

```
def tribonacci_recursive(n):
```

```
    if n == 0 or n == 1:
```

```
        return 0
```

```
    elif n == 2:
```

```
        return 1
```

```
    else:
```

```
        return tribonacci_recursive(n-3) + tribonacci_recursive(n-1) + tribonacci_recursive(n-2)
```

- 3 comparisons, 3 subtractions, 2 additions

let:

$n-k=0$

$k=n$

$$T(n) = T(n-3) + T(n-1) + T(n-2) + 3 + 3 + 2$$

$$= T(n-3) + T(n-1) + T(n-2) + 8$$

$$= 3^k \cdot T(n-k) + (3^k - 1) \cdot C$$

$$= 3^n \cdot T(n-n) + (3^n - 1) \cdot C$$

$$= 3^n \cdot T(0) \cdot (1 + C) - C$$

$$T(n) = 3^n$$

```
def tribonacci_iterative(n):
```

```
    if n == 0 or n == 1:
```

```
        return 0
```

```
    elif n == 2:
```

```
        return 1
```

```
    n3 = 0
```

```
    n2 = 1
```

```
    n1 = 1
```

```
    trib = n3 + n2 + n1
```

```
    for i in range(3, n-1):
```

```
        n3 = n2
```

```
        n2 = n1
```

```
        n1 = trib
```

```
        trib = n3 + n2 + n1
```

```
    return trib
```

$$T(n) = n + 3$$

Proof by properties of O :

$$T(n) = n + 3$$

$$= O(n + 3) \text{ (trivial)}$$

$$= O(n) \text{ (constant factor)}$$

$$T(n) = n + 3 \in O(n)$$

```
def tribonacci_memoized(n):
```

```
    C = [None] * max(n+1, 3)
```

```
    C[0] = 0
```

```
    C[1] = 0
```

```
    C[2] = 1
```

```
    trib = tribonacci_memoized_helper(n, C)
```

```
    return trib
```

```
def tribonacci_memoized_helper(n, C):
```

```
    if C[n] is not None:
```

```
        return C[n]
```

```
    else:
```

```
        result = tribonacci_memoized_helper(n-3, C) + tribonacci_memoized_helper(n-2, C) + tribonacci_memoized_helper(n-1, C)
```

```
        C[n] = result
```

```
    return result
```

$$T(n) = O(n)$$

```
def tribonacci_dynamic(n):
```

```
    A = [None] * max(n+1, 3)
```

```
    A[0] = 0
```

```
    A[1] = 0
```

```
    A[2] = 1
```

```
    for i in range(3, n+1):
```

```
        A[i] = A[i-3] + A[i-2] + A[i-1]
```

```
    return A[n]
```

$$T(n) = O(n)$$

- we are using the for loop to go through the arrays n times.