Discrete and continuous latent states of neural activity in *Caenorhabditis Elegans*

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July 25, 2016

Abstract

Recent advances in neural recording technologies have enabled simultaneous measurements of the majority of head ganglia neurons in immobilized C. elegans [1]. Moreover, since some neurons are known to reliably indicate the onset or offset of particular behaviors, like ventral and dorsal turns, behavioral state can be decoded from the simultaneous population recordings. These datasets provide unique visibility into the relationship between neural activity and behavior. While it seems clear that activity is inherently lower dimensional than the number of neurons due to strong correlations between cells, the nature of the latent brain state remains unclear. For example, is brain state better thought of as discrete or continuous, or perhaps a combination of the two? Does it obey linear or nonlinear dynamics? We propose a generative approach to probing these questions. We model the neural activity as a *switching linear dynamical system* (SLDS), with both discrete and continuous latent states, and conditionally linear dynamics. We then analyze the posterior distribution over states implied by the neural recordings and find that the discrete states correspond to stereotypical motor sequences. In contrast to previous work, these states are exposed in an entirely unsupervised manner.

1 Model

Assume the instantaneous neural activity at time t for a population of N neurons is represented as a vector, $\boldsymbol{y}_t \in \mathbb{R}^N$. In calcium imaging settings, the entries in this vector may be instaneous $\Delta F/F$ measurements, or another signal that captures neural activity. In this experiment, we use the smoothed time derivative of $\Delta F/F$. Over the course of an experiment, we measure a sequence of vectors, which we combine into a matrix denoted by $\boldsymbol{y}_{1:T}$.

Our model is based on the following assumptions: (i) the instantaneous neural activity, $\boldsymbol{y}_t$, reflects an underlying, low-dimensional latent state; (ii) this state has a discrete component, $z_t \in \{1, \ldots, K\}$, and a continuous component, $\boldsymbol{x}_t \in \mathbb{R}^D$; (iii) the continuous latent state has linear dynamics governed by the corresponding discrete latent state; and (iv) the observed neural activity is a linear function of the underlying states with additive Gaussian noise.

Discuss the motivation for these modeling assumptions.

These assumptions are combined in a switching linear dynamical system, which we formalize with the following generative model:

$$p(\boldsymbol{y}_{1:T}, \boldsymbol{x}_{1:T}, \boldsymbol{z}_{1:T} \,|\, \boldsymbol{\Theta}) = \prod_{t=1}^{T} p(z_t \,|\, z_{t-1}, \boldsymbol{\Theta})\, p(\boldsymbol{x}_t \,|\, z_{t-1}, \boldsymbol{x}_{t-1}, \boldsymbol{\Theta})\, p(\boldsymbol{y}_t \,|\, z_t, \boldsymbol{x}_t, \boldsymbol{\Theta}). \tag{1}$$

Our beliefs about the dynamics of these latent states are encoded in the form of the conditional distributions for z_t and $\boldsymbol{x}_t$. First, we assume the discrete states follow a Markov process,

$$p(z_t \,|\, z_{t-1}, \boldsymbol{\Theta}) \sim \text{Discrete}(\boldsymbol{\pi}^{(z_{t-1})}). \tag{2}$$

Next, we imbue the continuous latent states with linear Gaussian dynamics,

$$p(\boldsymbol{x}_t \,|\, \boldsymbol{x}_{t-1}, z_{t-1}, \boldsymbol{\Theta}) \sim \mathcal{N}(\boldsymbol{A}^{(z_{t-1})}\boldsymbol{x}_{t-1} + \boldsymbol{b}^{(z_{t-1})}, \boldsymbol{Q}^{(z_{t-1})}). \tag{3}$$

Finally, we impose the assumption of linear observations via the conditional distribution,

$$p(\boldsymbol{y}_t \,|\, \boldsymbol{x}_t, z_t, \boldsymbol{\Theta}) \sim \mathcal{N}(\boldsymbol{C}^{(z_t)}\boldsymbol{x}_t + \boldsymbol{d}^{(z_t)}, \boldsymbol{R}^{(z_t)}). \tag{4}$$

Thus, the parameters of the model are,

$$\boldsymbol{\Theta} = \left\{ \boldsymbol{A}^{(k)}, \boldsymbol{b}^{(k)}, \boldsymbol{Q}^{(k)}, \boldsymbol{C}^{(k)}, \boldsymbol{d}^{(k)}, \boldsymbol{R}^{(k)}, \boldsymbol{\pi}^{(k)} \right\}_{k=1}^{K}. \tag{5}$$

Discuss interpretation of the parameters. Set the stage for visualizing $\boldsymbol{A}^{(k)}$ in later sections.

2 Inference

Describe block Gibbs updates for $\boldsymbol{z}_{1:T} \,|\, \boldsymbol{x}_{1:T}, \boldsymbol{y}_{1:T}$ and $\boldsymbol{x}_{1:T} \,|\, \boldsymbol{z}_{1:T}, \boldsymbol{y}_{1:T}$. Given the latent states, the remaining parameters are sampled as in Bayesian linear regression.

3 Results

References

[1] Saul Kato, Harris S Kaplan, Tina Schrödel, Susanne Skora, Theodore H Lindsay, Eviatar Yemini, Shawn Lockery, and Manuel Zimmer. Global brain dynamics embed the motor command sequence of Caenorhabditis elegans. *Cell*, 163(3):656–669, 2015.