Bennet Sloan
Math 255 - HW9

1) Use Laplace Transforms to solve the IVP:

$$y'' + y = \begin{cases} 0 & 0 \le t < 2 \\ t-2 & 2 \le t < \infty \end{cases}, \quad y(0) = 0, \; y'(0) = 0$$

$$\mathcal{L}\{y'' + y\} = \mathcal{L}\{(t-2)h(t-2)\}$$

$$Y(s^2+1) = e^{-2s}\mathcal{L}\{t\}$$

$$Y = e^{-2s}s^{-2}(s^2+1)^{-1}$$

Partial Fraction Decomp.

$$\frac{1}{s^2(s^2+1)} = \frac{A}{s^2} + \frac{B}{s^2+1} \rightarrow (s^2+1)A + s^2B = 1$$

$$\longrightarrow \; s^2A + A + s^2B = 1 \rightarrow A = 1, \; B = -1$$

so, $Y = e^{-2s}\left[s^{-2} - (s+1)^{-1}\right]$

using $\mathcal{L}^{-1}\{(s+1)^{-1}\} = \sin(t)$

$$Y = e^{-2s}\mathcal{L}\{t - \sin(t)\}$$

$$y(t) = \mathcal{L}^{-1}\{e^{-2s}\mathcal{L}\{t\}\} - \mathcal{L}^{-1}\{e^{-2s}\mathcal{L}\{\sin(t)\}\}$$

$$y(t) = h(t-2)\left[(t-2) - \sin(t-2)\right]$$

2.) Let m & n be positive integers.
Use Convolution Thm. to evaluate integral.

$$\int_0^t (t-\tau)^m \tau^n \, d\tau \qquad \text{for } m, n \geq 0$$

$$= \mathcal{L}^{-1}\left\{ \mathcal{L}\{t^m\} \mathcal{L}\{t^n\} \right\} \qquad \text{By Conv. Thm.}$$

$$= \mathcal{L}^{-1}\left\{ \frac{n!}{s^{n+1}} \cdot \frac{m!}{s^{m+1}} \right\} \qquad \text{By table } t^n$$

$$= n!m! \, \mathcal{L}^{-1}\left\{ \frac{1}{s^{(n+m+1)+1}} \right\}$$

$$= n!m! \frac{t^{(n+m+1)}}{(n+m+1)!} \qquad \text{By table } \frac{1}{s^n}$$