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Chapter 1

Introduction

1.1 Laminated Composite Material

Composite materials offer improved strength, stiffness, corrosion resistance, etc. over conventional materials, and are widely used as alternative materials for applications in various industries ranging from electronic packaging to golf clubs, and medical equipment to homebuilding, making aircraft structure to space vehicles. The stacking sequence and fiber orientation of composite laminates give the designer additional 'degree of freedom' to tailor the design with respect to strength or stiffness. One widely known advantage of using composite material is can significantly reducing the weight of target structure, and many researchers attempted to improve the efficiency of using composite material by minimizing the thickness.

1.2 Classic Lamination Theory and Failure Theory

Classic lamination theory (CLT) is used to develop the stress-strain relationship of composite material under in-plane and out-of-plane loading. First, develop stress-strain relationships, elastic moduli, strengths of an angle ply based on a unidirectional lamina and the angle of the ply; second, because a laminate is consist of more than one lamina bonded together through their thickness, so the macromechanical analysis will be developed for a laminate based on applied loading. To check whether a designed lay-up is plausible or not, different failure theories have been developed.

1.3 Genetic Algorithm

In the design of composite material, gradient based optimization techniques are not applicable in this domain, because the design variables, such as fiber orientation, layer thickness, number of layers etc. are discrete. Genetic algorithm (GA) can be adopted in the optimization problem because it doesn't require the gradient information. Moreover, the GA has been proved a reliable technique and widely used in the design of composite material.

1.4 Artificial Neural Network

CLT is a classic analytical approach to obtain the stress and strain of composite material, the disadvantage of this method is quite cumbersome and in which involves compute matrix and integration operations. Artificial neural network (ANN) has been proved a reliable tool in modelling various engineering system in practice without

solving tricky equations and making ideal assumptions. In this thesis, the ANN is taken to approximate the numeric results based on CLT and failure theory.

1.5 Summary

In this thesis, first, we review the use of composite material in practice, then, the CLT to calculate the stress and strain under certain loading, last, the failure theories which are used to decide whether a composite material will failure or not. Second, the stochastic algorithm, GA, is studied and implemented in the design of composite material, two different cases are studied in which GA can be taken to obtain the optimal lay-up. At last, ANN is introduced to approximate the evaluation result of CLT, the reason for adopting ANN is to reduce the computation complexity based on CLT.

Chapter 2

Classic Lamination Theory and Failure Theory

2.1 Classic Lamination Theory

A laminate is consist of multiple laminae bonded together through thickness. In this chapter, first the stress-strain relationship is developed based on Hook's law for a single lamina; second, develop relationship of mechanical loads applied to a laminate to strains and stresses in each lamina, calculate the elastic moduli of laminate based on the elastic moduli of single laminate and the lay-up..

2.1.1 Analysis of stress and strain for composite material

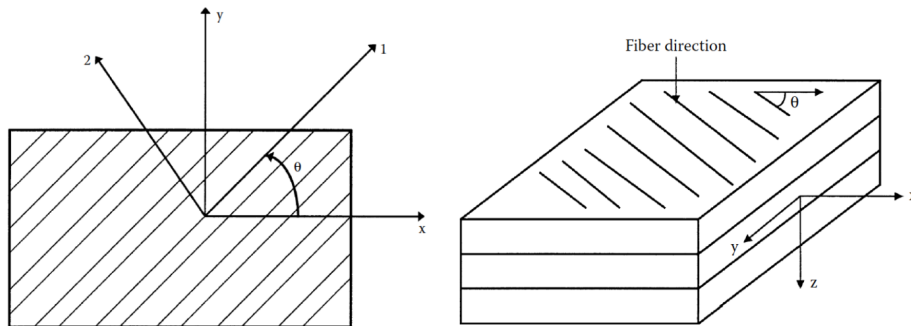


FIGURE 2.1: Lamina

2.1.2 Stress and Strain in a Lamina

For a single lamina has a small thickness under plane stress, and it's upper and lower surfaces of the lamina are free from external loads. According to the Hooke's Law, the three-dimensional stress-strain equations can be reduced to two-dimensional stress-strain equations. The stress-strain relation in local axis 1-2 is

$$\begin{bmatrix} \sigma_1 \\ \sigma_2 \\ \tau_{12} \end{bmatrix} = \begin{bmatrix} Q_{11} & Q_{12} & 0 \\ Q_{12} & Q_{22} & 0 \\ 0 & 0 & Q_{66} \end{bmatrix} \begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \gamma_{12} \end{bmatrix} \quad (2.1)$$

where Q_{ij} are the stiffnesses of the lamina that are related

to engineering elastic constants given by

$$\begin{aligned} Q_{11} &= \frac{E_1}{1 - \nu_{12}\nu_{21}}, \\ Q_{22} &= \frac{E_2}{1 - \nu_{12}\nu_{21}}, \\ Q_{66} &= G_{12}, \\ Q_{12} &= \frac{\nu_{21}E_2}{1 - \nu_{12}\nu_{21}}, \end{aligned} \quad (2.2)$$

where $E_1, E_2, \nu_{12}, G_{12}$ are four independent engineering elastic constants, which are defined as follows: E_1 is the longitudinal Young's modulus, E_2 is the transverse Young's modulus, ν_{12} is the major Poisson's ratio, and G_{12} is the in-plane shear modulus.

Stress strain relation in the global x-y axis is

$$\begin{bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{bmatrix} = \begin{bmatrix} \bar{Q}_{11} & \bar{Q}_{12} & \bar{Q}_{16} \\ \bar{Q}_{12} & \bar{Q}_{22} & \bar{Q}_{26} \\ \bar{Q}_{16} & \bar{Q}_{26} & \bar{Q}_{66} \end{bmatrix} \begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{bmatrix}, \quad (2.3)$$

where

$$\begin{aligned} \bar{Q}_{11} &= Q_{11}\cos^4\theta + Q_{22}\sin^4\theta + 2(Q_{12} + 2Q_{66})\sin^2\theta\cos^2\theta, \\ \bar{Q}_{12} &= (Q_{11} + Q_{22} - 4Q_{66})\sin^2\theta\cos^2\theta + Q_{12}(\cos^4\theta + \sin^4\theta), \\ \bar{Q}_{22} &= Q_{11}\sin^4\theta + Q_{22}\cos^4\theta + 2(Q_{12} + 2Q_{66})\sin^2\theta\cos^2\theta, \\ \bar{Q}_{16} &= (Q_{11} - Q_{12} - 2Q_{66})\cos^3\theta\sin\theta - (Q_{22} - Q_{12} - 2Q_{66})\sin^3\theta\cos\theta, \\ \bar{Q}_{26} &= (Q_{11} - Q_{12} - 2Q_{66})\cos\theta\sin^3\theta - (Q_{22} - Q_{12} - 2Q_{66})\cos^3\theta\sin\theta, \\ \bar{Q}_{66} &= (Q_{11} + Q_{22} - 2Q_{12} - 2Q_{66})\sin^2\theta\cos^2\theta + Q_{66}(\sin^4\theta + \cos^4\theta). \end{aligned} \quad (2.4)$$

The local and global stresses in an angle lamina are related to each other through the angle of the lamina θ , it can be written as

$$\begin{bmatrix} \sigma_1 \\ \sigma_2 \\ \tau_{12} \end{bmatrix} = [T] \begin{bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{bmatrix}, \quad (2.5)$$

where

$$[T] = \begin{bmatrix} \cos^2\theta & \sin^2\theta & 2\sin\theta\cos\theta \\ \sin^2\theta & \cos^2\theta & -2\sin\theta\cos\theta \\ -\sin\theta\cos\theta & \sin\theta\cos\theta & \cos^2\theta - \sin^2\theta \end{bmatrix}. \quad (2.6)$$

2.1.3 Stress and Strain in a Laminate

For forces and moment resultants acting on laminates, such as in plate and shell structures, the relationship between applied forces and moment and displacement can be given by

$$\begin{aligned}
\begin{bmatrix} N_x \\ N_y \\ N_{xy} \end{bmatrix} &= \begin{bmatrix} A_{11} & A_{12} & A_{16} \\ A_{12} & A_{22} & A_{26} \\ A_{16} & A_{26} & A_{66} \end{bmatrix} \begin{bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \end{bmatrix} \\
&+ \begin{bmatrix} B_{11} & B_{12} & B_{16} \\ B_{11} & B_{12} & B_{16} \\ B_{16} & B_{26} & B_{66} \end{bmatrix} \begin{bmatrix} k_x \\ k_y \\ k_{xy} \end{bmatrix}, \\
\begin{bmatrix} M_x \\ M_y \\ M_{xy} \end{bmatrix} &= \begin{bmatrix} B_{11} & B_{12} & B_{16} \\ B_{12} & B_{22} & B_{26} \\ B_{16} & B_{26} & B_{66} \end{bmatrix} \begin{bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \end{bmatrix} \\
&+ \begin{bmatrix} D_{11} & D_{12} & D_{16} \\ D_{11} & D_{12} & D_{16} \\ D_{16} & D_{26} & D_{66} \end{bmatrix} \begin{bmatrix} k_x \\ k_y \\ k_{xy} \end{bmatrix},
\end{aligned} \tag{2.7}$$

where

N_x, N_y - normal force per unit length;

N_{xy} - shear force per unit length;

M_x, M_y - bending moment per unit length;

M_{xy} - twisting moments per unit length;

ε^0, k - mid plane strains and curvature of a laminate in x-y coordinates.

The mid plane strain and curvature is given by

$$\begin{aligned}
A_{ij} &= \sum_{k=1}^n (\overline{Q}_{ij})_k (h_k - h_{k-1}) \quad i = 1, 2, 6, j = 1, 2, 6, \\
B_{ij} &= \frac{1}{2} \sum_{k=1}^n (\overline{Q}_{ij})_k (h_k^2 - h_{k-1}^2) \quad i = 1, 2, 6, j = 1, 2, 6, \\
D_{ij} &= \frac{1}{3} \sum_{k=1}^n (\overline{Q}_{ij})_k (h_k^3 - h_{k-1}^3) \quad i = 1, 2, 6, j = 1, 2, 6.
\end{aligned} \tag{2.8}$$

The $[A]$, $[B]$, and $[D]$ matrices are called the extensional, coupling, and bending stiffness matrices, respectively. The extensional stiffness matrix $[A]$ relates the resultant in-plane forces to the in-plane strains, and the bending stiffness matrix $[D]$ couples the resultant bending moments to the plane curvatures. The coupling stiffness matrix $[B]$ relates the force and moment terms to the midplane strains and midplane curvatures.

2.2 Failure Theory

Failure criteria for composite materials are more difficult to predict due to structural and material complexity in comparison to isotropic materials. The failure process of a composite materials can be regarded from microscopic and macroscopic points of view. Most popular criteria about the failure of an angle lamina are in terms of macroscopic failure criteria, which are based on the tensile, compressive and shear strengths. According to the failure surfaces, these criteria can be classified into two classes: one is called independent failure mode criteria which includes the maximum stress failure theory, maximum strain failure theory because their failure envelop are rectangle; another is called quadratic polynomial which includes Tsai-Wu, Chamis, Hoffman and Hill criteria because their failure surfaces are of ellipsoidal shape. In

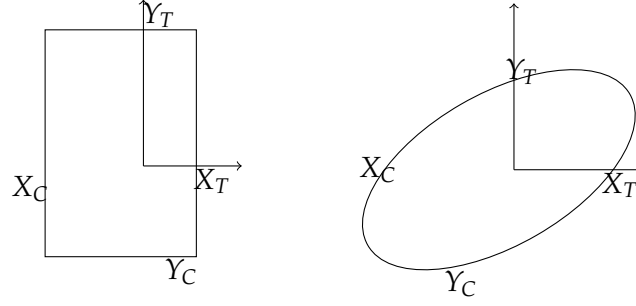


FIGURE 2.2: Schematic failure surfaces for maximum stress and quadratic failure criteria

the present study, two most reliable failure criteria is taken, Maximum stress and Tsai-wu. Both of these two failure criteria are based on the stresses in the local axes instead of principal normal stresses and maximum shear stresses, and four normal strength parameters and one shear stress for a unidirectional lamina are involved. The five strength parameters are

- $(\sigma_1^T)_{ult}$ = ultimate longitudinal tensile strength(in direction 1),
- $(\sigma_1^C)_{ult}$ = ultimate longitudinal compressive strength,
- $(\sigma_2^T)_{ult}$ = ultimate transverse tensile strength,
- $(\sigma_2^C)_{ult}$ = ultimate transverse compressive strength, and
- $(\tau_{12})_{ult}$ = and ultimate in-plane shear strength.

2.2.1 Tsai-wu failure criterion

The TW criterion is one of the most reliable static failure criteria which is derived from the von Mises yield criterion. A lamina is considered to fail if

$$H_1\sigma_1 + H_2\sigma_2 + H_6\tau_{12} + H_{11}\sigma_1^2 + H_{22}\sigma_2^2 + H_{66}\tau_{12}^2 + 2H_{12}\sigma_1\sigma_2 < 1 \quad (2.9)$$

is violated, where

$$\begin{aligned} H_1 &= \frac{1}{(\sigma_1^T)_{ult}} - \frac{1}{(\sigma_1^C)_{ult}}, \\ H_{11} &= \frac{1}{(\sigma_1^T)_{ult} (\sigma_1^C)_{ult}}, \\ H_2 &= \frac{1}{(\sigma_2^T)_{ult}} - \frac{1}{(\sigma_2^C)_{ult}}, \\ H_{22} &= \frac{1}{(\sigma_2^T)_{ult} (\sigma_2^C)_{ult}}, \\ H_{66} &= \frac{1}{(\tau_{12})_{ult}^2}, \\ H_{12} &= -\frac{1}{2} \sqrt{\frac{1}{(\sigma_1^T)_{ult} (\sigma_1^C)_{ult} (\sigma_2^T)_{ult} (\sigma_2^C)_{ult}}}. \end{aligned} \quad (2.10)$$

H_i is the strength tensors of the second order; H_{ij} is the strength tensors of the fourth order. σ_1 is the applied normal stress in direction 1; σ_2 is the applied normal stress in the direction 2; and τ_{12} is the applied in-plane shear stress.

2.2.2 Maximum Stress Failure Theory

Maximum stress(MS) failure theory consists of maximum normal stress theory proposed by Rankine and maximum shearing stress theory by Tresca. The stresses applied on a lamina can be resolved into the normal and shear stresses in the local axes. If any of the normal or shear stresses in the local axes of a lamina is equal or exceeds the corresponding ultimate strengths of the unidirectional lamina, the lamina is considered to be failed. That is

$$\begin{aligned}\sigma_1 &\geq (\sigma_1^T)_{ult} \text{ or } \sigma_1 \leq -(\sigma_1^C)_{ult}, \\ \sigma_2 &\geq (\sigma_2^T)_{ult} \text{ or } \sigma_2 \leq -(\sigma_2^C)_{ult}, \\ \tau_{12} &\geq (\tau_{12})_{ult} \text{ or } \tau_{12} \leq -(\tau_{12})_{ult}.\end{aligned}$$

where σ_1 and σ_2 are the normal stresses in the local axes 1 and 2, respectively; τ_{12} is the shear stress in the symmetry plane 1-2. The safety factor, or yield stress, is how much extra load beyond is intended a composite laminate will actually take, which is an indication of the material's load carrying capacity. If the value is less than 1.0, it means failure. The safety factor is defined as

$$SF = \frac{\text{Maximum Load Which Can Be Applied}}{\text{Load Applied}}. \quad (2.11)$$

The safety factor based on maximum stress theory is calculated by the following method: first, the principal stresses(σ_1^k, σ_2^k , and τ_{12}^k) are obtained by experiment; evaluate the safety factor along each direction according to equation 2.11; The minimum value among these safety factors are denoted as the safety factor of the lamina, SF_{MS}^k , it can be written as

$$SF_{MS}^k = \min \left\{ \begin{aligned} SF_X^k &= \begin{cases} \frac{X_t}{\sigma_{11}}, & \text{if } \sigma_{11} > 0 \\ \frac{X_c}{\sigma_{11}}, & \text{if } \sigma_{11} < 0 \end{cases} \\ SF_Y^k &= \begin{cases} \frac{Y_t}{\sigma_{22}}, & \text{if } \sigma_{22} > 0 \\ \frac{Y_c}{\sigma_{22}}, & \text{if } \sigma_{22} < 0 \end{cases} \\ SF_S^k &= \frac{S}{|\tau_{12}|} \end{aligned} \right. . \quad (2.12)$$

Assuming the composite laminate under a in-plane loading f , the corresponding stress on local stress in direction 1, local stress in direction 2, and shear stress for the k th lamina are $\sigma_1 SF_{TW}^k$, $\sigma_2 SF_{TW}^k$, and $\tau_{12} SF_{TW}^k$, respectively. Substitute them into the equation 2.9, the expression are given by

$$a(SF_{TW}^k)^2 + b(SF_{TW}^k) - 1 = 0,$$

where

$$a = H_{11}(\sigma_1)^2 + H_{22}(\sigma_2)^2 + H_{66}(\tau_{12})^2 + 2H_{12}\sigma_1\sigma_2,$$

$$b = H_1\sigma_1 + H_2\sigma_2 + H_6\tau_{12}.$$

Solve the above equation, the safety factor for the k th lamina is

$$SF_{TW}^k = \left| \frac{-b + \sqrt{b^2 + 4a}}{2a} \right|.$$

Then, the minimum of SF_{TW}^k is taken as the safety factor of the laminate which is written as

$$SF_{TW} = \min \text{ of } SF_{TW}^k \text{ for } k = 1, 2, \dots, m-1, m.$$

2.3 Summary

In this chapter, we review the CLT for composite material's analysis, then related failure theories are introduced to check whether a composite material would fail or not. In the following chapters, the CLT would be used to calculate the stress and strain under in-plane loading, it would also be used to generate the training data for stress and strain approximation based on neural network; in order to design a proper composite material, the failure theory are used to decide the feasibility of composite design.

Chapter 3

A New Genetic Algorithm Model for Constrained Problem

3.1 Genetic Algorithm Framework

GA is one of the most reliable stochastic algorithm, which has been widely used in discrete variables optimization problems.

3.2 Constrained Problem Optimization

3.3 A New Genetic Algorithm Model

3.3.1 Encoding

Due to the simplicity and efficiency of float representation, this encoding method is implemented to represent a possible solution. As shown in Figure 4.1 (a), these two chromosomes represent a $[+8_7 / -9_2]_s$ carbon T300/5308 laminated composite, and $[+19_4 / -36_6]_s$, respectively. Because the laminate adopted in this paper is symmetric to its mid-plane, so only half needs to be encoded.

3.3.2 Selection

The purpose of the selection operator is to choose mating pool to produce alternative solutions of better fitness. Traditional methods of selecting strategies only take the fitness of individuals into account, however, due to the existence of constraint, various selection schemes are implemented to select the mating set. Based on different selection schemes, the parents of next generation can be divided into three groups: proper groups, active groups, and potential groups according to different selecting methods.

Proper parents mean in which individual fulfills the constraints, which are chosen by the individual's fitness, individuals with better fitness are more likely to be chosen if they fit the constraint; active groups means that individual is supposed to be always exist in the parents during the GA, which are selected by fitness, ignoring the constraint; The individuals from active group may not correspond to feasible solutions, but their existence enriches the variety of the gene clips. Potential groups means that they are likely to turn into proper individual after a couple of generations, and potential individuals are chosen by constraint function, the more the individual fulfills the constraint, the more possibility it will be selected.

3.3.3 Crossover

The crossover operator happens among these three groups. the child of two proper groups are more likely to be a proper individual which can be used to obtain a alternative feasible solution. the child of an active individual and a potential individual can significantly change the gene of active individual's chromosome, which makes the individual evolve toward a new direction. The offspring of two active individuals are more likely to be an active individual, which can maintain the active group. The figure.4.1 (b) shows two children O_1

3.4 Summary

Chapter 4

Laminated Composite Material Optimization by New Genetic Algorithm Model

4.1 Case1: Maximum Strength Optimization

TABLE 4.1: Properties of T300/5308 carbon/epoxy composite

Property	Symbol	Unit	Graphite/Epoxy
Longitudinal elastic modulus	E_1	GPa	181
Traverse elastic modulus	E_2	GPa	10.3
Major Poisson's ratio	ν_{12}		0.28
Shear modulus	G_{12}	GPa	7.17
Ultimate longitudinal tensile strength	$(\sigma_1^T)_{ult}$	MP	1500
Ultimate longitudinal compressive strength	$(\sigma_1^C)_{ult}$	MP	1500
Ultimate transverse tensile strength	$(\sigma_2^T)_{ult}$	MPa	40
Ultimate transverse compressive strength	$(\sigma_2^C)_{ult}$	MPa	246
Ultimate in-plane shear strength	$(\tau_{12})_{ult}$	MPa	68

4.1.1 Methodology

4.1.2 Result and Discussion

4.2 Case2: Minimum Thickness Optimization

In this case, the mentioned NGAM in chapter three is taken to minimize thickness(or weight) of mid-plane symmetric composite laminate subject to in-plane loading. Fiber orientation and ply thickness are chosen as design variables, the proposed Tsai-wu and the maximum stress criteria are employed to determine whether load bearing capacity is exceeded or not.

4.2.1 Methodology

The optimization problem can be formulated as searching the optimal stacking sequence of composite laminate. There are two design variables in this problem, the angles in the laminate, and the number of layers that each fiber orientation has. The final objective function is given as

P_1 :	+7	+7	+7	+7	+7	+7	+7	-9	-9
P_2 :	+19	+19	+19	+19	-36	-36	-36	-36	-36

(a): Parents P_1 and P_2

O_1 :	+13	+13	+13	+13	+13	+13	-27	-27	-27
O_2 :	+22	+22	+22	+22	+22	+22	+22	+5	+5

(b): Offspring O_1 and O_2

O_1 :	+13	+13	+13	...	+13	+13	-27	...	-27
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(c): Offspring O_1 after lenght mutation

	+12	+12	+12	...	+12	+12	-26	...	-26
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(b): Offspring O_1 after angle mutation

FIGURE 4.1: GA Operators

$$\begin{aligned}
 F &= 2t_0 \sum_{k=1}^n n_k, \\
 SF_{MS} &\geq 1, \\
 SF_{TW} &\geq 1.
 \end{aligned} \tag{4.1}$$

The first term represents the total thickness of the composite laminates, t_0 is the ply thickness; n_k is the number of plies in the k th lamina, in which the fiber orientation is θ_k . The constraints here are two safety factors should not less than 1, which means $SF_{MS} \geq 1$, and $SF_{TW} \geq 1$, respectively.

4.2.2 Encoding

Due to the simplicity and efficiency of float representation, this encoding method is implemented to represent a possible solution. As shown in Figure 4.1 (a), these two chromosomes represent a $[+8_7 / -9_2]_s$ carbon T300/5308 laminated composite, and $[+19_4 / -36_6]_s$, respectively. Becasue the laminate adopted in this paper is symmetric to its mid-plane, so only half needs to be encoded.

4.2.3 Selection

The purpose of the selection operator is to chose mating pool to produce alternative solutions of better fitness. Traditional methods of selecting strategies only take the

fitness of individuals into account, however, due to the existence of constraint, various selection schemes are implemented to select the mating set. Based on different selection schemes, the parents of next generation can be divided into three groups: proper groups, active groups, and potential groups according to different selecting methods.

Proper parents mean in which individual fulfills the constraints, which are chosen by the individual's fitness, individuals with better fitness are more likely to be chosen if they fit the constraint; active groups means that individual is supposed to be always exist in the parents during the GA, which are selected by fitness, ignoring the constraint; The individuals from active group may not correspond to feasible solutions, but their existence enriches the variety of the gene clips. Potential groups means that they are likely to turn into proper individual after a couple of generations, and potential individuals are chosen by constraint function, the more the individual fulfills the constraint, the more possibility it will be selected.

4.2.4 Crossover

The crossover operator happens among these three groups. the child of two proper groups are more likely to be a proper individual which can be used to obtain a alternative feasible solution. the child of an active individual and a potential individual can significantly change the gene of active individual's chromosome, which makes the individual evolve toward a new direction. The offspring of two active individuals are more likely to be an active individual, which can maintain the active group. The figure.4.1 (b) shows two children O_1 and O_2 from two parents P_1 and P_2 , each angle C_a and its length C_l of a child are obtained by the following formula

$$\begin{cases} C_a = (P1_a + P2_a)/2 \\ C_l = (P1_l + P2_l)/2 \end{cases} \quad (4.2)$$

4.2.5 Mutation

A mutation direction is imposed on the mutation operator which to make sure the individual evolving toward the right direction. The mutation direction, denoted by md , is a n dimensional vector corresponding to the number of constraints, it is decided by the constraint thresholds CT_i and the current individual's constraint value, denoted as CV_i , The mutation vector can be obtained by the following formula

$$md = [CT_1, \dots, CT_{n-1}, CT_n] - [CV_0, \dots, CV_{n-1}, CV_n]$$

During the operator, the mutation procedure is consist of two phases: the length mutation of the chromosome, and the angle mutation of the chromosome. Because the chromosome's length is positive correlated with the individual's fitness, the coefficient of length mutation denoted by C_l , if $\sum_{i=1}^N CT_i$ great than $\sum_{i=1}^N CV_i$, the mutation length is restricted to the range $[0, (C_l \sum_{i=1}^N (CT_i - CV_i))/N]$, which means increase the chromosome's length; Assuming a $[+13_6 / -27_4]_s$ T300/5308 carbon/epoxy composite laminate under the loading $N_x = N_y = 10$ MPa m, its property as shown in table 4.1. According to CLT and failure theory, the two safety factors SF_{MS} and SF_{TW} are 0.0539, and 0.0540, respectively. So the mutation vector and is $[0.9461, 0.9460]$, assuming the length mutation coefficient is 20, so the mutation range is from 0 to 18. A random number is generated from the range $[0, 18]$, supposing the outcome is 13, then a length generator is used to a list, the its sum is 13, suppose the list is $[5, 8]$, the laminate after mutation is $[13_{11} / -27_{12}]_s$.

TABLE 4.2: The optimum lay-ups using two distinct fiber angles under various biaxial loading cases

Loading $N_x/N_y/N_{xy}$ (MPa m)	Optimum lay-up sequences	Laminate thickness	Safety factor for Tsai-wu	Safety factor for maximum stress
10/5/0	$[33_{29}/-39_{25}/-39]_s$	109	1.0074	1.0246
20/5/0	$[33_{22}/-31_{24}]_s$	92	1.0055	1.2065
40/5/0	$[29_{18}/-21_{23}/-21]_s$	83	1.0034	1.7350
80/5/0	$[-20_{27}/21_{25}/25]_s$	105	1.0029	1.2063
120/5/0	$[-18_{34}/17_{36}]_s$	140	1.0000	1.0898

If the $\sum_{i=1}^N CT_i$ less than $\sum_{i=1}^N CV_i$, the mutation length is restricted to the range $([\sum_{i=1}^N CT_i - CV_i]/N, 0]$, which means the individual's fitness exceeds the threshold value, and decrease the chromosome's length. Assuming a $[+33_{35}/-29_{26}]_s$ T300/5308 laminate is under loading $N_x = 10$ MPa, and $N_y = 5$ MPa, then, it's SF_{MS} constraint and SF_{TW} values are 1.0912, 1.0747, respectively. because the length mutation is 20, so the mutation range is from -2 to 0. This would decrease the chromosome's length.

$$LM = \begin{cases} [0, (C_l \sum_{i=1}^N (CT_i - CV_i))/N], & \text{if } \sum_{i=1}^N CT_i > \sum_{i=1}^N CV_i \\ [(C_l \sum_{i=1}^N (CT_i - CV_i))/N, 0], & \text{if } \sum_{i=1}^N CT_i < \sum_{i=1}^N CV_i \end{cases} \quad (4.3)$$

The relationship between the angles in the composite laminate and the chromosome's fitness is unclear, so the mutation direction of chromosome's angle is random. The coefficient angle mutation is C_a , the angle mutation range is $[0, C_a \sum_{i=1}^N (|CT_i - CV_i|)]$ or $[C_a \sum_{i=1}^N (-|CT_i - CV_i|), 0]$. It is can be written as

$$P(AM) = \begin{cases} 0.5, & AM = [0, C_a \sum_{i=1}^N (|CT_i - CV_i|)] \\ 0.5, & AM = [C_a \sum_{i=1}^N (-|CT_i - CV_i|), 0] \end{cases} \quad (4.4)$$

4.2.6 Result and Discussion

In present study, the T300/5308 graphite/epoxy material is used in the lay-up sequence optimization, and it's properties as shown in table.4.1. Two constraints are imposed on the composite laminates which are the safety factor SF_{MS} , and safety factor SF_{TW} , and the threshold values for both of them is 1. The constraint values of an individual are CV_1 and CV_2 . So the mutation vector here is a two dimensional vector $[1 - CV_1, 1 - CV_2]$, and the coefficient of length mutation C_l and angle mutation C_a are 20 and 10, respectively.

To verify the reliability of proposed method, two conditions are concerned: the first is only two distinct fiber orientation angles in the composite material; the second involves three distinct ply angles within the optimization process. In each situation, first, we present the search process by plotting relevant indicators, such as the fitness, strength ratio, and angle. Then, the optimum lay-ups under various loading cases is discussed.

Figure 4.2 (a) shows how the optimal individual's fitness and strength ratio evolves during the GA process. The solid curve shows the fitness value, the dashed curve shows the Tsai-wu safety factor, and the dotted curve shows MS safety factor. If the smaller strength ratio fulfills the constraint, this laminate must satisfy all the constraints, for simplicity, only the smaller strength ratio is presented in the figure 4.3 (a). The method to chose optimal individual considering two following situations, if

TABLE 4.3: The optimum lay-ups using three distinct fiber angles under various biaxial loading cases

Loading $N_x/N_y/N_{xy}$ (MPa m)	Optimum lay-up sequences	Laminate thickness	Safety factor for Tsai-wu	Safety factor for maximum stress
10/5/0	$[37_{27}/-38_{27}/-5]_s$	110	1.0023	1.0216
20/5/0	$[34_{24}/-32_{14}/-28_{11}]_s$	98	1.0237	1.2089
40/5/0	$[21_{28}/-32_{19}/23]_s$	100	1.0617	1.7076
80/5/0	$[-19_{24}/20_{27}/-17_{16}/-17]_s$	109	1.0056	1.2093
120/5/0	$[-19_{33}/12_{13}/16_{28}]_s$	148	1.0105	1.1014

TABLE 4.4: Comparison with the results of DSA

Loading	Akbulut and Sonmez'sakbulut2008optimum Study				Present Study			
$N_x/N_y/N_{xy}$ (MPa m)	Optimum lay-up sequences	laminate thickness	TW	MS	Optimum lay-up sequences	laminate thickness	TW	MS
10/5/0	$[37_{27}/-37_{27}]_s$	108	1.0068	1.0277	$[33_{29}/-39_{25}/-39]_s$	109	1.0074	1.0246
20/5/0	$[31_{23}/-31_{23}]_s$	92	1.0208	1.1985	$[33_{22}/-31_{24}]_s$	92	1.0055	1.2065
40/5/0	$[26_{20}/-26_{20}]_s$	80	1.0190	1.5381	$[29_{18}/-21_{23}/-21]_s$	83	1.0034	1.7350
80/5/0	$[21_{25}/-19_{28}]_s$	106	1.0113	1.2213	$[-20_{27}/21_{25}/25]_s$	105	1.0029	1.2063
120/5/0	$[17_{35}/-17_{35}]_s$	140	1.0030	1.0950	$[-18_{34}/17_{36}]_s$	140	1.0000	1.0898

no individual in the current population meets constraint, the one with biggest fitness is selected as the optimal individual; if there are one or multiple individuals fullfils requirement, the one with smallest fitness is chosen which means the smallest one has the biggest priority. Figure 4.2 (b) and 4.3 (b) show how the every fiber orientation changes , and Figure 4.2 (c) and 4.3 display how the number of each angles varies.

At the beginning of this GA process, the fitness curves increases very quickly, becasue individual's two strength ratios are very small, so the difference between the individual's fitness and the imposed constraint threshold is a big positive number, so the range of mutaion length is from 0 to $C_l(CT_0 - CV_0 + CT_1 - CV_1)/2$. The length of individual increases by n, which is random number between 0 and $C_l(CT_0 - CV_0 + CT_1 - CV_1)/2$. As can be seen from Figure 4.2 (a), both of optimal individual's fitness and strength ratio increases very quickly. The range of angle mutation is from 0 to $C_a(CT_0 - CV_0 + CT_1 - CV_1)/2$, and the number of each angle also changes violently. The Figure.4.2 (a) and 4.3 (a) show this property at the initial stage. During this stage, increasing individual's length playing a major role in increasing individual's fitness.

After a couple of generations, the optimal individual's fitness get bigger, and the difference between individual's fitness and constraint threshold get smaller. The range of mutation length turn smaller. At this stage, simply increase the individual's length doesn't make much difference in improve individual's fitnees, and a better composite laminates lay-up can dramatically change the optimal individual's fitness. That's why the fitness curve oscillated violently in this stage. At the same time, the strength ratio curve kept growing smoothly. But the growing speed got more smaller.

When GA comes to its last phase, GA finds individuals that meet all the constraints. Now the optimal individual's fitness is greater than the safety factor. The range of mutation length is from $C_l(CT_0 - CV_0 + CT_1 - CV_1)/2$ to 0. It means individuals need to decrease it's length and improve its internal structure to meet the constraint. That's why the fitness of optimal individual kept decreaing, however, the strength ratio curve still is greater then safety factor.

Table.4.4 shows the comparison with the result obtained by direct search simulated annealing(DSA) algorithm which is proposed by Akbulut and Sonmez**akbulut2008optimum**.

Both of variant GA and DSA are able to find feasible solution, but when loading is $N_x = 80$, $N_y = 5$ MPa m, variant GA gets a better solution than DSA. In the case that loadings are $N_x = 20$, $N_y = 5$ MPa m, and $N_x = 120$, $N_y = 5$ MPa m, the proposed GA offers an alternative. Compared with DSA method, the last advantage of variant GA is the number of layers doesn't have to be an even number.

4.3 Summary

In this paper, we reviewed the use of variant GA for the optimal design of composite laminated material under in-plane loading based on Tsai-wu and maximum stress failure criteria. GA is proposed to search the optimal lay-up for laminated composite under different loading cases. Two situations are considered under the same loading, a set of two distinct angles, and three distinct angles.

By setting the constant values of length mutation coefficient and angle mutation coefficient at the beginning, the convergence speed of search process can be controlled in an explicit way; During the optimization process, GA can adjust its length mutation range and angle mutation range based on the difference between individual's constraint values and constraint thresholds.

Finally, comparison of previous research and current result are presented. In some cases, the proposed GA in this paper is better off than DSA method. However, there is still many works to study within this GA, such as the fine-tuning of parameters taken in this GA.

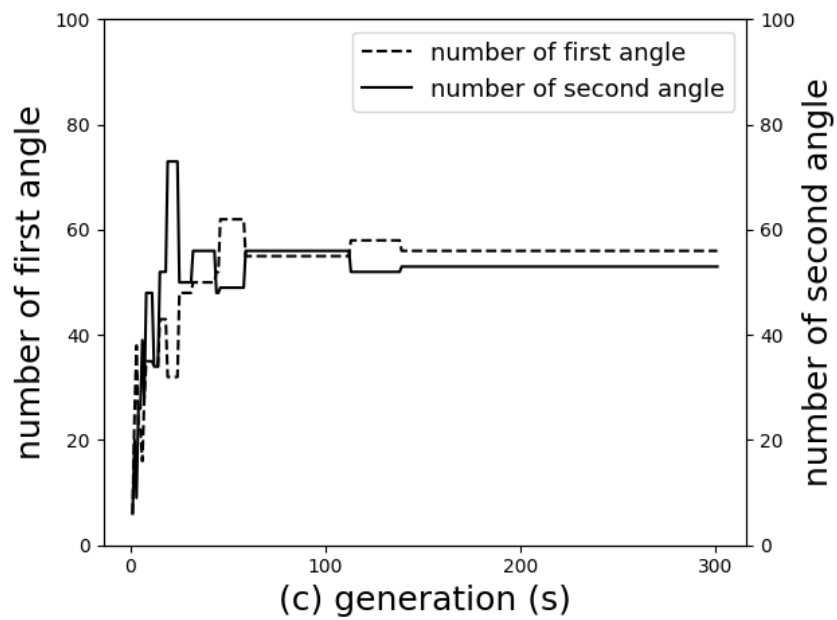
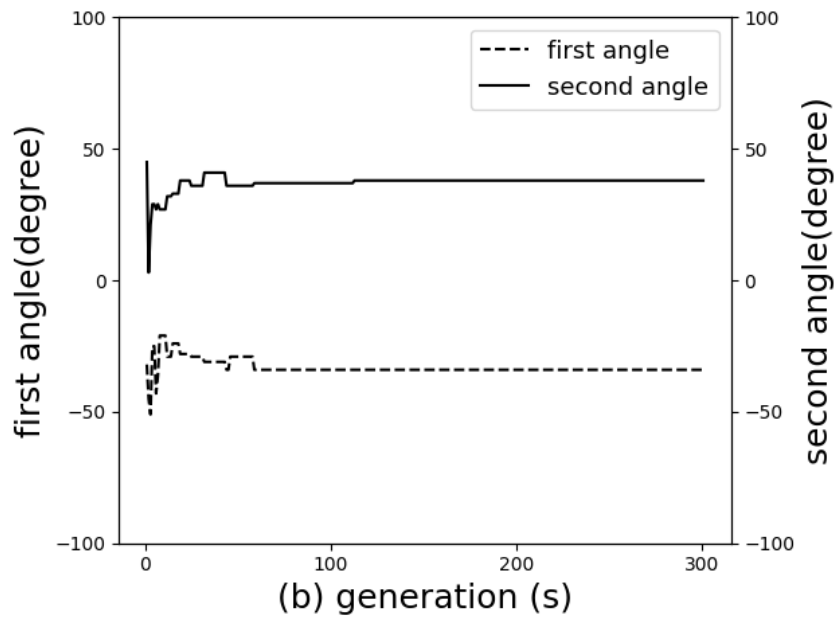
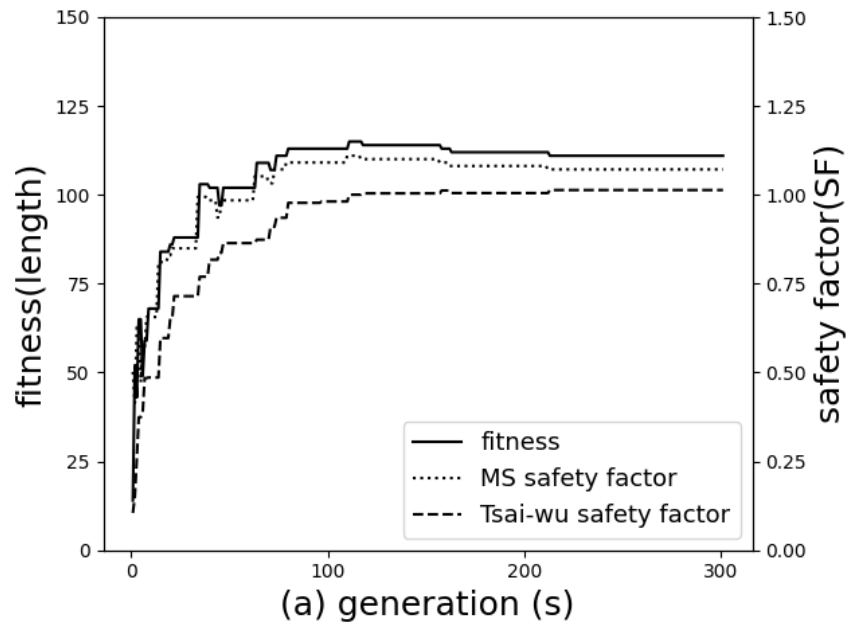


FIGURE 4.2: Two distinct angles

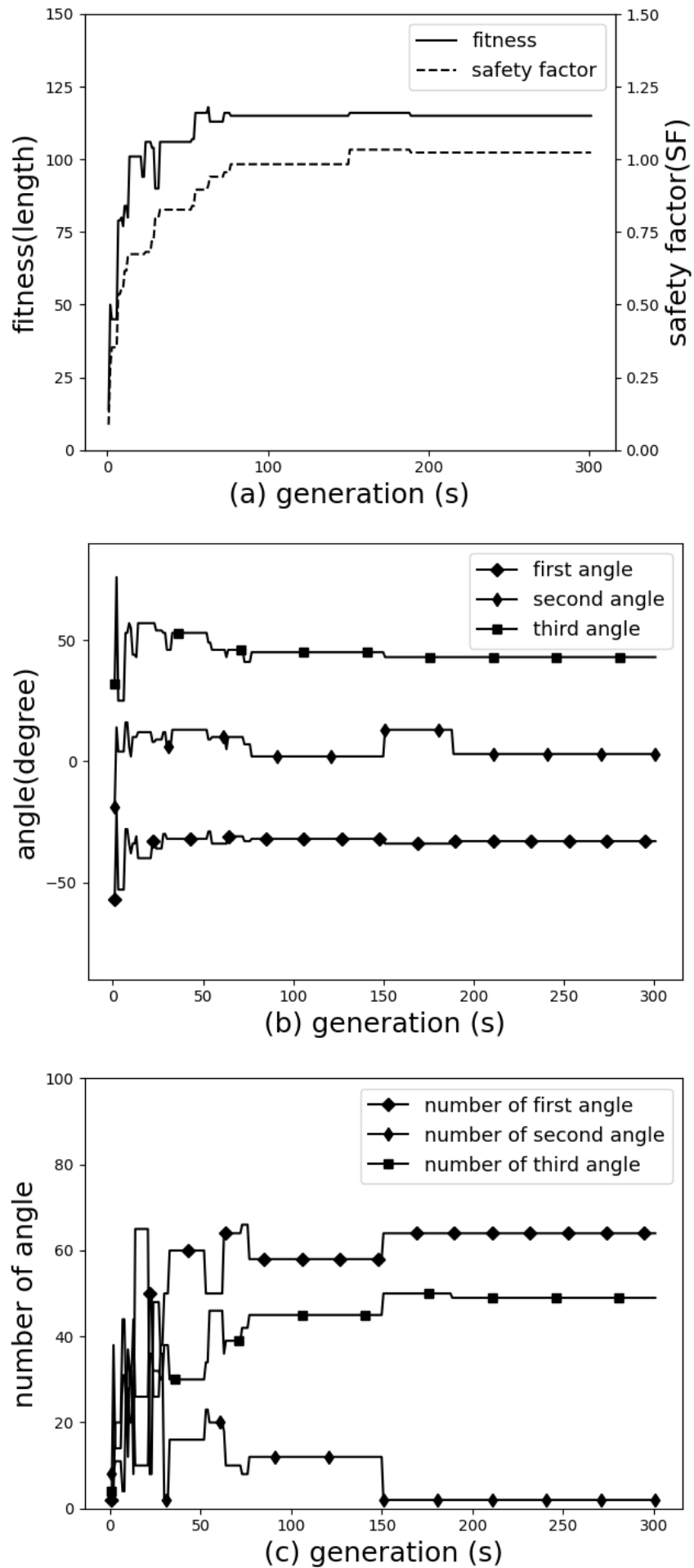


FIGURE 4.3: three distinct angles

Chapter 5

Approximation of CLT Based on Artificial Neural Network

5.1 Neural Network Design

5.1.1 Architecture

5.1.2 Discussion

5.2 Approximation Framework

5.2.1 Data Preparation

5.2.2 Training

5.2.3 Evaluation

5.2.4 Result and Discussion

5.3 Summary

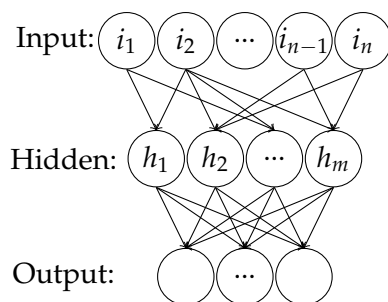


FIGURE 5.1: Neural Network Model

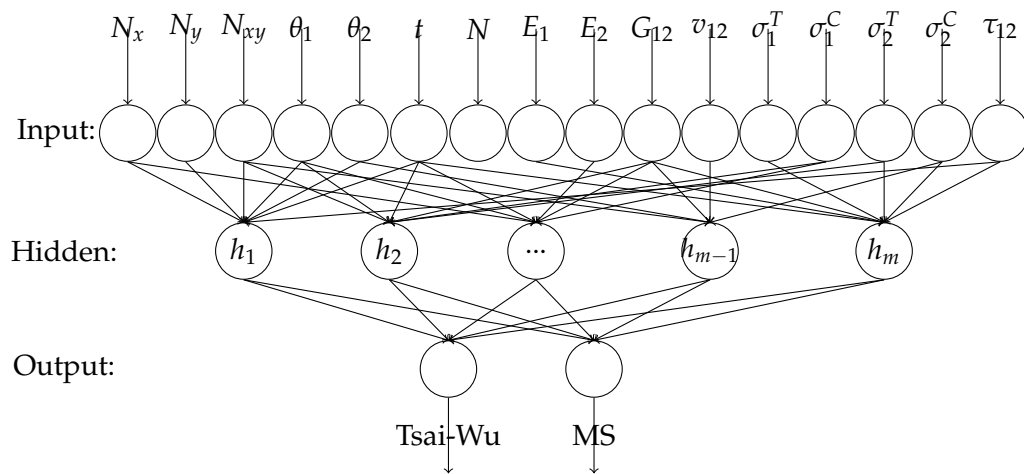


FIGURE 5.2: Neural Network Model

Chapter 6

Conclusion