Bennet Sloan

Mth 342

HW. 4

1.) Let $W = \{\vec{m} \in M_{3\times3} \mid \text{all rows/columns sum to } 0\}$

And $V = \{[\vec{w}]_{\mathcal{B}} \mid \vec{w} \in W\}$ so, $V = T(W)$

Let C be a matrix whose null space is V.

$$\text{RREF}(C) = \begin{bmatrix} \boxed{1} & 0 & 0 & 0 & -1 & -1 & 0 & -1 & -1 \\ 0 & \boxed{1} & 0 & 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & \boxed{1} & 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & \boxed{1} & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \boxed{1} & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Since $V \subset \mathbb{R}^9$, $V = T(W) = Nul(C)$

Where $T: M_{3\times3} \to \mathbb{R}^9$ is the isomorphism $T(\vec{m}) = [\vec{m}]_{\mathcal{B}}$

Then $\dim(V) = \dim \text{Ker} A + \text{rank} A$

$$= 4 + 5 = 9$$

Also, since $M_{3\times3} \cong \mathbb{R}^9$

$$|\mathcal{B}| = |S_{\mathbb{R}^9}|$$

$$|\mathcal{B}| = 9$$

Where $\mathcal{B}$ is any basis for $M_{3\times3}$.

Therefore $\dim(W) = 9$ since $W \subset M_{3\times3}$

2.) Let $A = \begin{bmatrix} 1 & 2 & 3 & 1 & 1 \\ 1 & 4 & 0 & 1 & 2 \\ 0 & 2 & -3 & 0 & 1 \\ 1 & 0 & 0 & 1 & 0 \end{bmatrix}$

If A_e is the echelon form of A,

then $A_e = \begin{bmatrix} \boxed{1} & 2 & 3 & 1 & 1 \\ 0 & \boxed{2} & -3 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \boxed{-6} & 1 & 0 \end{bmatrix}$ with pivots x_1, x_2, x_3

So a basis for Ram A is $\left\{ \begin{bmatrix} 1 \\ 1 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ 4 \\ 2 \\ 0 \end{bmatrix}, \begin{bmatrix} 3 \\ 0 \\ -3 \\ 0 \end{bmatrix} \right\}$.

Also, from the pivot rows of A_e we have

that a basis for Ram A^T is $\left\{ \begin{bmatrix} 1 \\ 2 \\ 3 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 2 \\ -3 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ -6 \\ 1 \\ 0 \end{bmatrix} \right\}$.

Now let A_{re} be the RREF of A.

Then $A_{re} = \begin{bmatrix} \boxed{1} & 0 & 0 & 0 & 0 \\ 0 & \boxed{1} & 0 & 1/4 & 1/2 \\ 0 & 0 & \boxed{1} & 1/6 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = 0$

so $x_h = x_3 \begin{bmatrix} 0 \\ -1/4 \\ 5/6 \\ 0 \end{bmatrix} + x_4 \begin{bmatrix} 0 \\ 1/2 \\ 0 \\ 1 \end{bmatrix}$

Meaning a basis for Nul(A) would be $\left\{ \begin{bmatrix} 0 \\ -1/4 \\ -1/6 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1/2 \\ 0 \\ 1 \end{bmatrix} \right\}$

3.) Let $A = \begin{bmatrix} 1 & i \\ i & -1 \end{bmatrix}$

Then $A_e = \begin{bmatrix} \boxed{1} & i \\ 0 & 0 \end{bmatrix}$ by $R_2 + (-i)R_1 \to R_2$

So a basis for Ran A would be $\left\{ \begin{bmatrix} 1 \\ i \end{bmatrix} \right\}$.

And a basis for RanA^T would be $\left\{ \begin{bmatrix} 1 \\ i \end{bmatrix} \right\}$.

Finaly a basis for Nul(A) would be $\left\{ \begin{bmatrix} -i \\ 1 \end{bmatrix} \right\}$

4.) Let basis $A = \{1, 1+t\}$

Let basis $B = \{1-t, 2t\}$

We are looking for $[I_{\mathbb{P}_1}]_{BA}$

Let $S = \{1, x\}$, the S.B. of $\mathbb{P}_1$

Then $[I_{\mathbb{P}_1}]_{SA} = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \overset{def}{=} A$

And, $[I_{\mathbb{P}_1}]_{SB} = \begin{bmatrix} 1 & 0 \\ -1 & 2 \end{bmatrix} \overset{def}{=} B$

Where $[I_{\mathbb{P}_1}]_{BS} = B^{-1} = \frac{1}{2}\begin{bmatrix} 2 & 0 \\ 1 & 1 \end{bmatrix}$

Now, $[I_{\mathbb{P}_1}]_{BA} = [I_{\mathbb{P}_1}]_{BS}[I_{\mathbb{P}_1}]_{SA}$

$= B^{-1} \cdot A$

$= \frac{1}{2}\begin{bmatrix} 2 & 0 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$

$[I_{\mathbb{P}_1}]_{BA} = \begin{bmatrix} 1 & 1 \\ 1/2 & 1 \end{bmatrix}$