Turn this in by making a pdf scan of your work and submitting it to Canvas by 11:59 pm on the due date.

How to write up homework. Please put your name and assignment number on the first page. You may use this page as a cover sheet. Show your work and explain it. In general, just writing down an answer without showing the steps to arriving at the answer is not sufficient for credit.

Current reading is Chapter 2 of Linear Algebra Done Wrong.

1. An $n \times n$ matrix is called symmetric if $A^T = A$. Let

$$V = \{A \in M_{n \times n} \mid A \text{ is symmetric}\}.$$

   (a) Show that V is a vector space, by showing that V is a subspace of $M_{n \times n}$. Use Lemma 4.3.2 on page 40 in the UC Davis text: show that

      (i) The additive identity belongs to V: $0_{n \times n} \in V$,
      (ii) V is closed under addition,
      (iii) V is closed under scalar multiplication.

      (This method of showing V is a vector space is less tedious than verifying that V satisfies all of the axioms of a vector space.)

   (b) Determine a basis for V. Hint: Let E_{ij} denote the matrix whose (i, j)- and (j, i)-entries are 1, with 0's elsewhere.

   (c) What is the dimension of V? (The dimension of a vector space V is the number of elements in a basis for V.)

2. Find the standard matrix for each linear transformation below:

   (a) $T : \mathbb{C}^3 \to \mathbb{C}^2$ given by $T(x, y, z)^T = (x + iy, x - iz)^T$.

      Here $(x, y, z)^T$ denotes the column vector $\begin{bmatrix} x \\ y \\ z \end{bmatrix}$.

   (b) $T : \mathbb{P}_4 \to \mathbb{P}_2$ given by $Tf(x) = f''(x)$. Use the standard basis $\{1, x, x^2, ..., x^n\}$ for $\mathbb{P}_n$.

3. Let $T_\theta : \mathbb{R}^2 \to \mathbb{R}^2$ be rotation by angle θ (counterclockwise from positive x-axis). In particular, $T_\alpha T_\beta = T_{\alpha + \beta}$. Deduce formulas for $\sin(\alpha + \beta)$ and $\cos(\alpha + \beta)$.

4. Read Chapter 2 of the UC Davis Text: Introduction to Complex Numbers. This should be a review. Then do the following problems from pages 24 - 25 (which should also be a review):

#1(a)(b)(c)(d)(f),

#2(a),

#3,

#4(b),

#6(a)(b).

(These are from the "Calculational Exercises" not the "Proof-Writing Exercises" for this Chapter.)