Bennet Sloan

Mth 342

Lab 5

3/14/17

1.) Let $A = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$ be the wire, in $\mathbb{R}^3$.

Then $P_{col(A)}\vec{x}$ is the point of collision,

And $P_{col(A)} = A(A^*A)^{-1}A^*$.

so, $P_{col(A)} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} \left(\begin{bmatrix} 1 & 2 & 3 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} \right)^{-1} \begin{bmatrix} 1 & 2 & 3 \end{bmatrix}$

$= \dfrac{1}{14} \begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 6 \\ 3 & 6 & 9 \end{bmatrix}$

2.) a.) β is the basis of V, $\dim(V) = |\beta| = 5$

b.) $[T]_{\beta\beta} = \begin{bmatrix} [T(\beta_1)]_\beta \dots [T(\beta_5)]_\beta \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & -2 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 2 & 0 & 0 \end{bmatrix}$

c.) $[T]_{\beta\beta}^* = [T]_{BB}^T$ since V is real.

so $[T]_{BB}^* = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 2 \\ 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & -2 & 0 & 0 \end{bmatrix}$

e.) T^* computes $\left(-\dfrac{d}{dx}\right)$ (negative of derivative)

This is an x-axis reflection.

f.) To show $(Tf, g) = (f, T^{*}g)$

Integration by parts is needed since

$\left[(f,g) = \int_{-\pi}^{\pi} f(x)\overline{g(x)}\,dx \right]$ multiplies periodic functions.

Let $v(x) = f(x) \rightarrow dv = f'(x)\,dx$

And $u(x) = g(x) \rightarrow du = g'(x)\,dx$

D.I.B.P. $\rightarrow \int_a^b u\,dv = uv\Big|_a^b - \int_a^b v\,du$

$$\int_{-\pi}^{\pi} f'(x)g(x)\,dx = f(x)g(x)\Big|_{-\pi}^{\pi} - \int_{-\pi}^{\pi} f(x)g'(x)\,dx$$

$\rightarrow$ since $f(x)g(x)$ can be represented as

a sum of sines & cosines with period 2π,

$$f(x)g(x)\Big|_{-\pi}^{\pi} = 0$$

So, $\int_{-\pi}^{\pi} f'(x)g(x)\,dx = \int_{-\pi}^{\pi} f(x)\left[-g'(x)\right]dx$

equally, $(Tf, g) = (f, T^{*}g)$ ∎