MTH 342 OSU Winter 2019
Friday. Feb. 8, Lab $\mathcal{I}$, done in class

Complete this and submit to Canvas by Monday, Feb. 11.

1. Let A be the following matrix. Compute its rank and find bases for the null space, the column space (which is the same as $\operatorname{Ran} A$), and the row space $\operatorname{Ran} A^T$.

$$A = \begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 1 & 0 \end{pmatrix}$$

2. Find the change of coordinates matrix that changes the coordinates in the basis $\{1, 1-t\}$ in $\mathbb{P}_1$ to the basis $\{1+t, 2t\}$.

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Mth 342
Lab I

2/11/19

1.) Let $A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 1 & 0 \end{bmatrix}$. Compute rank A and find

bases for the null space, column space and row space.

→ Let A_e be the echelon form of A.

$$A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 1 & 0 \end{bmatrix} \xrightarrow{-R_1 + R_3 \to R_3} \begin{bmatrix} \boxed{1} & 1 & 0 \\ 0 & \boxed{1} & 1 \\ 0 & 0 & 0 \end{bmatrix} = A_e$$

Therefore the pivot columns are columns 1 & 2.

So, a basis for the column space of A

is $\left\{ \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \right\}$.

Also, from the pivot rows of A_e we have

that a basis for the row space of A

is $\left\{ \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} \right\}$ (expressed as Ran A^T basis)

$\therefore$ rank $A \overset{def}{=} \dim \text{Ran } A$

$$\text{rank} A = \dim \left(\left\{ \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \right\} \right)$$

$$\text{rank} A = 2$$

Let A_{re} be the reduced echelon form of A.

$$A_e = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix} \xrightarrow{-R_2 + R_1 \to R_1} \begin{bmatrix} \boxed{1} & 0 & -1 \\ 0 & \boxed{1} & 1 \\ 0 & 0 & 0 \end{bmatrix} = A_{re}$$

Consider $Ax = 0$, then $x = \begin{pmatrix} x_3 \\ -x_3 \\ x_3 \end{pmatrix} = x_3 \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}$

Therefore a basis for the null space is $\left\{ \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix} \right\}$.

2.) Find the change of coordinates matrix that changes the coordinates in the basis $\{1, 1-t\}$ to the basis $\{1+t, 2t\}$.

Let basis $A = \{1, 1-t\}$

And basis $B = \{1+t, 2t\}$

We want to find $[I_{\mathbb{P}_1}]_{BA}$.

Let $S = \{1, x\}$, the standard basis of $\mathbb{P}_1$.

Then, $[I_{\mathbb{P}_1}]_{SA} = \begin{bmatrix} 1 & 1 \\ 0 & -1 \end{bmatrix} \overset{def}{=} A$

And, $[I_{\mathbb{P}_1}]_{SB} = \begin{bmatrix} 1 & 0 \\ 1 & 2 \end{bmatrix} \overset{def}{=} B$

where $[I_{\mathbb{P}_1}]_{BS} = B^{-1} = \frac{1}{2}\begin{bmatrix} 2 & 0 \\ -1 & 1 \end{bmatrix}$

Consider, $[I_{\mathbb{P}_1}]_{BA} = [I_{\mathbb{P}_1}]_{BS}[I_{\mathbb{P}_1}]_{SA}$

$$= B^{-1} \cdot A$$

$$= \frac{1}{2}\begin{bmatrix} 2 & 0 \\ -1 & 1 \end{bmatrix}\begin{bmatrix} 1 & 1 \\ 0 & -1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 \\ -\frac{1}{2} & \frac{1}{2} \end{bmatrix}\begin{bmatrix} 1 & 1 \\ 0 & -1 \end{bmatrix}$$

$$[I_{\mathbb{P}_1}]_{BA} = \begin{bmatrix} 1 & 1 \\ -\frac{1}{2} & -1 \end{bmatrix}$$