BGN: 2191A
P6
Q3

a. there exists a computable function f such that for all $x \in \text{Domain}(A)$, $x \in A \iff f(x) \in B$,

and the amount of work space required to compute f is logarithmic in the length of the input x

b. Assume $A \leq_L B$

$\therefore \exists f : \text{Dom}(A) \to \text{Dom}(B),\ \forall x \in \text{Dom}(A).\ x \in A \iff f(a) \in B$ (*)
f uses logarithmic working space

Assume $B \leq_L C$

$\therefore \exists g : \text{Dom}(B) \to \text{Dom}(C).\ \forall y \in \text{Dom}(B).\ y \in B \iff g(y) \in C$ (**)
g uses logarithmic working space

$\therefore$ consider gf.

$gf : \text{Dom}(A) \to \text{Dom}(C)$

gf uses logarithmic working space as logarithmic working space functions are closed under composition.
Let x arbitrary in $\text{Dom}(A)$
RTP: $x \in A \iff gf(x) \in C$

"⇒" first:

Assume $x \in A$
Let $y = f(x)$
$\therefore y \in B$ by (*)
$\therefore g(y) \in C$ by (**)
$\therefore gf(x) \in C$

"⇐" next:

Assume $gf(x) \in C$
Let $y = f(x)$
$\therefore g(y) \in C$
$\therefore y \in B$ by (**)
$\therefore f(x) \in B$
$\therefore x \in A$ by (*)

c. The Circuit-Value Problem (CVP) is P-complete

SAT is NP-complete under log-space reductions.

Reachability is NL-complete

VAL is co-NP-complete under log-space reductions

$A \leq_L B$?

Q. B \ A	CVP	SAT	Reachability	VAL
CVP	Yes	Unknown. Would imply $P = NP$	Yes	Unknown Would imply $P = co\text{-}NP$
SAT	Yes	Yes	Yes	Unknown Would imply $NP = co\text{-}NP$
Reachability	Unknown Would imply $P = NL$	Unknown Would imply $NP = NL = P$	Yes	Unknown Would imply $co\text{-}NP = NL = P$
VAL	Yes	Unknown would imply $NP = co\text{-}NP$	Yes	Yes