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***Abstract*—In the present study, a genetic algorithm methodological framework-based optimization procedure is proposed to minimize thickness(or weight) of midplane-symmetric composite laminate subject to in-plane loading. Fiber orientation and ply thickness are chosen as design variables, and a variant of GA is employed to search for the optimal design of composite laminates. To avoid spurious laminate designs, both the Tsai-wu and the maximum stress criteria are taken to determine whether load-bearing capacity is exceeded or not. Numerical results are obtained and presented under different loading cases.**

Keywords*—Genetic algorithm; Optimization; Classic lamination theory; Failure theory

I. Introduction

Composite materials offer improved strength, stiffness, corrosion resistance, etc. over conventional materials, and are widely used as alternative materials for applications in various industries ranging from electronic packaging to golf clubs, and medical equipment to homebuilding, making aircraft structure to space vehicles. The stacking sequence and fiber orientation of composite laminates give the designer additional degree of freedom to tailor the design with respect to strength or stiffness. One widely known advantage of using composite material is can significantly reducing the weight of target structure, and many researchers attempted to improve the efficiency of using composite material by minimizing the thickness[?], [?].

In practice, fiber orientations are restricted to a finite set of angles and ply thickness is a specific numeric value. Because the design variables are not continueous, a gradient-based optimization procedure, such as the gradient descent method, is not suitable to cope with such problems. Moreover, gradient-based optimization approach is very easily to get trapped in local minima, and many local optimum may exist in structural optimization problems. A stochastic optimization, such as the genetic algorithm(GA) and simulated annealing(SA), can deal with optimization problems with discrete variables. Besides, the stochastic method could escape from the local optimum, and obtain the global optimum. GA is one of the most reliably stochastic algorithms, which has been widely used in solving constraint design for composite laminate[?], [?], [?], [?], [?], [?], [?], [?], [?]. Although GA gains different advantages for solving discrete problems, many disadvantages exist within this approach. First, the optimization process of GA parameters, such as the population size, parent population,mutation percentage, etc., is very tedious; Second, the GA needs to evaluate the objective functions many times to achieve the optimization, and the computation cost is very high; the last problem within GA is the premature convergence. GA consists of five basic parts: the variable coding, selection scheme, crossover operator, mutation operator, and how the constraints are handled.

The first issue when implementing a GA is the representation of design variables, and an appropriate design representation is crucial to enhance the efficiency of GA. The canonical GA has always used binary strings to encode alternative solutions, however, some argued that the minimal cardinality, i.e., the binary representation, is not the best option.

Selection scheme plays a critical role in balancing the dilemma of exploration and exploitation inherent in GA, and various selection methods, for example, roulette wheel, elitist, and tournament, etc. have been proposed to overcome this issue. Both roulette selection and tournament selection are well-studied and widely employed in the optimization design of laminated composite due to their simplicity to code and efficiency for both nonparallel and parallel architectures.

Crossover is another crucial operator introduced into the GA methodology framework, in which the alternative solution is generated from the mating pool. multiple types of crossover operator have been utilized in the optimization design of composite structures, such as, one-point, two-point, and uniform crossover.

GA is originally proposed for unconstrained optimization. However, in order to deal with constrained design for composite laminate, some techniques were introduced into the GA. The first method is using of data structure, special data structure was developed to fulfill the symmetry constraint of the laminate, which consists of coding only half of the laminate and considering that each stack of the laminate is formed by two laminae with the same orientation but opposite signs[?], [?]. A penalty function is developed to convert a constrained problem into an unconstrained problem by adding a penalty term to the objective function. Another method to solve the constrained problem is introducing repair strategy by Todoroki and Haftka [?], which is aim to transform infeasible solutions to feasible solution by incorporating problem-specific knowledge.

Another major concern within GA is the convergence speed in terms of the time and computation cost needed to reach a solution of desired quality. The objective function based on the CLT is excessively time-consuming and complicate to evaluate, besides, the target function of GA needs to calculate many times. The traditional method to deal with this issue is by increasing the selection pressure to accelerate the convergence speed, however, in some cases, this approach does not achieve an ideal result. Because the GAs just provides a methodological framework to deal with tricky problems, which is heavily inspired by evolution of biology, it is unnecessary to exactly follow all the GA operation. It is possible to just perform one or more GA operations, and incorporate other techniques into GA. In the present study, a variant of mutation operator is introduced to accelerate the convergence process.

To check the feasibility of a laminate composite by imposing a strength constraint, various failure criterion have been proposed to decide whether it fails or not, such as maximum stress failure theory, maximum strain failure theory, Tsai-Hill Failure theory, and Tsai-Wu criterion. Each theory is proposed based on massive experiment data or complicate mathematical model, however single use any of them may lead to a false optimum design for some loading case due to the particular shape of its failure envelope. In order to overcome this disadvantage within every failure theory, two reliably failure criteria, maximum stress theory and Tsai-wu criterion are employed to check whether the composite laminate fullfills the constraint.

The rest of the paper is organized as follows. Section 2 explains the classical laminate theory and the failure criteria taken in the present study. Section 3 explains the proposed method of selection strategy and self-adaptative parameters for mutation during the GA process. Section 4 describes the result of the numerical experiments in different cases, and in the conclusion section, we dicuss the results.

II. Analysis of stress and strain for composite material

A. Stress and Strain in a Lamina

A single lamina has a small thickness under plane stress, and its upper and lower surfaces of lamina are free from external loads. According to Hooke's Law, the three-dimensional stress-strain equations can be reduced to two-dimensional stress-strain equations. The stress-strain relation in local axis 1-2 is

$$\sigma_1\sigma_2\tau_{12} = Q_{11}Q_{12}0Q_{12}Q_{22}000Q_{66}\varepsilon_1\varepsilon_2\gamma_{12}, \qquad (1)$$

where $Q_{ij}$are the stiffnesses of the lamina that are related

to engineering elastic constants given by

$$Q_{11} = \frac{E_1}{1-v_{12}v_{21}}, Q_{22} = \frac{E_2}{1-v_{12}v_{21}}, Q_{66} = G_{12}, Q_{12} = \frac{v_{21}E_2}{1-v_{12}v_{21}}, \qquad (2)$$

where E_1, E_2, v_{12}, G_{12} are four independent engineering elastic constants, which are defined as follows: E_1 is the longitudinal Young's modulus, E_2 is the transverse Young's modulus, v_{12} is the major Poisson's ratio, and G_{12} is the in-plane shear modulus.

Stress strain relation in the global x-y axis is

$$\begin{bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{bmatrix} = \begin{bmatrix} \bar{Q}_{11} & \bar{Q}_{12} & \bar{Q}_{16} \\ \bar{Q}_{12} & \bar{Q}_{22} & \bar{Q}_{26} \\ \bar{Q}_{16} & \bar{Q}_{26} & \bar{Q}_{66} \end{bmatrix} \begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{bmatrix}, \qquad (3)$$

where

$$\begin{aligned}
\bar{Q}_{11} &= Q_{11}cos^4\theta + Q_{22}sin^4\theta + 2\left(Q_{12}+2Q_{66}\right)sin^2\theta cos^2\theta, \\
\bar{Q}_{12} &= \left(Q_{11}+Q_{22}-4Q_{66}\right)sin^2\theta cos^2\theta + Q_{12}\left(cos^4\theta + sin^2\theta\right), \\
\bar{Q}_{22} &= Q_{11}sin^4\theta + Q_{22}cos^4\theta + 2\left(Q_{12}+2Q_{66}\right)sin^2\theta cos^2\theta, \\
\bar{Q}_{16} &= \left(Q_{11}-Q_{12}-2Q_{66}\right)cos^3\theta sin\theta - \left(Q_{22}-Q_{12}-2Q_{66}\right)sin^3\theta cos\theta, \\
\bar{Q}_{26} &= \left(Q_{11}-Q_{12}-2Q_{66}\right)cos\theta sin^3\theta - \left(Q_{22}-Q_{12}-2Q_{66}\right)cos^3\theta sin\theta, \\
\bar{Q}_{66} &= \left(Q_{11}+Q_{22}-2Q_{12}-2Q_{66}\right)sin\theta^2 cos\theta^2 + Q_{66}\left(sin\theta^4 + cos\theta^4\right).
\end{aligned} \qquad (4)$$

The local and global stresses in an angle lamina are related

to each other through the angle of the lamina θ, it can be written as

$$\begin{bmatrix} \sigma_1 \\ \sigma_2 \\ \tau_{12} \end{bmatrix} = [T] \begin{bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{bmatrix}, \qquad (5)$$

where

$$[T] = \begin{bmatrix} cos^2\theta & sin^2\theta & 2sin\theta cos\theta \\ sin^2\theta & cos^2\theta & -2sin\theta cos\theta \\ -sin\theta cos\theta & sin\theta cos\theta & cos^2\theta - sin^2\theta \end{bmatrix}. \qquad (6)$$

B. Stress and Strain in a Laminate

For forces and moment resultants acting on laminates, such as in plate and shell structures, the relationship between applied forces and moment and displacement can be given by

$$\begin{matrix} N_x \\ N_y \\ N_{xy} \end{matrix} = A_{11}A_{12}A_{16}A_{12}A_{22}A_{26}A_{16}A_{26}A_{66}\varepsilon_x^0\varepsilon_y^0\gamma_{xy}^0 + B_{11}B_{12}B_{16}B_{11}$$

M$_x$
M_y
M_{xy} = B$_{11}B_{12}B_{16}$
$B_{12}B_{22}B_{26}$
$B_{16}B_{26}B_{66}$ ε_x^0
ε_y^0
γ_{xy}^0
+ D$_{11}D_{12}D_{16}$
$D_{11}D_{12}D_{16}$
$D_{16}D_{26}D_{66}$ k$_x$
k_y
k_{xy}, (7)

where

N_x, N_y - normal force per unit length;

N_{xy} - shear force per unit length;

M_x, M_y - bending moment per unit length;

M_{xy} - twisting moments per unit length;

ε^0, k- mid plane strains and curvature of a laminate in x-y coordinates.

The mid plane strain and curvature is given by

$$A_{ij} = \sum_{k=1}^{n}(\overline{Q_{ij}})_k(h_k - h_{k-1}) i = 1,2,6, j = 1,2,6, B_{ij} = \frac{1}{2}\sum_{k=1}^{n}(\overline{Q_{ij}})_k(\qquad (8)$$

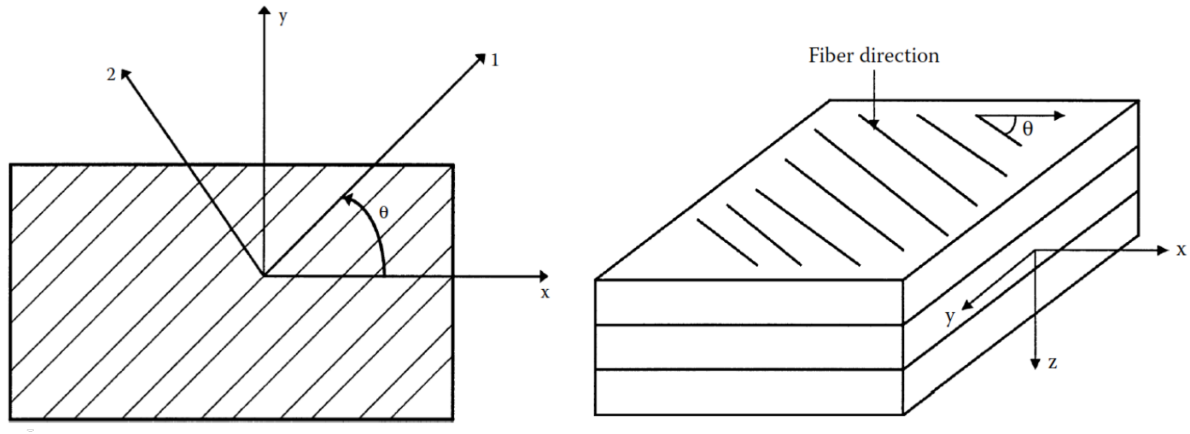

Fig. 1: Local and global axes of an angle lamina

The [A], [B], and [D] matrices are called the extensional, coupling, and bending stiffness matrices, respectively. The extensional stiffness matrix $[A]$ relates the resultant in-plane forces to the in-plain strains, and the bending stiffness matrix $[D]$ couples the resultant bending moments to the plane curvatures. The coupling stiffness matrix $[B]$ relates the force and moment terms to the midplane strains and midplane curvatures.

III. Failure criteria for a lamina

Failure criteria for composite materials are more difficult to predict due to structural and material complexity in comparison to isotropic materials. The failure process of composite materials can be regarded from microscopic and macroscopic points of view. Most popular criteria about the failure of an angle lamina are in terms of macroscopic failure criteria, which are based on the tensile, compressive and shear strengths. According to the failure surfaces, these criteria can be classified into two classes: one is called independent failure mode criteria which includes the maximum stress failure theory, maximum strain failure theory because their failure envelope are rectangle; another is called quadratic polynomial which includes Tsai-Wu, Chamis, Hoffman, and Hill criteria because their failure surfaces are of ellipsoidal shape. In the present study, the two most reliable failure criteria are taken, Maximum stress and Tsai-wu. Both of these two failure criteria are based on the stresses in the local axes instead of principal normal stresses and maximum shear stresses, and four normal strength parameters and one shear stress for a unidirectional lamina are involved. The five strength parameters are

$(\sigma_1^T)_{ult}$ = ultimate longitudinal tensile strength(in direction 1),

$(\sigma_1^C)_{ult}$ = ultimate longitudinal compressive strength,

$(\sigma_2^T)_{ult}$ = ultimate transverse tensile strength,

$(\sigma_2^C)_{ult}$ = ultimate transverse compressive strength, and

$(\tau_{12})_{ult}$ = and ultimate in-plane shear strength.

A. *Maximum stress failure criterion*

Maximum stress(MS) failure theory consists of maximum normal stress theory proposed by Rankine and maximum shearing stress theory by Tresca. The stresses applied on a lamina can be resolved into the normal and shear stresses in the local axes. If any of the normal or shear stresses in the local axes of a lamina is equal or exceeds the corresponding ultimate strengths of the unidirectional lamina, the lamina is considered to be failed. That is

$$\sigma_1 \geq (\sigma_1^T)_{ult} \text{ or } \sigma_1 \leq -(\sigma_1^C)_{ult},$$

$$\sigma_2 \geq (\sigma_2^T)_{ult} \text{ or } \sigma_2 \leq -(\sigma_2^C)_{ult},$$

$$\tau_{12} \geq (\tau_{12})_{ult} \text{ or } \tau_{12} \leq -(\tau_{12})_{ult}.$$

where σ_1 and σ_2 are the normal stresses in the local axes 1 and 2, respectively; τ_{12} is the shear stress in the symmetry plane 1-2.

B. *Tsai-wu failure criterion*

The TW criterion is one of the most reliable static failure criteria which is derived from the von Mises yield criterion. A lamina is considered to fail if

$$H_1\sigma_1+H_2\sigma_2+H_6\tau_{12}+H_{11}\sigma_1^2+H_{22}\sigma_2^2+H_{66}\tau_{12}^2+2H_{12}\sigma_1\sigma_2 < 1 \quad (9)$$

is violated, where

$$H_1 = \frac{1}{(\sigma_1^T)_{ult}} - \frac{1}{(\sigma_1^C)_{ult}}, H_{11} = \frac{1}{(\sigma_1^T)_{ult}(\sigma_1^C)_{ult}}, H_2 = \frac{1}{(\sigma_2^T)_{ult}} - \frac{1}{(\sigma_2^C)_{ult}} \quad (10)$$

H_i is the strength tensors of the second order; H_{ij} is the strength tensors of the fourth order. σ_1 is the applied normal stress in direction 1; σ_2 is the applied normal stress in the direction 2; and τ_{12} is the applied in-plane shear stress.

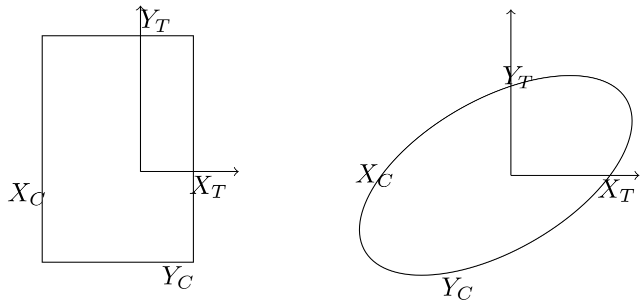

Fig. 2: Schematic failure surfaces for maximum stress and quadratic failure criteria

C. Failure Theories for a Laminate

If keep increasing the loading applied to a laminate, the laminate will fail. The failure process of a laminate is more complicated than lamina, because a laminate consists of multiple plies, and the fiber orientation, material, thickness of each ply may be different from the others. In most situations, some layer fails first and the remains continue to take more loads until all the plies fail. If one-ply fails, it means this lamina does not contribute to the load-carrying capacity of the laminate. The procedure for finding the first failure ply given follows the fully discounted method:

1) Compute the reduced stiffness matrix [Q] referred to as the local axis for each ply using its four engineering elastic constants E_1, E_2, E_{12}, and G_{12}.
2) Calculate the transformed reduced stiffness $[\bar{Q}]$ referring to the global coordinate system (x, y) using the reduced stiffness matrix [Q] obtained in step 1 and the ply angle for each layer.
3) Given the thickness and location of each layer, the three laminate stiffness matrices [A], [B], and [D] are determined.
4) Apply the forces and moments, $[N]_{xy}, [M]_{xy}$ solve Equation **??**, and calculate the middle plane strain $[\sigma^0]_{xy}$ and curvature $[k]_{xy}$.
5) Determine the local strain and stress of each layer under the applied load.
6) Use the ply-by-ply stresses and strains in the Tsai-wu failure theory to find the strength ratio, and the layer with smallest strength ratio is the first failed ply.

D. Safety factor

The safety factor, or yield stress, is how much extra load beyond is intended a composite laminate will actually take, which is an indication of the material's load-carrying capacity. If the value is less than 1.0, it means failure. The safety factor is defined as

$$SF = \frac{MaximumLoadWhichCanBeApplied}{LoadApplied}. \tag{11}$$

The safety factor based on maximum stress theory is calculated by the following method: first, the principal stresses(σ_1^k,σ_2^k, and τ_{12}^k) are obtained by experiment; evaluate

P_1:	+7	+7	+7	+7	+7	+7	+7	+7	-9	-9
P_2:	+19	+19	+19	+19	-36	-36	-36	-36	-36	-36

(a): Parents P_1 and P_2

O_1:	+13	+13	+13	+13	+13	+13	-27	-27	-27	-27
O_2:	+22	+22	+22	+22	+22	+22	+22	+5	+5	+5

(b): Offspring O_1 and O_2

O_1:	+13	+13	+13	$\cdots$	+13	+13	-27	$\cdots$	-27	-27
	11							7		

(c): Offspring O_1 after lenght mutation

+12	+12	+12	$\cdots$	+12	+12	-26	$\cdots$	-26	-26

(b): Offspring O_1 after angle mutation

Fig. 3: GA Operators

the safety factor along each direction according to equation **??**; The minimum value among these safety factors are denoted as the safety factor of the lamina, SF_{MS}^k, it can be written as

$\mathrm{SF}_{MS}^k = minof\,\{\,S\;F_X^k = \{\;X_t\sigma_{11}, if\sigma_{11} > 0$
$\frac{X_c}{\sigma_{11}}, if\sigma_{11} < 0$

$SF_Y^k = \{\;Y_t\sigma_{22}, if\sigma_{22} > 0$
$\frac{Y_c}{\sigma_{22}}, if\sigma_{22} < 0$

$SF_S^k = \{\;S|\tau_{12}|$

.

Assuming the composite laminate under an in-plane loading f, the corresponding stress on local stress in direction 1, local stress in direction 2, and shear stress for the kth lamina are $\sigma_1 SF_{TW}^k$, $\sigma_2 SF_{TW}^k$, and $\tau_{12} SF_{TW}^k$, respectively. Substitute them into equation **??**, the expression are given by

$$a(SF_{TW}^k)^2 + b(SF_{TW}^k) - 1 = 0,$$

where

$$a = H_{11}(\sigma_1)^2 + H_{22}(\sigma_2)^2 + H_{66}(\tau_{12})^2 + 2H_{12}\sigma_1\sigma_2,$$

$$b = H_1\sigma_1 + H_2\sigma_2 + H_6\tau_{12}.$$

Solve the above equation, the safety factor for the kth lamina is

$$SF_{TW}^k = |\tfrac{-b+\sqrt{b^2+4a}}{2a}|.$$

Then, the minimum of SF_{TW}^k is taken as the safety factor of the laminate which is written as

$$SF_{TW} = minofSF_{TW}^k fork = 1, 2, \cdots, m-1, m\ .$$

IV. Methodology

A. Objective function

The optimization problem can be formulated by searching the optimal stacking sequence of composite laminate. There are two design variables here, the angles in the laminate, and the number of layers that each fiber orientation has. The objective function is formulated as

$$F = 2t_0 \sum_{k=1}^{n} n_k, SF_{MS} \geq 1, SF_{TW} \geq 1. \quad (12)$$

The first term represents the total thickness of the composite laminates, t_0 is the ply thickness; n_k is the number of plies in the kth lamina, in which the fiber orientation is θ_k. The constraints here are two safety factors should not less than 1, which means $SF_{MS} \geq 1$, and $SF_{TW} \geq 1$, respectively.

B. Encoding

Due to the simplicity and efficiency of float representation, this encoding method is implemented to represent a possible solution. As shown in Figure **??** (a), these two chromsomes represent a $[+8_7/-9_2]_s$ carbon T300/5308 laminated composite, and $[+19_4/-36_6]_s$, respectively. Becasue the laminate adopted in this paper is symmetric to its mid-plane, so only half needs to be encoded.

C. Selection

The purpose of the selection operator is to choose mating pool to produce alternative solutions of better fitness. Traditional methods of selecting strategies only take the fitness of individuals into account, however, due to the existence of constraint, various selection schemes are implemented to select the mating set. Based on different selection schemes, the parents of next generation can be divided into three groups: proper groups, active groups, and potential groups according to different selecting methods.

Proper parents mean in which individual fullfills the constraints, which are chosen by the individual's fitness, individuals with better fitness are more likely to be chosen if they fit the constraint; active group means that individual within this group is supposed to always exist in the parents during the GA, which are selected by fitness, ignoring the constraint; The individuals from the active group may not correspond to feasible solutions, but their existence enriches the variety of the gene clips. Potential group means that individuals are likely to turn into proper individual after a couple of generations, and potential individuals are chosen by constraint function, the more the individual fulfills the constraint, the more possibility it will be selected.

D. Crossover

The crossover operator happens among these three groups. the child of two proper groups is more likely to be a proper individual which can be used to obtain an alternative feasible solution. the child of an active individual and a potential individual can significantly change the gene of an active individual's chromosome, which makes the individual evolve toward a new direction. The offspring of two active individuals are more likely to be an active individual, which can maintain the active group. The figure.**??** (b) shows two children O_1 and O_2 from two parents P_1 and P_2, each angle C_a and its length C_l of a child are obtained by the following formula

$\{ C_a = (P1_a + P2_a)/2$
$C_l = (P1_l + P2_l)/2$.

E. Mutation

A mutation direction is imposed on the mutation operator which to make sure the individual evolving toward the right direction. The mutation direction, denoted by md, is an n dimensional vector corresponding to the number of constraints, it is decided by the constraint thresholds CT_i and the current individual's constraint value, denoted as CV_i, The mutation vector can be obtained by the following formula

$$md = [CT_1, \cdots, CT_{n-1}, CT_n] - [CV_0, \cdots, CV_{n-1}, CV_n].$$

During this operator, the mutation procedure is consist of two phases: the length mutation of the chromosome, and the angle mutation of the chromosome. Because the chromosome's length is positively correlated with the individual's fitness, the coefficient of length mutation denoted by C_l, if $\sum_{i=1}^{N} CT_i$ great than $\sum_{i=1}^{N} CV_i$, the mutation length is restricted to the range $[0, (C_l \sum_{i=1}^{N} (CT_i - CV_i))/N]$, which means increase the chromosome's length; Assuming a $[+13_6/-27_4]_s$ T300/5308 carbon/epoxy composite laminate under the loading $N_x = N_y = 10$ MPa m, it's property as shown in table **??**. According to CLT and failure theory, the two safety factors SF_{MS} and SF_{TW} are 0.0539, and 0.0540, respectively. So the mutation vector and is $[0.9461, 0.9460]$, assuming the length mutation coefficient is 20, so the mutation range is from 0 to 18. A random number is generated from the range $[0, 18]$, supposing the outcome is 13, then a length generator is used to a list, the its sum is 13, suppose the list is [5, 8], the laminate after mutation is $[13_{11}/-27_{12}]_s$.

If the $\sum_{i=1}^{N} CT_i$ less than $\sum_{i=1}^{N} CV_i$, the mutation length is restricted to the range $([\sum_{i=1}^{N} CT_i - CV_i)/N, 0]$, which means the individual's fitness exceeds the threshold value, and decrease the chromosome's length. Assuming a $[+33_{35}/-29_{26}]_s$ T300/5308 laminate is under loading $N_x = 10$ MPa, and $N_y = 5$ MPa, then, it's SF_{MS} constraint and SF_{TW} values are 1.0912, 1.0747, respectively. because the length mutation is 20, so the mutation range is from -2 to 0. This would decrease the chromosome's length. LM = { .35![0,(C$_l \sum_{i=1}^{N} (CT_i - CV_i))/N], if \sum_{i=1}^{N} CT_i > \sum_{i=1}^{N} CV_i$
$[(C_l \sum_{i=1}^{N} (CT_i - CV_i))/N, 0], if \sum_{i=1}^{N} CT_i < \sum_{i=1}^{N} CV_i$

The relationship between the angles in the composite laminate and the chromosome's fitness is unclear, so the mutation direction of chromosome's angle is random. The coefficient angle mutation is C_a, the angle mutation range is $[0, C_a \sum_{i=1}^{N} (|CT_i - CV_i|)]$ or $[C_a \sum_{i=1}^{N} (-|CT_i - CV_i|), 0]$. It is can be written as

P(AM) = { 0 .5, $AM = [0, C_a \sum_{i=1}^{N} (|CT_i - CV_i|)]$
$0.5, AM = [C_a \sum_{i=1}^{N} (-|CT_i - CV_i|), 0]$.

V. Result and Discussion

In the present study, the T300/5308 graphite/epoxy material is used in the lay-up sequence optimization, and its properties as shown in table.**??**. Two constraints are imposed on the

TABLE I: Properties of T300/5308 carbon/epoxy composite

Property	Symbol	Unit	Graphite/Epoxy
Longitudinal elastic modulus	E_1	GPa	181
Traverse elastic modulus	E_2	GPa	10.3
Major Poisson's ratio	v_{12}		0.28
Shear modulus	G_{12}	GPa	7.17
Ultimate longitudinal tensile strength	$(\sigma_1^T)_{ult}$	MP	1500
Ultimate longitudinal compressive strength	$(\sigma_1^C)_{ult}$	MP	1500
Ultimate transverse tensile strength	$(\sigma_2^T)_{ult}$	MPa	40
Ultimate transverse compressive strength	$(\sigma_2^C)_{ult}$	MPa	246
Ultimate in-plane shear strength	$(\tau_{12})_{ult}$	MPa	68

composite laminates which are the safety factor SF_{MS} , and safety factor SF_{TW}, and the threshold values for both of them is 1. The constraint values of an individual are CV_1 and CV_2. So the mutation vector here is a two-dimensional vector $[1-CV_1, 1-CV_2]$, and the coefficient of length mutation C_l and angle mutation C_a are 20 and 10, respectively.

To verify the reliability of the proposed method, two conditions are concerned: the first is only two distinct fiber orientation angles in the composite material; the second involves three distinct ply angles within the optimization process. In each situation, first, we present the search process by plotting relevant indicators, such as the fitness, strength ratio, and angle. Then, the optimum lay-ups under various loading cases are discussed.

Figure **??**(a) shows how the optimal individual's fitness and strength ratio evolves during the GA process. The solid curve shows the fitness value, the dashed curve shows the Tsai-wu safety factor, and the dotted curve shows MS safety factor. If the smaller strength ratio fullfills the constraint, this laminate must satisfy all the constraints, for simplicity, only the smaller strength ratio is presented in the figure**??**(a). The method to chose optimal individual considering two following situations, if no individual in the current population meets constraint, the one with the biggest fitness is selected as the optimal individual; if there are one or multiple individuals fullfills requirement, the one with the smallest fitness is chosen which means the smallest one has the biggest priority. Figure **??** (b) and **??**(b) show how every fiber orientation changes, and Figure **??**(c) and **??**(c) display how the number of each angle varies.

At the beginning of this GA process, the fitness curves increases very quickly, because individual's two strength ratios are very small, so the difference between the individual's fitness and the imposed constraint threshold is a big positive number, so the range of mutation length is from 0 to $C_l(CT_0 - CV_0 + CT_1 - CV_1)/2$. The length of individual increases by n, which is a random number between 0 and $C_l(CT_0 - CV_0 + CT_1 - CV_1)/2$. As can be seen from Figure **??** (a), both of optimal individual's fitness and strength ratio increases very quickly. The range of angle mutation is from 0 to $C_a(CT_0 - CV_0 + CT_1 - CV_1)/2$, and the number of each angle also changes violently. The Figure.**??** (a) and **??** (a) show this property at the initial stage. During this stage, increasing an individual's length playing a major role in increasing its fitness.

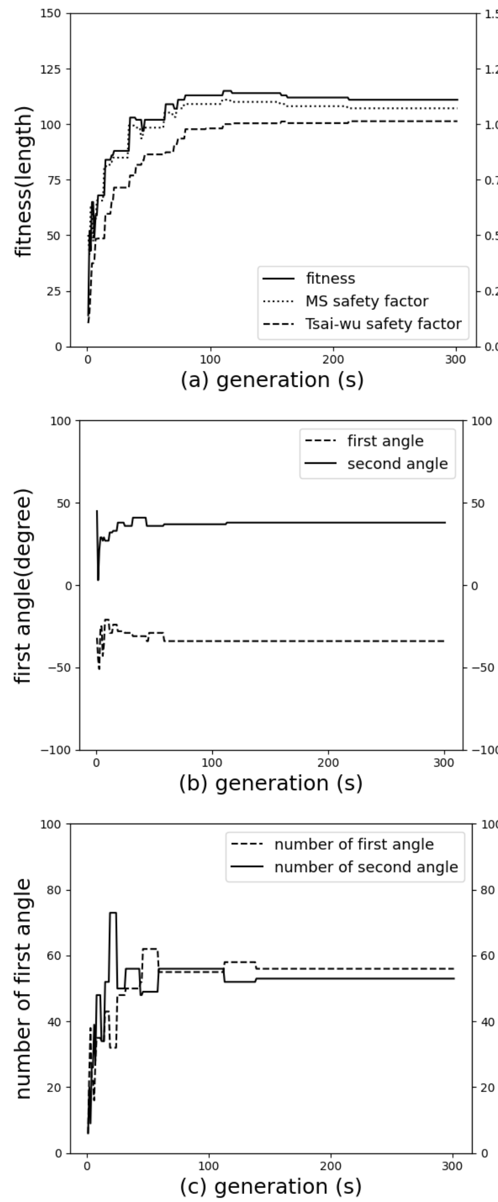

Fig. 4: Two distinct angles

TABLE II: The optimum lay-ups using two distinct fiber angles under various biaxial loading cases

Loading $N_x/N_y/N_{xy}$ (MPa m)	**Optimum lay-up sequences**	**Laminate thickness**	**Safety factor for Tsai-wu**	**Safety factor for maximum stress**
10/5/0	$[33_{29}/-39_{25}/-\bar{39}]_s$	109	1.0074	1.0246
20/5/0	$[33_{22}/-31_{24}]_s$	92	1.0055	1.2065
40/5/0	$[29_{18}/-21_{23}/-\bar{21}]_s$	83	1.0034	1.7350
80/5/0	$[-20_{27}/21_{25}/\bar{25}]_s$	105	1.0029	1.2063
120/5/0	$[-18_{34}/17_{36}]_s$	140	1.0000	1.0898

After a couple of generations, the optimal individual's fitness gets bigger, and the difference between individual's fitness and constraint threshold gets smaller. The range of mutation length turns smaller. At this stage, simply increase the individual's length doesn't make much difference in improve an individual's fitnees, and a better composite laminates lay-up can dramatically change the optimal individual's fitness. That's why the fitness curve oscillated violently in this stage. At the same time, the strength ratio curve keeps growing smoothly. But the growing speed gets more smaller.

When GA comes to its last phase, GA finds individuals that meet all the constraints. Now the optimal individual's fitness is greater than the safety factor. The range of mutation length is from $C_l(CT_0 - CV_0 + CT_1 - CV_1)/2$ to 0. It means individuals need to decrease its length and improve its internal structure to meet the constraint. That's why the fitness of optimal individual kept decreaing, however, the strength ratio curve still is greater than safety factor.

Table.**??** shows the comparison with the result obtained by direct search simulated annealing(DSA) algorithm which was proposed by Akbulut and Sonmez[**?**]. Both of variant GA and DSA are able to find feasible solution, but when loading is $N_x = 80$, $N_y = 5$ MPa m, variant GA got a better solution than DSA. In the case that loadings are $N_x = 20$, $N_y = 5$ MPa m, and $N_x = 120$, $N_y = 5$ MPa m, the proposed GA offered an alternative solution. Compared with DSA method, the last advantage of variant GA is the number of layers doesn't have to be even.

VI. Conclusion

In this paper, we reviewed the use of variant GA for the optimal design of composite laminated material under in-plane loading based on Tsai-wu and maximum stress failure criteria. GA is proposed to search the optimal lay-up for laminated composite under different loading cases. Two situations are considered under the same loading, a set of two distinct angles, and three distinct angles.

By setting the constant values of length mutation coefficient and angle mutation coefficient at the beginning, the convergence speed of the search process can be controlled in an explicit way; During the optimization process, GA can adjust its length mutation range and angle mutation range based on the difference between individual's constraint values and constraint thresholds.

Finally, comparison of previous research and current result are presented. In some cases, the proposed GA in this paper is better off than DSA method. However, there is still many works to study within this GA, such as the fine-tuning of parameters taken in this GA.

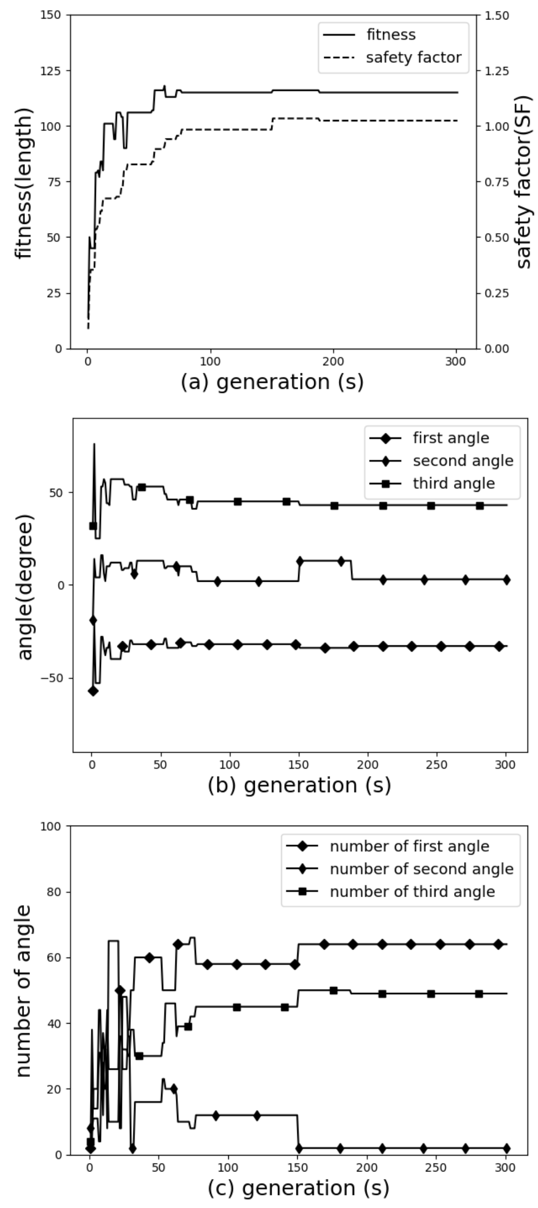

Fig. 5: Three distinct angles

TABLE III: The optimum lay-ups using three distinct fiber angles under various biaxial loading cases

Loading $N_x/N_y/N_{xy}$ (MPa m)	**Optimum lay-up sequences**	**Laminate thickness**	**Safety factor for Tsai-wu**	**Safety factor for maximum stress**
10/5/0	$[37_{27}/-38_{27}/-5]_s$	110	1.0023	1.0216
20/5/0	$[34_{24}/-32_{14}/-28_{11}]_s$	98	1.0237	1.2089
40/5/0	$[21_{28}/-32_{19}/2_3]_s$	100	1.0617	1.7076
80/5/0	$[-19_{24}/20_{27}/-17_{16}/-\bar{1}7]_s$	109	1.0056	1.2093
120/5/0	$[-19_{33}/12_{13}/16_{28}]_s$	148	1.0105	1.1014

TABLE IV: Comparison with the results of DSA

Loading	**Akbulut and Sonmez's[?] Study**				**Present Study**			
$N_x/N_y/N_{xy}$ (MPa m)	Optimum lay-up sequences	laminate thickness	TW	MS	Optimum lay-up sequences	laminate thickness	TW	MS
10/5/0	$[37_{27}/-37_{27}]_s$	108	1.0068	1.0277	$[33_{29}/-39_{25}/-\bar{3}9]_s$	109	1.0074	1.0246
20/5/0	$[31_{23}/-31_{23}]_s$	92	1.0208	1.1985	$[33_{22}/-31_{24}]_s$	92	1.0055	1.2065
40/5/0	$[26_{20}/-26_{20}]_s$	80	1.0190	1.5381	$[29_{18}/-21_{23}/-\bar{2}1]_s$	83	1.0034	1.7350
80/5/0	$[21_{25}/-19_{28}]_s$	106	1.0113	1.2213	$[-20_{27}/21_{25}/2\bar{5}]_s$	105	1.0029	1.2063
120/5/0	$[17_{35}/-17_{35}]_s$	140	1.0030	1.0950	$[-18_{34}/17_{36}]_s$	140	1.0000	1.0898

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REFERENCES

[1] L. A. Schmit and B. Farshi, "Optimum laminate design for strength and stiffness," *International Journal for Numerical Methods in Engineering*, vol. 7, no. 4, pp. 519–536, 1973.

[2] L. Schmit Jr and B. Farshi, "Optimum design of laminated fibre composite plates," *International journal for numerical methods in engineering*, vol. 11, no. 4, pp. 623–640, 1977.

[3] H. Fukunaga and G. Vanderplaats, "Strength optimization of laminated composites with respect to layer thickness and/or layer orientation angle," *Computers & Structures*, vol. 40, no. 6, pp. 1429–1439, 1991.

[4] C. M. Soares, V. F. Correia, H. Mateus, and J. Herskovits, "A discrete model for the optimal design of thin composite plate-shell type structures using a two-level approach," *Composite structures*, vol. 30, no. 2, pp. 147–157, 1995.

[5] R. Le Riche and R. Haftka, "Improved genetic algorithm for minimum thickness composite laminate design," *Composites Engineering*, vol. 5, no. 2, pp. 143–161, 1995.

[6] C. Jayatheertha, J. Webber, and S. Morton, "Application of artificial neural networks for the optimum design of a laminated plate," *Computers & structures*, vol. 59, no. 5, pp. 831–845, 1996.

[7] J. Wang and B. Karihaloo, "Optimum in situ strength design of composite laminates. part i: in situ strength parameters," *Journal of composite materials*, vol. 30, no. 12, pp. 1314–1337, 1996.

[8] S. Adali and V. E. Verijenko, "Minimum cost design of hybrid composite cylinders with temperature dependent properties," *Composite structures*, vol. 38, no. 1-4, pp. 623–630, 1997.

[9] V. F. Correia, C. M. Soares, and C. M. Soares, "Higher order models on the eigenfrequency analysis and optimal design of laminated composite structures," *Composite Structures*, vol. 39, no. 3-4, pp. 237–253, 1997.

[10] C. M. M. Scares, C. A. M. Soares, and V. M. F. Correia, "Optimization of multilaminated structures using higher-order deformation models," *Computer methods in applied mechanics and engineering*, vol. 149, no. 1-4, pp. 133–152, 1997.

[11] A. Y. Abu-Odeh and H. L. Jones, "Optimum design of composite plates using response surface method," *Composite structures*, vol. 43, no. 3, pp. 233–242, 1998.

[12] M. Lombardi and R. T. Haftka, "Anti-optimization technique for structural design under load uncertainties," *Computer methods in applied mechanics and engineering*, vol. 157, no. 1-2, pp. 19–31, 1998.

[13] R. Le Riche and J. Gaudin, "Design of dimensionally stable composites by evolutionary optimization," *Composite Structures*, vol. 41, no. 2, pp. 97–111, 1998.

[14] K. Sivakumar, N. Iyengar, and K. Deb, "Optimum design of laminated composite plates with cutouts using a genetic algorithm," *Composite Structures*, vol. 42, no. 3, pp. 265–279, 1998.

[15] S. Barakat and G. Abu-Farsakh, "The use of an energy-based criterion to determine optimum configurations of fibrous composites," *Composites science and technology*, vol. 59, no. 12, pp. 1891–1899, 1999.

[16] F. Richard and D. Perreux, "A reliability method for optimization of [+ ϕ,- ϕ] n fiber reinforced composite pipes," *Reliability Engineering & System Safety*, vol. 68, no. 1, pp. 53–59, 2000.

[17] J. Moita, J. I. Barbosa, C. M. Soares, and C. M. Soares, "Sensitivity analysis and optimal design of geometrically non-linear laminated plates and shells," *Computers & Structures*, vol. 76, no. 1-3, pp. 407–420, 2000.

[18] G. Soremekun, Z. Gürdal, R. Haftka, and L. Watson, "Composite laminate design optimization by genetic algorithm with generalized elitist selection," *Computers & structures*, vol. 79, no. 2, pp. 131–143, 2001.

[19] M. Walker and R. E. Smith, "A technique for the multiobjective optimisation of laminated composite structures using genetic algorithms and finite element analysis," *Composite structures*, vol. 62, no. 1, pp. 123–128, 2003.

[20] M. Di Sciuva, M. Gherlone, and D. Lomario, "Multiconstrained optimization of laminated and sandwich plates using evolutionary algorithms and higher-order plate theories," *Composite Structures*, vol. 59, no. 1, pp. 149–154, 2003.

[21] P. Kere, M. Lyly, and J. Koski, "Using multicriterion optimization for strength design of composite laminates," *Composite Structures*, vol. 62, no. 3-4, pp. 329–333, 2003.

[22] K. J. Callahan and G. E. Weeks, "Optimum design of composite laminates using genetic algorithms," *Composites Engineering*, vol. 2, no. 3, pp. 149–160, 1992.

[23] J. Park, J. Hwang, C. Lee, and W. Hwang, "Stacking sequence design of composite laminates for maximum strength using genetic algorithms," *Composite Structures*, vol. 52, no. 2, pp. 217–231, 2001.

[24] D. J. Deka, G. Sandeep, D. Chakraborty, and A. Dutta, "Multiobjective

optimization of laminated composites using finite element method and genetic algorithm," *Journal of reinforced plastics and composites*, vol. 24, no. 3, pp. 273–285, 2005.

[25] J. L. Pelletier and S. S. Vel, "Multi-objective optimization of fiber reinforced composite laminates for strength, stiffness and minimal mass," *Computers & structures*, vol. 84, no. 29-30, pp. 2065–2080, 2006.

[26] P. Jadhav and P. R. Mantena, "Parametric optimization of grid-stiffened composite panels for maximizing their performance under transverse loading," *Composite structures*, vol. 77, no. 3, pp. 353–363, 2007.

[27] J.-S. Kim, "Development of a user-friendly expert system for composite laminate design," *Composite Structures*, vol. 79, no. 1, pp. 76–83, 2007.

[28] C. H. Park, W. I. Lee, W. S. Han, and A. Vautrin, "Improved genetic algorithm for multidisciplinary optimization of composite laminates," *Computers & structures*, vol. 86, no. 19-20, pp. 1894–1903, 2008.

[29] N. Kogiso, L. T. Watson, Z. Gürdal, R. T. Haftka, and S. Nagendra, "Design of composite laminates by a genetic algorithm with memory," *MECHANICS OF COMPOSITE MATERIALS AND STRUCTURES An International Journal*, vol. 1, no. 1, pp. 95–117, 1994.

[30] A. Todoroki and R. T. Haftka, "Stacking sequence optimization by a genetic algorithm with a new recessive gene like repair strategy," *Composites Part B: Engineering*, vol. 29, no. 3, pp. 277–285, 1998.

[31] M. Akbulut and F. O. Sonmez, "Optimum design of composite laminates for minimum thickness," *Computers & Structures*, vol. 86, no. 21-22, pp. 1974–1982, 2008.