An Approximation method of Classic Lamination Theory based on Evolutionary Artificial Neural Network

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Abstract

In this study, a systematic methodological framework of artifical neural network(ANN) design is proposed to obtain the optimal architecture to solve practical engineering problems. Classic lamination theory(CLT) is widely used for predicting the behavior of composite materials under in-plane loading from a knowledge of the material properties of the individual layers and the laminate geometry. ANN is suggested to approximate the CLT, and genetic algorithm is introduced to optimize the architecture, parameters, and connections of the neural network.

Keywords: Classic Lamination Theory, Genetic Algorithm, Artificial neural network, Optimization

1. Introduction

Fiber-reinforced composite materials have been widely used in many industries because they offer improved mechanical stiffness, strength, and low specific gravity of fibers over conventional materials. The use of composite material

[⋆] Based on Evolving Artificial Neural Networks

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[1] Since 1880.

materials in structural application is range from electronic packaging, sports equipment, homebuilding, medical prosthetic devices, to high performance military structures. The stacking sequence, ply thickness, and fiber orientation of composite laminates give the designer additional 'degree of freedom' to tailor the design with respect to strength or stiffness. Classic lamination theory(CLT) is taken to predict the behavior of a laminate from a knowledge of the composite laminate properties of the individual layers and the laminate geometry.

Evolutionary artificial neural networks(EANN's) is a special class of artifical neural networks(ANN's) in which evolutionary algorithms(ES's) are introduced to learn the optimal ANN. EA's can be used in the ANN at three different levels: connection weights, architectures, input features, and learning rules. It is shown, the combinations of ANN's and EA's can significantly improve the performance of intelligent systems that rely's on ANN's or EA's alone.

The rest of the paper is organized as follows. Section 2 explains the classical laminate theory and the failure criteria taken in the present study. Section 3 explains the design of artifical neural network for mathmatical model approximation. Section 4 reviews the use of genetic algorithm in the design of neural network architecture and the parameters optimization during the training process of neural network. design Section 4 describes the result of the numerical experiments in different cases, and in the Conclusion section we dicuss the results.

2. Classic Lamination Theory

Classical lamination theory is based upon three simplifying engineering assumptions: (1) Each layer's thickness is very small and consist of homogeneous, orthotropic material, and these layers are prefectly bonded together; (2) The entire laminated composite is supposed to be under plane stress; (3) Normal cross sections of the entire laminate is normal to the deflected middle surface, and do not change in thickness.

2.1. Stress and Strain in a Lamina

For a single lamina has a small thickness under plane stress, and it's upper and lower surfaces of the lamina are free from external loads. According to the Hooke's Law, the three-dimensional stress-strain equations can be reduced to two-dimensional stress-strain equations. The stress-strain relation in local axis 1-2 is:

$$\begin{bmatrix} \sigma_1 \\ \sigma_2 \\ \tau_{12} \end{bmatrix} = \begin{bmatrix} Q_{11} & Q_{12} & 0 \\ Q_{12} & Q_{22} & 0 \\ 0 & 0 & Q_{66} \end{bmatrix} \begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \gamma_{12} \end{bmatrix} \tag{1}$$

where $Q_{ij}$are the stiffnesses of the lamina that are related

to engineering elastic constants given by

$$\begin{aligned} Q_{11} &= \frac{E_1}{1 - v_{12}v_{21}} \\ Q_{22} &= \frac{E_2}{1 - v_{12}v_{21}} \\ Q_{66} &= G_{12} \\ Q_{12} &= \frac{v_{21}E_2}{1 - v_{12}v_{21}} \end{aligned} \tag{2}$$

where E_1, E_2, v_{12}, G_{12} are four independent engineering elastic constants, which are defined as follows: E_1 is the longitudinal Young's modulus, E_2 is the transverse Young's modulus, v_{12} is the major Poisson's ratio, and G_{12} is the in-plane shear modulus.

Stress strain relation in the global x-y axis:

$$\begin{bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{bmatrix} = \begin{bmatrix} \bar{Q}_{11} & \bar{Q}_{12} & \bar{Q}_{16} \\ \bar{Q}_{12} & \bar{Q}_{22} & \bar{Q}_{26} \\ \bar{Q}_{16} & \bar{Q}_{26} & \bar{Q}_{66} \end{bmatrix} \begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{bmatrix} \tag{3}$$

where

$$\begin{gathered}
\bar{Q}_{11} = Q_{11}cos^4\theta + Q_{22}sin^4\theta + 2\left(Q_{12} + 2Q_{66}\right)sin^2\theta cos^2\theta \\
\bar{Q}_{12} = \left(Q_{11} + Q_{22} - 4Q_{66}\right)sin^2\theta cos^2\theta + Q_{12}\left(cos^4\theta + sin^2\theta\right) \\
\bar{Q}_{22} = Q_{11}sin^4\theta + Q_{22}cos^4\theta + 2\left(Q_{12} + 2Q_{66}\right)sin^2\theta cos^2\theta \\
\bar{Q}_{16} = \left(Q_{11} - Q_{12} - 2Q_{66}\right)cos^3\theta sin\theta - \left(Q_{22} - Q_{12} - 2Q_{66}\right)sin^3\theta cos\theta \\
\bar{Q}_{26} = \left(Q_{11} - Q_{12} - 2Q_{66}\right)cos\theta sin^3\theta - \left(Q_{22} - Q_{12} - 2Q_{66}\right)cos^3\theta sin\theta \\
\bar{Q}_{66} = \left(Q_{11} + Q_{22} - 2Q_{12} - 2Q_{66}\right)sin\theta^2 cos\theta^2 + Q_{66}\left(sin\theta^4 + cos\theta^4\right)
\end{gathered} \tag{4}$$

The local and global stresses in an angle lamina are related to each other through the angle of the lamina θ

$$\begin{bmatrix} \sigma_1 \\ \sigma_2 \\ \tau_{12} \end{bmatrix} = [T] \begin{bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{bmatrix} \tag{5}$$

where

$$[T] = \begin{bmatrix} cos^2\theta & sin^2\theta & 2sin\theta cos\theta \\ sin^2\theta & cos^2\theta & -2sin\theta cos\theta \\ -sin\theta cos\theta & sin\theta cos\theta & cos^2\theta - sin^2\theta \end{bmatrix} \tag{6}$$

2.2. Stress and Strain in a Laminate

For forces and moment resultants acting on laminates, such as in plate and shell structures, the relationship between applied forces and moment and displacement can be given by

$$
\begin{bmatrix} N_x \\ N_y \\ N_{xy} \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} & A_{16} \\ A_{12} & A_{22} & A_{26} \\ A_{16} & A_{26} & A_{66} \end{bmatrix} \begin{bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \end{bmatrix} + \begin{bmatrix} B_{11} & B_{12} & B_{16} \\ B_{11} & B_{12} & B_{16} \\ B_{16} & B_{26} & B_{66} \end{bmatrix} \begin{bmatrix} k_x \\ k_y \\ k_{xy} \end{bmatrix} \tag{7}
$$

$$
\begin{bmatrix} M_x \\ M_y \\ M_{xy} \end{bmatrix} = \begin{bmatrix} B_{11} & B_{12} & B_{16} \\ B_{12} & B_{22} & B_{26} \\ B_{16} & B_{26} & B_{66} \end{bmatrix} \begin{bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \end{bmatrix} + \begin{bmatrix} D_{11} & D_{12} & D_{16} \\ D_{11} & D_{12} & D_{16} \\ D_{16} & D_{26} & D_{66} \end{bmatrix} \begin{bmatrix} k_x \\ k_y \\ k_{xy} \end{bmatrix}
$$

N_x, N_y - normal force per unit length

N_{xy} - shear force per unit length

M_x, M_y - bending moment per unit length

M_{xy} - twisting moments per unit length

ε^0, k- mid plane strains and curvature of a laminate in x-y coordinates

The mid plane strain and curvature is given by

$$
\begin{aligned}
A_{ij} &= \sum_{k=1}^{n} (\overline{Q_{ij}})_k (h_k - h_{k-1}) i = 1, 2, 6, j = 1, 2, 6 \\
B_{ij} &= \frac{1}{2} \sum_{k=1}^{n} (\overline{Q_{ij}})_k (h_k^2 - h_{k-1}^2) i = 1, 2, 6, j = 1, 2, 6 \\
D_{ij} &= \frac{1}{3} \sum_{k=1}^{n} (\overline{Q_{ij}})_k (h_k^3 - h_{k-1}^3) i = 1, 2, 6, j = 1, 2, 6
\end{aligned} \tag{8}
$$

The [A], [B], and [D] matrices are called the extensional, coupling, and bending stiffness matrices, respectively. The extensional stiffness matrix $[A]$ relates the resultant in-plane forces to the in-plain strains, and the bending stiffness matrix $[D]$ couples the resultant bending moments to the plane curvatures. The coupling stiffness matrix $[B]$ relates the force and moment terms to the midplain strains and midplane curvatures.

3. Failure criteria for a lamina

Failure criteria for composite materials are more difficult to predict due to structural and material complexity in comparison to isotropic materials. The failure process of a composite materials can be regarded from microscopic and macroscopic points of view. Most popular criteria about the failure of an angle lamina are in terms of macroscopic failure criteria, which are based on the tensile, compressive and shear strengths. According to the failure surfaces, these criteria can be classified into two classes: one is called independent failure mode criteria which includes the maximum stress failure theory, maximum strain failure theory because their failure envelop are rectangle; another is called quadratic polynomial which includes Tsai-Wu, Chamis, Hoffman and Hill criteria because their failure surfaces are of ellipsoidal shape. In the present study, two most reliable failure criteria is taken, Maximum stress and Tsai-wu. Both of these two failure criteria are based on the stresses in the local axes instead of principal normal stresses and maximum shear stresses, and four normal strength parameters and one shear stress for a unidirectional lamina are involved. The five strength parameters are

$(\sigma_1^T)_{ult}$ = ultimate longitudinal tensile strength(in direction 1),

$(\sigma_1^C)_{ult}$ = ultimate longitudinal compressive strength,

$(\sigma_2^T)_{ult}$ = ultimate transverse tensile strength,

$(\sigma_2^C)_{ult}$ = ultimate transverse compressive strength, and

$(\tau_{12})_{ult}$ = and ultimate in-plane shear strength.

3.1. Maximum stress failure criterion

(MS) Maximum stress failure theory consists of maximum normal stress theory proposed by Rankine and maximum shearing stress theory by Tresca. The stresses applied on a lamina can be resolved into the normal and shear stresses in the local axes. If any of the normal or shear stresses in the local axes of a lamina is equal or exceeds the corresponding ultimate strengths of the unidirectional lamina, the lamina is considered to be failed. That is,

$\sigma_1 \geq (\sigma_1^T)_{ult}$ or $\sigma_1 \leq -(\sigma_1^C)_{ult}$

$\sigma_2 \geq (\sigma_2^T)_{ult}$ or $\sigma_2 \leq -(\sigma_2^C)_{ult}$

$\tau_{12} \geq (\tau_{12})_{ult}$ or $\tau_{12} \leq -(\tau_{12})_{ult}$

where σ_1 and σ_2 are the normal stresses in the local axes 1 and 2, respectively; τ_{12} is the shear stress in the symmetry plane 1-2.

3.2. Tsai-wu failure criterion

The TW criterion is one of the most reliable static failure criteria which is derived from the von Mises yield criterion. A lamina is considered to fail if

$$\begin{aligned} H_1\sigma_1 + H_2\sigma_2 + H_6\tau_{12} + H_{11}\sigma_1^2 + H_{22}\sigma_2^2 \\ + H_{66}\tau_{12}^2 + 2H_{12}\sigma_1\sigma_2 < 1 \end{aligned} \tag{9}$$

is violated, where

$$\begin{aligned} H_1 &= \frac{1}{\left(\sigma_1^T\right)_{ult}} - \frac{1}{\left(\sigma_1^C\right)_{ult}} \\ H_{11} &= \frac{1}{\left(\sigma_1^T\right)_{ult}\left(\sigma_1^C\right)_{ult}} \\ H_2 &= \frac{1}{\left(\sigma_2^T\right)_{ult}} - \frac{1}{\left(\sigma_2^C\right)_{ult}} \\ H_{22} &= \frac{1}{\left(\sigma_2^T\right)_{ult}\left(\sigma_2^C\right)_{ult}} \\ H_{66} &= \frac{1}{\left(\tau_{12}\right)_{ult}^2} \\ H_{12} &= -\frac{1}{2}\sqrt{\frac{1}{\left(\sigma_1^T\right)_{ult}\left(\sigma_1^C\right)_{ult}\left(\sigma_2^T\right)_{ult}\left(\sigma_2^C\right)_{ult}}} \end{aligned} \tag{10}$$

H_i is the strength tensors of the second order; H_{ij} is the strength tensors of the fourth order. σ_1 is the applied normal stress in direction 1; σ_2 is the applied normal stress in the direction 2; and τ_{12} is the applied in-plane shear stress.

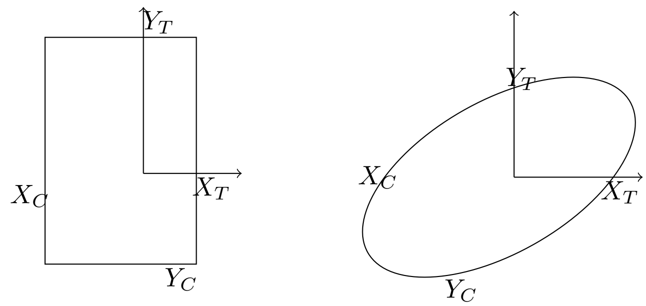

Figure 1: Schematic failure surfaces for maximum stress and quadratic failure criteria

3.3. Failure Theories for a Laminate

If keep increasing the loading applied to a laminate, the laminate will fails. The failure process of a laminate is more complicate than a lamina, because a laminate consists of multiple plies, and the fiber orientation, material, thickness of each ply maybe different from the others. In most situations, some layer fails first and the remains continue to take more loads until all the plies fail. If one ply fails, it means this lamina does not contribute to the load carrying capacity of the laminate. The procedure for finding the first failure ply given follows the fully discounted method:

1. Compute the reduced stiffness matrix [Q] referred to as the local axis for each ply using its four engineering elastic constants E_1, E_2, E_{12}, and G_{12}.
2. Calculate the transformed reduced stiffness $[\bar{Q}]$ referring to the global coordinate system (x, y) using the reduced stiffness matrix [Q] obtained in step 1 and the ply angle for each layer.
3. Given the thickness and location of each layer, the three laminate stiffness matrices [A], [B], and [D] are determined.
4. Apply the forces and moments, $[N]_{xy}, [M]_{xy}$ solve Equation 7, and calculate the middle plane strain $[\sigma^0]_{xy}$ and curvature $[k]_{xy}$.
5. Determine the local strain and stress of each layer under the applied load.
6. Use the ply-by-ply stresses and strains in the Tsai-wu failure theory to find the strength ratio, and the layer with smallest strenght ratio is the first failed ply.

3.4. Safety factor

The safety factor, or yield stress, is how much extra load beyond is intended a composite laminate will actually take. The safey factor is defined as

$$SR = \frac{\text{Maximum Load Which Can Be Applied}}{\text{Load Applied}} \tag{11}$$

.

4. Artificial Neural Network

Artificial neural networks(ANN) which heavily inspired by biology and psychology have been widely used to solve various practical engineering problems in such areas as pattern recognition, nonlinear regression, data mining, clustering and prediction. Methods based on complicated mathematical models is of intensive computation, approximation function evaluation techniques can be employed to accelerate the calculation process and reduce the computation cost. In order to solve practical engineering problems in composite material application, classic laminational theory(CLT) has been proposed which involves many matrix multiplication and integration calculation. ANN which has been proved is a reliable tool instead of complicate mathematical model. The design of neural network consists of three basic parts: neural network architecture, learning rules, and training techniques.

The weight training in ANN is to minimize the error function, such as the most widely used mean square error which calculate the difference between the desired and the prediction output values averaged over all examples. Backpropation algorithm has been successful applied to in many areas, and it's based on gradient descent. However, this class of algorithms are plagued by the possible existance of local minima or "flat spots" and "the curse of dimensionality". One method to overcome this problem is to adopt EANN's.

In this paper, the feedforward nueural network models are adopted in our system, because feedforward NNs are straigntforward and simply to code. For

Table 1: Different Activation Functions

Type	Description	Formula	Range
Linear	The output is proportional to the input	$f(x) = cx$	$(-\infty, +\infty)$
Sigmoid	A family of S-shaped functions	$f(x) = \frac{1}{1+e^{-cx}}$	$(0, 1)$
Gaussian	A coninuous bell-shaped curve	$f(x) = e^{-x^2}$	$(0, 1)$

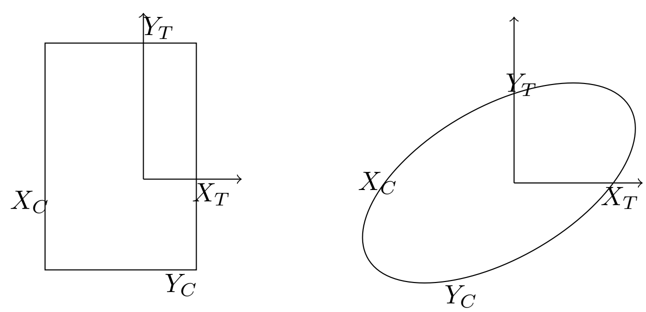

Figure 2: Schematic failure surfaces for maximum stress and quadratic failure criteria

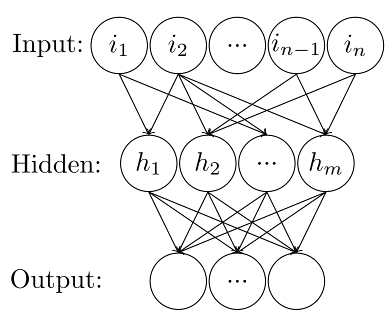

Figure 3: General Neural Network

funnction approximation, Cybenko demonstrated that a two-lay multilayer perceptron(MLP) is capbable of forming an arbitrarily close approximation to any continusous nonliner mapping[1]. Therefore, the we propose a GNN framework as shown in Figure 3. The search space is parametrized by: (i) the number of nodes m(possibly unbounded) in hidden layer, to narrow down the search space, m is less than n; (ii) the type of operation every nodes executes, e.g., sigmoid, linear, gaussian. (iii) the connection relationship between hidden nodes and inputs.

Figure 4: bbbbbbb

5. Evolution in EANN's

Evolution in EANN can be divided into four different levels: topology, learning rules, active functions, and connection weights. For the evolution of toplogy, the aim is to find an optimal ANN architecture for a specfic problem. The architecture of a neural network determines the information processing capability in application, which is the foundation of the ANN. Two critical issues are involved in the search process of an ANN architecture: the representation and the search operators.

The classic approach has always adopted binary strings to encode an alternative solutions.

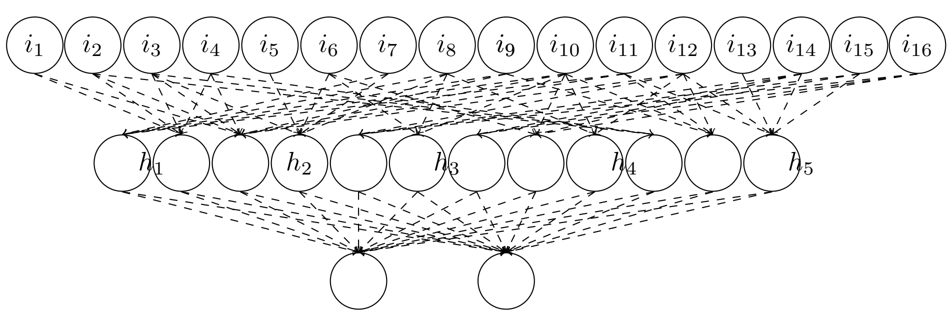

Figure 5: Parent 2

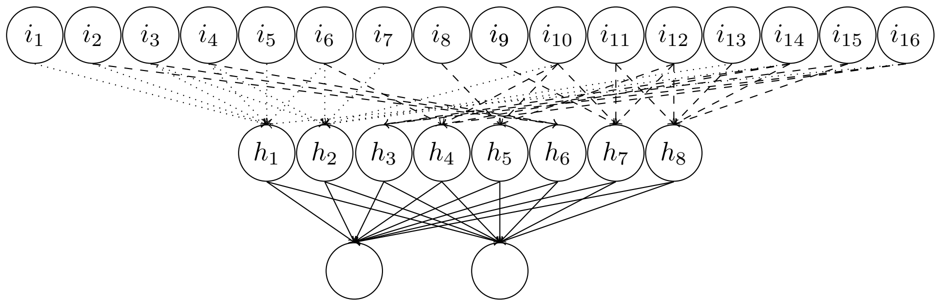

Figure 6: Child

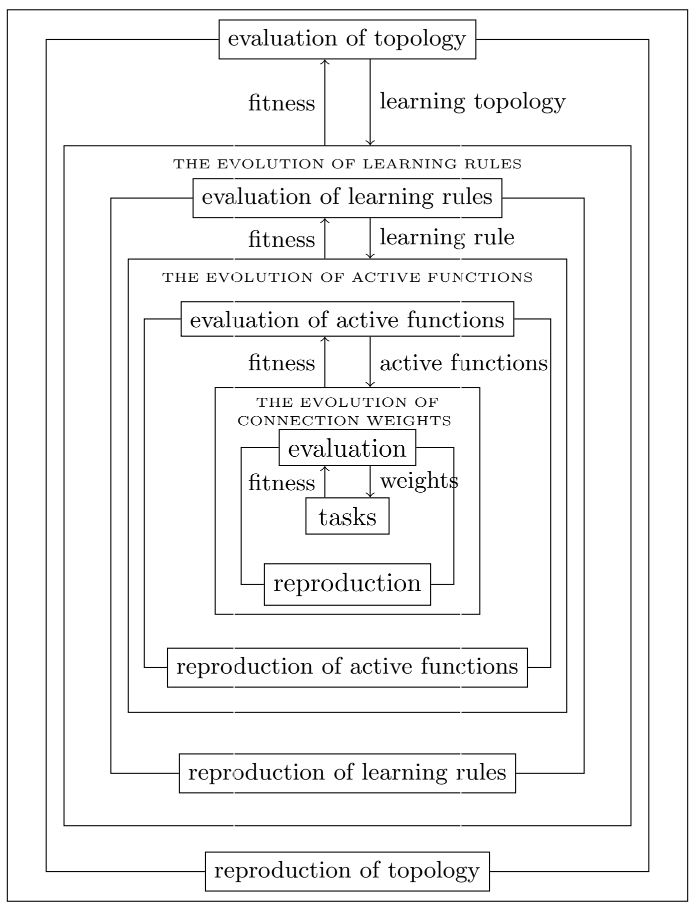

Figure 7: A general framwwork for EANN's

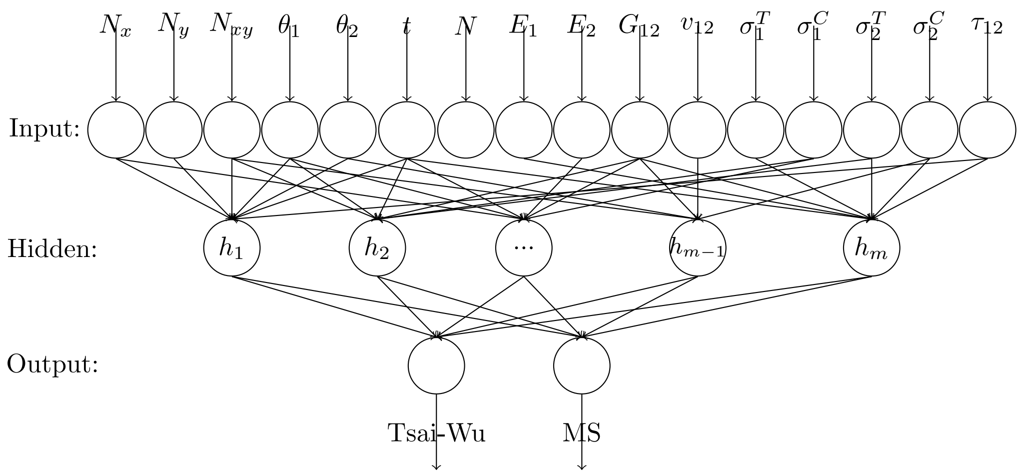

Figure 8: Neural Network Model

Tab.**??** gives an example of the binary representation of an ANN whose architecture is as shown in Fig.**??**. Each number in the digit denotes the connection relationship between input and nodes in hidden layer. It an connection exists, it's indicated by number one, otherwise, the number takes zero. The first sixteen digits denotes the connection relationship, and the last two digits are stand for the corresponding kernal function.

First, random initialize ANN population, partial training every ANN, For the evolution of the topology,

The inputs of the neural network is consist of four parts: in-plane loading N_x, N_y, and N_{xy}, design parameters of laminate, two distinct fiber orientation angle θ_1 and θ_2, ply thickness t, total number of plies N; five engineering constants of composite materials, E_1, E_2, ; five strength parameters of a unidirectional lamina. There are two outputs in the neural network, safety factors for MS theory and Tsai-Wu theory, respectively.

We have reviewed the use of GA and ANN for estimating a function based on CLT theory.

$+\theta$
$-\theta$
$\cdots$
$-\theta$
$+\theta$

Figure 9: Model for Angle ply laminate

References

References

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