a.

- i. Pareto improvement is when resources are reallocated such that the utility of one or more parties is improved without decreasing the utility of any other parties. For example, if there is a surplus of resources and two consumers, one of whom doesn't want any more of the resource and the other of whom wants more, allocating some of the surplus to the second consumer improves their utility without decreasing the utility of the other. This is a Pareto improvement.
- ii. A Pareto efficient allocation is a way of allocating resources such that no Pareto improvements can be made. For example, if one party owns all of the resources and another party wants some of the resources, there is no way to improve the second party's utility without decreasing that of the first. This is a Pareto efficient allocation. Pareto efficiency is a weak restriction, as it is not sufficient for an allocation to be even close to optimal.
- iii. Utility is a way of comparing preferences. It is an abstract concept to model the satisfaction (or removed dissatisfaction) gained by consuming a good or service. For example, a consumer may gain some utility from eating a twix, and they may gain more utility from eating a snickers. This model captures the fact that the consumer would prefer outcomes that result in them eating a snickers than those in which they eat a twix. Consumers are often modelled as rational agents who take actions which maximise their utility.
- b. The first theorem of welfare economics is that a market equilibrium is Pareto efficient. This is to say that when the price and quantity at which consumers are willing and able to purchase a good or service is the same as those at which producers are willing and able to sell them, there will be no way to increase the utility of anybody participating in the market without decreasing the utility of somebody else.

Furthermore, the second theorem of welfare economics states that any such allocation of resources in a market can be achieved using market forces provided that preferences are convex. Convexity of preferences is the condition that given three bundles of goods/services such that both of the first two have as least as much utility as the third, any weighted average of the first two will also have as least as much utility as the third. This roughly corresponds to the

theorem of diminishing marginal utility, which states that the more of a good/service consumed, the less utility gained by consuming an additional one.

Given this condition, the second theorem of welfare economics states that for any Pareto efficient allocation of resources, there exists an initial distribution of wealth such that market forces will result in that allocation.

However, Pareto efficiency does not imply justice, or that society as a whole is being treated fairly. To model this, welfare economics attempts to quantify societal welfare. It is non-trivial to turn utility into welfare, and so many approaches exist, each with their own problems. Two of these approaches are classical utilitarian welfare, and Rawlsian welfare.

Classical utilitarian welfare defines welfare as the sum of the utility of all of the members of society. This has the result that any Pareto improvement will improve welfare, which seems intuitive. However, not all Pareto efficient allocations maximise utilitarian welfare, since decreasing the utility of one person to increase that of another person to a greater extent increases welfare. Therefore, the second theorem of welfare economics implies that market forces alone are not guaranteed to maximise utilitarian welfare.

There are some issues with utilitarianism, such as the fact that a society in which some are incredibly happy, and some are incredibly sad may still have a high welfare. To combat this, Rawlsian welfare defined welfare as the minimum utility of all members of society – the utility of the most miserable person. An issue with this is that not all Pareto improvements will increase the welfare. For example, if there is a surplus good, there is no incentive to give it to anybody except the minimum-utility citizen, even if the second lowest-utility citizen would gain more utility from it.

Overall, it is incredibly difficult to define welfare to capture all of the intricacies of the human notion of "fairness". Maximising either classical utilitarian or Rawlsian welfare can lead to unfavourable outcomes. Therefore, instead of thinking of either of these as targets for maximisation, they are perhaps more suited to be thought of as heuristics to tell you which allocations require further investigation.

c. The free market achieves an equilibrium at which the price and quantity at which a firm is willing and able to produce goods/services is the same as the price and quantity at which consumers are willing and able to buy them. The first theorem of welfare economics states that this equilibrium will result in a

Pareto efficient allocation of resources. However, Pareto efficiency does not imply justice or high welfare. In fact, there are many Pareto efficient allocations which are incredibly unjust and unfair, and the second theorem of welfare economics states that any one of these could be the result of free market forces. This leads to the conclusion that we cannot expect the free market to result in a fair distribution of resources.

Furthermore, privacy is a very difficult resource for consumers to quantify. This is in part due to complex technical jargon which means that users of online services often don't know the extent to which their privacy is being violated. It is also due in part to the fact that the effects of having one's privacy violated are not immediately apparent, which makes it difficult for a consumer to gauge the value of the information they're agreeing to give up. As such, a consumer's utility function may underrepresent the value of privacy. This means that the market equilibrium may well result in more privacy violation than would be optimal if consumers had perfect information.

There are also some great benefits to giving up privacy. Data algorithms such as Facebook's recommendation engine are examples of how the more data a product can collect about you, the more useful or enjoyable the product becomes. This improved user experience combined with the difficulty of valuating the privacy that the user is giving up in return means that the free market will likely exacerbate the privacy problem.

Overall, it seems very unlikely that the free market will solve the privacy problem, because privacy is difficult to account for in a utility function, and there is no reason to believe that the free market will result in consumers being treated fairly.

a. An auction is an event in which many consumers compete for the opportunity to buy a good/service. There are many types of auctions including English auctions, Dutch auctions, first-price sealed-bid auctions, second-price sealed-bid auctions, and all-pay auctions.

An English action is one at which the seller starts the bidding at a low reserve price, and if a consumer is willing to pay more than that, they will bid more. This continues until nobody wants to outbid the current price, and then the top bidder pays their bid amount and receives the item. This price may be less than the winner's maximum budget, as long as it is higher than everybody else's.

A Dutch auction is one at which the seller starts the bidding at a high price, and then decreases the price until somebody is willing to pay that amount. This person then pays the price at which they bid and receives the item. This price will likely be the winner's maximum budget, unless they have reason to believe that everybody else's maximum budget is much lower.

A first-price sealed-bid auction is one at which each bidder is only allowed to bid one price. They do not know what anybody else has bid, and the highest bidder wins. Again, the price they bid is likely to be their maximum budget unless they have reason to believe that everybody else will bid much lower.

A second-price sealed-bid auction is also one at which each bidder is only allowed to bid one price and has no information about the other bids. However, the winner (with the highest bid) pays the price that was bid by the runner-up. This incentivises people to bid their honest valuation of the item, even if they believe that everybody else will bid lower.

An all-pay action sees every bidder pay at every round, each person dropping out when they decide not to pay any more money. The last person participating wins the goods. This plays into the sunk-cost fallacy, and thereby can cause the bidders to make irrational decisions.

All of these auction types have their pros and cons and deciding which one to use for a given good depends largely on the context in which it is being sold. Regardless of the type of auction, having more information about the true value of the item allows bidders to make better decisions.

b. In the online advertisement market, Google pioneered the idea of selling ad spaces using an auction. Rather than having a fixed price for the ad space, each potential advertiser bids a price, and then the platform selects a winner to optimise the revenue from the ad.

The most efficient way to select a winner is not simply to award the space to the highest bidder. This is because advertisers pay the platform on a perclick basis. Therefore, a better ad which gets more clicks could make the platform more revenue even if the cost per-click is lower. Total expected revenue can be calculated as the cost per-click multiplied by the expected number of clicks. To maximise this, a notion of ad "quality" can be estimated. This value should be proportional to the expected number of clicks, which itself is proportional to the ad relevance multiplied by the clickthrough rate. The space would be awarded to the advertiser with the expected revenue.

It can also be efficient to incentivise advertisers to increase their ad quality by reducing the cost per-click if their ad is much higher quality than their competitors. The actual price charged in the Google model is the winner's bid price multiplied by the ratio between the runner-up's ad quality and the winner's. This means that the higher your ad's quality compared to your competitor, the less you pay per-click. This is similar to a second-price sealed-bid auction.

I therefore believe that the most efficient way to allocate ad space is to award the ad to the bidder with highest product of bid value, relevance, and expected clickthrough rate, as this product will be proportional to the expected value. It may also increase revenue in the long run to charge less per-click if their ad is much higher quality than their competitor, as low-quality ads may reduce site visits and subsequently the click count over time.

c. Owners of advertising spaces, modelled as rational agents, are trying to maximise their profits. They will therefore award spaces to whomever they expect to result in the most revenue, often through an auction. For non-digital advertisements such as on billboards and in newspapers, the most reasonable way to price advertisements is a flat fee (i.e., the price does not depend on engagement, because this is difficult to measure). In these cases, it makes sense to simply award the space to the advertiser who bids the most money. If one political candidate wants to get an advertisement for

less money than their opponent, their best option is to do so with digital ads.

For online ads, the number of clicks can be counted, and so advertisers can be charged per-click. This means that the highest bidder is not necessarily the one who will bring in the most revenue; the expected number of clicks also needs to be considered. One way to account for this is to estimate the relevance of the ad. The political candidate might then choose to advertise on a website whose audience likely shares their values, making their ad more relevant.

However, the opposite may (and often does) prove more effective. A large part of internet ad revenue comes from so-called "anger clicks". If there is an ad on a website whose content is controversial, inflammatory, or offensive to the website's audience, the advertiser can expect to get a large number of clicks from users who want to see what it's all about, or perhaps want to leave a negative comment/review. In this way, it may also be worth the political candidate taking out a strongly-worded ad on a website whose audience aligns with their opponent.

Of course, while it is helpful to model the website owner as a rational agent, the field of behavioural economics tells us that agents can stray from their profit-maximising model for many reasons, one of which is ideology. They may charge more for advertisements that go against their values. It may then benefit the candidate to take out ads on websites run by their supporters, as they may get a discounted price.

Overall, the main ways for a political candidate to get ads at a lower cost than their opponent is to bid for ads on websites whose audience have strong political views. Those which align with the candidate will have high relevance, and so greater quality, and those which strongly clash with the candidate will have high engagement out of morbid curiosity and anger.

d. Bidding rings are a strategy for rigging an auction. It involves a group of bidders who collude to bid very low for the items. They would then have a private auction later and split the proceeds.

The risk of participating in a bidding ring is that everybody needs to agree to bid low. If one person breaks this agreement and bids slightly higher, they will receive the good at a heavily discounted price. In this way a prisoner's dilemma arises. This is to say that if everybody bids low as agreed,

everybody gets a medium amount of reward, if everybody bids higher, the auction proceeds as normal and each person gets a small amount of reward, but if everybody bids low and one person bids higher, the low-bidders receive nothing, and the high-bidder receives a large reward.

This means that it is only beneficial to engage in a bidding ring if you trust all of your co-conspirators to keep the agreement and bid low. That is, unless there are many consecutive auctions, in which case the optimal strategy for each individual is also optimal for the group. This strategy is tit-for-tat, in which the ring keeps the agreement in the first round and then in subsequent rounds they break the agreement if and only if someone broke it in the previous round. This removes the need for trust and makes bidding-rings viable through cooperation.

In conclusion, bidding rings in which everybody agrees to bid low in order to resell the items at a later private auction is a viable way to rig some types of auctions as long as there are many auctions in a row. If there is only one item, some bidders may benefit greatly by betraying the others.