Basic theory behind (X)PBD

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Contents

1 Introduction

1.1 What is (X)PBD?

PBD (Position Based Dynamics) proposed at [1] is a popular method because of its stability and ease of implementation. The reason for them is the same, PBD computes physical simulation only using positions inside the iterations and all we have to do is compute displacement and modify them. In other words, we don't have to use complicated numerical analysis, it sounds pretty good.

But, in contrast to ease of implementation, it isn't easy to understand PBD's background theory. This is the problem when modifying PBD depending on your purpose.

If you start your research from the original PBD paper[1], you will wonder how the authors derive constraints' formulations or why this solver works well. Or you start from XPBD [2], you will be confused by the suddenly appeared Lagrange multiplier or energy potential that we don't know how to handle. Unfortunately, we can't know much from them and it may be common in literature search, there is no clear path to learning them. Then, I decided to write a guidebook on the underlying theory of PBD.

1.2 Difference from existing PBD coursenote

Actually, there are some course notes on PBD written by authors who published papers on PBD and XPBD, e.g. [3]. These course notes describe the basic style of PBD and its extensions. But there is the same problem we saw in [1] and [2], that is, how to implement is described but why this method works well is not. Thus, I believe that this document isn't meaningless. Well then, let's start the journey to XPBD!

2 The history of PBD

I think starting from history is a good way to learn something because there are no leaps in logic and it will be easy to understand where we are. However, there is certainly redundancy, so you can skip this section to save time. I'll make an effort to write that you can understand everything even if you skipped this section.

2.1 PBD's chronicle

The history of PBD can be roughly divided into three parts. Let me name them pre-PBD, post-PBD and post-XPBD.

2.2 pre-PBD

The general flow of pre-PBD began from the appearance of constraint dynamics([4], [5] and [6]) through "*Large Steps in Cloth Simulation*"[7] simulates cloth with constraints, "*Advanced Character Physics*"[8] introduces position-based approach with the distance constraint and "*A Versatile and Robust Model for Geometrically Complex Deformable Solids*"[9] derives several constraints as energy functions. And, finally, with these benefits, "*Position Based Dynamics*"[1] having the advantage I mentioned above was published.

2.2.1 Pioneer scholars

- Andrew Witkin
- David Barraff
- Alan Barr
- Ronen Barzel
- Jhon Platt
- Matthias Müller
- Matthias Teschner

2.3 post-PBD

2.3.1 Pioneer scholars

- Matthias Müller
- Nuttapong Chentanez

2.4 post-XPBD

2.4.1 Pioneer scholars

- Miles Macklin

A Inportance of papers

A lower number means more important.

1. You must read these papers. But if you want to understand them completely, I recommend reading the others also.
   - "*Position Based Dynamics*" [1] is one of the main subjects of this document.
   - "*XPBD: position-based simulation of compliant constrained dynamics*" [2] addressed a numerical artifact that makes dependency between stiffness and iteration count or size of the time-step.
2. These papers are vital in understanding (X)PBD or have a strong impact on the field.
   -
3. These papers offer interesting discussions around PBD or deepen your understanding of PBD.
   - "*Constraint Methods for Neural Networks and Computer Graphics*" [6] Tdescribes the constraint methods for neural networks and computer graphics. It may not be easy to read because of 150 pages. But, if you have time, it's more worth reading this paper than some papers published before this one.
4. These papers have historical value, but deeper discussions are done in other papers.
   - "*Elastically deformable models*" [10] brought the formulation of elastic bodies to computer graphics.
   - "*Energy Constraints On Parameterized Models*" [11]The shape representations presented in this paper are quite different from the current ones and the constraints presented in this paper are slightly inconvenient to the current ones. Thus, we no longer have to read this.
   - "*A modeling system based on dynamic constraints*" [12] uses constraints as models' motion rather than to hold a model's detail and uses linear simultaneous equations when deriving forces.The points that are difficult to understand are that the paper doesn't describe the background of the equation derivation and that the symbols are scattered too much. Fortunately, we don't have to understand this paper completely to understand recent constrained dynamics because the style varies from the recent ones.
5. Not be classified yet.
   - "*Large steps in cloth simulation*" [7]
   - "*Advanced Character Physics*" [8]
   - "*A Versatile and Robust Model for Geometrically Complex Deformable Solids*" [9]
   - "*Meshless deformations based on shape matching*" [13]

- "*Interactive simulation of elastic deformable materials*" [14]
- "*Efficient simulation of inextensible cloth*" [15]
- "*Fast Simulation of Inextensible Hair and Fur*" [16]
- "*Long Range Attachments - A Method to Simulate Inextensible Clothing in Computer Games*" [17]
- "*Position Based Fluids*" [18]
- "*Position-based simulation of continuous materials*" [19]
- "*Strain Based Dynamics*" [20]
- "*Unified particle physics for real-time applications*" [21]
- "*Air Meshes for Robust Collision Handling*" [22]
- "*A survey on position based dynamics, 2017*" [3]
- "*Stable Constrained Dynamics*" [23]
- "*Small steps in physics simulation*" [24]
- "*Non-Smooth Newton Methods for Deformable Multi-Body Dynamics*" [25]
- "*Detailed Rigid Body Simulation with Extended Position Based Dynamics*" [26]
- "*A Constraint-based Formulation of Stable Neo-Hookean Materials*" [27]
- "*Physically Based Shape Matching*" [28]
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B Glossaly

B.1 symbols

B.2 terms

Glossary

iteration In computer science, iteration is the process of repeating a series of instructions multiple times. Especially at (X)PBD, the part of physical solver which treats constraints is called iteration.. 2

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