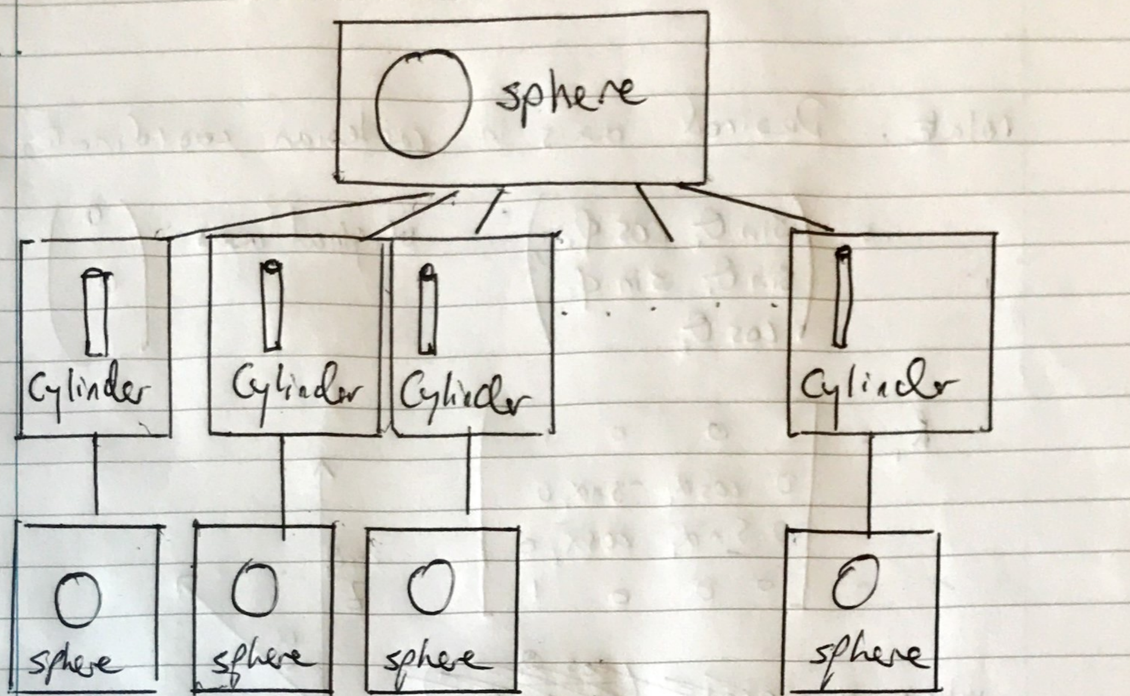
Question 4.
Paper 3
Candidate no. 2031B

a.



b. Transformation of Main large sphere = $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

Transformation of cylinder i with base-centre ϕ_i, θ_i:

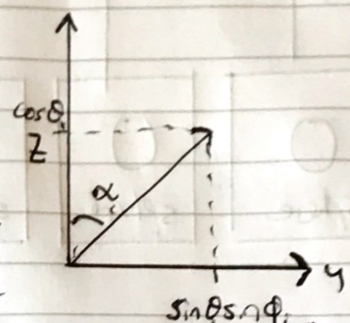
Assuming the base of the primitive is centred on the origin.

scale : $T_1 = \begin{bmatrix} 0.025 & 0 & 0 & 0 \\ 0 & 0.025 & 0 & 0 \\ 0 & 0 & 0.05 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

rotate : Desired axis in cartesian coordinates

is $\begin{pmatrix} \sin\theta_i \cos\phi_i \\ \sin\theta_i \sin\phi_i \\ \cos\theta_i \end{pmatrix}$ original axis: $\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$

$$R_1 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\alpha_i & -\sin\alpha_i & 0 \\ 0 & \sin\alpha_i & \cos\alpha_i & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$



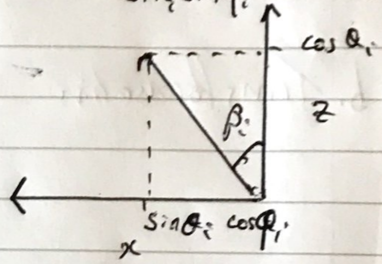
where $\alpha = \cancel{\arctan\left(\frac{\cos\theta_i}{\sin\theta_i \sin\phi_i}\right)}$

$= \cancel{\arctan(\tan\theta \operatorname{cosec}\phi_i)}$

$= \cancel{\arctan(\cot\theta_i \sin\phi_i)}$

$= \arctan(\tan\theta_i \sin\phi_i)$

$$R_2 = \begin{bmatrix} \cos\beta_i & 0 & -\sin\beta_i & 0 \\ 0 & 1 & 0 & 0 \\ \sin\beta_i & 0 & \cos\beta_i & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$



where $\beta_i = \arctan(\tan\theta_i \cos\phi_i)$

$R = T_2 = (R_2 R_1)^{-1}$

translate : $T_3 = \begin{bmatrix} 1 & 0 & 0 & \sin\theta_i \cos\phi_i \\ 0 & 1 & 0 & \sin\theta_i \sin\phi_i \\ 0 & 0 & 1 & \cos\theta_i \\ 0 & 0 & 0 & 1 \end{bmatrix}$

$\therefore$ total transformation of cylinder$_i$ =

$$\begin{bmatrix} 1 & 0 & 0 & \sin\theta_i \cos\phi_i \\ 0 & 1 & 0 & \sin\theta_i \sin\phi_i \\ 0 & 0 & 1 & \cos\theta_i \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \left(\begin{bmatrix} \cos\beta_i & 0 & -\sin\beta_i & 0 \\ 0 & 1 & 0 & 0 \\ \sin\beta_i & 0 & \cos\beta_i & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\alpha_i & -\sin\alpha_i & 0 \\ 0 & \sin\alpha_i & \cos\alpha_i & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \right)^{-1} \cdot \begin{bmatrix} 0.25 & 0 & 0 & 0 \\ 0 & 0.25 & 0 & 0 \\ 0 & 0 & 0.1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

where $\alpha_i = \arctan(\tan\theta_i \sin\phi_i)$

$\beta_i = \arctan(\tan\theta_i \cos\phi_i)$

Spike sphere :

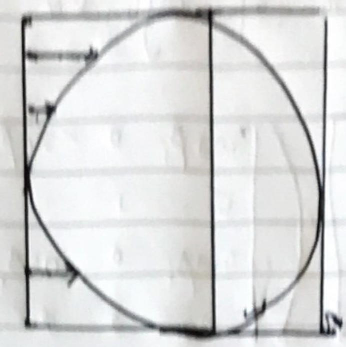
Scale : $\begin{bmatrix} 2 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

Translation : $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

i. Transformation:

$$\begin{bmatrix} 2 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

c.



By generating random points on the cylinder surrounding the sphere, and then mapping those points onto the sphere (with uniform z and θ)

Let rand() generate a uniform random float between 0 and 1

```
for (int i = 1; i ≤ N; i++) {
    theta[i] = 2π × rand()
    x = cos(theta[i])
    y = sin(theta[i])
    z = rand()
    r = √(1 - z²)
    x_s = xr
    y_s = yr
    phi[i] = arctan(r ÷ z)
}
```