Bennet Sloan Engr 203 5/24/19

7.1 If periodic, find the period

a) $f(t) = \cos(\pi t) + 2\cos(3\pi t) + 3\cos(5\pi t)$

$\because f(t) = 1^{st} + 3^{rd} + 5^{th}$ harmonics of cosine $\rightarrow$ $\boxed{\text{Periodic}}$

Also, $\omega_0 = \pi \rightarrow 2\pi f = \pi \rightarrow 2\pi \frac{1}{T} = \pi \rightarrow$ $\boxed{T = 2}$

b.) $y(t) = \sin(t) + 4\cos(2\pi t)$

$\because y(t) = \sin + \cos$

Now, for sin: $T = \frac{2\pi}{1} = 2\pi$

And for cos: $T = \frac{2\pi}{2\pi} = 1$

The LCM of $(1, 2\pi)$ DNE $\rightarrow$ $\boxed{\text{Not periodic}}$

c.) $h(t) = \cos^2(t)$

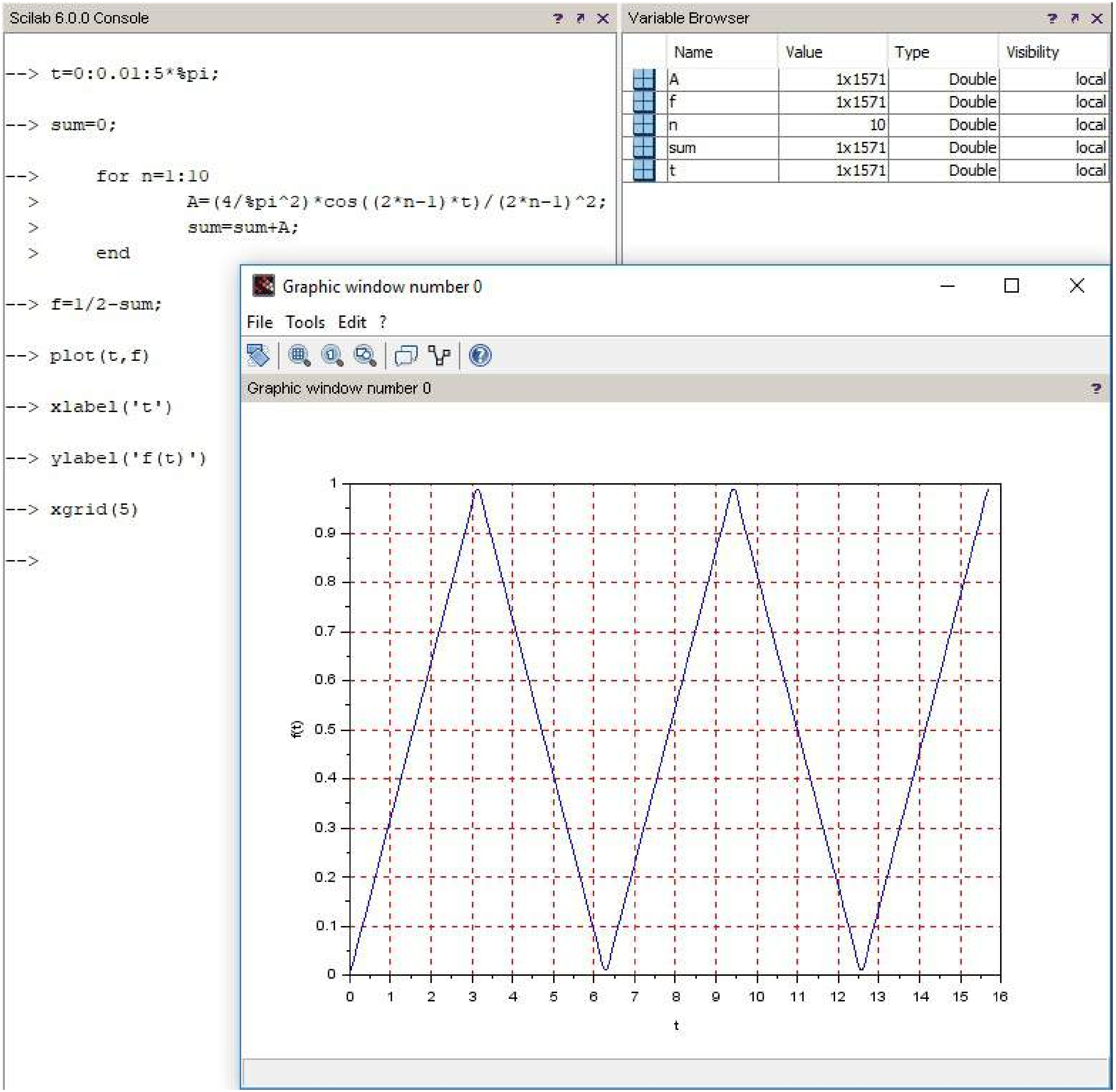
$\because h(t) = \cos^2(t) = \dfrac{1 + \cos(2t)}{2}$ by pythagorean identity.

so, $T = \frac{2\pi}{2} = \pi \rightarrow$ $\boxed{\text{Periodic}, \ T = \pi}$

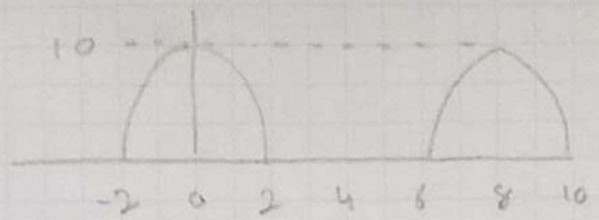
Alternatively

$\omega_0 = 2 \rightarrow 2\pi f = 2 \rightarrow 2\pi \frac{1}{T} = 2$

$\rightarrow T = \pi$ which checks again.


Scilab 6.0.0 Console
--> t=0:0.01:5*%pi;
--> sum=0;
--> for n=1:10
> A=(4/%pi^2)*cos((2*n-1)*t)/(2*n-1)^2;
> sum=sum+A;
> end
--> f=1/2-sum;
--> plot(t,f)
--> xlabel('t')
--> ylabel('f(t)')
--> xgrid(5)
-->
Variable Browser
Name Value Type Visibility
A 1x1571 Double local
f 1x1571 Double local
n 10 Double local
sum 1x1571 Double local
t 1x1571 Double local
Graphic window number 0
File Tools Edit ?
Graphic window number 0
f(t)
t
0 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1
0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16

2.3



$$f(t) = \begin{cases} 0 & , -6 < t < -2 \\ 10\cos\frac{\pi}{4}t & , -2 < t < 2 \\ 0 & , 2 < t < 6 \end{cases}$$

Since $f(t) = f(-t)$, $f(t)$ is an even function.

Also, $T = 8 - 0 = 8$, And $\omega_0 = \frac{2\pi}{T} = \frac{2\pi}{8} = \frac{\pi}{4}$

For DC component: $a_0 = \frac{1}{T}\int_0^T f(t)\,dt = \frac{2}{T}\int_0^{T/2} f(t)\,dt$

$= \frac{2}{8}\int_0^4 f(t)\,dt = \frac{2}{8}\left[\int_0^2 f(t)\,dt + \cancel{\int_2^4 f(t)\,dt}\right] = \frac{10}{4}\int_0^2 \cos\frac{\pi}{4}t\,dt$

$= \frac{10}{4}\left[\frac{4}{\pi}\sin\left(\frac{\pi}{4}t\right)\right]_0^2 = \frac{10}{\pi}(1-0) = \boxed{\frac{10}{\pi} = a_0}$

For a_n: $a_n = \frac{4}{T}\int_0^{T/2} f(t)\cos(n\omega_0 t)\,dt$, for even $f(t)$.

So, $= \frac{4}{8}\int_0^4 f(t)\cos(n\omega_0 t)\,dt = \frac{4}{8}\left[\int_0^2 f(t)\cos(n\omega_0 t) + \cancel{\int_2^4 f(t)\cos(n\omega_0 t)\,dt}\right]$

$= \frac{40}{8}\int_0^2 \cos\left(\frac{\pi}{4}t\right)\cos\left(\frac{n\pi}{4}t\right)dt = \underline{\frac{10}{2}\int_0^2 \cos\left(\frac{\pi}{4}t\right)\cos\left(\frac{n\pi}{4}t\right)dt = a_n}$

For $n=1$) $a_1 = \frac{10}{2}\int_0^2 \frac{1}{2}\left(1 + \cos\left(\frac{\pi t}{2}\right)\right)dt = \frac{10}{4}\left[t + \frac{2}{\pi}\sin\left(\frac{\pi}{2}t\right)\right]_0^2$

$= \frac{10}{4}\left[\left(2 + \frac{2}{\pi}\sin(\pi)\right) - \left(0 + \frac{2}{\pi}\sin(0)\right)\right] = \frac{10}{4}(2-0) = \boxed{5 = a_1}$

For $n>1$) $a_2 = \frac{10}{2}\int_0^2 \cos\left(\frac{\pi}{4}t\right)\cos\left(\frac{n\pi}{4}t\right)dt$, using $\cos A\cos B \rightarrow$

$= \frac{10}{2}\int_0^2 \frac{1}{2}\left\{\cos\left[\left(\frac{\pi}{4} + \frac{n\pi}{4}\right)t\right] + \cos\left[\left(\frac{\pi}{4} - \frac{n\pi}{4}\right)t\right]\right\}dt$

$= \underline{\frac{10}{4}\int_0^2 \cos\left[\left(\frac{\pi}{4} + \frac{n\pi}{4}\right)t\right] + \cos\left[\left(\frac{\pi}{4} - \frac{n\pi}{4}\right)t\right]dt, \quad n > 1}$

2.3 cont.

For $n=2$) $a_2 = \frac{10}{4}\int_0^2 \cos\left[\left(\frac{\pi}{4}+\frac{\pi}{2}\right)t\right] + \cos\left[\left(\frac{\pi}{4}-\frac{\pi}{2}\right)t\right] dt$

$= \frac{10}{4}\int_0^2 \cos\left(\frac{3\pi}{4}t\right) + \cos\left(\frac{\pi}{4}t\right) dt$, since $\cos(\theta) = \cos(-\theta)$

$= \frac{10}{4}\left[\frac{4}{3\pi}\sin\left(\frac{3\pi}{4}t\right) + \frac{4}{\pi}\sin\left(\frac{\pi}{4}t\right)\right]_0^2$

$= \frac{10}{4}\left[\left(\frac{4}{3\pi}\sin\left(\frac{3\pi}{2}\right) + \frac{4}{\pi}\sin\left(\frac{\pi}{2}\right)\right) - (0)\right]$

$= \frac{10}{4}\left[-\frac{4}{3\pi} + \frac{4}{\pi}\right] = \frac{10}{4}\left(\frac{8}{3\pi}\right) = \frac{80}{12\pi} = \boxed{\frac{20}{3\pi} = a_2}$

For $n=3$) $a_3 = \frac{10}{4}\int_0^2 \cos\left[\left(\frac{\pi}{4}+\frac{3\pi}{4}\right)t\right] + \cos\left[\left(\frac{\pi}{4}-\frac{3\pi}{4}\right)t\right] dt$

$= \frac{10}{4}\int_0^2 \cos(\pi t) + \cos(\pi t)\, dt$ since $\cos(\theta) = \cos(-\theta)$

$= \frac{10}{2}\int_0^2 \cos(\pi t)\, dt = 5\left[\frac{1}{\pi}\sin(\pi t)\right]_0^2 = 5[0-0] = 0$

So $\boxed{a_3 = 0}$

Lastly since $f(t)$ is even, it is purely cosines and therefore $\boxed{b_n = 0}$.

$\therefore$ $\boxed{\begin{aligned} a_0 &= \frac{10}{\pi} \\ a_1 &= 5 \\ a_2 &= \frac{20}{3\pi} \\ a_3 &= 0 \\ b_n &= 0 \end{aligned}}$

7.4 Determine if odd, even or neither

a.) $1+t$

consider $f(-t) = 1-t \rightarrow f(t) \neq f(-t)$

$-f(t) = -1-t \rightarrow f(-t) \neq -f(t)$ $\therefore$ $\boxed{\text{neither}}$

b.) t^2-1

consider $f(-t) = t^2-1 \rightarrow f(t) = f(-t)$ $\therefore$ $\boxed{\text{Even}}$

c.) $\cos(n\pi t)\sin(n\pi t)$

consider $f(-t) = \cos(-n\pi t)\sin(-n\pi t)$

$= -\cos(n\pi t)\sin(n\pi t) \rightarrow f(-t) = -f(t)$ $\boxed{\text{ODD}}$

d.) $\sin^2(\pi t)$

consider $f(-t) = \sin^2(-\pi t)$

$= \sin(-\pi t)\sin(-\pi t)$

$= [-\sin(\pi t)][-\sin(\pi t)]$

$= \sin^2(\pi t)$

$\rightarrow f(-t) = f(t)$ $\therefore$ $\boxed{\text{Even}}$