Some Constrained Simulations

・constraints for cloth representation

↓

Large steps in cloth simulation (1998)

・force-based implicit solver

↓

A versatile and Robust Model for Geometrically Complex Deformable Solids (2004)

・generalized constrained dynamics

Verlet's integration ('60)

↓

Advanced Character Phisics (2001)

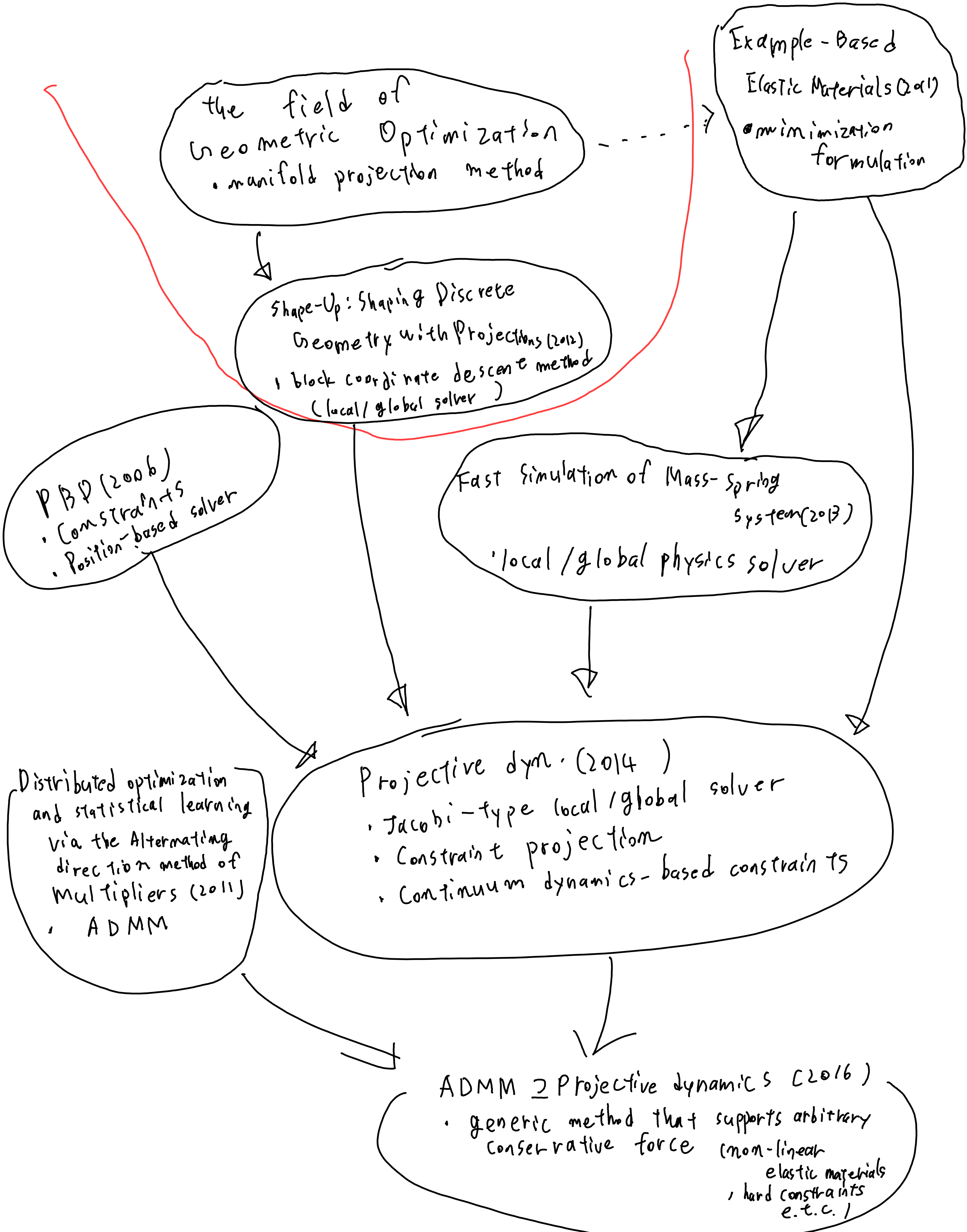
・Position-based method

↓

← force-based

position-based →

Position Based Dynamics (2006)



the field of Geometric Optimization
・manifold projection method
Example-Based Elastic Materials (2011)
・minimization formulation
Shape-Up: Shaping Discrete Geometry with Projections (2012)
・block coordinate descent method (local/global solver)
PBD (2006)
・Constraints
・Position-based solver
Fast Simulation of Mass-Spring System (2013)
・local/global physics solver
Distributed optimization and statistical learning via the Alternating direction method of multipliers (2011)
・ADMM
Projective dyn. (2014)
・Jacobi-type local/global solver
・Constraint projection
・Continuum dynamics-based constraints
ADMM ⊇ Projective dynamics (2016)
・generic method that supports arbitrary conservative force (non-linear elastic materials, hard constraints e.t.c.)

PBD
- position based method

Geometric, Variational Integrators for Computer Animation (2006)
- symplectic method

Efficient simulation of Inextensible cloth (2007)
- fast projection

Interactive simulation of Elastic Deformable Materials (2006)
- constraints with the phisically-based parameters
- compliant constraint formulation

Strain based Dynamics (2014)
- The constraints from strain tensor
- Decoupling stretch from shear resistance

XPBD has an advantage against to Strain based Dyn.

XPBD (2017)
- Physicall-based constraints
- total Lagrange multiplier ← This formulation resolves the iteration dependency

- Physics
  - Newtonian mechanics
  - Lagrangian mechanics
  - continuum Mechanics
    - what is Tensor
    - force, strain, and stress
    - some tensors
      - Kirchhoff stress tensor
      - Cauchy-Green deformation tensor
      - St. Venant strain tensor

      from dynamic deformables

- Numerical Integration
  - The linear solvers
  - Newton's method
  - Iterative solver and what is global solver
    - Itrative
      - Jacobi type
      - Gauss Seidel type
    - global
  - misc. topics
    - Lagrange Multiplier
    - LU decomposition
    - Schur decomposition

- Numerical physics
  - mass - spring system
  - Verlet's integration
  - Explicit / Implicit euler scheme
  - constrained Dynamics
  - shape matching

column

Newton, Euler, Lagrangian

Lagrange, Hamiltonian, Hessian

Jacobi, Newton-Raphthon, Newton-Euler equation

Jacobian

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- Interpret the papers
  - Position Based Dynamics
    - Large steps in Cloth Simulation (1998)
    - Advanced Character Physics (2001)
    - A versetile and Robust Model for Geometrically complex Deformable solids (2004)
    - Position Based Dynamics (2006)
  - XPBD
    - Geometric, Variational Integrators for Computer Animation (2006)
    - Interactive simulation of Elastic Deformable Materials (2006)
    - Efficient Simulation of Inextensible Cloth (2007)
    - XPBD (2017)
    - Small steps in Physics simulation (2018)
  - Projective Dynamics

⋮

⋮

- Projective Dymamics
  - Example-Based Elastic Materials (2011)
  - Fast simulation of Mass-Spring System (2013)
  - Projective Dynamics
  - ADMM ⊇ Projective Dynamics
- Post XPBD

XPBD p.3.

$$M(x^{n+1} - \tilde{x}) - \nabla C(x^{n+1})^T \lambda^{n+1} = 0 \quad \cdots ①$$

$$C(x^{n+1}) + \tilde{\alpha}\lambda^{n+1} = 0 \quad \cdots ②$$

$$\text{Let } x^{n+1} = x^n + \Delta x,\ \lambda^{n+1} = \lambda^n + \Delta\lambda \quad \cdots ③$$

and from ① $M(x^n + \Delta x - \tilde{x}) - \nabla C(x^{n+1})^T(\lambda^n + \Delta\lambda) = 0$,

$$M\Delta x - \nabla C(x^{n+1})^T\lambda^n + \nabla C(x^{n+1})^T\Delta\lambda = -M(x^n - \tilde{x})$$

$$M\Delta x - \{\nabla C(x^{n+1}) - \nabla C(x^n)\}\lambda^n - \nabla C(x^n)\lambda^n$$

$$- \{\nabla C(x^{n+1}) - \nabla C(x^n)\}\Delta\lambda - \nabla C(x^n)\Delta\lambda^n = -M(x^n - \tilde{x})$$

$$M\Delta x - \nabla C(x^n)\Delta\lambda^n - (\nabla C(x^{n+1}) - \nabla C(x^n))(\lambda^n + \Delta\lambda) = -g(x, \lambda)$$

$$M\Delta x - \nabla C(x^n)\Delta\lambda^n - \frac{\partial C(x)}{\partial x}(\lambda^n + \Delta\lambda)^T\Delta x = -g(x, \lambda)$$

$$\left(M - \frac{\partial C(x)}{\partial x}\right)\Delta x - \nabla C(x^n)\Delta\lambda^n - \frac{\partial C(x)}{\partial x}^T\Delta\lambda\Delta x = -g(x, \lambda)$$

drop ($\because \Delta\lambda\Delta x \ll 1$)

Hessian

$$\left(M - \frac{\partial C(x)}{\partial x}\right)\Delta x - \nabla C(x^n\Delta\lambda^n) = -g(x, \lambda) \quad \cdots ⑤$$

$K = \frac{\partial g}{\partial x}$ and from ③ into ②,

$$C(x^n + \Delta x) + \tilde{\alpha}(\lambda^n + \Delta\lambda)$$

$$\underbrace{C(x^n + \Delta x) - C(x_n)}_{\nabla(x_i)\Delta x} + C(x_n) + \tilde{\alpha}\Delta\lambda = - \quad \tilde{\alpha}\Delta\lambda \cdots ⑥$$

finally,

$$\begin{bmatrix} K & -\nabla C^T(x_i) \\ \nabla C(x_i) & \tilde{\alpha} \end{bmatrix}\begin{bmatrix} \Delta x \\ \Delta\lambda \end{bmatrix} = -\begin{bmatrix} g(x_i, \lambda_i) \\ h(x_i, \lambda_i) \end{bmatrix}$$