BGN 2031B
Paper 1
Question 16T

a. For a single die

Let X = the value of its face

$E[X] = \sum_i x_i p_i$

$E[X^2] = \sum_i x_i^2 p_i$

$V[X] = E[X^2] - (E[X])^2$

x_i	1	2	3	4	5	6	Σ
p_i	1/6	1/6	1/6	1/6	1/6	1/6	1
$x_i p_i$	1/6	2/6	3/6	4/6	5/6	6/6	21/6
$x_i^2 p_i$	1/6	4/6	9/6	16/6	25/6	36/6	91/6

$E[X] = 21/6 = \frac{7}{2}$

$V[X] = 91/6 - \frac{49}{4}$

$= \frac{182}{12} - \frac{147}{12}$

$= \frac{35}{12}$

Let X_i = the value on the face of the i^{th} die

Let $Y = X_1 + X_2 + X_3$

$E[Y] = E[X_1] + E[X_2] + E[X_3] = 3 \cdot \frac{7}{2} = \frac{21}{2}$

$$Var[Y] = V[X_1] + V[X_2] + V[X_3]$$

$$= 3 \cdot \frac{35}{12} = \frac{35}{4}$$

b i. $P(S) = \frac{1}{6} \cdot \frac{1}{6} = \frac{1}{36}$

ii. $P(D) = \frac{5}{6} \cdot \frac{4}{6} = \frac{10}{18} = \frac{5}{9}$

iii. $P(T) = \frac{1}{6} \cdot \frac{5}{6} \cdot \binom{3}{2}$

$= \frac{1}{6} \cdot \frac{5}{6} \cdot 3$

$= \frac{5}{12}$

iv. Each intersection point shows $X_1 + X_2$ and each outcome has probability $\frac{1}{36}$. Not all are shown

$X_2 \backslash X_1$	1	2	3	4	5	6
1	2	3	4	5		
2	3	4	5			
3	4	5				
4	5					
5						
6						

k	1	2	3	4	5
$P(X_1 + X_2 = k)$	0	$\frac{1}{36}$	$\frac{2}{36}$	$\frac{3}{36}$	$\frac{4}{36}$

~~Now each intersection point shows $P(X_1+X_2, X_3)$ for points at which $X_1+X_2+X_3$~~

$$P(X_1 + X_2 + X_3) \leq 5 = P(X_1 + X_2 = 2)\,P(X_3 \leq 3) + P(X_1 + X_2 = 3)\,P(X_3 \leq 2) + P(X_1 + X_2 = 4)\,P(X_3 \leq 1)$$

$$= \tfrac{1}{36} \cdot \tfrac{3}{36} + \tfrac{2}{36} \cdot \tfrac{2}{36} + \tfrac{3}{36} \cdot \tfrac{1}{36}$$

$$= \frac{3+4+3}{36} = \frac{10}{36} = \frac{5}{18}$$

c. $P(F|T) = \cancel{\dfrac{P(F \cap T)}{P(T)}} = \cancel{\dfrac{P(T|F)\,P(F)}{P(T)}} = \dfrac{P(T \text{ and } F)}{P(T)}$

Options satisfying T and F ~~(unordered)~~ unordered:

1, 1, 2

1, 1, 3

2, 2, 1

Each of these have $\tfrac{1}{36} \cdot \binom{3}{2} = \tfrac{3}{36} = \tfrac{1}{12}$ chance of occuring

$\therefore P(T \text{ and } F) = 3 \cdot \tfrac{1}{12} = \tfrac{1}{4}$

$\therefore P(F|T) = \dfrac{(1/4)}{(5/12)} = \dfrac{12}{5 \cdot 4} = \dfrac{3}{5}$