Candidate number: 2031B
Paper 1
Question 9

a.

i. The complexity of pushright(x) is O(n)
The complexity of popleft() is O(1)
The complexity of element_at(i) is O(i) which in general is O(n)

ii. This does not satisfy the fundamental inequality of amortized cost. The aggregate true cost of n calls to element_at(n) is kn^2 which is O(n^2) for some constant k. However the proposed aggregate amortized cost for the same actions would be O(n). Therefore there always exists some n for which the true aggregate cost is greater than the proposed aggregate amortized cost, and so the proposal must be incorrect.

b.

i.

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```
class RandomAccessQueue:
    array = empty array of length 8
    head_pointer = 0
    tail_pointer = 0

    def pushright(x):
        array[tail_pointer] = x
        tail_pointer++
        if (tail_pointer == array.length):
            new_array = empty array of length (array.length * 2)
            for (int i=0; i<tail_pointer-head_pointer; i++) {
                new_array[i] = array[head_pointer+i]
            }
            array = new_array
            tail_pointer -= head_pointer
            head_pointer = 0

    def popleft():
        if (head_pointer >= tail_pointer):
            throw Exception("The queue is empty")
            # or return null
        else:
            x = array[head_pointer]
            head_pointer++
            return x

    def element_at(i):
        index = head_pointer+i
        if (index >= tail_pointer):
            throw Exception("Index out of bounds")
            # or return null
        else:
            return array[index]
```

ii. The true cost of element_at(i) is O(1). The true cost of popleft() is O(1). For pushright(x), the cost of copying the array into a larger one is kn where n is the initial size of the array and k is a constant. After n calls to pushright, the total cost is n (for the inserting) + k(1+2+4+8+….+2^(floor(log_2(n-1))) which is strictly less than k(2n-3)+n which is O(n). Therefore the amortized cost of a single call to pushright(x) is O(1). Therefore all of the function calls have amortized cost O(1).