Lab 1: Resistors and Resistive Networks

ENGR 2420 | Olin College

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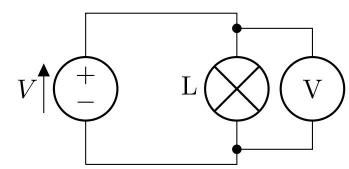
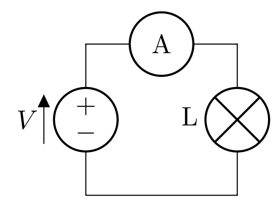
Prelab

1. **Electrical Measurement Concepts**

   (a) An ideal voltage meter has **infinite** internal resistance and should be connected in **parallel** with the circuit or device being tested.



   (b) An ideal current meter has **zero** internal resistance and should be connected in **series** with the circuit or device being tested.



2. **Resistive Divider Accuracy**

To find the coefficient of variation, CV, of a resistive divider, we start with the gain equation from a resistor divider.

$$A = \frac{R_2}{R_1 + R_2}$$

and plug it into the sensitivity equation like so.

$$S_{R_i}^{A} = R_i \frac{\delta \ln \frac{R_2}{R_1+R_2}}{\delta R_i}$$

Solving for the sensitivity with each resistor gives

$S_{R_1}^{A} = \frac{-R_1}{R_1+R_2}$ and $S_{R_2}^{A} = 1 - \frac{R_2}{R_1+R_2}$.

Now, we'll use this equation

$$CV_A = \sqrt{\sum_{i=1}^{n} (S_{R_i}^{A})^2 \cdot (CV_{R_i})^2}$$

and substitute in our resistor values and sensitivities like so.

$$CV_A = \sqrt{(\frac{-R_1}{R_1 + R_2})^2 \cdot CV_{R_1}^2 + (1 - \frac{R_2}{R_1 + R_2})^2 \cdot CV_{R_2}^2}$$

Simplifying, this gives us a final expression for the coefficient of variation as the following:

$$CV_A = \frac{R_1\sqrt{CV_1^2 + CV_2^2}}{R_1 + R_2}$$

3. **R-2R Ladder Network**

   One interesting attribute of the R-$2R$ ladder network is that at each node before a branch, the resistance of the remainder of the of ladder from that point is simply $2R$.

   Furthermore, at each branch the current, I_n, halves since it's being divided between the branch and the rest of the ladder

   Incorporating this information into $I = \frac{V}{R}$, we can quantify the sum of the current in the whole ladder as the following equation:

$$I_n = \frac{V}{2^{n+1} \cdot R}$$

   Therefore, the currents flowing through each of the $2R$ branches as a function of V and R are as follows:

$$I_1 = \frac{V}{4 \cdot R} \qquad I_n = \frac{V}{2^{n+1} \cdot R} \qquad I_{N-1} = \frac{V}{R} \qquad I_N = \frac{V}{2^{N+1} \cdot R}$$

4. **Accurate 2:1 Resistor Ratios**

   Two different implementations of a single unit in an R-$2R$ ladder network are shown below.

   This implementation uses 3 resistors for a single intermediary ladder branch and an additional 2 for the final branch, for a minimum of **5 resistors** in the smallest possible R-$2R$ ladder.

   This implementation uses less resistors, with 3 for a single intermediary ladder branch and only one additional resistor for the final branch, for a minimum of **4 resistors** in the smallest possible R-$2R$ ladder.