

Homework Assignment 9 [30 points]

STAT430 Unsupervised Learning - Fall 2023

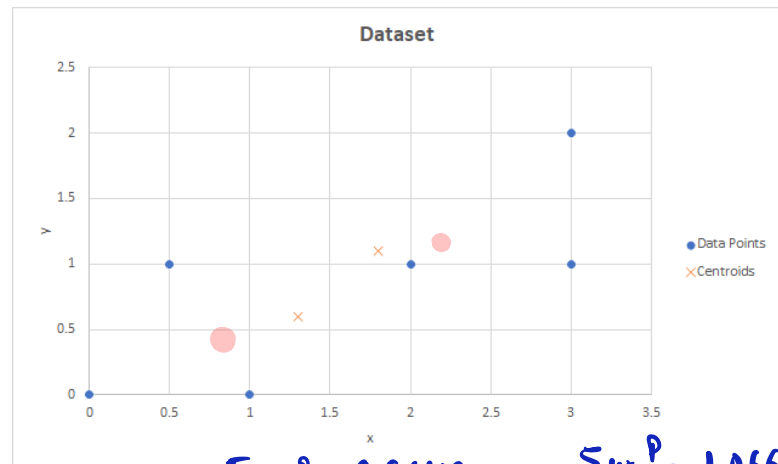
Problem	Points
1	0.5
2.1	0.25
2.2	2
2.3	1.5
2.4	1
3.1	1
3.2	3
4	0.5
5.1	0.75
5.2	0.75
5.3	0.75
5.4	0.75
5.5	1
6.1	1.5
6.2	2
6.3	0.5
6.4	1
6.5	1.5
6.6	0.75
6.7	0.5
6.8	1
6.9	0.75
6.1	1
6.11	1
6.12	1.5
6.13	0.5
7	2.5

Questions #1-#6: Answer the questions in the jupyter notebook.

Question #7:

We would like to cluster the dataset below using Fuzzy c-Means using $c = 2$ clusters and $p = 3$. The *current* centroids for the two cluster are (1.3, 0.6) and (1.8, 1.1). In the second table below, we have provided the squared distance that each object is to each of the *current* centroids. Determine what the *new* centroids will be in the next iteration of the algorithm. Show your work.

	Dataset	
	x	y
Object 1	0	0
Object 2	1	0
Object 3	0.5	1
Object 4	3	1
Object 5	3	2
Object 6	2	1



	Additional Information	
	Squared Distance to Centroid 1 (1.3, 0.6)	Squared Distance to Centroid 2 (1.8, 1.1)
Object 1	2.05	4.45
Object 2	0.45	1.85
Object 3	0.8	1.7
Object 4	3.05	1.45
Object 5	4.85	2.25
Object 6	0.65	0.05

$$\sum w_{ij}^p = 0.8649$$

Centroid 1
weighted
Distance

$$0.5956$$

$$0.6697$$

$$0.5931$$

$$0.4081$$

$$0.4052$$

$$0.2171$$

$$\sum w_{ij}^p = 1.0664$$

Centroid 2.
weighted
Distance

$$0.4044$$

$$0.3303$$

$$0.4069$$

$$0.5919$$

$$0.5948$$

$$0.7828$$

Example for object 1.

$$w_{11} = \frac{(1/2.05)^{1/3-1}}{(1/2.05)^{1/3-1} + (1/4.45)^{1/3-1}} = \frac{0.6984}{0.6984 + 0.4740} = 0.5956$$

$$w_{12} = \frac{(1/4.45)^{1/3-1}}{(1/2.05)^{1/3-1} + (1/4.45)^{1/3-1}} = \frac{0.4740}{0.6984 + 0.4740} = 0.4044$$

Example for C_{1x} .

$$C_{1x} = \frac{0.5956^3(0) + 0.6697^3(1) + 0.5931^3(0.5) + 0.4081^3(3) + 0.4052^3(3) + 0.2171^3(2)}{0.5956^3 + 0.6697^3 + 0.5931^3 + 0.4081^3 + 0.4052^3 + 0.2171^3} = 0.8286 / 0.8649 = 0.96$$

$$C_{1y} = 0.49$$

$$C_{2x} = 2.14$$

$$C_{2y} = 1.10$$

$$w_{ij} = \frac{(1/\text{dist}(x_i, c_j))^p}{\sum_{j=1}^c (1/\text{dist}(x_i, c_j))^p}$$

Centroid update.

$$C_k = \frac{\sum_{i=1}^n w_{ik}^p x_i}{\sum_{i=1}^n w_{ik}^p}$$

$$C_1 = (0.96, 0.49)$$

$$C_2 = (2.14, 1.10)$$