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A technique for constrained optimization of cross-ply laminates using a new variant of genetic algorithm

Huiyao Zhang[1] and Atsushi Yokoyama[2]

Abstract

The main challenge presented by the laminate composite design is the laminate layup, involving a set of fiber orientations, composite material systems, and stacking sequences. In nature, it is a combinatorial optimization problem that can be solved by the genetic algorithm (GA). In this present study, a new variant of the GA is proposed for the optimal design by modifying the selection strategy. To check the feasibility of a laminate subject to in-plane loading, the effect of the fiber orientation angles and material components on the first ply failure is studied. Then we compare the experimental results with works in other literature.

Keywords

Laminated composite, Classical lamination theory, Genetic algorithm, Optimal design

Introduction

Composite materials offer improved strength, stiffness, fatigue, corrosion resistance, etc. over conventional materials, and are widely used as materials for applications ranging from the automotive to shipbuilding industry, electronic packaging to golf clubs, and medical equipment to homebuilding. However, the high cost of fabrication of composites is a critical drawback to its application. For example, the graphite/epoxy composite part may cost as much as 650 to 900 per kilogram. In contrast, the price of glass/epoxy is about 2.5 times less. Manufacturing techniques such as sheet molding compounds and structural reinforcement injection molding are used to lower the costs for manufacturing automobile parts. An alternative approach is using hybrid composite materials.

The mechanical performance of a laminate composite is affected by a wide range of factors such as the thickness, material, and orientation of each lamina. Because of manufacturing limitations, all these variables are usually limited to a small set of discrete values. For example, the ply thickness is fixed, and ply orientation angles are limited to a set of angles such as 0, 45, and 90 degrees in practice. So the search process for the optimal design is a discrete optimization problem that can be solved by the GA. To tailor a laminate composite, the GA has been successfully applied to solve laminate design problems [1–11]. The GA simulates the process of natural evolution, including selection, crossover, and mutation according to Darwin's principle of "survival of the fittest". The known advantages of GAs are the following: (i): GAs are not easily trapped in local optima and can obtain the global optimum. (ii): GAs do not need gradient information and can be applied to discrete optimization problems. (iii): GAs can not only find the optimal value in the domain but also maintain a set of optimal solutions. However, the GA also has some disadvantages, for example, the GA needs to evaluate the target functions many times to achieve the optimization, and the cost of the search process is high. The GA consists of some basic parts, the coding of the design variable, the selection strategy, the crossover operator, the mutation operator, and how to deal with constraints. For the variable design part, there are two methods to deal with the representation of design variables, namely, binary string and real value representation [1,4]. Michalewicz [12] claimed that the performance of floating-point representation was better than binary representation in the numerical optimization problem. Selection strategy plays a critical role in the GA, which determines the convergence speed and the diversity of the population. To improve search ability and reduce search costs, various selection methods have been invented, and they can be divided into four classes: proportionate reproduction, ranking, tournament, and genitor(or "steady state") selection. In the optimization of laminate composite design, the roulette wheel [1,13], where the possibility of an individual to be chosen for the next generation is proportional to the fitness. Soremekun et al. [14] showed that the generalized elitist strategy outperformed a single individual elitism in some special cases.

Data structure, repair strategies, and penalty functions [15] are the most commonly used approaches to resolve constrained problems in the optimization of composite structures. Symmetric laminates are widely used in practical scenarios, and data structures can be used to fulfill symmetry constraints, which consists of coding half of the laminate and considering the rest with the opposite orientation. Todoroki [4] introduced a repair strategy that can scan the chromosome

[1]Department of Fiber Science and Engineering, Kyoto Institute of Technology
[2]Department of Fiber Science and Engineering, Kyoto Institute of Technology

Corresponding author:
Atsushi Yokoyama, Department of Fiber Science and Engineering, Kyoto Institute of Technology, Matsugasaki, Sakyo-ku, Kyoto, 606-8585,JAPAN
Email: yokoyama@kit.ac.jp

and repair the gene on the chromosome if it does not satisfy the contiguity constraint. The comparison of repair strategies in a permutation GA with the same orientation was presented by Liu et al.[5], and it showed that the Baldwinian repair strategy can substantially reduce the cost of constrained optimization. Haftka and Todoroki[1] used the GA to solve the laminate stacking sequence problem using a penalty function subject to buckling and strength constraints.

In typical engineering applications, composite materials are under very complicated loading conditions, not only inplane loading but also out-of-plane loading. Most of the studies on the optimization of the laminate composite material minimized the thickness[7,16], weight[17–19], and cost and weight[18,20], or maximized the static strength of the composite laminates for a targeted thickness[7,8,21,22]. In the present study, the cost and weight of laminates are minimized by modifying the objective function.

To check the feasibility of a laminate composite by imposing a strength constraint, failure analysis of a laminate is performed by applying suitable failure criteria. The failure criteria of laminated composites can be classified into three classes: non-interactive theories (e.g., maximum strain), interactive theories (e.g., Tsai-wu), and partially interactive theories (e.g., Puck failure criterion). Previous researchers adopted the first-ply-failure approach using Tsai-wu failure theory[17,20,23–28], Tsai-Hill[29,30], the maximum stress[31], or the maximum strain[31] static failure criteria. Akbulut[11] used the GA to minimize the thickness of composite laminates with Tsai-Hill and maximum stress failure criteria, and the advantage of this method is it avoids spurious optima. Naik et al.[32] minimized the weight of laminated composites under restrictions with a failure mechanism-based criterion based on the maximum strain and Tsai-wu. In the present study, Tsai-wu Static failure criteria are used to investigate the feasibility of a laminate composite.

Stress and Strain in a Laminate

A laminate structure consists of multiple lamina bonded together through their thickness. Considering a laminate composite plate that is subject to in-plane loading of extension, shear, bending and torsion, the classical lamination theory (CLT) is taken to calculate the stress and strain in the local and global axes of each ply, as shown in figure 1. Based on fiber orientation, material, and fiber thickness, there are a few special cases of laminate: the set of fiber angles in figure 2 only includes 0 and 90, which is called cross-ply laminate.

Stress and Strain in a Lamina

For a single lamina, the stress-strain relation in local axis 1-2 is:

$$\begin{bmatrix}\sigma_1\\ \sigma_2\\ \tau_{12}\end{bmatrix} = \begin{bmatrix}Q_{11} & Q_{12} & 0\\ Q_{12} & Q_{22} & 0\\ 0 & 0 & Q_{66}\end{bmatrix}\begin{bmatrix}\varepsilon_1\\ \varepsilon_2\\ \gamma_{12}\end{bmatrix}, \tag{1}$$

where Q_{ij} are the stiffnesses of the lamina that are related to engineering elastic constants given by

$$\begin{aligned} Q_{11} &= \frac{E_1}{1-v_{12}v_{21}},\\ Q_{22} &= \frac{E_2}{1-v_{12}v_{21}},\\ Q_{66} &= G_{12},\\ Q_{12} &= \frac{v_{21}E_2}{1-v_{12}v_{21}}, \end{aligned} \tag{2}$$

where E_1, E_2, v_{12}, G_{12} are four independent engineering elastic constants, which are defined as follows: E_1 is the longitudinal Young's modulus, E_2 is the transverse Young's modulus, v_{12} is the major Poisson's ratio, and G_{12} is the in-plane shear modulus.

Stress strain relation in the global x-y axis:

$$\begin{bmatrix}\sigma_x\\ \sigma_y\\ \tau_{xy}\end{bmatrix} = \begin{bmatrix}\bar{Q}_{11} & \bar{Q}_{12} & \bar{Q}_{16}\\ \bar{Q}_{12} & \bar{Q}_{22} & \bar{Q}_{26}\\ \bar{Q}_{16} & \bar{Q}_{26} & \bar{Q}_{66}\end{bmatrix}\begin{bmatrix}\varepsilon_x\\ \varepsilon_y\\ \gamma_{xy}\end{bmatrix}, \tag{3}$$

where

$$\begin{aligned} \bar{Q}_{11} &= Q_{11}c^4 + Q_{22}s^4 + 2\left(Q_{12}+2Q_{66}\right)s^2c^2,\\ \bar{Q}_{12} &= \left(Q_{11}+Q_{22}-4Q_{66}\right)s^2c^2 + Q_{12}\left(c^4+s^2\right),\\ \bar{Q}_{22} &= Q_{11}s^4 + Q_{22}c^4 + 2\left(Q_{12}+2Q_{66}\right)s^2c^2,\\ \bar{Q}_{16} &= \left(Q_{11}-Q_{12}-2Q_{66}\right)c^3s - \left(Q_{22}-Q_{12}-2Q_{66}\right)s^3c,\\ \bar{Q}_{26} &= \left(Q_{11}-Q_{12}-2Q_{66}\right)cs^3 - \left(Q_{22}-Q_{12}-2Q_{66}\right)c^3s,\\ \bar{Q}_{66} &= \left(Q_{11}+Q_{22}-2Q_{12}-2Q_{66}\right)s^2c^2 + Q_{66}\left(s^4+c^4\right). \end{aligned} \tag{4}$$

The c and s denote $cos\theta$ and $sin\theta$, respectively.

The local and global stresses in an angle lamina are related to each other through the angle of the lamina θ

$$\begin{bmatrix}\sigma_1\\ \sigma_2\\ \tau_{12}\end{bmatrix} = [T]\begin{bmatrix}\sigma_x\\ \sigma_y\\ \tau_{xy}\end{bmatrix} \tag{5}$$

where

$$[T] = \begin{bmatrix}c^2 & s^2 & 2sc\\ s^2 & c^2 & -2sc\\ -sc & sc & c^2-s^2\end{bmatrix}. \tag{6}$$

Stress and Strain in a Laminate

$$\begin{aligned} \begin{bmatrix}N_x\\ N_y\\ N_{xy}\end{bmatrix} &= \begin{bmatrix}A_{11} & A_{12} & A_{16}\\ A_{12} & A_{22} & A_{26}\\ A_{16} & A_{26} & A_{66}\end{bmatrix}\begin{bmatrix}\varepsilon_x^0\\ \varepsilon_y^0\\ \gamma_{xy}^0\end{bmatrix} + \begin{bmatrix}B_{11} & B_{12} & B_{16}\\ B_{11} & B_{12} & B_{16}\\ B_{16} & B_{26} & B_{66}\end{bmatrix}\begin{bmatrix}k_x\\ k_y\\ k_{xy}\end{bmatrix}\\ \begin{bmatrix}M_x\\ M_y\\ M_{xy}\end{bmatrix} &= \begin{bmatrix}B_{11} & B_{12} & B_{16}\\ B_{12} & B_{22} & B_{26}\\ B_{16} & B_{26} & B_{66}\end{bmatrix}\begin{bmatrix}\varepsilon_x^0\\ \varepsilon_y^0\\ \gamma_{xy}^0\end{bmatrix} + \begin{bmatrix}D_{11} & D_{12} & D_{16}\\ D_{11} & D_{12} & D_{16}\\ D_{16} & D_{26} & D_{66}\end{bmatrix}\begin{bmatrix}k_x\\ k_y\\ k_{xy}\end{bmatrix} \end{aligned} \tag{7}$$

N_x, N_y - normal force per unit length
N_{xy} - shear force per unit length
M_x, M_y - bending moment per unit length
M_{xy} - twisting moments per unit length

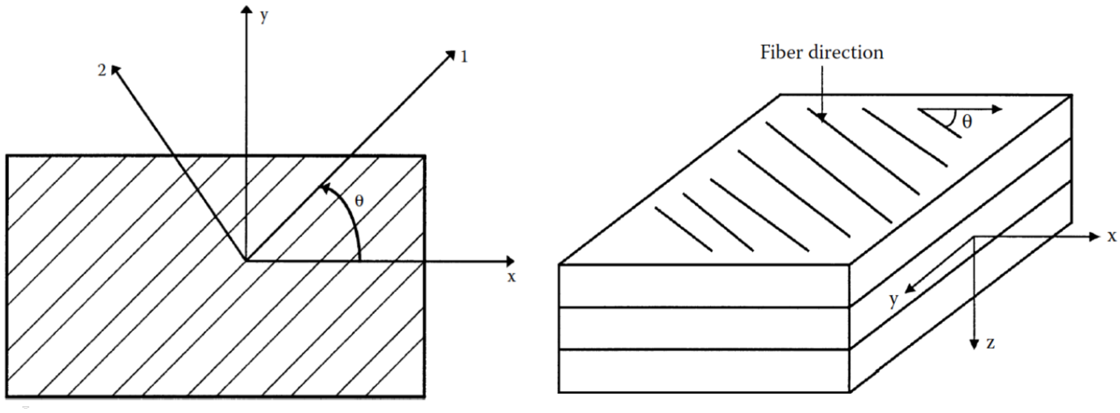

Figure 1. Local and global axes of an angle lamina.

Table 1. Comparison of the graphite/epoxy and glass/epoxy properties.

Property	Symbol	Unit	Graphite/Epoxy	Glass/Epoxy
Longitudinal elastic modulus	E_1	GPa	181	38.6
Traverse elastic modulus	E_2	GPa	10.3	8.27
Major Poisson's ratio	v_{12}		0.28	0.26
Shear modulus	G_{12}	GPa	7.17	4.14
Ultimate longitudinal tensile strength	$(\sigma_1^T)_{ult}$	MP	1500	1062
Ultimate longitudinal compressive strength	$(\sigma_1^C)_{ult}$	MP	1500	610
Ultimate transverse tensile strength	$(\sigma_2^T)_{ult}$	MPa	40	31
Ultimate transverse compressive strength	$(\sigma_2^C)_{ult}$	MPa	246	118
Ultimate in-plane shear strength	$(\tau_{12})_{ult}$	MPa	68	72
Density	ρ	g/cm^3	1.590	1.903
Cost			2.5	1

0
90
90
0
90

Figure 2. Model for cross-ply laminate.

ε^0, k- mid-plane strains and curvature of a laminate in x-y coordinates

The mid-plane strain and curvature is given by

$$\begin{aligned}
A_{ij} &= \sum_{k=1}^{n} (\overline{Q_{ij}})_k (h_k - h_{k-1}) i = 1,2,6, j = 1,2,6, \\
B_{ij} &= \frac{1}{2} \sum_{k=1}^{n} (\overline{Q_{ij}})_k (h_k^2 - h_{k-1}^2) i = 1,2,6, j = 1,2,6, \\
D_{ij} &= \frac{1}{3} \sum_{k=1}^{n} (\overline{Q_{ij}})_k (h_k^3 - h_{k-1}^3) i = 1,2,6, j = 1,2,6,
\end{aligned} \tag{8}$$

where the [A], [B], and [D] matrices are called the extensional, coupling, and bending stiffness matrices.

Failure Theory

Failure Process

A laminate will fail under increasing mechanical loading; however, the procedure of laminate failure may not be catastrophic. In some cases, some layers fail first, and the rest are able to continue to take additional loading until all the plies fail. A ply is fully discounted when a ply fails; then, the ply is replaced by a near-zero stiffness and strength. The procedure for finding the first ply failure in the present study follows the fully discounted method:

1. Compute the reduced stiffness matrix [Q] referred to as the local axis for each ply using its four engineering elastic constants E_1, E_2, E_{12}, and G_{12}.
2. Calculate the transformed reduced stiffness $[\bar{Q}]$ referring to the global coordinate system (x, y) using the reduced stiffness matrix [Q] obtained in step 1 and the ply angle for each layer.
3. Given the thickness and location of each layer, the three laminate stiffness matrices [A], [B], and [D] are determined.
4. Apply the forces and moments, $[N]_{xy}, [M]_{xy}$ solve Equation 7, and calculate the middle plane strain $[\sigma^0]_{xy}$ and curvature $[k]_{xy}$.

5. Determine the local strain and stress of each layer under the applied load.
6. Use the ply-by-ply stress-strain and related failure theories to determine the strength ratio.

Tsai-wu Failure Theory

Many different theories about the failure of an angle lamina have been developed for a unidirectional lamina, such as the maximum stress failure theory, maximum strain failure theory, Tsai-Hill failure theory, and Tsai-Wu failure theory. The failure theories of a lamina are based on the stresses in the local axes in the material. There are four normal strength parameters and one shear stress for a unidirectional lamina. The five strength parameters are:

$(\sigma_1^T)_{ult}$ = ultimate longitudinal tensile strength
$(\sigma_1^C)_{ult}$ = ultimate longitudinal compressive strength
$(\sigma_2^T)_{ult}$ = ultimate transverse tensile strength
$(\sigma_2^C)_{ult}$ = ultimate transverse compressive strength
$(\tau_{12})_{ult}$ = and ultimate in-plane shear strength

In the present study, Tsai-wu failure theory is taken to decide whether a lamina fails, because this theory is more general than the Tsai-Hill failure theory, which considers two different situations, the compression and tensile strengths of a lamina. A lamina is considered to fail if

$$\begin{aligned} H_1\sigma_1 + H_2\sigma_2 + H_6\tau_{12} + H_{11}\sigma_1^2 + H_{22}\sigma_2^2 \\ + H_{66}\tau_{12}^2 + 2H_{12}\sigma_1\sigma_2 < 1 \end{aligned} \tag{9}$$

is violated, where

$$SR = \frac{\text{Maximum Load}}{\text{Load Applied}} \tag{10}$$

The maximum load refers to that can be applied using Tsai-wu failure theory.

Genetic algorithm model

The GA starts with multiple individuals with limited chromosome lengths, in which maybe none of these individuals fulfill the constraints. The GA is assumed to derive appropriate offspring based on the initial population as the GA continues. The classic way to handle the constrained search of the GA is either to introduce repair strategies or use a penalty function. Figure 4 shows the classic flow chart of a GA framework which includes selection, crossover, and mutation operators. However, GA is originally proposed to solve unconstrained problems, therefore, a new approach is developed to address the constrained GA search problem in an unconstrained way.

Because of the existence of constraints, the population can be sorted by the fitness (obtained by the objective function) but can also be sorted by the constraint value obtained by the constraint function (assuming a constraint function exists), so the parents of the next generation can be chosen by the following three approaches. First, the population is sorted by fitness in ascending order, and individuals with smaller fitness are selected. These selected individuals form a group named as a proper group. Second, remove individual which satisfies constraints, and sort population by the difference between the individual's constraint value and the threshold of the constraint in descending order, and individuals with bigger differences are chosen to be the parents of the next generation. The group which forms are called potential group, and an individual from this group is referred to as a potential individual. Third, the population is sorted by fitness from low to high after removing individuals which fails to fit the constraints, select individuals with bigger fitness, and these individuals form the proper group. So the final parents' pool is consists of three groups, active group, potential group, and proper group. The number of active individuals, potential individuals, and proper individuals are called, respectively, active number, potential numbers, and proper number.

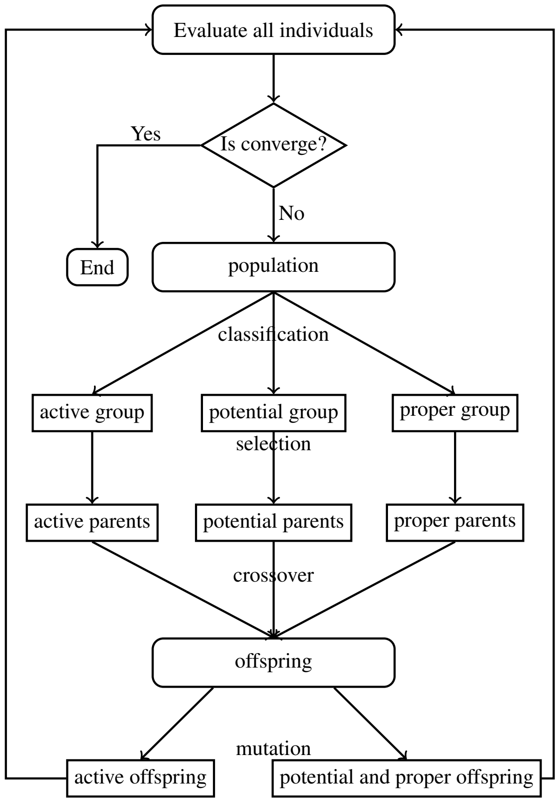

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Figure 3. General flowchart of proposed GA model in which the parents is consist of three different groups.

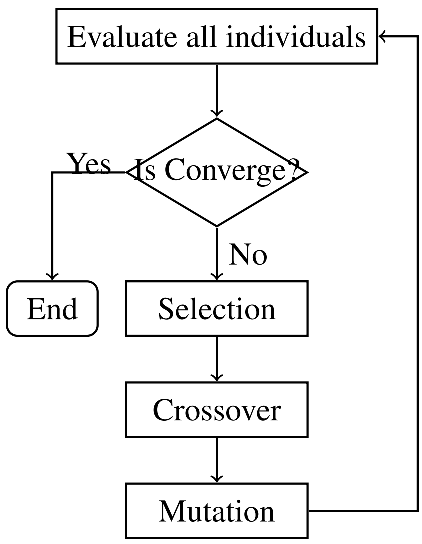

Figure 4. Traditional GA Model.

Each group in the parents' population has its role in the searching process. The problem within traditional GA is premature and has weak local search ability, therefore, traditional GAs are more likely to get stuck in a local optimum. To prevent the GA from experiencing early convergence and to improve the local search performance, the active group is proposed to overcome this problem. As its name suggests, this group would always live in the population. Because both active individual's fitness and constraint value are small, each individual can be treated as an independent gene clip. So their existence enriches the gene clip variety of the mating pool. The offspring of two active individuals are more likely to be an active individual, which can maintain the active group.

For an individual in the potential group, it doesn't satisfy the constraint, however, it's supposed to evolve into a proper individual after multiple generations by modifying its chromosome structure or length. The crossover operation could happen between a potential individual with an active individual, or a potential individual, or a proper individual. The child of an active individual and a potential individual is more likely to be a potential individual, and this active individual could inject a new gene clip into this potential individual, therefore providing a new evolution direction.

A proper individual means a feasible solution, it fulfills all the constraints. However, there are still some drawbacks within it, for example, its fitness is low. The crossover operation could happen between a proper individual and any other individuals.

The mutation operator for an active group is different from the potential group and proper group, because their roles in the searching process are different. The target of the potential group and proper group is to obtain a feasible solution, however, the role of the active group is to maintain the variety of gene clip in the mating pool.

Figure 3 shows the flow chart of the proposed GA. First, the population are divided into three groups, active group, potential group, and proper group by the above-mentioned method. Second, select an appropriate number of individuals from each group as parents, and the various selection scheme can be taken for each group.

The searching process can be divided into two phases according to whether proper individuals are generated. During the initial stage, no individual in the population is appropriate, which means the number of individuals in the proper group is zero. Both active group and potential group are full. After a couple of generations, some proper individuals could be produced. Then, GA comes to its second phase, the number of proper individuals begins to increase, finally, the number in the proper group reaches its upper bound. During the last phases,

Experiment

First, we formulate a constrained problem by searching the optimal stacking sequence of cross-ply laminate under in-plane loading whose strength ratio is not less than 2. Each lamina dimensions $1000 \times 1000 \times 0.165mm^3$ is adopted in this experiment, each graphite/epoxy, and glass/epoxy layer is assumed to cost 2.5 and 1 monetary units, respectively. The other material properties are shown in Table 1.

!htb

P_1:	90	90	0	0	0	90	90	90	90	90
P_2:	0	0	90	90	90	0	0	90	0	0

(a): Parents P_1 and P_2

O_1:	90	90	0	0	0	0	0	90	0	0
O_2:	0	0	90	90	90	90	90	90	90	90

(b): Offspring O_1 and O_2

O_1:	+13	90	90	$\cdots$	90	90	0	$\cdots$	0	0
		11						7		

(c): Offspring O_1 after lenght mutation

90	90	90	$\cdots$	90	90	0	$\cdots$	0	0

(b): Offspring O_1 after angle mutation

Figure 5. Examples of crossover, length mutation, angle mutation operator for proposed GA.

Problem formulation

In the present experiment, the optimal composite sequences, and the number of layers for a targeted strength ratio under in-plane loading conditions are investigated. The aim is to minimize the mass of a laminate composite for a targeted strength ratio based on the Tsai-wu failure theory. The design variables are the ply angles and the number of layers. Ply orientation restricted to a discrete set of angles (0, and 90 degrees). The problem can be formulated as the following equation

Find: $\{\theta_k, n\}$ $\theta_k \in \{0, 90\}$

Minimize: weight

Subject to: strength ratio

GA Operation

In the present study, floating encoding is adopted to represent a solution for the lay-up design of cross-ply laminate, Figure 5(a) shows two parents P_1 and P_2 which represent two cross-ply laminates, the corresponding laminates lay-ups are $[0_3/90_7]$ and $[0_6/90_4]$, respectively. Figure 5(b) shows two offspring of parents P_1 and P_2 which are consist of half of each parent's chromosome.

Figure 5 shows the mutation operations which consists of length mutation and angle mutation operation for the chromosome. For the chromosome length mutation, calculate the chromosome's strength ratio based on its sequence, if its strength ratio is less than the threshold, then increase its length. A length mutation coefficient is introduced to adjust the length mutation. As shown in figure 5(b), the strength ratio of O_1 chromosome is 0.0854, and the strength ratio threshold is 2. Suppose the length mutation coefficient takes 2, then the corresponding increase length is $2 \times (2 - 0.0854)$. For the angle mutation, randomly swap the gene from 0 to 90 in the chromosome, or the otherwise.

GA Parameters

Table 2 shows related GA parameters: the population is 40, and 50 percent is as the mating pool, so the parent population is 20; as shown in figure 6, the percentage of active individuals from the active group, potential individuals from the potential group, and proper individuals from the proper group are 0.3, 0.3, and 0.4, which means the corresponding number of these three types of individuals are 6, 6, and 8, respectively.

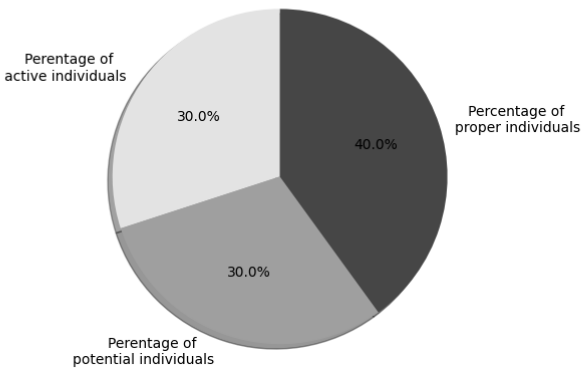

Figure 6. Percentage of active individuals from active group, potential individuals from potential group, and proper individuals from proper group in parent population.

Table 2. Parameters of proposed GA model.

Parameter	Value
Population	40
Initial length range	[3-15]
Encoding	Integer
Percentage of parent	0.5
Percentage of active group	0.3
Percentage of potential group	0.3
Percentage of proper group	0.4
Selection strategy for active group	Ranking
Selection strategy for potential group	Ranking
Selection strategy for proper group	Ranking
Crossover strategy	One-point
Mutation strategy	Mass mutation
Length mutation coefficient	5
Angle mutation rate	0.1

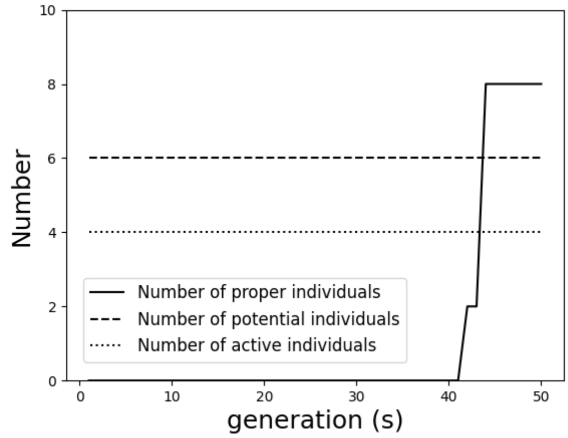

Figure 7. Number of individuals in each group as a function of generation.

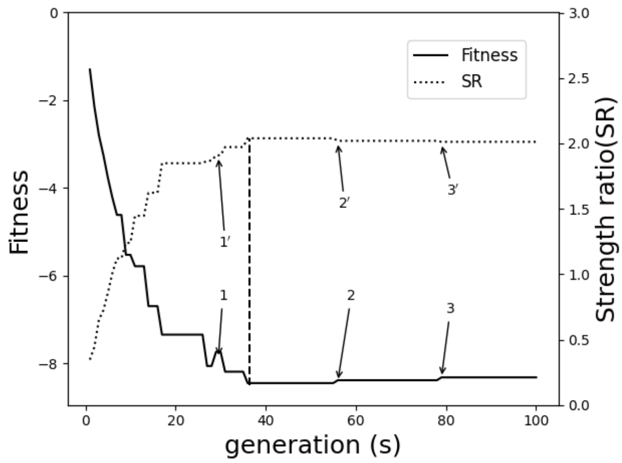

Figure 8. The fitness is the negation of the individual's mass. The solid curve is the fitness of the best individual in the population in respect to the generations; and the dotted line denotes its corresponding strength ratio. If no individuals in the population satisfy the constraint, the best individual is the one with the biggest strength ratio; if not, the best individual is the one with the smallest mass.

Numerical Results and Discussion

To figure out how the number of individuals in each group varies during the GA process, we conduct one-time experiment and show the number of individuals in each generation in respect of GA generation. Second, to verify its performance and stability, the GA is run one hundred times: the best, worst case, and average results are presented, respectively. Finally, we compare the results with the work in the other literature.

Figure 7 shows the number of individuals in each group during the one-time GA process. For both active group and potential groups, the number of individuals is fixed, and equal to its upper bound from the beginning to the end of the searching process. However, for the proper group, at the initial stage of GA, no individual fulfills the constraint, so the number of proper individuals is zero. As seen from figure 7, after forty generations, proper individuals appear, and its population increases very quickly up to its upper bound.

There are two approaches that the GA could obtain a better solution: the first is increasing the length of the chromosome; the second one is adjusting the internal structure of a chromosome. The GA process can be divided into two phases by whether there are proper individuals or not. Figure 8 shows the GA process in which the dashed vertical line is the watershed between the initial phase and the last phase. In the initial phase, no individual's strength ratio is over the specified threshold, and the main reason that the fitness gets smaller gradually is the increase of chromosome's length; At point 1 on the fitness curve, the fitness suddenly goes up, however, the corresponding strength ratio of point 1, denoted by the point $1'$ on the generation-strength ratio curve, also increases. This is because of the adjustment of a chromosome's lay-up. Then GA comes to its second phase. During this phase, the GA already found proper individuals which could satisfy the constraint, so the target in this stage is to improve fitness. This means GA needs to adjust its inner structure, at the point 2 and 3 on the generation-fitness curve, the fitness curves go up, and the corresponding strength ratio of these two points slightly go down, but both of them still satisfy the constraint.

Table 3 shows the searching results after conducting this experiment one hundred times in two length mutation coefficient cases for glass-epoxy and graphite-epoxy material, respectively. The best, worst case, and average experiment results are represented in this table. For the glass-epoxy material, if the coefficient value takes 1, the best and worst sequences are $[0_{80}/90_{52}]$, $0_{75}/90_{43}$, respectively; the average mass, cost, and the number of layers are 1.68, 123, 123. However, if the coefficient takes a relatively bigger value, the performance of GA is better than the GA with a smaller coefficient value. When the coefficient takes 1, the number of layers for the best and worst cases are 118 and 132, respectively. When the coefficient value si 5, the number of layers for them are 125 and 136, respectively. When graphite-epoxy was taken as the experiment material, similar experiment results were presented. This is because the mutation coefficient can control both the convergence speed and search performance, a small mutation coefficient would slow the convergence speed, however, it would lead to a small-grained search in the local.

Table 4 shows the optimal cross-ply sequences by the proposed GA and Choudhury and Mondal's[28] study. For the loading case $Nx = 1$ MPa m, the optimal lay-ups are a $[0_{68}/90_{72}]$ cross-ply laminate if glass-epoxy are taken, however, in the present study, a $[0_{75}/90_{43}]$ glass-epoxy cross-ply laminate has been found which significantly reduces both the cost and weight; and the constraint on the optimization is satisfied. Similarly, if graphite-epoxy is taken, compared with the $[0_{17}/90_{18}]$ cross-ply laminate, an alternative solution has been found, its lay-up is $[0_{17}/90_5]$. For both cases, the experiment results have shown that using the present proposed GA can obtain better results.

conclusions

In this paper, we reviewed the use of the proposed ga framework, classical lamination theory, and tsai-wu failure theory for the lay-up design for cross-ply laminate. Because GA is primarily used to solve an unconstrained problem, and it is not suitable for a constrained problem. In the present study, we deal with this constrained problem by mixing strategies of selection methods instead of adding punishment terms into the objective function. So the constraint problem can be solved in an unconstrained way.

This variant of the GA provides a new approach to address the constrained search for optimization of laminated composite, and this method can be easy to apply in other domains. At the same time, the proposed GA model is more complicated than the traditional GA model, which involves more parameters. To advance its performance, the fine-tuning of those parameters need more effort.

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Table 3. The optimum lay-ups for the loading $N_x = 1e^6$ N when changing the length mutation coefficient, the performance of the GA can be improved when the lenght mutation coefficient is smaller.

Length mutation coefficient	Material	case	Stacking sequence	Strength ratio	Mass	Cost	Layer
1	glass-epoxy	worst	$[0_{80}/90_{52}]$	2.010	8.58	132	132
		best	$[0_{75}/90_{43}]$	2.000	7.67	118	118
		average		2.012	7.83	123	123
	graphite-epoxy	worst	$[0_{17}/90_{15}]$	2.036	1.68	80	32
		best	$[0_{17}/90_{5}]$	2.005	1.15	55	22
		average		2.018	1.47	70	28
5	glass-epoxy	worst	$[0_{72}/90_{64}]$	2.009	8.84	136	136
		best	$[0_{72}/90_{53}]$	2.003	8.12	125	125
		average		2.008	8.55	131	131
	graphite-epoxy	worst	$[0_{18}/90_{24}]$	2.006	2.20	105	42
		best	$[0_{17}/90_{6}]$	2.001	1.20	57	23
		average		2.022	1.54	73	29

Table 4. Comparison of experiment results of Choudhury and Mondal's[28] and current study under in-plane loading $N_x = 1e6$ N. The results of present study is from previous experiment.

Cross Ply $[0_M/90_N]$	Choudhury and Mondal's study		Present study	
Material	Glass-Epoxy	Graphite-Epoxy	Glass-Epoxy	Graphite-Epoxy
M	68	17	75	17
N	72	18	43	5
no. of lamina(n)	140	35	118	22
SR	2.01	2.10	2.00	2.00
weight	9.10	1.84	7.67	1.15
cost	140	87.5	118	55

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