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A technique for constraint optimisation of symmetric laminate using a new varient of genetic algorithm

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Abstract

The main challenge presented by the design of laminate composite is the laminate layup, involving a set of fiber orientation, compoiste material system, and stacking sequence. In nature, it is a combinatorial optimization problem which can be solved by the genetic algorithm(GA). In this present study, a new varient of GA is introduced to search the optimal design by modifying the selection strategy. To improve the performance of this new GA, a speical group is maintained in the population during the optimization process. To check the feasibility of a laminate subject to in-plane loading, the effect of fiber orientation angles and materials component on the first ply failure is studied. A comparative study of basic GA and improved GA in laminate composite designing for a targeted safety factor is also studied. An optimal composite material and laminate layup is well developed for a targeted strength ratio which makes a compromise between weight and cost through improved genetic algorithm. Numberical results are obtained and presented for different loading cases.

Keywords

Genetic Algorithm, Laminate, Stacking Sequence, Hybrid Composite, Classical Lamination Theory

1. Introduction

Composite materials offer improved strength, stiffness, fatigue, and corrosion resistance, etc., over conventional materials, which are widely used as materials ranging from automotive to ship building industry, electronic packaging to golf clubs, medical equipment to home building. However, the high cost of fabrication of composites is a critical drawback for its application, for example, the graphite/epoxy composite part may cost as much as $650 to $900 per kilogram, while the price of glass/epoxy is about 2.5 times less. Manufacturing techniques such as sheet molding compound and structural reinforcement injection molding are taken to lower the cost in manufacturing automobile parts, and an alternative approach is using of hybrid compoiste material.

The mechanical performance of a laminate compoiste is affected by a wide range of factors, thickness, material, and orientation of each lamina. Beause of manufacturing limitation, all these variables are usually limited to a small set of discrete values. For example, ply thickness is fixed and ply orientation angles are limited to a set of angles such as 0,45,90 degrees in practice. So the search process for the optimal design is a discrete optimization problem which can be solved by GA. To tailor a laminate composite, GA has been successfully applied to solve laminate design problem[2,8,11,12,15,16,22,23,25,29,30]. GA simulates the process of natural evolutionary includes selection, crossover, and mutation according to Darwin's principal of "survival of the fittest". The known advantage of GA as the following: (i): GAs are not easily trapped in local optimum, and be able to obtain the global optimum. (ii): GA doesn't need gradient information and can be applied to discrete optimization problem. (iii): GA not only be able to find the optimal value in the domain, but also can maintain a set of optimal solutions. On the other hand, GA also has some disadvantages, for example, the GA needs to evaluate the target functions a lot of times to achieve the optimization, the cost of the search process is high. The GA consists of some basic parts, the coding of the design variable, selection strategy, crossover operator, mutation operator, and how to deal with constraints. For the variable design part, there are two methods to deal with the representation of design variables, binary string and real value representation[22,29]. Michalewicz[32] claimed the performance of floating point representation was better than binary representation in numerical optimization problem. Selection strategy plays a critical role in GA which decides the convergence speed and the diversity of the population. To improve search ability and reduce search cost, various selection methods have been invented, and it can be divided into four classes which are proportionate reproduction, ranking, tournament, and genitor(or "steady state") selection. In the optimization of laminate composite design, roulette wheel[22,24], where the possibility of an individual to be chosen for the next generation is proportional to it's fitness. Soremekun[28] showed generalized elitist strategy outperformed a single individual elitism in some special cases.

Data structure, repair strategies and penalty functions[10] are most common used approaches to resolve constraint problems in the optimization of composite structure.

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Symmetric laminates are widely used in practical scenario, data structure can be used to fulfil the symmetry constraint which consists of coding a half of the laminate and considering the rest with the opposite orientation. Todoroki[29] introduced a repair strategy which can scan the chromosome and repair the gene on the chromosome if it does satify the contiguity constraint. The comparsion of repair strategies in a permutation GA with the same orientation was presented by Liu[12], and it showed the Baldwinian repair strategy can significantly reduce the cost of constrained optimization. Haftka[22] used GA to solve laminate stacking sequence problem using a penalty function subject to buckling and strength constraints.

In typical engineering applications, composite materials are under very complicate loading conditions, not only in-plane loads but also out-of-plane loads. Most of the studies on the optimization of laminated composite materials was to mimimize the thickness[1,30], weight[5,6,19], cost and weight[5,18], or maximize the static strength of composite laminates for a targeted thickness[9,11,30]. In the present study, laminate cost and weight are minimized by modifying the objective function.

In order to check the feasibility of a composite laminate by imposing a strength constraint, failure analysis of a laminate is taken by applying suitable failure criteria. The failure criteria of laminated composites can be classified in three classes: non-interactive theories(e.g., Maximum strain), interactive theores(e.g., Tsai-wu), and parially interactive theories(e.g., Puck failure criterion). The previous researchers adopted the first-ply-failure approach using the Tsai-wu failure theory[3,6,7,14,18,20,21,27], Tsai-Hill[13,26], the maximum stress[7,18], or the maximum strain[31] static failure criteria. Akbulut[2] used GA to minimize the thickness of composite laminates with Tsai-Hill and maximum stress failure criterias, and the advantage of this method is to avoid spurious optima. Naik[17] minimized the weight of laminated composites under restrictions with a failure-mechanism-based criterion based on Maximum Strian and Tsai-wu. In the present study, Tsai-wu static failure criteria is used to investigated the feasibility of a composite laminate.

2. Stress and Strain in a Laminate

A laminated structure consists of multiple laminas bonded together through their thickness. Consider a laminated composite plate which is symmetric to its middle plane and subjected to in-plane loads of extension, shear, bending and torsion, the classical lamination theory(CLT) is taken to calculate the stresses and strains in the local and global axes of each ply. as shown in Fig.1.

2.1 Stress and Strian in a Lamina

For a single lamina, the stress strain relation in the local axis: 1-2

$$\begin{bmatrix}\sigma_1\\ \sigma_2\\ \tau_{12}\end{bmatrix} = \begin{bmatrix}Q_{11} & Q_{12} & 0\\ Q_{12} & Q_{22} & 0\\ 0 & 0 & Q_{66}\end{bmatrix}\begin{bmatrix}\varepsilon_1\\ \varepsilon_2\\ \gamma_{12}\end{bmatrix} \tag{1}$$

Where Q_{ij} are the stiffnesses of the lamina that are related to engineering elastic constants by

$$\begin{aligned} Q_{11} &= \frac{E_1}{1-v_{12}v_{21}} \\ Q_{22} &= \frac{E_2}{1-v_{12}v_{21}} \\ Q_{66} &= G_{12} \\ Q_{12} &= \frac{v_{21}E_2}{1-v_{12}v_{21}} \end{aligned} \tag{2}$$

Where, E_1, E_2, v_{12}, G_{12} are four independent engineering elastic constants, they are defined as

E_1 = longitudinal Young's modulus
E_2 = transverse Young's modulus
v_{12} = major Poisson's ratio
G_{12} = in-plane shear modulus
Stress strain relation in global axis: x-y

$$\begin{bmatrix}\sigma_x\\ \sigma_y\\ \tau_{xy}\end{bmatrix} = \begin{bmatrix}\bar{Q}_{11} & \bar{Q}_{12} & \bar{Q}_{16}\\ \bar{Q}_{12} & \bar{Q}_{22} & \bar{Q}_{26}\\ \bar{Q}_{16} & \bar{Q}_{26} & \bar{Q}_{66}\end{bmatrix}\begin{bmatrix}\varepsilon_x\\ \varepsilon_y\\ \gamma_{xy}\end{bmatrix} \tag{3}$$

where

$$\begin{aligned} \bar{Q}_{11} &= Q_{11}c^4 + Q_{22}s^4 + 2(Q_{12}+2Q_{66})s^2c^2 \\ \bar{Q}_{12} &= (Q_{11}+Q_{22}-4Q_{66})s^2c^2 + Q_{12}(c^4+s^2) \\ \bar{Q}_{22} &= Q_{11}s^4 + Q_{22}c^4 + 2(Q_{12}+2Q_{66})s^2c^2 \\ \bar{Q}_{16} &= (Q_{11}-Q_{12}-2Q_{66})c^3s - (Q_{22}-Q_{12}-2Q_{66})s^3c \\ \bar{Q}_{26} &= (Q_{11}-Q_{12}-2Q_{66})cs^3 - (Q_{22}-Q_{12}-2Q_{66})c^3s \\ \bar{Q}_{66} &= (Q_{11}+Q_{22}-2Q_{12}-2Q_{66})s^2c^2 + Q_{66}(s^4+c^4) \end{aligned} \tag{4}$$

The c and s denotes $cos\theta$, and $sin\theta$.

The local and global stresses in an angle lamina are related to each other through the angle of lamina θ

$$\begin{bmatrix}\sigma_1\\ \sigma_2\\ \tau_{12}\end{bmatrix} = [T]\begin{bmatrix}\sigma_x\\ \sigma_y\\ \tau_{xy}\end{bmatrix} \tag{5}$$

where

$$[T] = \begin{bmatrix}c^2 & s^2 & 2sc\\ s^2 & c^2 & -2sc\\ -sc & sc & c^2-s^2\end{bmatrix} \tag{6}$$

2.2 Stress and Strain in a Laminate

$$\begin{aligned} \begin{bmatrix}N_x\\ N_y\\ N_{xy}\end{bmatrix} &= \begin{bmatrix}A_{11} & A_{12} & A_{16}\\ A_{12} & A_{22} & A_{26}\\ A_{16} & A_{26} & A_{66}\end{bmatrix}\begin{bmatrix}\varepsilon_x^0\\ \varepsilon_y^0\\ \gamma_{xy}^0\end{bmatrix} + \begin{bmatrix}B_{11} & B_{12} & B_{16}\\ B_{11} & B_{12} & B_{16}\\ B_{16} & B_{26} & B_{66}\end{bmatrix}\begin{bmatrix}k_x\\ k_y\\ k_{xy}\end{bmatrix} \\ \begin{bmatrix}M_x\\ M_y\\ M_{xy}\end{bmatrix} &= \begin{bmatrix}B_{11} & B_{12} & B_{16}\\ B_{12} & B_{22} & B_{26}\\ B_{16} & B_{26} & B_{66}\end{bmatrix}\begin{bmatrix}\varepsilon_x^0\\ \varepsilon_y^0\\ \gamma_{xy}^0\end{bmatrix} + \begin{bmatrix}D_{11} & D_{12} & D_{16}\\ D_{11} & D_{12} & D_{16}\\ D_{16} & D_{26} & D_{66}\end{bmatrix}\begin{bmatrix}k_x\\ k_y\\ k_{xy}\end{bmatrix} \end{aligned} \tag{7}$$

N_x, N_y - normal force per unit length

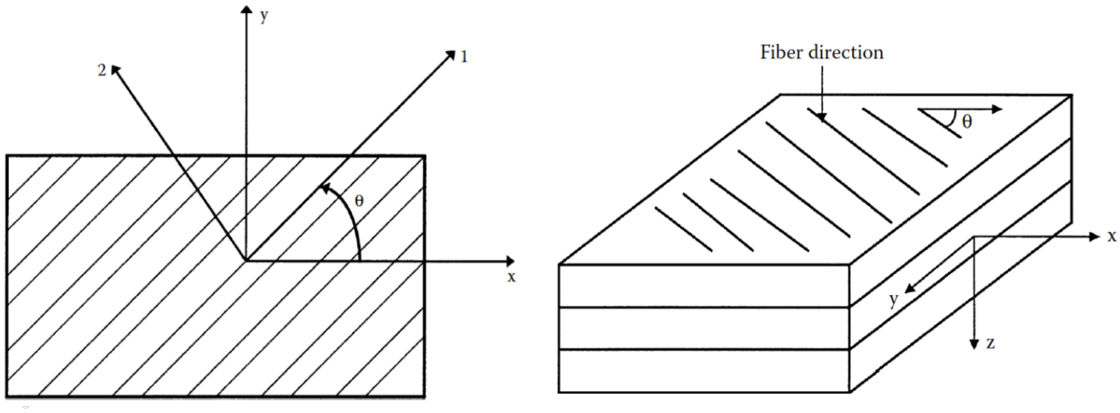

Figure 1. Lamina

N_{xy} - shear force per unit length
M_x, M_y - bending moment per unit length
M_{xy} - twisgint moments per unit length
ε^0, k - mid plane strains, and curvature of laminate in x-y coordinate

The mid plane strain and curvature is given by

$$\begin{bmatrix} \varepsilon^0 \\ k \end{bmatrix} = \begin{pmatrix} A & B \\ B & D \end{pmatrix}^{-1} \begin{Bmatrix} N \\ M \end{Bmatrix}$$

where

$$\begin{aligned} A_{ij} &= \sum_{k=1}^{n} (\overline{Q_{ij}})_k (h_k - h_{k-1}) i = 1,2,6, j = 1,2,6 \\ B_{ij} &= \frac{1}{2} \sum_{k=1}^{n} (\overline{Q_{ij}})_k (h_k - h_{k-1}) i = 1,2,6, j = 1,2,6 \\ D_{ij} &= \frac{1}{3} \sum_{k=1}^{n} (\overline{Q_{ij}})_k (h_k - h_{k-1}) i = 1,2,6, j = 1,2,6 \end{aligned} \tag{8}$$

The $[A]$, $[B]$, and $[D]$ matrices are called the extensional, coupling, and bending stiffness matrices.

3. Failure Theories

3.1 Failure Theories of an Angle Lamina

Many different theories about the failure of an angle lamina have been developed for a unidirectional lamina, such as maximum stress failure theory, maximum strain failure theory, Tsai-Hill failure theory, and Tsai-Wu failure theory. The failure theories of a lamina are based on the stresses in local axes in the material. There are four normal strength parameters and one shear stress for a unidirectional lamina. The five strength parameters are

$(\sigma_1^T)_{ult}$ = Ultimate longitudinal tensile strength,
$(\sigma_1^C)_{ult}$ = Ultimate longitudinal compressive strength,
$(\sigma_2^T)_{ult}$ = Ultimate transverse tensile strength,
$(\sigma_2^C)_{ult}$ = Ultimate transverse compressive strength, and
$(\tau_{12})_{ult}$ = Ultimate in-plane shear strength

In the present study, Tsai-wu failure theory is taken to decide whether a lamina is failed or not, the reason is chosen because this theory is more general than Tsai-Hill failure theory which consider two different situation, compressive and tensile strength of a lamina. A lamina is considered to be failed if

$$\begin{aligned} H_1\sigma_1 + H_2\sigma_2 + H_6\tau_{12} + H_{11}\sigma_1^2 + H_{22}\sigma_2^2 \\ + H_{66}\tau_{12}^2 + 2H_{12}\sigma_1\sigma_2 < 1 \end{aligned} \tag{9}$$

is violated. where

$$\begin{aligned} H_1 &= \frac{1}{(\sigma_1^T)_{ult}} - \frac{1}{(\sigma_1^C)_{ult}} \\ H_{11} &= \frac{1}{(\sigma_1^T)_{ult}(\sigma_1^C)_{ult}} \\ H_2 &= \frac{1}{(\sigma_2^T)_{ult}} - \frac{1}{(\sigma_2^C)_{ult}} \\ H_{22} &= \frac{1}{(\sigma_2^T)_{ult}(\sigma_2^C)_{ult}} \\ H_{66} &= \frac{1}{(\tau_{12})_{ult}^2} \\ H_{12} &= -\frac{1}{2}\sqrt{\frac{1}{(\sigma_1^T)_{ult}(\sigma_1^C)_{ult}(\sigma_2^T)_{ult}(\sigma_2^C)_{ult}}} \end{aligned} \tag{10}$$

The Equation 9 can determine whether a lamina failed or not, but it fails to give the information about how much load can be increased or decreased to keep the lamina safe. The strength ratio(SR) is to used to solve this problem, and defined as

$$SR = \frac{\text{Maximum Load Which Can Be Applied}}{\text{Load Applied}} \tag{11}$$

Substituting Equation 11 for SR into Equation 9,we obtain

$$\begin{aligned} (F_{11}\sigma_1^2 + F_{22}\sigma_2^2 + F_{66}\sigma_6^2 + 2F_{12}\sigma_1\sigma_2)SR^2 \\ + (F_1\sigma_1 + F_2\sigma_2)SR - 1 = 0 \end{aligned} \tag{12}$$

3.2 Failure Theories of a Laminate

A laminate will fail under increasing mechanical, however, the procedure of laminate failure may not be catastrophic. In some cases, some layer fail first and the rest is be able to continue to take more loads until all the plies fail. a ply is fully discount when a ply fails, then the ply is replaced by near zero stiffness and strength. The procedure for finding the first ply failure in the present study follows the fully discounted method[4]:

1. Compute the reduced stiffness matrix $[Q]$ referred to local axis for each ply using its four engineering elastic constants E_1, E_2, v_{12}, and G_{12}.
2. calculate the transformed reduced stiffness $[\bar{Q}]$ referred to global coordinate system (x, y) using reduced stiffness matrix $[Q]$ obtained in step 1 and ply angle for each layer.
3. Given the thickness t_k and the location of each layer, find out the three laminate stiffness matrices $[A]$, $[B]$, and $[D]$.
4. Apply forces and moments, $[N]_{xy}$, $[M]_{xy}$, solve the equation 7, calculate the middle plane strain $[\sigma^0]_{xy}$ and cruvature $[k]_{xy}$.
5. Find out the local strain and stress of each layer under the applied load.
6. Use the ply-by-ply stresses and strains in Tsai-wu failure theory to find out the strength ratio.

4. Optimum Design of Laminate Composite

4.1 Genetic Algorithm

The GA starts off with a bunch of individuals with limited chromosome length, in which maybe none of these individuals fulfil the safety factor constraint. The GA are supposed to derive appropriate offspring based on initial population as the GA goes on. The classic way to handle constraint search of GA are either introducing repair strategies or using a penaly function, here a new approach is come up with to deal with constraint GA search problem by modifying the selection strategy.

Because of the existence of constraint, it means that the population not only can be sorted by fitness, but also can sorted by the constraint value obtained by the constraint function(assuming constraint functions exists), so the parents of next generation can be chosen by the following two approaches. First, sort out the population by the absolute difference between the individual's constraint value and the constraint threshold in an ascending order, and individual with smaller difference is more like to be chosen. Individuals obtained by this method are called potential individuals. Second, sort out the population by fitness from low to high after remove the unproper individuals, and an individual is proper which means it fulfils the constraint, and individuals obtained by this way are called proper individuals. So the final parents consists of two parts, potential individuals and proper individuals, and the number of potential individuals and proper individual are called, respectively, potential number and proper number. For example, assumming the parent population is 20, 60 percent of them is potential individuals, and the rest is proper individuals. So the potential number is 12, and the proper number is 8.

At the beginning of the GA, no individual in the population is appropriate, which means the number of proper individuals nearly zero. So the GA can be divided into two stages according to whether proper individual are generated during the search process. During the initial stages, the number of potential individuals gradually decreases from maximum(which is parent population) to the potential number, while the number of proper individuals increases from zero to the proper number as the GA goes on. After the initial stage, both of the number of two groups converge to pre-assigned number. In order to differentiate the current selection methods from the following, the current GA is called basic GA. In the following experiment, 50 percent of the parent are constraint individuals, and 50 percent of the parent are proper individuals.

The problem with this basic GA is premature and weak local search ability, basic GA are more likely to get stuck in local optimum. Therefore, to prevent the GA from early convergence and improve the local search performance, a new selection method is proposed, which is ignoring whether the individual satisfy the constraint requirement or not, and ranking individuals by their fitness. Individuals selected by this method are called active individuals, because they are supposed to be always in the population. So in the improved GA, parents consists of three parts: active individuals, constraint individuals, and proper individuals. In the following experiment, 20 percent of the parent population are active individuals, 30 percent of the parent are constraint individuals, and the rest is proper individuals.

In the present study, the relevant parameters of GA are as shown in Table 1. The design variables are the materials, number of layers, and ply orientation restricted to a discrete set of angles (0, ±45 and 90 degrees). The possible materials are graphite/epoxy, caron/epoxy, and glass/epoxy and are represented by the codes 0, 1 and 2, respectively.

”LRIC” denotes the length range of initial chromosome.

The laminate chromosomes are represented by double-gene string which can be divided into two parts, one part represents the angles, the other part represents the materials(as shown in Figure 2(P_1)) . To maintain the diversity of the population, single-point crossover is taken during the evolution process. The break point in the string are chosen randomly, and one of the offspring of parent 1(as shown in Figure 2(P_1)) and parent 2(as shown in Figure 2(P_2)) is obtained by combining the gene segments $P1_o$ and $P2_o$, $P1_m$ and $P2_m$, respectively. The gene code of the offspring laminate is $[+45, -45, -45, -45, -45, -45, -45, 0, 1, 0, 1, 1, 0, 1, 0]$.

To prevent the search from getting stuck in a local optimum, mutation is used to random change the gene in the chromosome, the offspring after mutation operator is as shown in Figure 2

The GA is a stochastic procedure which heavily depends on the generator of pseudo random numbers. In the present study, the standard Wichmann-Hill generator is used in the algorithm, which combines three pure multiplicative congruential generators of modulus 30269, 30307 and 30323.

4.2 Design Problem I

The aim is to minimize the mass of a composite laminate for a targeted strength ratio by Tsai-wu failure theory. The design variable are the ply angles and the number of layers.

Find: $\{\theta_k, n\}$ $\theta_k \in \{0, +45, -45, 90\}$
Minimize: weight
Subject to: strength ratio and first ply failure constraint

4.3 Design Problem II

The aim is to mimimize the combined cost and weight of hybrid composite laminate under various loading cases, so

Table 1. GA parameters

Parameter	Seed	Population size	LRIC	Encoding	Crossover Strategy	Mutation strategy
Value	1	10	[3-15]	Integer	One-point	Mass mutation

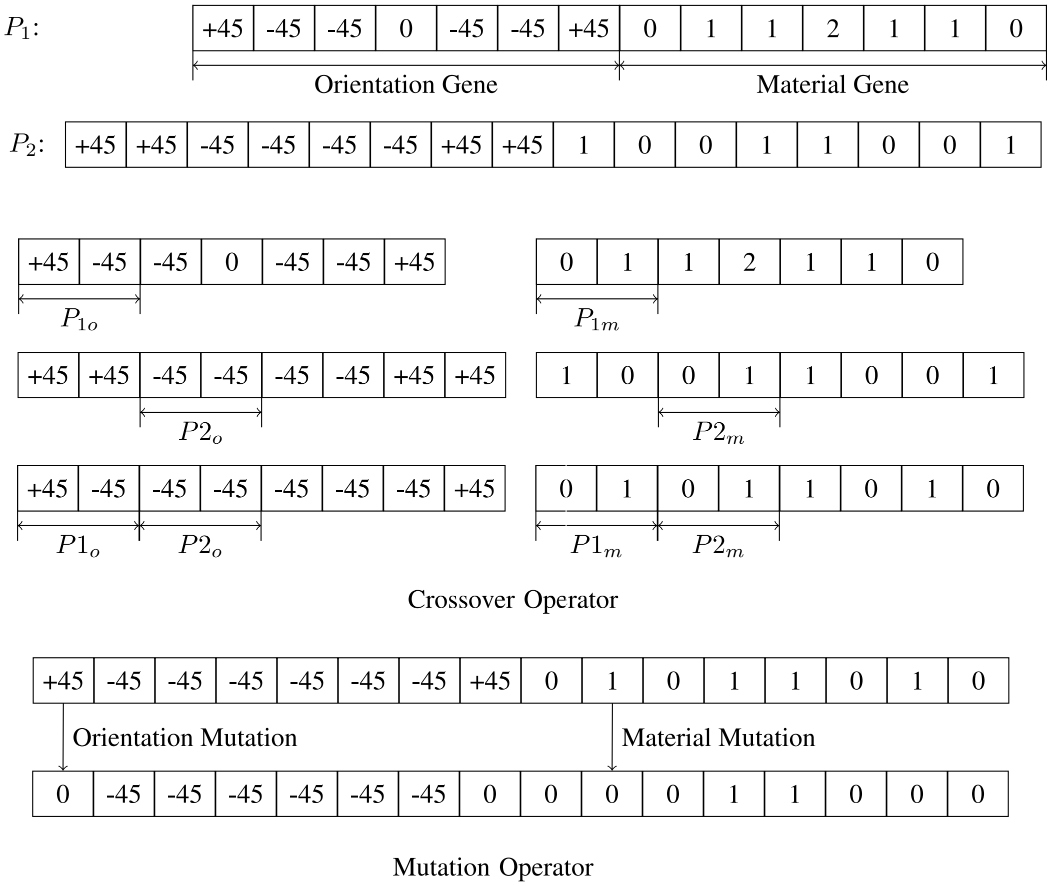

Figure 2. GA Operators

the design variable not only include the ply angles and number of layers, but also the material of each lamina.

Find: $\{\theta_k, \text{mat}_k, n\}$ $\quad \theta_k \in \{0, +45, -45, 90\}$ $\quad \text{mat}_k \in \{CA, GR, GL\}$

Minimize:

$$F = \frac{\text{Cost}}{C_{\text{min}}} + \frac{\text{Weight}}{W_{\text{min}}} \tag{13}$$

Subject to: strength ratio and first ply failure constraint

Here CA, GF, and GL represent carbon/epoxy, graphite/epoxy, and glass/epoxy, C_{min} and W_{min} represent the cost and weight corresponding to the laminates with minimum cost and minimum weight obtained from previous problem.

5. Numberical Results and Discussion

A composite laminate with dimensions $1000 \times 1000 \times 0.165mm^3$ of each lamina is under various loading cases, and each CA, GF, and GL layer is assumed to cost 8, 2.5 and 1 monetary units, respectively. The other used material properties are as shown in Table 2. In the present experiment, the optimum composite system, layup, thickness, and number of layers for a targeted strength ratio(2 in this paper) under two different in-plane loading is investigated.

The programming language Python3 is employed to implement the genetic algorithm, which is a high-level object-oriented language, and it's grammar is very concise and easy to pick up.

Subscript "cr" denotes a carbon/epoxy ply, "gr" denotes graphite/epoxy ply, "gl" denotes a glass/epoxy ply

The GA process can be divided into two phases by whether there are individuals which are appropriate or not. During the initial phase, no individual's strength ratio is over the specified threshold, so individuals with bigger fitness are more likely to be chosen as parents, that's why the strength ratio curves go all the way up to the specified threshold during the first stage; After the initial phase, the GA produces a bunch of appropriate individuals, and then the target function comes to play, as you can see from Fig.3, the fitness curves are trending to go down, but the strength ratio curves are keep to greater the specified threshold.

In the first experiment, the applied stress are $N_x = N_y = 1e6$ N. As shown in the Figure3, the Figure 3(a), (b), and (c) were the experiment results for single material, Figure 3(d) is for hybrid composite material. For the single materials, both of the basic GA and improved GA method obtained the optimal value, but the improved GA converged

Table 2. Comparsion of carbon/epoxy, graphite/epoxy, and glass/epoxy properties

Property	Symbol	Unit	Carbon/Epoxy	Graphite/Epoxy	Glass/Epoxy
Longitudinal elastic modulus	E_1	GPa	116.6	181	38.6
Traverse elastic modulus	E_2	GPa	7.67	10.3	8.27
Major Poisson's ratio	v_{12}		0.27	0.28	0.26
Shear modulus	G_{12}	GPa	4.17	7.17	4.14
Ultimate longitudinal tensile strength	$(\sigma_1^T)_{ult}$	MP	2062	1500	1062
Ultimate longitudinal compressive strength	$(\sigma_1^C)_{ult}$	MP	1701	1500	610
Ultimate transverse tensile strength	$(\sigma_2^T)_{ult}$	MPa	70	40	31
Ultimate transverse compressive strength	$(\sigma_2^C)_{ult}$	MPa	240	246	118
Ultimate in-plane shear strength	$(\tau_{12})_{ult}$	MPa	105	68	72
Density	ρ	g/cm^3	1.605	1.590	1.903
Cost			8	2.5	1

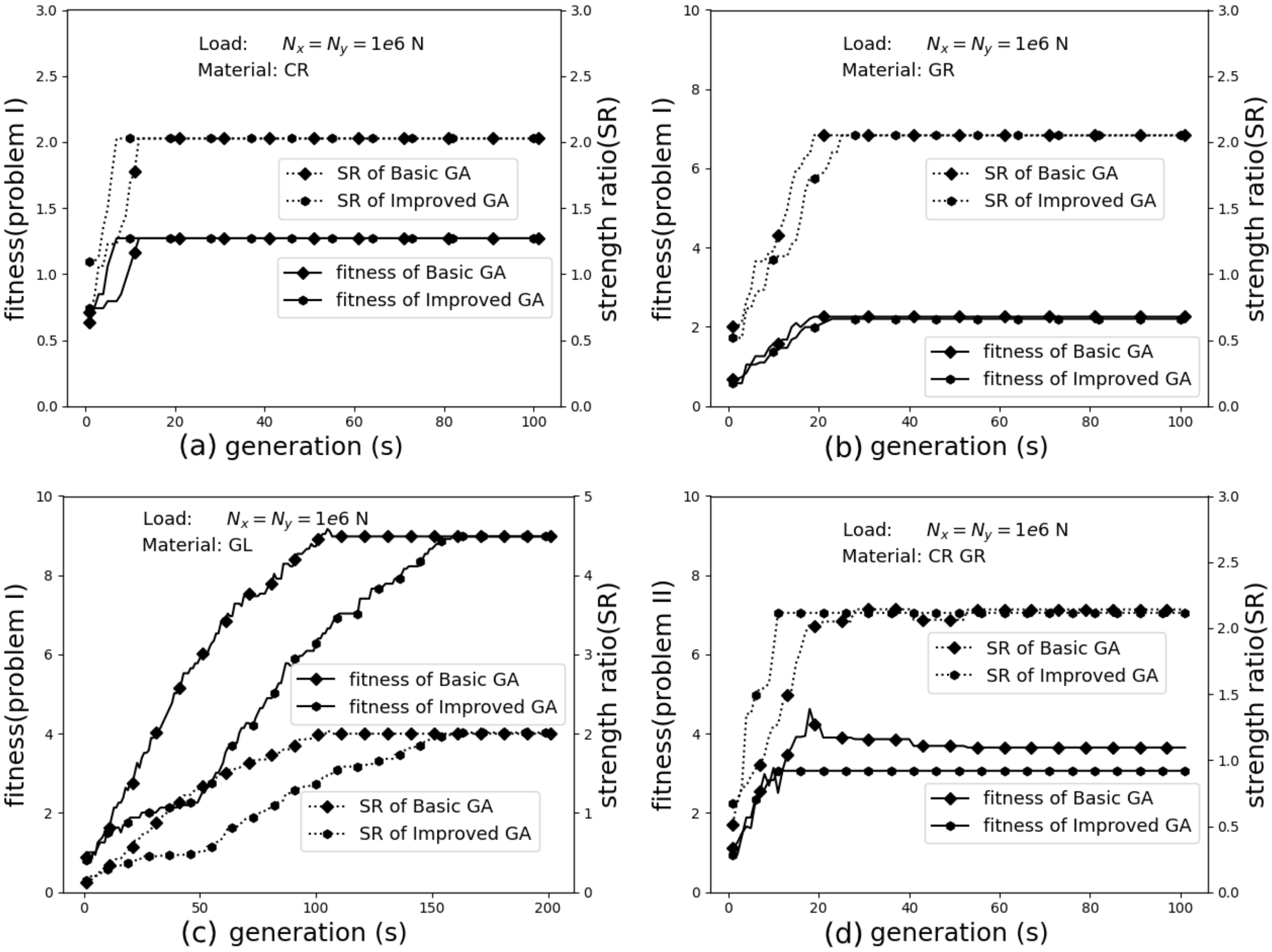

Figure 3. GA process under load $N_x = N_y$ = 1e6 N

Table 3. Optimization results

Load(N)	Problem	Algorithm	Stacking sequence	Strength ratio	Mass	Cost	Layer
	I	GA	$[\text{-}45_6^{cr}/\text{+}45_6^{cr}]_s$	2.026	1.271	192.0	24
	I	IGA	$[\text{-}45_6^{cr}/\text{+}45_6^{cr}]_s$	2.026	1.271	192.0	24
	I	GA	$[0_6^{gr}/\text{-}45_4^{gr}/\text{+}45_4^{gr}/90_7^{gr}/\bar{9}0^{gr}]_s$	2.051	2.256	107.5	43
$N_x = 1e6$ $N_y = 1e6$	I	IGA	$[\text{+}45_{10}^{gr}/\text{-}45_{10}^{gr}/\text{-}\bar{4}5^{gr}]_s$	2.024	2.151	102.5	41
	I	GA	$[\text{-}45_{35}^{gl}/\text{+}45_{36}^{gl}/\text{+}\bar{4}5^{gl}]_s$	2.001	8.980	143.0	143
	I	IGA	$[\text{-}45_{35}^{gl}/\text{+}45_{36}^{gl}/\text{+}\bar{4}5^{gl}]_s$	2.001	8.980	143.0	143
	II	GA	$[\text{-}45_{12}^{gr}/\text{+}45_5^{cr}/\text{+}45_7^{gr}]_s$	2.141	2.523	175	48
	II	IGA	$[\text{-}45_9^{gr}/\text{+}45_9^{gr}/\text{-}45_2^{cr}/\text{+}45_2^{cr}]_s$	2.054	2.313	154	44

more slowly than the basic GA. As it can be seen from Table 3, a $[\text{-}45_6/\text{+}45_6]_s$ carbon/epoxy laminate has the least weight, denoted by W_{min}, and a $[\text{-}45_{35}/\text{+}45_{73}/\text{+}45_{35}]$ graphite/epoxy laminate has the lowest cost, denoted by

C_{min}. The W_{min} and C_{min} were used to evaluate the fitness of the second problem, which is the layup design of hybrid composite material , as shown in sub-figure d,the improved GA obtained a more appropriate system layup, whose strenght ratio is greater than the specified safety factor, and weight and cost is less than the result obtained by basic GA method, as shown in Table 3. Compared with basic GA, the improved GA method showed more powerful global search ability in the initial phase.

In the second case, the applied stress were $N_x = N_y = N_z = 1e6$ N, the experiment results were as shown in the Figure 4. In the first experiment, as can be seen from Figure 4(a), the improved GA got a better system layup then result obtained by basic GA; In the second experiment, as shown in the Figure4(b), during the initial phase, the fitness curves of basic GA and improved GA went all the way up to the previous specified threshold, however, the improved GA converged more slowly then the basic GA which means the search cost of improved GA is greater then basic GA. After the initial phase, the fitness curve of basic didn't change anymore, it got trapped in local. However, the fitness curve of improved GA was gradually going down, at the same time, the strength ratio curve of improved GA were keep to be greater then the threshold. It means the improved GA was able to get out of optimum and obtained a much better system layup. The improved GA offered more powerful local search ability. In the third experiment, as shown in Figure 4, both of basic GA and improved GA obtained the same result, but the improved GA converged more slowly than the basic GA. From these three experiment for single material, we knew a $[+45_6^{cr}]_s$ carbon/epoxy laminate has the least mass, and a $[+45_{11}^{gl}/+\bar{4}5^{gl}]_s$ glass laminate has the least cost. In the last experiment, the improved GA obtained a little bit better result than the basic GA, as it shown in Table 4, Compared with the $[+45_{12}^{cr}]$ laminate, the weight of a $[+45_8^{gr}/+\bar{4}5^{gl}]_s$ laminate increases 41.8%, however, the cost decreases 56%.

6. Conclusions

In this paper, a combination of CLT and a variant of GA are employed to minimize the weight and cost of a single-material and hybrid composite laminate, respectively, under various in-plane loading cases. Results are presented in two sections, stacking sequence optimization for a single material laminate, and weighted mass and cost optimization of a carbon/epoxy, graphite/epoxy, and glass/epoxy hybrid laminate. Furthermore, the performance of the basic GA is improved by changing the selection strategy.

This variant of GA provides a new approach to deal with constraint search in laminate composite optimization, and this method is very easy to extend for solving multiple constraints search problem in other domain. The problem with the current method is to adjust the parameters in the GA to obtain the best performance.

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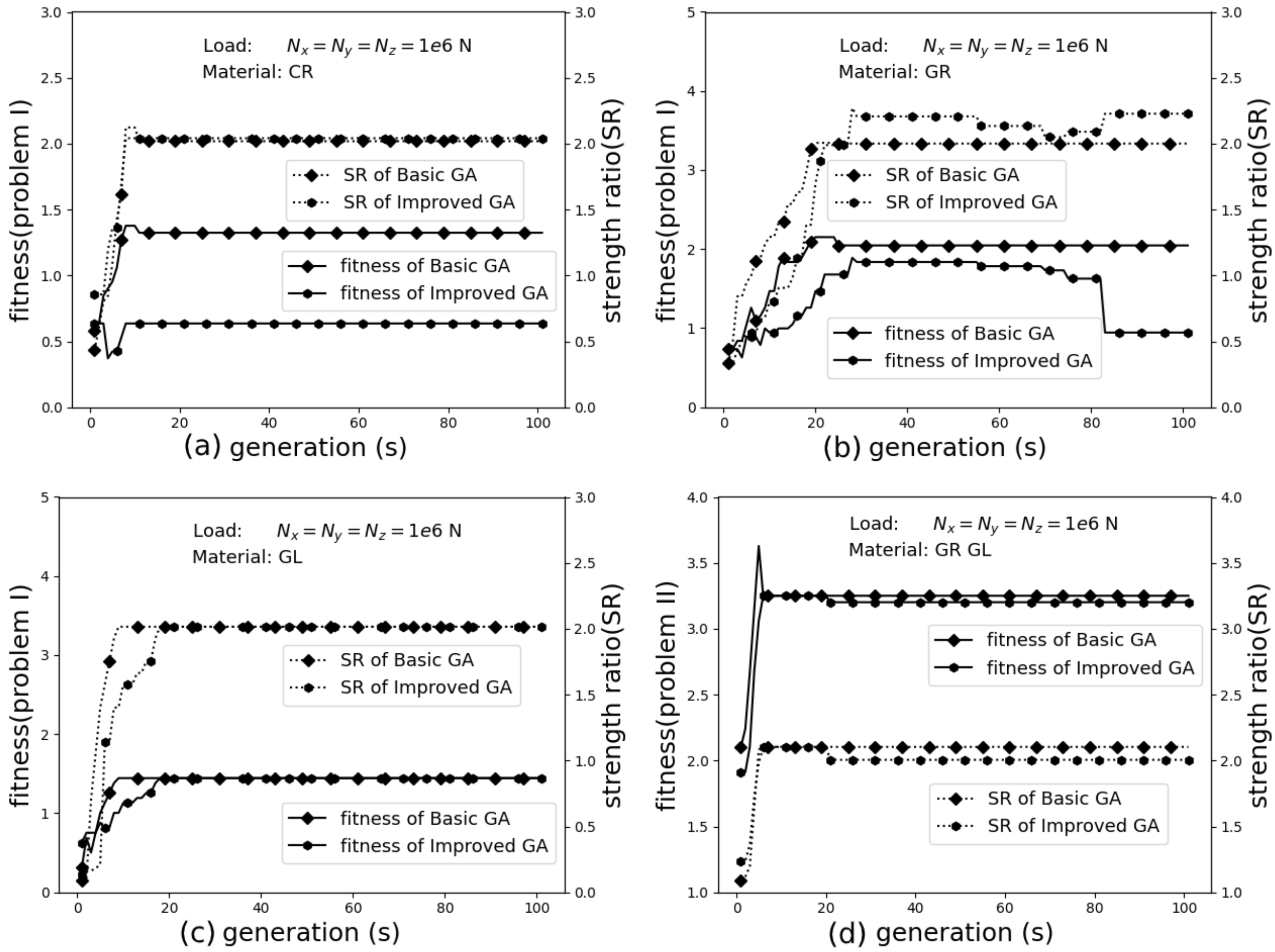

Figure 4. GA process under load $N_x = N_y = N_z$ = 1e6 N

Table 4. Optimization results

Load(N)	Problem	Algorithm	Stacking sequence	Strength ratio	Mass	Cost	Layer
$N_x = 1e6$ $N_y = 1e6$ $N_{xy} = 1e6$	I	GA	$[+45^{cr}_{11}/-45^{cr}/+\bar{4}5^{cr}]_s$	2.018	1.324	200.0	25
	I	IGA	$[+45^{cr}_{6}]_s$	2.041	0.636	96.0	12
	I	GA	$[0^{gr}_{4}/+45^{gr}_{12}/90^{gr}_{3}/+\bar{4}5]_s$	2.001	2.046	97.5	39
	I	IGA	$[+45^{gr}_{9}]_s$	2.227	0.945	45.0	18
	I	GA	$[+45^{gl}_{11}/+\bar{4}5^{gl}]_s$	2.015	1.444	23.0	23
	I	IGA	$[+45^{gl}_{11}/+\bar{4}5^{gl}]_s$	2.015	1.444	23.0	23
	II	GA	$[+45^{gl}/+45^{gr}_{8}]_s$	2.031	0.965	42.0	18
	II	IGA	$[+45^{gr}_{8}/+\bar{4}5^{gl}]_s$	2.005	0.902	41.0	17

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