Turn this in by making a pdf scan of your work and submitting it to Canvas by 11:59 pm on the due date.

How to write up homework. Please put your name and assignment number on the first page. You may use this page as a cover sheet. Show your work and explain it. In general, just writing down an answer without showing the steps to arriving at the answer is not sufficient for credit.

Current reading is Chapter 5 of Linear Algebra Done Wrong (LADW).

If you use Matlab on any of these problems, or if you use an online matrix inversion calculator, show and explain your work as usual.

1. Let $V = \mathbb{R}^n$. Show that the *taxicab norm* on V (the p-norm with $p = 1$)

$$\|\mathbf{x}\|_1 = \sum_{i=1}^{n} |x_i|$$

satisfies the defining properties of a norm (see p. 123 of LADW). Use that fact that ordinary absolute value $|\cdot|$ on $\mathbb{R}$ satisfies the triangle inequality.

2. Use the formula for projection given in Chapter 5, Section 4.2 of LADW to find the matrix of orthogonal projection P onto the column space of the matrix

$$A = \begin{pmatrix} 1 & 1 \\ 2 & -1 \\ -2 & 4 \end{pmatrix}$$

Assume that the column space is a real vector space (all scalars are real numbers).

(a) What is the projection matrix P?

(b) What is the size of P?

(c) Why is the dimension of the column space of A different than the number of rows of the projection matrix P?

(d) What is the geometric meaning of $\|\mathbf{x} - P\mathbf{x}\|$? (In an inner product space $\|\cdot\|$ denotes the norm induced by the inner product.)

3. Find the parabola ($y = ax^2 + bx + c$ in the xy-plane) of best fit given the data:

$$\{(-1, 1), (0, 0), (1, 1), (2, 3)\}.$$

Find a least squares solution. Is the solution unique?

4. Let A be an $m \times n$ matrix. Show that $\ker(A) = \ker(A^*A)$.

Hints:

(a) In general, if you want to show that two sets B and C are equal, you can show $B \subset C$ and $C \subset B$. In this case you can establish the desired equality by showing

$$\ker(A) \subset \ker(A^*A) \quad \text{and} \quad \ker(A^*A) \subset \ker(A).$$

(b) $\|A\mathbf{x}\|^2 = (A\mathbf{x}, A\mathbf{x}) = (A^*A\mathbf{x}, \mathbf{x})$.

Using Matlab

In Matlab, A' gives the conjugate transpose of a matrix A (so A' in Matlab is what we call A^*). For example,

```
>> A = [2 i; 0 3];
>> A

A =

        2          +     0i                 0          +     1i
        0          +     0i                 3          +     0i

>> A'

ans =

        2          +     0i                 0          +     0i
        0          -     1i                 3          +     0i
```

Of course if A is a real matrix, then A' is just the transpose of A.