

Maths Supervision Work 10

7. $xyz + x^3 + y^4 + z^5 = 0$

$$\therefore \frac{\partial}{\partial y} \Big|_z (xyz + x^3 + y^4 + z^5) = 0$$

$$\therefore z \frac{\partial}{\partial y} \Big|_z (xy) + 3x^2 \frac{\partial x}{\partial y} \Big|_z + 4y^3 = 0$$

$$\therefore z \left(\frac{\partial x}{\partial y} \Big|_z y + x \right) + 3x^2 \frac{\partial x}{\partial y} \Big|_z + 4y^3 = 0$$

$$(3x^2 + yz) \frac{\partial x}{\partial y} \Big|_z + xz + 4y^3 = 0$$

$$\therefore \frac{\partial x}{\partial y} \Big|_z = \frac{-xz - 4y^3}{3x^2 + yz}$$

ok

you can also use directly the differential

$d() = \dots dx + \dots dy + \dots dz$ then pick the one you are interested in

$$\frac{\partial}{\partial z} \Big|_x (xyz + x^3 + y^4 + z^5) = 0$$

$$\therefore x \frac{\partial}{\partial z} \Big|_x (yz) + 4y^3 \frac{\partial y}{\partial z} \Big|_x + 5z^4 = 0$$

$$\therefore x \left(\frac{\partial y}{\partial z} \Big|_x \cdot z + y \right) + 4y^3 \frac{\partial y}{\partial z} \Big|_x + 5z^4 = 0$$

$$\therefore (xz + 4y^3) \frac{\partial y}{\partial z} \Big|_x + xy + 5z^4 = 0$$

$$\therefore \frac{\partial y}{\partial z} \Big|_x = \frac{-xy - 5z^4}{xz + 4y^3}$$

$$\frac{\partial}{\partial x} \Big|_y (xyz + x^3 + y^4 + z^5) = 0$$

$$\therefore y \frac{\partial}{\partial x} \Big|_y (xz) + 3x^2 + 5z^4 \frac{\partial z}{\partial x} \Big|_y = 0$$

$$\therefore y \left(z + x \frac{\partial z}{\partial x} \Big|_y \right) + 3x^2 + 5z^4 \frac{\partial z}{\partial x} \Big|_y = 0$$

$$\therefore (yz + 5z^4) \frac{\partial z}{\partial x} \Big|_y + 3x^2 + yz = 0$$

$$\therefore \frac{\partial z}{\partial x} \Big|_y = \frac{-3x^2 - yz}{yz + 5z^4}$$

$$\begin{aligned}
 & \therefore \frac{\partial z}{\partial y} \bigg|_z = \frac{\partial y}{\partial z} \bigg|_x \cdot \frac{\partial z}{\partial x} \bigg|_y \\
 & = \left(\frac{-xz - 4y^3}{3x^2 + yz} \right) \left(\frac{-xy - 5z^4}{xz + 4y^3} \right) \left(\frac{-3x^2 - yz}{xy + 5z^4} \right) \\
 & = - \frac{(xz + 4y^3)(xy + 5z^4)(3x^2 + yz)}{(3x^2 + yz)(xz + 4y^3)(xy + 5z^4)} \\
 & = -1
 \end{aligned}$$

8. $\hat{i} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$ $\hat{j} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$ are basis vectors in x - y space

$\hat{u} = \begin{pmatrix} \cos\theta \\ \sin\theta \end{pmatrix}$ $\hat{v} = \begin{pmatrix} -\sin\theta \\ \cos\theta \end{pmatrix}$ are basis vectors in u - v space
 you have to show how you pick these ones, diagram maybe??

Consider a point $\vec{p} = x\hat{i} + y\hat{j} = u\hat{u} + v\hat{v}$

$$\begin{aligned}
 u &= \vec{p} \cdot \hat{u} = (x\hat{i} + y\hat{j}) \cdot \hat{u} = \begin{pmatrix} x \\ y \end{pmatrix} \cdot \begin{pmatrix} \cos\theta \\ \sin\theta \end{pmatrix} = x\cos\theta + y\sin\theta \\
 v &= \vec{p} \cdot \hat{v} = (x\hat{i} + y\hat{j}) \cdot \hat{v} = \begin{pmatrix} x \\ y \end{pmatrix} \cdot \begin{pmatrix} -\sin\theta \\ \cos\theta \end{pmatrix} = -x\sin\theta + y\cos\theta
 \end{aligned}$$

$$\begin{aligned}
 x &= \vec{p} \cdot \hat{i} = (u\hat{u} + v\hat{v}) \cdot \hat{i} = \begin{pmatrix} u\cos\theta - v\sin\theta \\ u\sin\theta + v\cos\theta \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 0 \end{pmatrix} \\
 &= u\cos\theta - v\sin\theta
 \end{aligned}$$

$$\begin{aligned}
 y &= \vec{p} \cdot \hat{j} = \begin{pmatrix} u\cos\theta - v\sin\theta \\ u\sin\theta + v\cos\theta \end{pmatrix} \cdot \begin{pmatrix} 0 \\ 1 \end{pmatrix} \\
 &= u\sin\theta + v\cos\theta
 \end{aligned}$$

$$\frac{\partial f}{\partial u} = \frac{\partial f}{\partial x} \cdot \frac{\partial x}{\partial u} + \frac{\partial f}{\partial y} \cdot \frac{\partial y}{\partial u}$$

$$= \frac{\partial f}{\partial x} \cos \theta + \frac{\partial f}{\partial y} \sin \theta$$

$$\frac{\partial^2 f}{\partial u^2} = \cos \theta \left(\frac{\partial^2 f}{\partial x^2} \cdot \frac{\partial x}{\partial u} + \frac{\partial^2 f}{\partial x \partial y} \cdot \frac{\partial y}{\partial u} \right) + \sin \theta \left(\frac{\partial^2 f}{\partial y^2} \cdot \frac{\partial y}{\partial u} + \frac{\partial^2 f}{\partial x \partial y} \cdot \frac{\partial x}{\partial u} \right)$$

$$= \cos^2 \theta \frac{\partial^2 f}{\partial x^2} + \sin^2 \theta \frac{\partial^2 f}{\partial y^2} + 2 \cos \theta \sin \theta \frac{\partial^2 f}{\partial x \partial y}$$

$$\frac{\partial f}{\partial v} = \frac{\partial f}{\partial x} \cdot \frac{\partial x}{\partial v} + \frac{\partial f}{\partial y} \cdot \frac{\partial y}{\partial v}$$

$$= \frac{\partial f}{\partial x} (-\sin \theta) + \frac{\partial f}{\partial y} \cdot \cos \theta$$

$$\frac{\partial^2 f}{\partial v^2} = -\sin \theta \left(\frac{\partial^2 f}{\partial x^2} \cdot \frac{\partial x}{\partial v} + \frac{\partial^2 f}{\partial x \partial y} \cdot \frac{\partial y}{\partial v} \right) + \cos \theta \left(\frac{\partial^2 f}{\partial y^2} \cdot \frac{\partial y}{\partial v} + \frac{\partial^2 f}{\partial x \partial y} \cdot \frac{\partial x}{\partial v} \right)$$

$$= \sin^2 \theta \frac{\partial^2 f}{\partial x^2} + \cos^2 \theta \frac{\partial^2 f}{\partial y^2} - 2 \cos \theta \sin \theta \frac{\partial^2 f}{\partial x \partial y}$$

$$\frac{\partial^2 f}{\partial u^2} + \frac{\partial^2 f}{\partial v^2} = (\cos^2 \theta + \sin^2 \theta) \frac{\partial^2 f}{\partial x^2} + (\cos^2 \theta + \sin^2 \theta) \frac{\partial^2 f}{\partial y^2} + (2 \cos \theta \sin \theta - 2 \cos \theta \sin \theta) \frac{\partial^2 f}{\partial x \partial y}$$

$$= \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2}$$

ok