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Laminated Composite Plate Optimization by Genetic Algorithm

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Abstract Failure analysis of laminated composite plates under different mechanical loads for different stacking sequences, fiber orientation, and composite material system is studied in this paper. An optimum composite material and laminate layup is studied for a targeted strength ratio which makes a compromise between weight and cost through genetic algorithm.

Keywords Genetic Algorithm · Laminates · Stacking Sequence · Hybrid Composites

1 Introduction

Composites material offer improved strength, stiffness, fatigue, and corrosion resistance, etc over conventional materials, which is widely used in automotive, aerospace, and ship building industry. However, the high cost of fabrication of composites is a critical drawback for its application, for example, the graphite/epoxy composite part may cost as much as \$650 to \$900 per kilogram. The mechanical performance of a composite is affected by a wide range of factors, fiber length, fiber orientation, fiber shape, and the matrix etc.

Genetic algorithms(GAs) simulate the process of natural evolutionary includes selection, crossover ,and mutation according to Darwin's principal of "survival of the fittest".

According to T Back [**?**],the selection mechanism is one of the primary means of controlling the GA's convergence rate and its likelihood of finding global optima.

2 Stress and Strain in a Laminate

A laminated structure is consisting of multiple laminas bonded together through their thickness. Consider a laminated composite plate which is symmetric to its middle plane and subjected to in-plane loads of extension, shear, bending and torsion, the classical lamination theory(CLT) is taken to calculate the stresses and strains in the local and global axes of each ply. as shown in Fig.1.

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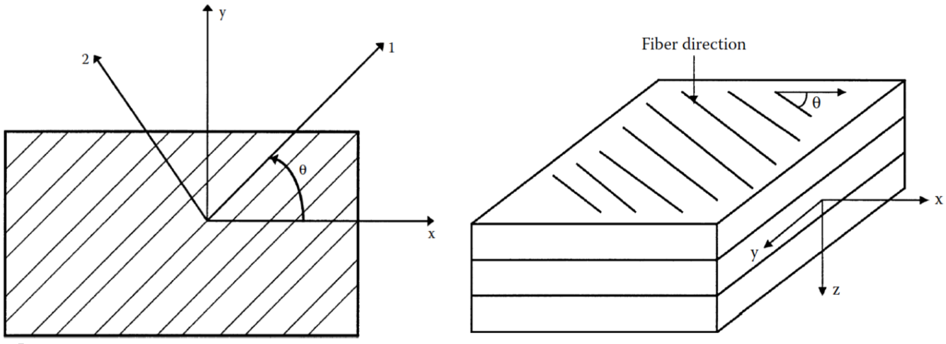

Fig. 1: Lamina

2.1 Stress and Strian in a Lamina

For a single lamina, the stress strain relation in the local axis.

$$\begin{bmatrix}\sigma_1\\ \sigma_2\\ \tau_{12}\end{bmatrix} = \begin{bmatrix}Q_{11} & Q_{12} & 0\\ Q_{12} & Q_{22} & 0\\ 0 & 0 & Q_{66}\end{bmatrix}\begin{bmatrix}\varepsilon_1\\ \varepsilon_2\\ \gamma_{12}\end{bmatrix} \tag{1}$$

Where Q_{ij} are the stiffnesses of the lamina that are related to engineering elastic constants by

$$\begin{aligned}Q_{11} &= \frac{E_1}{1-v_{12}v_{21}}\\ Q_{22} &= \frac{E_2}{1-v_{12}v_{21}}\\ Q_{66} &= G_{12}\\ Q_{12} &= \frac{v_{21}E_2}{1-v_{12}v_{21}}\end{aligned} \tag{2}$$

Where, E_1, E_2, v_{12}, G_{12} are four independent engineering elastic constants, they are defined as

E_1 = longitudinal Young's modulus(in direction 1)
E_2 = transverse Young's modulus(in direction 1)
v_{12} = major Poisson's ratio
G_{12} = in-plane shear modulus (in plane 1-2)

Stress strain relation in global axis are

$$\begin{bmatrix}\sigma_x\\ \sigma_y\\ \tau_{xy}\end{bmatrix} = \begin{bmatrix}\bar{Q}_{11} & \bar{Q}_{12} & \bar{Q}_{16}\\ \bar{Q}_{12} & \bar{Q}_{22} & \bar{Q}_{26}\\ \bar{Q}_{16} & \bar{Q}_{26} & \bar{Q}_{66}\end{bmatrix}\begin{bmatrix}\varepsilon_x\\ \varepsilon_y\\ \gamma_{xy}\end{bmatrix} \tag{3}$$

where

$$\begin{aligned}
\bar{Q}_{11} &= Q_{11}c^4 + Q_{22}s^4 + 2\left(Q_{12}+2Q_{66}\right)s^2c^2 \\
\bar{Q}_{12} &= \left(Q_{11}+Q_{22}-4Q_{66}\right)s^2c^2 + Q_{12}\left(c^4+s^2\right) \\
\bar{Q}_{22} &= Q_{11}s^4 + Q_{22}c^4 + 2\left(Q_{12}+2Q_{66}\right)s^2c^2 \\
\bar{Q}_{16} &= \left(Q_{11}-Q_{12}-2Q_{66}\right)c^3s - \left(Q_{22}-Q_{12}-2Q_{66}\right)s^3c \\
\bar{Q}_{26} &= \left(Q_{11}-Q_{12}-2Q_{66}\right)cs^3 - \left(Q_{22}-Q_{12}-2Q_{66}\right)c^3s \\
\bar{Q}_{66} &= \left(Q_{11}+Q_{22}-2Q_{12}-2Q_{66}\right)s^2c^2 + Q_{66}\left(s^4+c^4\right)
\end{aligned} \tag{4}$$

The local and global stresses in an angle lamina are related to each other through the angle of lamina θ

$$\begin{bmatrix} \sigma_1 \\ \sigma_2 \\ \tau_{12} \end{bmatrix} = [T] \begin{bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{bmatrix} \tag{5}$$

where

$$[T] = \begin{bmatrix} c^2 & s^2 & 2sc \\ s^2 & c^2 & -2sc \\ -sc & sc & c^2-s^2 \end{bmatrix} \tag{6}$$

2.2 Stress and Strain in a Laminate

$$\begin{aligned}
\begin{bmatrix} N_x \\ N_y \\ N_{xy} \end{bmatrix} &= \begin{bmatrix} A_{11} & A_{12} & A_{16} \\ A_{12} & A_{22} & A_{26} \\ A_{16} & A_{26} & A_{66} \end{bmatrix} \begin{bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \end{bmatrix} + \begin{bmatrix} B_{11} & B_{12} & B_{16} \\ B_{11} & B_{12} & B_{16} \\ B_{16} & B_{26} & B_{66} \end{bmatrix} \begin{bmatrix} k_x \\ k_y \\ k_{xy} \end{bmatrix} \\
\begin{bmatrix} M_x \\ M_y \\ M_{xy} \end{bmatrix} &= \begin{bmatrix} B_{11} & B_{12} & B_{16} \\ B_{12} & B_{22} & B_{26} \\ B_{16} & B_{26} & B_{66} \end{bmatrix} \begin{bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \end{bmatrix} + \begin{bmatrix} D_{11} & D_{12} & D_{16} \\ D_{11} & D_{12} & D_{16} \\ D_{16} & D_{26} & D_{66} \end{bmatrix} \begin{bmatrix} k_x \\ k_y \\ k_{xy} \end{bmatrix}
\end{aligned} \tag{7}$$

where

$$\begin{aligned}
A_{ij} &= \sum_{k=1}^{n} (\overline{Q_{ij}})_k (h_k - h_{k-1}) \\
B_{ij} &= \frac{1}{2} \sum_{k=1}^{n} (\overline{Q_{ij}})_k (h_k - h_{k-1}) \\
D_{ij} &= \frac{1}{3} \sum_{k=1}^{n} (\overline{Q_{ij}})_k (h_k - h_{k-1})
\end{aligned} \tag{8}$$

The $[A]$, $[B]$, and $[D]$ matrices are called the extensional, coupling, and bending stiffness matrices.

3 Failure Theories of an Angle Lamina

3.1 Failure Theories of an Angle Lamina

Many different theories about the failure of an angle lamina have been developed for a unidirectional lamina, such as maximum stress failure theory, maximum strain failure theory, Tsai-Hill failure theory, and Tsai-Wu failure theory. The failure theories of a lamina are based on the stresses in local axes in the material. There are four normal strength parameters and one shear stress for a unidirectional lamina. The five strength parameters are

$(\sigma_1^T)_{ult}$ = Ultimate longitudinal tensile strength(in direction 1),
$(\sigma_1^C)_{ult}$ = Ultimate longitudinal compressive strength(in direction 1),
$(\sigma_2^T)_{ult}$ = Ultimate transverse tensile strength(in direction 2),
$(\sigma_2^C)_{ult}$ = Ultimate transverse compressive strength(in direction 2), and
$(\tau_{12})_{ult}$ = Ultimate in-plane shear strength

In this paper, Tsai-wu failure theory is taken to decide whether a lamina is failed or not, the reason is chosen because this theory is more general than Tsai-Hill failure theory which consider two different situation, compressive and tensile strength of a lamina. A lamina is considered to be failed if

$$H_1\sigma_1 + H_2\sigma_2 + H_6\tau_{12} + H_{11}\sigma_1^2 + H_{22}\sigma_2^2 + H_{66}\tau_{12}^2 + 2H_{12}\sigma_1\sigma_2 < 1 \tag{9}$$

is violated. where

$$\begin{aligned}
H_1 &= \frac{1}{(\sigma_1^T)_{ult}} - \frac{1}{(\sigma_1^C)_{ult}} \\
H_{11} &= \frac{1}{(\sigma_1^T)_{ult}(\sigma_1^C)_{ult}} \\
H_2 &= \frac{1}{(\sigma_2^T)_{ult}} - \frac{1}{(\sigma_2^C)_{ult}} \\
H_{22} &= \frac{1}{(\sigma_2^T)_{ult}(\sigma_2^C)_{ult}} \\
H_{66} &= \frac{1}{(\tau_{12})_{ult}^2} \\
H_{12} &= -\frac{1}{2}\sqrt{\frac{1}{(\sigma_1^T)_{ult}(\sigma_1^C)_{ult}(\sigma_2^T)_{ult}(\sigma_2^C)_{ult}}}
\end{aligned} \tag{10}$$

The Equation 9 can determin whether a lamina failed or not, but it failed to give the information about how much load can be increased or decreased to keep the lamina safe. The strength ratio(SR) is to used to solve this problem, and defined as

$$SR = \frac{\text{Maximum Load Which Can Be Applied}}{\text{Load Applied}} \tag{11}$$

Substituting Equation 11 for SR into Equation 9,we obtain

$$(F_{11}\sigma_1^2 + F_{22}\sigma_2^2 + F_{66}\sigma_6^2 + 2F_{12}\sigma_1\sigma_2)SR^2 + (F_1\sigma_1 + F_2\sigma_2)SR - 1 = 0 \tag{12}$$

3.2 Failure Theories of a Laminate

1. Compute the reduced stiffness matrix $[Q]$ referred to local axis for each ply using its four engineering elastic constants E_1, E_2, v_{12}, and G_{12}.
2. calculate the transformed reduced stiffness $[\bar{Q}]$ referred to global coordinate system (x, y) using reduced stiffness matrix $[Q]$ obtained in step 1 and ply angle for each layer.
3. Given the thickness t_k and the location of each layer, find out the three laminate stiffness matrices $[A]$, $[B]$, and $[D]$.
4. Apply forces and moments, $[N]_{xy}$, $[M]_{xy}$, solve the equation 7, calculate the middle plane strain $[\sigma^0]_{xy}$ and cruvature $[k]_{xy}$.
5. Find out the local strain and stress of each layer under the applied load.
6. Use the ply-by-ply stresses and strains in Tsai-wu failure theory to find out the strenght ratio.

4 Genetic Algorithm Procedure

Objective Function: $\theta_k, n,$ material
Minimize: Weight
Minimize: Cost
Minimize: Cost/Mini Cost + Weight/Mini Weight
Subject to: Safety Factor

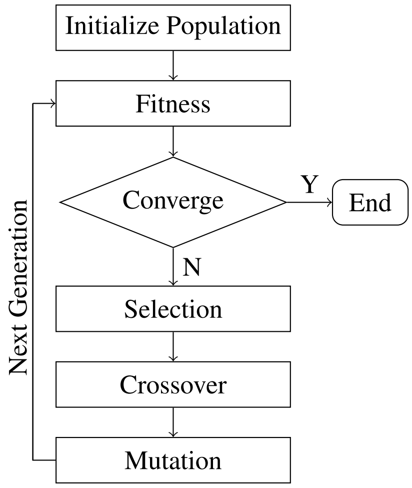

Fig. 2: GA Procedure with Share Function

5 Results and Discussion

Table 1: Typical Properties of a Unidirectional Lamina(SI System of Units)

Property	Symbol	Unit	Glass/Epoxy	Graphite/Epoxy
Fiber volume fraction	V_f		0.45	0.70
Longitudinal elastic modulus	E_1	GPa	38.6	181
Traverse elastic modulus	E_2	GPa	8.27	10.3
Major Poisson's ratio	v_{12}		0.26	0.28
Shear modulus	G_{12}	GPa	4.14	7.17
Ultimate longitudinal tensile strength	$(\sigma_1^T)_{ult}$	MPa	1062	1500
Ultimate longitudinal compressive strength	$(\sigma_1^C)_{ult}$	MPa	610	1500
Ultimate transverse tensile strength	$(\sigma_2^T)_{ult}$	MPa	31	40
Ultimate transverse compressive strength	$(\sigma_2^C)_{ult}$	MPa	118	246
Ultimate in-plane shear strength	$(\tau_{12})_{ult}$	MPa	72	68

Table 2: Comparative study of different composite materials for a defined strenght ratio

Load	Objective Function	Stacking sequence	Strength ratio	Mass	Cost	Layer
	min{cost}	$[0_{gl}]_{6s}$	2.103	0.780	12.0	12
$N_x = 1e6$ N	min{mass}	$[0_{gr}]_9$	2.227	0.472	22.5	9
	min{cost+mass}	$[0_{gr5}/0_{gl}]_s$	2.082	0.550	22.0	10

Load	Objective Function	Stacking sequence	Strength ratio	Mass	Cost	Layer
$N_x = 1e6$ N $N_y = 0.5e6$ N	min{cost}	$[0_{gl}]_{6s}$	2.103	0.780	12.0	12
	min{mass}	$[0_{gr}]_9$	2.227	0.472	22.5	9
	min{cost+mass}	$[0_{gr5}/0_{gl}]_s$	2.082	0.550	22.0	10
Load	**Type of composite**	**Stacking sequence**	**Strength ratio**	**Mass**	**Cost**	**Height**
$N_{xy} = 1e6$ N	Glass/Epoxy	$[0]_{6s}$	2.103	0.707	12.0	1.980
	Graphite/Epoxy	$[0]_9$	2.227	0.481	22.5	1.485
	Hybrid composite	$[Gl\text{-}E/Gr\text{-}E_5]_s$	2.082	0.545	22.0	1.650
Load	**Objective**	**Stacking sequence**	**Strength ratio**	**Mass**	**Cost**	**Height**
$N_x = N_y =$ $N_{xy} = 1e6$ N	min mass	$[0]_{6s}$	2.103	0.707	12.0	1.980
	min cost	$[0]_9$	2.227	0.481	22.5	1.485
	min cost + weight	$[Gl\text{-}E/Gr\text{-}E_5]_s$	2.082	0.545	22.0	1.650

Table 3: GA-parameters

parameter	value
population size	20
encoding method	float encoding
selection strategy	roulette wheel
crossover strategy	one-point
mutation strategy	mass mutation

6 Concluding Remarks

7 Acknowledgements

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The previous researchers adopted the first-ply-failure approach using the Tsai-wu failure theory [9, 15, 4, 19, 14, 5, 12]
minimize thickness [1, 21],weight[4, 3, 13], cost and weight[3, 12]
Genetic Algorithm has been successfully applied to composite design optimization[16, 11, 17, 20, 8, 18, 21, 7, 6, 10, 2]

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