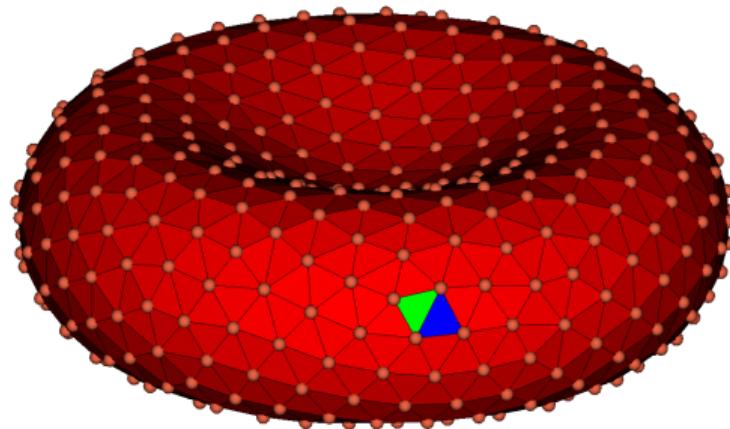
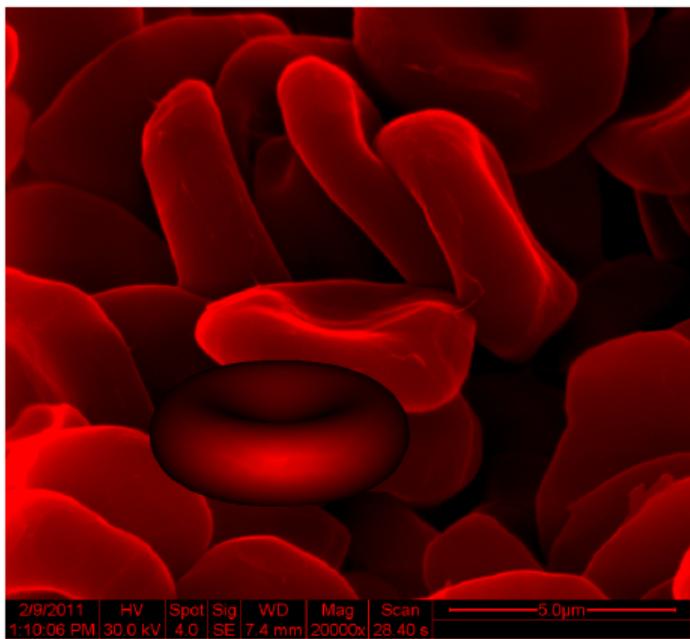


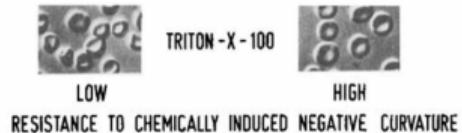
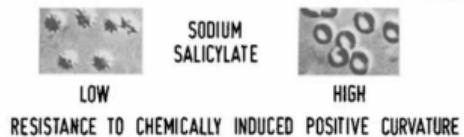
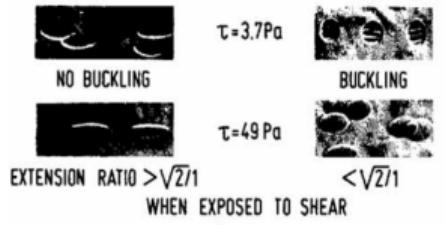
Bending models with spontaneous curvature and area-difference elasticity

S. Litvinov,
collaborators: X. Bian, P. Koumoutsakos

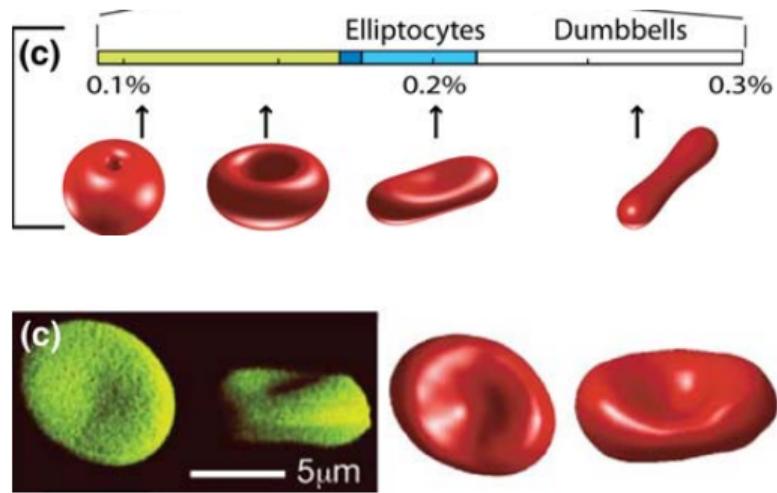
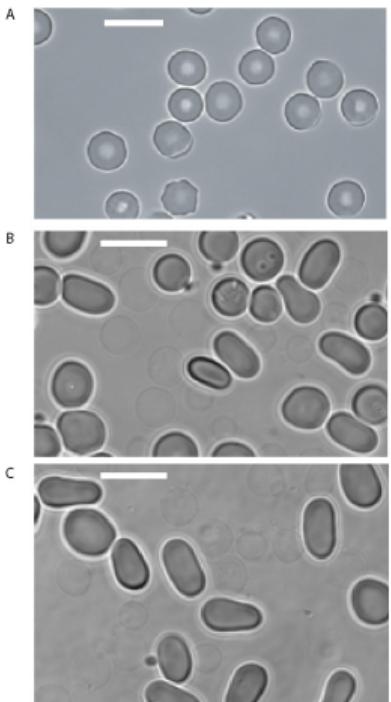
cse-lab.ethz.ch

Model

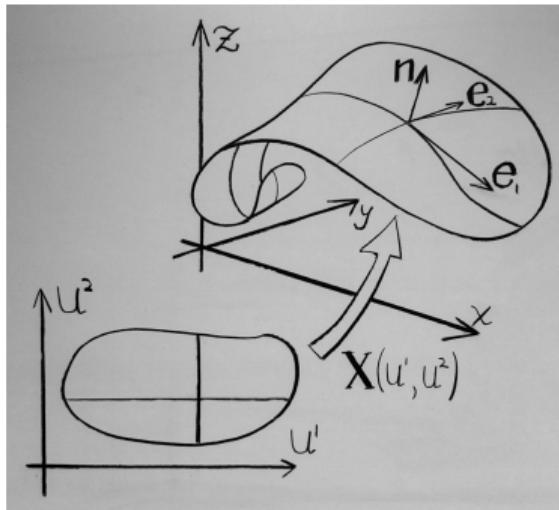




Elliptocytes or ovalocytes



Curvature



$$\begin{aligned} \mathbf{x} &= \mathbf{x}(u^1, u^2) & \mathbf{e}_\alpha &= \mathbf{x}_{,\alpha} & g_{\alpha\beta} &= \mathbf{e}_\alpha \mathbf{e}_\beta & \mathbf{n} &\propto \mathbf{e}_1 \times \mathbf{e}_2 \\ H_{\alpha\beta} &= \mathbf{e}_\alpha \mathbf{n}_{,\beta} & H &= \text{trace}(\mathbf{H}) = H_{\alpha\beta} g^{\alpha\beta} \end{aligned}$$

Energy

$$G \propto \int H^2 dA, \quad M \propto \int H dA$$

Minimal G

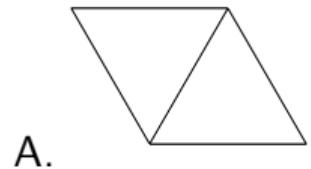
SC G and M

BC $G, M = M_0$

ADE G and $(M - M_0)^2$

SC-ADE G and M and $(M - M_0)^2$

Models

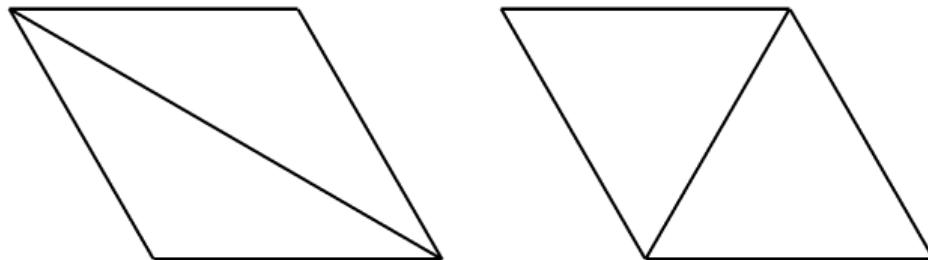


C. operators for energy

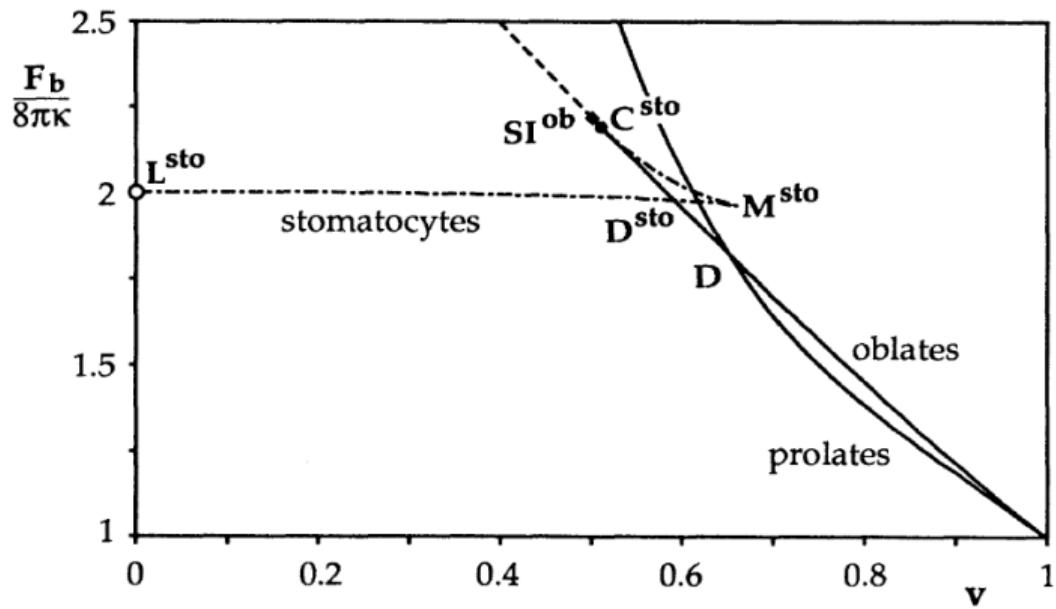
D. operators for force

Regularization

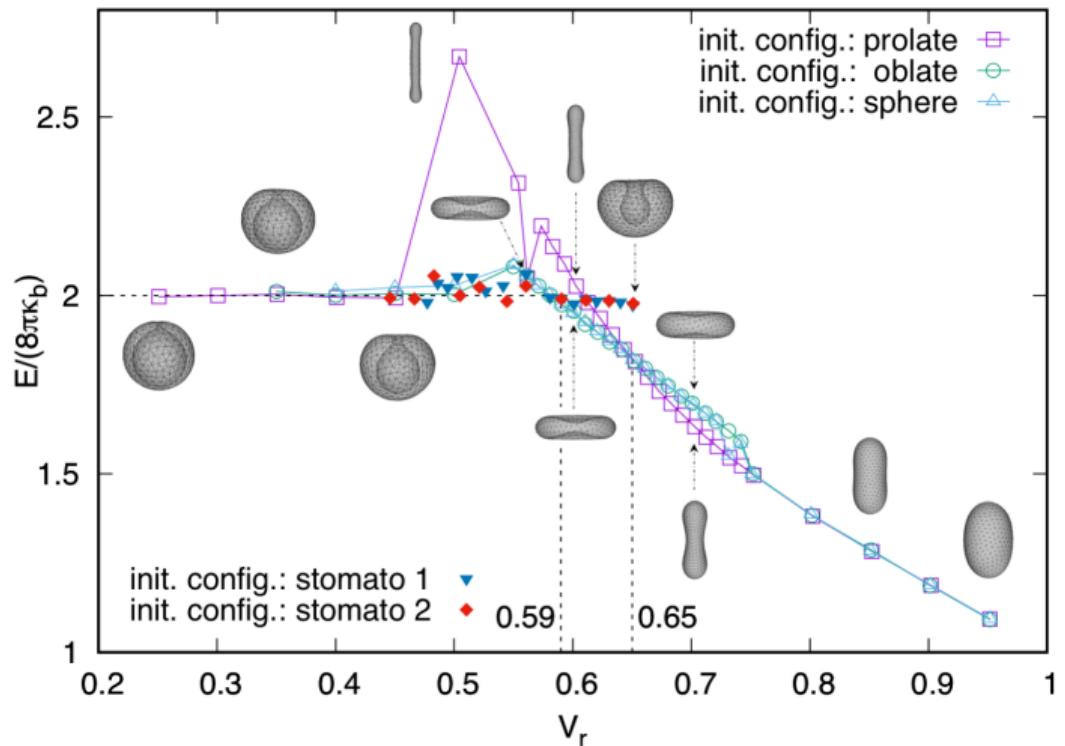
- Constrain area of each triangle
- “Edge rotation”



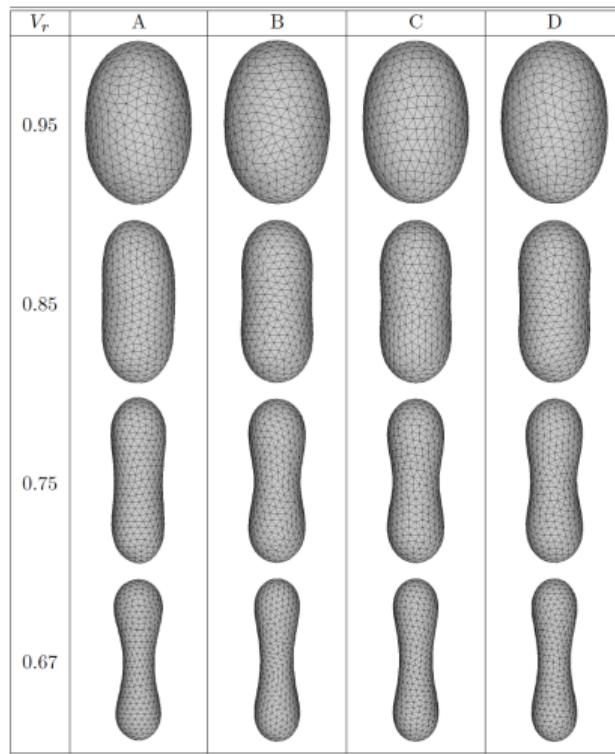
Minimal



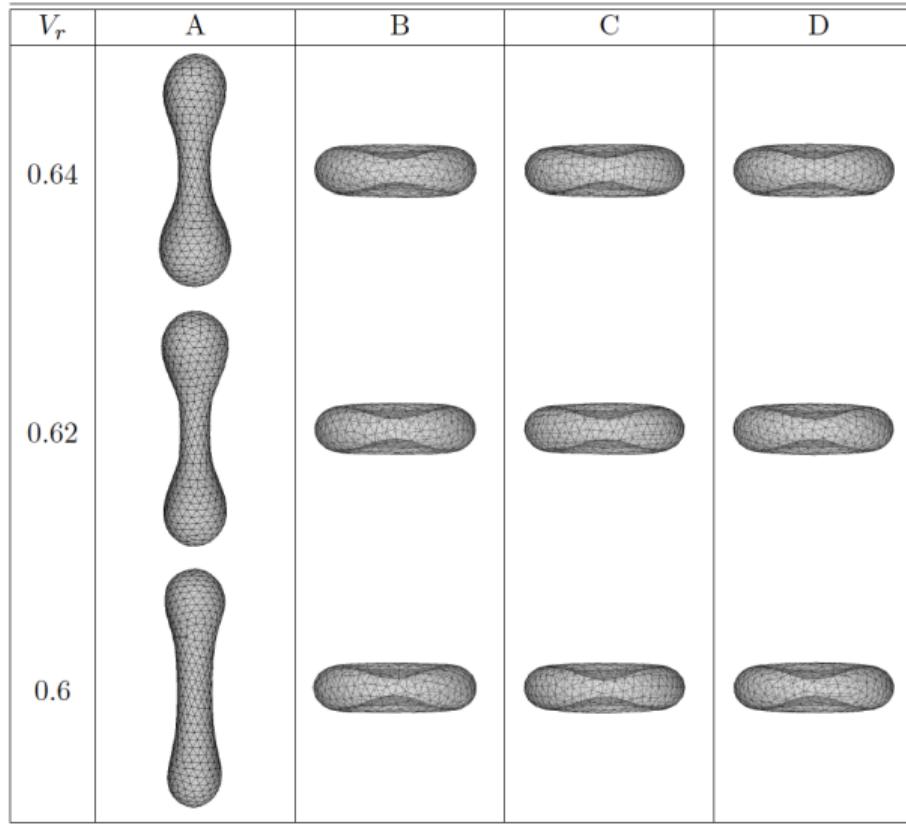
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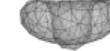
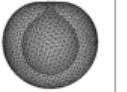
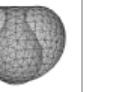
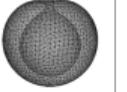
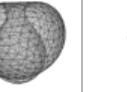
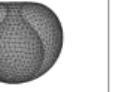
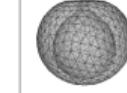
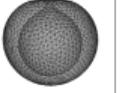
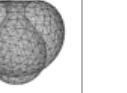
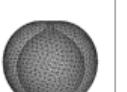
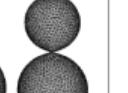
Minimal



Minimal



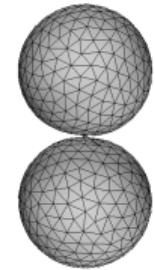
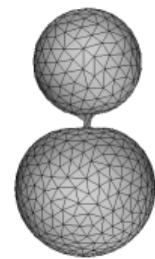
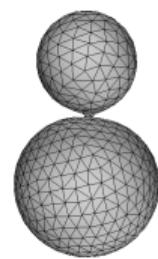
Minimal

V_r	B: $N_t = 320$	B: $N_t = 1280$	B: $N_t = 5120$	C: $N_t = 1280$	C: $N_t = 5120$	D: $N_t = 1280$	D: $N_t = 5120$
0.45							
0.4						NA	
0.35						NA	
0.3						NA	
0.25						NA	NA

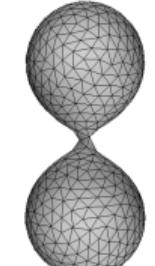
SC

V_r 0.736 0.718 0.707 0.65 0.57 0.52

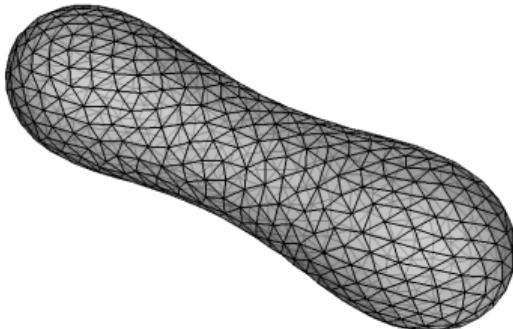
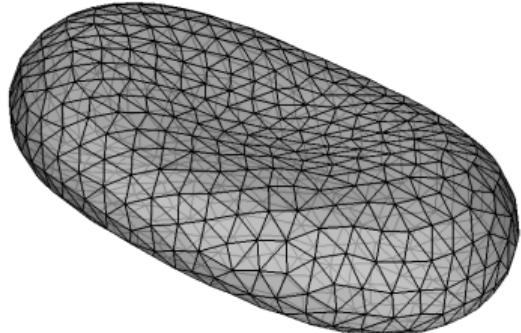
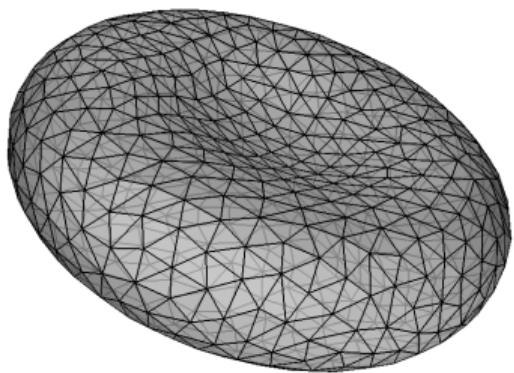
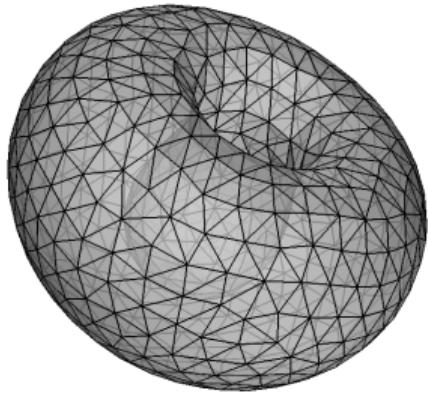
A



C



ADE



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