

Maths Supervision Work 11

$$H = U + pV$$

~~$$dH = \left. \frac{\partial H}{\partial U} \right|_p dU + \left. \frac{\partial H}{\partial S} \right|_p dS$$
$$= \left. \frac{\partial}{\partial U} (U + pV) \right|_p$$~~

~~$$dH = \frac{\partial H}{\partial U} dU + \frac{\partial H}{\partial p} dp + \frac{\partial H}{\partial V} dV$$~~

$$= \frac{\partial}{\partial U} (U + pV) dU + \frac{\partial}{\partial p} (U + pV) dp + \frac{\partial}{\partial V} (U + pV) dV$$

$$= dU + V dp + p dV$$

you can do that straight away and apply the differential from $H = U + pV$

$$= T dS - p dV + V dp + p dV$$

$$= T dS + V dp$$

$$\left. \frac{\partial T}{\partial p} \right|_S = \left. \frac{\partial V}{\partial S} \right|_p$$

$$U = H - pV$$

$$\frac{\partial U}{\partial p} = \frac{\partial H}{\partial p} - V = T \frac{\partial S}{\partial p} + V - V = T \frac{\partial S}{\partial p}$$

you have to specify if it is a p, S, V const !!!

$$\frac{\partial U}{\partial V \partial p} = \frac{\partial T}{\partial V} \frac{\partial S}{\partial p}$$

starting with $dU = T dS - p dV$

what about the d/dV of dS/dp ???

$$\frac{\partial^2 U}{\partial V} = \frac{\partial H}{\partial V} - p = T \frac{\partial S}{\partial V} - p$$

$$\frac{\partial^2 U}{\partial p \partial V} = \frac{\partial T}{\partial p} \frac{\partial S}{\partial V} - 1 = \frac{\partial T}{\partial V} \frac{\partial S}{\partial p}$$

$$\left. \frac{\partial S}{\partial V} \right|_p \left. \frac{\partial T}{\partial p} \right|_V - \left. \frac{\partial S}{\partial p} \right|_V \left. \frac{\partial T}{\partial V} \right|_p = 1$$

ok

11. ~~$\frac{\partial T}{\partial V} = \frac{\partial p}{\partial S}$~~ ~~$\frac{\partial p}{\partial S} = \frac{\partial T}{\partial V}$~~ ~~$\frac{\partial V}{\partial T} = \frac{\partial S}{\partial p}$~~ ~~$\frac{\partial S}{\partial p} = \frac{\partial V}{\partial T}$~~

~~$\frac{\partial G}{\partial p} = V$~~
 ~~$G = \int V dp$~~
~~constant~~

$$dU = TdS - pdV$$

$$\frac{\partial U}{\partial S} = T$$

$$\frac{\partial^2 U}{\partial T \partial S} = 1 \Rightarrow \frac{\partial U}{\partial T} = S + C(V)$$

$$\frac{\partial U}{\partial V} = -p$$

$$\frac{\partial^2 U}{\partial p \partial V} = -1 \Rightarrow \frac{\partial U}{\partial p} = -V + D(S)$$

$$dU = (-V + D(S))dp + (S + C(V))dT$$

Let $G = -U$ ohhhh !!! with $C(V) = 0 = D(S) \quad \forall V, S$
 why $G = -U$???

$$dG = Vdp - SdT$$

$$dG = Vdp - SdT$$

$$dU = TdS - pdV$$

$$\frac{\partial V}{\partial T} = -\frac{\partial S}{\partial p}$$

$$dG - dU = d(-ST) + d(pV)$$

$$\text{so } G = U + pV - ST$$