Maths Supervision

6. $f(x) = x^2$ for $-1 \leq x \leq 1$, and by periodicity elsewhere

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} \left(a_n \cos\frac{n\pi x}{1} + b_n \sin\frac{n\pi x}{1}\right)$$

$$a_0 = \int_{-1}^{1} f(x)\,dx = \int_{-1}^{1} x^2\,dx = \left[\frac{x^3}{3}\right]_{-1}^{1} = 2/3$$

$$a_n = \int_{-1}^{1} \cos(n\pi x) x^2\,dx = 2\int_{0}^{1} \cos(n\pi x) x^2\,dx$$ (even integrand)

$$= \frac{4(-1)^n}{n^2\pi^2}$$

$$b_n = \int_{-1}^{1} \sin(n\pi x) x^2\,dx = 0$$ (odd integrand)

$$\therefore f(x) = 1/3 + \sum_{n=1}^{\infty} \frac{4(-1)^n}{n^2\pi^2}\cos(n\pi x)$$ as required.

b. $$f(1) = 1/3 + \sum_{n=1}^{\infty} \frac{4(-1)^n}{n^2\pi^2}\cos(n\pi)$$

$$= \frac{1}{3} + \frac{4}{\pi^2}\sum_{n=1}^{\infty} \frac{(-1)^n(-1)^n}{n^2}$$

$$= \frac{1}{3} + \frac{4}{\pi^2}\sum_{n=1}^{\infty} \frac{1}{n^2}$$

$$= 1^2 = 1$$

$$\therefore 2/3 = \frac{4}{\pi^2}\sum_{n=1}^{\infty} \frac{1}{n^2}$$

$$\therefore \sum_{n=1}^{\infty} \frac{1}{n^2} = \frac{\pi^2}{6}$$ as required.