BGN 2031 B
Paper 1
Question 11W

a. i. $(1/2 + 2i)^2 = 1/4 - 4 + 2i$

$= -15/4 + 2i$

ii $(1/2 + 2i)^4 = (-15/4 + 2i)^2$

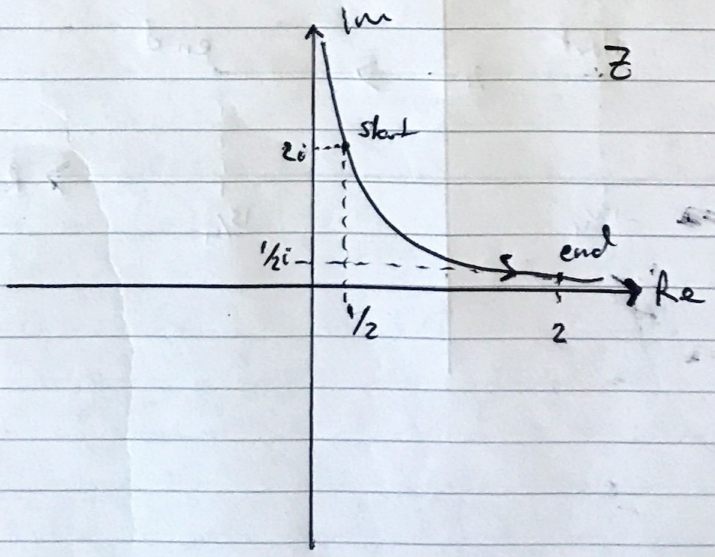
$= 15^2/16 - 4 - 15i$

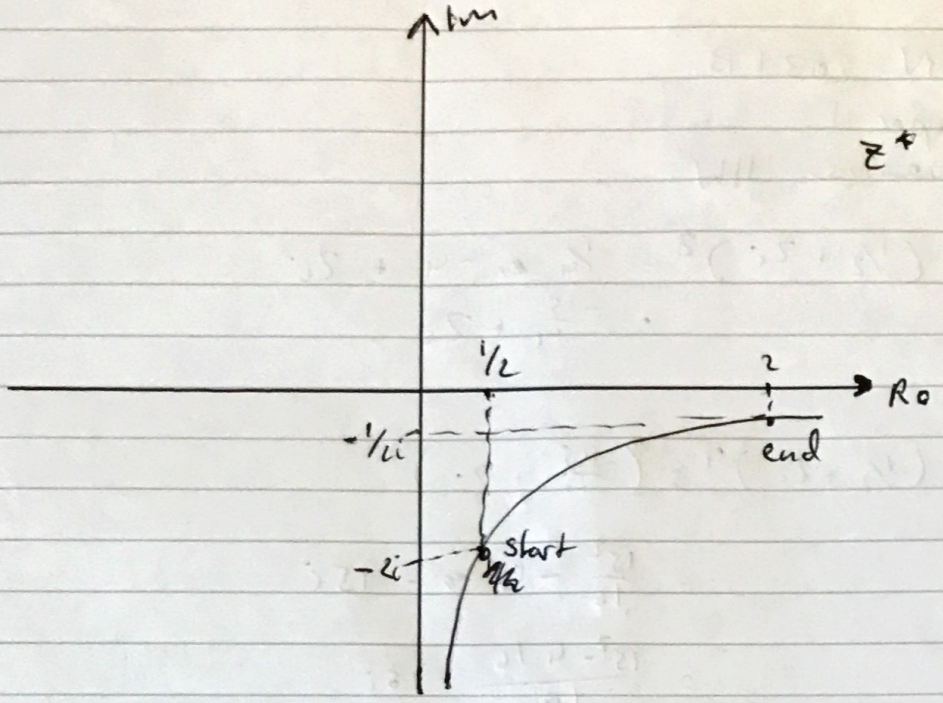
$= \frac{15^2 - 4 \cdot 16}{16} - 15i$

$= \frac{12 \cdot 16 - 15}{16} - 15i$

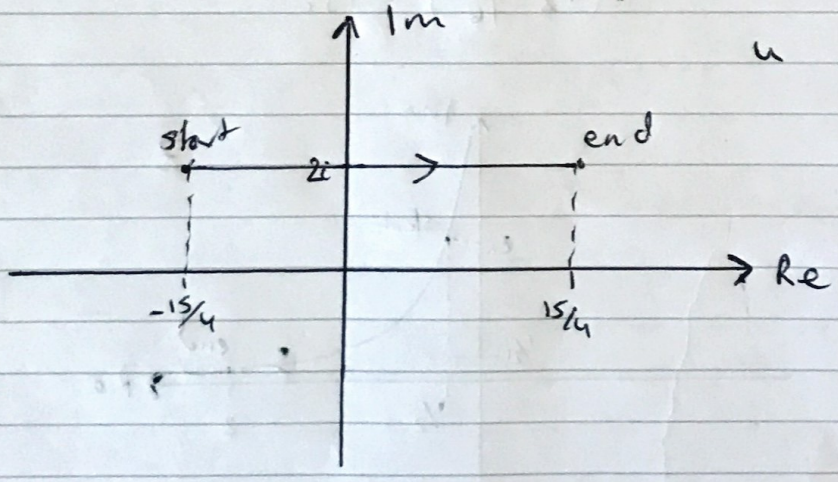
$= (12 - 15/16) - 15i$

b. i.





ii. $z^2 = (t + \frac{1}{t})^2 = t^2 - \frac{1}{t^2} + 2i$



iii. $v = z^4 = \left(t^2 - \frac{1}{t^2} + 2i\right)^2$

$= \left(t^2 - \frac{1}{t^2}\right)^2 - 4 + 4\left(t^2 - \frac{1}{t^2}\right)i$

$v|_{t=1/2} = \left(12 - \frac{15}{16}\right) - 15i$

$v|_{t=2} = \left(12 - \frac{15}{16}\right) + 15i$

For $\mathrm{Im}(v) = 0$:

$4\left(t^2 - \frac{1}{t^2}\right) \stackrel{!}{=} 0$

$\therefore t^2 - \frac{1}{t^2} = 0$

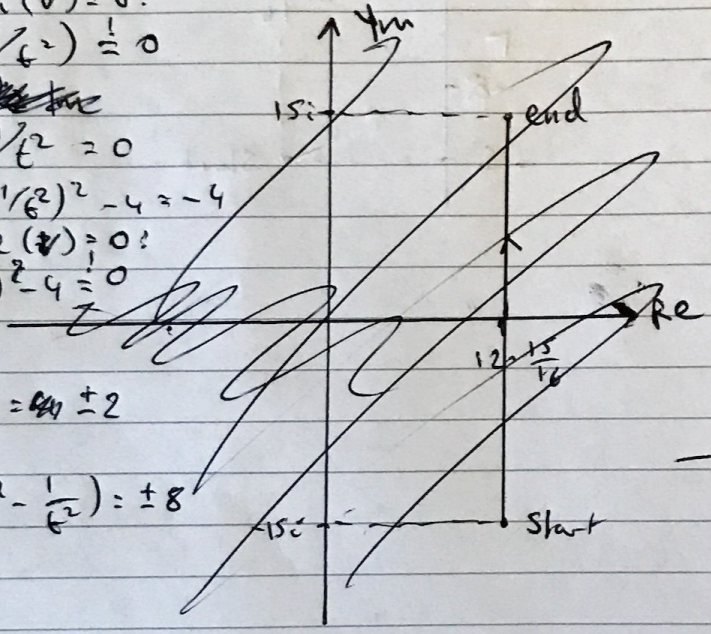
$\therefore \left(t^2 - \frac{1}{t^2}\right)^2 - 4 = -4$

For $\mathrm{Re}(v) = 0$:

$\left(t^2 - \frac{1}{t^2}\right)^2 - 4 \stackrel{!}{=} 0$

$\therefore t^2 - \frac{1}{t^2} = \pm 2$

$\therefore 4\left(t^2 - \frac{1}{t^2}\right) = \pm 8$



c. $v = z^4 = \left(t^2 - \frac{1}{t^2}\right)^2 - 4 + 4\left(t^2 - \frac{1}{t^2}\right)i$

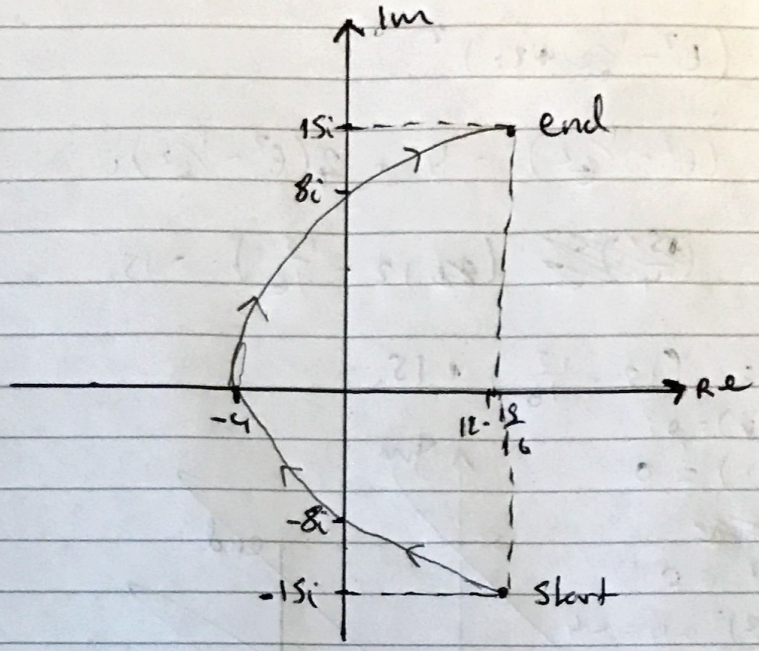
Let $x = \mathrm{Re}(v) = \left(t^2 - \frac{1}{t^2}\right)^2 - 4$

$y = \mathrm{Im}(v) = 4\left(t^2 - \frac{1}{t^2}\right)$

$t^2 - \frac{1}{t^2} = \frac{y}{4}$

$\therefore x = \left(\frac{y}{4}\right)^2 - 4 = \frac{1}{16}y^2 - 4$

graph overleaf


Im
15i
end
8i
Re
-4
12.18
16
-8i
-15i
start