Fall-2023 5304 LecN1 Notes

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November 6, 2023

Topics: Introduction; Types of problems seen in this course; Math background; Matrices; Eigenvalues and Eigencevectors; Null space and range; Rank; Types of matrices; Special Matrices.

1 Matrix Multiplication

Matrix-Matrix Multiplication

Matrix-Vector Multiplication, 2 Ways

Form 1: Dot product view

Form 2: Linear combination view In this view, vector can be treated as a place storing the coefficients of the column of matrix A.

Vector-Matrix Multiplication, 2 Ways

Vector-Vector Multiplication, 2 Ways

Form 1: Inner product view This is a scalar.
Form 2: Outer product view Will produce rank-1 matrix.

2 Rank

Rank + Nullity Theorem

Some Conclusions/Facts

1, A full rank matrix X, and a nonzero vector v, $Xv \neq 0$
2, If A is a nonsingular, square matrix, full rank, then A^{-1} is full rank. $R(A) = R(A^{-1}) = n$
3, If we have a matrix A and a full rank matrix Q, then R(A) = R(QA) = R(AQ). i.e., useing Q left-multiply a matrix or right-multiply a matrix will not change the rank of the original matrix.

3 Special Matrices

Diaguonal Matrix

Vandermond

Hermitan

Unitary

Orthogonal

Symmetric and Skew-Symmetric

Def od SM: $A^T = A$

Def of SSM: $A^T = -A$

Skew-Symmetric