<u>Lab Section 2: Second Order Circuits</u>

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ECE 203-01

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Abstract

In the case of constant resistance loads, the dissipation of energy in capacitors and inductors is proportional to the instantaneous voltage and current respectively. Therefore, when both these elements exist in the same circuit, a second order ODE arises. Furthermore, this results in energy oscillations which dissipate when resistance is accounted for. Analysis of this dampening can be determined simply by the characteristic polynomial of the ODE. If the discriminant is negative, it is underdamped, if it is zero, it is critically damped, and if it is positive the oscillation is overdamped. In this experiment a series RLC configuration will be tested to explore the effect of resistance dampening and to ascertain the ODE experimentally.

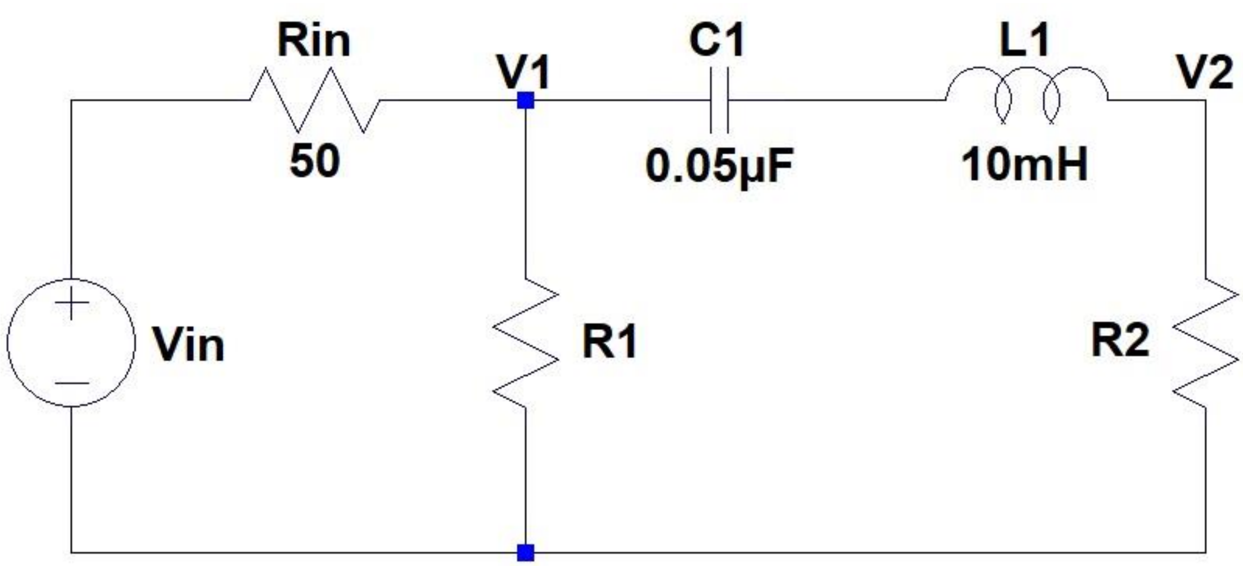
Equipment

- LTspiceXVII
- MATLAB/Excel

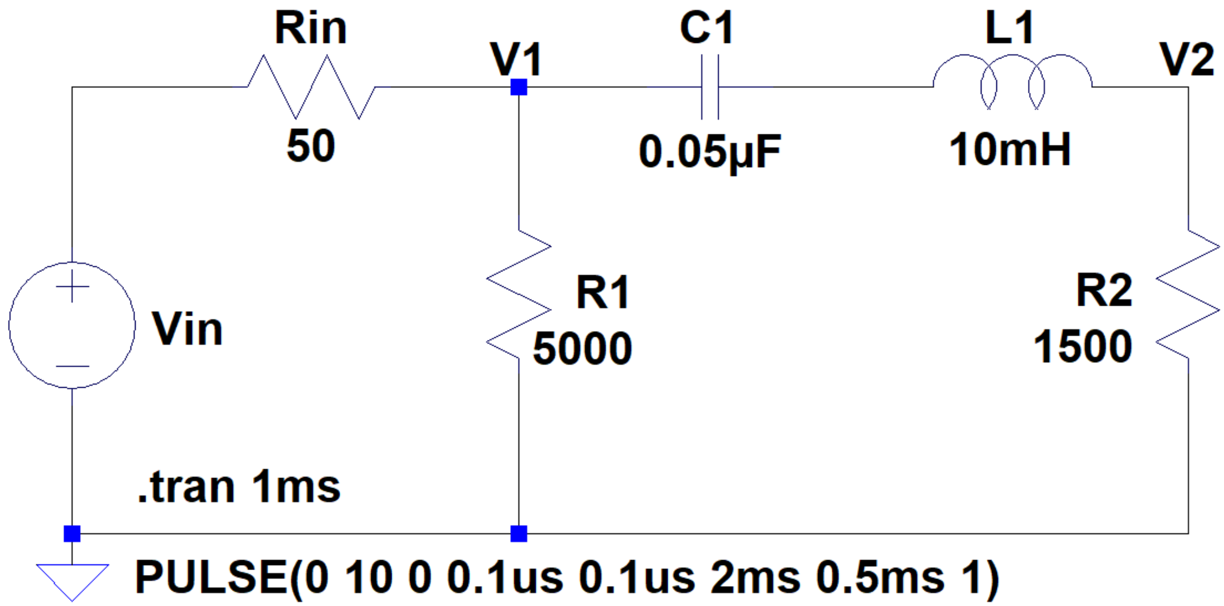
Procedure

1. Simulate circuit 1 for V2 with R1= 5000 and R2=1500 (overdamped) at 2kHz.
2. Isolate the exponential portion and plot on semi-log graph.
3. Find the time constants and note the effects of changing R2's value.
4. Simulate circuit 2 for V2 with R1= 100 and R2=80 (underdamped) at 600Hz.
5. Calculate the frequency of the voltage response.
6. Plot the envelope on semi-log graph and find time constants.
7. Note the effects of changing R2's value.

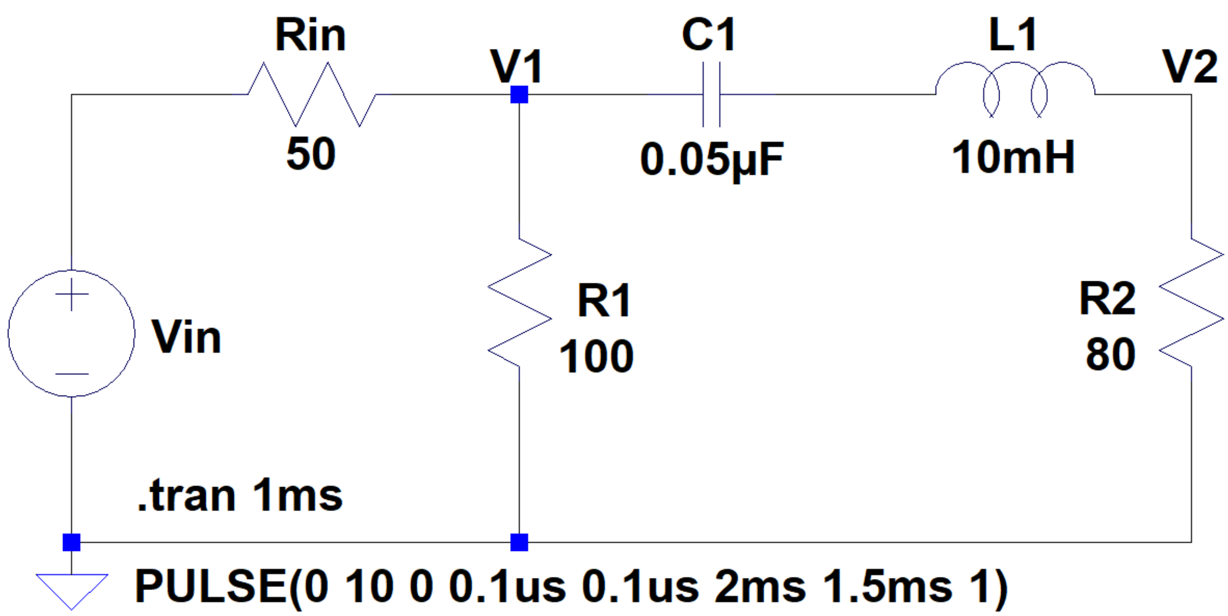
Circuits



Circuit 0 - Series RLC template



Circuit 1 - Overdamped series RLC



Circuit 2 - Underdamped series RLC

Effects of R2 in Overdamped case

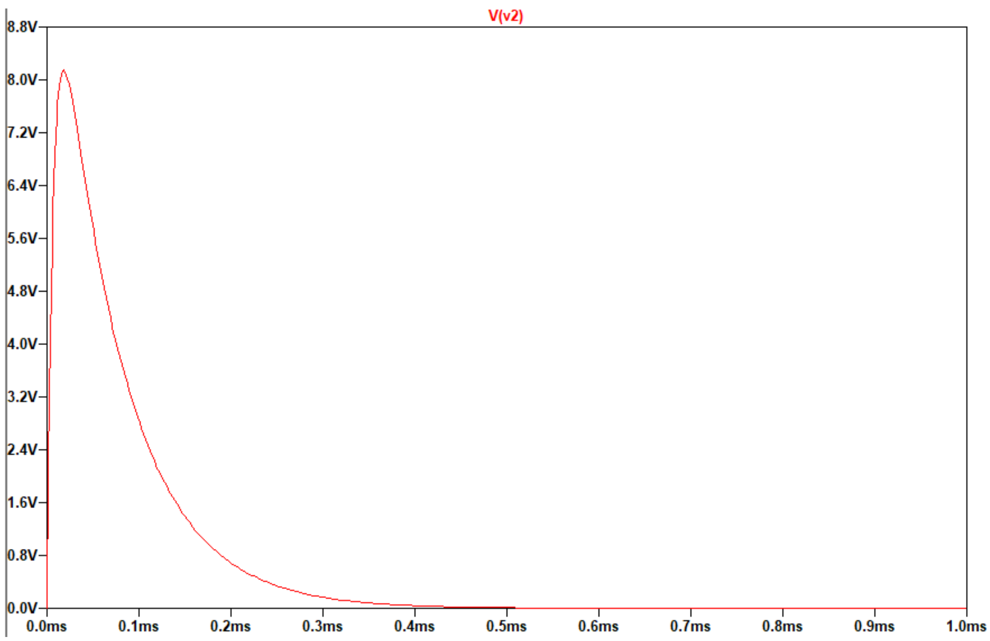


Figure 1 - Overdamped with R2 = 1500

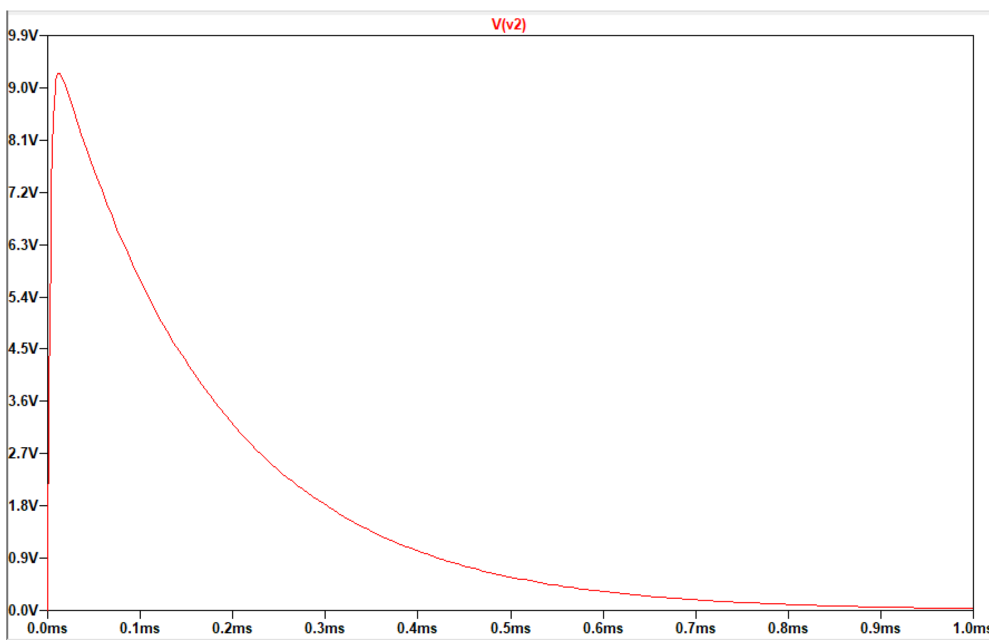


Figure 2 - Overdamped with R2 = 3500

Effects of R2 in Underdamped case

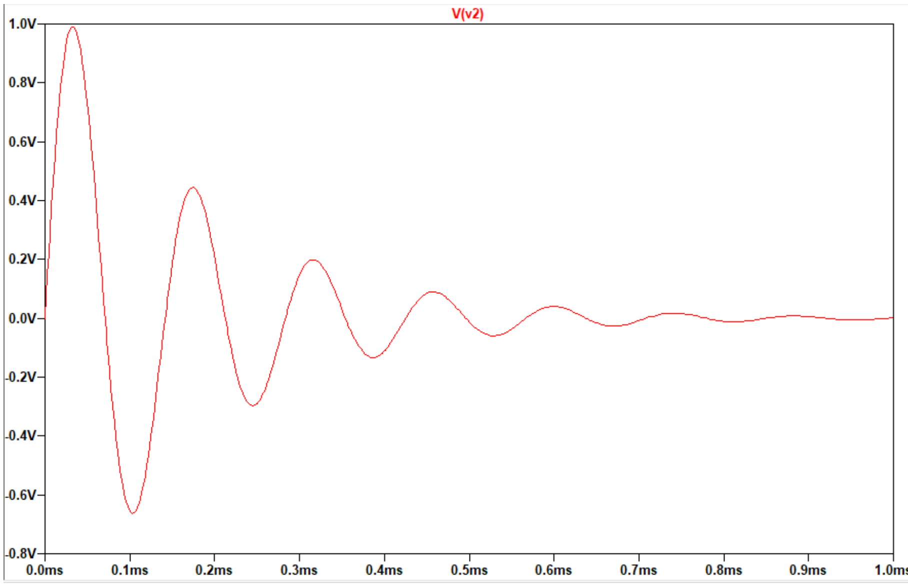


Figure 3 - Underdamped with R2 = 80

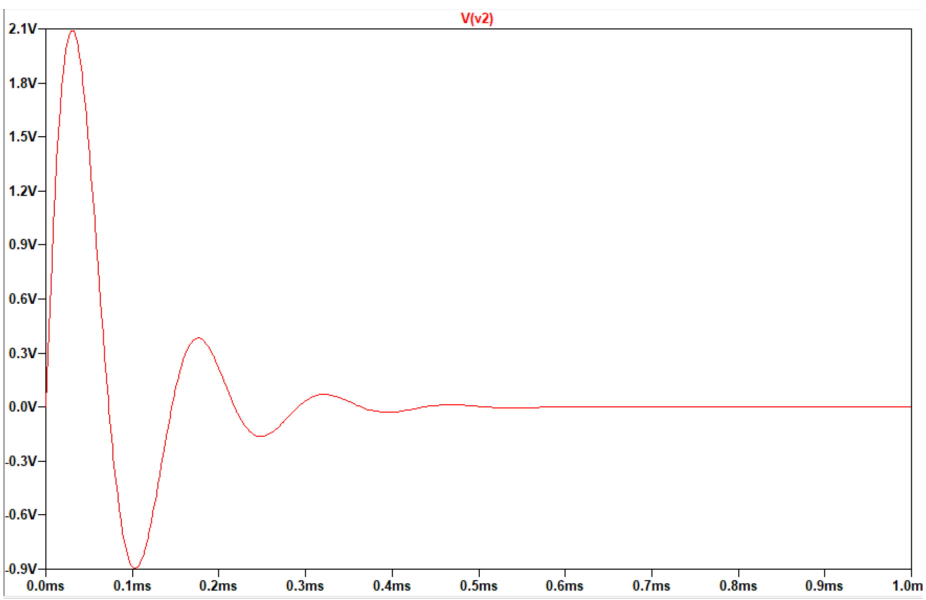


Figure 4 - Underdamped with R2 = 200

Calculations

Overdamped series case

We know, $i(t) = A_1 e^{s_1 t} + A_2 e^{s_2 t} + k_\infty, \; A_1 = -A_2$

Where $s_{1,2} = -\alpha \pm \sqrt{\alpha^2 - {\omega_0}^2}$

$\alpha = \frac{R_1 + R_2}{2L} = \frac{6500}{.02} = 325{,}000$

$\omega_0 = \frac{1}{\sqrt{LC}} = 44{,}721.36$

So, $s_1 = -3{,}091.628$

$s_2 = -646{,}908.372$

Now, $i(t) = A_1 e^{-3{,}091.628t} + A_2 e^{-646{,}908.372t}$

Since one exponential will decay rapidly, a data point after the decay will be chosen to find A1.

So, using point from data: (0.0007073575, 0.0005070079)

It follows, $0.0005070079 = 0.112A_1 + (0)A_2$

$A_1 = 0.004516$

$A_2 = -0.004516$

So $i(t) = 0.004516e^{-3{,}091.628t} - 0.004516e^{-646{,}908.372t}$

And for v(t), it is negated $v(t) = 0.004516e^{-646{,}908.372t} - 0.004516e^{-3{,}091.628t}$

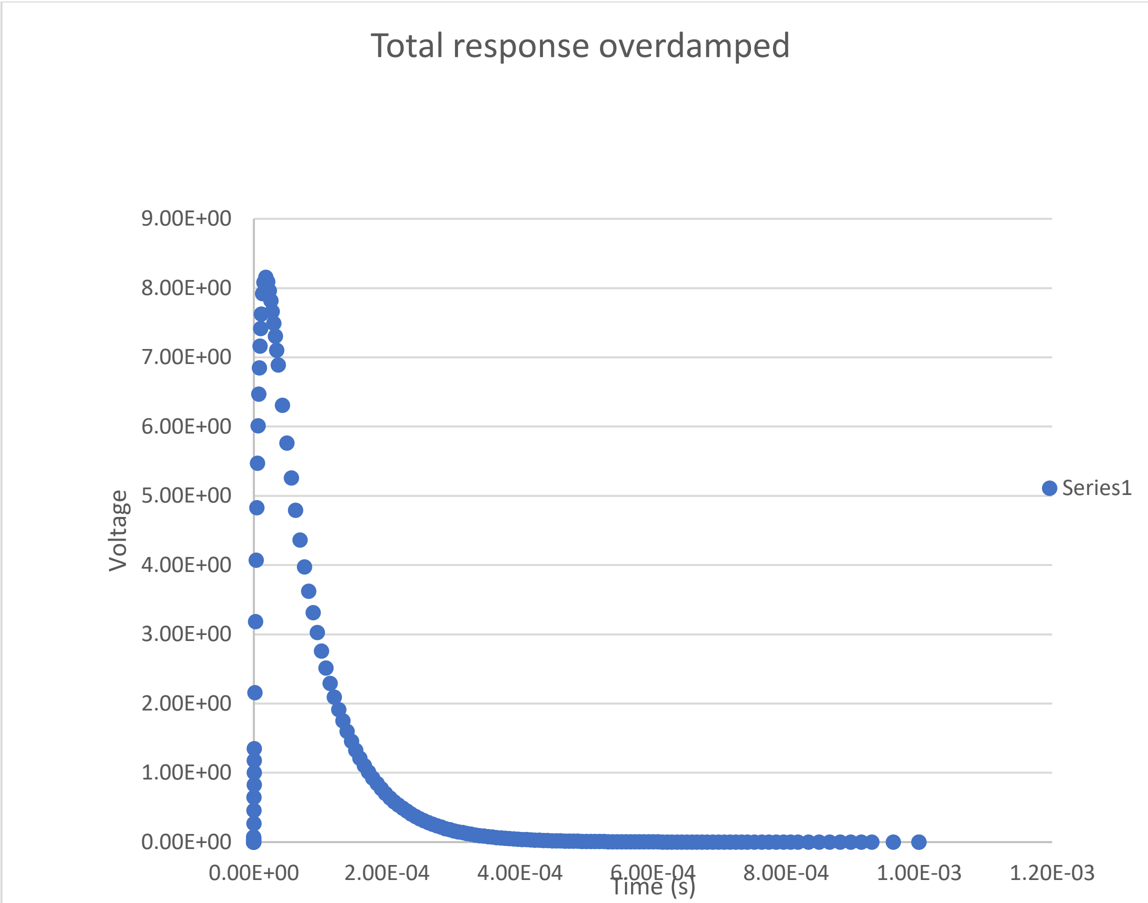


Figure 5 - Natural response at V2

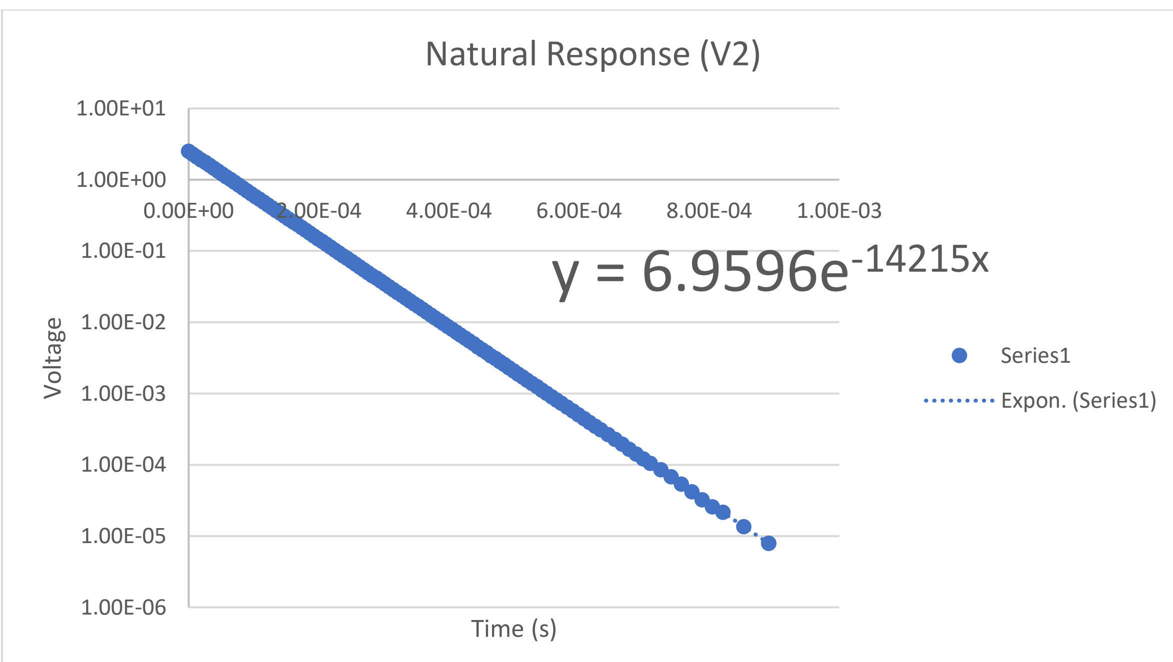


Figure 6 - shifted by.01ms

Underdamped series case

From the data, the frequency is about 7040 Hz

We know that $i(t) = e^{-\alpha t}(A_1 \cos(\omega_d t) + A_2 \sin(\omega_d t))$

Since i(0)=0, $A_1 = 0 \rightarrow i(t) = A_2 e^{-\alpha t} \sin(\omega_d t)$

From fig.8, $A_2 = 1.19, \;\; \alpha = 5665, \;\; \omega_d = 2\pi * 7040 = 44{,}233$

So, $i(t) = 1.19e^{-5665t} \sin(44233t)$

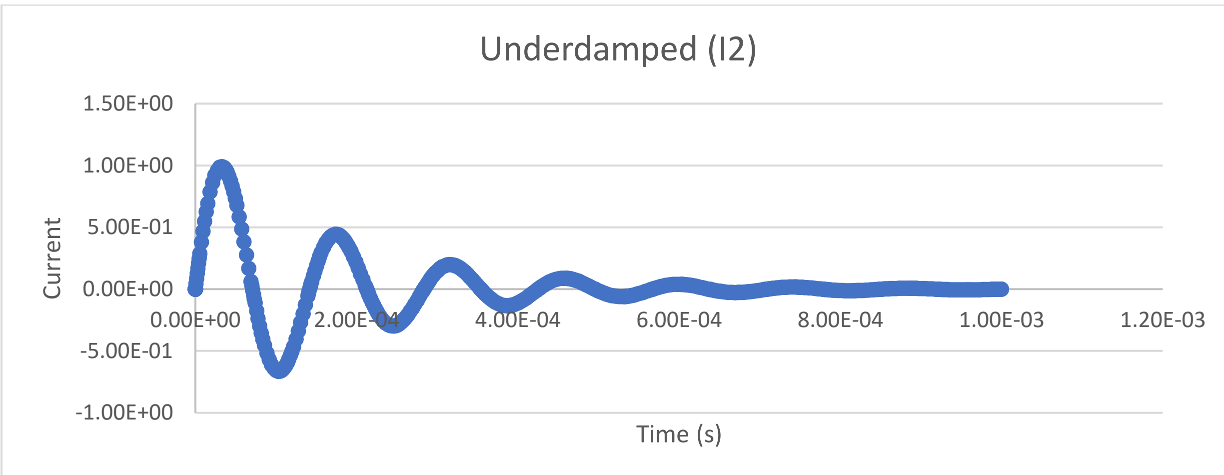


Figure 7 - Raw current data for underdamped (I2)

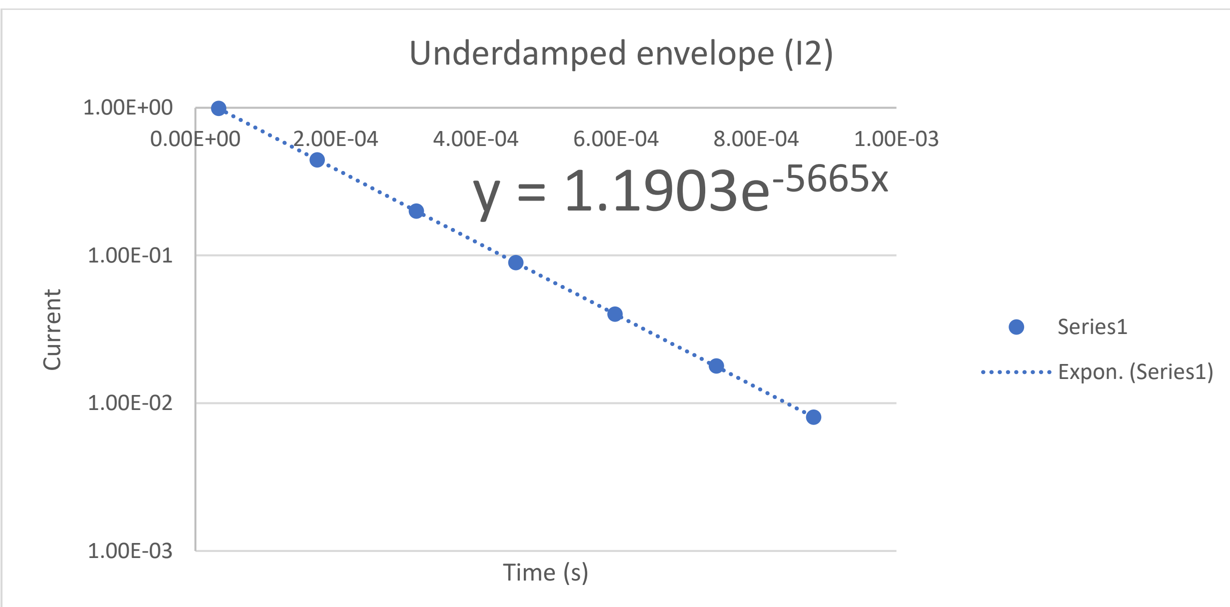


Figure 8 - Envelope from peaks of current data

Conclusions

The constants obtained from the data reflect the amount of resistance present. In the overdamped case, additional resistance increases the transient response while in the underdamped case it is the opposite, although both converge slower than a critically damped circuit. This can be interpreted as a function producing a Cauchy sequence in a complete normed vector space with negative eigenvalues as poles. As we saw, the type of dampening is dependent on the sign of the discriminant of the characteristic polynomial. Without resistance, and without a forcing voltage, the system would be simple harmonic, so the parallels to spring mass systems are apparent.