Lab 3: Resistors and Diodes

ENGR 2420 | Olin College

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Prelab

1. Forward-Active Bipolar Transistor Characteristics

   (a) We have equations relating both α and β to I_s, so we can setup an equation and simplify as shown.

   $$\frac{I_b\beta}{e^{V_{be}/U_T}} = \frac{I_e\alpha}{e^{V_{be}/U_T}}$$
   $$I_b\beta = I_e\alpha$$

   (b) From the lab handout, we're given a derivation for r_d in terms of I_b as shown:

   $$r_b = \frac{\delta V_b}{\delta I_b}$$
   $$r_b = (\frac{I_s}{\beta} \cdot e^{V_{be}/U_T} \cdot \frac{1}{U_T})^{-1}$$
   $$r_d = \frac{U_T}{I_b}$$

   (c) We can find an expression for g_m in terms of I_c as follows.

   $$g_m = \frac{\delta I_c}{\delta V_b}$$
   $$g_m = I_s e^{\frac{V_{be}}{U_T}} \frac{1}{U_T}$$
   $$g_m = \frac{I_c}{U_T}$$

   (d) r_d and g_m are inversely proportional to each other as a function of β, as shown.

   $$g_m \frac{e^{\frac{V_b}{V_e}}}{U_T} \rightarrow \frac{I_c}{U_T} \rightarrow \frac{\beta I_b}{U_T}$$
   $$r_d = \frac{U_T}{I_b},\ g_m = \frac{\beta I_b}{U_T}$$
   $$r_d = \frac{\beta}{g_m}$$

2. Emitter-Degenerated Bipolar Transistor

   (a) We can represent r_b in terms of I as shown.

$$R_b = \frac{\delta V_b}{\delta I_b} = r_b + (\beta_F + 1) \cdot R_x$$
$$r_b = R_b - (\beta_F + 1) \cdot R_x$$
$$r_b = R_b - (\frac{I_s}{I_b} e^{\frac{V_{be}}{U_T}} + 1) \cdot R_x$$

   (b) We can derive an equation for G_m in terms of I_e as follows:

$$G_m = \frac{\delta I_e}{\delta V_b}$$
$$G_m = \frac{g_m}{1 + R_x / r_e}$$
$$G_m = \frac{g_m}{1 + R_x / \frac{r_b}{\beta_F + 1}}$$
$$G_m = g_m (1 + R_x \frac{R_b - \frac{I_s}{I_b} e^{V_{be}/U_T}}{\frac{I_s}{I_b} e^{V_{be}/U_t} + 1})^{-1}$$

   (c) We can represent δV_e in terms of δV_b as follows:

$$I_e + \delta I_e = \frac{I_s}{\alpha} \cdot e^{\frac{V_{be}}{U_T}} \cdot e^{\frac{\delta V_b - \delta V_e}{U_T}}$$
$$I_e + \delta I_e = \frac{I_s}{\alpha} \cdot e^{\frac{V_{be}}{U_T}} (1 + \frac{\delta V_b - \delta V_e}{U_T})$$
$$I_e + \delta I_e = I_e \cdot (1 + \frac{\delta V_b - \delta V_e}{U_T})$$
$$I_e + \delta I_e = I_e + I_e \cdot (\frac{\delta V_b - \delta V_e}{U_T})$$
$$\delta I_e = I_e \cdot (\frac{\delta V_b - \delta V_e}{U_T})$$
$$\frac{\delta V_e}{R} = I_e \cdot (\frac{\delta V_b - \delta V_e}{U_T})$$
$$\delta V_e U_T = R \cdot I_e \cdot (\delta V_b - \delta V_e)$$
$$\delta V_e = \frac{\delta R V_b I_e}{U_T + R I_e}$$
$$\delta V_e = \frac{\delta V_b I_e}{\frac{U_T}{R} + I_e}$$

   (d) We can represent $I_o n$ as a function of R as follows:

$$\delta V_b = \delta I_b \cdot \frac{U_T \beta}{I_{on}} + \frac{1}{2} \delta V_b$$
$$\frac{1}{2} (\delta I_b \frac{U_T \beta}{I_{on}} + \delta I_b \cdot (\beta + 1) \cdot R = \frac{\delta I_b U_T \beta}{I_{on}}$$
$$\frac{1}{2} (\frac{U_T \beta}{I_{on}} + (\beta + 1) \cdot R = \frac{U_T \beta}{I_{on}}$$
$$\frac{1}{2} (\frac{U_T \beta}{I_{on}}) + \frac{1}{2} R (\beta + 1) = \frac{U_T \beta}{I_{on}}$$
$$R(\beta + 1) = \frac{U_T \beta}{I_{on}}$$
$$I_{on} = \frac{U_T \alpha}{R}$$

(e) We can simplify an expression for V_{on} as a function of I_{on} as follows.

$$I = I_{on} = I_s e^{V/U_T} - 1$$
$$V_{on} = U_T \log \frac{I_{on}}{I_s} + 1$$

(f) We can express the relationship between I_c and V_b in terms of V_on and U_T.

$$\frac{V_e}{R} = I_c + \frac{I_c}{\beta}$$
$$\frac{V_b - V_{be}}{R} = I_c + \frac{I_c}{\beta}$$
$$I_{on}\frac{V_b - V_{be}}{U_T \cdot \alpha} = I_c + \frac{I_c}{\beta}$$
$$I_s \cdot e^{\frac{V_{on}}{U_T}} \cdot \frac{V_b - V_{be}}{U_T \cdot \alpha} = I_c(1 + \frac{1}{\beta})$$
$$I_s e^{\frac{V_{on}}{U_T}} \cdot \frac{V_b}{V_{be}} = I_s e^{\frac{V_{be}}{U_T}} \cdot \frac{1}{\alpha}$$
$$e^{\frac{V_{on}}{U_T}} \cdot \frac{V_b}{V_{be}} = e^{\frac{V_{be}}{U_T}} \cdot \frac{1}{\alpha}$$
$$e^{\frac{V_{on}}{U_T}} \cdot \frac{V_b}{V_{be}} = U_T e^{\frac{V_{be}}{U_T}}$$
$$V_b - Vbe = U_T e^{\frac{V_{be} - V_{on}}{U_T}}$$
$$V_b = U_T + Vbe + U_T e^{\frac{V_{be} - V_{on}}{U_T}}$$
$$V_b = V_{be}\frac{I_c}{g_m} e^{\frac{V_{be} - V_{on}}{U_T}}$$
$$V_b - V_{on} = V_{be} - V_{on} + \frac{I_c}{g_m} e^{\frac{V_{be} - V_{on}}{U_T}}$$

(g) We can approximate the previous relationship when $V_{be} < V_{on}$ as $V_b = V_{be}$ because as V_{on} approaches a large number (infinity), the entire exponential term above will go to zero because e will be raised to a negative power.

Because we factor out the term containing I_c, we can't generate an expression relating it to V_b. This makes sense because the transistor should effectively not pass a current to the collector when the V_{be} is less than the turn on voltage.

(h) We can approximate the relationship when $V_{be} > V_{on}$ as $V_b = V_{be} + \frac{I_c}{g_m} e^{\frac{V_{be}}{U_T}}$
Solving for I_c, we get the following:

$$V_b = V_{be} + \frac{I_c}{g_m} e^{\frac{V_{be}}{U_T}}$$
$$V_b - V_{be} = \frac{I_c}{g_m} e^{\frac{V_{be}}{U_T}}$$
$$g_m(V_b - V_{be}) = I_c e^{\frac{V_{be}}{U_T}}$$
$$g_m \frac{V_b - V_{be}}{e^{\frac{V_{be}}{U_T}}} = I_c$$