Complete this and submit it to Canvas by the posted due date: Wednesday, Feb. 20.

Recall that we can view matrix multiplication in the following way (p. 19 LADW):

If $\mathbf{b}_1$, $\mathbf{b}_2$, ..., $\mathbf{b}_r$ are the columns of B , then $A\mathbf{b}_1$, $A\mathbf{b}_2$, ..., $A\mathbf{b}_r$ are the columns of AB.

Useful notation: Let $\{\vec{e}_1, \vec{e}_2, \ldots, \vec{e}_n\}$ denote the standard basis for $\mathbf{R}^n$. Then the $n \times n$ identity matrix can be written

$$I = [\vec{e}_1\, \vec{e}_2\, \cdots\, \vec{e}_n].$$

1. Suppose A is an $n \times n$ matrix such that $A = SDS^{-1}$ where $D = \mathrm{diag}\{d_1, d_2, \ldots, d_n\}$ is a diagonal matrix, and S is an invertible matrix. Show that the columns of S are eigenvectors of A with corresponding eigenvalues being the diagonal entries of D.

Hint: Let the columns of S be $\vec{v}_1$, $\vec{v}_2$, ...,$\vec{v}_n$ (without assuming anything about them). Compute $A[\vec{v}_1\, \vec{v}_2\, \cdots\, \vec{v}_n]$ in two different ways and compare the results.

2. Show that if a matrix P satisfies $P^2 = P$, and if λ is an eigenvalue of P, then $\lambda = 0$ or $\lambda = 1$.