BGN 2031B
Paper 2
Question 11W

a. $|r_1(1) - r_2(-1)|$

$= |a_1 + n_1 - a_2 + n_2|$

$= \left| \begin{pmatrix} 3 \\ 2 \\ 0 \end{pmatrix} \right|$

$= \sqrt{9+4} = \sqrt{13}$

b. $n_1 \times n_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{vmatrix}$

$= \hat{i}(1) - \hat{j}(1) + \hat{k}(0)$

$= \begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix}$

$\therefore a_1 + \lambda n_1 - a_2 - t n_2 = \frac{s}{\sqrt{2}} \begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix}$

$\therefore \begin{pmatrix} 1+\lambda-t \\ \lambda - t \\ -1-t \end{pmatrix} = \frac{s}{\sqrt{2}} \begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix}$

$\therefore t = -1 \quad \therefore 2 + \lambda = \frac{s}{\sqrt{2}}, \quad 1 + \lambda = \frac{-s}{\sqrt{2}}$

$\therefore 1 = \frac{2s}{\sqrt{2}} \quad \therefore s = \frac{\sqrt{2}}{2}$

c ii $\vec{d_0} = a_2 + t n_2$

$\therefore \cancel{\cdots} = \begin{pmatrix} 0 + t \\ 0 + t \\ 1 + t \end{pmatrix} = \begin{pmatrix} t \\ t \\ 1+t \end{pmatrix} \cancel{\cdots} = \begin{pmatrix} x \\ y \\ z \end{pmatrix}$

$\therefore x = y$

$z = x - 1 = y - 1$

$\therefore \vec{d_0} = \begin{pmatrix} x \\ x \\ x-1 \end{pmatrix}$

$\vec{d_0} \cdot n_2 = \cancel{\cdot} \; 3x - 1 = 0$

$\therefore x = \frac{1}{3}$

$\therefore \vec{d_0} = \begin{pmatrix} 1/3 \\ 1/3 \\ -2/3 \end{pmatrix}$

$\therefore d_0 = |\vec{d_0}| = \sqrt{\frac{1}{9} + \frac{1}{9} + \frac{4}{9}} = \sqrt{\frac{6}{9}} = \sqrt{2/3}$

i $\vec{d_0} = a_2 + t n_2$

$\vec{d_0} \cdot n_2 = (a_2 + t n_2) \cdot n_2 = 0$

$\therefore a_2 \cdot n_2 + t |n_2|^2 = 0$

$\therefore t = \frac{a_2 \cdot n_2}{|n_2|^2}$

$\therefore \vec{d_0} = a_2 + \frac{a_2 \cdot n_2}{|n_2|^2} n_2$

$\therefore d_0 = |\vec{d_0}| = \left| a_2 + \frac{a_2 \cdot n_2}{|n_2|^2} n_2 \right|$

di. Let $\vec{d}(t)$ be the shortest line from $r_1$ to $r_2(t)$

$$\vec{d}(t) = k\big((a_1 + \lambda n_1) \times (a_2 + t n_2)\big)$$

$$= k\left(\begin{pmatrix} 1+\lambda \\ \lambda \\ 0 \end{pmatrix} \times \begin{pmatrix} t \\ t \\ 1+t \end{pmatrix}\right)$$

$$= k' \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1+\lambda & \lambda & 0 \\ t & t & 1+t \end{vmatrix}$$

$$= k\Big(\hat{i}(\lambda + \lambda t) - \hat{j}\,(1+t)(1+\lambda) + \hat{k}\,(t)\Big)$$

$$= k \begin{pmatrix} \lambda + \lambda t \\ (1+t)(1+\lambda) \\ t \end{pmatrix}$$

$$\begin{pmatrix} 1+\lambda \\ \lambda \\ 0 \end{pmatrix} + k \begin{pmatrix} \lambda + \lambda t \\ (1+t)(1+\lambda) \\ t \end{pmatrix} = \begin{pmatrix} t \\ t \\ 1+t \end{pmatrix}$$

$\cancel{kt = 1+t}$

$\therefore\ kt = 1 + t$

$\therefore\ k = \frac{1}{t} + 1$

$\therefore\ 1+\lambda + \left(\frac{1}{t} + 1\right)(\lambda + \lambda t) = t$

$\therefore\ 1 + \lambda + \frac{\lambda}{t} + \lambda + \lambda + \lambda t = t$

$$1 + 3\lambda + \lambda t + \frac{\lambda}{t} = t$$

$$\therefore \lambda\left(3 + t + \frac{1}{t}\right) = t - 1$$

$$\therefore \lambda = \frac{t-1}{3+t+\frac{1}{t}} = \frac{t^2-t}{3t+t^2+1}$$

$$\therefore \alpha(t) = \left(\frac{1}{t} + 1\right) \div \left| \begin{pmatrix} \frac{(t^2-t)(1+t)}{3t+t^2+1} \\ \frac{(1+t)\left(1+\frac{t^2-t}{3t+t^2+1}\right)}{t} \end{pmatrix} \right|$$