

Missing 7?

Morgan
Saville

Page 1

Supervision 1 Questions

- i) A variable which can exist in one of two possible states 0 or 1. On or Off, High or Low
- ii) A function which accepts ~~one~~ ^{one} or more boolean values as inputs, and outputs a boolean value
- iii) A function, usually defined, in terms of logic gates, which accepts one or more boolean values as inputs, and outputs one or more boolean values

2 3 fundamental gates:

AND $x \Rightarrow y \Rightarrow z$

X	Y	Z
0	0	0
0	1	0
1	0	0
1	1	1

$$z = x * y$$

$$= xy$$

OR $x \Rightarrow y \Rightarrow z$

X	Y	Z
0	0	0
0	1	1
1	0	1
1	1	1

$$z = x + y$$

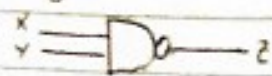
NOT $x \Rightarrow \neg x \Rightarrow z$

X	Z
0	1
1	0

$$z = \bar{x}$$

4 compound gates

NAND



$$z = \overline{xy}$$

X	Y	Z
0	0	1
0	1	1
1	0	1
1	1	0

V

NOR

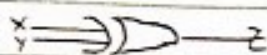


$$z = \overline{x+y}$$

X	Y	Z
0	0	1
0	1	0
1	0	0
1	1	0

V

XOR



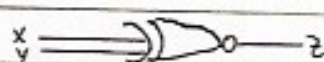
$$z = x\bar{y} + \bar{x}y$$

$$= x \oplus y$$

X	Y	Z
0	0	0
0	1	1
1	0	1
1	1	0

V

XNOR



$$z = xy + \bar{x}\bar{y}$$

X	Y	Z
0	0	1
0	1	0
1	0	0
1	1	1

V

3 Suppose there is a beeper in a car which should beep when either (or both)

- The car is moving, the passenger's seat has a person in it, and the passenger's seat belt is not done up
- The car is moving and there is an object near it.

Let c = the car is moving

p = someone is in the passenger's seat

b = the passenger's seat belt is done up

ω = there is an object near the car

(lowercase omega used because o for object looks too much like 0)

B = beeper is beeping

$B = c(b \cdot p + \omega)$

c	b	p	ω	B
0	0	0	0	0
0	0	0	1	0
0	0	1	0	0
0	0	1	1	0
0	1	0	0	0
0	1	0	1	0
0	1	1	0	0
0	1	1	1	0
1	0	0	0	0
1	0	0	1	1
1	0	1	0	0
1	0	1	1	1
1	1	0	0	1
1	1	0	1	1
1	1	1	0	0
1	1	1	1	1

4. 1.

a	b	\bar{a}	$\bar{a} \cdot b$	\bar{b}	$a \bar{b}$	x
0	0	1	1	1	1	0
0	1	1	0	0	1	1
1	0	0	1	1	0	1
1	1	0	1	0	1	0

a	b	\bar{a}	$\bar{a} + b$	\bar{b}	$\overline{a+b}$	x
0	0	1	0	1	0	1
0	1	1	0	0	1	0
1	0	0	1	1	0	0
1	1	0	0	0	0	1

2. i) $abc + ab\bar{c} = ab(c + \bar{c}) = ab(1) = ab$ ✓

ii) $a(\bar{a} + b) = a\bar{a} + ab = 0 + ab = ab$ ✓

iii) $ab + \bar{a}c =$

~~$ab + \bar{a}c + ab\bar{c} + ab\bar{c}$~~

~~$= ab + \bar{a}c + ab\bar{c} + ab\bar{c}$~~

~~$= ab(\bar{a} + a) + \bar{a}c + ab\bar{c}$~~

~~$= ab + ab\bar{c} + \bar{a}c$~~

$= (ab + \bar{a})(ab + c)$

$= ((\bar{a} + a)(\bar{a} + b))(a + c)(b + c)$

$= (1)(\bar{a} + b)(a + c)(b + c)$

$= (a + c)(\bar{a} + b)(b + c)$

$= (a + c)(\bar{a}b + \bar{a}c + bb + bc)$

$= (a + c)(b(\bar{a} + 1) + \bar{a}c)$

$= (a + c)(\bar{a}c + b)$

$= a\bar{a}c + ab + \bar{a}cc + bc$

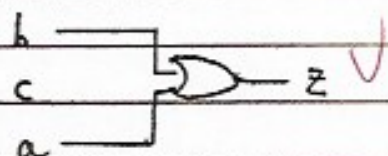
$= 0 + \bar{a}c + ab + bc$

$= a\bar{a} + \bar{a}c + ab + bc$

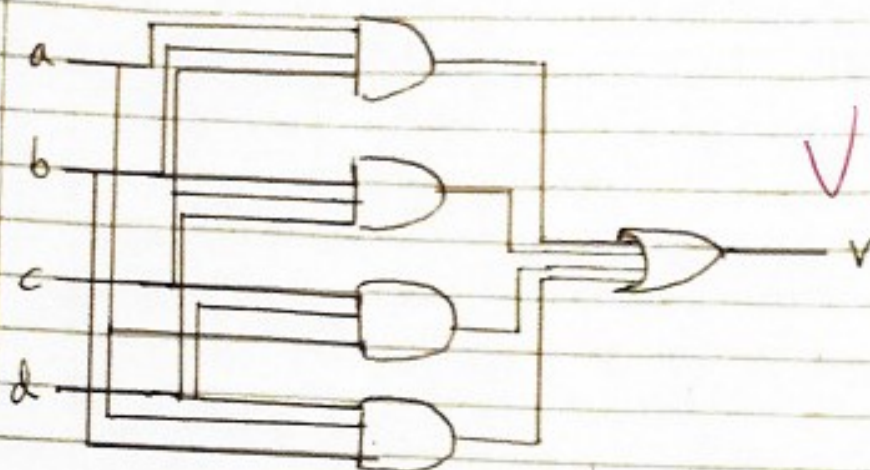
$= (a + c)(\bar{a} + b)$ ✓

$$\begin{aligned} \text{iv) } & (a+c)(a+d)(b+c)(b+d) \\ &= (a+cd)(b+cd) \\ &= ab+cd \end{aligned} \quad \checkmark$$

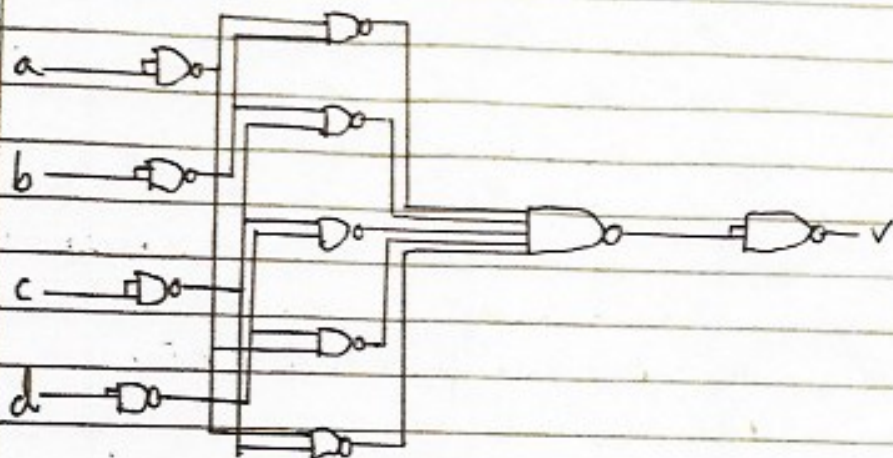
$$\begin{aligned} 3. \quad z &= b + (abc + \bar{a}) \\ &= b + (\overline{abc} \cdot a) \\ &= \cancel{b + \bar{a} + \bar{b} + \bar{c}} \\ &= b + a(\bar{a} + \bar{b} + \bar{c}) \\ &= b + a\bar{a} + a\bar{b} + a\bar{c} \\ &= b + 0 + a(\bar{b} + \bar{c}) \\ &= \cancel{b + a + \bar{a} + \bar{b} + \bar{c}} \quad b + a(\bar{b} + \bar{c}) \\ &= \cancel{b + a + \bar{a} + \bar{b} + \bar{c}} \quad a(\bar{b} + \bar{c}) \\ &= \cancel{b + a + \bar{a} + \bar{b} + \bar{c}} \quad a(\bar{b} + \bar{c}) \\ &= \cancel{b + a + \bar{a} + \bar{b} + \bar{c}} \quad a(\bar{b} + \bar{c}) \\ &= \cancel{b + a + \bar{a} + \bar{b} + \bar{c}} \quad a(\bar{b} + \bar{c}) \\ &= abc + ab\bar{c} + \bar{a}bc + \bar{a}b\bar{c} + a\bar{b}c + a\bar{b}\bar{c} + ab\bar{c} + a\bar{b}\bar{c} \\ &= abc + ab\bar{c} + \bar{a}bc + \bar{a}b\bar{c} + a\bar{b}c + a\bar{b}\bar{c} \\ &= a(bc + b\bar{c} + \bar{b}c + \bar{b}\bar{c}) + \bar{a}(b\bar{c} + bc) \\ &= a + \bar{a}b \\ &= (a + \bar{a})(a + b) \\ &= a + b \quad \checkmark \end{aligned}$$



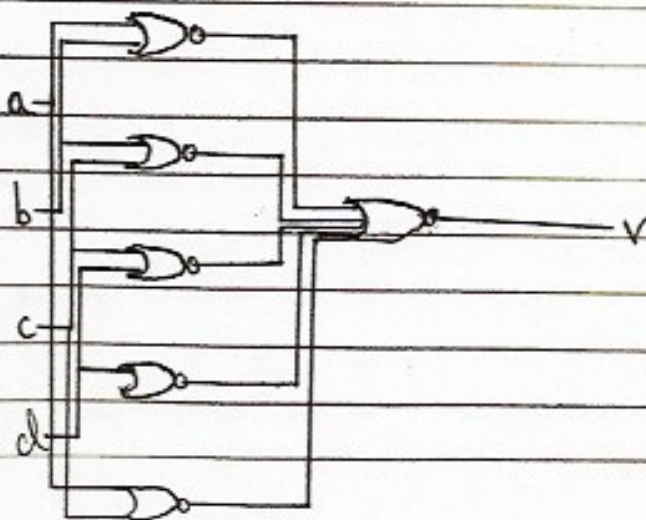
$$4. \quad v = abc + abd + acd + bcd \quad \checkmark$$



5a $V = (\overline{a}b)(\overline{a}c)(\overline{a}d)(\overline{b}c)(\overline{c}d)$



b $V = (\overline{a+b}) + (\overline{a+c}) + (\overline{a+d}) + (\overline{b+c}) + (\overline{c+d})$ ✓



6. $f = \bar{a}\bar{d} + b\bar{c} + \bar{a}b\bar{c}d$

		b			
		ab			
d	cd	00	01	11	10
	00	1	1	0	1
	01	1	1	0	1
	11	0	0	0	0
	10	1	1	0	0

we don't
need to
include X

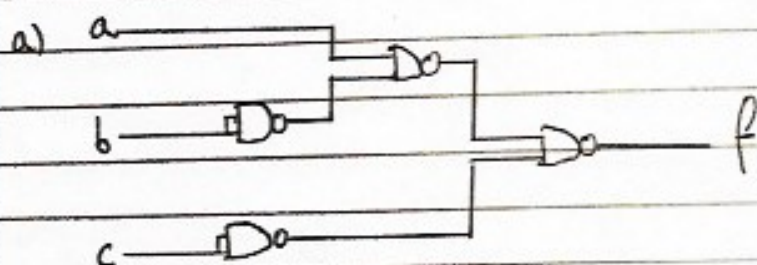
$f = \bar{a}\bar{c} + \bar{a}\bar{d} + b\bar{c}$ ✓

$\bar{f} = ab + cd + ac \Rightarrow f = \overline{ab + cd + ac} = \bar{a}\bar{b} + \bar{c}\bar{d} + \bar{a}\bar{c}$

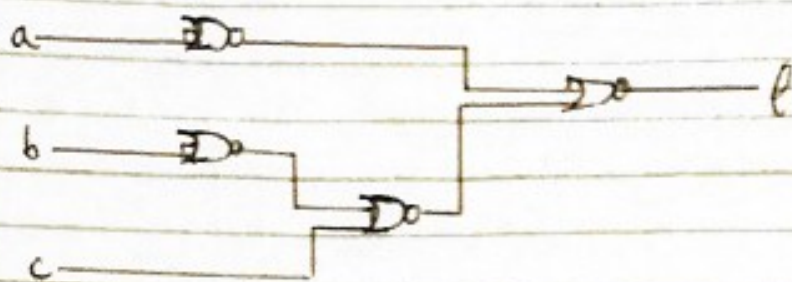
7.

		b			
		ab			
c	0	0	0	0	1
	1	0	0	1	1

$f = ab + ac = (\bar{a}\bar{b})\bar{c} \Rightarrow \bar{a}\bar{b} = \bar{a}\bar{c}$



$\bar{f} = \bar{a} + b\bar{c} \Leftrightarrow f = \bar{a} + b\bar{c} = \bar{a} + (\bar{b} + c)$ ✓



5. a) i) $BD + AC + AB \checkmark$ Canonical = $\bar{a} b \bar{c} d + \bar{a} b c d + \dots$
 ii) $A(B+C) + BD$

b) i) $BD + AB + A\bar{C}\bar{D}$
 ii) $B(A+D)$

c) i) ~~$\bar{B}D + A\bar{C}\bar{D} + \bar{A}\bar{B}C + \bar{A}\bar{C}D$~~
 $\bar{B}D + A\bar{C}\bar{D} + \bar{A}\bar{B}C + \bar{A}\bar{C}D \checkmark$
 ii) $\bar{C}(A\bar{D} + D\bar{A}) + \bar{B}(\bar{A}C + D)$
 $= \bar{C}(A \oplus D) + \bar{B}(\bar{A}C + D)$

d) i) $BD + \bar{B}\bar{D} \checkmark$
 ii) $\bar{B} \oplus D$

iii) $f = \overline{AB + CD + \bar{B}D} \checkmark$
 $\therefore f = \overline{AB + CD + \bar{B}D}$
 $= (\overline{AB})(\overline{CD})(\overline{\bar{B}D})$
 $= (\bar{A} + \bar{B})(\bar{C} + \bar{D})(B + D) \checkmark$

1 1 1 x

ii. Prime implicants : $\bar{A}BD, AC\bar{D}, \bar{B}\bar{D}, C\bar{D}, A\bar{D}, BD, BC, AB$

	0	1	5	6	7	8	10	12	13	14	15
$\bar{A}BD$			✓		✓						
$AC\bar{D}$							✓			✓	
$\bar{B}\bar{D}$	✓	✓				✓	✓				
$C\bar{D}$		✓		✓			✓			✓	
$A\bar{D}$			✓		✓	✓	✓	✓		✓	
$\bar{B}D$			✓		✓				✓		✓
BC				✓	✓					✓	✓
AB								✓	✓	✓	✓