FEM

primitive's parameters

$\varepsilon = Bu$ strain–displacement

$\sigma = D\varepsilon$ stress–strain

strain energy : $\frac{V}{2}\sigma^T\varepsilon = \frac{V}{2}u^T B^T D B u$ (V: volume)

external force : $u^T f$

total energy $\frac{V}{2}u^T B^T D B u - u^T f$

minimum parameter

$K = VB^T D B$

$Ku = f$

what we solve u

If the object has fixed point, the correspond coodinates are fixed.

$Ax = b$

This type equation is usually solved by a iterative method

e.g.) CG method

Newton phys.

mass

$M\ddot{x} = f$

$v = v_0 + \int_{t_0}^{t} f(t)/m \, dt$

$(\dot{x} = v)$

We want to know next step's configulation x_{t+1} and v_{t+1} when we have x_t, v_t and f_t.

One could write $\begin{cases} v_{t+1} = v_t + \Delta v \\ x_{t+1} = x_t + \frac{v_{t+1}}{\Delta t} \end{cases}$ (Δt: the time step)

And then, we can use explicit / implicit integration method to solve them on the computer

explicit integration

e.g. forward Euler method, Runge-Kutta Integration and Verlet Integration)

$\begin{pmatrix} \Delta x \\ \Delta v \end{pmatrix} = h \begin{pmatrix} v \\ M^{-1} f_t \end{pmatrix}$

we just assign past values.

implicit integration

$(\text{something})\,\Delta v = (\text{something})$

PBD

constraints

$x \leftarrow x + \Delta x$

XPBD