Morgan
Saville

Supervision 1 Questions

1 i) A variable which can exist in one of two possible states: 0 or 1; On or Off; High or Low

ii) A function which accepts one or more boolean values as inputs, and outputs a boolean value

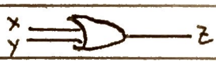
iii) A function, usually defined in terms of logic gates, which accepts one or more boolean values as inputs, and outputs one or more boolean values

2 3 fundamental gates:

AND 

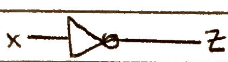
$Z = X * Y$
$= XY$

X	Y	Z
0	0	0
0	1	0
1	0	0
1	1	1

OR 

$Z = X + Y$

X	Y	Z
0	0	0
0	1	1
1	0	1
1	1	1

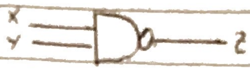
NOT 

$Z = \overline{X}$

X	Z
0	1
1	0

4 compound gates

NAND



$Z = \overline{XY}$

X	Y	Z
0	0	1
0	1	1
1	0	1
1	1	0

NOR

$Z = \overline{X+Y}$

X	Y	Z
0	0	~~0~~ 1
0	1	0
1	0	0
1	1	0

XOR

$Z = X\bar{Y} + \bar{X}Y$
$= X \oplus Y$

X	Y	Z
0	0	0
0	1	1
1	0	1
1	1	0

XNOR

$Z = XY + \bar{X}\bar{Y}$

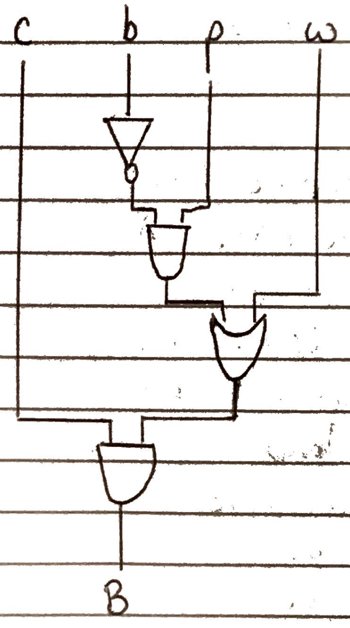
X	Y	Z
0	0	1
0	1	0
1	0	0
1	1	1

3. Suppose there is a beeper in a car which should beep when either (or both)

- The car is moving, the passenger's seat has a person in it, and the passenger's seat belt is not done up
- The car is moving and there is an object near it.

Let c = the car is moving
p = someone is in the passenger's seat
b = the passenger's seat belt is done up
ω = there is an object near the car
(lowercase omega used because o for object looks too much like 0)
B = beeper is beeping

$B = c(\bar{b} \cdot p + \omega)$



c	p	b	ω	B
0	0	0	0	0
0	0	0	1	0
0	0	1	0	0
0	0	1	1	0
0	1	0	0	0
0	1	0	1	0
0	1	1	0	0
0	1	1	1	0
1	0	0	0	0
1	0	0	1	1
1	0	1	0	0
1	0	1	1	1
1	1	0	0	1
1	1	0	1	1
1	1	1	0	0
1	1	1	1	1

4\. 1.

a	b	$\bar a$	$\overline{\bar a \cdot b}$	$\bar b$	$\overline{a \cdot \bar b}$	x
0	0	1	1	1	1	0
0	1	1	0	0	1	1
1	0	0	1	1	0	1
1	1	0	1	0	1	0

a	b	$\bar a$	$\overline{\bar a + b}$	$\bar b$	$\overline{a + \bar b}$	x
0	0	1	0	1	0	1
0	1	1	0	0	1	0
1	0	0	1	1	0	0
1	1	0	0	0	0	1

2\. i) $abc + ab\bar c = ab(c+\bar c) = ab(1) = ab$

ii) $a(\bar a + b) = a\bar a + ab = 0 + ab = ab$

iii) $ab + \bar a c = \cancel{abc + ab\bar c + \bar a b c + \bar a \bar b c}$

$= \cancel{[illegible]}$

$= \cancel{\bar a bc + ab\bar c + abc + \bar a \bar b c}$

$= \cancel{bc(a+\bar a) + ab\bar c + \bar a \bar b c}$

$\cancel{= bc + ab\bar c + \bar a \bar b c}$

$= (ab + \bar a)(ab + c)$

$= ((\bar a + a)(\bar a + b))((a+c)(b+c))$

$= (1)(\bar a + b)(a+c)(b+c)$

$= (a+c)(\bar a + b)(b+c)$

$= (a+c)(\bar a b + \bar a c + bb + bc)$

$= (a+c)(b(\bar a + c + 1) + \bar a c\ \cancel{[illegible]})$

$= (a+c)(\bar a c + b\ \cancel{[illegible]})$

$= a\bar a c + ab + \bar a c c + bc$

$= 0 + \bar a c + ab + bc$

$= a\bar a + \bar a c + ab + bc$

$= (a+c)(\bar a + b)$

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iv) $(a+c)(a+d)(b+c)(b+d)$

$= (a+cd)(b+cd)$

$= ab + cd$

3. $z = b + \overline{(abc + \bar{a})}$

$= b + (\overline{abc} \cdot a)$

$\cancel{= b + abc}$

$= b + a(\bar{a} + \bar{b} + \bar{c})$

$= b + a\bar{a} + a\bar{b} + a\bar{c}$

$= b + 0 + a(\bar{b} + \bar{c})$

$= \cancel{b + a \cdot bc}\ b + a(\bar{b} + \bar{c})$

$\cancel{= b + ab\ a(b+\bar{b})(\bar{b}+\bar{c})}$

$\cancel{= b + a(b\bar{b} + b\bar{c} + \bar{b}\bar{b} + \bar{b}\bar{c})}$

$\cancel{= b + a(0 + b\bar{c} + \bar{b} + \bar{b}\bar{c})}$

$\cancel{= b + a(\bar{b} + \bar{c}(b+\bar{b}))}$

$= abc + ab\bar{c} + \bar{a}bc + \bar{a}b\bar{c} + a\bar{b}c + a\bar{b}\bar{c} + ab\bar{c} + a\bar{b}\bar{c}$

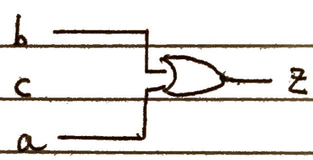
$= abc + ab\bar{c} + \bar{a}bc + \bar{a}b\bar{c} + a\bar{b}c + a\bar{b}\bar{c}$

$= a(bc + b\bar{c} + \bar{b}c + \bar{b}\bar{c}) + \bar{a}(b\bar{c} + bc)$

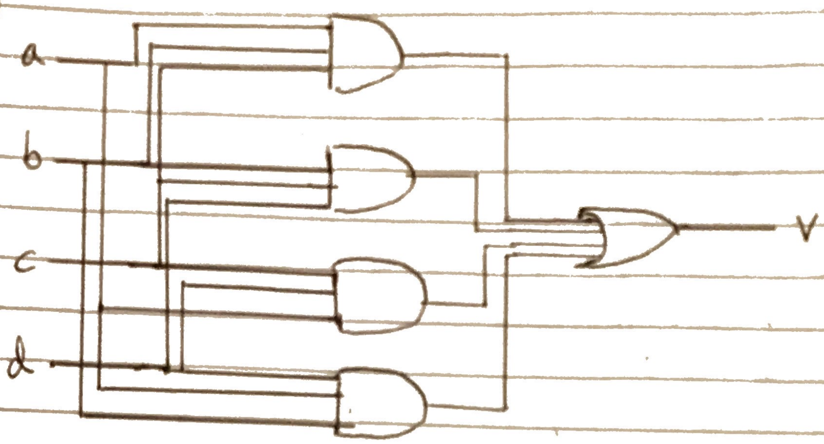
$= a + \bar{a}b$

$= (a+\bar{a})(a+b)$

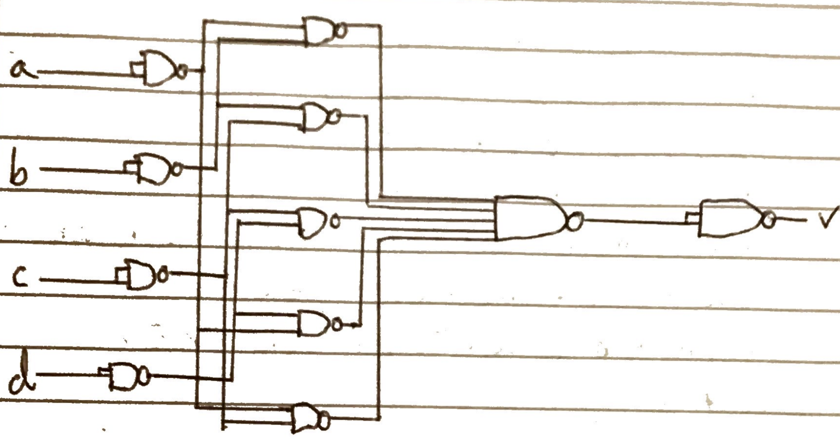
$= a + b$



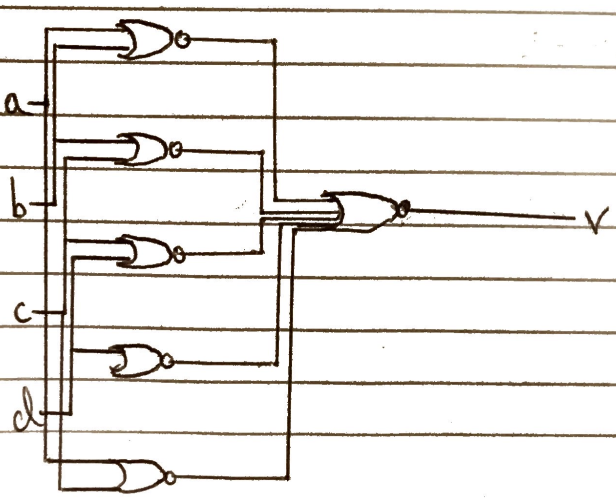
4 $v = abc + abd + acd + bcd$



5a $v = \overline{\overline{(\bar{a}\bar{b})}\,\overline{(\bar{a}\bar{c})}\,\overline{(\bar{a}\bar{d})}\,\overline{(\bar{b}\bar{c})}\,\overline{(\bar{c}\bar{d})}}$



b $v = \overline{\overline{(a+b)} + \overline{(a+c)} + \overline{(a+d)} + \overline{(b+c)} + \overline{(c+d)}}$



6. $f = \bar a \bar d + \bar b \bar c + \bar a b \bar c d$

cd \ ab	00	01	11	10
00	1	1	0	1
01	1	1	0	1
11	0	0	0	0
10	1	1	0	X

$f = \bar a \bar c + \bar a \bar d + \bar b \bar c$

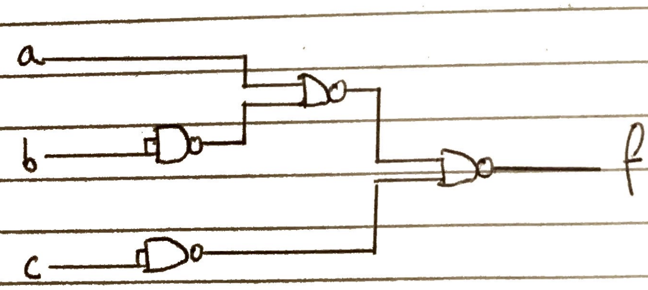
$\bar f = ab + cd + ac$

7.

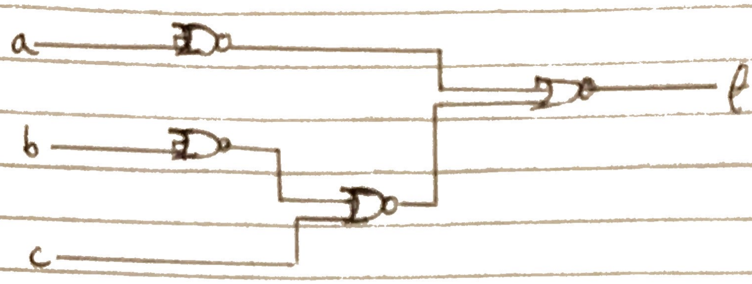
c \ ab	00	01	11	10
0	0	0	0	1
1	0	0	1	1

$f = a\bar b + c = \overline{\overline{(a\bar b)}\,\bar c}$

a)



$\bar f = \bar a + b\bar c \Leftrightarrow f = \overline{\bar a + b\bar c} = \overline{\bar a + \overline{(\bar b + c)}}$



5\. a) i) $BD + AC + AB$

ii) $A(B+C) + BD$

b) i) $BD + AB$

ii) $B(A+D)$

c) i) $\cancel{\bar{B}D + A\bar{B}\bar{C} + \bar{A}\bar{B}C + \bar{A}\bar{C}D}$

$\bar{B}D + A\bar{C}\bar{D} + \bar{A}\bar{B}C + \bar{A}\bar{C}D$

ii) $\bar{C}(A\bar{D} + D\bar{A}) + \bar{B}(\bar{A}C + D)$

$= \bar{C}(A \oplus D) + \bar{B}(\bar{A}C + D)$

d) i) $BD + \bar{B}\bar{D}$

ii $\overline{B \oplus D}$

iii) $\bar{f} = AB + CD + \bar{B}D$

$\therefore f = \overline{AB + CD + \bar{B}D}$

$= (\overline{AB})(\overline{CD})(\overline{\bar{B}D})$

$= (\bar{A}+\bar{B})(\bar{C}+\bar{D})(B+\bar{D})$

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6.

index	A B C D		
0	0 0 0 0	0 0 x 0	x 0 x 0
		x 0 0 0	~~[illegible]~~
2	0 0 1 0		
8	1 0 0 0	0 x 1 0	x x 1 0
		x 0 1 0	1 x x 0
5	0 1 0 1	1 0 x 0	
6	0 1 1 0	1 x 0 0	x 1 x 1
			x 1 1 x
10	1 0 1 0		1 1 x x
12	1 1 0 0	0 1 x 0	
		x 1 0 1	
7	0 1 1 1	0 1 1 x	
13	1 1 0 1	x 1 1 0	
14	1 1 1 0	1 x 1 0	
		1 1 0 x	
15	1 1 1 1	1 1 x 0	
		x 1 1 1	
		1 1 x 1	
		1 1 1 x	

ii Prime implicants : $\bar{A}BD$, $AC\bar{D}$, $\bar{B}\bar{D}$, $C\bar{D}$, $A\bar{D}$, BD, BC, AB

	0	2	5	6	7	8	10	12	13	14	15
$\bar{A}BD$			✓		✓						
$AC\bar{D}$							✓			✓	
$\bar{B}\bar{D}$	✓	✓				✓	✓				
$C\bar{D}$		✓		✓			✓			✓	
$A\bar{D}$						✓	✓	✓		✓	
BD			✓		✓				✓		✓
BC				✓	✓					✓	✓
AB								✓	✓	✓	✓