Complete this and submit it to Canvas by the posted due date: Tuesday, Feb. 19.

1. It is a fact that if A is an $n \times n$ matrix with eigenvalues $\lambda_1, \lambda_2, \ldots \lambda_n$ listed according to algebraic multiplicity, then $\sum_{i=1}^{n} \lambda_i = \mathrm{tr}(A)$. The purpose of this exercise to to make this plausible (it is not a proof).

Let A be a 3×3 matrix. Let λ_1, λ_2 and λ_3 be the eigenvalues of A (repetition allowed to count algebraic multiplicity). The characteristic polynomial of A is $f(\lambda) = \det(A - \lambda I)$.

(a) Why is $f(\lambda) = (\lambda_1 - \lambda)(\lambda_2 - \lambda)(\lambda_3 - \lambda)$?

(b) What is the coefficient of the λ^2 term in $f(\lambda)$? Expand f and compute this coefficient.

(c) Now suppose that $A = \begin{pmatrix} 3 & * & * \\ * & 4 & * \\ * & * & 5 \end{pmatrix}$, $* =$ any number.

Let us write $f(\lambda)$ in a different way (for this specific A):

$$\begin{aligned} f(\lambda) &= \det \begin{pmatrix} 3-\lambda & * & * \\ * & 4-\lambda & * \\ * & * & 5-\lambda \end{pmatrix} \\ &= (3-\lambda)\det\begin{pmatrix} 4-\lambda & * \\ * & 5-\lambda \end{pmatrix} + \text{ terms of degree at most } \underline{\hspace{2em}} \\ &= (3-\lambda)(4-\lambda)(5-\lambda) + \text{ terms of degree at most } \underline{\hspace{2em}}. \end{aligned}$$

Use this different way writing f to again compute the coefficient of λ^2.

(d) By comparing answers from parts (b) and (d) what can you conclude about the sum of the eigenvalues of A and the trace of A (for our specific A)?

2. Why isn't the computation we did in part 1 a proof? List some reasons. Think about how you could you turn this specific calculation into a general proof. In other words, what could you do to prove the following statement:

For any $n \times n$ matrix A (with eigenvalues $\lambda_1, \ldots \lambda_n$ listed according to multiplicity) we have

$$\sum_{i=1}^{n} \lambda_i = \operatorname{tr}(A).$$

3. Prove the following statement:

The eigenvalues of an $n \times n$ triangular matrix are the entries on the main diagonal.

(You can use the fact that the determinant of a triangular matrix is the product of its entries on the main diagonal.)