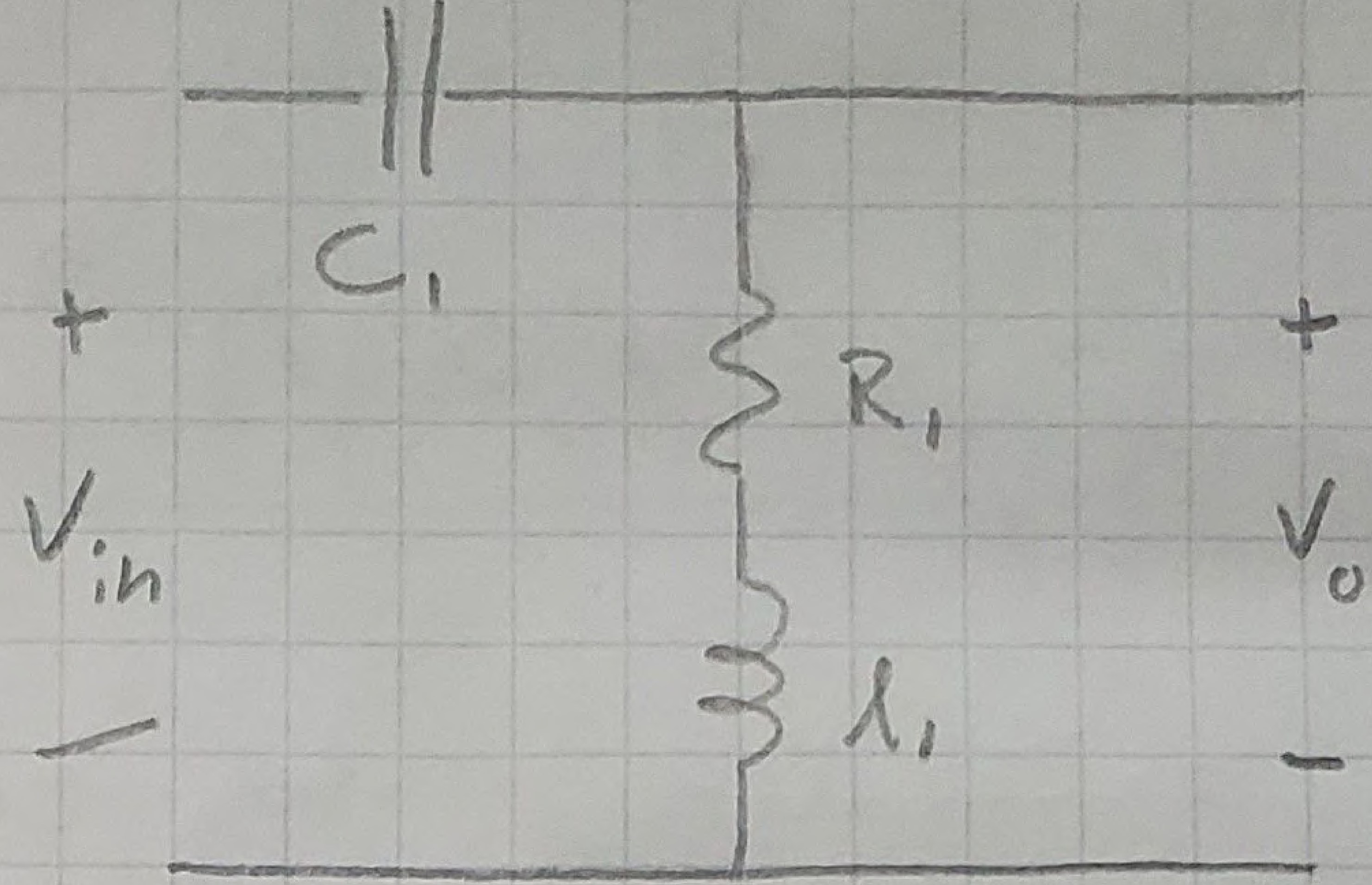
Bennet Sloan   Engr 203   4/24/19

3.1 Find $H_{V_o}(s)$.



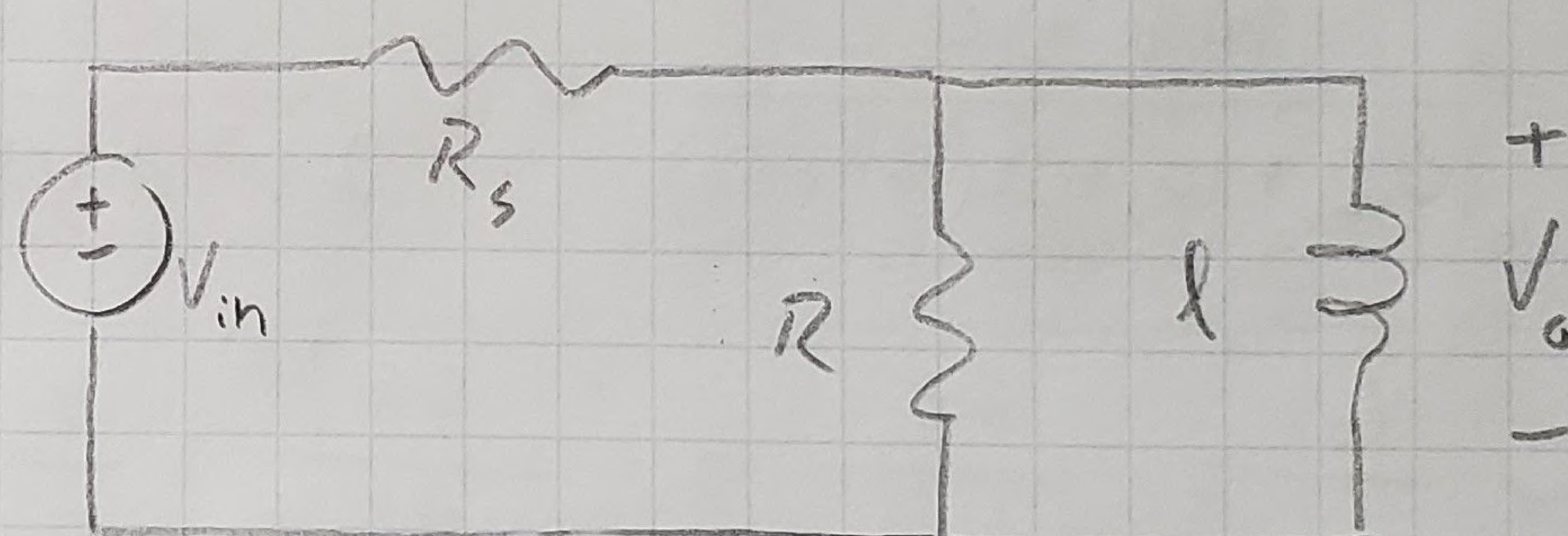
Voltage Divider $(Z_R + Z_l)$

$$H_{j\omega} = \frac{Z_R + Z_l}{Z_C + Z_l + Z_R}$$

$$H_s = \frac{R + sl}{\frac{1}{sC} + sl + R} \circ \frac{l^{-1}}{l^{-1}} = \frac{R/l + s}{\frac{1}{sCl} + s + \frac{R}{l}} \circ \frac{s}{s} = \frac{s^2 + s\frac{R}{l}}{s^2 + s\frac{R}{l} + \frac{1}{lC}}$$

$$H(s) = \boxed{\frac{s\left(s + \frac{R}{l}\right)}{s^2 + s\frac{R}{l} + \frac{1}{lC}}}$$

3.2 Find $H_{V_o}(s)$.

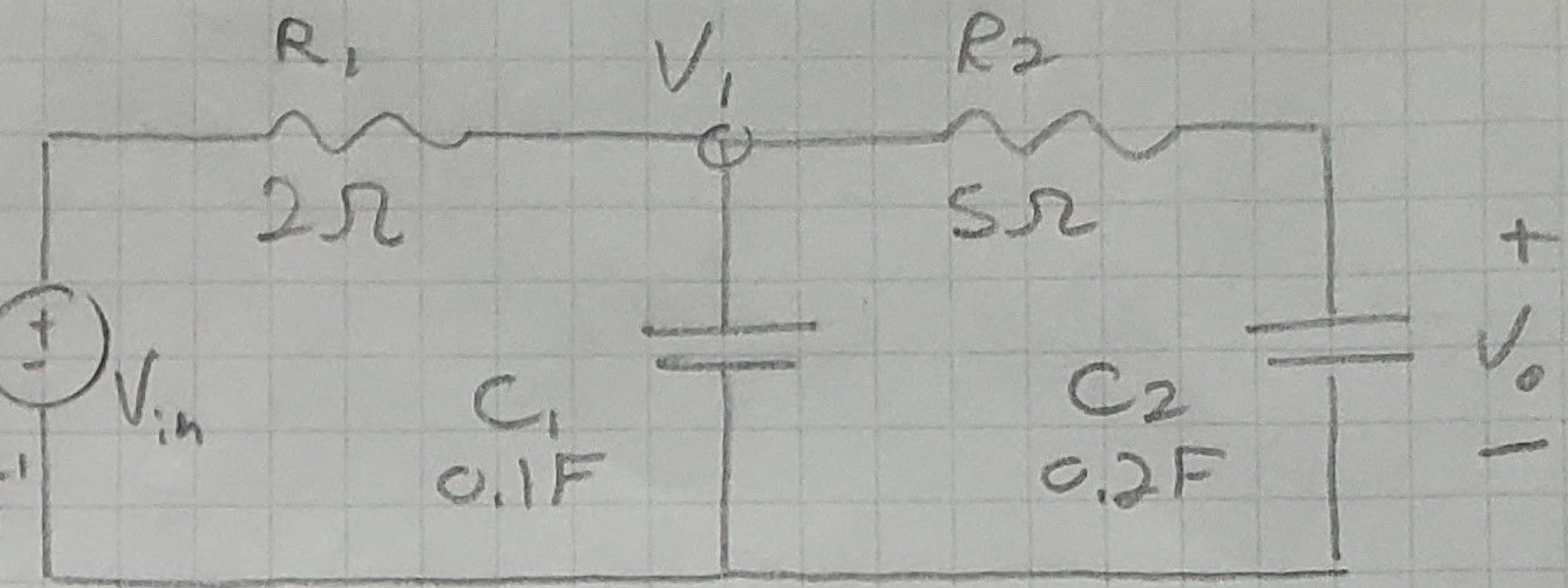


Voltage Divider $(Z_R || Z_l)$

$$Z_R || Z_l = \frac{Z_R Z_l}{Z_R + Z_l} = \frac{Rsl}{R + sl} \circ \frac{l^{-1}}{l^{-1}} = \frac{sR}{s + \frac{R}{l}}$$

$$\text{so, } H_{j\omega} = \frac{Z_R || Z_l}{Z_R || Z_l + Z_{R_s}} \longrightarrow H_s = \frac{sR\left[s + \frac{R}{l}\right]^{-1}}{sR\left[s + \frac{R}{l}\right]^{-1} + R_s} \circ \frac{s + \frac{R}{l}}{s + \frac{R}{l}}$$

$$= \frac{sR}{sR + R_s\left[s + \frac{R}{l}\right]} \circ \frac{R^{-1}}{R^{-1}} = \frac{s}{s + s\frac{R_s}{R} + \frac{R_s}{l}} = \boxed{\frac{s}{s\left(1 + \frac{R_s}{R}\right) + \frac{R_s}{l}}}$$



3.3 Find $H_{V_o}(s)$.

$$H_s = \frac{V_1}{V_{in}} \circ \frac{V_o}{V_1}, \quad C_1 = 10s^{-1}, \; C_2 = 5s^{-1}$$

Let $Z_{eq} = C_1 \| (R_2 + C_2)$, so, $Z_{eq} = \dfrac{(10s^{-1})(5+5s^{-1})}{10s^{-1}+5s^{-1}+5} \circ \dfrac{s}{s}$

$$= \frac{10(5s^{-1}+5)}{5s+15} \circ \frac{5^{-1}}{5^{-1}} = \frac{10(s^{-1}+1)}{s+3} \circ \frac{s}{s} = \frac{10s+10}{s^2+3s}.$$

Then, $\dfrac{V_1}{V_{in}} = \dfrac{Z_{eq}}{Z_{eq}+R_1} = \dfrac{(10s+10)(s^2+3s)^{-1}}{(10s+10)(s^2+3s)^{-1}+2} \circ \dfrac{s^2+3s}{s^2+3s}$

$$= \frac{10s+10}{10s+10+2(s^2+3s)} = \frac{10s+10}{2s^2+16s+10}.$$

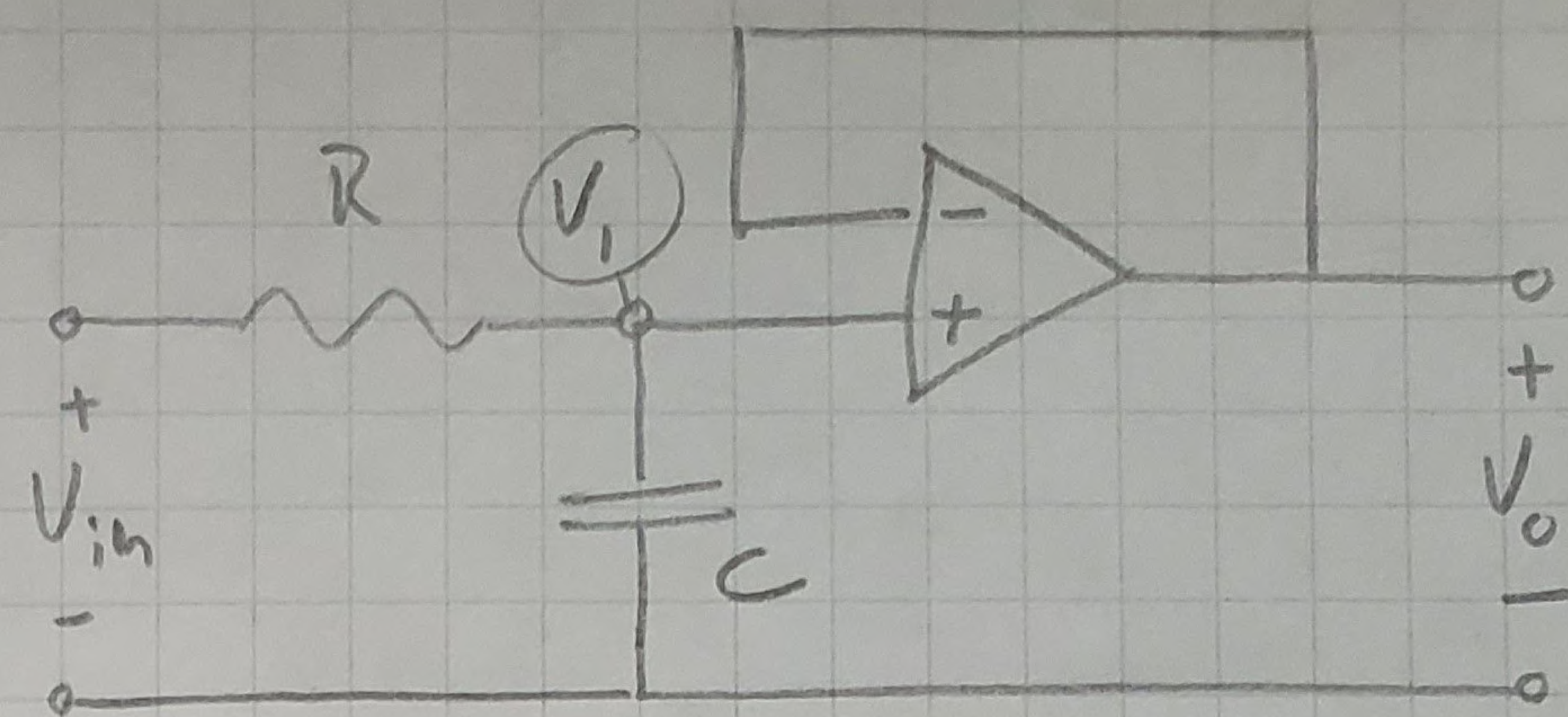
Now, $\dfrac{V_o}{V_1} = \dfrac{C_2}{C_2+R_2} = \dfrac{5s^{-1}}{5s^{-1}+5} \circ \dfrac{s}{s} = \dfrac{5}{5s+5} \circ \dfrac{5^{-1}}{5^{-1}} = \dfrac{1}{s+1}$

Therefore $H_s = \dfrac{V_1}{V_{in}} \circ \dfrac{V_o}{V_1} = \dfrac{10s+10}{2s^2+16s+10} \circ \dfrac{1}{s+1}$

$$= \frac{10\cancel{(s+1)}}{2s^2+16s+10} \circ \frac{1}{\cancel{s+1}} = \frac{10}{2s^2+16s+10} \circ \frac{2^{-1}}{2^{-1}}$$

$$\boxed{H(s) = \frac{5}{s^2+8s+5}}$$

3.4 Find $H_{V_0}(s)$.



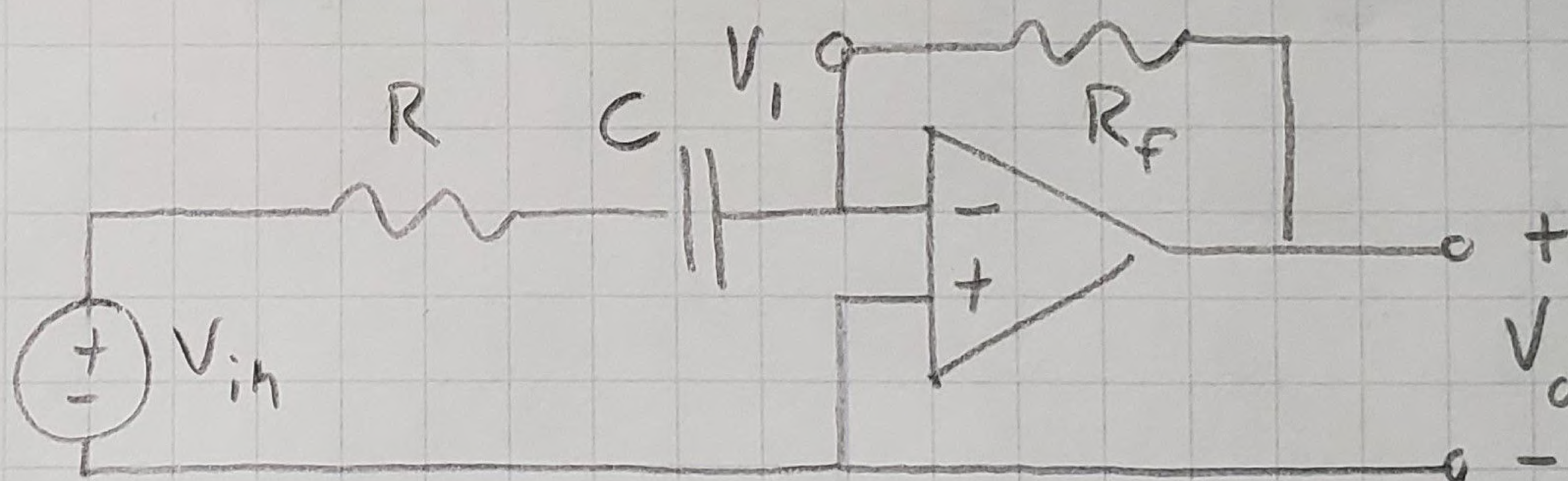
Since $V_+ = V_-$,

Then $V_1 = V_o$.

So, $$\frac{V_1}{V_{in}} = \frac{Z_C}{Z_R + Z_C} = \frac{(sC)^{-1}}{R + (sC)^{-1}} \circ \frac{s}{s}$$

$$= \frac{C^{-1}}{sR + C^{-1}} \circ \frac{R^{-1}}{R^{-1}} = \boxed{\frac{(RC)^{-1}}{s + (RC)^{-1}} = H(s)}$$

3.5 Find $H_{V_0}(s)$.



Since $V_+ = V_-$,

Then $V_1 = 0V$.

Nodal Analysis (V_1): $$\frac{V_{in} - 0}{Z_R + Z_C} - \frac{0 - V_o}{Z_{R_f}} = 0$$

$$\rightarrow \frac{V_{in}}{Z_R + Z_C} = \frac{-V_o}{Z_{R_f}} \longrightarrow \frac{V_o}{V_{in}} = \frac{-Z_R}{Z_R + Z_C} = \frac{-R_f}{R + (sC)^{-1}} \circ \frac{s}{s}$$

$$= \frac{(-R_f)\,s}{Rs + C^{-1}} \circ \frac{R^{-1}}{R^{-1}} = \boxed{\frac{\left(-\frac{R_f}{R}\right) s}{s + (RC)^{-1}} = H(s)}$$