BGN: 2191A

P6

Q9

a. DPLL:

1. Write the formula in clause form

2. Delete tautological clauses (clauses containing both a literal and its negation)

3. If there are any unit clauses, set a flag f and delete them, removing its negation from any other clauses.

4. If any clauses contain pure literals, delete them and set f.

5. If the empty clause is reached, the formula is neither satisfiable nor valid

6. If all clauses are deleted, the formula is satisfiable if f is set, valid otherwise.

7. Perform a case split on a literal and reapply the algorithm to both cases. If ~~either case~~ is valid or satisfiable, the formula is satisfiable. Otherwise, it is neither valid nor satisfiable.

Advantages: contains less redundancy than BDD, so less memory usage.

BDD:

1. Identify a top-level connective of the formula

2. Recursively convert both sides of the connective to BDDs

3. Combine the BDDs using the rules for that connective

4. If the resulting BDD is 1, the formula is valid.
   If it is 0, the formula is neither valid nor satisfiable
   Otherwise, the formula is satisfiable.

Advantages: Considerably more efficient than DPLL b.i. It is logically correct in the sense that any satisfying assignment of the original formula will be a satisfying assignment of the modified one, and vice versa, but validity is not presered in this way.

ii. This is likely to significantly decrease the runtime of the DPLL operation, as the algorithm will have to perform fewer case splits and can instead rely more on the non-recursive operations.

c.i. The formula is true iff f has a fixed point and is not the identity.

$$
\dfrac{
\dfrac{P(a) \Rightarrow \qquad
\dfrac{\dfrac{\dfrac{x \mapsto f(x)}{\neg P(f(x)), \neg P(x) \Rightarrow}}{\neg P(f(x)) \land \neg P(x)}\;(\land l)}{\exists x(\neg P(f(x)) \lor \neg P(x)) \Rightarrow}\;(\exists l)
}{P(a) \lor \exists x(\neg P(f(x)) \land \neg P(x)) \Rightarrow}\;(\lor l)
\qquad
\dfrac{\dfrac{\dfrac{x \not\mapsto f(x)}{P(x), \neg P(f(x)) \Rightarrow}}{P(x) \land \neg P(f(x)) \Rightarrow}\;(\land l)}{\exists x(P(x) \land \neg P(f(x))) \Rightarrow}\;(\exists l)
}{[\neg(\neg P(a) \land \forall x(P(f(x)) \lor P(x))] \lor \exists x(P(x) \land \neg P(f(x))) \Rightarrow}\;(\lor l)
$$

$$\cancel{\neg P(f(t)) \Rightarrow}$$

$(\forall l)$

$$\cancel{\neg P(a), \forall x(\neg P(f(x))) \Rightarrow}$$

$(\land l)$

$$\cancel{(\neg P(a) \land \forall x(\neg P(f(x))), \neg P(x) \Rightarrow}$$

$(\land l)$

$$\cancel{(\neg P(a) \land \forall x(\neg P(f(x))) \land \neg P(x) \Rightarrow}$$

$$\cancel{\neg[\neg(\neg P(a) \land \forall x(\neg P(f(x))) \lor P(x)] \Rightarrow \quad \exists x(P(x) \land \neg P(f(x))) \Rightarrow}$$

$(\lor l)$

$$\cancel{\neg[\neg(\neg P(a) \land \forall x(\neg P(f(x))) \lor P(x)] \lor \exists x(P(x) \land \neg P(f(x))) \Rightarrow}$$

$$\cancel{[\neg P(a) \land \forall x(\neg P(f(x)) \to P(x)] \to \exists x(P(x) \land \neg P(f(x))) \Rightarrow}$$

ii. False. $\neg P(x)$ for all x is a falsifying interpretation.

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{P(t), Q(u), \neg P(x)}{P(t), \forall x Q(x), \neg P(x)}(\forall L)}{\forall x P(x), \forall x Q(x), \neg P(x) \Rightarrow}(\forall L) \qquad \dfrac{\dfrac{P(t), Q(u), Q(x)}{P(t), \forall x Q(x), Q(x)}(\forall L)}{\forall x P(x), \forall x Q(x), Q(x) \Rightarrow}(\forall L)}{\forall x P(x), \forall x Q(x), (\neg P(x) \vee Q(x)) \Rightarrow}(\vee L)}{(\forall x P(x)) \wedge (\forall x Q(x)), (\neg P(x) \vee Q(x)) \Rightarrow}(\wedge L)}{(\forall x P(x)) \wedge (\forall x Q(x)), \exists x(\neg P(x) \vee Q(x)) \Rightarrow}(\exists L)}{(\forall x P(x)) \wedge (\forall x Q(x)) \vee \exists x (P(x) \to \neg Q(x)) \Rightarrow}(\vee L)$$