BGN 2031B

Paper 2

Question 15Y

a. $\cancel{\frac{dy}{dx} + \frac{y}{x}}$

$$x^2 \frac{dy}{dx} + xy = x^2 + y^2$$

$$\therefore \cancel{x\left(\frac{dy}{dx} - 1\right) + xy = y^2}$$

$$\therefore x\frac{dy}{dx} + y = x + \frac{y^2}{x}$$

$$\therefore \frac{d}{dx}(xy) = x + \frac{y^2}{x}$$

b. i $\frac{d^2v}{dt^2} = \frac{d}{dt}\frac{dv}{dt} = \frac{du}{dt} = 4v+2$

$\therefore \frac{d^2v}{dt^2} = 4v+2$

ii $\therefore \frac{dv}{dt} = \int 4v+2 \, dt = 2v^2+2v+C$

$v = \int 2v^2+2v+\cancel{C}C \, dt$

$= \frac{2}{3}v^3+v^2+\cancel{C}Cv+D$

iii. $\cancel{u = \frac{dv}{dt} = 2v^2+2v+C}$

$\frac{d^2u}{dt^2} = \frac{d}{dt}\frac{du}{dt} = \frac{d}{dt}(4v+2)$

$= 4\frac{dv}{dt}$

$= 4u$

$\therefore \frac{du}{dt} = \int 4u \, dt = 2u^2 + \cancel{A}A$

$u = \int 2u^2+A \, dt = \frac{2}{3}u^3+Au+B$

c. Homogeneous version:

$\frac{d^2y}{dx^2} + 2\frac{dy}{dx} = 0$

Assume $y = e^{\lambda x}$

$\therefore \lambda^2 + 2\lambda = 0$

$\therefore \lambda(\lambda+2) = 0$

$\therefore \lambda_1 = 0, \quad \lambda_2 = -2$

$$y = \cancel{\quad} + Ae^{-2x} + B$$

trial
Specific integral: $y = Cx + D$

$$\frac{dy}{dx} = C$$

$$\frac{d^2y}{dx^2} = 0$$

$$\therefore \frac{d^2y}{dx^2} + 2\frac{dy}{dx} = 2C \overset{!}{=} 3x \quad \text{which doesn't work}$$

Try: $y = Cx^2 + Dx$

$$\frac{dy}{dx} = 2Cx + D$$

$$\frac{d^2y}{dx^2} = 2C$$

$$\frac{d^2y}{dx^2} + 2\frac{dy}{dx} = 2C + 4Cx + 2D \overset{!}{=} 3x$$

$$\therefore 4C = 3$$

$$\therefore C = 3/4$$

$$2C + 2D = 3/2 + 2D = 0$$

$$\therefore D = -C = -3/4$$

$\therefore$ General solution.

$$y = Ae^{-2x} + B + 3/4\,x^2 - 3/4\,x$$

ii. $y(0) = A + B \stackrel{!}{=} 1$

$\cancel{y(x)=}$ $y'(x) = -2Ae^{-2x} + \frac{3}{2}x - \frac{3}{4}$

$\therefore y'(0) = -2A - \frac{3}{4} = 0$

$\therefore 2A = -\frac{3}{4}$

$\therefore A = -\frac{3}{8}$

$B = 1 + \frac{3}{8} = \frac{11}{8}$

$\therefore y = -\frac{3}{8}e^{-2x} + \frac{11}{8} + \frac{3}{4}x^2 - \frac{3}{4}x$