Morgan Saville

Maths Supervision Work 6

IS.

n	$f^{(n)}(x)$	$f^{(n)}(0)$	$\frac{f^{(n)}(0)}{n!}x^n$
0	$\tan^{-1}(x)$	0	0
1	$\frac{1}{1+x^2}$	1	x
2	$\frac{-2x}{(1+x^2)^2}$	0	0
3	$\frac{2(3x^2-1)}{(x^2+1)^3}$	$\cancel{4}$ -2	$-x^3/3$
4	$\frac{-24x(x^2-1)}{(x^2+1)^4}$	0	0
5	$\frac{24(5x^4-10x^2+1)}{(x^2+1)^5}$	$\cancel{120}$ 24	$x^5/5$

$\therefore \tan^{-1}(x) \simeq x - x^3/3 + x^5/5$

~~$\pi/4 \approx 0.785398163$~~

~~Number of terms — Approximation~~

~~$f(x) = \sum_{n=0}^{N} \frac{(-1)^n x^{2n+1}}{2n+1}$~~

~~$= x \sum_{n=0}^{N} \frac{(-x^2)^n}{2n+1}$~~

a. $\pi \simeq 3.14159\ 26536$

To be accurate to 10dp the magnitude of the $(n+1)^{th}$ term must be less than 10^{-10}

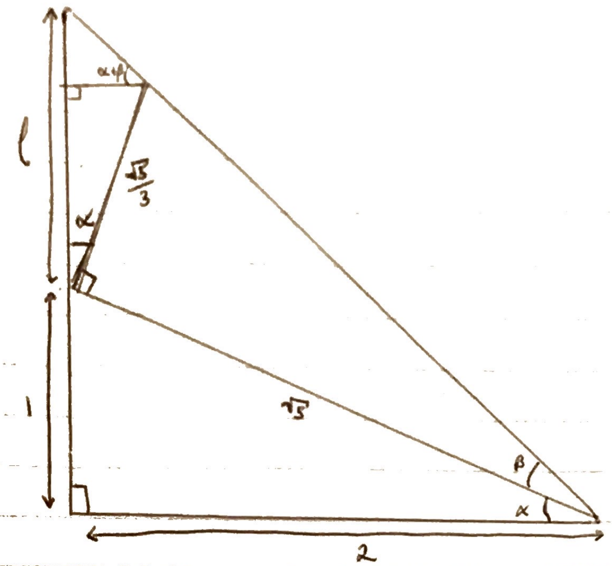
$\therefore \frac{1}{2n+3} < 10^{-10}$ $\therefore 2n+3 > 10^{10}$

$\therefore n > 5\cdot 10^{9} - 3/5$

b. $\cancel{\tan^{-1}(1/2) = 1/2 - \frac{(1/2)^3}{3} + \frac{(1/2)^5}{5} - \dots}$

$\cancel{\tan^{-1}(1/3) = 1/3 - \frac{(1/3)^3}{3} + \frac{(1/3)^5}{5} - \dots}$

$\cancel{\tan^{-1}(1/2) + \tan^{-1}(1/3) =}$



$\alpha = \arctan(1/2)$

$\beta = \arctan(1/3)$

$$\tan(\alpha+\beta) = \frac{1+l}{2} = \frac{l - \frac{\sqrt{5}}{3}\cos\alpha}{\frac{\sqrt{5}}{3}\sin\alpha} = \frac{l - \frac{\sqrt{5}}{3}\cdot\frac{2}{\sqrt{5}}}{\frac{\sqrt{5}}{3}\cdot\frac{1}{\sqrt{5}}}$$

$$\therefore \frac{1+l}{2} = \frac{l - 2/3}{1/3} = 3l - 2$$

$\therefore 1 + l = 6l - 4$

$\therefore 5l = 5$

$\therefore l = 1$

$\therefore \tan(\alpha+\beta) = \frac{2}{2} = 1$

$\therefore \alpha+\beta = \arctan(1/2) + \arctan(1/3) = \arctan(1) = \frac{\pi}{4}$

$$\tan^{-1}(1/2) = 1/2 - \frac{1}{3\cdot 2^3} + \frac{1}{5\cdot 2^5} - \cdots$$

$$\tan^{-1}(1/3) = 1/3 - \frac{1}{3\cdot 3^3} + \frac{1}{5\cdot 3^5} -$$

$$\therefore \frac{\pi}{4} = 1/2 + 1/3 - \frac{1}{3\cdot 2^3} - \frac{1}{3\cdot 3^3} + \frac{1}{5\cdot 2^5} + \frac{1}{5\cdot 3^5} - \cdots$$

$$\therefore \pi = \sum_{n=0}^{\infty} \frac{1}{(2n+1)2^{2n-1}} + \frac{4}{(2n+1)3^{2n+1}}$$

6a. $p + p + p + p + 2p + p/2 = 1$

$$\therefore p\left(13/2\right) = 1$$

$$\therefore p = 2/13$$

b. $\langle x \rangle = p/2 + 2p + 3p + 4p + 5p + 6\cdot 2p$

$$= 2/13\,(1/2 + 2 + 3 + 4 + 5 + 12)$$

$$= 2/13 \cdot \frac{53}{2}$$

$$= 53/13$$

c. $P(X > \langle x \rangle) = P(X=5) + P(X=6)$

$$= p + 2p$$

$$= 6/13$$

d. $\sigma^2 = \frac{p}{2}\left(1 - \frac{53}{13}\right)^2 + p\left(2 - \frac{53}{13}\right)^2 + p\left(3 - \frac{53}{13}\right)^2 + p\left(4 - \frac{53}{13}\right)^2$

$$+ p\left(5 - \frac{53}{13}\right)^2 + 2p\left(6 - \frac{53}{13}\right)^2$$

$$= \frac{480}{169}$$

e. $\langle x^2 \rangle = \frac{p}{2} + 4p + 9p + 16p + 25p + 36 \cdot 2p$

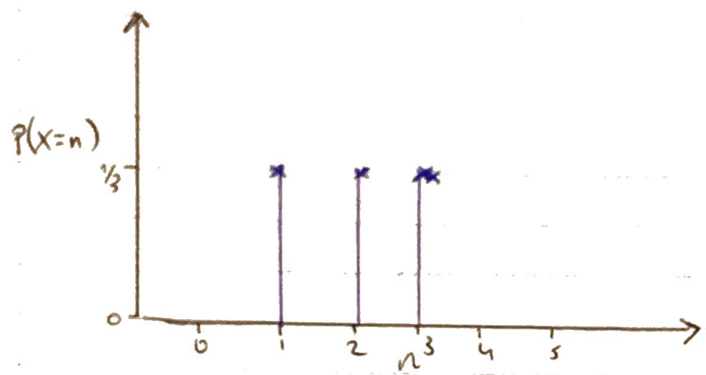
$= \frac{253}{13}$

$\langle x^2 \rangle - \langle x \rangle^2 = \frac{253}{13} - \left(\frac{53}{13}\right)^2 = \frac{480}{169}$

K1. $P(X=1) = 1/3$

$P(X=2) = 2/3 \cdot 1/2 = 1/3$

$P(X=3) = 2/3 \cdot 1/2 = 1/3$



$X \propto U(1,3)$

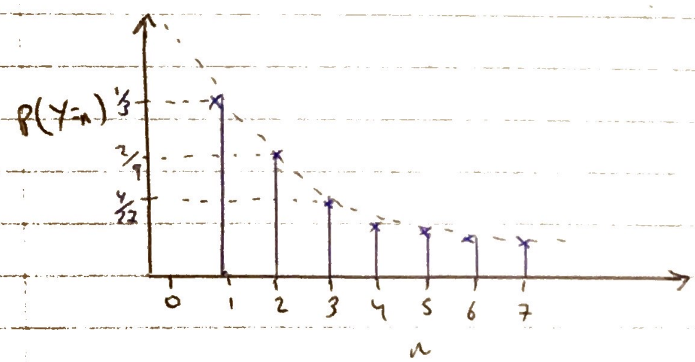
$P(Y=1) = 1/3$

$P(Y=2) = 2/3 \cdot 1/3$

$P(Y=3) = 2/3 \cdot 2/3 \cdot 1/3$

$P(Y=n) = (2/3)^{n-1} \cdot 1/3$

$Y \propto Geo(1/3)$



$$\langle x \rangle = \frac{1}{3}(1+2+3) = 2$$

$$\langle y \rangle = \cancel{\sum_{n=0}^{\infty} \left(\frac{2}{3}\right)^{n} \cdot \frac{n+1}{3}}$$

$$= \sum_{n=1}^{\infty} \left(\frac{2}{3}\right)^{n-1} \cdot \frac{n}{3}$$

$$= \frac{1}{3}\left(1 + 2\left(\frac{2}{3}\right) + 3\left(\frac{2}{3}\right)^2 + 4\left(\frac{2}{3}\right)^3 \ldots\right)$$

$$= \frac{1}{3}\left(1-\frac{2}{3}\right)^{-2}$$

$$= \frac{1}{3}\cdot\left(\frac{1}{3}\right)^{-2}$$

$$= 3$$