Project 1 Applied Linear Optimization
SF 2812

Group 1B2
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Abstract

This report covers the first project, part B, assigned by the course SF2812 Applied Linear Optimization. The main assignment is to compute the optimal strategy for Green Destruct on transporting waste material from factories and at which location to destruct it. The linear optimization and stochastic optimization model are calculated by GAMs.

Background and Problem Description

A transportation problem concerning providing demand under the restriction of availability, while making the maximum profit, is in this assignment modelled as an optimization problem.
As a consequence in the production of wooden floors a hazardous waste is created. The company Green Destruct AB performs environmentally friendly processes to destruct such waste. The company is situated at four different locations, these locations and their capacity of destruction, in tons, per week is presented in the table below.

Destruction capacity table				
Location	Arboga	Fagersta	Ludvika	Nyköping
Capacity	7	8	9	8

The company producing the wooden floors has eight factories, the factories are producing different amounts of floor and therefore their respectively waste created differs. In the following table the locations of the factory's and their amount of waste created, in tons, per week is presented.

Amount of waste table								
Location	Eskilstuna	Falun	Gävle	Norrköping	Stockholm	Uppsala	Västerås	Örebro
Waste	2	2	4	3	9	6	3	2

Once per week all of the hazardous waste is transported from the factories to Green Destruct, for this service they are payed 250 kr/kilo. The transportation company takes a cost of 1 kr/kilo per kilometer in the total transported weight is at most 1500 kilos, above this maximum weight the cost is increased to 1.4 kr/kilo. The last step of the process is to destruct the waste, which is at a cost of 100 kr/kilo.
The optimization problem takes the distance from each factory and the locations of the destruction plants under consideration, these distances, in kilometers, are presented in the table below.

Distance table								
Factory / Plant	Eskilstuna	Falun	Gävle	Norrköping	Stockholm	Uppsala	Västerås	Örebro
Arboga	45	165	195	110	156	129	52	40
Fagersta	95	88	126	187	176	117	66	103
Ludvika	137	63	150	220	222	163	112	112
Nyköping	82	260	257	59	106	168	128	135

Basic Exercise

The basic exercise aims to solve the optimization problem in order to find the optimal transporting of the material and at which location it should be destructed. A program is created and solved by GAMS. Green Destruct are also interested in an analysis of excess capacity. The results are presented under section *Results and Analysis*.

Advanced Exercise

A new contract is to be drawn up by Green Destruct to maximize their profit further. Previously Green Destruct took care of a fixed amount of waste but as for the new contract they will instead take care of the actual amount of waste produced at each factory. The optimization problem corresponding to the new contract will be model as a stochastic optimization problem.

The production of waste will be normally distributed using expected values as the previous fixed amount of waste and a standard deviation equal to 5 %. The previous cut-off weight determining the transportation cost per kilogram will be altered with the new contract. The cheaper class should be fixed, but the 40 % more expensive class may vary from week to week. In case of a larger total production of waste than destruction capacity, it is possible for Green Destruct to expand their capacity, at a cost of 200 kr/excess kilo. This change in strategy and the resulting outcome is presented under section *Results and Analysis*.

Mathematical Formulation

Basic Exercise

The optimization problem to be solved is expressed as (P)

$$
\begin{array}{lll}
(P) & \text{maximize} & \sum_i \sum_j (x_{ij} + e_{ij}) \cdot (inc - c_{destr}) - \sum_i \sum_j l_{ij} \cdot (x_{ij} \cdot c_{trans0} + e_{ij} \cdot c_{trans1}) \\
 & \text{s.t.} & \sum_i (x_{ij} + e_{ij}) = d_j \\
 & & \sum_j (x_{ij} + e_{ij}) \leq a_i \\
 & & 0 \leq x_{ij} \leq 1500 \\
 & & e_{ij} \geq 0, \ i = 1, ..., 4, \ j = 1, ..., 8
\end{array}
\tag{1}
$$

obtained with the variables:

$$
\begin{aligned}
x_{ij} &= \text{kilos of waste transported from factory } j \text{ to plant } i \text{ that less than 1500 kilos} \\
e_{ij} &= \text{kilos of waste transported from factory } j \text{ to plant } i \text{ that more than 1500 kilos} \\
inc &= \text{income per kilo in kr} \\
l_{ij} &= \text{distance in kilometers from factory } j \text{ to plant } i \\
c_{trans0} &= \text{original transportation cost per kilo per kilometer in kr} \\
c_{trans1} &= \text{increased transportation cost per kilo per kilometer in kr} \\
c_{destr} &= \text{destruction cost per kilo in kr} \\
d_j &= \text{demand of factory } j \text{ in kilos} \\
a_i &= \text{destruction capacity of plant } i \text{ in kilos}
\end{aligned}
$$

The optimal solution of (P) should maximize the objective function, the maximal profit which is calculated by the total income with subtraction of the total transportation cost and the total destruction cost. The problem contains two constraints, due to demand and availability.

Advanced Exercise

As for the advanced exercise, the optimization problem to be solved is expressed as (RP)

$$
\begin{aligned}
(SP) \qquad \max \quad & \sum_i \sum_j x_{ij} \cdot c_{trans0} + \sum_k \left[\sum_j p_k \cdot d_{jk} \cdot (inc - c_{destr}) - \sum_i \sum_j p_k \cdot l_{ij} \cdot e_{ijk} \cdot c_{trans1} \right. \\
& \left. - \sum_i p_k \cdot c^{'}_{destr} \cdot r_{ik} \right] \\
\text{s.t.} \quad & \sum_i (x_{ij} + e_{ijk}) \geq d_{jk} \\
& \sum_j (x_{ij} + e_{ijk}) \leq a_i + r_{ik} \\
& 0 \leq x_{ij} \leq 1500 \\
& e_{ijk} \geq 0 \\
& r_{ik} \geq 0, \; i = 1, ..., 4, \; j = 1, ..., 8
\end{aligned} \tag{2}
$$

obtained with the variables:

$$
\begin{aligned}
p_k &= \text{probability of scenario k} \\
x_{ij} &= \text{fixed kilos of waste transported from factory } j \text{ to plant } i \text{ that less than 1500 kilos} \\
e_{ijk} &= \text{kilos of waste transported from factory } j \text{ to plant } i \text{ that more than 1500 kilos in scenario k} \\
inc &= \text{income per kilo in kr} \\
l_{ij} &= \text{distance in kilometers from factory } j \text{ to plant } i \\
c_{trans0} &= \text{original transportation cost per kilo per kilometer in kr} \\
c_{trans1} &= \text{increased transportation cost per kilo per kilometer in kr} \\
c_{destr} &= \text{destruction cost per kilo in kr} \\
c^{'}_{destr} &= \text{extra destruction cost per kilo in kr} \\
d_{jk} &= \text{demand of factory } j \text{ in kilos in scenario k} \\
a_i &= \text{destruction capacity of plant } i \text{ in kilos} \\
r_{ik} &= \text{excess of the nominal capacity of plant } i \text{ in kilos in scenario k}
\end{aligned}
$$

The optimal solution of (SP) should maximize the objective function, the maximal expected profit which is calculated by the total income with subtraction of the total transportation cost and the total destruction cost. The total transportation cost is calculated by the cheaper part which is fixed per scenario and the 40 % more expensive part which may vary. The total destruction is calculated by the part below nominal capacities and the excess part which will be 100 kr/kilo more expensive. For different scenario, the total transportation cost and the total destruction cost will changed due to different waste production. The problem contains two constraints, due to demand and availability. The problem produces a stochastic two-stage programming.

Results and Analysis

Basic exercise

The model created in GAMS computes the optimal way of transporting the waste from all factories to the destruction plants in order to maximize the profit for Green Destruct AB. The following table presents the distribution of waste in kilograms.

Distribution table

Plant \ Factory	Eskilstuna	Falun	Gävle	Norrköping	Stockholm	Uppsala	Västerås	Örebro
Arboga	2000	0	0	0	1500	1500	500	1500
Fagersta	0	0	2000	0	1500	3000	1500	0
Ludvika	0	2000	2000	0	1000	1500	1000	500
Nyköping	0	0	0	3000	5000	0	0	0

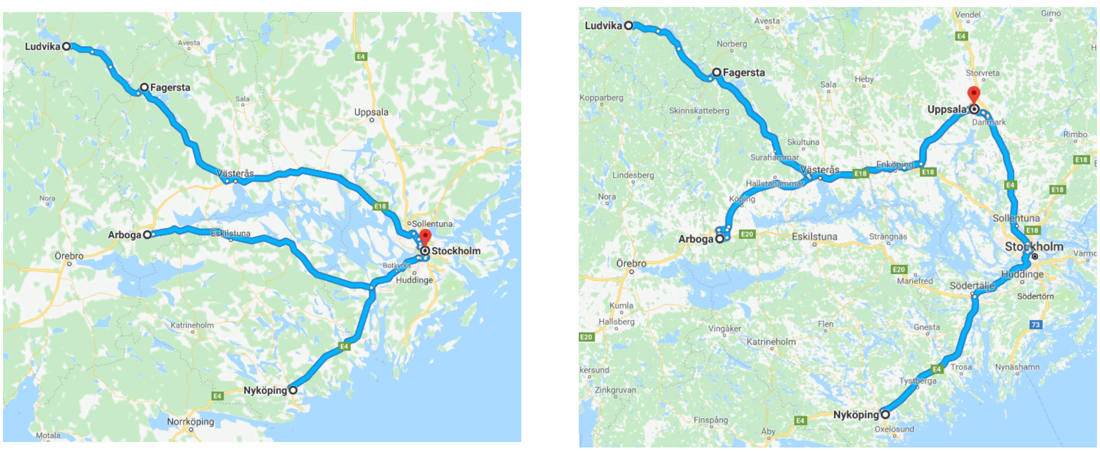
The total profit for Green Destruct per week is 982 200 kr.

Analyzing the excess capacity, presented in the table below, the plant located in Ludvika has the capacity to destruct 1000 kilograms above the calculate result. Ludvika is the destruction plant with the largest destruction capacity.

Excess capacity table

Location	Arboga	Fagersta	Ludvika	Nyköping
Capacity	7000	8000	9000	8000
Result	7000	8000	8000	8000
Excess	0	0	1000	0

Analyzing the locations graphically gives a intuitive explanation to which plant holds the excess capacity since the route to Ludvika often passes the plant at Fagersta, see Figure 1 a) and b). The wood-floor producers largest production sites are Stockholm and Uppsala, who's routes to Ludvika are the second longest, according to the distance table.



(a) From Stockholm

(b) From Uppsala

Figure 1: Transportation routes

Advanced exercise

In the advanced part, with the new contract, the stochastic optimization problem calculates the new, fixed part of, distribution of waste at each destruction plant from each factory, presented in the table below. The locations of the plants destructing the factories waste concur with the result from the basic exercise, but the amount of waste slightly altered.

Distribution table, fixed part								
Plant \ Factory	Eskilstuna	Falun	Gävle	Norrköping	Stockholm	Uppsala	Västerås	Örebro
Arboga	1500	0	0	0	1500	1500	425.808	1500
Fagersta	0	0	1500	0	1500	1500	1500	0
Ludvika	0	1500	1500	0	932.902	1500	1020.170	451.403
Nyköping	0	0	0	1500	1500	0	0	0

The expected total profit for Green Destruct per week is 977 958.856 kr.
The expected profit for the stochastic optimization problem is lower than for the optimization problem in the basic exercise.

An example of scenario k

The varying part of waste may differ from week to week and is presented as an example of a scenario k. The results, provided by GAMs, from different scenarios implies that Green Destruct would expand their capacities at Arboga and Nyköping.

Amount of waste table for scenario k								
Location	Eskilstuna	Falun	Gävle	Norrköping	Stockholm	Uppsala	Västerås	Örebro
Waste	1937.889	2095.150	4023.030	2982.849	8982.645	5971.305	3042.314	1951.403

Distribution table, extra part for scenario k								
Plant \ Factory	Eskilstuna	Falun	Gävle	Norrköping	Stockholm	Uppsala	Västerås	Örebro
Arboga	437.889	0	0	0	32.593	7.374	96.335	0
Fagersta	0	0	536.069	0	0	1463.931	0	0
Ludvika	0	595.150	486.961	0	0	0	0	0
Nyköping	0	0	0	1482.849	3517.151	0	0	0

No excess of the nominal capacities.

In addition, the advanced model doesn't take correlation between the different wood-producing plants into account since the standard to measure it cannot be found. To make the model more accurate, if the demand is similar at all sites, the correlation will be positive.

Summary

There are several model improvements to consider. A large vulnerability is the extra transport cost due to waste weight above 1500 kilos. Green destruct could reduce this cost by increasing the service cost for the factory for their corresponding extra waste.
To further reduce the transportation costs Green destruct could set up an extra destruction plant close to their largest suppliers of waste, preferably between Stockholm and Uppsala. If the cost of

setting up an extra plant is too large compared to the long term increase in profit, another possibility is to relocate the destruction capacities by increasing the capacity at Arvika and decreasing at Ludvika. In Figure 2 the locations of the factories are marked with blue pins and the plants are marked with red pins, to graphically demonstrate the distances.

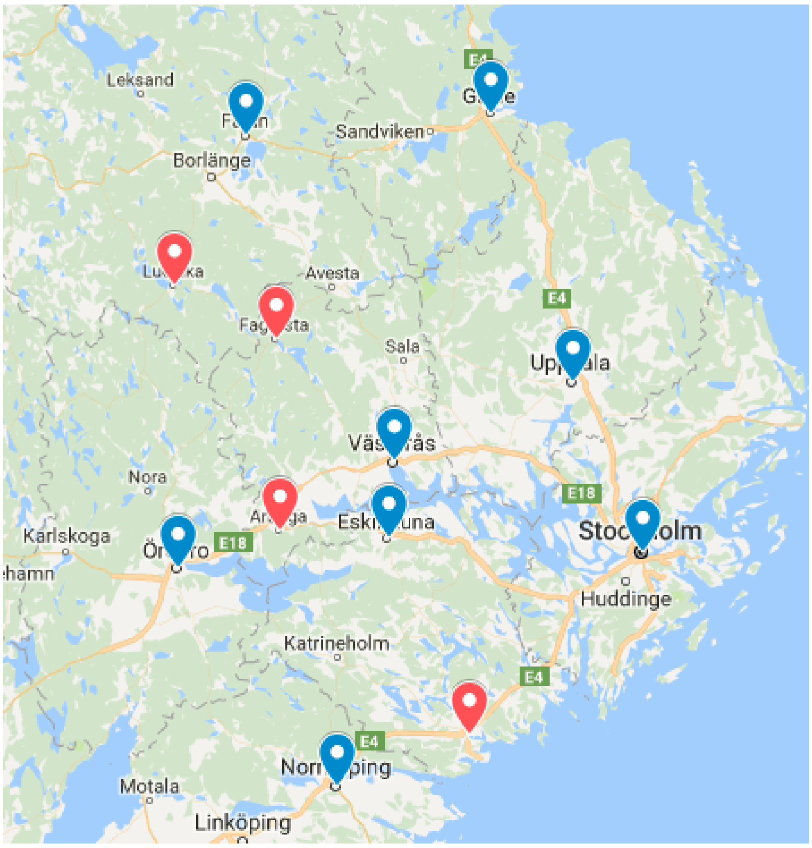


Figure 2: Locations of factories and plants