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Maths Supervision Work 1

5a. Seperable

$$\frac{1}{y^2+1}\frac{dy}{dx} = \frac{1}{\cos^2 x}$$

$$\therefore \int \frac{1}{y^2+1}\frac{dy}{dx}\,dx = \int \frac{1}{\cos^2 x}\,dx$$

$$\therefore \arctan(y) = \tan(x) + C$$ ok

$$\therefore y = \tan(\tan x + C)$$

$$y(0) = \tan(\tan 0 + c)$$
$$= \tan C$$
$$= 0$$

$\therefore$ $C = \arctan 0 = k\pi$ for some integer k, w.l.o.g. take $k = 0$

$$\therefore C = 0$$

$$\therefore y = \tan(\tan(x))$$

b. Seperable

$$\frac{dy}{dx} + 4xy = 2x(y^2+1)$$

$$\therefore \frac{dy}{dx} = 2x(y^2+1) - 4xy$$
$$= 2x(y^2+1-2y)$$
$$= 2x(y-1)^2$$

$$\therefore \frac{1}{(y-1)^2}\frac{dy}{dx} = 2x$$

$$\therefore \int \frac{1}{(y-1)^2}\frac{dy}{dx}\,dx = \int 2x\,dx$$

$$\therefore -\frac{1}{y-1} = x^2 + C$$

$$\therefore y - 1 = -\frac{1}{x^2+C}$$

$$y = 1 - \frac{1}{x^2+C}$$

$y(0) = 1 - \frac{1}{c} = 0$

$\therefore \frac{1}{c} = 1$

$\therefore c = 1$

ok

$\therefore y = 1 - \frac{1}{x^2+1}$

c. Linear

$x\frac{dy}{dx} + (x-1)y = e^{-x}$

$\therefore \frac{dy}{dx} + \frac{x-1}{x}y = \frac{1}{x}e^{-x}$

Integrating factor $= e^{\int \frac{x-1}{x}dx} = e^{\int (1-\frac{1}{x})dx} = e^{x - \ln|x|} = \frac{e^x}{|x|}$

$\therefore \frac{e^x}{|x|}\frac{dy}{dx} + \frac{e^x}{|x|}\cdot\frac{x-1}{x}y = \frac{e^x}{|x|}\cdot\frac{1}{x}e^{-x}$

$\therefore \frac{d}{dx}\left(\frac{e^x}{|x|}y\right) = \frac{1}{x|x|}$

$\therefore \frac{e^x}{|x|}y = \int \frac{1}{x|x|}dx = C - \frac{1}{|x|}$

$\therefore e^x y = C|x| - 1$

$\therefore y = \frac{c|x|-1}{e^x}$

$y(1) = \frac{C-1}{e} = 0$

$\therefore C = 1$

$\therefore y = \frac{|x|-1}{e^x}$

ok...try also finding a general solution to
the initial equation, then a particular solution and justify
the choices you make

d. ~~Linear~~

$$\cancel{(1+x^3)\frac{dy}{dx} - x^2 y = x^2}$$

$$\cancel{\therefore \frac{dy}{dx} = \frac{-x^2}{1+x^3} y = \frac{x^2}{1+x^3}}$$

Seperable

$$(1+x^3)\frac{dy}{dx} = x^2(1+y)$$

$$\therefore \frac{1}{1+y}\frac{dy}{dx} = \frac{x^2}{1+x^3} \quad \text{or } y=-1$$

$$\therefore \int \frac{1}{1+y}\frac{dy}{dx}\,dx = \int \frac{x^2}{1+x^3}\,dx \quad \text{or } y=-1$$

$$\therefore \ln|1+y| = \tfrac{1}{3}\ln|1+x^3| + C \quad \text{or } y=-1$$

$$\therefore 1+y = \pm e^{\ln|(1+x^3)^{1/3}|+C} \quad \text{or } y=-1$$

$$= A|(1+x^3)^{1/3}|$$

$$\therefore y = A|(1+x^3)^{1/3}| - 1$$

$$y(0) = A|1^{1/3}| - 1$$
$$= A-1$$
$$= 0$$

$$\therefore A = 1$$

ok

$$\therefore y = |(1+x^3)^{1/3}| - 1$$

6. $(y-x)\frac{dy}{dx} + (2x+3y) = 0$

$$\therefore \left(\frac{y}{x}-1\right)\frac{dy}{dx} + \left(2+3\frac{y}{x}\right) = 0$$

$$\therefore \frac{dy}{dx} = \frac{-2-3\frac{y}{x}}{y/x - 1} = \frac{2+3\frac{y}{x}}{1-y/x}$$

Let $u = y/x$, $\therefore \frac{dy}{dx} = \frac{d}{dx}ux = u + x\frac{du}{dx}$

$$\therefore u + x\frac{du}{dx} = \frac{2+3u}{1-u}$$

$$\therefore x\frac{du}{dx} = \frac{2+3u}{1-u} - \frac{u-u^2}{1-u} = \frac{2+2u+u^2}{1-u}$$

$$\therefore \frac{1-u}{2+2u+u^2}\frac{du}{dx} = \frac{1}{x}$$ ok

$$\therefore \int \frac{1-u}{2+2u+u^2}\frac{du}{dx}\,dx = \int \frac{1}{x}\,dx$$

Let $I = \int \frac{1-u}{2+2u+u^2}\,du$

$$= \int \frac{2}{2+2u+u^2}\,du - \tfrac{1}{2}\ln|2+2u+u^2|$$

$$= \int \frac{2}{(u+1)^2+1}\,du - \tfrac{1}{2}\ln|2+2u+u^2|$$

$$= 2\arctan(u+1) - \tfrac{1}{2}\ln|2+2u+u^2| + 2C$$ ok

$$\therefore 2\arctan(u+1) - \tfrac{1}{2}\ln|2+2u+u^2| = \ln|x| + 2C$$

$$\therefore 2\arctan(u+1) = 2C + \ln|x(2+2u+u^2)^{1/2}|$$

$$\cancel{= 2C + \ln|(2x^2+2x+1)^{1/2}|}$$

$$\cancel{= 2C + \ln|x+1|}$$

$$\cancel{\therefore u+1 = \tan(C + \tfrac{1}{2}\ln|x+1|)}$$

$$\cancel{\therefore u = \tan(C + \tfrac{1}{2}\ln|x+1|) - 1}$$

$$\cancel{\therefore y = x\tan(C + \tfrac{1}{2}\ln|x+1|) - x}$$

$$\therefore 2\arctan(u+1) = 2C + \ln|(2x^2+2xy+y^2)^{1/2}|$$

$$\therefore \arctan(u+1) = C + \tfrac{1}{4}\ln|2x^2+2xy+y^2|$$

well done

$$\therefore \arctan(y/x+1) = C + \tfrac{1}{4}\ln|2x^2+2xy+y^2|$$