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Laminated Composite Plate Optimization by Genetic Algorithm

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Abstract Failure analysis of laminated composite plates under different mechanical loads for different stacking sequences, fiber orientation, and composite material system is studied in this paper. An optimum composite material and laminate layup is studied for a targeted strength ratio which makes a compromise between weight and cost through genetic algorithm.

Keywords Genetic Algorithm · Laminates · Stacking Sequence · Hybrid Composites

1 Introduction

Composites material offer improved strength, stiffness, fatigue, and corrosion resistance, etc over conventional materials, which is widely used in automotive, aerospace, and ship building industry. However, the high cost of fabrication of composites is a critical drawback for its application, for example, the graphite/epoxy composite part may cost as much as \$650 to \$900 per kilogram. The mechanical performance of a composite is affected by a wide range of factors, fiber length, fiber orientation, fiber shape, and the matrix etc.

Genetic algorithms(GAs) simulate the process of natural evolutionary includes selection, crossover ,and mutation according to Darwin's principal of "survival of the fittest".

According to T Back [**?**],the selection mechanism is one of the primary means of controlling the GA's convergence rate and its likelihood of finding global optima.

Four evaluation criteria are used. The first is normalized cost per genetic search,

$$C_n$$

cost is determinted by the following formula.

$$C_n = N_g P / R$$

where P is population size, R is Apparent reliability.

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2 Objective Function

Given the safety factor, The objective function is minimum the weight of the laminate.

$$obj = min(weight)$$

3 Stress-Strain Relationship for a Lamina

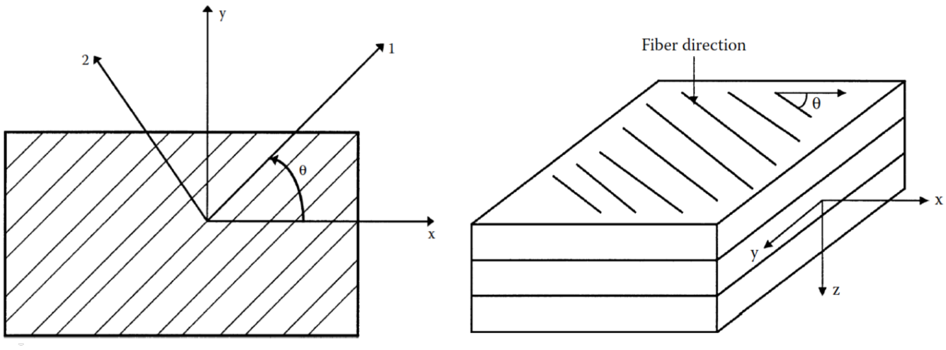

Fig. 1: Lamina

$$\begin{bmatrix} \sigma_1 \\ \sigma_2 \\ \tau_{12} \end{bmatrix} = \begin{bmatrix} Q_{11} & Q_{12} & 0 \\ Q_{12} & Q_{22} & 0 \\ 0 & 0 & Q_{66} \end{bmatrix} \begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \gamma \end{bmatrix} \tag{1}$$

Where Q_{ij} are the stiffnesses of the lamina that are related to the compliance matrix compnents and elastic constants by

$$\begin{aligned} Q_{11} &= \frac{E_1}{1-\mu_{12}\mu_{21}} \\ Q_{22} &= \frac{E_2}{1-\mu_{12}\mu_{21}} \\ Q_{66} &= G_{12} \\ Q_{12} &= \frac{\mu_{21}E_2}{1-\mu_{12}\mu_{21}} \end{aligned} \tag{2}$$

The local and global stresses in an angle lamina are related to each other through the angle of lamina θ

$$\begin{bmatrix} \sigma_1 \\ \sigma_2 \\ \tau_{12} \end{bmatrix} = [T] \begin{bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{bmatrix} \tag{3}$$

$$[T] = \begin{bmatrix} c^2 & s^2 & 2sc \\ s^2 & c^2 & -2sc \\ -sc & sc & c^2-s^2 \end{bmatrix} \tag{4}$$

$$\begin{bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{bmatrix} = \begin{bmatrix} \bar{Q}_{11} & \bar{Q}_{12} & \bar{Q}_{16} \\ \bar{Q}_{12} & \bar{Q}_{22} & \bar{Q}_{26} \\ \bar{Q}_{16} & \bar{Q}_{26} & \bar{Q}_{66} \end{bmatrix} \begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{bmatrix} \tag{5}$$

$$\begin{aligned}\bar{Q}_{11} &= Q_{11}c^4 + Q_{22}s^4 + 2\left(Q_{12} + 2Q_{66}\right)s^2c^2 \\ \bar{Q}_{12} &= \left(Q_{11} + Q_{22} - 4Q_{66}\right)s^2c^2 + Q_{12}\left(c^4 + s^2\right) \\ \bar{Q}_{22} &= Q_{11}s^4 + Q_{22}c^4 + 2\left(Q_{12} + 2Q_{66}\right)s^2c^2\end{aligned} \tag{6}$$

$$\begin{aligned}\bar{Q}_{16} &= \left(Q_{11} - Q_{12} - 2Q_{66}\right)c^3s - \left(Q_{22} - Q_{12} - 2Q_{66}\right)s^3c \\ \bar{Q}_{26} &= \left(Q_{11} - Q_{12} - 2Q_{66}\right)cs^3 - \left(Q_{22} - Q_{12} - 2Q_{66}\right)c^3s \\ \bar{Q}_{66} &= \left(Q_{11} + Q_{22} - 2Q_{12} - 2Q_{66}\right)s^2c^2 + Q_{66}\left(s^4 + c^4\right)\end{aligned} \tag{7}$$

4 Failure Theories of an Angle Lamina

Many different theories about the failure of an angle lamina have been developed for a unidirectional lamina, such as maximum stress failure theory, maximum strain failure theory, Tsai-Hill failure theory, and Tsai-Wu failure theory. The failure theories of a lamina are based on the stresses in local axes in the material. There are four normal strength parameters and one shear stress for a unidirectional lamina. The five strength parameters are

$(\sigma_1^T)_{ult}$ = Ultimate longitudinal tensile strength(in direction 1),
$(\sigma_1^C)_{ult}$ = Ultimate longitudinal compressive strength(in direction 1),
$(\sigma_2^T)_{ult}$ = Ultimate transverse tensile strength(in direction 2),
$(\sigma_2^C)_{ult}$ = Ultimate transverse compressive strength(in direction 2), and
$(\tau_{12})_{ult}$ = Ultimate in-plane shear strength

In this paper, Tsai-wu failure theory is taken to decide whether a lamina is failed or not, the reason is chosen because this theory is more general than Tsai-Hill failure theory which consider two different situation, compressive and tensile strength of a lamina. A lamina is considered to be failed if

$$H_1\sigma_1 + H_2\sigma_2 + H_6\tau_{12} + H_{11}\sigma_1^2 + H_{22}\sigma_2^2 + H_{66}\tau_{12}^2 + 2H_{12}\sigma_1\sigma_2 < 1 \tag{8}$$

is violated. where

$$H_1 = \frac{1}{\left(\sigma_1^T\right)_{ult}} - \frac{1}{\left(\sigma_1^C\right)_{ult}} \tag{9}$$

$$H_{11} = \frac{1}{\left(\sigma_1^T\right)_{ult}\left(\sigma_1^C\right)_{ult}} \tag{10}$$

$$H_2 = \frac{1}{\left(\sigma_2^T\right)_{ult}} - \frac{1}{\left(\sigma_2^C\right)_{ult}} \tag{11}$$

$$H_{22} = \frac{1}{\left(\sigma_2^T\right)_{ult}\left(\sigma_2^C\right)_{ult}} \tag{12}$$

$$H_{66} = \frac{1}{\left(\tau_{12}\right)_{ult}^2} \tag{13}$$

$$H_{12} = -\frac{1}{2}\sqrt{\frac{1}{\left(\sigma_1^T\right)_{ult}\left(\sigma_1^C\right)_{ult}\left(\sigma_2^T\right)_{ult}\left(\sigma_2^C\right)_{ult}}} \tag{14}$$

The Equation 8 can determin whether a lamina failed or not, but it failed to give the information about how much load can be increased or decreased to keep the lamina safe. The strength ratio(SR) is to used to solve this problem, and defined as

$$SR = \frac{\text{Maximum Load Which Can Be Applied}}{\text{Load Applied}} \tag{15}$$

Substituting Equation 15 for SR into Equation 8,we obtain

$$(F_{11}\sigma_1^2 + F_{22}\sigma_2^2 + F_{66}\sigma_6^2 + 2F_{12}\sigma_1\sigma_2)SR^2 + (F_1\sigma_1 + F_2\sigma_2)SR - 1 = 0 \tag{16}$$

5 Stress-Strain Relationship for a Laminate

The transformation of the equation*sof the off-axis coordinates to the principal axis of the material stress tensor as

$$[T] = \begin{bmatrix} \cos^2\theta & \sin^2\theta & 2\sin 2\theta \\ \sin^2\theta & \cos^2\theta & -2\sin 2\theta \\ -\frac{1}{2}\sin 2\theta & \frac{1}{2}\sin 2\theta & \cos 2\theta \end{bmatrix} \tag{17}$$

$$\begin{bmatrix} N_x \\ N_y \\ N_{xy} \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} & A_{16} \\ A_{12} & A_{22} & A_{26} \\ A_{16} & A_{26} & A_{66} \end{bmatrix} \begin{bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \end{bmatrix} + \begin{bmatrix} B_{11} & B_{12} & B_{16} \\ B_{11} & B_{12} & B_{16} \\ B_{16} & B_{26} & B_{66} \end{bmatrix} \begin{bmatrix} k_x \\ k_y \\ k_{xy} \end{bmatrix} \tag{18}$$

$$\begin{bmatrix} M_x \\ M_y \\ M_{xy} \end{bmatrix} = \begin{bmatrix} B_{11} & B_{12} & B_{16} \\ B_{12} & B_{22} & B_{26} \\ B_{16} & B_{26} & B_{66} \end{bmatrix} \begin{bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \end{bmatrix} + \begin{bmatrix} D_{11} & D_{12} & D_{16} \\ D_{11} & D_{12} & D_{16} \\ D_{16} & D_{26} & D_{66} \end{bmatrix} \begin{bmatrix} k_x \\ k_y \\ k_{xy} \end{bmatrix}$$

where

$$\begin{aligned} A_{ij} &= \sum_{k=1}^{n} (\overline{Q_{ij}})_k (h_k - h_{k-1}) \\ B_{ij} &= \frac{1}{2}\sum_{k=1}^{n} (\overline{Q_{ij}})_k (h_k - h_{k-1}) \\ D_{ij} &= \frac{1}{3}\sum_{k=1}^{n} (\overline{Q_{ij}})_k (h_k - h_{k-1}) \end{aligned} \tag{19}$$

The $[A]$, $[B]$, and $[D]$ matrices are called the extensional, coupling, and bending stiffness matrices.
where R is the strength ratio,σ_{ia} is allowable stress,σ_i is the stress under loading
substituting Eq. 2 for σ into Eq. 5, we obtain

$$\begin{aligned} &F_1\sigma_{1(a)} + F_2\sigma_{2(a)} + F_{11}\sigma_{1(a)}^2 + F_{22}\sigma_{2(a)}^2 \\ &\quad + F_{66}\sigma_{6(a)}^2 + 2F_{12}\sigma_{1(a)}\sigma_{2(a)} = 1 \end{aligned}$$

For orthoropic materials with three planes of symmetry,

$$\begin{aligned} &(F_{11}\sigma_1^2 + F_{22}\sigma_2^2 + F_{66}\sigma_6^2 + 2F_{12}\sigma_1\sigma_2)R^2 \\ &+ (F_1\sigma_1 + F_2\sigma_2)R - 1 = 0 \end{aligned}$$

5.1 Solution by a genetic algorithm

as required. Don’t forget to give each section and subsection a unique label).

6 Results and Discussion

Table 1: Typical Properties of a Unidirectional Lamina(SI System of Units)

Property	Symbol	Unit	Glass/Epoxy	Graphite/Epoxy
Fiber volume fraction	V_f		0.45	0.70
Longitudinal elastic modulus	E_1	GPa	38.6	181
Traverse elastic modulus	E_2	GPa	8.27	10.3
Major Poisson's ratio	v_{12}		0.26	0.28
Shear modulus	G_{12}	GPa	4.14	7.17
Ultimate longitudinal tensile strength	$(\sigma_1^T)_{ult}$	MPa	1062	1500
Ultimate longitudinal compressive strength	$(\sigma_1^C)_{ult}$	MPa	610	1500
Ultimate transverse tensile strength	$(\sigma_2^T)_{ult}$	MPa	31	40
Ultimate transverse compressive strength	$(\sigma_2^C)_{ult}$	MPa	118	246
Ultimate in-plane shear strength	$(\tau_{12})_{ult}$	MPa	72	68

Table 2: Comparative study of different composite materials for a defined strenght ratio

Load	Type of composite	Stacking sequence	Strength ratio	Mass	Cost	Height
$N_x = 1e6$ N	Glass/Epoxy	$[0]_{6s}$	2.103	0.707	12.0	1.980
	Graphite/Epoxy	$[0]_9$	2.227	0.481	22.5	1.485
	Hybrid composite	$[Gl\text{-}E/Gr\text{-}E_5]_s$	2.082	0.545	22.0	1.650
Load	**Type of composite**	**Stacking sequence**	**Strength ratio**	**Mass**	**Cost**	**Height**
$N_y = 1e6$ N	Glass/Epoxy	$[0]_{6s}$	2.103	0.707	12.0	1.980
	Graphite/Epoxy	$[0]_9$	2.227	0.481	22.5	1.485
	Hybrid composite	$[Gl\text{-}E/Gr\text{-}E_5]_s$	2.082	0.545	22.0	1.650
Load	**Type of composite**	**Stacking sequence**	**Strength ratio**	**Mass**	**Cost**	**Height**
$N_{xy} = 1e6$ N	Glass/Epoxy	$[0]_{6s}$	2.103	0.707	12.0	1.980
	Graphite/Epoxy	$[0]_9$	2.227	0.481	22.5	1.485
	Hybrid composite	$[Gl\text{-}E/Gr\text{-}E_5]_s$	2.082	0.545	22.0	1.650
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Table 3: GA-parameters

parameter	value
population size	20
encoding method	float encoding
selection strategy	roulette wheel
crossover strategy	one-point
mutation strategy	mass mutation

7 Concluding Remarks

8 Acknowledgements

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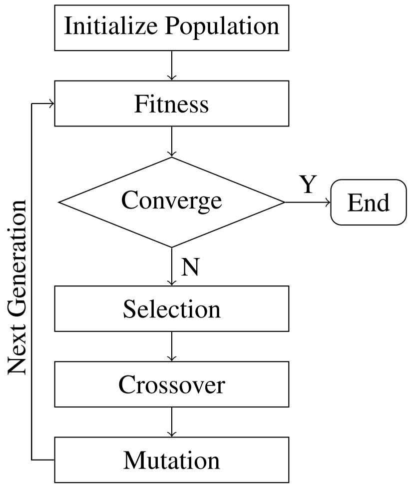

Fig. 2: GA Procedure with Share Function

A[9] [8] [1] [3] [2] [11] [6] [5] [4] [10] [7]

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