# Computational Physics
# Problem Set 8

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In this problem set we simulate the 2D Ising model in a rectangular lattice with no external magnetic field and periodic boundary conditions using the Metropolis Monte Carlo method.

$$E = -J \sum_i \sum_{j \in \text{neighbors}(i)} \sigma_i \sigma_j; \qquad \sigma = \begin{cases} +1 \\ -1 \end{cases} \tag{1}$$

To speed up the algorithm, the spins can be updated in a checkerboard pattern with vectorized caclulations[1], as demonstrated in figure 1.

Notes:

1. Relaxation time varies with the temperature and the size of the lattice, but it is generally small compared to my simulation times. I took the relaxation time to be a tenth of the simulation time.

2. In small lattices, the correlation length is hard to measure and diverges easily, so the data for lattice sidelengths 8 and 16 is omitted.

3. Due to the "freezing" effect, outliers are common in low temperatures and ruin the data. For large sidelengths, an enormous amount of data is needed to correct the effect of the outliers. Because of this, I smoothed out the curves for the sidelengths 256 and 512 by hand.

Also, I made some `gifs` which can be found under the `anim` directory.

[1]Source: Joshua, R.; Mauro, B.; Massimiliano, F.; Massimo, B.; *A Performance Study of the 2D Ising Model on GPUs* arXiv.org link

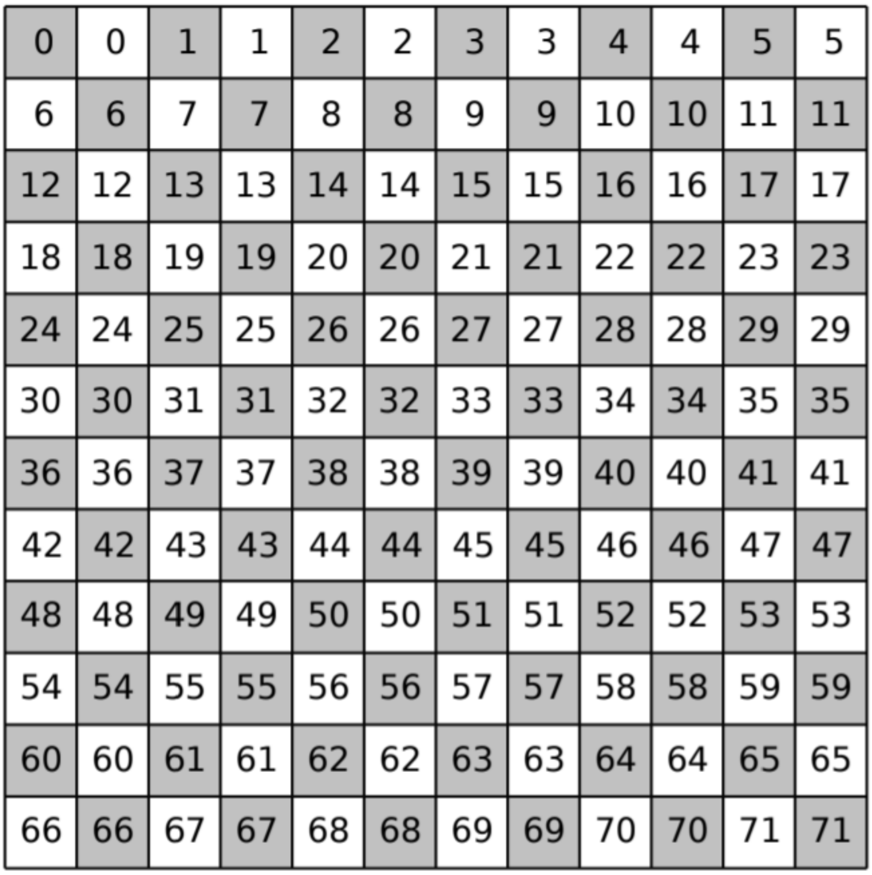



Figure 1: the cells of each "color" do not interact with each other, so they can be updated all at the same time

Table 1: For calculating $\xi$, only sidelengths 32, 64, 128, and 256 were used. For calculating $\chi$ sidelength 512 was not used due to low accuracy. The actual values (coming directly from theory) for $\nu$, $\gamma$, and $\beta$ are 1, $7/4 = 1.75$, and $1/8 = 0.125$. So as you can see, the calculated values are fairly close to the actual theoretical values.

| Relation | Critical Exponent | | $T_c(\infty)$ |
|---|---|---|---|
| $\xi \sim L \sim \lvert T_c - T\rvert^{-\nu}$ | $\nu$ | 0.91 | 2.264 |
| $C_V \sim c_0 \ln \lvert T_c - T\rvert$ | $c_0$ | -0.56 | 2.265 |
| $\chi \sim \lvert T_c - T\rvert^{-\gamma}$ | $\gamma$ | 1.70 | 2.264 |
| $\langle\lvert m\rvert\rangle \sim \lvert T_c - T\rvert^{\beta}$ | $\beta$ | 0.123 | 2.270 |

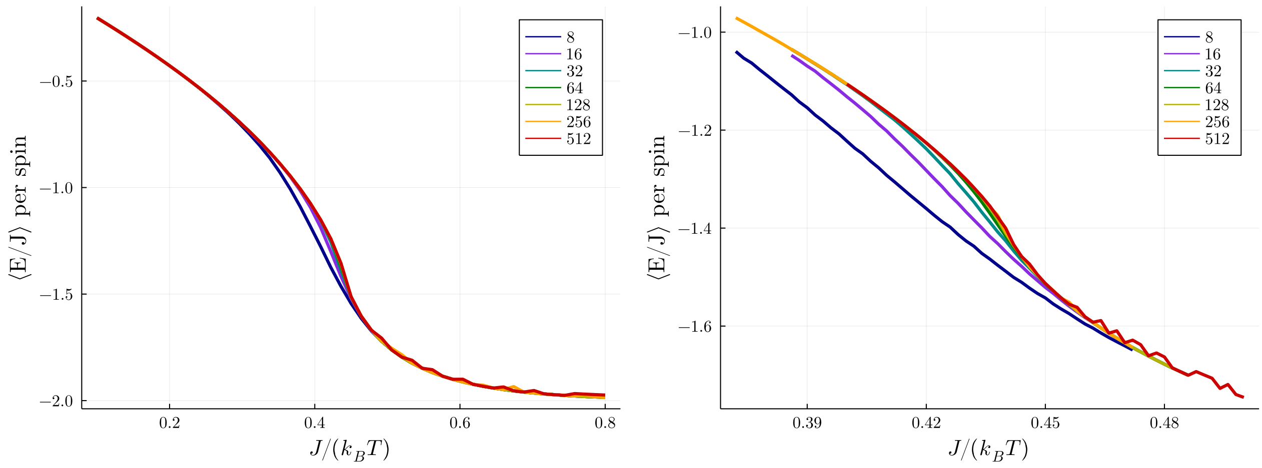



Figure 2: Energy

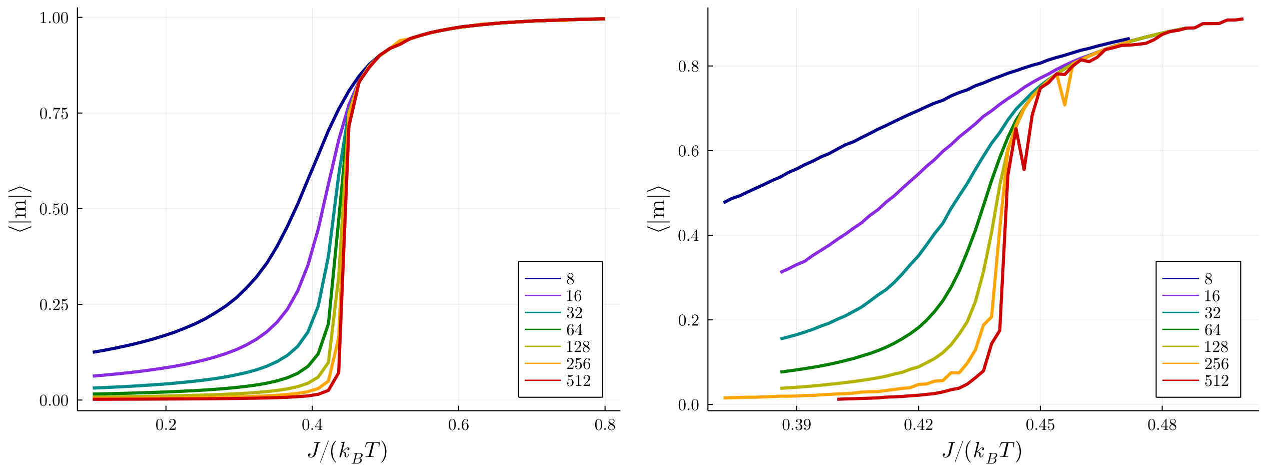



Figure 3: Magnetization (mean spin)

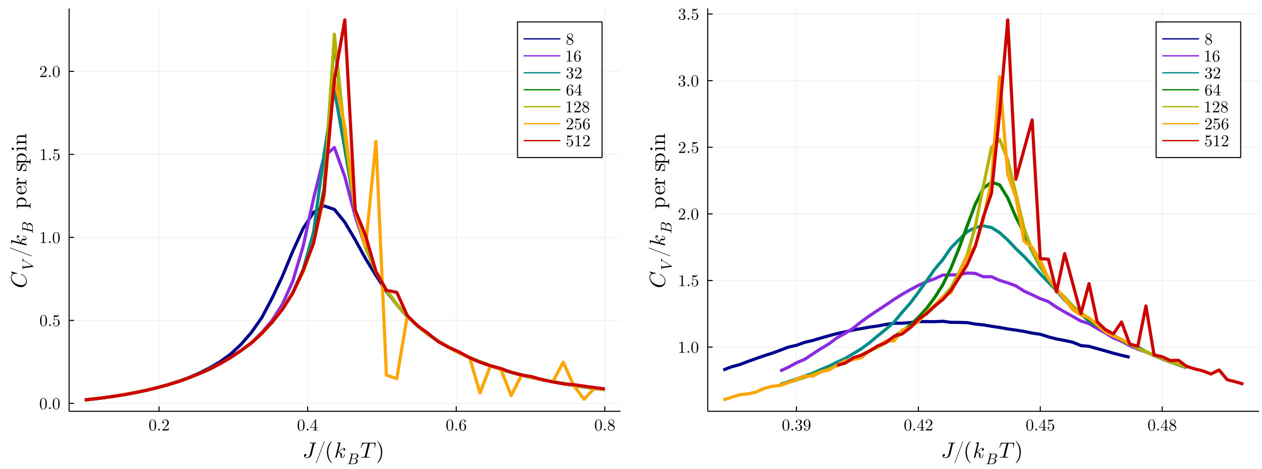



Figure 4: Heat Capacity $C_V$

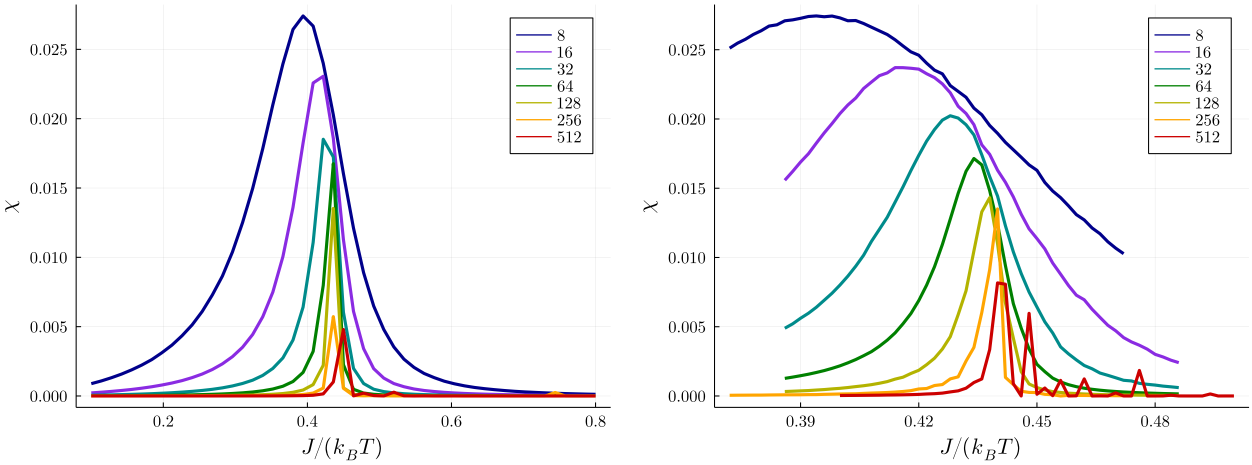



Figure 5: Magnetic Susceptibility $\chi$

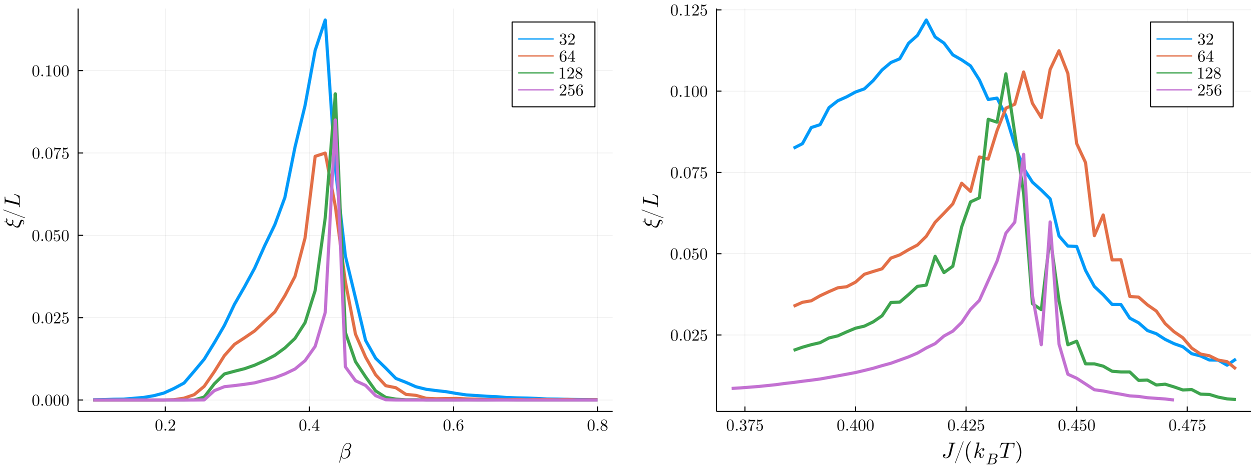
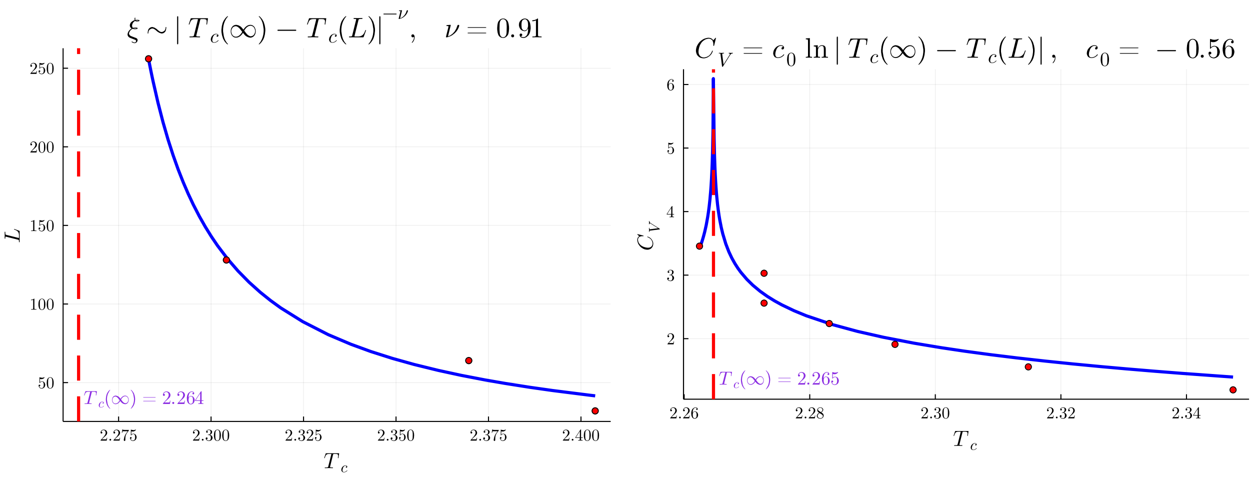
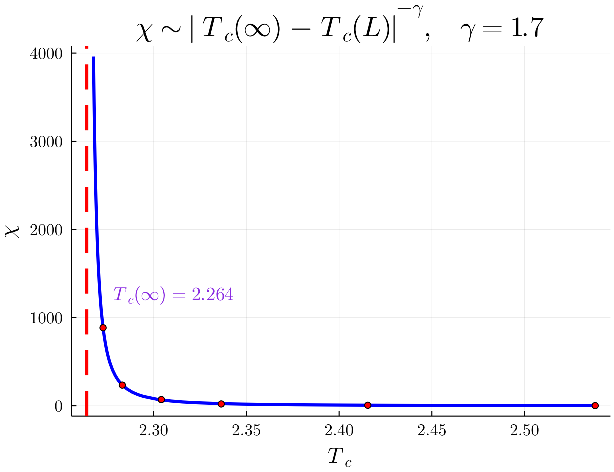
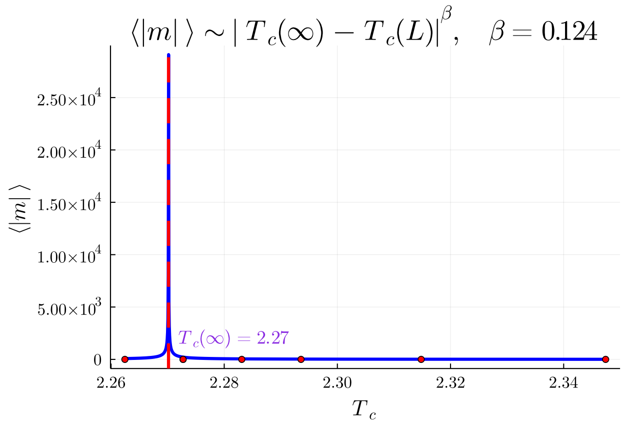
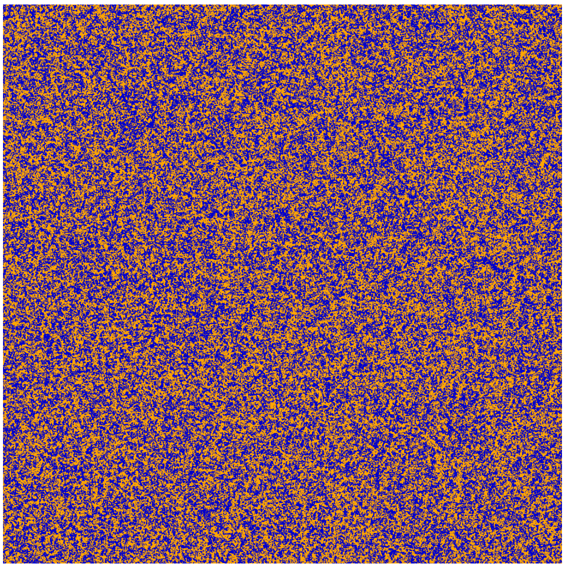



Figure 6: Correlation Length $\xi$

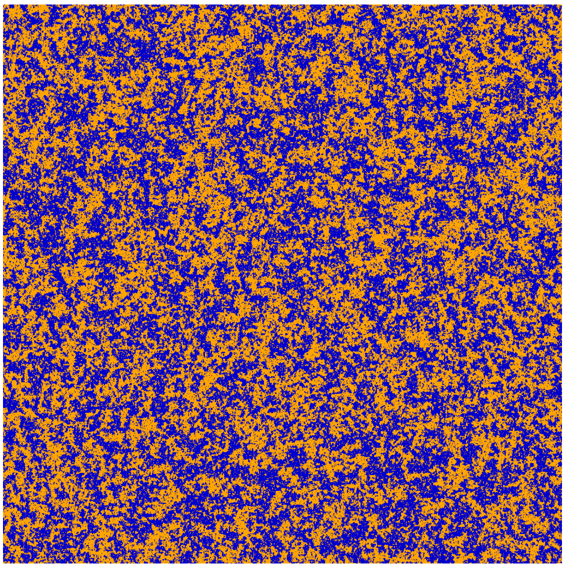




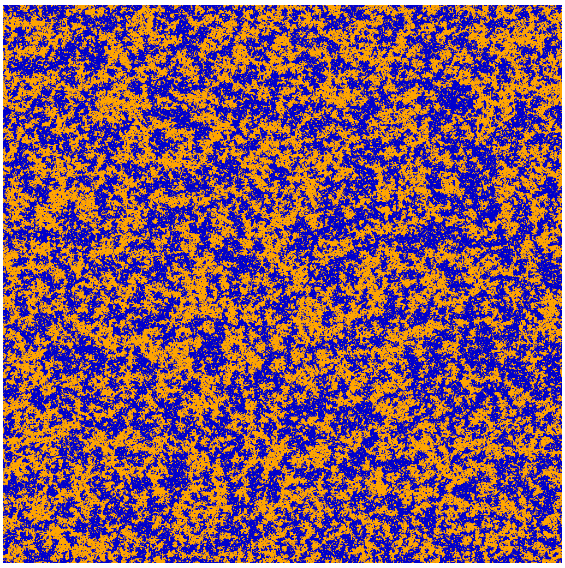



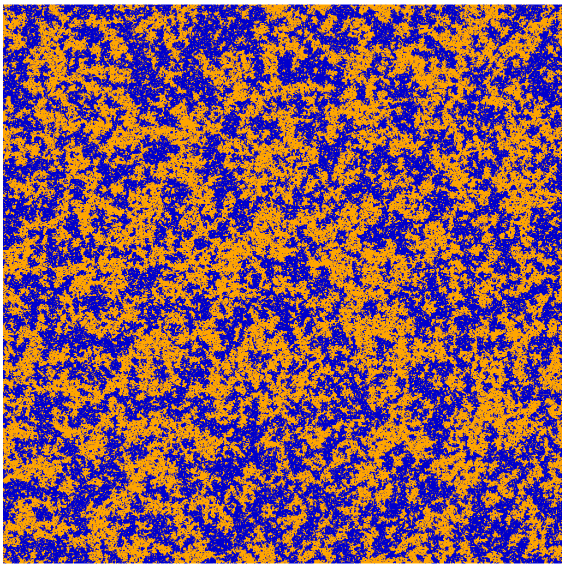
$J/(k_B T) = 0.3$



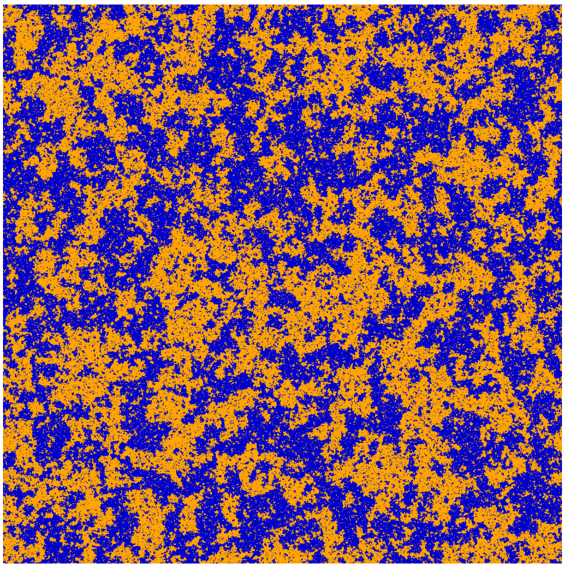
$J/(k_B T) = 0.4$



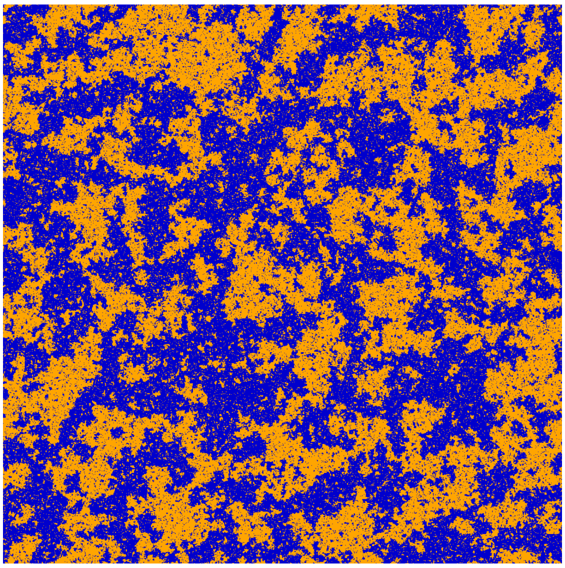
$J/(k_B T) = 0.41$



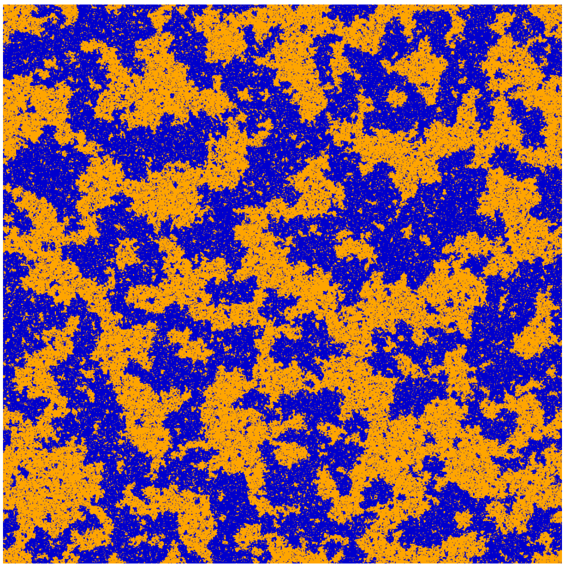
$J/(k_B T) = 0.42$



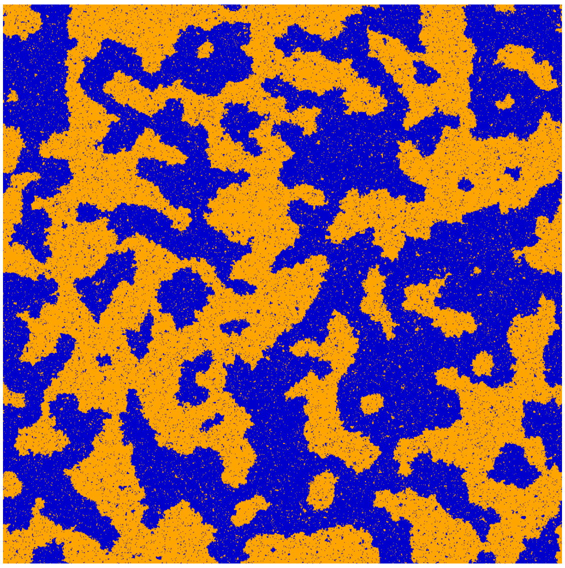
$J/(k_B T) = 0.43$



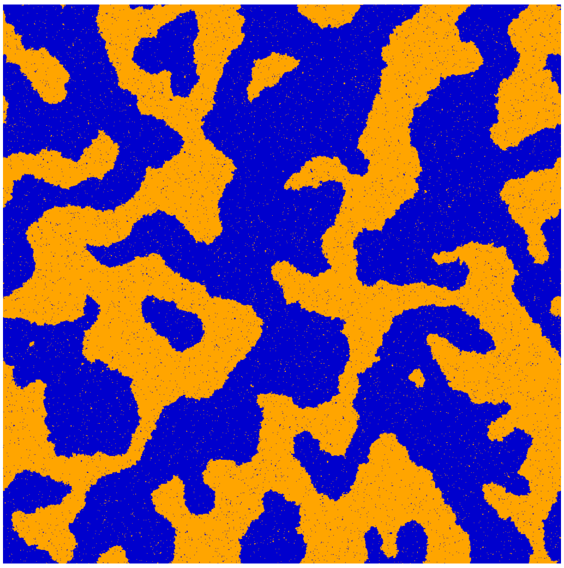
$J/(k_B T) = 0.44$



$J/(k_B T) = 0.45$



$J/(k_B T) = 0.5$



$J/(k_B T) = 0.6$