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## Chapter 1

# Introduction

### 1.1 Laminated Composite Material

Composite materials offer improved strength, stiffness, corrosion resistance, etc. over conventional materials, and are widely used as alternative materials for applications in various industries ranging from electronic packaging to golf clubs, and medical equipment to homebuilding, making aircraft structure to space vehicles. The stacking sequence and fiber orientation of composite laminates give the designer additional 'degree of freedom' to tailor the design with respect to strength or stiffness. One widely known advantage of using composite material is can significantly reducing the weight of target structure, and many researchers attempted to improve the efficiency of using composite material by minimizing the thickness.

### 1.2 Classic Lamination Theory and Failure Theory

Classic lamination theory (CLT) is used to develop the stress-strain relationship of composite material under in-plane and out-of-plane loading. First, develop stress-strain relationships, elastic moduli, strengths of an angle ply based on a unidirectional lamina and the angle of the ply; second, because a laminate is consist of more than one lamina bonded together through their thickness, so the macromechanical analysis will be developed for a laminate based on applied loading. To check whether a designed lay-up is plausible or not, different failure theories have been developed.

### 1.3 Genetic Algorithm

In the design of composite material, gradient based optimization techniques are not applicable in this domain, because the design variables, such as fiber orientation, layer thickness, number of layers etc. are discrete. Genetic algorithm (GA) can be adopted in the optimization problem because it doesn't require the gradient information. Moreover, the GA has been proved a reliable technique and widely used in the design of composite material.

### 1.4 Artificial Neural Network

CLT is a classic analytical approach to obtain the stress and strain of composite material, the disadvantage of this method is quite cumbersome and in which involves compute matrix and integration operations. Artificial neural network (ANN) has been proved a reliable tool in modelling various engineering system in practice without

solving tricky equations and making ideal assumptions. In this thesis, the ANN is taken to approximate the numeric results based on CLT and failure theory.

## **1.5 Summary**

In this thesis, first, we review the use of composite material in practice, then, the CLT to calculate the stress and strain under certain loading, last, the failure theories which are used to decide whether a composite material will failure or not. Second, the stochastic algorithm, GA, is studied and implemented in the design of composite material, two different cases are studied in which GA can be taken to obtain the optimal lay-up. At last, ANN is introduced to approximate the evaluation result of CLT, the reason for adopting ANN is to reduce the computation complexity based on CLT.

## Chapter 2

# Classic Lamination Theory and Failure Theory

## 2.1 Classic Lamination Theory

A laminate is consist of multiple laminae bonded together through thickness. In this chapter, first the stress-strain relationship is developed based on Hook's law for a single lamina; second, develop relationship of mechanical loads applied to a laminate to strains and stresses in each lamina, calculate the elastic moduli of laminate based on the elastic moduli of single laminate and the lay-up..

### 2.1.1 Analysis of stress and strain for composite material

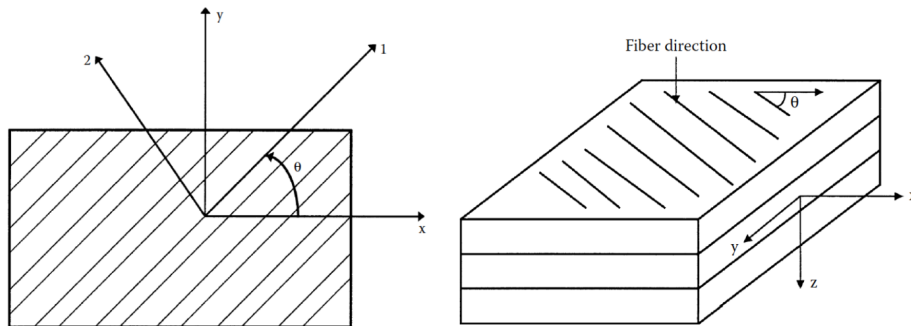


FIGURE 2.1: Lamina

### 2.1.2 Stress and Strain in a Lamina

For a single lamina has a small thickness under plane stress, and it's upper and lower surfaces of the lamina are free from external loads. According to the Hooke's Law, the three-dimensional stress-strain equations can be reduced to two-dimensional stress-strain equations. The stress-strain relation in local axis 1-2 is

$$\begin{bmatrix} \sigma_1 \\ \sigma_2 \\ \tau_{12} \end{bmatrix} = \begin{bmatrix} Q_{11} & Q_{12} & 0 \\ Q_{12} & Q_{22} & 0 \\ 0 & 0 & Q_{66} \end{bmatrix} \begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \gamma_{12} \end{bmatrix} \quad (2.1)$$

where  $Q_{ij}$  are the stiffnesses of the lamina that are related

to engineering elastic constants given by

$$\begin{aligned} Q_{11} &= \frac{E_1}{1 - \nu_{12}\nu_{21}}, \\ Q_{22} &= \frac{E_2}{1 - \nu_{12}\nu_{21}}, \\ Q_{66} &= G_{12}, \\ Q_{12} &= \frac{\nu_{21}E_2}{1 - \nu_{12}\nu_{21}}, \end{aligned} \quad (2.2)$$

where  $E_1, E_2, \nu_{12}, G_{12}$  are four independent engineering elastic constants, which are defined as follows:  $E_1$  is the longitudinal Young's modulus,  $E_2$  is the transverse Young's modulus,  $\nu_{12}$  is the major Poisson's ratio, and  $G_{12}$  is the in-plane shear modulus.

Stress strain relation in the global x-y axis is

$$\begin{bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{bmatrix} = \begin{bmatrix} \bar{Q}_{11} & \bar{Q}_{12} & \bar{Q}_{16} \\ \bar{Q}_{12} & \bar{Q}_{22} & \bar{Q}_{26} \\ \bar{Q}_{16} & \bar{Q}_{26} & \bar{Q}_{66} \end{bmatrix} \begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{bmatrix}, \quad (2.3)$$

where

$$\begin{aligned} \bar{Q}_{11} &= Q_{11}\cos^4\theta + Q_{22}\sin^4\theta + 2(Q_{12} + 2Q_{66})\sin^2\theta\cos^2\theta, \\ \bar{Q}_{12} &= (Q_{11} + Q_{22} - 4Q_{66})\sin^2\theta\cos^2\theta + Q_{12}(\cos^4\theta + \sin^4\theta), \\ \bar{Q}_{22} &= Q_{11}\sin^4\theta + Q_{22}\cos^4\theta + 2(Q_{12} + 2Q_{66})\sin^2\theta\cos^2\theta, \\ \bar{Q}_{16} &= (Q_{11} - Q_{12} - 2Q_{66})\cos^3\theta\sin\theta - (Q_{22} - Q_{12} - 2Q_{66})\sin^3\theta\cos\theta, \\ \bar{Q}_{26} &= (Q_{11} - Q_{12} - 2Q_{66})\cos\theta\sin^3\theta - (Q_{22} - Q_{12} - 2Q_{66})\cos^3\theta\sin\theta, \\ \bar{Q}_{66} &= (Q_{11} + Q_{22} - 2Q_{12} - 2Q_{66})\sin^2\theta\cos^2\theta + Q_{66}(\sin^4\theta + \cos^4\theta). \end{aligned} \quad (2.4)$$

The local and global stresses in an angle lamina are related to each other through the angle of the lamina  $\theta$ , it can be written as

$$\begin{bmatrix} \sigma_1 \\ \sigma_2 \\ \tau_{12} \end{bmatrix} = [T] \begin{bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{bmatrix}, \quad (2.5)$$

where

$$[T] = \begin{bmatrix} \cos^2\theta & \sin^2\theta & 2\sin\theta\cos\theta \\ \sin^2\theta & \cos^2\theta & -2\sin\theta\cos\theta \\ -\sin\theta\cos\theta & \sin\theta\cos\theta & \cos^2\theta - \sin^2\theta \end{bmatrix}. \quad (2.6)$$

### 2.1.3 Stress and Strain in a Laminate

For forces and moment resultants acting on laminates, such as in plate and shell structures, the relationship between applied forces and moment and displacement can be given by

$$\begin{aligned}
\begin{bmatrix} N_x \\ N_y \\ N_{xy} \end{bmatrix} &= \begin{bmatrix} A_{11} & A_{12} & A_{16} \\ A_{12} & A_{22} & A_{26} \\ A_{16} & A_{26} & A_{66} \end{bmatrix} \begin{bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \end{bmatrix} \\
&+ \begin{bmatrix} B_{11} & B_{12} & B_{16} \\ B_{11} & B_{12} & B_{16} \\ B_{16} & B_{26} & B_{66} \end{bmatrix} \begin{bmatrix} k_x \\ k_y \\ k_{xy} \end{bmatrix}, \\
\begin{bmatrix} M_x \\ M_y \\ M_{xy} \end{bmatrix} &= \begin{bmatrix} B_{11} & B_{12} & B_{16} \\ B_{12} & B_{22} & B_{26} \\ B_{16} & B_{26} & B_{66} \end{bmatrix} \begin{bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \end{bmatrix} \\
&+ \begin{bmatrix} D_{11} & D_{12} & D_{16} \\ D_{11} & D_{12} & D_{16} \\ D_{16} & D_{26} & D_{66} \end{bmatrix} \begin{bmatrix} k_x \\ k_y \\ k_{xy} \end{bmatrix},
\end{aligned} \tag{2.7}$$

where

$N_x, N_y$  - normal force per unit length;

$N_{xy}$  - shear force per unit length;

$M_x, M_y$  - bending moment per unit length;

$M_{xy}$  - twisting moments per unit length;

$\varepsilon^0, k$  - mid plane strains and curvature of a laminate in x-y coordinates.

The mid plane strain and curvature is given by

$$\begin{aligned}
A_{ij} &= \sum_{k=1}^n (\overline{Q}_{ij})_k (h_k - h_{k-1}) \quad i = 1, 2, 6, j = 1, 2, 6, \\
B_{ij} &= \frac{1}{2} \sum_{k=1}^n (\overline{Q}_{ij})_k (h_k^2 - h_{k-1}^2) \quad i = 1, 2, 6, j = 1, 2, 6, \\
D_{ij} &= \frac{1}{3} \sum_{k=1}^n (\overline{Q}_{ij})_k (h_k^3 - h_{k-1}^3) \quad i = 1, 2, 6, j = 1, 2, 6.
\end{aligned} \tag{2.8}$$

The  $[A]$ ,  $[B]$ , and  $[D]$  matrices are called the extensional, coupling, and bending stiffness matrices, respectively. The extensional stiffness matrix  $[A]$  relates the resultant in-plane forces to the in-plane strains, and the bending stiffness matrix  $[D]$  couples the resultant bending moments to the plane curvatures. The coupling stiffness matrix  $[B]$  relates the force and moment terms to the midplane strains and midplane curvatures.

## 2.2 Failure Theory

Failure criteria for composite materials are more difficult to predict due to structural and material complexity in comparison to isotropic materials. The failure process of a composite materials can be regarded from microscopic and macroscopic points of view. Most popular criteria about the failure of an angle lamina are in terms of macroscopic failure criteria, which are based on the tensile, compressive and shear strengths. According to the failure surfaces, these criteria can be classified into two classes: one is called independent failure mode criteria which includes the maximum stress failure theory, maximum strain failure theory because their failure envelop are rectangle; another is called quadratic polynomial which includes Tsai-Wu, Chamis, Hoffman and Hill criteria because their failure surfaces are of ellipsoidal shape. In

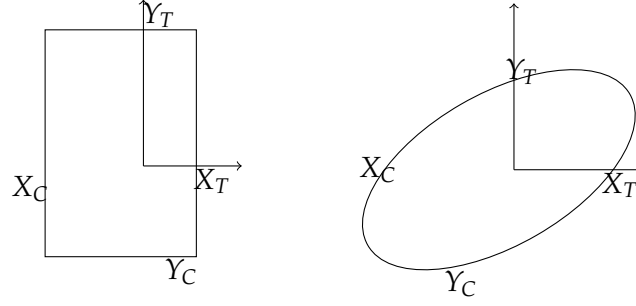


FIGURE 2.2: Schematic failure surfaces for maximum stress and quadratic failure criteria

the present study, two most reliable failure criteria is taken, Maximum stress and Tsai-wu. Both of these two failure criteria are based on the stresses in the local axes instead of principal normal stresses and maximum shear stresses, and four normal strength parameters and one shear stress for a unidirectional lamina are involved. The five strength parameters are

- $(\sigma_1^T)_{ult}$  = ultimate longitudinal tensile strength(in direction 1),
- $(\sigma_1^C)_{ult}$  = ultimate longitudinal compressive strength,
- $(\sigma_2^T)_{ult}$  = ultimate transverse tensile strength,
- $(\sigma_2^C)_{ult}$  = ultimate transverse compressive strength, and
- $(\tau_{12})_{ult}$  = and ultimate in-plane shear strength.

### 2.2.1 Tsai-wu failure criterion

The TW criterion is one of the most reliable static failure criteria which is derived from the von Mises yield criterion. A lamina is considered to fail if

$$H_1\sigma_1 + H_2\sigma_2 + H_6\tau_{12} + H_{11}\sigma_1^2 + H_{22}\sigma_2^2 + H_{66}\tau_{12}^2 + 2H_{12}\sigma_1\sigma_2 < 1 \quad (2.9)$$

is violated, where

$$\begin{aligned} H_1 &= \frac{1}{(\sigma_1^T)_{ult}} - \frac{1}{(\sigma_1^C)_{ult}}, \\ H_{11} &= \frac{1}{(\sigma_1^T)_{ult} (\sigma_1^C)_{ult}}, \\ H_2 &= \frac{1}{(\sigma_2^T)_{ult}} - \frac{1}{(\sigma_2^C)_{ult}}, \\ H_{22} &= \frac{1}{(\sigma_2^T)_{ult} (\sigma_2^C)_{ult}}, \\ H_{66} &= \frac{1}{(\tau_{12})_{ult}^2}, \\ H_{12} &= -\frac{1}{2} \sqrt{\frac{1}{(\sigma_1^T)_{ult} (\sigma_1^C)_{ult} (\sigma_2^T)_{ult} (\sigma_2^C)_{ult}}}. \end{aligned} \quad (2.10)$$



$H_i$  is the strength tensors of the second order;  $H_{ij}$  is the strength tensors of the fourth order.  $\sigma_1$  is the applied normal stress in direction 1;  $\sigma_2$  is the applied normal stress in the direction 2; and  $\tau_{12}$  is the applied in-plane shear stress.

### 2.2.2 Maximum Stress Failure Theory

Maximum stress(MS) failure theory consists of maximum normal stress theory proposed by Rankine and maximum shearing stress theory by Tresca. The stresses applied on a lamina can be resolved into the normal and shear stresses in the local axes. If any of the normal or shear stresses in the local axes of a lamina is equal or exceeds the corresponding ultimate strengths of the unidirectional lamina, the lamina is considered to be failed. That is

$$\begin{aligned}\sigma_1 &\geq (\sigma_1^T)_{ult} \text{ or } \sigma_1 \leq -(\sigma_1^C)_{ult}, \\ \sigma_2 &\geq (\sigma_2^T)_{ult} \text{ or } \sigma_2 \leq -(\sigma_2^C)_{ult}, \\ \tau_{12} &\geq (\tau_{12})_{ult} \text{ or } \tau_{12} \leq -(\tau_{12})_{ult}.\end{aligned}$$

where  $\sigma_1$  and  $\sigma_2$  are the normal stresses in the local axes 1 and 2, respectively;  $\tau_{12}$  is the shear stress in the symmetry plane 1-2. The safety factor, or yield stress, is how much extra load beyond is intended a composite laminate will actually take, which is an indication of the material's load carrying capacity. If the value is less than 1.0, it means failure. The safety factor is defined as

$$SF = \frac{\text{Maximum Load Which Can Be Applied}}{\text{Load Applied}}. \quad (2.11)$$

The safety factor based on maximum stress theory is calculated by the following method: first, the principal stresses( $\sigma_1^k, \sigma_2^k$ , and  $\tau_{12}^k$ ) are obtained by experiment; evaluate the safety factor along each direction according to equation 2.11; The minimum value among these safety factors are denoted as the safety factor of the lamina,  $SF_{MS}^k$ , it can be written as

$$SF_{MS}^k = \min \left\{ \begin{aligned} SF_X^k &= \begin{cases} \frac{X_t}{\sigma_{11}}, & \text{if } \sigma_{11} > 0 \\ \frac{X_c}{\sigma_{11}}, & \text{if } \sigma_{11} < 0 \end{cases} \\ SF_Y^k &= \begin{cases} \frac{Y_t}{\sigma_{22}}, & \text{if } \sigma_{22} > 0 \\ \frac{Y_c}{\sigma_{22}}, & \text{if } \sigma_{22} < 0 \end{cases} \\ SF_S^k &= \frac{S}{|\tau_{12}|} \end{aligned} \right. . \quad (2.12)$$

Assuming the composite laminate under a in-plane loading  $f$ , the corresponding stress on local stress in direction 1, local stress in direction 2, and shear stress for the  $k$ th lamina are  $\sigma_1 SF_{TW}^k$ ,  $\sigma_2 SF_{TW}^k$ , and  $\tau_{12} SF_{TW}^k$ , respectively. Substitute them into the equation 2.9, the expression are given by

$$a(SF_{TW}^k)^2 + b(SF_{TW}^k) - 1 = 0,$$

where

$$a = H_{11}(\sigma_1)^2 + H_{22}(\sigma_2)^2 + H_{66}(\tau_{12})^2 + 2H_{12}\sigma_1\sigma_2,$$

$$b = H_1\sigma_1 + H_2\sigma_2 + H_6\tau_{12}.$$

Solve the above equation, the safety factor for the  $k$ th lamina is

$$SF_{TW}^k = \left| \frac{-b + \sqrt{b^2 + 4a}}{2a} \right|.$$

Then, the minimum of  $SF_{TW}^k$  is taken as the safety factor of the laminate which is written as

$$SF_{TW} = \min \text{ of } SF_{TW}^k \text{ for } k = 1, 2, \dots, m-1, m.$$

## 2.3 Summary

In this chapter, we review the CLT for composite material's analysis, then related failure theories are introduced to check whether a composite material would fail or not. In the following chapters, the CLT would be used to calculate the stress and strain under in-plane loading, it would also be used to generate the training data for stress and strain approximation based on neural network; in order to design a proper composite material, the failure theory are used to decide the feasibility of composite design.

## Chapter 3

# A New Genetic Algorithm Model for Constrained Problem

### 3.1 Genetic Algorithm Framework

GA is one of the most reliable stochastic algorithm, which has been widely used in discrete variables optimization problems [17, 18, 7, 20, 11, 8, 24, 2, 5, 16, 1, 13, 12, 19, 4, 15, 14, 21, 23, 6, 9]. In practice, fiber orientations are restricted to a finite set of angles, and ply thickness is a specific numeric value. Because the design variables are not continuous, a gradient based optimization procedure, such as gradient descent method, is not suitable to cope with such problems. Moreover, gradient based optimization approach is very easy to get trapped in local minima, and many local optimum may exist in structural optimization problems. A stochastic algorithm, such as GA, is able to deal with optimization problems with discrete design variables. Besides, stochastic method could escape from local optimum, and obtain global optimum.

Although GA gains different advantages for solving discrete problems, many disadvantages exists within this method. First, the optimization process of GA parameters, such as the population size, parent population, mutation percentage, etc., is very tedious; Second, the GA needs to evaluate the objective function many times to acheive the optimization, and the computation cost is very high; the last problem within GA is the premature convergence. GA consists of five basic parts: the variable encoding method, selection scheme, crossover operator, mutation operator, and how the constraints are handled. A typical GA process is show in Figure 3.1.

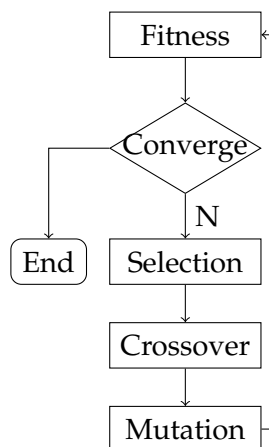


FIGURE 3.1: GA process

The first issue when implementing a GA is the representation of design variables, and an appropriate design representation is crucial to enhance the efficiency of GA. The canonical GA has always used binary strings to encode alternative solutions, however, some argued that the minimal cardinality, i.e., the binary representation, are not the best option. Real value string has been widely employed in

Selection scheme plays a critical role in balancing the dilemma of exploration and exploitation inherent in GA, and various selection methods, for example, roulette wheel, elitist, and tournament etc., have been proposed to overcome this issue. Both of roulette selection and tournament selection are well-studied and widely employed in the optimization design of laminated composite due to their simplicity to code and efficiency for both nonparallel and parallel architectures.

Crossover is another crucial operator introduced into the GA methodology framework, in which the alternative solution is generated from the mating pool. multiple types of crossover operator has been utilized in the optimization design of composite structures, such as: one-point, two-point, and uniform crossover. .

## 3.2 Constrained Problem Optimization

However, GA is originally proposed for unconstrained optimization problem, in order to deal with constrained design for composite laminate, some techniques are introduced into GA. The first method is using of data structure, special data structure has been developed to fulfils the corresponding constraint, for example, in order to fulfill the symmetry constraint of a laminate, the chromosome is consist of coding only half of the laminate and considering that each stack of the laminate is formed by two laminate with the same orientation but opposite signs [11, 10]. The second approach is reformulating the objective function. A penalty function is developed to convert a constrained problem into an unconstrained problem by adding penalty term to the objective funtion. Another method to solve constrained problem is introducing repair strategy by Todoroki and Haftka [22], which is aim to transform infeasible solutions to feasible solution by incorporating problem-specific knowledge.

## 3.3 A New Genetic Algorithm Model

### 3.3.1 Classification

The population is randomly generated, for every individual in this population the corresponding constraint numeric value can be obtained. As shown in Figure 3.2, The population can be divided into three groups according to constraint value, which are active group, potential group and proper group

### 3.3.2 Selection

The purpose of the selection operator is to chose mating pool to produce alternative solutions of better fitness. Traditional methods of selecting strategies only take the fitness of individuals into account, however, due to the existance of constraint, various selection schemes are implemented to selecet the mating set. Based on different selection schemes, the parents of next generation can be divided into three groups: proper groups, active groups, and potential groups according to different selecting methods.

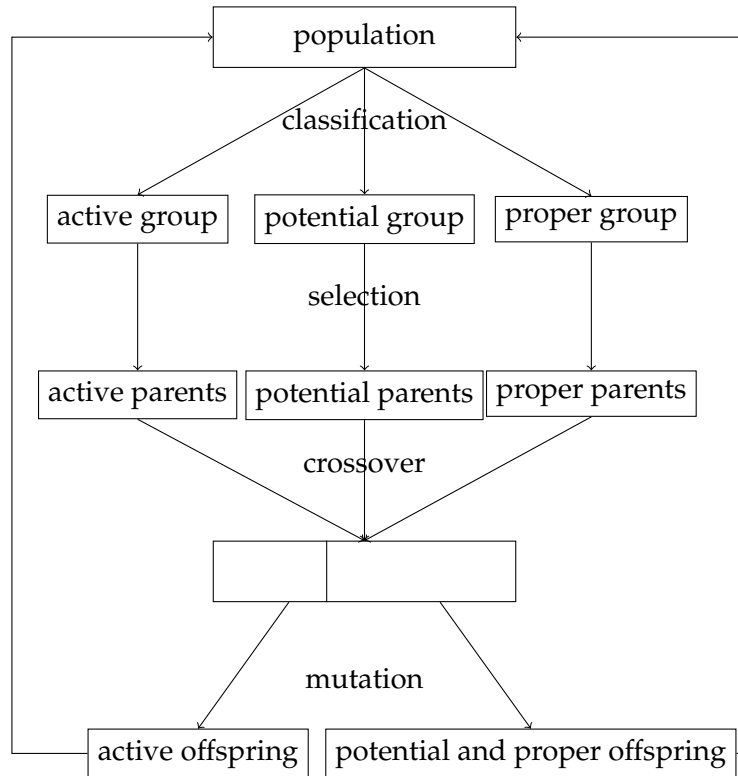


FIGURE 3.2: GA Model

Proper parents mean in which individual fulfills the constraints, which are chosen by the individual's fitness, individuals with better fitness are more likely to be chosen if they fit the constraint; active groups means that individual is supposed to be always exist in the parents during the GA, which are selected by fitness, ignoring the constraint; The individuals from active group may not correspond to feasible solutions, but their existence enriches the variety of the gene clips. Potential groups means that they are likely to turn into proper individual after a couple of generations, and potential individuals are chosen by constraint function, the more the individual fulfills the constraint, the more possibility it will be selected.

### 3.3.3 Crossover

The crossover operator happens among these three groups. the child of two proper groups are more likely to be a proper individual which can be used to obtain a alternative feasible solution. the child of an active individual and a potential individual can significantly change the gene of active individual's chromosome, which makes the individual evolve toward a new direction. The offspring of two active individuals are more likely to be an active individual, which can maintain the active group. The figure.4.1 (b) shows two children  $O_1$

### 3.3.4 Mutation

### **3.4 Summary**

In this chapter, first, we review the application of traditional GA in the design of composite material; then A new GA framework has been come up with, in the following chapters, this NGAM will be adopted to directed the lay-up design of composite material.

## Chapter 4

# Laminated Composite Material Optimization by New Genetic Algorithm Model

### 4.1 Case1: Maximum Strength Optimization

TABLE 4.1: Comparison of the carbon/epoxy, graphite/epoxy, and glass/epoxy properties

Property	Symbol	Unit	Carbon/Epoxy	Graphite/Epoxy	Glass/Epoxy
Longitudinal elastic modulus	$E_1$	GPa	116.6	181	38.6
Traverse elastic modulus	$E_2$	GPa	7.67	10.3	8.27
Major Poisson's ratio	$\nu_{12}$		0.27	0.28	0.26
Shear modulus	$G_{12}$	GPa	4.17	7.17	4.14
Ultimate longitudinal tensile strength	$(\sigma_1^T)_{ult}$	MP	2062	1500	1062
Ultimate longitudinal compressive strength	$(\sigma_1^C)_{ult}$	MP	1701	1500	610
Ultimate transverse tensile strength	$(\sigma_2^T)_{ult}$	MPa	70	40	31
Ultimate transverse compressive strength	$(\sigma_2^C)_{ult}$	MPa	240	246	118
Ultimate in-plane shear strength	$(\tau_{12})_{ult}$	MPa	105	68	72
Density	$\rho$	$g/cm^3$	1.605	1.590	1.903
Cost			8	2.5	1

#### 4.1.1 Methodology

#### 4.1.2 Result and Discussion

### 4.2 Case2: Minimum Thickness Optimization

In this case, the mentioned NGAM in chapter three is taken to minimize thickness(or weight) of mid-plane symmetric composite laminate subject to in-plane loading. Fiber orientation and ply thickness are chosen as design variables, the proposed Tsai-wu and the maximum stress criteria are employed to determine whether load bearing capacity is exceeded or not.

#### 4.2.1 Methodology

The optimization problem can be formulated as searching the optimal stacking sequence of composite laminate. There are two design variables in this problem, the angles in the laminate, and the number of layers that each fiber orientation has. The final objective function is given as

$$\begin{aligned}
F &= 2t_0 \sum_{k=1}^n n_k, \\
SF_{MS} &\geq 1, \\
SF_{TW} &\geq 1.
\end{aligned} \tag{4.1}$$

The first term represents the total thickness of the composite laminates,  $t_0$  is the ply thickness;  $n_k$  is the number of plies in the  $k$ th lamina, in which the fiber orientation is  $\theta_k$ . The constraints here are two safety factors should not less than 1, which means  $SF_{MS} \geq 1$ , and  $SF_{TW} \geq 1$ , respectively.

### 4.2.2 Encoding

Due to the simplicity and efficiency of float representation, this encoding method is implemented to represent a possible solution. As shown in Figure 4.1 (a), these two chromosomes represent a  $[+8_7 / -9_2]_s$  carbon T300/5308 laminated composite, and  $[+19_4 / -36_6]_s$ , respectively. Because the laminate adopted in this paper is symmetric to its mid-plane, so only half needs to be encoded.

### 4.2.3 Selection

The purpose of the selection operator is to choose mating pool to produce alternative solutions of better fitness. Traditional methods of selecting strategies only take the fitness of individuals into account, however, due to the existence of constraint, various selection schemes are implemented to select the mating set. Based on different selection schemes, the parents of next generation can be divided into three groups: proper groups, active groups, and potential groups according to different selecting methods.

Proper parents mean in which individual fulfills the constraints, which are chosen by the individual's fitness, individuals with better fitness are more likely to be chosen if they fit the constraint; active groups means that individual is supposed to be always exist in the parents during the GA, which are selected by fitness, ignoring the constraint; The individuals from active group may not correspond to feasible solutions, but their existence enriches the variety of the gene clips. Potential groups means that they are likely to turn into proper individual after a couple of generations, and potential individuals are chosen by constraint function, the more the individual fulfills the constraint, the more possibility it will be selected.

### 4.2.4 Crossover

The crossover operator happens among these three groups. the child of two proper groups are more likely to be a proper individual which can be used to obtain an alternative feasible solution. the child of an active individual and a potential individual can significantly change the gene of active individual's chromosome, which makes the individual evolve toward a new direction. The offspring of two active individuals are more likely to be an active individual, which can maintain the active group. The figure.4.1 (b) shows two children  $O_1$  and  $O_2$  from two parents  $P_1$  and  $P_2$ , each angle  $C_a$  and its length  $C_l$  of a child are obtained by the following formula



$P_1$ :	+7	+7	+7	+7	+7	+7	+7	-9	-9
$P_2$ :	+19	+19	+19	+19	-36	-36	-36	-36	-36

(a): Parents  $P_1$  and  $P_2$ 

$O_1$ :	+13	+13	+13	+13	+13	+13	-27	-27	-27
$O_2$ :	+22	+22	+22	+22	+22	+22	+22	+5	+5

(b): Offspring  $O_1$  and  $O_2$ 

$O_1$ :	+13	+13	+13	...	+13	+13	-27	...	-27

(c): Offspring  $O_1$  after lenght mutation

	+12	+12	+12	...	+12	+12	-26	...	-26
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(b): Offspring  $O_1$  after angle mutation

FIGURE 4.1: GA Operators

$$\begin{cases} C_a &= (P1_a + P2_a)/2 \\ C_l &= (P1_l + P2_l)/2 \end{cases} \quad (4.2)$$

#### 4.2.5 Mutation

A mutation direction is imposed on the mutation operator which to make sure the individual evolving toward the right direction. The mutation direction, denoted by  $md$ , is a  $n$  dimensional vector corresponding to the number of constraints, it is decided by the constraint thresholds  $CT_i$  and the current individual's constraint value, denoted as  $CV_i$ . The mutation vector can be obtained by the following formula

$$md = [CT_1, \dots, CT_{n-1}, CT_n] - [CV_0, \dots, CV_{n-1}, CV_n]$$

During the operator, the mutation procedure is consist of two phases: the length mutation of the chromosome, and the angle mutation of the chromosome. Because the chromosome's length is positive correlated with the individual's fitness, the coefficient of length mutation denoted by  $C_l$ , if  $\sum_{i=1}^N CT_i$  great than  $\sum_{i=1}^N CV_i$ , the mutation length is restricted to the range  $[0, (C_l \sum_{i=1}^N (CT_i - CV_i)) / N]$ , which means increase the chromosome's length; Assuming a  $[+13_6 / -27_4]_s$  T300/5308 carbon/epoxy composite laminate under the loading  $N_x = N_y = 10$  MPa m, it's property as shown in table ???. According to CLT and failure theory, the two safety factors  $SF_{MS}$  and  $SF_{TW}$  are 0.0539, and 0.0540, respectively. So the mutation vector and is  $[0.9461, 0.9460]$ , assuming the length mutation coefficient is 20, so the mutation range is from 0 to 18. A random number is generated from the range  $[0, 18]$ , supposing the outcome is 13, then a length generator is used to a list, the it's sum is 13, suppose the list is  $[5, 8]$ , the laminate after mutation is  $[13_{11} / -27_{12}]_s$ .

If the  $\sum_{i=1}^N CT_i$  less than  $\sum_{i=1}^N CV_i$ , the mutation length is restricted to the range  $[(\sum_{i=1}^N CT_i - CV_i) / N, 0]$ , which means the individual's fitness exceeds the threshold value, and decrease the chromosome's length. Assuming a  $[+33_{35} / -29_{26}]_s$  T300/5308 laminate is under loading  $N_x = 10$  MPa, and  $N_y = 5$  MPa, then, it's  $SF_{MS}$  constraint and  $SF_{TW}$  values are 1.0912, 1.0747, respectively. because the length mutation is 20, so the mutation range is from -2 to 0. This would decrease the chromosome's length.

$$LM = \begin{cases} [0, (C_l \sum_{i=1}^N (CT_i - CV_i)) / N], & \text{if } \sum_{i=1}^N CT_i > \sum_{i=1}^N CV_i \\ [(C_l \sum_{i=1}^N (CT_i - CV_i)) / N, 0], & \text{if } \sum_{i=1}^N CT_i < \sum_{i=1}^N CV_i \end{cases} \quad (4.3)$$

The relationship between the angles in the composite laminate and the chromosome's fitness is unclear, so the mutation direction of chromosome's angle is random. The coefficient angle mutation is  $C_a$ , the angle mutation range is  $[0, C_a \sum_{i=1}^N (|CT_i - CV_i|)]$  or  $[C_a \sum_{i=1}^N (-|CT_i - CV_i|), 0]$ . It is can be written as

$$P(AM) = \begin{cases} 0.5, & AM = [0, C_a \sum_{i=1}^N (|CT_i - CV_i|)] \\ 0.5, & AM = [C_a \sum_{i=1}^N (-|CT_i - CV_i|), 0] \end{cases} \quad (4.4)$$

TABLE 4.2: The optimum lay-ups using two distinct fiber angles under various biaxial loading cases

Loading $N_x/N_y/N_{xy}$ (MPa m)	Optimum lay-up sequences	Laminate thickness	Safety factor for Tsai-wu	Safety factor for maximum stress
10/5/0	$[33_{29}/-39_{25}/-39]_s$	109	1.0074	1.0246
20/5/0	$[33_{22}/-31_{24}]_s$	92	1.0055	1.2065
40/5/0	$[29_{18}/-21_{23}/-21]_s$	83	1.0034	1.7350
80/5/0	$[-20_{27}/21_{25}/25]_s$	105	1.0029	1.2063
120/5/0	$[-18_{34}/17_{36}]_s$	140	1.0000	1.0898

#### 4.2.6 Result and Discussion

In present study, the T300/5308 graphite/epoxy material is used in the lay-up sequence optimization, and it's properties as shown in table.???. Two constraints are imposed on the composite laminates which are the safety factor  $SF_{MS}$ , and safety factor  $SF_{TW}$ , and the threshold values for both of them is 1. The constraint values of an individual are  $CV_1$  and  $CV_2$ . So the mutation vector here is a two dimensional vector  $[1 - CV_1, 1 - CV_2]$ , and the coefficient of length mutation  $C_l$  and angle mutation  $C_a$  are 20 and 10, respectively.

To verify the reliability of proposed method, two conditions are concerned: the first is only two distinct fiber orientation angles in the composite material; the second involves three distinct ply angles within the optimization process. In each situation, first, we present the search process by plotting relevant indicators, such as the fitness, strength ratio, and angle. Then, the optimum lay-ups under various loading cases is discussed.

Figure 4.2 (a) shows how the optimal individual's fitness and strength ratio evolves during the GA process. The solid curve shows the fitness value, the dashed curve shows the Tsai-wu safety factor, and the dotted curve shows MS safety factor. If the smaller strength ratio fulfills the constraint, this laminate must satisfy all the constraints, for simplicity, only the smaller strength ratio is presented in the figure 4.3 (a). The method to choose optimal individual considering two following situations, if no individual in the current population meets constraint, the one with biggest fitness is selected as the optimal individual; if there are one or multiple individuals fulfill requirement, the one with smallest fitness is chosen which means the smallest one has the biggest priority. Figure 4.2 (b) and 4.3 (b) show how the every fiber orientation changes, and Figure 4.2 (c) and 4.3 display how the number of each angles varies.

At the beginning of this GA process, the fitness curves increase very quickly, because individual's two strength ratios are very small, so the difference between the individual's fitness and the imposed constraint threshold is a big positive number, so the range of mutation length is from 0 to  $C_l(CT_0 - CV_0 + CT_1 - CV_1)/2$ . The length of individual increases by  $n$ , which is a random number between 0 and  $C_l(CT_0 - CV_0 + CT_1 - CV_1)/2$ . As can be seen from Figure 4.2 (a), both of optimal individual's fitness and strength ratio increase very quickly. The range of angle mutation is from 0 to  $C_a(CT_0 - CV_0 + CT_1 - CV_1)/2$ , and the number of each angle also changes violently. The Figure 4.2 (a) and 4.3 (a) show this property at the initial stage. During this stage, increasing individual's length plays a major role in increasing individual's fitness.

After a couple of generations, the optimal individual's fitness gets bigger, and the difference between individual's fitness and constraint threshold gets smaller. The range of mutation length turns smaller. At this stage, simply increasing the individual's length doesn't make much difference in improving individual's fitness, and a better

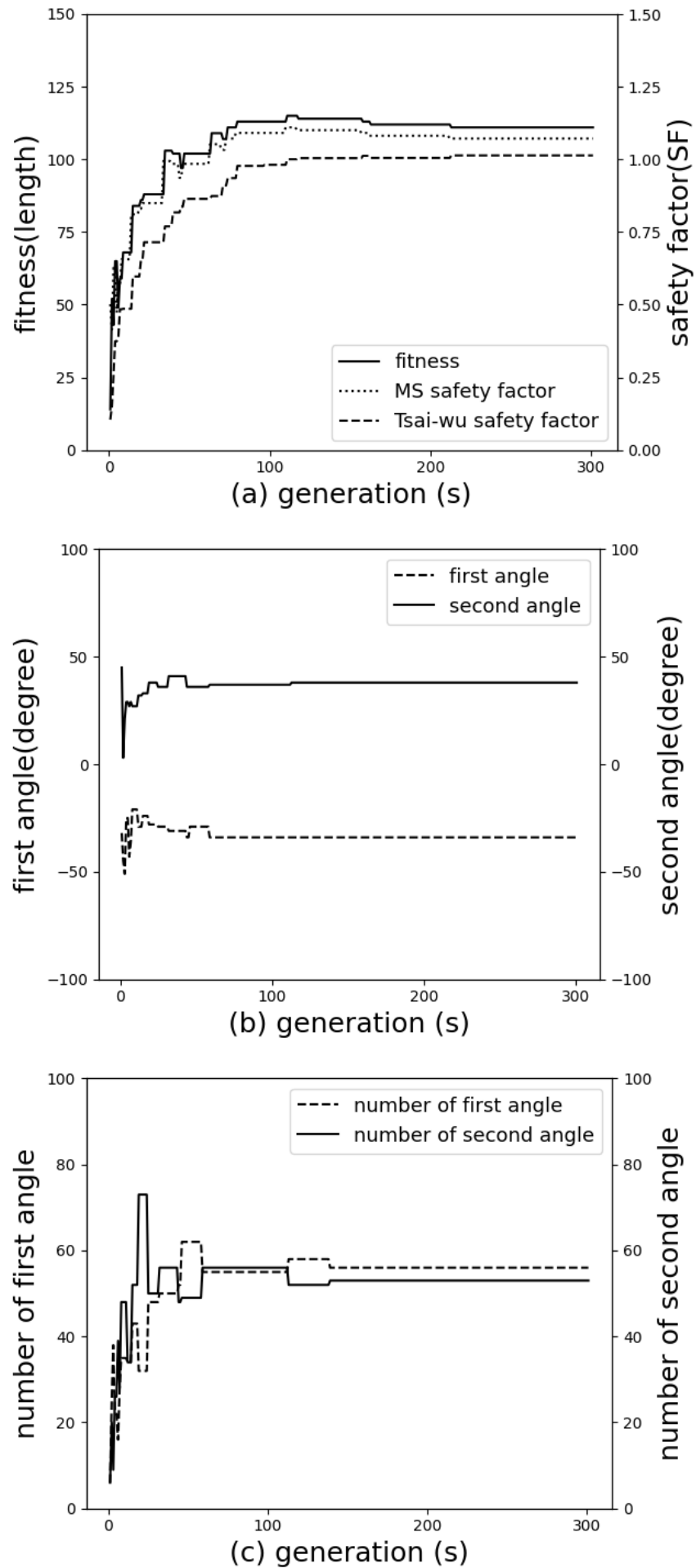


FIGURE 4.2: Two distinct angles

TABLE 4.3: The optimum lay-ups using three distinct fiber angles under various biaxial loading cases

Loading $N_x/N_y/N_{xy}$ (MPa m)	Optimum lay-up sequences	Laminate thickness	Safety factor for Tsai-wu	Safety factor for maximum stress
10/5/0	$[37_{27}/-38_{27}/-5]_s$	110	1.0023	1.0216
20/5/0	$[34_{24}/-32_{14}/-28_{11}]_s$	98	1.0237	1.2089
40/5/0	$[21_{28}/-32_{19}/23]_s$	100	1.0617	1.7076
80/5/0	$[-19_{24}/20_{27}/-17_{16}/-17]_s$	109	1.0056	1.2093
120/5/0	$[-19_{33}/12_{13}/16_{28}]_s$	148	1.0105	1.1014

TABLE 4.4: Comparison with the results of DSA

Loading $N_x/N_y/N_{xy}$ (MPa m)	Akbulut and Sonmez's[3] Study				Present Study			
	Optimum lay-up sequences	laminate thickness	TW	MS	Optimum lay-up sequences	laminate thickness	TW	MS
10/5/0	$[37_{27}/-37_{27}]_s$	108	1.0068	1.0277	$[33_{29}/-39_{25}/-39]_s$	109	1.0074	1.0246
20/5/0	$[31_{23}/-31_{23}]_s$	92	1.0208	1.1985	$[33_{22}/-31_{24}]_s$	92	1.0055	1.2065
40/5/0	$[26_{20}/-26_{20}]_s$	80	1.0190	1.5381	$[29_{18}/-21_{23}/-21]_s$	83	1.0034	1.7350
80/5/0	$[21_{25}/-19_{28}]_s$	106	1.0113	1.2213	$[-20_{27}/21_{25}/25]_s$	105	1.0029	1.2063
120/5/0	$[17_{35}/-17_{35}]_s$	140	1.0030	1.0950	$[-18_{34}/17_{36}]_s$	140	1.0000	1.0898

composite laminates lay-up can dramatically change the optimal individual's fitness. That's why the fitness curve oscillated violently in this stage. At the same time, the strength ratio curve kept growing smoothly. But the growing speed got more smaller.

When GA comes to its last phase, GA finds individuals that meet all the constraints. Now the optimal individual's fitness is greater than the safety factor. The range of mutation length is from  $C_l(CT_0 - CV_0 + CT_1 - CV_1)/2$  to 0. It means individuals need to decrease it's length and improve its internal structure to meet the constraint. That's why the fitness of optimal individual kept decreasing, however, the strength ratio curve still is greater than safety factor.

Table.4.4 shows the comparison with the result obtained by direct search simulated annealing(DSA) algorithm which is proposed by Akbulut and Sonmez[3]. Both of variant GA and DSA are able to find feasible solution, but when loading is  $N_x = 80$ ,  $N_y = 5$  MPa m, variant GA gets a better solution than DSA. In the case that loadings are  $N_x = 20$ ,  $N_y = 5$  MPa m, and  $N_x = 120$ ,  $N_y = 5$  MPa m, the proposed GA offers an alternative. Compared with DSA method, the last advantage of variant GA is the number of layers doesn't have to be an even number.

### 4.3 Summary

In this paper, we reviewed the use of variant GA for the optimal design of composite laminated material under in-plane loading based on Tsai-wu and maximum stress failure criteria. GA is proposed to search the optimal lay-up for laminated composite under different loading cases. Two situations are considered under the same loading, a set of two distinct angles, and three distinct angles.

By setting the constant values of length mutation coefficient and angle mutation coefficient at the beginning, the convergence speed of search process can be controlled in an explicit way; During the optimization process, GA can adjust its length mutation range and angle mutation range based on the difference between individual's constraint values and constraint thresholds.

Finally, comparison of previous research and current result are presented. In some cases, the proposed GA in this paper is better off than DSA method. However, there is still many works to study within this GA, such as the fine-tuning of parameters taken in this GA.

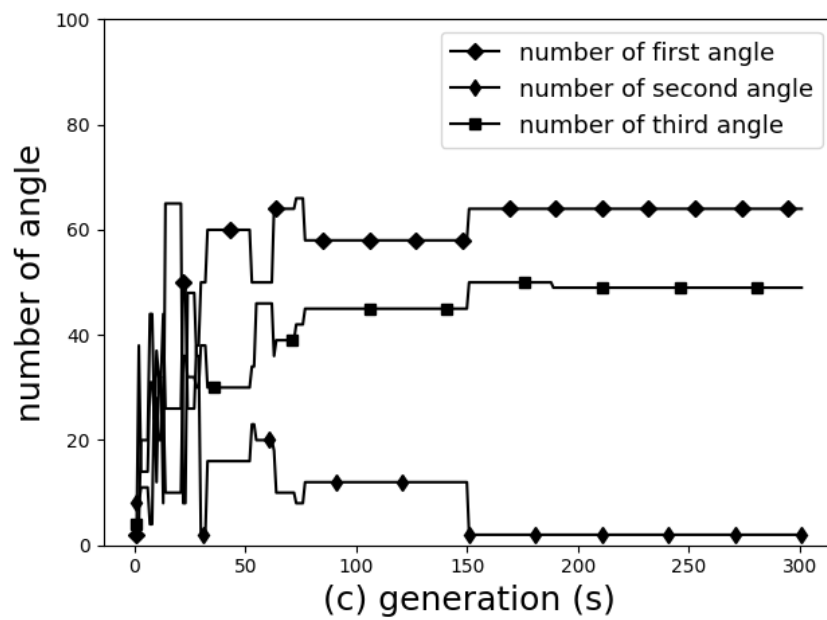
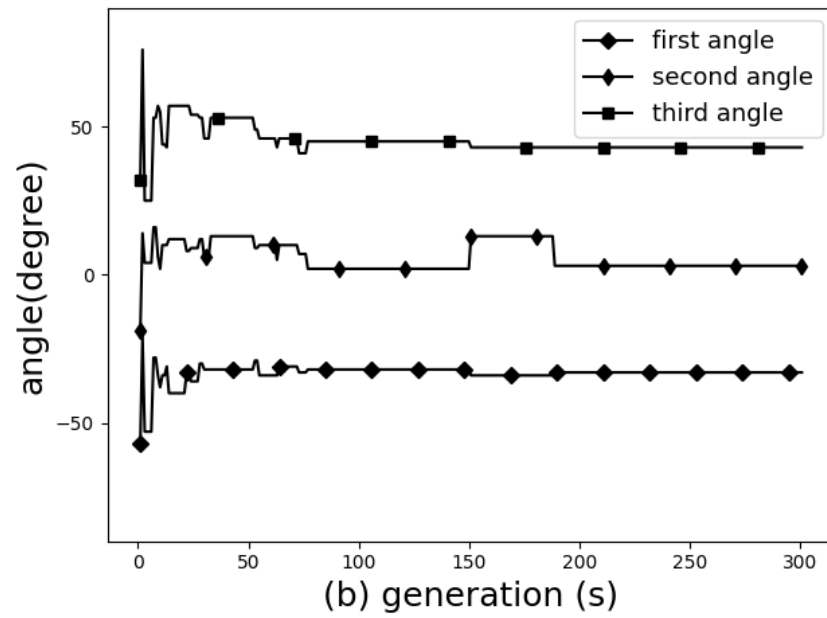
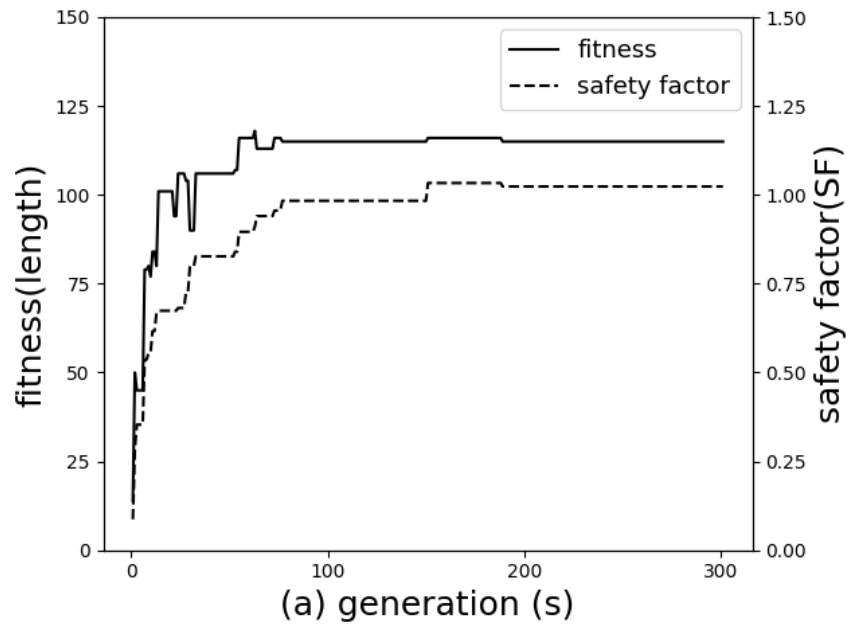


FIGURE 4.3: three distinct angles





## Chapter 5

# Approximation of CLT Based on Artificial Neural Network

## 5.1 Neural Network Design

### 5.1.1 Architecture

### 5.1.2 Discussion

## 5.2 Approximation Framework

### 5.2.1 Data Preparation

### 5.2.2 Training

### 5.2.3 Evaluation

### 5.2.4 Result and Discussion

## 5.3 Summary

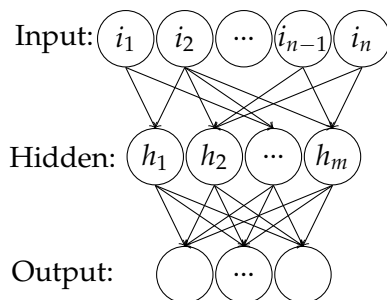


FIGURE 5.1: Neural Network Model

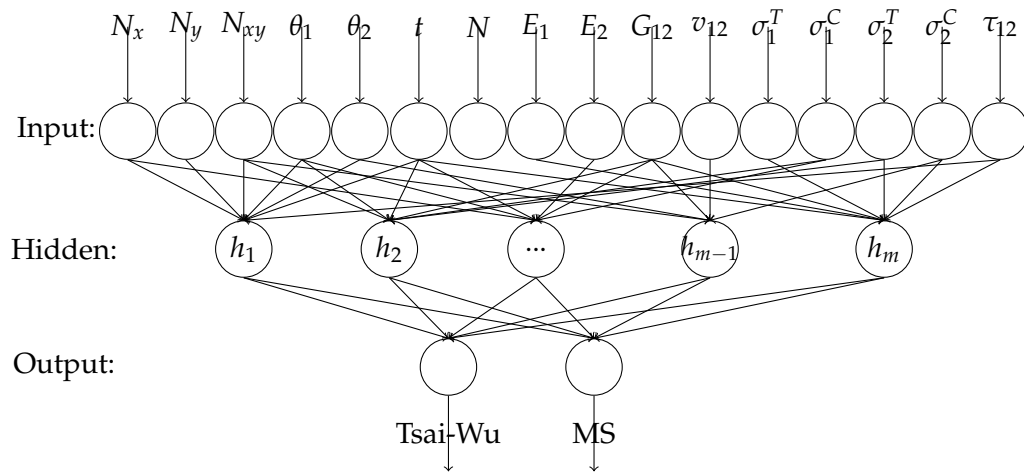


FIGURE 5.2: Neural Network Model

## **Chapter 6**

# **Conclusion**



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