Bennet Sloan

Mth 256 - HW5 10/29/18

1.) Determine all real numbers $b > 1$ so that $x^2y'' + y = 0$ has a nonzero solution $y(x)$ on $1 \le x \le b$ such that $y(1) = 0$ & $y(b) = 0$.

* Cauchy-Euler eq. : $ax^2y'' + bxy' + cy = f(x)$, $a \ne 0$

$\therefore$ soln : $a\frac{d^2y}{dt^2} + (b-a)\frac{dy}{dt} + cy = 0$, $t = \ln(x)$

So, $a = 1$, $b = 0$, $c = 1$

$$\frac{d^2y}{dt^2} - \frac{dy}{dt} + y = 0$$

$$r^2 - r + 1 = 0$$

$$r = \frac{1 \pm \sqrt{-3}}{2} = \frac{1}{2} \pm \frac{\sqrt{3}}{2}i$$

complex roots $\rightarrow$ $y = x^{\frac{1}{2}}\left[c_1\cos\left(\frac{\sqrt{3}}{2}\ln x\right) + c_2\sin\left(\frac{\sqrt{3}}{2}\ln x\right)\right]$

For $y(1) = 0$: $0 = 1\left[c_1\cos(0) + c_2\sin(0)\right] \rightarrow c_1 = 0$

For $y(b) = 0$: $0 = c_2\sin\left(\frac{\sqrt{3}}{2}\ln(b)\right)b^{\frac{1}{2}}$

$$0 = \sin\left(\frac{\sqrt{3}}{2}\ln(b)\right)$$

$\because \sin(\lambda) = 0$ for $\lambda = k\pi$, $k \in \mathbb{Z}$

So, $\frac{\sqrt{3}}{2}\ln b = k\pi \rightarrow b = e^{\frac{2}{\sqrt{3}}k\pi}$

2.) Suppose A is a constant such that $y = Ae^{2x}$ is a solution to the DE $y'' + 4y' + 3y = e^{2x}$

Given $y = Ae^{2x}$	$r^2 + 4r + 3 = 0$
$y' = 2Ae^{2x}$	$(r+3)(r+1) \rightarrow r = -3, -1$
$y'' = 4Ae^{2x}$	$y_c(x) = c_1 e^{-3x} + c_2 e^{-x}$

so, $4Ae^{2x} + 4(2Ae^{2x}) + 3(Ae^{2x}) = e^{2x}$

$4A + 8A + 3A = 1$

$A = 1/15 \longrightarrow y_p(x) = \frac{1}{15} e^{2x}$

$\therefore\ y(x) = y_c(x) + y_p(x)$

$y(x) = c_1 e^{-3x} + c_2 e^{-x} + \frac{1}{15} e^{2x}$