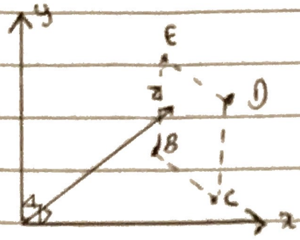
Morgan Saville
Page 1

Maths Supervision Questions 2

E1 a) $\cancel{\vec{n} = \vec{BC} \times \vec{BD}}$



$$\vec{n} = \vec{BC} \times \vec{BE}$$

$$= \left(\begin{pmatrix}5\\0\\3\end{pmatrix} - \begin{pmatrix}2\\0\\7\end{pmatrix}\right) \times \left(\begin{pmatrix}2\\4\\7\end{pmatrix} - \begin{pmatrix}2\\0\\7\end{pmatrix}\right)$$

$$= \begin{pmatrix}3\\0\\-4\end{pmatrix} \times \begin{pmatrix}0\\4\\0\end{pmatrix}$$

$$= \begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 3 & 0 & -4\\ 0 & 4 & 0\end{vmatrix}$$

$$= \begin{pmatrix}-16\\0\\12\end{pmatrix}$$

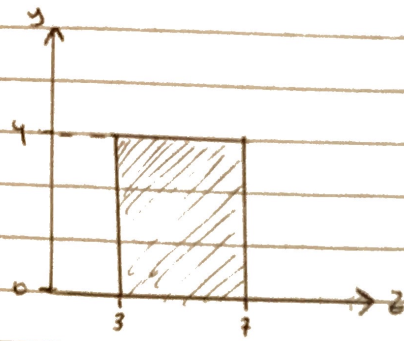
$$|\vec{n}| = |\vec{BC}||\vec{BE}|\sin\frac{\pi}{2} = |\vec{BC}||\vec{BE}| = A$$
$$= \sqrt{16^2 + 12^2}$$
$$= \sqrt{400}$$
$$= 20$$

$$\therefore \hat{n} = \frac{1}{|\vec{n}|}\vec{n} = \frac{1}{20}\begin{pmatrix}-16\\0\\12\end{pmatrix} = \frac{1}{5}\begin{pmatrix}-4\\0\\3\end{pmatrix}$$

b) $\vec{S} = A\hat{n} = |\vec{n}|\hat{n} = \vec{n} = \cancel{\begin{pmatrix}-4\\0\\3\end{pmatrix}} \begin{pmatrix}-16\\0\\12\end{pmatrix}$

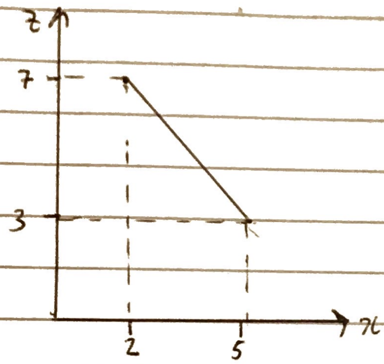
$$= 20\hat{n}$$

c) $S_x = -16$



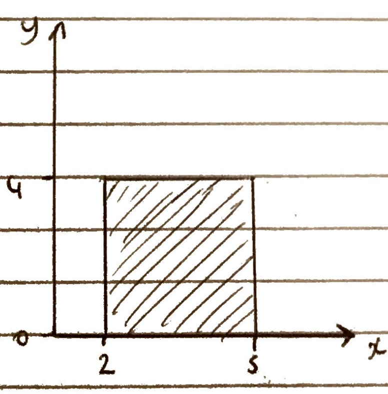
$|S_x| = (7-3) \times (4-0)$
$= 16$

d) $S_y = 0$



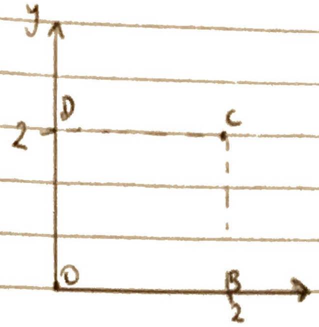
$|S_y| = 0$ by observation

e) $S_z = 12$



$|S_z| = (5-2) \times (4-0)$
$= 12$

2. a)



Area = $2 \times 2 = 4$
Since the square lies in the x-y plane, the unit normal is $\hat{k}$

$\therefore \vec{S} = 4\hat{k}$

S_p = Component of $\vec{S}$ projected onto the plane
$= \vec{S} \cdot \hat{n}$

$$= \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ 0 \\ 4 \end{pmatrix} \cdot \begin{pmatrix} 0 \\ -1 \\ 1 \end{pmatrix} = \frac{1}{\sqrt{2}} \cdot 4 = 2\sqrt{2}$$

b) Taking the 'upper' surface of a pyramid to mean the entire pyramid excluding the base (i.e. $\hat{k}$ points upwards):

Let $\vec{S_t}$ = total vector area of pyramid
$\vec{S_b}$ = vector area of base
$\vec{S}$ = vector area of upper surface

$\vec{S} + \vec{S_b} = \vec{S_t} = 0$
$\therefore \vec{S} = -\vec{S_b} = 4\hat{k}$ ($\vec{S_b}$ is just $\vec{S}$ from part a), reversed to point outwards from the pyramid)

If instead "upper" surface means the triangle DCE (i.e. $\hat{j}$ points upwards):

$\vec{S} = \frac{1}{2}|\vec{DC}||\vec{DE}|\cos\theta \ \hat{n} = \cancel{\frac{1}{2}\vec{DC} \times \vec{DE}} \ \cancel{\frac{1}{2}\vec{DE} \times \vec{DC}}$
$= -\frac{1}{2}\vec{DC} \times \vec{DE}$

$$= -\frac{1}{2}\left(\begin{pmatrix}2\\2\\0\end{pmatrix}-\begin{pmatrix}0\\2\\0\end{pmatrix}\right)\times\left(\begin{pmatrix}1\\1\\1\end{pmatrix}-\begin{pmatrix}0\\2\\0\end{pmatrix}\right)$$

$$= -\frac{1}{2}\begin{pmatrix}2\\0\\0\end{pmatrix}\times\begin{pmatrix}1\\-1\\1\end{pmatrix}$$

$$= -\frac{1}{2}\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 2 & 0 & 0\\ 1 & -1 & 1\end{vmatrix}$$

$$= -\frac{1}{2}\begin{pmatrix}0\\-2\\-2\end{pmatrix}$$

$$= \begin{pmatrix}0\\1\\1\end{pmatrix}$$

c) Taking the x-y plane to be horizontal

Let $\vec{S}$ = vector area of lampshade
Let $\vec{S}_b$ = vector area of base
Let $\vec{S}_t$ = vector area of top

$$\vec{S}_b = -\pi r_b^2 \hat{k} = -16\pi\hat{k}$$

$$\vec{S}_t = \pi r_t^2 \hat{k} = 9\pi\hat{k}$$

$$\vec{S} + \vec{S}_b + \vec{S}_t = 0$$

$$\therefore \vec{S} = -\vec{S}_b - \vec{S}_t = 16\pi\hat{k} - 9\pi\hat{k}$$
$$= 7\pi\hat{k}$$

irrespective of the height of the lampshade