Bennet Sloan Engr 203 5/3/19

4.1 $\mathcal{L}\{\cosh(\alpha t)\} \rightarrow \int_0^\infty \left[\frac{1}{2}(e^{\alpha t} + e^{-\alpha t}) \cdot e^{-st}\right] dt$

$= \frac{1}{2}\left[\int_0^\infty e^{-(s-\alpha)t} dt + \int_0^\infty e^{-(s+\alpha)t} dt\right]$

$= -\frac{1}{2}\left\{\frac{1}{s-\alpha}\left[e^{-(s-\alpha)t}\right]_0^\infty + \frac{1}{s+\alpha}\left[e^{-(s+\alpha)t}\right]_0^\infty\right\}$

$= -\frac{1}{2}\left\{\frac{1}{s-\alpha}\left[e^{-\infty} - e^{0}\right] + \frac{1}{s+\alpha}\left[e^{-\infty} - e^{0}\right]\right\}$

$= -\frac{1}{2}\left[-\frac{1}{s-\alpha} - \frac{1}{s+\alpha}\right] = \frac{1}{2}\left[\frac{1}{s-\alpha} + \frac{1}{s+\alpha}\right]$

$= \frac{1}{2}\left[\frac{s+\alpha}{(s-\alpha)(s+\alpha)} + \frac{s-\alpha}{(s-\alpha)(s+\alpha)}\right] = \frac{1}{2}\left[\frac{2s}{s^2-\alpha^2}\right] = \boxed{\frac{s}{s^2-\alpha^2}}$

4.2 $\mathcal{L}\{e^{-2t}\sin(4t)u(t)\} \rightarrow \int_0^\infty \left[e^{-2t}\sin(4t)u(t)\right]e^{-st} dt$

$= \int_0^\infty e^{-(s+2)t}\sin(4t)\, dt = \mathcal{L}\{\sin(4t)\}_{s=s+2} = \left[\frac{4}{s^2+4^2}\right]_{s=s+2}$

$= \boxed{\frac{4}{(s+2)^2+4^2}}$

4.3 $\mathcal{L}\{5u(\frac{t}{2})\}$. Let $\lambda = \alpha t = \frac{1}{2}t$, then $\dot{\lambda} = \frac{1}{2} \rightarrow dt = \frac{d\lambda}{\alpha}$

$\rightarrow \int_0^\infty 5u(\lambda)e^{-\lambda\frac{s}{\alpha}}\frac{d\lambda}{\alpha} = \frac{1}{\alpha}\int_0^\infty 5u(\lambda)e^{-\lambda\frac{s}{\alpha}} d\lambda = \frac{5}{\alpha}\mathcal{L}\{\alpha\}$

$= 10 \cdot \frac{1}{2s} = \boxed{\frac{5}{s}}$

**4.4** $\mathcal{L}\{e^{-2t}\cosh(4t)u(t)\} \to \int_0^\infty \cosh(4t)e^{-(s+2)t}\,dt$

$$= \mathcal{L}\{e^{-(s+2)t}\cosh(4t)\} = \mathcal{L}\{\cosh(4t)\}_{s=s+2}$$

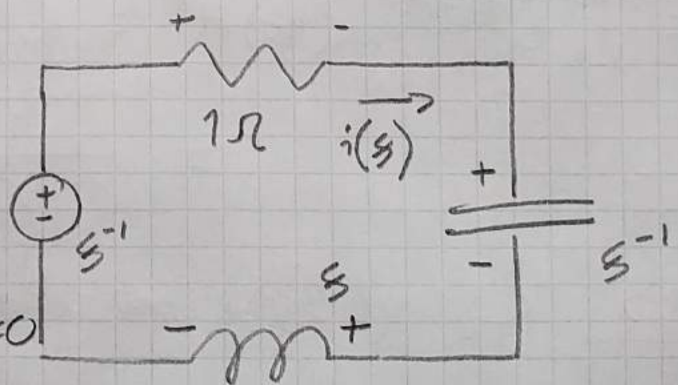
$$= \left[\frac{s}{s^2-4^2}\right]_{s=s+2} = \boxed{\frac{(s+2)}{(s+2)^2-4^2}}$$

**4.5** $\mathcal{L}\{(t-4)u(t-2)\} \to \mathcal{L}\{(t-2)u(t-2)\} - 2\mathcal{L}\{u(t-2)\}$

$$= \left[\frac{1}{s^2}\circ e^{-2s}\right] - 2\left[\frac{1}{s}\, e^{-2s}\right]$$

$$\boxed{= e^{-2s}\left[\frac{1}{s^2} - \frac{2}{s}\right]}$$

**4.6** Find $i(t)$.




From KVL:

$$-s^{-1} + i(s)(1\Omega) + i(s)(s^{-1}) + i(s)s = 0$$

$$i(s)\left[1+s+s^{-1}\right] = s^{-1} \to i(s)\,\frac{s^{-1}}{s+1+s^{-1}}\circ\frac{s}{s} = \frac{1}{s^2+s+1}$$

C.T.S. $\to i(s) = \left[s+\frac{1}{2} \pm j\frac{\sqrt{3}}{2}\right]^{-2} = \frac{A}{s+\frac{1}{2}+j\frac{\sqrt{3}}{2}} + \frac{-A}{s+\frac{1}{2}-j\frac{\sqrt{3}}{2}}$

4.6 cont.)

$$A = \left[s + \tfrac{1}{2} + j\tfrac{\sqrt{3}}{2}\right] F(s)\Big|_{s=-\frac{1}{2}-j\frac{\sqrt{3}}{2}}$$

$$= \left[-\cancel{\tfrac{1}{2}} - j\frac{\sqrt{3}}{2} + \cancel{\tfrac{1}{2}} - j\frac{\sqrt{3}}{2}\right]^{-1} = \left[-j\sqrt{3}\right]^{-1}$$

but $-\frac{1}{j} = j$, so $A = j\frac{1}{\sqrt{3}}$,

Now, $$i(s) = \frac{j\frac{1}{\sqrt{3}}}{s + \left(\frac{1}{2} + j\frac{\sqrt{3}}{2}\right)} - \frac{j\frac{1}{\sqrt{3}}}{s + \left(\frac{1}{2} - j\frac{\sqrt{3}}{2}\right)}$$

We know in general, $X(s) = \frac{A_1}{s+P_1} + \frac{A_1^*}{s+P_1^*}$

Moreover $\mathcal{L}^{-1}\{X(s)\} = A_1 e^{-P_1 t} + A_1^* e^{-P_1^* t} = 2\,\mathrm{Re}\left[A_1 e^{P_1 t}\right]$

So, $\mathcal{L}^{-1}\{i(s)\} = 2\,\mathrm{Re}\left[\frac{s}{\sqrt{3}} e^{-\left(\frac{1}{2} + j\frac{\sqrt{3}}{2}\right)t}\right]$

if $P_1 = \alpha + j\beta$, $\rightarrow 2|A_1| e^{-\alpha t}\cos(\beta t + \tan(\not\alpha))$

$$= (2)\sqrt{0^2 + \tfrac{1}{\sqrt{3}}^2}\, e^{-\frac{1}{2}t}\cos\left(\tfrac{\sqrt{3}}{2}t + 90^\circ\right) \rightarrow \boxed{i(t) = 1.1547 e^{-0.5t}\cos(0.866t - 90^\circ) \text{ Amps}}$$