Bennet Sloan 10/15/2018
Mth 256 - HW 3

1) Solve $(3x^2 - 2xy + 3y^2)\frac{dy}{dx} = y^2 - 6xy - 3x^2$

$$(-y^2 + 6xy + 3x^2)\,dx + (3x^2 - 2xy + 3y^2)\,dy = 0$$

So,
$$M(x,y) = -y^2 + 6xy + 3x^2$$
$$N(x,y) = 3x^2 - 2xy + 3y^2$$

And
$$M_y = -2y + 6x$$
$$N_x = -2y + 6x$$
$\therefore$ Exact

Consider $\int N(x,y)\,dy = \varphi(x,y)$
$$\int 3x^2 - 2xy + 3y^2\,dy = \varphi(x,y)$$
$$3x^2y - xy^2 + y^3 + c(x) = \varphi(x,y)$$

Also, $\frac{d}{dx}\left[\varphi(x,y)\right] = 6xy - y^2 + c'(x) = \varphi_x(x,y)$

Since $\varphi_x = M(x,y) = -y^2 + 6xy + 3x^2$,

Then, $6xy - y^2 + c'(x) = -y^2 + 6xy + 3x^2$
$$c'(x) = 3x^2$$

And,
$$c(x) = \int 3x^2\,dx$$
$$c(x) = x^3 + C$$

Therefore, $x^3 + 3x^2y - xy^2 + y^3 = C$

2.) Consider the nonlinear DE

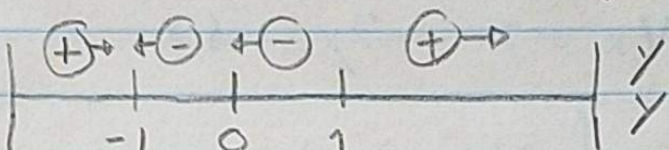
$$\frac{dy}{dt} = y^2(y^2-1)$$

a.) classify the equilibrium solutions

For $\frac{dy}{dt} = 0$: $\quad y^2(y^2-1) = 0$

$$y = 0, 1, -1$$

Consider:



So, $\quad y = -1 \rightarrow$ stable

$y = 0 \rightarrow$ semi-stable

$y = 1 \rightarrow$ unstable

b.) Suppose $y(t)$ is a solution to this DE such that $y(0) = -\frac{1}{2}$. Determine $\lim_{t \to \infty} y(t)$

Consider $\quad y' = \left(-\frac{1}{2}\right)^2\left(\left(-\frac{1}{2}\right)^2 - 1\right)$

$$= \frac{1}{4}\left(-\frac{3}{4}\right) < 0$$

Therefore closest stable equilibrium is $y = -1$.

$$\therefore \lim_{t \to \infty} y(t) = -1$$