Morgan Saville

Probability Supervision 1

2.1 $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$= 2/3 + 1/6 - 1/9$

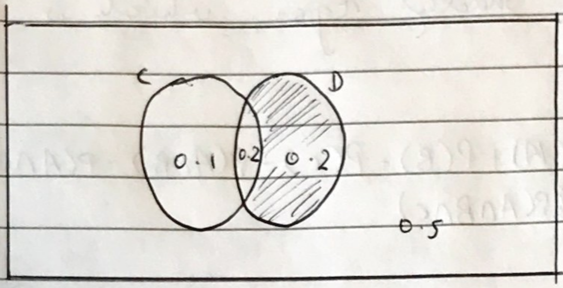
$= 13/18$

2.2 $P(E^c \cap F^c) = P((E \cup F)^c) = 1 - P(E \cup F)$

$= 1 - 3/4$

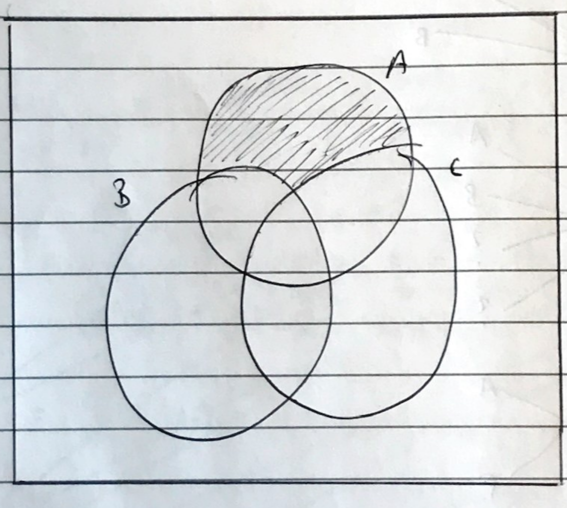
$= 1/4$

2.3



$P(C^c \cap D) = 0.2$

2.4.



$P(A \cap B^c \cap C^c) = P((A^c \cup B \cup C)^c)$

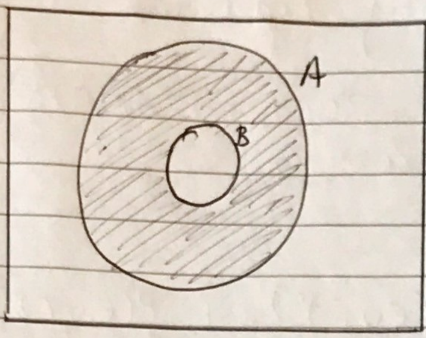
$= 1 - P(A^c \cup B \cup C)$

$= 1 - P(A^c) - P(B \cup C) + P(A^c \cap (B \cup C))$

$= P(A) + P(A^c \cap (B \cup C)) - P(B) - P(C) + P(B \cap C)$

$= P(A \cup B \cup C) - P(B) - P(C) + P(B \cap C)$

2.5

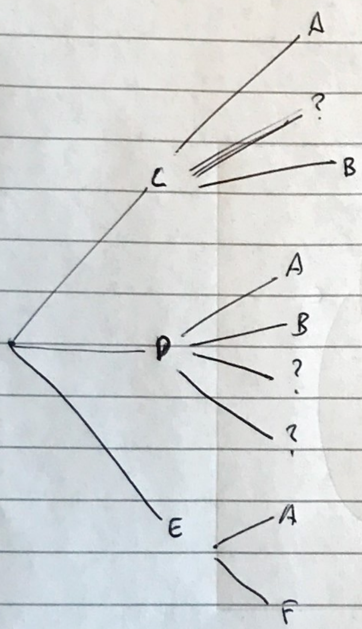


$B \subset A$

$P(A \setminus B)$ is the area of the shaded region, which is clearly $P(A) - P(B)$

2.15 $P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(A \cap C) - P(B \cap C) + P(A \cap B \cap C)$

3.1



$\therefore P(B \text{ after two selections}) = 2/9$

3.2a $P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{2/36}{11/36} = 2/11$

b. $P(A) = 3/36 = 1/12 \neq \frac{2}{11} = P(A|B)$

The probability of A is affected by B so they are not independent

3.3a. $P(S_1) = 1/4$

$P(S_2|S_1) = \frac{12}{51} = \frac{4}{17}$

$P(S_2|S_1^c) = 13/51$

b. $P(S_2) = P(S_2|S_1)P(S_1) + P(S_2|S_1^c)P(S_1^c)$

$= 4/17 \cdot 1/4 + 13/51 \cdot 3/4$

$= 1/17 + 13/68$

$= 1/4$

~~3.4~~ $\cancel{P(B|T)} = \cancel{\frac{P(T|B)P(B)}{P(T)}} = \cancel{\frac{P(T|B)P(B)}{P(T|B)P(B) + P(T|B^c)P(B^c)}}$

$= \cancel{\frac{\frac{7}{10}\cdot\frac{13}{10^6}}{\frac{7}{10}\frac{13}{10^5} + \frac{1}{10}(1-\frac{13}{10^5})}}$ $\cancel{\frac{0.7 \cdot 1.3 \cdot 10^5}{0.7 \cdot 1.3 \cdot 10^5 + 0.1 \cdot (1 - 1.3 \cdot 10^5)}}$

$\cancel{= 9.1 \cdot 10^{-5}}$

$\cancel{P(B|T^c)} =$

$P(B|\cancel{T})P(T) + P(B|\cancel{T^c})P(\cancel{T^c}) = P(B)$

$\therefore P(B|T)$ $\cancel{P(T|B)}$

$$P(T) = P(T \mid B)P(B) + P(T \mid B^c)P(B^c)$$
$$= 0.7 \cdot 1.3 \cdot 10^{-5} + 0.1(1 - 1.3 \cdot 10^{-5})$$
$$= 0.1$$

$$P(B \mid T) = \frac{P(T \mid B)P(B)}{P(T)} = \frac{0.7 \cdot 1.3 \cdot 10^{-5}}{0.1}$$
$$= 9.1 \cdot 10^{-5}$$

$$P(B \mid T)P(T) + P(B \mid T^c)P(T^c) = P(B)$$
$$\therefore P(B \mid T^c) = \frac{P(B) - P(B \mid T)P(T)}{1 - P(T)}$$
$$= \frac{1.3 \cdot 10^{-5} - 9.1 \cdot 10^{-5} \cdot 0.1}{0.9}$$
$$= 4.3 \cdot 10^{-6}$$

6.1 $Z \sim B(2, 1/6)$

$$P(Z = k) = \binom{2}{k} \left(\frac{1}{6}\right)^k \left(\frac{5}{6}\right)^{2-k}$$
$$= \frac{2}{k!(2-k)!} \frac{5^{2-k}}{36} \quad \text{for } 0 \leq k \leq 2$$
$$= \frac{5^{2-k}}{18\, k!\,(2-k)!}$$

a	0	1	2
$p(a)$	$\frac{25}{36}$	$\frac{10}{36}$	$\frac{1}{36}$

b $\{M = 2, Z = 0\} = \{(1,2), (2,1)\}$ with probability $2/36 = 1/18$

$\{S = 5, Z = 1\} = \emptyset$ with probability 0

$\{S = 8, Z = 1\} = \{(2,6), (6,2)\}$ with probability $1/18$

c. $P(M=2) = \frac{4}{36} = 1/9$

$P(M=2 \wedge Z=0) = \frac{4}{36} = 1/9$

$\therefore P(M=2 \mid Z=0) = \frac{P(M=2 \wedge Z=0)}{P(Z=0)} = \frac{(1/9)}{(\frac{25}{36})} = \frac{4}{25} \neq 1/9$

$\therefore$ The probability that $M=2$ is affected by knowing whether $Z=0$
so they are <u>not</u> independent

Q. 2a

a	0	1	4
$p(a)$	1/4	1/4	1/2

with $p(a)=0$ for all other a

b.

a	1	3/4	$\pi - 3$
$F_X(a)$	1/2	3/8	3/8
$F_Y(a)$	1/2	1/4	1/4