Bennet C. Sloan 10/8/2018
Mth 256 - HW 2

1.) The tangent line to a curve at any point $P(x, y)$ has an x-intercept equal to $\frac{1}{2}x$. This curve also passes through the point $(1,2)$. Find the equation of the curve.

We know $m = \frac{y - y_0}{x - x_0}$ where $m = \frac{dy}{dx}$.

Given x-intercept at any point equals $\frac{x}{2}$, Moreover $(\frac{x}{2}, 0)$ lies on the curve.

$$\text{So, } \frac{dy}{dx} = \frac{y - (0)}{x - (\frac{x}{2})}$$

$$\frac{dy}{dx} = \frac{y}{\frac{x}{2}}$$

$$\frac{dy}{dx} = \frac{2y}{x}$$

$$\frac{1}{2}\frac{dy}{y} = \frac{dx}{x}$$ ∘ Separable ODE

$$\frac{1}{2}\int \frac{1}{y}\,dy = \int \frac{1}{x}\,dx$$

$$\frac{1}{2}\ln|y| = \ln|x| + c$$

$$\ln|\sqrt{y}| = \ln|x| + c$$

$$\sqrt{y} = cx$$

$$y = cx^2$$

$$\boxed{\therefore y(x) = 2x^2}$$

Using Point $(1,2)$ for c: $2 = c1^2 \rightarrow c = 2$

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2.) Solve the separable DE

$$y(t+3)\frac{dy}{dt} = 3 + 2y - y^2$$

$$\left[y \cdot dy\right]\left[(t+3) \cdot \frac{1}{dt}\right] = 3 + 2y - y^2$$

$$\frac{y}{3+2y-y^2}\,dy = \frac{1}{t+3}\,dt$$

$$\frac{y}{(1+y)(3-y)}\,dy = \frac{1}{t+3}\,dt$$

Partial Fraction Expansion

$$\frac{y}{(1+y)(3-y)} = \frac{A}{1+y} + \frac{B}{3-y}$$

$$y = A(3-y) + B(1+y)$$

For $y = 3$: $3 = B(4) \longrightarrow B = \frac{3}{4}$

For $y = -1$: $-1 = A(4) \longrightarrow A = -\frac{1}{4}$

So, $\int \left[\frac{-\frac{1}{4}}{1+y} + \frac{\frac{3}{4}}{3-y}\right] dy = \int \left[\frac{1}{t+3}\right] dt$

$$-\frac{1}{4} \cdot \ln|1+y| + \frac{3}{4}\ln|3-y| = \ln|t+3| + C$$

$$e^{\left[\ln\left(\frac{|3-y|^{3/4}}{|1+y|^{1/4}}\right)\right]} = e^{\left[\ln|t+3| + C\right]}$$

$$\therefore \frac{|3-y|^{\frac{3}{4}}}{|1+y|^{\frac{1}{4}}} = C(t+3)$$