Drawing Images With Equations

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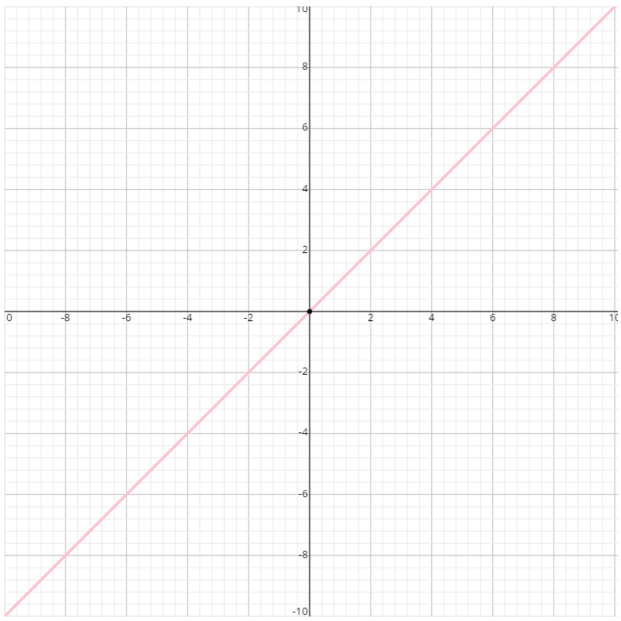
1 Introduction

This paper introduces a method of combining mathematical equations in various ways so as to create a graphical effect. This is a continuation of my previous paper, *Algorithmic Functions as Mathematical Expressions*, as it uses several of the functions designed therein. This paper was inspired by the Batman Curve [1] which uses parts of various geometric functions to create a function whose graph is the Batman symbol. This paper describes a method of converting similar images (that is to say, those made out of sections of geometric shapes) into a single equation whose graph is that image.

2 Basic Transformations

2.1 Shifting

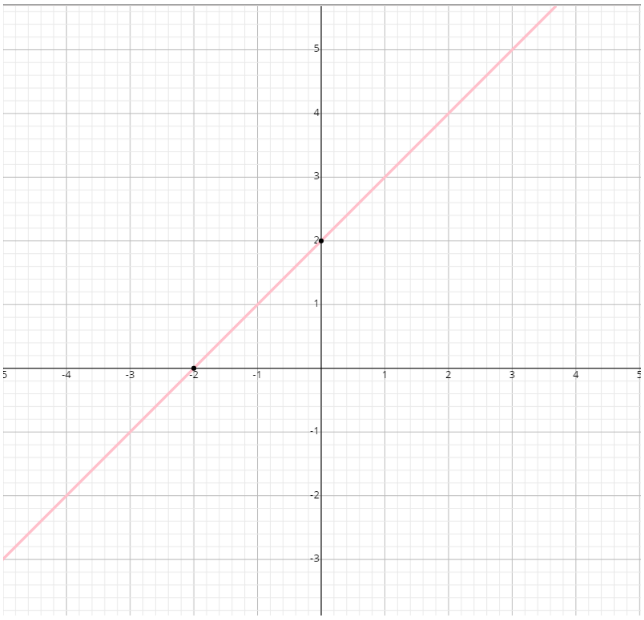
The first step in manipulating a function to appear how you wish is translation. This includes modifying a function so that the graph shifts horizontally or vertically. To demonstrate, we will first look at the graph of the equation $y = x$.



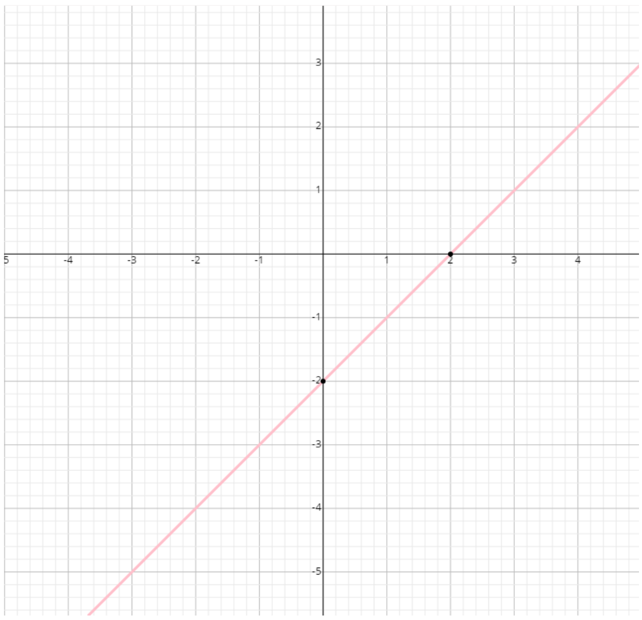
If we wanted to shift this line Δ places to the left, we have to replace every mention of x with $x + \Delta$. For example, to shift 2 places to the left, we change the equation to $y = x + 2$, which

[1] http://mathworld.wolfram.com/BatmanCurve.html

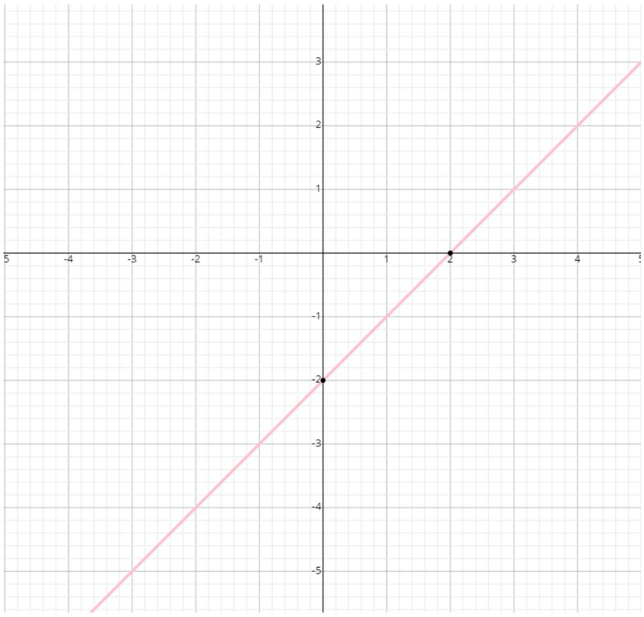
looks like



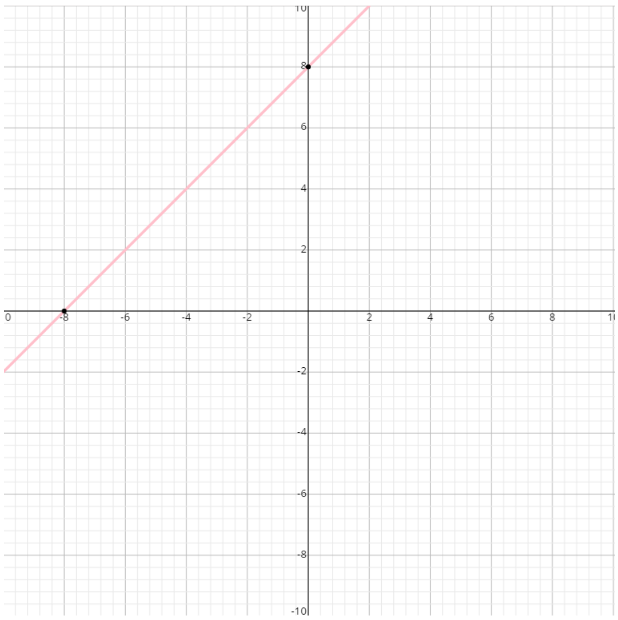
If we want to shift the line 2 places to the right (-2 places to the left), we change the equation to $y = x - 2$



Similarly, a vertical shift of Δ places downwards would require replacing every mention of y with $y + \Delta$, thus making our equation $y + \Delta = x$. A shift of 2 downwards would be $y + 2 = x$, which would look like

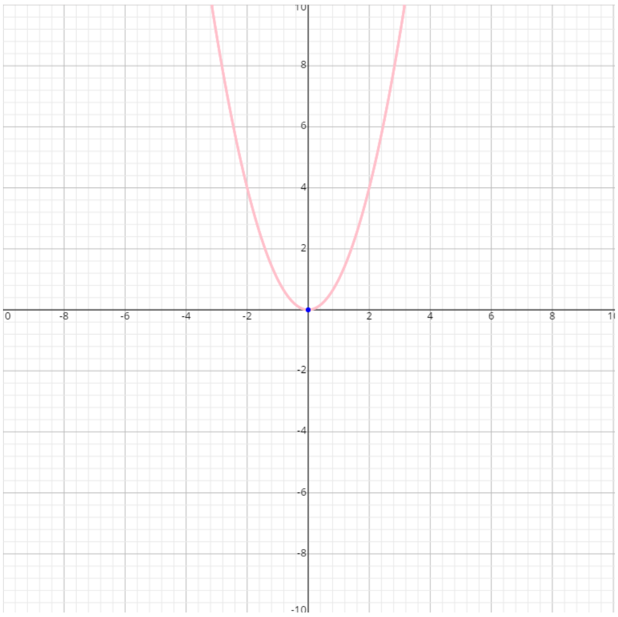


These transformations can be applied in conjunction with each other. For example, to shift the line 3 places upwards and 5 to the left would require changing the equation to $y - 3 = x + 5$. Its graph would look like

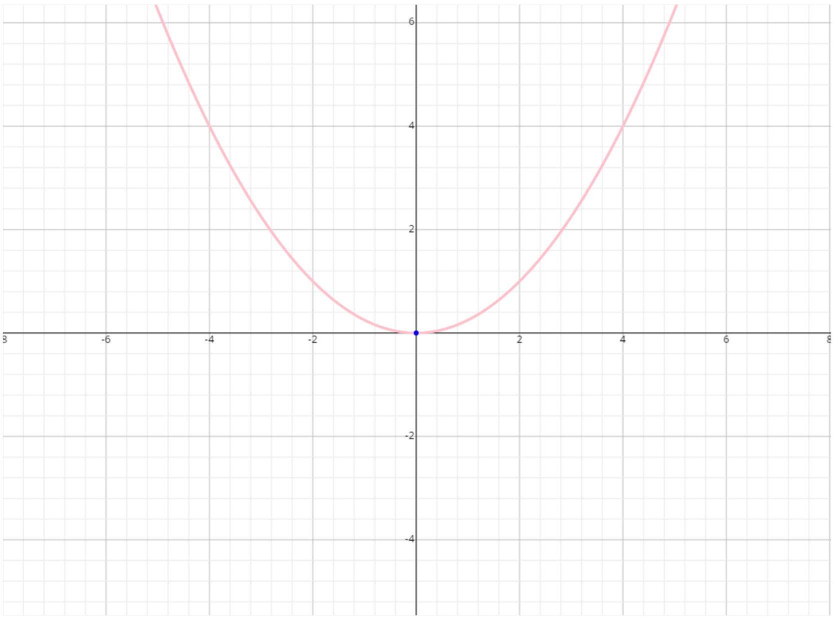


2.2 Stretching

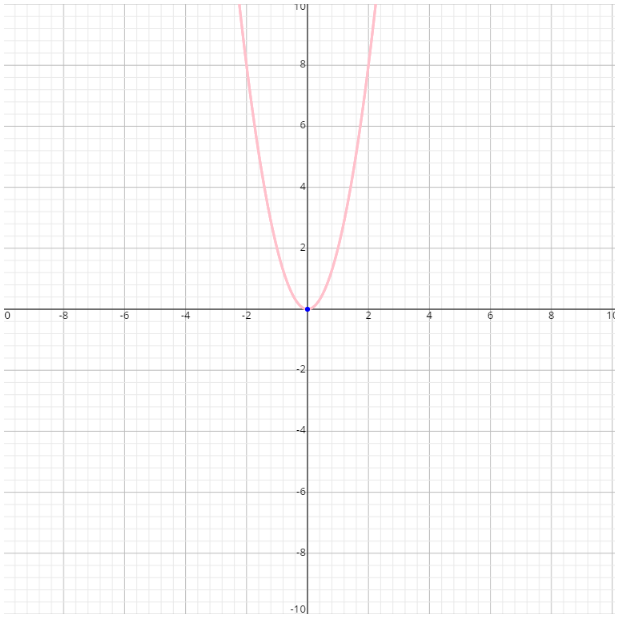
Another basic transformation is stretching. To demonstrate this, consider the function $y = x^2$. Its graph looks like this



To stretch this line by a factor of α horizontally, You must modify the equation so that every mention of x is replaced with $\frac{x}{\alpha}$. For example, to stretch by a factor of 2, the function would become $y = (\frac{x}{2})^2$. Its graph would look like

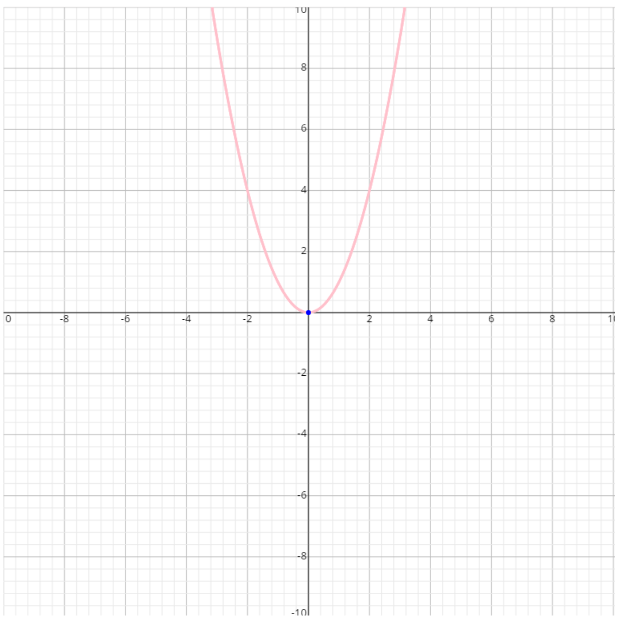


Similarly, to stretch by a factor of α in the vertical direction, the equation must be changed in such a way that every mention of y is replaced with $\frac{y}{\alpha}$. To stretch vertically by a factor of 2, the function would become $\frac{y}{2} = x^2$, which looks like

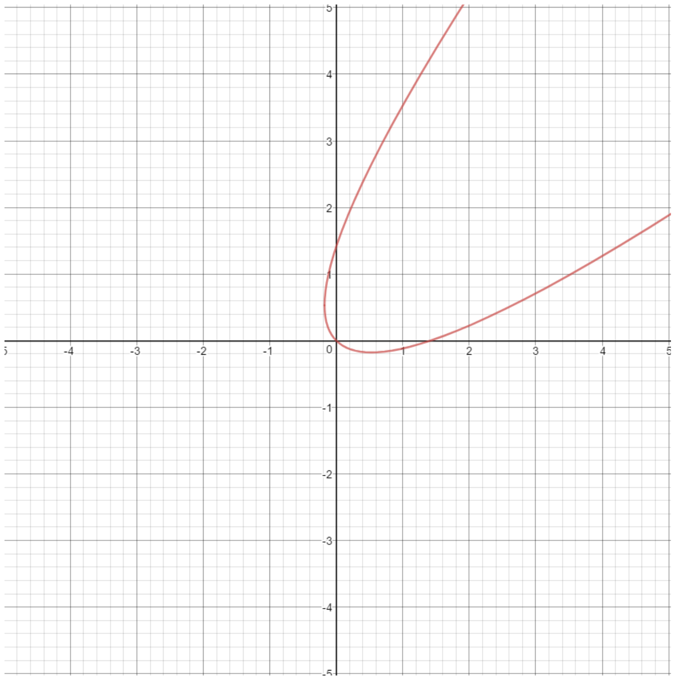


2.3 Rotation

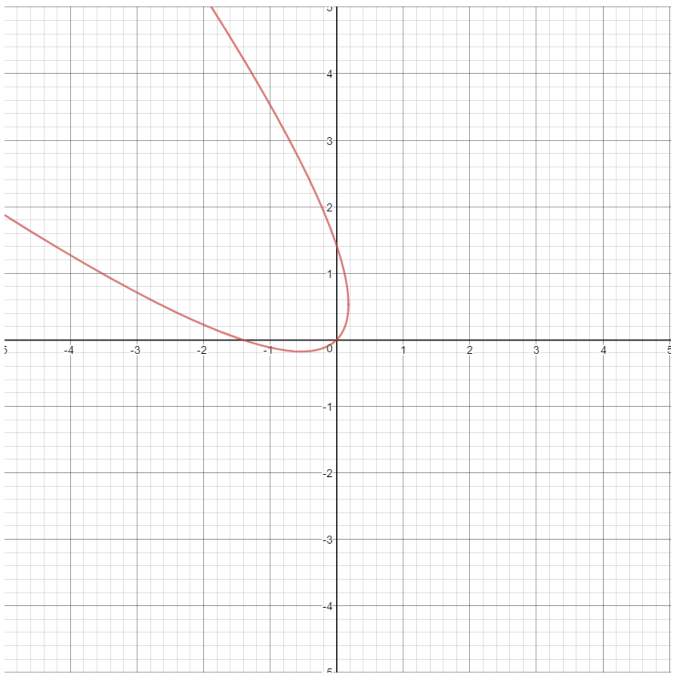
Rotation is slightly more difficult. Imagine some coordinate (a, b) such that a and b have some relationship. For example the relationship could be $a = b$ or $b = a^2$ or $a^2 + b^2 = 1$, etc. Now, we need to remodel these coordinates as a single number on the complex plane with a real part of a and an imaginary part of b. Imagine another complex number $x + yi$. This number is equal to $a + bi$, rotated clockwise by an angle of θ around 0. We can calculate $a + bi$ in terms of $x + yi$ by rotating $x + yi$ anticlockwise by an angle of θ around 0. This is done by multiplying by $e^{i\theta}$, which equals $cos\theta + i \cdot sin\theta$. This means that $a + bi$ is equal to $(x \cdot cos\theta - y \cdot sin\theta) + (x \cdot sin\theta + y \cdot cos\theta)i$. We can convert this back into coordinates. If the point at (a, b) rotates clockwise around $(0, 0)$ by an angle of θ to give the point (x, y), then $a = x \cdot cos\theta - y \cdot sin\theta$ and $b = x \cdot sin\theta + y \cdot cos\theta$. It follows that whatever relationship was met by a and b is also met by $x \cdot cos\theta - y \cdot sin\theta$ and $x \cdot sin\theta + y \cdot cos\theta$. Therefore, if you have an equation linking x and y and you want to modify it so that its graph has been rotated clockwise by an angle of θ around (0,0), then replace every mention of x with $x \cdot cos\theta - y \cdot sin\theta$ and every mention of y with $x \cdot sin\theta + y \cdot cos\theta$. For example, if we again take the the equation $y = x^2$ which looks like this



and we want to rotate it clockwise around (0,0) by $\frac{\pi}{4}$ radians, then the equation becomes $x \cdot sin\frac{\pi}{4} + y \cdot cos\frac{\pi}{4} = (x \cdot cos\frac{\pi}{4} - y \cdot sin\frac{\pi}{4})^2$

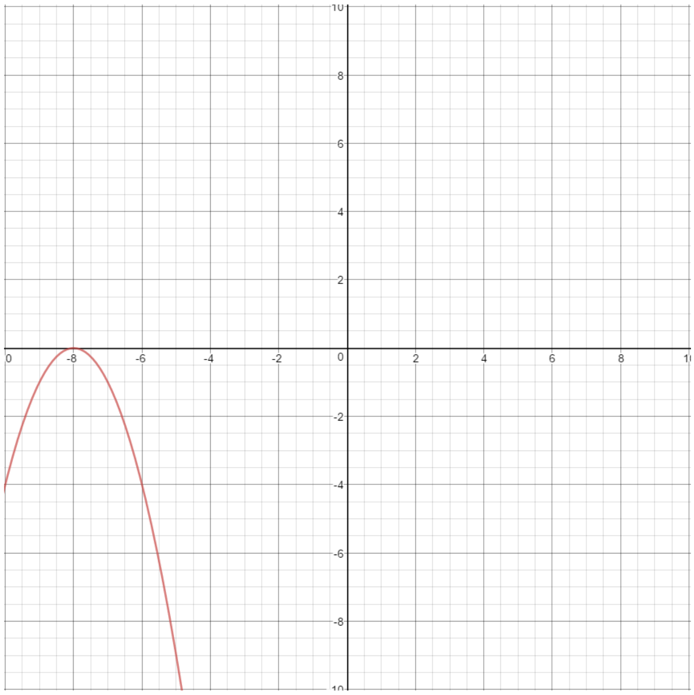


Similarly, a rotation of θ anticlockwise is equivalent to a rotation of $-\theta$ clockwise. To rotate our original $y = x^2$ equation by $\frac{\pi}{4}$ radians anticlockwise, we rotate clockwise by $\frac{-\pi}{4}$ radians, making the equation $x \cdot \sin\left(\frac{-\pi}{4}\right) + y \cdot \cos\left(\frac{-\pi}{4}\right) = \left(x \cdot \cos\left(\frac{-\pi}{4}\right) - y \cdot \sin\left(\frac{-\pi}{4}\right)\right)^2$. Its graph looks like

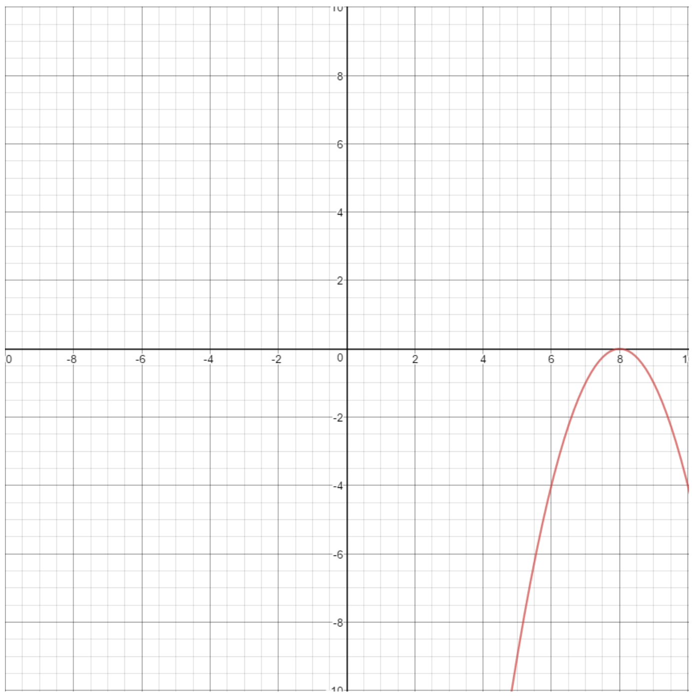


2.4 Order of Operations

When applying these transformations, it is important to bear in mind the order in which they are applied. For example, if we take our equation $y = x^2$ and perform a translation of 8 to the right followed by a rotation of π radians clockwise, the equation becomes $x \cdot sin\pi + y \cdot cos\pi = (x \cdot cos\pi - y \cdot sin\pi - 8)^2$ and its graph would look like



Whereas, if you apply the rotation first, followed by the transformation, the equation becomes $(x-8)sin\pi + y \cdot cos\pi = ((x-8)cos\pi - y \cdot sin\pi)^2$ and its graph instead looks like



The same principle applies to all types of transformation. You can apply any number of transformations to an equation, but the order in which you apply them will usually change the appearance of the graph plotted by the resulting equation.

3 Binary Expressions

The next step in creating equations from images is converting equations into so called *binary expressions*. This is a topic covered in my previous paper, and it allows equations to be combined in various ways. An equation is either true or false, whereas a binary expression is either equal to 1 or 0. As such, the binary expression b generated from any equation can be plugged into the equation $b = 1$ which plots the same graph as the original equation. The method for converting an equation into a binary expression is quite intuitive, but it is different for every type of equation.

3.1 Equals

In this section and the subsequent sections, we will use LHS to represent the expression on the left hand side of the equation and RHS to represent the expression on the right hand side. For an equation $LHS = RHS$, its binary expression looks like $1 - \lceil \frac{|(LHS)-(RHS)|}{|(LHS)-(RHS)|+1} \rceil$. If $LHS = RHS$ then that binary expression will equal 1, otherwise it will equal 0. For example, the equation $x = y$ is equivalent to the equation $1 - \lceil \frac{|x-y|}{|x-y|+1} \rceil = 1$.

3.2 Not Equals

The binary expression for $LHS \neq RHS$ is equivalent to $1-$ the binary expression for $LHS = RHS$, making it $\lceil \frac{|(LHS)-(RHS)|}{|(LHS)-(RHS)|+1} \rceil$.

3.3 Less Than or Equal To

Saying that $LHS \leq RHS$ is the same as saying that $RHS - LHS$ is positive or zero. In other words $RHS - LHS$ is equal to its absolute value. Therefore, the equation $LHS \leq RHS$ is equivalent to the equation $RHS - LHS = |RHS - LHS|$. This is now a basic equality, and so its binary expression is $1 - \lceil \frac{|(RHS-LHS)-(|RHS-LHS|)|}{|(RHS-LHS)-(|RHS-LHS|)|+1} \rceil$.

3.4 Greater Than or Equal To

LHS is greater than or equal to RHS if and only if RHS is less than or equal to LHS. This means that its binary expression is the same as above, except with LHS and RHS switched around. It would look like $1 - \lceil \frac{|(LHS-RHS)-(|LHS-RHS|)|}{|(LHS-RHS)-(|LHS-RHS|)|+1} \rceil$.

3.5 Less Than

LHS is less than RHS means the same as LHS is not greater than or equal to RHS. This means that its binary expression would be equal to $1-$ the binary expression for a *greater than or equal to* equation. Its binary expression is $\lceil \frac{|(LHS-RHS)-(|LHS-RHS|)|}{|(LHS-RHS)-(|LHS-RHS|)|+1} \rceil$.

3.6 Greater Than

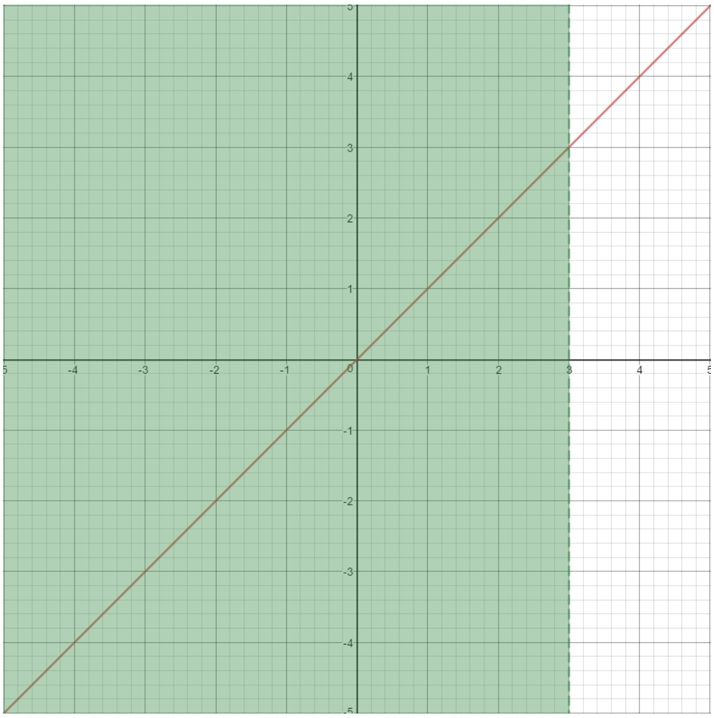
Similarly, the binary expression for $LHS > RHS$ is $1-$ the binary expression for $LHS \leq RHS$, which is $\lceil \frac{|(RHS-LHS)-(|RHS-LHS|)|}{|(RHS-LHS)-(|RHS-LHS|)|+1} \rceil$.

4 Logical Operations

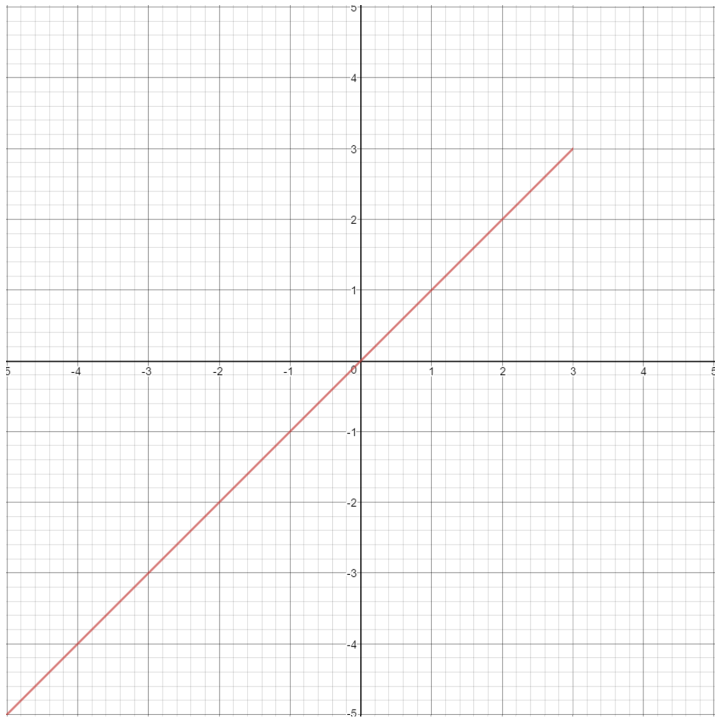
In order to create an equation which plots a combination of two lines, first we need to convert both equations into binary expressions and perform a logical operation on them.

4.1 AND operator

If we think of both equations as the set of the points which are plotted by them, an AND operator is equivalent to a intersection operator. In other words, if we take the binary expressions of two equations and perform an AND operation on them, the resulting expression is the binary expression for an equation which plots only the intersections of the two lines. An AND operator is simply multiplication, i.e. the product of two binary expressions. For example, take the equations $x = y$ and $x < 3$. Plotted on a graph together, they look like this:

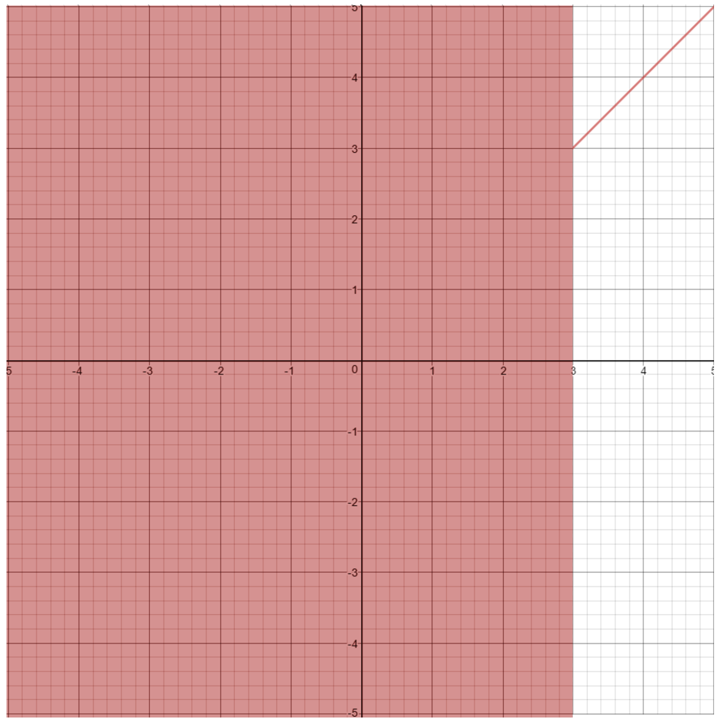


In order to create an equation which plots only their intersections, we must first find the binary expressions for both of them. The binary expression for $x = y$ is $1 - \lceil \frac{|x-y|}{|x-y|+1} \rceil$ and the binary expression for $x < 3$ is $\lceil \frac{|x-3-|x-3||}{|x-3-|x-3||+1} \rceil$. Now we multiply those together to get $(1 - \lceil \frac{|x-y|}{|x-y|+1} \rceil)(\lceil \frac{|x-3-|x-3||}{|x-3-|x-3||+1} \rceil)$. For any x and y for which this expression equals 1, the point (x, y) is plotted by both equations, and so the equation $(1 - \lceil \frac{|x-y|}{|x-y|+1} \rceil)(\lceil \frac{|x-3-|x-3||}{|x-3-|x-3||+1} \rceil) = 1$ plots the following graph:



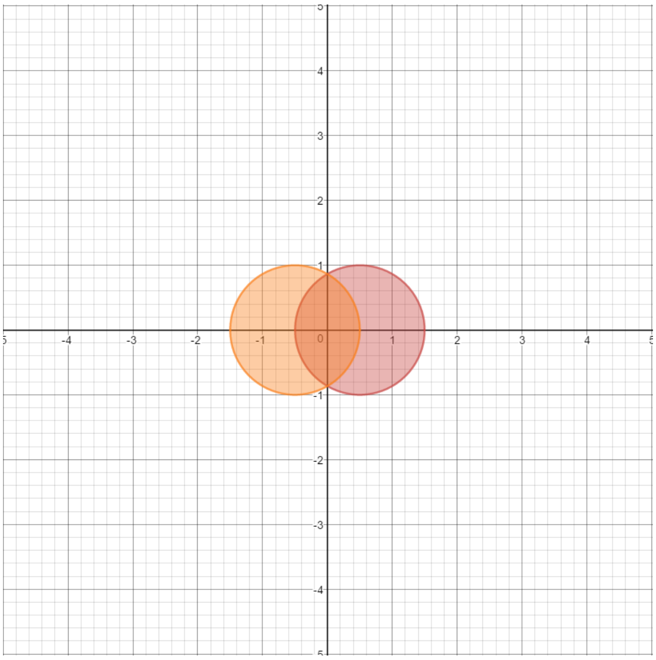
4.2 OR Operator

The OR operator is equivalent to a union operator in set theory. If we have two binary expressions a and b, the OR operation takes their sum and then subtracts their product, $a + b - ab$. If you set this equal to 1, it plots the points which appear on either or both of the two lines. For example, take the two previous equations $x = y$ and $x < 3$. Again, their binary expressions are $1 - \lceil \frac{|x-y|}{|x-y|+1} \rceil$ and $\lceil \frac{|x-3-|x-3||}{|x-3-|x-3||+1} \rceil$. Performing an OR operation on these results in a binary expression $1 - \lceil \frac{|x-y|}{|x-y|+1} \rceil + \lceil \frac{|x-3-|x-3||}{|x-3-|x-3||+1} \rceil - (1 - \lceil \frac{|x-y|}{|x-y|+1} \rceil)(\lceil \frac{|x-3-|x-3||}{|x-3-|x-3||+1} \rceil)$. If we set this expression equal to 1, we get the equation $1 - \lceil \frac{|x-y|}{|x-y|+1} \rceil + \lceil \frac{|x-3-|x-3||}{|x-3-|x-3||+1} \rceil - (1 - \lceil \frac{|x-y|}{|x-y|+1} \rceil)(\lceil \frac{|x-3-|x-3||}{|x-3-|x-3||+1} \rceil) = 1$. This equation plots the following graph:



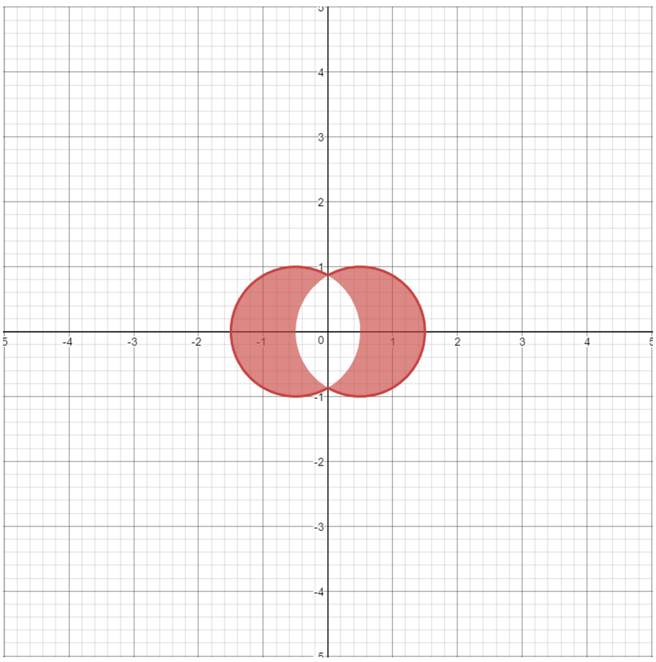
4.3 XOR Operator

The XOR operation plots points which lie on exactly one of the lines, but not both. It is similar to the OR operator except that instead of subtracting the product from the sum of the binary expressions, you subtract double the product. For example, take the two equations $(x+0.5)^2+y^2 \leq 1$ (in orange below) and $(x-0.5)^2+y^2 \leq 1$ (in red below). If you plot these two equations on the same graph it looks like



Their respective binary expressions are $1-\lceil\frac{\left|(1-((x+0.5)^2+y^2))-\left(\left|1-((x+0.5)^2+y^2)\right|\right)\right|}{|(1-((x+0.5)^2+y^2))-(|1-((x+0.5)^2+y^2)|)|+1}\rceil$ and $1-\lceil\frac{\left|(1-((x-0.5)^2+y^2))-\left(\left|1-((x-0.5)^2+y^2)\right|\right)\right|}{|(1-((x-0.5)^2+y^2))-(|1-((x-0.5)^2+y^2)|)|+1}\rceil$. If you apply an XOR operation, it becomes

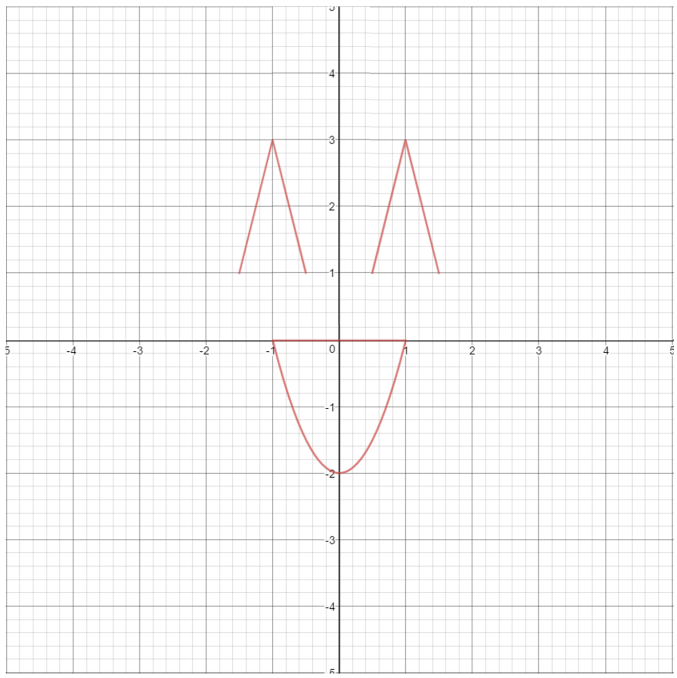
$1-\lceil\frac{\left|(1-((x+0.5)^2+y^2))-\left(\left|1-((x+0.5)^2+y^2)\right|\right)\right|}{|(1-((x+0.5)^2+y^2))-(|1-((x+0.5)^2+y^2)|)|+1}\rceil + 1-\lceil\frac{\left|(1-((x-0.5)^2+y^2))-\left(\left|1-((x-0.5)^2+y^2)\right|\right)\right|}{|(1-((x-0.5)^2+y^2))-(|1-((x-0.5)^2+y^2)|)|+1}\rceil - 2(1-\lceil\frac{\left|(1-((x+0.5)^2+y^2))-\left(\left|1-((x+0.5)^2+y^2)\right|\right)\right|}{|(1-((x+0.5)^2+y^2))-(|1-((x+0.5)^2+y^2)|)|+1}\rceil)(1-\lceil\frac{\left|(1-((x-0.5)^2+y^2))-\left(\left|1-((x-0.5)^2+y^2)\right|\right)\right|}{|(1-((x-0.5)^2+y^2))-(|1-((x-0.5)^2+y^2)|)|+1}\rceil)$. If you set this equal to 1 and plot it, it makes the following graph:



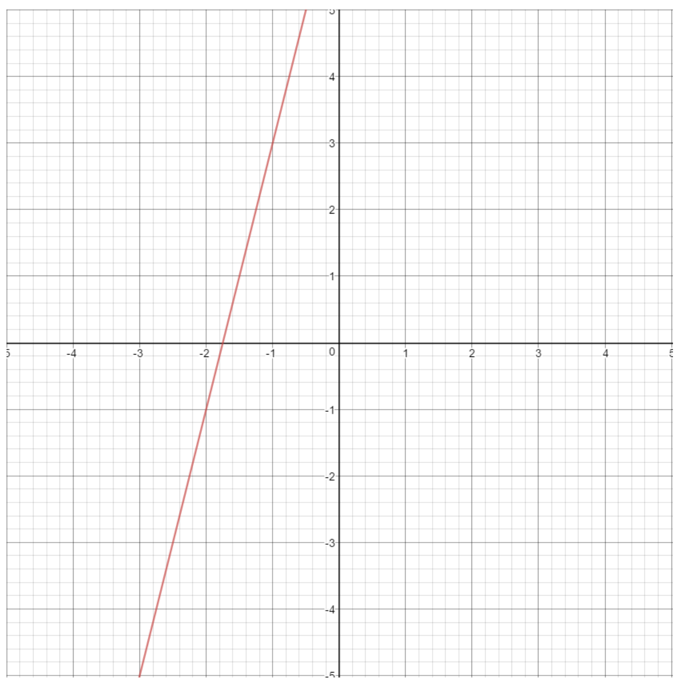
You can join any number of equations using any combination of logical operators. As with the transformations, the order in which and manner with which you join these equations may still make an impact on the final graph they produce.

5 Example

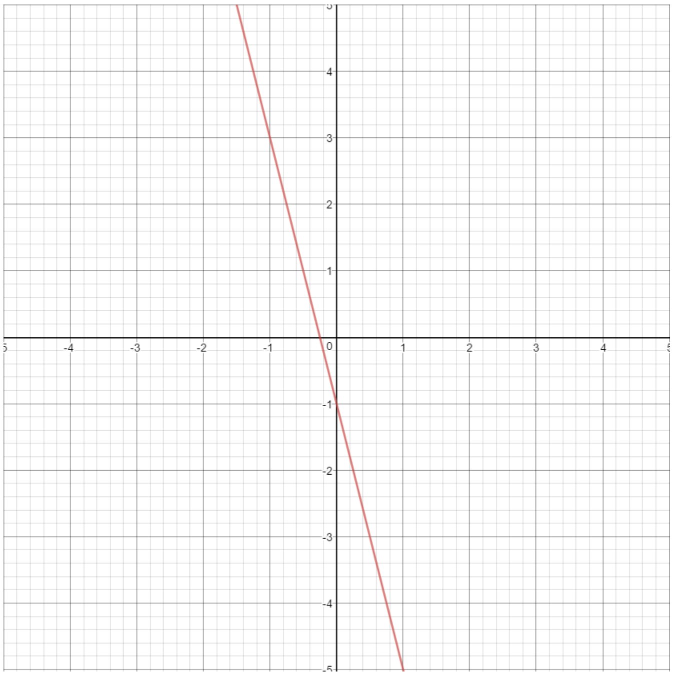
In this section, I will demonstrate some of the topics contained within the previous sections. I will do so by creating an equation which plots a smiley face on a graph as shown below.



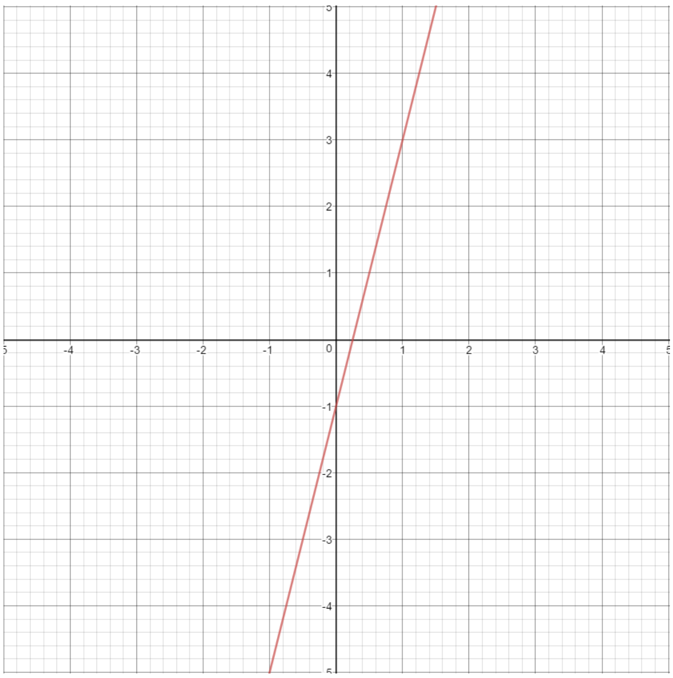
The first step is to split the graph up into its individual shapes. The eyes are composed of 4 straight lines and the mouth is composed of a straight line and a parabola. The first straight line is the following:



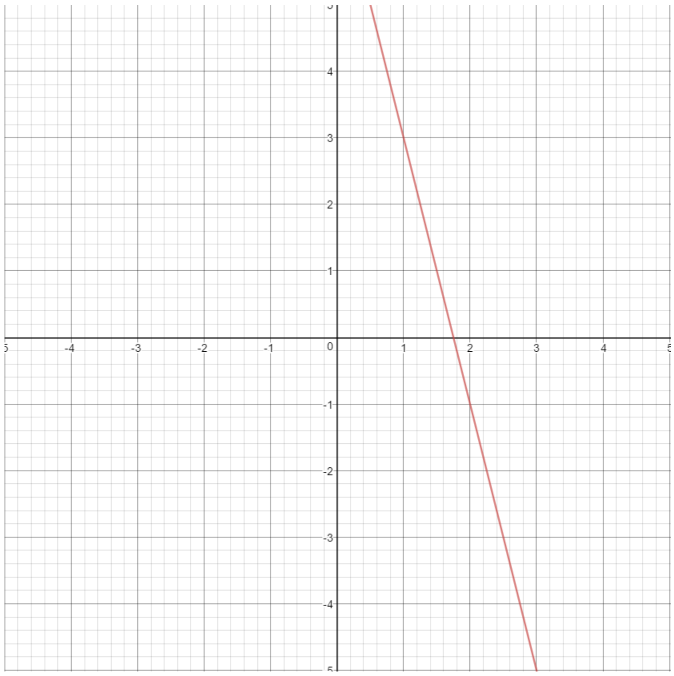
The equation for this line is $y = 4x + 7$. Only the parts of this line between $y = 1$ and $y = 3$ need to be plotted, but those same conditions need to be applied to the other 3 lines of the eye, so they can be implemented later. The binary expression for this line is $1 - \lceil \frac{|4x+7-y|}{|4x+7-y|+1} \rceil$. From now on this expression will be referred to as a. The second line of the eyes is below.



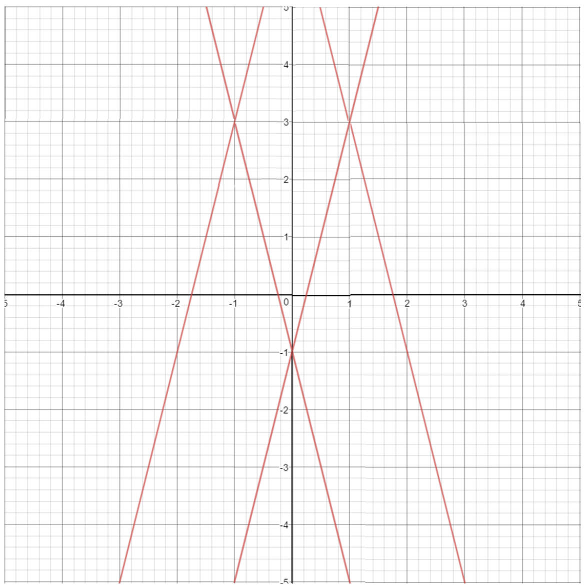
The equation for this line is $y = -4x - 1$ and its binary expression $b = 1 - \lceil \frac{|-4x-1-y|}{|-4x-1-y|+1} \rceil$. The third line from the eyes is below.



The equation for this line is $y = 4x - 1$ and its binary expression $c = 1 - \lceil \frac{|4x-1-y|}{|4x-1-y|+1} \rceil$. The last line from the eyes is below.



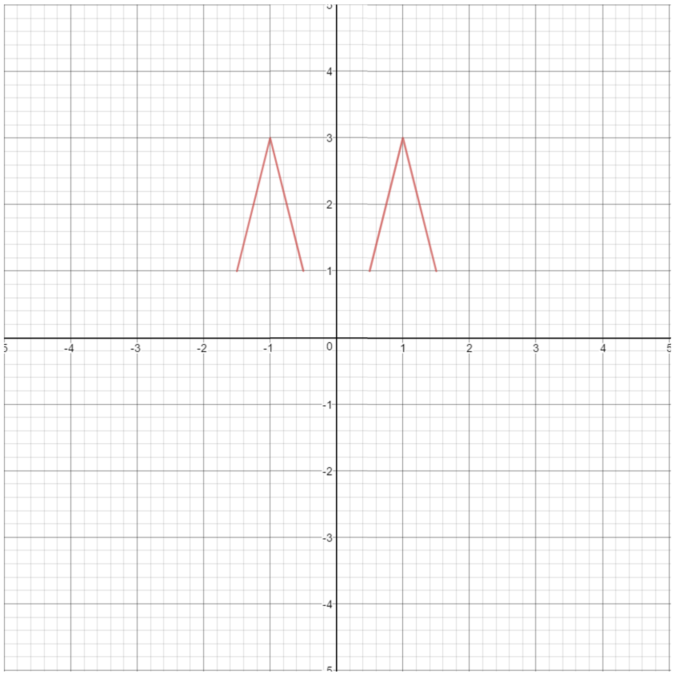
The equation for this line is $y = -4x + 7$ and its binary expression $d = 1 - \lceil \frac{|-4x+7-y|}{|-4x+7-y|+1} \rceil$ If we apply 3 OR operators, joining these 4 expressions, we get a new expression $((a + b - ab) + c - (a + b - ab)c) + d - ((a + b - ab) + c - (a + b - ab)c)d$. If we set this equal to 1 and plot it, it looks like



As previously mentioned, we need only the parts of these lines where y is between 1 and 3. In other words, where $y \geq 1$ and $y \leq 3$. The binary expressions for these are $1 - \lceil \frac{|y-1-|y-1||}{|y-1-|y-1||+1} \rceil$ and $1 - \lceil \frac{|3-y-|3-y||}{|3-y-|3-y||+1} \rceil$ respectively. We need to join these two and the expression for the 4 eye lines above using two AND operations. The resulting expression is

$$(1 - \lceil \frac{|y-1-|y-1||}{|y-1-|y-1||+1} \rceil)(1 - \lceil \frac{|3-y-|3-y||}{|3-y-|3-y||+1} \rceil)(((a + b - ab) + c - (a + b - ab)c) + d - ((a + b - ab) + c - (a + b - ab)c)d)$$

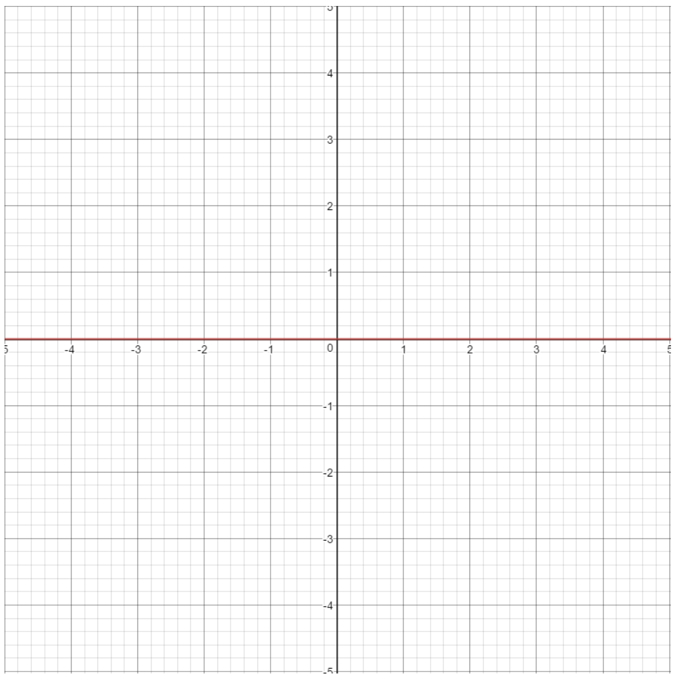
If we set this equal to 1 and plot it it becomes



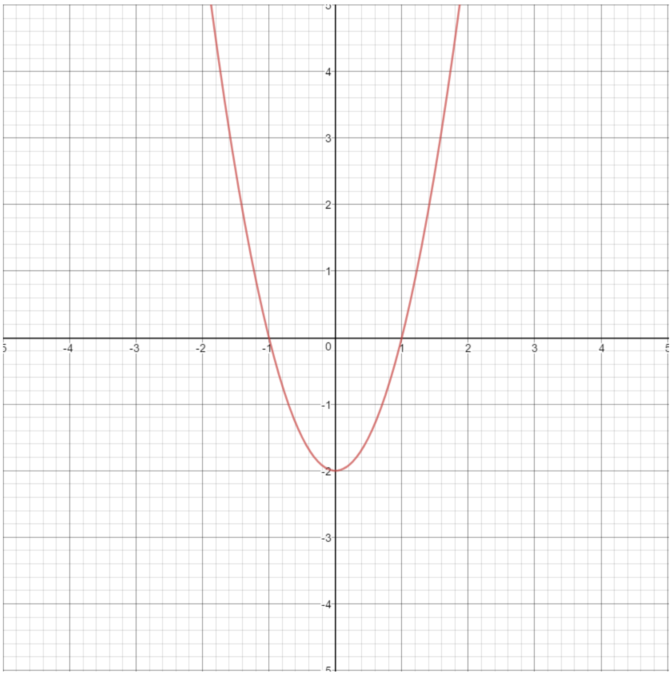
We can substitute a, b, c and d in with what they were defined as above. This makes the expression

$$(1-\lceil\frac{|y-1-|y-1||}{|y-1-|y-1||+1}\rceil)(1-\lceil\frac{|3-y-|3-y||}{|3-y-|3-y||+1}\rceil)((((1-\lceil\frac{|4x+7-y|}{|4x+7-y|+1}\rceil)+(1-\lceil\frac{|-4x-1-y|}{|-4x-1-y|+1}\rceil)-(1-\lceil\frac{|4x+7-y|}{|4x+7-y|+1}\rceil)(1-\lceil\frac{|-4x-1-y|}{|-4x-1-y|+1}\rceil))+(1-\lceil\frac{|4x-1-y|}{|4x-1-y|+1}\rceil)-((1-\lceil\frac{|4x+7-y|}{|4x+7-y|+1}\rceil)+(1-\lceil\frac{|-4x-1-y|}{|-4x-1-y|+1}\rceil)-(1-\lceil\frac{|4x+7-y|}{|4x+7-y|+1}\rceil)(1-\lceil\frac{|-4x-1-y|}{|-4x-1-y|+1}\rceil))(1-\lceil\frac{|4x-1-y|}{|4x-1-y|+1}\rceil))+(1-\lceil\frac{|-4x+7-y|}{|-4x+7-y|+1}\rceil)-(((1-\lceil\frac{|4x+7-y|}{|4x+7-y|+1}\rceil)+(1-\lceil\frac{|-4x-1-y|}{|-4x-1-y|+1}\rceil)-(1-\lceil\frac{|4x+7-y|}{|4x+7-y|+1}\rceil)(1-\lceil\frac{|-4x-1-y|}{|-4x-1-y|+1}\rceil))+(1-\lceil\frac{|4x-1-y|}{|4x-1-y|+1}\rceil)-((1-\lceil\frac{|4x+7-y|}{|4x+7-y|+1}\rceil)+(1-\lceil\frac{|-4x-1-y|}{|-4x-1-y|+1}\rceil)-(1-\lceil\frac{|4x+7-y|}{|4x+7-y|+1}\rceil)(1-\lceil\frac{|-4x-1-y|}{|-4x-1-y|+1}\rceil))(1-\lceil\frac{|4x-1-y|}{|4x-1-y|+1}\rceil))(1-\lceil\frac{|-4x+7-y|}{|-4x+7-y|+1}\rceil))$$

We can call this expression e and if you set it equal to 1 and plot it, it makes the same graph as above. Next we need the equation for the roof of the mouth. The line looks like



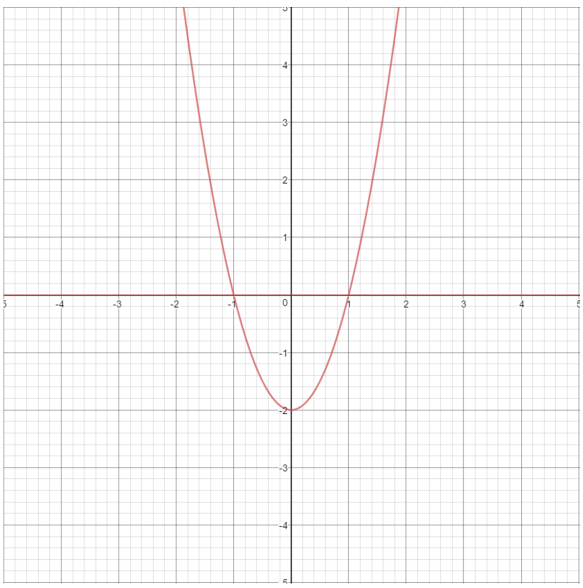
and the equation is $y = 0$. Although we only need the section between $x = -1$ and $x = 1$, this same condition applies to the other part of the mouth as well, so we can apply it later. The boolean expression for $y = 0$ is $1 - \lceil \frac{|y|}{|y|+1} \rceil$. We can call this f. The curve of the mouth looks like this:



The equation for it is $y = 2x^2 - 2$ and its binary expression is $1 - \lceil \frac{|2x^2-2-y|}{|2x^2-2-y|+1} \rceil$. We can call this g. If we join these with an OR operator we get the expression $f + g - fg$, and if we substitute in f and g the expression becomes

$$(1 - \lceil \frac{|y|}{|y|+1} \rceil) + (1 - \lceil \frac{|2x^2-2-y|}{|2x^2-2-y|+1} \rceil) - (1 - \lceil \frac{|y|}{|y|+1} \rceil)(1 - \lceil \frac{|2x^2-2-y|}{|2x^2-2-y|+1} \rceil)$$

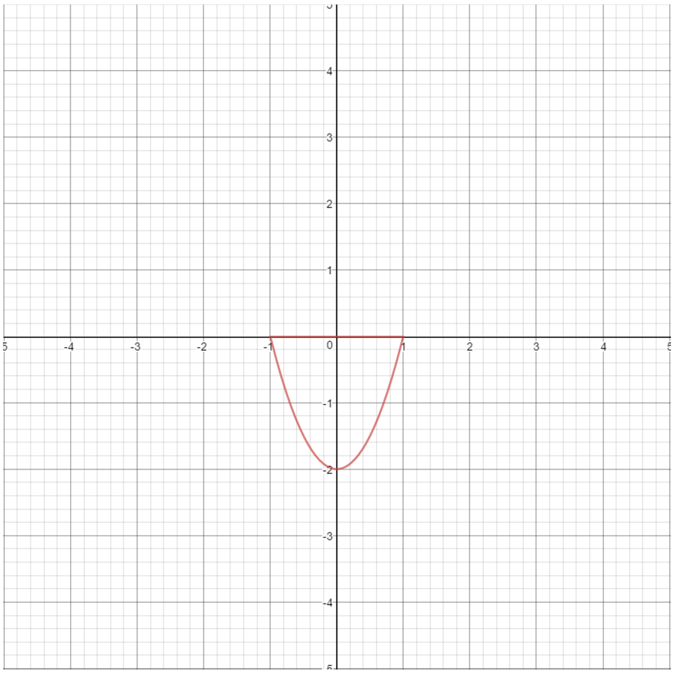
And if we set this equal to 1 and plot it, the graph is



However, we only need the parts of these lines where $y \geq -1$ and $y \leq 1$. The binary expressions for these are $1 - \lceil \frac{|y+1-|y+1||}{|y+1-|y+1||+1} \rceil$ and $1 - \lceil \frac{|y-1-|y-1||}{|y-1-|y-1||+1} \rceil$ respectively. If we multiply these together and multiply the result by the expression above, we get

$$(1 - \lceil \frac{|y+1-|y+1||}{|y+1-|y+1||+1} \rceil)(1 - \lceil \frac{|y-1-|y-1||}{|y-1-|y-1||+1} \rceil)((1 - \lceil \frac{|y|}{|y|+1} \rceil) + (1 - \lceil \frac{|2x^2-2-y|}{|2x^2-2-y|+1} \rceil) - (1 - \lceil \frac{|y|}{|y|+1} \rceil)(1 - \lceil \frac{|2x^2-2-y|}{|2x^2-2-y|+1} \rceil))$$

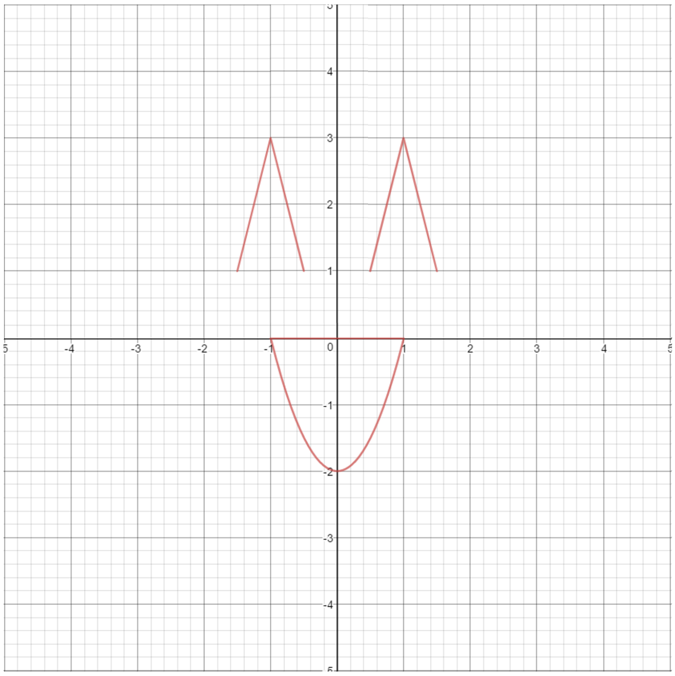
We can call this h and when we set it equal to 1 and plot, it looks like



If we use an OR operation on h and e, we get the expression $h+e-he$ and if we substitute in h and e, we get the expression

$$((1-\lceil\frac{|y+1-|y+1||}{|y+1-|y+1||+1}\rceil)(1-\lceil\frac{|y-1-|y-1||}{|y-1-|y-1||+1}\rceil)((1-\lceil\frac{|y|}{|y|+1}\rceil)+(1-\lceil\frac{\left|2x^2-2-y\right|}{|2x^2-2-y|+1}\rceil)-(1-\lceil\frac{|y|}{|y|+1}\rceil)(1-\lceil\frac{\left|2x^2-2-y\right|}{|2x^2-2-y|+1}\rceil)))+((1-\lceil\frac{|y-1-|y-1||}{|y-1-|y-1||+1}\rceil)(1-\lceil\frac{|3-y-|3-y||}{|3-y-|3-y||+1}\rceil)((((1-\lceil\frac{|4x+7-y|}{|4x+7-y|+1}\rceil)+(1-\lceil\frac{|-4x-1-y|}{|-4x-1-y|+1}\rceil)-(1-\lceil\frac{|4x+7-y|}{|4x+7-y|+1}\rceil)(1-\lceil\frac{|-4x-1-y|}{|-4x-1-y|+1}\rceil))+(1-\lceil\frac{|4x-1-y|}{|4x-1-y|+1}\rceil)-((1-\lceil\frac{|4x+7-y|}{|4x+7-y|+1}\rceil)+(1-\lceil\frac{|-4x-1-y|}{|-4x-1-y|+1}\rceil)-(1-\lceil\frac{|4x+7-y|}{|4x+7-y|+1}\rceil)(1-\lceil\frac{|-4x-1-y|}{|-4x-1-y|+1}\rceil))(1-\lceil\frac{|4x-1-y|}{|4x-1-y|+1}\rceil))+(1-\lceil\frac{|-4x+7-y|}{|-4x+7-y|+1}\rceil)-(((1-\lceil\frac{|4x+7-y|}{|4x+7-y|+1}\rceil)+(1-\lceil\frac{|-4x-1-y|}{|-4x-1-y|+1}\rceil)-(1-\lceil\frac{|4x+7-y|}{|4x+7-y|+1}\rceil)(1-\lceil\frac{|-4x-1-y|}{|-4x-1-y|+1}\rceil))+(1-\lceil\frac{|4x-1-y|}{|4x-1-y|+1}\rceil)-((1-\lceil\frac{|4x+7-y|}{|4x+7-y|+1}\rceil)+(1-\lceil\frac{|-4x-1-y|}{|-4x-1-y|+1}\rceil)-(1-\lceil\frac{|4x+7-y|}{|4x+7-y|+1}\rceil)(1-\lceil\frac{|-4x-1-y|}{|-4x-1-y|+1}\rceil))(1-\lceil\frac{|4x-1-y|}{|4x-1-y|+1}\rceil))(1-\lceil\frac{|-4x+7-y|}{|-4x+7-y|+1}\rceil)))-((1-\lceil\frac{|y+1-|y+1||}{|y+1-|y+1||+1}\rceil)(1-\lceil\frac{|y-1-|y-1||}{|y-1-|y-1||+1}\rceil)((1-\lceil\frac{|y|}{|y|+1}\rceil)+(1-\lceil\frac{\left|2x^2-2-y\right|}{|2x^2-2-y|+1}\rceil)-(1-\lceil\frac{|y|}{|y|+1}\rceil)(1-\lceil\frac{\left|2x^2-2-y\right|}{|2x^2-2-y|+1}\rceil)))((1-\lceil\frac{|y-1-|y-1||}{|y-1-|y-1||+1}\rceil)(1-\lceil\frac{|3-y-|3-y||}{|3-y-|3-y||+1}\rceil)((((1-\lceil\frac{|4x+7-y|}{|4x+7-y|+1}\rceil)+(1-\lceil\frac{|-4x-1-y|}{|-4x-1-y|+1}\rceil)-(1-\lceil\frac{|4x+7-y|}{|4x+7-y|+1}\rceil)(1-\lceil\frac{|-4x-1-y|}{|-4x-1-y|+1}\rceil))+(1-\lceil\frac{|4x-1-y|}{|4x-1-y|+1}\rceil)-((1-\lceil\frac{|4x+7-y|}{|4x+7-y|+1}\rceil)+(1-\lceil\frac{|-4x-1-y|}{|-4x-1-y|+1}\rceil)-(1-\lceil\frac{|4x+7-y|}{|4x+7-y|+1}\rceil)(1-\lceil\frac{|-4x-1-y|}{|-4x-1-y|+1}\rceil))(1-\lceil\frac{|4x-1-y|}{|4x-1-y|+1}\rceil))+(1-\lceil\frac{|-4x+7-y|}{|-4x+7-y|+1}\rceil)-(((1-\lceil\frac{|4x+7-y|}{|4x+7-y|+1}\rceil)+(1-\lceil\frac{|-4x-1-y|}{|-4x-1-y|+1}\rceil)-(1-\lceil\frac{|4x+7-y|}{|4x+7-y|+1}\rceil)(1-\lceil\frac{|-4x-1-y|}{|-4x-1-y|+1}\rceil))+(1-\lceil\frac{|4x-1-y|}{|4x-1-y|+1}\rceil)-((1-\lceil\frac{|4x+7-y|}{|4x+7-y|+1}\rceil)+(1-\lceil\frac{|-4x-1-y|}{|-4x-1-y|+1}\rceil)-(1-\lceil\frac{|4x+7-y|}{|4x+7-y|+1}\rceil)(1-\lceil\frac{|-4x-1-y|}{|-4x-1-y|+1}\rceil))(1-\lceil\frac{|4x-1-y|}{|4x-1-y|+1}\rceil))(1-\lceil\frac{|-4x+7-y|}{|-4x+7-y|+1}\rceil)))$$

If we set this expression equal to 1 and plot it on a graph, it makes the entire smiley face equation shown below



Therefore our final equation which plots the smiley face is

$$((1-\lceil\frac{|y+1-|y+1||}{|y+1-|y+1||+1}\rceil)(1-\lceil\frac{|y-1-|y-1||}{|y-1-|y-1||+1}\rceil)((1-\lceil\frac{|y|}{|y|+1}\rceil)+(1-\lceil\frac{|2x^2-2-y|}{|2x^2-2-y|+1}\rceil)-(1-\lceil\frac{|y|}{|y|+1}\rceil)(1-\lceil\frac{|2x^2-2-y|}{|2x^2-2-y|+1}\rceil)))+((1-\lceil\frac{|y-1-|y-1||}{|y-1-|y-1||+1}\rceil)(1-\lceil\frac{|3-y-|3-y||}{|3-y-|3-y||+1}\rceil)((((1-\lceil\frac{|4x+7-y|}{|4x+7-y|+1}\rceil)+(1-\lceil\frac{|-4x-1-y|}{|-4x-1-y|+1}\rceil)-(1-\lceil\frac{|4x+7-y|}{|4x+7-y|+1}\rceil)(1-\lceil\frac{|-4x-1-y|}{|-4x-1-y|+1}\rceil))+(1-\lceil\frac{|4x-1-y|}{|4x-1-y|+1}\rceil)-((1-\lceil\frac{|4x+7-y|}{|4x+7-y|+1}\rceil)+(1-\lceil\frac{|-4x-1-y|}{|-4x-1-y|+1}\rceil)-(1-\lceil\frac{|4x+7-y|}{|4x+7-y|+1}\rceil)(1-\lceil\frac{|-4x-1-y|}{|-4x-1-y|+1}\rceil))(1-\lceil\frac{|4x-1-y|}{|4x-1-y|+1}\rceil))+(1-\lceil\frac{|-4x+7-y|}{|-4x+7-y|+1}\rceil)-(((1-\lceil\frac{|4x+7-y|}{|4x+7-y|+1}\rceil)+(1-\lceil\frac{|-4x-1-y|}{|-4x-1-y|+1}\rceil)-(1-\lceil\frac{|4x+7-y|}{|4x+7-y|+1}\rceil)(1-\lceil\frac{|-4x-1-y|}{|-4x-1-y|+1}\rceil))+(1-\lceil\frac{|4x-1-y|}{|4x-1-y|+1}\rceil)-((1-\lceil\frac{|4x+7-y|}{|4x+7-y|+1}\rceil)+(1-\lceil\frac{|-4x-1-y|}{|-4x-1-y|+1}\rceil)-(1-\lceil\frac{|4x+7-y|}{|4x+7-y|+1}\rceil)(1-\lceil\frac{|-4x-1-y|}{|-4x-1-y|+1}\rceil))(1-\lceil\frac{|4x-1-y|}{|4x-1-y|+1}\rceil))(1-\lceil\frac{|-4x+7-y|}{|-4x+7-y|+1}\rceil)))-((1-\lceil\frac{|y+1-|y+1||}{|y+1-|y+1||+1}\rceil)(1-\lceil\frac{|y-1-|y-1||}{|y-1-|y-1||+1}\rceil)((1-\lceil\frac{|y|}{|y|+1}\rceil)+(1-\lceil\frac{|2x^2-2-y|}{|2x^2-2-y|+1}\rceil)-(1-\lceil\frac{|y|}{|y|+1}\rceil)(1-\lceil\frac{|2x^2-2-y|}{|2x^2-2-y|+1}\rceil)))((1-\lceil\frac{|y-1-|y-1||}{|y-1-|y-1||+1}\rceil)(1-\lceil\frac{|3-y-|3-y||}{|3-y-|3-y||+1}\rceil)((((1-\lceil\frac{|4x+7-y|}{|4x+7-y|+1}\rceil)+(1-\lceil\frac{|-4x-1-y|}{|-4x-1-y|+1}\rceil)-(1-\lceil\frac{|4x+7-y|}{|4x+7-y|+1}\rceil)(1-\lceil\frac{|-4x-1-y|}{|-4x-1-y|+1}\rceil))+(1-\lceil\frac{|4x-1-y|}{|4x-1-y|+1}\rceil)-((1-\lceil\frac{|4x+7-y|}{|4x+7-y|+1}\rceil)+(1-\lceil\frac{|-4x-1-y|}{|-4x-1-y|+1}\rceil)-(1-\lceil\frac{|4x+7-y|}{|4x+7-y|+1}\rceil)(1-\lceil\frac{|-4x-1-y|}{|-4x-1-y|+1}\rceil))(1-\lceil\frac{|4x-1-y|}{|4x-1-y|+1}\rceil))+(1-\lceil\frac{|-4x+7-y|}{|-4x+7-y|+1}\rceil)-(((1-\lceil\frac{|4x+7-y|}{|4x+7-y|+1}\rceil)+(1-\lceil\frac{|-4x-1-y|}{|-4x-1-y|+1}\rceil)-(1-\lceil\frac{|4x+7-y|}{|4x+7-y|+1}\rceil)(1-\lceil\frac{|-4x-1-y|}{|-4x-1-y|+1}\rceil))+(1-\lceil\frac{|4x-1-y|}{|4x-1-y|+1}\rceil)-((1-\lceil\frac{|4x+7-y|}{|4x+7-y|+1}\rceil)+(1-\lceil\frac{|-4x-1-y|}{|-4x-1-y|+1}\rceil)-(1-\lceil\frac{|4x+7-y|}{|4x+7-y|+1}\rceil)(1-\lceil\frac{|-4x-1-y|}{|-4x-1-y|+1}\rceil))(1-\lceil\frac{|4x-1-y|}{|4x-1-y|+1}\rceil))(1-\lceil\frac{|-4x+7-y|}{|-4x+7-y|+1}\rceil)))=1$$

6 Conclusion

The contents of this paper show that for any image composed of any number of parts of any geometric shapes, there is a single equation which, when plotted on a graph, produces that image. While the framework for creating such an equation described in this paper may not produce the simplest or shortest equation which plots the image, it does prove that one will always exist for any geometric graph. This process is also very easy to automate with code, as in order to combine any equations, there is no need to evaluate them or even calculate anything, only enter addition signs, subtraction signs and brackets. Similarly, you do not need to evaluate an equation in order to convert it into a binary expression. You only need to check which type of equation it is (e.g. equals, less than, greater than) and split it into the left hand side and the right hand side, and then insert them into the binary expression templates defined in the previous sections.