Contents

Chapter 1

Introduction

1.1 Laminated Composite Material

Composite materials offer improved strength, stiffness, corrosion resistance, etc. over conventional materials, and are widely used as alternative materials for applications in various industries ranging from electronic packaging to golf clubs, and medical equipment to homebuilding, making aircraft structure to space vechicles. The stacking sequence and fiber orientation of composite laminates give the designer additional 'degree of freedom' to tailor the design with respect to strength or stiffness. One widely known advange of using composite material is can significantly reducing the weight of target structure, and many researchers attempted to improve the efficiency of using composite materail by mimimizing the thickness.

1.2 Classic Lamination Theory and Failure Theory

Classic lamination theory(CLT) is used to develop the stress-strain relationship of composite material under in-plane and out-of-plane loading. First, develop stress-strain relationships, elastic moduli, strengths of an angle ply based on a unidirectional lamina and the angle of the ply; second, becasue a laminate is consist of more than one lamina bonded together through their thickness, so the macromechianical analysis will be developed for a laminate based on applied loading. To check whether a designed lay-up is plausible or not, different failure theories have been developed.

1.3 Genetic Algorithm

In the design of composite material, gradient based optimization techiques are not appliable in this domain, because the design varaibles, such as fiber orientation, layer thickness, number of layers etc. are discrete. Genetic algorithm(GA) can be adopted in the optimization problem becasue it doesn't require the gradient information. Moreover, the GA has been proved a reliable techique and widely used in the design of composite material.

1.4 Artificial Neural Network

CLT is an classic analytical approach to obtain the stress and stain of composite material, the disadvantage of this method is quiet cubersome and in which involves compliate matrix and integration operations. Artificial neural network(ANN) has been proved a reliable tool in modelling various engineering system in practice without solving tricky equations and making ideal assumptions. In this thesis, the ANN is taken to approximate the numberic results based on CLT and failure theory.

1.5 Summary

In this thesis, first, we review the use of composite material in practice, then, the CLT to calculate the stress and strain under certain loading, last, the failure theories which are used to decide whether a composite material will failure or not. Second, the stochastic algorith, GA, is studied and implemented in the desgin of composite material, two different cases are studied in which GA can be taken to obtain the optimal lay-up. At last, ANN is introduced to approximate the evaluation result of CLT, the reason for adopting ANN is to reduce the computation complexity based on CLT.

Chapter 2

Classic Lamination Theory and Failure Theory

2.1 Classic Lamination Theory

A laminate is consist of multiple laminas bonded together through thickness. In this chapter, first the stress-strain relationship is developed based on Hook's law for a single lamina; second, develop relationship of mechanical loads applied to a laminate to strains and stresses in each lamina, calculate the elastic moduli of laminate based on the elastic moduli of single laminate and the lay-up..

2.1.1 Analysis of stress and strain for composite material

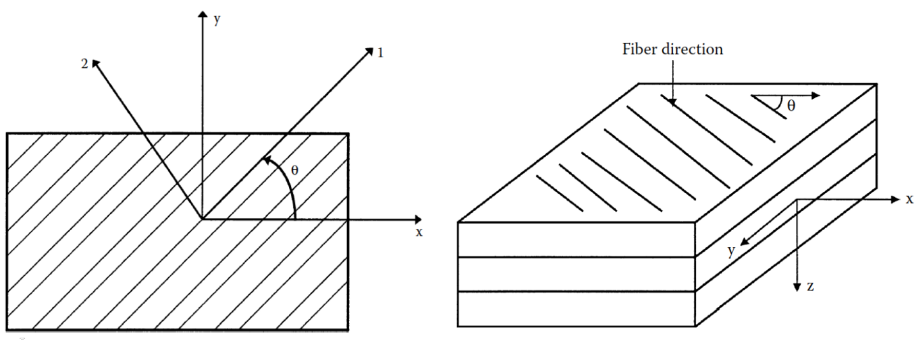

FIGURE 2.1: Lamina

2.1.2 Stress and Strain in a Lamina

For a single lamina has a small thickness under plane stress, and it's upper and lower surfaces of the lamina are free from external loads. According to the Hooke's Law, the three-dimensional stress-strain equations can be reduced to two-dimensional stress-strain equations. The stress-strain relation in local axis 1-2 is

$$\begin{bmatrix} \sigma_1 \\ \sigma_2 \\ \tau_{12} \end{bmatrix} = \begin{bmatrix} Q_{11} & Q_{12} & 0 \\ Q_{12} & Q_{22} & 0 \\ 0 & 0 & Q_{66} \end{bmatrix} \begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \gamma_{12} \end{bmatrix}. \tag{2.1}$$

where $Q_{ij}$are the stiffnesses of the lamina that are related to engineering elastic constants given by

$$\begin{aligned}
Q_{11} &= \frac{E_1}{1-v_{12}v_{21}}, \\
Q_{22} &= \frac{E_2}{1-v_{12}v_{21}}, \\
Q_{66} &= G_{12}, \\
Q_{12} &= \frac{v_{21}E_2}{1-v_{12}v_{21}},
\end{aligned} \tag{2.2}$$

where E_1, E_2, v_{12}, G_{12} are four independent engineering elastic constants, which are defined as follows: E_1 is the longitudinal Young's modulus, E_2 is the transverse Young's modulus, v_{12} is the major Poisson's ratio, and G_{12} is the in-plane shear modulus.

Stress strain relation in the global x-y axis is

$$\begin{bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{bmatrix} = \begin{bmatrix} \bar{Q}_{11} & \bar{Q}_{12} & \bar{Q}_{16} \\ \bar{Q}_{12} & \bar{Q}_{22} & \bar{Q}_{26} \\ \bar{Q}_{16} & \bar{Q}_{26} & \bar{Q}_{66} \end{bmatrix} \begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{bmatrix}, \tag{2.3}$$

where

$$\begin{aligned}
\bar{Q}_{11} &= Q_{11}cos^4\theta + Q_{22}sin^4\theta + 2\left(Q_{12}+2Q_{66}\right)sin^2\theta cos^2\theta, \\
\bar{Q}_{12} &= \left(Q_{11}+Q_{22}-4Q_{66}\right)sin^2\theta cos^2\theta + Q_{12}\left(cos^4\theta + sin^2\theta\right), \\
\bar{Q}_{22} &= Q_{11}sin^4\theta + Q_{22}cos^4\theta + 2\left(Q_{12}+2Q_{66}\right)sin^2\theta cos^2\theta, \\
\bar{Q}_{16} &= \left(Q_{11}-Q_{12}-2Q_{66}\right)cos^3\theta sin\theta - \left(Q_{22}-Q_{12}-2Q_{66}\right)sin^3\theta cos\theta, \\
\bar{Q}_{26} &= \left(Q_{11}-Q_{12}-2Q_{66}\right)cos\theta sin^3\theta - \left(Q_{22}-Q_{12}-2Q_{66}\right)cos^3\theta sin\theta, \\
\bar{Q}_{66} &= \left(Q_{11}+Q_{22}-2Q_{12}-2Q_{66}\right)sin\theta^2 cos\theta^2 + Q_{66}\left(sin\theta^4 + cos\theta^4\right).
\end{aligned} \tag{2.4}$$

The local and global stresses in an angle lamina are related
to each other through the angle of the lamina θ, it can be written as

$$\begin{bmatrix} \sigma_1 \\ \sigma_2 \\ \tau_{12} \end{bmatrix} = [T] \begin{bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{bmatrix}, \tag{2.5}$$

where

$$[T] = \begin{bmatrix} cos^2\theta & sin^2\theta & 2sin\theta cos\theta \\ sin^2\theta & cos^2\theta & -2sin\theta cos\theta \\ -sin\theta cos\theta & sin\theta cos\theta & cos^2\theta - sin^2\theta \end{bmatrix}. \tag{2.6}$$

2.1.3 Stress and Strain in a Laminate

For forces and moment resultants acting on laminates, such as in plate and shell structures, the relationship between applied forces and moment and displacement can be given by

$$\begin{bmatrix} N_x \\ N_y \\ N_{xy} \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} & A_{16} \\ A_{12} & A_{22} & A_{26} \\ A_{16} & A_{26} & A_{66} \end{bmatrix} \begin{bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \end{bmatrix} + \begin{bmatrix} B_{11} & B_{12} & B_{16} \\ B_{11} & B_{12} & B_{16} \\ B_{16} & B_{26} & B_{66} \end{bmatrix} \begin{bmatrix} k_x \\ k_y \\ k_{xy} \end{bmatrix},$$

$$\begin{bmatrix} M_x \\ M_y \\ M_{xy} \end{bmatrix} = \begin{bmatrix} B_{11} & B_{12} & B_{16} \\ B_{12} & B_{22} & B_{26} \\ B_{16} & B_{26} & B_{66} \end{bmatrix} \begin{bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \end{bmatrix} + \begin{bmatrix} D_{11} & D_{12} & D_{16} \\ D_{11} & D_{12} & D_{16} \\ D_{16} & D_{26} & D_{66} \end{bmatrix} \begin{bmatrix} k_x \\ k_y \\ k_{xy} \end{bmatrix}, \tag{2.7}$$

where
N_x, N_y - normal force per unit length;
N_{xy} - shear force per unit length;
M_x, M_y - bending moment per unit length;
M_{xy} - twisting moments per unit length;
ε^0, k- mid plane strains and curvature of a laminate in x-y coordinates.
The mid plane strain and curvature is given by

$$\begin{aligned} A_{ij} &= \sum_{k=1}^{n} (\overline{Q_{ij}})_k (h_k - h_{k-1}) i = 1,2,6, j = 1,2,6, \\ B_{ij} &= \frac{1}{2} \sum_{k=1}^{n} (\overline{Q_{ij}})_k (h_k^2 - h_{k-1}^2) i = 1,2,6, j = 1,2,6, \\ D_{ij} &= \frac{1}{3} \sum_{k=1}^{n} (\overline{Q_{ij}})_k (h_k^3 - h_{k-1}^3) i = 1,2,6, j = 1,2,6. \end{aligned} \tag{2.8}$$

The [A], [B], and [D] matrices are called the extensional, coupling, and bending stiffness matrices, respectively. The extensional stiffness matrix $[A]$ relates the resultant in-plane forces to the in-plain strains, and the bending stiffness matrix $[D]$ couples the resultant bending moments to the plane curvatures. The coupling stiffness matrix $[B]$ relates the force and moment terms to the midplain strains and midplane curvatures.

2.2 Failure Theory

Failure criteria for composite materials are more difficult to predict due to structural and material complexity in comparison to isotropic materials. The failure process of a composite materials can be regarded from microscopic and macroscopic points of view. Most popular criteria about the failure of an angle lamina are in terms of macroscopic failure criteria, which are based on the tensile, compressive and shear strengths. According to the failure surfaces, these criteria can be classified into two classes: one is called independent failure mode criteria which includes the maximum stress failure theory, maximum strain failure theory because their failure envelop are rectangle; another is called quadratic polynomial which includes Tsai-Wu, Chamis, Hoffman and Hill criteria because their failure surfaces are of ellipsoidal shape. In

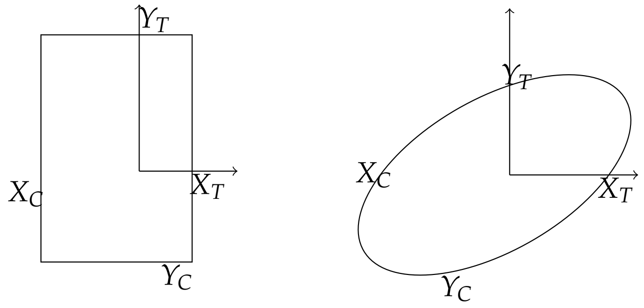

FIGURE 2.2: Schematic failure surfaces for maximum stress and quadratic failure criteria

the present study, two most reliable failure criteria is taken, Maximum stress and Tsai-wu. Both of these two failure criteria are based on the stresses in the local axes instead of principal normal stresses and maximum shear stresses, and four normal strength parameters and one shear stress for a unidirectional lamina are involved. The five strength parameters are

$(\sigma_1^T)_{ult}$ = ultimate longitudinal tensile strength(in direction 1),
$(\sigma_1^C)_{ult}$ = ultimate longitudinal compressive strength,
$(\sigma_2^T)_{ult}$ = ultimate transverse tensile strength,
$(\sigma_2^C)_{ult}$ = ultimate transverse compressive strength, and
$(\tau_{12})_{ult}$ = and ultimate in-plane shear strength.

2.2.1 Tsai-wu failure criterion

The TW criterion is one of the most reliable static failure criteria which is derived from the von Mises yield criterion. A lamina is considered to fail if

$$\begin{aligned} H_1\sigma_1 + H_2\sigma_2 + H_6\tau_{12} + H_{11}\sigma_1^2 + H_{22}\sigma_2^2 \\ + H_{66}\tau_{12}^2 + 2H_{12}\sigma_1\sigma_2 < 1 \end{aligned} \tag{2.9}$$

is violated, where

$$\begin{aligned} H_1 &= \frac{1}{(\sigma_1^T)_{ult}} - \frac{1}{(\sigma_1^C)_{ult}}, \\ H_{11} &= \frac{1}{(\sigma_1^T)_{ult}(\sigma_1^C)_{ult}}, \\ H_2 &= \frac{1}{(\sigma_2^T)_{ult}} - \frac{1}{(\sigma_2^C)_{ult}}, \\ H_{22} &= \frac{1}{(\sigma_2^T)_{ult}(\sigma_2^C)_{ult}}, \\ H_{66} &= \frac{1}{(\tau_{12})_{ult}^2}, \\ H_{12} &= -\frac{1}{2}\sqrt{\frac{1}{(\sigma_1^T)_{ult}(\sigma_1^C)_{ult}(\sigma_2^T)_{ult}(\sigma_2^C)_{ult}}}. \end{aligned} \tag{2.10}$$

H_i is the strength tensors of the second order; H_{ij} is the strength tensors of the fourth order. σ_1 is the applied normal stress in direction 1; σ_2 is the applied normal stress in the direction 2; and τ_{12} is the applied in-plane shear stress.

2.2.2 Maximum Stress Failure Theory

Maximum stress(MS) failure theory consists of maximum normal stress theory proposed by Rankine and maximum shearing stress theory by Tresca. The stresses applied on a lamina can be resolved into the normal and shear stresses in the local axes. If any of the normal or shear stresses in the local axes of a lamina is equal or exceeds the corresponding ultimate strengths of the unidirectional lamina, the lamina is considered to be failed. That is

$\sigma_1 \geq (\sigma_1^T)_{ult}$ or $\sigma_1 \leq -(\sigma_1^C)_{ult}$,
$\sigma_2 \geq (\sigma_2^T)_{ult}$ or $\sigma_2 \leq -(\sigma_2^C)_{ult}$,
$\tau_{12} \geq (\tau_{12})_{ult}$ or $\tau_{12} \leq -(\tau_{12})_{ult}$.

where σ_1 and σ_2 are the normal stresses in the local axes 1 and 2, respectively; τ_{12} is the shear stress in the symmetry plane 1-2. The safety factor, or yield stress, is how much extra load beyond is intended a composite laminate will actually take, which is an indication of the material's load carrying capacity. If the value is less then 1.0, it means failure. The safey factor is defined as

$$SF = \frac{\text{Maximum Load Which Can Be Applied}}{\text{Load Applied}}. \tag{2.11}$$

The safety factor based on maximum stress theory is calculated by the following method: first, the principal stresses(σ_1^k,σ_2^k, and τ_{12}^k) are obtained by experiment; evaluate the safety factor along each direction according to equation 5.10; The minimum value among these safety factors are denoted as the safety factor of the lamina, SF_{MS}^k, it can be written as

$$SF_{MS}^k = \text{min of} \begin{cases} SF_X^k = \begin{cases} \frac{X_t}{\sigma_{11}}, & \text{if } \sigma_{11} > 0 \\ \frac{X_c}{\sigma_{11}}, & \text{if } \sigma_{11} < 0 \end{cases} \\ SF_Y^k = \begin{cases} \frac{Y_t}{\sigma_{22}}, & \text{if } \sigma_{22} > 0 \\ \frac{Y_c}{\sigma_{22}}, & \text{if } \sigma_{22} < 0 \end{cases} \\ SF_S^k = \begin{cases} \frac{S}{|\tau_{12}|} \end{cases} \end{cases}. \tag{2.12}$$

Assuming the composite laminate under a in-plane loading f, the corresponding stress on local stress in direction 1, local stress in direction 2, and shear stress for the kth lamina are $\sigma_1 SF_{TW}^k$, $\sigma_2 SF_{TW}^k$, and $\tau_{12} SF_{TW}^k$, respectively. Substitute them into the equation 5.8, the expression are given by

$a(SF_{TW}^k)^2 + b(SF_{TW}^k) - 1 = 0$,

where

$a = H_{11}(\sigma_1)^2 + H_{22}(\sigma_2)^2 + H_{66}(\tau_{12})^2 + 2H_{12}\sigma_1\sigma_2$,
$b = H_1\sigma_1 + H_2\sigma_2 + H_6\tau_{12}$.

Solve the above equation, the safety factor for the kth lamina is

$SF_{TW}^k = |\frac{-b+\sqrt{b^2+4a}}{2a}|$.

Then, the minimum of SF_{TW}^k is taken as the safety factor of the laminate which is written as

$SF_{TW} =$ min of SF_{TW}^k for $k = 1, 2, \cdots, m-1, m$.

2.3 Summary

In this chapter, we review the CLT for composite material's analysis, then related failure theories are introduced to check whether a composite material would fail or not. In the following chapters, the CLT would be used to calculate the stress and strain under in-plane loading, it would also be used to generate the training data for stress and strain approximation based on neural network; in order to design a proper composite material, the failure theory are used to decide the feasibility of composite design.

Chapter 3

A New Genetic Algorithm Model for Constrained Problem

3.1 Genetic Algorithm Framework

GA is one of the most reliable stochastic algorithm, which has been widely used in discrete variables optimization problems [36, 37, 13, 39, 20, 15, 44, 2, 8, 35, 1, 25, 21, 38, 5, 34, 28, 41, 43, 11, 17]. In practice, fiber orientations are restricted to a finite set of angles, and ply thickness is a specific numeric value. Because the design variables are not continueous, a gradient based optimization procedure, such as gradient descent method, is not suitable to cope with such problems. Moreover, gradient based optimization approach is very eazy to get trapped in local minima, and many local optimum may exist in structural optimization problems. A stochastic algorithm, such as GA, is able to deal with optimization problems with discrete design variables. Besides, stochastic method could escape from local optimum, and obtain global optimum.

Although GA gains different advantages for solving discrete problems, many disadvantages exists within this method. First, the optimization process of GA parameters, such as the population size, parent population, mutation percentage, etc., is very tedious; Second, the GA needs to evaluate the objective function many times to acheive the optimization, and the computation cost is very high; the last problem within GA is the premature convergence. GA consists of five basic parts: the variable encoding method, selection scheme, crossover operator, mutation operator, and how the constraints are handled. A typical GA process is show in Figure 3.1.

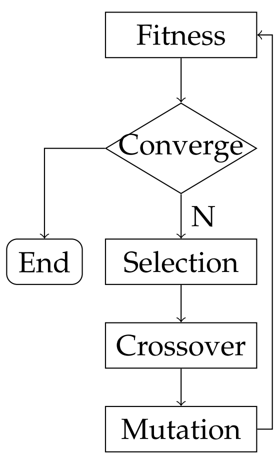

FIGURE 3.1: GA process

The first issue when implementing a GA is the representation of design variables, and an appropriate design representation is crucial to enhance the efficiency of GA. The canonical GA has always used binary strings to encode alternative solutions, however, some argued that the minimal cardinality, i.e., the binary representation, are not the best option. Real value string has been widely employed in

Selection scheme plays a critical role in balancing the dilemma of exploration and exploitation inherented in GA, and various selection methods, for example, roulette wheel, elitist, and tournament etc., have been proposed to overcome this issue. Both of roulette selection and tournament selection are well-studied and widely employed in the optimization design of laminated composite due to their simplicity to code and efficiency for both nonparallel and parallel architectures.

Crossover is another crucial operator introduced into the GA methodology framework, in which the alternative solution is generated from the mating pool. multiple types of crossover operator has been utilized in the optimization design of composite structures, such as: one-point, two-point, and uniform crossover. .

3.2 Constrained Problem Optimization

However, GA is originally proposed for unconstrained optimization problem, in order to deal with constrained design for composite laminate, some techniques are introduced into GA. The first method is using of data structure, special data structure has been developed to fulfils the corresponding constraint, for example, in order to fulfill the symmetry constraint of a laminate, the chromsome is consist of coding only half of the laminate and considering that each stack of the laminate is formed by two laminate with the same orientation but opposite signs [20, 19]. The second approach is reformulating the objective function. A penalty function is developed to convert a constrained problem into an unconstrained problem by adding penalty term to the objective funtion. Another method to solve constrained problem is introducing repair strategy by Todoroki and Haftka [42], which is aim to transform infeasible solutions to feasible solution by incorporating problem-specific knowledge.

3.3 A New Genetic Algorithm Model

3.3.1 Classification

The population is randomly generated, for every individual in this population the corresponding constraint numberic value can be obtained. As shown in Figure 3.2,The population can be divided into three groups according to constraint value, which are active group, potential group and proper group

3.3.2 Selection

The purpose of the selection operator is to chose mating pool to produce alternative solutions of better fitness. Traditional methods of selecting strategies only take the fitness of individuals into acount, however, due to the existance of constraint, various selection schemes are implemented to selecet the mating set. Based on different selection schemes, the parents of next generation can be divided into three groups: proper groups, active groups, and potential groups according to different selecting methods.

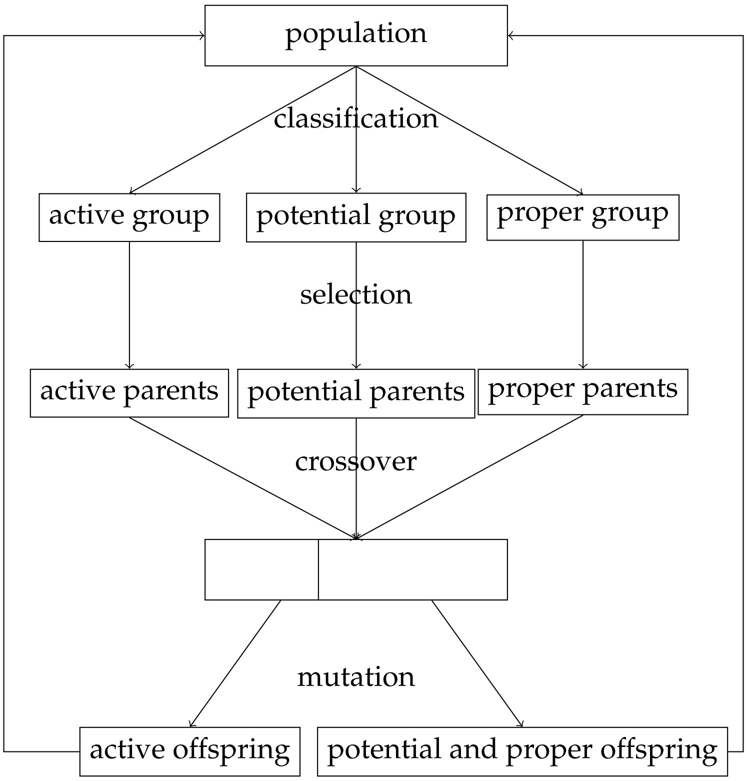

FIGURE 3.2: GA Model

Proper parents mean in which individual fullfils the constraints, which are chosen by the individual's fitnees, individuals with better fitness are more likely to be chosen if they fit the constraint; active groups means that individual is supposed to be always exist in the parents during the GA, which are selected by fitness, ignoring the constraint; The individuals from active group may not correspond to feasible solutions, but their existance enriches the variety of the gene clips. Potential groups means that they are likely to turn into proper individual after a couple of generations, and potential individuals are chosen by constraint function, the more the individual fulfils the constraint, the more possiblity it will be selected.

3.3.3 Crossover

The crossover operator happens among these three groups. the child of two proper groups are more likely to be a proper individual which can be used to obtain a alternative feasible solution. the child of an active individual and a potential individual can significantly change the gene of active individual's chromsome, which makes the individual evolve toward a new direction. The offspring of two active individuals are more likely to be an active individual, which can maitain the active group. The figure.4.10 (b) shows two children O_1

3.3.4 Mutation

i

3.4 Summary

In this chapter, frist, we review the application of tradional GA in the design of composite material; then A new GA framework has been come up with, in the following chapters, this NGAM will be adopted to directed the lay-up design of composite material.

Chapter 4

Laminated Composite Material Optimization by New Genetic Algorithm Model

4.1 Case1: Maximum Strength Optimization

4.2 Introduction

Composite materials offer improved strength, stiffness, fatigue, corrosion resistance, etc., over conventional materials, and are widely used as materials for applications ranging from the automotive to shipbuilding industry, electronic packaging to golf clubs, and medical equipment to homebuilding. However, the high cost of fabrication of composites is a critical drawback to its application. For example, the graphite/epoxy composite part may cost as much as 650 to 900 per kilogram. In contrast, the price of glass/epoxy is about 2.5 times less. Manufacturing techniques such as sheet molding compounds and structural reinforcement injection molding are used to lower the costs for manufacturing automobile parts. An alternative approach is using hybrid composite materials.

The mechanical performance of a laminate composite is affected by a wide range of factors such as the thickness, material, and orientation of each lamina. Because of manufacturing limitations, all these variables are usually limited to a small set of discrete values. For example, the ply thickness is fixed, and ply orientation angles are limited to a set of angles such as 0, 45, and 90 degrees in practice. So the search process for the optimal design is a discrete optimization problem that can be solved by the GA. To tailor a laminate composite, the GA has been successfully applied to solve laminate design problems[**riche1993optimization**, **nagendra1996improved**, **sadagopan1998application**, **liu2000permutation**, **lin2004stacking**, **murugan2007target**, 42, 38, 43, 16, 3]. The GA simulates the process of natural evolution, including selection, crossover, and mutation according to Darwin's principle of "survival of the fittest". The known advantages of GAs are the following: (i): GAs are not easily trapped in local optima and can obtain the global optimum. (ii): GAs do not need gradient information and can be applied to discrete optimization problems. (iii): GAs can not only find the optimal value in the domain but also maintain a set of optimal solutions. However, the GA also has some disadvantages, for example, the GA needs to evaluate the target functions many times to achieve the optimization, and the cost of the search process is high. The GA consists of some basic parts, the coding of the design variable, the selection strategy, the crossover operator, the mutation operator, and how to deal with constraints. For the variable design part, there are two methods to deal with the representation of design variables, namely, binary string and real value representation[**riche1993optimization**, 42]. Michalewicz[**zbigniew1996genetic**] claimed that the performance of floating-point representation was better than binary representation in the numerical optimization problem. Selection strategy plays a critical role in the GA, which determines the convergence speed and the diversity of the population. To improve search ability and reduce search costs, various selection methods have been invented, and they can be divided into four classes: proportionate reproduction, ranking, tournament, and genitor(or "steady state") selection. In the optimization of laminate composite design, the roulette wheel[**riche1993optimization**, **seresta2007optimal**], where the possibility of an individual to be chosen for the next generation is proportional to the fitness. Soremekun et al.[41] showed that the generalized elitist strategy outperformed a single individual elitism in some special cases.

Data structure, repair strategies, and penalty functions[20] are the most commonly used approaches to resolve constrained problems in the optimization of composite structures. Symmetric laminates are widely used in practical scenarios, and data structures can be used to fulfill symmetry constraints, which consists of coding half of the laminate and considering the rest with the opposite orientation. Todoroki[42] introduced a repair strategy that can scan the chromosome and repair the gene on the chromosome if it does not satisfy the contiguity constraint. The comparison of repair strategies in a permutation GA with the same orientation was presented by Liu et al.[**liu2000permutation**], and it showed that the Baldwinian repair strategy can substantially reduce the cost of constrained optimization. Haftka and Todoroki[**riche1993optimization**] used the GA to solve the laminate stacking sequence problem using a penalty function subject to buckling and strength constraints.

In typical engineering applications, composite materials are under very complicated loading conditions, not only in-plane loading but also out-of-plane loading. Most of the studies on the optimization of the laminate composite material minimized the thickness[1, 43], weight[12, 10, 30], and cost and weight[10, 29], or maximized the static strength of the composite laminates for a targeted thickness[**lin2004stacking**, **gholami2020multi**, 43, 18]. In the present study, the cost and weight of laminates are minimized by modifying the objective function.

To check the feasibility of a laminate composite by imposing a strength constraint, failure analysis of a laminate is performed by applying suitable failure criteria. The failure criteria of laminated composites can be classified into three classes: non-interactive theories (e.g., maximum strain), interactive theories (e.g., Tsai-wu), and partially interactive theories (e.g., Puck failure criterion). Previous researchers adopted the first-ply-failure approach using Tsai-wu failure theory[27, 33, 12, 40, 32, 14, 29, 7], Tsai-Hill[26, 39], the maximum stress[45], or the maximum strain[45] static failure criteria. Akbulut[3] used the GA to minimize the thickness of composite laminates with Tsai-Hill and maximum stress failure criteria, and the advantage of this method is it avoids spurious optima. Naik et al.[**naik2008design**] minimized the weight of laminated composites under restrictions with a failure mechanism-based criterion based on the maximum strain and Tsai-wu. In the present study, Tsai-wu Static failure criteria are used to investigate the feasibility of a laminate composite.

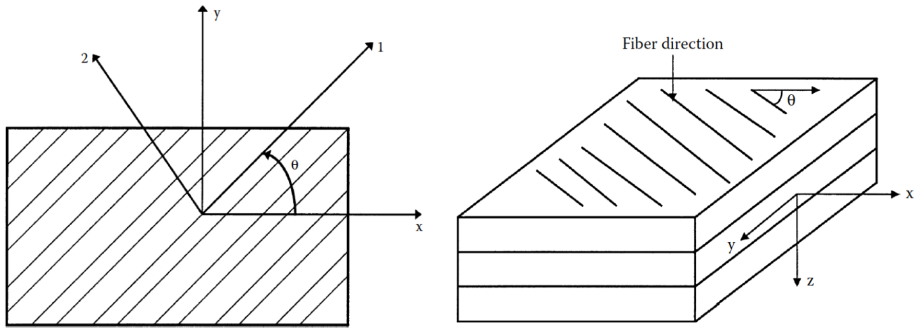

FIGURE 4.1: Local and global axes of an angle lamina.

0
90
90
0
!tb 90

FIGURE 4.2: Model for cross-ply laminate.

4.3 Stress and Strain in a Laminate

A laminate structure consists of multiple lamina bonded together through their thickness. Considering a laminate composite plate that is subject to in-plane loading of extension, shear, bending and torsion, the classical lamination theory (CLT) is taken to calculate the stress and strain in the local and global axes of each ply, as shown in Fig. 4.9. Based on fiber orientation, material, and fiber thickness, there are a few special cases of laminate: the set of fiber angles in Fig. 4.2 only includes 0 and 90, which is called cross-ply laminate.

TABLE 4.1: Comparison of the graphite/epoxy and glass/epoxy properties.

Property	Symbol	Unit	Graphite/Epoxy	Glass/Epoxy
Longitudinal elastic modulus	E_1	GPa	181	38.6
Traverse elastic modulus	E_2	GPa	10.3	8.27
Major Poisson's ratio	v_{12}		0.28	0.26
Shear modulus	G_{12}	GPa	7.17	4.14
Ultimate longitudinal tensile strength	$(\sigma_1^T)_{ult}$	MP	1500	1062
Ultimate longitudinal compressive strength	$(\sigma_1^C)_{ult}$	MP	1500	610
Ultimate transverse tensile strength	$(\sigma_2^T)_{ult}$	MPa	40	31
Ultimate transverse compressive strength	$(\sigma_2^C)_{ult}$	MPa	246	118
Ultimate in-plane shear strength	$(\tau_{12})_{ult}$	MPa	68	72
Density	ρ	g/cm^3	1.590	1.903
Cost			2.5	1

4.3.1 Stress and Strain in a Lamina

For a single lamina, the stress-strain relation in local axis 1-2 is:

$$\begin{bmatrix} \sigma_1 \\ \sigma_2 \\ \tau_{12} \end{bmatrix} = \begin{bmatrix} Q_{11} & Q_{12} & 0 \\ Q_{12} & Q_{22} & 0 \\ 0 & 0 & Q_{66} \end{bmatrix} \begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \gamma_{12} \end{bmatrix}, \tag{4.1}$$

where Q_{ij} are the stiffnesses of the lamina that are related
to engineering elastic constants given by

$$\begin{aligned} Q_{11} &= \frac{E_1}{1 - v_{12}v_{21}}, \\ Q_{22} &= \frac{E_2}{1 - v_{12}v_{21}}, \\ Q_{66} &= G_{12}, \\ Q_{12} &= \frac{v_{21}E_2}{1 - v_{12}v_{21}}, \end{aligned} \tag{4.2}$$

where E_1, E_2, v_{12}, G_{12} are four independent engineering elastic constants, which are defined as follows: E_1 is the longitudinal Young's modulus, E_2 is the transverse Young's modulus, v_{12} is the major Poisson's ratio, and G_{12} is the in-plane shear modulus.

Stress strain relation in the global x-y axis:

$$\begin{bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{bmatrix} = \begin{bmatrix} \bar{Q}_{11} & \bar{Q}_{12} & \bar{Q}_{16} \\ \bar{Q}_{12} & \bar{Q}_{22} & \bar{Q}_{26} \\ \bar{Q}_{16} & \bar{Q}_{26} & \bar{Q}_{66} \end{bmatrix} \begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{bmatrix}, \tag{4.3}$$

where

$$\begin{aligned} \bar{Q}_{11} &= Q_{11}c^4 + Q_{22}s^4 + 2\left(Q_{12} + 2Q_{66}\right)s^2c^2, \\ \bar{Q}_{12} &= \left(Q_{11} + Q_{22} - 4Q_{66}\right)s^2c^2 + Q_{12}\left(c^4 + s^2\right), \\ \bar{Q}_{22} &= Q_{11}s^4 + Q_{22}c^4 + 2\left(Q_{12} + 2Q_{66}\right)s^2c^2, \\ \bar{Q}_{16} &= \left(Q_{11} - Q_{12} - 2Q_{66}\right)c^3s - \left(Q_{22} - Q_{12} - 2Q_{66}\right)s^3c, \\ \bar{Q}_{26} &= \left(Q_{11} - Q_{12} - 2Q_{66}\right)cs^3 - \left(Q_{22} - Q_{12} - 2Q_{66}\right)c^3s, \\ \bar{Q}_{66} &= \left(Q_{11} + Q_{22} - 2Q_{12} - 2Q_{66}\right)s^2c^2 + Q_{66}\left(s^4 + c^4\right). \end{aligned} \tag{4.4}$$

The c and s denote $cos\theta$ and $sin\theta$, respectively.

The local and global stresses in an angle lamina are related
to each other through the angle of the lamina θ

$$\begin{bmatrix} \sigma_1 \\ \sigma_2 \\ \tau_{12} \end{bmatrix} = [T] \begin{bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{bmatrix} \tag{4.5}$$

where

$$[T] = \begin{bmatrix} c^2 & s^2 & 2sc \\ s^2 & c^2 & -2sc \\ -sc & sc & c^2 - s^2 \end{bmatrix}. \tag{4.6}$$

4.3.2 Stress and Strain in a Laminate

$$\begin{aligned}
\begin{bmatrix} N_x \\ N_y \\ N_{xy} \end{bmatrix} &= \begin{bmatrix} A_{11} & A_{12} & A_{16} \\ A_{12} & A_{22} & A_{26} \\ A_{16} & A_{26} & A_{66} \end{bmatrix} \begin{bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \end{bmatrix} \\
&+ \begin{bmatrix} B_{11} & B_{12} & B_{16} \\ B_{11} & B_{12} & B_{16} \\ B_{16} & B_{26} & B_{66} \end{bmatrix} \begin{bmatrix} k_x \\ k_y \\ k_{xy} \end{bmatrix} \\
\begin{bmatrix} M_x \\ M_y \\ M_{xy} \end{bmatrix} &= \begin{bmatrix} B_{11} & B_{12} & B_{16} \\ B_{12} & B_{22} & B_{26} \\ B_{16} & B_{26} & B_{66} \end{bmatrix} \begin{bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \end{bmatrix} \\
&+ \begin{bmatrix} D_{11} & D_{12} & D_{16} \\ D_{11} & D_{12} & D_{16} \\ D_{16} & D_{26} & D_{66} \end{bmatrix} \begin{bmatrix} k_x \\ k_y \\ k_{xy} \end{bmatrix}
\end{aligned} \tag{4.7}$$

N_x, N_y - normal force per unit length
N_{xy} - shear force per unit length
M_x, M_y - bending moment per unit length
M_{xy} - twisting moments per unit length
ε^0, k- mid-plane strains and curvature of a laminate in x-y coordinates
The mid-plane strain and curvature is given by

$$\begin{aligned}
A_{ij} &= \sum_{k=1}^{n} (\overline{Q_{ij}})_k (h_k - h_{k-1}) i = 1,2,6, j = 1,2,6, \\
B_{ij} &= \frac{1}{2} \sum_{k=1}^{n} (\overline{Q_{ij}})_k (h_k^2 - h_{k-1}^2) i = 1,2,6, j = 1,2,6, \\
D_{ij} &= \frac{1}{3} \sum_{k=1}^{n} (\overline{Q_{ij}})_k (h_k^3 - h_{k-1}^3) i = 1,2,6, j = 1,2,6,
\end{aligned} \tag{4.8}$$

where the [A], [B], and [D] matrices are called the extensional, coupling, and bending stiffness matrices.

4.4 Failure Theory

4.4.1 Failure Process

A laminate will fail under increasing mechanical loading; however, the procedure of laminate failure may not be catastrophic. In some cases, some layers fail first, and the rest are able to continue to take additional loading until all the plies fail. A ply is fully discounted when a ply fails; then, the ply is replaced by a near-zero stiffness and strength. The procedure for finding the first ply failure in the present study follows the fully discounted method:

1. Compute the reduced stiffness matrix [Q] referred to as the local axis for each ply using its four engineering elastic constants E_1, E_2, E_{12}, and G_{12}.
2. Calculate the transformed reduced stiffness $[\bar{Q}]$ referring to the global coordinate system (x, y) using the reduced stiffness matrix [Q] obtained in step 1 and the ply angle for each layer.

3. Given the thickness and location of each layer, the three laminate stiffness matrices [A], [B], and [D] are determined.

4. Apply the forces and moments, $[N]_{xy}$, $[M]_{xy}$ solve Equation 5.5, and calculate the middle plane strain $[\sigma^0]_{xy}$ and curvature $[k]_{xy}$.

5. Determine the local strain and stress of each layer under the applied load.

6. Use the ply-by-ply stress-strain and related failure theories to determine the strength ratio.

4.4.2 Tsai-wu Failure Theory

Many different theories about the failure of an angle lamina have been developed for a unidirectional lamina, such as the maximum stress failure theory, maximum strain failure theory, Tsai-Hill failure theory, and Tsai-Wu failure theory. The failure theories of a lamina are based on the stresses in the local axes in the material. There are four normal strength parameters and one shear stress for a unidirectional lamina. The five strength parameters are:
$(\sigma_1^T)_{ult}$ = ultimate longitudinal tensile strength
$(\sigma_1^C)_{ult}$ = ultimate longitudinal compressive strength
$(\sigma_2^T)_{ult}$ = ultimate transverse tensile strength
$(\sigma_2^C)_{ult}$ = ultimate transverse compressive strength
$(\tau_{12})_{ult}$ = and ultimate in-plane shear strength

In the present study, Tsai-wu failure theory is taken to decide whether a lamina fails, because this theory is more general than the Tsai-Hill failure theory, which considers two different situations, the compression and tensile strengths of a lamina. A lamina is considered to fail if

$$\begin{aligned} H_1\sigma_1 + H_2\sigma_2 + H_6\tau_{12} + H_{11}\sigma_1^2 + H_{22}\sigma_2^2 \\ + H_{66}\tau_{12}^2 + 2H_{12}\sigma_1\sigma_2 < 1 \end{aligned} \tag{4.9}$$

is violated, where

$$SR = \frac{\text{Maximum Load}}{\text{Load Applied}} \tag{4.10}$$

The maximum load refers to that can be applied using Tsai-wu failure theory.

4.5 Genetic algorithm model

The genetic algorithm starts with multiple individuals with limited chromosome lengths, in which maybe none of these individuals fulfill the constraints. The GA is assumed to derive appropriate offspring based on the initial population as the GA continues. The traditional way to handle the constrained search of the GA is either to introduce repair strategies or to use a penalty function. Fig. 4.4 shows the classic flow chart of a GA framework, which includes selection, crossover, and mutation operators. However, GA is originally proposed to solve unconstrained problems; therefore, we suggest a new approach to address the constrained GA search problem in an unconstrained way.

Because of the existence of constraints, the population not only can be sorted by the fitness (obtained by the objective function) but also sorted by the constraint value obtained by the constraint function (assuming a constraint function exists), so

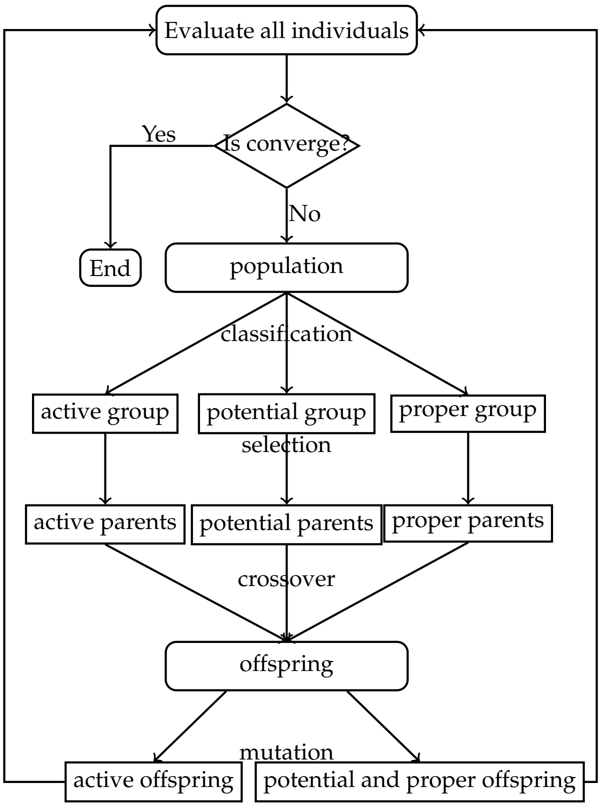

FIGURE 4.3: General flowchart of proposed GA model in which the parents is consist of three different groups.

the parents of the next generation can be chosen by the following three approaches. First, the population is sorted by fitness in ascending order, and individuals with smaller fitness are selected. These selected individuals form a group named as a proper group. Second, remove individual which satisfies constraints, and sort population by the difference between the individual's constraint value and the threshold of the constraint in descending order, and individuals with bigger differences are chosen to be the parents of the next generation. The group which forms are called potential group, and an individual from this group is referred to as a potential individual. Third, the population is sorted by fitness from low to high after removing individuals which fails to fit the constraints, select individuals with bigger fitness, and these individuals form the proper group. So the final parents' pool is consists of three groups, active group, potential group, and proper group. The number of active individuals, potential individuals, and proper individuals are called, respectively, active number, potential numbers, and proper number.

Each group in the parents' population has its role in the searching process. The problem within traditional GA is premature and has weak local search ability, therefore, traditional GAs are more likely to get stuck in a local optimum. To prevent the GA from experiencing early convergence and to improve the local search performance, the active group is proposed to overcome this problem. As its name suggests, this group would always live in the population. Because both active individual's fitness and constraint value are small, each individual can be treated as an independent gene clip. So their existence enriches the gene clip variety of the mating pool. The offspring of two active individuals are more likely to be an active individual, which can maintain the active group.

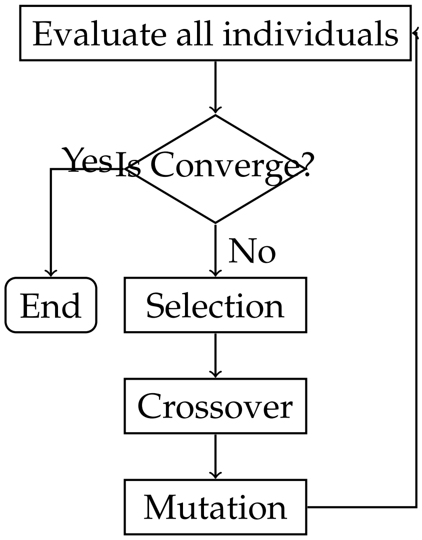

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FIGURE 4.4: Traditional GA Model.

For an individual in the potential group, it doesn't satisfy the constraint, however, it's supposed to evolve into a proper individual after multiple generations by modifying its chromosome structure or length. The crossover operation could happen between a potential individual with an active individual, or a potential individual, or a proper individual. The child of an active individual and a potential individual is more likely to be a potential individual, and this active individual could inject a new gene clip into this potential individual, therefore providing a new evolution direction.

A proper individual means a feasible solution, which fulfills all the constraints. However, there are still some drawbacks within it, for example, its fitness is low. The crossover operation could happen between a proper individual and any other individuals.

The mutation operator for an active group is different from the potential group and proper group because their roles in the searching process are different: the target of the potential group and proper group is to obtain a feasible solution; however, the role of the active group is to maintain the variety of gene clips in the mating pool.

Fig.4.3 shows the flow chart of the proposed GA. First, the population is divided into three groups, active group, potential group, and proper group by the above-mentioned method. Second, select an appropriate number of individuals from each group as parents, and the various selection scheme can be taken for each group.

The searching process can be divided into two phases according to whether proper individuals are generated or not. During the initial stage, no individual in the population is appropriate, which means the number of individuals in the proper group is zero. Both active group and potential group are full. After a couple of generations, some proper individuals could be produced. Then, the GA comes to its second phase, the number of proper individuals begins to increase, finally, the number in the proper group reaches its upper bound.

4.6 Experiment

First, we formulate a constrained problem by searching the optimal stacking sequence of cross-ply laminate under in-plane loading under the constraint whose

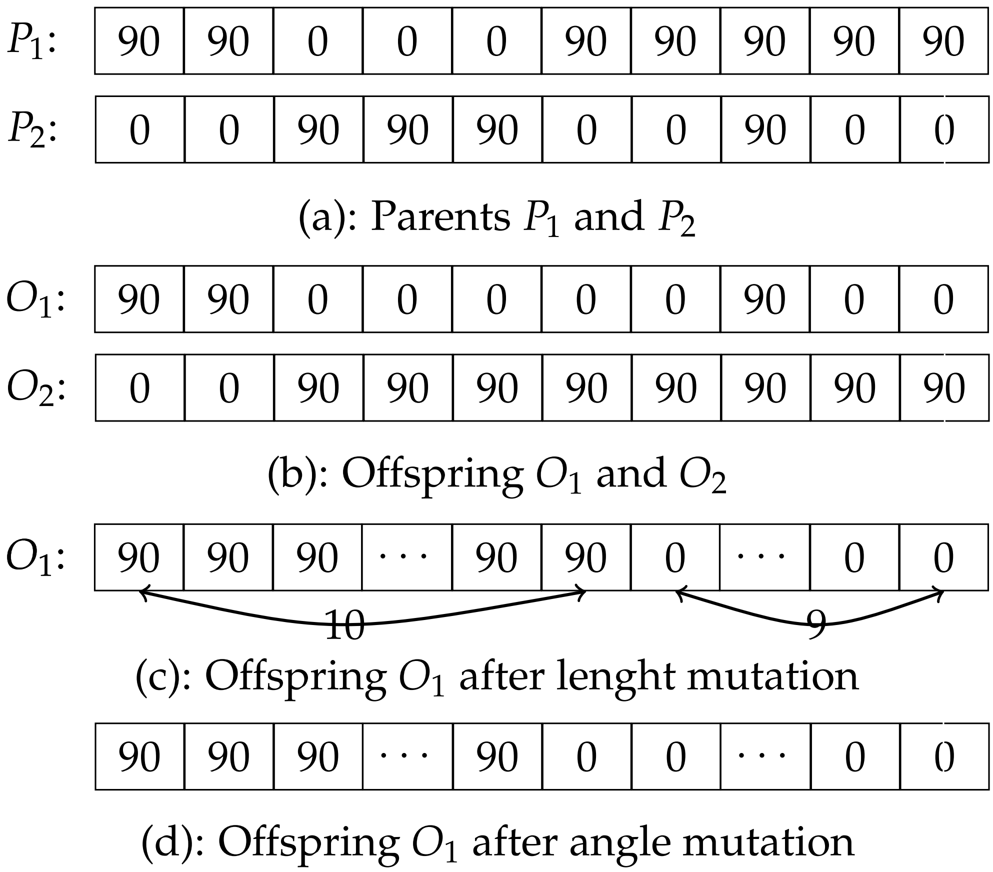

FIGURE 4.5: Examples of crossover, length mutation, angle mutation operator for proposed GA.

strength ratio is not less than two. Each lamina dimensions $1000 \times 1000 \times 0.165mm^3$ is adopted in this experiment, each graphite/epoxy, and glass/epoxy layer is assumed to cost 2.5 and 1 monetary units, respectively. The other material properties are shown in Tab. 4.1.

4.6.1 Problem formulation

In the present experiment, the optimal composite sequences, and the number of layers for a targeted strength ratio under in-plane loading conditions are investigated. The aim is to minimize the mass of a laminate composite for a targeted strength ratio based on the Tsai-wu failure theory. The design variables are the ply angles and the number of layers. Ply orientation restricted to a discrete set of angles (0, and 90 degrees). The problem can be formulated as the following equation:

Find: $\{\theta_k, n\}$ $\theta_k \in \{0, 90\}$,
Minimize: weight,
Subject to: strength ratio.

4.6.2 GA Operation

In the present study, floating encoding is adopted to represent a solution for the layup design of cross-ply laminate, Fig. 4.10(a) shows two parents P_1 and P_2 represent two cross-ply laminates, the corresponding laminates layups are $[0_3/90_7]$ and

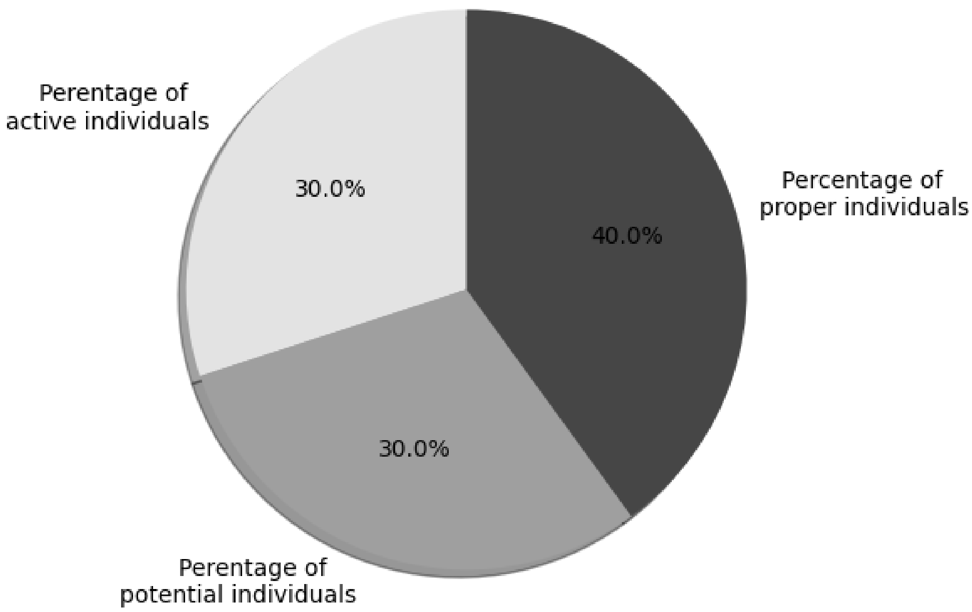

FIGURE 4.6: Percentage of active individuals from active group, potential individuals from potential group, and proper individuals from proper group in parent population.

$[0_6/90_4]$, respectively; Fig. 4.10(b) shows two offspring of parents P_1 and P_2 are consist of half of each parent's chromosome; Fig. 4.10(c) and 4.10(d) display the offspring after length and angle mutation, respectively.

For the chromosome length mutation, calculate the chromosome's strength ratio based on its sequence, if its strength ratio is less than the threshold, then increase its length; otherwise, reduce it. We introduce the term length mutation coefficient to control the length mutation. As shown in figure 4.10(b), the strength ratio of O_1 chromosome is 0.0854, and the strength ratio threshold is five. Suppose the length mutation coefficient takes two, then the corresponding increase length is the multiplication result of length mutation coefficient and the difference between current strength ratio and the threshold: the result is $5 \times (2 - 0.0854)$, round this number to its closest integer, which is 9. So the length of offspring O_1 changes from 10 to 19 after length mutation. For the angle mutation, randomly swap the gene from 0 to 90 in the chromosome, or the otherwise.

4.6.3 GA Parameters

Tab. 4.2 shows related GA parameters: the population is 40, and 50 percent is as the mating pool, so the parent population is 20; as shown in Fig. 4.6, the percentage of active individuals from the active group, potential individuals from the potential group, and proper individuals from the proper group are 0.3, 0.3, and 0.4, which means the corresponding number of these three types of individuals are 6, 6, and 8, respectively.

TABLE 4.2: Parameters of proposed GA model.

Parameter	Value
Population	40
Initial length range	[3-15]
Encoding	Integer
Percentage of parent	0.5
Percentage of active group	0.3
Percentage of potential group	0.3
Percentage of proper group	0.4
Selection strategy for active group	Ranking
Selection strategy for potential group	Ranking
Selection strategy for proper group	Ranking
Crossover strategy	One-point
Mutation strategy	Mass mutation
Length mutation coefficient	5
Angle mutation rate	0.1

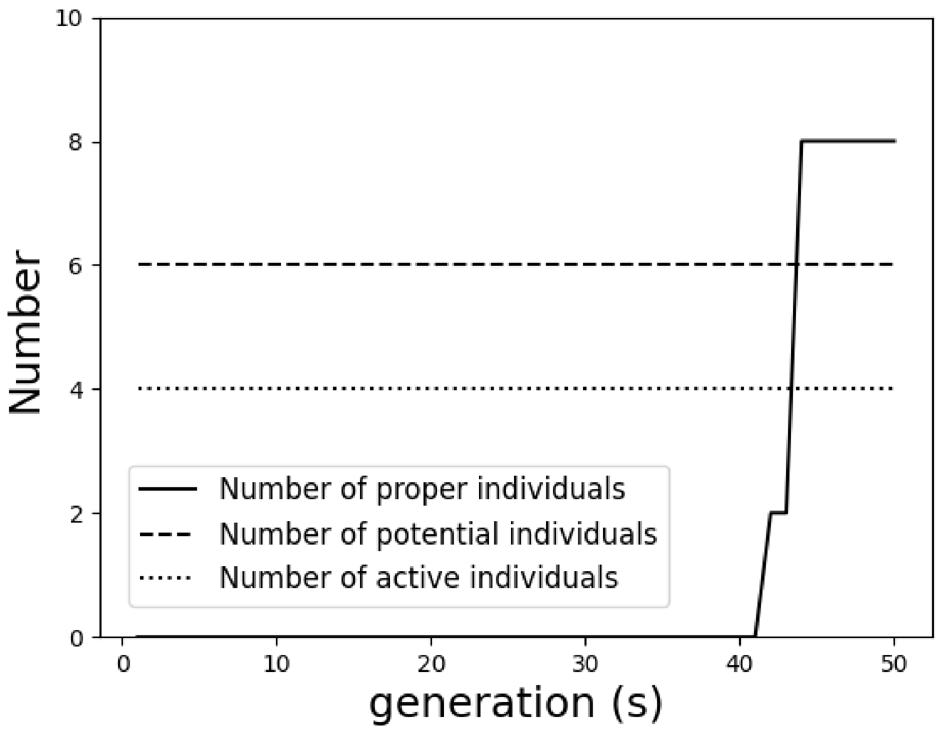

FIGURE 4.7: Number of individuals in each group as a function of generation.

4.7 Numerical Results and Discussion

To figure out how the number of individuals in each group varies during the GA process, we conduct one-time experiment and show the number of individuals in each generation in respect of GA generation. Second, to verify its performance and stability, the GA is run one hundred times: the best, worst case, and average results are presented, respectively. Finally, we compare the results with the work in the other literature.

Fig. 4.7 shows the number of individuals in each group during the one-time GA process. For both active group and potential groups, the number of individuals is fixed, and equal to its upper bound from the beginning to the end of the searching process. However, for the proper group, at the initial stage of GA, no individual

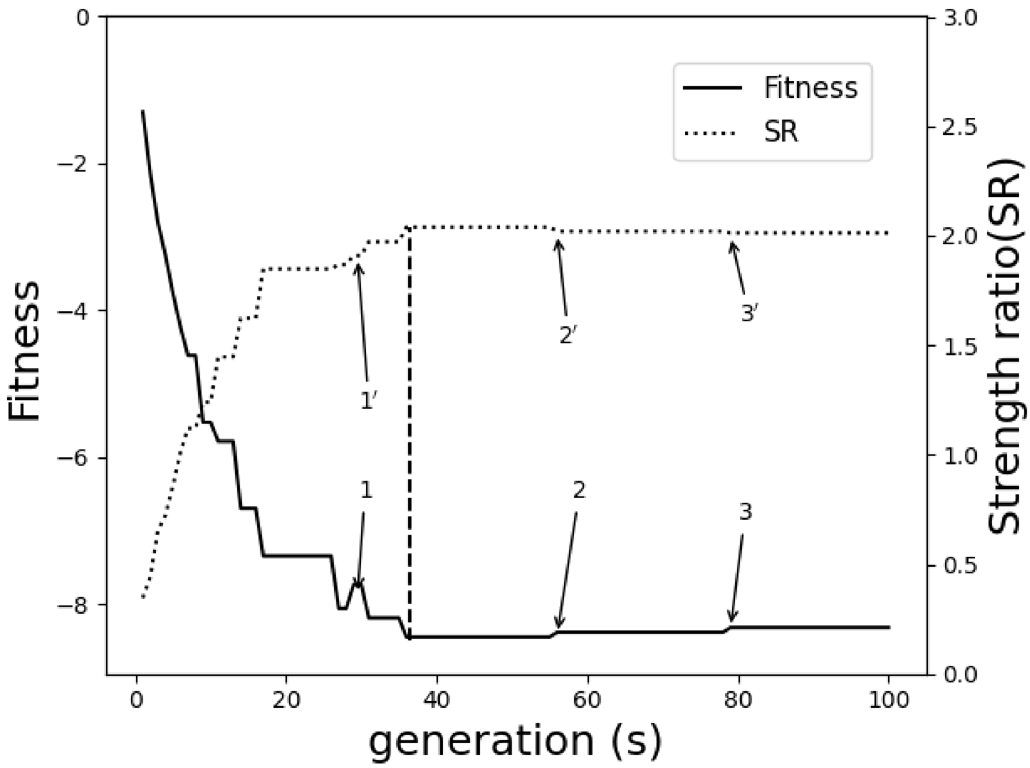

FIGURE 4.8: The fitness is the negation of the individual's mass. The solid curve is the fitness of the best individual in the population in respect to the generations; and the dotted line denotes its corresponding strength ratio. If no individuals in the population satisfy the constraint, the best individual is the one with the biggest strength ratio; if not, the best individual is the one with the smallest mass.

fulfills the constraint, so the number of proper individuals is zero. As seen from Fig. 4.7, after forty generations, proper individuals appear, and its population increases very quickly up to its upper bound.

There are two approaches that the GA could obtain a better solution: the first is increasing the length of the chromosome; the second one is adjusting the internal structure of a chromosome. The GA process can be divided into two phases by whether there are proper individuals or not. Fig. 4.8 shows the GA process in which the dashed vertical line is the watershed between the initial phase and the last phase. In the initial phase, no individual's strength ratio is over the specified threshold, and the main reason that the fitness gets smaller gradually is the increase of chromosome's length; At point 1 on the fitness curve, the fitness suddenly goes up, however, the corresponding strength ratio of point 1, denoted by the point 1′ on the generation-strength ratio curve, also increases. it is because of the adjustment of a chromosome's layup. Then GA comes to its second phase. During this phase, the GA already found proper individuals which could satisfy the constraint, so the target in this stage is to improve fitness. This means GA needs to adjust its inner structure, at the point 2 and 3 on the generation-fitness curve, the fitness curves go up, and the corresponding strength ratio of these two points slightly go down, but both of them still satisfy the constraint.

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TABLE 4.3: The optimum layup for the loading $N_x = 1e^6$ N when changing the length mutation coefficient, the performance of the GA can be improved when the lenght mutation coefficient is smaller.

Length mutation coefficient	Material	case	Stacking sequence	Strength ratio	Mass	Cost	Layer
6*1	3*glass-epoxy	worst	$[0_{40}/90_{26}]_s$	2.010	8.58	132	132
		best	$[90_{24}/0_{38}/\bar{90}]_s$	2.078	8.12	125	125
		average		2.012	7.83	123	123
	3*graphite-epoxy	worst	$[0_9/90_4/\bar{0}]_s$	2.17	1.41	68	27
		best	$[0_9/90_1/\bar{0}]_s$	2.15	1.10	53	21
		average		2.018	1.47	70	28
6*5	3*glass-epoxy	worst	$[0_{36}/90_{32}]_s$	2.009	8.84	136	136
		best	$[0_{36}/90_{26}/\bar{90}]_s$	2.003	8.12	125	125
		average		2.008	8.55	131	131
	3*graphite-epoxy	worst	$[0_9/90_{12}]_s$	2.006	2.20	105	42
		best	$[0_8/90_3/\bar{0}]_s$	2.001	1.20	57	23
		average		2.022	1.54	73	29

TABLE 4.4: Comparison of experiment results of Choudhury and Mondal's[7] and current study under in-plane loading $N_x = 1e6$ N. The results of present study is from previous experiment.

Cross Ply $[0_M/90_N]$	Choudhury and Mondal's study		Present study	
Material	Glass-Epoxy	Graphite-Epoxy	Glass-Epoxy	Graphite-Epoxy
M	68	17	76	19
N	72	18	49	2
no. of lamina(n)	140	35	125	21
SR	2.01	2.10	2.078	2.15
weight	9.10	1.84	8.12	1.10
cost	140	87.5	125	53

Tab. 4.3 shows the searching results after conducting this experiment one hundred times in two length mutation coefficient cases for glass-epoxy and graphite-epoxy material, respectively. The best, worst case, and average experiment results are showed in this table. For the glass-epoxy material, if the length mutation coefficient takes one, the best and worst sequences are $[0_{40}/90_{26}]_s$, $[90_{24}/0_{38}/\bar{90}]_s$; the average mass, cost, and number of layers are 7.83, 123, 123. If we increase its length mutation coefficient, suppose it is five, the number of layers for best and worst cases are 125 and 136; the average mass, cost, and number of layers are 8.55, 131, 131. When graphite-epoxy is taken as the experiment material, similar experiment results are found. Comparing these two results, we see that a bigger value of length mutation coefficient can improve this GA's performance. This is because the mutation coefficient can control both the convergence speed and search performance, a small mutation coefficient would slow the convergence speed, however, it would lead to a small-grained exploitation in the local space.

Tab. 4.4 shows the optimal cross-ply sequences by the proposed GA and Choudhury and Mondal's[7] study. For the loading case $Nx = 1$ MPa m, the optimal layups are a $[0_{68}/90_{72}]$ cross-ply laminate if glass-epoxy is taken; however, in the present study, a $[90_{24}/0_{38}/\bar{90}]_s$ glass-epoxy cross-ply laminate is found which significantly reduces both the cost and weight, and it satisfies the constraint. Similarly, if graphite-epoxy is taken, compared with the $[0_{17}/90_{18}]$ cross-ply laminate, an alternative solution is found, its layup is $[0_9/90_1/\bar{0}]_s$. For both cases, we can see that the experiment results show that using the present proposed GA can obtain better results.

4.8 conclusions

In this paper, we reviewed the use of the proposed ga framework, classical lamination theory, and tsai-wu failure theory for the layup design for cross-ply laminate. Because GA is primarily used to solve an unconstrained problem, and it is not suitable for a constrained problem. In the present study, we deal with this constrained problem by mixing strategies of selection methods instead of adding punishment terms into the objective function. So the constraint problem can be solved in an unconstrained way.

This variant of the GA provides a new approach to address the constrained search for optimization of laminated composite, and this method can be easy to apply in other domains. At the same time, the proposed GA model is more complicated than the traditional GA model, which involves more parameters. To advance its performance, the fine-tuning of those parameters need more effort.

Acknowledgment

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4.9 Case2: Minimum Thickness Optimization

4.10 Introduction

Composite materials offer improved strength, stiffness, corrosion resistance, etc. over conventional materials, and are widely used as alternative materials for applications in various industries ranging from electronic packaging to golf clubs, and medical equipment to homebuilding, making aircraft structure to space vehicles. The stacking sequence and fiber orientation of composite laminates give the designer additional degree of freedom to tailor the design with respect to strength or stiffness. One widely known advantage of using composite material is can significantly reducing the weight of target structure, and many researchers attempted to improve the efficiency of using composite material by minimizing the thickness[36, 37, 13, 39, 20, 15, 44, 2, 8, 35, 1, 25, 21, 38, 5, 34, 28, 41, 43, 11, 17].

In practice, fiber orientations are restricted to a finite set of angles and ply thickness is a specific numeric value. Because the design variables are not continueous, a gradient-based optimization procedure, such as the gradient descent method, is not suitable to cope with such problems. Moreover, gradient-based optimization approach is very easily to get trapped in local minima, and many local optimum may exist in structural optimization problems. A stochastic optimization, such as the genetic algorithm(GA) and simulated annealing(SA), can deal with optimization problems with discrete variables. Besides, the stochastic method could escape from the local optimum, and obtain the global optimum. GA is one of the most reliably stochastic algorithms, which has been widely used in solving constraint design for composite laminate[6, 41, 31, 43, 10, 32, 14, 18, 30]. Although GA gains different advantages for solving discrete problems, many disadvantages exist within this approach. First, the optimization process of GA parameters, such as the population size, parent population,mutation percentage, etc., is very tedious; Second, the GA needs to evaluate the objective functions many times to achieve the optimization, and the computation cost is very high; the last problem within GA is the premature convergence. GA consists of five basic parts: the variable coding, selection scheme, crossover operator, mutation operator, and how the constraints are handled.

The first issue when implementing a GA is the representation of design variables, and an appropriate design representation is crucial to enhance the efficiency of GA. The canonical GA has always used binary strings to encode alternative solutions, however, some argued that the minimal cardinality, i.e., the binary representation, is not the best option.

Selection scheme plays a critical role in balancing the dilemma of exploration and exploitation inherent in GA, and various selection methods, for example, roulette wheel, elitist, and tournament, etc. have been proposed to overcome this issue. Both roulette selection and tournament selection are well-studied and widely employed in the optimization design of laminated composite due to their simplicity to code and efficiency for both nonparallel and parallel architectures.

Crossover is another crucial operator introduced into the GA methodology framework, in which the alternative solution is generated from the mating pool. multiple types of crossover operator have been utilized in the optimization design of composite structures, such as, one-point, two-point, and uniform crossover.

GA is originally proposed for unconstrained optimization. However, in order to deal with constrained design for composite laminate, some techniques were introduced into the GA. The first method is using of data structure, special data structure was developed to fulfill the symmetry constraint of the laminate, which consists of coding only half of the laminate and considering that each stack of the laminate is formed by two laminae with the same orientation but opposite signs[20, 19]. A penalty function is developed to convert a constrained problem into an unconstrained problem by adding a penalty term to the objective function. Another method to solve the constrained problem is introducing repair strategy by Todoroki and Haftka [42], which is aim to transform infeasible solutions to feasible solution by incorporating problem-specific knowledge.

Another major concern within GA is the convergence speed in terms of the time and computation cost needed to reach a solution of desired quality. The objective function based on the CLT is excessively time-consuming and complicate to evaluate, besides, the target function of GA needs to calculate many times. The traditional method to deal with this issue is by increasing the selection pressure to accelerate the convergence speed, however, in some cases, this approach does not achieve an ideal result. Because the GAs just provides a methodological framework to deal with tricky problems, which is heavily inspired by evolution of biology, it is unnecessary to exactly follow all the GA operation. It is possible to just perform one or more GA operations, and incorporate other techniques into GA. In the present study, a variant of mutation operator is introduced to accelerate the convergence process.

To check the feasibility of a laminate composite by imposing a strength constraint, various failure criterion have been proposed to decide whether it fails or not, such as maximum stress failure theory, maximum strain failure theory, Tsai-Hill

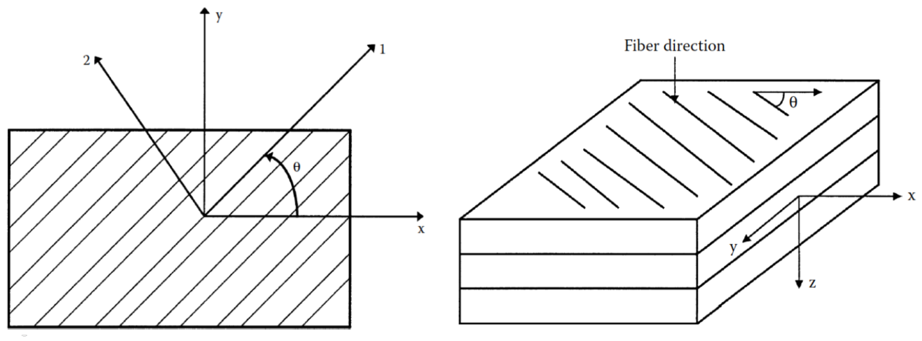

FIGURE 4.9: Local and global axes of an angle lamina

Failure theory, and Tsai-Wu criterion. Each theory is proposed based on massive experiment data or complicate mathematical model, however single use any of them may lead to a false optimum design for some loading case due to the particular shape of its failure envelope. In order to overcome this disadvantage within every failure theory, two reliably failure criteria, maximum stress theory and Tsai-wu criterion are employed to check whether the composite laminate fullfills the constraint.

The rest of the paper is organized as follows. Section 2 explains the classical laminate theory and the failure criteria taken in the present study. Section 3 explains the proposed method of selection strategy and self-adaptative parameters for mutation during the GA process. Section 4 describes the result of the numerical experiments in different cases, and in the conclusion section, we dicuss the results.

4.11 Analysis of stress and strain for composite material

4.11.1 Stress and Strain in a Lamina

A single lamina has a small thickness under plane stress, and its upper and lower surfaces of lamina are free from external loads. According to Hooke's Law, the three-dimensional stress-strain equations can be reduced to two-dimensional stress-strain equations. The stress-strain relation in local axis 1-2 is

$$\begin{bmatrix} \sigma_1 \\ \sigma_2 \\ \tau_{12} \end{bmatrix} = \begin{bmatrix} Q_{11} & Q_{12} & 0 \\ Q_{12} & Q_{22} & 0 \\ 0 & 0 & Q_{66} \end{bmatrix} \begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \gamma_{12} \end{bmatrix}, \tag{4.11}$$

where $Q_{ij}$are the stiffnesses of the lamina that are related

to engineering elastic constants given by

$$\begin{aligned} Q_{11} &= \frac{E_1}{1 - v_{12}v_{21}}, \\ Q_{22} &= \frac{E_2}{1 - v_{12}v_{21}}, \\ Q_{66} &= G_{12}, \\ Q_{12} &= \frac{v_{21}E_2}{1 - v_{12}v_{21}}, \end{aligned} \tag{4.12}$$

where E_1, E_2, v_{12}, G_{12} are four independent engineering elastic constants, which are defined as follows: E_1 is the longitudinal Young's modulus, E_2 is the transverse Young's modulus, v_{12} is the major Poisson's ratio, and G_{12} is the in-plane shear modulus.

Stress strain relation in the global x-y axis is

$$\begin{bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{bmatrix} = \begin{bmatrix} \bar{Q}_{11} & \bar{Q}_{12} & \bar{Q}_{16} \\ \bar{Q}_{12} & \bar{Q}_{22} & \bar{Q}_{26} \\ \bar{Q}_{16} & \bar{Q}_{26} & \bar{Q}_{66} \end{bmatrix} \begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{bmatrix}, \tag{4.13}$$

where

$$\begin{aligned}
\bar{Q}_{11} &= Q_{11}cos^4\theta + Q_{22}sin^4\theta + 2\left(Q_{12} + 2Q_{66}\right)sin^2\theta cos^2\theta, \\
\bar{Q}_{12} &= \left(Q_{11} + Q_{22} - 4Q_{66}\right)sin^2\theta cos^2\theta + Q_{12}\left(cos^4\theta + sin^2\theta\right), \\
\bar{Q}_{22} &= Q_{11}sin^4\theta + Q_{22}cos^4\theta + 2\left(Q_{12} + 2Q_{66}\right)sin^2\theta cos^2\theta, \\
\bar{Q}_{16} &= \left(Q_{11} - Q_{12} - 2Q_{66}\right)cos^3\theta sin\theta - \left(Q_{22} - Q_{12} - 2Q_{66}\right)sin^3\theta cos\theta, \\
\bar{Q}_{26} &= \left(Q_{11} - Q_{12} - 2Q_{66}\right)cos\theta sin^3\theta - \left(Q_{22} - Q_{12} - 2Q_{66}\right)cos^3\theta sin\theta, \\
\bar{Q}_{66} &= \left(Q_{11} + Q_{22} - 2Q_{12} - 2Q_{66}\right)sin\theta^2 cos\theta^2 + Q_{66}\left(sin\theta^4 + cos\theta^4\right).
\end{aligned} \tag{4.14}$$

The local and global stresses in an angle lamina are related
to each other through the angle of the lamina θ, it can be written as

$$\begin{bmatrix} \sigma_1 \\ \sigma_2 \\ \tau_{12} \end{bmatrix} = [T] \begin{bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{bmatrix}, \tag{4.15}$$

where

$$[T] = \begin{bmatrix} cos^2\theta & sin^2\theta & 2sin\theta cos\theta \\ sin^2\theta & cos^2\theta & -2sin\theta cos\theta \\ -sin\theta cos\theta & sin\theta cos\theta & cos^2\theta - sin^2\theta \end{bmatrix}. \tag{4.16}$$

4.11.2 Stress and Strain in a Laminate

For forces and moment resultants acting on laminates, such as in plate and shell structures, the relationship between applied forces and moment and displacement can be given by

$$
\begin{bmatrix} N_x \\ N_y \\ N_{xy} \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} & A_{16} \\ A_{12} & A_{22} & A_{26} \\ A_{16} & A_{26} & A_{66} \end{bmatrix} \begin{bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \end{bmatrix} + \begin{bmatrix} B_{11} & B_{12} & B_{16} \\ B_{11} & B_{12} & B_{16} \\ B_{16} & B_{26} & B_{66} \end{bmatrix} \begin{bmatrix} k_x \\ k_y \\ k_{xy} \end{bmatrix},
$$

$$
\begin{bmatrix} M_x \\ M_y \\ M_{xy} \end{bmatrix} = \begin{bmatrix} B_{11} & B_{12} & B_{16} \\ B_{12} & B_{22} & B_{26} \\ B_{16} & B_{26} & B_{66} \end{bmatrix} \begin{bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \end{bmatrix} + \begin{bmatrix} D_{11} & D_{12} & D_{16} \\ D_{11} & D_{12} & D_{16} \\ D_{16} & D_{26} & D_{66} \end{bmatrix} \begin{bmatrix} k_x \\ k_y \\ k_{xy} \end{bmatrix}, \tag{4.17}
$$

where
N_x, N_y - normal force per unit length;
N_{xy} - shear force per unit length;
M_x, M_y - bending moment per unit length;
M_{xy} - twisting moments per unit length;
ε^0, k- mid plane strains and curvature of a laminate in x-y coordinates.
The mid plane strain and curvature is given by

$$
\begin{aligned}
A_{ij} &= \sum_{k=1}^{n} (\overline{Q_{ij}})_k (h_k - h_{k-1}) i = 1,2,6, j = 1,2,6, \\
B_{ij} &= \frac{1}{2} \sum_{k=1}^{n} (\overline{Q_{ij}})_k (h_k^2 - h_{k-1}^2) i = 1,2,6, j = 1,2,6, \\
D_{ij} &= \frac{1}{3} \sum_{k=1}^{n} (\overline{Q_{ij}})_k (h_k^3 - h_{k-1}^3) i = 1,2,6, j = 1,2,6.
\end{aligned} \tag{4.18}
$$

The [A], [B], and [D] matrices are called the extensional, coupling, and bending stiffness matrices, respectively. The extensional stiffness matrix $[A]$ relates the resultant in-plane forces to the in-plain strains, and the bending stiffness matrix $[D]$ couples the resultant bending moments to the plane curvatures. The coupling stiffness matrix $[B]$ relates the force and moment terms to the midplane strains and midplane curvatures.

Failure criteria for composite materials are more difficult to predict due to structural and material complexity in comparison to isotropic materials. The failure process of composite materials can be regarded from microscopic and macroscopic points of view. Most popular criteria about the failure of an angle lamina are in terms of macroscopic failure criteria, which are based on the tensile, compressive and shear strengths. According to the failure surfaces, these criteria can be classified into two classes: one is called independent failure mode criteria which includes the maximum stress failure theory, maximum strain failure theory because their failure envelope are rectangle; another is called quadratic polynomial which includes Tsai-Wu, Chamis, Hoffman, and Hill criteria because their failure surfaces are of ellipsoidal shape. In the present study, the two most reliable failure criteria are taken, Maximum stress and Tsai-wu. Both of these two failure criteria are based on the stresses in the local axes instead of principal normal stresses and maximum shear stresses, and four normal strength parameters and one shear stress for a unidirectional lamina are involved. The five strength parameters are

$(\sigma_1^T)_{ult}$ = ultimate longitudinal tensile strength(in direction 1),
$(\sigma_1^C)_{ult}$ = ultimate longitudinal compressive strength,
$(\sigma_2^T)_{ult}$ = ultimate transverse tensile strength,
$(\sigma_2^C)_{ult}$ = ultimate transverse compressive strength, and
$(\tau_{12})_{ult}$ = and ultimate in-plane shear strength.

4.11.3 Maximum stress failure criterion

Maximum stress(MS) failure theory consists of maximum normal stress theory proposed by Rankine and maximum shearing stress theory by Tresca. The stresses applied on a lamina can be resolved into the normal and shear stresses in the local axes. If any of the normal or shear stresses in the local axes of a lamina is equal or exceeds the corresponding ultimate strengths of the unidirectional lamina, the lamina is considered to be failed. That is

$\sigma_1 \geq (\sigma_1^T)_{ult}$ or $\sigma_1 \leq -(\sigma_1^C)_{ult}$,
$\sigma_2 \geq (\sigma_2^T)_{ult}$ or $\sigma_2 \leq -(\sigma_2^C)_{ult}$,
$\tau_{12} \geq (\tau_{12})_{ult}$ or $\tau_{12} \leq -(\tau_{12})_{ult}$.

where σ_1 and σ_2 are the normal stresses in the local axes 1 and 2, respectively; τ_{12} is the shear stress in the symmetry plane 1-2.

4.11.4 Tsai-wu failure criterion

The TW criterion is one of the most reliable static failure criteria which is derived from the von Mises yield criterion. A lamina is considered to fail if

$$\begin{aligned} &H_1\sigma_1 + H_2\sigma_2 + H_6\tau_{12} + H_{11}\sigma_1^2 + H_{22}\sigma_2^2 \\ &\quad + H_{66}\tau_{12}^2 + 2H_{12}\sigma_1\sigma_2 < 1 \end{aligned} \tag{4.19}$$

is violated, where

$$\begin{aligned} H_1 &= \frac{1}{(\sigma_1^T)_{ult}} - \frac{1}{(\sigma_1^C)_{ult}}, \\ H_{11} &= \frac{1}{(\sigma_1^T)_{ult}(\sigma_1^C)_{ult}}, \\ H_2 &= \frac{1}{(\sigma_2^T)_{ult}} - \frac{1}{(\sigma_2^C)_{ult}}, \\ H_{22} &= \frac{1}{(\sigma_2^T)_{ult}(\sigma_2^C)_{ult}}, \\ H_{66} &= \frac{1}{(\tau_{12})_{ult}^2}, \\ H_{12} &= -\frac{1}{2}\sqrt{\frac{1}{(\sigma_1^T)_{ult}(\sigma_1^C)_{ult}(\sigma_2^T)_{ult}(\sigma_2^C)_{ult}}}. \end{aligned} \tag{4.20}$$

H_i is the strength tensors of the second order; H_{ij} is the strength tensors of the fourth order. σ_1 is the applied normal stress in direction 1; σ_2 is the applied normal stress in the direction 2; and τ_{12} is the applied in-plane shear stress.

4.11.5 Safety factor

The safety factor, or yield stress, is how much extra load beyond is intended a composite laminate will actually take, which is an indication of the material's load-carrying capacity. If the value is less than 1.0, it means failure. The safety factor is defined as

$$SF = \frac{\text{Maximum Load Which Can Be Applied}}{\text{Load Applied}}. \tag{4.21}$$

The safety factor based on maximum stress theory is calculated by the following method: first, the principal stresses(σ_1^k,σ_2^k, and τ_{12}^k) are obtained by experiment; evaluate the safety factor along each direction according to equation 5.10; The minimum value among these safety factors are denoted as the safety factor of the lamina, SF_{MS}^k, it can be written as

$$SF_{MS}^k = \text{min of} \begin{cases} SF_X^k = \begin{cases} \frac{X_t}{\sigma_{11}}, \text{ if } \sigma_{11} > 0 \\ \frac{X_c}{\sigma_{11}}, \text{ if } \sigma_{11} < 0 \end{cases} \\ SF_Y^k = \begin{cases} \frac{Y_t}{\sigma_{22}}, \text{ if } \sigma_{22} > 0 \\ \frac{Y_c}{\sigma_{22}}, \text{ if } \sigma_{22} < 0 \end{cases} \\ SF_S^k = \begin{cases} \frac{S}{|\tau_{12}|} \end{cases} \end{cases} . \tag{4.22}$$

Assuming the composite laminate under an in-plane loading f, the corresponding stress on local stress in direction 1, local stress in direction 2, and shear stress for the kth lamina are $\sigma_1 SF_{TW}^k$, $\sigma_2 SF_{TW}^k$, and $\tau_{12} SF_{TW}^k$, respectively. Substitute them into equation 5.8, the expression are given by

$a(SF_{TW}^k)^2 + b(SF_{TW}^k) - 1 = 0$,

where

$a = H_{11}(\sigma_1)^2 + H_{22}(\sigma_2)^2 + H_{66}(\tau_{12})^2 + 2H_{12}\sigma_1\sigma_2$,

$b = H_1\sigma_1 + H_2\sigma_2 + H_6\tau_{12}$.

Solve the above equation, the safety factor for the kth lamina is

$SF_{TW}^k = |\frac{-b+\sqrt{b^2+4a}}{2a}|$.

Then, the minimum of SF_{TW}^k is taken as the safety factor of the laminate which is written as

$SF_{TW} = \text{ min of } SF_{TW}^k \text{ for } k = 1, 2, \cdots, m-1, m$.

4.12 Methodology

4.12.1 Objective function

The optimization problem can be formulated by searching the optimal stacking sequence of composite laminate. There are two design variables here, the angles in the laminate, and the number of layers that each fiber orientation has. The objective function is formulated as

P_1:	+7	+7	+7	+7	+7	+7	+7	+7	-9	-9
P_2:	+19	+19	+19	+19	-36	-36	-36	-36	-36	-36

(a): Parents P_1 and P_2

O_1:	+13	+13	+13	+13	+13	+13	-27	-27	-27	-27
O_2:	+22	+22	+22	+22	+22	+22	+22	+5	+5	+5

(b): Offspring O_1 and O_2

$$O_1: \underbrace{+13 \;|\; +13 \;|\; +13 \;|\; \cdots \;|\; +13 \;|\; +13}_{11} \;|\; \underbrace{-27 \;|\; \cdots \;|\; -27 \;|\; -27}_{7}$$

(c): Offspring O_1 after lenght mutation

+12	+12	+12	· · ·	+12	+12	-26	· · ·	-26	-26

(d): Offspring O_1 after angle mutation

FIGURE 4.10: GA Operators

$$F = 2t_0 \sum_{k=1}^{n} n_k, \\ SF_{MS} \geq 1, \\ SF_{TW} \geq 1. \tag{4.23}$$

The first term represents the total thickness of the composite laminates, t_0 is the ply thickness; n_k is the number of plies in the kth lamina, in which the fiber orientation is θ_k. The constraints here are two safety factors should not less than 1, which means $SF_{MS} \geq 1$, and $SF_{TW} \geq 1$, respectively.

4.12.2 Encoding

Due to the simplicity and efficiency of float representation, this encoding method is implemented to represent a possible solution. As shown in Figure 4.10 (a), these two chromsomes represent a $[+8_7/-9_2]_s$ carbon T300/5308 laminated composite, and $[+19_4/-36_6]_s$, respectively. Becasue the laminate adopted in this paper is symmetric to its mid-plane, so only half needs to be encoded.

4.12.3 Selection

The purpose of the selection operator is to choose mating pool to produce alternative solutions of better fitness. Traditional methods of selecting strategies only take the fitness of individuals into account, however, due to the existence of constraint, various selection schemes are implemented to select the mating set. Based on different selection schemes, the parents of next generation can be divided into three groups: proper groups, active groups, and potential groups according to different selecting methods.

Proper parents mean in which individual fullfills the constraints, which are chosen by the individual's fitness, individuals with better fitness are more likely to be chosen if they fit the constraint; active group means that individual within this group is supposed to always exist in the parents during the GA, which are selected by fitness, ignoring the constraint; The individuals from the active group may not correspond to feasible solutions, but their existence enriches the variety of the gene clips. Potential group means that individuals are likely to turn into proper individual after a couple of generations, and potential individuals are chosen by constraint function, the more the individual fulfills the constraint, the more possibility it will be selected.

4.12.4 Crossover

The crossover operator happens among these three groups. the child of two proper groups is more likely to be a proper individual which can be used to obtain an alternative feasible solution. the child of an active individual and a potential individual can significantly change the gene of an active individual's chromosome, which makes the individual evolve toward a new direction. The offspring of two active individuals are more likely to be an active individual, which can maintain the active group. The figure.4.10 (b) shows two children O_1 and O_2 from two parents P_1 and

P_2, each angle C_a and its length C_l of a child are obtained by the following formula

$$\begin{cases} C_a &= (P1_a + P2_a)/2 \\ C_l &= (P1_l + P2_l)/2 \end{cases}. \tag{4.24}$$

TABLE 4.5: Properties of T300/5308 carbon/epoxy composite

Property	Symbol	Unit	Graphite/Epoxy
Longitudinal elastic modulus	E_1	GPa	181
Traverse elastic modulus	E_2	GPa	10.3
Major Poisson's ratio	v_{12}		0.28
Shear modulus	G_{12}	GPa	7.17
Ultimate longitudinal tensile strength	$(\sigma_1^T)_{ult}$	MP	1500
Ultimate longitudinal compressive strength	$(\sigma_1^C)_{ult}$	MP	1500
Ultimate transverse tensile strength	$(\sigma_2^T)_{ult}$	MPa	40
Ultimate transverse compressive strength	$(\sigma_2^C)_{ult}$	MPa	246
Ultimate in-plane shear strength	$(\tau_{12})_{ult}$	MPa	68

4.12.5 Mutation

A mutation direction is imposed on the mutation operator which to make sure the individual evolving toward the right direction. The mutation direction, denoted by md, is an n dimensional vector corresponding to the number of constraints, it is decided by the constraint thresholds CT_i and the current individual's constraint value, denoted as CV_i, The mutation vector can be obtained by the following formula

$$\text{md} = [CT_1, \cdots, CT_{n-1}, CT_n] - [CV_0, \cdots, CV_{n-1}, CV_n].$$

During this operator, the mutation procedure is consist of two phases: the length mutation of the chromosome, and the angle mutation of the chromosome. Because the chromosome's length is positively correlated with the individual's fitness, the coefficient of length mutation denoted by C_l, if $\sum_{i=1}^{N} CT_i$ great than $\sum_{i=1}^{N} CV_i$, the mutation length is restricted to the range $[0, (C_l \sum_{i=1}^{N} (CT_i - CV_i))/N]$, which means increase the chromosome's length; Assuming a $[+13_6/-27_4]_s$ T300/5308 carbon/epoxy composite laminate under the loading $N_x = N_y = 10$ MPa m, it's property as shown in table 4.5. According to CLT and failure theory, the two safety factors SF_{MS} and SF_{TW} are 0.0539, and 0.0540, respectively. So the mutation vector and is $[0.9461, 0.9460]$, assuming the length mutation coefficient is 20, so the mutation range is from 0 to 18. A random number is generated from the range $[0, 18]$, supposing the outcome is 13, then a length generator is used to a list, the its sum is 13, suppose the list is [5, 8], the laminate after mutation is $[13_{11}/-27_{12}]_s$.

If the $\sum_{i=1}^{N} CT_i$ less than $\sum_{i=1}^{N} CV_i$, the mutation length is restricted to the range $([\sum_{i=1}^{N} CT_i - CV_i)/N, 0]$, which means the individual's fitness exceeds the threshold value, and decrease the chromsome's length. Assuming a $[+33_{35}/-29_{26}]_s$ T300/5308

TABLE 4.6: The optimum lay-ups using two distinct fiber angles under various biaxial loading cases

Loading $N_x/N_y/N_{xy}$ (MPa m)	Optimum lay-up sequences	Laminate thickness	Safety factor for Tsai-wu	Safe for max
10/5/0	$[33_{29}/\text{-}39_{25}/\text{-}\bar{39}]_s$	109	1.0074	1
20/5/0	$[33_{22}/\text{-}31_{24}]_s$	92	1.0055	1
40/5/0	$[29_{18}/\text{-}21_{23}/\text{-}\bar{21}]_s$	83	1.0034	1
80/5/0	$[\text{-}20_{27}/21_{25}/\bar{25}]_s$	105	1.0029	1
120/5/0	$[\text{-}18_{34}/17_{36}]_s$	140	1.0000	1

laminate is under loading $N_x = 10$ MPa, and $N_y = 5$ MPa, then, it's SF_{MS} constraint and SF_{TW} values are 1.0912, 1.0747, respectively. because the length mutation is 20, so the mutation range is from -2 to 0. This would decrease the chromosome's length.

$$\mathrm{LM} = \begin{cases} [0, (C_l \sum_{i=1}^{N} (CT_i - CV_i))/N], \text{ if } \sum_{i=1}^{N} CT_i > \sum_{i=1}^{N} CV_i \\ [(C_l \sum_{i=1}^{N} (CT_i - CV_i))/N, 0], \text{ if } \sum_{i=1}^{N} CT_i < \sum_{i=1}^{N} CV_i \end{cases} \tag{4.25}$$

The relationship between the angles in the composite laminate and the chromosome's fitness is unclear, so the mutation direction of chromosome's angle is random. The coefficient angle mutation is C_a, the angle mutation range is $[0, C_a \sum_{i=1}^{N} (|CT_i - CV_i|)]$ or $[C_a \sum_{i=1}^{N} (-|CT_i - CV_i|), 0]$. It is can be written as

$$\mathrm{P(AM)} = \begin{cases} 0.5, \ \mathrm{AM} = [0, C_a \sum_{i=1}^{N} (|CT_i - CV_i|)] \\ 0.5, \ \mathrm{AM} = [C_a \sum_{i=1}^{N} (-|CT_i - CV_i|), 0] \end{cases} . \tag{4.26}$$

4.13 Result and Discussion

In the present study, the T300/5308 graphite/epoxy material is used in the lay-up sequence optimization, and its properties as shown in table.4.5. Two constraints are imposed on the composite laminates which are the safety factor SF_{MS} , and safety factor SF_{TW}, and the threshold values for both of them is 1. The constraint values of an individual are CV_1 and CV_2. So the mutation vector here is a two-dimensional vector $[1 - CV_1, 1 - CV_2]$, and the coefficient of length mutation C_l and angle mutation C_a are 20 and 10, respectively.

To verify the reliability of the proposed method, two conditions are concerned: the first is only two distinct fiber orientation angles in the composite material; the second involves three distinct ply angles within the optimization process. In each situation, first, we present the search process by plotting relevant indicators, such as the fitness, strength ratio, and angle. Then, the optimum lay-ups under various loading cases are discussed.

Figure 4.11(a) shows how the optimal individual's fitness and strength ratio evolves during the GA process. The solid curve shows the fitness value, the dashed curve shows the Tsai-wu safety factor, and the dotted curve shows MS safety factor. If the smaller strength ratio fullfills the constraint, this laminate must satisfy all the constraints, for simplicity, only the smaller strength ratio is presented in the figure4.12(a). The method to chose optimal individual considering two following situations, if no individual in the current population meets constraint, the one with

TABLE 4.7: The optimum lay-ups using three distinct fiber angles under various biaxial loading cases

Loading $N_x/N_y/N_{xy}$ (MPa m)	Optimum lay-up sequences	Laminate thickness	Safety factor for Tsai-wu	Safe for max
10/5/0	$[37_{27}/\text{-}38_{27}/\text{-}5]_s$	110	1.0023	1
20/5/0	$[34_{24}/\text{-}32_{14}/\text{-}28_{11}]_s$	98	1.0237	1
40/5/0	$[21_{28}/\text{-}32_{19}/2_3]_s$	100	1.0617	1
80/5/0	$[\text{-}19_{24}/20_{27}/\text{-}17_{16}/\text{-}\bar{1}7]_s$	109	1.0056	1
120/5/0	$[\text{-}19_{33}/12_{13}/16_{28}]_s$	148	1.0105	1

the biggest fitness is selected as the optimal individual; if there are one or multiple individuals fullfills requirement, the one with the smallest fitness is chosen which means the smallest one has the biggest priority. Figure 4.11 (b) and 4.12(b) show how every fiber orientation changes, and Figure 4.11(c) and 4.12(c) display how the number of each angle varies.

At the beginning of this GA process, the fitness curves increases very quickly, because individual's two strength ratios are very small, so the difference between the individual's fitness and the imposed constraint threshold is a big positive number, so the range of mutation length is from 0 to $C_l(CT_0 - CV_0 + CT_1 - CV_1)/2$. The length of individual increases by n, which is a random number between 0 and $C_l(CT_0 - CV_0 + CT_1 - CV_1)/2$. As can be seen from Figure 4.11 (a), both of optimal individual's fitness and strength ratio increases very quickly. The range of angle mutation is from 0 to $C_a(CT_0 - CV_0 + CT_1 - CV_1)/2$, and the number of each angle also changes violently. The Figure.4.11 (a) and 4.12 (a) show this property at the initial stage. During this stage, increasing an individual's length playing a major role in increasing its fitness.

After a couple of generations, the optimal individual's fitness gets bigger, and the difference between individual's fitness and constraint threshold gets smaller. The range of mutation length turns smaller. At this stage, simply increase the individual's length doesn't make much difference in improve an individual's fitnees, and a better composite laminates lay-up can dramatically change the optimal individual's fitness. That's why the fitness curve oscillated violently in this stage. At the same time, the strength ratio curve keeps growing smoothly. But the growing speed gets more smaller.

When GA comes to its last phase, GA finds individuals that meet all the constraints. Now the optimal individual's fitness is greater than the safety factor. The range of mutation length is from $C_l(CT_0 - CV_0 + CT_1 - CV_1)/2$ to 0. It means individuals need to decrease its length and improve its internal structure to meet the constraint. That's why the fitness of optimal individual kept decreaing, however, the strength ratio curve still is greater than safety factor.

Table.4.8 shows the comparison with the result obtained by direct search simulated annealing(DSA) algorithm which was proposed by Akbulut and Sonmez[3]. Both of variant GA and DSA are able to find feasible solution, but when loading is $N_x = 80$, $N_y = 5$ MPa m, variant GA got a better solution than DSA. In the case that loadings are $N_x = 20$, $N_y = 5$ MPa m, and $N_x = 120$, $N_y = 5$ MPa m, the proposed GA offered an alternative solution. Compared with DSA method, the last advantage of variant GA is the number of layers doesn't have to be even.

TABLE 4.8: Comparison with the results of DSA

Loading	Akbulut and Sonmez's[3] Study				Present Study		
$N_x/N_y/N_{xy}$ (MPa m)	Optimum lay-up sequences	laminate thickness	TW	MS	Optimum lay-up sequences	laminate thickness	TW
10/5/0	$[37_{27}/\text{-}37_{27}]_s$	108	1.0068	1.0277	$[33_{29}/\text{-}39_{25}/\text{-}\bar{3}9]_s$	109	1.0074
20/5/0	$[31_{23}/\text{-}31_{23}]_s$	92	1.0208	1.1985	$[33_{22}/\text{-}31_{24}]_s$	92	1.0055
40/5/0	$[26_{20}/\text{-}26_{20}]_s$	80	1.0190	1.5381	$[29_{18}/\text{-}21_{23}/\text{-}\bar{2}1]_s$	83	1.0034
80/5/0	$[21_{25}/\text{-}19_{28}]_s$	106	1.0113	1.2213	$[\text{-}20_{27}/21_{25}/2\bar{5}]_s$	105	1.0029
120/5/0	$[17_{35}/\text{-}17_{35}]_s$	140	1.0030	1.0950	$[\text{-}18_{34}/17_{36}]_s$	140	1.0000

4.14 Conclusion

In this paper, we reviewed the use of variant GA for the optimal design of composite laminated material under in-plane loading based on Tsai-wu and maximum stress failure criteria. GA is proposed to search the optimal lay-up for laminated composite under different loading cases. Two situations are considered under the same loading, a set of two distinct angles, and three distinct angles.

By setting the constant values of length mutation coefficient and angle mutation coefficient at the beginning, the convergence speed of the search process can be controlled in an explicit way; During the optimization process, GA can adjust its length mutation range and angle mutation range based on the difference between individual's constraint values and constraint thresholds.

Finally, comparison of previous research and current result are presented. In some cases, the proposed GA in this paper is better off than DSA method. However, there is still many works to study within this GA, such as the fine-tuning of parameters taken in this GA.

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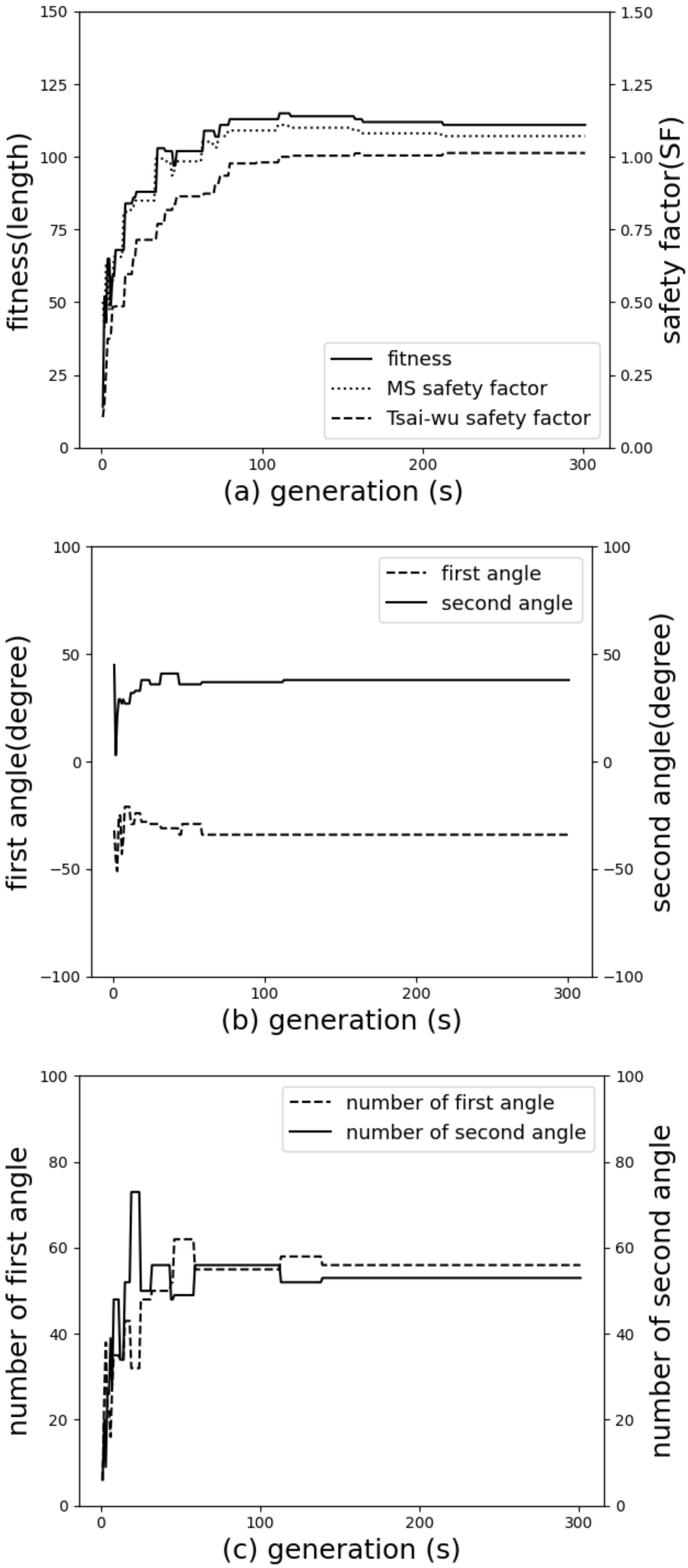


FIGURE 4.11: Two distinct angles

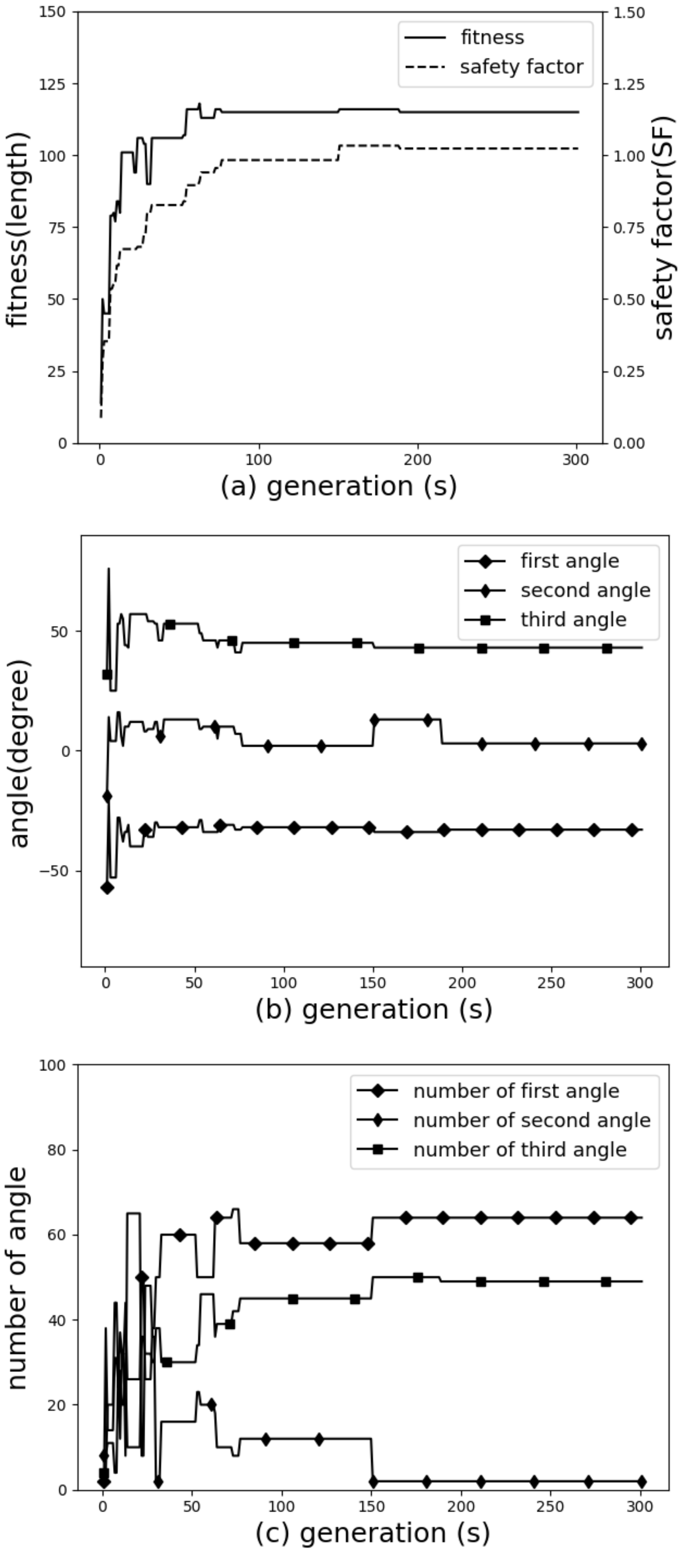


FIGURE 4.12: Three distinct angles

Chapter 5

Approximation of CLT Based on Artificial Neural Network

5.1 Introduction

Fiber-reinforced composite materials have been widely used in a variety of applications, which include electronic packaging, sports equipment, homebuilding, medical prosthetic devices, high-performance military structures, etc. because they offer improved mechanical stiffness, strength, and low specific gravity of fibers over conventional materials. The stacking sequence, ply thickness, and fiber orientation of composite laminates give the designer an additional 'degree of freedom' to tailor the design with respect to strength or stiffness. Classical lamination theory(CLT) and failure theory, e.g., Tsai-Wu failure criteria, is usually taken to predict the behavior of a laminate from a knowledge of the composite laminate properties of the individual layers and the laminate geometry.

However, the use of CLT needs intensive computation which takes an analytical method to solve the problem, since it involves massive matrix multiplication and integration calculation. Techniques of function approximation can accelerate the calculation process and reduce the computation cost. Artificial neural network(ANN), heavily inspired by biology and psychology, is a reliable tool instead of a complicated mathematical model. ANN has been widely used to solve various practical engineering problems in applications, such as pattern recognition, nonlinear regression, data mining, clustering, prediction, etc. Evolutionary artificial neural networks is a special class of artificial neural networks, in which evolutionary algorithms are introduced to design the topology of an ANN, and can be used at four different levels: connection weights, architectures, input features, and learning rules. It is shown that the combinations of ANN's and evolutionary algorithm [24] can significantly improve the performance of intelligent systems than that rely's on ANN's or evolutionary algorithms alone.

The rest of this paper is organized as the following: section II introduces the CLT and the failure criteria, which is used to check whether the composite material fails or not in the present study; section III covers the design of artificial neural network for a function approximation; section IV reviews the use of the genetic algorithm in the design of neural network architecture, and the techniques of parameters optimization during the training process; section V presents the result of the numerical experiments in different cases; in the conclusion part, we present and discuss the experiment results.

5.2 Classic lamination theory and Failure theory

5.2.1 Classic Lamination Theory

CLT is based upon three simplifying engineering assumptions: Each layer's thickness is small and consists of homogeneous, orthotropic material, and these layers are perfectly bonded together through the thickness; The entire laminated composite is supposed to be under in-plane loading; Normal cross-sections of the laminate is normal to the deflected middle surface, and do not change in thickness. Fig. 5.1 shows the coordinate system used for showing an angle lamina. The axis in the 1-2 coordinate system are called the local axis or the material axis, and the axis in the x-y coordinate system are called global axis.

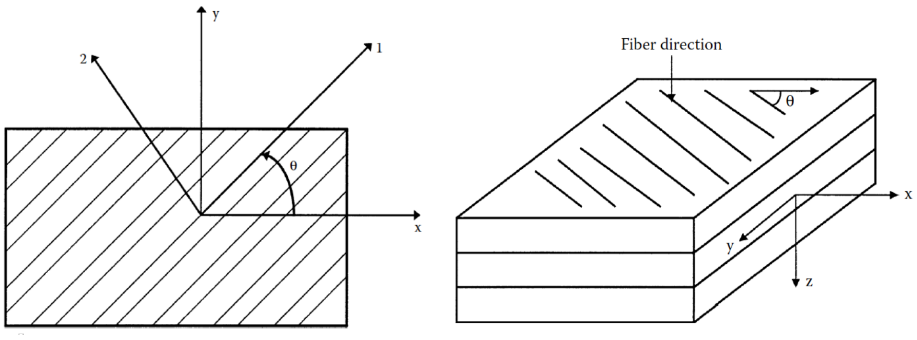


FIGURE 5.1: The left diagram shows the local and global axis of an angle lamina, which is from a laminate as shown in the right diagram.

Special cases of laminates, i.e., symmetric laminates, cross-ply laminates, are important in the design of laminated structures. A laminate is called an angle ply laminate if it has plies of the same material and thickness and only oriented at $+\theta$ and $-\theta$ directions. A model of an angle ply laminate is as shown in Fig. 5.2.

Stress and Strain in a Lamina

For a single lamina under in-plane loading whose thickness is relatively small, suppose the upper and lower surfaces of the lamina are free from external loading. According to Hooke's law, the three-dimensional stress-strain equations can be reduced to two-dimensional stress-strain equations in the composite material. The stress-strain relation in local axis 1-2 is

$$\begin{bmatrix} \sigma_1 \\ \sigma_2 \\ \tau_{12} \end{bmatrix} = \begin{bmatrix} Q_{11} & Q_{12} & 0 \\ Q_{12} & Q_{22} & 0 \\ 0 & 0 & Q_{66} \end{bmatrix} \begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \gamma_{12} \end{bmatrix} \tag{5.1}$$

where Q_{ij} are the stiffnesses of the lamina. And they are related to engineering elastic constants as follows:

$$\begin{aligned} Q_{11} &= \frac{E_1}{1-v_{12}v_{21}}, \\ Q_{22} &= \frac{E_2}{1-v_{12}v_{21}}, \\ Q_{66} &= G_{12}, \\ Q_{12} &= \frac{v_{21}E_2}{1-v_{12}v_{21}}, \end{aligned} \tag{5.2}$$

$+\theta$
$-\theta$
$\cdots$
$-\theta$
$+\theta$

FIGURE 5.2: Model for angle ply laminate

where E_1, E_2, v_{12}, G_{12} are four independent engineering elastic constants, which are defined as follows: E_1 is the longitudinal Young's modulus, E_2 is the transverse Young's modulus, v_{12} is the major Poisson's ratio, and G_{12} is the in-plane shear modulus.

Stress strain relation in the global x-y axis is

$$\begin{bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{bmatrix} = \begin{bmatrix} \bar{Q}_{11} & \bar{Q}_{12} & \bar{Q}_{16} \\ \bar{Q}_{12} & \bar{Q}_{22} & \bar{Q}_{26} \\ \bar{Q}_{16} & \bar{Q}_{26} & \bar{Q}_{66} \end{bmatrix} \begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{bmatrix} \tag{5.3}$$

where

$$\begin{aligned}
\bar{Q}_{11} &= Q_{11} cos^4\theta + Q_{22} sin^4\theta + 2\left(Q_{12} + 2Q_{66}\right) sin^2\theta cos^2\theta, \\
\bar{Q}_{12} &= \left(Q_{11} + Q_{22} - 4Q_{66}\right) sin^2\theta cos^2\theta + Q_{12}\left(cos^4\theta + sin^2\theta\right), \\
\bar{Q}_{22} &= Q_{11} sin^4\theta + Q_{22} cos^4\theta + 2\left(Q_{12} + 2Q_{66}\right) sin^2\theta cos^2\theta, \\
\bar{Q}_{16} &= \left(Q_{11} - Q_{12} - 2Q_{66}\right) cos^3\theta sin\theta - \left(Q_{22} - Q_{12} - 2Q_{66}\right) sin^3\theta cos\theta, \\
\bar{Q}_{26} &= \left(Q_{11} - Q_{12} - 2Q_{66}\right) cos\theta sin^3\theta - \left(Q_{22} - Q_{12} - 2Q_{66}\right) cos^3\theta sin\theta, \\
\bar{Q}_{66} &= \left(Q_{11} + Q_{22} - 2Q_{12} - 2Q_{66}\right) sin\theta^2 cos\theta^2 + Q_{66}\left(sin\theta^4 + cos\theta^4\right).
\end{aligned} \tag{5.4}$$

Stress and Strain in a Laminate

For forces and moment resultants acting on laminates, such as in plate and shell structures, the relationship between applied forces and moment and displacement can be given by

$$\begin{aligned}
\begin{bmatrix} N_x \\ N_y \\ N_{xy} \end{bmatrix} &= \begin{bmatrix} A_{11} & A_{12} & A_{16} \\ A_{12} & A_{22} & A_{26} \\ A_{16} & A_{26} & A_{66} \end{bmatrix} \begin{bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \end{bmatrix} \\
&+ \begin{bmatrix} B_{11} & B_{12} & B_{16} \\ B_{11} & B_{12} & B_{16} \\ B_{16} & B_{26} & B_{66} \end{bmatrix} \begin{bmatrix} k_x \\ k_y \\ k_{xy} \end{bmatrix},
\end{aligned} \tag{5.5}$$

where N_x, N_y refers to the normal force per unit length; N_{xy} means shear force per unit length; ε^0 and k_{xy} denotes mid plane strains and curvature of a laminate in x-y coordinates The mid-plane strain and curvature is given by

$$
\begin{aligned}
A_{ij} &= \sum_{k=1}^{n} (\overline{Q_{ij}})_k (h_k - h_{k-1}) i = 1,2,6, j = 1,2,6, \\
B_{ij} &= \frac{1}{2} \sum_{k=1}^{n} (\overline{Q_{ij}})_k (h_k^2 - h_{k-1}^2) i = 1,2,6, j = 1,2,6, \\
D_{ij} &= \frac{1}{3} \sum_{k=1}^{n} (\overline{Q_{ij}})_k (h_k^3 - h_{k-1}^3) i = 1,2,6, j = 1,2,6.
\end{aligned}
\tag{5.6}
$$

The [A], [B], and [D] matrices are called the extensional, coupling, and bending stiffness matrices, respectively. The extensional stiffness matrix $[A]$ relates the resultant in-plane forces to the in-plain strains, and the bending stiffness matrix $[D]$ couples the resultant bending moments to the plane curvatures. The coupling stiffness matrix $[B]$ relates the force and moment terms to the midplain strains and midplane curvatures.

5.2.2 Failure criteria for a lamina

Failure criteria for composite materials are more difficult to predict due to structural and material complexity in comparison to isotropic materials. The failure process of composite materials can be regarded from microscopic and macroscopic points of view. Most popular criteria about the failure of an angle lamina are in terms of macroscopic failure criteria, which are based on the tensile, compressive, and shear strengths. According to the failure surfaces, these criteria [27, 33, 12, 40, 32, 14, 29, 7], can be classified into two classes: one is called independent failure mode criteria which includes the maximum stress failure theory[45], maximum strain failure theory because their failure envelop are rectangle; another is called quadratic polynomial which includes Tsai-Wu[26, 39], Chamis, Hoffman and Hill criteria because their failure surfaces are of ellipsoidal shape. In the present study, the two most reliable failure criteria are taken, Maximum stress and Tsai-wu. Both of these two failure criteria are based on the stresses in the local axis instead of principal normal stresses and maximum shear stresses, and four normal strength parameters and one shear stress for a unidirectional lamina are involved. The five strength parameters are

$(\sigma_1^T)_{ult}$ = ultimate longitudinal tensile strength(in direction 1),
$(\sigma_1^C)_{ult}$ = ultimate longitudinal compressive strength,
$(\sigma_2^T)_{ult}$ = ultimate transverse tensile strength,
$(\sigma_2^C)_{ult}$ = ultimate transverse compressive strength, and
$(\tau_{12})_{ult}$ = and ultimate in-plane shear strength.

Maximum stress(MS) failure criterion

Maximum stress failure theory consists of maximum normal stress theory proposed by Rankine and maximum shearing stress theory proposed by Tresca. The stress applied on a lamina can be resolved into the normal and shear stress in the local axis. If any of the normal or shear stresses in the local axis of a lamina is equal or exceeds the corresponding ultimate strengths of the unidirectional lamina, the lamina is considered to be failed. That is,

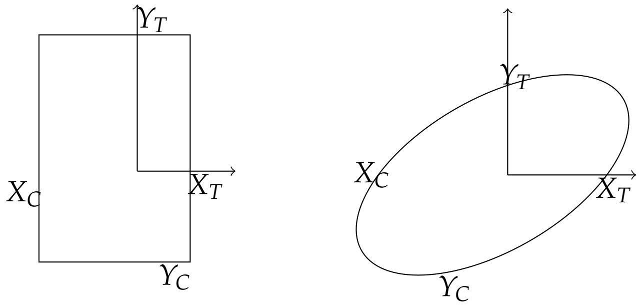


FIGURE 5.3: Schematic failure surfaces for maximum stress and quadratic failure criteria

$$\begin{aligned} \sigma_1 &\geq (\sigma_1^T)_{ult} \quad or \quad \sigma_1 \leq -(\sigma_1^C)_{ult}, \\ \sigma_2 &\geq (\sigma_2^T)_{ult} \quad or \quad \sigma_2 \leq -(\sigma_2^C)_{ult}, \\ \tau_{12} &\geq (\tau_{12})_{ult} \quad or \quad \tau_{12} \leq -(\tau_{12})_{ult}, \end{aligned} \tag{5.7}$$

where σ_1 and σ_2 are the normal stresses in the local axis 1 and 2; τ_{12} is the shear stress in the symmetry plane 1-2.

Tsai-wu failure criterion

The TW criterion is one of the most reliable static failure criteria derived from the von Mises yield criterion. A lamina is considered to fail if

$$\begin{aligned} &H_1\sigma_1 + H_2\sigma_2 + H_6\tau_{12} + H_{11}\sigma_1^2 + H_{22}\sigma_2^2 \\ &\quad + H_{66}\tau_{12}^2 + 2H_{12}\sigma_1\sigma_2 < 1 \end{aligned} \tag{5.8}$$

is violated, where

$$\begin{aligned} H_1 &= \frac{1}{(\sigma_1^T)_{ult}} - \frac{1}{(\sigma_1^C)_{ult}}, \\ H_{11} &= \frac{1}{(\sigma_1^T)_{ult}(\sigma_1^C)_{ult}}, \\ H_2 &= \frac{1}{(\sigma_2^T)_{ult}} - \frac{1}{(\sigma_2^C)_{ult}}, \\ H_{22} &= \frac{1}{(\sigma_2^T)_{ult}(\sigma_2^C)_{ult}}, \\ H_{66} &= \frac{1}{(\tau_{12})_{ult}^2}, \\ H_{12} &= -\frac{1}{2}\sqrt{\frac{1}{(\sigma_1^T)_{ult}(\sigma_1^C)_{ult}(\sigma_2^T)_{ult}(\sigma_2^C)_{ult}}}. \end{aligned} \tag{5.9}$$

H_i is the strength tensors of the second-order; H_{ij} is the strength tensors of the fourth-order. σ_1 is the applied normal stress in direction 1; σ_2 is the applied normal stress in direction 2; τ_{12} is the applied in-plane shear stress.

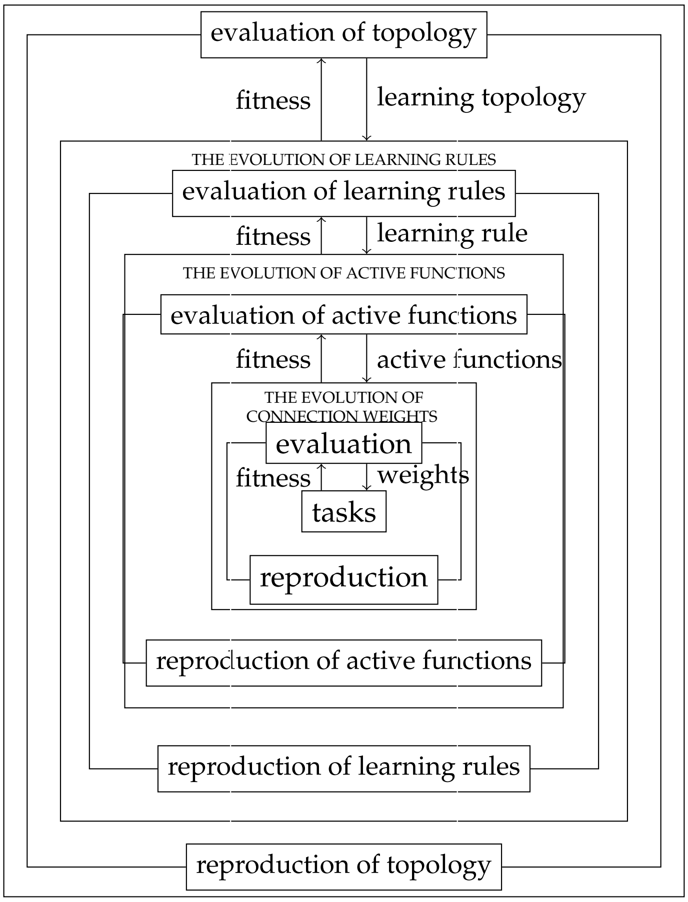


FIGURE 5.4: A general framwwork for EANN, in which fitness refers to the corresponding value of objective function.

Strength ratio

The safety factor, or yield stress, is how much extra load beyond is intended a composite laminate will take. The strength ratio is defined as

$$SR = \frac{\text{Maximum Load Which Can Be Applied}}{\text{Load Applied}} \tag{5.10}$$

.

5.3 Evolutionary Artificial Neural Network

5.3.1 General neural network

In this paper, the feedforward ANN is adopted in the current study, since it is straightforward and simple to code. For function approximation through an ANN, Cybenko demonstrated that a two-layer perceptron can form an arbitrarily close approximation to any continuous nonlinear mapping[9]. Therefore, a two-layer feedforward ANN is proposed in the present study. Fig. 5.5 shows a general framework for a two-layer NN, in which the number of nodes in the hidden layer and the connection with inputs, are critical in the design of an ANN. For nodes in the hidden layer, we can think of them as feature extractors or detectors. Therefore, nodes within it should partially be connected with the inputs of an ANN, since the unnecessary connections would increase the model's complicacy, which will reduce an ANN's performance. Because we treat the nodes in the hidden layer as feature

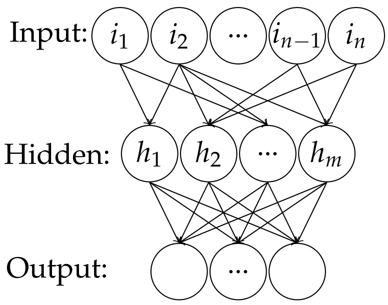


FIGURE 5.5: Network diagram for the two-layer neural network. The input, hidden, and output variables are represented by nodes, and the weight parameters are represented by links between the nodes. Arrows denote the direction of information flow through the network during forward propagation.

TABLE 5.1: Examples of widely used activation functions in the design of artificial neural network.

Type	Description	Formula	Range	Encoding
Linear	The output is proportional to the input	$f(x) = cx$	$(-\infty, +\infty)$	00
Sigmoid	A family of S-shaped functions	$f(x) = \frac{1}{1+e^{-cx}}$	$(0, 1)$	01
ReLU	A piece-wise function	$f(x) = max\{0, x\}$	$(0, +\infty)$	10
Softplus	A family of S-shaped functions	$f(x) = ln(1 + e^x)$	$(0, +\infty)$	11

extractors, so the number of nodes in this layer should be less than the number of inputs. For the nodes in the last layer, every node should be fully connected with nodes in the previous layer, since we think of the nodes in the hidden layer as features. The rest, which affects a NN's performance, are activation function, and ANN's training method. In the following section, we denote the ith node in the input layer, and the hidden layer, as i_i, and h_i, respectively.

5.3.2 Activation function

The activation function is one of the critical parts of an ANN. Liu [23] et al. claims that the performance of neural networks with different activation functions is different, even if they have the same architecture. A generalized activation function can be written as

$$y_i = f_i(\sum_{j=1}^{n} w_{ij}x_j - \theta) \tag{5.11}$$

where y_i is the output of the node i, x_j is the jth input to the node, and w_{ij} is the connection weight between adjacent nodes i and j. Tab. 5.1 displays the most widely adopted activation functions in the design of an ANN, which is used for the current study.

5.3.3 Weights learning

The weight training in an ANN is to minimize the error function, such as the most widely used mean square error function, which calculates the difference between the desired and the prediction output values averaged overall examples. Gradient descent algorithm is widely adopted to reduce the value of an error function, which

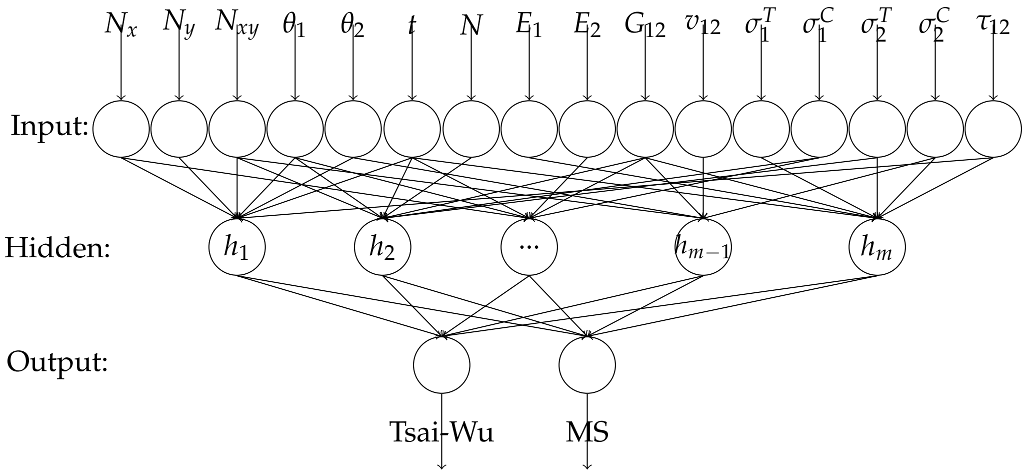


FIGURE 5.6: Diagram for modeling the target function of strength ratio calculating for an angle ply laminate.

has been successfully applied in many practical areas. However, this class of algorithms is plagued by the possible existence of local minima or "flat spots" and "the curse of dimensionality." One method to overcome this problem is to adopt a genetic algorithm(GA)

5.4 Methodology

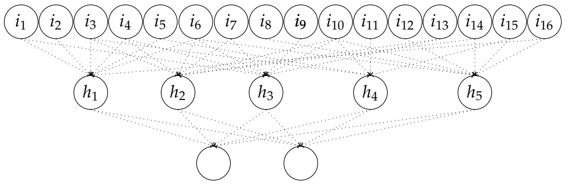
For an angle ply laminate, given the laminate's lay-up, material properties, in-plane loading, etc., we can compute its strength ratio based on Tsai-Wu failure theory or maximum stress theory. To model this function, we propose an ANN framework shown in Fig. 5.4, which derives from the previous two-layer model. There are sixteen inputs of this ANN, which are in-plane loading N_x, N_y, and N_{xy}; design parameters of a laminate, two fiber orientation θ_1 and θ_2, ply thickness t, total number of plies N; five engineering constants of composite materials, E_1, E_2, G_{12}, and v_{12}; five strength parameters of a unidirectional lamina. Two outputs are strength ratio according to MS theory and strength ratio according to Tsai-Wu theory.

The work involved in the evolution process of ANN consists of three parts: search space, which includes the ANN's topology, transfer function, etc.; search strategy, which details how to explore the search space; performance estimation strategy refers to the process of estimating this performance.

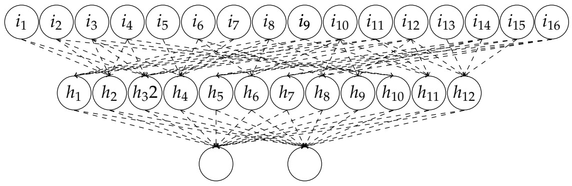
5.4.1 Search Space

We propose a GNN framework as shown in Fig. 5.5. The search space is parametrized by: (i) the number of nodes m(possibly unbounded) in the hidden layer, to narrow down the search space, the assumption is that m less than n; (ii) the type of operation every node executes, e.g., sigmoid, linear, gaussian. (iii) the connection relationship between hidden nodes and inputs; (IV) if a connection exists, the weight value in the connection.

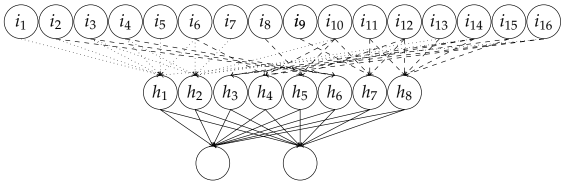
Therefore, evolution in EANN can be divided into four different levels: topology, learning rules, active functions, and connection weights. For the evolution of topology, the aim is to find an optimal ANN architecture for a specific problem. The architecture of a neural network determines the information processing capability in



(A) Parent 1



(B) Parent 2



(C) Child

FIGURE 5.7: Examples of three ANNs, with (a) and (b) as parent ANNs, and (c) as the child of (a) and (b). child c inherits the connection relationship part from parent 1 denoted by the darker dashed lines,and the rest from parent 2 denoted by the gray dashed line.

an application, which is the foundation of the ANN. Two critical issues are involved in the search process of an ANN architecture: the representation and the search operators. Fig. 5.4 summarizes these four levels of evolution in an ANN.

5.4.2 Search Strategy

To use the GA method in this work, we need to represent the ANN, devise a fitness function that determines how good a solution is, and decides the genetic search operators, including selection, mutation, and crossover.

For the representation of an ANN, encode the h_i node as an eighteen digits binary string. The initial sixteen digits in the string correspond to the connections between i_i and h_i, with '1' implying there exists a connection between them, with '0' implying no connection exists. The last two digits in the string refer to an activation function, such as "01" which means a sigmoid function. Tab. 5.2 are examples of the binary representation of ANNs whose architecture is as shown in Fig. 5.7.

For the objective function, treat the multiplicative inverse of the mean squared error, which is the difference between the target and actual output averaged overall examples, as the fitness function.

The crossover between individuals results in exploiting the area between the given two parent solutions. In the present study, we search the local area by combining the genes of half number of nodes from both parents. Fig. 5.7 illustrates the crossover operator: Fig. 5.7 (c) is the child of Fig. 5.7 (a) and Fig. 5.7 (a), the

TABLE 5.2: The binary representation of parent 1, parent 2, and child corresponding to Fig.5.5(a), (b) and (c), with $i_1, i_2, \cdots, I_{16}$ denote sixteen inputs and $h_1, h_2, \cdots, h_{12}$ refer to nodes in the hidden layer. 1 represents an edge from the input node to hidden node, and 0 represents no edge from input nodes to hidden node.

Hidden	Nodes	i_1	i_2	i_3	i_4	i_5	i_6	i_7	i_8	i_9	i_{10}	i_{11}	i_{12}	i_{13}	i_{14}	i_{15}	i_{16}	f	f
5*P1	h_1	1	1	1	1	1	1	0	0	0	0	0	0	0	0	1	1	0	0
	h_2	0	1	1	1	0	0	0	1	0	0	1	1	0	0	0	0	1	1
	h_3	1	0	0	1	0	1	1	0	1	1	0	0	1	0	0	0	0	0
	h_4	0	0	1	0	1	0	0	0	0	1	0	1	0	0	1	0	0	1
	h_5	0	0	0	0	0	1	0	1	0	1	0	1	0	1	1	1	0	1
12*P2	h_1	0	0	0	0	0	0	1	1	1	1	1	0	0	0	0	0	1	0
	h_2	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	h_3	1	1	1	1	0	0	0	0	1	1	1	1	0	0	0	0	1	1
	h_4	0	0	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0
	h_5	0	0	0	0	0	0	0	0	0	0	0	1	0	1	1	1	0	1
	h_6	0	0	0	0	0	1	0	1	0	1	0	1	0	1	1	1	0	1
	h_7	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	0	0
	h_8	0	0	0	0	0	1	0	0	0	1	0	0	0	1	0	1	0	0
	h_9	0	0	0	0	0	1	0	1	0	1	0	1	0	0	0	0	0	1
	h_{10}	0	1	1	1	0	0	0	0	1	1	1	1	0	0	0	0	1	1
	h_{11}	0	0	0	0	0	0	0	0	1	1	1	1	0	0	0	0	1	1
	h_{12}	0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	0	1	1
8*Child	h_1	1	1	1	1	1	1	0	0	0	0	0	0	0	0	1	1	0	0
	h_2	0	1	1	1	0	0	0	1	0	0	1	1	0	0	0	0	1	1
	h_1	0	0	0	0	0	0	1	1	1	1	1	0	0	0	0	0	1	0
	h_2	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	h_3	1	1	1	1	0	0	0	0	1	1	1	1	0	0	0	0	1	1
	h_4	0	0	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0
	h_5	0	0	0	0	0	0	0	0	0	0	0	1	0	1	1	1	0	1
	h_6	0	0	0	0	0	1	0	1	0	1	0	1	0	1	1	1	0	1

connection relationship of hidden nodes with inputs are from both parents, and the corresponding activation functions are also from both parents. In the binary representation Tab. 5.2, we can see that the first two rows of the child are the same as the first two rows of parent P_1, and the last six rows of the child are the same as the first six rows of parent P_2.

5.4.3 Performance estimation strategy

The simplest approach to this problem is to perform a standard training and validation of the architecture on a dataset, however, this method is inefficient and computationally intensive. Therefore, much recent research[4] focuses on developing methods that reduce the cost of performance estimation. In this work, during the GA process, we adopt the following straightforward and efficient method to estimate the performance of an ANN: first, train a neural network one hundred times on the training dataset; second, do the validation test; estimate the neural network's performance according to its fitness of objective function on the test dataset.

TABLE 5.3: Examples of the training data

Input				Output	
Load	Laminate Structure	Material Property	Failure Property	MS	Tsai-Wu
-70,-10,-40,	90,-90,4,1.27,	38.6,8.27,0.26,4.14,	1062.0,610.0,31,118,72,	0.0102,	0.0086
-10,10,0,	-86,86,80,1.27,	181.0,10.3,0.28,7.17,	1500.0,1500.0,40,246,68,	0.4026,	2.5120
-70,-50,80,	-38,38,4,1.27,	116.6,7.67,0.27,4.173,	2062.0,1701.0,70,240,105,	0.0080,	0.0325
-70,80,-40,	90,-90,48,1.27,	38.6,8.27,0.26,4.14,	1062.0,610.0,31,118,72,	0.0218,	0.1028
-20,-30,0,	-86,86,60,1.27,	181.0,10.3,0.28,7.17,	1500.0,1500.0,40,246,68,	0.6481,	0.9512
0,-40,0,	74,-74,168,1.27,	181.0,10.3,0.28,7.17,	1500.0,1500.0,40,246,68,	1.3110,	3.9619

5.5 Experiment

In the previous section, we present the details of our strategies for designing an ANN. In this section, we explain the details of the preparation of the training dataset, and validation dataset.

5.5.1 Dataset Preparation

For composite material, it is impossible to obtain massive training data from the practical scenario. Therefore, we use classical lamination theory and failure theory, which follows a two-step procedure: first, evaluate the stress and strain according to classic lamination theory; second, substitute them into the corresponding equation to get the strength ratio. We repeat this procedure to yield 14000 points uniformly distributed over the domain space. We define the domain of the corresponding inputs as follows: the range of in-plane loading varies from 0 to 120; the range of fiber orientation θ is from -90 to 90; ply thickness t is 1.27mm, the number of plies range N is from 4 to 120. Three different composite material is used in this experiment, as shown in Tab. 4.1. Fig. 5.3 shows part of the training data, which are randomly selected from the generated training dataset. To speeds up the learning and accelerate convergence, the input atttributes of the dataset are rescaled to between 0 and 1.0 by a linear function.

5.5.2 ANN training and validation

The ANN training procedure is carried out by optimising the multinomial logistic regression objective using mini-batch gradient descent[22] with momentum. The batch size is set to 1000, momentum to 0.9. the learning rate is set to 10^{-2}. As for the training dataset and validation dataset, We follow the 70/30 rule, with 70% of the entire data for training and 30% for validation.

5.5.3 Genetic algorithm

The genetic algorithm involves the evolution of an artificial neural network's topology, activation function, etc., in the optimizing process. The corresponding parameters are as the following. The population is 10, the percentage of parents in the population is 40%; the strategy of selecting parents is rank-based; the mutation rate of the offspring is 0.3.

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Chapter 6

Conclusion

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