

Quiz 2 - To be discussed during lecture on Friday July 10.

Work on these problems **before** the Friday lecture, ready to discuss your approach and solution with other students. I will break the class into small groups and allow you to correct your answers if you see fit. You will need to upload your answers by 5:00 PM Friday July 10.

Problem 1: Newton's method has been used to find a root α of a given function $f(x)$. It is known that $1 \leq \alpha \leq 2$. Assuming that the error satisfies

$$|x_{n+1} - \alpha| \leq 5 \times |x_n - \alpha|^2,$$

how close should x_0 be taken to α , so that after 5 iterates, the error is less than 10^{-4} ?

Problem 3: The root α of the equation $f(x) = 0$ has been approximated using the bisection, Newton's and secant method. Also a fixed point method of the form $x = x - f(x)$ has been used. The output of n versus x_n for each of the methods is the following.

n	Method 1	Method 2	Method 3	Method 4
1	8.0000000e-01	5.0000000e-01	8.0000000e-01	5.0000000e-01
2	8.3264391e-01	8.0831796e-01	8.5000000e-01	8.0061425e-01
3	8.4293081e-01	8.4722487e-01	8.2500000e-01	8.4124474e-01
4	8.4633468e-01	8.4806168e-01	8.3750000e-01	8.4788696e-01
5	8.4747837e-01	8.4806208e-01	8.4375000e-01	8.4806141e-01
6	8.4786458e-01	8.4806208e-01	8.4687500e-01	8.4806208e-01
7	8.4799523e-01	8.4806208e-01	8.4843750e-01	8.4806208e-01

An estimation of the error is given by $|x_{n+1} - x_n|$ with the following outcome.

n	Method 1	Method 2	Method 3	Method 4
1	3.2643909e-02	3.0831796e-01	5.0000000e-02	3.0061425e-01
2	1.0286900e-02	3.8906912e-02	2.5000000e-02	4.0630483e-02
3	3.4038748e-03	8.3681436e-04	1.2500000e-02	6.6422282e-03
4	1.1436828e-03	3.9681341e-07	6.2500000e-03	1.7444122e-04
5	3.8621710e-04	8.9261931e-14	3.1250000e-03	6.7270528e-07
6	1.3064538e-04	0.0000000e+00	1.5625000e-03	6.6783135e-11

Part I: Identify the output corresponding to each of the four methods used. Justify your answer.

Part II: For each of the methods, how many more iterates would be needed so that the error in the approximation is smaller than 10^{-15} . You need to justify your answer.

Part III: In a common set of coordinate axis, sketch the graph of n versus $-\log_{10}(|x_n - x_{n-1}|)$. Explain your reasoning, in particular how did you determined the slope of some of the graphs.
