MECHANICAL ENGINEERING Prof. S. Gutschmidt S2 2025	Dynamics & Vibrations (ENME 203) Project Assignment	UC UNIVERSITY OF CANTERBURY Te Whare Wānanga o Waitaha CHRISTCHURCH NEW ZEALAND

Assigned on: 29 September 2025	**Due Date: 17 October 2025 (5 pm)**

Part 3 - Vibration Analysis of an Airfoil Section [4+4+2 %]

This part of the project focuses on the motion response of the wing section for different flight speeds U. As derived in Project Parts 1 & 2, the analysis is based on the two governing equations, describing the flexural bending and torsional motion of the wing section

$$m\,\ddot{y} - ml_\theta\,\ddot{\theta} + d\,\dot{y} + k\,y = L\,, \qquad I^F\,\ddot{\theta} - ml_\theta\,\ddot{y} + d_\theta\,\dot{\theta} + k_\theta\,\theta = l_\alpha L\,.$$

Note, that these equations were linearized about the wing's stable horizontal equilibrium position $[y = y_{eq}, \theta_{eq} = 0]$. Parameters of the wing section are as defined in Parts 1 & 2 and given below. Distinctive points G, F and A mark the centre of mass, flexural axis and aerodynamic centres, while $y = y_F$ and θ are the variables representing bending and torsion, respectively.

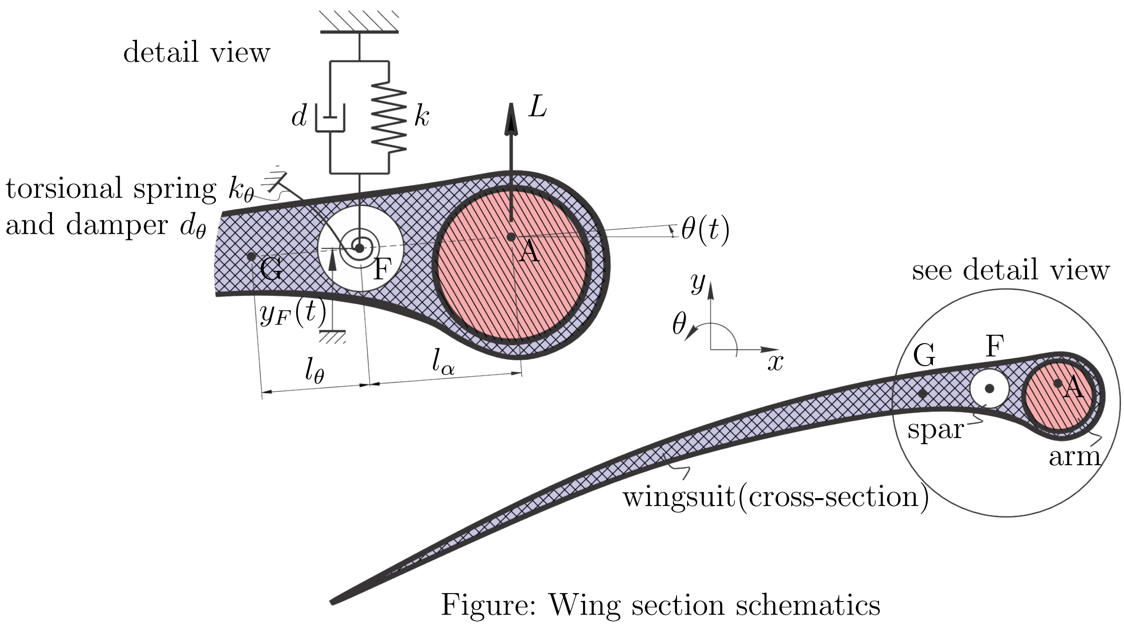

Figure: Wing section schematics

Given parameters:
$k = 1500$ N/m, $k_\theta = 200$ Nm/rad, $d = 150$ Ns/m, $d_\theta = 0.03$ Nms/rad, $l_\theta = 0.1$ m, $l_\alpha = 0.05$ m, $m = 2$ kg, $I^F = 0.13$ kgm^2, $C_\theta = 0.4$ kg/m, $C_y = 0.6$ kg/m, $0 \leq U \leq 550$ km/h, $0 \leq t \leq 1$ s.

The lift function L is given by $L = C_\theta U^2\theta + C_y U\dot{y}$. U is the airspeed. These parameter values are loosely based on a wingsuit, for a 0.25 m section of arm wing.

a) Consider the case $l_\theta = 0$, $C_y = 0$ for which the equations decouple. At a critical value of airspeed U the wing section's torsional mode will become statically unstable, which is known as divergence. Derive the expression of the critical airspeed value U_{div} and compute it for the given set of parameters. Plot the natural angular frequency $\omega = f(U)$ as a function of air speed U to validate this critical value.

b) Consider another case for which the equations decouple, namely for $l_\theta = 0$, $C_\theta = 0$. For this case and at another critical value of the airspeed U, the wing's bending mode becomes dynamically unstable, which is known as flutter instability. Derive the expression of the critical airspeed value $U_{flutter}$ and compute it for the given set of parameters. Plot the modal damping $\zeta\omega = f(U)$ as a function of the air speed U to validate this critical value.

   Furthermore, using a numerical integration solver of your choice (e.g. Euler, Newmark-β, Runge Kutta), simulate the motion of the wing section y for $U < U_{flutter}$ and $U > U_{flutter}$ to confirm the critical flutter airspeed.

c) Consider the fully coupled system and carry out a modal analysis (e.g. using the command `polyeig` in MATLAB or `numpy.linalg.eig` in PYTHON). Plot modal damping and frequency (from roots) as a function of airspeed U for $U \in [0\ 500]$ km. Identify and discuss all instability events in comparison to the scenarios of the decoupled systems. (Re-plot graphs from a) and b) in the same diagrams for comparison.)

d) Modify any *design* parameter(s) of the wing section to ensure a fully stable flight behaviour for airspeeds up to 330 km/h. Re-plot modal frequency and damping characteristics, together with a time-response simulation(s) to confirm a stable performance. Briefly describe what this change of parameter(s) may entail for the wingsuit and/or wingsuit operator.