Exercise 3

1. Permutation

formula: $P^n_r = \dfrac{n!}{(n-r)!}$

n = amount
r = how you want it arranged.

$$P^8_4 = \frac{8!}{(8-4)!} = \frac{8!}{4!} = \frac{8 \cdot 7 \cdot 6 \cdot 5 \cdot \cancel{4!}}{\cancel{4!}} = 1680$$

2. Combination

formula: $C^n_r = \cancel{\dfrac{n!}{r!(n-r)!}} \; \dfrac{n!}{(n-r)!\,r!}$

n = amount
r = items randomly ~~selete~~ selected

$$C^7_4 = \cancel{\frac{7!}{4!(7-4)!}} \; \frac{7!}{(7-4)!\,4!} = \frac{7!}{3!\,4!} = 35$$

3. Combination

formula: $C^n_r = \dfrac{n!}{(n-r)!\,r!}$

$$C^{10}_3 = \frac{10!}{(10-3)!\,3!} = \frac{10!}{7!\,3!} = \frac{10 \cdot 9 \cdot 8 \cdot \cancel{7!}}{\cancel{7!}\;3 \cdot 2} = \frac{720}{6} = 120$$

$$C^{15}_2 = \frac{15!}{(15-2)!\,2!} = \frac{15!}{13!\,2!} = \frac{15 \cdot 14 \cdot \cancel{13!}}{\cancel{13!}\;2} = \frac{15 \cdot 14}{2} = \frac{210}{2} = 105$$

$$C^{25}_5 = \frac{25!}{(25-5)!\,5!} = \frac{25!}{20!\,5!} = \frac{25 \cdot 24 \cdot 23 \cdot 22 \cdot 21 \cdot \cancel{20!}}{\cancel{20!}\;5 \cdot 4 \cdot 3 \cdot 2 \cdot 1} = 53\,130$$

$$P = \frac{120 \times 105}{53\,130} = \frac{12\,600}{53\,130} = 0.2372 \approx 23.72\%$$

Exercise 2

1.

$Q_1 = 18$

$Q_2 = 30$

$Q_3 = 42$

$$\text{Tmean} = \frac{Q_1 + 2Q_2 + Q_3}{4} = \frac{18 + 60 + 42}{4}$$

$$= 30$$

2.

$$GM = \left(\prod_{i=1}^{n} (1+gi) \right)^{\frac{1}{n}}$$

$$= (1.05) \cdot (1.10) \cdot (0.97) \cdot (1.06)$$

$$= \sqrt[4]{1.05 \times 1.10 \times 0.97 \times 1.06}$$

$$= 1.044 - 1 = 0.044 = 4.4\% \text{ per year.}$$

3.

$\cancel{68}$, 70, 72, 75, 80, 85, 90, 92, $\cancel{100}$

$$\frac{70 + 72 + 75 + 80 + 85 + 90 + 92}{8}$$

10% = 82.375

Finding SD

Formula

$$SD = \sqrt{\frac{1}{n} \sum_{i=1}^{n} (x_i - \bar{x})^2}$$

-> use n for regular SD (Population)

use n-1 for t-test (sample)

x_i = every of the data

$\bar{x}$ = mean of ~~every~~ all data.

Exercise 4

1.

Year 1 : ~~0.0~~ 0.10 + 1 = 1.10

Year 2 : 0.15 + 1 = 1.15

Year 3 : -0.05 + 1 = 0.95

Year 4 : 0.08 + 1 = 1.08

Year 5 : 0.12 + 1 = 1.12

$$\sqrt[5]{(1.10)\cdot(1.15)\cdot(0.95)\cdot(1.08)\cdot(1.12)}$$

$= 1.078$

$1.078 - 1 = 0.078$

$0.078 \times 100 = 7.8\%$

2.

Group A

min : 7

25th : 9

50th : 13

75th : 15

max : 16

Exercise 7

1. one way ANOVA = F-Table

1. Make the hypothsis

H_0 : All means for differet fertilizers are the same

H_1 : There is at least one difference

2. Find mean for A, B & C

$\bar{X}_a = 15.4$

$\bar{X}_b = 20.4$

$\bar{X}_c = 26$

$\bar{X} : 19.53$

3. sum of squares

$SST = \Sigma(\bar{x}_{ij} - \bar{X})^2$

$\rightarrow$ All the data minus w overall mean trs u squared & add them all

$(15 - 19.53)^2 + \ldots . (24 - 19.53)^2 = t$

$SSB = n\Sigma(\bar{x}_i - \bar{x})^2$

$\rightarrow$ ~~overall~~ mean - overall mean trs squared

$3[(15.4 - 19.53)^2 + (20.4 - 19.53)^2 + (26 - 19.53)^2] = b$

$SSE = t - B = e$

4. degree of freedom

$df_B = n - 1 = 3 - 1 = 2$

$dfe = N - n = 15 - 3 = 12$

$\rightarrow MSB = \frac{b}{dfb} = mb$

$\rightarrow MSE = \frac{e}{dfe} = me$

$F = \frac{mb}{me}$

5. Crit Value

df_B & $dfe = 2$ & 12

$C =$ ~~2&1~~ 3.8853

If F value is far value is far beyond the crit val, reject H0.

2. Chi-squared

① Make hypothesis

H_0 : The observed matches expected

H_1 : observed does not match expectation

② formula :

$$\chi^2 = \sum \frac{(O_{ij} - E_{ij})^2}{E_{ij}}$$

O : observed

E : expected

$$E_{ij} = \frac{\underbrace{\Sigma R}_{\text{row}} \times \underbrace{\Sigma C}_{\text{column}}}{\Sigma G} = \frac{40 \times 30}{90} = \cancel{\frac{1200}{90}} =$$

$\frac{1200}{90}$	$\frac{1400}{90}$	$\frac{1000}{90}$
$\frac{900}{90}$	$\frac{1050}{90}$	$\frac{750}{90}$
$\frac{600}{90}$	$\frac{700}{90}$	$\frac{500}{90}$

O_{ij} = Data

$$\chi^2 = \left(\frac{10 - \frac{1200}{90}}{\frac{1200}{90}}\right)^2 + \dots \left(\frac{10 - \frac{500}{90}}{\frac{500}{90}}\right)^2 = \chi^2$$

test score

③ degree of freedom

$$df = \underbrace{(r-1)}_{\text{row}}\underbrace{(c-1)}_{\text{column}} = (3-1)(3-1) = (2)(2) = 4$$

④ Crit val by chi-squared table

C = 9.488

⑤ conclusion

if χ^2 is more than the the crit val, reject H_0.

klo ga failure to reject.

3. Two way ANOVA

① Define hypothesis

H_0 = Lang & Stud Method does not have any effect on the variables & have no interaction

H_1 = (opposite H_0)

②
$$\overline{X} = \frac{78+82+85+90 \ldots 75+80}{6 \times 3} =$$

$$\overline{X}_{R1} = \frac{78+82+85+90+88+92}{6}$$

$\overline{X}_{R2}$ = count per row

$\overline{X}_{R3}$ = — " " —

$$\overline{X}_{C1} = \frac{78+82+85+72+72 \ldots +68+78}{9}$$

$\overline{X}_{C2}$ = — " —

③ $SST = \sum (x_1 - \overline{x}) = (78-\overline{x})^2 + \ldots (80-\overline{x})^2 = SST$

④ SSB = R → numb of row $\cdot \sum(\overline{X}_R - \overline{x})^2 = 2\left[(\overline{x}_{R1} - \overline{x})^2 + \ldots (\overline{x}_{R3} - \overline{x})^2\right]$

C → numb of column $\cdot \sum(\overline{x}_c - \overline{x})^2 = 3\left[(\overline{x}_{c1} - \overline{x})^2 + (\overline{x}_{c2} - \overline{x})^2\right]$

I → $\sum\sum n_{ij} (\overline{x_{ij}} - \overline{x_i} - \overline{x_j} + \overline{x})^2$

$$= 3\left(\frac{78+82+85}{3} - \underbrace{\overline{X}_{Ri_1}}_{\text{mean of row 1}} - \underbrace{\overline{X}_{Ci}}_{\text{mean of Column 1}} + \overline{X}\right)^2 + 3\left(\frac{90+88+92}{3} - \underbrace{\overline{X}_{R1}}_{\text{mean of Row 1}} - \underbrace{\overline{X}_{C2}}_{\text{mean of Column 2}} + \overline{X}\right)^2$$

$$+ \ldots 3\left(\frac{78+75+80}{3} - \overline{X}_{R3} - \overline{X}_{Ci_2} + \overline{X}\right)^2 =$$

$$SSE = \sum (X_{ijk} - \overline{X_0})^2$$

$$= \left(78 - \frac{78+82+85}{3}\right)^2 + \left(82 - \frac{78+82+85}{3}\right)^2 + \ldots \left(80 - \frac{78+75+8}{3}\right.$$

Find degrees of freedom

df Row : $r-1 = 2$

df Column : $c-1 = 1$

df Interactions : $(r-1)(c-1) = 2 \cdot 1 = 2$

df Error : N − row × column) = $18 - (3 \cdot 2) = 12$

df Total : numb of observation $-1 = 18 - 1 = 17$ (bvp data)

$$MSB_R = \frac{SSB\ row}{dfr}$$

$$MSB_C = \frac{SSB\ column}{dfc}$$

$$MSB_I = \frac{SSB\ I}{df\ I}$$

$$MSB_E = \frac{SSE}{dfE}$$

Crit val

C_R = dfR & dfe = 3.89
2 & 12

C_C = dfc & dfe = 4.75
1 & 12

C_I = dfi & dfe = 3.89
2 & 12

F-val

$$F_R = \frac{MSB_R}{MSE}$$

$$F_B = \frac{MSB_C}{MSE}$$

$$F_i = \frac{MSB_I}{MSE}$$

compare Lan & Stud Method both.

Conclude

Compare F-val & crit

if F val is more than crit val, ~~acce~~ reject H_0

Exercise 4

1. Percentage returns.

2.

3. and : multipy

or : either one

$P(A \& B) = P(A) \cdot P(B)$

$P(A / B) = P(A) + P(B) - P(A \& B)$

$$\frac{4}{52} \times \frac{1}{2} = \frac{4}{104} = \frac{2}{52}$$

4.

X		Y
9 7 4 2	1	3 6 8
8 6 4 1	2	0 3 5 7 8
2 0	3	9 3

5. $P = C^{n}_{x} \cdot p^{x} q^{n-x}$

n = numb of experiments

x = numb of success

p = Probability of success

q = Probability of fail $(1 - p)$

$$C^{5}_{3} \cdot \left(\frac{1}{2}\right)^{3} \left(\frac{1}{2}\right)^{5-3}$$

$$= \frac{5!}{3!\,2!} \cdot \frac{1}{8} \cdot \frac{1}{4} = \frac{10}{32} = \frac{5}{16}$$