SF2812 Applied Linear Optimization, 2017/2018
Project assignment 2B
Due Tuesday February 27 2018 23.59

Discussion between the groups is encouraged, but each group must individually solve the assignments. It is *not* allowed to use solutions made by others in any form. Please see the course web page for more detailed information on the rules for the assignments.
Instructions on how to present the project assignments can be found at the course web page.
The exercises are divided into basic exercises and advanced exercises. Sufficient treatment of the basic exercises gives a passing grade. Inclusion of the advanced exercises is necessary for the higher grades (typically A-C). A member of a group who has not worked on the advanced exercises says so in the self assessment form.
Instructions for the report:

- The report should have a leading title page where the project name and the group members' names, personal number and e-mail addresses are clearly stated.
- The report should be written using a suitable word processor.
- The contents should be such that another student in the course, who is not familiar with the project, should be able to read the report and easily understand:
  1. What is the problem? What it the problem background? This does *not* mean a copy of the project description, but rather a suitable summary of necessary information needed in order to understand the problem statement.
  2. How has the group chosen to formulate the problem mathematically? What assumptions have been made? If these assumptions affect the solution, this should be noted.
  3. What is the meaning of constraints, variables and objective function in the mathematical formulation?
  4. What is the solution of the formulated optimization problem? If suitable, refer the mathematical solution to the terminology of the (non-mathematical) problem formulation. (There could be more than one optimization problem.)
- Most project descriptions contain a number of questions to be answered in the report. The report *must* contain the answers to these questions. They should, however, in a natural way be part of the content of the report and not be given in a "list of answers". The purpose of the questions is to suggest suitable issues to consider in the part of the report where the results are interpreted and analyzed. Additional interpretations are encouraged as well as generalizations and other ways of modeling the problem.
- A suggested outline of the report is as follows:
  1. Possibly a short abstract.
  2. Problem description and background information.
  3. Mathematical formulation.
  4. Results and analysis (interpretation of results).
  5. A concluding section with summary and conclusions.

Deviations from the outline can of course be done.

- GAMS code should not be part of the report, and should not be referred to in the report.
- Each group should upload the following documents via the Canvas page of the course no later than by the deadline of the assignment:
  - The report as a pdf file.
  - GAMS files.

  Please upload your documents as individual pdf and gms files, and not as zip files.
- Each student should fill out a paper copy of the self assessment form and hand in at the beginning of the presentation lecture.

This exercise concerns control of a mobile robot. The robot is moving in a twodimensional space, where the coordinates are given by z_1 and z_2. The state vector x is fourdimensional, consisting of z_1, z_2, and their time derivatives $\dot{z}_1$, $\dot{z}_2$ according to

$$x = \begin{pmatrix} z_1 & \dot{z}_1 & z_2 & \dot{z}_2 \end{pmatrix}^T.$$

We further consider a time-discretized model, so that the state vector at time step k is denoted

$$x(k) = \begin{pmatrix} x_1(k) & x_2(k) & x_3(k) & x_4(k) \end{pmatrix}^T, \quad k = 0, 1, \ldots,$$

where $x_1(k)$ gives the z_1-coordinate at time kh, and $x_2(k)$ gives the velocity in the z_1-direction at time kh, where h is the sampling interval. The analogous information in the z_2 direction is contained in $x_3(k)$ and $x_4(k)$.
The state equesion for the robot is updated according to the following time-discrete model:

$$x(k+1) = \Phi x(k) + \Gamma u(k), \quad u(k) \in \mathcal{U},$$

where

$$\Phi = \begin{pmatrix} 1 & h & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & h \\ 0 & 0 & 0 & 1 \end{pmatrix}, \qquad \Gamma = \begin{pmatrix} h^2/2 & 0 \\ h & 0 \\ 0 & h^2/2 \\ 0 & h \end{pmatrix},$$

and the limitiations on the control signal are given by

$$\mathcal{U} = \left\{ u = \begin{pmatrix} u_1 \\ u_2 \end{pmatrix} : -\alpha \leq u_j \leq \alpha, \ j = 1, 2 \right\}.$$

In the above formulas, h is the sampling time and α is a positive number. The above dynamics may be interpreted as the time-discrete control of the system

$$\begin{aligned} \ddot{z}_1 &= u_1, \quad -\alpha \leq u_1 \leq \alpha, \\ \ddot{z}_2 &= u_2, \quad -\alpha \leq u_2 \leq \alpha. \end{aligned}$$

Consequently, our robot is described by Newton's equation in z_1-direction and z_2.direction respectively. This may appear to an unrealistic simplification, but it can be shown that under suitable assumptions, the above model gives a correct description of the dynamics of the robot.

Basic exercises

1. Let $h = 1$ and let $\alpha = 2$. Determine, by formulating and solving a mixed-interger linear program, a control signal which in minimal time (i.e., in a minimum number of time steps), steers the robot from rest in the origin to the set

$$X_E = \{x : 5.9 \leq x_1 \leq 6.1,\ -0.1 \leq x_2 \leq 0.1,\ 6.9 \leq x_3 \leq 7.1,\ -0.1 \leq x_4 \leq 0.1\}\,.$$

2. Study how the solution is changed if the sampling time h and the control signal limit α are decreased.

Advanced exercise

3. We now have the task of steering the robot from rest in the origin to the set X_E in such a way that the robot all the time is in the corridor marked in Figure 1. Model, solve and analyze this problem in an analgous fashion as for the basic exercises.

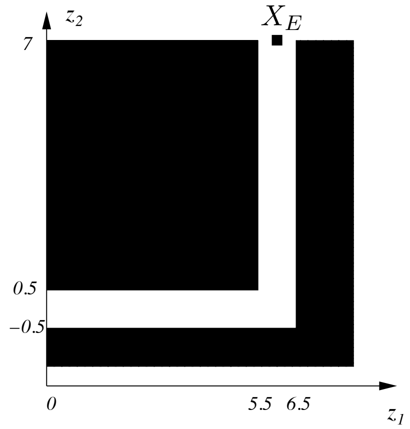

Figure 1: The robot has to stay in the corridor until reaching X_E.

Good luck!