SF2955 Computer Intensive Methods in Mathematical Statistics

Home Assignment 1: Sequential Monte Carlo-based mobility tracking in cellular networks

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1 A hidden Markov model for mobility tracking

1.1 Motion Model

The motion model is as follow:

$$X_{n+1} = \Phi X_n + \Psi_z Z_n + \Psi_w W_{n+1} \tag{1.1}$$

Based on the motion model, we are able to simulate a trajectory $\{(X_n^1, X_n^2)\}_{n=0}^m$. However, before such simulation could occur, we must prove the following variables are Markov Chain.

1.1.1 $\{X_n\}_{n\in N}$ is not a Markov Chain

The random variables in the motion model are Z_n and W_{n+1}. W_{n+1} is $N(0_{2\times 1}, \sigma^2 I)$ distributed. Hence, it is independent of X_n,...,X_0. However, Z_n depends on Z_{n-1}. Hence, we can conclude that Z_n is not independent of X_n,...,X_0 which makes $\{X_n\}_{n\in N}$ not a Markov Chain.

1.1.2 $\{\widetilde{X}_n\}_{n\in N}$ is a Markov Chain

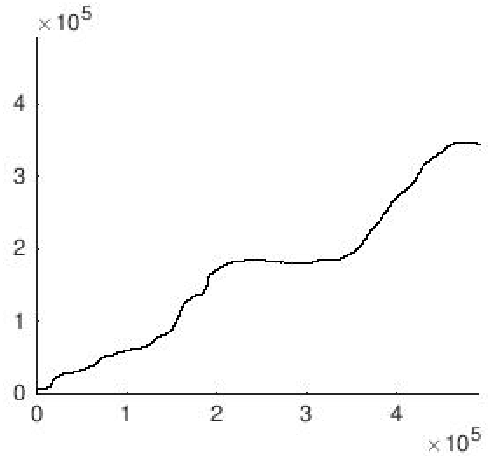
The equation is as follow:

$$\{\widetilde{X}_n\} = (X_n^T, Z_n^T)^T \tag{1.2}$$

The random variables in this equation would be X_n and Z_n. As seen in (1.3), X_n is a Markov chain. Furthermore, Z_n id independent of X_n,...,X_0. We can conclude that $\{\widetilde{X}_n\}_{n\in N}$ is a Markov Chain.

1.1.3 Simulation of Trajectory

We would now simulate a trajectory $\{(X_n^1, X_n^2)\}_{n=0}^m$ of some arbitrary length m. For this assignment, we would set m as 100000. Based on (1.2), we would obtain the following trajectory plot:



1.2 Observation Model

When the target is moving, its location can be detected by RSSI(*received signal strength indication*) from 6 BS(*basis station*), whose position is denoted by $\{\pi_l\}_{l=1}^s$. We can calculate the RSSI received form the l^{th} BS at time n by:

$$Y_n^l = v - 10\eta log_{10}||(X_n^1, X_n^2)^T - \pi_l|| + V_n^l \tag{1.3}$$

where v and η are constant and V_n^l are independent Gaussian noise variables with mean zero and standard deviation $\zeta = 1.5$.

1.2.1 $\{(\widetilde{X}_n, Y_n)\}_{n\in N}$ is a hidden Markov Chain

The random variable in the RSSI are X_n^1, X_n^2 and V_n^l where π_l is known. V_n^l is $N(0, \zeta^2)$ hence it is independent of X_n,...,X_0.
As we have known, $\{\widetilde{X}_n\}_{n\in N}$ is a Markov Chain, and $(X_n^1, X_n^2)^T$ is a Markov Chain($\{X_n\}_{n\in N}$ is a Markov Chain).
Thus,

$$P(Y_0, ..., Y_n|\widetilde{X_0}, ..., \widetilde{X_n}) = P(Y_0|\widetilde{X_0}) \cdots P(Y_n|\widetilde{X_n}) \tag{1.4}$$

1.2.2 The transition density

$$p(y_n|\widetilde{x}_n) = p(y_n|x_n, z_n) = p(y_n|x_n) = p(y_n|(x_n^1, x_n^2)) \tag{1.5}$$

As seen in (1.5),

$$(Y_n^l|X_n) \sim N(v - 10\eta log_{10}||(X_n^1, X_n^2)^T - \pi_l||, \zeta^2) \tag{1.6}$$

Thus,

$$(Y_n|X_n) \sim N((v - 10\eta log_{10}||(X_n^1, X_n^2)^T - \pi_1||, ..., v - 10\eta log_{10}||(X_n^1, X_n^2)^T - \pi_6||), \zeta^2 I_{6\times 6}) \tag{1.7}$$

The transition density of $(Y_n|\widetilde{X}_n)$ is the density of Multivariate Gaussian distribution with the expectation $(v - 10\eta log_{10}||(X_n^1, X_n^2)^T - \pi_1||, ..., v - 10\eta log_{10}||(X_n^1, X_n^2)^T - \pi_6||)$ and the standard deviation $\zeta^2 I_{6\times 6}$.

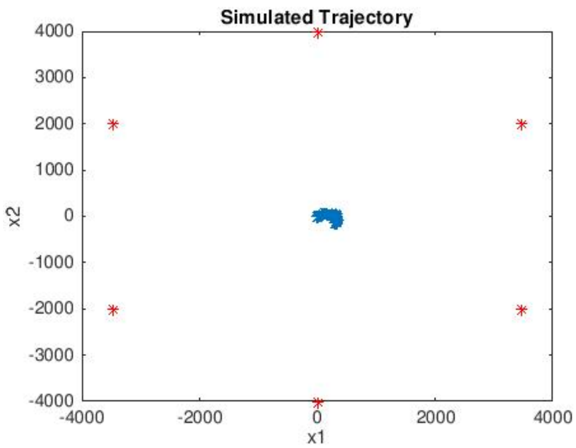
2 Mobility Tracking Using SMC Methods

Our aim is to estimate, by processing the measurements in succession, the positions of the target by means of optimal filtering. This could be achieved by sequential importance sampling(SIS) or sequential importance sampling with resampling(SISR).

2.1 Sequential Importance Sampling (SIS)

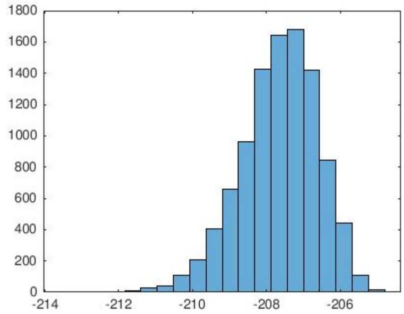
We would now implement the sequential importance sampling (SIS) algorithm for sampling from $f(\widetilde{x}_n|y_{0:n})$, n=0,1,...,501 for the observation stream in RSSI-measurements.mat.In our algorithm, we have set our particle sample size N as 100000.

Based on this algorithm, we would then be able to achieve estimates of $\{(T_n^1, T_n^2)\}_{n=0}^{500}$. In order to facilitate better visualization, we have plot these estimates in the plane together with the locations of the basis stations. The plot is as follow:

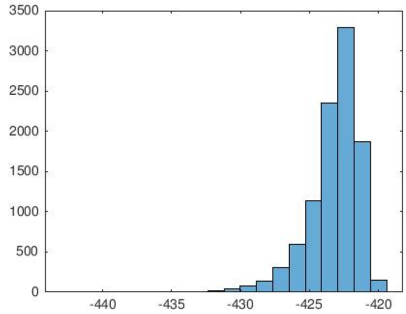


In order to get the efficient sample sizes, we have plot histogram with varying importance weight at n=20,40,60. Our goal was to produce a sequence of weighted samples representing $f(\widetilde{x}_n|y_{0:n})$ which is a normal distribution.

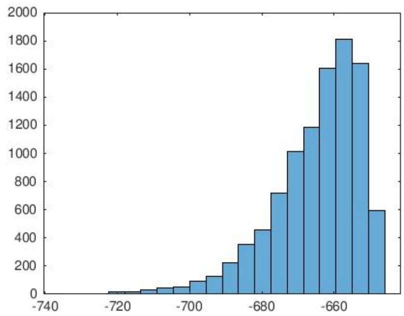
From Figure (a) to (c), we are able to observe that a n=60 would produce a histogram which better represent a normal distribution curve.



(a) n=20



(b) n=40

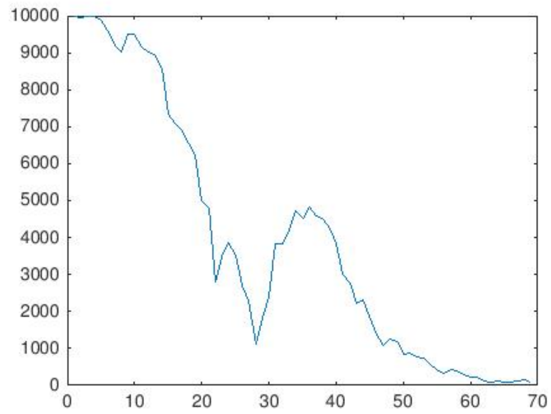


(c) n=60

Another way to obtain the efficient sample size is by measuring the coefficient of variation using the following formula:

$$CV_n = \sqrt{\frac{1}{N}\sum_{i=1}^{N}(N\frac{\omega_k^i}{\Omega_n} - 1)^2} \tag{2.1}$$

The lower the coefficient of variation, the more efficient the sample size is. With that in mind, we would then plot a graph as shown in the figure below:

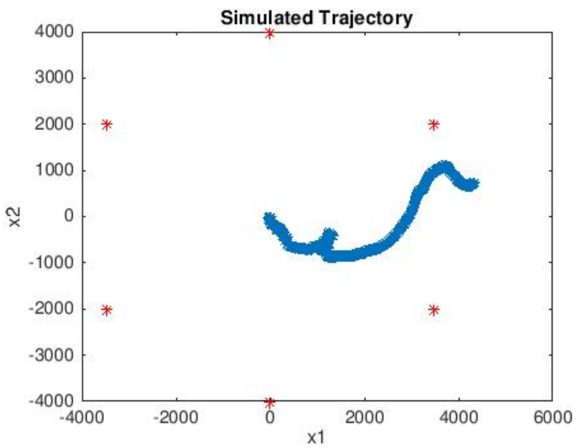


From the above plot and some calculations, we are able to observe that the efficient sample size is 65.

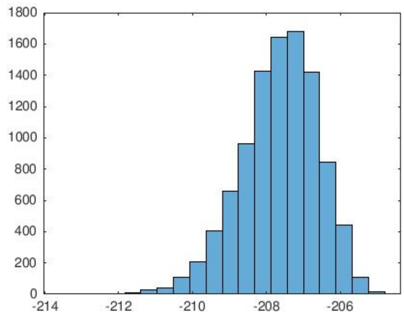
2.2 Sequential Importance Sampling with Resampling(SISR)

We would now implement the sequential importance sampling with resampling (SISR) algorithm for sampling from the same flow of densities by adding a selection step to the algorithm designed for Sequential Importance Sampling. In our algorithm, we have set our particle sample size N as 100000.

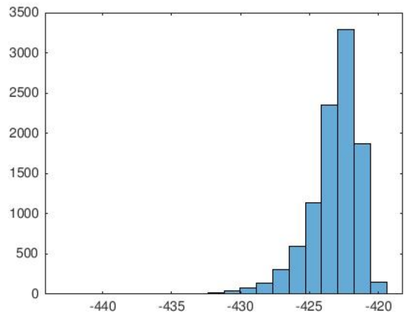
Based on this algorithm, we would then be able to achieve estimates of $\{(T_n^1, T_n^2)\}_{n=0}^{500}$. In order to facilitate better visualization, we have plot these estimates in the following figure



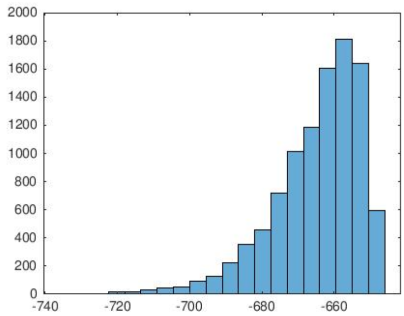
From Figure (a) to (c), we are able to observe that a n=200 would produce a histogram which better represent a normal distribution curve.



(a) n=100



(b) n=150



(c) n=200

Another way to obtain the efficient sample size is by measuring the coefficient of variation using the following formula:

$$CV_n = \sqrt{\frac{1}{N}\sum_{i=1}^{N}(N\frac{\omega_k^i}{\Omega_n} - 1)^2} \tag{2.2}$$

The lower the coefficient of variation, the more efficient the sample size is. After some calculations, we are able to observe that the efficient sample size is 221.

3 SMC-based model calibration

To calibrate the unknown $\zeta \in (0, 3)$, we maximize the normalized log-likelihood function:

$$l_m(\zeta, y_{0:m}) = m^{-1} ln L_m(\zeta, y_{0:m}) \tag{3.1}$$

However, it is intractable. So we designed grid $\{\zeta_j\}$ in the parameter space $(0, 3)$ then we choosed the one that maximize the corresponding $l_m^N(\zeta_j)$ for te given data input $y_{0:m}$. To estimate the likelihood function, we used:

$$c_{N,m}^{SISR} = \frac{1}{N^{m+1}} \prod_{k=0}^{m} \Omega_k \tag{3.2}$$

$$where \quad \Omega_k = \sum_{i=1}^{N} \omega_k^i \tag{3.3}$$

Thus, the log likelihood function for a fixed ζ_j can be estimated by:

$$l_m^N = m^{-1} ln c_{N,m}^{SISR} = \frac{1}{m} \sum_{k=0}^{m} ln\Omega_k - \frac{m+1}{m} lnN = \frac{1}{m} \sum_{k=0}^{m} ln(\sum_{i=1}^{N} \omega_k^i) - \frac{m+1}{m} lnN \tag{3.4}$$

where $\{\omega_k^i\}$ are gotten by SISR depending on ζ_j.

We used an increment of 0.2 from 0.5 to 3 for standard deviation. The $\widehat{\zeta}$ which maximizes the log-likelihood we got is **2.5** and we can plot the estimations of $\{(T_n^1, T_n^2)\}_{n=0}^{500}$ with this $\widehat{\zeta}$ as following figure