Morgan Saville

Maths Supervision Work 8

59.a. Try $y = e^{\lambda x}$

$\lambda^2 - 5\lambda + 6 = 0$

$\therefore (\lambda - 2)(\lambda - 3) = 0$

$\therefore \lambda = 2$ or 3

$\therefore y = Ae^{2x} + Be^{3x}$

b. Try $y = e^{\lambda x}$

$2\lambda^2 - 3\lambda - 2 = 0$

$\lambda = \frac{3 \pm \sqrt{3^2 + 4 \cdot 2 \cdot 2}}{2 \cdot 2} = \frac{3 \pm \sqrt{25}}{4} = \frac{3 \pm 5}{4} = -1/2$ or 2

$\therefore y = Ae^{-x/2} + Be^{2x}$

c. Try $y = e^{\lambda x}$

~~$\lambda^2 - 9\lambda = 0$~~

~~$\therefore \lambda(\lambda - 3) = 0$~~

~~$\therefore \lambda = 0$ or 3~~

~~$\therefore y = (A + Bx)e^{3x}$~~

$\lambda^2 - 9 = 0$

$\therefore \lambda = \pm 3$

$\therefore y = A\cosh 3x + B\sinh 3x$

d. Try $y = e^{\lambda x}$

$\lambda^2 + 4 = 0$

$\therefore \lambda = \pm 2i$

$\therefore y = A\sin x + B\cos x$

e. Try $y = e^{\lambda x}$

$\lambda^2 + 2\lambda + 5 = 0$

$\therefore \lambda = \frac{-2 \pm \sqrt{2^2 - 4 \cdot 5}}{2} = -1 \pm \sqrt{1-5} = -1 \pm 2i$

$\therefore y = e^{-x}(A\cos 2x + B\sin 2x)$

10a. Try $y = \alpha x + \beta$

$y' = \alpha, \quad y'' = 0$

$\therefore y'' - 5y' + 6y = -5\alpha + 6\alpha x + 6\beta = x$

Comparing coefficients

$6\alpha = 1 \rightarrow \alpha = 1/6$

$6\beta - 5\alpha = 0$

$\therefore \beta = \frac{5}{6}\alpha = \frac{5}{36}$

$\therefore y = \frac{1}{6}x + \frac{5}{36}$

b. Try $y = \alpha e^{2x}$

$\therefore y' = 2\alpha e^{2x}$, $y'' = 4\alpha e^{2x}$

$\therefore y'' - 9y = 4\alpha e^{2x} - 9\alpha e^{2x}$
$= -5\alpha e^{2x}$
$= e^{2x}$

$\therefore -5\alpha = 1$
$\therefore \alpha = -1/5$

$\therefore y = -\frac{1}{5} e^{2x}$

c. Try $y = \alpha e^{3x}$

$\therefore y' = 3\alpha e^{3x}$, $y'' = 9\alpha e^{3x}$

$\therefore y'' - 9y = 9\alpha e^{3x} - 9\alpha e^{3x}$
$= 0$

Try $y = \alpha x e^{3x}$

$y' = \alpha(e^{3x} + 3xe^{3x}) = \alpha(3x+1)e^{3x}$
$y'' = \alpha(3e^{3x} + 3(3x+1)e^{3x}) = \alpha(9x+6)e^{3x}$

$\therefore y'' - 9y = \alpha(9x+6)e^{3x} - 9\alpha x e^{3x}$
$= 6\alpha e^{3x}$
$= e^{3x}$

$\therefore 6\alpha = 1$
$\therefore \alpha = 1/6$
$\therefore y = \frac{1}{6} x e^{3x}$

d. ~~$Ae^{2x} + Be^{3x}$~~

By linearity and b) and c),

~~$y = -\frac{2}{5}e^{2x} + \frac{1}{6}xe^{3x}$~~

$y = -\frac{2}{5}e^{2x} + \frac{1}{6}xe^{3x}$

e. Try $y = A\sin x + B\cos x$

$y' = A\cos x - B\sin x$

$y'' = -A\sin x - B\cos x$

$y'' - 5y' + 6y = -A\sin x - B\cos x - 5A\cos x + 5B\sin x + 6A\sin x + 6B\cos x$

$= (5A + 5B)\sin x + (5B - 5A)\cos x$

$= \sin x$

$\therefore 5B - 5A = 0$

$\therefore A = B$

$5A + 5B = 10A = 1$

$A = B = \frac{1}{10}$

$\therefore y = \frac{1}{10}(\sin x + \cos x)$