Q2

You have been tasked with modelling a hypothetical mosquito outbreak in the theme park. Based on (possibly fictional) research done on "mosquito outbreaks in maths-themed theme parks", the mosquitos would have the following population parameters:

Vector entry	Age (days)	Birth Rate	Survival Rate
x_1	0-5	0	0.15
x_2	5-10	0	0.30
x_3	10-15	0	0.50
x_4	15-20	100	0.40
x_5	20-25	100	0.25
x_6	25-30	100	0.25
x_7	30-35	100	

These age groups correspond to specific stages of a mosquito's life, where x_1 are eggs, x_2 are larvae, x_3 are pupae, and x_{4-7} are adults.

a) Apply the Leslie Model to the mosquito population and model 50 iterations (250 days) of population growth, starting with a population of $\boldsymbol{x}^{(\mathbf{0})} = [0, 0, 0, 10, 0, 0, 0]^T$. Plot the mosquito populations for each iteration as stacked bar charts. (You may find the following links helpful for plotting stacked bar charts in Python or Matlab.)

b) Comment on whether you think that the mosquito population distribution has approached the theoretical long-term distribution after the 50 iterations of population growth in (a). Clearly explain your answer.

c) One of the potential options for preventing an outbreak is to effectively lower the birth rate of eggs by culling x_1. Determine the "harvest" rate required for stopping population growth while only culling x_1. Confirm this answer by finding the dominant eigenvalue of L_h, the Leslie matrix with effective birth rates. (You may find pg. 262-267 of Week 5 notes helpful)

Question 2

The Leslie matrix L is created by placing the birth rates along the first row, and then the survival rates along the sub-diagonal. This produces the following matrix:

$$L = \begin{pmatrix} 0 & 0 & 0 & 100 & 100 & 100 & 100 \\ 0.15 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0.3 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.5 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.4 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.25 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0.25 & 0 \end{pmatrix}$$

To compute the $(k+1)$th population vector, we left-multiply the kth population vector by L. We can do this iteratively in Python, starting from an initial population vector $x = (0 \quad 0 \quad 0 \quad 10 \quad 0 \quad 0 \quad 0)^T$ using the following code snippet, which also saves the results in an array.

```
for i in range(1,51):
        x = L @ x
        results.append(x)

    return np.array(results)
```

These results can be displayed with a stacked bar chart. Figure 1 below shows the population evolving over the first 50 iterations (250 days). It is clear from the graph that the population is growing exponentially. Figure 2 shows the same population data, but normalised to total population so that each bar shows the distribution of the population. This plot shows that the population seems to be tending towards a steady state distribution. The total population has not reached a steady state, as it is still growing (and will continue to do so as long as this model holds), but the distribution has approximately reached a steady state.

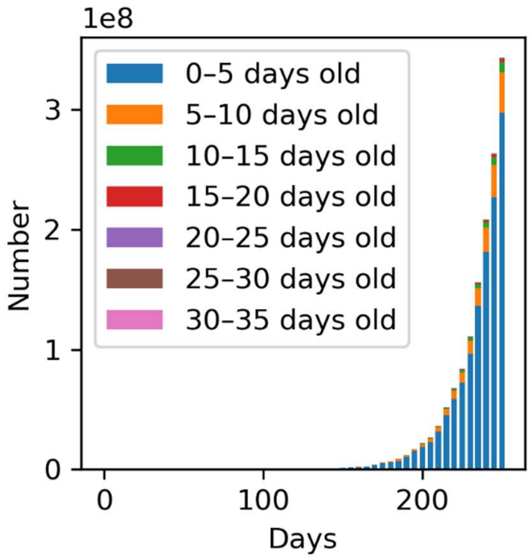

Figure 1: Mosquito population growth

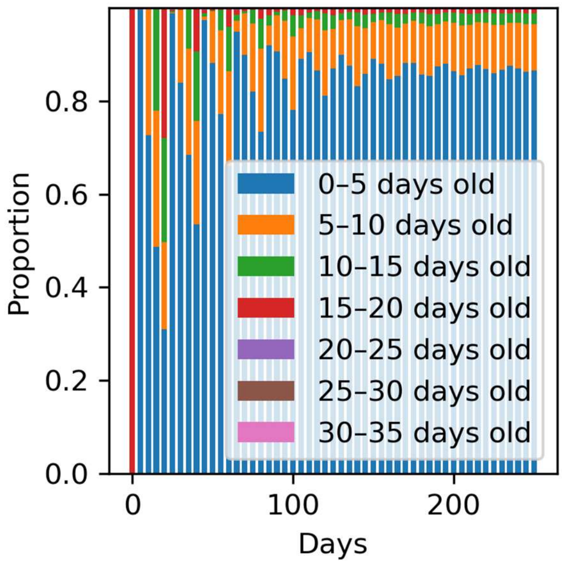

Figure 2: Mosquito population proportion

We can confirm our observations by using `numpy.linalg.eig(L)` to find the eigenvalues and eigenvectors of L. This is shown below, along with some useful scaling and post-processing.

```
>>> vals, vecs = np.linalg.eig(L)
>>> vals[0]
1.32493708+0.j
>>> np.absolute(vals)
array([1.3249, 1.2279, 1.2279, 1.1224, 0.3288, 0.2762, 0.2762])
>>> np.real(np.abs(vecs[:, 0]) / np.linalg.norm(vecs[:, 0], 1))
array([0.8680, 0.0983, 0.0223, 0.0084, 0.0025, 0.0005, 0.0001])
```

L has a dominant eigenvalue approximately equal to 1.325. This is larger than 1, which indicates that this population will grow over time, as shown in the simulation. The dominant eigenvector is given by scaling the corresponding eigenvector by its 1-norm. This matches the distribution estimated by the simulation, with approximately 87% in the first age class (eggs), 10% in the second, 2% in the third age class, and then almost imperceptibly small proportions in older age classes.

To prevent an outbreak, we could attempt to cull (harvest) enough eggs to reduce the dominant eigenvalue of L to below 1. To accomplish this, we can multiply the second row of L by $(1-h)$, where $h = \frac{R-1}{R}$ and

$$R = b_1 + b_2 s_1 + b_3 s_1 s_2 + \cdots + b_n s_1 s_2 \ldots s_{n-1}$$

Python gives $R = 3.43125$ and $h = 0.709$, which then gives us the new Leslie matrix,

$$L_h = \begin{pmatrix} 0 & 0 & 0 & 100 & 100 & 100 & 100 \\ \color{red}{0.0437} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0.3 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.5 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.4 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.25 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0.25 & 0 \end{pmatrix},$$

with the new survival rate highlighted in red.

Using the same method as earlier, Python now calculates the dominant eigenvalue as 1, which indicates that this population will remain steady. If we round the harvesting rate up to 71% and compute another 50 iterations, we get Figure 3. It is clear that the population has stabilised, and we would expect it to gradually decline over time. If we chose a higher calue of h, we would expect the population to decline at a greater rate.

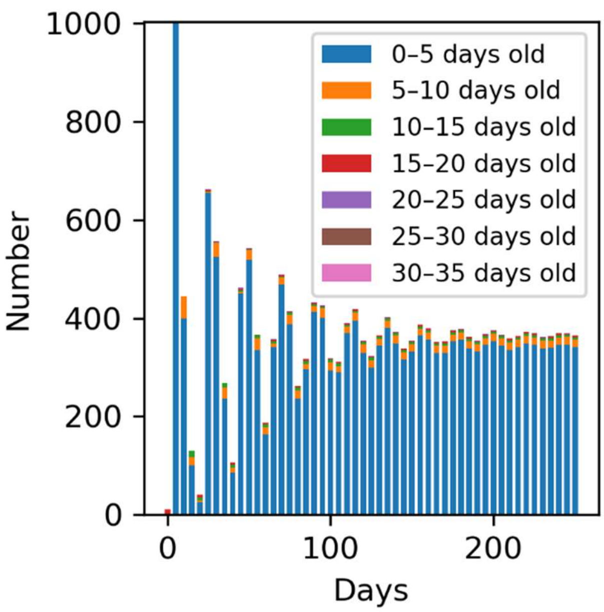

Figure 3: Culled mosquito population