SIMON FRASER UNIVERSITY
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Faculty of Applied Science

School of Mechatronics Systems Engineering

MSE 211 - Computational Methods for Engineers

Instructor : Dr. Ahad Armin

Project - Mathematical Modeling and Simulation

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Due date:

April 11th, 2023

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Abstract

The modeling and simulation of the oscillation of a pendulum with consideration of the frictional force is done on Matlab. This is achieved by using the Runge-Kutta 4th order method and ODEs to evaluate a function for different values of θ and λ. With the help of Matlab, the change in the displacement and velocity over time are observed and plotted on graphs where they are compared to the analytical solutions which are found from a predefined function.

Introduction

The modeling and simulation of the oscillation of a pendulum, with consideration of friction is the focus of this report. The first problem is understanding how the angle θ of the pendulum changes over time. The second problem is observing how velocity changes over time. This is done by using various analytical and numerical methods such as Runge-Kutta 4th order method and Ordinary Differential Equations (ODEs) which are iterated through with the help of Matlab.

Results and Discussion

Displacement and Velocity Analysis

The displacement of a pendulum is simulated in Matlab where Runge-Kutta 4th order method is used to determine the change in the displacement over a time interval. For this project, the chosen time interval is (0,8). The function in Figure 1 was used in Matlab to determine the analytical values for the displacement of the pendulum where $\theta_0 = [0, 10, 15]$ and $\lambda = [0.05, 0.03, 0.07, 0.09]$. The analytical values for the displacement were obtained from Matlab, shown in Figure 2, where ft1, ft2, and ft3 are the values of the function evaluated when θ = 0, 10, and 15 respectively. Furthermore, the function is evaluated with the same values of θ for different values of λ.

Figure 1: Equation used to find the analytical values of the displacement

$$\theta(t) = \theta_0 \, exp\left(\frac{-\lambda t}{2m}\right)\left(cos(\alpha t) + \frac{\lambda}{2m\alpha} sin(\alpha t)\right)$$

Figure 2: Analytical values of displacement of a pendulum

lambda	theta1	theta2	theta3
0.03	0	4.7198	7.0797
0.05	0	3.6367	5.455
0.07	0	2.8021	4.2031
0.09	0	2.159	3.2385

The experimental values of the displacement and velocity of the pendulum are determined by using the Runge-Kutta 4th order method in Matlab. Figure 3 shows the results of the Runge-Kutta method evaluated at the same values for θ_0 and λ. The results of the graphs show that as λ increases, the displacement and the velocity of the pendulum approach a value of zero at a faster rate. On the other hand, as λ decreases it takes more time for the displacement and velocity to approach zero. In order to obtain the analytical values for the velocity of the pendulum, the time derivative of the function in Figure 1 is taken, thus yielding the equation in Figure 4. The analytical values of the velocity are plotted on Matlab as shown in Figure 5. Upon further analysis of the experimental values, they agree with the analytical solutions.

Figure 3: Graphs of the displacement and velocity of a pendulum evaluated at different values of θ and λ

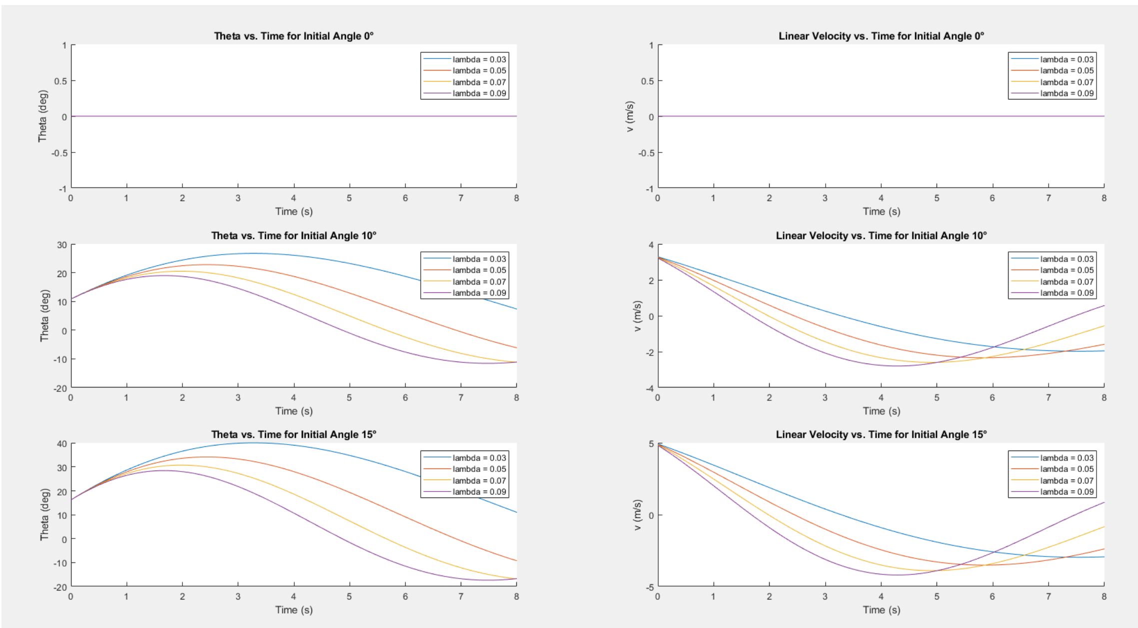
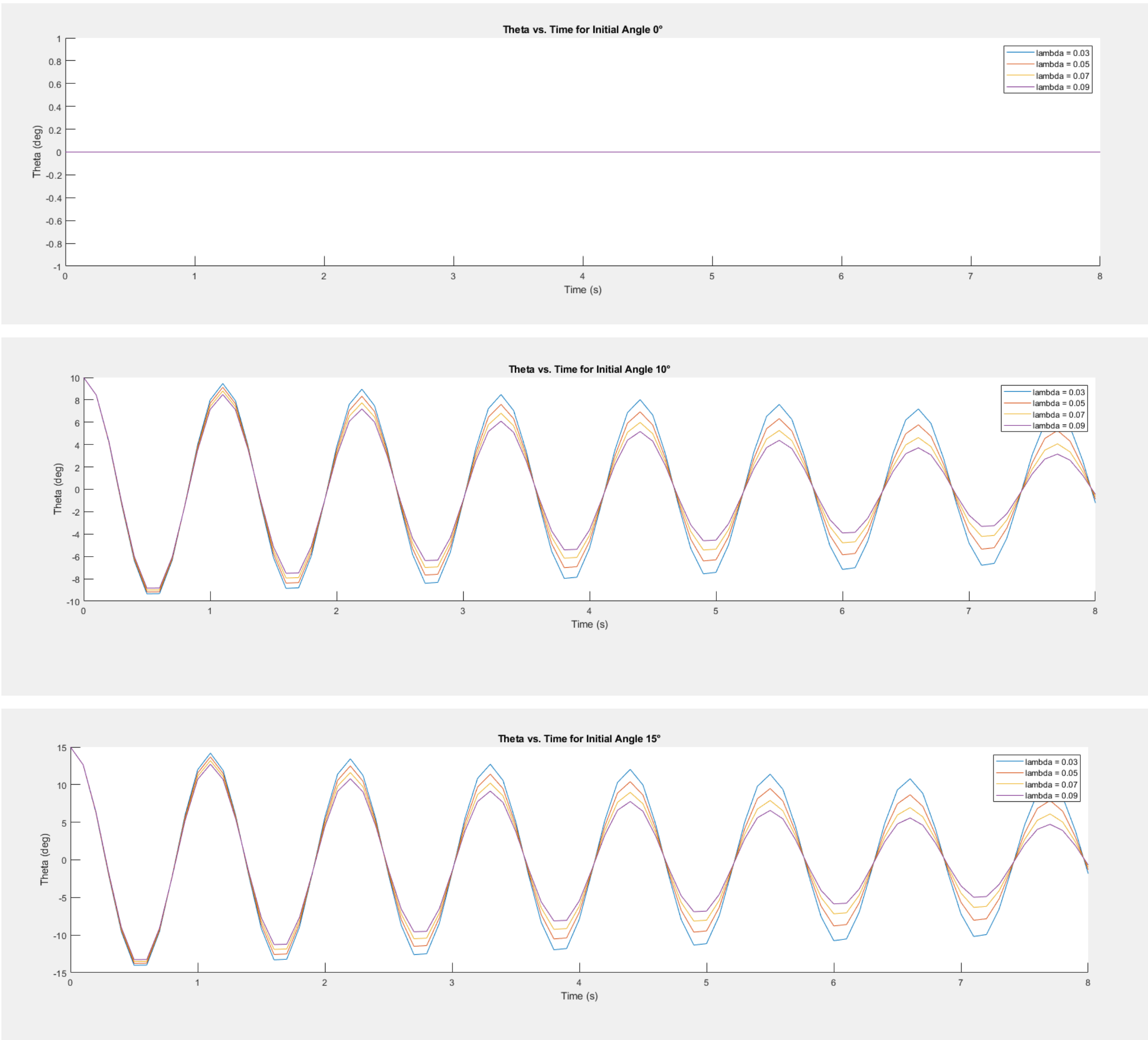


Figure 4: First derivative of Eqn(11) from the lab manual, evaluated by hand using Product Rule

$$\theta(t) = \theta_0 e^{\left(\frac{-t}{2m}\right)}\left[\cos(\alpha t) + \frac{\lambda}{2m\alpha}\sin(\alpha t)\right]$$

$$\frac{d\theta}{dt} = -\alpha\theta e^{\left(-\frac{\lambda t}{2m}\right)}\sin(\alpha t) - \left(\frac{\lambda}{2m}\right)\theta e^{\left(-\frac{\lambda t}{2m}\right)}\cos(\alpha t) + \left(\frac{\lambda}{2m\alpha}\right)\alpha\theta e^{\left(-\frac{\lambda t}{2m}\right)}\cos(\alpha t) - \left(\frac{\lambda}{2m}\right)\left(\frac{\lambda}{2m\alpha}\right)\theta e^{\left(-\frac{\lambda t}{2m}\right)}\sin(\alpha t)$$

Figure 5: Graphs of analytical values for the velocity



Conclusion

The kinematic behavior of a pendulum can be modeled with a 2nd-order differential equation. By increasing the initial angle, the velocity also increases. Changing the friction coefficient, λ, changes how quickly the waveform attenuates to 0. Having a larger λ attenuates the waveform to zero sooner.