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Maths Supervision Work 9

S1a. $f = x^3 - 3x^2y + 3xy^2 + 8y^3 - 3y$

$\frac{\partial f}{\partial x} = 3x^2 - 6xy + 3y^2$

$\frac{\partial f}{\partial y} = -3x^2 + 6xy + 24y^2 - 3$

$\frac{\partial^2 f}{\partial x^2} = 6x - 6y$

$\frac{\partial^2 f}{\partial y^2} = 6x + 48y$

$\frac{\partial}{\partial y}\left(\frac{\partial f}{\partial x}\right) = -6x + 6y$

$\frac{\partial}{\partial x}\left(\frac{\partial f}{\partial y}\right) = -6x + 6y$

b. $f = e^{-x^2y^2}$

$\frac{\partial f}{\partial x} = -2xy^2 e^{-x^2y^2}$

$\frac{\partial f}{\partial y} = -2x^2y e^{-x^2y^2}$

$\frac{\partial^2 f}{\partial x^2} = -2y^2\left(e^{-x^2y^2} - 2xy^2 e^{-x^2y^2}\right)$

$= -2y^2(1 - 2xy^2)e^{-x^2y^2}$

$\frac{\partial^2 f}{\partial y^2} = -2x^2\left(e^{-x^2y^2} - 2x^2y e^{-x^2y^2}\right)$

$= -2x^2(1 - 2x^2y)e^{-x^2y^2}$

$\frac{\partial}{\partial y}\left(\frac{\partial f}{\partial x}\right) = -2x\left(2y e^{-x^2y^2} - 2x^2y^3 e^{-x^2y^2}\right)$

$= -4xy(1 - x^2y^2)e^{-x^2y^2}$

$\frac{\partial}{\partial x}\left(\frac{\partial f}{\partial x}\right) = -2y\left(2x e^{-x^2y^2} - 2x^3y^2 e^{-x^2y^2}\right)$

$= -4xy(1 - x^2y^2)e^{-x^2y^2}$

53.a.

c. $f = \frac{1}{x^2+xy+2y^2}$

$$\frac{\partial f}{\partial x} = -\left(\frac{2x+y}{(x^2+xy+2y^2)^2}\right)$$

$$\frac{\partial f}{\partial y} = -\left(\frac{x+4y}{(x^2+xy+2y^2)^2}\right)$$

$$\frac{\partial^2 f}{\partial x^2} = -\left(\frac{2(x^2+xy+2y^2)^2-(2x+y)\cdot 2(x^2+xy+2y^2)\cdot(2x+y)}{(x^2+xy+2y^2)^4}\right)$$

$$= -\left(\frac{2(x^2+xy+2y^2)-2(2x+y)^2}{(x^2+xy+2y^2)^3}\right)$$

$$\frac{\partial^2 f}{\partial y^2} = -\left(\frac{4(x^2+xy+2y^2)^2-(x+4y)\cdot 2(x^2+xy+2y^2)\cdot(x+4y)}{(x^2+xy+2y^2)^4}\right)$$

$$= -\left(\frac{4(x^2+xy+2y^2)-2(x+4y)^2}{(x^2+xy+2y^2)^3}\right)$$

$$\frac{\partial}{\partial y}\left(\frac{\partial f}{\partial x}\right) = -\left(\frac{(x^2+xy+2y^2)^2-(2x+y)\cdot 2(x^2+xy+2y^2)(x+4y)}{(x^2+xy+2y^2)^4}\right)$$

$$= -\left(\frac{(x^2+xy+2y^2)-2(2x+y)(x+4y)}{(x^2+xy+2y^2)^3}\right)$$

$$\frac{\partial}{\partial x}\left(\frac{\partial f}{\partial y}\right) = -\left(\frac{(x^2+xy+2y^2)^2-(x+4y)\cdot 2(x^2+xy+2y^2)(2x+y)}{(x^2+xy+2y^2)^4}\right)$$

$$= -\left(\frac{(x^2+xy+2y^2)-2(2x+y)(x+4y)}{(x^2+xy+2y^2)^3}\right)$$

S3. a. $f = x^3 - 3x^2y + 3xy^2 + 8y^3 - 3y$

Taking $\frac{\partial f}{\partial x} = 3x^2 - 6xy + 3y^2 = 0$,

$x^2 - 2xy + y^2 = 0$

$\therefore (x-y)^2 = 0$

$\therefore x = y$

Taking $\frac{\partial f}{\partial y} = -3x^2 + 6xy + 24y^2 - 3 = 0$

$\therefore x^2 + 2xy + 8y^2 - 1 = 0$

Substituting $x = y$,

$x^2 + 2x^2 + 8x^2 = 1$

$\therefore 11x^2 = 1$

$\therefore x = y = \pm \frac{1}{\sqrt{11}}$

$\therefore$ Stationary points are $\left(\frac{1}{\sqrt{11}}, \frac{1}{\sqrt{11}}\right)$ and $\left(-\frac{1}{\sqrt{11}}, -\frac{1}{\sqrt{11}}\right)$

b. $f = e^{-x^2y^2}$

Taking $\frac{\partial f}{\partial x} = -2xy^2 e^{-x^2y^2} = 0$

$\therefore x = 0$ or $y = 0$

Taking $\frac{\partial f}{\partial y} = -2x^2y\, e^{-x^2y^2} = 0$

$\therefore x = 0$ or $y = 0$

$\therefore$ Stationary points are $\forall x \forall y$. $(0, y)$ or $(x, 0)$

c. $f = \frac{1}{x^2+xy+2y^2}$

Taking $\frac{\partial f}{\partial x} = -\left(\frac{2x+y}{(x^2+xy+2y^2)^2}\right) = 0$

$\therefore 2x+y=0$

$\therefore y=-2x \Rightarrow \frac{\partial f}{\partial x} = \frac{0}{0}$ $\therefore$ There are no solutions

~~Taking $\frac{\partial f}{\partial y} = -\left(\frac{x+4y}{(x^2+xy+2y^2)^2}\right) = 0$~~

~~$\therefore x+4y=0$~~

~~$\therefore x+4(-2x)=0$~~

~~$\therefore -7x=0$~~

~~$\therefore x=0$~~

~~$\therefore y=0$~~

~~$\therefore$ Stationary point is $(0,0)$~~

d. $f = e^{-\frac{1}{x+y}}$

Taking $\frac{\partial f}{\partial x} = \frac{1}{(x+y)^2} e^{-\frac{1}{x+y}} = 0$

There are no solutions.

e. $f = \frac{\sinh x}{\sinh y}$

Taking $\frac{\partial f}{\partial x} = \frac{\cosh x}{\sinh y} = 0$

$\therefore \cosh x = 0$ $\therefore$ There are no solutions

f. $f = (x^2+y^2)^{1/2}$

Taking $\frac{\partial f}{\partial x} = \frac{1}{2}(x^2+y^2)^{-1/2} \cdot 2x = 0$

$\therefore x = 0$

Taking $\frac{\partial f}{\partial y} = \frac{1}{2}(x^2+y^2)^{-1/2} \cdot 2y = 0$

$\therefore y = 0$

$\therefore \frac{\partial f}{\partial y} = \frac{0}{0}$ $\therefore$ there are no solutions

g. $f = \arctan(\cancel{y}/x)$

Taking $\frac{\partial f}{\partial x} = \frac{-y}{(1+\frac{y^2}{x^2})x^2} = \frac{-y}{x^2+y^2} = 0$

$\therefore y = 0$

Taking $\frac{\partial f}{\partial y} = \frac{1}{(1+\frac{y^2}{x^2})x} = \frac{1}{(x+\frac{y^2}{x})} = 0$

$\therefore$ There are no solutions.

h. $f = x^y$

Taking $\frac{\partial f}{\partial x} = yx^{y-1} = 0$

$\therefore y = 0$ or $x = 0$

Taking $\frac{\partial f}{\partial y} = \ln x \, x^y = 0$

Case 1: $y = 0 \Rightarrow \ln x = 0 \Rightarrow x = 1$

Case 2: $x = 0 \Rightarrow$ Indeterminate form $\rightarrow$ no solutions

$\therefore$ The stationary point is at $(1, 0)$