Contents

Chapter 1

Introduction

1.1 Laminated Composite Material

Composite materials offer improved strength, stiffness, corrosion resistance, etc. over conventional materials, and are widely used as alternative materials for applications in various industries ranging from electronic packaging to golf clubs, and medical equipment to homebuilding, making aircraft structure to space vechicles. The stacking sequence and fiber orientation of composite laminates give the designer additional 'degree of freedom' to tailor the design with respect to strength or stiffness. One widely known advange of using composite material is can significantly reducing the weight of target structure, and many researchers attempted to improve the efficiency of using composite materail by mimimizing the thickness.

1.2 Classic Lamination Theory and Failure Theory

Classic lamination theory(CLT) is used to develop the stress-strain relationship of composite material under in-plane and out-of-plane loading. First, develop stress-strain relationships, elastic moduli, strengths of an angle ply based on a unidirectional lamina and the angle of the ply; second, becasue a laminate is consist of more than one lamina bonded together through their thickness, so the macromechianical analysis will be developed for a laminate based on applied loading. To check whether a designed lay-up is plausible or not, different failure theories have been developed.

1.3 Genetic Algorithm

In the design of composite material, gradient based optimization techiques are not appliable in this domain, because the design varaibles, such as fiber orientation, layer thickness, number of layers etc. are discrete. Genetic algorithm(GA) can be adopted in the optimization problem becasue it doesn't require the gradient information. Moreover, the GA has been proved a reliable techique and widely used in the design of composite material.

1.4 Artificial Neural Network

CLT is an classic analytical approach to obtain the stress and stain of composite material, the disadvantage of this method is quiet cubersome and in which involves compliate matrix and integration operations. Artificial neural network(ANN) has been proved a reliable tool in modelling various engineering system in practice without solving tricky equations and making ideal assumptions. In this thesis, the ANN is taken to approximate the numberic results based on CLT and failure theory.

1.5 Summary

In this thesis, first, we review the use of composite material in practice, then, the CLT to calculate the stress and strain under certain loading, last, the failure theories which are used to decide whether a composite material will failure or not. Second, the stochastic algorith, GA, is studied and implemented in the desgin of composite material, two different cases are studied in which GA can be taken to obtain the optimal lay-up. At last, ANN is introduced to approximate the evaluation result of CLT, the reason for adopting ANN is to reduce the computation complexity based on CLT.

Chapter 2

Classic Lamination Theory and Failure Theory

2.1 Classic Lamination Theory

A laminate is consist of multiple laminas bonded together through thickness. In this chapter, first the stress-strain relationship is developed based on Hook's law for a single lamina; second, develop relationship of mechanical loads applied to a laminate to strains and stresses in each lamina, calculate the elastic moduli of laminate based on the elastic moduli of single laminate and the lay-up..

2.1.1 Analysis of stress and strain for composite material

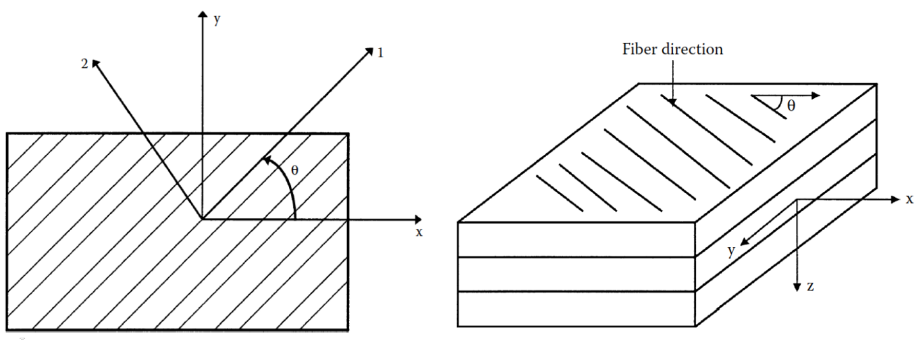

FIGURE 2.1: Lamina

2.1.2 Stress and Strain in a Lamina

For a single lamina has a small thickness under plane stress, and it's upper and lower surfaces of the lamina are free from external loads. According to the Hooke's Law, the three-dimensional stress-strain equations can be reduced to two-dimensional stress-strain equations. The stress-strain relation in local axis 1-2 is

$$\begin{bmatrix} \sigma_1 \\ \sigma_2 \\ \tau_{12} \end{bmatrix} = \begin{bmatrix} Q_{11} & Q_{12} & 0 \\ Q_{12} & Q_{22} & 0 \\ 0 & 0 & Q_{66} \end{bmatrix} \begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \gamma_{12} \end{bmatrix}. \tag{2.1}$$

where $Q_{ij}$are the stiffnesses of the lamina that are related to engineering elastic constants given by

$$\begin{aligned} Q_{11} &= \frac{E_1}{1 - v_{12} v_{21}}, \\ Q_{22} &= \frac{E_2}{1 - v_{12} v_{21}}, \\ Q_{66} &= G_{12}, \\ Q_{12} &= \frac{v_{21} E_2}{1 - v_{12} v_{21}}, \end{aligned} \tag{2.2}$$

where E_1, E_2, v_{12}, G_{12} are four independent engineering elastic constants, which are defined as follows: E_1 is the longitudinal Young's modulus, E_2 is the transverse Young's modulus, v_{12} is the major Poisson's ratio, and G_{12} is the in-plane shear modulus.

Stress strain relation in the global x-y axis is

$$\begin{bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{bmatrix} = \begin{bmatrix} \bar{Q}_{11} & \bar{Q}_{12} & \bar{Q}_{16} \\ \bar{Q}_{12} & \bar{Q}_{22} & \bar{Q}_{26} \\ \bar{Q}_{16} & \bar{Q}_{26} & \bar{Q}_{66} \end{bmatrix} \begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{bmatrix}, \tag{2.3}$$

where

$$\begin{aligned} \bar{Q}_{11} &= Q_{11} cos^4\theta + Q_{22} sin^4\theta + 2\left(Q_{12} + 2Q_{66}\right) sin^2\theta cos^2\theta, \\ \bar{Q}_{12} &= \left(Q_{11} + Q_{22} - 4Q_{66}\right) sin^2\theta cos^2\theta + Q_{12}\left(cos^4\theta + sin^2\theta\right), \\ \bar{Q}_{22} &= Q_{11} sin^4\theta + Q_{22} cos^4\theta + 2\left(Q_{12} + 2Q_{66}\right) sin^2\theta cos^2\theta, \\ \bar{Q}_{16} &= \left(Q_{11} - Q_{12} - 2Q_{66}\right) cos^3\theta sin\theta - \left(Q_{22} - Q_{12} - 2Q_{66}\right) sin^3\theta cos\theta, \\ \bar{Q}_{26} &= \left(Q_{11} - Q_{12} - 2Q_{66}\right) cos\theta sin^3\theta - \left(Q_{22} - Q_{12} - 2Q_{66}\right) cos^3\theta sin\theta, \\ \bar{Q}_{66} &= \left(Q_{11} + Q_{22} - 2Q_{12} - 2Q_{66}\right) sin\theta^2 cos\theta^2 + Q_{66}\left(sin\theta^4 + cos\theta^4\right). \end{aligned} \tag{2.4}$$

The local and global stresses in an angle lamina are related
to each other through the angle of the lamina θ, it can be written as

$$\begin{bmatrix} \sigma_1 \\ \sigma_2 \\ \tau_{12} \end{bmatrix} = [T] \begin{bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{bmatrix}, \tag{2.5}$$

where

$$[T] = \begin{bmatrix} cos^2\theta & sin^2\theta & 2sin\theta cos\theta \\ sin^2\theta & cos^2\theta & -2sin\theta cos\theta \\ -sin\theta cos\theta & sin\theta cos\theta & cos^2\theta - sin^2\theta \end{bmatrix}. \tag{2.6}$$

2.1.3 Stress and Strain in a Laminate

For forces and moment resultants acting on laminates, such as in plate and shell structures, the relationship between applied forces and moment and displacement can be given by

$$\begin{bmatrix} N_x \\ N_y \\ N_{xy} \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} & A_{16} \\ A_{12} & A_{22} & A_{26} \\ A_{16} & A_{26} & A_{66} \end{bmatrix} \begin{bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \end{bmatrix} + \begin{bmatrix} B_{11} & B_{12} & B_{16} \\ B_{11} & B_{12} & B_{16} \\ B_{16} & B_{26} & B_{66} \end{bmatrix} \begin{bmatrix} k_x \\ k_y \\ k_{xy} \end{bmatrix},$$
$$\begin{bmatrix} M_x \\ M_y \\ M_{xy} \end{bmatrix} = \begin{bmatrix} B_{11} & B_{12} & B_{16} \\ B_{12} & B_{22} & B_{26} \\ B_{16} & B_{26} & B_{66} \end{bmatrix} \begin{bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \end{bmatrix} + \begin{bmatrix} D_{11} & D_{12} & D_{16} \\ D_{11} & D_{12} & D_{16} \\ D_{16} & D_{26} & D_{66} \end{bmatrix} \begin{bmatrix} k_x \\ k_y \\ k_{xy} \end{bmatrix}, \tag{2.7}$$

where
N_x, N_y - normal force per unit length;
N_{xy} - shear force per unit length;
M_x, M_y - bending moment per unit length;
M_{xy} - twisting moments per unit length;
ε^0, k- mid plane strains and curvature of a laminate in x-y coordinates.
The mid plane strain and curvature is given by

$$A_{ij} = \sum_{k=1}^{n} (\overline{Q_{ij}})_k (h_k - h_{k-1}) i = 1,2,6, j = 1,2,6,$$
$$B_{ij} = \frac{1}{2} \sum_{k=1}^{n} (\overline{Q_{ij}})_k (h_k^2 - h_{k-1}^2) i = 1,2,6, j = 1,2,6, \tag{2.8}$$
$$D_{ij} = \frac{1}{3} \sum_{k=1}^{n} (\overline{Q_{ij}})_k (h_k^3 - h_{k-1}^3) i = 1,2,6, j = 1,2,6.$$

The [A], [B], and [D] matrices are called the extensional, coupling, and bending stiffness matrices, respectively. The extensional stiffness matrix $[A]$ relates the resultant in-plane forces to the in-plain strains, and the bending stiffness matrix $[D]$ couples the resultant bending moments to the plane curvatures. The coupling stiffness matrix $[B]$ relates the force and moment terms to the midplain strains and midplane curvatures.

2.2 Failure Theory

Failure criteria for composite materials are more difficult to predict due to structural and material complexity in comparison to isotropic materials. The failure process of a composite materials can be regarded from microscopic and macroscopic points of view. Most popular criteria about the failure of an angle lamina are in terms of macroscopic failure criteria, which are based on the tensile, compressive and shear strengths. According to the failure surfaces, these criteria can be classified into two classes: one is called independent failure mode criteria which includes the maximum stress failure theory, maximum strain failure theory because their failure envelop are rectangle; another is called quadratic polynomial which includes Tsai-Wu, Chamis, Hoffman and Hill criteria because their failure surfaces are of ellipsoidal shape. In

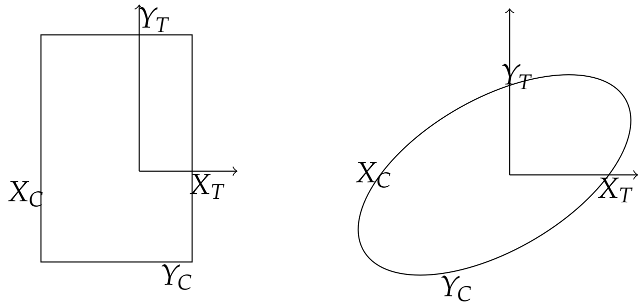

FIGURE 2.2: Schematic failure surfaces for maximum stress and quadratic failure criteria

the present study, two most reliable failure criteria is taken, Maximum stress and Tsai-wu. Both of these two failure criteria are based on the stresses in the local axes instead of principal normal stresses and maximum shear stresses, and four normal strength parameters and one shear stress for a unidirectional lamina are involved. The five strength parameters are

$(\sigma_1^T)_{ult}$ = ultimate longitudinal tensile strength(in direction 1),
$(\sigma_1^C)_{ult}$ = ultimate longitudinal compressive strength,
$(\sigma_2^T)_{ult}$ = ultimate transverse tensile strength,
$(\sigma_2^C)_{ult}$ = ultimate transverse compressive strength, and
$(\tau_{12})_{ult}$ = and ultimate in-plane shear strength.

2.2.1 Tsai-wu failure criterion

The TW criterion is one of the most reliable static failure criteria which is derived from the von Mises yield criterion. A lamina is considered to fail if

$$\begin{aligned} &H_1\sigma_1 + H_2\sigma_2 + H_6\tau_{12} + H_{11}\sigma_1^2 + H_{22}\sigma_2^2 \\ &+ H_{66}\tau_{12}^2 + 2H_{12}\sigma_1\sigma_2 < 1 \end{aligned} \tag{2.9}$$

is violated, where

$$\begin{aligned} H_1 &= \frac{1}{(\sigma_1^T)_{ult}} - \frac{1}{(\sigma_1^C)_{ult}}, \\ H_{11} &= \frac{1}{(\sigma_1^T)_{ult}(\sigma_1^C)_{ult}}, \\ H_2 &= \frac{1}{(\sigma_2^T)_{ult}} - \frac{1}{(\sigma_2^C)_{ult}}, \\ H_{22} &= \frac{1}{(\sigma_2^T)_{ult}(\sigma_2^C)_{ult}}, \\ H_{66} &= \frac{1}{(\tau_{12})_{ult}^2}, \\ H_{12} &= -\frac{1}{2}\sqrt{\frac{1}{(\sigma_1^T)_{ult}(\sigma_1^C)_{ult}(\sigma_2^T)_{ult}(\sigma_2^C)_{ult}}}. \end{aligned} \tag{2.10}$$

H_i is the strength tensors of the second order; H_{ij} is the strength tensors of the fourth order. σ_1 is the applied normal stress in direction 1; σ_2 is the applied normal stress in the direction 2; and τ_{12} is the applied in-plane shear stress.

2.2.2 Maximum Stress Failure Theory

Maximum stress(MS) failure theory consists of maximum normal stress theory proposed by Rankine and maximum shearing stress theory by Tresca. The stresses applied on a lamina can be resolved into the normal and shear stresses in the local axes. If any of the normal or shear stresses in the local axes of a lamina is equal or exceeds the corresponding ultimate strengths of the unidirectional lamina, the lamina is considered to be failed. That is

$\sigma_1 \geq (\sigma_1^T)_{ult}$ or $\sigma_1 \leq -(\sigma_1^C)_{ult}$,
$\sigma_2 \geq (\sigma_2^T)_{ult}$ or $\sigma_2 \leq -(\sigma_2^C)_{ult}$,
$\tau_{12} \geq (\tau_{12})_{ult}$ or $\tau_{12} \leq -(\tau_{12})_{ult}$.

where σ_1 and σ_2 are the normal stresses in the local axes 1 and 2, respectively; τ_{12} is the shear stress in the symmetry plane 1-2. The safety factor, or yield stress, is how much extra load beyond is intended a composite laminate will actually take, which is an indication of the material's load carrying capacity. If the value is less then 1.0, it means failure. The safey factor is defined as

$$SF = \frac{\text{Maximum Load Which Can Be Applied}}{\text{Load Applied}}. \tag{2.11}$$

The safety factor based on maximum stress theory is calculated by the following method: first, the principal stresses(σ_1^k,σ_2^k, and τ_{12}^k) are obtained by experiment; evaluate the safety factor along each direction according to equation 2.11; The minimum value among these safety factors are denoted as the safety factor of the lamina, SF_{MS}^k, it can be written as

$$SF_{MS}^k = \text{min of} \begin{cases} SF_X^k = \begin{cases} \frac{X_t}{\sigma_{11}}, & \text{if } \sigma_{11} > 0 \\ \frac{X_c}{\sigma_{11}}, & \text{if } \sigma_{11} < 0 \end{cases} \\ SF_Y^k = \begin{cases} \frac{Y_t}{\sigma_{22}}, & \text{if } \sigma_{22} > 0 \\ \frac{Y_c}{\sigma_{22}}, & \text{if } \sigma_{22} < 0 \end{cases} \\ SF_S^k = \begin{cases} \frac{S}{|\tau_{12}|} \end{cases} \end{cases}. \tag{2.12}$$

Assuming the composite laminate under a in-plane loading f, the corresponding stress on local stress in direction 1, local stress in direction 2, and shear stress for the kth lamina are $\sigma_1 SF_{TW}^k$, $\sigma_2 SF_{TW}^k$, and $\tau_{12} SF_{TW}^k$, respectively. Substitute them into the equation 2.9, the expression are given by

$a(SF_{TW}^k)^2 + b(SF_{TW}^k) - 1 = 0,$

where

$a = H_{11}(\sigma_1)^2 + H_{22}(\sigma_2)^2 + H_{66}(\tau_{12})^2 + 2H_{12}\sigma_1\sigma_2,$
$b = H_1\sigma_1 + H_2\sigma_2 + H_6\tau_{12}.$

Solve the above equation, the safety factor for the kth lamina is

$SF_{TW}^k = |\frac{-b+\sqrt{b^2+4a}}{2a}|.$

Then, the minimum of SF_{TW}^k is taken as the safety factor of the laminate which is written as

$SF_{TW} = \text{ min of } SF_{TW}^k \text{ for } k = 1, 2, \cdots, m-1, m\ .$

2.3 Summary

In this chapter, we review the CLT for composite material's analysis, then related failure theories are introduced to check whether a composite material would fail or not. In the following chapters, the CLT would be used to calculate the stress and strain under in-plane loading, it would also be used to generate the training data for stress and strain approximation based on neural network; in order to design a proper composite material, the failure theory are used to decide the feasibility of composite design.

Chapter 3

A New Genetic Algorithm Model for Constrained Problem

3.1 Genetic Algorithm Framework

GA is one of the most reliable stochastic algorithm, which has been widely used in discrete variables optimization problems [17, 18, 7, 20, 11, 8, 24, 2, 5, 16, 1, 13, 12, 19, 4, 15, 14, 21, 23, 6, 9]. In practice, fiber orientations are restricted to a finite set of angles, and ply thickness is a specific numeric value. Because the design variables are not continueous, a gradient based optimization procedure, such as gradient descent method, is not suitable to cope with such problems. Moreover, gradient based optimization approach is very eazy to get trapped in local minima, and many local optimum may exist in structural optimization problems. A stochastic algorithm, such as GA, is able to deal with optimization problems with discrete design variables. Besides, stochastic method could escape from local optimum, and obtain global optimum.

Although GA gains different advantages for solving discrete problems, many disadvantages exists within this method. First, the optimization process of GA parameters, such as the population size, parent population, mutation percentage, etc., is very tedious; Second, the GA needs to evaluate the objective function many times to acheive the optimization, and the computation cost is very high; the last problem within GA is the premature convergence. GA consists of five basic parts: the variable encoding method, selection scheme, crossover operator, mutation operator, and how the constraints are handled. A typical GA process is show in Figure 3.1.

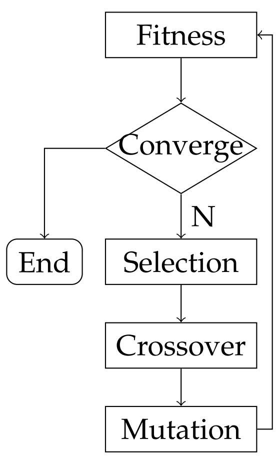

FIGURE 3.1: GA process

The first issue when implementing a GA is the representation of design variables, and an appropriate design representation is crucial to enhance the efficiency of GA. The canonical GA has always used binary strings to encode alternative solutions, however, some argued that the minimal cardinality, i.e., the binary representation, are not the best option. Real value string has been widely employed in

Selection scheme plays a critical role in balancing the dilemma of exploration and exploitation inherented in GA, and various selection methods, for example, roulette wheel, elitist, and tournament etc., have been proposed to overcome this issue. Both of roulette selection and tournament selection are well-studied and widely employed in the optimization design of laminated composite due to their simplicity to code and efficiency for both nonparallel and parallel architectures.

Crossover is another crucial operator introduced into the GA methodology framework, in which the alternative solution is generated from the mating pool. multiple types of crossover operator has been utilized in the optimization design of composite structures, such as: one-point, two-point, and uniform crossover. .

3.2 Constrained Problem Optimization

However, GA is originally proposed for unconstrained optimization problem, in order to deal with constrained design for composite laminate, some techniques are introduced into GA. The first method is using of data structure, special data structure has been developed to fulfils the corresponding constraint, for example, in order to fulfill the symmetry constraint of a laminate, the chromsome is consist of coding only half of the laminate and considering that each stack of the laminate is formed by two laminate with the same orientation but opposite signs [11, 10]. The second approach is reformulating the objective function. A penalty function is developed to convert a constrained problem into an unconstrained problem by adding penalty term to the objective funtion. Another method to solve constrained problem is introducing repair strategy by Todoroki and Haftka [22], which is aim to transform infeasible solutions to feasible solution by incorporating problem-specific knowledge.

3.3 A New Genetic Algorithm Model

3.3.1 Classification

The population is randomly generated, for every individual in this population the corresponding constraint numberic value can be obtained. As shown in Figure 3.2,The population can be divided into three groups according to constraint value, which are active group, potential group and proper group

3.3.2 Selection

The purpose of the selection operator is to chose mating pool to produce alternative solutions of better fitness. Traditional methods of selecting strategies only take the fitness of individuals into acount, however, due to the existance of constraint, various selection schemes are implemented to selecet the mating set. Based on different selection schemes, the parents of next generation can be divided into three groups: proper groups, active groups, and potential groups according to different selecting methods.

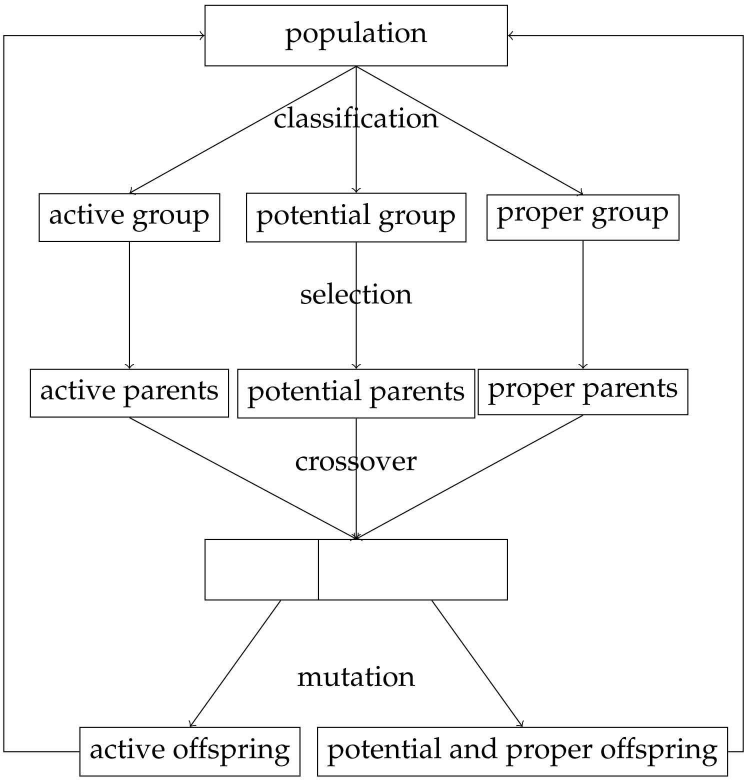

FIGURE 3.2: GA Model

Proper parents mean in which individual fullfils the constraints, which are chosen by the individual's fitnees, individuals with better fitness are more likely to be chosen if they fit the constraint; active groups means that individual is supposed to be always exist in the parents during the GA, which are selected by fitness, ignoring the constraint; The individuals from active group may not correspond to feasible solutions, but their existance enriches the variety of the gene clips. Potential groups means that they are likely to turn into proper individual after a couple of generations, and potential individuals are chosen by constraint function, the more the individual fulfils the constraint, the more possiblity it will be selected.

3.3.3 Crossover

The crossover operator happens among these three groups. the child of two proper groups are more likely to be a proper individual which can be used to obtain a alternative feasible solution. the child of an active individual and a potential individual can significantly change the gene of active individual's chromsome, which makes the individual evolve toward a new direction. The offspring of two active individuals are more likely to be an active individual, which can maitain the active group. The figure.4.2 (b) shows two children O_1

3.3.4 Mutation

i

3.4 Summary

In this chapter, frist, we review the application of tradional GA in the design of composite material; then A new GA framework has been come up with, in the following chapters, this NGAM will be adopted to directed the lay-up design of composite material.

Chapter 4

Laminated Composite Material Optimization by New Genetic Algorithm Model

4.1 Case1: Maximum Strength Optimization

TABLE 4.1: Comparison of the carbon/epoxy, graphite/epoxy, and glass/epoxy properties

Property	Symbol	Unit	Carbon/Epoxy	Graphite/Epoxy	Glass/Epoxy
Longitudinal elastic modulus	E_1	GPa	116.6	181	38.6
Traverse elastic modulus	E_2	GPa	7.67	10.3	8.27
Major Poisson's ratio	v_{12}		0.27	0.28	0.26
Shear modulus	G_{12}	GPa	4.17	7.17	4.14
Ultimate longitudinal tensile strength	$(\sigma_1^T)_{ult}$	MP	2062	1500	1062
Ultimate longitudinal compressive strength	$(\sigma_1^C)_{ult}$	MP	1701	1500	610
Ultimate transverse tensile strength	$(\sigma_2^T)_{ult}$	MPa	70	40	31
Ultimate transverse compressive strength	$(\sigma_2^C)_{ult}$	MPa	240	246	118
Ultimate in-plane shear strength	$(\tau_{12})_{ult}$	MPa	105	68	72
Density	ρ	g/cm^3	1.605	1.590	1.903
Cost			8	2.5	1

TABLE 4.2: The optimum lay-ups for the loading $N_x = 1e6$ N

Cross Ply $[0_M/90_N]$	Previous Research		Current Research	
Material	Glass-Epoxy	Graphite-Epoxy	Glass-Epoxy	Graphite-Epoxy
M	68	17	78	18
N	72	18	28	8
no. of lamina(n)	140	35	106	26
SR	2.01	2.10	2.03	2.16
weight	9.10	1.84	6.89	102.5

0
90
90
0
90

FIGURE 4.1: Model for cross ply laminate

4.1.1 Methodology

4.1.2 Result and Discussion

4.2 Case2: Minimum Thickness Optimization

In this case, the mentioned NGAM in chapter three is taken to minimize thickness(or weight) of mid-plane symmetric composite laminate subject to in-plane loading. Fiber orientation and ply thickness are chosen as design variables, the proposed Tsai-wu and the maximum stress criteria are employed to determine whether load bearing capacity is exceeded or not.

4.2.1 Methodology

The optimization problem can be formulated as searching the optimal stacking sequence of composite laminate. There are two design variables in this problem, the angles in the laminate, and the number of layers that each fiber orientation has. The final objective function is given as

$$\begin{aligned} F &= 2t_0 \sum_{k=1}^{n} n_k, \\ SF_{MS} &\geq 1, \\ SF_{TW} &\geq 1. \end{aligned} \tag{4.1}$$

The first term represents the total thickness of the composite laminates, t_0 is the ply thickness; n_k is the number of plies in the kth lamina, in which the fiber orientation is θ_k. The constraints here are two safety factors should not less than 1, which means $SF_{MS} \geq 1$, and $SF_{TW} \geq 1$, respectively.

4.2.2 Encoding

Due to the simplicity and efficiency of float representation, this encoding method is implemented to represent a possible solution. As shown in Figure 4.2 (a), these two chromsomes represent a $[+8_7/-9_2]_s$ carbon T300/5308 laminated composite, and $[+19_4/-36_6]_s$, respectively. Becasue the laminate adopted in this paper is symmetric to its mid-plane, so only half needs to be encoded.

4.2.3 Selection

The purpose of the selection operator is to chose mating pool to produce alternative solutions of better fitness. Traditional methods of selecting strategies only take the fitness of individuals into acount, however, due to the existance of constraint, various selection schemes are implemented to selecet the mating set. Based on different selection schemes, the parents of next generation can be divided into three groups: proper groups, active groups, and potential groups according to different selecting methods.

Proper parents mean in which individual fullfils the constraints, which are chosen by the individual's fitnees, individuals with better fitness are more likely to be chosen if they fit the constraint; active groups means that individual is supposed to be always exist in the parents during the GA, which are selected by fitness, ignoring the constraint; The individuals from active group may not correspond to feasible

P_1:	+7	+7	+7	+7	+7	+7	+7	+7	-9	-9
P_2:	+19	+19	+19	+19	-36	-36	-36	-36	-36	-36

(a): Parents P_1 and P_2

O_1:	+13	+13	+13	+13	+13	+13	-27	-27	-27	-27
O_2:	+22	+22	+22	+22	+22	+22	+22	+5	+5	+5

(b): Offspring O_1 and O_2

O_1:	+13	+13	+13	$\cdots$	+13	+13	-27	$\cdots$	-27	-27
	11						7			

(c): Offspring O_1 after lenght mutation

+12	+12	+12	$\cdots$	+12	+12	-26	$\cdots$	-26	-26

(b): Offspring O_1 after angle mutation

FIGURE 4.2: GA Operators

solutions, but their existance enriches the variety of the gene clips. Potential groups means that they are likely to turn into proper individual after a couple of generations, and potential individuals are chosen by constraint function, the more the individual fulfils the constraint, the more possiblity it will be selected.

4.2.4 Crossover

The crossover operator happens among these three groups. the child of two proper groups are more likely to be a proper individual which can be used to obtain a alternative feasible solution. the child of an active individual and a potential individual can significantly change the gene of active individual's chromsome, which makes the individual evolve toward a new direction. The offspring of two active individuals are more likely to be an active individual, which can maitain the active group. The figure.4.2 (b) shows two children O_1 and O_2 from two parents P_1 and P_2, each angle C_a and it's length C_l of a child are obtained by the following formula

$$\begin{cases} C_a & = (P1_a + P2_a)/2 \\ C_l & = (P1_l + P2_l)/2 \end{cases}. \tag{4.2}$$

4.2.5 Mutation

A mutation direction is imposed on the mutation operator which to make sure the individual evolving toward the right direction. The mutation direction, denoted by md, is a n dimensional vector corresonding to the number of constraints, it is decided by the constraint thresholds CT_i and the current individual's constraint value, denoted as CV_i, The mutation vector can be obtained by the following formula

$$\text{md} = [CT_1, \cdots, CT_{n-1}, CT_n] - [CV_0, \cdots, CV_{n-1}, CV_n]$$

During the operator, the mutation procedure is consist of two phases: the length mutation of the chromsome, and the angle mutation of the chromsome. Becasue the chromsome's length is positive correlated with the individual's fitness, the coefficient of length mutation denoted by C_l, if $\sum_{i=1}^{N} CT_i$ great than $\sum_{i=1}^{N} CV_i$, the mutation length is restricted to the range $[0, (C_l \sum_{i=1}^{N} (CT_i - CV_i))/N]$, which means increase the chromsome's length;Assuming a $[+13_6/-27_4]_s$ T300/5308 carbon/epoxy composite laminate under the loading $N_x = N_y = 10$ MPa m, it's property as shown in table **??**. According to CLT and failure theory, the two safety factors SF_{MS} and SF_{TW} are 0.0539, and 0.0540, respectively. So the mutation vector and is $[0.9461, 0.9460]$, assuming the length mutation coefficient is 20, so the mutation range is from 0 to 18. A random number is generated from the range $[0, 18]$, supposing the outcome is 13, then a length generator is used to a list, the it's sum is 13, suppose the list is [5, 8], the laminate after mutation is $[13_{11}/-27_{12}]_s$.

If the $\sum_{i=1}^{N} CT_i$ less than $\sum_{i=1}^{N} CV_i$, the mutation length is restricted to the range $([\sum_{i=1}^{N} CT_i - CV_i)/N, 0]$, which means the individual's fitness exceeds the threshold value, and decrease the chromsome's length. Assuming a $[+33_{35}/-29_{26}]_s$ T300/5308 laminate is under loading $N_x = 10$ MPa, and $N_y = 5$ MPa, then, it's SF_{MS} constraint and SF_{TW} values are 1.0912, 1.0747, respectively. because the length mutation is 20, so the mutation range is from -2 to 0. This would decrease the chromsome's length.

$$\text{LM} = \begin{cases} [0, (C_l \sum_{i=1}^{N} (CT_i - CV_i))/N], \text{ if } \sum_{i=1}^{N} CT_i > \sum_{i=1}^{N} CV_i \\ [(C_l \sum_{i=1}^{N} (CT_i - CV_i))/N, 0], \text{ if } \sum_{i=1}^{N} CT_i < \sum_{i=1}^{N} CV_i \end{cases} \tag{4.3}$$

TABLE 4.3: The optimum lay-ups using two distinct fiber angles under various biaxial loading cases

Loading $N_x/N_y/N_{xy}$ (MPa m)	Optimum lay-up sequences	Laminate thickness	Safety factor for Tsai-wu	Safety factor for maximum stress
10/5/0	$[33_{29}/\text{-}39_{25}/\text{-}\bar{3}9]_s$	109	1.0074	1.0246
20/5/0	$[33_{22}/\text{-}31_{24}]_s$	92	1.0055	1.2065
40/5/0	$[29_{18}/\text{-}21_{23}/\text{-}\bar{2}1]_s$	83	1.0034	1.7350
80/5/0	$[\text{-}20_{27}/21_{25}/2\bar{5}]_s$	105	1.0029	1.2063
120/5/0	$[\text{-}18_{34}/17_{36}]_s$	140	1.0000	1.0898

The relationship between the angles in the composite laminate and the chromsome's fitness is unclear, so the mutation direction of chromsome's angle is random. The coefficient angle mutation is C_a, the angle mutation range is $[0, C_a \sum_{i=1}^{N} (|CT_i - CV_i|)]$ or $[C_a \sum_{i=1}^{N} (-|CT_i - CV_i|), 0]$. It is can be written as

$$\text{P(AM)} = \begin{cases} 0.5, \ \text{AM} = [0, C_a \sum_{i=1}^{N} (|CT_i - CV_i|)] \\ 0.5, \ \text{AM} = [C_a \sum_{i=1}^{N} (-|CT_i - CV_i|), 0] \end{cases} . \tag{4.4}$$

4.2.6 Result and Discussion

In present study, the T300/5308 graphite/epoxy material is used in the lay-up sequence optimization, and it's properties as shown in table.**??**. Two constraints are imposed on the composite laminates which are the safety factor SF_{MS} , and safety factor SF_{TW}, and the threshold values for both of them is 1. The constraint values of an individual are CV_1 and CV_2. So the mutation vector here is a two dimensional vector $[1 - CV_1, 1 - CV_2]$, and the coefficient of length mutation C_l and angle mutation C_a are 20 and 10 ,respectively.

To verify the reliablity of proposed method, two conditions are concerned: the first is only two distinct fiber orientation angles in the composite material; the second involves three distinct ply angles within the optimization process. In each situation, first, we present the search process by plotting revelvant indicators, such as the fitness, strength ratio, and angle. Then, the optimum lay-ups under various loading cases is discussed.

Figure 4.3 (a) shows how the optimal individual's fitness and strength ratio evolves during the GA process. The solid curve shows the fitness value, the dashed curve shows the Tsai-wu safety factor, and the dotted curve shows MS safety factor. If the smaller strength ratio fullfils the constraint, this laminate must satisfy all the constraints, for simplicity, only the smaller strength ratio is presented in the figure4.4 (a). The method to chose optimal individual considering two following situations, if no individual in the current population meets constraint, the one with biggest fitness is selected as the optimal individual; if there are one or multiple individuals fullfils requirement, the one with smallest fitness is chosen which means the smallest one has the biggest priority. Figure 4.3 (b) and 4.4 (b) show how the every fiber orientation changes , and Figure 4.3 (c) and 4.4 display how the number of each angles varies.

At the beginning of this GA process, the fitness curves increases very quickly, becasue individual's two strength ratios are very small, so the difference between the individual's fitness and the imposed constraint threshold is a big positive number, so the range of mutaion length is from 0 to $C_l(CT_0 - CV_0 + CT_1 - CV_1)/2$. The length of individual increases by n, which is random number between 0 and

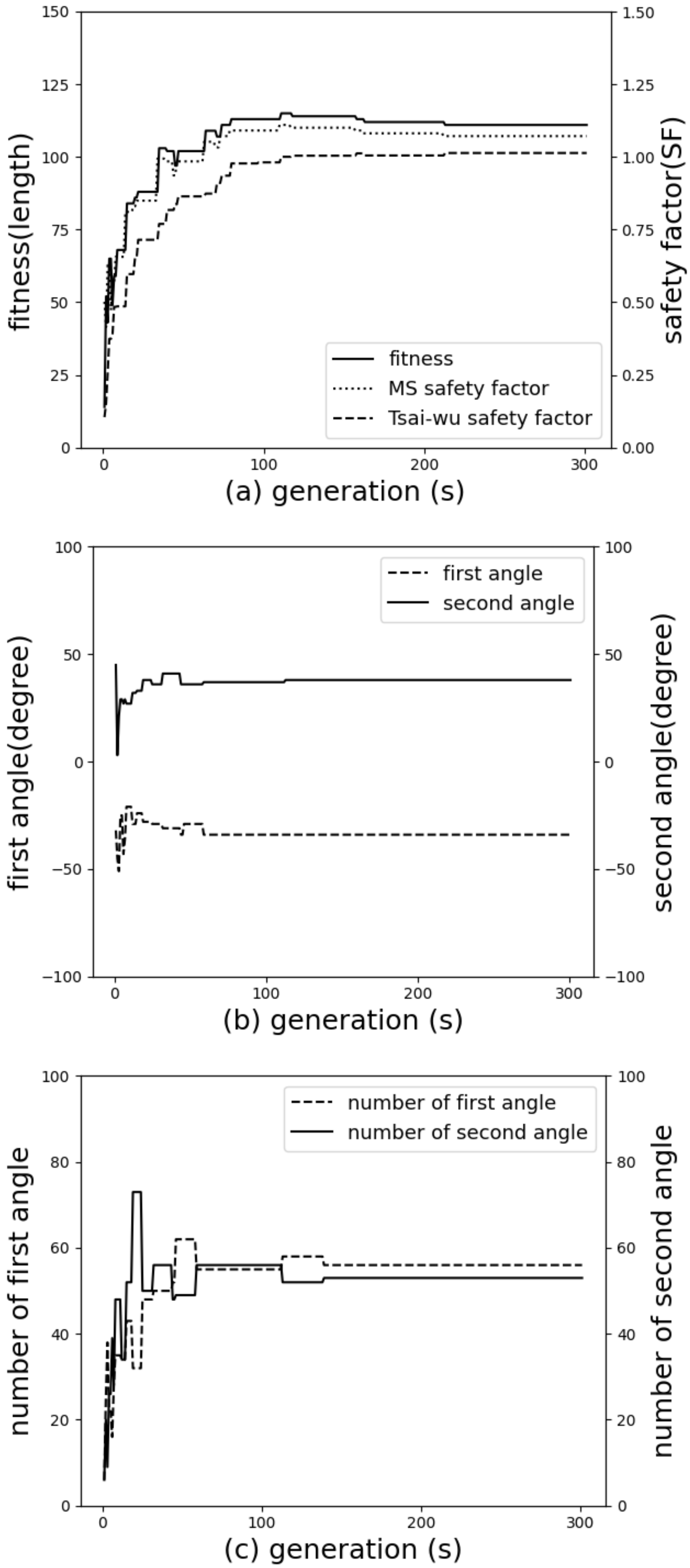

FIGURE 4.3: Two distinct angles

TABLE 4.4: The optimum lay-ups using three distinct fiber angles under various biaxial loading cases

Loading $N_x/N_y/N_{xy}$ (MPa m)	Optimum lay-up sequences	Laminate thickness	Safety factor for Tsai-wu	Safety factor for maximum stress
10/5/0	$[37_{27}/\text{-}38_{27}/\text{-}5]_s$	110	1.0023	1.0216
20/5/0	$[34_{24}/\text{-}32_{14}/\text{-}28_{11}]_s$	98	1.0237	1.2089
40/5/0	$[21_{28}/\text{-}32_{19}/2_3]_s$	100	1.0617	1.7076
80/5/0	$[\text{-}19_{24}/20_{27}/\text{-}17_{16}/\text{-}\bar{1}7]_s$	109	1.0056	1.2093
120/5/0	$[\text{-}19_{33}/12_{13}/16_{28}]_s$	148	1.0105	1.1014

TABLE 4.5: Comparison with the results of DSA

Loading	Akbulut and Sonmez's[3] Study				Present Study			
$N_x/N_y/N_{xy}$ (MPa m)	Optimum lay-up sequences	laminate thickness	TW	MS	Optimum lay-up sequences	laminate thickness	TW	MS
10/5/0	$[37_{27}/\text{-}37_{27}]_s$	108	1.0068	1.0277	$[33_{29}/\text{-}39_{25}/\text{-}\bar{3}9]_s$	109	1.0074	1.0246
20/5/0	$[31_{23}/\text{-}31_{23}]_s$	92	1.0208	1.1985	$[33_{22}/\text{-}31_{24}]_s$	92	1.0055	1.2065
40/5/0	$[26_{20}/\text{-}26_{20}]_s$	80	1.0190	1.5381	$[29_{18}/\text{-}21_{23}/\text{-}\bar{2}1]_s$	83	1.0034	1.7350
80/5/0	$[21_{25}/\text{-}19_{28}]_s$	106	1.0113	1.2213	$[\text{-}20_{27}/21_{25}/\bar{2}5]_s$	105	1.0029	1.2063
120/5/0	$[17_{35}/\text{-}17_{35}]_s$	140	1.0030	1.0950	$[\text{-}18_{34}/17_{36}]_s$	140	1.0000	1.0898

$C_l(CT_0 - CV_0 + CT_1 - CV_1)/2$. As can be seen from Figure 4.3 (a), both of optimal individual's fitness and strength ratio increases very quickly. The range of angle mutation is from 0 to $C_a(CT_0 - CV_0 + CT_1 - CV_1)/2$, and the number of each angle also changes violently. The Figure.4.3 (a) and 4.4 (a) show this property at the initial stage. During this stage, increasing individual's length playing a major role in increasing individual's fitness.

After a couple of generations, the optimal individual's fitness get bigger, and the difference between individual's fitness and constraint threshold get smaller. The range of mutation length turn smaller. At this stage, simply increase the individual's length doesn't make much difference in improve individual's fitnees, and a better composite laminates lay-up can dramaticly change the optimal individual's fitness. That's why the fitness curve oscillated violently in this stage. At the same time, the strength ratio curve kept growing smoothly. But the growing speed got more smaller.

When GA comes to its last phase, GA finds individuals that meet all the constraints. Now the optimal individual's fitness is greater than the safety factor. The range of mutation length is from $C_l(CT_0 - CV_0 + CT_1 - CV_1)/2$ to 0. It means individuals need to decrease it's length and improve its internal structure to meet the constraint. That's why the fitness of optimal individual kept decreasing, however, the strength ratio curve still is greater then safety factor.

Table.4.5 shows the comparison with the result obtained by direct search simulated annealing(DSA) algorithm which is proposed by Akbulut and Sonmez[3]. Both of variant GA and DSA are able to find feasible solution, but when loading is $N_x = 80$, $N_y = 5$ MPa m, variant GA gets a better solution than DSA. In the case that loadings are $N_x = 20$, $N_y = 5$ MPa m, and $N_x = 120$, $N_y = 5$ MPa m, the proposed GA offers an alternative. Compared with DSA method, the last advantage of variant GA is the number of layers doesn't have to be an even number.

4.3 Summary

In this paper, we reviewed the use of variant GA for the optimal design of composite laminated material under in-plane loading based on Tsai-wu and maximum stress

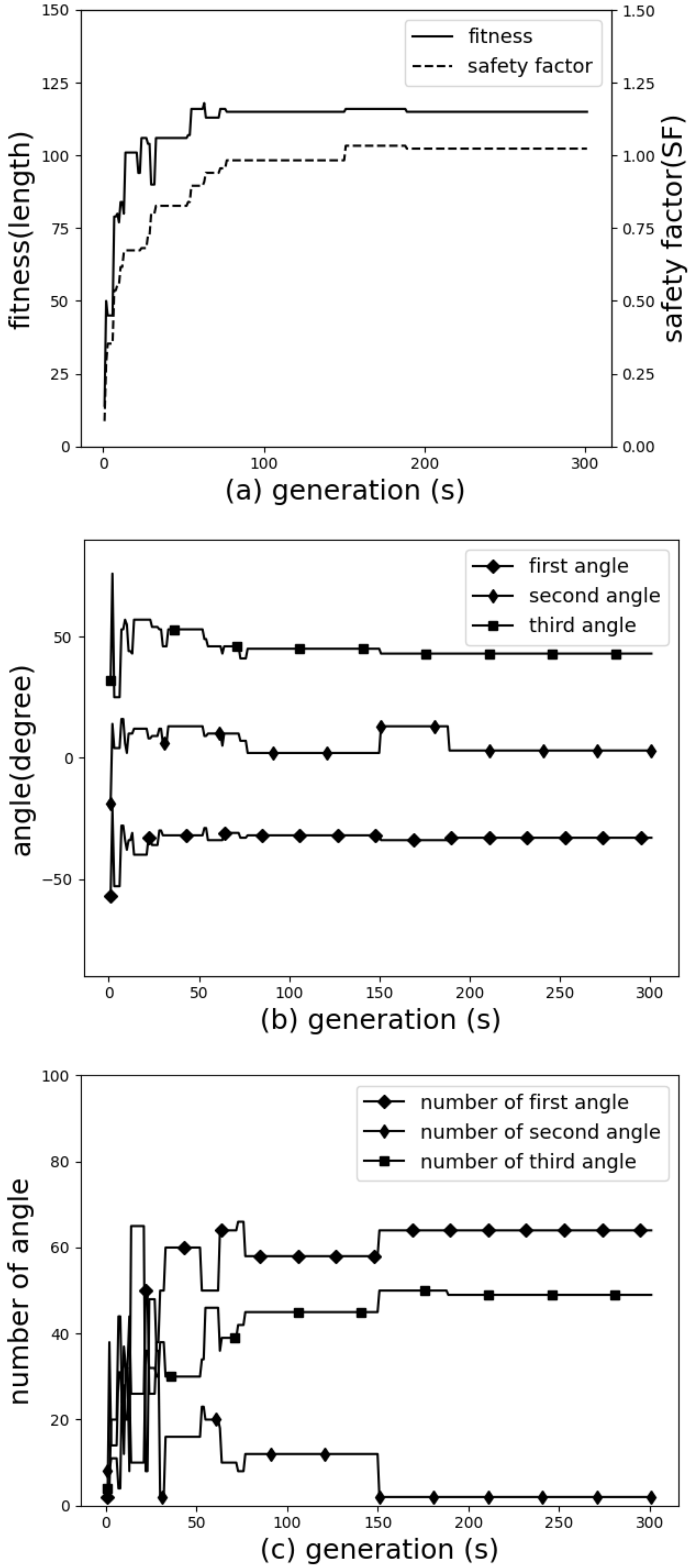

FIGURE 4.4: three distinct angles

failure criteria. GA is proposed to search the optimal lay-up for laminated composite under different loading cases. Two situations are considered under the same loading, a set of two distinct angles, and three distinct angles.

By setting the constant values of length mutaion coefficient and angle mutation coefficient at the beginning, the convergence speed of search process can be controlled in an explicit way; During the optimization process, GA can adjust its length mutation range and angle mutation range based on the difference between individual's constraint values and constraint thresholds.

Finally, comparison of previous research and current result are presented. In some cases, the proposed GA in this paper is better off than DSA method. However, there is still many works to study within this GA, such as the fine-tuning of parameters taken in this GA.

Chapter 5

Approximation of CLT Based on Artificial Neural Network

5.1 Neural Network Design

Artificial neural networks(ANN) which heavily inspired by biology and psychology have been widely used to solve various practical engineering problems in such areas as pattern recognition, nonlinear regression, data mining, clustering and prediction. Methods based on complicated mathematical models is of intensive computation, approximation function evaluation techniques can be employed to accelerate the calculation process and reduce the computation cost. In order to solve practical engineering problems in composite material application, classic laminational theory(CLT) has been proposed which involves many matrix multiplication and integration calculation. ANN which has been proved is a reliable tool instead of complicate mathematical model. The design of neural network consists of three basic parts: neural network architecture, learning rules, and training techniques.

The weight training in ANN is to minimize the error function, such as the most widely used mean square error which calculate the difference between the desired and the prediction output values averaged over all examples. Backpropation algorithm has been successful applied to in many areas, and it's based on gradient descent. However, this class of algorithms are plagued by the possible existance of local minima or "flat spots" and "the curse of dimensionality". One method to overcome this problem is to adopt EANN's

5.1.1 Architecture

The inputs of the neural network is consist of four parts: in-plane loading N_x, N_y, and N_{xy}, design parameters of laminate, two distinct fiber orientation angle θ_1 and θ_2, ply thickness t, total number of plies N; five engineering constants of composite materials, E_1, E_2, ; five strength parameters of a unidirectional lamina. There are two outputs in the neural network, safety factors for MS theory and Tsai-Wu theory, respectively.

5.2 Approximation Framework

5.2.1 Data Preparation

It's implausible to obatain the training data from practial experiments, so CLT is taken to generate the training data and evaluation data. Three composite materials are used, as shown in Table 4.1. The fiber orientation range is from -90 to 90, the

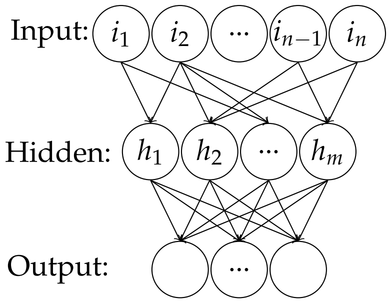

FIGURE 5.1: Neural Network Model

$+\theta$
$-\theta$
$\cdots$
$-\theta$
$+\theta$

FIGURE 5.2: Model for Angle ply laminate

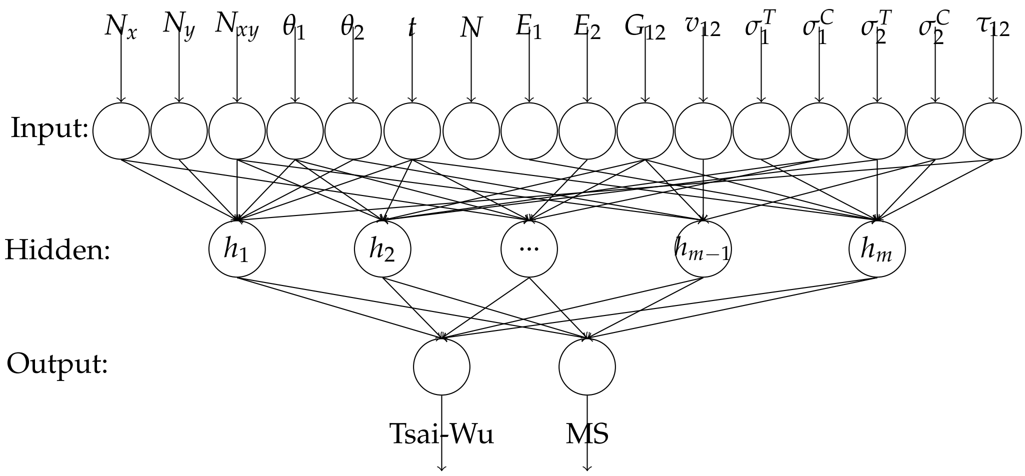

FIGURE 5.3: Neural Network Model

Input				**Output**	
Load	Laminate Structure	Material Property	Failure Property	MS	Tsai-Wu
-70,-10,-40,	90,-90,4,1.27,	38.6,8.27,0.26,4.14,	1062.0,610.0,31,118,72,	0.0102,	0.0086
-10,10,0,	-86,86,80,1.27,	181.0,10.3,0.28,7.17,	1500.0,1500.0,40,246,68,	0.4026,	2.5120
-70,-50,80,	-38,38,4,1.27,	116.6,7.67,0.27,4.173,	2062.0,1701.0,70,240,105,	0.0080,	0.0325
-70,80,-40,	90,-90,48,1.27,	38.6,8.27,0.26,4.14,	1062.0,610.0,31,118,72,	0.0218,	0.1028
-20,-30,0,	-86,86,60,1.27,	181.0,10.3,0.28,7.17,	1500.0,1500.0,40,246,68,	0.6481,	0.9512
0,-40,0,	74,-74,168,1.27,	181.0,10.3,0.28,7.17,	1500.0,1500.0,40,246,68,	1.3110,	3.9619

TABLE 5.1: Comparsion between practical and simulation

Input				Output			
Load	Laminate Structure	Material Property	Failure Property	CLT MS	CLT Tsai-Wu	ANN MS	ANN Tsai-Wu
120,5,0	10,-10,8,1.27	38.6,8.27,0.26,4.14	1062.0,610.0,31,118,72	0.068	0.062	0	0
120,5,0	10,-10,2,1.27	38.6,8.27,0.26,4.14	1062.0,610.0,31,118,72	1.69	2.18	0	0
...	...	...	...	...	...	0	0
120,5,0	10,-10,14,1.27	38.6,8.27,0.26,4.14	1062.0,610.0,31,118,72	1.70	1.56	0	0
120,5,0	10,-10,8,1.27	181,10.3,0.28,7.17	1500.0,1500.0,40,246,68	0.072	0.024	0	0

number of layers range is from 5 to 200. Assuming the composite material only subjects to in-plane loading, each component range is from -90 to 90;

we sample this function to yield 140000 points distributed uniformly over aboved mentioned ranges as the training data.

5.2.2 Training

5.2.3 Evaluation

5.2.4 Result and Discussion

5.3 Summary

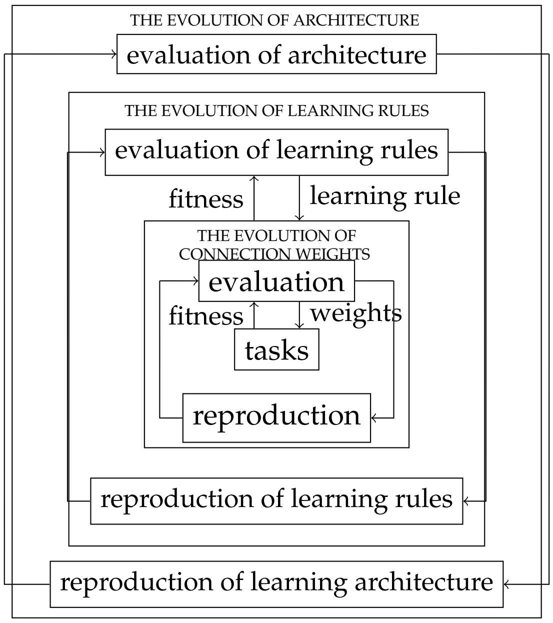

FIGURE 5.4: Genetic algorithm and artificial neural network

Chapter 6

Conclusion

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