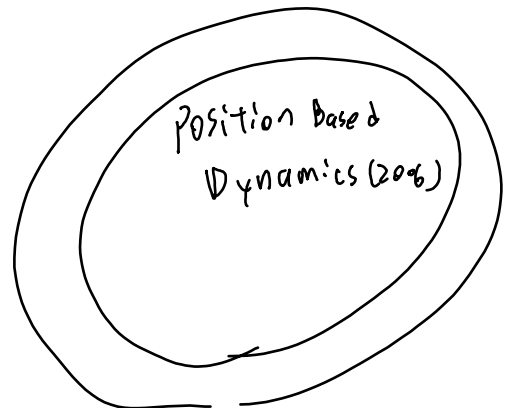
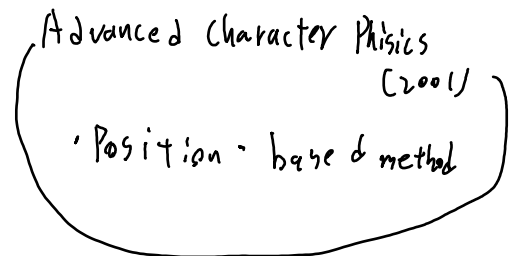
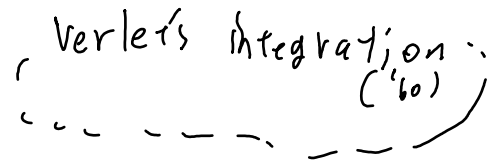


← force-based

→ position-based



the field of  
Geometric Optimization  
• manifold projection method

Example - Based  
Elastic Materials QoI  
• minimization  
formulation

Shape-Up: Shaping Discrete  
Geometry with Projections (2012)  
• block coordinate descent method  
(local/global solver)

PBP (2006)  
• Constraints  
• Position-based solver

Fast Simulation of Mass-Spring  
System (2013)  
• local/global physics solver

Distributed optimization  
and statistical learning  
via the Alternating  
direction method of  
Multipliers (2011)  
• ADMM

Projective dym. (2014)  
• Jacobi-type local/global solver  
• Constraint projection  
• Continuum dynamics-based constraints

ADMM  $\supset$  Projective dynamics (2016)  
• generic method that supports arbitrary  
conservative force (non-linear  
elastic materials  
• hard constraints  
e.t.c.)

PBD  
• position based method

Efficient simulation  
of Inextensible cloth (2007)  
• fast projection

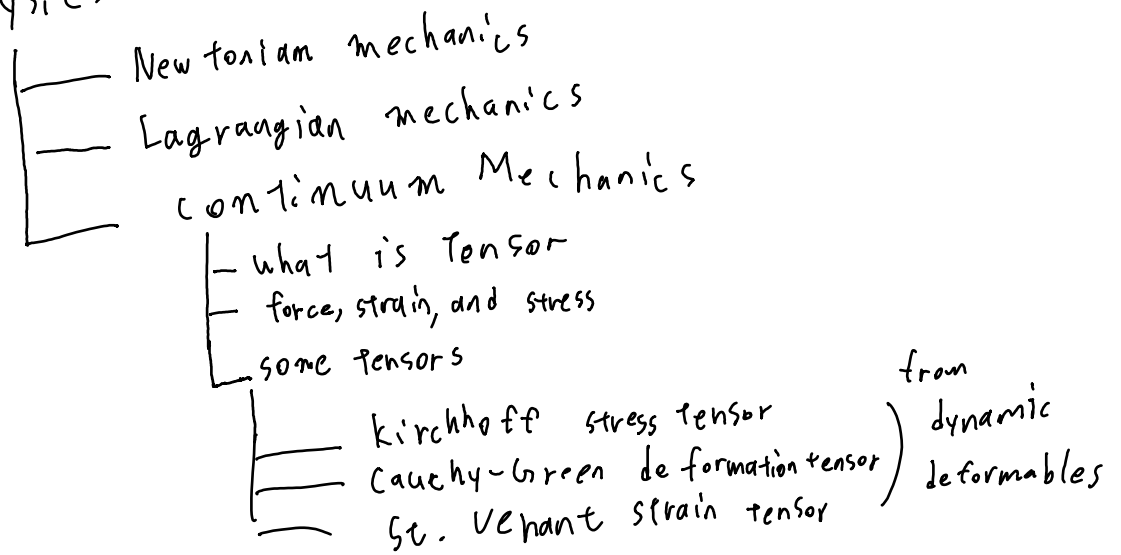
Interactive simulation  
of Elastic Deformable Materials  
(2006)  
• constraints with the physically-based  
parameters  
• compliant constraint formulation

Strain based Dynamics (2014)  
• The constraints from strain tensor  
• Decoupling stretch from shear resistance

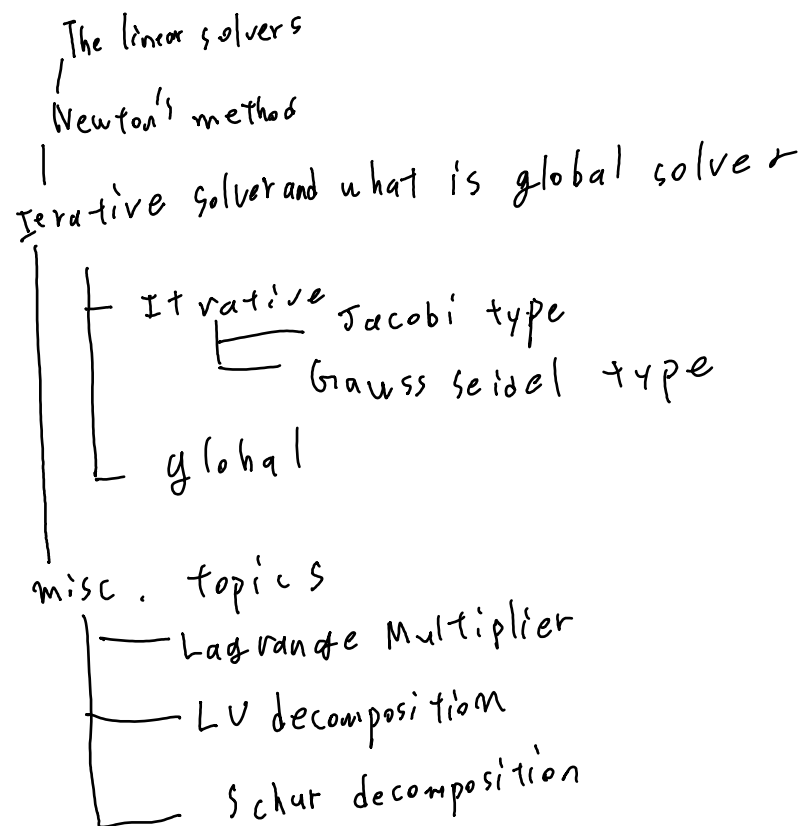
\*  
XPBD has an advantage  
against ST to Strain based Dyn.

XPBD (2017)  
• Physical-based constraints  
• total Lagrange multiplier ← (This formulation  
resolves the iteration  
dependency)

## • Physics



## • Numerical Integration



- Numerical physics

- └ mass-spring system
- └ Verlet's integration
- └ Explicit/Implicit euler scheme

- └ constrained Dynamics
- └ shape matching

column

Newton, Euler, Lagrangian

- Interpret the papers
- └ Lagrange, Hamiltonian, Messian
- └ Jacobi, Newton-Raphson, Newton-Euler equation
- └ Position Based Dynamics

Jacobian

- └ Large steps in Cloth Simulation (1998)

- └ Advanced Character Physics (2001)

- └ A versatile and Robust Model for Geometrically complex deformable solids (2004)

- └ Position Based Dynamics (2006)

- └ XPBD

- └ Geometric, Variational Integrators for Computer Animation (2006)

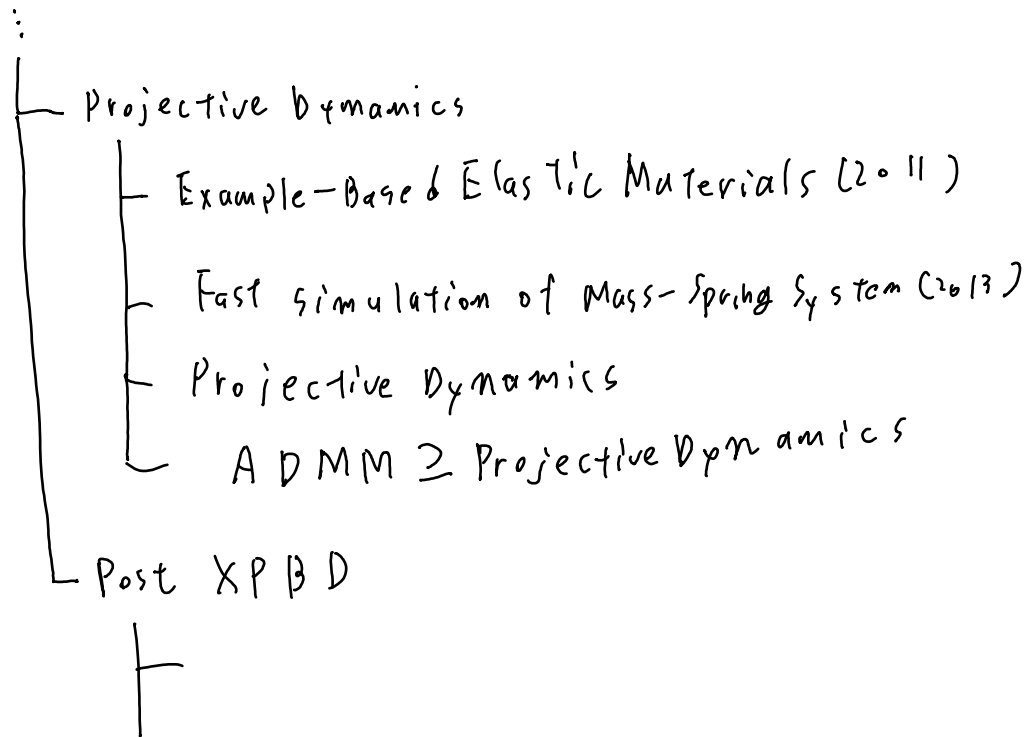
- └ Interactive simulation of Elastic Deformable Materials (2006)

- └ Efficient simulation of Inextensible cloth (2007)

- └ XPBD (2017)

- └ Small steps in physics simulation (2018)

- └ Projective Dynamics



XPBD p.3.

$$M(x^{n+1} - \bar{x}) - \nabla C(x^{n+1})^T z^{n+1} = 0 \quad \dots \textcircled{1}$$

$$C(x^{n+1}) + \bar{\alpha} z^{n+1} = 0 \quad \dots \textcircled{2}$$

$$\text{Let } x^{n+1} = x^n + \Delta x, \quad z^{n+1} = z^n + \Delta z \quad \dots \textcircled{3}$$

$$\text{and from } \textcircled{1} \quad M(x^n + \Delta x - \bar{x}) - \nabla C(x^{n+1})^T (z^n + \Delta z) = 0,$$

$$\begin{aligned} & \underbrace{M \Delta x - \nabla C(x^{n+1})^T z^n + \nabla C(x^{n+1})^T \Delta z}_{M \Delta x - \{\nabla C(x^{n+1}) - \nabla C(x^n)\} z^n - \nabla C(x^n) z^n} = -M(x^n - \bar{x}) \\ & \rightarrow M \Delta x - \{\nabla C(x^{n+1}) - \nabla C(x^n)\} z^n - \nabla C(x^n) z^n \end{aligned}$$

$$- \{\nabla C(x^{n+1}) - \nabla C(x^n)\} \Delta z - \nabla C(x^n) \Delta z^n = -M(x^n - \bar{x})$$

$$\widetilde{M \Delta x - \nabla C(x^n) \Delta z^n} - (\nabla C(x^{n+1}) - \nabla C(x^n)) (z^n + \Delta z) = -g(x, z)$$

$$- \widetilde{M \Delta x - \nabla C(x^n) \Delta z^n} - \frac{\partial C(x)}{\partial x} (z^n + \Delta z)^T \Delta x = -g(x, z)$$

$$- \left( M - \frac{\partial C(x)}{\partial x} \right) \Delta x - \nabla C(x^n) \Delta z^n - \frac{\partial C(x)}{\partial x} \Delta z \Delta x = -g(\bar{x}, z)$$

drop ( $\because \Delta z \Delta x \ll 1$ )

$$\rightarrow \left( \underbrace{M - \frac{\partial C(x)}{\partial x}}_{\text{Hessian}} \right) \Delta x - \nabla C(x^n) \Delta z^n = -g(x, z) \quad \dots \textcircled{5}$$

$k = \frac{\partial g}{\partial x}$  and from  $\textcircled{3}$  into  $\textcircled{2}$ ,

$$C(x^n + \Delta x) + \bar{\alpha} (z^n + \Delta z)$$

$$C(x^n + \Delta x) = C(x_n) + C(x_n) + \bar{\alpha} \Delta z = - \bar{\alpha} \Delta z \quad \textcircled{6}$$

$$\underbrace{\nabla C(x_i) \Delta x}_{\text{finally,}} \left[ \begin{array}{cc} K - \nabla C^T(x_i) \\ \nabla C(x_i) & \bar{\alpha} \end{array} \right] \begin{bmatrix} \Delta x \\ \Delta z \end{bmatrix} = - \begin{bmatrix} g(x_i, z_i) \\ h(x_i, z_i) \end{bmatrix}$$