Lab 2: Resistors and Diodes

ENGR 2420 | Olin College

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Prelab

1. **Diode-Connected Transistor Characteristics**

(a) If we force a current greater than a nanoamp into the diode-connected transistor, we could represent it's characteristics with the following equation.

$$\begin{aligned} 2nA &= 3fA \cdot e^{\frac{V}{U_T}} - 1 \\ \frac{2 \times 10^-9A}{2 \times 10^-15A} &= e^{\frac{V}{U_T}} - 1 \\ 1000000A &= e^{\frac{V}{U_T}} - 1 \\ 1000000 + 1 &= e^{\frac{V}{U_T}} \end{aligned}$$

When we simplify, we see that the term, + 1, does not have much of an influence given that I_s is so small. Therefore we can conclude that the equation $I = I_s(e^{\frac{V}{U_T}})$ a good approximation of the transistor's characteristics. We'll use it for the remainder of the questions in this part of the lab.

(b) In general, an increase of I by a factor n yields a voltage generalized by $V = U_T \ln{(n)} + U_T \ln{(\frac{I}{I_s})}$.

Increasing the current by e fold gives:

$$\begin{gathered} V = U_T(\ln{(e \cdot I)} - \ln{I_s}) \\ \downarrow \\ V = U_T \ln{e} + U_T \ln{\frac{I}{I_s}} \\ \downarrow \\ V = U_T + U_T \ln{\frac{I}{I_s}} \end{gathered}$$

Increasing the current by a decade yields:

$$\begin{gathered} V = U_T(\ln{(10 \cdot I)} - \ln{I_s}) \\ \downarrow \\ V = U_T \ln{(10)} + U_T \ln{\frac{I}{I_s}} \end{gathered}$$

(c) The equation for *incremental diode resistance* can be simplified as show.

$$r_d = \frac{\delta V}{\delta I} = \frac{\delta}{\delta I}(U_T \ln{I} - U_T \ln{I_s}) = \frac{U_T}{I}$$

(d) Given what we've established above, a voltage across the transistor would result in the same behavior since V and I are linked in the approximated ideal diode equation, $I = I_s(e^{\frac{V}{U_T}})$.

(e) To start, we can take the approximated ideal diode equation, $I = I_s e^{\frac{V}{U_T}}$, and take the natural log of both sides as shown.

$$\ln{I} = \ln{e^{\frac{V}{U_T}}} + \ln{I_s}$$

If we plot the above on a semilog-y plot, we effectively have an equation in $y = mx + b$ form, which makes the above analogous to the following.

$$V(I) = \frac{I}{U_T} + I_s$$

.

We can now compute linear regression on the data, extracting U_T from the slope of the best fit line and I_s as its intercept.

2. **Characteristics of a Resistor and a Diode in Series**

   (a) Equations for the voltage across each component are as follows.

$$V_R = IR$$

$$V_T = U_T \ln{(\frac{I}{I_s})}$$

$$V_{in} = IR + U_T \ln{(\frac{I}{I_s})}$$

   (b) Equations for the change in voltage across each component are as follows.

$$\delta V_R = \frac{\delta}{\delta I}(IR) = R$$

$$\delta V_T = \frac{\delta}{\delta I}(U_T \ln{(\frac{I}{I_s})}) = \frac{U_T}{I}$$

$$\delta V_{in} = \frac{\delta}{\delta I}(U_T \ln{(\frac{I}{I_s})} + IR) = \frac{U_T}{I} + R$$

   (c) The derived equation for the *turn on current* of the transistor as a function of R is as follows.

$$I_{on} = \frac{U_T}{R}$$

   (d) We can express V_{on} as the following equation.

$$V_{on} = U_T \ln{(\frac{I_{on}}{I_s})}$$

   (e) We can represent the fractions of δV_{in} that appear across the transistor and resistor as follows.

$$\begin{array}{cc} \frac{\delta V_T}{\delta V_{in}} = \frac{\frac{U_T}{I}}{\frac{U_T}{I}+R} & \frac{\delta V_R}{\delta V_{in}} = \frac{R}{\frac{U_T}{I}+R} \\ \downarrow & \downarrow \\ \frac{\delta V_T}{\delta V_{in}} = \frac{I_{on}}{I_{on}+I} & \frac{\delta V_R}{\delta V_{in}} = 1 - \frac{I_{on}}{I_{on}+I} \end{array}$$

   (f)
   - When $I \ll I_{on}$, δV_T, almost all of δV_{in} is coming from δV_T.
   - V_{in} will change logarithmically with respect to I

   - When $I \gg I_{on}$, δV_T, contributes a nothing to δV_{in}
   - V_T is not changing much with respect to V_{in}
   - V_{in} changes linearly with respect to I

   (g)
   - V would change linearly with V_{in} when $V_{in} < V_{on}$ by more than a few U_T
   - I does not depend on V_{in} in these circumstances given the current response of the transistor

   - V would change exponentially with V_{in} when $V_{in} > V_{on}$ by more than a few U_T
   - I would correlate exponentially with V_{in}