Basic theory behind (X)PBD

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Contents

1 Introduction

1.1 What is (X)PBD?

PBD (Position Based Dynamics) proposed at [4] is a popular method because of its stability and ease of implementation. The reason for them is the same, PBD computes physical simulation only using positions inside the iterations and all we have to do is compute displacement and modify them. In other words, we don't have to use complicated numerical analysis theories, it sounds pretty good.

But, in contrast to ease of implementation, it isn't easy to understand PBD's background theory. This is the problem when modifying PBD depending on your purpose.

If you start your research from the original PBD paper[4], you will wonder how the authors derive constraints' formulations or why this solver works well. Or you start from XPBD [7], you will be confused by the suddenly appeared Lagrange multiplier or energy potential that we don't know how to handle. Unfortunately, we can't know much from them and it may be common in literature search, there is no clear path to learning them. Then, I decided to write a guidebook on the underlying theory of PBD.

1.2 Difference from existing PBD coursenote

Actually, there are some course notes on PBD written by authors who published papers on PBD and XPBD, e.g. [13]. These course notes describe the basic style of PBD and its extensions. But there is the same problem we saw in [4] and [7], that is, how to implement is described but why this method works well is not. Thus, I believe that this document isn't meaningless. Well then, let's start the journey to XPBD!

2 The history of PBD

I think starting from history is a good way to learn something because there are no leaps in logic and it will be easy to understand where we are. However, there is certainly redundancy, so you can skip this section to save time. I'll make an effort to write that you can understand everything even if you skipped this section.

2.1 PBD's chronicle

The history of PBD can be roughly divided into three parts. Let them are pre-PBD, post-PBD and post-XPBD.

2.2 Pre-PBD

The flow of the pre-PBD began from the appearance of constraint dynamics([14], [15] and [16]) through "*Large* Steps in Cloth Simulation"[1] used constraints as shape representation and simulates cloth with energy form constraints' gradients, "*Advanced* Character Physics"[2] introduces position-based approach derived from Verlet's integration scheme with the distance constraints and "*A* Versatile and Robust Model for Geometrically Complex Deformable Solids"[3] generalize [1]'s method. And, finally,

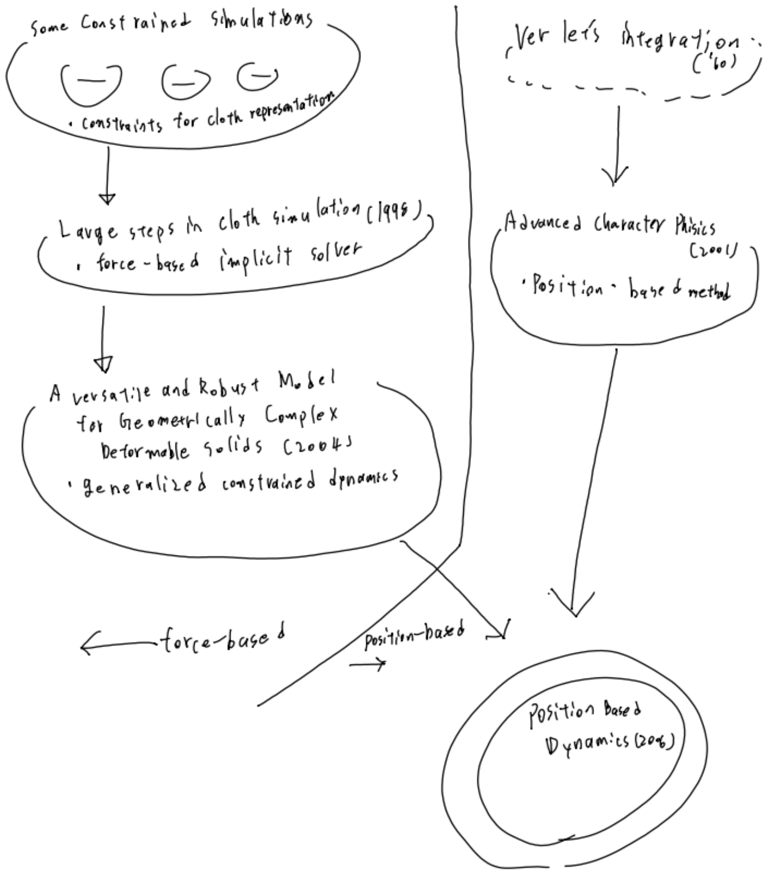

Fig. 1 history of pre-PBD

"*Position* Based Dynamics"[4] introduced the generalized constraints method from [3] into [2]'s position-based simulation to use various constraints.

2.3 Post-PBD

2.4 Post-XPBD

3 Physics

3.1 Newtonial mechanics

3.2 Lagrangian mechanics

3.3 Continuum mechanics

3.3.1 What is tensor?

3.3.2 Force, strain, and stress

3.4 Exambles of tensor

3.4.1 Kirchhoff stress tensor

3.4.2 Cauchy-Green deformation tensor

3.4.3 St. Venant strain tensor

4 Numerical integration

4.1 The linear solvers

4.2 Newton's method

4.3 Iterative solvers

4.4 What is the local/global solver

4.5 Misc. topics

4.5.1 Lagrange Multiplier

4.5.2 LU decomposition

4.5.3 Schur decomposition

5 Numerical physics

5.1 verlet's integration

5.2 mass-spring system

5.3 Explicit/implicit euler scheme

5.4 Constrained Dynamics

5.5 shape matching

6 Column : In the terminology mess

6.1 Newton*

6.2 Euler*

6.3 Jacobi*

6.4 Lagrange*, Hamilton*, Hesse*

7 Interplet the papers

7.1 To PBD

7.1.1 *Large steps in cloth simulation*[1]

7.1.2 *Advanced Character Physics*[2]

7.1.3 *A Versatile and Robust Model for Geometrically Complex Deformable Solids*[3]

7.1.4 *Position Based Dynamics*[4]

7.2 To XPBD

7.2.1 *Geometric, Variational Integrators for Computer Animation*[5]

7.2.2 *Interactive simulation of elastic deformable materials*[6]

7.2.3 *XPBD : position-based simulation of compliant constrained dynamics*[7]

7.2.4 *Small steps in physics simulation*[8]

7.3 Bonus section : Projective Dynamics

7.3.1 *Example-based elastic materials*[9]

7.3.2 *Fast simulation of mass-spring systems*[10]

7.3.3 *Projective Dynamics: Fusing Constraint Projections for Fast Simulation*[11]

7.3.4 *ADMM $\subseteq$ projective dynamics: fast simulation of general constitutive models*[12]

7.4 Post-XPBD

A Inportance of papers

A lower number means more important. The papers' names are aligned in the lexicographic order as possible as I can.

1. You must read these papers if you want to understand (X)PBD. But if you want to understand them completely, I recommend reading the others also.
   - "*Position* Based Dynamics"[4] is one of the main subjects of this document.
   - "*XPBD* : position-based simulation of compliant constrained dynamics"[7] addressed a numerical artifact that makes dependency between stiffness and iteration count or size of the time-step.
2. These papers are vital in understanding (X)PBD or have a strong impact on the field.
   - "*Advanced* Character Physics"[2] introduced position based simulation method derived from Verlet's integration scheme and combined distance constraints. The framework of PBD can be seen here. Specifically,using distance and angular constraints, modifying verticies' position directoly and solving constraints with some iterations. This paper is easier to read than PBD, but the ideas appear here and pseudo codes are presented. I recommend reading this before read PBD.
   - "*Large* steps in cloth simulation"[1] solved implicit integration with constraints by conjugate gradient method for off-line simulation. The scheme isn't used at PBD, but this one gives us a perspective of present physical simulations.
   - "*Projective* Dynamics: Fusing Constraint Projections for Fast Simulation"[11] provided a position-based method that uses energy formulation and local/global solver. The method has a local Jacobi-like solver and a linear global equation one. The solvers enable robust and fast simulation without safeguards against singular or indefinite Hessians. If you want to understand state-of-the-art physics simulation methods, including PBD, it's better to read this.
   - "*ADMM* $\subseteq$ projective dynamics: fast simulation of general constitutive models"[12] is worth reading because ADMM is actively researched now because of its robustness, parallelizability, and simplicity.
3. These papers offer interesting discussions around PBD, deepen your understanding of PBD, or description of basic physical simulation scheme.
   - "*Constraint* Methods for Neural Networks and Computer Graphics"[16] describes the constraint methods for neural networks and computer graphics. It may not be easy to read because of 150 pages. But, if you have time, it's more worth reading this paper than some papers published before this one.
   - "*Example-based* elastic materials"[9] provides a concept of the elastic manifold and optimization method for deforming into an artist-desirable state. However, this paper lacks reference to the manifold projection methods that are apparently well-known in the geometric optimization field.This paper helps us understand Projective dynamics, but I don't know Projective Dynamics's relevance with (X)PBD for now.

- "*Fast* simulation of mass-spring systems"[10] describes the local/global type solver to the mass-spring system clearly. Thus this paper helps us understand the variant solvers.
- "*Interactive* simulation of elastic deformable materials"[6] introduces physical parameters into constrained dynamics; the spirit is inherited by XPBD and there is an interesting discussion about the integrator. But I think this paper does explain the concept poorly,e. g. the integrator provided here, equation (20) lack of explanation, etc. Therefore, this one classified here.
- "*Nucleus:* Towards a unified dynamics solver for computer graphics"[17] is a good introduction to constrained dynamics because it shows its implementation aspect. However, it doesn't describe how constraints are resolved, so it isn't full-contained.
- "*Robust* treatment of collisions, contact and friction for cloth animation"[18]'s scheme separates physical simulation into internal parts and external parts. Therefore, we can choose the internal modeling(e.g. mass-spring) and the external modeling(e.g. collision repulsion, friction, or gravity) independently. However, the scheme is not directly related to PBD.
- "*Strain* Based Dynamics"[19]. The formulation of the strain tensor in XPBD seems to be based on "Interactive Simulation of Elastic Deformable Materials"(2006) rather than this paper. However, the formulations described here are easy to understand if you are familiar with PBD and strain tensor.
- "*Geometric,* Variational Integrators for Computer Animation"[5] presents the variational integrator from the Lagrangian/Hamiltonian physics that preserves linear/angular momenta. Its conservative quantities will be more important in fields such as robotics or accurate physical computation. But, this knowledge may be useless if you only want to understand (X)PBD.
- "*Efficient* simulation of inextensible cloth"[20] treated the constrained system as globally linearized form and solved with a direct approach at each iteration.

4. These ones have historical value, but deeper discussions are done in other papers.

   - "*Elastically* deformable models"[21] brought the formulation of elastic bodies to computer graphics.
   - "*Energy* Constraints On Parameterized Models"[**?**] uses shape representations that quite different from the current ones and the constraints presented in this paper are slightly inconvenient to the current ones. Thus, we no longer have to read this.
   - "*A* modeling system based on dynamic constraints"[22] uses constraints as models' motion rather than to hold a model's detail and uses linear simultaneous equations when deriving forces.The points that are difficult to understand are that the paper doesn't describe the background of the equation derivation and that the symbols are scattered too much. Fortunately, we don't have to read this paper completely to understand present constrained dynamics because the style varies from the recent ones.

5. Not be classified yet.

- “*A* Versatile and Robust Model for Geometrically Complex Deformable Solids” [3]
- “*Meshless* deformations based on shape matching” [23]
- “*Fast* Simulation of Inextensible Hair and Fur” [24]
- “*Long* Range Attachments - A Method to Simulate Inextensible Clothing in Computer Games” [25]
- “*Position* Based Fluids” [26]
- “*Position-based* simulation of continuous materials” [27]
- “*Unified* particle physics for real-time applications” [28]
- “*Air* Meshes for Robust Collision Handling” [29]
- “*A* survey on position based dynamics, 2017” [13]
- “*Stable* Constrained Dynamics” [30]
- “*Small* steps in physics simulation” [8]
- “*Non-Smooth* Newton Methods for Deformable Multi-Body Dynamics” [31]
- “*Detailed* Rigid Body Simulation with Extended Position Based Dynamics” [32]
- “*A* Constraint-based Formulation of Stable Neo-Hookean Materials” [33]
- “*Physically* Based Shape Matching” [34]
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B Glossaly

B.1 symbols

B.2 terms

Glossary

conjugate gradient method : This method, abbreviated as CG method is used linear equations that form is $A\boldsymbol{x} = \boldsymbol{b}$. This method often has high computational cost . 7
constraint : At the (X)PBD, constraint is relationship among arbitrary vertices what we want to keep during simulation. It can express not only between two vertices relation, e.g. distance a vertex to another one, but among more than three vertices, e.g. volume of the tetrahedron . 3, 4, 7, 10

iteration : In computer science, iteration is the process of repeating a series of instructions multiple times. Especially at (X)PBD, the part of physical solver which treats constraints is called iteration . 3, 7

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