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Stacking Sequence Optimization by an Improved Genetic Algorithm

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Abstract An improved genetic algorithm is used to obtain the stacking sequence of the laminate that reach the maximum strength

Keywords Genetic Algorithm · Laminates · Stacking Sequence

1 Introduction

Fiber-reinfored composites are widely used in automotive, aerospace, shipbuilding, and other branches of engineering because of their high specific strength and stiffness.

Genetic algorithms(GAs) simulate the process of natural evolutionary includes selection, crossover ,and mutation according to Darwin's principal of "survival of the fittest".

According to T Back [**?**],the selection mechanism is one of the primary means of controlling the GA's convergence rate and its likelihood of finding global optima.

Four evaluation criteria are used. The first is normalized cost per genetic search,

$$C_n$$

cost is determinted by the following formula.

$$C_n = N_g P/R$$

where P is population size, R is Apparent reliability.

2 Failure Theores

For orthoropic materials with three planes of symmetry,

$$F_1\sigma_1 + F_2\sigma_2 + F_{11}\sigma_1^2 + F_{22}\sigma_2^2 + F_{66}\sigma_6^2 + 2F_2F_2 = 1 \tag{1}$$

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3 Objective Function

Given the safety factor, The objective function is minimum the weight of the laminate.

$$obj = min(weight)$$

Substituting Eq. 9 for R into Eq. 4,we obtain

$$\begin{aligned}&(F_{11}\sigma_1^2 + F_{22}\sigma_2^2 + F_{66}\sigma_6^2 + 2F_{12}\sigma_1\sigma_2)R^2 \\ &+ (F_1\sigma_1 + F_2\sigma_2)R - 1 = 0\end{aligned}$$

4 Stress-Strain Relationship for a Laminate

$$\begin{bmatrix}\sigma_1\\ \sigma_2\\ \tau_{12}\end{bmatrix} = \begin{bmatrix}Q_{11} & Q_{12} & 0\\ Q_{12} & Q_{22} & 0\\ 0 & 0 & Q_{66}\end{bmatrix}\begin{bmatrix}\varepsilon_1\\ \varepsilon_2\\ \gamma\end{bmatrix} \tag{2}$$

Where Q_{ij} are the stiffnesses of the lamina that are related to the compliance matrix compnents and elastic constants by

$$\begin{aligned}Q_{11} &= \frac{E_1}{1-\mu_{12}\mu_{21}}\\ Q_{22} &= \frac{E_2}{1-\mu_{12}\mu_{21}}\\ Q_{66} &= G_{12}\\ Q_{12} &= \frac{\mu_{21}E_2}{1-\mu_{12}\mu_{21}}\end{aligned} \tag{3}$$

The transformation of the equation*sof the off-axis coordinates to the principal axis of the material stress tensor as

$$[T] = \begin{bmatrix}\cos^2\theta & \sin^2\theta & 2\sin 2\theta\\ \sin^2\theta & \cos^2\theta & -2\sin 2\theta\\ -\frac{1}{2}\sin 2\theta & \frac{1}{2}\sin 2\theta & \cos 2\theta\end{bmatrix} \tag{4}$$

$$\begin{bmatrix}N_x\\ N_y\\ N_{xy}\end{bmatrix} = \begin{bmatrix}A_{11} & A_{12} & A_{16}\\ A_{12} & A_{22} & A_{26}\\ A_{16} & A_{26} & A_{66}\end{bmatrix}\begin{bmatrix}\varepsilon_x^0\\ \varepsilon_y^0\\ \gamma_{xy}^0\end{bmatrix} + \begin{bmatrix}B_{11} & B_{12} & B_{16}\\ B_{11} & B_{12} & B_{16}\\ B_{16} & B_{26} & B_{66}\end{bmatrix}\begin{bmatrix}k_x\\ k_y\\ k_{xy}\end{bmatrix} \tag{5}$$

$$\begin{bmatrix}M_x\\ M_y\\ M_{xy}\end{bmatrix} = \begin{bmatrix}B_{11} & B_{12} & B_{16}\\ B_{12} & B_{22} & B_{26}\\ B_{16} & B_{26} & B_{66}\end{bmatrix}\begin{bmatrix}\varepsilon_x^0\\ \varepsilon_y^0\\ \gamma_{xy}^0\end{bmatrix} + \begin{bmatrix}D_{11} & D_{12} & D_{16}\\ D_{11} & D_{12} & D_{16}\\ D_{16} & D_{26} & D_{66}\end{bmatrix}\begin{bmatrix}k_x\\ k_y\\ k_{xy}\end{bmatrix}$$

where

$$\begin{aligned}A_{ij} &= \sum_{k=1}^{n}(\overline{Q_{ij}})_k(h_k - h_{k-1})\\ B_{ij} &= \frac{1}{2}\sum_{k=1}^{n}(\overline{Q_{ij}})_k(h_k - h_{k-1})\\ D_{ij} &= \frac{1}{3}\sum_{k=1}^{n}(\overline{Q_{ij}})_k(h_k - h_{k-1})\end{aligned} \tag{6}$$

The $[A]$, $[B]$, and $[D]$ matrices are called the extensional, coupling, and bending stiffness matrices.

$$\begin{aligned} F_1 &= \frac{1}{X_t} - \frac{1}{X_c}, \quad F_{11} = \frac{1}{X_t X_c} \\ F_2 &= \frac{1}{Y_t} - \frac{1}{Y_c}, \quad F_{22} = \frac{1}{Y_t Y_c} \\ F_{66} &= \frac{1}{S^2} \end{aligned} \tag{7}$$

$$R = \frac{\sigma_{i(\alpha)}}{\sigma_i}$$

where R is the strength ratio,σ_{ia} is allowable stress,σ_i is the stress under loading
substituting Eq. 9 for σ into Eq. 4, we obtain

$$\begin{aligned} &F_1\sigma_{1(a)} + F_2\sigma_{2(a)} + F_{11}\sigma^2_{1(a)} + F_{22}\sigma^2_{2(a)} \\ &\quad + F_{66}\sigma^2_{6(a)} + 2F_{12}\sigma_{1(a)}\sigma_{2(a)} = 1 \end{aligned}$$

For orthoropic materials with three planes of symmetry,

$$\begin{aligned} &(F_{11}\sigma_1^2 + F_{22}\sigma_2^2 + F_{66}\sigma_6^2 + 2F_{12}\sigma_1\sigma_2)R^2 \\ &+ (F_1\sigma_1 + F_2\sigma_2)R - 1 = 0 \end{aligned}$$

4.1 Solution by a genetic algorithm

as required. Don't forget to give each section and subsection a unique label).

5 Results and Discussion

Table 1: Typical Properties of a Unidirectional Lamina(SI System of Units)

Property	Symbol	Unit	Glass/Epoxy	Graphite/Epoxy
Fiber volume fraction	V_f		0.45	0.70
Longitudinal elastic modulus	E_1	GPa	38.6	181
Traverse elastic modulus	E_2	GPa	8.27	10.3
Major Poisson's ratio	ν_{12}		0.26	0.28
Shear modulus	G_{12}	GPa	4.14	7.17
Ultimate longitudinal tensile strength	$(\sigma_1^T)_{ult}$	MPa	1062	1500
Ultimate longitudinal compressive strength	$(\sigma_1^C)_{ult}$	MPa	610	1500
Ultimate transverse tensile strength	$(\sigma_2^T)_{ult}$	MPa	31	40
Ultimate transverse compressive strength	$(\sigma_2^C)_{ult}$	MPa	118	246
Ultimate in-plane shear strength	$(\tau_{12})_{ult}$	MPa	72	68

Table 2: Comparative study of different composite materials for a defined strenght ratio

Type of composite	Stacking sequence	Strength ratio	Mass	Cost	Height
Glass/Epoxy	$[0]_{6s}$	2.103	0.707	12.0	1.980
Graphite/Epoxy	$[0]_9$	2.227	0.481	22.5	1.485
Hybrid composite	$[Gl\text{-}E/Gr\text{-}E_5]_s$	2.082	0.545	22.0	1.650

Type of composite	Stacking sequence	Strength ratio	Mass	Cost	Height
Glass/Epoxy	$[90/-90]$	2.103	0.707	12.0	1.980
Graphite/Epoxy	$[90/-90]$	2.227	0.481	22.5	1.485
Hybrid composite	$[Gl\text{-}E/Gr\text{-}E_5]_s$	2.082	0.545	22.0	1.650

Table 3: GA-parameters

parameter	value
population size	20
encoding method	float encoding
selection strategy	roulette wheel
crossover strategy	one-point
mutation strategy	mass mutation

6 Concluding Remarks

7 Acknowledgements

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The target function is

$$p = \begin{cases} 1 \; E\left(x_{new}\right) < E\left(x_{old}\right) \\ \exp\left(-\frac{E(x_{new})-E(x_{old})}{T}\right) E\left(x_{new}\right) > E\left(x_{old}\right) \end{cases} \tag{8}$$

$$f(r) = \sin(r)/r + 1$$
$$\text{where } r = \sqrt{(x-50)^2 + (y-50)^2} + 2.71828$$

we define R is the strength ratio,

$$R = \frac{\sigma_{i(\alpha)}}{\sigma_i} \tag{9}$$

where $\sigma_{i\sigma}$ is allowable stress and σ_i is the stress under loading,R is the ratio of the component of allowable stress and the stress component under stress. substituting $\sigma_{i(\alpha)}$ for σ_i into Eq. 4, we obtain

$$\begin{aligned} F_1\sigma_{1(\alpha)} + F_2\sigma_{2(\alpha)} + F_{11}\sigma^2_{1(\alpha)} + F_{22}\sigma^2_{2(\alpha)} \\ + F_{66}\sigma^2_{6(\alpha)} + 2F_{12}\sigma_{1(\alpha)}\sigma_{2(\alpha)} = 1 \end{aligned} \tag{10}$$

Substituting $\sigma_{i(\sigma)} = R\sigma_i$ into Eq. 4,we obtain

$$\begin{aligned} (F_{11}\sigma_1^2 + F_{22}\sigma_2^2 + F_{66}\sigma_6^2 + 2F_{12}\sigma_1\sigma_2)R^2 \\ + (F_1\sigma_1 + F_2\sigma_2)R - 1 = 0 \end{aligned} \tag{11}$$

This is a quadric equation about R, the value of $(R-1)$ is the multiples that the stress can be increased The objective function to be maximized is the strenght of the laminate.

$$F = max(\frac{1}{R(i)-1}) \tag{12}$$

where i is the layer number
A[9] [8] [1] [3] [2] [11] [6] [5] [4] [10] [7]

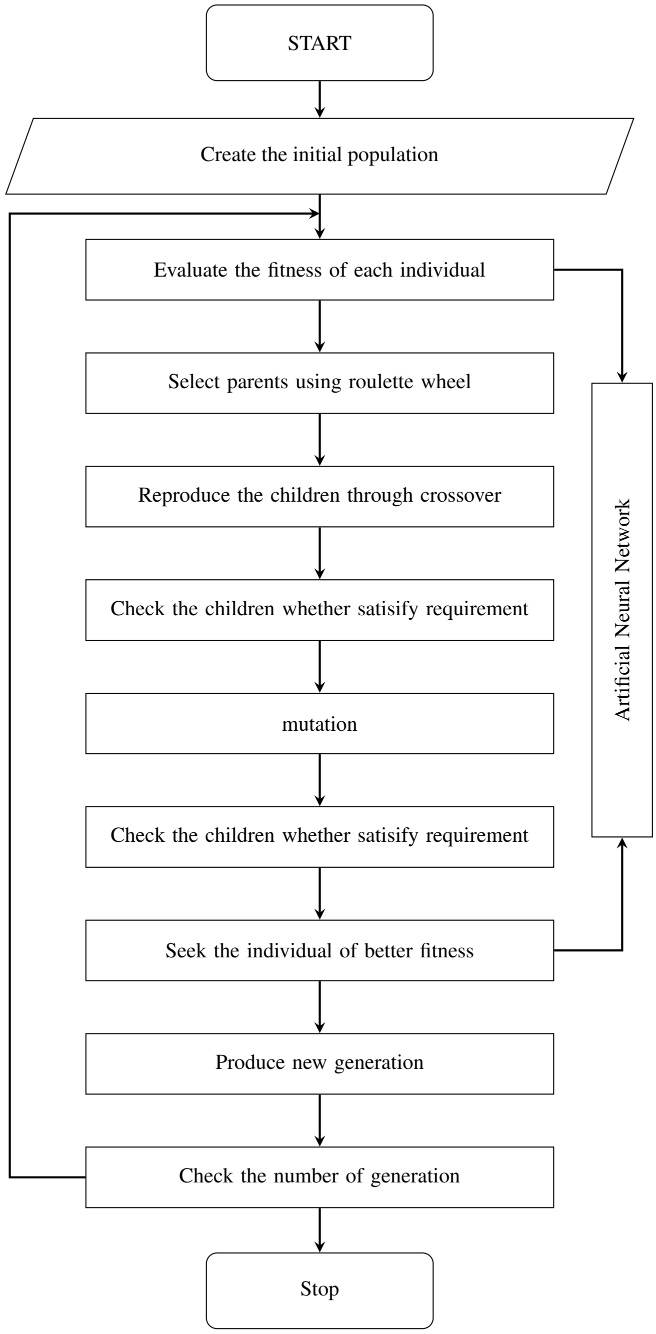

Fig. 1: the flowchart of Genetic Algorithm

References

1. Mustafa Akbulut and Fazil O Sonmez. Design optimization of laminated composites using a new variant of simulated annealing. *Computers & structures*, 89(17-18):1712–1724, 2011.
2. Jing-Fen Chen, Evgeny V Morozov, and Krishnakumar Shankar. Simulating progressive failure of composite laminates including in-ply and delamination damage effects. *Composites Part A: Applied Science and Manufacturing*, 61:185–200, 2014.
3. Isaac M Daniel, Ori Ishai, Issac M Daniel, and Ishai Daniel. *Engineering mechanics of composite materials*, volume 3. Oxford university press New York, 1994.
4. Ji Genlin. Survey on genetic algorithm [j]. *Computer Applications and Software*, 2(1):69–73, 2004.
5. David E Goldberg and Kalyanmoy Deb. A comparative analysis of selection schemes used in genetic algorithms. In *Foundations of genetic algorithms*, volume 1, pages 69–93. Elsevier, 1991.
6. David E Goldberg and John Henry Holland. Genetic algorithms and machine learning. 1988.
7. Seyedali Mirjalili. Genetic algorithm. In *Evolutionary algorithms and neural networks*, pages 43–55. Springer, 2019.
8. Ozden O Ochoa and John J Engblom. Analysis of progressive failure in composites. *Composites Science and Technology*, 28(2):87–102, 1987.
9. JN Reddy and AK Pandey. A first-ply failure analysis of composite laminates. *Computers & Structures*, 25(3):371–393, 1987.
10. Eugene Semenkin and Maria Semenkina. Self-configuring genetic algorithm with modified uniform crossover operator. In *International Conference in Swarm Intelligence*, pages 414–421. Springer, 2012.
11. U Taetragool, PH Shah, VA Halls, JQ Zheng, and RC Batra. Stacking sequence optimization for maximizing the first failure initiation load followed by progressive failure analysis until the ultimate load. *Composite Structures*, 180:1007–1021, 2017.