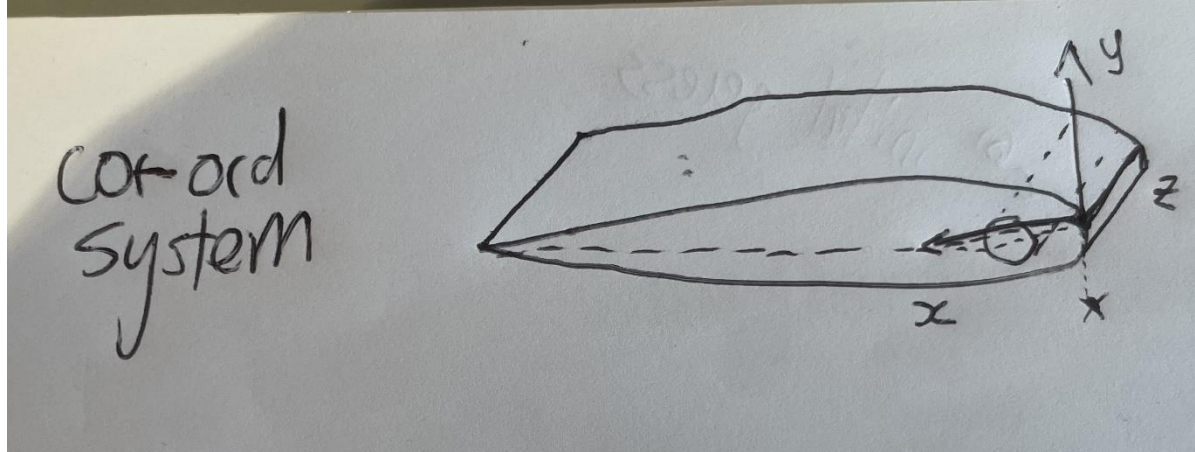


ENME203 Project Part 1 – Summary Answer Sheet

a)

$$G_{com} = (0.419, 0, 0.125)$$



$t = 0.12$ $l_0 = 0.1$ $l_x = 0.05$ $m = 2 \text{ kg}$ $\mu = 24.48 \text{ kg/m}^2$
 $A = \int_0^1 y_{up}(x) - y_{low}(x) dx = \int_0^1 2y_{up}(x) dx$
 $A = 1.2 \int_0^1 0.2969\sqrt{x} - 0.126x - 0.3516x^2 + 0.2843x^3 - 0.1036x^4 dx$
 $= 1.2 \left[0.2969 \left(\frac{2}{3} \right) - 0.1260 \left(\frac{1}{2} \right) - 0.3516 \left(\frac{1}{3} \right) + 0.2843 \left(\frac{1}{4} \right) - 0.1036 \left(\frac{1}{5} \right) \right]$
 $= 0.0817 \text{ m}^2$
 $\bar{x} = \frac{1}{A} \int_0^1 x \cdot 2y_{up}(x) dx$
 $= \frac{1.2}{0.0817} \int_0^1 x \left[0.2969\sqrt{x} - 0.126x - 0.3516x^2 + 0.2843x^3 - 0.1036x^4 \right] dx$
 $= \frac{1.2}{0.0817} \left[0.2969 \left(\frac{2}{5} \right) - 0.1260 \left(\frac{1}{3} \right) - 0.3516 \left(\frac{1}{4} \right) + 0.2843 \left(\frac{1}{5} \right) - 0.1036 \left(\frac{1}{6} \right) \right]$
 $= \frac{1.2}{0.0817} \times 0.02845 = 0.4186 \text{ m}$
 $\bar{y} = 0$ due to symmetry
 $\bar{z} = 0.125 \left(\frac{0.250}{2} \right)$ due to symmetry
 $G_{com} = (0.419, 0, 0.125)$

b)

$$G_{com} = (0.357, 0, 0.125)$$

b density ρ arm density 4ρ arm $r = 0.05$
 wing surface volume = $V_w = 0.0817 \times 0.250$ arm centre 0.15 from leading edge
 $= 0.0204$
 $= (0.0817 - \frac{9}{4} \times 0.1^2) \times 0.250 = 0.07384 \text{ m}^3$
 $\times 0.250 = 0.01846$

$$\text{arm volume} = \frac{\pi}{4} (0.1)^2 \times 0.250 = 0.00464 \text{ m}^3$$

total $2 = A_{wing} \cdot \rho \cdot w + A_{arm} \cdot 4\rho \cdot w$
 $M_{wing} = 0.0739 \cdot 75.9 \cdot 0.250$
 $= 1.404$
 $M_{arm} = 0.00464 \cdot 4 \cdot 75.9 \cdot 0.250$
 $= 0.546 \text{ kg}$
 $\bar{x} = \frac{(M_{wing} \cdot \bar{x}_{wing} + M_{arm} \cdot \bar{x}_{arm})}{(M_{wing} + M_{arm})} = 0.379 \text{ m}$

$$= (0.379, 0, 0.125)$$

b $M_{total} = 0.0817\rho + 3\pi r^2\rho$

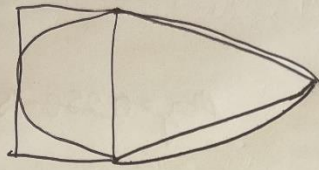
$$\frac{\sum m_i x_i}{\sum m_i} = \frac{0.0817 \times 0.416 + 3\pi r^2 \times 0.15}{0.0817 + 3\pi r^2} = 0.357 \text{ m}$$

$$COM = (0.357, 0, 0.125)$$

c)

$$I_{zz}^G = (0.106003 \text{ kgm}^2) \quad I_{zz}^F = (0.127624 \text{ kgm}^2)$$

c



$$t(x) = \begin{cases} t_{\max} & (0 \leq x < a) \\ t_{\max} \left(1 - \frac{x-a}{1-a}\right) & a \leq x < 1 \end{cases} \quad (q=0.3)$$

$$A = \int_0^1 f(x) dx = t_{\max} a + \frac{t_{\max}(1-a)}{2} = t_{\max} \frac{a+1}{2}$$

$$M_x = \int_0^1 x t_{\max}(x) dx = t_{\max} \frac{a^2 + a + 1}{6}$$

$$\frac{x_c}{A} = \frac{M_x}{A} = \frac{a^2 + a + 1}{3(a+1)}$$

$$I_o = \mu(I_1 + I_2)$$

$$I_1 = \int_0^1 x^2 t(x) dx$$

$$\begin{aligned} I_2 &= \int_0^1 \int_{-h(x)}^{h(x)} y^2 dy dx \\ &= \int_0^1 \frac{2}{3} h(x)^3 dx \\ &= \frac{1}{12} \int_0^1 t(x)^3 dx \end{aligned}$$

$$I_o = \mu \left[t_{\max} \frac{a^3 + a^2 + a + 1}{12} + t_{\max}^3 \frac{3a+1}{48} \right]$$

$$\begin{aligned} I_{G_{zz}} &= I_o - m(x_g^2 + y_g^2) \\ &= 0.106003 \text{ kg} \cdot \text{m}^2 \end{aligned}$$

$$\begin{aligned} I_{F_{zz}} &= I_{G_{zz}} + m(x_F - x_G)^2 \\ &= 0.127624 \text{ kg} \cdot \text{m}^2 \end{aligned}$$

d)

Calculated	CAD
$I_{ZZ}^G = (0.106003 \text{ kgm}^2)$	$I_{ZZ}^G = (0.110387 \text{ kgm}^2)$
$I_{ZZ}^F = (0.127624 \text{ kgm}^2)$	$I_{ZZ}^F = (0.129552 \text{ kgm}^2)$
Com = (0.419,0,0.125)	Com = (0.4178,0,0.125)

Enme203

Options...

Override Mass Properties...

Recalculate

☒ Include hidden bodies/components

☐ Show weld bead mass

Report coordinate values relative to: -- default --

Mass properties of Enme203

Configuration: Default

Coordinate system: -- default --

Mass (user-overridden) = 2.00000000 kilograms

Volume = 0.02042651 cubic meters

Surface area = 0.67329941 square meters

Center of mass: (meters)

X = 0.41788837

Y = 0.00000000

Z = 0.12500000

Principal axes of inertia and principal moments of inertia: (kilograms * square m

Taken at the center of mass.

lx = (1.00000000, 0.00000000, 0.00000000)

ly = (0.00000000, 0.00000000, -1.00000000)

lz = (0.00000000, 1.00000000, 0.00000000)

Px = 0.01207553

Py = 0.11038774

Pz = 0.11914554

Moments of inertia: (kilograms * square meters)

Taken at the center of mass and aligned with the output coordinate system. (U

Lxx = 0.01207553

Lyy = 0.00000000

Lzz = 0.00000000

Lxy = 0.00000000

Lyx = 0.11914554

Lyz = 0.00000000

Lzx = 0.00000000

Lzy = 0.00000000

Lzz = 0.11038774

Moments of inertia: (kilograms * square meters)

Taken at the output coordinate system. (Using positive tensor notation.)

lxx = 0.04332553

lxy = 0.00000000

lzx = 0.10447209

lyx = 0.00000000

lyy = 0.49965692

lzy = 0.00000000

lzx = 0.10447209

lzy = 0.00000000

lzz = 0.45964912

Help

Print...

Copy to Clipboard

Enme203

Options...

Override Mass Properties...

Recalculate

☒ Include hidden bodies/components

☐ Show weld bead mass

Report coordinate values relative to: Coordinate System1

Mass properties of Enme203

Configuration: Default

Coordinate system: Coordinate System1

Mass (user-overridden) = 2.00000000 kilograms

Volume = 0.02042651 cubic meters

Surface area = 0.67329941 square meters

Center of mass: (meters)

X = 0.09788837

Y = 0.00000000

Z = 0.00000000

Principal axes of inertia and principal moments of inertia: (kilograms * square m

Taken at the center of mass.

lx = (1.00000000, 0.00000000, 0.00000000)

ly = (0.00000000, 0.00000000, -1.00000000)

lz = (0.00000000, 1.00000000, 0.00000000)

Px = 0.01207553

Py = 0.11038774

Pz = 0.11914554

Moments of inertia: (kilograms * square meters)

Taken at the center of mass and aligned with the output coordinate system. (U

Lxx = 0.01207553

Lyy = 0.00000000

Lzz = 0.00000000

Lxy = 0.00000000

Lyx = 0.11914554

Lyz = 0.00000000

Lzx = 0.00000000

Lzy = 0.00000000

Lzz = 0.11038774

Moments of inertia: (kilograms * square meters)

Taken at the output coordinate system. (Using positive tensor notation.)

lxx = 0.01207553

lxy = 0.00000000

lzx = 0.00000000

lyx = 0.00000000

lyy = 0.13830981

lzy = 0.00000000

lzx = 0.00000000

lzy = 0.00000000

lzz = 0.12955200

Help

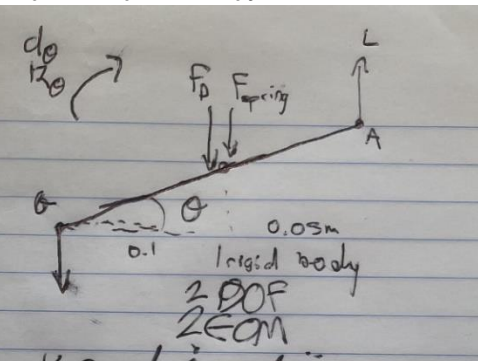
Print...

Copy to Clipboard

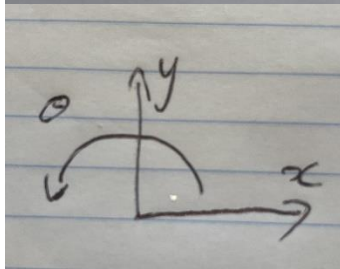
e)

$$m\ddot{y} = C_\theta U^2 \theta + C_y U \dot{y} - mg - d\dot{y} - ky$$

$$I^F \ddot{\theta} - C_\theta U^2 \theta l_\alpha - C_y U \dot{y} L_\alpha - mgl_\theta + K_\theta \theta + d_\theta \dot{\theta} - ml_\theta \ddot{y} = 0$$



$\Sigma F = m\ddot{y} = FL - F_D - F_{\text{spring}} - mg$
 $FL = C_\theta U^2 \theta + C_y U \dot{y}$
 $- m\ddot{y} = C_\theta U^2 \theta + C_y U \dot{y} - mg - d\dot{y} - ky$
 $I^F \ddot{\theta} = (I_\theta U^2 \theta + C_y U \dot{y}) l_\alpha + mgl_\theta - K_\theta \theta - d_\theta \dot{\theta} + ml_\theta \ddot{y}$
 $- I^F \ddot{\theta} - C_\theta U^2 \theta l_\alpha - C_y U \dot{y} l_\alpha - mgl_\theta + K_\theta \theta + d_\theta \dot{\theta} - ml_\theta \ddot{y} = 0$
 $m\ddot{y} - C_y U \dot{y} + d\dot{y} + ky = C_\theta U^2 \theta = -mg$



Appendix

(Put your full working here, or merge working pages to the PDF submission)

You can make small adjustments the answer boxes to fit more working/evidence if needed, provided that the question order remains consistent.

In the summary answers, show the key steps of your working (e.g. final integration equations), note the methods used (e.g. numerical solvers, Symbolab, Desmos, etc), any assumptions, and the final answers.

$t = 0.12$ $b = 0.1$ $h = 0.05$ $m = 2 \text{ kg}$ $M = 24.48 \text{ kg/m}^3$
 $A = \int_0^1 (y_{\text{top}}(x) - y_{\text{bot}}(x)) dx = \int_0^1 2y_{\text{top}}(x) dx$
 $A = 1.2 \int_0^1 (0.2969\sqrt{x} - 0.126x - 0.3516x^2 + 0.2843x^3 - 0.1036x^4) dx$
 $= 1.2 [0.2969(\frac{2}{3}) - 0.126(\frac{1}{2}) - 0.3516(\frac{1}{3}) + 0.2843(\frac{1}{4}) - 0.1036(\frac{1}{5})]$
 $= 0.0817 \text{ m}^2$
 $\bar{x} = \frac{1}{A} \int_0^1 x \cdot 2y_{\text{top}}(x) dx$
 $= \frac{1}{0.0817} \int_0^1 x [0.2969\sqrt{x} - 0.126x - 0.3516x^2 + 0.2843x^3 - 0.1036x^4] dx$
 $= \frac{1}{0.0817} [0.2969(\frac{2}{5}) - 0.126(\frac{1}{3}) - 0.3516(\frac{1}{4}) + 0.2843(\frac{1}{5}) - 0.1036(\frac{1}{6})]$
 $= \frac{1}{0.0817} \times 0.02845 = 0.4136 \text{ m}$
 $\bar{y} = 0$ due to symmetry $= (0.419, 0, 0.125)$
 $\bar{z} = 0.125$ ($\frac{0.250}{2}$) due to symmetry

b density ρ arm density ρ_p arm $r = 0.05$
 wing area $V_{\text{wing}} = V_{\text{arm}} = 0.020425$
 $V_{\text{arm}} = \pi r^2 L = \pi (0.05)^2 \times 0.250 = 0.0049087$
 $\rho_p = \frac{M}{V_{\text{arm}}} = \frac{2}{0.0049087} = 407.4 \text{ kg/m}^3$
 $M_{\text{wing}} = 0.0734 \times 75.9 \times 0.250 = 1.404$
 $M_{\text{arm}} = 0.0049087 \times 407.4 = 2.0$
 $\bar{x} = \frac{M_{\text{wing}} \cdot \bar{x}_{\text{wing}} + M_{\text{arm}} \cdot \bar{x}_{\text{arm}}}{M_{\text{wing}} + M_{\text{arm}}} = \frac{1.404 \times 0.419 + 2.0 \times 0.125}{1.404 + 2.0} = 0.279 \text{ m}$

c
 $f(x) = \begin{cases} t_{\text{max}}(1 - \frac{x}{a}) & 0 \leq x \leq a \\ 0 & x > a \end{cases}$ ($q = 0.3$)
 $A = \int_0^a f(x) dx = t_{\text{max}} a - \frac{t_{\text{max}}}{2} \frac{a^2}{a} = t_{\text{max}} \frac{a}{2}$
 $M_x = \int_0^a x f(x) dx = t_{\text{max}} \frac{a^3}{6}$
 $\bar{x} = \frac{M_x}{A} = \frac{a^2/6}{a/2} = \frac{a}{3}$
 $I_0 = u(I_1 + I_2)$ $I_1 = \int_0^a x^2 f(x) dx = \int_0^a x^2 (t_{\text{max}}(1 - \frac{x}{a})) dx = t_{\text{max}} \int_0^a (x^2 - \frac{x^3}{a}) dx = t_{\text{max}} [\frac{x^3}{3} - \frac{x^4}{4a}]_0^a = t_{\text{max}} [\frac{a^3}{3} - \frac{a^4}{4a}] = t_{\text{max}} \frac{a^3}{12}$
 $I_2 = \int_0^a y^2 f(x) dy = \int_0^a [\int_0^y f(x) dx] y^2 dy = \int_0^a [\int_0^y t_{\text{max}}(1 - \frac{x}{a}) dx] y^2 dy = \int_0^a t_{\text{max}} [\frac{xy}{2} - \frac{x^2}{4a}]_0^y y^2 dy = \int_0^a t_{\text{max}} [\frac{y^3}{2} - \frac{y^4}{4a}] dy = t_{\text{max}} [\frac{y^4}{8} - \frac{y^5}{20a}]_0^a = t_{\text{max}} [\frac{a^4}{8} - \frac{a^5}{20a}] = t_{\text{max}} \frac{a^4}{40}$
 $I_{\text{c.m.}} = I_0 - m(\bar{x}^2 + \bar{y}^2)$ $I_{\text{c.m.}} = I_{\text{c.m.}} + m(\bar{x}^2 + \bar{y}^2)$
 $= 0.108003 \text{ kg} \cdot \text{m}^2$ $= 0.127624 \text{ kg} \cdot \text{m}^2$

$LZ^{(1)} = x^{(1)}$
 20

b $\bar{x} = \frac{m_{\text{wing}} \cdot \bar{x}_{\text{wing}} + m_{\text{arm}} \cdot \bar{x}_{\text{arm}}}{(m_{\text{wing}} + m_{\text{arm}})}$

$m_{\text{wing}} = 0.250 \times 75.9 \times 0.020425$

total volume = 0.020425
 volume wing = $\pi \times 0.1^2 \times 0.250 = \frac{1}{4000}$
 0.020425

$$\Sigma F = m\ddot{y} = FL - F_0 - F_{\text{spring}} - mg$$

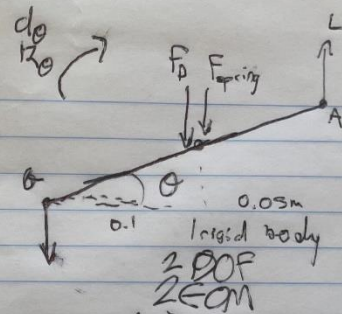
$$FL = C_0 U^2 \theta + c_y U \dot{y}$$

$$- m\ddot{y} = C_0 U^2 \theta + c_y U \dot{y} - mg - d\dot{y} - k_y y$$

$$\ddot{\theta} \quad I_{\theta}^E = (I_0 U^2 \theta + c_y U \dot{y}) l_a + mg l_0 - K_0 \theta - d_0 \dot{\theta} + m l_0 \ddot{y}$$

$$- I_{\theta}^E \ddot{\theta} - C_0 U^2 \theta l_a - c_y U \dot{y} l_a - mg l_0 + K_0 \theta + d_0 \dot{\theta} - m l_0 \ddot{y} = 0$$

$$m\ddot{y} - c_y U \dot{y} + d\dot{y} + k_y y - C_0 U^2 \theta = -mg$$



d)

$$a_{10} = a_c + \dot{\omega}$$

$$= \begin{pmatrix} 0 \\ -a \\ 0 \end{pmatrix}$$

trans

$$m_{tot} \ddot{y} + (d - c_y U) \dot{y} + k_y - C_0 U^2 \theta = 0$$

rot

$$I_{\theta}^E \ddot{\theta} + d_0 \dot{\theta} + (K_0 - I_A C_0 U^2) \theta - I_A c_y U \dot{y} + m_{tot} I_0 \ddot{y} = 0$$