Maths Supervision Work 11

$H = U + pV$

$$\cancel{dH = \left.\frac{\partial H}{\partial U}\right|_p dU + \left.\frac{\partial H}{\partial S}\right|_p dS}$$

$$\cancel{= \left.\frac{\partial}{\partial U}\right|_p (U + pV)}$$

$$dH = \frac{\partial H}{\partial U} dU + \frac{\partial H}{\partial p} dp + \frac{\partial H}{\partial V} dV$$

$$= \frac{\partial}{\partial U}(U + pV)\, dU + \frac{\partial}{\partial p}(U + pV)\, dp + \frac{\partial}{\partial V}(U + pV)\, dV$$

$$= dU + V dp + p dV$$

$$= T dS - p dV + V dp + p dV$$

$$= T dS + V dp$$

$$\therefore \left.\frac{\partial T}{\partial p}\right|_S = \left.\frac{\partial V}{\partial S}\right|_p$$

$U = H - pV$

$$\frac{\partial U}{\partial p} = \frac{\partial H}{\partial p} - V = T\frac{\partial S}{\partial p} + V - V = T\frac{\partial S}{\partial p}$$

$$\frac{\partial^2 U}{\partial V \partial p} = \frac{\partial T}{\partial V} \cdot \frac{\partial S}{\partial p}$$

$$\frac{\partial^2 U}{\partial V} = \frac{\partial H}{\partial V} - p = T\frac{\partial S}{\partial V} - p$$

$$\frac{\partial^2 U}{\partial p \partial V} = \frac{\partial T}{\partial p}\frac{\partial S}{\partial V} - 1 = \frac{\partial T}{\partial V} \cdot \frac{\partial S}{\partial p}$$

$$\therefore \left.\frac{\partial S}{\partial V}\right|_p \cdot \left.\frac{\partial T}{\partial p}\right|_V - \left.\frac{\partial S}{\partial p}\right|_V \left.\frac{\partial T}{\partial V}\right|_p = 1$$

11. $\cancel{\frac{\partial T}{\partial V} = -\frac{\partial p}{\partial S}} \quad \cancel{\frac{\partial V}{\partial T} = \frac{\partial S}{\partial p}}$

$\cancel{\frac{\partial G}{\partial p} = V}$

$\therefore \cancel{G = \int V\,dp + \text{constant}}$

$dU = T\,dS - p\,dV$

$\therefore \frac{\partial U}{\partial S} = T$

$\frac{\partial^2 U}{\partial T \partial S} = 1 \implies \frac{\partial U}{\partial T} = S + C(V)$

$\frac{\partial U}{\partial V} = -p$

$\therefore \frac{\partial^2 U}{\partial p \partial V} = -1 \implies \frac{\partial U}{\partial p} = -V + D(S)$

$\therefore dU = (-V + D(S))\,dp + (S + C(V))\,dT$

$\therefore$ Let $G = -U$ with $C(V) = 0 = D(S) \;\forall V, S$

$\therefore dG = V\,dp - S\,dT$

$\therefore \frac{\partial V}{\partial T} = -\frac{\partial S}{\partial p}$