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A technique for constrained optimization of symmetric laminates using a new variant of genetic algorithm

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Abstract

The main challenge presented by the laminate composite design is the laminate layup, involving a set of fiber orientations, composite material systems, and stacking sequences. In nature, it is a combinatorial optimization problem that can be solved by the genetic algorithm (GA). In this present study, a new variant of the GA is proposed for the optimal design by modifying the selection strategy. To check the feasibility of a laminate subject to in-plane loading, the effect of the fiber orientation angles and material components on the first ply failure is studied. An optimal composite material and laminate layup is well-established for a targeted strength ratio, which compromises weight and cost through an improved genetic algorithm. The numerical results are obtained and presented for different loading cases.

Keywords

Laminate composite, Classical lamination theory, Genetic Algorithm, Optimal design

Introduction

Composite materials offer improved strength, stiffness, fatigue, corrosion resistance, etc. over conventional materials, and are widely used as materials for applications ranging from the automotive to shipbuilding industry, electronic packaging to golf clubs, and medical equipment to homebuilding. However, the high cost of fabrication of composites is a critical drawback to its application. For example, the graphite/epoxy composite part may cost as much as 650 to 900 per kilogram. In contrast, the price of glass/epoxy is about 2.5 times less. Manufacturing techniques such as sheet molding compounds and structural reinforcement injection molding is used to lower the costs for manufacturing automobile parts. An alternative approach is using hybrid composite materials.

The mechanical performance of a laminate composite is affected by a wide range of factors such as the thickness, material, and orientation of each lamina. Because of manufacturing limitations, all these variables are usually limited to a small set of discrete values. For example, the ply thickness is fixed and ply orientation angles are limited to a set of angles such as 0, 45, and 90 degrees in practice. So the search process for the optimal design is a discrete optimization problem that can be solved by the GA. To tailor a laminate composite, the GA has been successfully applied to solve laminate design problems [1–11]. The GA simulates the process of natural evolution, including selection, crossover, and mutation according to Darwin's principal of "survival of the fittest". The known advantages of GAs are the following: (i): GAs are not easily trapped in local optima and can obtain the global optimum. (ii): GAs do not need gradient information and can be applied to discrete optimization problems. (iii): GAs can not only find the optimal value in the domain but also maintain a set of optimal solutions. However, the GA also has some disadvantages, for example, the GA needs to evaluate the target functions many times to achieve the optimization, and the cost of the search process is high. The GA consists of some basic parts, the coding of the design variable, the selection strategy, the crossover operator, the mutation operator, and how to deal with constraints. For the variable design part, there are two methods to deal with the representation of design variables, namely, binary string and real value representation [1,4]. Michalewicz [12] claimed that the performance of floating-point representation was better than binary representation in the numerical optimization problem. Selection strategy plays a critical role in the GA, which determines the convergence speed and the diversity of the population. To improve search ability and reduce search costs, various selection methods have been invented, and they can be divided into four classes: proportionate reproduction, ranking, tournament, and genitor(or "steady state") selection. In the optimization of laminate composite design, the roulette wheel [1,13], where the possibility of an individual to be chosen for the next generation is proportional to the fitness. Soremekun et al. [14] showed that the generalized elitist strategy outperformed a single individual elitism in some special cases.

Data structure, repair strategies and penalty functions [15] are the most commonly used approaches to resolve constrained problems in the optimization of composite structures. Symmetric laminates are widely used in practical scenarios, and data structures can be used to fulfill symmetry constraints, which consists of coding half of the laminate and

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considering the rest with the opposite orientation. Todoroki[4] introduced a repair strategy that can scan the chromosome and repair the gene on the chromosome if it does not satisfy the contiguity constraint. The comparison of repair strategies in a permutation GA with the same orientation was presented by Liu et al.[5], and it showed that the Baldwinian repair strategy can substantially reduce the cost of constrained optimization. Haftka and Todoroki[1] used the GA to solve the laminate stacking sequence problem using a penalty function subject to buckling and strength constraints.

In typical engineering applications, composite materials are under very complicated loading conditions, not only inplane loading but also out-of-plane loading. Most of the studies on the optimization of the laminate composite material minimized the thickness[7,16], weight[17–19], and cost and weight[18,20], or maximized the static strength of the composite laminates for a targeted thickness[7,8,21,22]. In the present study, the cost and weight of laminates are minimized by modifying the objective function.

To check the feasibility of a laminate composite by imposing a strength constraint, failure analysis of a laminate is performed by applying suitable failure criteria. The failure criteria of laminated composites can be classified into three classes: non-interactive theories (e.g., maximum strain), interactive theories (e.g., Tsai-wu), and partially interactive theories (e.g., Puck failure criterion). Previous researchers adopted the first-ply-failure approach using Tsai-wu failure theory[17,20,23–28], Tsai-Hill[29,30], the maximum stress[31], or the maximum strain[31] static failure criteria. Akbulut[11] used the GA to minimize the thickness of composite laminates with Tsai-Hill and maximum stress failure criteria, and the advantage of this method is it avoid spurious optima. Naik et al.[32] minimized the weight of laminated composites under restrictions with a failure mechanism-based criterion based on the maximum strain and Tsai-wu. In the present study, Tsai-wu Static failure criteria are used to investigate the feasibility of a laminate composite.

Stress and Strain in a Laminate

A laminate structure consists of multiple lamina bonded together through their thickness. Considering a laminate composite plate that is symmetric to its middle plane subject to in-plane loading of extension, shear, bending and torsion, the classical lamination theory (CLT) is taken to calculate the stress and strain in the local and global axes of each ply, as shown in Fig. 1.

Stress and Strain in a Lamina

For a single lamina, the stress-strain relation in local axis 1-2 is:

$$\begin{bmatrix}\sigma_1\\ \sigma_2\\ \tau_{12}\end{bmatrix} = \begin{bmatrix}Q_{11} & Q_{12} & 0\\ Q_{12} & Q_{22} & 0\\ 0 & 0 & Q_{66}\end{bmatrix}\begin{bmatrix}\varepsilon_1\\ \varepsilon_2\\ \gamma_{12}\end{bmatrix} \tag{1}$$

where $Q_{ij}$are the stiffnesses of the lamina that are related to engineering elastic constants given by

$$\begin{aligned} Q_{11} &= \frac{E_1}{1-v_{12}v_{21}}\\ Q_{22} &= \frac{E_2}{1-v_{12}v_{21}}\\ Q_{66} &= G_{12}\\ Q_{12} &= \frac{v_{21}E_2}{1-v_{12}v_{21}} \end{aligned} \tag{2}$$

where E_1, E_2, v_{12}, G_{12} are four independent engineering elastic constants, which are defined as follows: E_1 is the longitudinal Young's modulus, E_2 is the transverse Young's modulus, v_{12} is the major Poisson's ratio, and G_{12} is the in-plane shear modulus.

Stress strain relation in the global x-y axis:

$$\begin{bmatrix}\sigma_x\\ \sigma_y\\ \tau_{xy}\end{bmatrix} = \begin{bmatrix}\bar{Q}_{11} & \bar{Q}_{12} & \bar{Q}_{16}\\ \bar{Q}_{12} & \bar{Q}_{22} & \bar{Q}_{26}\\ \bar{Q}_{16} & \bar{Q}_{26} & \bar{Q}_{66}\end{bmatrix}\begin{bmatrix}\varepsilon_x\\ \varepsilon_y\\ \gamma_{xy}\end{bmatrix} \tag{3}$$

where

$$\begin{aligned} \bar{Q}_{11} &= Q_{11}c^4 + Q_{22}s^4 + 2\left(Q_{12}+2Q_{66}\right)s^2c^2\\ \bar{Q}_{12} &= \left(Q_{11}+Q_{22}-4Q_{66}\right)s^2c^2 + Q_{12}\left(c^4+s^2\right)\\ \bar{Q}_{22} &= Q_{11}s^4 + Q_{22}c^4 + 2\left(Q_{12}+2Q_{66}\right)s^2c^2\\ \bar{Q}_{16} &= \left(Q_{11}-Q_{12}-2Q_{66}\right)c^3s - \left(Q_{22}-Q_{12}-2Q_{66}\right)s^3c\\ \bar{Q}_{26} &= \left(Q_{11}-Q_{12}-2Q_{66}\right)cs^3 - \left(Q_{22}-Q_{12}-2Q_{66}\right)c^3s\\ \bar{Q}_{66} &= \left(Q_{11}+Q_{22}-2Q_{12}-2Q_{66}\right)s^2c^2 + Q_{66}\left(s^4+c^4\right) \end{aligned} \tag{4}$$

The c and s denotes $cos\theta$ and $sin\theta$.

The local and global stresses in an angle lamina are related to each other through the angle of the lamina θ

$$\begin{bmatrix}\sigma_1\\ \sigma_2\\ \tau_{12}\end{bmatrix} = [T]\begin{bmatrix}\sigma_x\\ \sigma_y\\ \tau_{xy}\end{bmatrix} \tag{5}$$

where

$$[T] = \begin{bmatrix}c^2 & s^2 & 2sc\\ s^2 & c^2 & -2sc\\ -sc & sc & c^2-s^2\end{bmatrix} \tag{6}$$

Stress and Strain in a Laminate

$$\begin{aligned} \begin{bmatrix}N_x\\ N_y\\ N_{xy}\end{bmatrix} &= \begin{bmatrix}A_{11} & A_{12} & A_{16}\\ A_{12} & A_{22} & A_{26}\\ A_{16} & A_{26} & A_{66}\end{bmatrix}\begin{bmatrix}\varepsilon_x^0\\ \varepsilon_y^0\\ \gamma_{xy}^0\end{bmatrix} + \begin{bmatrix}B_{11} & B_{12} & B_{16}\\ B_{11} & B_{12} & B_{16}\\ B_{16} & B_{26} & B_{66}\end{bmatrix}\begin{bmatrix}k_x\\ k_y\\ k_{xy}\end{bmatrix}\\ \begin{bmatrix}M_x\\ M_y\\ M_{xy}\end{bmatrix} &= \begin{bmatrix}B_{11} & B_{12} & B_{16}\\ B_{12} & B_{22} & B_{26}\\ B_{16} & B_{26} & B_{66}\end{bmatrix}\begin{bmatrix}\varepsilon_x^0\\ \varepsilon_y^0\\ \gamma_{xy}^0\end{bmatrix} + \begin{bmatrix}D_{11} & D_{12} & D_{16}\\ D_{11} & D_{12} & D_{16}\\ D_{16} & D_{26} & D_{66}\end{bmatrix}\begin{bmatrix}k_x\\ k_y\\ k_{xy}\end{bmatrix} \end{aligned} \tag{7}$$

N_x, N_y - normal force per unit length
N_{xy} - shear force per unit length
M_x, M_y - bending moment per unit length
M_{xy} - twisting moments per unit length

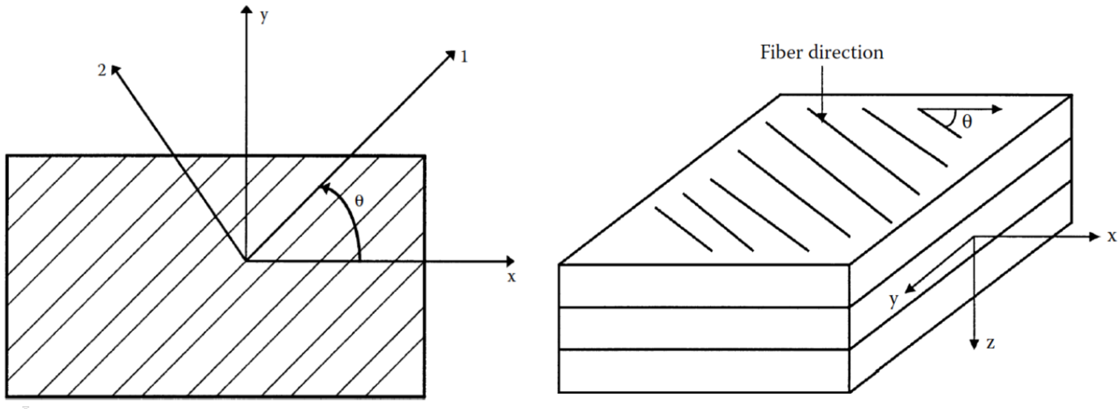


Figure 1. Lamina

0
90
90
0
90

Figure 2. Model for cross ply laminate

ε^0, k- mid plane strains and curvature of a laminate in x-y coordinates

The mid plane strain and curvature is given by

$$\begin{aligned} A_{ij} &= \sum_{k=1}^{n} (\overline{Q_{ij}})_k (h_k - h_{k-1}) i = 1,2,6, j = 1,2,6 \\ B_{ij} &= \frac{1}{2} \sum_{k=1}^{n} (\overline{Q_{ij}})_k (h_k^2 - h_{k-1}^2) i = 1,2,6, j = 1,2,6 \\ D_{ij} &= \frac{1}{3} \sum_{k=1}^{n} (\overline{Q_{ij}})_k (h_k^3 - h_{k-1}^3) i = 1,2,6, j = 1,2,6 \end{aligned} \tag{8}$$

The [A], [B], and [D] matrices are called the extensional, coupling, and bending stiffness matrices.

Failure Theory

Failure Process

A laminate will fail under increasing mechanical loading; however, the procedure of laminate failure may not be catastrophic. In some cases, some layers fail first, and the rest are able to continue to take additional loading until all the plies fail. A ply is fully discounted when a ply fails; then, the ply is replaced by a near zero stiffness and strength. The procedure for finding the first ply failure in the present study follows the fully discounted method:

1. Compute the reduced stiffness matrix [Q] referred to as the local axis for each ply using its four engineering elastic constants E_1, E_2, E_{12}, and G_{12}.
2. Calculate the transformed reduced stiffness $[\bar{Q}]$ referring to the global coordinate system (x, y) using the reduced stiffness matrix [Q] obtained in step 1 and the ply angle for each layer.
3. Given the thickness and location of each layer, the three laminate stiffness matrices [A], [B], and [D] are determined.
4. Apply the forces and moments, $[N]_{xy}, [M]_{xy}$ solve Equation 7, and calculate the middle plane strain $[\sigma^0]_{xy}$ and curvature $[k]_{xy}$.
5. Determine the local strain and stress of each layer under the applied load.
6. Use the ply-by-ply stress-strain and related failure theories to determine the strength ratio.

Tsai-wu Failure Theory

Many different theories about the failure of an angle lamina have been developed for a unidirectional lamina, such as the maximum stress failure theory, maximum strain failure theory, Tsai-Hill failure theory, and Tsai-Wu failure theory. The failure theories of a lamina are based on the stresses in the local axes in the material. There are four normal strength parameters and one shear stress for a unidirectional lamina. The five strength parameters are:

$(\sigma_1^T)_{ult}$ = ultimate longitudinal tensile strength
$(\sigma_1^C)_{ult}$ = ultimate longitudinal compressive strength
$(\sigma_2^T)_{ult}$ = ultimate transverse tensile strength
$(\sigma_2^C)_{ult}$ = ultimate transverse compressive strength
$(\tau_{12})_{ult}$ = and ultimate in-plane shear strength

In the present study, Tsai-wu failure theory is taken to decide whether a lamina fails, because this theory is more general than the Tsai-Hill failure theory, which considers two different situations, the compression and tensile strengths of a lamina. A lamina is considered to fail if

$$\begin{aligned} H_1\sigma_1 + H_2\sigma_2 + H_6\tau_{12} + H_{11}\sigma_1^2 + H_{22}\sigma_2^2 \\ + H_{66}\tau_{12}^2 + 2H_{12}\sigma_1\sigma_2 < 1 \end{aligned} \tag{9}$$

is violated, where

$$SR = \frac{\text{Maximum Load Which Can Be Applied}}{\text{Load Applied}} \tag{10}$$

Genetic algorithm model

The GA starts off with multiple individuals with limited chromosome lengths, in which maybe none of these

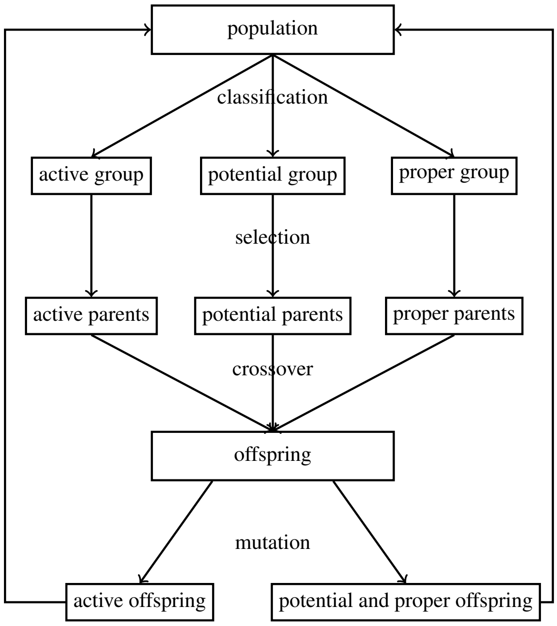


Figure 3. New GA Model

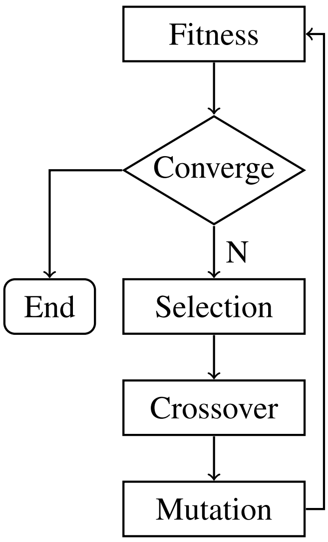


Figure 4. Traditional GA Model

individuals fulfill the constraints. The GA is assumed to derive appropriate offspring based on the initial population as the GA continues. The classic way to handle the constrained search of the GA is either to introduce repair strategies or use a penalty function. Figure 4 shows the classic flow chart of a GA framework which includes selection, crossover and mutation operators. However, GA is originally proposed to solve unconstrained problems, therefore, a new approach is developed to address the constrained GA search problem in an unconstrained way.

Because of the existence of constraints, the population can be sorted by the fitness (obtained by the objective function) but can also be sorted by the constraint value obtained by the constraint function (assuming a constraint function exists), so the parents of the next generation can be chosen by the following three approaches. First, the population is sorted by fitness in an ascending order, and individuals with smaller fitness are selected. These selected individuals form a group named as proper group. Second, remove individual which satisfies constraints, and sort population by the difference between the individual's constraint value and the threshold of the constraint in a descending order, and individuals with bigger differences are chosen to be the parents of next generation. The group which forms are called potential group, and individual from this group is refered as potential individual. Third, the population is sorted by fitness from low to high after removing individuals which fails to fit the constraints, select individuals with bigger fitness, and these individuals form the proper group. So the final parents pool is consists of three groups, active group, potential group and proper group. The number of active individuals, potential individuals and proper individuals are called, respectively, active number, potential numbers and proper number.

Each group in the parents population has its own role in the searching process. The problem within traditional GA is premature and has weak local search ability, therefore, traditional GAs are more likely to get stuck in a local optimum. To prevent the GA from experiencing early convergence and to improve the local search performance, the active group is proposed to overcome this problem. As its name suggests, this group would always live in the population. Because both active individual's fitness and constraint value are small, each individual can be treated as a independent gene clip. So their existance enriches the gene clip varity of mating pool. The offspring of two active individuals are more likely to be an active individual, which can maintain the active group.

For an individual in the potential group, it doesn't satisfy constraint, however, it's supposed to evolve into a proper individual after multiple generations by modifying its chromosome structure or length. The crossover operation could happen between a potential individual with an active individual, or a potential individual or a proper individual. The child of an active individual and a potential is more likely to be a potential individual, and this active individual could inject new gene clip into this potential individual, therefore providing a new evolution direction.

An proper individual means a feasible solution, it fulfills all the constraints. However, there are still some drawbacks within it, for example, its fitness is low. The crossover operation could happen between a proper individual and any other individuals.

The mutation operator for active group is different from the potential group and proper group, becasue their roles in the searching process are different. The target of the potential group and proper group are to obtain a feasible solution, however, the role of active group is to maintain the variety of gene clip in the mating pool.

Figure 3 shows the flow chart of new GA. First, the population are divided into three groups, active group, potential group, and proper group by above mentioned method. Second, select appropriate number of individuals from each group as parents, and the various selection scheme can be taken for each group.

The searching process can be divided into two phases according to whether proper individuals are generated. During the initial stage, no individual in the population is appropriate, which means the number of individuals in the proper group is zero. Both active group and potential group are full. After a couple of generations, some proper

P_1:	90	90	0	0	0	90	90	90	90	90
P_2:	0	0	90	90	90	0	0	90	0	0

(a): Parents P_1 and P_2

O_1:	90	90	0	0	0	0	0	90	0	0
O_2:	0	0	90	90	90	90	90	90	90	90

(b): Offspring O_1 and O_2

O_1:	+13	90	90	$\cdots$	90	90	0	$\cdots$	0	0
			11					7		

(c): Offspring O_1 after lenght mutation

90	90	90	$\cdots$	90	90	0	$\cdots$	0	0

(b): Offspring O_1 after angle mutation

Figure 5. GA Operators

individuals could be produced. Then, GA comes to its second phase, the number of proper individuals begins to increase, finally, the number in the proper group reaches its upper bound. During the last phases,

Experiment

In the present study, the relevant parameters of the GA are shown in Table 1. The design variables are the materials, number of layers, and ply orientation restricted to a discrete set of angles ($0, \pm 45$ and 90 degrees). The possible materials are graphite/epoxy, carbon/epoxy, and glass/epoxy and are represented by codes 0, 1 and 2, respectively.

Figure 5 (b) shows

Table 1. GA parameters

Parameter	Value
Population size	40
Initial length range	[3-15]
Encoding	Integer
Percentage of active group	0.3
Percentage of potential group	0.3
Percentage of proper group	0.4
Selection strategy for active group	Ranking
Selection strategy for potential group	Ranking
Selection strategy for proper group	Ranking
Crossover strategy	One-point
Mutation strategy	Mass mutation

The laminate chromosome is represented by a double-gene string that can be divided into two parts: one part represents the angles, and the other part represents the materials (as shown in Figure 5(P_1)) . To maintain the diversity of the population, single-point crossover is taken during the evolution process. The break points in the string are randomly chosen, and one of the offspring of parent 1 (as shown in Figure 5(P_1)) and parent 2 (as shown in Figure 5(P_2)) is obtained by combining the gene segments $P1_o$ and $P2_o$ and $P1_m$ and $P2_m$, respectively. The gene code of the offspring laminate is $[+45, -45, -45, -45, -45, -45, -45, 0, 1, 0, 1, 1, 0, 1, 0]$.

To prevent the search from becoming stuck in a local optimum, mutation is used to randomly change the gene in the chromosome, and the offspring after the mutation operator is as shown in Figure 5

The GA is a stochastic procedure that heavily depends on the generator of pseudorandom numbers. In the present study, the standard Wichmann-Hill generator is used in the algorithm, which combines three pure multiplicative congruent generators of moduli 30269, 30307 and 30323. The seed used in this paper is 1.

Design Problem I

The aim is to minimize the mass of a laminate composite for a targeted strength ratio based on the Tsai-wu failure theory. The design variable are the ply angles and the number of layers.

Find: $\{\theta_k, n\}$ $\theta_k \in \{0, +45, -45, 90\}$
Minimize: weight
Subject to: strength ratio and first ply failure constraint

Design Problem II

The aim is to minimize the weighted cost and weight of hybrid composite laminates under various loading cases, so the design variable not only includes the ply angles and number of layers but also the material of each lamina.

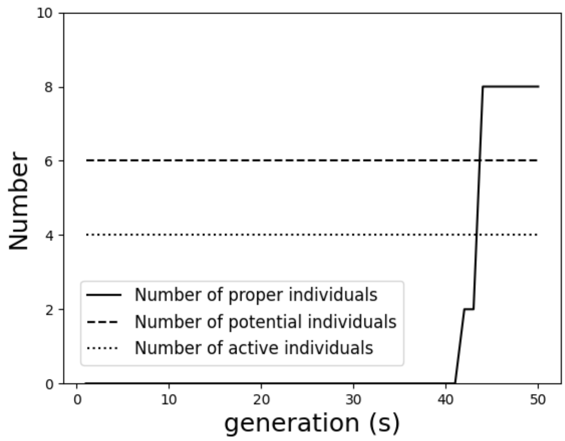
Find: $\{\theta_k, \mathrm{mat}_k, n\}$ $\theta_k \in \{0, +45, -45, 90\}$ $\mathrm{mat}_k \in \{CA, GR, GL\}$
Minimize:

$$F = \frac{\text{Cost}}{C_{\min}} + \frac{\text{Weight}}{W_{\min}} \tag{11}$$

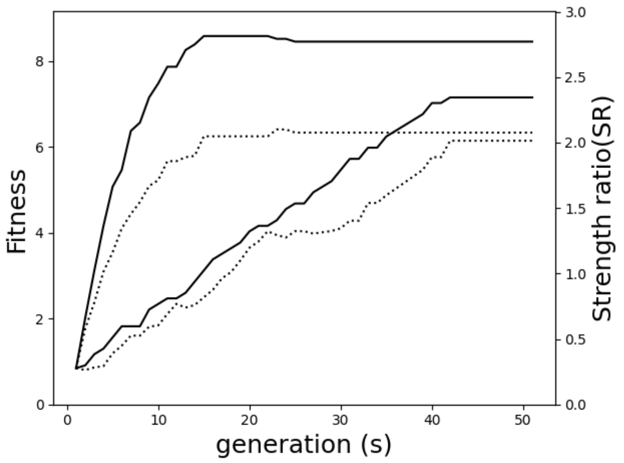
Subject to: strength ratio and first ply failure constraint

Here, CA, GF, and GL represent carbon/epoxy, graphite/epoxy, and glass/epoxy, and $C_{\min}$ and $W_{\min}$ represent the cost and weight corresponding to the laminates with a minimum cost and minimum weight obtained from previous problem.

Numerical Results and Discussion



A laminate composite with dimensions $1000 \times 1000 \times 0.165 mm^3$ for each lamina is under various loading cases, and each CA, GF, and GL layer is assumed to cost 8, 2.5 and 1 monetary units, respectively. The other material properties



are shown in Table 2. A carbon/epoxy ply is represented by "cr", a graphite/epoxy ply by "gl", and a glass/epoxy ply by "gl". In the present experiment, the optimal composite system, layup, thickness, and number of layers for a targeted strength ratio (2 in this paper) under two different in-plane loading conditions is investigated.

Table 3 shows the optimal lay-up sequences by the variant GA and Choudhury and Mondal's[28] study. For the loading case $Nx = 1$ MPa m, the optimal lay-ups are a $[0_{68}/90_{72}]$ cross ply laminate if glass-epoxy are taken, however, in present study, a $[0_{78}/90_{28}]$ glass-epoxy cross ply laminate has been found which significantly reduces both the cost and weight with increasing laminate's SR; If graphite-epoxy was used, compared with the $[0_{17}/90_{18}]$ cross ply laminate, an alternative cross ply laminate is found, its lay-up is $[0_{18}/90_{8}]$.

The GA process can be divided into two phases by whether there are individuals that are appropriate or not. During the initial phase, no individual's strength ratio is over the specified threshold, so individuals with larger fitness are more likely to be chosen as parents, which is why the strength ratio curves go all the way up to the specified threshold during the first stage. After the initial phase, the GA produces many appropriate individuals, and then the target function is utilized, and as shown in Fig. **??**, the fitness curves tend to decrease, but the strength ratio curves remain greater than the specified threshold.

In the first experiment, the applied stress is $N_x = N_y = 1e6$ N. As shown in Figure **??**, Figures **??**(a), (b), and (c) show the experimental results for a single material, Figure **??**(d) shows the results for the hybrid composite material. For the single materials, both the basic GA and improved GA method obtained the optimal value, but the improved GA converged more slowly than the basic GA. As seen from Table 4, a [-45_6/+$45_6]_s$ carbon/epoxy laminate has the least weight, denoted by W_{min}, and a [-45_{35}/+45_{73}/+45_{35}] graphite/epoxy laminate has the lowest cost, denoted by C_{min}. W_{min} and C_{min} were used to evaluate the fitness of the second problem, which is the layup design of the hybrid composite material. As shown in subfigure d, the improved GA obtained a more appropriate system layup, whose strength ratio was greater than the specified safety factor, and the weight and cost are less than the result obtained by the basic GA method, as shown in Table 4. Compared with the basic GA, the improved GA method showed more powerful global search ability in the initial phase.

In the second case, the applied stress was $N_x = N_y = N_z = 1e6$ N, and the experimental results were as shown in the Figure **??**. In the first experiment, as seen from Figure **??**(a), the improved GA obtained a better system layup than the result obtained by the basic GA. In the second experiment, as shown in Figure **??**(b), during the initial phase, the fitness curves of the basic GA and improved GA went all the way up to the previous specified threshold; however, the improved GA converged more slowly than the basic GA, which means that the search cost of the improved GA was greater than that of the basic GA. After the initial phase, the fitness curve of the basic GA did not change anymore, it got trapped in the local domain. However, the fitness curve of the improved GA gradually decreased; at the same time, the strength ratio curve of the improved GA was maintained to be greater than the threshold. This means the improved GA was able to get out of the optimum and obtain a much better system layup. The improved GA offered more powerful local search ability. In the third experiment, as shown in Figure **??**, both the basic GA and improved GA obtained the same result, but the improved GA converged more slowly than the basic GA. From these three experiments for a single material, we know that a [+$45_6^{cr}]_s$ carbon/epoxy laminate has the least mass, and a [+45_{11}^{gl}/+$\bar{4}5^{gl}]_s$ glass laminate has the least cost. In the last experiment, the improved GA obtained a slightly better result than the basic GA, as shown in Table **??**. Compared with the [+45_{12}^{cr}] laminate, the weight of a [+45_8^{gr}/+$\bar{4}5^{gl}]_s$ laminate increased 41.8%, however, the cost decreased 56%.

Conclusions

In this paper, a combination of CLT and a variant of the GA are employed to minimize the weight and cost of a single-material and hybrid composite laminate, respectively, under various in-plane loading cases. The results are presented in two sections: stacking sequence optimization for a single material laminate, and weighted mass and cost optimization of a carbon/epoxy, graphite/epoxy, and glass/epoxy hybrid laminates. Furthermore, the performance of the basic GA is improved by changing the selection strategy.

This variant of the GA provides a new approach to address the search constraint in laminate composite optimization, and this method is very easy to extend for solving the multiple constraint search problem in other domain. The problem with the current method involves adjusting the parameters in the GA to obtain the best performance.

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Table 2. Comparison of the carbon/epoxy, graphite/epoxy, and glass/epoxy properties

Property	Symbol	Unit	Carbon/Epoxy	Graphite/Epoxy	Glass/Epoxy
Longitudinal elastic modulus	E_1	GPa	116.6	181	38.6
Traverse elastic modulus	E_2	GPa	7.67	10.3	8.27
Major Poisson's ratio	v_{12}		0.27	0.28	0.26
Shear modulus	G_{12}	GPa	4.17	7.17	4.14
Ultimate longitudinal tensile strength	$(\sigma_1^T)_{ult}$	MP	2062	1500	1062
Ultimate longitudinal compressive strength	$(\sigma_1^C)_{ult}$	MP	1701	1500	610
Ultimate transverse tensile strength	$(\sigma_2^T)_{ult}$	MPa	70	40	31
Ultimate transverse compressive strength	$(\sigma_2^C)_{ult}$	MPa	240	246	118
Ultimate in-plane shear strength	$(\tau_{12})_{ult}$	MPa	105	68	72
Density	ρ	g/cm^3	1.605	1.590	1.903
Cost			8	2.5	1

Table 3. The optimum lay-ups for the loading $N_x = 1e6$ N

Cross Ply $[0_M/90_N]$	Previous Research		Current Research	
Material	Glass-Epoxy	Graphite-Epoxy	Glass-Epoxy	Graphite-Epoxy
M	68	17	78	18
N	72	18	28	8
no. of lamina(n)	140	35	106	26
SR	2.01	2.10	2.03	2.16
weight	9.10	1.84	6.89	102.5

Table 4. The optimum lay-ups for the loading $N_x = N_y = 1e6$ N

coefficient	Material	case	Stacking sequence	Strength ratio	Mass	Cost	Layer
0.1	glass-epoxy	worst	$[0_{80}/90_{52}]$	2.010	8.58	132	132
		best	$[0_{75}/90_{43}]$	2.000	7.67	118	118
		average		2.012	7.83	123	123
	graphite-epoxy	worst	$[0_{17}/90_{15}]$	2.036	1.68	80	32
		best	$[0_{17}/90_{5}]$	2.005	1.15	55	22
		average		2.018	1.47	70	28
0.1	glass-epoxy	worst	$[0_{72}/90_{64}]$	2.009	8.84	136	136
		best	$[0_{72}/90_{53}]$	2.003	8.12	125	125
		average		2.008	8.55	131	131
	graphite-epoxy	worst	$[0_{18}/90_{24}]$	2.006	2.20	105	42
		best	$[0_{17}/90_{6}]$	2.001	1.20	57	23
		average		2.022	1.54	73	29

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