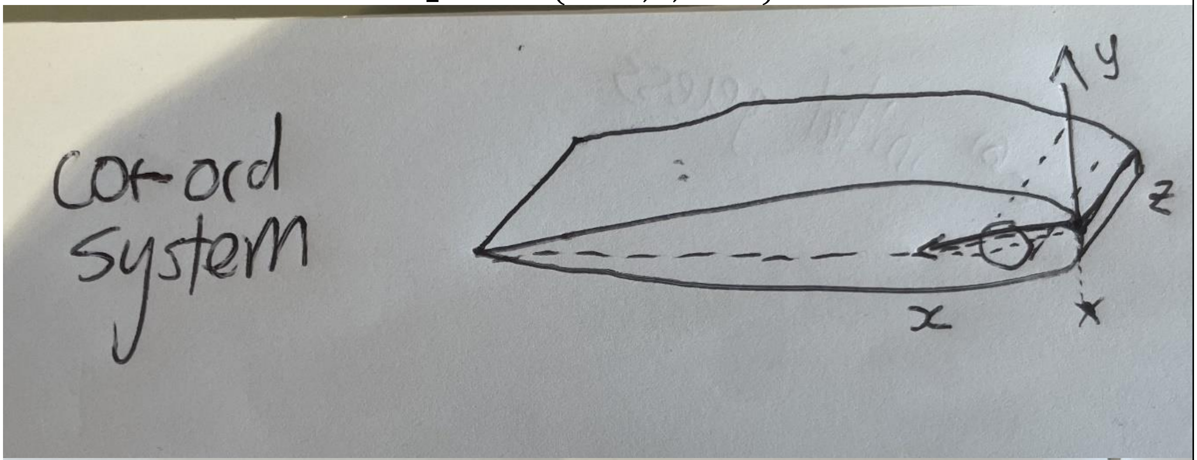
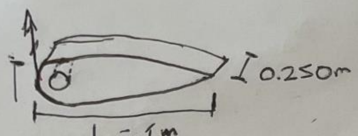
ENME203 Project Part 1 – Summary Answer Sheet

a)

$$G\_com = (0.419, 0, 0.125)$$



a



$t = 0.12$ $l_\theta = 0.1$, $l_\alpha = 0.05$ $m = 2kg$, $\mu = 24.48\ kg/m^2$

$$A = \int_0^1 y_{up}(x) - y_{low}(x)\,dx = \int_0^1 2y_{up}(x)\,dx$$

$$A = 1.2\int_0^1 0.2969\sqrt{x} - 0.126x - 0.3516x^2 + 0.2843x^3 - 0.1036x^4\,dx$$

$$= 1.2\left[0.2969(2/3) - 0.1260(1/2) - 0.3516(1/3) + 0.2843(1/4) - 0.1036(1/5)\right]$$

$$= 0.0817m^2$$

$$\bar{x} = 1/A \int_0^1 x \cdot 2y_{up}(x)\,dx$$

$$= \frac{1.2}{0.0817}\int_0^1 x\left[0.2969\sqrt{x} - 0.126x - 0.3616x^2 + 0.2843x^3 - 0.1036x^4\right]dx$$

$$= 1.2\,0.2969(2/5) - 0.1260(1/3) - 0.3516(1/4) + 0.2843(1/5) - 0.1036(1/6)$$

$$= 1.2\,0.02845$$

$$= \frac{1.2}{A} \times 0.02845 = 0.4186\,m$$

$\bar{y} = 0$ due to symmetry $= \underline{(0.419, 0, 0.125)}$

$\bar{z} = 0.125$ $\left(\frac{0.250}{2}\right)$ due to symmetry b)

G_com = (0.357, 0, 0.125)

b

b density ρ arm density 4ρ arm $r = 0.05$

arm centre 0.15 fro[m] leading

Wingconcept volume $= V_w = \cancel{0.0817 \times 0.250}$

$= \cancel{0.0204}$

$= \left(0.0817 - 9\frac{\pi}{4} 0.1^2\right) = 0.07384\,m^2$

$\times 0.250 = 0.01846$

arm volume $= \frac{\pi}{4}(0.1)^2 \times 0.250 = 0.001964\,m^3$

$2 = A_{wing} \cdot \rho \cdot w + A_{arm} \cdot 4\rho \cdot w$

$M_{wing} = 0.0739 \cdot 75.9 \cdot 0.250$

$= 1.404$

$= \rho = \left(\frac{(A_{wing} \cdot w + A_{arm} \cdot 4 \cdot w)}{2} \right)^{-1} = 75.9\ kg/m^3$

$M_{arm} = 0.007854 \cdot 4 \cdot 75.9 \cdot 0.250$

$= 0.596\,kg$

$\bar{x} = \frac{(M_{wing} \cdot \bar{x}_{wing} + M_{arm} \cdot \bar{x}_{arm})}{(M_{wing} + M_{arm})} = \cancel{0.379\,m}$

$\cancel{= (0.379, 0, 0.125)}$

b $M_{total} = 0.0817\rho + 3\pi r^2 \rho$

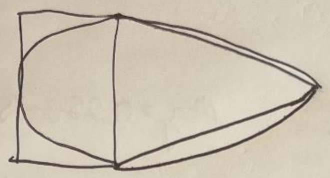
$\frac{\sum m_i x_i}{\sum m_i} = \frac{0.0817 \times 0.416 + 3\pi r^2 \times 0.15}{0.0817 + 3\pi r^2} = 0.357\,m$

COM $= (0.357, 0, 0.125)$

c)

$$I_{zz}^{G} = (0.106003 \text{ kgm}^\wedge 2)\ I_{zz}^{F} = (0.127624 \text{ kgm}^\wedge 2)$$

C



$$t(x) = \begin{cases} t_{max} & (0 \le x < a) \\ t_{max}\left(1 - \frac{x-a}{1-a}\right) & a < x < 1 \end{cases} \qquad (a = 0.3)$$

$$A_{tot} = \int_0^1 f(x)\,dx = t_{max}\,a + \frac{t_{max}(1-a)}{2} = t_{max}\frac{a+1}{2}$$

$$M_x = \int_0^1 x\,t(x)\,dx = t_{max}\frac{a^2+a+1}{6}$$

$$\underline{x_c} = \frac{M_x}{A} = \frac{a^2+a+1}{3(a+1)}$$

$$I_o = \mu(I_1 + I_2) \qquad I_1 = \int_0^1 x^2\,t(x)\,dx \qquad I_2 = \int_0^1 \int_{-h(x)}^{h(x)} y^2\,dy\,dx$$

$$= \int_0^1 \frac{2}{3}h(x)^3\,dx$$

$$= \frac{1}{12}\int t(x)^3\,dx$$

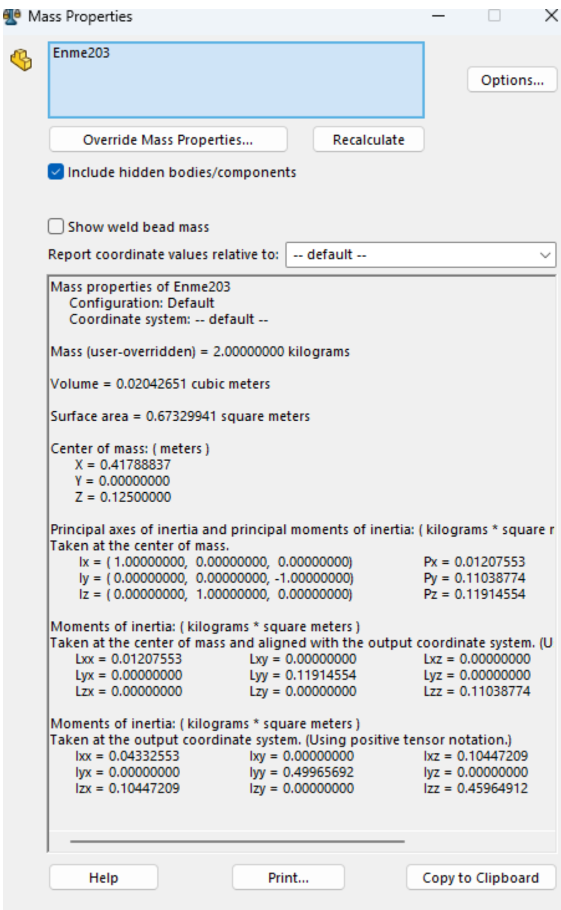
$$\therefore\ I_o = \mu\left[t_{max}\frac{a^3+a^2+a+1}{12} + t_{max}^3\frac{3a+1}{48}\right]$$

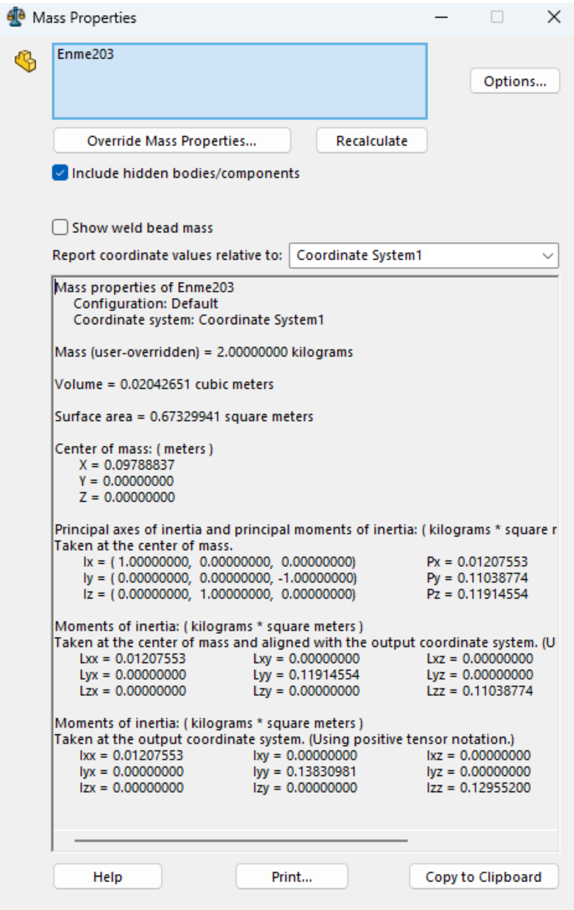
$$I_{G_{zz}} = I_o - m(x_g^2 + y_g^2) \qquad I_{F_{zz}} = I_{G_{zz}} + m(x_F - x_G)^2$$

$$\therefore = 0.106003\ \text{kg}\cdot\text{m}^2 \qquad = 0.127624\ \text{kg}\,\text{m}^2$$

d)

Calculated	CAD
$I_{zz}^{G} = (0.106003 \text{ kgm}^\wedge 2)$	$I_{zz}^{G} = (0.110387 \text{ kgm}^\wedge 2)$
$I_{zz}^{F} = (0.127624 \text{ kgm}^\wedge 2)$	$I_{zz}^{F} = (0.129552 \text{ kgm}^\wedge 2)$
$\text{Com} = (0.419,0,0.125)$	$\text{Com} = (0.4178,0,0.125)$





e)

$$m\ddot{y} = C_\theta U^2\theta + C_y U_{\dot{y}} - mg - d\dot{y} - ky$$

$$\mathrm{I}^{\mathrm{F}}\ddot{\theta} - C_\theta U^2\theta l_\alpha - C_y U_{\dot{y}} L_\alpha - mgl_\theta + K_\theta\theta + d_\theta\dot{\theta} - ml_\theta\ddot{y} = 0$$

$\Sigma F = m\ddot{y} = F_L - F_D - F_{spring} - mg$

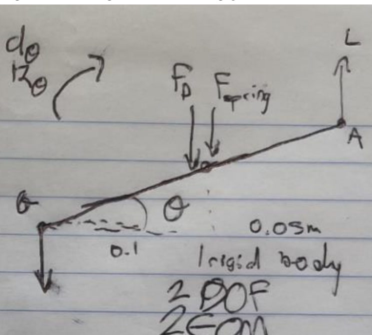
$\cancel{F_L = C_\theta U^2\theta + C_y U_{\dot{y}}}$

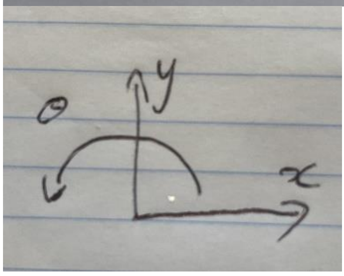
$m\ddot{y} = C_\theta U^2\theta + C_y U_{\dot{y}} - mg - d\dot{y} - ky$

$\ddot{\phi}$ $\quad I^F_\theta = (C_\theta U^2\theta + C_y U_{\dot{y}})l_\alpha + mgl_\theta - K_\theta\theta - d_\theta\dot{\theta} + ml_\theta\ddot{y}$

$I^F\ddot{\theta} - C_\theta U^2\theta l_\alpha - C_y U_{\dot{y}} l_\alpha - mgl_\theta + K_\theta\theta + d_\theta\dot{\theta} - ml_\theta\ddot{y} = 0$

$m\ddot{y} - C_y U_{\dot{y}} + d\dot{y} + ky - C_\theta U^2\theta = -mg$





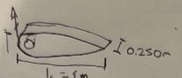
Appendix

(Put your full working here, or merge working pages to the PDF submission)

You can make small adjustments the answer boxes to fit more working/evidence if needed, provided that the question order remains consistent.

In the summary answers, show the key steps of your working (e.g. final integration equations), note the methods used (e.g. numerical solvers, Symbolab, Desmos, etc), any assumptions, and the final answers.

a



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$= 1.2\left[0.2969(2/3) - 0.1260(1/2) - 0.3516(1/3) + 0.2843(1/4) - 0.1036(1/5)\right]$

$= 0.0817m^2$

$\bar{x} = 1/A\int_0^1 x \cdot 2y_{up}(x)\,dx$

$= \frac{\cancel{1.2}}{0.0817}\int_0^1 x\left[0.2969\sqrt{x} - 0.126x - 0.3616x^2 + 0.2843x^3 - 0.1036x^4\right]dx$

$= \cancel{1.2}\,0.2969(2/5) - 0.1260(1/3) - 0.3516(1/4) + 0.2843(1/5) - 0.1036(1/6)$

$= \cancel{1.2}\,0.02845$

$= \frac{1.2}{A} \times 0.02845 = 0.4186m$

$\bar{y} = 0$ due to symmetry $= (0.419, 0, 0.125)$

$\bar{z} = 0.125 \ \left(\frac{0.250}{2}\right)$ due to symmetry

b

b density ρ arm density 4ρ arm $r = 0.05$

arm centre 0.15 from leading

wingwould Volume $= V_w = \cancel{0.0817 \times 0.250}$

$= \cancel{0.0204}$

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$\rho = \left(\frac{(A_{wing} \cdot w + A_{arm} \cdot 4 \cdot w)}{2}\right)^{-1} = 75.9\ kg/m^3$

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$\bar{x} = \frac{(M_{wing} \cdot \bar{x}_{wing} + M_{arm} \cdot \bar{x}_{arm})}{(M_{wing} + M_{arm})} = \cancel{0.379m}$

$= \cancel{(0.379, 0, 0.125)}$

coord system

~~semi ellipse area ... = 0.0628~~

~~triangle area = 1/2 bh = 0.042~~

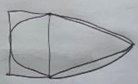
b

$M_{total} = 0.0817\rho + 3\pi r^2\rho$

$\frac{\sum m_i x_i}{\sum m_i} = \frac{0.0817 \times 0.416 + 3\pi r^2 \times 0.15}{0.0817 + 3\pi r^2} = 0.357m$

COM $= (0.357, 0, 0.125)$

c



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$A_{tot} = \int_0^1 f(x)\,dx = t_{max}a + \frac{t_{max}(1-a)}{2} = t_{max}\frac{a+1}{2}$

$M_x = \int_0^1 x\,t_{max}(x)\,dx = t_{max}\frac{a^2+a+1}{6}$

$\underline{x_c} = \frac{M_x}{A} = \frac{a^2+a+1}{3(a+1)}$

$I_0 = \mu(I_1 + I_2) \qquad I_1 = \int_0^1 x^2 t(x)\,dx \qquad I_2 = \int_0^1\int_{-h(x)}^{h(x)} y^2\,dy\,dx$

$= \int_0^1 2/3\ h(x)^3\,dx$

$= 1/12\int t(x)^3\,dx$

$\therefore$

$I_0 = \mu\left[t_{max}\frac{a^3+a^2+a+1}{12} + t_{max}^3\frac{3a+1}{48}\right]$

$I_{G_{zz}} = I_0 - m(x_g^2 + y_g^2) \qquad I_{P_{zz}} = I_{G_{zz}} + m(x_P - x_G)^2$

$\therefore = 0.10003\ kg \cdot m^2 \qquad = 0.127624\ kg\,m^2$

$L\mu x$

$\cancel{z}\,y^{(1)}$

$Lz^{(1)} = x^{(0)}$

2 0

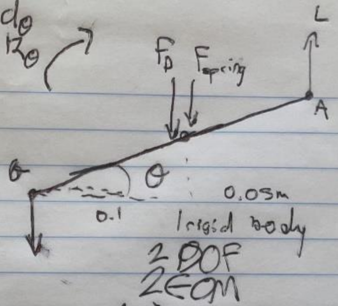
b $\bar{x} = \frac{M_{wing} \cdot \bar{x}_{wing} + M_{arm} \cdot \bar{x}_{arm}}{(M_{wing} + M_{arm})}$

~~$M_{wing} = 0.250 \cdot 75.9 \cdot 0.01846$~~

total volume $= 0.020425$

volume wing $= \pi \times 0.1^2 \times 0.250 = \frac{1}{400\pi}$

0.020425



$$\sum F = m\ddot{y} = L - F_D - F_{spring} - mg$$

$$\cancel{L = C_\theta U^2\theta + C_y U\dot{y}}$$

$$m\ddot{y} = C_\theta U^2\theta + C_y U\dot{y} - mg - d\dot{y} - ky$$

$$\ddot{\phi} \quad I^{E}_{\theta} = (C_\theta U^2\theta + C_y U\dot{y})l_\alpha + mg\,l_\theta - K_\theta\theta - d_\theta\dot{\theta} + m l_\theta\ddot{y}$$

$$I^{E}\ddot{\theta} - C_\theta U^2\theta l_\alpha - C_y U\dot{y} l_\alpha - mg\,l_\theta + K_\theta\theta + d_\theta\dot{\theta} - m l_\theta\ddot{y}_t = 0$$

$$m\ddot{y} - C_y U\dot{y} + d\dot{y} + ky - C_\theta U^2\theta = -mg$$

d)

$$a_b = a_c + \dot{\omega}$$

$$= \begin{pmatrix} 0 \\ -g \\ 0 \end{pmatrix}$$

Done

trans: $m_{tot}\ddot{y} + (d - C_y U)\dot{y} + ky - C_\theta U^2\theta = 0$

rot: $I^{E}_{EZ}\ddot{\theta} + d_\theta\dot{\theta} + (K_\theta - l_A C_\theta U^2)\theta - l_A C_y U\dot{y} + m_{tot} l_G\ddot{y} = 0$