

Laminated Composite Plate Optimization by Improved Genetic Algorithm

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Abstract

Failure analysis of laminated composite plates under different mechanical loads for different stacking sequences, fiber orientation, and composite material system is studied in this paper. An optimum composite material and laminate layout is well investigated for a targeted strength ratio which makes a compromise between weight and cost through improved genetic algorithm.

Keywords

Genetic Algorithm, Laminates, Stacking Sequence, Hybrid Composites

Introduction

Composite materials offer improved strength, stiffness, fatigue, and corrosion resistance, etc over conventional materials, which is widely used in automotive, aerospace, and ship building industry. However, the high cost of fabrication of composites is a critical drawback for its application, for example, the graphite/epoxy composite part may cost as much as \$650 to \$900 per kilogram, while the price of glass/epoxy is about 2.5 times less, and the mechanical performance of a composite material is affected by a wide range of factors, thickness, material, and orientation of each lamina.

In typical engineering applications, composite materials are under very complicate loading conditions, not only in-plane loads but also out-of-plane loads. Most of the studies on the optimization of laminated composite materials was to minimize the thickness [Abu-Odeh and Jones \(1998\)](#); [Walker and Smith \(2003\)](#), weight [Fang and Springer \(1993\)](#); [Deka et al. \(2005\)](#); [Park et al. \(2008\)](#), cost and weight [Deka et al. \(2005\)](#); [Omkar et al. \(2008\)](#), or maximize the static strength of composite laminates for a targeted thickness [Walker and Smith \(2003\)](#); [Lin and Lee \(2004\)](#); [Kim \(2007\)](#). In the present study, laminate cost and weight are minimized by modifying the objective function.

To tailor a laminate composite, genetic algorithm (GA) has been successfully applied to solve laminate design problem [Riche and Haftka \(1993\)](#); [Nagendra et al. \(1996\)](#); [Sadagopan and Pitchumani \(1998\)](#); [Todoroki and Haftka \(1998\)](#); [Liu et al. \(2000\)](#); [Sivakumar et al. \(1998\)](#); [Walker and Smith \(2003\)](#); [Lin and Lee \(2004\)](#); [Kang and Kim \(2005\)](#); [Murugan et al. \(2007\)](#); [Akbulut and Sonmez \(2008\)](#). GA simulates the process of natural evolutionary includes selection, crossover, and mutation according to Darwin's principal of "survival of the fittest". Selection is the most important operator of GA which decides the diversity of the population. In this step, if the selection pressure increases, the converge speed of the population increases, however, the diversity of the population decreases. To improve the search ability and reduce the search cost, various selection

methods have been invented, and the selection schemes can be divided into four classes which are proportionate reproduction, ranking, tournament, and genitor (or "steady state") selection. In the optimization of laminated composite structure, roulette wheel [Riche and Haftka \(1993\)](#); [Seresta et al. \(2007\)](#) and tournament [Lin and Lee \(2004\)](#) have been applied. The pros of GA as the following: (i): GAs are not easily trapped in local optima, and be able to obtain the global optimal. (ii): GA doesn't need gradient information and can be applied to discrete optimization problem. (iii): GA not only be able to find the optimal value in the domain, but also can maintain a set of optimal solutions.

In order to check the feasibility of a composite laminate by imposing a strength constraint, failure analysis of a laminate is taken by applying suitable failure criteria. The previous researchers adopted the first-ply-failure approach using the Tsai-wu failure theory [Massard \(1984\)](#); [Reddy and Pandey \(1987\)](#); [Fang and Springer \(1993\)](#); [Soeiro et al. \(1994\)](#); [Pelletier and Vel \(2006\)](#); [Jadhav and Mantena \(2007\)](#); [Omkar et al. \(2008\)](#); [Choudhury et al. \(2019\)](#), Tsai-Hill [Martin \(1987\)](#); [Soares et al. \(1995\)](#), the maximum stress [Jadhav and Mantena \(2007\)](#); [Omkar et al. \(2008\)](#), or the maximum strain [Watkins and Morris \(1987\)](#) static failure criteria. In the present study, Tsai-wu static failure criteria is used to investigated the feasibility of a composite laminate.

Stress and Strain in a Laminate

A laminated structure is consisting of multiple laminas bonded together through their thickness. Consider a laminated composite plate which is symmetric to its middle plane and subjected to in-plane loads of extension, shear,

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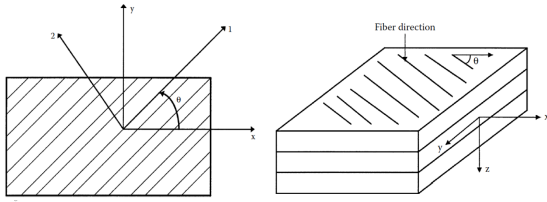


Figure 1. Lamina

bending and torsion, the classical lamination theory (CLT) is taken to calculate the stresses and strains in the local and global axes of each ply, as shown in Fig. 1.

Stress and Strian in a Lamina

For a single lamina, the stress strain relation in the local axis.

$$\begin{bmatrix} \sigma_1 \\ \sigma_2 \\ \tau_{12} \end{bmatrix} = \begin{bmatrix} Q_{11} & Q_{12} & 0 \\ Q_{12} & Q_{22} & 0 \\ 0 & 0 & Q_{66} \end{bmatrix} \begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \gamma_{12} \end{bmatrix} \quad (1)$$

Where Q_{ij} are the stiffnesses of the lamina that are related to engineering elastic constants by

$$\begin{aligned} Q_{11} &= \frac{E_1}{1 - \nu_{12}\nu_{21}} \\ Q_{22} &= \frac{E_2}{1 - \nu_{12}\nu_{21}} \\ Q_{66} &= G_{12} \\ Q_{12} &= \frac{\nu_{21}E_2}{1 - \nu_{12}\nu_{21}} \end{aligned} \quad (2)$$

Where, $E_1, E_2, \nu_{12}, G_{12}$ are four independent engineering elastic constants, they are defined as

E_1 = longitudinal Young's modulus E_2 = transverse Young's modulus ν_{12} = major Poisson's ratio G_{12} = in-plane shear modulus Stress strain relation in global axis are

$$\begin{bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{bmatrix} = \begin{bmatrix} \bar{Q}_{11} & \bar{Q}_{12} & \bar{Q}_{16} \\ \bar{Q}_{12} & \bar{Q}_{22} & \bar{Q}_{26} \\ \bar{Q}_{16} & \bar{Q}_{26} & \bar{Q}_{66} \end{bmatrix} \begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{bmatrix} \quad (3)$$

where

$$\begin{aligned} \bar{Q}_{11} &= Q_{11}c^4 + Q_{22}s^4 + 2(Q_{12} + 2Q_{66})s^2c^2 \\ \bar{Q}_{12} &= (Q_{11} + Q_{22} - 4Q_{66})s^2c^2 + Q_{12}(c^4 + s^4) \\ \bar{Q}_{22} &= Q_{11}s^4 + Q_{22}c^4 + 2(Q_{12} + 2Q_{66})s^2c^2 \\ \bar{Q}_{16} &= (Q_{11} - Q_{12} - 2Q_{66})c^3s - (Q_{22} - Q_{12} - 2Q_{66})s^3c \\ \bar{Q}_{26} &= (Q_{11} - Q_{12} - 2Q_{66})cs^3 - (Q_{22} - Q_{12} - 2Q_{66})c^3s \\ \bar{Q}_{66} &= (Q_{11} + Q_{22} - 2Q_{12} - 2Q_{66})s^2c^2 + Q_{66}(s^4 + c^4) \end{aligned} \quad (4)$$

The c and s denotes $\cos\theta$, and $\sin\theta$.

The local and global stresses in an angle lamina are related to each other through the angle of lamina θ

$$\begin{bmatrix} \sigma_1 \\ \sigma_2 \\ \tau_{12} \end{bmatrix} = [T] \begin{bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{bmatrix} \quad (5)$$

where

$$[T] = \begin{bmatrix} c^2 & s^2 & 2sc \\ s^2 & c^2 & -2sc \\ -sc & sc & c^2 - s^2 \end{bmatrix} \quad (6)$$

Stress and Strain in a Laminate

$$\begin{bmatrix} N_x \\ N_y \\ N_{xy} \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} & A_{16} \\ A_{12} & A_{22} & A_{26} \\ A_{16} & A_{26} & A_{66} \end{bmatrix} \begin{bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \end{bmatrix} + \begin{bmatrix} B_{11} & B_{12} & B_{16} \\ B_{12} & B_{22} & B_{26} \\ B_{16} & B_{26} & B_{66} \end{bmatrix} \begin{bmatrix} k_x \\ k_y \\ k_{xy} \end{bmatrix} \quad (7)$$

$$\begin{bmatrix} M_x \\ M_y \\ M_{xy} \end{bmatrix} = \begin{bmatrix} B_{11} & B_{12} & B_{16} \\ B_{12} & B_{22} & B_{26} \\ B_{16} & B_{26} & B_{66} \end{bmatrix} \begin{bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \end{bmatrix} + \begin{bmatrix} D_{11} & D_{12} & D_{16} \\ D_{12} & D_{22} & D_{26} \\ D_{16} & D_{26} & D_{66} \end{bmatrix} \begin{bmatrix} k_x \\ k_y \\ k_{xy} \end{bmatrix}$$

where

$$\begin{aligned} A_{ij} &= \sum_{k=1}^n (\bar{Q}_{ij})_k (h_k - h_{k-1}) \\ B_{ij} &= \frac{1}{2} \sum_{k=1}^n (\bar{Q}_{ij})_k (h_k - h_{k-1}) \\ D_{ij} &= \frac{1}{3} \sum_{k=1}^n (\bar{Q}_{ij})_k (h_k - h_{k-1}) \end{aligned} \quad (8)$$

The $[A]$, $[B]$, and $[D]$ matrices are called the extensional, coupling, and bending stiffness matrices.

Failure Theories of an Angle Lamina

Failure Theories of an Angle Lamina

Many different theories about the failure of an angle lamina have been developed for a unidirectional lamina, such as maximum stress failure theory, maximum strain failure theory, Tsai-Hill failure theory, and Tsai-Wu failure theory. The failure theories of a lamina are based on the stresses in local axes in the material. There are four normal strength parameters and one shear stress for a unidirectional lamina.

The five strength parameters are

$$\begin{aligned} (\sigma_1^T)_{ult} &= \text{Ultimate longitudinal tensile strength,} \\ (\sigma_1^C)_{ult} &= \text{Ultimate longitudinal compressive strength,} \\ (\sigma_2^T)_{ult} &= \text{Ultimate transverse tensile strength,} \\ (\sigma_2^C)_{ult} &= \text{Ultimate transverse compressive strength, and} \\ (\tau_{12})_{ult} &= \text{Ultimate in-plane shear strength} \end{aligned}$$

In the present study, Tsai-wu failure theory is taken to decide whether a lamina is failed or not, the reason is chosen because this theory is more general than Tsai-Hill failure theory which consider two different situation, compressive and tensile strength of a lamina. A lamina is considered to be failed if

$$H_1\sigma_1 + H_2\sigma_2 + H_6\tau_{12} + H_{11}\sigma_1^2 + H_{22}\sigma_2^2 + H_{66}\tau_{12}^2 + 2H_{12}\sigma_1\sigma_2 < 1 \quad (9)$$

is violated. where

$$\begin{aligned}
H_1 &= \frac{1}{(\sigma_1^T)_{ult}} - \frac{1}{(\sigma_1^C)_{ult}} \\
H_{11} &= \frac{1}{(\sigma_1^T)_{ult}(\sigma_1^C)_{ult}} \\
H_2 &= \frac{1}{(\sigma_2^T)_{ult}} - \frac{1}{(\sigma_2^C)_{ult}} \\
H_{22} &= \frac{1}{(\sigma_2^T)_{ult}(\sigma_2^C)_{ult}} \\
H_{66} &= \frac{1}{(\tau_{12})_{ult}^2} \\
H_{12} &= -\frac{1}{2} \sqrt{\frac{1}{(\sigma_1^T)_{ult}(\sigma_1^C)_{ult}(\sigma_2^T)_{ult}(\sigma_2^C)_{ult}}}
\end{aligned} \quad (10)$$

The Equation 9 can determine whether a lamina failed or not, but it fails to give the information about how much load can be increased or decreased to keep the lamina safe. The strength ratio(SR) is used to solve this problem, and defined as

$$SR = \frac{\text{Maximum Load Which Can Be Applied}}{\text{Load Applied}} \quad (11)$$

Substituting Equation 11 for SR into Equation 9, we obtain

$$\begin{aligned}
&(F_{11}\sigma_1^2 + F_{22}\sigma_2^2 + F_{66}\sigma_6^2 + 2F_{12}\sigma_1\sigma_2)SR^2 \\
&+ (F_1\sigma_1 + F_2\sigma_2)SR - 1 = 0
\end{aligned} \quad (12)$$

Failure Theories of a Laminate

1. Compute the reduced stiffness matrix $[Q]$ referred to local axis for each ply using its four engineering elastic constants E_1 , E_2 , ν_{12} , and G_{12} .
2. calculate the transformed reduced stiffness $[\bar{Q}]$ referred to global coordinate system (x, y) using reduced stiffness matrix $[Q]$ obtained in step 1 and ply angle for each layer.
3. Given the thickness t_k and the location of each layer, find out the three laminate stiffness matrices $[A]$, $[B]$, and $[D]$.
4. Apply forces and moments, $[N]_{xy}$, $[M]_{xy}$, solve the equation 7, calculate the middle plane strain $[\sigma^0]_{xy}$ and cruvature $[k]_{xy}$.
5. Find out the local strain and stress of each layer under the applied load.
6. Use the ply-by-ply stresses and strains in Tsai-wu failure theory to find out the strength ratio.

Optimum Design of laminated composites

Improved Genetic Algorithm

In the present study, the relevant parameters of GA are as shown in Table ???. The design variables are the materials, number of layers, and ply orientation restricted to a discrete set of angles (0, ± 45 and 90 degrees). The possible materials are graphite/epoxy, caron/epoxy, and glass/epoxy and are represented by the codes 0, 1 and 2, respectively.

The selection method of the basic GA is sorting the population according to the absolute difference of individual's strength ratio and specified safety factor in an incresing order, individuals with smaller difference are more likely to be selected; However, because of premature and weak local search ability, basic GA are more likely to get stuck in local optimum. Therefore, to prevent the GA from early convergence and improve the

local search performance, an improved GA is proposed by chosing individual according to fitness, ignoring whether the individual satisfy the constraint requirement. by combing basic GA existing selection strategy with this new selection approach, Individuals are selected from the two approaches forming the parents of next generation.

The laminate chromosomes are represented by double-gene string which can be divided into two parts, one part represents the angles, the other part represents the materials(as shown in Figure 4). To maintain the diversity of the population, single-point crossover is taken during the evolution process. The break point in the string are chosen randomly, and one of the offspring of parent 1(as shown in Figure 4) and parent 2(as shown in Figure 3) is obtained by combining the gene segments $P1_o$ and $P2_o$, $P1_m$ and $P2_m$, respectively. The gene code of the offspring laminate is $[+45, -45, -45, -45, -45, -45, -45, 0, 1, 0, 1, 0, 1, 0]$.

To prevent the search from getting stuck in a local optimum, mutation is used to random change the gene in the chromosome, the offspring after mutation operator is as shown in Figure 5

Design Problem I

The aim is to minimize the mass of a composite laminate for a targeted strength ratio by Tsai-wu failure theory. The design variable are the ply angles and the number of layers.

Find: $\{\theta_k, n\}$ $\theta_k \in \{0, +45, -45, 90\}$

Minimize: weight

Subject to: strength ratio and first ply failure constraint

Design Problem II

The aim is to minimize the combined cost and weight of hybrid composite laminate under various loading cases, so the design variable not only include the ply angles and number of layers, but also the material of each lamina.

Find: $\{\theta_k, \text{mat}_k, n\}$ $\theta_k \in \{0, +45, -45, 90\}$ $\text{mat}_k \in \{CA, GR, GL\}$

Minimize:

$$F = \frac{\text{Cost}}{C_{\min}} + \frac{\text{Weight}}{W_{\min}} \quad (13)$$

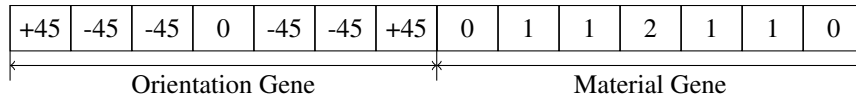
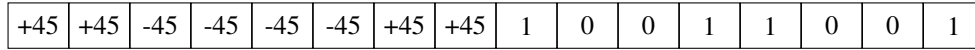
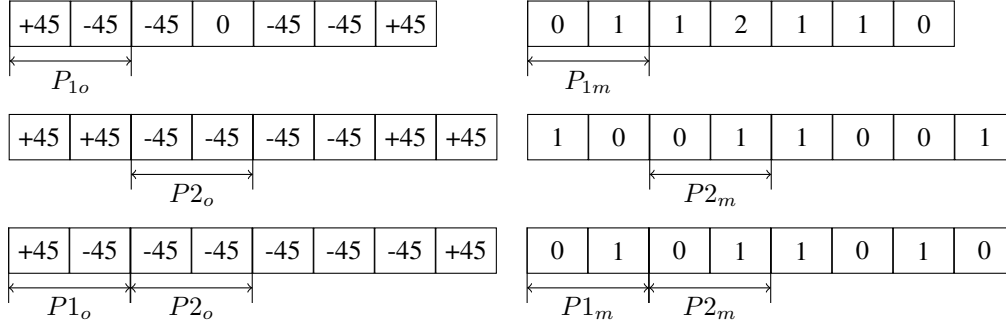
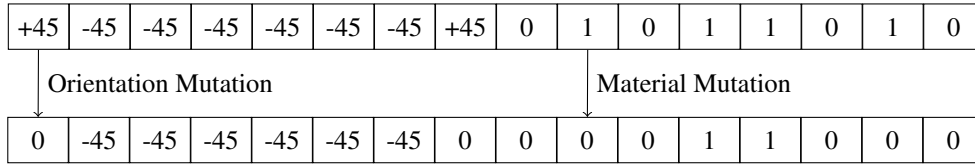
Subject to: strength ratio and first ply failure constraint

Here CA, GF, and GL represent carbon/epoxy, graphite/epoxy, and glass/epoxy, C_{\min} and W_{\min} represent the cost and weight corresponding to the laminates with minimum cost and minimum weight obtained from previous problem.

numerical results and discussion

A composite laminate with dimensions $1000 \times 1000 \times 0.165\text{mm}^3$ of each lamina is under various loading cases, and each CA, GF, and GL layer is assumed to cost 8, 2.5 and 1 monetary units, respectively. The other used material properties are as shown in Table 1. In the present experiment, the optimum composite system, layup, thickness, and number of layers for a targeted strength ratio(2 in this paper) under two different in-plane loading is investigated.

The GA process can be divided into two phases by whether there are individuals which are appropriate or not, an appropriate individual means it can satisfy the constraint.

**Figure 2.** Parent 1**Figure 3.** Parent 2**Figure 4.** Crossover Operation**Figure 5.** Mutation**Figure 6.** Example header text.

During the initial phase, no individual's strength ratio is over the specified threshold, so individuals with bigger fitness are more likely to be chosen as parents, that's why the strength ratio curves go all the way up to the specified threshold during the first stage; After the initial phase, the GA produces a bunch of appropriate individuals, and then the target function comes to play, as you can see from Fig.7, the fitness curves are trending to go down, but the strength ratio curves are keep to greater the specified threshold.

In the first experiment, the applied stress are $N_x = N_y = 1e6$ N, and the safety factor is 2. As shown in the lower left sub-figure of Fig.7, both of the basic GA and improved GA methods obtained the optimal value, but the improved GA converged more slowly than the basic GA. As it can be seen from Table 2, a $[-45_6/+45_6]_s$ carbon/epoxy laminate has the least weight, and a $[-45_{35}/+45_{73}/+45_{35}]$ graphite/epoxy laminate has the lowest cost.

In the second experiment, the applied stress are $N_x = N_y = N_z = 1e6$ N, and the safety factor is 2. As shown in the upper right sub-figure of Fig.8, both of the two GA approaches reached to certain mature states, however, the improved GA offers more powerful local search ability, in which basic GA got stuck in local optimum, and improved GA was able to get out of the local optimum and obtained global optimum value. As it shown in Table 3, Compared with the $[+45_{12}^{CT}]$ laminate, the weight of a $[+45_8^{gr}/+45_5^{gl}]_s$ laminate increases 41.8%, however, the cost decreases 56%.

Conclusions

In this paper, a combination of CLT and improved GA are employed to minimize the weight and cost of a single-material and hybrid composite laminate, respectively, under various in-plane loading cases. Improved GA is proposed to obtain the global optimum design, results are presented in two sections, stacking sequence optimization for a single material laminate, and combined weight and cost optimization of a carbon/epoxy, graphite/epoxy, and glass/epoxy hybrid laminate. The improved GA significantly improves the search performance, and is not easily trapped in local optima.

Figures and tables

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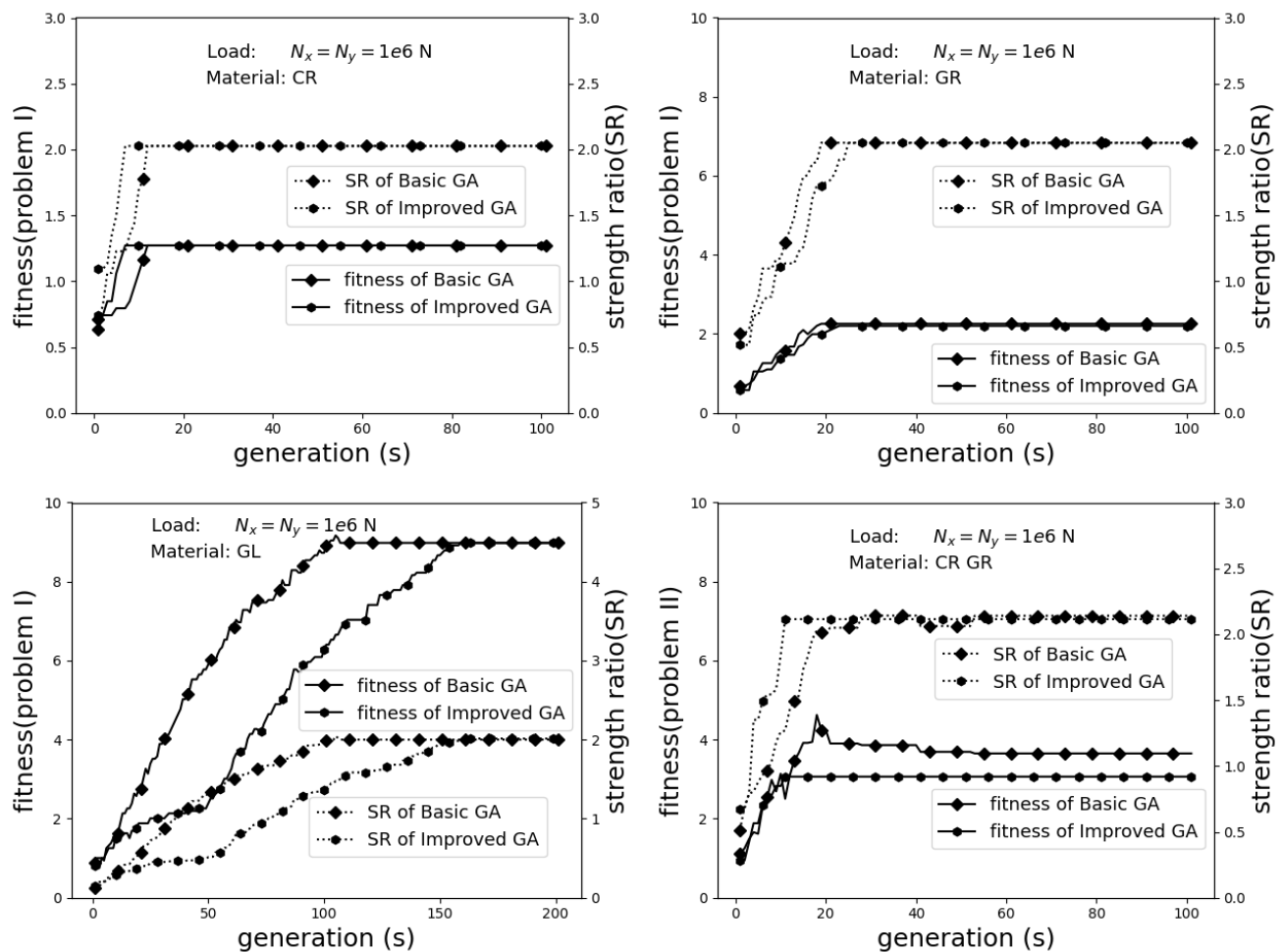
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Table 1. Comparison of carbon/epoxy, graphite/epoxy, and glass/epoxy properties

Property	Symbol	Unit	Carbon/Epoxy	Graphite/Epoxy	Glass/Epoxy
Longitudinal elastic modulus	E_1	GPa	116.6	181	38.6
Transverse elastic modulus	E_2	GPa	7.67	10.3	8.27
Major Poisson's ratio	ν_{12}		0.27	0.28	0.26
Shear modulus	G_{12}	GPa	4.17	7.17	4.14
Ultimate longitudinal tensile strength	$(\sigma_1^T)_{ult}$	MP	2062	1500	1062
Ultimate longitudinal compressive strength	$(\sigma_1^C)_{ult}$	MP	1701	1500	610
Ultimate transverse tensile strength	$(\sigma_2^T)_{ult}$	MPa	70	40	31
Ultimate transverse compressive strength	$(\sigma_2^C)_{ult}$	MPa	240	246	118
Ultimate in-plane shear strength	$(\tau_{12})_{ult}$	MPa	105	68	72
Density	ρ	g/cm^3	1.605	1.590	1.903
Cost			8	2.5	1

**Figure 7.** GA process under load $N_x = N_y = 1e6$ N

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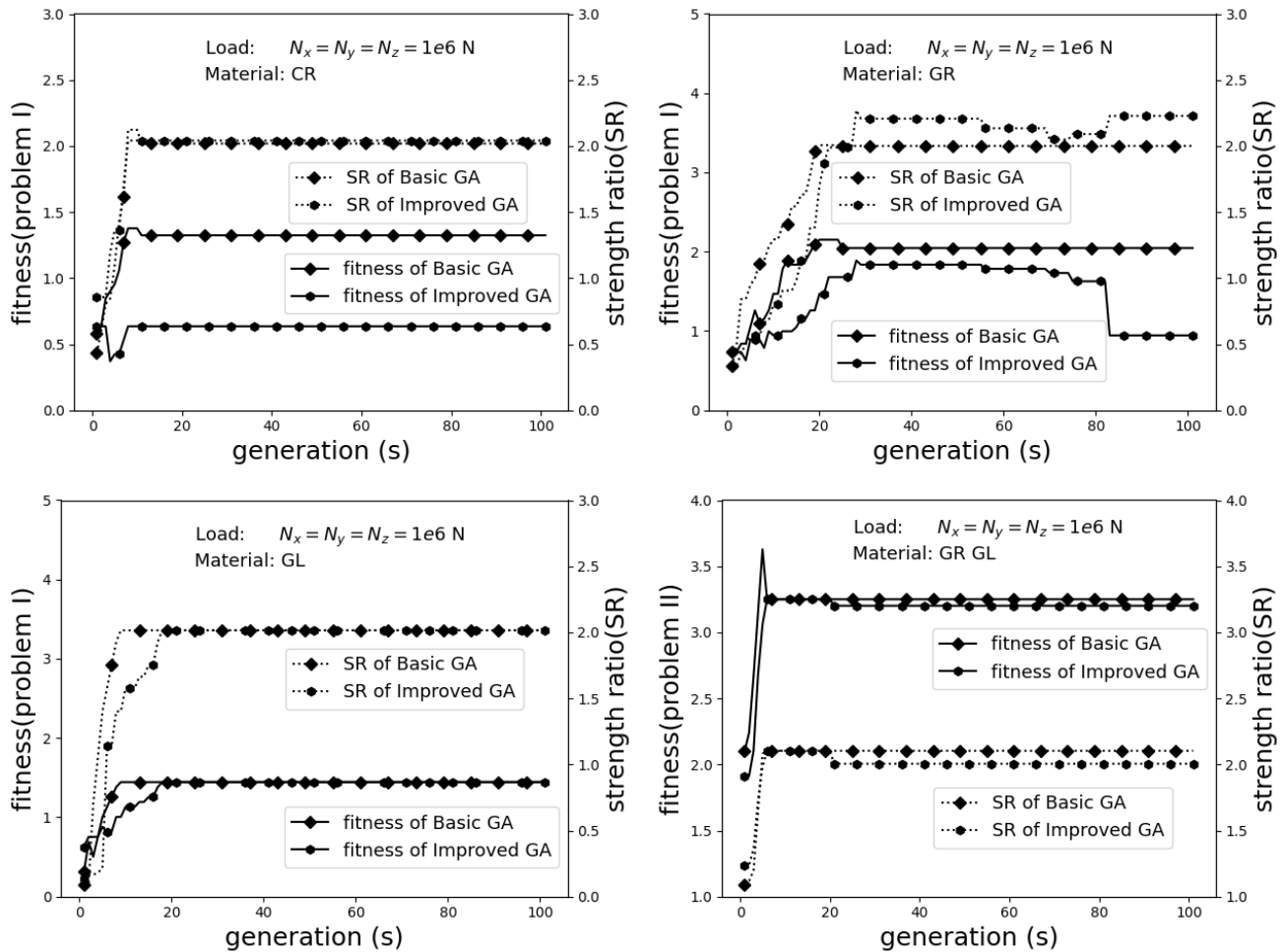
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Table 2. Optimization results

Load(N)	Problem	Algorithm	Stacking sequence	Strength ratio	Mass	Cost	Layer
$N_x = 1e6$ $N_y = 1e6$	I	GA	$[-45_{6^{cr}}^{cr}/+45_{6^{cr}}^{cr}]_s$	2.026	1.271	192.0	24
	I	IGA	$[-45_{6^{cr}}^{cr}/+45_{6^{cr}}^{cr}]_s$	2.026	1.271	192.0	24
	I	GA	$[0_6^{gr}/-45_4^{gr}/+45_4^{gr}/90_7^{gr}/90_7^{gr}]_s$	2.051	2.256	107.5	43
	I	IGA	$[+45_{10}^{gr}/-45_{21}^{gr}/+45_{10}^{gr}]_s$	2.024	2.151	102.5	41
	I	GA	$[-45_{35}^{gl}/+45_{73}^{gl}/+45_{35}^{gl}]_s$	2.001	8.980	143.0	143
	I	IGA	$[-45_{35}^{gl}/+45_{73}^{gl}/+45_{35}^{gl}]_s$	2.001	8.980	143.0	143
	II	GA	$[-45_{12}^{gr}/+45_5^{cr}/+45_7^{gr}]_s$	2.141	2.523	175	48
	II	IGA	$[-45_9^{gr}/+45_9^{gr}/-45_2^{cr}/+45_2^{cr}]_s$	2.054	2.313	154	44

Subscript "cr" denotes a carbon/epoxy ply, "gr" denotes graphite/epoxy ply, "gl" denotes a glass/epoxy ply

**Figure 8.** GA process under load $N_x = N_y = N_z = 1e6$ N**Table 3.** Optimization results

Load(N)	Problem	Algorithm	Stacking sequence	Strength ratio	Mass	Cost	Layer
$N_x = 1e6$ $N_y = 1e6$ $N_{xy} = 1e6$	I	GA	$[+45_{11}^{cr}/-45_{11}^{cr}/+45_{11}^{cr}]_s$	2.018	1.324	200.0	25
	I	IGA	$[+45_{12}^{cr}]_s$	2.041	0.636	96.0	12
	I	GA	$[0_4^{gr}/+45_{12}^{gr}/90_3^{gr}/+45]_s$	2.001	2.046	97.5	39
	I	IGA	$[+45_{18}^{gr}]_s$	2.227	0.945	45.0	18
$N_x = 1e6$ $N_y = 1e6$ $N_{xy} = 1e6$	I	GA	$[+45_{23}^{gl}]_s$	2.015	1.444	23.0	23
	I	IGA	$[+45_{23}^{gl}]_s$	2.015	1.444	23.0	23
	II	GA	$[+45_{16}^{gl}/+45_{16}^{gr}/+45_{16}^{gl}]_s$	2.031	0.965	42.0	18
	II	IGA	$[+45_8^{gr}/+45_8^{gl}]_s$	2.005	0.902	41.0	17

Subscript "cr" denotes a carbon/epoxy ply, "gr" denotes graphite/epoxy ply, "gl" denotes a glass/epoxy ply

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