MATH 161, Homework Set 3

Name:

Write out the partial fraction decomposition you would use for the following rational expressions (but you don't have to solve it – so if the problem is $\frac{1}{(x+2)(x+3)}$, just write $\frac{1}{(x+2)(x+3)} = \frac{A}{x+2} + \frac{B}{x+3}$, but don't actually solve for A and B).

1) $\frac{1}{(x+4)(x+3)^2}$

2) $\frac{x}{(x^2+6x+8)}$

3) $\frac{x}{(x^2+5)(x-3)}$

4) $\frac{-1}{(x^2-1)(x+5)}$

5) $\frac{2-x}{x(x^2+1)^2}$

6) $\frac{3}{(x^2+4)(x^2+6)}$

7) $\frac{1}{(x^2-4)(x+1)^2}$

8) $\frac{x}{(x^2+9)(x-7)^2}$

9) $\frac{7}{x^3+3x^2+2x}$

10) $\frac{-4x}{x^2+9x+20}$

Evaluate the following improper integrals, using proper limit notation.

11) $\int_1^\infty \frac{1}{t^4}dt$

12) $\int_0^\infty xe^{-x^2}dx$

13) $\int_3^\infty \frac{2x+7}{x^2+7x+10}dx$

14) $\int_{-\infty}^{-1} \frac{1}{y^5}dy$

15) $\int_{-\infty}^0 e^{2x}dx$

16) $\int_{-\infty}^\infty x^3e^{-x^4}dx$

Determine whether the following improper integrals converge or diverge. If the integral converges, evaluate it. If the integral diverges, justify that assertion.

17) $\int_1^\infty \frac{1}{z}dz$

18) $\int_{-1}^1 \frac{1}{x^3}dx$

19) $\int_0^9 \frac{1}{\sqrt{x}}dx$

20) $\int_{-\infty}^\infty \frac{1}{s^5}ds$

21) $\int_1^\infty \frac{1}{x^{1/3}}dx$

22) $\int_0^e \frac{1}{x}dx$

Evaluate the following integrals, showing all work.

23) $\int \frac{4x+5}{x^2+x-2}dx$

24) $\int \frac{u-11}{u^2-u-12}du$

25) $\int \frac{x^2-x+3}{(x^2+4)(x+1)}dx$

26) $\int \frac{-12\cos(\theta)}{\sin(\theta)^2+2\sin(\theta)-8}d\theta$

27) $\int \frac{6t^2+18t+6}{t^3+5t^2+6t}dt$

28) $\int \frac{1}{(x+1)^2(x+2)}dx$

29) $\int \frac{u}{(1+u^2)^2}du$

30) $\int \frac{x^2+x+4}{x^3+4x}dx$

31) How would you simplify the expression $\left(\frac{(n+1)!}{n!}\right)^2$?

32) How would you simplify the expression $\frac{(n-2)!}{(n+2)!}$?

33) How would you simplify the expression $\frac{(2n+1)!}{(2n+3)!}$?

34) How would you simplify the expression $\frac{n!}{(n+2)!}$?

35) How would you simplify the expression $\frac{(n-1)!(n+1)!}{(n!)^2}$?

36) How would you simplify the expression $\frac{(2n)!}{(2n+2)!}$?

37) Use the fact that $\int_0^\infty x^{20}e^{-x}dx = 20!$ to show that $\int_0^\infty x^{21}e^{-x}dx = 21!$. (You don't have to show your work for the L'Hoptial's rule).

38) Use the fact that $\int_0^\infty x^{45}e^{-x}dx = 45!$ to show that $\int_0^\infty x^{46}e^{-x}dx = 46!$. (You don't have to show your work for the L'Hoptial's rule).