Bennet Sloan
Mth 256 - HW8

1.) $\mathcal{L}^{-1}\left\{\frac{1}{s^2+3s+3}\right\}$

$$\mathcal{L}^{-1}\left\{\frac{1}{s^2+3s+(3/2)^2-(3/2)^2+3}\right\}$$

$$\mathcal{L}^{-1}\left\{\frac{1}{(s+3/2)^2+3/4}\right\}$$

$$\frac{2}{\sqrt{3}}\mathcal{L}^{-1}\left\{\frac{\sqrt{3}/2}{(s+3/2)^2+(\sqrt{3}/2)^2}\right\}$$

$\lambda = -\frac{3}{2}$
$\omega = \frac{\sqrt{3}}{2}$

using $e^{\lambda t}\sin\omega t \longleftrightarrow \frac{\omega}{(s-\lambda)^2+\omega^2}, (s>\lambda)$

$$\frac{2}{\sqrt{3}}\mathcal{L}^{-1}\{F(s)\} = f(t)$$

$$\frac{2}{\sqrt{3}}e^{-\frac{3}{2}t}\sin\left(\frac{\sqrt{3}}{2}t\right) = f(t)$$

2.) $3y'' + 6y' + 3y = 9$, $y(0)=0$, $y'(0)=6$

Let $Y(s) = \mathcal{L}\{y(t)\}$

Then $\mathcal{L}\{y'\} = sY - y(0)$

$\mathcal{L}\{y''\} = s^2Y - sy(0) - y'(0)$

so, $\left(3s^2Y - \cancel{3sy(0)} - 3y'(0)\right) + \left(6sY - \cancel{6y(0)}\right) + (3Y) = \frac{9}{s}$

$\left(3s^2Y + 6sY + 3Y\right) - 3(6) = \frac{9}{s}$

$Y\left(3s^2 + 6s + 3\right) = \frac{9}{s} + 18$

$Y = \frac{9}{s(3s^2+6s+3)} + \frac{18}{(3s^2+6s+3)}$

$Y = \frac{3}{s(s+1)^2} + \frac{6}{(s+1)^2}$

Partial Fraction Decomposition

$\frac{3}{s(s+1)^2} = \frac{A}{s} + \frac{B}{s+1} + \frac{C}{(s+1)^2}$

$3 = A(s+1)^2 + B(s+1)s + Cs$

$3 = s^2(A+B) + s(2A+B+C) + A$

so, $A + B = 0$ | $A = 3$

$2A + B + C = 0$ | $B = -3$

$A = 3$ | $C = -3$

2. continued.)

$$Y = \frac{3}{s} - \frac{3}{s+1} - \frac{3}{(s+1)^2} + \frac{6}{(s+1)^2}$$
$$= \frac{3}{s} - \frac{3}{s+1} + \frac{3}{(s+1)^2}$$

using $\mathcal{L}^{-1}\left\{\frac{n!}{(s-a)^{n+1}}\right\} = t^n e^{at}$, $a=-1$, $n=0$, $n=1$

$$\mathcal{L}^{-1}\{Y\} = \mathcal{L}^{-1}\left\{\frac{3}{s}\right\} - \mathcal{L}^{-1}\left\{\frac{3}{s+1}\right\} + \mathcal{L}^{-1}\left\{\frac{3}{(s+1)^2}\right\}$$

$$y(t) = 3 - 3e^{-t} + 3te^{-t}$$