS [140]. This method performs the numerical simulation in chronological order. Sequential MCS is not common in PLF but is used in other applications that require time series, such as stability analysis [27,141,142].

MCS is in general able to yield high accuracy in PLF analyses. This method often acts as a reference for other PLF computation methods. MCS also supports any type of PDF and is relatively easy to implement. In addition, MCS is also able to handle correlations between inputs. However, the main drawback of the MCS method is a massive computational burden, as it needs to iterate a large number of calculations.

5.2. Analytical

In contrast to the numerical methods, which depend on iterations, the analytical methods solve the problem by performing arithmetic operations on the input PDFs. In order to do PLF computations using an analytical method, a mathematical simplification of the power flow equations such as linearization is needed. Analytical methods can be classified into two groups, convolution and cumulant methods.

The first generation of analytical-based PLF methods utilised a conventional convolution method [14,143]. However, this requires an expensive convolution calculation for PLF even after utilizing a discrete convolution such as discrete convolution in frequency domain by performing Fourier Transform [144]. In recent studies, convolution method are rarely deployed. Several recent attempts to increase the efficiency of the convolution method were done with Gaussian mixture models [43] and sequence operation theory [145].

To avoid the computational burden and convolution calculation, the concept of the cumulants was introduced. The idea behind these methods is replacing the moments of the probability distribution by the cumulants of the distribution and performing the calculation based on the cumulants as given in Fig. 8. Then, the input-output sensitivity of the system should be made, usually using several deterministic load flow simulations. This sensitivity is then used to calculate the cumulants of the output [27].

To determine the shape of the output distribution from the cumulants of the output, several types of expansion series are commonly used, such as the Cornish-Fisher, Gram-Charlier, and Edgeworth expansions [9,18,38,64]. The combined cumulant and Gram-Charlier method have also been utilised for time series PLF in Ref. [146]. In that paper, the day-ahead PLF was generated using forecasted input data. Another expansion series called Legendre series has been used in Ref. [147] to reconstruct the PDF of the PV harmonic current in a harmonic load flow study. This series performed better in coping with the randomness of the magnitude and phase angle of the harmonic current. There is a recently introduced method to generate a polynomial distribution based on the method of moments [60]. This method is applicable for cumulant-based PLF analysis but its effectiveness in PLF is yet to be investigated.

Initially, cumulant methods were unable to handle correlations. To remedy this, the extended cumulant method was introduced [119]. However, this method has difficulties in handling a large number of correlations and, thus, to cope with microgrid systems that has high correlation between its inputs, efficiently [20]. The simplification in the PLF process using these methods could also lead to less accurate results. A more recently published study proposed the LH sampling method to obtain the cumulants of the input variables [44]. The LH sampling used

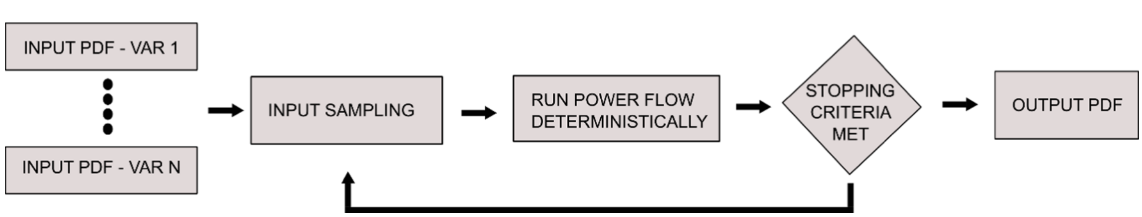

Fig. 7. Conceptual framework of numerical methods for PLF computation.

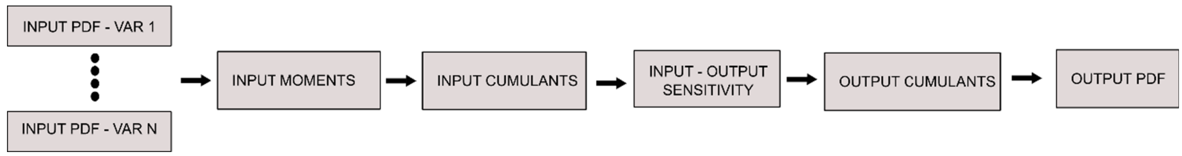

Fig. 8. Conceptual framework of analytical cumulant-based methods for PLF computation.

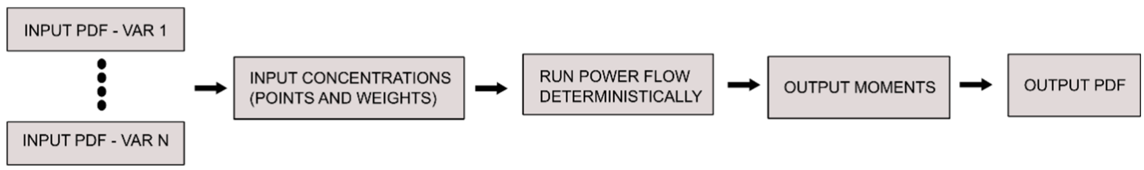

Fig. 9. Conceptual framework of approximation methods for PLF computation.

in this study has an ability to accommodate the correlation efficiently.

To determine the PLF for an extended time frame using analytical method, one study [45] proposed a method called general analytical technique (GAT). In that study, GAT method successfully calculated weekly statistical characterizations for each month.

5.3. Approximation

The idea behind the approximation methods is to utilize approximations based on a small number of sample points and their weights, corresponding to the input PDFs, to estimate the statistics of the output as illustrated in Fig. 9. These methods have the ability to perform a nonlinear analysis based on a few numerical-statistical properties gained from the inputs. The goal of these methods is to generate minimum number of samples from the input variables that contain sufficient information about the PDFs of the inputs. The approximation methods that have been used in PLF can be divided into two distinct groups: point estimation (PE) and modified PE methods.

PE utilizes certain statistical properties of the input variables, such as mean and variance, and does not require complete information about the distribution, which makes it computationally efficient. For n number of random variables, various numbers of deterministic load flow are needed, depends on the PE scheme. PE $2n + 1$ scheme, for example, requires $2n + 1$ deterministic load flow calculations. Several PE schemes used in literature are $2n$, $3n$, $2n + 1$, $4n + 1$ [4]. However, PE has a low accuracy when dealing with higher statistical moments, and it does not provide the PDF of the results [20]. PE is common in PLF computation for systems with PV and EV [21,85].

Approximation method by default is not suitable for correlated variables. Hence, several attempts to include correlation in approximation has been developed. A combined modified Harr $2n$ PE and Gauss-quadrature method showed its ability to produce the sample points that cope with correlated variables using the decomposition method in PLF simulations [21,22]. Another modified Hong $2n + 1$ PE method used rotational transformation to include the correlation by transforming the correlated random inputs into uncorrelated random inputs based on the eigenvectors of the covariance matrix [148].

The unscented transformation (UT) method was developed using several mathematical assumptions and complex algorithm to deterministically generates an appropriate number of input variable samples that the mean and covariance of the samples has the same values as the input PDF [149]. The main advantage of UT is its ability to represent the correlations in the model properly using the covariance matrix with less mathematical burden [39,150]. Therefore, several studies have used this method in PLF with correlation modelling [20,39].

6. Application of PLF for the system with PV and EV

Although there have been several studies on power system uncertainty modelling, the application of PLF on distribution systems with PV generation and EV charging is relatively rare. PLF studies considering VRE more often include wind rather than PV. The uncommon inclusion of EV charging in PLF is also responsible for the small number of studies concerning the PV-EV combination. However, with a growing number of PV and EVs in distribution grids, this trend is expected to change. This section characterizes recent PLF studies that do include the combination of PV generation and EV charging in terms of probability distributions used, correlation modelling, computational methods, and network models. Table 5 summarises the main findings.

6.1. Probability distribution

There are numerous probability distributions for the load, as shown in Table 5. However, recent PLF studies with PV and EV employed only the normal distribution and empirical PDFs. The studies that used the normal distribution considered it common practice in power system analysis [37]. As has been seen in Section 3.1, the normal distribution is not suitable, and rarely used, for modelling of residential load. Nonetheless, the PLF studies with PV and EV have preferred to model the load distribution as a normally distributed total load, even though this approach may not be accurate for distribution systems with a high share of residential loads. Other studies preferred using empirical PDFs, which is accurate for a specific location, but difficult to generalize and implement on other systems, and the availability of data is challenging.

PV distribution modelling based on irradiance or clear-sky index are both found in PLF studies with the PV and EV combination. There is no consensus on which method is preferred, but the model based on irradiance is more popular. Ref. [21] used both an irradiance-based model and historical data for different cases but did not state which one has optimal performance. More recently proposed probability distribution models, such as the triple normal or two-state clear-sky index, which are arguably more realistic, have so far never been employed for this scenario. There is also no distinction between the probability distributions of centralized and distributed PV system in these studies. Neither has there been any attempt at directly modelling the probability distribution for the net load from PV and load for each house.

For the EV modelling, several different model assumptions have been made in the reviewed PLF studies. Refs. [20,21,85] assumed that all of the EVs were PHEVs and took into account the uncertainties from PHEV operating status. Other studies [37,45,86] assumed all of the EVs were fully electric. Another approach considered both EV types [17]. There is no agreement on what the most likely future types of EVs are. The assumed charging location also varied widely among the previous studies. Some models [20,21] assumed that the charging takes place in the charging station, while another [17] included both residential charging and charging station. One study [85] focused on the effect of time-of-use electricity tariffs at home and, hence, considered residential charging only. Refs. [45,86] assumed charging ports were available in

Table 5
Summary of previous PLF studies on systems with PV and EVs.

Ref	PLF methods			Probability distribution			Probability distribution with correlation modelling						System model	
	N	AN	AP	Load	PV	EV	Load-load	PV-PV	EV-EV	Load-PV	Load-EV	PV-EV	Network	Model
[17]	MC*	EC	–	Normal	I: beta T: Normal	SPOS: normal BC: normal M: lognormal CAT: exponential CST: exponential	LD	LD	LD	–	–	–	Tr&D	Tr: Ward-Hale 6-bus, IEEE 14-bus D: modified IEEE 69-bus, 33-bus
[20]	MC*	–	RBFNN	Normal	I: beta	SPOS: normal BC: normal M: lognormal CAT: exponential CST: exponential	UT	–	–	UT			MG	Wood and woollenberg 6-bus IEEE 14-bus
[85]	MC*	–	3 PE	EPDF	I:Gamma	M: lognormal A: poisson							D	PG&E 69 bus
[21]	MC*	SEC*	GQPE	Normal	I:beta P: EPDF	A: exponential CST: exponential M: lognormal TC: Rayleigh, hist	CD						Tr	IEEE 118-bus system
[37]	MC	–	–	Normal	I:beta	M: lognormal	–	–	–	–	–	–	Tr	IEEE 30-bus,57-bus,118-bus
[45]	MC*	GAT	–	EPDF	CSI: Gamma	M: lognormal A: GRV	–	GC	GC	GC	GC	GC	D	ENDE 100 RDS
[86]	MC*	SEC	–	EPDF	CSI: Gamma	M: lognormal A: GRV	–	GC	GC	GC	GC	GC	D	IEEE-33 bus

N: Numerical
AN: Analytical
AP: Approximation
MC: Monte Carlo
EC: Extended cumulant
SEC: Series expansion based cumulant
GAT: General Analytical Technique
RBFNN: Radial-basis function neural network
PE: Point estimate
GQPE: Gauss quadrature PE
I: Irradiance
T: Temperature
P: PV Power
EPDF: empirical PDF
CSI: Clear Sky Index
SPOS: Single PHEV Operation Status
BC: Battery capacity
M: Mileage
CAT: Customer arrival time
CST: Customer service time
A: Arrival process
TC: Total charging
GRV: Gauss random variable
LD: Linear dependance
GC: Gaussian Copula
UT: Unscented transformation
CD: Cholesky Decomposition
Tr: Transmission
D: Distribution
MG: Microgrid
* : as comparison.

parking areas.

Every EV charging model in Table 5 used the mileage of individual EVs as the main basis of calculation and all of them employed the lognormal distribution for mileage. As seen in Table 5, there is no large variation among other marginal distributions such as customer service time, battery capacity, and PHEV charging status. However, there was more variation among models used for arrival patterns. The most popular arrival pattern models used in PLF studies with PV and EVs are using the Poisson process for arrivals or, equivalently, assuming exponentially distributed times between arrivals, and Gaussian random variables. However, it is good to note that for EV charging modelling, one review paper [28] has argued that the model accuracy depends more on the model assumptions than the actual modelling technique.

6.2. Correlation modelling

Some PLF studies that considered both PV and EVs also modelled correlation between the input variables. Ref. [17] modelled the intra-variable correlation only (load-load, PV-PV, EV-EV), using a linear dependency approach. The study in Ref. [21] considered the load-load correlation only, using Cholesky decomposition, while Ref. [20] modeled load-PV correlation using UT. Two studies [45,86] did a complete correlation modelling for load-PV, load-EV, and PV-EV using the Gaussian copula.

As seen in Table 4, however, there are several more options for correlation modelling that have been used for other applications but never in PLF with PV and EVs. Several types of copulas have been used for load-load correlation modelling, but never, to the best of the authors' knowledge, in PLF studies considering PV and EVs. Similarly, although many copulas have been successfully used to model correlations between spatially distributed PV systems, they have not been applied to PLF that included PV and EVs except Gaussian copula.

Because EV charging is usually considered as a separate input variable apart from other loads, correlation modelling requires a method that could couple many variables. As discussed in Section 4, a pair copula should be able to handle the multiple variables required in this case.

6.3. Computational method

The most common method for PLF computations is MCS, which is known for its accuracy and reliability. All of the studies summarised in Table 5 utilized MCS as one of the PLF computational methods. However, because of its computational burden, this method is usually used only as benchmark or for comparison. Recent studies focus on finding a method that has similar accuracy as MCS but faster computational speed. Several methods from the analytical and approximation groups have been employed for PLF with PV and EVs.

All of the analytical methods used for this application utilized the extended cumulant method with series expansion to approximate the probability distributions of the PLF output. The extended cumulant method extends the conventional cumulant method to consider input variable correlations. Among the available expansion series, the Cornish-Fisher expansion is the most commonly used one. This series is considered to have better performance for estimating non-normal distributions, which are common in PLF outputs [22].

There is a wider variety of methods in the approximation computational group. Ref. [85] utilized the $3n$ PE method for PLF with PV systems and plug-in EVs. More PE-based methods were used in Ref. [21], including three, five, and seven PEMs, and were compared with the newly introduced Gauss-quadrature-based method. The study claimed that the new method performed better than the previous PEMs. Another approach, shown in Ref. [20], combined neural networks and UT for PLF including PV, wind, and PHEVs.

6.4. Network model

PLF can be implemented both for transmission and distribution systems. The transmission network here is described as a system where multiple loads are seen as single aggregated units connected at substations. The distribution network is defined as the system that connects individual customers. Power flow modelling in the transmission system is generally simpler and often assumes that the system is balanced. However, in the radial and meshed distribution system, the system is usually unbalanced and more complicated. The complexity of the distribution system is increasing with the introduction of distributed PV generation and EV charging. Hence, even though PLF is usually performed for transmission networks, PLF studies with PV and EVs is also becoming more common for distribution systems, as shown by Refs. [17,45,85,86].

The introduction of distributed generation has also led to an increase in popularity of the microgrid concept. There are several definitions of microgrids in previous studies, but generally it can be described as a local cluster of interconnected distributed generators and loads, the operation of which can be managed without connecting to a larger grid system [151]. The microgrid network requires an energy management system, smart sensors, data concentrators for communication between energy management system and smart sensors, and a data management system. In microgrids, PV is often considered as the main type of generation. Hence, several microgrid models have used a wide range of methods for including the uncertainties of PV generation [151]. However, the inclusion of EVs in microgrid PLF analyses is still not common. One of the rare studies of PLF analysis including PV and EV in microgrids is Ref. [20].

7. Concluding discussion

Based on all of the summarised papers in this work, PLF analysis for distribution systems with PV generation and EV charging has a great potential for improvement and for playing a vital role in the future distribution system. The improvement of the analysis could come from combining the current state-of-the-art in this study and to utilize applicable knowledge from other areas for every stage of the PLF analysis. Future PLF studies should also, to a greater extent, consider distribution systems with PV generation and EV charging.

First, in terms of input probabilistic distribution modelling, the next step for this research is implementing the currently developing spatio-temporal models for load, PV, and EV to represent more realistic input variables. As regards the loads, using skewed PDFs, e.g., the Gamma and lognormal distributions, is recommended over the normal distribution. This is further recommended if the load represents the load of a few consumers, i.e., systems that are small or not aggregated. A bottom-up approach to load modelling should also be considered for systems dominated by household loads. This approach is supported by well-developed models of individual loads. Even though EV charging is often modelled separately from load, for a household with residential charging and PV, the net load has proven to give a significant difference in terms of distribution shape compared to individual household load only, and is suitable for implementation in this bottom-up approach. For PV distribution modelling, one challenge is to develop a realistic model for a system with a large portion of distributed PV. The indirect approach of PV distribution modelling based on the clear-sky index is preferred because it represents the weather conditions better. One potential improvement in this approach is to include a more recent realistic irradiance model that takes into account the sky condition. The presence of EVs could be disruptive for the future distribution system and, hence, PLF calculations should consider EV charging on a regular basis. However, the uncertainty of the future EV development also seems higher than for load and PV. The difference in the assumptions between different studies is wide, as reflected in Ref. [9]. Some important notes from the literature to improve EV charging models are to state the type of EV in the models, correctly model the EV charging infrastructure, and use a more realistic distribution for trip mileage, i.e., lognormal and Weibull distributions.

As regards the correlation modelling, several studies reviewed here have shown the importance of considering correlation for the PLF analysis, not least for distribution systems. There are multiple methods for including correlations in PLF analysis. The copula is found to be reliable and applicable as it can be used for all computational methods. The presence of PV and EV in the distribution system, however, increases the complexity of the computational methods with correlation modelling. Future PLF studies need to focus on correlations between multiple input variables applicable to several scenarios of Load-PV-EV interactions.

Finally, for PLF computation, each method shown here has its own advantages and disadvantages. The numerical methods are proven to be the best accuracy-wise and often act as the benchmark, but are computationally expensive. The cumulant-based analytical methods have less computational burden with acceptable accuracy but are struggling to represent highly correlated variables accurately. Meanwhile, the approximation method has an acceptable computational demand, is able to include correlation in the modified groups, but often struggling to reach acceptable accuracy in higher statistical moments.

The current objective of PLF computation remains the same: to find a more computationally efficient numerical method, or a sufficiently accurate non-numerical method. However, the need for much faster computational methods increases as PLF is used for increasingly complex task, such as smart grid related scheduling and time-series operations. The computational method should also be able to cope with the increasing number of input variables in the future distribution power systems where more technologies will presumably be integrated. The need for more data is also crucial. Currently, the input distributions and correlation models are rarely compared with real measured data. The PLF analysis also should be tested on real network data more often rather than conventional model data. This will help ensure that PLF analysis is realistic and accurate when applying it to real networks in the future.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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