In this study, an self-adaptative genetic algorithm(SAGA) is proposed to minimize thickness(or weight) of laminated composite subject to in-plane loading. Fiber orientation angles and ply thickness are chosen as design variables. By imposing safety factor as the constraint, SAGA is used to search the optimal design of a laminated composite under virous loading cases. To check the feasibility of a laminated lay-up, Tsai-wu failure criterion is taken to calculate the safety factor.

1 Introduction

Composite materials offer improved strength, stiffness, corrosion resistance, etc. over conventional materials, and are widely used as alternative materials for applications in various industries ranging from electronic packaging to golf clubs, and medical equipment to homebuilding, making aircraft structure to space vechicles. One widely known advange of using composite material is can significantly reducing the weight of target structure.

thickness[1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21]

In practice, fiber orientations are restricted to a finite set of angles, and layer thickness is a specific numberic value.

2 Analysis of

2.1 Stress and Strain in a Lamina

For a single lamina has a small thickness under plane stress, and it's upper and lower surfaces of the lamina are free from external loads. According to the Hooke's Law, the three-dimensional stress-strain equations can be reduced to two-dimensional stress-strain equations. The stress-strain relation in local axis 1-2 is:

$$\begin{bmatrix} \sigma_1 \\ \sigma_2 \\ \tau_{12} \end{bmatrix} = \begin{bmatrix} Q_{11} & Q_{12} & 0 \\ Q_{12} & Q_{22} & 0 \\ 0 & 0 & Q_{66} \end{bmatrix} \begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \gamma_{12} \end{bmatrix} \tag{1}$$

where $Q_{ij}$are the stiffnesses of the lamina that are related

to engineering elastic constants given by

$$\begin{aligned} Q_{11} &= \frac{E_1}{1 - v_{12} v_{21}} \\ Q_{22} &= \frac{E_2}{1 - v_{12} v_{21}} \\ Q_{66} &= G_{12} \\ Q_{12} &= \frac{v_{21} E_2}{1 - v_{12} v_{21}} \end{aligned} \tag{2}$$

where E_1, E_2, v_{12}, G_{12} are four independent engineering elastic constants, which are defined as follows: E_1 is the longitudinal Young's modulus, E_2 is the transverse Young's modulus, v_{12} is the major Poisson's ratio, and G_{12} is the in-plane shear modulus.

Stress strain relation in the global x-y axis:

$$\begin{bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{bmatrix} = \begin{bmatrix} \bar{Q}_{11} & \bar{Q}_{12} & \bar{Q}_{16} \\ \bar{Q}_{12} & \bar{Q}_{22} & \bar{Q}_{26} \\ \bar{Q}_{16} & \bar{Q}_{26} & \bar{Q}_{66} \end{bmatrix} \begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{bmatrix} \tag{3}$$

where

$$\begin{aligned}
\bar{Q}_{11} &= Q_{11}c^4 + Q_{22}s^4 + 2\left(Q_{12} + 2Q_{66}\right)s^2c^2 \\
\bar{Q}_{12} &= \left(Q_{11} + Q_{22} - 4Q_{66}\right)s^2c^2 + Q_{12}\left(c^4 + s^2\right) \\
\bar{Q}_{22} &= Q_{11}s^4 + Q_{22}c^4 + 2\left(Q_{12} + 2Q_{66}\right)s^2c^2 \\
\bar{Q}_{16} &= \left(Q_{11} - Q_{12} - 2Q_{66}\right)c^3s - \left(Q_{22} - Q_{12} - 2Q_{66}\right)s^3c \\
\bar{Q}_{26} &= \left(Q_{11} - Q_{12} - 2Q_{66}\right)cs^3 - \left(Q_{22} - Q_{12} - 2Q_{66}\right)c^3s \\
\bar{Q}_{66} &= \left(Q_{11} + Q_{22} - 2Q_{12} - 2Q_{66}\right)s^2c^2 + Q_{66}\left(s^4 + c^4\right)
\end{aligned} \tag{4}$$

The c and s denotes $cos\theta$ and $sin\theta$.

The local and global stresses in an angle lamina are related to each other through the angle of the lamina θ

$$\begin{bmatrix} \sigma_1 \\ \sigma_2 \\ \tau_{12} \end{bmatrix} = [T] \begin{bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{bmatrix} \tag{5}$$

where

$$[T] = \begin{bmatrix} c^2 & s^2 & 2sc \\ s^2 & c^2 & -2sc \\ -sc & sc & c^2 - s^2 \end{bmatrix} \tag{6}$$

2.2 Stress and Strain in a Laminate

$$\begin{aligned}
\begin{bmatrix} N_x \\ N_y \\ N_{xy} \end{bmatrix} &= \begin{bmatrix} A_{11} & A_{12} & A_{16} \\ A_{12} & A_{22} & A_{26} \\ A_{16} & A_{26} & A_{66} \end{bmatrix} \begin{bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \end{bmatrix} \\
&+ \begin{bmatrix} B_{11} & B_{12} & B_{16} \\ B_{11} & B_{12} & B_{16} \\ B_{16} & B_{26} & B_{66} \end{bmatrix} \begin{bmatrix} k_x \\ k_y \\ k_{xy} \end{bmatrix} \\
\begin{bmatrix} M_x \\ M_y \\ M_{xy} \end{bmatrix} &= \begin{bmatrix} B_{11} & B_{12} & B_{16} \\ B_{12} & B_{22} & B_{26} \\ B_{16} & B_{26} & B_{66} \end{bmatrix} \begin{bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \end{bmatrix} \\
&+ \begin{bmatrix} D_{11} & D_{12} & D_{16} \\ D_{11} & D_{12} & D_{16} \\ D_{16} & D_{26} & D_{66} \end{bmatrix} \begin{bmatrix} k_x \\ k_y \\ k_{xy} \end{bmatrix}
\end{aligned} \tag{7}$$

N_x, N_y - normal force per unit length

N_{xy} - shear force per unit length

M_x, M_y - bending moment per unit length
M_{xy} - twisting moments per unit length
ε^0, k- mid plane strains and curvature of a laminate in x-y coordinates
The mid plane strain and curvature is given by

$$\begin{aligned}
A_{ij} &= \sum_{k=1}^{n} (\overline{Q_{ij}})_k (h_k - h_{k-1}) i = 1,2,6, j = 1,2,6 \\
B_{ij} &= \frac{1}{2} \sum_{k=1}^{n} (\overline{Q_{ij}})_k (h_k^2 - h_{k-1}^2) i = 1,2,6, j = 1,2,6 \\
D_{ij} &= \frac{1}{3} \sum_{k=1}^{n} (\overline{Q_{ij}})_k (h_k^3 - h_{k-1}^3) i = 1,2,6, j = 1,2,6
\end{aligned} \tag{8}$$

The [A], [B], and [D] matrices are called the extensional, coupling, and bending stiffness matrices, respectively. The extensional stiffness matrix $[A]$ relates the resultant in-plane forces to the in-plain strains, and the bending stiffness matrix $[D]$ couples the resultant bending moments to the plane curvatures. The coupling stiffness matrix $[B]$ relates the force and moment terms to the midplain strains and midplane curvatures.

3 Failure Theory for a lamina

Many different theories about the failure of an angle lamina have been developed for a unidirectional lamina, such as the maximum stress failure theory, maximum strain failure theory, Tsai-Hill failure theory, and Tsai-Wu failure theory. The failure theories of a lamina are based on the stresses in the local axes in the material. There are four normal strength parameters and one shear stress for a unidirectional lamina. The five strength parameters are:
$(\sigma_1^T)_{ult}$ = ultimate longitudinal tensile strength
$(\sigma_1^C)_{ult}$ = ultimate longitudinal compressive strength
$(\sigma_2^T)_{ult}$ = ultimate transverse tensile strength
$(\sigma_2^C)_{ult}$ = ultimate transverse compressive strength
$(\tau_{12})_{ult}$ = and ultimate in-plane shear strength

In the present study, Tsai-wu failure theory is taken to decide whether a lamina fails, because this theory is more general than the Tsai-Hill failure theory, which considers two different situations, the compression and tensile strengths of a lamina. A lamina is considered to fail if

$$\begin{gathered}
H_1\sigma_1 + H_2\sigma_2 + H_6\tau_{12} + H_{11}\sigma_1^2 + H_{22}\sigma_2^2 \\
+ H_{66}\tau_{12}^2 + 2H_{12}\sigma_1\sigma_2 < 1
\end{gathered} \tag{9}$$

is violated, where

$$SR = \frac{\text{Maximum Load Which Can Be Applied}}{\text{Load Applied}} \tag{10}$$

3.1 Failure Theories for a Laminate

If keep increasing the loading applied to a laminate, the laminate will fails. The failure process of a laminate is more complicate than a lamina, because a laminate consists of multiple plies, and the fiber orientation, material, thickness of each ply maybe different from the others. In most situations, some layer fails first and the remains continue to take more loads until all the plies fail. If one ply fails, it means this lamina does not contribute to the load carrying capacity of the laminate. The procedure for finding the first failure ply given follows the fully discounted method:

1. Compute the reduced stiffness matrix [Q] referred to as the local axis for each ply using its four engineering elastic constants E_1, E_2, E_{12}, and G_{12}.

2. Calculate the transformed reduced stiffness $[\bar{Q}]$ referring to the global coordinate system (x, y) using the reduced stiffness matrix [Q] obtained in step 1 and the ply angle for each layer.

3. Given the thickness and location of each layer, the three laminate stiffness matrices [A], [B], and [D] are determined.

4. Apply the forces and moments, $[N]_{xy}, [M]_{xy}$ solve Equation 7, and calculate the middle plane strain $[\sigma^0]_{xy}$ and curvature $[k]_{xy}$.

5. Determine the local strain and stress of each layer under the applied load.

6. Use the ply-by-ply stresses and strains in the Tsai-wu failure theory to find the strength ratio, and the layer with smallest strenght ratio is the first failed ply.

4 Methodology

4.1 Objective function

There are two design variables here, the angles in the laminate, and the number of layers that each fiber orientation has. The objective function is as

$F = 2t_0 \sum_{k=1}^{n} n_k$

The first term represent the total thickness of the composite laminates, t_0 is ply thickness; n_k is the number of plies in the kth lamina, in which the fiber orientation is θ_k.

The only constraint is the safety factor of the material under certain loading, and it should greater than 1.

4.2 Selection

The purpose of the selection operator is how to chose parents to produce children of better fitness. Traditional methods of selecting strategies only take the fitness of the individual into acount, however, becasue of the existance of constraint, the selection strategies have to change a little bit. The parents of next generation consists of three groups: proper groups, active groups, and potential groups.

Proper parents mean individual fullfils the constraint, which are chosen by the individual's fitnees, individuals with better fitness are more likely to be chosen if they fit the constraint; active groups means that individual is supposed to be always exist in the parents during the GA, which are selected by fitness, ignoring the constraint; potential groups means that they are likely to turn into proper individual after a couple of generations, and potential individuals are chosen by constraint function, the more the individual fulfils the constraint, the more possiblity it will be selected.

4.3 Crossover

The crossover operator happens among these three groups. the child of two proper groups are more likely to be a proper individual which can be used to obtain a better individual. the child of an active individual and a potential individual can significantly change the gene of active individual's chromsome, which lets the individual evolved toward a new direction. The offspring of two active individuals are more likely to be an active individual, which can maitain the active group.

4.4 Mutation

A mutation direction is imposed on the mutation operator which to make sure the individual evolving toward the right direction. The mutation direction, denoted by md, is a n dimensional vector corresonding to number of constraints, it is decided by the constraint thresholds CT_i and the current individual's constraint value, denoted as CV_i, The mutation vector can be obtained by the following formula

md $= [CT_1, \cdots, CT_{n-1}, CT_n] - [CV_0, \cdots, CV_{n-1}, CV_n]$

During the operator, the mutation consists of three parts, the length of the chromsome, the angle of the chromsome, and the number of each angle. Becasue the chromsome's length is positive correlated with the individual's fitness, the coefficient of length mutation denoted by C_l, if $\sum_{i=1}^{N} CT_i$ great than zero, the mutation length is restricted to the range $[0, C_l \sum_{i=1}^{N} CT_i]$; if the $\sum_{i=1}^{N} CT_i$ less than zero, the mutation length is restricted to the range $[0, \sum_{i=1}^{N} CT_i]$; Assuming a $[13_6/-27_4]_s$ carbon T300/5308 composite laminate under the loading $N_{xx} = N_{yy} = 10$ MPa m, the only constraint is the safety factor greater than 1. Accoring to the Tsai-Wu criterion, its safety factor is 0.0539. So the mutation vector is [0.941], assuming the coefficient is 20, so the mutation range is from 0 to 18. A random number is generated from the range $[0, 18]$, supposing the outcome is 13, then a length generator is used to a list, the it's sum is 13, suppose the list is [5, 8], the laminate after mutation is $[13_{11}/-27_{12}]_s$.

The relationship between the angles in the composite laminate and the chromsome's fitness is unclear, so the mutation direction of chromsome's angle is random. The coefficient angle mutation is C_a, $[0, C_a \sum_{i=1}^{N} CT_i]$

5 Result

In this experiment, only one constraint is imposed on the composite laminates which is the safety factor CT_1, and its value is 1. The constraint value of individual is CV_1. So the mutation vector here is a one dimensional vector $[1 - CV_1]$, and the coefficient of length mutation C_l and angle mutation C_a, respectively, chosen here is 20 and 10.

Figure 1 (a) shows how the optimal individual's fitness and strength ratio vary during the GA process. The method to chose optimal individual considering two following situations, if no individual in the current population meets constraint, the one with biggest fitness is selected as the optimal individual; if there are one or multiple individuals fullfils requirement, the one with smallest fitness is chosen. Figure 1 (b) shows how the two distinct fiber orientation changes at the same time, and Figure 1 (c) how the number of each angles change.

At the beginning of this GA process, the fitness curves increased very quickly, becasue of individual's strength ratio CT_0 is very small, so the difference between the individual's fitness and the imposed constraint threshold is a big positive number, so the range of mutaion length is from 0 to $C_l(CT_0 - CV_0)$. The length of individual increases by n, which is random number between 0 and $C_l(CT_0 - CV_0)$. As can be seen from Figure 1 (a), both of optimal individual's fitness and strength ratio increases very quickly. The range of mutaion angle is from 0 to $C_a(CT_0 - CV_0)$, and the number of every angle also change violently. During this stage, increasing individual's length playing a major role in increasing individual's fitness.

After a couple of generations, the optimal individual's fitness get bigger, and the difference between individual's fitness and constraint threshold get smaller. The range of mutaion length $[0, C_l(CT_0 - CV_0)]$ turn smaller. At this stage, simply increase the individual's length doesn't make much difference in improve individual's fitnees, and a better composite laminates lay-up can dramaticly change the optimal individual's fitness. That's why the fitness curve oscillated violently in this stage. At the same time, the strength ratio curve kept growing smoothly. But the growing speed got more smaller.

When GA comes to its last phase, GA found individuals that meet the constraint. Now the optimal individual's fitness is greater than the safety factor. The range of mutation length is from $C_l(CT_0 - CV_0)$ to 0. It means individuals need to decrease it's length and improve its internal structure to meet the constraint. That's why the fitness of optimal individual kept decreaing, however, the strength ratio curve still is greater then safety factor.

6 Conclusion

In this paper, SAGA is proposed to search the optimal lay-up for laminated composite under different loading. Two situations are considered under the same loading, a set of two distinct angles, and three distinct angles, SAGA

Table 1: An Example of a Table

Property	Symbol	Unit	Graphite/Epoxy
Longitudinal elastic modulus	E_1	GPa	181
Traverse elastic modulus	E_2	GPa	10.3
Major Poisson's ratio	v_{12}		0.28
Shear modulus	G_{12}	GPa	7.17
Ultimate longitudinal tensile strength	$(\sigma_1^T)_{ult}$	MP	1500
Ultimate longitudinal compressive strength	$(\sigma_1^C)_{ult}$	MP	1500
Ultimate transverse tensile strength	$(\sigma_2^T)_{ult}$	MPa	40
Ultimate transverse compressive strength	$(\sigma_2^C)_{ult}$	MPa	246
Ultimate in-plane shear strength	$(\tau_{12})_{ult}$	MPa	68
Density	ρ	g/cm^3	1.590

Table 2: The optimum lay-ups using two distinct fiber angles under various biaxial loading cases

Loading $N_x/N_y/N_{xy}$ (MPa m)	Optimum lay-up sequences	laminate thickness	Safety factor
10/5/0	$[33_{29}/\text{-}39_{25}/\text{-}\bar{3}9]_s$	109	1.0074
20/5/0	$[33_{22}/\text{-}31_{24}]_s$	92	1.0055
40/5/0	$[29_{18}/\text{-}21_{23}/\text{-}\bar{2}3]_s$	83	1.0034
80/5/0	$[\text{-}20_{27}/21_{25}/\bar{2}5]_s$	105	1.0029
120/5/0	$[\text{-}18_{34}/17_{36}]_s$	140	1.0000

Table 3: The optimum lay-ups using three distinct fiber angles under various biaxial loading cases

Loading $N_x/N_y/N_{xy}$ (MPa m)	Optimum lay-up sequences	laminate thickness	Safety factor
10/5/0	$[37_{27}/\text{-}38_{27}/\text{-}5]_s$	110	1.0023
20/5/0	$[34_{24}/\text{-}32_{14}/\text{-}28_{11}]_s$	98	1.0237
40/5/0	$[21_{28}/\text{-}32_{19}/2_3]_s$	100	1.0788
80/5/0	$[\text{-}21_{25}/\text{-}16_3/21_{26}]_s$	108	1.0128
120/5/0	$[\text{-}18_{34}/17_{36}]_s$	140	1.0000

can adjust its convergence speed depending on the difference of individual's constraint value and constraint threshold.

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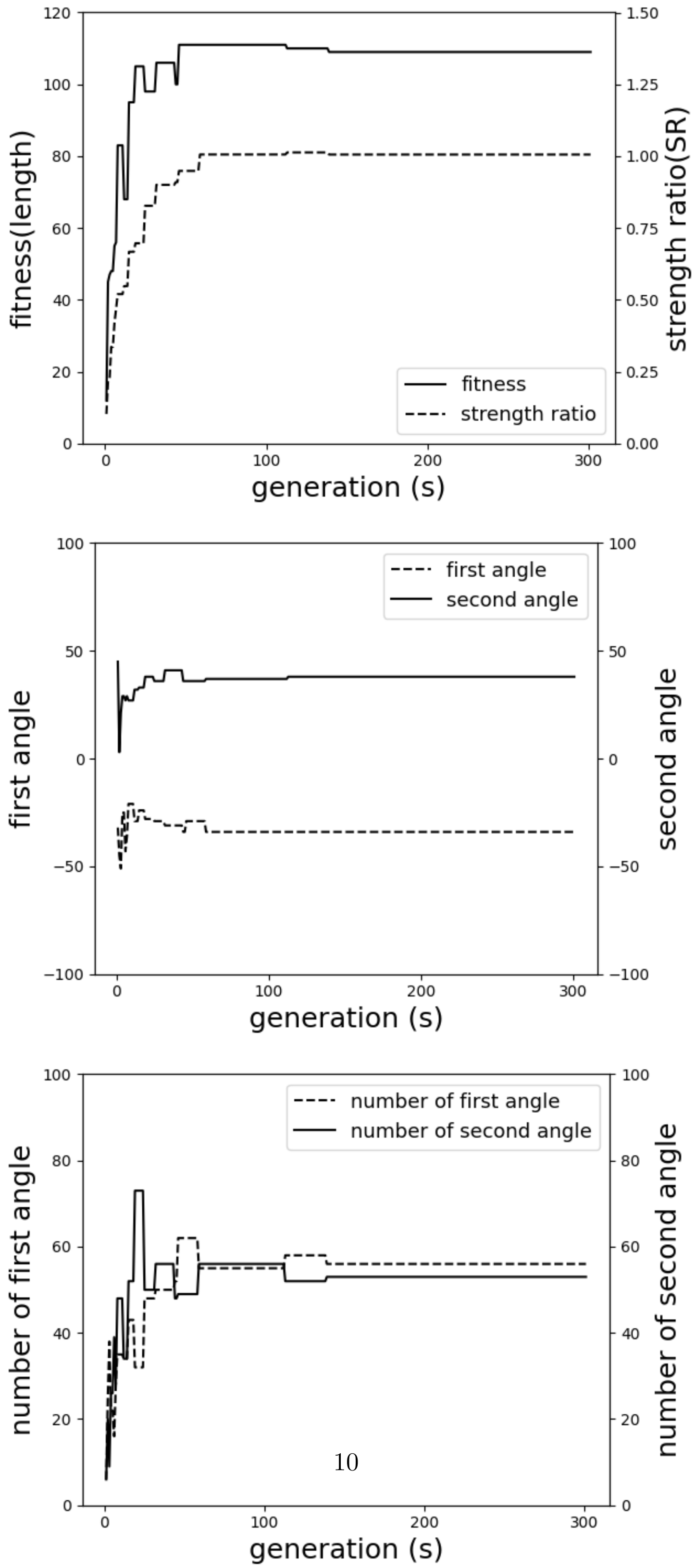

Figure 1: Two distinct angles

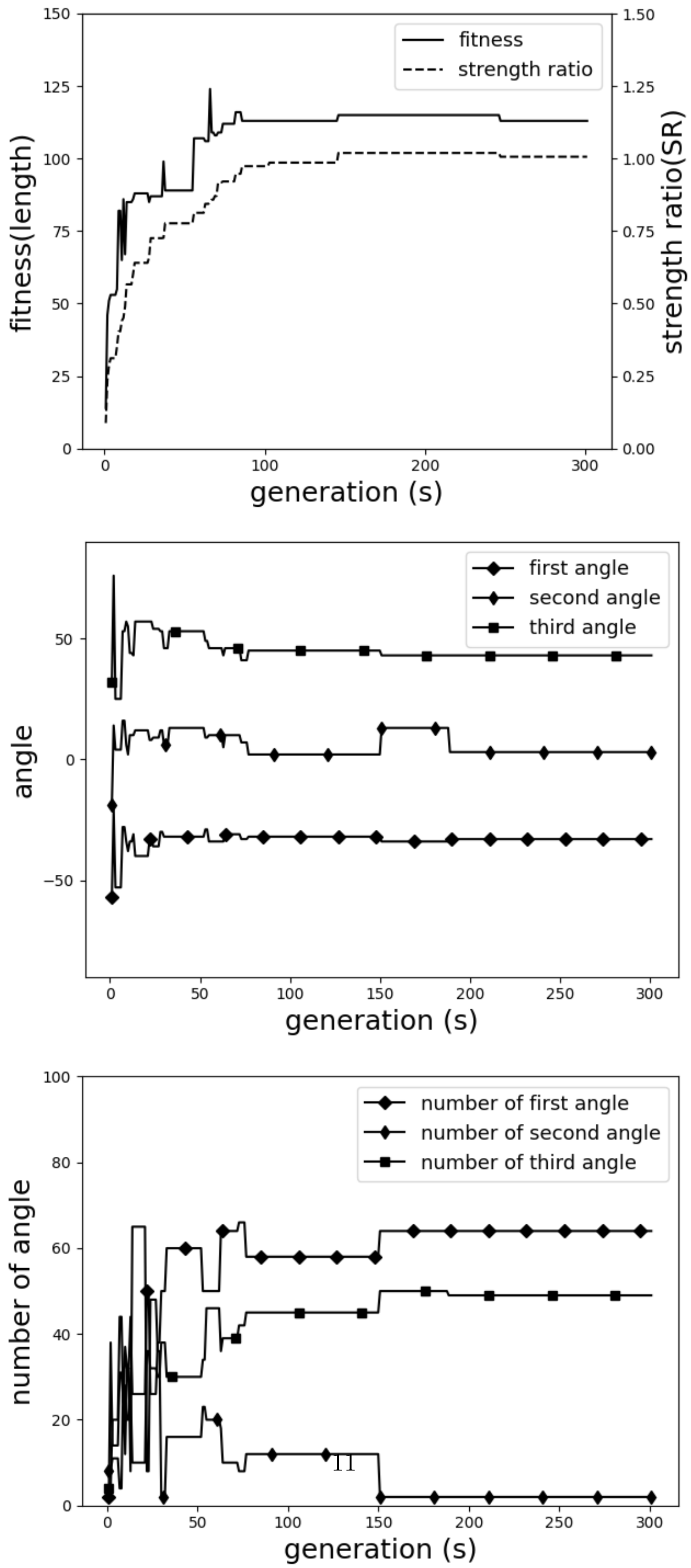

Figure 2: Three distinct angles