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# Laminated Composite Plate Optimization by Genetic Algorithm

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**Abstract** Failure analysis of laminated composite plates under different mechanical loads for different stacking sequences, fiber orientation, and composite material system is studied in this paper. An optimum composite material and laminate layup is studied for a targeted strength ratio which makes a compromise between weight and cost through genetic algorithm.

**Keywords** Genetic Algorithm · Laminates · Stacking Sequence · Hybrid Composites

## 1 Introduction

Composites material offer improved strength, stiffness, fatigue, and corrosion resistance, etc over conventional materials, which is widely used in automotive, aerospace, and ship building industry. However, the high cost of fabrication of composites is a critical drawback for its application, for example, the graphite/epoxy composite part may cost as much as \$650 to \$900 per kilogram, while the price of glass/epoxy is about 2.5 times less. The mechanical performance of a composite is affected by a wide range of factors, thickness, material,number and orientation of a lamina.

In typical engineering applications, composite materials are under very complicate loading conditions, not only in-plane loads but also out-of-plane loads. Most of the studies on the optimization of laminated composite materials was to mimimize the thickness [1, 26], weight[4, 5, 16], cost and weight[4, 15], or maximize the static strength of composite laminates for a targeted thickness[8, 9, 26]. In the present study, laminate cost and weight are minimized by modifying the objective function.

To tailor a laminate composite, genetic algorithm(GA) has been successfully applied to solve laminate design problem[2, 7, 9, 10, 13, 14, 19, 20, 22, 25, 26]. GA simulates the process of natural evolutionary includes selection, crossover, and mutation according to Darwin's principal of "survival of the fittest". Selection is the most important operator of GA which decides the diversity of the population. In this step, if the selection pressure increases, the converge speed of the population increases, however, the diversity of the population decrease. To improve the search ability and reduce the search cost, various selection

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methods has been invented, and the selection schemes can be divided into four classes which are proportionate reproduction, ranking, tournament, and genitor(or ”steady state”) selection. In the optimization of laminated composite structure, roulette wheel[19, 21] and tournament[9] have been applied. The pros of GA as the following: (i): GAs are not easily trapped in local optima, and be able to obtain the global optimal. (ii): GA doesn’t need gradient information and can be applied to discrete optimization problem. (iii): GA not only be able to find the optimal value in the domain, but also can maintain a set of optimal solutions

In order to check the feasibility of a composite laminate by imposing a strength constraint, failure analysis of a laminate is taken by applying suitable failure criteria. The previous researchers adopted the first-ply-failure approach using the Tsai-wu failure theory [3, 5, 6, 12, 15, 17, 18, 24], Tsai-Hill[11, 23], the maximum stress[6, 15], or the maximum strain[27] static failure criteria. In the present study, Tsai-wu static failure criteria is used to investigated the feasibility of a composite laminate.

## 2 Stress and Strain in a Laminate

A laminated structure is consisting of multiple laminas bonded together through their thickness. Consider a laminated composite plate which is symmetric to its middle plane and subjected to in-plane loads of extension, shear, bending and torsion, the classical lamination theory(CLT) is taken to calculate the stresses and strains in the local and global axes of each ply. as shown in Fig.1.

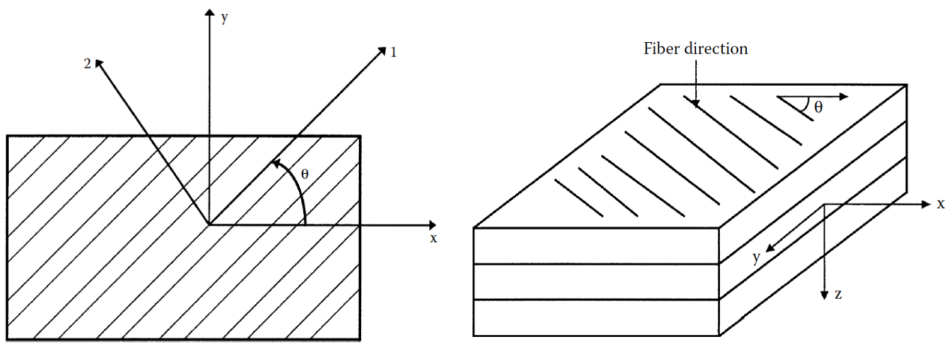


Fig. 1: Lamina

### 2.1 Stress and Strian in a Lamina

For a single lamina, the stress strain relation in the local axis.

$$\begin{bmatrix} \sigma_1 \\ \sigma_2 \\ \tau_{12} \end{bmatrix} = \begin{bmatrix} Q_{11} & Q_{12} & 0 \\ Q_{12} & Q_{22} & 0 \\ 0 & 0 & Q_{66} \end{bmatrix} \begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \gamma_{12} \end{bmatrix} \tag{1}$$

Where $Q_{ij}$ are the stiffnesses of the lamina that are related to engineering elastic constants by

$$\begin{aligned}
Q_{11} &= \frac{E_1}{1-v_{12}v_{21}} \\
Q_{22} &= \frac{E_2}{1-v_{12}v_{21}} \\
Q_{66} &= G_{12} \\
Q_{12} &= \frac{v_{21}E_2}{1-v_{12}v_{21}}
\end{aligned} \tag{2}$$

Where, $E_1, E_2, v_{12}, G_{12}$ are four independent engineering elastic constants, they are defined as

$E_1$ = longitudinal Young's modulus(in direction 1)
$E_2$ = transverse Young's modulus(in direction 1)
$v_{12}$ = major Poisson's ratio
$G_{12}$ = in-plane shear modulus (in plane 1-2)

Stress strain relation in global axis are

$$\begin{bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{bmatrix} = \begin{bmatrix} \bar{Q}_{11} & \bar{Q}_{12} & \bar{Q}_{16} \\ \bar{Q}_{12} & \bar{Q}_{22} & \bar{Q}_{26} \\ \bar{Q}_{16} & \bar{Q}_{26} & \bar{Q}_{66} \end{bmatrix} \begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{bmatrix} \tag{3}$$

where

$$\begin{aligned}
\bar{Q}_{11} &= Q_{11}c^4 + Q_{22}s^4 + 2\left(Q_{12} + 2Q_{66}\right)s^2c^2 \\
\bar{Q}_{12} &= \left(Q_{11} + Q_{22} - 4Q_{66}\right)s^2c^2 + Q_{12}\left(c^4 + s^2\right) \\
\bar{Q}_{22} &= Q_{11}s^4 + Q_{22}c^4 + 2\left(Q_{12} + 2Q_{66}\right)s^2c^2 \\
\bar{Q}_{16} &= \left(Q_{11} - Q_{12} - 2Q_{66}\right)c^3s - \left(Q_{22} - Q_{12} - 2Q_{66}\right)s^3c \\
\bar{Q}_{26} &= \left(Q_{11} - Q_{12} - 2Q_{66}\right)cs^3 - \left(Q_{22} - Q_{12} - 2Q_{66}\right)c^3s \\
\bar{Q}_{66} &= \left(Q_{11} + Q_{22} - 2Q_{12} - 2Q_{66}\right)s^2c^2 + Q_{66}\left(s^4 + c^4\right)
\end{aligned} \tag{4}$$

The local and global stresses in an angle lamina are related to each other through the angle of lamina $\theta$

$$\begin{bmatrix} \sigma_1 \\ \sigma_2 \\ \tau_{12} \end{bmatrix} = [T] \begin{bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{bmatrix} \tag{5}$$

where

$$[T] = \begin{bmatrix} c^2 & s^2 & 2sc \\ s^2 & c^2 & -2sc \\ -sc & sc & c^2 - s^2 \end{bmatrix} \tag{6}$$

## 2.2 Stress and Strain in a Laminate

$$\begin{aligned}
\begin{bmatrix} N_x \\ N_y \\ N_{xy} \end{bmatrix} &= \begin{bmatrix} A_{11} & A_{12} & A_{16} \\ A_{12} & A_{22} & A_{26} \\ A_{16} & A_{26} & A_{66} \end{bmatrix} \begin{bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \end{bmatrix} + \begin{bmatrix} B_{11} & B_{12} & B_{16} \\ B_{11} & B_{12} & B_{16} \\ B_{16} & B_{26} & B_{66} \end{bmatrix} \begin{bmatrix} k_x \\ k_y \\ k_{xy} \end{bmatrix} \\
\begin{bmatrix} M_x \\ M_y \\ M_{xy} \end{bmatrix} &= \begin{bmatrix} B_{11} & B_{12} & B_{16} \\ B_{12} & B_{22} & B_{26} \\ B_{16} & B_{26} & B_{66} \end{bmatrix} \begin{bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \end{bmatrix} + \begin{bmatrix} D_{11} & D_{12} & D_{16} \\ D_{11} & D_{12} & D_{16} \\ D_{16} & D_{26} & D_{66} \end{bmatrix} \begin{bmatrix} k_x \\ k_y \\ k_{xy} \end{bmatrix}
\end{aligned} \tag{7}$$

where

$$
\begin{aligned}
A_{ij} &= \sum_{k=1}^{n} (\overline{Q_{ij}})_k (h_k - h_{k-1}) \\
B_{ij} &= \frac{1}{2} \sum_{k=1}^{n} (\overline{Q_{ij}})_k (h_k - h_{k-1}) \\
D_{ij} &= \frac{1}{3} \sum_{k=1}^{n} (\overline{Q_{ij}})_k (h_k - h_{k-1})
\end{aligned} \tag{8}
$$

The $[A]$, $[B]$, and $[D]$ matrices are called the extensional, coupling, and bending stiffness matrices.

## 3 Failure Theories of an Angle Lamina

### 3.1 Failure Theories of an Angle Lamina

Many different theories about the failure of an angle lamina have been developed for a unidirectional lamina, such as maximum stress failure theory, maximum strain failure theory, Tsai-Hill failure theory, and Tsai-Wu failure theory. The failure theories of a lamina are based on the stresses in local axes in the material. There are four normal strength parameters and one shear stress for a unidirectional lamina. The five strength parameters are

$(\sigma_1^T)_{ult}$ = Ultimate longitudinal tensile strength(in direction 1),
$(\sigma_1^C)_{ult}$ = Ultimate longitudinal compressive strength(in direction 1),
$(\sigma_2^T)_{ult}$ = Ultimate transverse tensile strength(in direction 2),
$(\sigma_2^C)_{ult}$ = Ultimate transverse compressive strength(in direction 2), and
$(\tau_{12})_{ult}$ = Ultimate in-plane shear strength

In the present study, Tsai-wu failure theory is taken to decide whether a lamina is failed or not, the reason is chosen because this theory is more general than Tsai-Hill failure theory which consider two different situation, compressive and tensile strength of a lamina. A lamina is considered to be failed if

$$
H_1\sigma_1 + H_2\sigma_2 + H_6\tau_{12} + H_{11}\sigma_1^2 + H_{22}\sigma_2^2 + H_{66}\tau_{12}^2 + 2H_{12}\sigma_1\sigma_2 < 1 \tag{9}
$$

is violated. where

$$
\begin{aligned}
H_1 &= \frac{1}{(\sigma_1^T)_{ult}} - \frac{1}{(\sigma_1^C)_{ult}} \\
H_{11} &= \frac{1}{(\sigma_1^T)_{ult}(\sigma_1^C)_{ult}} \\
H_2 &= \frac{1}{(\sigma_2^T)_{ult}} - \frac{1}{(\sigma_2^C)_{ult}} \\
H_{22} &= \frac{1}{(\sigma_2^T)_{ult}(\sigma_2^C)_{ult}} \\
H_{66} &= \frac{1}{(\tau_{12})_{ult}^2} \\
H_{12} &= -\frac{1}{2}\sqrt{\frac{1}{(\sigma_1^T)_{ult}(\sigma_1^C)_{ult}(\sigma_2^T)_{ult}(\sigma_2^C)_{ult}}}
\end{aligned} \tag{10}
$$

The Equation 9 can determin whether a lamina failed or not, but it failed to give the information about how much load can be increased or decreased to keep the lamina safe. The strength ratio(SR) is to used to solve this problem, and defined as

$$SR = \frac{\text{Maximum Load Which Can Be Applied}}{\text{Load Applied}} \tag{11}$$

Substituting Equation 11 for $SR$ into Equation 9,we obtain

$$(F_{11}\sigma_1^2 + F_{22}\sigma_2^2 + F_{66}\sigma_6^2 + 2F_{12}\sigma_1\sigma_2)SR^2 + (F_1\sigma_1 + F_2\sigma_2)SR - 1 = 0 \tag{12}$$

3.2 Failure Theories of a Laminate

1. Compute the reduced stiffness matrix $[Q]$ referred to local axis for each ply using its four engineering elastic constants $E_1$, $E_2$, $v_{12}$, and $G_{12}$.
2. calculate the transformed reduced stiffness $[\bar{Q}]$ referred to global coordinate system (x, y) using reduced stiffness matrix $[Q]$ obtained in step 1 and ply angle for each layer.
3. Given the thickness $t_k$ and the location of each layer, find out the three laminate stiffness matrices $[A]$, $[B]$, and $[D]$.
4. Apply forces and moments, $[N]_{xy}$, $[M]_{xy}$, solve the equation 7, calculate the middle plane strain $[\sigma^0]_{xy}$ and cruvature $[k]_{xy}$.
5. Find out the local strain and stress of each layer under the applied load.
6. Use the ply-by-ply stresses and strains in Tsai-wu failure theory to find out the strength ratio.

## 4 Genetic Algorithm Procedure

Find: $\{\theta_k, \text{mat}_k, n\}$ $\theta_k \in \{0, +45, -45, 90\}$ $\text{mat}_k \in \{CA, GR, GL\}$

Minimize: weight

Subject to: safety factor and first ply failure constraint

Each CA, GR and GL layer is assumed to cost 8, 2.5 and 1 monetary units, respectively.

Table 1: GA-parameters

| parameter | value |
|---|---|
| population size | 20 |
| encoding method | float encoding |
| selection strategy | roulette wheel |
| crossover strategy | one-point |
| mutation strategy | mass mutation |

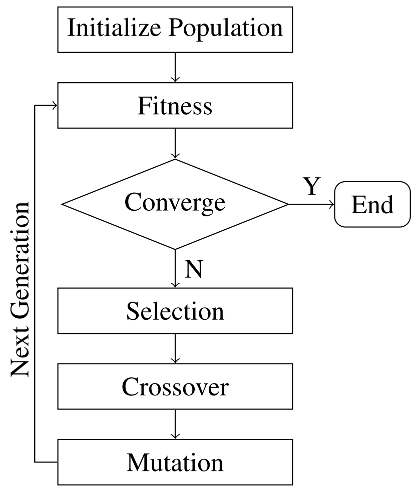


Fig. 2: GA Procedure with Share Function

## 5 Results and Discussion

Table 2: Typical Properties of a Unidirectional Lamina(SI System of Units)

| Property | Symbol | Unit | Carbon/Epoxy | Graphite/Epoxy | Glass/Epoxy |
|---|---|---|---|---|---|
| Longitudinal elastic modulus | $E_1$ | GPa | 116.6 | 181 | 38.6 |
| Traverse elastic modulus | $E_2$ | GPa | 7.67 | 10.3 | 8.27 |
| Major Poisson's ratio | $v_{12}$ | | 0.27 | 0.28 | 0.26 |
| Shear modulus | $G_{12}$ | GPa | 4.17 | 7.17 | 4.14 |
| Ultimate longitudinal tensile strength | $(\sigma_1^T)_{ult}$ | MP | 2062 | 1500 | 1062 |
| Ultimate longitudinal compressive strength | $(\sigma_1^C)_{ult}$ | MP | 1701 | 1500 | 610 |
| Ultimate transverse tensile strength | $(\sigma_2^T)_{ult}$ | MPa | 70 | 40 | 31 |
| Ultimate transverse compressive strength | $(\sigma_2^C)_{ult}$ | MPa | 240 | 246 | 118 |
| Ultimate in-plane shear strength | $(\tau_{12})_{ult}$ | MPa | 105 | 68 | 72 |
| Density | $\rho$ | $g/cm^3$ | 1.605 | 1.590 | 1.903 |
| Cost | | | 8 | 2.5 | 1 |

Table 3: Comparative study of different composite materials for a defined strength ratio

| Load | Objective Function | Stacking sequence | Strength ratio | Mass | Cost | Layer |
|---|---|---|---|---|---|---|
| $N_x = 1e6$ N | | $[0_{cr3}]_s$ | 2.041 | 0.318 | 48.0 | 6 |
| | | $[0_{gr4}/0_{gr}]_s$ | 2.227 | 0.472 | 22.5 | 9 |
| | | $[0_{gl6}]_s$ | 2.103 | 0.753 | 12.0 | 12 |
| | | $[0_{gr4}/0_{gl}]_s$ | 2.031 | 0.482 | 21.0 | 9 |
| Load | Objective Function | Stacking sequence | Strength ratio | Mass | Cost | Layer |
| $N_y = 1e6$ N | | $[90_{cr3}]_s$ | 2.041 | 0.318 | 48.0 | 6 |
| | | $[90_{gr4}/90_{gr}]_s$ | 2.227 | 0.472 | 22.5 | 9 |
| | | $[90_{gl6}]_s$ | 2.103 | 0.753 | 12.0 | 12 |
| | | $[90_{gr4}/90_{gl}]_s$ | 2.031 | 0.482 | 21.0 | 9 |

| Load | Function | Stacking sequence | Strength ratio | Mass | Cost | Layer |
|---|---|---|---|---|---|---|
| $N_{xy} = 1e6$ N | | $[-45_{cr6} + 45_{cr7}]_s$ | 2.013 | 1.377 | 208.0 | 26 |
| | | $[-45_{gr5} / + 45_{gr23}]_s$ | 2.105 | 1.732 | 82.5 | 33 |
| | | $[-45_{gl17} + 45_{gl_3 8}]_s$ | 2.011 | 6.908 | 110.0 | 110 |
| | | $[-45_{ca3} / + 45_{ca4} / + 45_{gr6} / + 45_{ca}]_s$ | 2.074 | 1.424 | 150 | 27 |

| Load | Function | Stacking sequence | Strength ratio | Mass | Cost | Layer |
|---|---|---|---|---|---|---|
| $N_x = 1e6$ N<br>$N_y = 1e6$ N | | $[-45_{cr6} + 45_{cr6}]_s$ | 2.026 | 1.271 | 192.0 | 24 |
| | | $[+45_{gr10} / - 45_{gr21} / + 45_{gr10}]$ | 2.024 | 2.151 | 102.5 | 41 |
| | | $[-45_{gl35} + 45_{gl_7 3} / + 45_{gl35}]$ | 2.001 | 8.980 | 143.0 | 143 |

| Load | Function | Stacking sequence | Strength ratio | Mass | Cost | Layer |
|---|---|---|---|---|---|---|
| $N_x = 1e6N$<br>$N_y = 1e6$<br>$N_{xy} = 1e6$ | | $[+45_{cr12}]$ | 2.041 | 0.636 | 96.0 | 12 |
| | | $[+45_{gr18}]$ | 2.227 | 0.945 | 45.0 | 45 |
| | | $[+45_{gl23}]$ | 2.015 | 1.444 | 23.0 | 23 |
| | | $[+45_{gl} / + 45_{gr16} / + 45_{gr}]$ | 2.031 | 0.965 | 42.0 | 18 |

## 6 Concluding Remarks

## 7 Acknowledgements

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