Candidate Number: 2031B
Paper 3
Question 7

a.

i. Betweenness centrality of a node V is the proportion of shortest paths between each pair of nodes which go through V.

ii. B and F are “gatekeeper” nodes i.e. to get from the top section of the graph to the bottom section, you must go through B and F. E and H have betweenness centrality of 0 because there are no nodes for which to reach them from another node the path must pass through E or H.

b.

i. The diameter of a network is the longest length of the shortest path between any pair of nodes in the graph (the maximum distance between any two nodes). The diameter of the undirected graph is 6 because this is the length of the shortest path from both C to H and E to H. There is no pair of nodes for which the shortest path between them is 6 nodes long.

ii. They do change. For example in the directed graph, there are no paths from the bottom row of the graph to the top. However in the undirected graph, these paths do exist and some of the options for the shortest paths from F, G, and H to C go through A, increasing A’s betweenness centrality. Also the shortest path from D or E to A no longer need to pass through C in the undirected graph, decreasing C’s betweenness centrality.

iii.

i. This is false. Consider S is:
A -> B
A -> C
B -> D
D -> B
D -> E
E -> D
E -> C
C -> C
And T is as defined. Let X be node A. Here X_T has greater betweenness centrality than X_S

ii. This is false. Consider S is:

A -> B

B -> C

C -> A

C -> D

D -> B

And T is as defined. Let X be node B. Here X_S has a greater betweenness centrality than X_T

iii. This is false for the same reason as in ii.

c.

i. High in-degree means that lots of people ask this person for fishing advice. High out-degree means that this person asks many other people for fishing advice.

ii. No. For example, there is no path from F to A.

iii. A graph with greater in- and out-degrees is more likely to be strongly connected. In the extreme case in which the in-degree and out-degree of every node is equal to the number of nodes in the graph minus 1, then the graph is fully connected which is a special case of being strongly connected. In the other extreme in which the in-degree and out-degree of every node is 0, then the graph is fully unconnected.