A point (a, b, c) lies on the side of the cylinder iff it satisfies the following equations

① $z_b \leq c \leq z_b + h$

② $(a - x_b)^2 + (b - y_b)^2 = r^2$

where r is the radius of the cylinder

$\therefore$ Let (x_p, y_p, z_p) be a point on the ray with parameter s which lies on the cylinder

$$x_p = x_0 + s x_d$$
$$y_p = y_0 + s y_d$$
$$z_p = z_0 + s z_d$$

From ②,

$$(x_0 + s x_d - x_b)^2 + (y_0 + s y_d - y_b)^2 = r^2$$

$$\therefore x_0^2 + s^2 x_d^2 + x_b^2 + 2 s x_0 x_d - 2 s x_d x_b - 2 x_0 x_b$$
$$+ y_0^2 + s^2 y_d^2 + y_b^2 + 2 s y_0 y_d - 2 s y_d y_b - 2 y_0 y_b = r^2$$

$$\therefore (x_d^2 + y_d^2) s^2 + (2 x_0 x_d - 2 x_d x_b + 2 y_0 y_d - 2 y_d y_b) s + (-2 x_0 x_b - 2 y_b y_b - r^2$$
$$= 0$$

$$\therefore s = \frac{-\beta \pm \sqrt{\beta^2 - 4\alpha\gamma}}{2\gamma} \quad \text{where } \alpha = x_d^2 + y_d^2$$

$$\beta = 2 x_0 x_d - 2 x_d x_b + 2 y_0 y_d - 2 y_d y_b$$

$$\gamma = -2 x_0 x_b - 2 y_0 y_b - r^2$$

And the solution is valid iff $s \geq 0 \wedge 0 \leq z_0 + s z_d - z_b \leq h$