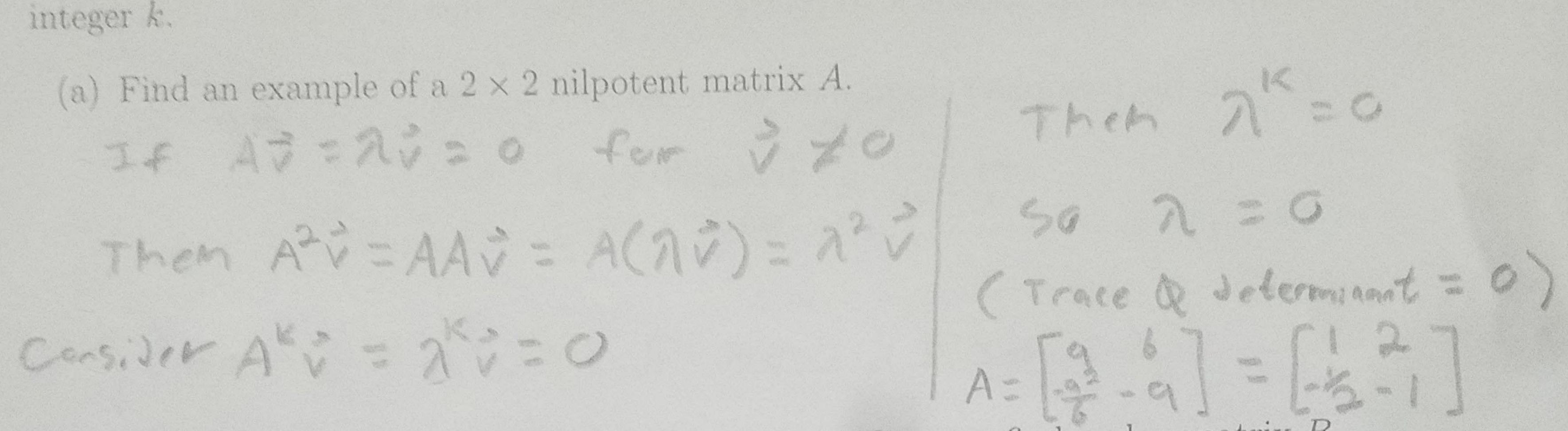
1. Find the (complex) eigenvalues of the rotation matrix

The (complex) eigenvalues of the totation matrix
$$\begin{bmatrix} \cos \alpha & -\sin \alpha \\ \sin \alpha & \cos \alpha \end{bmatrix}$$

$$(\cos \alpha - n)(\cos \alpha - n) - (-\sin \alpha)(\sin \alpha)$$

$$(\cos \alpha - n)^2 + \sin^2 \alpha$$

$$(\cos \alpha + n)^2 + \sin^2$$



(b) Find an example of a 3×3 nilpotent matrix B. Can you find such a matrix B having the property that $B^2 \neq 0$?

2. An $n \times n$ matrix A is called nilpotent if $A^k = 0$ (the zero matrix) for some positive

If trace
$$B = Jet D = 0$$
 but $B^2 \neq 0$,

Then B could have a zero row and column, zero diagonal.

 $ex: B = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}, B^2 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \neq 0, B^3 = 0$

(c) Verify that 0 is an eigenvalue of your 2×2 matrix A. This means find a nonzero vector \vec{v} such that $A\vec{v} = 0\vec{v} = \vec{0}$.

$$A\overrightarrow{v} = 0 \longrightarrow \begin{bmatrix} 1 & 2 \\ -5 & -1 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$50 \quad 0 = -26$$

$$A\overrightarrow{v} = 0 \longrightarrow \begin{bmatrix} 1 & 2 \\ -5 & -1 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$5 = \begin{bmatrix} -2 \\ 1 \end{bmatrix}$$

(d) Prove that if an $n \times n$ matrix A is nilpotent, and λ is an eigenvalue of A, then $\lambda = 0$. (An eigenvalue can be zero, but the zero vector is never an eigenvector.)

If $A^{K}\vec{v} = \vec{\lambda}\vec{v} = 0$, $A^{K} = 0$ since $\vec{v} \neq 0$.

The only scalar $A^{K} = 0$ is A = 0.