Contents

Chapter 1

Introduction

1.1 Laminated Composite Material

Composite materials offer improved strength, stiffness, corrosion resistance, etc. over conventional materials, and are widely used as alternative materials for applications in various industries ranging from electronic packaging to golf clubs, and medical equipment to homebuilding, making aircraft structure to space vechicles. The stacking sequence and fiber orientation of composite laminates give the designer additional 'degree of freedom' to tailor the design with respect to strength or stiffness. One widely known advange of using composite material is can significantly reducing the weight of target structure, and many researchers attempted to improve the efficiency of using composite materail by mimimizing the thickness.

1.2 Classic Lamination Theory and Failure Theory

Classic lamination theory(CLT) is used to develop the stress-strain relationship of composite material under in-plane and out-of-plane loading. First, develop stress-strain relationships, elastic moduli, strengths of an angle ply based on a unidirectional lamina and the angle of the ply; second, becasue a laminate is consist of more than one lamina bonded together through their thickness, so the macromechianical analysis will be developed for a laminate based on applied loading. To check whether a designed lay-up is plausible or not, different failure theories have been developed.

1.3 Genetic Algorithm

In the design of composite material, gradient based optimization techiques are not appliable in this domain, because the design varaibles, such as fiber orientation, layer thickness, number of layers etc. are discrete. Genetic algorithm(GA) can be adopted in the optimization problem becasue it doesn't require the gradient information. Moreover, the GA has been proved a reliable techique and widely used in the design of composite material.

1.4 Artificial Neural Network

CLT is an classic analytical approach to obtain the stress and stain of composite material, the disadvantage of this method is quiet cubersome and in which involves compliate matrix and integration operations. Artificial neural network(ANN) has been proved a reliable tool in modelling various engineering system in practice without solving tricky equations and making ideal assumptions. In this thesis, the ANN is taken to approximate the numberic results based on CLT and failure theory.

1.5 Summary

In this thesis, first, we review the use of composite material in practice, then, the CLT to calculate the stress and strain under certain loading, last, the failure theories which are used to decide whether a composite material will failure or not. Second, the stochastic algorith, GA, is studied and implemented in the desgin of composite material, two different cases are studied in which GA can be taken to obtain the optimal lay-up. At last, ANN is introduced to approximate the evaluation result of CLT, the reason for adopting ANN is to reduce the computation complexity based on CLT.

Chapter 2

Classic Lamination Theory and Failure Theory

2.1 Classic Lamination Theory

A laminate is consist of multiple laminas bonded together through thickness. In this chapter, first the stress-strain relationship is developed based on Hook's law for a single lamina; second, develop relationship of mechanical loads applied to a laminate to strains and stresses in each lamina, calculate the elastic moduli of laminate based on the elastic moduli of single laminate and the lay-up..

2.1.1 Analysis of stress and strain for composite material

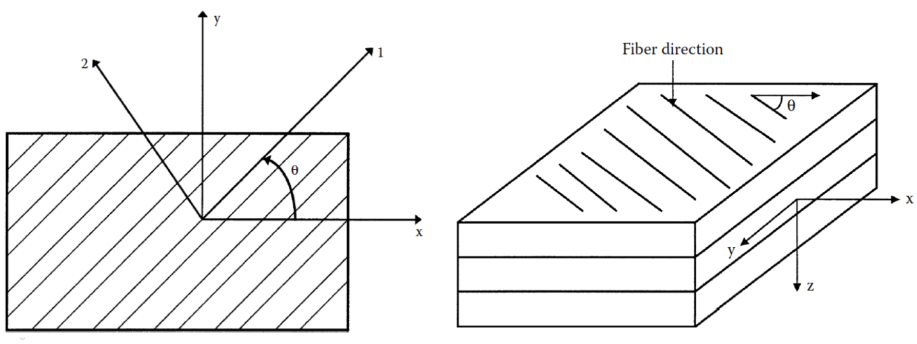

FIGURE 2.1: Lamina

2.1.2 Stress and Strain in a Lamina

For a single lamina has a small thickness under plane stress, and it's upper and lower surfaces of the lamina are free from external loads. According to the Hooke's Law, the three-dimensional stress-strain equations can be reduced to two-dimensional stress-strain equations. The stress-strain relation in local axis 1-2 is

$$\begin{bmatrix} \sigma_1 \\ \sigma_2 \\ \tau_{12} \end{bmatrix} = \begin{bmatrix} Q_{11} & Q_{12} & 0 \\ Q_{12} & Q_{22} & 0 \\ 0 & 0 & Q_{66} \end{bmatrix} \begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \gamma_{12} \end{bmatrix}. \tag{2.1}$$

where $Q_{ij}$are the stiffnesses of the lamina that are related to engineering elastic constants given by

$$\begin{aligned} Q_{11} &= \frac{E_1}{1 - v_{12}v_{21}}, \\ Q_{22} &= \frac{E_2}{1 - v_{12}v_{21}}, \\ Q_{66} &= G_{12}, \\ Q_{12} &= \frac{v_{21}E_2}{1 - v_{12}v_{21}}, \end{aligned} \tag{2.2}$$

where E_1, E_2, v_{12}, G_{12} are four independent engineering elastic constants, which are defined as follows: E_1 is the longitudinal Young's modulus, E_2 is the transverse Young's modulus, v_{12} is the major Poisson's ratio, and G_{12} is the in-plane shear modulus.

Stress strain relation in the global x-y axis is

$$\begin{bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{bmatrix} = \begin{bmatrix} \bar{Q}_{11} & \bar{Q}_{12} & \bar{Q}_{16} \\ \bar{Q}_{12} & \bar{Q}_{22} & \bar{Q}_{26} \\ \bar{Q}_{16} & \bar{Q}_{26} & \bar{Q}_{66} \end{bmatrix} \begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{bmatrix}, \tag{2.3}$$

where

$$\begin{aligned} \bar{Q}_{11} &= Q_{11}cos^4\theta + Q_{22}sin^4\theta + 2\left(Q_{12} + 2Q_{66}\right) sin^2\theta cos^2\theta, \\ \bar{Q}_{12} &= \left(Q_{11} + Q_{22} - 4Q_{66}\right) sin^2\theta cos^2\theta + Q_{12}\left(cos^4\theta + sin^2\theta\right), \\ \bar{Q}_{22} &= Q_{11}sin^4\theta + Q_{22}cos^4\theta + 2\left(Q_{12} + 2Q_{66}\right) sin^2\theta cos^2\theta, \\ \bar{Q}_{16} &= \left(Q_{11} - Q_{12} - 2Q_{66}\right) cos^3\theta sin\theta - \left(Q_{22} - Q_{12} - 2Q_{66}\right) sin^3\theta cos\theta, \\ \bar{Q}_{26} &= \left(Q_{11} - Q_{12} - 2Q_{66}\right) cos\theta sin^3\theta - \left(Q_{22} - Q_{12} - 2Q_{66}\right) cos^3\theta sin\theta, \\ \bar{Q}_{66} &= \left(Q_{11} + Q_{22} - 2Q_{12} - 2Q_{66}\right) sin\theta^2 cos\theta^2 + Q_{66}\left(sin\theta^4 + cos\theta^4\right). \end{aligned} \tag{2.4}$$

The local and global stresses in an angle lamina are related
to each other through the angle of the lamina θ, it can be written as

$$\begin{bmatrix} \sigma_1 \\ \sigma_2 \\ \tau_{12} \end{bmatrix} = [T] \begin{bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{bmatrix}, \tag{2.5}$$

where

$$[T] = \begin{bmatrix} cos^2\theta & sin^2\theta & 2sin\theta cos\theta \\ sin^2\theta & cos^2\theta & -2sin\theta cos\theta \\ -sin\theta cos\theta & sin\theta cos\theta & cos^2\theta - sin^2\theta \end{bmatrix}. \tag{2.6}$$

2.1.3 Stress and Strain in a Laminate

For forces and moment resultants acting on laminates, such as in plate and shell structures, the relationship between applied forces and moment and displacement can be given by

$$
\begin{aligned}
\begin{bmatrix} N_x \\ N_y \\ N_{xy} \end{bmatrix} &= \begin{bmatrix} A_{11} & A_{12} & A_{16} \\ A_{12} & A_{22} & A_{26} \\ A_{16} & A_{26} & A_{66} \end{bmatrix} \begin{bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \end{bmatrix} \\
&+ \begin{bmatrix} B_{11} & B_{12} & B_{16} \\ B_{11} & B_{12} & B_{16} \\ B_{16} & B_{26} & B_{66} \end{bmatrix} \begin{bmatrix} k_x \\ k_y \\ k_{xy} \end{bmatrix}, \\
\begin{bmatrix} M_x \\ M_y \\ M_{xy} \end{bmatrix} &= \begin{bmatrix} B_{11} & B_{12} & B_{16} \\ B_{12} & B_{22} & B_{26} \\ B_{16} & B_{26} & B_{66} \end{bmatrix} \begin{bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \end{bmatrix} \\
&+ \begin{bmatrix} D_{11} & D_{12} & D_{16} \\ D_{11} & D_{12} & D_{16} \\ D_{16} & D_{26} & D_{66} \end{bmatrix} \begin{bmatrix} k_x \\ k_y \\ k_{xy} \end{bmatrix},
\end{aligned}
\tag{2.7}
$$

where
N_x, N_y - normal force per unit length;
N_{xy} - shear force per unit length;
M_x, M_y - bending moment per unit length;
M_{xy} - twisting moments per unit length;
ε^0, k- mid plane strains and curvature of a laminate in x-y coordinates.
The mid plane strain and curvature is given by

$$
\begin{aligned}
A_{ij} &= \sum_{k=1}^{n} (\overline{Q_{ij}})_k (h_k - h_{k-1}) i = 1,2,6, j = 1,2,6, \\
B_{ij} &= \frac{1}{2} \sum_{k=1}^{n} (\overline{Q_{ij}})_k (h_k^2 - h_{k-1}^2) i = 1,2,6, j = 1,2,6, \\
D_{ij} &= \frac{1}{3} \sum_{k=1}^{n} (\overline{Q_{ij}})_k (h_k^3 - h_{k-1}^3) i = 1,2,6, j = 1,2,6.
\end{aligned}
\tag{2.8}
$$

The [A], [B], and [D] matrices are called the extensional, coupling, and bending stiffness matrices, respectively. The extensional stiffness matrix $[A]$ relates the resultant in-plane forces to the in-plain strains, and the bending stiffness matrix $[D]$ couples the resultant bending moments to the plane curvatures. The coupling stiffness matrix $[B]$ relates the force and moment terms to the midplain strains and midplane curvatures.

2.2 Failure Theory

Failure criteria for composite materials are more difficult to predict due to structural and material complexity in comparison to isotropic materials. The failure process of a composite materials can be regarded from microscopic and macroscopic points of view. Most popular criteria about the failure of an angle lamina are in terms of macroscopic failure criteria, which are based on the tensile, compressive and shear strengths. According to the failure surfaces, these criteria can be classified into two classes: one is called independent failure mode criteria which includes the maximum stress failure theory, maximum strain failure theory because their failure envelop are rectangle; another is called quadratic polynomial which includes Tsai-Wu, Chamis, Hoffman and Hill criteria because their failure surfaces are of ellipsoidal shape. In

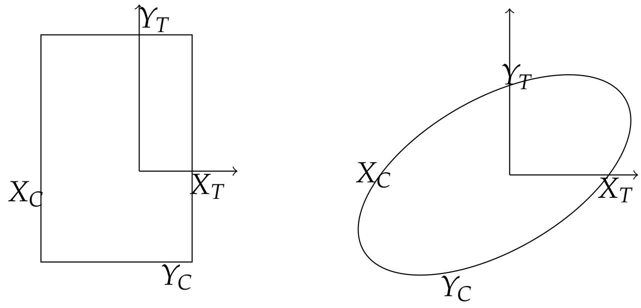

FIGURE 2.2: Schematic failure surfaces for maximum stress and quadratic failure criteria

the present study, two most reliable failure criteria is taken, Maximum stress and Tsai-wu. Both of these two failure criteria are based on the stresses in the local axes instead of principal normal stresses and maximum shear stresses, and four normal strength parameters and one shear stress for a unidirectional lamina are involved. The five strength parameters are
$(\sigma_1^T)_{ult}$ = ultimate longitudinal tensile strength(in direction 1),
$(\sigma_1^C)_{ult}$ = ultimate longitudinal compressive strength,
$(\sigma_2^T)_{ult}$ = ultimate transverse tensile strength,
$(\sigma_2^C)_{ult}$ = ultimate transverse compressive strength, and
$(\tau_{12})_{ult}$ = and ultimate in-plane shear strength.

2.2.1 Tsai-wu failure criterion

The TW criterion is one of the most reliable static failure criteria which is derived from the von Mises yield criterion. A lamina is considered to fail if

$$\begin{aligned} &H_1\sigma_1 + H_2\sigma_2 + H_6\tau_{12} + H_{11}\sigma_1^2 + H_{22}\sigma_2^2 \\ &+ H_{66}\tau_{12}^2 + 2H_{12}\sigma_1\sigma_2 < 1 \end{aligned} \tag{2.9}$$

is violated, where

$$\begin{aligned} H_1 &= \frac{1}{(\sigma_1^T)_{ult}} - \frac{1}{(\sigma_1^C)_{ult}}, \\ H_{11} &= \frac{1}{(\sigma_1^T)_{ult}(\sigma_1^C)_{ult}}, \\ H_2 &= \frac{1}{(\sigma_2^T)_{ult}} - \frac{1}{(\sigma_2^C)_{ult}}, \\ H_{22} &= \frac{1}{(\sigma_2^T)_{ult}(\sigma_2^C)_{ult}}, \\ H_{66} &= \frac{1}{(\tau_{12})_{ult}^2}, \\ H_{12} &= -\frac{1}{2}\sqrt{\frac{1}{(\sigma_1^T)_{ult}(\sigma_1^C)_{ult}(\sigma_2^T)_{ult}(\sigma_2^C)_{ult}}}. \end{aligned} \tag{2.10}$$

H_i is the strength tensors of the second order; H_{ij} is the strength tensors of the fourth order. σ_1 is the applied normal stress in direction 1; σ_2 is the applied normal stress in the direction 2; and τ_{12} is the applied in-plane shear stress.

2.2.2 Maximum Stress Failure Theory

Maximum stress(MS) failure theory consists of maximum normal stress theory proposed by Rankine and maximum shearing stress theory by Tresca. The stresses applied on a lamina can be resolved into the normal and shear stresses in the local axes. If any of the normal or shear stresses in the local axes of a lamina is equal or exceeds the corresponding ultimate strengths of the unidirectional lamina, the lamina is considered to be failed. That is

$\sigma_1 \geq (\sigma_1^T)_{ult}$ or $\sigma_1 \leq -(\sigma_1^C)_{ult}$,
$\sigma_2 \geq (\sigma_2^T)_{ult}$ or $\sigma_2 \leq -(\sigma_2^C)_{ult}$,
$\tau_{12} \geq (\tau_{12})_{ult}$ or $\tau_{12} \leq -(\tau_{12})_{ult}$.

where σ_1 and σ_2 are the normal stresses in the local axes 1 and 2, respectively; τ_{12} is the shear stress in the symmetry plane 1-2. The safety factor, or yield stress, is how much extra load beyond is intended a composite laminate will actually take, which is an indication of the material's load carrying capacity. If the value is less then 1.0, it means failure. The safey factor is defined as

$$SF = \frac{\text{Maximum Load Which Can Be Applied}}{\text{Load Applied}}. \tag{2.11}$$

The safety factor based on maximum stress theory is calculated by the following method: first, the principal stresses(σ_1^k,σ_2^k, and τ_{12}^k) are obtained by experiment; evaluate the safety factor along each direction according to equation 2.11; The minimum value among these safety factors are denoted as the safety factor of the lamina, SF_{MS}^k, it can be written as

$$SF_{MS}^k = \text{min of} \begin{cases} SF_X^k = \begin{cases} \frac{X_t}{\sigma_{11}}, & \text{if } \sigma_{11} > 0 \\ \frac{X_c}{\sigma_{11}}, & \text{if } \sigma_{11} < 0 \end{cases} \\ SF_Y^k = \begin{cases} \frac{Y_t}{\sigma_{22}}, & \text{if } \sigma_{22} > 0 \\ \frac{Y_c}{\sigma_{22}}, & \text{if } \sigma_{22} < 0 \end{cases} \\ SF_S^k = \left\{ \frac{S}{|\tau_{12}|} \right. \end{cases}. \tag{2.12}$$

Assuming the composite laminate under a in-plane loading f, the corresponding stress on local stress in direction 1, local stress in direction 2, and shear stress for the kth lamina are $\sigma_1 SF_{TW}^k$, $\sigma_2 SF_{TW}^k$, and $\tau_{12} SF_{TW}^k$, respectively. Substitute them into the equation 2.9, the expression are given by

$a(SF_{TW}^k)^2 + b(SF_{TW}^k) - 1 = 0,$

where

$a = H_{11}(\sigma_1)^2 + H_{22}(\sigma_2)^2 + H_{66}(\tau_{12})^2 + 2H_{12}\sigma_1\sigma_2,$
$b = H_1\sigma_1 + H_2\sigma_2 + H_6\tau_{12}.$

Solve the above equation, the safety factor for the kth lamina is

$SF_{TW}^k = |\frac{-b+\sqrt{b^2+4a}}{2a}|.$

Then, the minimum of SF_{TW}^k is taken as the safety factor of the laminate which is written as

$SF_{TW} =$ min of SF_{TW}^k for $k = 1, 2, \cdots, m-1, m$.

2.3 Summary

In this chapter, we review the CLT for composite material's analysis, then related failure theories are introduced to check whether a composite material would fail or not. In the following chapters, the CLT would be used to calculate the stress and strain under in-plane loading, it would also be used to generate the training data for stress and strain approximation based on neural network; in order to design a proper composite material, the failure theory are used to decide the feasibility of composite design.

Chapter 3

A New Genetic Algorithm Model for Constrained Problem

3.1 Genetic Algorithm Framework

GA is one of the most reliable stochastic algorithm, which has been widely used in discrete variables optimization problems.

3.2 Constrained Problem Optimization

3.3 A New Genetic Algorithm Model

3.3.1 Encoding

Due to the simplicity and efficiency of float representation, this encoding method is implemented to represent a possible solution. As shown in Figure **??** (a), these two chromsomes represent a $[+8_7/-9_2]_s$ carbon T300/5308 laminated composite, and $[+19_4/-36_6]_s$, respectively. Becasue the laminate adopted in this paper is symmetric to its mid-plane, so only half needs to be encoded.

3.3.2 Selection

The purpose of the selection operator is to chose mating pool to produce alternative solutions of better fitness. Traditional methods of selecting strategies only take the fitness of individuals into acount, however, due to the existance of constraint, various selection schemes are implemented to selecet the mating set. Based on different selection schemes, the parents of next generation can be divided into three groups: proper groups, active groups, and potential groups according to different selecting methods.

Proper parents mean in which individual fullfils the constraints, which are chosen by the individual's fitnees, individuals with better fitness are more likely to be chosen if they fit the constraint; active groups means that individual is supposed to be always exist in the parents during the GA, which are selected by fitness, ignoring the constraint; The individuals from active group may not correspond to feasible solutions, but their existance enriches the variety of the gene clips. Potential groups means that they are likely to turn into proper individual after a couple of generations, and potential individuals are chosen by constraint function, the more the individual fulfils the constraint, the more possiblity it will be selected.

3.3.3 Crossover

The crossover operator happens among these three groups. the child of two proper groups are more likely to be a proper individual which can be used to obtain a alternative feasible solution. the child of an active individual and a potential individual can significantly change the gene of active individual's chromsome, which makes the individual evolve toward a new direction. The offspring of two active individuals are more likely to be an active individual, which can maitain the active group. The figure.**??** (b) shows two children O_1

3.4 Summary

Chapter 4

Laminated Composite Material Optimization by New Genetic Algorithm Model

4.1 Case1: Maximum Strength Optimization

TABLE 4.1: Properties of T300/5308 carbon/epoxy composite

Property	Symbol	Unit	Graphite/Epoxy
Longitudinal elastic modulus	E_1	GPa	181
Traverse elastic modulus	E_2	GPa	10.3
Major Poisson's ratio	v_{12}		0.28
Shear modulus	G_{12}	GPa	7.17
Ultimate longitudinal tensile strength	$(\sigma_1^T)_{ult}$	MP	1500
Ultimate longitudinal compressive strength	$(\sigma_1^C)_{ult}$	MP	1500
Ultimate transverse tensile strength	$(\sigma_2^T)_{ult}$	MPa	40
Ultimate transverse compressive strength	$(\sigma_2^C)_{ult}$	MPa	246
Ultimate in-plane shear strength	$(\tau_{12})_{ult}$	MPa	68

TABLE 4.2: The optimum lay-ups using two distinct fiber angles under various biaxial loading cases

Loading $N_x/N_y/N_{xy}$ (MPa m)	Optimum lay-up sequences	Laminate thickness	Safety factor for Tsai-wu	Safety factor for maximum stress
10/5/0	$[33_{29}/-39_{25}/-\bar{3}9]_s$	109	1.0074	1.0246
20/5/0	$[33_{22}/-31_{24}]_s$	92	1.0055	1.2065
40/5/0	$[29_{18}/-21_{23}/-\bar{2}1]_s$	83	1.0034	1.7350
80/5/0	$[-20_{27}/21_{25}/\bar{2}5]_s$	105	1.0029	1.2063
120/5/0	$[-18_{34}/17_{36}]_s$	140	1.0000	1.0898

TABLE 4.3: The optimum lay-ups using three distinct fiber angles under various biaxial loading cases

Loading $N_x/N_y/N_{xy}$ (MPa m)	Optimum lay-up sequences	Laminate thickness	Safety factor for Tsai-wu	Safety factor for maximum stress
10/5/0	$[37_{27}/\text{-}38_{27}/\text{-}5]_s$	110	1.0023	1.0216
20/5/0	$[34_{24}/\text{-}32_{14}/\text{-}28_{11}]_s$	98	1.0237	1.2089
40/5/0	$[21_{28}/\text{-}32_{19}/2_{3}]_s$	100	1.0617	1.7076
80/5/0	$[\text{-}19_{24}/20_{27}/\text{-}17_{16}/\text{-}\bar{1}7]_s$	109	1.0056	1.2093
120/5/0	$[\text{-}19_{33}/12_{13}/16_{28}]_s$	148	1.0105	1.1014

TABLE 4.4: Comparison with the results of DSA

Loading	Akbulut and Sonmez'sakbulut2008optimum Study				Present Study			
$N_x/N_y/N_{xy}$ (MPa m)	Optimum lay-up sequences	laminate thickness	TW	MS	Optimum lay-up sequences	laminate thickness	TW	MS
10/5/0	$[37_{27}/\text{-}37_{27}]_s$	108	1.0068	1.0277	$[33_{29}/\text{-}39_{25}/\text{-}\bar{3}9]_s$	109	1.0074	1.0246
20/5/0	$[31_{23}/\text{-}31_{23}]_s$	92	1.0208	1.1985	$[33_{22}/\text{-}31_{24}]_s$	92	1.0055	1.2065
40/5/0	$[26_{20}/\text{-}26_{20}]_s$	80	1.0190	1.5381	$[29_{18}/\text{-}21_{23}/\text{-}\bar{2}1]_s$	83	1.0034	1.7350
80/5/0	$[21_{25}/\text{-}19_{28}]_s$	106	1.0113	1.2213	$[\text{-}20_{27}/21_{25}/\bar{2}5]_s$	105	1.0029	1.2063
120/5/0	$[17_{35}/\text{-}17_{35}]_s$	140	1.0030	1.0950	$[\text{-}18_{34}/17_{36}]_s$	140	1.0000	1.0898

4.1.1 Methodology

4.1.2 Result and Discussion

4.2 Case2: Minimum Thickness Optimization

4.2.1 Methodology

4.2.2 Result and Discussion

4.3 Summary

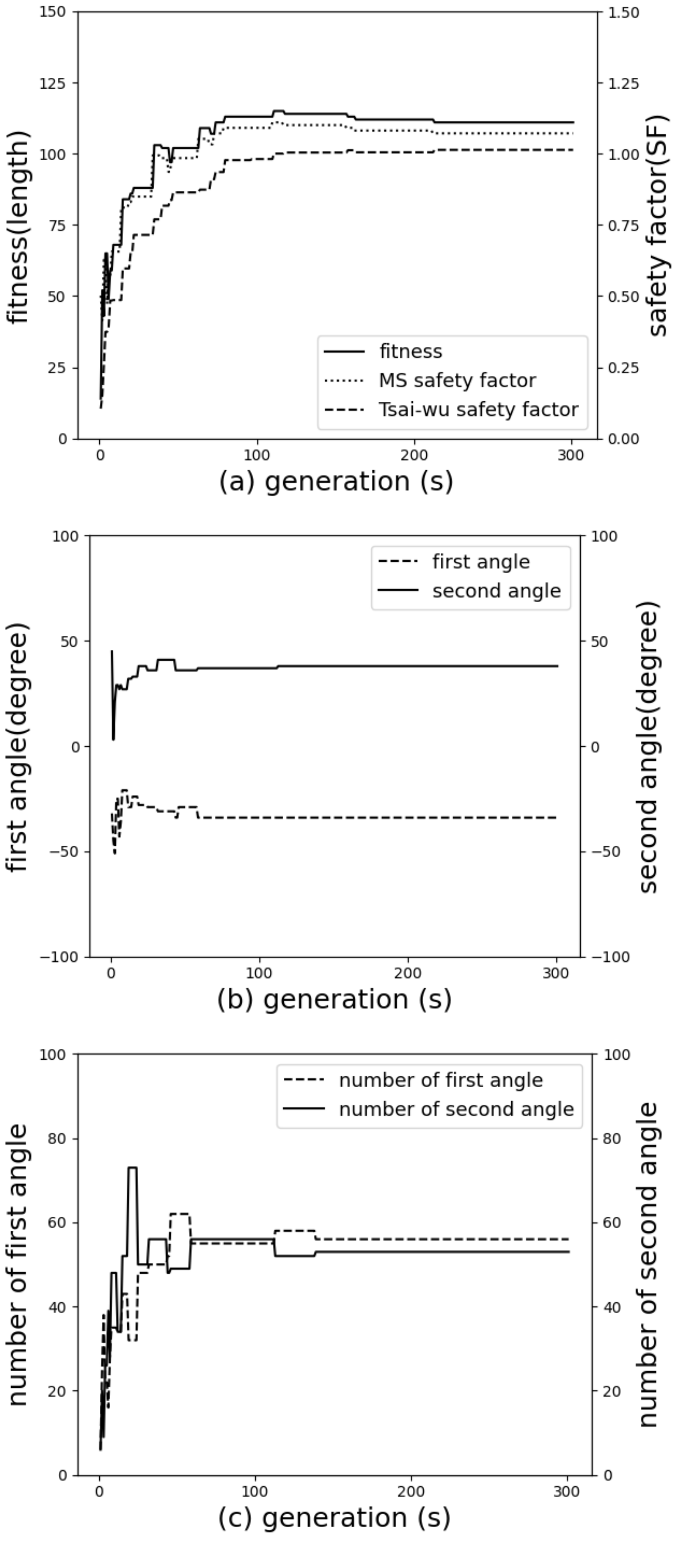

FIGURE 4.1: Two distinct angles

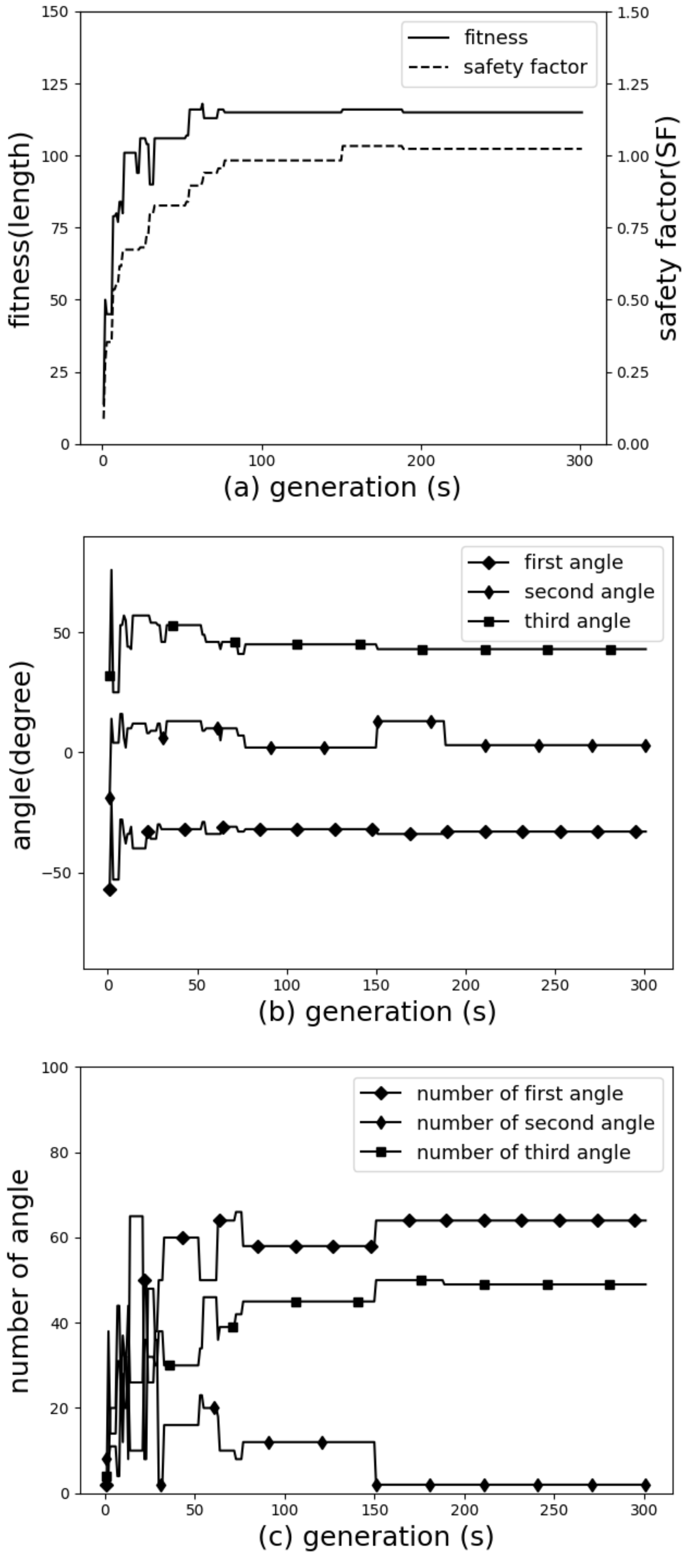

FIGURE 4.2: three distinct angles

Chapter 5

Approximation of CLT Based on Artificial Neural Network

5.1 Neural Network Design

5.1.1 Architecture

5.1.2 Discussion

5.2 Approximation Framework

5.2.1 Data Preparation

5.2.2 Training

5.2.3 Evaluation

5.2.4 Result and Discussion

5.3 Summary

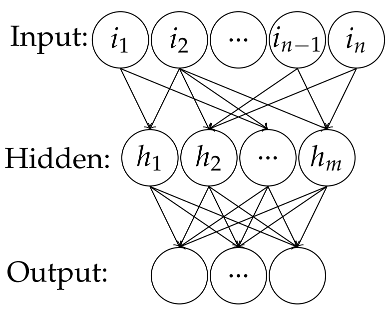

FIGURE 5.1: Neural Network Model

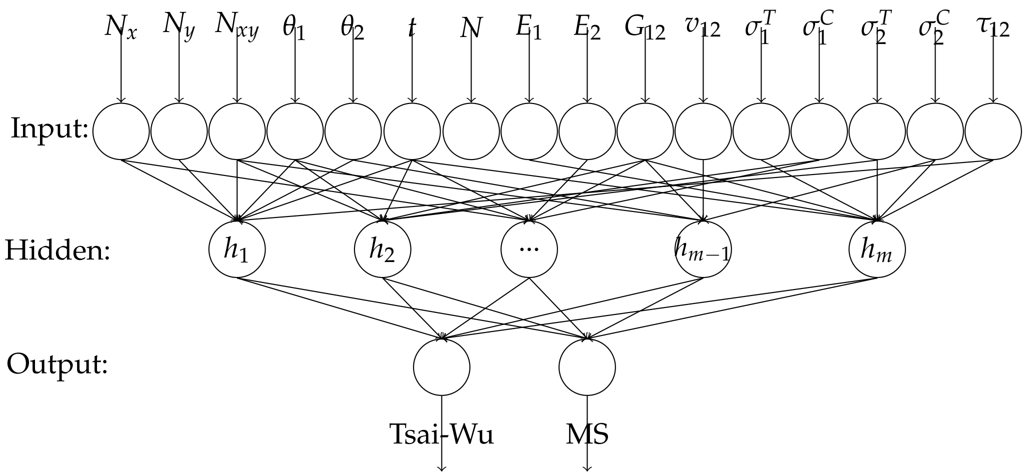

FIGURE 5.2: Neural Network Model

Chapter 6

Conclusion