BGN: 2031B
Paper 1
Question 5.

a. i. $H \sim Bin(300, 0.8)$

$E(H) = 300 \cdot 0.8 = 240$

$V(H) = 300 \cdot 0.8 \cdot 0.2 = 48$

ii. $H$ is approximately $\sim N(240, 48)$

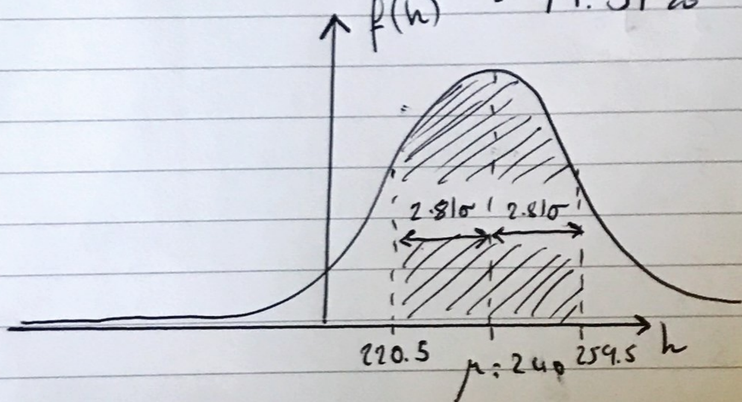
$E(H) = 240$

$V(H) = 48$

iii. $\cancel{P(219.5 \leq H \leq 260)}$

$P(220.5 \leq H \leq 259.5) \approx 0.9951$

$= 99.51\%$




iv. $X \sim N(0.8, 5.\overline{33})$

$E(X) = 0.8$

$V(X) = 5.\overline{33} = \frac{1}{1875}$

b. i. $\displaystyle\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x,y)\,dy\,dx \stackrel{!}{=} 1$

$$\therefore \int_0^1\int_0^x f(x,y)\,dy\,dx$$

$$= \int_0^1\int_0^x cx\,dy\,dx$$

$$= \int_0^1 \left[cxy\right]_0^x dx$$

$$= \int_0^1 cx^2\,dx$$

$$= \left[\frac{cx^3}{3}\right]_0^1$$

$$= \frac{c}{3}$$

$$\stackrel{!}{=} 1$$

$\therefore c = 3$

ii. $\displaystyle f_X(x) = \int_{-\infty}^{\infty} f(x,y)\,dy$

$$= \int_0^x cx\,dy \qquad \text{for } 0<x<1$$

$$= \left[cxy\right]_0^x$$

$$= cx^2 = 3x^2$$

$$f_Y(y) = \int_{-\infty}^{\infty} f(x,y)\,dx$$

$$= \int_y^1 cx\,dx \qquad \text{for } 0<y<1$$

$$= \left[\frac{cx^2}{2}\right]_y^1$$

$$= \frac{c}{2} - \frac{cy^2}{2}$$

$$= \frac{c}{2}(1-y^2) = \tfrac{3}{2}(1-y^2)$$

$$\therefore f_X(x) = \begin{cases} 3x^2 & \text{if } 0<x<1 \\ 0 & \text{otherwise} \end{cases}$$

$$f_Y(y) = \begin{cases} \tfrac{3}{2}(1-y^2) & \text{if } 0<y<1 \\ 0 & \text{otherwise} \end{cases}$$

iii. $f_X(x) f_Y(y) = 3x^2\left(\tfrac{3}{2}(1-y^2)\right)$ for $0<y<x<1$

$$= \frac{9}{2}x^2(1-y^2)$$

$$\neq 3x$$

$$= f(x,y)$$

$\therefore$ X and Y are not independent