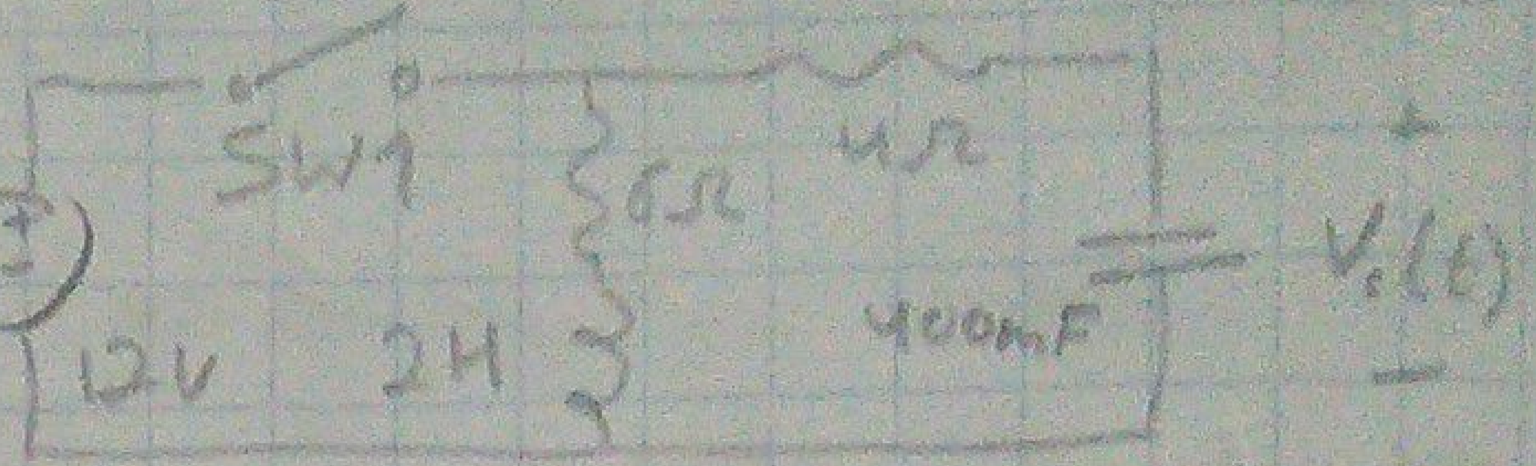
Bennet Sloan Engr 203 4/8/17

2.1 SW1 opens at $t=0$. Find I.C.'s & end behavior for $v_c(t)$.



Assume $\dot{v}_c(0^-)=0$, then $v_c(0^-)=12V$

Assume $\dot{i}_L(0^-)=0$, then $i_L(0^-)=\frac{12V}{6\Omega}=2A$

Since $\dot{v}_c, \dot{i}_L < \infty$, $\boxed{v_c(0^+)=12V \ \& \ i_L(0^+)=2A}$

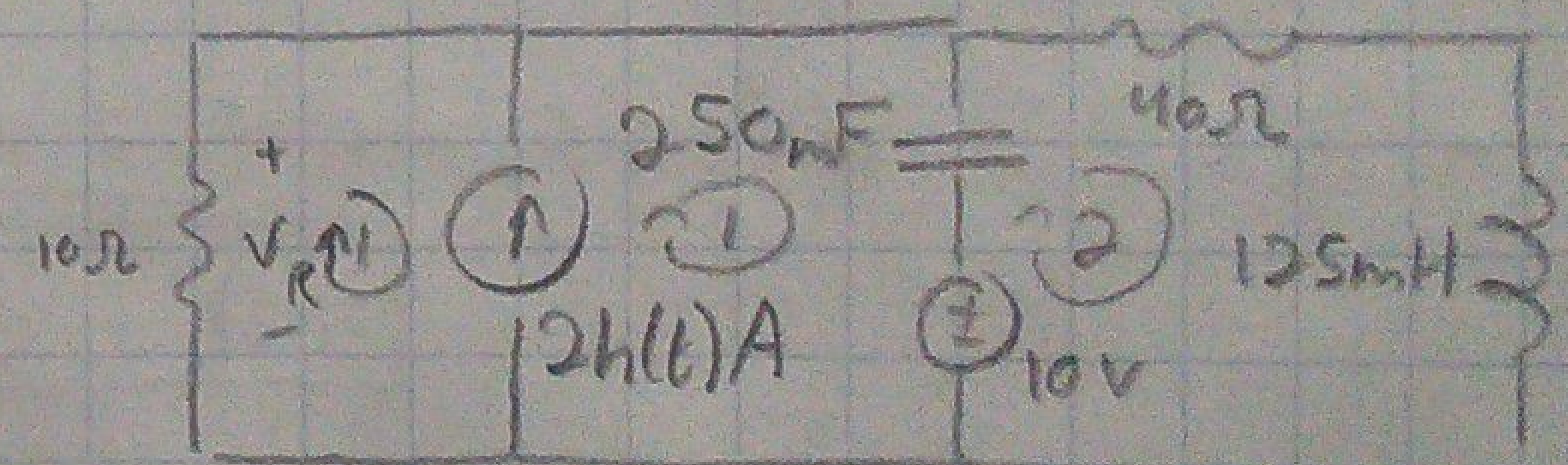
For $t \geq 0$: $i_c(0^+)=-i_L(0^+)$, so $C\dot{v}_c(0^+)=-2 \rightarrow \boxed{\dot{v}_c(0^+)=-5 \ V/s}$

From KVL: $-v_L(0^+)-i_L(0^+)6\Omega-i_L(0^+)4\Omega+v_c(0^+)=0 \rightarrow -v_L(0^+)-(2A)10\Omega+12V=0$

$\rightarrow v_L(0^+)=-8$. So, $L\dot{i}_L(0^+)=-8 \rightarrow \boxed{\dot{i}_L(0^+)=-4 \ A/s}$

Because the steady state is source-free, $\boxed{i_L(\infty)=0A \ \& \ v_c(\infty)=0V}$

2.2 Find the I.C.'s & end behavior.



Assume $\dot{v}_c(0^-)=\dot{i}_L(0^-)=0 \therefore \dot{v}_c, \dot{i}_L < \infty$,

So $v_c(0^-)=-10$, $i_L(0^-)=0$, $v_R(0^-)=0 \rightarrow \boxed{v_c(0^+)=-10V, \ i_L(0^+)=0A, \ v_R(0^+)=0V}$

For $t \geq 0$: $C\dot{v}_c(0^+)=2A \rightarrow \boxed{\dot{v}_c(0^+)=8 \ V/s}$

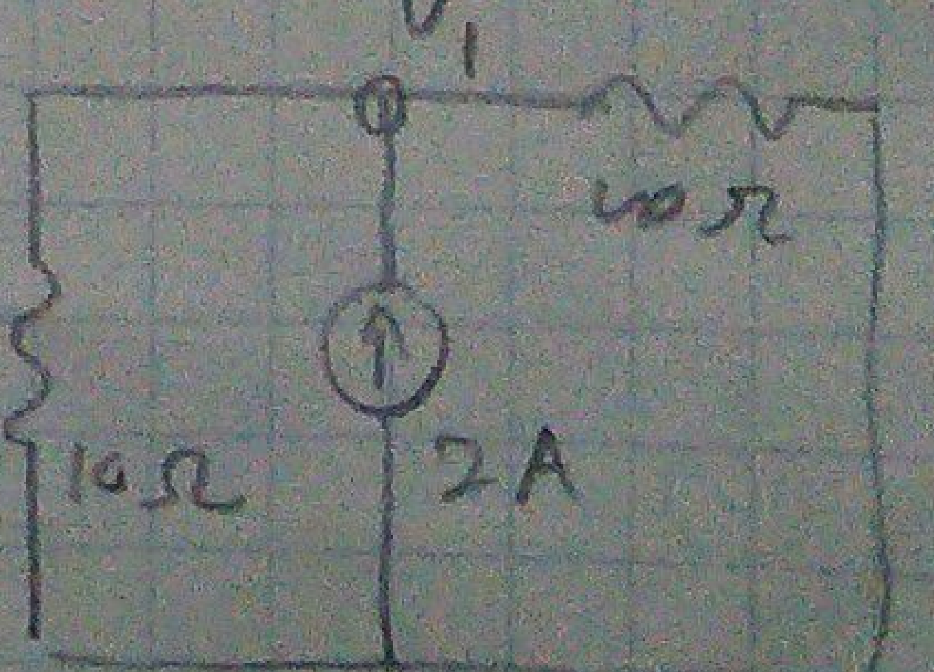
From KVL(2): $\cancel{-10+10}+i_L(0^+)40\Omega+v_L=0 \rightarrow v_L=0 \rightarrow \boxed{\dot{i}_L(0^+)=0 \ A/s}$

From KVL(1): $-v_R(0^+)+v_c(0^+)+10=0 \xrightarrow{\frac{\partial}{\partial t}} -\dot{v}_R(0^+)+\dot{v}_c(0^+)=0$

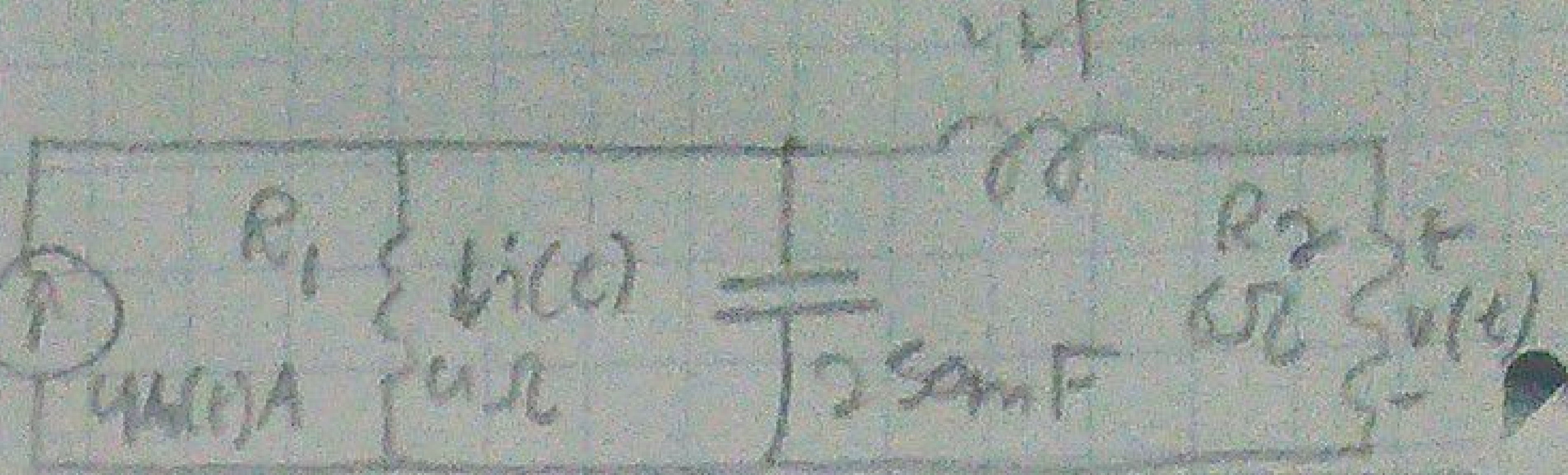
So $\dot{v}_R(0^+)=\dot{v}_c(0^+) \rightarrow \boxed{\dot{v}_R(0^+)=8 \ V/s}$

$v_1=(10 \| 40)2A=16V$, so $v_c(\infty)=16-10=6V$

And $i_L(\infty)=\frac{16V}{40\Omega}=0.40A$ $\boxed{\begin{array}{l} v_c(\infty)=6V \\ i_L(\infty)=0.40A \end{array}}$

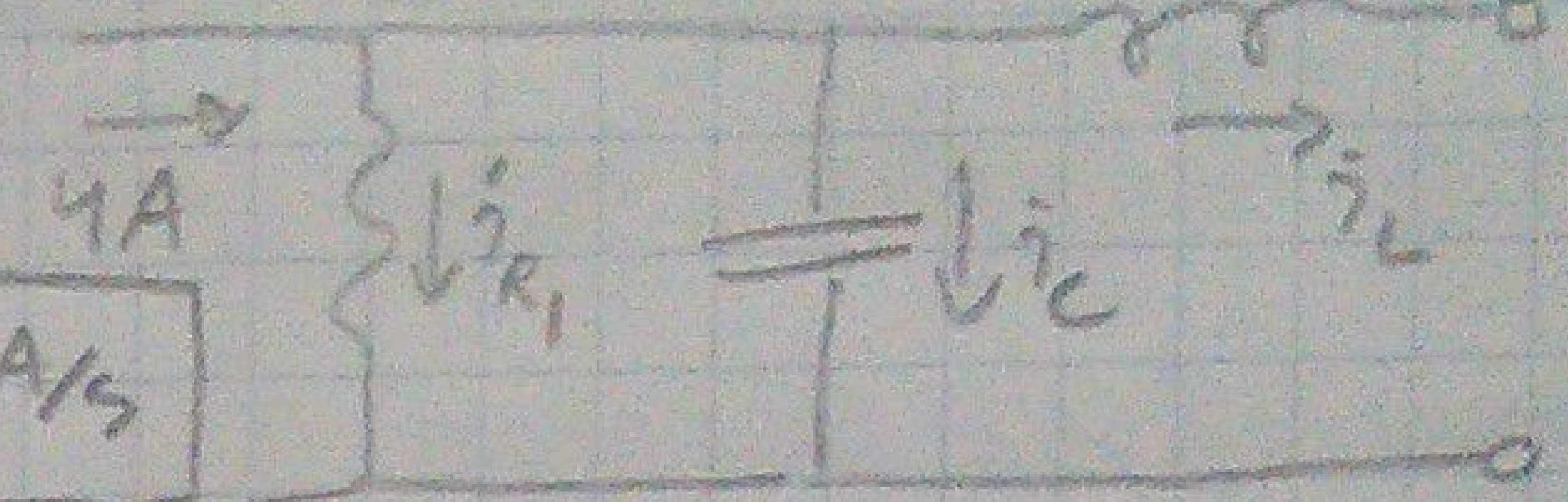


2.3 Find the I.C.'s & end behavior



Circuit starts at zero charge, so $v_c(0^+) = \boxed{v_{R_2}(0^+) = 0V}$, $i_L(0^+) = \boxed{i_{R_1}(0^+) = 0A}$

For $t \geq 0$: $i_c(0^+) = 4A \to \dot{v}_c(0^+) = \frac{4}{0.25} = 16 \, V/s$



Therefore $i_{R_1} = \frac{v_c}{4\Omega} \to \dot{i}_{R_1} = \frac{\dot{v}_c}{4} \to \boxed{\dot{i}(0^+) = 4 \, A/s}$

From KVL: $v_L(0^+) + i_L(0^+)6\Omega - v_c(0^+) = 0 \to v_L(0^+) = 0 \to \dot{i}_L(0^+) = 0$

Since $v_{R_2} = i_L \cdot 6\Omega$, then $\boxed{\dot{v}_{R_1}(0^+) = 0}$

For end behavior: $i_L(\infty) = 1.6A$, $v_c(\infty) = (6 \| 4) 4A = 9.6V$.

$\boxed{\begin{array}{l} v_{R_2}(\infty) = 9.6V \\ i_{R_1}(\infty) = 2.4A \end{array}}$

So, $v_{R_2}(\infty) = i_L(\infty) 6\Omega = 9.6V$ & $i_{R_1}(\infty) = \frac{1}{4\Omega} v_c(\infty) = 2.4A$

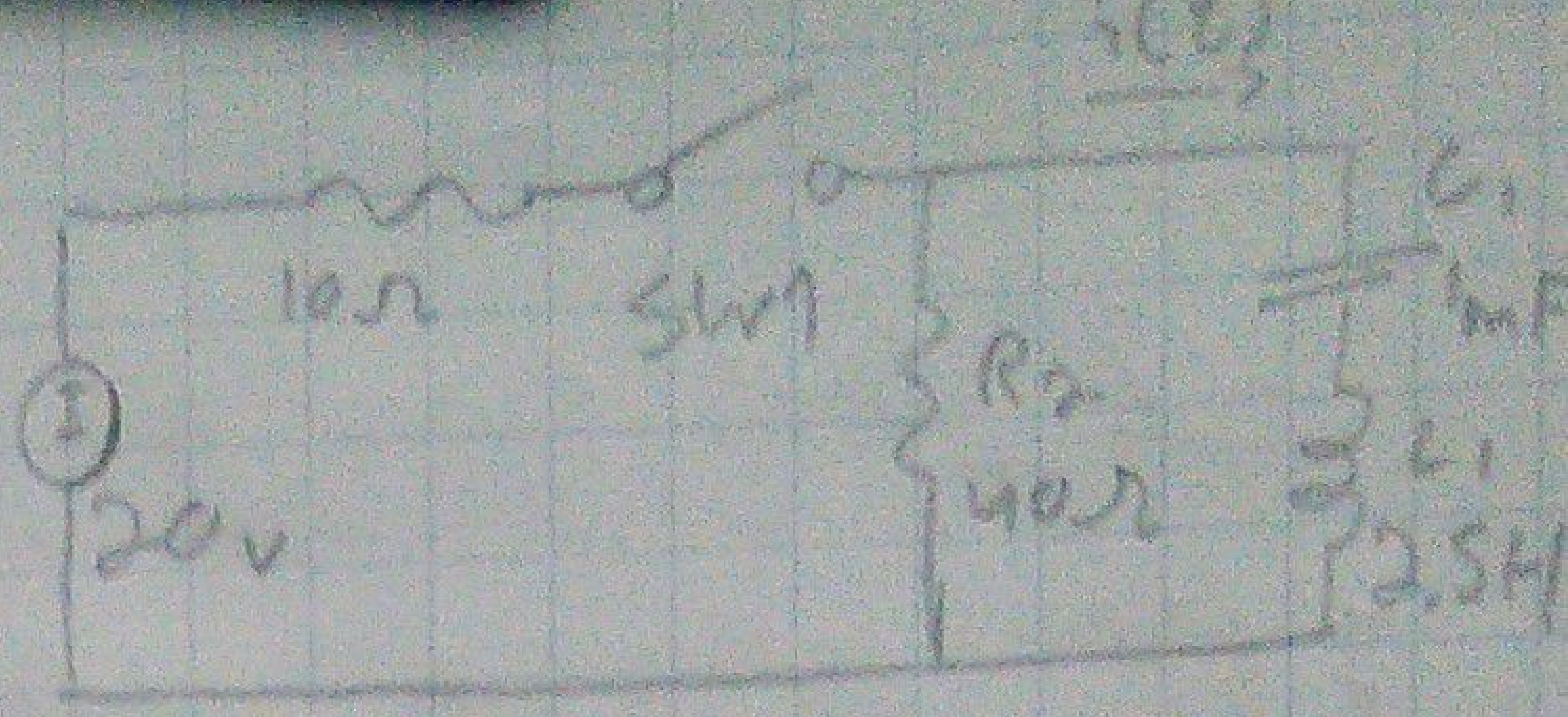
2.4 Consider $\ddot{i} + 10\dot{i} + 25i = 0$, $i(0) = 10$, $\dot{i}(0) = 0$. Find $i(t)$, $t > 0$

char. eq. $\to r^2 + 10r + 25 = 0 \to r = -5, -5 \to i(t) = c_1 e^{-5t} + c_2 t e^{-5t}$

Using I.C.'s: $i(0) = c_1 \to c_1 = 10$. Also $\dot{i}(0) = -5c_1 + c_2 \to c_2 = 50$

$\therefore \boxed{i(t) = e^{-5t}(10 + 50t) \, A}$

2.5 SW1 opens at $t=0$. Find $i(t)$.



$i(0^+) = 0V \to v_c(0^+) = \frac{40}{50}20 = 16V$

For $t \geq 0$: From KVL: $v_L + i(0^+)40\Omega + v_c(0^+) = 0 \to v_L = -16V$

So $L\dot{i}(0^+) = -16 \to \dot{i}(0^+) = -6.4\ A/s$

From KVL: $v_R + v_c + v_L = 0 \to Ri(t) + C^{-1}\int_0^t v(\tau)d\tau + v_0 + L\dot{i} = 0$

$\to L\ddot{i} + R\dot{i} + C^{-1}i = 0 \to \ddot{i} + RL^{-1}\dot{i} + (LC)^{-1}i = 0$

Stability: $\alpha = R(2L)^{-1} = 8$, $\omega_0 = (LC)^{-\frac{1}{2}} = 20 \to$ Underdamped

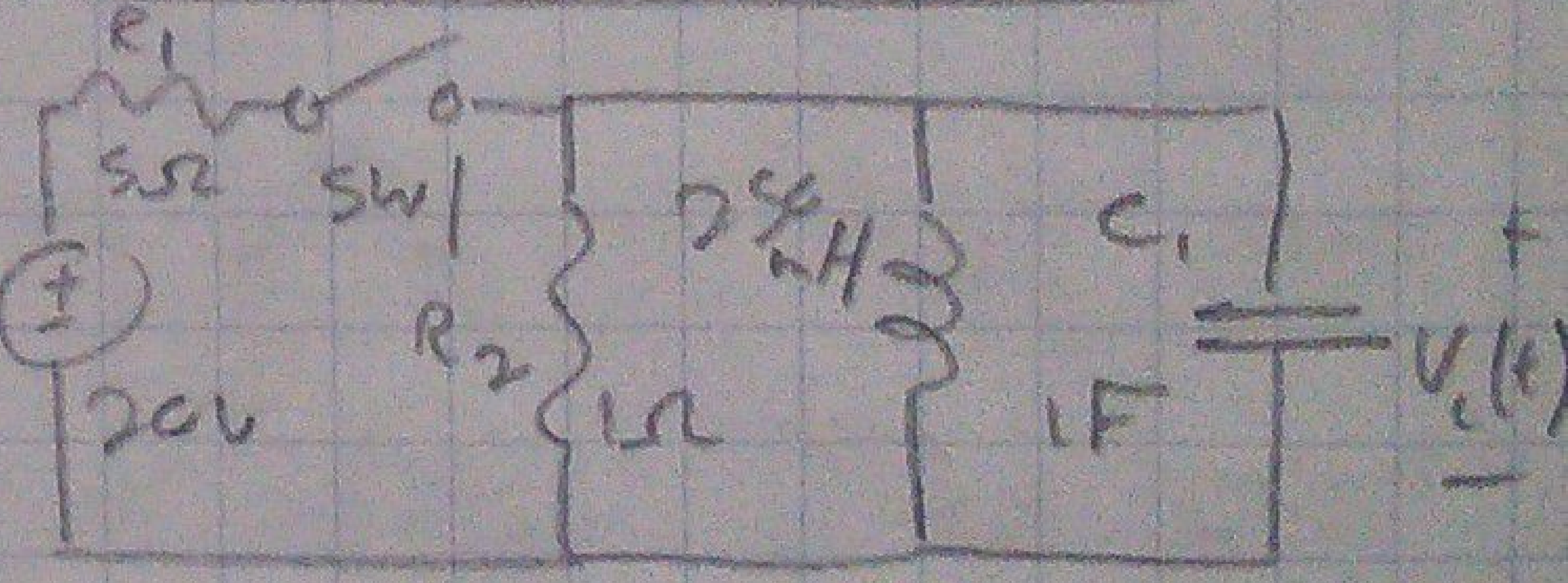
$\omega_d = (\omega_d^2 - \alpha^2)^{\frac{1}{2}} = 18.33 \to i(t) = e^{-8t}(C_1\cos(\omega_d t) + C_2\sin(\omega_d t))$

$i(0) = 0 = C_1 \to \dot{i}(0^+) = -8e^0(C_2\sin(\omega_d t)) + e^0(C_2(\omega_d)\cos(\omega_d t))$

$\to \dot{i}(0^+) = \omega_d C_2 = -6.4 \to C_2 = -0.349$

$\therefore \boxed{i(t) = e^{-8t}(-0.349\sin(18.33A))\ A}$

2.6 SW1 closes at $t=0$, Find $v_c(t)$



Uncharged $\to i_L(0^+) = 0A$, $v_c(0^+) = 0V$

And $i_L(\infty) = 4A$, $v_c(\infty) = 0V$.

From KCL: $i_R + i_L + i_C = i_s \to v_c(\frac{1}{R_2}) + L^{-1}\int_{-\infty}^{0} v(\tau)d\tau + C\dot{v}_c = \frac{V_s - V_1}{R_1}$

$\to v_c(\frac{1}{R_1} + \frac{1}{R_2}) + L^{-1}\int_{-\infty}^{0} v(\tau)d\tau + C\dot{v}_c = \frac{V_s}{R_1} \to \ddot{v}_c + (C(\frac{1}{R_1}+\frac{1}{R_2}))^{-1}\dot{v} + (LC)^{-1}v = 0$

Stability: $\alpha = [2C(\frac{1}{R_1}+\frac{1}{R_2})]^{-1} = 0.6$, $\omega_0 = (LC)^{-\frac{1}{2}} = 2 \to$ Underdamped

So $\omega_d = (\omega_0^2 - \alpha^2)^{\frac{1}{2}} = 1.91$, for $v_c(t) = e^{-2t}(C_1\cos(\omega_d t) + C_2\sin(\omega_d t))$

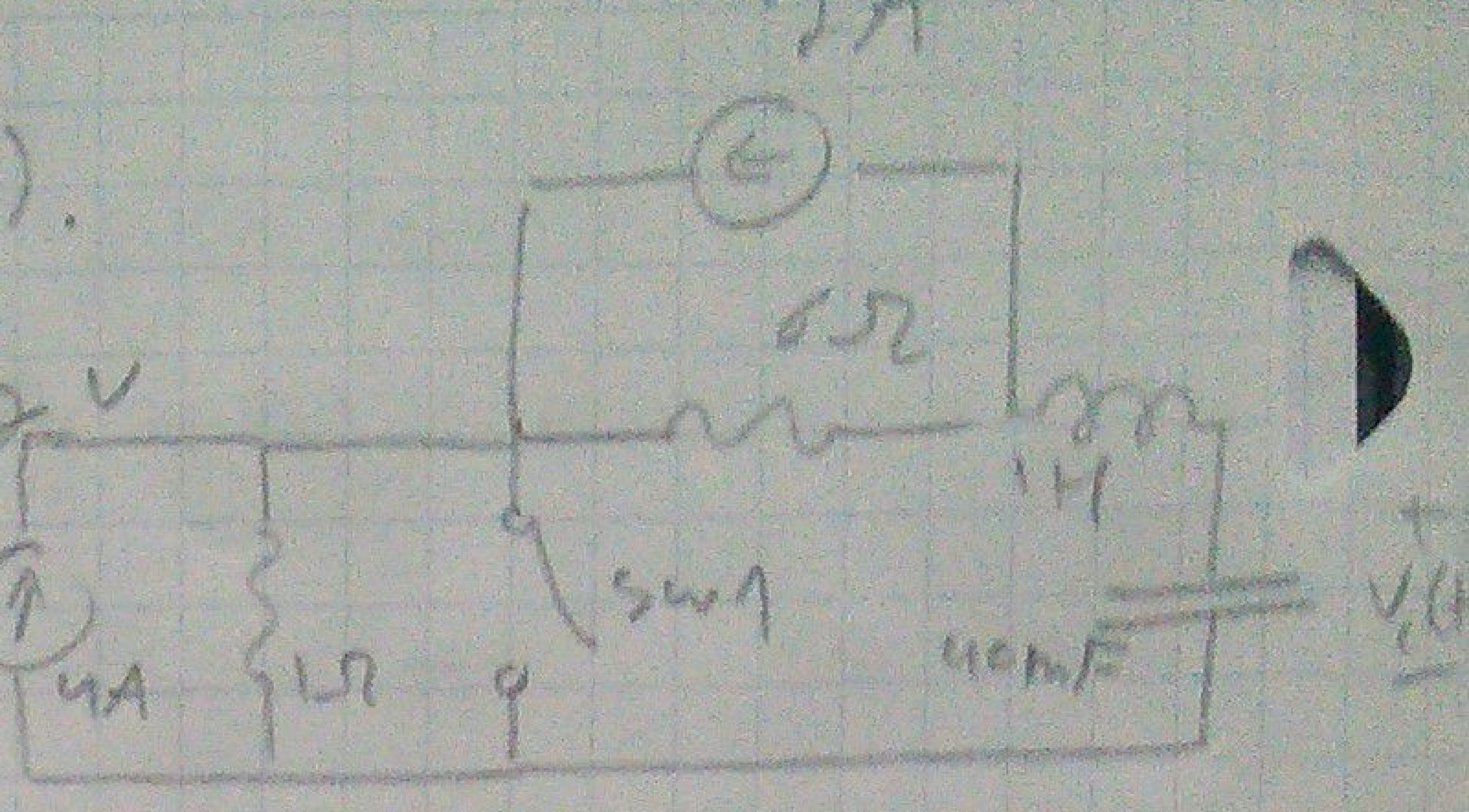
Now, $v_c(0^+) = 0 = C_1$, And $\dot{v}_c = -0.6e^{-0.6t}(C_2\sin(\omega_d t)) + e^{-0.6t}(\omega_d C_2\cos(\omega_d t))$

But $i_c(0^+)C^{-1} = \dot{v}(0^+) \to i_c = i_s = 4A \to \dot{v}(0^+) = \frac{4A}{1F} = 4\ V/s$

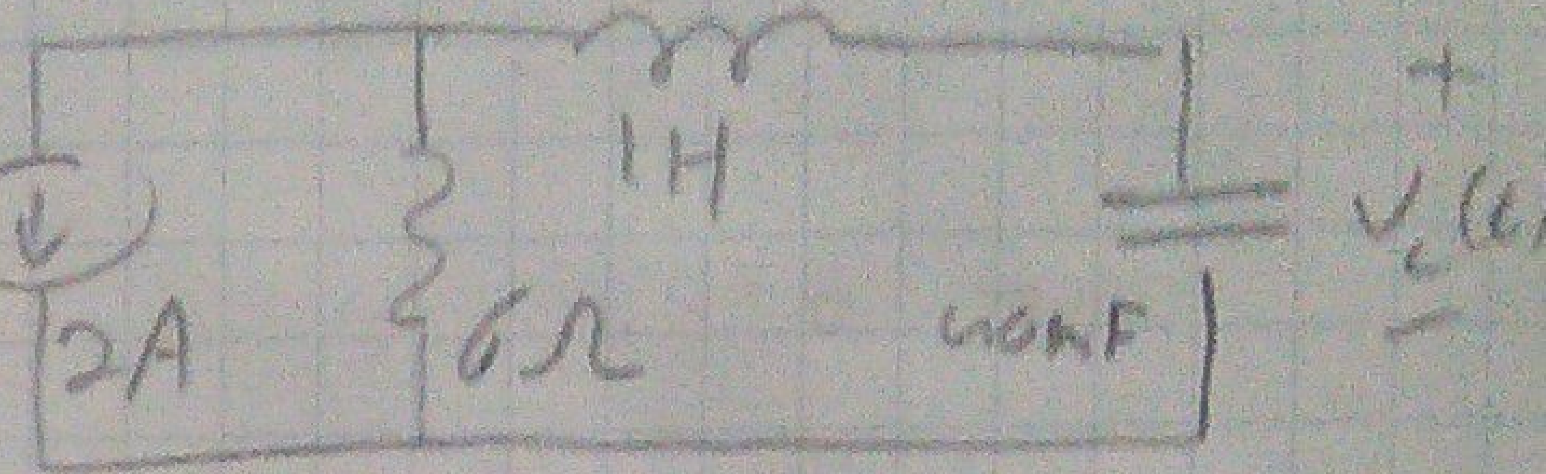
So, $\dot{v}(0^+) = 4 = \omega_d C_2 \to C_2 = \frac{4}{\omega_d} = 2.097$. $\boxed{v(t) = e^{-0.6t}(2.097\sin(1.9t))}$

2.7 SW1 Closes at $t=0$. Find $v(t)$.

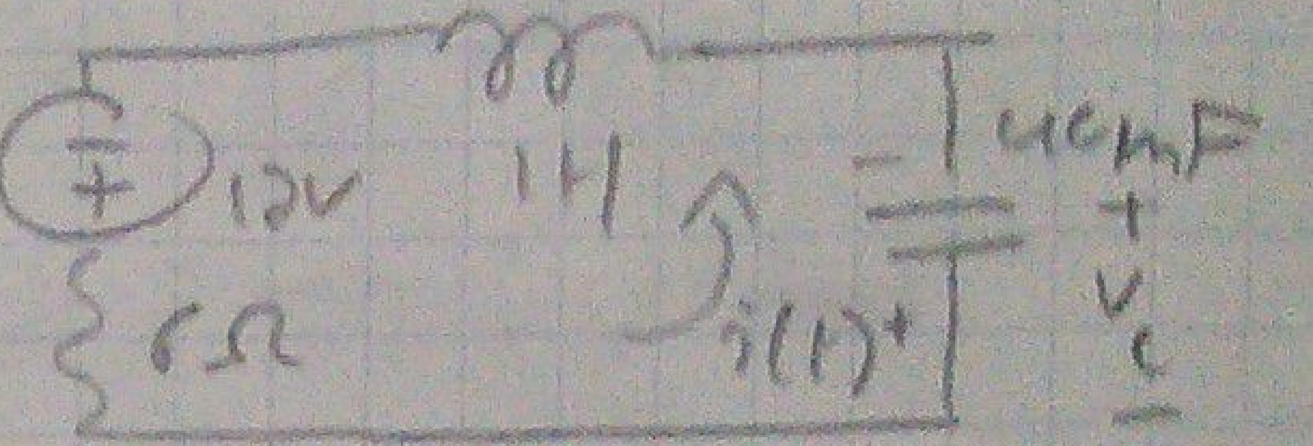
I.C.'s: $i_L(0^-)=0A$, $v_c(0^-)=-8V$, $v_c(\infty)=-12$ V



For $t \geq 0$:



... source transformation



From KVL: $-V_c + L\dot{i} - V_s + Ri = 0$

But $i = C\dot{v}$ & $v_c = -v_{KVL}$, so:

$$v + L[C\dot{v}]' + RC\dot{v} - V_s = 0 \to \ddot{v} + RL^{-1}\dot{v} + (LC)^{-1}v = (LC)^{-1}V_s$$

stability: $\alpha = (2L)^{-1}R = 3$, $\omega_0 = (LC)^{-\frac{1}{2}} = 5 \to$ underdamped

So $v_c(t) = v_T + v_{ss} = v_T + v(\infty) = v_T + (-v_c\infty) = v_T + 12$

For v_T: $v_T = e^{-3t}(c_1\cos(\omega_d t) + c_2\sin(\omega_d t))$, where $\omega_d = (\omega_0^2-\alpha^2)^{\frac{1}{2}} = 4$.

using I.C.'s: $-v_c(0) = v_{ss} + c_1 \to 8 = 12 + c_1 \to c_1 = -4$

so $v(t) = e^{-3t}(-4\cos(\omega_d t) + c_2\sin(\omega_d t)) + v_{ss}$

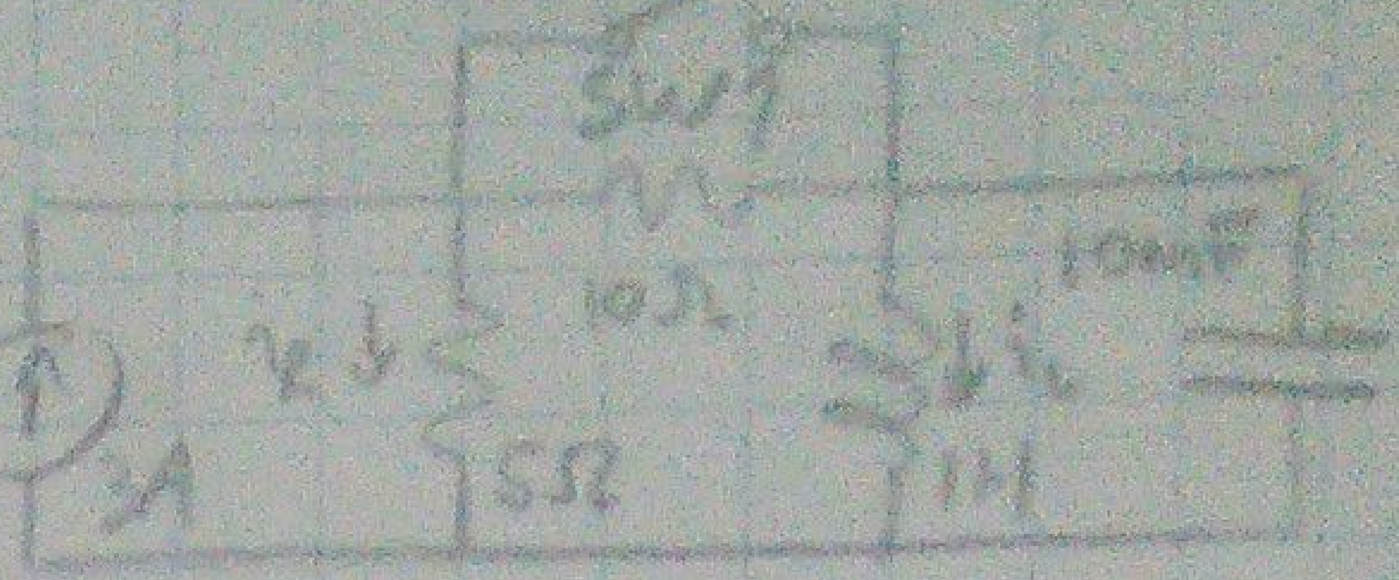
Therefore: $\dot{v}(0) = -3e^0(-4+0) + e^0(0+4c_2)\ \dot{v}_{ss}$

$\to \dot{v}(0) = 12 + 4c_2 + 0$, but $\dot{v}(0) = C^{-1}i(0) = 0$

$\to c_2 = -3$. Also, $v_{KVL} = -v_c$.

$\therefore\ v_c(t) = \left[e^{-3t}(4\cos(4t) + 3\sin(4t)) - 12\right]$ V

$\underline{2.8}$ SW1 closes at $t=0$. Find $v_c(t)$.



$i_L(0^-) = \frac{5\Omega}{15\Omega} 3A = 1A$, $v_c(0^-) = 0v$, $i_L(\infty) = 3A$, $v_c(\infty) = 0v$

For $t \geq 0$) $i_R + i_L + i_C = 3 \xrightarrow{\text{Voltage}} \frac{v}{R_1} + L^{-1}\int_{-\infty}^{t} v(\tau)d\tau + C\dot{v} = 3$

$\rightarrow \frac{\partial}{\partial t} \rightarrow R_1^{-1}\dot{v} + L^{-1}v + C\ddot{v} = 0 \rightarrow \underline{\ddot{v} + (R_1C)^{-1}\dot{v} + (LC)^{-1}v = 0}$

Stability) $\omega_0 = 10$, $\alpha = (2RC)^{-1} = 10 \rightarrow$ Crit. damp.

$\therefore v(t) = v_{ss} + v_T \quad \because v_{ss} = v(\infty) = 0 \rightarrow v(t) = e^{-10}(C_1 t + C_2)$

Since $v(0^+) = 0$, $0 = e^{0}(C_1(0) + C_2) \rightarrow C_2 = 0$

Also, $\dot{v}(0^+) = C^{-1} i_c(0^+) = 200 \text{ V/s}$

So, $200 = [C_1 t e^{-10t}]'_0 = C_1 e^{0} - 10 C_1 (0) e^{0} = C_1 \rightarrow C_1 = 200$

$$\boxed{\therefore v_c(t) = e^{-10t}(200t) \text{ v}}$$