MTH351 **Introduction to Numerical Analysis** **Summer 2020**

Homework 1 - Due Monday June 29

Problem 1: Consider the evaluation of the following integral

$$I = \int_0^1 \frac{1 - e^{-t}}{t} \mathrm{d}t$$

which can not be obtained in terms of simple functions.

Part I: Using the Taylor series of $f(t) = e^{-t}$ and term by term integration, obtain a series that converges to I.

Part II: Denoting by S_n the partial sums corresponding to the series you determined in Part I, find n so that

$$|I - S_n| < 10^{-6}$$

Give a brief justification for your answer.

Problem 2: Let

$$\mathrm{Ci}(x) = \frac{1}{x} \int_0^x \frac{1 - \cos t}{t^2} \mathrm{d}t$$

Part I: Using the Taylor series of $\cos t$ obtain a power series representation of Ci.

Part II: Using the ratio test, check that the series you obtained in Part I converges for all $x \in \mathbf{R}$.

Part III: Denote by $S_n(x)$ the polynomial consisting of the first n terms of the series you determined in Part I. Determine n so that

$$|\mathrm{Ci}(x) - S_n(x)| \leq 10^{-6}$$

for all $|x| \leq 1$.
