1. Let V be the vector space $\mathbb{P}_2(\mathbb{R})$ (polynomials of degree ≤ 2 with real coefficients). This becomes an inner product space when equipped with the inner product defined by

$$(f, g) = \int_0^1 f(x)\overline{g(x)}dx = \int_0^1 f(x)g(x)dx \quad \text{for every } f, g \in \mathbb{P}_2(\mathbb{R}).$$

Apply the Gram-Schmidt procedure to the standard basis $\{1, x, x^2\}$ in order to produce an orthonormal basis for $\mathbb{P}_2(\mathbb{R})$.

If $v_1, \dots, v_n$ is a list of linearly independent vectors in an inner product space V, then there exists an orthonormal list $e_1, \dots, e_n$ such that

$$\operatorname{span}(v_1, \dots, v_k) = \operatorname{span}(e_1, \dots, e_k), \text{ for all } k = 1, 2, \dots, n.$$

Gram-Schmidt procedure: Set $e_1 = \dfrac{v_1}{\|v_1\|}$. Next set

$$e_2 = \frac{v_2 - (v_2, e_1)e_1}{\|v_2 - (v_2, e_1)e_1\|},$$

$$e_3 = \frac{v_3 - (v_3, e_1)e_1 - (v_3, e_2)e_2}{\|v_3 - (v_3, e_1)e_1 - (v_3, e_2)e_2\|},$$

and so on:

$$e_k = \frac{v_k - (v_k, e_1)e_1 - (v_k, e_2)e_2 - \cdots - (v_k, e_{k-1})e_{k-1}}{\|v_k - (v_k, e_1)e_1 - (v_k, e_2)e_2 - \cdots - (v_k, e_{k-1})e_{k-1}\|}.$$