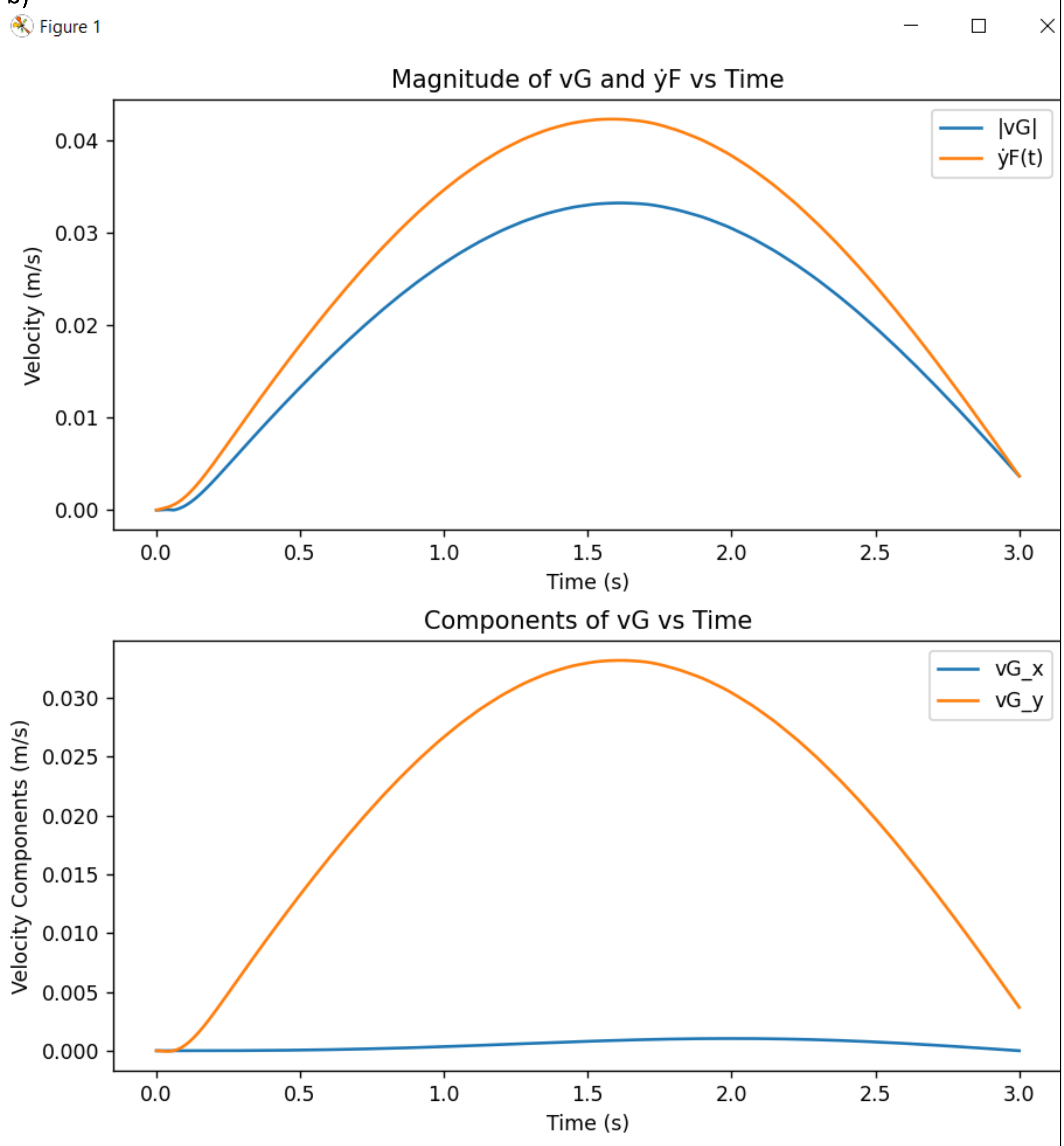
ENME203 Project Part 1 – Summary Answer Sheet

a)

$$v_G = \begin{pmatrix} l_\theta\, \dot{\theta}\, \sin\theta \\ \dot{y}_F(t) \; - \; l_\theta\, \dot{\theta}\, \cos\theta \\ 0 \end{pmatrix} \quad , |\boldsymbol{v_G}| = \sqrt{\left(l_\theta\, \dot{\theta}\, \sin\theta\right)^2 + \left(\dot{y}_F - l_\theta\, \dot{\theta}\, \cos\theta\right)^2}$$

b)

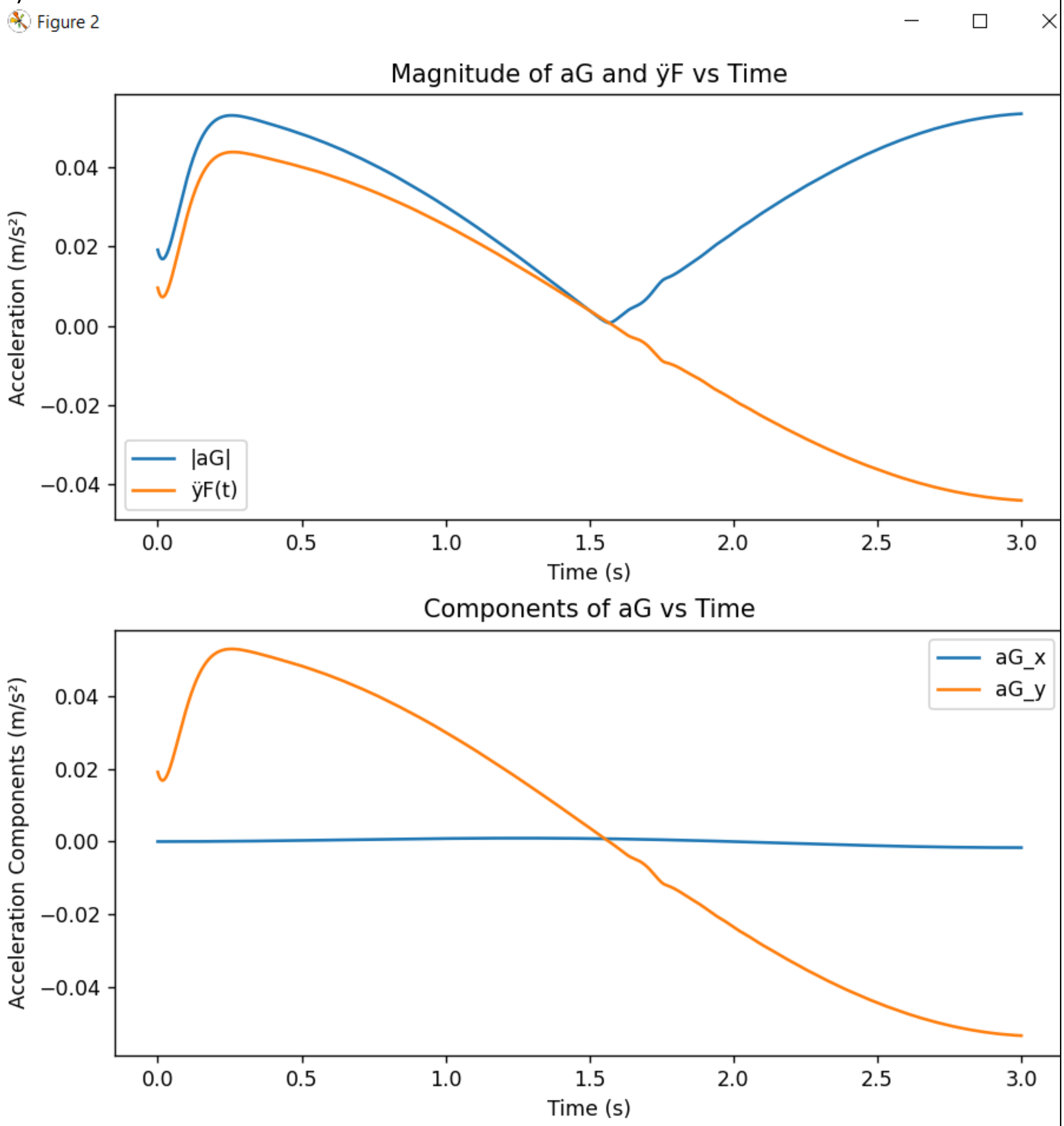


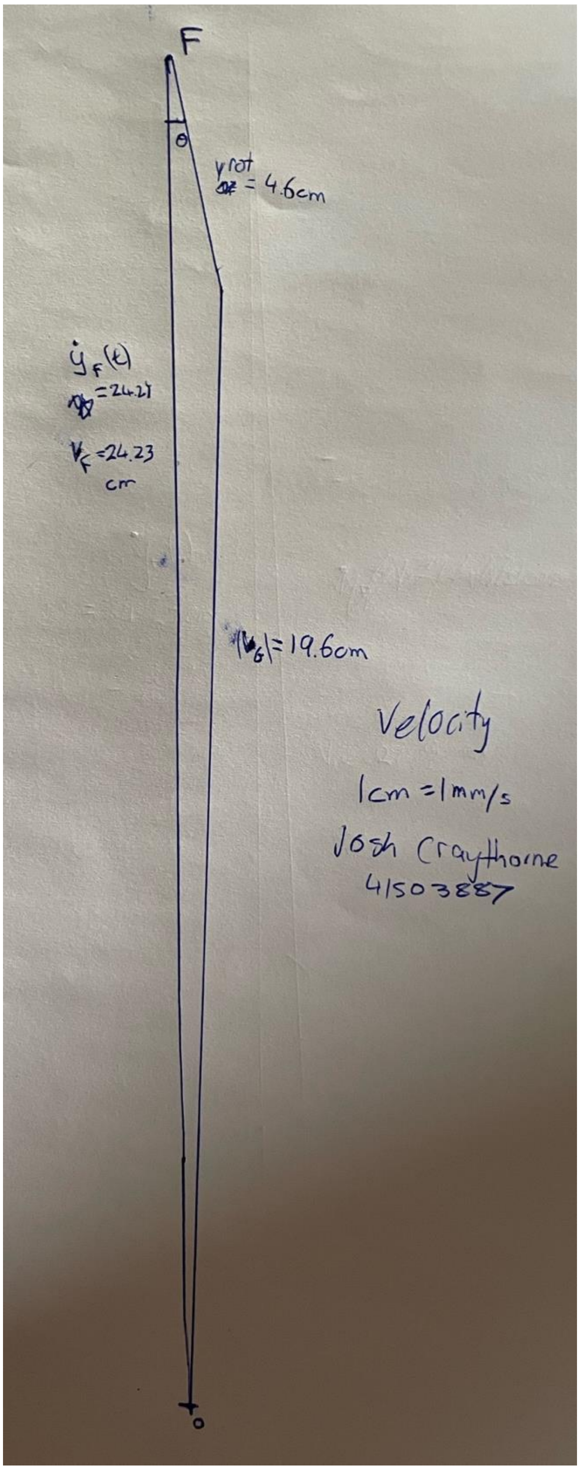
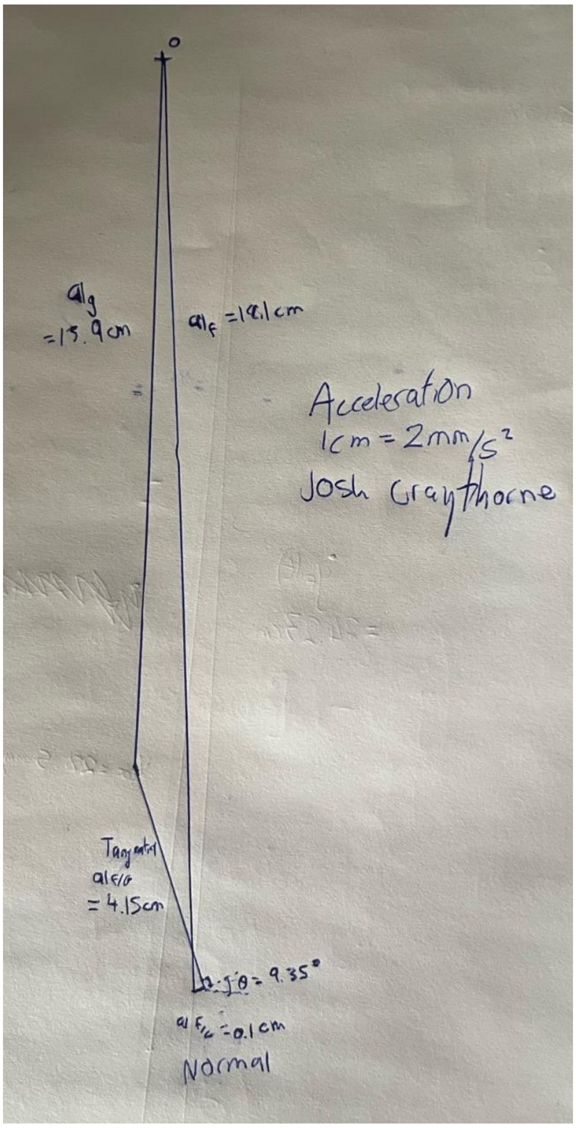
c)

$$a_G = \begin{pmatrix} l_\theta\left(\ddot{\theta}\ \sin\theta + \dot{\theta}^2\ \cos\theta\right) \\ \ddot{y}_F(t)\ -\ l_\theta\ddot{\theta}\ \cos\theta + l_\theta\dot{\theta}^2\ \sin\theta \\ 0 \end{pmatrix}$$

$$|\boldsymbol{a_G}| = \sqrt{\left[l_\theta\left(\ddot{\theta}\ \sin\theta + \dot{\theta}^2\ \cos\theta\right)\right]^2 + \left[\ddot{y}_F(t)\ -\ l_\theta\ddot{\theta}\ \cos\theta + l_\theta\dot{\theta}^2\ \sin\theta\right]^2}$$

d)



e) Velocity diagram	e) Acceleration diagram
	

Appendix

(Put your work here, or merge working pages to the PDF submission)

WORKING

F origin

$$\underline{r}_g(t) = \begin{pmatrix} l_a \\ y_F(t) \\ 0 \end{pmatrix} + R(\theta(t))\begin{pmatrix} l_\theta \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} l_a \\ y_F(t) \\ 0 \end{pmatrix} + \begin{pmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 0 \end{pmatrix}\begin{pmatrix} l_\theta \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} l_\theta\cos\theta \\ l_\theta\sin\theta \\ 0 \end{pmatrix}$$

$$V_G = \dot{r}_G = \begin{pmatrix} 0 \\ \dot{y}_F(t) \end{pmatrix} + \begin{pmatrix} -l_\theta\sin\theta \\ l_\theta\cos\theta \end{pmatrix}\dot{\theta} = \begin{pmatrix} -l_\theta\dot{\theta}\sin\theta \\ \dot{y}_F + l_\theta\dot{\theta}\cos\theta \end{pmatrix} \Rightarrow \text{Z axis is zero}$$

$$V_G = \begin{pmatrix} +l_\theta\dot{\theta}\sin(\theta(t)) \\ \dot{y}_F - l_\theta\dot{\theta}\cos(\theta) \\ 0 \end{pmatrix}$$

$$\underline{\alpha}(t) = \begin{pmatrix} l_\theta\ddot{\theta}(t)\sin(\theta(t)) + l_\theta\dot{\theta}(t)^2\cos(\theta(t)) \\ \ddot{y}_F - l_\theta\ddot{\theta}(t)\cos\theta(t) + l_\theta\dot{\theta}(t)^2\sin(\theta(t)) \\ 0 \end{pmatrix}$$

$$a(t) = \frac{d}{dt}(V_G(t)) \quad \text{checked via both methods}$$

$$\underline{r}_g(t) \quad \dot{r}_g(t) = \underline{V}_G(t) \quad \dot{V}_G(t) = a(t)$$

$$V_G = V_F + \omega_{GF} \times r_{G/F} \qquad a_G = a_{rF} + \dot{\omega}_{GF} \times r_{G/F} + \omega_{GF} \times (\omega_{GF} \times r_{GF})$$

$$a = \begin{pmatrix} 0 \\ \ddot{y}_F \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ \ddot{\theta} \end{pmatrix} \times \begin{pmatrix} -l_\theta\cos(\theta(t)) \\ -l_\theta\sin(\theta(t)) \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ \dot{\theta} \end{pmatrix} \times \left(\begin{pmatrix} 0 \\ 0 \\ \dot{\theta} \end{pmatrix} \times \begin{pmatrix} -l_\theta\cos(\theta(t)) \\ -l_\theta\sin(\theta(t)) \\ 0 \end{pmatrix}\right)$$

$$= \begin{pmatrix} 0 \\ \ddot{y}_F \\ 0 \end{pmatrix} + \begin{pmatrix} \ddot{\theta}(t)\, l_\theta\sin(\theta(t)) \\ -\ddot{\theta}\, l_\theta\cos(\theta(t)) \\ 0 \end{pmatrix} + \begin{pmatrix} \dot{\theta}(t)^2\, l_\theta\cos(\theta(t)) \\ \dot{\theta}(t)^2\, l_\theta\sin(\theta(t)) \end{pmatrix}$$

F origin

$$r_G = \begin{matrix} -l_\theta\cos\theta(t) \\ -l_\theta\sin\theta(t) \end{matrix}$$

$$V_G = V_F + \omega \times r_{G/F}$$

$$r_{G/F} = \begin{pmatrix} -l_\theta\cos\theta \\ -l_\theta\sin\theta \end{pmatrix} \Rightarrow \begin{pmatrix} -l_\theta\cos\theta \\ -l_\theta\sin\theta \\ 0 \end{pmatrix}$$

$$\omega \times r = \begin{pmatrix} 0 \\ 0 \\ \dot{\theta} \end{pmatrix} \times \begin{pmatrix} r_x \\ r_y \\ 0 \end{pmatrix} = \begin{pmatrix} -\dot{\theta} r_y \\ \dot{\theta} r_x \\ 0 \end{pmatrix} \Rightarrow \begin{pmatrix} -\dot{\theta}(-l_\theta\sin\theta) \\ \dot{\theta}(-l_\theta\cos\theta) \\ 0 \end{pmatrix}$$

counting bending 2 times mistake

$$V_G = \begin{pmatrix} 0 \\ \dot{y}_F \\ 0 \end{pmatrix} + \begin{pmatrix} -\dot{\theta}(-l_\theta\sin\theta(t)) \\ \dot{\theta}(-l_\theta\cos\theta(t)) \\ 0 \end{pmatrix}$$

$$\underline{V}_G = \begin{pmatrix} +l_\theta\dot{\theta}(t)\sin\theta(t) \\ \dot{y}_F - l_\theta\dot{\theta}(t)\cos\theta(t) \\ 0 \end{pmatrix}$$

$$\dot{\omega} = \begin{pmatrix} 0 \\ 0 \\ \ddot{\theta} \end{pmatrix}$$

$$r_{G/F}(t) = \begin{pmatrix} -l_\theta \cos\theta \\ -l_\theta \sin\theta \end{pmatrix}$$

$$v_g(t) = V_F + \omega_{GF} \times r_{G/F}$$

$$= \begin{pmatrix} 0 \\ \dot{y}_F(t) \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ \dot{\theta}(t) \end{pmatrix} \times \begin{pmatrix} -l_\theta \cos(\theta(t)) \\ -l_\theta \sin(\theta(t)) \\ 0 \end{pmatrix}$$

$$v_g(t) = \begin{pmatrix} l_\theta \dot{\theta}(t) \sin(\theta(t)) \\ \dot{y}_F(t) - l_\theta \dot{\theta} (\cos(\theta(t))) \\ 0 \end{pmatrix}$$

$$\frac{d}{dt}\left(v_g(t)\right) = \begin{pmatrix} l_\theta \ddot{\theta}(t) \sin(\theta(t)) + l_\theta \dot{\theta}(t)^2 \cos(\theta(t)) \\ \ddot{y}_F(t) - (l_\theta \ddot{\theta}(t) \cos(\theta(t)) - l_\theta \dot{\theta}(t)^2 \sin(\theta(t))) \\ 0 \end{pmatrix}$$

$$a_g(t) = \begin{pmatrix} l_\theta \left(\ddot{\theta}(t) \sin(\theta(t)) + \dot{\theta}(t)^2 \cos(\theta(t)) \right) \\ \ddot{y}_F(t) - l_\theta \left[\ddot{\theta}(t) \cos(\theta(t)) + l_\theta \dot{\theta}(t)^2 \sin(\theta(t)) \right] \\ 0 \end{pmatrix}$$

<u>or</u>

$$a_g(t) = a_{ref} + \dot{\omega}_{GF} \times r_{G/F} + \omega_{GF} \times (\omega_{GF} \times r_{G/F})$$

$$= \begin{pmatrix} 0 \\ \ddot{y}_F(t) \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ \ddot{\theta}(t) \end{pmatrix} \times \begin{pmatrix} -l_\theta \cos(\theta(t)) \\ -l_\theta \sin(\theta(t)) \\ 0 \end{pmatrix} + \left(\begin{pmatrix} 0 \\ 0 \\ \dot{\theta} \end{pmatrix} \times \left(\begin{pmatrix} 0 \\ 0 \\ \dot{\theta} \end{pmatrix} \times \begin{pmatrix} -l_\theta \cos(\theta(t)) \\ -l_\theta \sin(\theta(t)) \\ 0 \end{pmatrix} \right) \right)$$

$$a_g(t) = \begin{pmatrix} 0 \\ \ddot{y}_F(t) \\ 0 \end{pmatrix} + \begin{pmatrix} \ddot{\theta}(t)\, l_\theta \sin(\theta(t)) \\ -\ddot{\theta}(t)\, l_\theta \cos(\theta(t)) \\ \end{pmatrix} + \begin{pmatrix} \dot{\theta}(t)^2 l_\theta \cos(\theta(t)) \\ \dot{\theta}(t)^2 l_\theta \sin(\theta(t)) \\ 0 \end{pmatrix}$$

CODE

```
import numpy as np

import matplotlib.pyplot as plt


data = np.loadtxt('C:\COSC\wing_kinematics.csv', delimiter=',')

t = data[:, 0]

yFdot = data[:, 2]

yFddot = data[:, 3]


epsilon = 0.175

Omega = np.pi / 3

l_theta = 0.1



theta = (epsilon / 2) * (1 - np.cos(Omega * t))

theta_dot  = (epsilon / 2) * Omega * np.sin(Omega * t)

theta_ddot  = (epsilon / 2) * (Omega**2) * np.cos(Omega * t)



Vx = l_theta * theta_dot * np.sin(theta)

Vy = yFdot - l_theta * theta_dot * np.cos(theta)
```

```
vG_mag = np.hypot(Vx, Vy)



Ax= l_theta * (theta_ddot * np.sin(theta) + theta_dot**2 * np.cos(theta))
Ay = yFddot - l_theta * (-theta_ddot * np.cos(theta) + theta_dot**2 * np.sin(theta))
aG_mag = np.hypot(Ax, Ay)

fig1, (ax1, ax2) = plt.subplots(2, 1, figsize=(8, 8))

ax1.plot(t, vG_mag, label='|vG|')
ax1.plot(t, yFdot, label='ẏF(t)')
ax1.set_xlabel('Time (s)')
ax1.set_ylabel('Velocity (m/s)')
ax1.legend()
ax1.set_title('Magnitude of vG and ẏF vs Time')



ax2.plot(t, Vx, label='vG_x')
ax2.plot(t, Vy, label='vG_y')
ax2.set_xlabel('Time (s)')
ax2.set_ylabel('Velocity Components (m/s)')
ax2.legend()
ax2.set_title('Components of vG vs Time')

plt.tight_layout()
plt.show()

fig2, (ax3, ax4) = plt.subplots(2, 1, figsize=(8, 8))
```

```
ax3.plot(t, aG_mag, label='|aG|')
ax3.plot(t, yFddot, label='ÿF(t)')
ax3.set_xlabel('Time (s)')
ax3.set_ylabel('Acceleration (m/s²)')
ax3.legend()
ax3.set_title('Magnitude of aG and ÿF vs Time')

ax4.plot(t, Ax, label='aG_x')
ax4.plot(t, Ay, label='aG_y')
ax4.set_xlabel('Time (s)')
ax4.set_ylabel('Acceleration Components (m/s²)')
ax4.legend()
ax4.set_title('Components of aG vs Time')
plt.show()
```