MTH 342 OSU Winter 2019 Wed., Jan. 30, Lab G, done in class.
Complete this and submit it to Canvas by the posted due date: Friday, Feb. 1.

1. Let $M_{3\times 3}$ denote the vector space of all 3×3 matrices with real entries. Let

$$W = \{\vec{m} \in M_{3\times 3} \mid \text{ all row and column sums of } \vec{m} \text{ are zero}\}.$$

Note that we are writing 3×3 matrices as vectors (they belong to the vector space $M_{3\times 3}$).

1. Find a basis $\mathcal{B}$ for $M_{3\times 3}$. Make $\mathcal{B}$ as simple as possible.

2. Find and n so that $M_{3\times 3}$ is isomorphic to $\mathbb{R}^n$. (An isomorphism from $M_{3\times 3}$ to $\mathbb{R}^n$ is $T : M_{3\times 3} \to \mathbb{R}^n$ defined by $T(\vec{m}) = [\vec{m}]_{\mathcal{B}}$.)

3. Let $V \subset \mathbb{R}^n$ be defined by

$$V = \{[\vec{w}]_{\mathcal{B}} \mid \vec{w} \in W\}$$

In other words, $V = T(W)$. Can you write V as the Null space of some matrix C? Hint: think about the constraints that define W. What is the size of C?

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Mth 342
2/1/19

1.) $W = \{ \vec{m} \in M_{3\times3} \mid \Sigma \text{ row} = \Sigma \text{ col} = 0 \}$

A basis for $M_{3\times3}$ would be $\underline{9}$ single entry matrices.

$$B = \left\{ \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \right\}$$

2.) $A \circ \begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \\ a_5 \\ a_6 \\ a_7 \\ a_8 \\ a_9 \end{bmatrix}$

Given $M_{3\times3} \cong \mathbb{R}^n$

Therefore $T(\vec{m}) = [\vec{m}]_B$

so, $n = 9$ (bijective)

3.) $V = \{ [\vec{w}]_B \mid \vec{w} \in W \}$

If $V = T(W)$, then $[\vec{w}]_B = T(\vec{w})$

Let $\vec{m} = \begin{bmatrix} x_{11} & x_{12} & x_{13} \\ x_{21} & x_{22} & x_{23} \\ x_{31} & x_{32} & x_{33} \end{bmatrix}$ And $[\vec{m}]_B = \begin{bmatrix} x_{11} \\ x_{12} \\ x_{13} \\ x_{21} \\ x_{22} \\ x_{23} \\ x_{31} \\ x_{32} \\ x_{33} \end{bmatrix}$

There are 6 constraints:

$$\begin{aligned} x_{11} + x_{12} + x_{13} &= 0 & \quad x_{11} + x_{21} + x_{31} &= 0 \\ x_{21} + x_{22} + x_{23} &= 0 & \quad x_{12} + x_{22} + x_{32} &= 0 \\ x_{31} + x_{32} + x_{33} &= 0 & \quad x_{13} + x_{23} + x_{33} &= 0 \end{aligned}$$

Therefore V is the nullspace of C,

Where C is defined by the 6 constraints:

$$\underbrace{\begin{array}{c} \begin{array}{ccccccccc} x_{11} & x_{12} & x_{13} & x_{21} & x_{22} & x_{23} & x_{31} & x_{32} & x_{33} \end{array} \\ \begin{bmatrix} 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 \\ 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 \end{bmatrix} \end{array}}_{\substack{C \\ (6 \times 9)}} \underbrace{\begin{bmatrix} x_{11} \\ x_{12} \\ x_{13} \\ x_{21} \\ x_{22} \\ x_{23} \\ x_{31} \\ x_{32} \\ x_{33} \end{bmatrix}}_{\substack{[\vec{m}]_B \\ (9 \times 1)}} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$