

Basic theory behind (X)PBD

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1 Introduction

1.1 What is (X)PBD?

PBD (Position Based Dynamics) proposed at [1] is a popular method because of its stability and ease of implementation. The reason for them is the same, PBD computes physical simulation only using positions inside the [iterations](#) and all we have to do is compute displacement and modify them. In other words, we don't have to use complicated numerical analysis, it sounds pretty good.

But, in contrast to ease of implementation, it isn't easy to understand PBD's background theory. This is the problem when modifying PBD depending on your purpose.

If you start your research from the original PBD paper[1], you will wonder how the authors derive constraints' formulations or why this solver works well. Or you start from XPBD [2], you will be confused by the suddenly appeared Lagrange multiplier or energy potential that we don't know how to handle. Unfortunately, we can't know much from them and it may be common in literature search, there is no clear path to learning them. Then, I decided to write a guidebook on the underlying theory of PBD.

1.2 Difference from existing PBD coursenote

Actually, there are some course notes on PBD written by authors who published papers on PBD and XPBD, e.g. [3]. These course notes describe the basic style of PBD and its extensions. But there is the same problem we saw in [1] and [2], that is, how to implement is described but why this method works well is not. Thus, I believe that this document isn't meaningless. Well then, let's start the journey to XPBD!

2 The history of PBD

I think starting from history is a good way to learn something because there are no leaps in logic and it will be easy to understand where we are. However, there is certainly redundancy, so you can skip this section to save time. I'll make an effort to write that you can understand everything even if you skipped this section.

2.1 PBD's chronicle

The history of PBD can be roughly divided into three parts. Let me name them pre-PBD, post-PBD and post-XPBD.

2.2 pre-PBD

The general flow of pre-PBD began from the appearance of constraint dynamics([4], [5] and [6]) through “*Large Steps in Cloth Simulation*”[7] used [constraints](#) as shape representation and simulates cloth with energy form [constraints](#)' gradients, “*Advanced Character Physics*”[8] introduces position-based approach derived from Verlet's integration scheme with the distance constraints and “*A Versatile and Robust Model for Geometrically Complex Deformable Solids*”[9] generalize [7]'s method. And, finally,

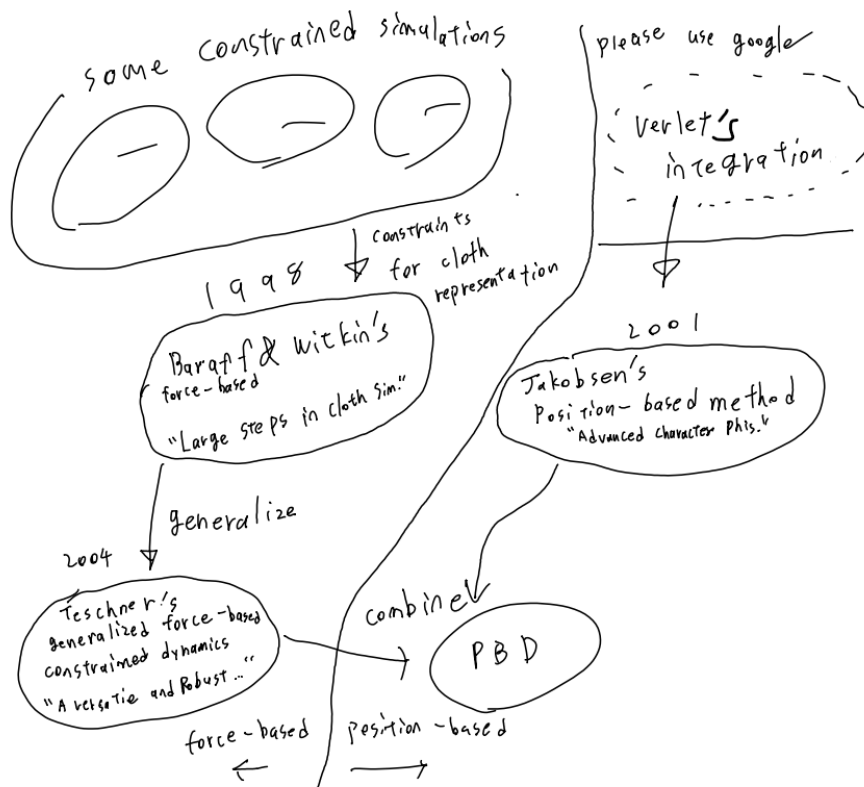


Fig. 1 history of pre-PBD

“Position Based Dynamics” [1] introduced the generalized constraints method from [9] into [8]’s position-based simulation to use various constraints.

2.2.1 Pioneer scholars

- Andrew Witkin
- David Baraff
- Alan Barr
- Ronen Barzel
- Jhon Platt
- Matthias Müller
- Matthias Teschner

2.3 post-PBD

2.3.1 Pioneer scholars

- Matthias Müller
- Nuttapong Chentanez

2.4 post-XPBD

2.4.1 Pioneer scholars

- Miles Macklin

A Importance of papers

A lower number means more important.

1. You must read these papers. But if you want to understand them completely, I recommend reading the others also.
 - “*Position Based Dynamics*”[1] is one of the main subjects of this document.
 - “*XPBD : position-based simulation of compliant constrained dynamics*”[2] addressed a numerical artifact that makes dependency between stiffness and iteration count or size of the time-step.
2. These papers are vital in understanding (X)PBD or have a strong impact on the field.
 - “*Advanced Character Physics*”[8] introduced position based simulation method derived from Verlet’s integration scheme and combined distance constraints. The framework of PBD can be seen here. Specifically, using distance and angular constraints, modifying vertices’ position directly and solving constraints with some iterations. This paper is easier to read than PBD, but the ideas appear here and pseudo codes are presented. I recommend reading this before read PBD.
 - “*Large steps in cloth simulation*”[7] solved implicit integration with constraints by conjugate gradient method for off-line simulation. The scheme isn’t used at PBD, but this one gives us a perspective of present physical simulations.
3. These papers offer interesting discussions around PBD or deepen your understanding of PBD.
 - “*Constraint Methods for Neural Networks and Computer Graphics*”[6] Describes the constraint methods for neural networks and computer graphics. It may not be easy to read because of 150 pages. But, if you have time, it’s more worth reading this paper than some papers published before this one.
4. These ones have historical value, but deeper discussions are done in other papers.
 - “*Elastically deformable models*”[10] brought the formulation of elastic bodies to computer graphics.
 - “*Energy Constraints On Parameterized Models*”[11] The shape representations presented in this paper are quite different from the current ones and the constraints presented in this paper are slightly inconvenient to the current ones. Thus, we no longer have to read this.
 - “*A modeling system based on dynamic constraints*”[12] uses constraints as models’ motion rather than to hold a model’s detail and uses linear simultaneous equations when deriving forces. The points that are difficult to understand are that the paper doesn’t describe the background of the equation derivation and that the symbols are scattered too much. Fortunately, we don’t have to read this

paper completely to understand present constrained dynamics because the style varies from the recent ones.

5. Not be classified yet.

- “A Versatile and Robust Model for Geometrically Complex Deformable Solids” [9]
- “Meshless deformations based on shape matching” [13]
- “Interactive simulation of elastic deformable materials” [14]
- “Efficient simulation of inextensible cloth” [15]
- “Fast Simulation of Inextensible Hair and Fur” [16]
- “Long Range Attachments - A Method to Simulate Inextensible Clothing in Computer Games” [17]
- “Position Based Fluids” [18]
- “Position-based simulation of continuous materials” [19]
- “Strain Based Dynamics” [20]
- “Unified particle physics for real-time applications” [21]
- “Air Meshes for Robust Collision Handling” [22]
- “A survey on position based dynamics, 2017” [3]
- “Stable Constrained Dynamics” [23]
- “Small steps in physics simulation” [24]
- “Non-Smooth Newton Methods for Deformable Multi-Body Dynamics” [25]
- “Detailed Rigid Body Simulation with Extended Position Based Dynamics” [26]
- “A Constraint-based Formulation of Stable Neo-Hookean Materials” [27]
- “Physically Based Shape Matching” [28]

- “” □
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B Glossary

B.1 symbols

B.2 terms

References

- [1] Müller, M., Heidelberger, B., Hennix, M., Ratcliff, J.: Position based dynamics. *J. Vis. Commun. Image Represent.* **18**(2), 109–118 (2007) <https://doi.org/10.1016/j.jvcir.2007.01.005>
- [2] Macklin, M., Müller, M., Chentanez, N.: Xpbd: position-based simulation of compliant constrained dynamics. In: *Proceedings of the 9th International Conference on Motion in Games. MIG '16*, pp. 49–54. Association for Computing Machinery, New York, NY, USA (2016). <https://doi.org/10.1145/2994258.2994272> . <https://doi.org/10.1145/2994258.2994272>
- [3] Bender, J., Müller, M., Macklin, M.: A survey on position based dynamics, 2017. In: *Proceedings of the European Association for Computer Graphics: Tutorials. EG '17*. Eurographics Association, Goslar, DEU (2017). <https://doi.org/10.2312/egt.20171034> . <https://doi.org/10.2312/egt.20171034>
- [4] Witkin, A., Fleischer, K., Barr, A.: Energy constraints on parameterized models. *SIGGRAPH Comput. Graph.* **21**(4), 225–232 (1987) <https://doi.org/10.1145/37402.37429>
- [5] Barzel, R., Barr, A.H.: A modeling system based on dynamic constraints. *SIGGRAPH Comput. Graph.* **22**(4), 179–188 (1988) <https://doi.org/10.1145/378456.378509>
- [6] Platt, J.C.: Constraint methods for neural networks and computer graphics. PhD thesis, USA (1990). UMI Order No: GAX90-00594
- [7] Baraff, D., Witkin, A.: Large steps in cloth simulation. In: *Proceedings of the 25th Annual Conference on Computer Graphics and Interactive Techniques. SIGGRAPH '98*, pp. 43–54. Association for Computing Machinery, New York, NY, USA (1998). <https://doi.org/10.1145/280814.280821> . <https://doi.org/10.1145/280814.280821>

- [8] Jakobsen, T.P.: Advanced character physics. (2003). <https://api.semanticscholar.org/CorpusID:5102710>
- [9] Teschner, M., Heidelberger, B., Muller, M., Gross, M.: A versatile and robust model for geometrically complex deformable solids. In: Proceedings Computer Graphics International, 2004., pp. 312–319 (2004). <https://doi.org/10.1109/CGI.2004.1309227>
- [10] Terzopoulos, D., Platt, J., Barr, A., Fleischer, K.: Elastically deformable models. SIGGRAPH Comput. Graph. **21**(4), 205–214 (1987) <https://doi.org/10.1145/37402.37427>
- [11] Witkin, A., Fleischer, K., Barr, A.: Energy constraints on parameterized models. SIGGRAPH Comput. Graph. **21**(4), 225–232 (1987) <https://doi.org/10.1145/37402.37429>
- [12] Barzel, R., Barr, A.H.: A modeling system based on dynamic constraints. SIGGRAPH Comput. Graph. **22**(4), 179–188 (1988) <https://doi.org/10.1145/378456.378509>
- [13] Müller, M., Heidelberger, B., Teschner, M., Gross, M.: Meshless deformations based on shape matching. ACM Trans. Graph. **24**(3), 471–478 (2005) <https://doi.org/10.1145/1073204.1073216>
- [14] Servin, M., Lacoursière, C., Melin, N.: Interactive simulation of elastic deformable materials. (2006). <https://api.semanticscholar.org/CorpusID:16851165>
- [15] Goldenthal, R., Harmon, D., Fattal, R., Bercovier, M., Grinspun, E.: Efficient simulation of inextensible cloth. ACM Trans. Graph. **26**(3), 49 (2007) <https://doi.org/10.1145/1276377.1276438>
- [16] Müller, M., Kim, T., Chentanez, N.: Fast simulation of inextensible hair and fur. (2012). <https://doi.org/10.2312/PE/vrphys/vrphys12/039-044>
- [17] Kim, T., Chentanez, N., Müller, M.: Long range attachments - a method to simulate inextensible clothing in computer games, pp. 305–310 (2012). <https://doi.org/10.2312/SCA/SCA12/305-310>
- [18] Macklin, M., Müller, M.: Position based fluids. ACM Trans. Graph. **32**(4) (2013) <https://doi.org/10.1145/2461912.2461984>
- [19] Bender, J., Koschier, D., Charrier, P., Weber, D.: Position-based simulation of continuous materials. Computers & Graphics **44**, 1–10 (2014) <https://doi.org/10.1016/j.cag.2014.07.004>
- [20] Müller, M., Chentanez, N., Kim, T.-Y., Macklin, M.: Strain based dynamics. In: Proceedings of the ACM SIGGRAPH/Eurographics Symposium on Computer

- Animation. SCA '14, pp. 149–157. Eurographics Association, Goslar, DEU (2015)
- [21] Macklin, M., Müller, M., Chentanez, N., Kim, T.-Y.: Unified particle physics for real-time applications. *ACM Trans. Graph.* **33**(4) (2014) <https://doi.org/10.1145/2601097.2601152>
 - [22] Müller, M., Chentanez, N., Kim, T.-Y., Macklin, M.: Air meshes for robust collision handling. *ACM Trans. Graph.* **34**(4) (2015) <https://doi.org/10.1145/2766907>
 - [23] Tournier, M., Nesme, M., Gilles, B., Faure, F.: Stable constrained dynamics. *ACM Trans. Graph.* **34**(4) (2015) <https://doi.org/10.1145/2766969>
 - [24] Macklin, M., Storey, K., Lu, M., Terdiman, P., Chentanez, N., Jeschke, S., Müller, M.: Small steps in physics simulation. In: *Proceedings of the 18th Annual ACM SIGGRAPH/Eurographics Symposium on Computer Animation. SCA '19*. Association for Computing Machinery, New York, NY, USA (2019). <https://doi.org/10.1145/3309486.3340247> . <https://doi.org/10.1145/3309486.3340247>
 - [25] Macklin, M., Erleben, K., Müller, M., Chentanez, N., Jeschke, S., Makoviychuk, V.: Non-smooth newton methods for deformable multi-body dynamics. *ACM Trans. Graph.* **38**(5) (2019) <https://doi.org/10.1145/3338695>
 - [26] Müller, M., Macklin, M., Chentanez, N., Jeschke, S., Kim, T.-Y.: Detailed rigid body simulation with extended position based dynamics. In: *Proceedings of the ACM SIGGRAPH/Eurographics Symposium on Computer Animation. SCA '20*. Eurographics Association, Goslar, DEU (2020). <https://doi.org/10.1111/cgf.14105> . <https://doi.org/10.1111/cgf.14105>
 - [27] Macklin, M., Muller, M.: A constraint-based formulation of stable neo-hookean materials. In: *Proceedings of the 14th ACM SIGGRAPH Conference on Motion, Interaction and Games. MIG '21*. Association for Computing Machinery, New York, NY, USA (2021). <https://doi.org/10.1145/3487983.3488289> . <https://doi.org/10.1145/3487983.3488289>
 - [28] Müller, M., Macklin, M., Chentanez, N., Jeschke, S.: Physically based shape matching. *Computer Graphics Forum* **41**, 1–7 (2023) <https://doi.org/10.1111/cgf.14618>