Morgan
Saville

Probability Supervision 1

2.1 $4! = 24$ elements of the sample space

2.2 "It is certainly not true that neither John nor Mary is to blame": $((J \cup M)^c)^c$ $(\dagger)$

or equivalently $(J^c \cap M^c)^c$

"John or Mary is to blame, or both": $J \cup M$ $(\ddagger)$

Starting with $(\dagger)$, $(J^c \cap M^c)^c = ((J \cup M)^c)^c$
$= J \cup M = (\ddagger)$

2.3. $P(\text{Jan}) = P(\text{Mar}) = P(\text{May}) = P(\text{July}) = P(\text{Aug}) = P(\text{oct}) = P(\text{Dec})$

$= \frac{3}{4} \cdot \frac{31}{365} + \frac{1}{4} \cdot \frac{31}{366} = \frac{93}{1460} + \frac{31}{1464} = \frac{181412}{2137440}$

$= \frac{45353}{534360}$

$P(\text{Apr}) = P(\text{June}) = P(\text{Sept}) = P(\text{Nov})$

$= \frac{3}{4} \cdot \frac{30}{365} + \frac{1}{4} \cdot \frac{30}{366} = \frac{9}{146} + \frac{5}{244} = \frac{1463}{17812}$

$P(\text{Feb}) = \frac{3}{4} \cdot \frac{28}{365} + \frac{1}{4} \cdot \frac{29}{366} = \frac{21}{365} + \frac{29}{1464} = \frac{41329}{534360}$

2.4 Assuming that $P(M) = \frac{1}{12}$ $\forall$ months M,

$P(L) = \frac{7}{12}$ $P(R) = \frac{8}{12} = \frac{2}{3}$

2.5

$P((a_i,a_j))$	1	2	3	4	5	6	i
1	p_1	0	0	0	0	0	
2	0	p_2	0	0	0	0	
3	0	0	p_3	0	0	0	
4	0	0	0	p_4	0	0	
5	0	0	0	0	p_5	0	
6	0	0	0	0	0	p_6	

j

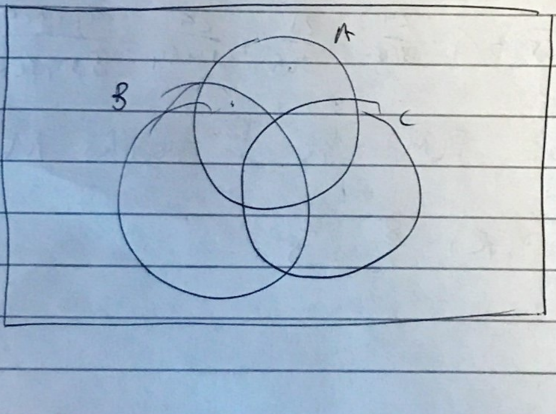
$\therefore \sum_{i,j} P((a_i,a_j)) = \sum_i p_i = 1$

and $\forall_{i,j}\ 0 \le P((a_i,a_j)) \le 1$

$\therefore$ P is a probability function

The outcome of the second experiment is always the same as that of the first experiment

2.15 $P(A \vee B \vee C) = P(A) + P(B) + P(C) - P(A \wedge B) - P(A \cap C) - P(B \wedge C) + 3P(A \wedge B \cap C)$



3.1 $P(N|L) = \frac{P(N \wedge L)}{P(L)} = \frac{(3/12)}{(7/12)} = 3/7$

3.2 $P(C|A) = \frac{P(C \wedge A)}{P(A)} = \frac{(1/3)}{(1/3)} = 1$

$P(C^c | A \cup B) = 1 - P(C | A \cup B)$

$= 1 - \frac{P(C \wedge (A \cup B))}{P(A \cup B)} = 1 - \frac{(1/3)}{(2/3)} = 1 - 1/2 = 1/2$

3.3 $P(A|C) = \frac{P(A \wedge C)}{P(C)} = \frac{(1/3)}{(2/3)} = 1/2$

$P(A^c|C) = \frac{P(A^c \wedge C)}{P(C)} = \frac{(1/3)}{(2/3)} = 1/2$

$\therefore P(A|C) + P(A^c|C) = 1/2 + 1/2 = 1$

3.4 $P(R_3 | R_4^c) = \frac{P(R_3 \wedge R_4^c)}{1 - P(R_4)} = \left(\int_3^4 e^{-t} dt\right) \cdot \frac{1}{1-e^{-4}}$

$= \left[-e^{-t}\right]_3^4 \cdot \frac{1}{1-e^{-4}}$

$= (-e^{-4} + e^{-3}) \cdot \frac{1}{1-e^{-4}}$

$= \frac{e^{-3} - e^{-4}}{1-e^{-4}} = \frac{e-1}{e^4-1}$

4.1 $(\omega_1, \omega_2) \in \{(2,6), (3,5), (4,4), (5,3)(6,2)\}$

4.2. $P(S=4) = 3/36 = 1/12$

$P(S=5) = 4/36 = 1/9$

$P(S=6) = 5/36$

$P(S=7) = 6/36 = 1/6$

$P(S=8) = 5/36$

$P(S=9) = 4/36 = 1/9$

$P(S=10) = 3/36 = 1/12$

$P(S=11) = 2/36 = 1/18$

$P(S=12) = 1/36$