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Effect Sizes: Definitional Formulae

See:
Rosenthal, R. (1994). Parametric measures of effect size. In H. Cooper & L. V. Hedges (Eds.), *The handbook of research synthesis* (pp. 231-244). New York: Russell Sage Foundation.
Rosenthal, R. Rosnow, R. L., & Rubin, D. B. (2000) Contrasts and effect sizes in behavioral research: A correlational approach. Cambridge, UK: Cambridge University Press.

Please feel free to contact me if any of these formulae seem incorrect – it is possible that typographical errors may have been made.

Effect sizes based on correlations

$$r = \Phi = r_{pb} = \frac{\sum Z_x Z_y}{n} = \frac{\sum\left(\frac{X-\overline{X}}{\sqrt{\frac{\sum\left(X-\overline{X}\right)^2}{n}}}\right)\left(\frac{Y-\overline{Y}}{\sqrt{\frac{\sum\left(Y-\overline{Y}\right)^2}{n}}}\right)}{n} = \frac{\frac{\sum(X-\overline{X})(Y-\overline{Y})}{n}}{s_x s_y} = \frac{\mathrm{covariance}_{xy}}{s_x s_y}$$

NOTE: These formulae are for the population correlation between x and y. You may also see these formulae with n-1 rather than n, which provides the sample estimate of the population correlation. Whichever form is chosen, be sure to use either n or n-1 in all parts of the formula.

$$z_r = r' = \tfrac{1}{2}\log_e\left[\frac{1+r}{1-r}\right] = \tfrac{1}{2}\left[\log_e(1+r) - \log_e(1-r)\right] \quad \textit{where } \log_e \textit{ is the natural log function}$$

(LN on some calculators)

To transform back from z_r (r') metric to r metric

$$r = \frac{e^{2z_r} - 1}{e^{2z_r} + 1}, \quad \textit{where e is the exponent function } (e^x \textit{ on some calculators})$$

$$\text{Cohen's } q = Z_{r1} - Z_{r2}$$

Effect sizes a la d

$$\text{Cohen's d} = \frac{\overline{X}_1 - \overline{X}_2}{\sigma_{\text{pooled}}}$$

$$\text{Hedges's g} = \frac{\overline{X}_1 - \overline{X}_2}{s_{\text{pooled}}}$$

$$\text{Glass's } \Delta = \frac{\overline{X}_1 - \overline{X}_2}{s_{\text{control group}}}$$

Where

$$\sigma_{\text{pooled}} = \sqrt{\frac{(n_1)\,\sigma_1^2 + (n_2)\,\sigma_2^2}{n_1 + n_2}} \quad \text{and} \quad s_{\text{pooled}} = \sqrt{\frac{(n_1 - 1)\,s_1^2 + (n_2 - 1)\,s_2^2}{n_1 + n_2 - 2}}$$

$$\text{and} \quad \sigma_{\text{pooled}} = s_{\text{pooled}} \sqrt{\frac{n_1 + n_2 - 2}{n_1 + n_2}}$$

You may also see σ_{pooled} *referred to as* σ_{within}, *and* s_{pooled} *referred to as* s_{within} *or as* $\sqrt{MS_{within}}$

Effect sizes for proportions

$$\text{Cohen's } g = p - .50$$

where p estimates a population proportion

$$d' = p_1 - p_2$$

where p_1 and p_2 are estimates of the population proportions

$$\text{Cohen's } h = \arcsin p_1 - \arcsin p_2$$

$$\text{Probit } d' = Z_{p_1} - Z_{p_2}$$

where Z_{p_1} and Z_{p_2} are standard normal deviate transformed estimates of population proportions

$$\text{Logit } d' = \log_e\left[\frac{p_1}{1-p_1}\right] - \log_e\left[\frac{p_2}{1-p_2}\right]$$

Transforming between Effect Sizes

Computing r

$$r = \sqrt{\frac{d^2}{d^2 + 4}} \quad \textit{– if you assume that the two populations are equally numerous}$$

$$r = \sqrt{\frac{d^2}{d^2 + \frac{1}{PQ}}} \quad \textit{– if the populations are clearly different in size}$$

where P is the proportion of Group 1 individuals in the combined Group 1 + Group 2 population and Q = 1 - P.

$$r = \sqrt{\frac{g^2 n_1 n_2}{g^2 n_1 n_2 + (n_1 + n_2)df}}$$

where df = n1 + n2 - 2 in the 2 - group case and n - 1 in the 1 - group case

Computing d

$$d = \frac{2r}{\sqrt{1 - r^2}}$$

$$d = g\sqrt{\frac{n_1 + n_2}{df}}$$

Computing g

$$g = \frac{r}{\sqrt{1 - r^2}} \sqrt{\frac{df\,(n_1 + n_2)}{n_1 n_2}}$$

$$g = \frac{d}{\sqrt{\frac{n_1 + n_2}{df}}}$$

Transforming between Effect Sizes (continued)

transform between eta squared (η^2) and omega squared (ω^2)

$$\text{eta}^2 \text{ for an effect} = \eta^2 = \frac{\left(df_{effect} + \frac{n_i n_j \omega^2}{1-\omega^2}\right)\left(\frac{1}{df_{error}}\right)}{\left(df_{effect} + \frac{n_i n_j \omega^2}{1-\omega^2}\right)\left(\frac{1}{df_{error}}\right) + 1}$$

$$\text{omega2 for an effect} = \omega^2 = \frac{\frac{df_{error}\eta^2}{1-\eta^2} - df_{effect}}{\left(\frac{df_{error}\eta^2}{1-\eta^2} - df_{effect}\right) + n_i n_j}$$

Computing significance tests from effect sizes

Recall, Inferential test statistic = Effect size X Size of Study

$$\chi^2_{(1)} = Z^2 = r^2 N$$

$$Z = \sqrt{\chi^2_{(1)}} = r\sqrt{N}$$

$$t = \frac{r}{\sqrt{1-r^2}}\sqrt{df}$$

$$t = d\left(\frac{\sqrt{n_1 n_2}}{(n_1+n_2)}\right)\sqrt{df} \quad \textit{for the independent sample t - test and unequal sample sizes}$$

$$t = d\frac{\sqrt{df}}{2} \quad \textit{for the independent sample t - test and equal sample sizes}$$

$$t = d\sqrt{df} = \frac{\overline{X}_{sample} - \mu_{population}}{\sqrt{\frac{s^2_{sample}}{n_{sample}}}} \quad \textit{for the one sample t - test}$$

$$t = g\left(\frac{1}{\sqrt{\frac{1}{n_1}+\frac{1}{n_2}}}\right) = g\left(\sqrt{\frac{n_1 n_2}{n_1+n_2}}\right) = \frac{\overline{X}_1 - \overline{X}_2}{\sqrt{\frac{s_1^2}{n_1}+\frac{s_2^2}{n_2}}} \textit{ or } \frac{\overline{X}_1 - \overline{X}_2}{\sqrt{\left(\frac{1}{n_1}+\frac{1}{n_2}\right)s^2_{pooled}}} \quad \textit{for the independent sample t - test}$$

$$F = t^2 = \frac{r^2}{1-r^2}(df_{error}) = g^2\left(\frac{n_1 n_2}{n_1+n_2}\right) = d^2\left(\frac{df_{error}}{4}\right) \quad \textit{for an F test } \boldsymbol{with\ numerator\ df = 1}$$

$$F = \frac{s^2_{betweenmeans}}{s^2_{withinmeans}}(n) \quad \textit{assuming equal n's - for F tests with} \geq \textit{1 numerator df}$$

$$F = \frac{eta^2}{1-eta^2}\left(\frac{df_{error}}{df_{means}}\right) \quad \textit{for F tests with} \geq \textit{1 numerator df}$$

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Computing Effect Sizes from significance tests

$$r = \Phi = r_{pb} = \sqrt{\frac{\chi^2_{(1)}}{n}} = \frac{Z}{\sqrt{n}}$$

$$r = \sqrt{\frac{t^2}{t^2 + df}} = \sqrt{\frac{F}{F + df_{error}}}$$ *for F tests with numerator df = 1*

$$d = t\left(\frac{n_1 + n_2}{\sqrt{df}\sqrt{n_1 n_2}}\right)$$ *which simplifies to* $d = \frac{2t}{\sqrt{df}}$ *if the groups have equal n*

$$d = \sqrt{F}\left(\frac{n_1 + n_2}{\sqrt{df}\sqrt{n_1 n_2}}\right)$$ *which simplifies to* $d = \frac{2\sqrt{F}}{\sqrt{df}}$ *if the groups have equal n.*

(For F tests with numerator df = 1)

$$g = t\left(\frac{\sqrt{n_1 + n_2}}{\sqrt{n_1 n_2}}\right)$$ *which simplifies to* $g = \frac{2t}{\sqrt{n}}$ *if the groups have equal n*

$$g = \sqrt{F}\left(\frac{\sqrt{n_1 + n_2}}{\sqrt{n_1 n_2}}\right)$$ *which simplifies to* $g = \frac{2\sqrt{F}}{\sqrt{n}}$ *if the groups have equal n.*

(For F tests with numerator df = 1)

$\underline{\text{For F tests with any numerator degrees of freedom:}}$

$$\text{eta}^2 \text{ for an effect} = \eta^2 = \frac{\frac{F_{effect}(df_{effect})}{df_{error}}}{\frac{F_{effect}(df_{effect})}{df_{error}} + 1}$$

$$\text{omega}^2 \text{ for an effect} = \omega^2 = \frac{(F_{effect} - 1)(df_{effect})}{(F_{effect} - 1)(df_{effect}) + n_i n_j}$$