Fall-2023 5304 PrEx04

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This exercise is related to LecN5 and LecN6.

1 True or False

a: If A and B are SPD then A+B is also SPD

True. Using the operation property of inner product
$\langle (A+B)u, u\rangle = \langle Au, u\rangle + \langle Bu, u\rangle > 0$

b: When A is SPD then its inverse is also SPD

True. There are 2 ways to show.

Way1: Matrix is SPD if it is symmetric and its eigenvalues are positive.
Then, use the fact that the eigenvalues of A^{-1} are (1/eigenvalues of A).

Way2: Use the definition of SPD and the property of inner product.
Known that $\langle Au, u\rangle > 0$ for all $u \neq 0$
Let $v = A^{-1}u$, then $u = Av$ and $v = A^{-1}u \neq 0$
Thus,
$\left\langle A^{-1}u, u\right\rangle = \langle v, Av\rangle = \langle Av, v\rangle > 0.$