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Review

A comprehensive review on uncertainty modeling methods in modern power systems

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ABSTRACT

In recent years, modern power systems integrated with renewable energy sources and communication technologies have been rapidly developed. However, the randomness and intermittency of renewable energy sources, together with the unexpected disturbances in the vulnerable cyber layer, introduce uncertainties into power systems, which pose huge threats to the stable operation of modern power systems and should be carefully considered. Moreover, there is a lack of a comprehensive review of uncertainty modeling in modern power systems, which is one of the significant tasks in quantifying uncertainties. In this review, the methods and respective characteristics of uncertainty modeling in modern power systems are analyzed. According to the outputs of methods, they are categorized as model-driven and data-driven methods. The existing uncertainties in modern power systems and the respective modeling methods are investigated from the aspects of the physical layer, cyber layer, and economic and social layer. The applications of uncertainty modeling methods in modern power system operation are summarized from the aspects of steady state, dynamic, and risk analysis of modern power systems. Also, the prospective future research topics are recommended.

1. Introduction

In power systems, inherent uncertainties are widespread, e.g., load fluctuation, electricity price variation, and system interruption induced by the change of weather conditions or equipment faults. Moreover, with the construction of modern power systems, more uncertainties are introduced. On the generation side, since sustainable development and carbon emission reduction are concerned worldwide, governments have issued numerous policies to stimulate renewable energy generation [1,2], which has been extensively installed in power systems. In 2024, the worldwide installed generation capacity of renewable energy sources is expected to increase by 540 GW [3]. However, renewable power generations dominated by wind power and photovoltaic (PV) possess the characteristics of randomness and fluctuation due to the change of weather conditions, which injects uncertainties into modern power systems. Meanwhile, on the demand side, electric vehicles (EVs) have been developing rapidly, with more than 40 million EVs on roads worldwide until 2023 [4]. However, due to the randomness of EV charging, massive uncertainties will be introduced. Also, novel communication and computing technologies are coupled to physical power systems, contributing to the transformation from traditional power systems into modern cyber-physical power systems (CPPSs) [1]. Although the introduction of the cyber layer is beneficial for achieving more flexible and effective coordinated control, substantial uncertainties such as communication congestion, time delay, and even cyber-attacks exist in the cyber layer of CPPS. One of the most severe cyber-attack incidents occurred on the Ukrainian power grid in 2015, resulting in the power outage of 22.5 thousand consumers [5]. In general, there are increasing uncertainties in the physical and cyber layers, which have posed a tremendous threat to the safety and stability of modern power systems and need to be seriously investigated.

Typically, two tasks are mainly needed to quantify the impacts of uncertainties on power systems, which are uncertainty modeling and

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Abbreviations

AI	Artificial intelligence
ASAI	Average service availability index
ASUI	Average service unavailability index
BNN	Bayesian neural network
BR	Backward reduction
CDF	Cumulative distribution function
CNN	Convolution neural network
CPPS	Cyber physical power system
DoS	Denial-of-service
EENS	Expected energy not supplied
EUE	Expected unserved energy
EV	Electric vehicle
FDI	False data injection
FFS	Fast forward selection
FOR	Forced outage ratio
GAN	Generative adversarial network
GMM	Gaussian mixture model
GRU	Gate recurrent unit
KDE	Kernel density estimation
LHS	Latin hypercube sampling
LODF	Line outage distribution factor
LOLE	Loss of load expectation
LOLF	Loss of load frequency
LOLP	Loss of load probability
LRA	Low-rank approximation
LSTM	Long short-term memory
MCS	Monte Carlo simulation
MPPT	Maximum power point tracking
MTTF	Mean time to fault
MTTR	Mean time to recovery
PCE	Polynomial chaos expansion
PDF	Probability density function
PEM	Point estimate method
PV	Photovoltaic
RNN	Recurrent neural network
RoCoF	Rate of change of frequency
SAIDI	System average interruption duration index
SAIFI	System average interruption frequency index
SDE	Stochastic differential equation
SoC	State of charge
TSI	Transient stability index

uncertainty propagation analysis. Uncertainty modeling methods characterize the randomness or fluctuations of uncertainties. The randomness of uncertainties in this paper focuses on the statistical characteristic of possible values of uncertainties at a certain moment. And the fluctuation of uncertainties focuses on the time-varying property of uncertainties. It should be noted that whether to consider the randomness or the fluctuation of uncertainties depends on the specific studied issue. By comparison, uncertainty propagation analysis methods investigate the relationship between the uncertainties and the concerned system outputs. For example, in the probabilistic power flow analysis considering the uncertainty of wind, the uncertainty modeling describes the randomness of wind speed, and the uncertainty propagation analysis method calculates the probability distribution of system power flow based on the results from uncertainty modeling. To analyze the impact of uncertainties on the operation of modern power systems, both two tasks are essential. More specifically, the outputs of uncertainty modeling methods are the inputs of uncertainty propagation analysis methods. Some reviews have summarized the existing methods for quantifying uncertainties in modern power systems. The probabilistic and possibilistic uncertainty quantification methods have been summarized in [6]. Authors in [7] have covered more uncertainty quantification methods, including interval analysis methods and robust optimization methods for uncertainties. In [8], in addition to the summary of uncertainty quantification methods, the uncertainties existing in power systems also have been investigated. However, the above reviews discuss the uncertainty modeling and propagation analysis methods together without distinguishing them, and only a part of the uncertainty modeling methods are covered. Additionally, some reviews have studied uncertainties and applications of uncertainty quantification methods in power system operation. In [9], the uncertainty modeling methods for characterizing the uncertainties of PV and EV have been studied. The uncertainties existing in the physical layer of power systems have been summarized in [10]. Authors in [11] have investigated the applications of uncertainty quantification methods in power flow analysis. In [12], the applications of uncertainty quantification methods in power system optimization operations have been discussed. Authors in [13] have studied the influence of uncertainties on power system stability. However, only limited uncertainties and applications have been investigated, and recent studies are not covered in the above reviews. Thus, a comprehensive review containing the uncertainty modeling methods in power systems, as well as the uncertainty uncertainties and applications, is lacking.

In view of the above facts, it is vital to summarize the uncertainty modeling methods to provide a reference for the research on the operation of modern power systems. The main contributions of this review are as follows: 1) This review distinguishes uncertainty modeling methods from propagation analysis methods and conducts a detailed classification, comparison, and summary of uncertainty modeling methods, which is valuable for relevant industrial technicians to have a deep understanding of uncertainty modeling. By comparison, in the current reviews, the uncertainty modeling methods are mostly neglected or partly integrated into uncertainty propagation analysis methods. 2) The review of the uncertainties in modern power systems covers the uncertainties existing in traditional power systems, as well as the uncertainties induced by the penetration of renewable energy sources, the integration of cyber networks, and electricity markets. By comparison, many existing studies usually only summarize the uncertainties caused by the penetration of renewable energy sources while ignoring other uncertainties. 3) This review comprehensively investigates the applications of uncertainty modeling methods in modern power systems. Not only power flow analysis, operation optimization, and probabilistic stability analysis, which are involved in the existing reviews, but also the applications in stochastic stability and risk analysis are studied.

The remainder of the paper is structured as follows. Section 2 classifies uncertainty modeling methods from the perspectives of the outputs of uncertainty modeling methods and summarizes the characteristics of different methods. In Section 3, the uncertainties in modern power systems are categorized from the physical layer, cyber layer, and economic and social layer. Moreover, the characteristics of different uncertainties and corresponding modeling methods are summarized. The applications of uncertainty modeling methods in modern power system operations are addressed in Section 4. Section 5 recommends promising future research topics. Section 6 contains concluding remarks.

2. Uncertainty modeling methods

Uncertainty modeling methods are typically generic and not specifically designed to handle uncertainties in modern power systems. And modeling the uncertainties in modern power systems is just one of the

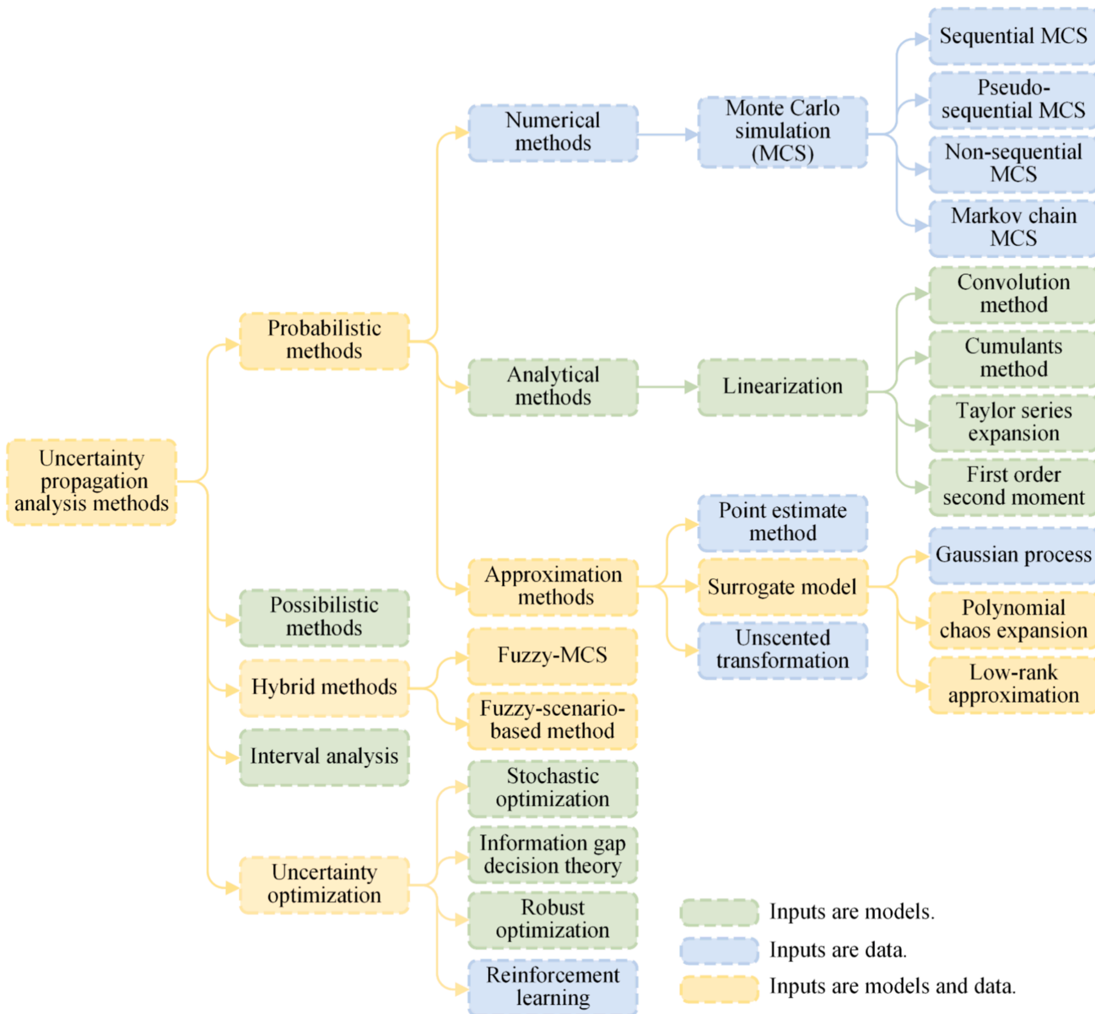

Fig. 1. Summary of uncertainty propagation analysis methods.

scenarios where these uncertainty modeling methods are used. This means that the uncertainties in modern power systems are not the origins of uncertainty modeling methods but their utilization scenarios. Also, since this paper focuses on uncertainty modeling methods rather than uncertainties, uncertainty modeling methods need to be highlighted and introduced at the beginning. Thus, uncertainty modeling methods are introduced in this section, and uncertainties in modern power systems are summarized in Section 3 for a clear logic and a concise paper structure. Moreover, though this paper mainly focuses on the uncertainty modeling methods, based on the facts that the uncertainty propagation analysis and modeling methods are not clearly split in the existing reviews, uncertainty propagation analysis methods in modern power systems are re-summarized according to [6–17], as shown in Fig. 1.

According to the different inputs of propagation analysis methods, i.e., the outputs of the uncertainty modeling methods, this paper classifies uncertainty modeling methods into two categories, namely, model-driven methods and data-driven methods. Thus, if the outputs of uncertainty modeling methods are data, they are defined as data-driven methods in this paper; otherwise, they are defined as model-driven methods. There are only model-driven methods and data-driven methods till now since the outputs of existing uncertainty modeling methods are either data or models. However, it should be noted that the model/data dual-driven uncertainty modeling methods may be designed for specific purposes in the future. Additionally, regarding some propagation analysis methods, such as cumulant methods, the probabilistic distribution of uncertainties, such as Weibull distribution or Gaussian distribution with given parameters, is required, where model-driven uncertainty modeling methods are employed, e.g., probabilistic density function (PDF) and cumulative distribution function (CDF). Also, there are some propagation analysis methods requiring the probabilistic results acquired via simulations, such as Monte Carlo simulation (MCS) and point estimate method (PEM). Data-driven uncertainty modeling methods will be adopted to generate the required scenarios using sampling techniques, historical data, or data generation technologies. Besides, for the conventional polynomial chaos expansion (PCE) and low-rank approximation (LRA) methods, the PDFs of uncertainties and sampling data are needed, where both data-driven and model-driven methods are required.

2.1. Model-driven methods

2.1.1. Probabilistic density function / Cumulative distribution function methods

2.1.1.1. Single probabilistic density function. The most common method is to use the single PDF to model uncertainties. The types and parameters of PDFs are determined by fitting the collected historical data in most current research. For a random variable X, the PDF $f(.)$ at the value x can be defined as [18]

$$f(x) = \lim_{\Delta x \to 0} P\{x \leqslant X \leqslant x + \Delta x\}/\Delta x \quad (1)$$

Table 1
Typical continuous PDFs.

Type of PDF	Form of PDF	Parameter description	Examples of uncertainties
Beta	$f_{Be}(x) = \Gamma(\alpha_{Be1} + \alpha_{Be2})x^{\alpha_{Be1}-1}(1-x)^{\alpha_{Be2}-1} / (\Gamma(\alpha_{Be1})\Gamma(\alpha_{Be2}))$, $0 < x < 1$	α_{Be1}, α_{Be2}: Shape parameters; $\Gamma(.)$: Γ function.	Solar irradiance [19–22] PV [23–28]
Exponential	$f_{Ex}(x) = \exp(-x/\alpha_{Ex})/\alpha_{Ex}$, $x > 0$	α_{Ex}: Mean.	EV [29]
Gamma	$f_{Ga}(x) = \alpha_{Ga2}^{\alpha_{Ga1}} x^{\alpha_{Ga1}-1}\exp(-\alpha_{Ga2}x)/\Gamma(\alpha_{Ga1})$, $x > 0$	α_{Ga1}: Shape parameter; α_{Ga2}: Scale parameter.	Clearness index [30] Wind power generation [31] Conventional load [31]
Gaussian	$f_{Gau}(x) = \exp(-(x-\alpha_{Gau2})^2/(2\alpha_{Gau1}^2))/(\sqrt{2\pi}\alpha_{Gau1})$, $-\infty < x < +\infty$	α_{Gau1}: Standard deviation; α_{Gau2}: Mean.	Conventional generation [32] Wind speed [33] Wind power generation [26,31] Conventional load [22–25,27–29,31–42] EV [29,43] Operational parameter [23,44] Communication error [42] Carbon dioxide emission [45] Cumulative energy demand [45]
Lognormal	$f_{Lo}(x) = \exp(-(\ln x - \alpha_{Gau2})^2/(2\alpha_{Gau1}^2))/(\sqrt{2\pi}\alpha_{Lo1}x)$, $x > 0$	α_{Lo1}: Standard deviation of logarithmic values; α_{Lo2}: Mean of logarithmic values.	Solar irradiance [37] Conventional load [46] EV [19] Windstorm [47] Carbon dioxide emission [45]
Rayleigh	$f_{Ra}(x) = x\exp(-x^2/(2\alpha_{Ra}^2))/\alpha_{Ra}^2$, $x > 0$	α_{Ra}: Scale parameter.	Wind speed [19,48]
Uniform	$f_{Un}(x) = 1/(\alpha_{Un1} - \alpha_{Un2})$, $\alpha_{Un1} < x < \alpha_{Un2}$	α_{Un1}: Lower bound; α_{Un2}: Upper bound.	Conventional generation [35] Component fault [49] Electricity price [45] Discount rate [45] Energy price [45]
Weibull	$f_{We}(x) = \alpha_{We2}(x/\alpha_{We1})^{\alpha_{We2}-1}\exp(-(x/\alpha_{We1})^{\alpha_{We2}})/\alpha_{We1}$, $x \geqslant 0$	α_{We1}: Scale parameter; α_{We2}: Shape parameter.	Wind speed [23,29,30,32,36–38,50–54] Solar irradiance [39] Wind power generation [25,27] EV [19] Windstorm [47]

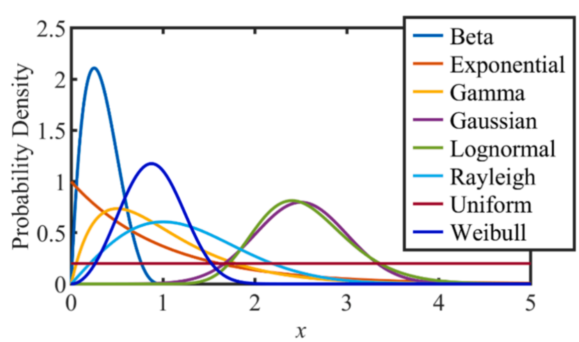

Fig. 2. Curves of typical continuous PDFs.

where Δx is the infinitesimal; $P\{x \le X \le x + \Delta x\}$ is the probability of random variable x in $[x, x + \Delta x]$.

1) Continuous PDFs.

Some typical continuous PDFs widely utilized in power system uncertainty modeling are summarized in Table 1, the curves of which are displayed in Fig. 2.

2) Discrete PDFs.

Apart from continuous uncertainties, discrete or approximate discrete uncertainties also exist in power systems. For such variables, discrete single PDFs are usually utilized. Typical discrete PDFs are presented in Table 2, the curves of which are drawn in Fig. 3.

Additionally, to adequately characterize the uncertainties in specific scenarios, some empirical PDFs are developed. Typical PDF developing methods include forming a versatile PDF [63] and introducing the truncated interval [64].

Table 2
Typical discrete PDFs.

Type of PDF	Form of PDF	Parameter description	Examples of uncertainties
Binomial	$f_{Be}(x) = \begin{cases} \alpha_{Be} & , x = 1 \\ 1 - \alpha_{Be} & , x = 0 \end{cases}$	α_{Be}: Probability when the test result is $x = 1$.	Conventional generation [34,55] Generation fault [56,57] EV [32] Component fault [50,57] Ice storm [58] Earthquake [58] Denial-of-service (DoS) attack [59]
Bernoulli	$f_{Bi}(x) = C_{\alpha_{Bi1}}^{x}\alpha_{Bi2}^{x}(1-\alpha_{Bi2})^{\alpha_{Bi1}-x}$, $x = 0, 1, 2, \ldots, \alpha_{Bi1}$	α_{Bi}: Probability when the test result is $x = 1$.	Generation fault [56]
Poisson	$f_{Po}(x) = \alpha_{Po}^{x}\exp(-\alpha_{Po})/x!$, $x = 0, 1, 2, \ldots, +\infty$	α_{Po}: Mean.	EV [29] Component fault [60] Windstorm [61] Ice storm [62]

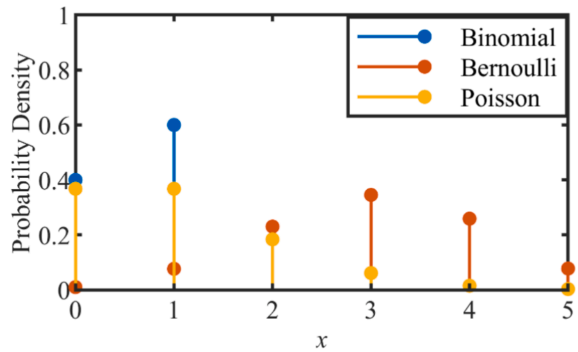

Fig. 3. Curves of typical discrete PDFs.

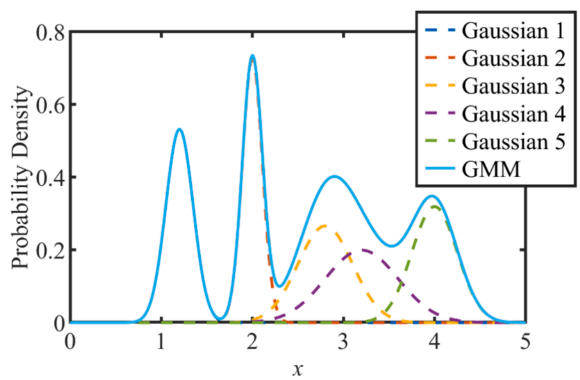

Fig. 4. Gaussian mixture model combined with 5 Gaussian functions.

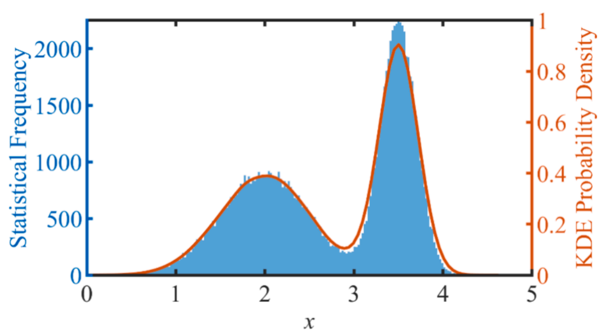

Fig. 5. KDE result of sampling data.

2.1.1.2. Mixed probabilistic density function. 1) Gaussian mixture model.

The above single PDF/CDF methods assume that the historical data follow a certain single distribution. If the probability distribution of uncertainties is complex, especially when it presents multi-peak characteristics, a single PDF cannot adequately reflect the characteristics of uncertainties. Gaussian mixture model (GMM) is not limited to the above assumption. And the PDF of complex uncertainties is characterized by the linear combination of several Gaussian PDFs. The PDF of GMM $f_{GMM}(x)$ can be expressed as follows [65]. GMM mixed by 5 Gaussian PDFs is drawn in Fig. 4 as an example.

$$f_{GMM}(x) = \sum_{n=1}^{N} \omega_n \exp\left(-(x-\alpha_{Gaun2})^2/(2\alpha_{Gaun1}^2)\right)/(\sqrt{2\pi}\alpha_{Gaun1}) \tag{2}$$

where N is the number of Gaussian PDFs; α_{Gaun1}, α_{Gaun2} are the standard deviation and the mean of n-th Gaussian PDF, respectively; ω_n is the weight of the n-th Gaussian PDF and meets the following conditions

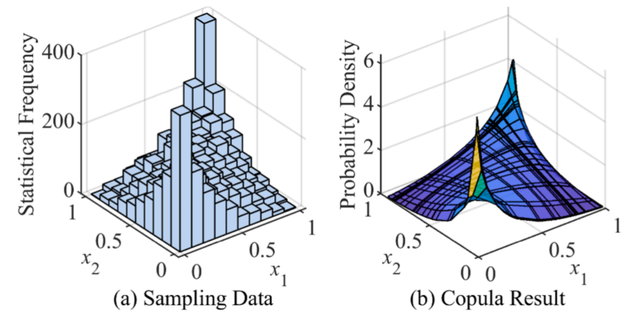

Fig. 6. Copula result of sampling data.

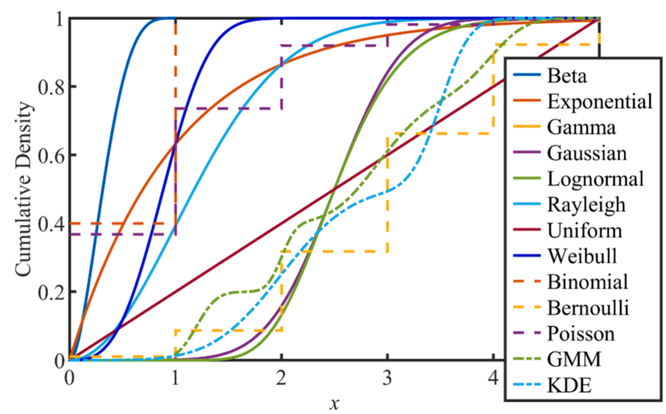

Fig. 7. CDF curves.

$$0 < \omega_n \leqslant 1, \ \sum_{n=1}^{N} \omega_n = 1. \tag{3}$$

2) Kernel density estimation method

GMM and single PDF/CDF methods assume that historical data follow a certain probability distribution, where the subjective priori knowledge is added. By comparison, kernel density estimation (KDE) needs no priori knowledge and is completely based on historical data. KDE function $f_{KDE}(x)$ can be expressed as follows.

$$f_{KDE}(x) = \sum_{m=1}^{M} g((x-x_m)/d)/(Md) \tag{4}$$

where M is the number of samples; d is the bandwidth; x_m is the m-th sample; $g(.)$ is the kernel function. According to the theory of KDE [66], when $M \to +\infty$, $h \to 0$, $Mh \to +\infty$, $f_{KDE}(x)$ will converge in probability to the PDF of actual historical data. An example of using KDE to describe a group of sampling data is shown in Fig. 5.

3) Copula method.

In all the above methods, PDF is adopted to model a single random variable. If there are several uncertainties with correlations, a joint PDF can be used. However, in power systems, the joint PDF is difficult to obtain directly, whereas the marginal PDFs are relatively easier to acquire. Copula method is usually utilized to obtain the equivalent joint PDF when the marginal PDFs $f_{xi}(x_i)$ are obtained. The Copula function $f_{Cop}(x_1, x_2, \dots, x_P)$ is shown as follows [67]

$$f_{Cop}(x_1, x_2, \cdots, x_P) = \frac{\partial^P \varphi(F_{x_1}^{-1}(x_1), F_{x_2}^{-1}(x_2), \cdots, F_{x_P}^{-1}(x_P))}{\partial F_{x_1}(x_1)\partial F_{x_2}(x_2)\cdots\partial F_{x_P}(x_P)} \prod_{i=1}^{P} f_{x_i}(x_i) \tag{5}$$

where $F_{xi}(x_i)$ is the integral of $f_{xi}(x_i)$; P is the number of random variables; $\varphi(.)$ is the generator function. An example of using the Copula method to describe sampling data is shown in Fig. 6.

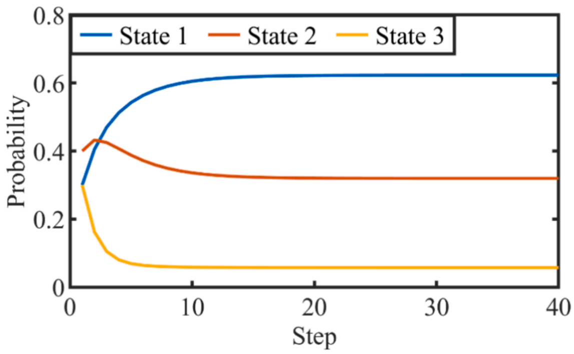

Fig. 8. Markov chain with 3 states.

2.1.1.3. Cumulative distribution function. CDF $F(x)$ is the integral of PDF, which can be expressed as [18]:

$$F(x) = \int_{-\infty}^{x} f(u)du. \tag{6}$$

Due to the simple transformation relationship illustrated in (6), the uncertainties modeled by PDFs can also be modeled by CDFs. The CDFs corresponding to the PDFs in Fig. 2-Fig. 5 are illustrated in Fig. 7.

2.1.2. Stochastic process methods

The PDF/CDF methods focus on modeling the steady-state characteristics of random variables at the specific time point. By comparison, time-varying characteristics cannot be reflected by them directly and cannot be ignored in some concerned issues, which can be modeled by stochastic process methods. In the uncertainty modeling of modern power systems, Markov chain and stochastic differential equation (SDE) are commonly adopted.

2.1.2.1. Markov chain. For a stochastic process $X(t)$ in any time $t_1 < t_2 < \cdots < t_n$ with possible state set $\boldsymbol{I}$, if it satisfies the following relationship, $X(t)$ is a Markov process and has Markov property.

$$\begin{aligned} P\{X(t_n)\leqslant x_n|X(t_1) = x_1,\ X(t_2) = x_2,\ \cdots,\ X(t_{n-1}) = x_{n-1}\} \\ = P\{X(t_n)\leqslant x_n|X(t_{n-1}) = x_{n-1}\} \end{aligned} \tag{7}$$

where $x_i \in \boldsymbol{I},\ i = 1,\ 2,\ \cdots,\ n-1$.

And the Markov process with discrete time and possible states is a Markov chain, denoted as $\{X_n = X(n),\ n = 0,\ 1,\ 2,\ \cdots\}$ with possible states $\boldsymbol{I} = \{I_1,\ I_2,\ \cdots\}$. Thus, for $\forall m,\ n$, there is:

$$\begin{aligned} & P\{X_{m+n} = I_j|X_0 = I_1,\ X_0 = I_2,\ \cdots,\ X_m = I_i\} \\ & = P\{X_{m+n} = I_j|X_m = I_i\}\doteq P_{ij}(m,\ m+n) \end{aligned} \tag{8}$$

Since a Markov chain will certainly transform into one of the states in $\boldsymbol{I}$ at time $m+n$ from the state I_i at time n, there is:

$$\sum_{j=1}^{+\infty} P_{ij}(m,\ m+n) = 1 \tag{9}$$

Additionally, the matrix composed of $P_{ij}(m,\ m+n)$ is defined as the n-th step state transition matrix $\boldsymbol{P}(m,\ m+n)$ [68]. And if $P_{ij}(m,\ m+n)$ is only determined by i, j, and n, denoted as $P_{ij}(n)$, the Markov chain is homogeneous or called stationary. Thus, for a stationary Markov chain, the state transition process can be formulated as follows. An example of the state transition of a Markov chain with 3 states is shown in Fig. 8.

$$\boldsymbol{p}(n) = \boldsymbol{p}(0)\boldsymbol{P}(n) = \boldsymbol{p}(0)\boldsymbol{P}^n(1) \tag{10}$$

where $\boldsymbol{p}(n)$ denotes the probability distribution of Markov chain states at n-th step.

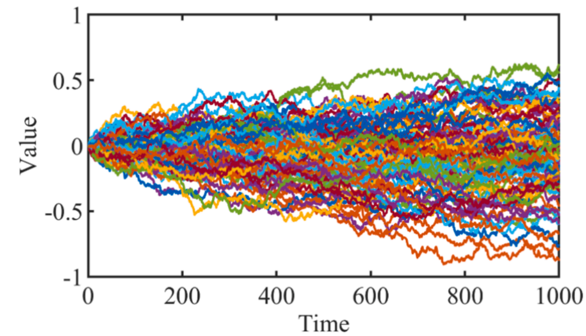

Fig. 9. Winner process.

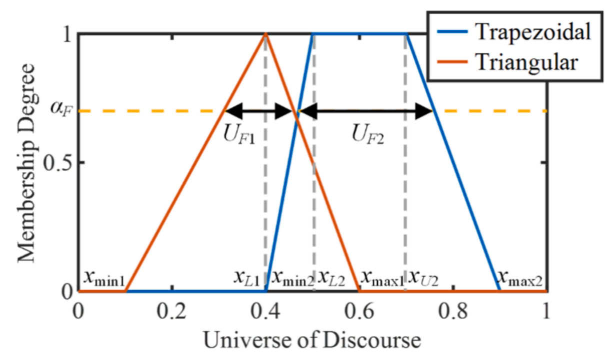

Fig. 10. Trapezoidal and triangular fuzzy sets.

2.1.2.2. Stochastic differential equation. As a typical control system, the dynamic response of the power system is normally modeled as differential equations. After the introduction of stochastic processes, the differential equations will transform into SDEs [69]. Ito process $x_S(t)$ is usually considered in power systems, which has the following form

$$dx_S(t) = \mu_S(x_S(t))dt + \sigma_S(x_S(t))dW(t) \tag{11}$$

where $\mu_S(x_S(t))$ and $\sigma_S(x_S(t))$ are the drift and diffusion terms, respectively, which can be derived from:

$$\begin{cases} \mu_S(x_S(t)) = \mathbb{E}[x_S(t)] \\ \sigma_S^2(x_S(t)) = \mathrm{Var}[x_S(t)] = \mathbb{E}\{[x_S(t) - \mu_S(x_S(t))]^2\} \end{cases} \tag{12}$$

where $\mathrm{Var}(\cdot)$ and $\mathbb{E}(\cdot)$ are variance and expectation operators, respectively.

$W(t)$ is commonly the Winner process, which has the following characteristics:

(1) A second-order moment independent increment process, which means that the second-order moment $\mathbb{E}[\ W^2(t)]$ of the Winner process $\{W(t),\ t\geqslant 0\}$ always exists when $t\geqslant 0$, and for $0\leqslant t_0 < t_1 < t_2 < \cdots < t_n$, $W(t_1) - W(t_0),\ W(t_2) - W(t_1),\ \cdots,\ W(t_n) - W(t_{n-1})$ are independent.

(2) $W(0) = 0$.

(3) For all $0 \le t_1 < t_2$, $W(t_2)$-$W(t_1) \sim N(0, t_2 - t_1)$, where $N(0, t_2 - t_1)$ denotes Gaussian distribution with the mean of 0 and the variance of $t_2 - t_1$.

An example group of Winner processes is shown in Fig. 9.

2.1.3. Set-based methods

2.1.3.1. Fuzzy set. In the aforementioned methods, there are parameters to be determined based on the historical data. However, sometimes no sufficient historical data are available. Also, as the operation conditions change, it may be ineffective to depict the changes with deterministic parameters. Thus, the fuzzy set methods are introduced for

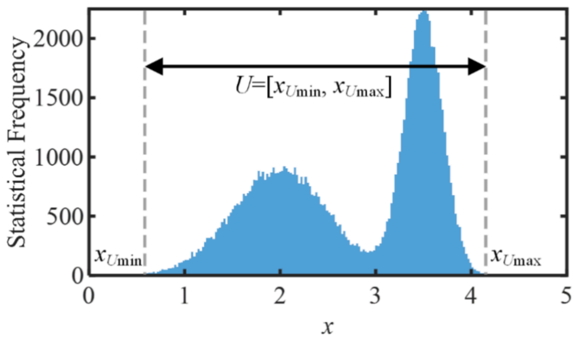

Fig. 11. Uncertainty set of sampling data.

modeling these epistemic uncertainties in the above scenarios [70]. In the fuzzy set methods, the universe of discourse $U_{FD} = (x_{\min}, x_L, x_U, x_{\max})$ is introduced to characterize the range of the uncertainty. The membership degree function $f_F(x)$ is applied to quantify the degree of x belonging to the uncertainty. And the α-cut U_F related to the membership degree level α_F is used to model the uncertainty, which can be formulated as [7]:

$$U_F = \{x \in U_{FD} | f_D(x) \geqslant \alpha_F,\ 0 \leqslant \alpha_F \leqslant 1\}. \tag{13}$$

Additionally, U_F can also be expressed by the lower limit $U_{F\min}$ and the upper limit of $U_{F\max}$:

$$U_F = [U_{F\min},\ U_{F\max}] \tag{14}$$

In power system uncertainty modeling, trapezoidal and triangular membership degree functions are typically applied, which are drawn in Fig. 10.

2.1.3.2. Uncertainty set. During the operation of power systems, sometimes operators may pay more attention to the boundary of uncertainties since they need to ensure that the adopted dispatching methods are effective at the range of possible values of uncertainties or in the worst scenarios, which usually correspond to interval optimization belonging to interval analysis and robust optimization, respectively. The interval optimization problem can be generally formulated as [71]:

$$\begin{aligned} &\min_{\boldsymbol{X}_I} g_I(\boldsymbol{X}_I,\ \boldsymbol{Z}_I) \\ &s.t. \begin{cases} h_I(\boldsymbol{X}_I,\ \boldsymbol{Z}_I) \geqslant b_I \\ \boldsymbol{X}_I \in \boldsymbol{U}_I \\ \boldsymbol{Z}_I \in \boldsymbol{Q}_I \end{cases} \end{aligned} \tag{15}$$

where $g_I(\cdot)$ and $h_I(\cdot)$ are the objective function and the constraints of the interval optimization problem; $\boldsymbol{X}_I$ and $\boldsymbol{U}_I$ denote the uncertainties and the boundary of uncertainties; $\boldsymbol{Z}_I$ and $\boldsymbol{Q}_I$ express the decision variables and the boundary of decision variables; b_I is the boundary of constraints.

By comparison, the robust optimization problem is generally formulated as [72]:

$$\min_{\boldsymbol{X}_R} \max_{\boldsymbol{Z}R} g_R(\boldsymbol{X}_R,\ \boldsymbol{Z}_R) s.t. \begin{cases} h_R(\boldsymbol{X}_R,\ \boldsymbol{Z}_R) \geqslant 0 \\ \boldsymbol{X}_R \in \boldsymbol{U}_R \\ \boldsymbol{Z}_R \in \boldsymbol{Q}_R \end{cases} \tag{16}$$

where $g_R(\cdot)$ and $h_R(\cdot)$ are the objective function and the constraints of the robust optimization problem; $\boldsymbol{X}_R$ and $\boldsymbol{U}_R$ denote the uncertainties and the boundary of uncertainties; $\boldsymbol{Z}_R$ and $\boldsymbol{Q}_R$ express the decision variables and the boundary of decision variables.

To solve the interval optimization problem formulated as (15), the interval possibility degree or the theory of direct interval matching is usually introduced to transform the interval optimization problem into deterministic optimization problems [71]. Also, to solve the robust optimization problem, (16) is usually transformed into the master problem and the subproblem [72]. All these solving procedures need the analytical expression of uncertainty boundaries. Thus, the uncertainty set methods are introduced to characterize the boundary of uncertainties [73]. For example, for the uncertainty shown in Fig. 5, it can also be modeled by the uncertainty set with the lower limit and the upper limit as presented in Fig. 11. And the typical uncertainty sets adopted in modern power systems for modeling single and multiple uncertainties are summarized in Table 3.

2.2. Data-driven methods

2.2.1. Sampling methods

2.2.1.1. Random sampling. For the uncertainty propagation analysis methods with data as input, more available data usually contribute to more accurate results. However, constrained by the calculation time and resources, only limited data are sampled and analyzed. Reflecting the probability distribution of historical data or PDFs as accurately as possible through limited sampling data is an issue to be tackled in sampling methods. Among them, the random sampling method is the most conventional and simple one. However, since the computer can only produce pseudo-random numbers, alternative methods are proposed to improve sampling efficiency.

2.2.1.2. Latin hypercube sampling. Latin hypercube sampling (LHS) method is a hierarchical sampling method that contains two steps, i.e., sampling and permutation. For K random variables $x_1, x_2, \ldots, x_k, \ldots, x_K$, and the CDF of x_k, i.e., $y_k = F_k(x_k)$, LHS method can be summarized as:

(1) Sampling: The longitudinal axis of the CDF curve is divided into H equally spaced intervals. The midpoint of each interval is chosen as the sampling value of y_k. And the inverse function of CDF is adopted to calculate the sampling value of h-th x_k [74].

$$x_{kn} = F_k^{-1}((h - 0.5)/H) \tag{17}$$

where H is the number of samples.

(2) Permutation: After forming the $K \times H$-order initial sampling matrix, each row of the initial sampling matrix is reordered, where the

Table 3
Comparison of uncertainty sets.

Type of set	Form of set	Parameter description	Advantages	Drawbacks
Box set	$U_B = \{\boldsymbol{x} : \boldsymbol{x}_{Blow} \leqslant \boldsymbol{x} \leqslant \boldsymbol{x}_{Bupp}\}$	$\boldsymbol{x}$: Random variables; $\boldsymbol{x}_{Blow}$: Lower bound of $\boldsymbol{x}$; $\boldsymbol{x}_{Bupp}$: Upper bound of $\boldsymbol{x}$.	• Low complexity • Straightforward to model	• Unable to describe correlations of multiple random variables • Not precise
Budget set	$U_G = \{\boldsymbol{x} : x_{Glowi} \leqslant x_i \leqslant x_{Guppi},\ \sum_{x_i \in \Omega} \left\lvert \frac{x_i - \mu_{Gi}}{x_{Guppi} - x_{Glowi}} \right\rvert \leqslant \Gamma_G\}$	x_{Glowi}: Lower bound of x_i; x_{Guppi}: Upper bound of x_i; μ_{Gi}: Mean of x_i; Γ_G: Budget; Ω: Set of x_i.	• Weakening conservation of box sets • More accurate than box sets	• Unable to describe correlations of multiple random variables
Ellipsoidal set	$U_E = \{\boldsymbol{x} : (\boldsymbol{x} - \boldsymbol{\mu}_E)^T \boldsymbol{\rho}_E^{-1} (\boldsymbol{x} - \boldsymbol{\mu}_E) \leqslant \gamma_E\}$	$\boldsymbol{\mu}_E$: Mean of $\boldsymbol{x}$; $\boldsymbol{\rho}_E$: Covariance matrix of $\boldsymbol{x}$; γ_E: Radius of set.	• Considering correlations of multiple random variables	• High complexity
Data-driven set	$U_D = \{\boldsymbol{x} : Pr(\boldsymbol{x}_{Dlow} \leqslant \boldsymbol{x} \leqslant \boldsymbol{x}_{Dupp}) \leqslant 1 - \beta_D\}$	$\boldsymbol{x}_{Dlow}$: Lower bound of $\boldsymbol{x}$; $\boldsymbol{x}_{Dupp}$: Upper bound of $\boldsymbol{x}$; β_D: Confidence interval.	• Narrowing ranges of sets • Weakening conservation of sets	• Requiring additional data

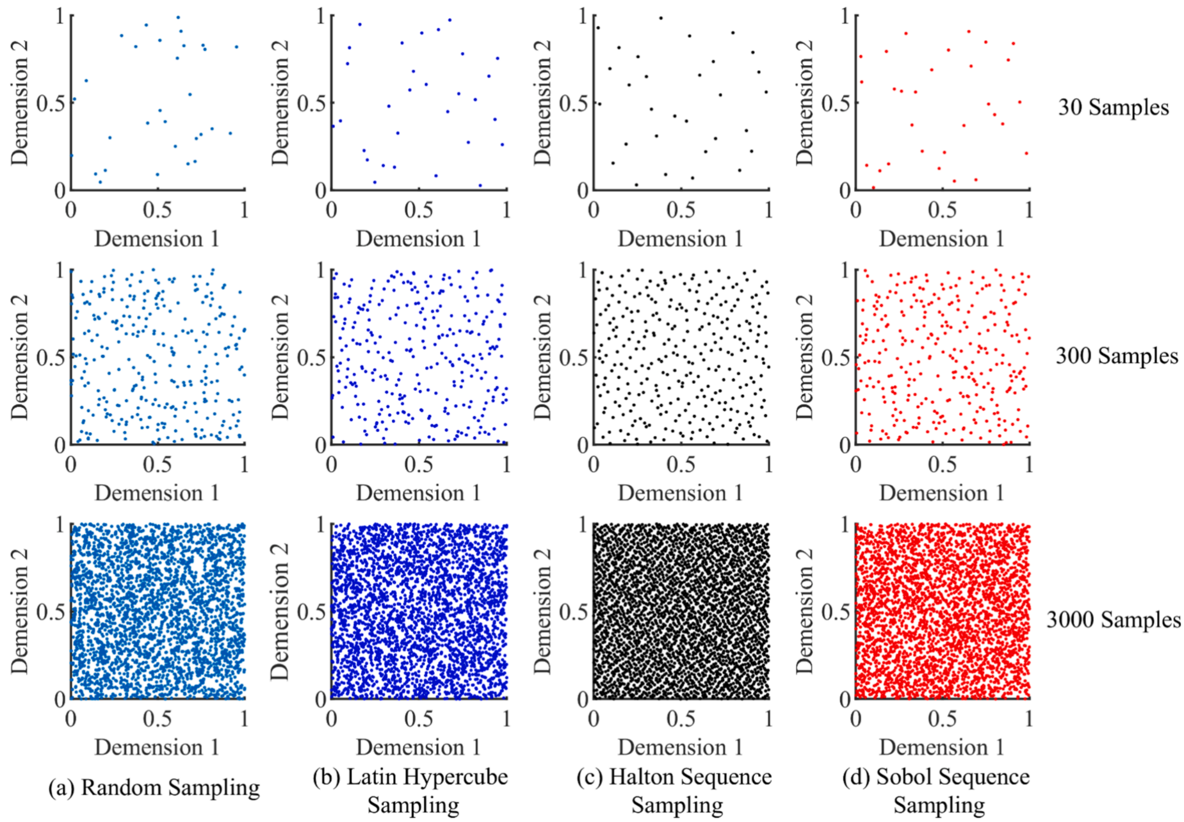

Fig. 12. Sampling performance of sampling methods.

sequence orthogonalization method can be used to minimize the correlation of each row of the sampling matrix.

In [34], the authors have pointed out that for two independent random variables, the mean of the coverage space percentage of random sampling is $[(H-1)/(H+1)]^2$, whereas LHS is $[(H-1)/H]^2$, which illustrates that LHS has a larger sampling space under the same number of samples. Also, some improved LHS methods are proposed, such as discrete LHS [55] and LHS combined with Cholesky decomposition [34], which improve the performance of LHS from the perspective of reducing calculation time, data storage space, and the correlation between variables.

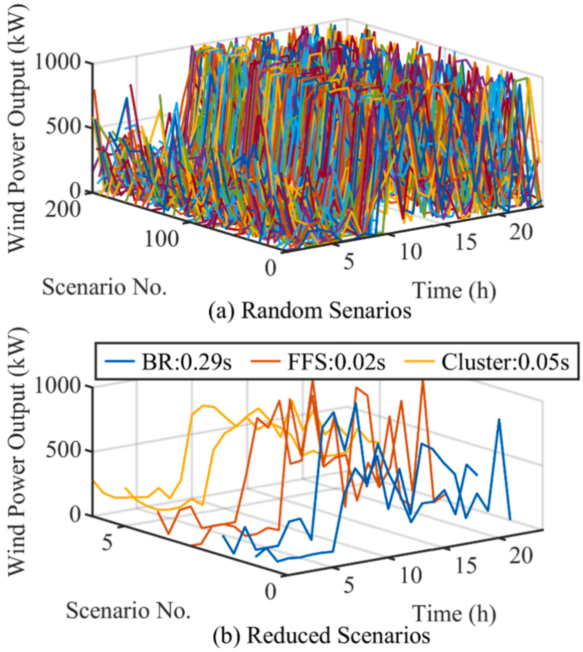

Fig. 13. Comparison of different scenario reduction methods.

2.2.1.3. Low discrepancy sequence sampling. Another method for improving the sampling performance is to sample data according to the sequences with a high degree of randomness, named as low discrepancy sequences. The core of generating the low discrepancy sequence is that if the low discrepancy sequence has already had numbers $q-1$, the q-th number will be inserted into the largest interval of the existing sequence so as to avoid the aggregation of the number series in local space. Halton sequence [75] and Sobol sequence [76] are two common sequences. Sobol sequence reorders Halton sequence by recursively generating direction numbers, which performs better in the space coverage of high-dimensional variables. The sampling performance of random sampling, LHS, Halton sequence sampling, and Sobol sequence sampling in 2D space is demonstrated in Fig. 12.

2.2.2. Scenario reduction methods

Scenario reduction methods select or generate limited representative scenarios to reflect all scenarios formed by historical time series. If the initial scenario is S, the distance of every scenario s_i and s_j is $l(s_i, s_j)$, the scenario set deleted by the scenario reduction method is D, the reserved set is R, the objective function of the scenario reduction method is [77]

$$\min \sum_{s_i \in D} p_i \min \sum_{s_j \in R} l(s_i,\ s_j) \tag{18}$$

where p_i is the weighted probability of scenario s_i. When the distance between the deleted set D and the reserved set R is minimized by scenario reduction, the reserved set R can reflect the original scenarios S as typical scenarios. The typical scenario reduction methods used in

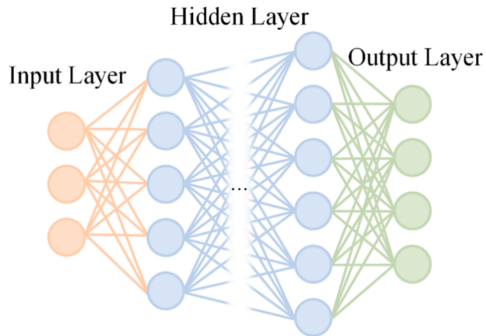

Fig. 14. Structure of fully-connected neural network.

modern power systems are the backward reduction (BR) method, the fast forward selection (FFS) method, and the cluster method.

2.2.2.1. Backward reduction. The procedure of the BR method can be summarized as: (1) Set the initial scenario set S as the reserved set R; (2) Calculate the sum of distances between all scenarios in the reserved set R and other scenarios; (3) Find the scenario s_i with the smallest sum of distances; (4) Eliminate it into the deleted set D; (5) Find the scenario s_j closest to the chosen scenario s_i in the reserved set R; (6) Add the probability p_i of scenario s_i to the probability of scenario s_j to ensure that the total probability in the reserved set R is 1. Repeat the deletion and probability superposition steps until the number of scenarios in the reserved set R meets the requirements [78].

2.2.2.2. Fast forward selection. The FFS method is essentially a simplified inverse algorithm of the BR method. The core of FFS is to treat the initial scenarios S as the deleted set D and select reserved set R from the deleted set. Generally, the number of scenarios in reserved set R is much less than that in deleted set D. Thus, the iteration number of FFS is much less than that of BR, but the performance will decline [78].

2.2.2.3. Cluster. The above two scenario selection reduction methods select the existing scenarios as the typical scenarios. However, sometimes, when the existing scenarios are not typical enough, cluster methods can be adopted to generate the typical scenarios. The core of the cluster method is to calculate the distance among all scenarios and classify them with small distances into one cluster [78]. And the centers of clusters are regarded as the typical scenarios.

For an example group of randomly generated output power of wind generation, scenario reduction methods are carried out and presented in Fig. 13, where the FFS method takes the least time, whereas the BR method takes the longest time.

2.2.3. Neural network methods

The uncertainties in power systems are sometimes affected by multiple factors, and there is an interactive relationship among these factors, which is difficult to explicitly characterize. Since neural networks can express complex nonlinear relationships, they have been widely applied in the uncertainty modeling of modern power systems in recent years. The conventional fully-connected neural network structure is given in Fig. 14.

A fully-connected neural network with one hidden layer can be expressed as:

$$\hat{\boldsymbol{y}} = a_{Out}(\boldsymbol{W}_{Out}a_{Hid}(\boldsymbol{W}_{Hid}\boldsymbol{x} + \boldsymbol{b}_{Hid}) + \boldsymbol{b}_{Out}) \tag{19}$$

where $\boldsymbol{x}$ is the input variable; $\hat{\boldsymbol{y}}$ is the output variable; $\boldsymbol{W}_{Hid}$ and $\boldsymbol{W}_{out}$ are the weights of the hidden layer and the output layer, respectively; $\boldsymbol{b}_{Hid}$ and $\boldsymbol{b}_{Out}$ denote the bias of the hidden layer and the output layer, separately; a_{Hid} and a_{Out} express the activation functions of the hidden layer and the output layer, respectively. Apart from the fully-connected neural network, the typical neural network methods used in modern power systems include convolution neural network (CNN), recurrent neural network (RNN), Bayesian neural network (BNN), and generative adversarial network (GAN) due to the advantages of graphic feature extraction, feature extraction of time series, uncertainty handling, and data generation, respectively. These different neural network methods are basic neural network frameworks, which can be individually used or in combination.

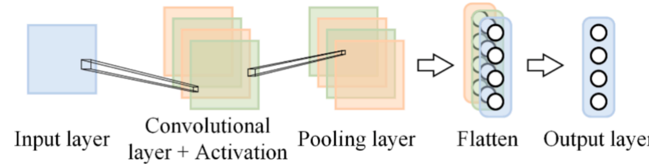

Fig. 15. Structure of CNN.

2.2.3.1. Convolution neural network. One of the most essential properties of CNN is that a neuron is only connected with part of adjacent neurons. By weight sharing, the complexity of network calculation is reduced. The characteristics of input variables are learned and extracted in parallel through convolution and pooling [79], which is suitable for image processing. A typical CNN structure is presented in Fig. 15.

2.2.3.2. Recurrent neural network. RNN is designed for time series, the neurons of which have both feedforward and feedback connections. In RNN, the hidden state $\boldsymbol{h}_t$ at time t is determined by both the input $\boldsymbol{x}_t$ at time t and the hidden state $\boldsymbol{h}_{t-1}$ at time $t-1$, which can be formulated as:

$$\boldsymbol{h}_t = \tanh(\boldsymbol{W}_{RNN}[\boldsymbol{h}_{t-1}, \ \boldsymbol{x}_t] + \boldsymbol{b}_{RNN}) \tag{20}$$

where $\boldsymbol{W}_{RNN}$ and $\boldsymbol{b}_{RNN}$ denote the weight and the bias of RNN, respectively.

However, when the network structure is deep, there will be vanishing or exploding gradient. To overcome the above problems, long short-term memory (LSTM) neural network adds the input gate, forget gate, and output gate on the basis of RNN. Specifically, the input gate $\boldsymbol{i}_t$ of LSTM can be expressed as:

$$\begin{cases} \boldsymbol{i}_t = \sigma(\boldsymbol{W}_i[\boldsymbol{h}_{t-1}, \ \boldsymbol{x}_t] + \boldsymbol{b}_i) \\ \widetilde{\boldsymbol{C}}_t = \tanh(\boldsymbol{W}_c[\boldsymbol{h}_{t-1}, \ \boldsymbol{x}_t] + \boldsymbol{b}_c) \end{cases} \tag{21}$$

where $\widetilde{\boldsymbol{C}}_t$ is the current memory unit; $\boldsymbol{W}_i$ and $\boldsymbol{W}_c$ are the weights of the input gate and the memory unit, respectively; $\boldsymbol{b}_i$ and $\boldsymbol{b}_c$ are the biases of the input gate and the memory unit, respectively.

The forget gate $\boldsymbol{f}_t$ of LSTM can be expressed as:

$$\boldsymbol{f}_t = \sigma(\boldsymbol{W}_f[\boldsymbol{h}_{t-1}, \ \boldsymbol{x}_t] + \boldsymbol{b}_f) \tag{22}$$

where $\boldsymbol{W}_f$ and $\boldsymbol{b}_f$ denote the weight and the bias of the forget gate, respectively.

The output gate $\boldsymbol{o}_t$ of LSTM can be expressed as:

$$\begin{cases} \boldsymbol{o}_t = \sigma(\boldsymbol{W}_o[\boldsymbol{h}_{t-1}, \ \boldsymbol{x}_t] + \boldsymbol{b}_o) \\ \boldsymbol{h}_t = \boldsymbol{o}_t{}^*\tanh(\boldsymbol{C}_t) \end{cases} \tag{23}$$

where $\boldsymbol{W}_o$ and $\boldsymbol{b}_o$ denote the weight and the bias of the output gate, respectively.

Gate recurrent unit (GRU) neural network simplifies LSTM and only introduces reset gate $\boldsymbol{z}_t$ and the update gate $\boldsymbol{r}_t$, reducing network complexity [80], which can be formulated as:

$$\begin{cases} \boldsymbol{z}_t = \sigma(\boldsymbol{W}_z[\boldsymbol{h}_{t-1}, \ \boldsymbol{x}_t] + \boldsymbol{b}_z) \\ \boldsymbol{r}_t = \sigma(\boldsymbol{W}_r[\boldsymbol{h}_{t-1}, \ \boldsymbol{x}_t] + \boldsymbol{b}_r) \\ \widetilde{\boldsymbol{h}}_t = \tanh(\boldsymbol{W}_h[\boldsymbol{r}_t{}^*\boldsymbol{h}_{t-1}, \ \boldsymbol{x}_t] + \boldsymbol{b}_h) \\ \boldsymbol{h}_t = (1 - \boldsymbol{z}_t){}^*\boldsymbol{h}_{t-1} + \boldsymbol{z}_t{}^*\widetilde{\boldsymbol{h}}_t \end{cases} \tag{24}$$

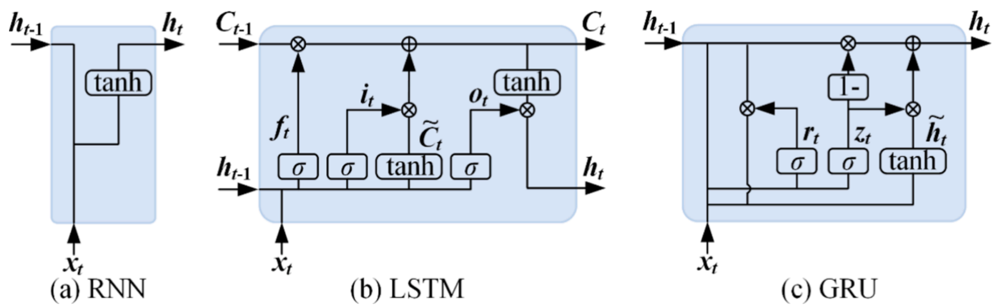

Fig. 16. Structure comparison of RNN, LSTM, and GRU.

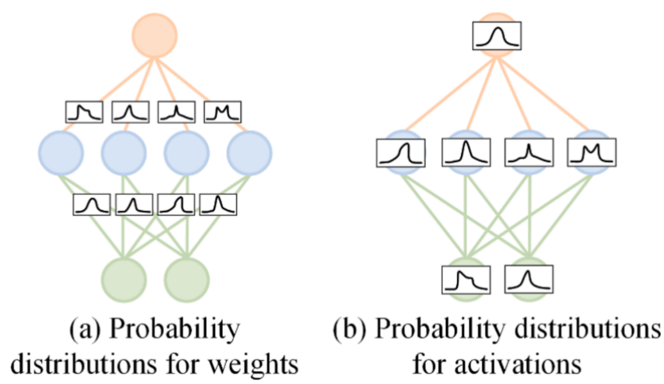

Fig. 17. Typical BNN structures [82].

where $\boldsymbol{W}_z$, $\boldsymbol{W}_r$, and $\boldsymbol{W}_h$ denote the weights of the reset gate, the update gate, and the hidden state, respectively; $\boldsymbol{b}_z$, $\boldsymbol{b}_r$, and $\boldsymbol{b}_h$ denote the biases of the reset gate, the update gate, and the hidden state, respectively.

The structure comparison of RNN, LSTM, and GRU is illustrated in Fig. 16.

2.2.3.3. Bayesian neural network. One of the major differences between BNN and other neural network methods is that the parameters of BNN are not fixed but follow certain probability distributions, i.e., epistemic uncertainties. Thus, the calculation of BNN parameters θ is actually to determine the parameters of their probability distributions. Moreover, the measurement errors of data are considered in BNN. Finally, every output of BNN also follows a certain probability distribution. Two typical BNNs are presented in Fig. 17. To calculate θ, the following loss function $\mathrm{L}_{BNN}(\theta)$ is usually used to train BNNs in practice [81]:

$$\mathrm{L}_{BNN}(\theta) = \frac{1}{N_{BNN}}\sum_{i=1}^{N_{BNN}}\frac{1}{2\sigma_i^2}\sqrt{\left(y_{BNNi}-\widehat{y}_{BNNi}\right)^2}+\frac{1}{2}\log\sigma_i^2 \tag{25}$$

where N_{BNN} is the data quantity of dataset D for training BNN; $y_{BNN.}$ and $\widehat{y}_{BNN.}$ are measured data and outputs of BNN, respectively; $\sigma_.$ denotes the measurement errors of data.

Also, the variance of the output of BNN can be estimated as [81]:

$$\mathrm{Var}(\widehat{y}|x,\ \mathrm{D}) = \mathbb{E}_{\theta|\mathrm{D}}[\mathrm{Var}(\widehat{y}|x,\ \theta)] + \mathrm{Var}_{\theta|\mathrm{D}}[\mathbb{E}(\widehat{y}|x,\ \theta)] \tag{26}$$

2.2.3.4. Generative adversarial network. Different from the above neural network methods, GAN is composed of two neural networks, i.e., a discriminator and a generator. The discriminator is used to assess whether the input data is historical data or generated data. And the generator is used to generate data with the same distribution as the historical data [61]. Since there are two networks that need to be trained simultaneously, the training of GAN is more difficult to converge than other neural network methods. A typical GAN structure is presented in Fig. 18.

According to the summary in Section 2, the uncertainty modeling methods are illustrated in Fig. 19. The characteristics of uncertainty modeling methods are summarized in Table 4. The matching matrix of uncertainty modeling and propagation analysis methods are presented in Table 5.

It should be noted that, as discussed above, the uncertainty modeling methods are categorized according to the outputs of the modeling methods. However, it can be noticed in Table 5 that the outputs of some modeling methods are not consistent with the inputs of the propagation analysis methods. For example, the outputs of PDF/CDF modeling methods are models, while the inputs of numerical propagation analysis methods are data. The reason is that data-driven sampling methods are employed between these two methods to convert the models into data, which means there are two steps during uncertainty modeling. Firstly, PDF/CDF modeling method is used to obtain probability distributions of uncertainties. Then, the sampling method is utilized to attain limited sampling data.

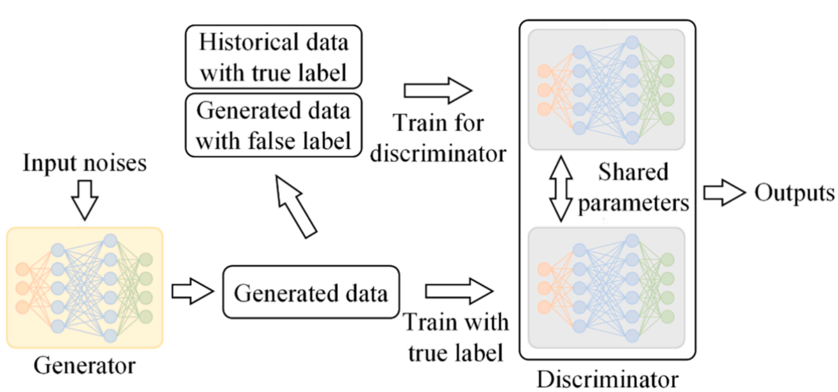

Fig. 18. Structure of GAN.

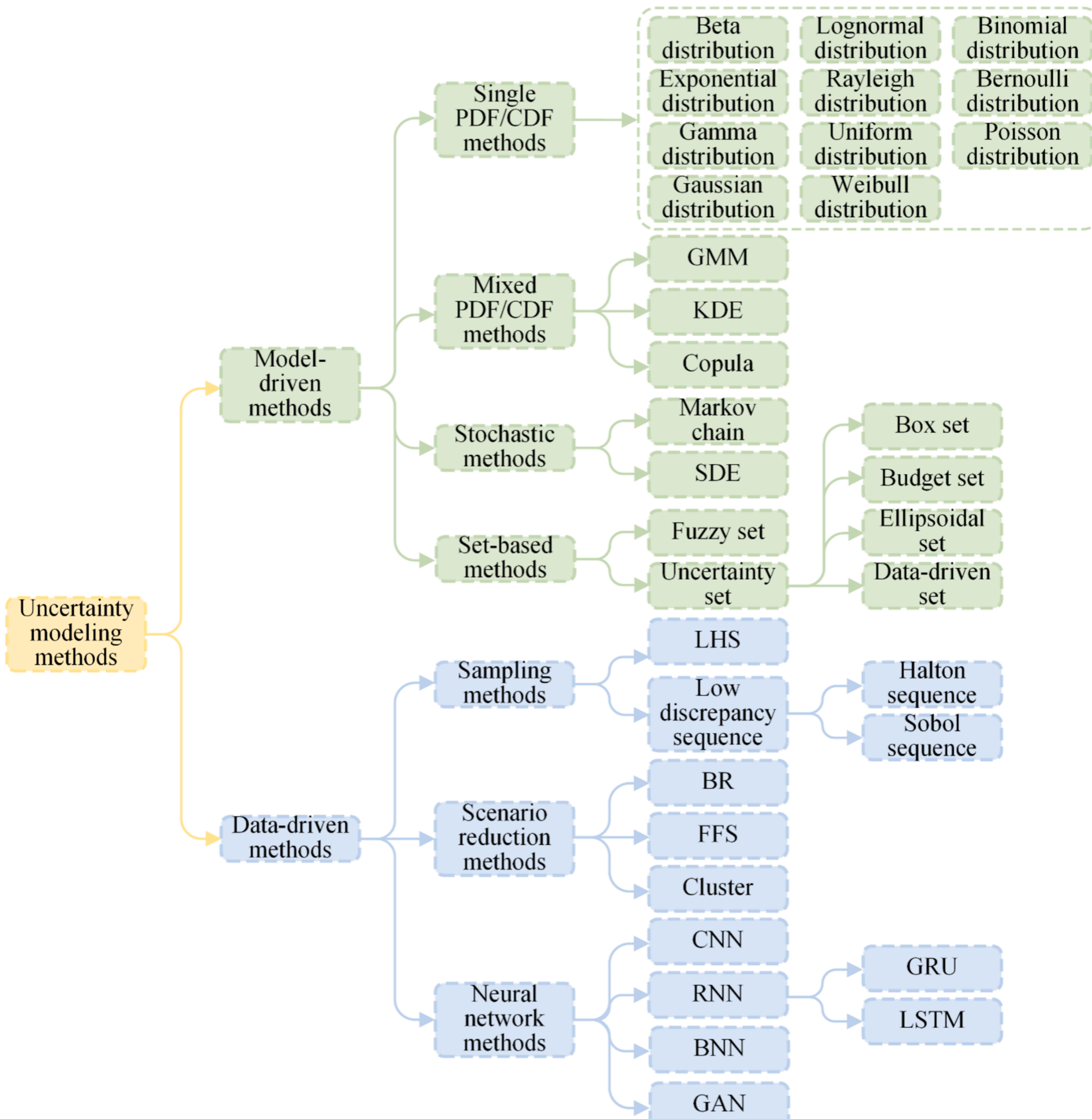

Fig. 19. Summary of uncertainty modeling methods.

Table 4
Characteristic summary of uncertainty modeling methods.

Uncertainty modeling methods		Advantages	Drawbacks
Model-driven methods	**PDF/CDF methods**	• Extensive optional types of PDFs/CDFs • Modeling Simply	• Relying on historical data • Unable to reflect temporal fluctuations of uncertainties
	Stochastic process methods	• Reflecting steady-state randomness of uncertainties • Reflecting temporal fluctuations of uncertainties	• Relying on historical data
	Set-based methods	• Getting rid of dependence on historical data • Reducing calculation amount	• Relying on experience • Insufficiently fine boundary • Unable to reflect temporal fluctuations of uncertainties
Data-driven methods	**Sampling methods**	• Reflecting steady-state randomness of uncertainties through a small amount of data • Reduce computational cost of propagation analysis	• Unable to reflect temporal fluctuations of uncertainties
	Scenario reduction methods	• Reflecting temporal fluctuations of uncertainties • Reducing computational cost of propagation analysis	• Relying on experience
	Neural network methods	• Describing characteristics of uncertainties induced by multiple factors • Reflecting steady-state randomness of uncertainties • Reflecting temporal fluctuations of uncertainties	• Relying on a large amount of historical data • Time-consuming in training neural networks

Table 5
Matching matrix of modeling methods and propagation analysis methods.

Uncertainty modeling methods		Probabilistic methods			Possibilistic methods	Hybrid methods		Interval analysis	Uncertainty optimization		
		Numerical methods	Analytical methods	Approximation methods		Fuzzy-MCS	Fuzzy-scenario-based method		Stochastic optimization	IGDT	Robust optimization
Model-driven methods	PDF/CDF methods	Single PDF/CDF [20,22–24,29,31,36,37,40,43,45,47,48,50,56,83–85] Mixed PDF/CDF [86–89]	Single PDF/CDF [19,25,26,38,41,49,52,53,57,90–94] Mixed PDF/CDF [38,95]	Single PDF/CDF [27,28,33,35,39] Mixed PDF/CDF [96–99]				Single PDF/CDF [100]	Single PDF/CDF [20,21,41,45,46,84,101–104] Mixed PDF/CDF [86,88,97]		Single PDF/CDF [105] Mixed PDF/CDF [106]
	Stochastic process methods	SDE [16,69,107–110]							SDE [108,111]		
	Set-based methods		Uncertainty set [94]		Fuzzy set [112–115]	Fuzzy set [85,116]	Fuzzy set [48]	Uncertainty set [100,117–119]	Fuzzy set [120]	Uncertainty set [121–124]	Uncertainty set [21,105,106,125–139]
Data-driven methods	Sampling methods	Random sampling [36,83,87,140] LHS sampling [88] Low discrepancy sequence sampling [29,86]		LHS sampling [28]					LHS sampling [88,141,142] Low discrepancy sequence sampling [86]		
	Scenario reduction methods	BR [84] FFS [84] Cluster [20]							BR [51,78,84] FFS [78,84] Cluster [20,78,142,143]		
	Neural network methods								CNN [51,79] GAN [51,79,144]		

3. Uncertainties in modern power systems

There are many uncertainty classifications, such as (1) aleatory uncertainty and epistemic uncertainty [17,145], (2) technical uncertainty and economic uncertainty [6,12,14], (3) operational uncertainty and disturbance uncertainty [13], (4) input (or called external) uncertainty and system (or called internal) uncertainty [10,146], (5) decision-dependent uncertainty and decision-independent uncertainty [147], (6) parameter uncertainty, operational state uncertainty, measurement uncertainty, prediction uncertainty, and price uncertainty [146], (7) subjective uncertainty and objective uncertainty [146], and (8) long-term uncertainty, medium-term uncertainty, short-term uncertainty, and real-time uncertainty [146]. Thus, following any one of these classifications cannot comprehensively summarize the uncertainties in power systems. Moreover, though the existing uncertainty classifications are different, all these studies about uncertainty classifications point out the fundamental physical origins of uncertainties. From the perspective of the structure of modern power systems, the uncertainties can be classified into physical and cyber layer uncertainty based on the position of uncertainties in power systems. For the physical layer, power systems can be split into generation, consumption, and network. Also, the changes of economic and social factors in electricity markets will bring uncertainties to power systems. Therefore, this section summarizes the uncertainties in the physical layer, the cyber layer, and the economic and social layer of modern power systems. Additionally, it should be noted that only the uncertainties in power systems that can be regarded as independent variables in the power system operation analysis are considered in this paper and are modeled with uncertainty modeling methods. For the uncertainties regarded as dependent variables, e.g., decision-dependent uncertainties, they are the results to be determined by using uncertainty propagation analysis, are not modeled with uncertainty modeling methods, and are not included. For example, for the probabilistic power flow analysis considering wind power generation, both the wind speed and the operation states of power systems, e.g., bus voltage, are random. However, the randomness of wind speed leads to the randomness of bus voltage. Thus, wind speed is regarded as the independent variable, and bus voltage is regarded as the dependent variable in this issue. And only the wind speed is regarded as the uncertainty and modeled by the uncertainty modeling method, e.g., PDF method, rather than both the wind speed and the bus voltage.

3.1. Physical layer uncertainty

3.1.1. Generation uncertainty

3.1.1.1. Conventional generation uncertainty. In power systems, operation states and outputs of generators may change frequently. For the uncertainties of operation states, the forced outage ratio (FOR) is usually obtained by analyzing historical data firstly, and the current state probability of generators is expressed by up state and outage state modeled by binomial distribution or Bernoulli distribution [34]. In addition, some more refined multi-state models are also adopted [56]. In the sequential analysis of generator operation states, mean time to fault (MTTF) and mean time to recovery (MTTR) are introduced and combined with Markov chain [148]. For output uncertainties, in the actual situation, generation outputs may be constrained rather than change continuously. The possibility of output is commonly described by sampling the historical data [140]. In addition, generators may be affected by small disturbances during operation, which will affect the output and dynamic performance. These disturbances are usually modeled as SDE [109,149]. Also, the measurement errors of generation operation data are usually considered as Gaussian PDFs [150]. Additionally, some parameters of generators are unknown due to the limited measurement conditions, which are treated as epistemic uncertainties and may affect the power system operation, where PDF methods are used to model these uncertainties [151].

3.1.1.2. Wind power uncertainty. With the development of renewable power generation technologies, the penetration of renewable energy sources in power systems continues to increase. Wind power generation and PV have become the main uncertainties on the generation side due to their wide application and intermittency and randomness affected by weather conditions.

The impact of wind power uncertainties on power systems is reflected in the steady-state output and dynamic performance. For steady-state outputs, the relationship between output power and wind speed can be established by using piecewise functions [15] according to maximum power point tracking (MPPT). And the probability distribution of the wind power generation output can be derived based on the probability distribution of wind speed and the relationship between output power and wind speed [23,24,29,30,33,37–39,50,52–54,83,85,87,89,90,92,94,98,105,152,153], which is usually expressed by using the following equations [15]:

$$P_w(V_w) = \frac{1}{2}\rho_w A_w C_p V_w^3 \tag{27}$$

$$P_w(V_w) = \begin{cases} 0 & \text{, for } V_w < V_i \text{ and } V_w > V_o \\ P_{wr}(V_w^k - V_i^k)/(V_r^k - V_i^k) & \text{, for } V_i \leqslant V_w \leqslant V_r \\ P_{wr} & \text{, for } V_r < V_w \leqslant V_o \end{cases} , k = 1, 2, 3 \tag{28}$$

where V_w is the wind speed; P_w denotes the output power of wind power generation; ρ_w is the air density; A_w expresses the swept area of the rotor; C_p is wind energy utilization coefficient; P_{wr} denotes the rated output power; V_i, V_r, and V_o are the cut-in, rated, and cut-out wind speeds of wind power generation, respectively.

In terms of timescale, the probability distribution of short-term wind speed mostly follows the Beta distribution, whereas the medium and long-term distribution is dominated by the Weibull distribution [83]. However, there are many factors that may affect wind speed, including meteorological and geographical conditions, such as the distance between the wind farm and the coast [154]. Also, the wind speeds of wind turbines close to each other are correlated, and the wake effect in wind farms will also affect the outputs of wind power generations [155]. To accurately depict the uncertainties of wind power generation outputs, two common types of methods have been developed. One is to take the relevant factors into account and assess the randomness of wind speed, thereby describing output uncertainties of wind power generations, where copula methods [95,97] and neural network methods [156] are generally applied. The other is to directly model the uncertainties of wind power generation outputs, where scenario reduction methods [51,78,141,142,144,157,158] and set-based methods [121,124,126,127,130–135,138,159] are commonly employed. For dynamic performance, similar to conventional power generation, the output also fluctuates when considering time-varying small disturbances. Stochastic processes are usually adopted to describe these uncertainties [16,110,160]. Also, BNN methods and PDF methods are usually adopted to characterize measurement errors and epistemic features of wind power generation uncertainties [161,162].

3.1.1.3. Photovoltaic uncertainty. Similar to wind power generation, PV outputs are also affected by weather conditions, and the uncertainty modeling procedure is similar. Single PDF/CDF methods are usually applied to describe the probability distribution of solar irradiance [19–21,23,37,39,85,87,98,104,152], and then the uncertainty of PV output is described based on the relationship between solar irradiance and the output, which is typically formulated as [15]:

$$P_{PV}(G_{PV}) = \zeta_{PV} A_{PV} G_{PV} \tag{29}$$

$$P_{PV}(G_{PV}) = \begin{cases} P_{PVr} G_{PV}/G_{std} & \text{, for } 0 < G_{PV} \leqslant G_{std} \\ P_{PVr} & \text{, for } G_{PV} \geqslant G_{std} \end{cases} \tag{30}$$

$$P_{PV}(G_{PV}) = \begin{cases} P_{PVr} G_{PV}^2/(G_{std} X_c) & \text{, for } 0 < G_{PV} \leqslant X_c \\ P_{PVr} G_{PV}/G_{std} & \text{, for } G_{PV} \geqslant X_c \end{cases} \tag{31}$$

where G_{PV} is the solar irradiance; P_{PV} denotes the output power of PV; ζ_{PV} expresses the generation efficiency; A_{PV} is the area of the PV module; P_{PVr} denotes the rated output power of PV; G_{std} and X_c are the solar irradiance under standard environment conditions and a specific irradiance point, respectively.

For timescale, the probability distribution of short-term solar irradiance mostly follows the Beta distribution, whereas the long-term distribution is dominated by the Gaussian distribution [163]. In addition, PV outputs are also related to geographical location, cloud coverage, and other factors. Mixed PDF/CDF methods are often utilized when considering the correlation among multiple factors [164]. Also, neural network methods [51,144,161] and set-based methods [124,125,127,131,132,137,138,165] are usually used when directly describing uncertainties of PV outputs, including measurement errors and epistemic features.

3.1.2. Consumption uncertainty

3.1.2.1. Conventional load uncertainty. Conventional loads are highly correlated with time and also fluctuate with the environment, weather, electricity price, user behavior, etc. It is challenging to use explicit functions to describe the relationship between loads and related factors. Hence, modeling methods are usually applied to directly characterize the uncertainty of conventional loads, and their measurement errors and epistemic features can be considered by using BNN [161]. When historical data are lacking, fuzzy set methods are usually applied [48,120]. By comparison, single PDF/CDF or mixed PDF/CDF methods [22–25,27–29,31–33,35,37–42,46,85,87,89,93,94,97–102,104,166] are adopted to describe the probability distributions of loads when historical data are available. When the temporal fluctuation of loads needs to be considered, sampling methods and scenario reduction methods are often applied [51,59,88,141–143,157,158,167–169]. In addition, the fluctuation of different timescale loads and different load types are various. Some studies focus on uncertainties of specific loads, i.e., short-term [170,171] and long-term loads [172], residential [171] and industrial loads [170].

3.1.2.2. Electric vehicle uncertainty. In recent years, with the increasing popularity of EV, it has become one of the main uncertainties of loads. The impact of uncertainties of EVs on modern power systems is mainly reflected in the randomness of the charging state, which is related to the current battery state of EVs, user behaviors, and other factors. Single PDF/CDF or mixed PDF/CDF methods are commonly utilized to characterize the uncertainties of EVs from three aspects, i.e., daily arrival time [43], initial battery state of charge (SoC) [43], and traveling distance [19,29,43,97]. The daily arrival time can be utilized to calculate the start charging time of EVs, and the consumed power, i.e., the power needed to be charged, can be calculated based on the initial battery SoC and traveling distance. When charging fluctuations need to be considered, sampling methods and scenario reduction methods are often adopted [152]. When the research focuses on the uncertainty of the charging power itself rather than relevant factors, the randomness of charging power can also be directly modeled [38,85].

3.1.3. Network uncertainty

Network uncertainties include the parameter and topology changes. Single PDF/CDF methods are usually applied to characterize parameter changes [23,42,44]. Other network uncertainties are generally introduced by topology changes, which are mainly caused by equipment faults in networks. Similar to describing the fault uncertainty of power generation equipment, the faults of lines and transformers in networks are typically modeled by using single PDF/CDF methods [60,94] or SDE methods [69]. Among the causes of faults, disaster is one of the factors that cannot be overlooked. Disasters concerned in power systems include windstorms [47,173], earthquakes [58], ice storms [58,62], etc. Single PDF/CDF methods are usually adopted to model the occurrence probability of disasters, and then the relationship between disaster occurrence and fault occurrence is characterized [58,62]. Some more detailed models consider uncertainties in the intensity [47,173] and movement [47,173] of disasters.

3.2. Cyber layer uncertainty

With the wide application of communication technologies in modern power systems, CPPS has emerged and developed fast in recent years. However, due to the network transmission distance, hardware restrictions, and other factors, CPPS communication delay would occur. Meanwhile, due to the existence of cyber layers, CPPS is more vulnerable to frequent cyber-attack incidents compared with traditional power systems. Thus, communication time delays and cyber-attacks are two main uncertainties in the cyber layer. For time delay, stochastic process methods are applied to characterize the change of time delay [40,174,175]. For the uncertainties of cyber-attacks, there are 2 forms of cyber-attack that are widely studied in CPPSs, i.e., the DoS attack and the false data injection (FDI) attack. DoS attacks will result in temporary communication interruption or termination, while FDI attacks manipulate system operational data. Two methods are often adopted to model the uncertainty of DoS attacks. One is to use discrete PDF/CDF methods to describe the packet loss rate caused by DoS attacks [59], and the other is to use Markov chain methods to depict the change of DoS attacks [68,176]. For FDI attacks, the probability of the injected false random data is commonly described by the single PDF/CDF method [103], and the change of the false data can be characterized by the stochastic process methods [107,177].

3.3. Economic and social layer uncertainty

From the aspects of the economic and social layer, uncertainties, such as fuel supply, cost of production, economic growth, and discount rate, have impacts on power system operations [12]. The uncertainties of these factors can be modeled by single PDF/CDF methods [45]. However, there is a complex correlation relationship among these factors. Thus, a more common way is to reflect social and economic uncertainties on the uncertainties of electricity prices and then investigate the influence of electricity prices on the operation of power systems. Although there are numerous types of electricity prices, e.g., spot price and marginal price, the typical methods adopted for modeling different electricity price uncertainties are similar, where single PDF/CDF methods [84] and uncertainty set methods [123] are widely applied. When addressing the fluctuation of electricity prices, scenario reduction methods [157] and neural network methods [178] are often utilized. Recently, with the widespread concern for sustainable development, the uncertainty of carbon emission factors has also been included in the uncertainty analysis of power systems [45,179].

The uncertainties in modern power systems modeled by uncertainty modeling methods are summarized in Table 6. Also, uncertainties modeled by PDF/CDF methods have been summarized in Table 1 and Table 2 in detail.

4. Applications of uncertainty modeling in the analysis of modern power system operation

The uncertainties in modern power systems increase the complexity of power system operation states and put forward high requirements for power system operation analysis. Thus, this section focuses on which issues about power system operation the uncertainty modeling methods are applied to solve and how these uncertainty modeling methods can be combined with other methods to solve them, whereas Section 3 focuses on uncertainties themselves. And this section summarizes the application of uncertainty modeling methods in power system operation from the aspect of steady state, dynamic, and risk analysis.

4.1. Applications in steady-state analysis

4.1.1. Probabilistic power flow analysis

The traditional power flow analysis of power systems is deterministic, where the network topology, parameters, and injected power of each node are fixed, and the power flow results are also determined. However, due to numerous uncertainties in systems, the results of power flow are probabilistic. If the deterministic power flow analysis method is still used, it is necessary to calculate all possible situations, which considerably increases the computational cost. Probabilistic power flow analysis can directly obtain the probability distribution of power flow according to the probability model of uncertainties, which greatly reduces the computational cost.

Uncertainty modeling methods are often adopted to describe the probability distributions of generations [19,24–27,30,32,34,94,95,153,181,182] and consumption sources [19,24,25,27,32–34,38,94] and then calculate the probability distribution of power flow. Three types of uncertainty modeling methods are extensively applied, including single PDF/CDF, mixed PDF/CDF, and sampling methods. In probabilistic power flow analysis, there are commonly three types of approaches combined modeling methods with propagation analysis methods. One is to calculate the power flow results at sampling points through the sampling method and MCS after obtaining the historical data or PDF/CDF of power injection uncertainties [59,181]. This approach takes the highest time cost. Another is to acquire the PDF/CDF of uncertainties and then use the linearization propagation analysis methods to attain the probabilistic power flow results [19,25,38,94,95]. For the third approach, the time cost is between those of the first two approaches. After obtaining the historical data or PDF/CDF of uncertainties, a small number of sampling data are collected by sampling method to calculate the simplified surrogate model of power systems, and then the surrogate model is applied to acquire the probabilistic power flow results [27,182].

4.1.2. Probabilistic economic operation analysis

The intrusion of uncertainties into modern power systems will also have an impact on the economic operation. The traditional economic operation analysis is a deterministic optimization problem, which is not suitable for the scenarios where uncertainties exist. Economic operation analysis involves wide optimization issues, including unit commitment [78,86,105,121,134,139,144], economic dispatch [22,46,51,101,103,104,120,127,128,130–132,134,135,137,138,141,158,167,168,180,183], siting [39,142], capacity sizing [39,45,88,108,125,142,143,147,157,169], bidding strategy [21,79,102,123], electricity purchasing [122], reconfiguration [41,136], etc. According to Fig. 1, there are three typical optimization propagation analysis methods, which also correspond to different uncertainty analysis procedures. Corresponding to stochastic optimization methods, there are usually three types of uncertainty analysis procedures. One is to obtain the mean value according to the PDF/CDF of uncertainties or attain the sampling points through sampling methods. Then, stochastic optimization calculation is conducted at the limited sampling points or the mean value to obtain the PDF/CDF or the mean value of the objective [21,45,86,104,167]. The other is based on the historical data or acquired PDF/CDF of uncertainties. The typical scenarios are obtained by sampling methods and scenario reduction methods, and then stochastic optimization is conducted under typical scenarios

Table 6
Summary of uncertainties modeled by uncertainty modeling methods.

Uncertainty modeling methods		Physical layer uncertainty			Cyber layer uncertainty	Economic and social layer uncertainty
		Generation uncertainty	Consumption uncertainty	Network uncertainty		
Model-driven methods	**PDF/CDF methods**	Conventional generation [32,34,35,55,151] Wind speed [19,23,24,29,32,33,36–39,48,50,52–54,85–87,89–92,94,95,98,153,180] Solar irradiance [19–22,37–39,85,87,98,104,164,180] Clearness index [30] Wind power generation [25–27,31,88,97,101,102,166] PV [23–28,96,97,101,181] Generator fault [56,57]	Conventional load [22–25,27–29,32–42,46,85,87–89,94,98–102,104,166,182] EV charging state [32] EV simultaneously charging number [19] EV daily driven mile [19,29] EV recharging power [19] Number of charging EV per day [29] EV start charging time [43] EV SoC [43] EV charging service time [29] EV charging load [38] EV departure time [43]	Operational parameter [23,44] Component fault [49,50,57,60,94] Windstorm [47] Ice storm [58,62] Earthquake [58]	Communication error [42] DoS attack [59] FDI attack [103]	Electricity price [45,84] Discount rate [45] Energy price [45] Carbon dioxide emission [45] Cumulative energy demand [45]
	Stochastic process methods	Conventional generation [109,149] Wind speed [110,160,175] Solar irradiance [110] Wind power generation [16] PV [111]	Conventional load [149]	Component fault [69]	Communication time delay [40,174,175] FDI attack [107] DoS attack [68,176]	Electricity price [108] Fuel price [39]
	Set-based methods	Conventional generation [100] Wind power generation [105,121,124,126,127,130–132,134,135,138,159] PV [124,125,127,131,132,137,138,165] Generator parameter [117] Operational parameter [85]	Conventional load [21,48,112–114,119,120,122,124–126,128,131,132,136–139,159] EV charging power [153]	Operational parameter [118] Component fault [94,159]		Electricity price [122,123,157] Installed capacity [48]
Data-driven methods	**Sampling methods**	Conventional generation [34,55,140] Wind speed [29,36,83,86,87,141,182] Solar irradiance [87] Wind power generation [59,88,142] PV [28,59,96,142]	Conventional load [28,29,34,36,59,87,88,142] EV daily driven mile [29] Number of charging EV per day [29] EV charging service time [29]			Electricity price [84]
	Scenario reduction methods	Solar irradiance [169] Wind power generation [51,78,141,142,144,157,158,180] PV [20,51,142,144,157,168,180]	Conventional load [51,142,143,157,158,167–169] EV charging state [85]			Electricity price [178] Low carbon factor [179]
	Neural network methods	Conventional generation [150] Wind speed [156] Wind power generation [51,61,80,144,161] PV [51,61,144,161]	Conventional load [51,161,171] EV arrival time [79] EV departure time [79]			Electricity price [178]

Table 7
Application summary of uncertainty modeling methods in modern power system.

Uncertainty modeling methods		Steady-state analysis		Dynamic analysis		Risk analysis	
		Probabilistic power flow analysis	**Probabilistic economic operation analysis**	**Probabilistic stability analysis**	**Stochastic stability analysis**	**Reliability analysis**	**Resilience analysis**
Model-driven methods	**PDF/CDF methods**	Probabilistic voltage [19,24–27,30,32,38,94,181] Probabilistic active power [19,32–34,59,94,153] Probabilistic reactive power [19,32,33] Percentage of different load types [95]	Unit commitment [86,105] Economic dispatch [22,46,97,101,103,104,106,180] Bidding strategy [21,102] Siting [39] Capacity sizing [39,45,88] Reconfiguration [41]	Small signal stability [23,29,35–37,40,44,52,53,64,92] Rotor angle stability [28,36,96,98,99] Voltage stability [36,83,87,89,96] Frequency stability [36,42,54,59,90,91,151]		Hazardous effect assessment [49,50,55,56,93] Reliability improvement strategy design [60]	Hazardous effect assessment [58,173] Resilience improvement strategy design [62]
	Stochastic process methods		Economic dispatch [84] Siting [39] Capacity sizing [39,108]		Rotor angle fluctuation [16,69] Voltage fluctuation [149] Frequency fluctuation [160] Stability improvement strategy design [40,68,174–176] Stable region [107] Stable probability [69]	Hazardous effect assessment [109]	Response strategy [177]
	Set-based methods	Probabilistic voltage [94,119] Probabilistic active power [94,119,153] Probabilistic reactive power [119]	Unit commitment [105,121,134,139] Economic dispatch [106,120,124,127–132,135,137,138] Bidding strategy [123] Capacity sizing [115,125] Electricity purchasing [122] Reconfiguration [136]			Hazardous effect assessment [126]	Response strategy [133]
Data-driven methods	**Sampling methods**	Probabilistic voltage [182]	Unit commitment [86] Economic dispatch [141] Capacity sizing [88]	Small signal stability [140] Rotor angle stability [96] Voltage stability [36,83,87,96]			Hazardous effect assessment [173]
	Scenario reduction methods		Unit commitment [78,144] Economic dispatch [51,141,158,167,168,180] Siting [142] Capacity sizing [142,143,157,169]				
	Neural network methods		Unit commitment [144] Economic dispatch [51,161] Bidding strategy [79]				

[51,78,123,141–144,168,169,180]. The third one is to set the probability of optimization constraints with uncertainty parameters being higher than the predetermined confidence level and transform the problem into chance constrained programming [46,101–103,183]. Besides, the analysis procedure corresponding to IGDT and robust optimization is to model uncertainties by adopting uncertainty set methods and to integrate the model into the optimization constraints [21,105,121,122,124,125,127,128,130–132,134–139]. The difference is that the set boundary of IGDT is uncertain, and it is regarded as having a positive or negative impact on the expected target and is included in the objective function. In contrast, the set boundary of robust optimization is determined.

4.2. Applications in dynamic analysis

4.2.1. Probabilistic stability analysis

Power system stability analysis can be viewed as examining the ability to sustain the operating equilibrium after small or large disturbances. For probabilistic stability analysis, the stability issues caused by uncertainties in the physical layer, such as power generation [23,28,29,31,35,36,42,52–54,83,87,89–92,96,98,99,140,159], load [14,23,29,31,35–37,40,42,53,87,89,98,99,159] and operating parameters [23,42,44] are usually studied. These uncertainties are usually considered to be time-invariant within the time period to be analyzed. Thus, the probabilistic system stability is affected due to the change of steady states of power systems.

When analyzing probabilistic small disturbance stability, PDF/CDF methods and sampling methods are commonly adopted to model uncertainties, and then MCS methods [23,36,37,40,87,140], linearization methods [52,53,92], or surrogate model methods [35] are utilized to calculate the stability indices related to the eigenvalues of system state-space equations. The typical indices include critical eigenvalues [40,44,117,140], the real part of critical eigenvalue [29,36,37,52,53,92], damping ratios [23,35,37,40], and participation factors [140]. Additionally, rotor angle stability, voltage stability, and frequency stability are widely concerned. The rotor angle stability represents the ability of the generator to maintain synchronous operation after being disturbed. Voltage stability is related to the ability of power systems to maintain the bus voltage at an acceptable level, which usually reflects the balance of reactive power of power systems. Frequency stability refers to the ability of the system to maintain the frequency to the allowable range without collapse, which often illustrates the balance of the active power. Transient stability index (TSI) is the typical index of probabilistic rotor angle stability [28,36,98]. For common indices of probabilistic voltage stability, there are load margin [31,36], probability of voltage instability [31], load increase limit [35], conservation voltage reduction capability [87], P-V curve [83,89], and Q-V curve [83,89]. And the rate of change of frequency (RoCoF) [54,90,91] and frequency nadir/vertex [36,91] are typical indices of probabilistic frequency stability.

4.2.2. Stochastic stability analysis

In the probabilistic stability analysis, system state-space equations are considered as time-invariant, which are ordinary differential equations. By comparison, in stochastic stability analysis, the influence of time-varying stochastic disturbances on system stability is considered, where system state-space equations will be time-varying, which are SDEs. In the stochastic stability analysis of power systems, two types of problems are mainly studied. One is to analyze the influence of uncertainties on systems, i.e., whether the system is stable or not when there are uncertainties in systems, including wind speed [160], conventional generation [109,149], and conventional load fluctuation [149] modeled by SDE methods, and faults expressed by Markov chain methods. Another is how to design control strategies to improve the stochastic stability of systems when there are uncertainties in systems, where the uncertainties in the cyber layer are normally explored and modeled by Markov chain methods, including stochastic attacks and communication time delays [68,107,174,176].

4.3. Applications in risk analysis

4.3.1. Reliability analysis

The potential risks are considered in the design of power systems, and the corresponding margin will be reserved to permit normal operation states of power systems under faults. Reliability analysis focuses on the security of power systems during accidents and the ability to avoid chain faults, resulting in systems out of control and large-area power outages. Uncertainty faults in systems are broadly involved, including generator, transmission line, and equipment faults [49,50,56,57,60]. Discrete PDF/CDF methods are used for modeling, and the assessment indices of reliability are calculated, including line outage distribution factor (LODF) [57], loss of load expectation (LOLE) [56,126], loss of load probability (LOLP) [49,55,56], loss of load frequency (LOLF) [49], expected energy not supplied (EENS) [49,50], expected unserved energy (EUE) [55], average service availability index (ASAI) [50], average service unavailability index (ASUI) [50], system average interruption duration index (SAIDI) [50], system average interruption frequency index (SAIFI) [50], and clearing time [93], by cooperating with linearization methods, sampling methods, and MCS propagation analysis methods, thereby conducting reliability analysis. In addition, there are studies focusing on designing protection schemes to improve system reliability with the reliability assessment indices as the objectives [60].

4.3.2. Resilience analysis

Resilience analysis focuses on assessing the restoration ability of systems suffering unexpected extreme incidents [184]. Compared with reliability analysis, there are overlaps between the faults concerned in reliability analysis and those concerned in resilience analysis. The difference is that reliability analysis focuses on faults with high probability and relatively limited impact on power systems, whereas resilience analysis focuses on faults with low probability and great impact. In resilience analysis of power systems, the impact of disasters and cyber-attacks is generally assessed [185]. The analysis procedure and indices are similar to those of reliability analysis. The probability of extreme events is commonly modeled by PDF/CDF methods, and then the assessment indices are calculated by MCS methods [47,173]. Also, the response strategies during the extreme events modeled by Markov chains and the recovery strategies have been concerned in resilience analysis [177].

The application of modeling methods adopted in modern power systems is summarized in Table 7.

5. Current research limitations and recommendations for future research

The above review reveals that the uncertainty modeling methods have provided effective solutions for analyzing the impact of uncertainties on the operation of modern power systems. However, with the development of modern power systems, the existing modeling methods may not be effective for the potential issues in the future and need to be further improved. The following research studies are considered to be promising and recommended.

5.1. Generalization of uncertainty modeling methods

Most current uncertainty modeling methods can only reflect one of the characteristics of uncertainties. For example, PDF methods reflect the statistical characteristic of possible values of uncertainties at a certain moment. Stochastic process methods reflect the time-varying characteristics of uncertainties. And uncertainty set methods reflect the boundary of possible values of uncertainties. Thus, if different characteristics of uncertainties are considered simultaneously, multiple uncertainty modeling methods are needed. Moreover, since different uncertainty modeling methods are suitable for different uncertainty propagation analysis methods, the overall framework for analyzing the impact of different characteristics of uncertainties on modern power systems will be complicated. Although there are some studies focusing on proposing general uncertainty methods for tackling different characteristics, e.g., different timescales [186,187], these methods cannot comprehensively handle various typical characteristics of uncertainties till now.

To fill this research gap, it deserves to integrate existing uncertainty modeling methods into a general form, which is challenging and may need new theories. Also, superior methods in other research areas can be investigated and modified to combine existing uncertainty modeling methods. Moreover, the flexibility of data-driven uncertainty modeling methods can be explored, e.g., building multiple neural networks for modeling different characteristics of uncertainties and integrating them into the unified one.

5.2. Accuracy and efficiency improvement of uncertainty modeling methods

Most existing uncertainty modeling methods rely on a large number of data to ensure high modeling accuracy, which are usually time-consuming. Thus, how to balance accuracy and efficiency and reduce the required data is worth studying. Moreover, there is a lack of a specific standard to illustrate the acceptable accuracy of uncertainty modeling methods. Additionally, though there are some studies using data augmentation technologies to reduce the required quantity of historical data in uncertainty modeling [51,79,144], how to ensure that the quality of generated data is within an acceptable degree also needs to be addressed.

To fill this research gap, the acceptable accuracy of uncertainty modeling methods needs to be determined based on the common practice in the industry. Also, the existing uncertainty modeling methods and data augmentation technologies can be improved to ensure accuracy improvement under the same historical data quantity. Moreover, the analytical relationship between the historical data quantity and the accuracy and efficiency of uncertainty modeling methods can be obtained through detailed derivation. By selecting the appropriate historical data quantity based on the derived analytical relationship, a balanced improvement of accuracy and efficiency can be achieved.

5.3. Development of uncertainty modeling methods suitable for novel uncertainties in modern power systems

With the wide utilization of renewable power generation, apart from wind power and PV, novel types of renewable power generation have been gradually integrated into modern power systems. Meanwhile, on the demand side, the increasing number of controllable devices are connected to power systems. Also, the integration of the cyber layer into power systems introduces massive new components together with potential uncertainties. Although there are some studies focusing on the uncertainty modeling of demand response resources [188,189], there will be massive novel types of uncertainties with unique characteristics due to the increasing complexity of modern power systems, which require to be carefully modeled.

To fill this research gap, the latest development of modern power systems requires to be continuously paid attention to. And the possible types and characteristics of uncertainties of new components connected to power systems can be analyzed based on their operation states, where the corresponding uncertainty modeling method can be used to characterize the uncertainties. If the characteristics of novel uncertainties cannot be effectively characterized by existing uncertainty modeling methods, the development of novel uncertainty modeling methods suitable for specific uncertainties is needed.

5.4. Development of uncertainty modeling methods suitable for novel tasks to tackle emerging issues

With the rapid development of modern power systems, emerging issues simultaneously occur. For example, in the research area of power system stability, there are only issues related to rotor angle stability, voltage stability, and frequency stability [190]. However, due to the high proportion of power electronics in modern power systems, new stability issues occur, including resonance stability and converter-driven stability [191]. It deserves to be discussed and investigated whether these new issues require uncertainty analysis and what specific tasks uncertainty modeling methods will be applied to.

To fill this research gap, sensitivity analysis can be conducted to quantify the impact of uncertainties characterized by uncertainty modeling methods on emerging issues in modern power systems, thereby revealing the necessity of considering uncertainties in these issues. And uncertainty modeling methods, together with corresponding uncertainty analysis frameworks, suitable for these emerging issues need to be developed.

6. Conclusions

In this paper, the uncertainty modeling methods applied in modern power systems are comprehensively reviewed. Firstly, this review classifies uncertainty modeling methods according to the outputs of uncertainty modeling methods as model-driven and data-driven methods and analyzes the characteristics of these methods. Then, uncertainties in modern power systems are summarized from physical, cyber, and economic and social layers. Moreover, the typical procedures of characterizing these uncertainties by adopting uncertainty modeling methods are studied. Also, the application of uncertainty modeling methods in power system operation is investigated. Meanwhile, with the construction of modern power systems, the current uncertainty modeling methods may be ineffective in the future. Thus, the potential areas of further research are recommended, including generalization of uncertainty modeling methods, accuracy and efficiency improvement of uncertainty modeling methods, and development of uncertainty modeling methods suitable for novel uncertainties and tasks.

CRediT authorship contribution statement

Zhaoyuan Wang: Writing – original draft, Validation, Methodology, Investigation. **Siqi Bu:** Writing – review & editing, Validation, Supervision, Project administration, Investigation, Funding acquisition, Conceptualization. **Jiaxin Wen:** Writing – original draft, Validation, Investigation. **Can Huang:** Validation.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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Data availability

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