MTH351 Introduction to Numerical Analysis Summer 2020

Quiz 3 - To be discussed in Lecture Friday July 24 - Typo corrected

Give yourself 30 - 45 minutes to work on these problems. Write down your own solutions and be ready to discuss them in small group during lecture on Friday July 24. You will have a chance to rewrite your answer if needed. You will need to upload your corrected answers by 6:00 PM, Friday July 24.

A typo in Problem 2 noted during discussion in class is corrected in this version. Absolute values were missing!!!

Problem 1: A table of values of $f(x) = \ln(x), 1 \leq x \leq 5$, is to be constructed with the values of $\ln(x)$ given at points $x_j = 1 + j * h$ where $h = 4/n$. If linear interpolation is used in this table, what is the smallest value of n one can use so that the resulting interpolation error is less than $5 * 10^{-6}$.

Problem 2: In lecture we have discussed the oscillations that the polynomial $\Psi_n(x) = \prod_{j=0}^{n}(x - x_j)$ has when the nodes x_j are equidistant, that is $x_j = x_0 + j \times h$ where h is a fixed positive parameter. This problem is to illustrate these oscillations by computing the maximum of this function for $n = 2$ and $n = 3$. Note that these polynomials will occur in the error estimate when quadratic or cubic interpolation is being used.

Part I: With a slight change of notation, let $\Psi_2(x) = (x + h)x(x - h)$ corresponding to $x_0 = -h, x_1 = 0, x_2 = h.$. Find analytically the maximum of $|\Psi_2|$ on the interval $[-h, h]$.

Part II: Consider now $\Psi_3(x) = (x + 3h/2)(x + h/2)(x - h/2)(x - 3h/2)$ corresponding to the nodes $x_0 = -3h/2, x_1 = h/2, x_2 = h/2, x_3 = 3h/2$. Find analytically the maximum of $|\Psi_3|$ on the interval $[-3h/2, 3h/2]$.

Part III: Take $h = 1$ and find the ratio of the maximum of $|\Psi_3|$ divided by the maximum of $|\Psi_2|$. Make comments on the impact on the error estimate in polynomial interpolation.