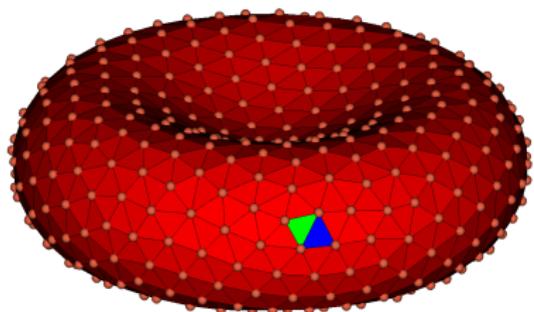
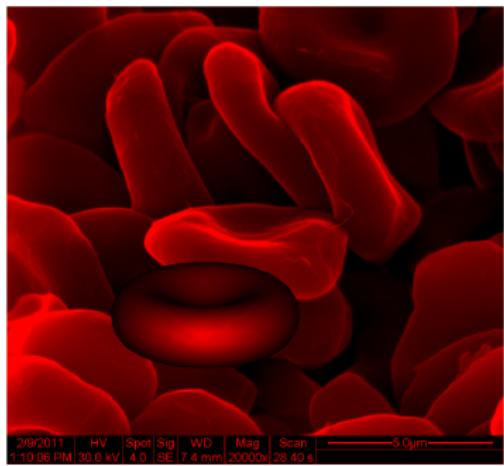
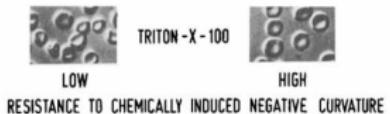
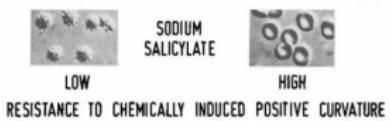
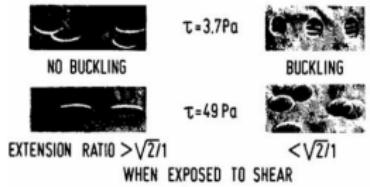

Bending models with spontaneous curvature and area-difference elasticity

S. Litvinov,
collaborators: X. Bian, P. Koumoutsakos

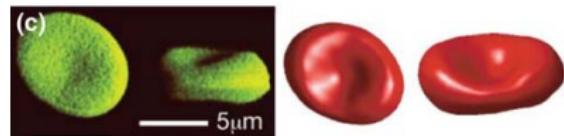
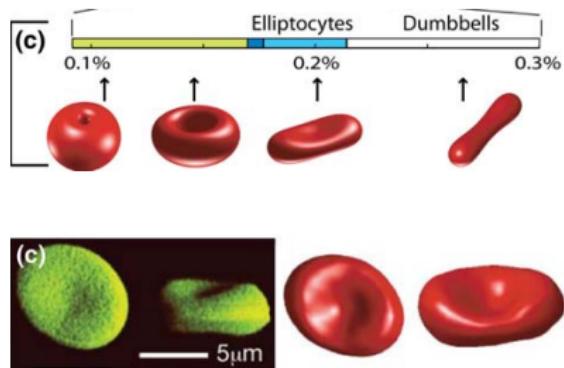
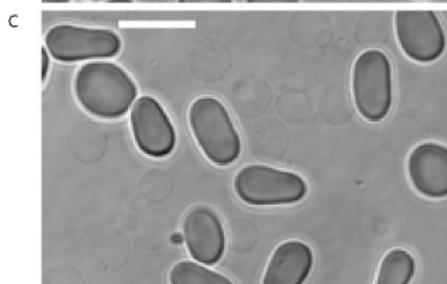
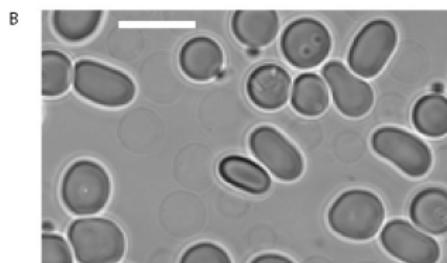
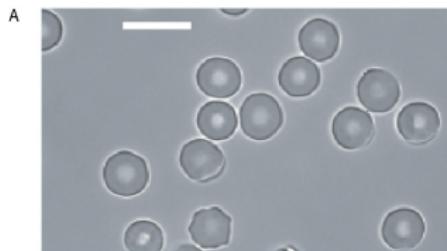
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Model

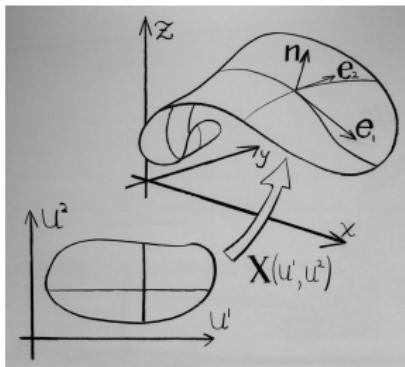




Elliptocytes or ovalocytes



Curvature



$$\begin{aligned} \mathbf{x} &= \mathbf{x}(u^1, u^2) & \mathbf{e}_\alpha &= \mathbf{x}_{,\alpha} & g_{\alpha\beta} &= \mathbf{e}_\alpha \mathbf{e}_\beta & \mathbf{n} &\propto \mathbf{e}_1 \times \mathbf{e}_2 \\ H_{\alpha\beta} &= \mathbf{e}_\alpha \mathbf{n}_{,\beta} & H &= \text{trace}(\mathbf{H}) = H_{\alpha\beta} g^{\alpha\beta} \end{aligned}$$

Energy

$$G \propto \int H^2 dA, \quad M \propto \int H dA$$

Minimal G

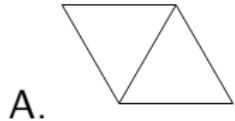
SC G and M

BC $G, M = M_0$

ADE G and $(M - M_0)^2$

SC-ADE G and M and $(M - M_0)^2$

Models

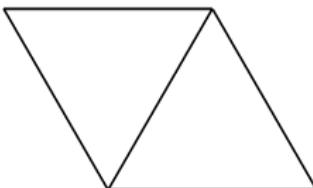
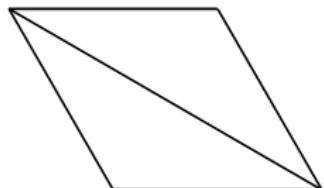


C. operators for energy

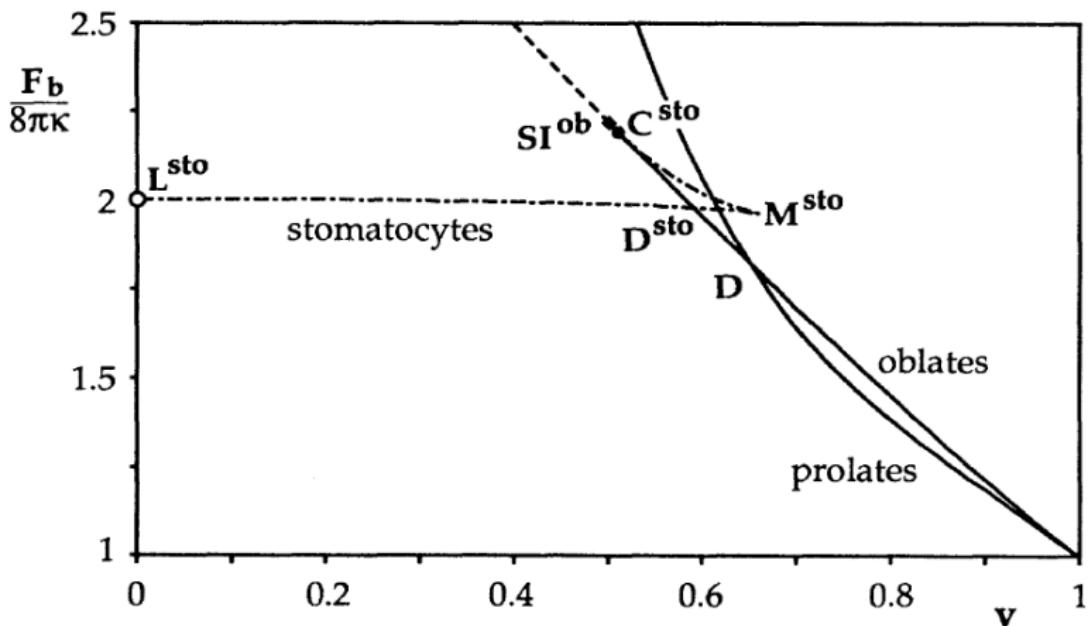
D. operators for force

Regularization

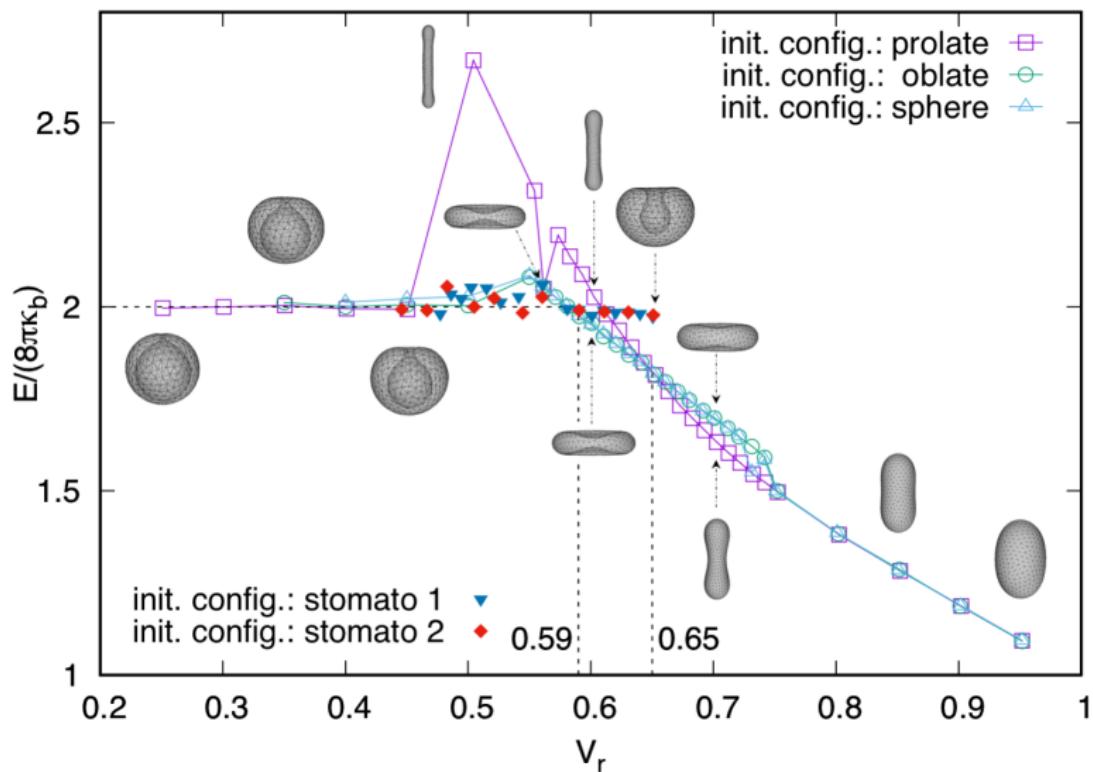
- Constrain area of each triangle
- “Edge rotation”



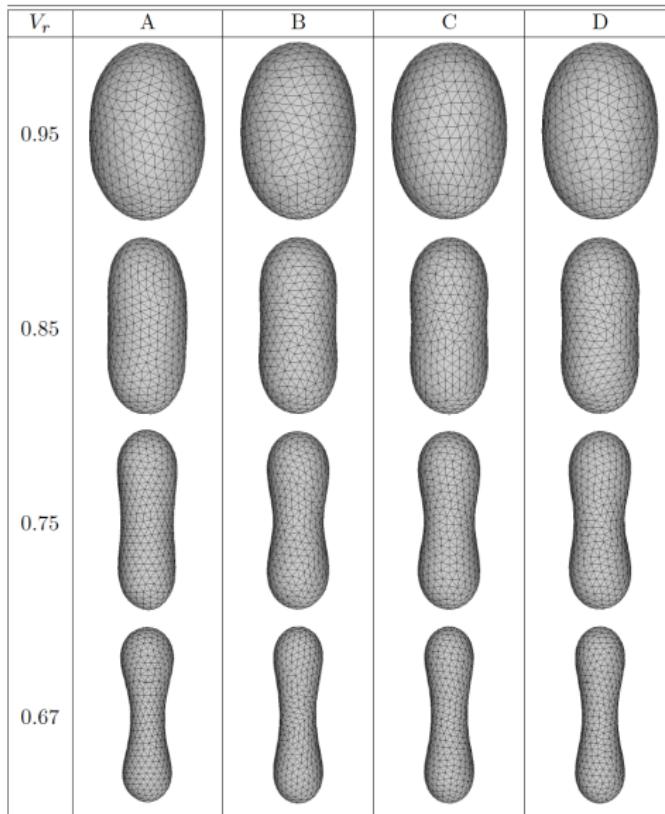
Minimal



Minimal



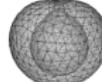
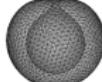
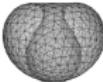
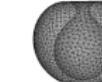
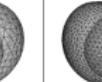
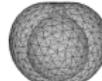
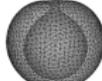
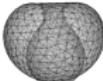
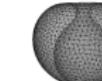
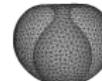
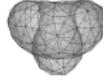
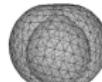
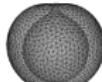
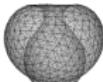
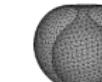
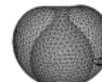
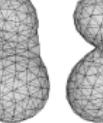
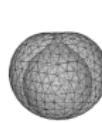
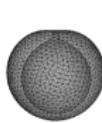
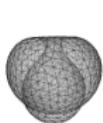
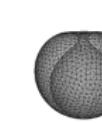
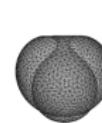
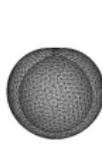
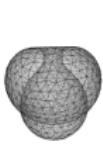
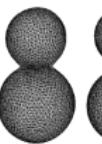
Minimal



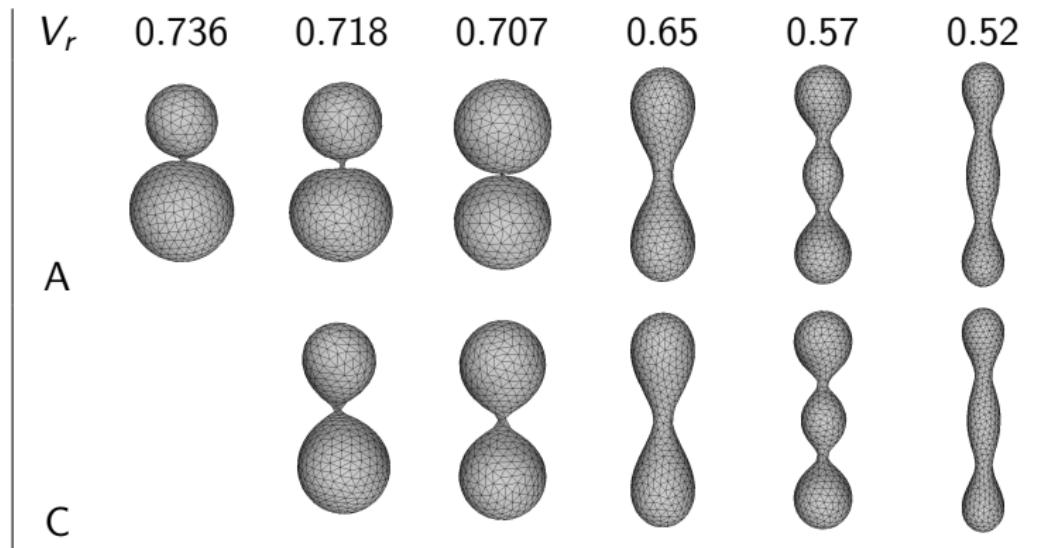
Minimal

| V_r | A | B | C | D |
|-------|---|---|---|--|
| 0.64 |  |  |  |  |
| 0.62 |  |  |  |  |
| 0.6 |  |  |  |  |

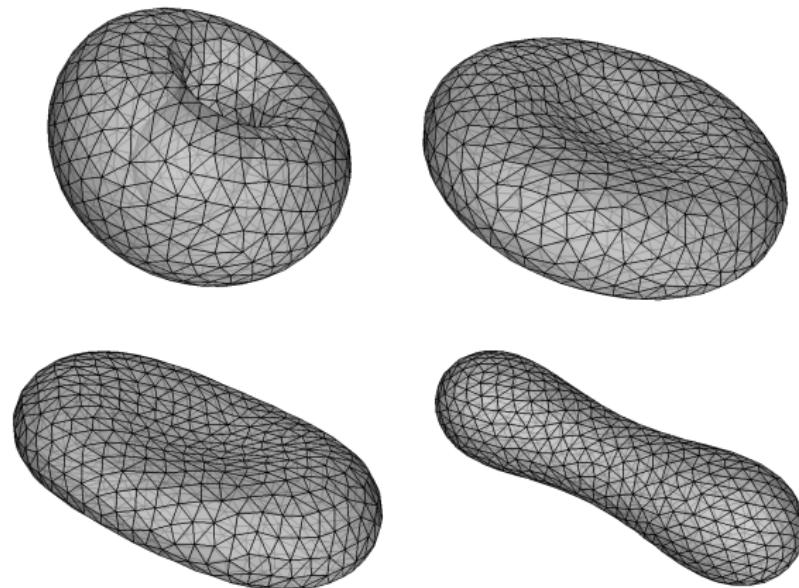
Minimal

| V_r | B: $N_t = 320$ | B: $N_t = 1280$ | B: $N_t = 5120$ | C: $N_t = 1280$ | C: $N_t = 5120$ | D: $N_t = 1280$ | D: $N_t = 5120$ |
|-------|---|---|---|---|---|--|---|
| 0.45 |  |  |  |  |  |  |  |
| 0.4 |  |  |  |  |  | NA |  |
| 0.35 |  |  |  |  |  | NA |  |
| 0.3 |  |  |  |  |  | NA |  |
| 0.25 |  |  |  |  |  | NA | NA |

SC



ADE



-  Schmid-Schönbein, H and Grebe, R and Heidtmann, Hannelore, A new membrane concept for viscous RBC deformation in shear, *Annals of the New York Academy of Sciences*, 416 (1983), 225–254
-  Deserno, M Fluid lipid membranes: From differential geometry to curvature stresses *Chemistry and physics of lipids*, 185 (2015), 11-45
-  Khairy K., Howard J. Minimum-energy vesicle and cell shapes calculated using spherical harmonics parameterization *Soft Matter* 7.5 (2011) 2138-2143.