Optimum design of laminated composites for minimum thickness by a variant of genetic algorithm

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***Abstract*—In the present study, a genetic algoirthm methodological framework based optimization procedure is proposed to minimizae thickness(or weight) of midplane-symmetric composite laminate subject to in-plane loading. Fiber orientation and ply thickness are chosen as design variables, and a variant of GA is employed to search the optimal design of composite laminates. In order to avoid spurious laminate designs, both the Tsai-wu and the maximum stress criteria are taken to determine whether load bearing capacity is exceeded or not. Numberical results are obtained and presented under different loading cases.**

Keywords*—*Genetic Algorithm; Optimization; Classic Lamination Theory; Failure Theory

I. Introduction

Composite materials offer improved strength, stiffness, corrosion resistance, etc. over conventional materials, and are widely used as alternative materials for applications in various industries ranging from electronic packaging to golf clubs, and medical equipment to homebuilding, making aircraft structure to space vechicles. The stacking sequence and fiber orientation of composite laminates give the designer additional 'degree of freedom' to tailor the design with respect to strength or stiffness. One widely known advange of using composite material is can significantly reducing the weight of target structure, and many researchers attempted to improve the efficiency of using composite materail by mimimizing the thickness[1], [2], [3], [4], [5], [6], [7], [8], [9], [10], [11], [12], [13], [14], [15], [16], [17], [18], [19], [20], [21].

In practice, fiber orientations are restricted to a finite set of angles, and ply thickness is a specific numberic value. Because the design variables are not continueous, a gradient based optimization procedure, such as gradient descent method, is not suitable to cope with such problems. Moreover, gradient based optimization approach is very eazily to get trapped in local minima, and many local optimum may exist in structural optimization problems. A stochastic optimization, such as genetic algorithm(GA) and simulated annealing(SA), is able to deal with optimization problem with discrete variables. Besides, stochastic method could escape from local optimum, and obtain global optimum. GA is one of the most reliably stochastic algorithm, which has been widely used in solving constraint desgin for composite laminate[22], [18], [23], [19], [24], [25], [26], [27], [28]. Although GA gains different advantages for solving discrete problems, many disadvantages exists within this approach. First, the optimization process of GA parameters, such as the population size, parent population,mutation percentage, etc., is very tedious; Second, the GA needs to evaluate the objective functions many times to acheive the optimization, and the compuation cost is very high; the last problem within GA is the premature convergence. GA consists of five basic parts: the variable coding, selection scheme, crossover operator, mutation operator and how the constraints are handled.

The first issue when implementing a GA is the representation of design variables, and an appropriate design representation is crucial to enhance the efficiency of GA. The canonical GA has always used binary strings to encode alternative solutions, however, some argued that the minimal cardinality, i.e., the binary representation, are not the best option. Real value string has been widely employed in

Selection scheme plays a critical role in balancing the dilemma of exploration and exploitation inherented in GA, and various selection methods, for example, roulette wheel, elitist, and tournament etc., have been proposed to overcome this issue. Both of roulette selection and tournament selection are well-studied and widely employed in the optimization design of laminated composite due to their simplicity to code and efficiency for both nonparallel and parallel architectures.

Crossover is another crucial operator introduced into the GA methodology framework, in which the alternative solution is generated from the mating pool. multiple types of crossover operator has been utilized in the optimization design of composite structures, such as: one-point, two-point, and uniform crossover.

GA is originally proposed for unconstrained optimization. However, in order to deal with constrained design for composite laminate, some techiques were introduced into the GA. The first method is using of data structure, special data structure was developed to fulfils the symmetry constraint of the laminate, which consists of coding only half of the laminate and considering that each stack of the laminate is formed by two laminae with the same orientation but opposite signs[5], [29]. A penalty function is developed to convert a constrained problem into an unconstrained problem by adding penalty term to the objective funtion. Another method to solve constrained problem is introducing repair strategy by Todoroki and Haftka [30], which is aim to transform infeasible solutions to feasible solution by incorporating problem-specific knowledge.

Another major concern within GA is the convergence speed in terms of the time and computation cost needed to reach a solution of desired quality. The objective function based on the CLT is excessively time-consuming and complicate to evaluate, in addition, the target function of GA needs to be calculate many times. The traditional method to deal with this issue is by increasing the selection pressure to accelerate the convergence speed, however, in some cases, this approach does not acheive an ideal result. Becasue the GAs just provides a methodological framework to deal with trickey problems, which is heavily inspired by evolution of biology, it is unnecessary to exactly follow all the GA operation. It is possible to just perform one or more GA operations, and incorporate other techiques into GA. In present study, a variant of mutation operator is introduced to accelerate the convergence process.

To check the feasibility of a laminate composite by imposing a strength constraint, various failure criterion have been proposed to decide whether it fails or not, such as maximum stress failure theory, maximum strain failure theory, Tsai-Hill Failure theory and Tsai-Wu criterion. Each theory is proposed based on massive experiment data or complicate mathematical model, however single use any of them may lead to false optimum design for some loading case due to the particular shape of its failure envelope. In order to overcome this disadvantage within every failure theory, two reliably failure criteria, maximum stress theory and Tsai-wu criterion are employed to check whether the composite laminate fullfils the constraint.

The rest of the paper is organized as follows. Section 2 explains the classical laminate theory and the failure criteria taken in the present study. Section 3 explains the proposed method of selection strategy and self-adaptative parameters for mutation during the GA process. Section 4 describes the result of the numerical experiments in different cases, and in the Conclusion section we dicuss the results.

II. Analysis of

A. Stress and Strain in a Lamina

For a single lamina has a small thickness under plane stress, and it's upper and lower surfaces of the lamina are free from external loads. According to the Hooke's Law, the three-dimensional stress-strain equations can be reduced to two-dimensional stress-strain equations. The stress-strain relation in local axis 1-2 is:

$$\begin{bmatrix}\sigma_1\\ \sigma_2\\ \tau_{12}\end{bmatrix} = \begin{bmatrix}Q_{11} & Q_{12} & 0\\ Q_{12} & Q_{22} & 0\\ 0 & 0 & Q_{66}\end{bmatrix}\begin{bmatrix}\varepsilon_1\\ \varepsilon_2\\ \gamma_{12}\end{bmatrix} \tag{1}$$

where $Q_{ij}$are the stiffnesses of the lamina that are related

to engineering elastic constants given by

$$\begin{aligned} Q_{11} &= \frac{E_1}{1-v_{12}v_{21}}\\ Q_{22} &= \frac{E_2}{1-v_{12}v_{21}}\\ Q_{66} &= G_{12}\\ Q_{12} &= \frac{v_{21}E_2}{1-v_{12}v_{21}} \end{aligned} \tag{2}$$

where E_1, E_2, v_{12}, G_{12} are four independent engineering elastic constants, which are defined as follows: E_1 is the longitudinal Young's modulus, E_2 is the transverse Young's modulus, v_{12} is the major Poisson's ratio, and G_{12} is the in-plane shear modulus.

Stress strain relation in the global x-y axis:

$$\begin{bmatrix}\sigma_x\\ \sigma_y\\ \tau_{xy}\end{bmatrix} = \begin{bmatrix}\bar{Q}_{11} & \bar{Q}_{12} & \bar{Q}_{16}\\ \bar{Q}_{12} & \bar{Q}_{22} & \bar{Q}_{26}\\ \bar{Q}_{16} & \bar{Q}_{26} & \bar{Q}_{66}\end{bmatrix}\begin{bmatrix}\varepsilon_x\\ \varepsilon_y\\ \gamma_{xy}\end{bmatrix} \tag{3}$$

where

$$\begin{aligned} \bar{Q}_{11} &= Q_{11}cos^4\theta + Q_{22}sin^4\theta + 2\left(Q_{12}+2Q_{66}\right)sin^2\theta cos^2\theta\\ \bar{Q}_{12} &= \left(Q_{11}+Q_{22}-4Q_{66}\right)sin^2\theta cos^2\theta + Q_{12}\left(cos^4\theta + sin^2\theta\right)\\ \bar{Q}_{22} &= Q_{11}sin^4\theta + Q_{22}cos^4\theta + 2\left(Q_{12}+2Q_{66}\right)sin^2\theta cos^2\theta\\ \bar{Q}_{16} &= \left(Q_{11}-Q_{12}-2Q_{66}\right)cos^3\theta sin\theta - \left(Q_{22}-Q_{12}-2Q_{66}\right)sin^3\theta cos\theta\\ \bar{Q}_{26} &= \left(Q_{11}-Q_{12}-2Q_{66}\right)cos\theta sin^3\theta - \left(Q_{22}-Q_{12}-2Q_{66}\right)cos^3\theta sin\theta\\ \bar{Q}_{66} &= \left(Q_{11}+Q_{22}-2Q_{12}-2Q_{66}\right)sin\theta^2 cos\theta^2 + Q_{66}\left(sin\theta^4 + cos\theta^4\right) \end{aligned} \tag{4}$$

The local and global stresses in an angle lamina are related to each other through the angle of the lamina θ

$$\begin{bmatrix}\sigma_1\\ \sigma_2\\ \tau_{12}\end{bmatrix} = [T]\begin{bmatrix}\sigma_x\\ \sigma_y\\ \tau_{xy}\end{bmatrix} \tag{5}$$

where

$$[T] = \begin{bmatrix} cos^2\theta & sin^2\theta & 2sin\theta cos\theta\\ sin^2\theta & cos^2\theta & -2sin\theta cos\theta\\ -sin\theta cos\theta & sin\theta cos\theta & cos^2\theta - sin^2\theta \end{bmatrix} \tag{6}$$

B. Stress and Strain in a Laminate

For forces and moment resultants acting on laminates, such as in plate and shell structures, the relationship between applied forces and moment and displacement can be given by

$$\begin{aligned} \begin{bmatrix}N_x\\ N_y\\ N_{xy}\end{bmatrix} &= \begin{bmatrix}A_{11} & A_{12} & A_{16}\\ A_{12} & A_{22} & A_{26}\\ A_{16} & A_{26} & A_{66}\end{bmatrix}\begin{bmatrix}\varepsilon_x^0\\ \varepsilon_y^0\\ \gamma_{xy}^0\end{bmatrix}\\ &+ \begin{bmatrix}B_{11} & B_{12} & B_{16}\\ B_{11} & B_{12} & B_{16}\\ B_{16} & B_{26} & B_{66}\end{bmatrix}\begin{bmatrix}k_x\\ k_y\\ k_{xy}\end{bmatrix}\\ \begin{bmatrix}M_x\\ M_y\\ M_{xy}\end{bmatrix} &= \begin{bmatrix}B_{11} & B_{12} & B_{16}\\ B_{12} & B_{22} & B_{26}\\ B_{16} & B_{26} & B_{66}\end{bmatrix}\begin{bmatrix}\varepsilon_x^0\\ \varepsilon_y^0\\ \gamma_{xy}^0\end{bmatrix}\\ &+ \begin{bmatrix}D_{11} & D_{12} & D_{16}\\ D_{11} & D_{12} & D_{16}\\ D_{16} & D_{26} & D_{66}\end{bmatrix}\begin{bmatrix}k_x\\ k_y\\ k_{xy}\end{bmatrix} \end{aligned} \tag{7}$$

N_x, N_y - normal force per unit length

N_{xy} - shear force per unit length

M_x, M_y - bending moment per unit length

M_{xy} - twisting moments per unit length

ε^0, k- mid plane strains and curvature of a laminate in x-y coordinates

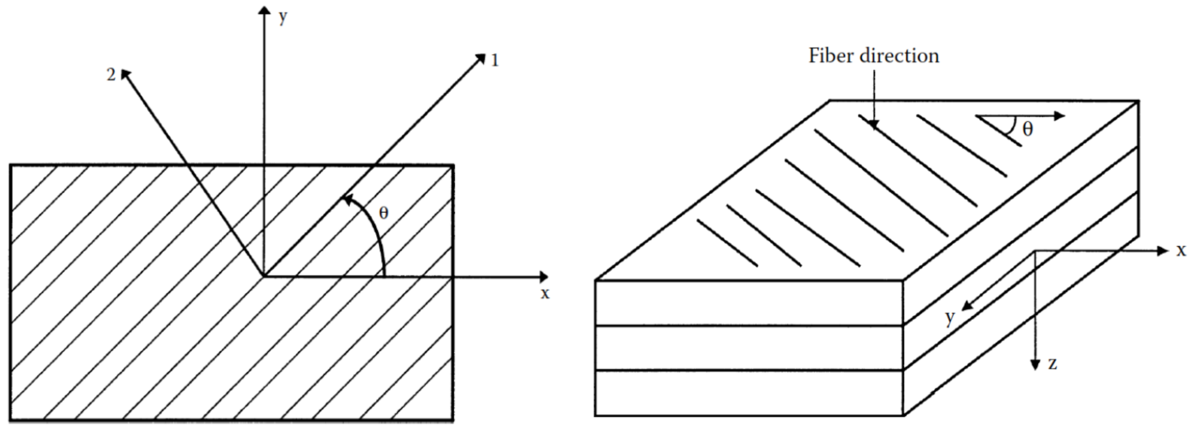

Fig. 1: Lamina

The mid plane strain and curvature is given by

$$
\begin{aligned}
A_{ij} &= \sum_{k=1}^{n} (\overline{Q_{ij}})_k (h_k - h_{k-1}) i = 1,2,6, j = 1,2,6 \\
B_{ij} &= \frac{1}{2} \sum_{k=1}^{n} (\overline{Q_{ij}})_k (h_k^2 - h_{k-1}^2) i = 1,2,6, j = 1,2,6 \\
D_{ij} &= \frac{1}{3} \sum_{k=1}^{n} (\overline{Q_{ij}})_k (h_k^3 - h_{k-1}^3) i = 1,2,6, j = 1,2,6
\end{aligned}
\tag{8}
$$

The [A], [B], and [D] matrices are called the extensional, coupling, and bending stiffness matrices, respectively. The extensional stiffness matrix $[A]$ relates the resultant in-plane forces to the in-plain strains, and the bending stiffness matrix $[D]$ couples the resultant bending moments to the plane curvatures. The coupling stiffness matrix $[B]$ relates the force and moment terms to the midplain strains and midplane curvatures.

III. Failure criteria for a lamina

Failure criteria for composite materials are more difficult to predict due to structural and material complexity in comparison to isotropic materials. The failure process of a composite materials can be regarded from microscopic and macroscopic points of view. Most popular criteria about the failure of an angle lamina are in terms of macroscopic failure criteria, which are based on the tensile, compressive and shear strengths. According to the failure surfaces, these criteria can be classified into two classes: one is called independent failure mode criteria which includes the maximum stress failure theory, maximum strain failure theory because their failure envelop are rectangle; another is called quadratic polynomial which includes Tsai-Wu, Chamis, Hoffman and Hill criteria because their failure surfaces are of ellipsoidal shape. In the present study, two most reliable failure criteria is taken, Maximum stress and Tsai-wu. Both of these two failure criteria are based on the stresses in the local axes instead of principal normal stresses and maximum shear stresses, and four normal strength parameters and one shear stress for a unidirectional lamina are involved. The five strength parameters are

$(\sigma_1^T)_{ult}$ = ultimate longitudinal tensile strength(in direction 1),

$(\sigma_1^C)_{ult}$ = ultimate longitudinal compressive strength,

$(\sigma_2^T)_{ult}$ = ultimate transverse tensile strength,

$(\sigma_2^C)_{ult}$ = ultimate transverse compressive strength, and

$(\tau_{12})_{ult}$ = and ultimate in-plane shear strength.

A. Maximum stress failure criterion

(MS) Maximum stress failure theory consists of maximum normal stress theory proposed by Rankine and maximum shearing stress theory by Tresca. The stresses applied on a lamina can be resolved into the normal and shear stresses in the local axes. If any of the normal or shear stresses in the local axes of a lamina is equal or exceeds the corresponding ultimate strengths of the unidirectional lamina, the lamina is considered to be failed. That is,

$\sigma_1 \geq (\sigma_1^T)_{ult}$ or $\sigma_1 \leq -(\sigma_1^C)_{ult}$

$\sigma_2 \geq (\sigma_2^T)_{ult}$ or $\sigma_2 \leq -(\sigma_2^C)_{ult}$

$\tau_{12} \geq (\tau_{12})_{ult}$ or $\tau_{12} \leq -(\tau_{12})_{ult}$

where σ_1 and σ_2 are the normal stresses in the local axes 1 and 2, respectively; τ_{12} is the shear stress in the symmetry plane 1-2.

B. Tsai-wu failure criterion

The TW criterion is one of the most reliable static failure criteria which is derived from the von Mises yield criterion. A lamina is considered to fail if

$$
\begin{aligned}
H_1\sigma_1 + H_2\sigma_2 + H_6\tau_{12} + H_{11}\sigma_1^2 + H_{22}\sigma_2^2 \\
+ H_{66}\tau_{12}^2 + 2H_{12}\sigma_1\sigma_2 < 1
\end{aligned}
\tag{9}
$$

is violated, where

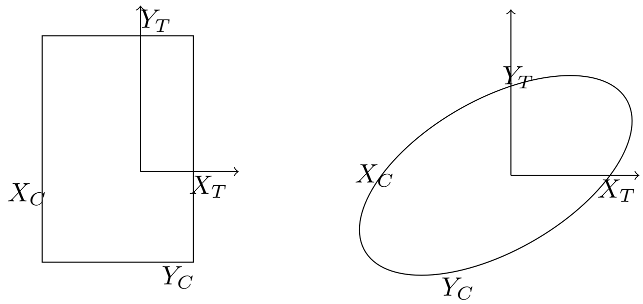

Fig. 2: Schematic failure surfaces for maximum stress and quadratic failure criteria

$$\begin{aligned}
H_1 &= \frac{1}{\left(\sigma_1^T\right)_{ult}} - \frac{1}{\left(\sigma_1^C\right)_{ult}} \\
H_{11} &= \frac{1}{\left(\sigma_1^T\right)_{ult}\left(\sigma_1^C\right)_{ult}} \\
H_2 &= \frac{1}{\left(\sigma_2^T\right)_{ult}} - \frac{1}{\left(\sigma_2^C\right)_{ult}} \\
H_{22} &= \frac{1}{\left(\sigma_2^T\right)_{ult}\left(\sigma_2^C\right)_{ult}} \\
H_{66} &= \frac{1}{\left(\tau_{12}\right)_{ult}^2} \\
H_{12} &= -\frac{1}{2}\sqrt{\frac{1}{\left(\sigma_1^T\right)_{ult}\left(\sigma_1^C\right)_{ult}\left(\sigma_2^T\right)_{ult}\left(\sigma_2^C\right)_{ult}}}
\end{aligned} \tag{10}$$

H_i is the strength tensors of the second order; H_{ij} is the strength tensors of the fourth order. σ_1 is the applied normal stress in direction 1; σ_2 is the applied normal stress in the direction 2; and τ_{12} is the applied in-plane shear stress.

C. Failure Theories for a Laminate

If keep increasing the loading applied to a laminate, the laminate will fails. The failure process of a laminate is more complicate than a lamina, because a laminate consists of multiple plies, and the fiber orientation, material, thickness of each ply maybe different from the others. In most situations, some layer fails first and the remains continue to take more loads until all the plies fail. If one ply fails, it means this lamina does not contribute to the load carrying capacity of the laminate. The procedure for finding the first failure ply given follows the fully discounted method:

1) Compute the reduced stiffness matrix [Q] referred to as the local axis for each ply using its four engineering elastic constants E_1, E_2, E_{12}, and G_{12}.
2) Calculate the transformed reduced stiffness $[\bar{Q}]$ referring to the global coordinate system (x, y) using the reduced stiffness matrix [Q] obtained in step 1 and the ply angle for each layer.
3) Given the thickness and location of each layer, the three laminate stiffness matrices [A], [B], and [D] are determined.
4) Apply the forces and moments, $[N]_{xy}, [M]_{xy}$ solve Equation 7, and calculate the middle plane strain $[\sigma^0]_{xy}$ and curvature $[k]_{xy}$.
5) Determine the local strain and stress of each layer under the applied load.
6) Use the ply-by-ply stresses and strains in the Tsai-wu failure theory to find the strength ratio, and the layer with smallest strenght ratio is the first failed ply.

D. Safety factor

The safety factor, or yield stress, is how much extra load beyond is intended a composite laminate will actually take, which is an indication of the material's load carrying capacity. If the value is less then 1.0, it means failure. The safey factor is defined as

$$SR = \frac{\text{Maximum Load Which Can Be Applied}}{\text{Load Applied}} \tag{11}$$

.

The safety factor based on maximum stress theory is calculated by the following method: first, the principal stresses(σ_1^k,σ_2^k, and τ_{12}^k) are obtained by experiment; evaluate the safety factor along each direction according to equation 11; The minimum value among these safety factors are denoted as the safety factor of the lamina, SF_{MS}^k.

$$SF_{MS}^k = \min \text{ of} \begin{cases} SF_X^k = \begin{cases} \frac{X_t}{\sigma_{11}}, & \text{if } \sigma_{11} > 0 \\ \frac{X_c}{\sigma_{11}}, & \text{if } \sigma_{11} < 0 \end{cases} \\ SF_Y^k = \begin{cases} \frac{Y_t}{\sigma_{22}}, & \text{if } \sigma_{22} > 0 \\ \frac{Y_c}{\sigma_{22}}, & \text{if } \sigma_{22} < 0 \end{cases} \\ SF_S^k = \begin{cases} \frac{S}{|\tau_{12}|} \end{cases} \end{cases} \tag{12}$$

Assuming the composite laminate under a in-plane loading f, the corresponding stress on local stress in direction 1, local stress in direction 2, and shear stress for the kth lamina are $\sigma_1 SF_{TW}^k$, $\sigma_2 SF_{TW}^k$, and $\tau_{12} SF_{TW}^k$, respectively. Substitute them into the equation 9, the expression are given by

$$a(SF_{TW}^k)^2 + b(SF_{TW}^k) - 1 = 0$$

where

$$a = H_{11}(\sigma_1)^2 + H_{22}(\sigma_2)^2 + H_{66}(\tau_{12})^2 + 2H_{12}\sigma_1\sigma_2$$

$$b = H_1\sigma_1 + H_2\sigma_2 + H_6\tau_{12}$$

Solve the above equation, the safety factor for the kth lamina is

$$SF_{TW}^k = \left|\frac{-b+\sqrt{b^2+4a}}{2a}\right|.$$

Then, the minimum of SF_{TW}^k is taken as the safety factor of the laminate which is written as

$$SF_{TW} = \text{ min of } SF_{TW}^k \text{ for } k = 1, 2, \cdots, m-1, m \text{ .}$$

P_1:	+7	+7	+7	+7	+7	+7	+7	+7	-9	-9
P_2:	+19	+19	+19	+19	-36	-36	-36	-36	-36	-36

(a): Parents P_1 and P_2

O_1:	+13	+13	+13	+13	+13	+13	-27	-27	-27	-27
O_2:	+22	+22	+22	+22	+22	+22	+22	+5	+5	+5

(b): Offspring O_1 and O_2

O_1:	+13	+13	+13	$\cdots$	+13	+13	-27	$\cdots$	-27	-27

11 7

	+22	+13	+13	$\cdots$	+13	+13	-27	$\cdots$	-27	-27

(b): Offspring O_1 and O_2 after mutation

Fig. 3: GA Operators

IV. Methodology

A. Objective function

The optimization problem can be formulated as searching the optimal stacking sequence of composite laminate. There are two design variables here, the angles in the laminate, and the number of layers that each fiber orientation has. The objective function is formulated as

$$\begin{aligned} F &= 2t_0 \sum_{k=1}^{n} n_k \\ SF_{MS} &\geq 0 \\ SF_{TW} &\geq 0 \end{aligned} \tag{13}$$

The first term represent the total thickness of the composite laminates, t_0 is ply thickness; n_k is the number of plies in the kth lamina, in which the fiber orientation is θ_k.

The constraints are the safety factor of the material under certain loading, and it should greater than 1.

B. Encoding

Due to the simplicity and efficiency of float representation, this encoding method is implemented to represent a possible solution.

C. Selection

The purpose of the selection operator is to chose mating pool to produce alternative solutions of better fitness. Traditional methods of selecting strategies only take the fitness of individuals into acount, however, due to the existance of constraint, various selection schemes are implemented to selecet the mating set. Then, the parents of next generation consists of three groups: proper groups, active groups, and potential groups according to different selecting methods.

Proper parents mean individual fullfils the constraint, which are chosen by the individual's fitnees, individuals with better fitness are more likely to be chosen if they fit the constraint; active groups means that individual is supposed to be always exist in the parents during the GA, which are selected by fitness, ignoring the constraint; The individuals from active group may not correspond to feasible solutions, but their existance enriches the variety of the gene clips. Potential groups means that they are likely to turn into proper individual after a couple of generations, and potential individuals are chosen by constraint function, the more the individual fulfils the constraint, the more possiblity it will be selected.

D. Crossover

The crossover operator happens among these three groups. the child of two proper groups are more likely to be a proper individual which can be used to obtain a better individual. the child of an active individual and a potential individual can significantly change the gene of active individual's chromsome, which lets the individual evolved toward a new direction. The offspring of two active individuals are more likely to be an active individual, which can maitain the active group.

E. Mutation

A mutation direction is imposed on the mutation operator which to make sure the individual evolving toward the right direction. The mutation direction, denoted by md, is a n dimensional vector corresonding to number of constraints, it is decided by the constraint thresholds CT_i and the current individual's constraint value, denoted as CV_i, The mutation vector can be obtained by the following formula

$$\text{md} = [CT_1, \cdots, CT_{n-1}, CT_n] - [CV_0, \cdots, CV_{n-1}, CV_n]$$

During the operator, the mutation consists of three parts, the length of the chromsome, the angle of the chromsome, and the number of each angle. Becasue the chromsome's length is positive correlated with the individual's fitness, the coefficient of length mutation denoted by C_l, if $\sum_{i=1}^{N} CT_i$ great than zero, the mutation length is restricted to the range $[0, C_l \sum_{i=1}^{N} CT_i]$; if the $\sum_{i=1}^{N} CT_i$ less than zero, the mutation length is restricted to the range $[0, \sum_{i=1}^{N} CT_i]$; Assuming a $[13_6/-27_4]_s$ carbon T300/5308 composite laminate under the loading $N_{xx} = N_{yy} = 10$ MPa m, the only constraint is the safety factor greater than 1. Accoring to the Tsai-Wu criterion, its safety factor is 0.0539. So the mutation vector is $[0.941]$, assuming the coefficient is 20, so the mutation range is from 0 to 18. A random number is generated from the range $[0, 18]$, supposing the outcome is 13, then a length generator is used to a list, the it's sum is 13, suppose the list is [5, 8], the laminate after mutation is $[13_{11}/-27_{12}]_s$.

The relationship between the angles in the composite laminate and the chromsome's fitness is unclear, so the mutation direction of chromsome's angle is random. The coefficient angle mutation is C_a, $[0, C_a \sum_{i=1}^{N} CT_i]$

V. Result

In this experiment, two constraints are imposed on the composite laminates which are the safety factor CT_1 for Tsai-Wu, and safety factor CT_2 for maximum stress. Both of them

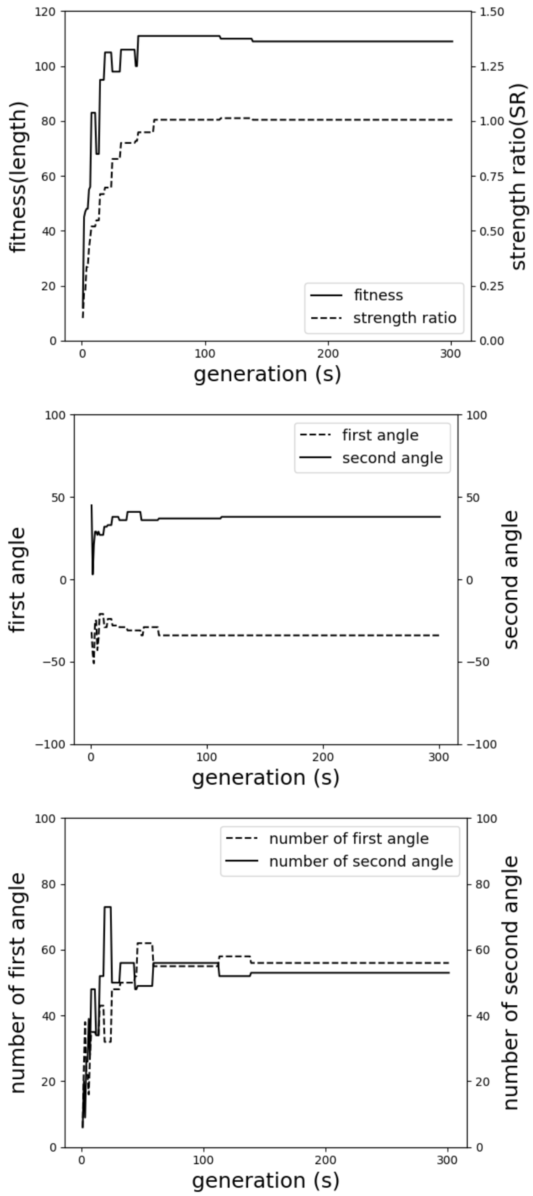

Fig. 4: Two distinct angles

value is 1. The constraint values of an individual are CV_1 and CV_2. So the mutation vector here is a two dimensional vector $[1-CV_1, 1-CV_2]$, and the coefficient of length mutation C_l and angle mutation C_a, respectively, chosen here is 20 and 10.

Figure 4 (a) shows how the optimal individual's fitness and strength ratio vary during the GA process. The method to chose optimal individual considering two following situations, if no individual in the current population meets constraint, the one with biggest fitness is selected as the optimal individual; if there are one or multiple individuals fullfils requirement, the one with smallest fitness is chosen. Figure 4 (b) shows how the two distinct fiber orientation changes at the same time, and Figure 4 (c) how the number of each angles change.

At the beginning of this GA process, the fitness curves increased very quickly, becasue of individual's strength ratio CT_0 is very small, so the difference between the individual's fitness and the imposed constraint threshold is a big positive number, so the range of mutaion length is from 0 to $C_l(CT_0 - CV_0)$. The length of individual increases by n, which is random number between 0 and $C_l(CT_0 - CV_0)$. As can be seen from Figure 4 (a), both of optimal individual's fitness and strength ratio increases very quickly. The range of mutaion angle is from 0 to $C_a(CT_0 - CV_0)$, and the number of every angle also change violently. During this stage, increasing individual's length playing a major role in increasing individual's fitness.

After a couple of generations, the optimal individual's fitness get bigger, and the difference between individual's fitness and constraint threshold get smaller. The range of mutaion length $[0, C_l(CT_0 - CV_0)]$ turn smaller. At this stage, simply increase the individual's length doesn't make much difference in improve individual's fitnees, and a better composite laminates lay-up can dramaticly change the optimal individual's fitness. That's why the fitness curve oscillated violently in this stage. At the same time, the strength ratio curve kept growing smoothly. But the growing speed got more smaller.

When GA comes to its last phase, GA found individuals that meet the constraint. Now the optimal individual's fitness is greater than the safety factor. The range of mutation length is from $C_l(CT_0 - CV_0)$ to 0. It means individuals need to decrease it's length and improve its internal structure to meet the constraint. That's why the fitness of optimal individual kept decreaing, however, the strength ratio curve still is greater then safety factor.

VI. Conclusion

In this paper, we reviewed GA, and CLT for the optimal design of composite laminated materail. SAGA is proposed to search the optimal lay-up for laminated composite under different loading. Two situations are considered under the same loading, a set of two distinct angles, and three distinct angles, SAGA can adjust its convergence speed depending on the difference of individual's constraint value and constraint threshold.

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TABLE I: An Example of a Table

Property	Symbol	Unit	Graphite/Epoxy
Longitudinal elastic modulus	E_1	GPa	181
Traverse elastic modulus	E_2	GPa	10.3
Major Poisson's ratio	v_{12}		0.28
Shear modulus	G_{12}	GPa	7.17
Ultimate longitudinal tensile strength	$(\sigma_1^T)_{ult}$	MP	1500
Ultimate longitudinal compressive strength	$(\sigma_1^C)_{ult}$	MP	1500
Ultimate transverse tensile strength	$(\sigma_2^T)_{ult}$	MPa	40
Ultimate transverse compressive strength	$(\sigma_2^C)_{ult}$	MPa	246
Ultimate in-plane shear strength	$(\tau_{12})_{ult}$	MPa	68
Density	ρ	g/cm^3	1.590

TABLE II: The optimum lay-ups using two distinct fiber angles under various biaxial loading cases

Loading $N_x/N_y/N_{xy}$ (MPa m)	Optimum lay-up sequences	laminate thickness	Safety factor	MS
10/5/0	$[33_{29}/\text{-}39_{25}/\text{-}\bar{3}9]_s$	109	1.0074	1.0246
20/5/0	$[33_{22}/\text{-}31_{24}]_s$	92	1.0055	1.2065
40/5/0	$[29_{18}/\text{-}21_{23}/\text{-}\bar{2}1]_s$	83	1.0034	1.7350
80/5/0	$[\text{-}20_{27}/21_{25}/\bar{2}5]_s$	105	1.0029	1.2063
120/5/0	$[\text{-}18_{34}/17_{36}]_s$	140	1.0000	1.0898

TABLE III: The optimum lay-ups using three distinct fiber angles under various biaxial loading cases

Loading $N_x/N_y/N_{xy}$ (MPa m)	Optimum lay-up sequences	laminate thickness	Safety factor
10/5/0	$[37_{27}/\text{-}38_{27}/\text{-}5]_s$	110	1.0023
20/5/0	$[34_{24}/\text{-}32_{14}/\text{-}28_{11}]_s$	98	1.0237
40/5/0	$[21_{28}/\text{-}32_{19}/2_3]_s$	100	1.0788
80/5/0	$[\text{-}21_{25}/\text{-}16_3/21_{26}]_s$	108	1.0128
120/5/0	$[\text{-}18_{34}/17_{36}]_s$	140	1.0000

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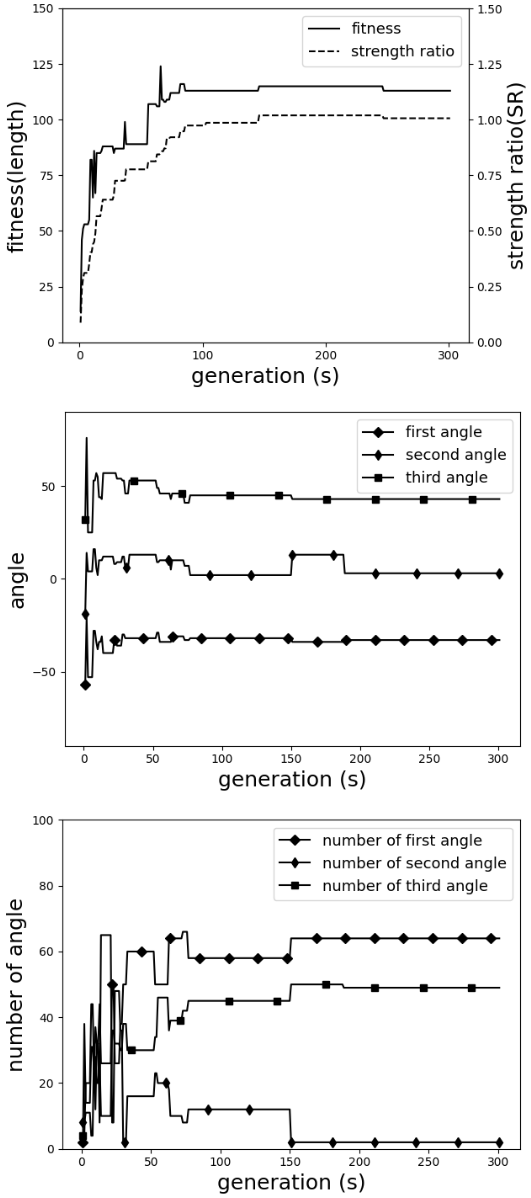

Fig. 5: Three distinct angles

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