Optimum design of laminated composites for minimum thickness by a self-adaptative genetic algorithm

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Content

1. Classic Lamination Theory

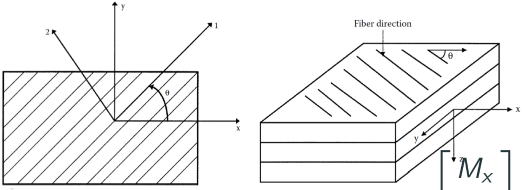


Figure 1: Composite Material

$$
\begin{bmatrix} N_x \\ N_y \\ N_{xy} \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} & A_{16} \\ A_{12} & A_{22} & A_{26} \\ A_{16} & A_{26} & A_{66} \end{bmatrix} \begin{bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \end{bmatrix} + \begin{bmatrix} B_{11} & B_{12} & B_{16} \\ B_{11} & B_{12} & B_{16} \\ B_{16} & B_{26} & B_{66} \end{bmatrix} \begin{bmatrix} k_x \\ k_y \\ k_{xy} \end{bmatrix}
$$

$$
\begin{bmatrix} M_x \\ M_y \\ M_{xy} \end{bmatrix} = \begin{bmatrix} B_{11} & B_{12} & B_{16} \\ B_{12} & B_{22} & B_{26} \\ B_{16} & B_{26} & B_{66} \end{bmatrix} \begin{bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \end{bmatrix} + \begin{bmatrix} D_{11} & D_{12} & D_{16} \\ D_{11} & D_{12} & D_{16} \\ D_{16} & D_{26} & D_{66} \end{bmatrix} \begin{bmatrix} k_x \\ k_y \\ k_{xy} \end{bmatrix} \tag{1}
$$

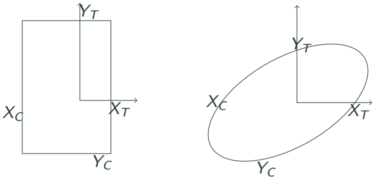


Figure 2: Schematic failure surfaces for maximum stress and quadratic failure criteria

- Maximum stress failure

$$SF_{MS}^k = \text{min of} \begin{cases} SF_X^k = \begin{cases} \frac{X_t}{\sigma_{11}}, & \text{if } \sigma_{11} > 0 \\ \frac{X_c}{\sigma_{11}}, & \text{if } \sigma_{11} < 0 \end{cases} \\ SF_Y^k = \begin{cases} \frac{Y_t}{\sigma_{22}}, & \text{if } \sigma_{22} > 0 \\ \frac{Y_c}{\sigma_{22}}, & \text{if } \sigma_{22} < 0 \end{cases} \\ SF_S^k = \begin{cases} \frac{S}{|\tau_{12}|} \end{cases} \end{cases}.$$

- Tsai-wu failure theory

$$H_1\sigma_1 + H_2\sigma_2 + H_6\tau_{12} + H_{11}\sigma_1^2 + H_{22}\sigma_2^2 + H_{66}\tau_{12}^2 + 2H_{12}\sigma_1\sigma_2 < 1$$

3. Self-adaptative GA

- Modifying selection strategy: in order to handle the constraint search
- Self-adaptative mutation direction of fiber orientation and laminate thickness: random change the length, and the angle in the laminate.
- The self-adaptative parameters don't refer to parent's proportion, mutation probability.

3. Self-adaptative GA: selection operator

- acitve group: individual is used to increase the diversity of the population
- potential group: individual doesn't fulfill constraint
- proper group: individual meet constraint

3. Self-adaptative GA: mutation operator

$$\text{md} = [CT_1, \cdots, CT_{n-1}, CT_n] - [ICV_0, \cdots, ICV_{n-1}, ICV_n]$$

- md means mutation direction.
- CT_i denotes the i-th constraint, such as weight, safety factor.
- ICV_i denotes individual's i-th constraint value, such as, weight, safety factor of current individual.

4. Self-adaptative GA: mutation operator

- length mutation =

$$\begin{cases} LMC * [0, \sum_{i=1}^{N} md_i] & \text{if } \sum_{i=1}^{N} md_i > 0 \\ LMC * [\sum_{i=1}^{N} md_i, 0] & \text{if } \sum_{i=1}^{N} md_i < 0 \end{cases}$$

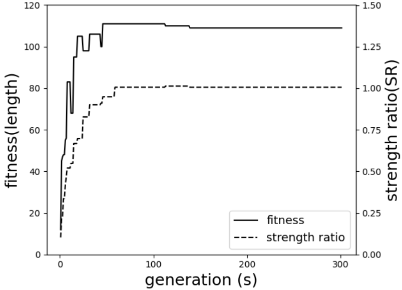
  LMC stands for length mutation coefficient, it's a positive integer.

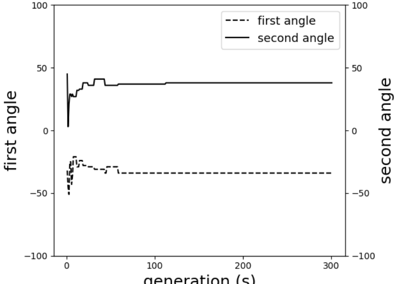
- angle mutation =

$$\begin{cases} AMC * [0, \sum_{i=1}^{N} md_i] & \text{if } \sum_{i=1}^{N} md_i > 0 \\ AMC * [\sum_{i=1}^{N} md_i, 0] & \text{if } \sum_{i=1}^{N} md_i < 0 \end{cases}$$

  AMC stands for angle mutation coefficient, it's sign is unclear.

5. Result: Loading $N_x = 10, N_y = 5$ MPa m





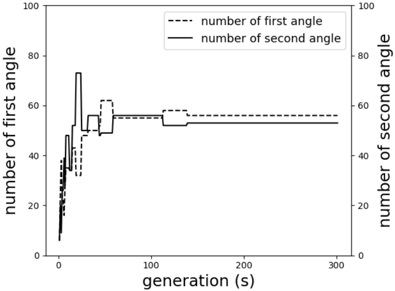


Figure 3: Two distinct angles in the laminate

6. Comparison with related research

Table 1: Comparison with the results of DSA

Loading	Akbulut and Sonmez's Study				Present Study			
$N_x/N_y/N_{xy}$ (MPa m)	Optimum lay-up sequences	laminate thickness	TW	MS	Optimum lay-up sequences	laminate thickness	TW	MS
10/5/0	$[37_{27}/\text{-}37_{27}]_s$	108	1.0068	1.0277	$[33_{29}/\text{-}39_{25}/\text{-}\overline{39}]_s$	109	1.0074	1.0246
20/5/0	$[31_{23}/\text{-}31_{23}]_s$	92	1.0208	1.1985	$[33_{22}/\text{-}31_{24}]_s$	92	1.0055	1.2065
40/5/0	$[26_{20}/\text{-}26_{20}]_s$	80	1.0190	1.5381	$[29_{18}/\text{-}21_{23}/\text{-}\overline{21}]_s$	83	1.0034	1.7350
80/5/0	$[21_{25}/\text{-}19_{28}]_s$	106	1.0113	1.2213	$[\text{-}20_{27}/21_{25}/\overline{25}]_s$	105	1.0029	1.2063
120/5/0	$[17_{35}/\text{-}17_{35}]_s$	140	1.0030	1.0950	$[\text{-}18_{34}/17_{36}]_s$	140	1.0000	1.0898