2.) Find the standard matrix

a.) $T: \mathbb{C}^3 \to \mathbb{C}^2$ given by $T(x,y,z)^T = (x+iy, x-iz)^T$

Let $A = \begin{bmatrix} a & b & c \\ d & e & f \end{bmatrix}$ be the standard matrix

Then $\begin{bmatrix} a & b & c \\ d & e & f \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} ax+by+cz \\ dx+ey+fz \end{bmatrix} = \begin{bmatrix} x+iy+0z \\ x+0y-iz \end{bmatrix}$

So, $M = \begin{bmatrix} 1 & i & 0 \\ 1 & 0 & -i \end{bmatrix}$

b.) $T: \mathbb{P}_4 \to \mathbb{P}_2$ given by $Tf(x) = f''(x)$.

Let $f(x) = a_0 + a_1 x + a_2 x^2 + a_3 x^3 + a_4 x^4$

$f'(x) = a_1 + 2a_2 x + 3a_3 x^2 + 4a_4 x^3$

$f''(x) = 2a_2 + 6a_3 x + 12a_4 x^2$

Then $\begin{bmatrix} u_{11} & \cdots & u_{15} \\ u_{21} & \ddots & \vdots \\ u_{31} & \cdots & u_{35} \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \\ a_5 \end{bmatrix} = \begin{bmatrix} 2a_2 \\ 6a_3 \\ 12a_4 \end{bmatrix}$

solution $\to$ $u_{13} = 2$, $u_{24} = 6$, $u_{35} = 12$

$\therefore$ standard matrix $= \begin{bmatrix} 0 & 0 & 2 & 0 & 0 \\ 0 & 0 & 0 & 6 & 0 \\ 0 & 0 & 0 & 0 & 12 \end{bmatrix}$

3.) Let $T_\theta : \mathbb{R}^2 \to \mathbb{R}^2$ be rotation by angle θ, in particular, $T_a T_b = T_{a+b}$

Deduce formulas for $\sin(\alpha+\beta)$ & $\cos(\alpha+\beta)$.

Let $T_\alpha = \begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$ & $T_\beta = \begin{bmatrix} \cos\beta & -\sin\beta \\ \sin\beta & \cos\beta \end{bmatrix}$

Then $T_\alpha T_\beta = \begin{bmatrix} \cos\alpha\cos\beta - \sin\alpha\sin\beta & -(\sin\alpha\cos\beta + \cos\alpha\sin\beta) \\ \sin\alpha\cos\beta + \cos\alpha\sin\beta & \cos\alpha\cos\beta - \sin\alpha\sin\beta \end{bmatrix}$

$= T_{\alpha+\beta} \quad \begin{bmatrix} \cos(\alpha+\beta) & -\sin(\alpha+\beta) \\ \sin(\alpha+\beta) & \cos(\alpha+\beta) \end{bmatrix}$

so, $\sin(\alpha+\beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$

$\cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$

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Homework 1

1.) $V = \{A \in M_{n \times n} \mid A^T = A\}$

i) Let $a_{ij} = 0$ for all i, j

Then $a_{ij} = a_{ji}$.

so $\forall A \in M_{n \times n}$

$0_{n\times n} + A_{n\times n} = A_{n\times n} + 0_{n \times n} = A_{n\times n}$, $\therefore 0_{n \times n} \in V$ ✓

ii) Let A & $B \in V$

where $A = (a_{ij})_{n\times n}$, $B = (b_{ij})_{n\times n}$

Consider $A + B = (a_{ij})_{n\times n} + (b_{ij})_{n\times n} = (c_{ij})_{n\times n}$

$\because A^T = A$ & $B^T = B$ so:

$a_{ij} + b_{ij} = c_{ij} = a_{ji} + b_{ji} = c_{ji} \quad \forall ij$

so $c = (c_{ij})_{n\times n}$ is symmetric

$\therefore c \in V$ & V is closed under addition.

iii) Let $A \in V$ & $c \in R$

Consider $cA = c(a_{ij})_{n\times n} = (cb_{ij})_{n\times n}$

so, $cA \in V$ & V is closed under scalar mult.

Moreover, V is a subspace of $M_{n\times n}$ $\therefore V$ is a vector space.

6.) Let E_{ij} be the matrix whose (i,j) & (j,i) entries are 1 with 0's elsewhere.

Let $A = (a_{ij}) \in V$ be any index

Then A_{ij} has value 1 at a_{ij} & a_{ji} withe zero's elsewhere

consider $A = A_{11}E_{11} + A_{12}E_{12} + \dots + A_{1n}E_{1n}$
$+ A_{22}E_{22} + A_{23}E_{23} + \dots + A_{2n}E_{2n}$
$+ A_{33}E_{33} + \dots + A_{nn}E_{nn}$

Then $\{E_{11}, E_{12}, \dots E_{1n}, E_{22}, \dots E_{2n}, E_{33}, \dots, E_{3n}, E_{44}, \dots E_{4n} \dots E_{nn}\}$ is a basis for V.

Moreover $\dim(V) = \frac{n(n+1)}{2}$