

An approximation method of strength ratio calculation of laminated composite material based on evolutionary artificial neural network

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Abstract—Traditionally, classic lamination theory (CLT) is widely used to compute properties of composite materials under in-plane and out-of-plane loading from a knowledge of the material properties of the individual layers and the laminate geometry. In this study, a systematic procedure is proposed to design an artificial neural network (ANN) for a practical engineering problem, which is applied to calculate the strength ratio of a laminated composite material under in-plane loading, in which the genetic algorithm is proposed to optimize the search process at four different levels: the architecture, parameters, connections of the neural network, and active functions.

Keywords—Classic Lamination Theory; Genetic Algorithm; Artificial neural network; Optimization

I. INTRODUCTION

Fiber-reinforced composite materials have been widely used in a variety of applications, which include electronic packaging, sports equipment, homebuilding, medical prosthetic devices, high-performance military structures, etc. because they offer improved mechanical stiffness, strength, and low specific gravity of fibers over conventional materials. The stacking sequence, ply thickness, and fiber orientation of composite laminates give the designer an additional 'degree of freedom' to tailor the design with respect to strength or stiffness. CLT and failure theory, e.g., Tsai-Wu failure criteria, is usually taken to predict the behavior of a laminate from a knowledge of the composite laminate properties of the individual layers and the laminate geometry.

However, the use of CLT needs intensive computation which takes an analytical method to solve the problem, since it involves massive matrix multiplication and integration calculation. Techniques of function approximation can accelerate the calculation process and reduce the computation cost. Artificial neural network (ANN), heavily inspired by biology and psychology, is a reliable tool instead of a complicated mathematical model. ANN has been widely used to solve various practical engineering problems in applications, such as pattern recognition, nonlinear regression, data mining, clustering, prediction, etc. Evolutionary artificial neural networks (EANN's) is a special class of artificial neural networks (ANN's), in which evolutionary algorithms are introduced to design the topology of an ANN, and can be used at four different levels:

connection weights, architectures, input features, and learning rules. It is shown that the combinations of ANN's and EA's can significantly improve the performance of intelligent systems than that rely's on ANN's or evolutionary algorithms alone.

The rest of the paper is organized as the following: chapter two explains the classical laminate theory and the failure criteria taken in the present study; chapter three explains the design of artificial neural network for mathematical model approximation; chapter four reviews the use of genetic algorithm in the design of neural network architecture and the parameters optimization during the training process of neural network design; chapter five describes the result of the numerical experiments in different cases; in the conclusion section we discuss the results.

II. CLASSIC LAMINATION THEORY AND FAILURE THEORY

A. Classic Lamination Theory

Classical lamination theory is based upon three simplifying engineering assumptions: (1) Each layer's thickness is very small and consist of homogeneous, orthotropic material, and these layers are perfectly bonded together; (2) The entire laminated composite is supposed to be under plane stress; (3) Normal cross sections of the entire laminate is normal to the deflected middle surface, and do not change in thickness.

1) *Stress and Strain in a Lamina*: For a single lamina has a small thickness under plane stress, and it's upper and lower surfaces of the lamina are free from external loads. According to the Hooke's Law, the three-dimensional stress-strain equations can be reduced to two-dimensional stress-strain equations. The stress-strain relation in local axis 1-2 is

$$\begin{bmatrix} \sigma_1 \\ \sigma_2 \\ \tau_{12} \end{bmatrix} = \begin{bmatrix} Q_{11} & Q_{12} & 0 \\ Q_{12} & Q_{22} & 0 \\ 0 & 0 & Q_{66} \end{bmatrix} \begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \gamma_{12} \end{bmatrix}, \quad (1)$$

where Q_{ij} are the stiffnesses of the lamina. And they are related to engineering elastic constants as follows:

$$\begin{aligned} Q_{11} &= \frac{E_1}{1 - \nu_{12}\nu_{21}}, \\ Q_{22} &= \frac{E_2}{1 - \nu_{12}\nu_{21}}, \\ Q_{66} &= G_{12}, \\ Q_{12} &= \frac{\nu_{21}E_2}{1 - \nu_{12}\nu_{21}}, \end{aligned} \quad (2)$$

where $E_1, E_2, \nu_{12}, G_{12}$ are four independent engineering elastic constants, which are defined as follows: E_1 is the longitudinal Young's modulus, E_2 is the transverse Young's modulus, ν_{12} is the major Poisson's ratio, and G_{12} is the in-plane shear modulus.

Stress strain relation in the global x-y axis:

$$\begin{bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{bmatrix} = \begin{bmatrix} \bar{Q}_{11} & \bar{Q}_{12} & \bar{Q}_{16} \\ \bar{Q}_{12} & \bar{Q}_{22} & \bar{Q}_{26} \\ \bar{Q}_{16} & \bar{Q}_{26} & \bar{Q}_{66} \end{bmatrix} \begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{bmatrix} \quad (3)$$

where

$$\begin{aligned} \bar{Q}_{11} &= Q_{11} \cos^4 \theta + Q_{22} \sin^4 \theta + 2(Q_{12} + 2Q_{66}) \sin^2 \theta \cos^2 \theta \\ \bar{Q}_{12} &= (Q_{11} + Q_{22} - 4Q_{66}) \sin^2 \theta \cos^2 \theta + Q_{12} (\cos^4 \theta + \sin^4 \theta) \\ \bar{Q}_{22} &= Q_{11} \sin^4 \theta + Q_{22} \cos^4 \theta + 2(Q_{12} + 2Q_{66}) \sin^2 \theta \cos^2 \theta \\ \bar{Q}_{16} &= (Q_{11} - Q_{12} - 2Q_{66}) \cos^3 \theta \sin \theta - (Q_{22} - Q_{12} - 2Q_{66}) \sin^3 \theta \cos \theta \\ \bar{Q}_{26} &= (Q_{11} - Q_{12} - 2Q_{66}) \cos \theta \sin^3 \theta - (Q_{22} - Q_{12} - 2Q_{66}) \cos^3 \theta \sin \theta \\ \bar{Q}_{66} &= (Q_{11} + Q_{22} - 2Q_{12} - 2Q_{66}) \sin^2 \theta \cos^2 \theta \end{aligned} \quad (4)$$

2) *Stress and Strain in a Laminate*: For forces and moment resultants acting on laminates, such as in plate and shell structures, the relationship between applied forces and moment and displacement can be given by

$$\begin{bmatrix} N_x \\ N_y \\ N_{xy} \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} & A_{16} \\ A_{12} & A_{22} & A_{26} \\ A_{16} & A_{26} & A_{66} \end{bmatrix} \begin{bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \end{bmatrix} + \begin{bmatrix} B_{11} & B_{12} & B_{16} \\ B_{12} & B_{22} & B_{26} \\ B_{16} & B_{26} & B_{66} \end{bmatrix} \begin{bmatrix} k_x \\ k_y \\ k_{xy} \end{bmatrix} \quad (5)$$

N_x, N_y - normal force per unit length

N_{xy} - shear force per unit length

ε^0, k - mid plane strains and curvature of a laminate in x-y coordinates

The mid plane strain and curvature is given by

$$\begin{aligned} A_{ij} &= \sum_{k=1}^n (\bar{Q}_{ij})_k (h_k - h_{k-1}) \quad i = 1, 2, 6, j = 1, 2, 6 \\ B_{ij} &= \frac{1}{2} \sum_{k=1}^n (\bar{Q}_{ij})_k (h_k^2 - h_{k-1}^2) \quad i = 1, 2, 6, j = 1, 2, 6 \\ D_{ij} &= \frac{1}{3} \sum_{k=1}^n (\bar{Q}_{ij})_k (h_k^3 - h_{k-1}^3) \quad i = 1, 2, 6, j = 1, 2, 6 \end{aligned} \quad (6)$$

The $[A]$, $[B]$, and $[D]$ matrices are called the extensional, coupling, and bending stiffness matrices, respectively. The extensional stiffness matrix $[A]$ relates the resultant in-plane forces to the in-plane strains, and the bending stiffness matrix $[D]$ couples the resultant bending moments to the plane curvatures. The coupling stiffness matrix $[B]$ relates the force and moment terms to the midplane strains and midplane curvatures.

$+\theta$
$-\theta$
\dots
$-\theta$
$+\theta$

Fig. 1: Model for Angle ply laminate

B. Failure criteria for a lamina

Failure criteria for composite materials are more difficult to predict due to structural and material complexity in comparison to isotropic materials. The failure process of a composite materials can be regarded from microscopic and macroscopic points of view. Most popular criteria about the failure of an angle lamina stem from microscopic failure criteria, which are based on the tensile, compressive and shear strengths. According to the failure surfaces, these criteria [9], [12], [5], [12], [11], [10], [10], [23] can be classified into two classes: one is called independent failure mode criteria which includes the maximum stress failure theory [15], maximum strain failure theory because their failure envelop are rectangle; another is called quadratic polynomial which includes Tsai-Wu [8], [13], Chamis, Hoffman and Hill criteria because their failure surfaces are of ellipsoidal shape. In the present study, two most reliable failure criteria is taken, Maximum stress and Tsai-wu. Both of these two failure criteria are based on the stresses in the local axes instead of principal normal stresses and maximum shear stresses, and four normal strength parameters and one shear stress for a unidirectional lamina are involved. The five strength parameters are

$(\sigma_1^T)_{ult}$ = ultimate longitudinal tensile strength (in direction 1),

$(\sigma_1^C)_{ult}$ = ultimate longitudinal compressive strength,

$(\sigma_2^T)_{ult}$ = ultimate transverse tensile strength,

$(\sigma_2^C)_{ult}$ = ultimate transverse compressive strength, and

$(\tau_{12})_{ult}$ = and ultimate in-plane shear strength.

1) Maximum stress failure criterion: (MS)

Maximum stress failure theory consists of maximum normal stress theory proposed by Rankine and maximum shearing stress theory by Tresca. The stresses applied on a lamina can be resolved into the normal and shear stresses in the local axes. If any of the normal or shear stresses in the local axes of a lamina is equal or exceeds the corresponding ultimate strengths of the unidirectional lamina, the lamina is considered to be failed. That is,

$$\sigma_1 \geq (\sigma_1^T)_{ult} \text{ or } \sigma_1 \leq -(\sigma_1^C)_{ult}$$

$$\sigma_2 \geq (\sigma_2^T)_{ult} \text{ or } \sigma_2 \leq -(\sigma_2^C)_{ult}$$

$$\tau_{12} \geq (\tau_{12})_{ult} \text{ or } \tau_{12} \leq -(\tau_{12})_{ult}$$

where σ_1 and σ_2 are the normal stresses in the local axes 1 and 2, respectively; τ_{12} is the shear stress in the symmetry plane 1-2.

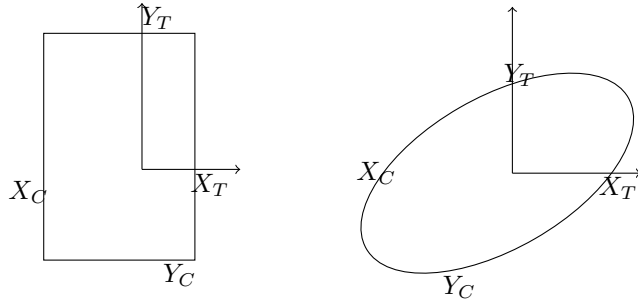


Fig. 2: Schematic failure surfaces for maximum stress and quadratic failure criteria

2) *Tsai-wu failure criterion*: The TW criterion is one of the most reliable static failure criteria which is derived from the von Mises yield criterion. A lamina is considered to fail if

$$H_1\sigma_1 + H_2\sigma_2 + H_6\tau_{12} + H_{11}\sigma_1^2 + H_{22}\sigma_2^2 + H_{66}\tau_{12}^2 + 2H_{12}\sigma_1\sigma_2 < 1 \quad (7)$$

is violated, where

$$\begin{aligned} H_1 &= \frac{1}{(\sigma_1^T)_{ult}} - \frac{1}{(\sigma_1^C)_{ult}} \\ H_{11} &= \frac{1}{(\sigma_1^T)_{ult}(\sigma_1^C)_{ult}} \\ H_2 &= \frac{1}{(\sigma_2^T)_{ult}} - \frac{1}{(\sigma_2^C)_{ult}} \\ H_{22} &= \frac{1}{(\sigma_2^T)_{ult}(\sigma_2^C)_{ult}} \\ H_{66} &= \frac{1}{(\tau_{12})_{ult}^2} \\ H_{12} &= -\frac{1}{2} \sqrt{\frac{1}{(\sigma_1^T)_{ult}(\sigma_1^C)_{ult}(\sigma_2^T)_{ult}(\sigma_2^C)_{ult}}} \end{aligned} \quad (8)$$

H_i is the strength tensors of the second order; H_{ij} is the strength tensors of the fourth order. σ_1 is the applied normal stress in direction 1; σ_2 is the applied normal stress in the direction 2; and τ_{12} is the applied in-plane shear stress.

3) *Strength ratio*: The safety factor, or yield stress, is how much extra load beyond is intended a composite laminate will actually take. The safety factor is defined as

$$SR = \frac{\text{Maximum Load Which Can Be Applied}}{\text{Load Applied}} \quad (9)$$

III. EVOLUTIONARY ARTIFICIAL NEURAL NETWORK

A. General neural network

In this paper, the feedforward neural network models are adopted in our system, because feedforward NNs are straightforward and simply to implement. For function approximation, Cybenko demonstrated that a two-layer multilayer

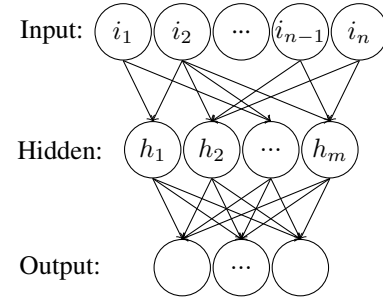


Fig. 3: General Neural Network

TABLE I: Different Activation Functions

Type	Description
Linear	The output is proportional to the input
Sigmoid	A family of S-shaped functions
tanh	A family of Hyperbolic functions
Gaussian	A continuous bell-shaped curve
ReLU	A piece-wise function
Softplus	A family of S-shaped functions

perceptron (MLP) is capable of forming an arbitrarily close approximation to any continuous nonlinear mapping [3]. Therefore, a two layers feedforward neural network is proposed in present study, for nodes in the hidden layer, they are in essence feature extractors and detectors. Therefore, every nodes in the hidden layer should be partially connected within the inputs, the unnecessary connections would increase the model's complexity which would reduce the ANN's performance. For the nodes in the last layer, every node should be fully connected with nodes in previous. The design of neural network consists of three basic parts: neural network architecture, learning rules, and training techniques.

B. Transfer function

The transfer function has been shown to be one of the critical parts of the architecture. Liu [7] et al. have claimed that ANNs with different active functions play an important role in the architecture's performance. A generalized transfer function can be written as

$$y_i = f_i\left(\sum_{j=1}^n w_{ij}x_j - \theta\right) \quad (10)$$

where y_i is the output of the node i , x_j is the j th input to the node, and w_{ij} is the connection weight between adjacent nodes i and j . Most widely transfer function f_i is listed in table I

C. Weights learning

The weight training in ANN is to minimize the error function, such as the most widely used mean square error which calculate the difference between the desired and the prediction output values averaged over all examples. Back-propagation algorithm has been successful applied to in many areas, and it's based on gradient descent. However, this class of algorithms are plagued by the possible existance of local minima or "flat spots" and "the curse of dimensionality". One method to overcome this problem is to adopt EANN's[16] through a systematic search process[4] to build the topology of ANN.

IV. METHODOLOGY

The works involved in the evolution process of ANN can be categorized into three parts: search space which defines the architecture can be represented in principle; search strategy which details how to explore the search space; performance estimation strategy that refers to the process of estimating this performance.

A. Search Space

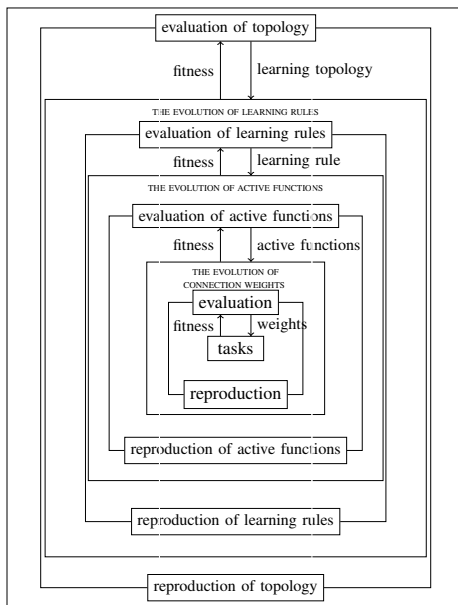
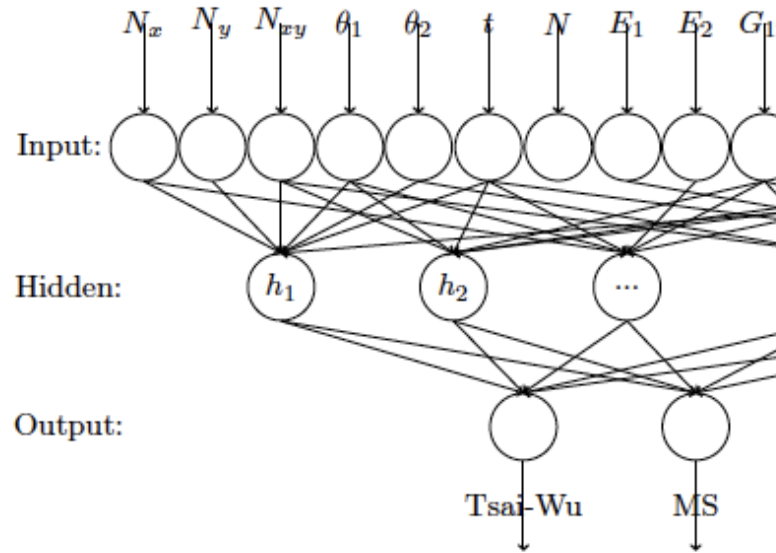


Fig. 4: A general framwwork for EANN's

we propose a GNN framework as shown in Figure 3. The search space is parametrized by: (i) the number of nodes m (possibly unbounded) in hidden layer, to narrow down the search space, the assumptions is that m less than n ; (ii) the type of operation every nodes executes, e.g., sigmoid, linear, gaussian. (iii) the connection relationship between hidden nodes and inputs; (IV) if a connection exists, the weight value in the connection.

Therefore, evolution in EANN can be divided into four different levels: topology, learning rules, active functions, and connection weights. For the evolution of topology, the aim is to find an optimal ANN architecture for a specific problem. The architecture of a neural network determines the information



processing capability in application, which is the foundation of the ANN. Two critical issues are involved in the search process of an ANN architecture: the representation and the search operators. Figure 4 summarizes different these four levels of evolution in ANN's.

The inputs of the neural network is consist of four parts: in-plane loading N_x , N_y , and N_{xy} , design parameters of laminate, two distinct fiber orientation angle θ_1 and θ_2 , ply thickness t , total number of plies N ; five engineering constants of composite materials, E_1 , E_2 , ; five strength parameters of a unidirectional lamina. There are two outputs in the neural network, safety factors for MS theory and Tsai-Wu theory, respectively.

B. Search Strategy

The classic approach has always adopted binary strings to encode an alternative solutions.

Tab.II gives an example of the binary representation of an ANN whose architecture is as shown in Fig.???. Each number in the digit denotes the connection relationship between input and nodes in hidden layer. It an connection exists, it's indicated by number one, otherwise, the number takes zero. The first sixteen digits denotes the connection relationship, and the last two digits are stand for the corresponding kernel function.

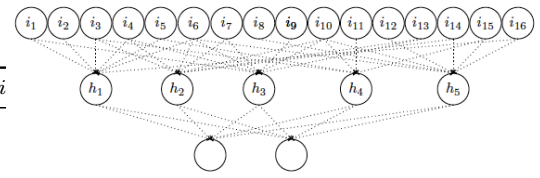
First, random initialize ANN population, partial training every ANN, For the evolution of the topology,

C. Performance estimation strategy

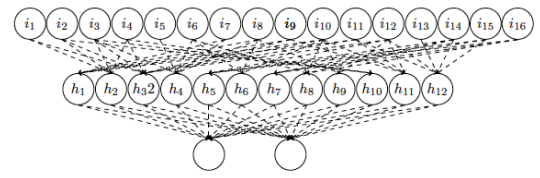
The simplest approach to this problem is to perform a standard training and validation of the architecture on dataset, however, this method is inefficient and computational intensive. Therefore, much recent research[1] focus on developing methods that reduce the cost of performance estimation.

TABLE II: Binary representation of Parent 1

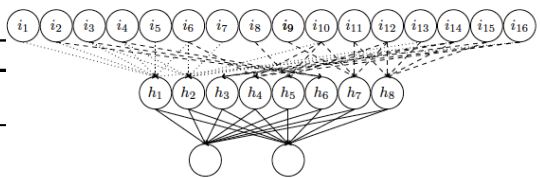
Hidden	Nodes	i_1	i_2	i_3	i_4	i_5	i_6	i_7	i_8	i_9	i_{10}	i
P1	h_1	1	1	1	1	1	1	0	0	0	0	
	h_2	0	1	1	1	0	0	0	1	0	0	
	h_3	1	0	0	1	0	1	1	0	1	1	
	h_4	0	0	1	0	1	0	0	0	0	1	
	h_5	0	0	0	0	0	1	0	1	0	1	
P2	h_1	0	0	0	0	0	0	1	1	1	1	
	h_2	1	1	1	1	0	0	0	0	0	0	
	h_3	1	1	1	1	0	0	0	0	1	1	
	h_4	0	0	1	1	1	1	1	0	0	0	
	h_5	0	0	0	0	0	0	0	0	0	0	
	h_6	0	0	0	0	0	1	0	1	0	1	
	h_7	0	0	0	0	0	0	0	0	0	0	
	h_8	0	0	0	0	0	1	0	0	0	1	
	h_9	0	0	0	0	0	1	0	1	0	1	
	h_{10}	0	1	1	1	0	0	0	0	1	1	
	h_{11}	0	0	0	0	0	0	0	0	1	1	
	h_{12}	0	0	0	0	0	0	0	0	0	0	



0	1	0	0	1	0	1
0	1	0	1	0	1	1



0	1	0	0	0	0	1
1	1	0	0	0	0	1



1062.0,610.0,31,118,72,	0.0102,	0.0086
1500.0,1500.0,40,246,68,	0.4026,	2.5120

Input		
Load	Laminate Structure	Material Property
-70,-10,-40,	90,-90,4,1.27,	38.6,8.27,0.26,4.14,
-10,10,0,	-86,86,80,1.27,	181.0,10.3,0.28,7.17,
-70,-50,80,	-38,38,4,1.27,	116.6,7.67,0.27,4.173,
-70,80,-40,	90,-90,48,1.27,	38.6,8.27,0.26,4.14,
-20,-30,0,	-86,86,60,1.27,	181.0,10.3,0.28,7.17,
0,-40,0,	74,-74,168,1.27,	181.0,10.3,0.28,7.17,

V. EXPERIMENT

We applied this search strategy to dataset generated by the classic lamination theory and failure theories. In this dataset, sixteen attributes and two actual values are given.

A. Dataset Preparation

Equation 3 takes an analytical approach to model the relationship between stress and strain. We sample this function to yield 14000 points uniformly distributed over the domain space.

The range of in-plane loading is from 0 to 120; the range of fiber orientation θ is from -90 to 90; ply thickness t is 1.27mm, number of plies range N is from 4 to 120; Three different material is used in this experiment, as shown in table III. Figure ?? shows part of the training data.

In order to speeds up the learning and accerlate convergence, the input attributes of the data set are rescaled to between 0 and 1.0 by a linear function.

VI. RESULT AND DISCUSSION

We have conducted experiment by the use of the generated data set. This data set is randomly partitioned into a training set and a test.

Figure ?? shows five ANNs with different topologies, quite different results have been observed when different

architectures are adopted. It is clear that architecture whose mean of average difference is less than the rest.

Table IV shows part of the valudation.

VII. CONCLUSION

We have reviewed the use of GA and EANN's as an alternative for calculatating the strength ratio based on MS and Tsai-wu failure theories. To find a near-optimal ANN architecture automatically, four levels evolution are proposed to be introduced into EANN's evolution process for approximating a target function. Experiment studies were carried out for the training data and indicated that evolved GNNs did generalise the target function.

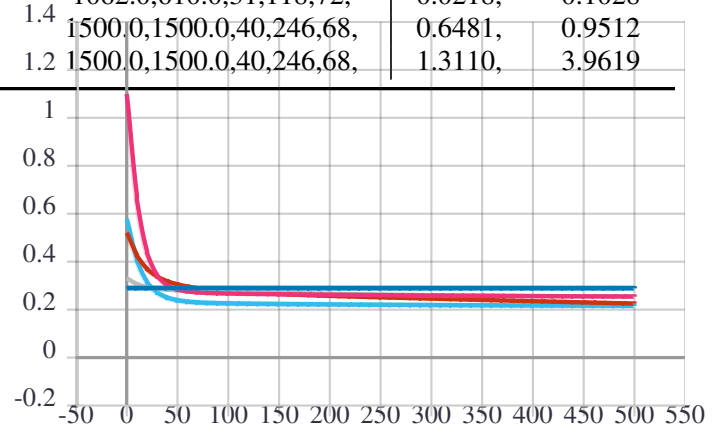
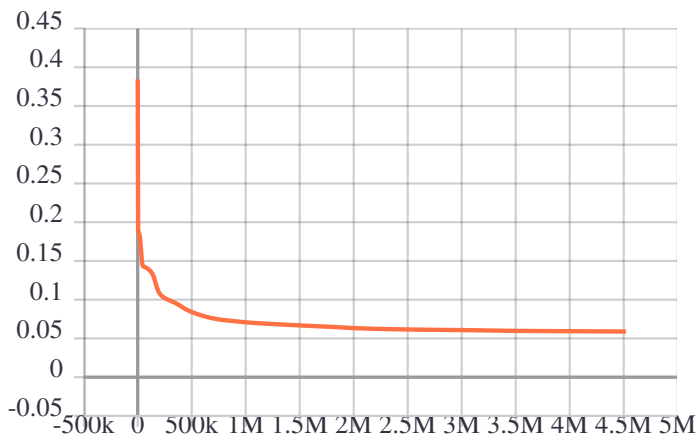


TABLE III: Comparison of the carbon/epoxy, graphite/epoxy, and glass/epoxy properties

Property	Symbol	Unit	Carbon/Epoxy	Graphite/Epoxy	Glass/Epoxy
Longitudinal elastic modulus	E_1	GPa	116.6	181	38.6
Traverse elastic modulus	E_2	GPa	7.67	10.3	8.27
Major Poisson's ratio	ν_{12}		0.27	0.28	0.26
Shear modulus	G_{12}	GPa	4.17	7.17	4.14
Ultimate longitudinal tensile strength	$(\sigma_1^T)_{ult}$	MP	2062	1500	1062
Ultimate longitudinal compressive strength	$(\sigma_1^C)_{ult}$	MP	1701	1500	610
Ultimate transverse tensile strength	$(\sigma_2^T)_{ult}$	MPa	70	40	31
Ultimate transverse compressive strength	$(\sigma_2^C)_{ult}$	MPa	240	246	118
Ultimate in-plane shear strength	$(\tau_{12})_{ult}$	MPa	105	68	72
Density	ρ	g/cm^3	1.605	1.590	1.903
Cost			8	2.5	1

TABLE IV: Comparison between practical and simulation

Input			Output			
Load	Laminate Structure	Material Property	Failure Property	GLT	MS-Termin	MS-Termin
-10,40,20	26,-26,168,1.27	116.6,7.67,0.27,4.17	2062.0,1701.0,70.0,240.0,105.0	0.342	0.476	0.351
20,-70,-30	10,-10,196,1.27	181.0,10.3,0.28,7.17	1500.0,1500.0,40.0,246.68	0.653	0.489	0.612
60,-20,0	82 -82,128,1.27	181.0,10.3,0.28,7.17	1500.0,1500.0,40.0,246.68	1.663	0.112	1.673



There are more improvements we can make over our the search strategy and it's application in predicting composite material's properties. The future work is to develop a more sophisticated ANN, which not only is able to predict the properties for angle ply laminate, but also for the other type of laminated composite material.

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