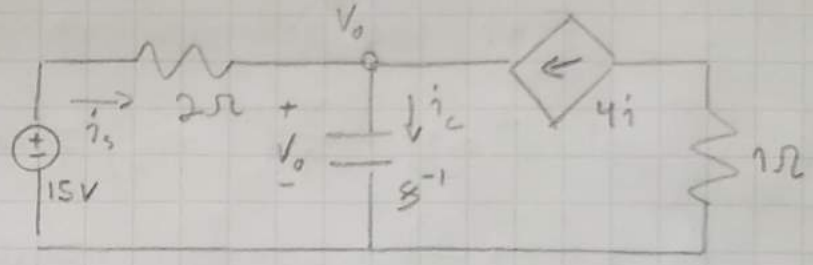
Bennet Sloan Engr 203 5/10/19

5.1 Find $v_o(t)$



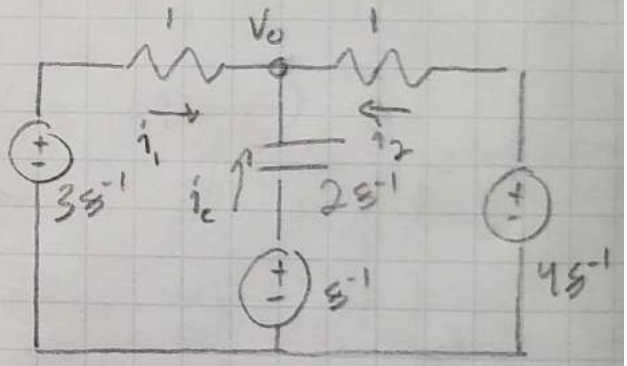
$i_c = 4i + i = 5i$

So, $i_c = 5i_s$ where $i_c = \frac{V_o}{5^{-1}}$, $i_s = \frac{15 - V_o}{2}$

$\rightarrow V_o s = \left[\frac{15 - V_o}{2}\right]5 \rightarrow V_o s = \frac{5}{2}15 - \frac{5}{2}V_o \rightarrow V_o s + \frac{5}{2}V_o = 37.5$

$\rightarrow V_o\left(s + \frac{5}{2}\right) = 37.5 \rightarrow V_o(s) = \frac{37.5}{s + 2.5} \xrightarrow{\mathcal{L}^{-1}} \boxed{v(t) = 37.5\, e^{-2.5t}\, u(t)\ V}$

5.2 Find $V_o(s)$ using node-voltage



N.V. at V_o) $i_1 + i_c + i_2 = 0$

For i_1) $i_1 = \frac{3s^{-1} - V_o}{1}$

For i_c) $i_c = \frac{s^{-1} - V_o}{2s^{-1}}$ For $i_2 = \frac{4s^{-1} - V_o}{1}$

So, $(3s^{-1} - V_o) + \left(\frac{s^{-1} - V_o}{2s^{-1}}\right) + (4s^{-1} - V_o) = 0$

$\rightarrow V_o\left(-2 - \frac{s}{2}\right) + 3s^{-1} + \frac{s^{-1}}{2s^{-1}} + 4s^{-1} = 0$

$\rightarrow -V_o\left(2 + \frac{s}{2}\right) = -\left(3s^{-1} + 4s^{-1} + \frac{1}{2}\right) \rightarrow V_o = \frac{3s^{-1} + 4s^{-1} + \frac{1}{2}}{2 + \frac{s}{2}} \cdot \frac{s}{s}$

$\rightarrow V_o = \frac{7 + \frac{s}{2}}{\frac{s^2}{2} + 2s} \cdot \frac{2}{2} \rightarrow \boxed{V_o = \frac{s + 14}{s^2 + 4s}}$

5.3 Find the s-domain mesh currents

For i_1) $-10s^{-1} + i_1 + \frac{s}{4}(i_1 - i_2) = 0$

$\rightarrow i_1\left(1 + \frac{s}{4}\right) + i_2\left(-\frac{s}{4}\right) = 10s^{-1}$

For i_2) $\frac{s}{4}(i_2 - i_1) + i_2(4 + s) = 0 \rightarrow i_1\left(-\frac{s}{4}\right) + i_2\left(s + 4 + \frac{s}{4}\right) = 0$

$$\text{So, } \begin{bmatrix} 1+\frac{s}{4} & -\frac{s}{4} \\ -\frac{s}{4} & s+4+\frac{s}{4} \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} 10s^{-1} \\ 0 \end{bmatrix} \rightarrow i_1 = \left(5 + \frac{16}{s}\right) i_2$$

$$\rightarrow \left[i_2\left(1 + \frac{s}{4}\right)\left(5 + \frac{16}{s}\right) - i_2\left(\frac{s}{4}\right) = \frac{10}{s} \right] \cdot 4$$

$$\rightarrow \left[i_2\left[\left(4 + s\right)\left(5 + \frac{16}{s}\right) - 5\right] \right] = 40s^{-1}$$

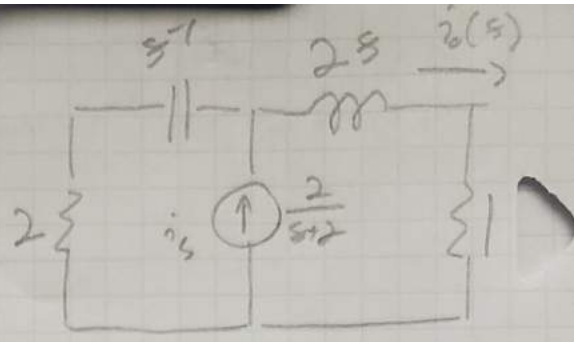
$$\rightarrow i_2\left(36 + \frac{64}{s} + 4s\right) = 40s^{-1} \rightarrow i_2 = \frac{40s^{-1}}{4s + 64s^{-1} + 36} \cdot \frac{s}{s}$$

$$\rightarrow \boxed{i_2 = \frac{10}{s^2 + 9s + 16}}$$

$$\text{Also, } i_1 = \frac{\left(5 + \frac{16}{s}\right)10}{s^2 + 9s + 16} \rightarrow \frac{50 + 160s^{-1}}{s^2 + 9s + 16}$$

$$\rightarrow i_1 = \frac{50s + 160}{s^3 + 9s^2 + 16s}$$

5.4 Find the s-domain current $i_o(t)$



i_o divider) $i_o = i_s \dfrac{2 + s^{-1}}{2 + s^{-1} + 2s + 1} \cdot \dfrac{s}{s}$

$\rightarrow \dfrac{2s+1}{2s^2+3s+1} \cdot \dfrac{2}{s+2} \rightarrow i_o = \dfrac{2(s+\frac{1}{2})}{2(s^2+\frac{3}{2}+\frac{1}{2})} \cdot \dfrac{2}{s+2} = \dfrac{s+\frac{1}{2}}{(s+1)(s+\frac{1}{2})} \cdot$

$\rightarrow \dfrac{2}{(s+2)(s+1)} = \dfrac{A}{s+2} + \dfrac{B}{s+1}$

s.t. $A = s+2\, i_o\big|_{s=-2} = \dfrac{2}{-2+1} = \underline{-2}$

$B = s+1\, i_o\big|_{s=-1} = \dfrac{2}{-1+2} = \underline{2}$

$\therefore i_o = \dfrac{2}{s+1} - \dfrac{2}{s+2} \rightarrow \mathcal{L}^{-1}\{i_o(s)\} = \mathcal{L}^{-1}\left\{\dfrac{2}{s+1} - \dfrac{2}{s+2}\right\}$

$$\boxed{i_o(t) = u(t)\, 2(e^{-t} - e^{-2t})\ A, \quad t \geq 0}$$