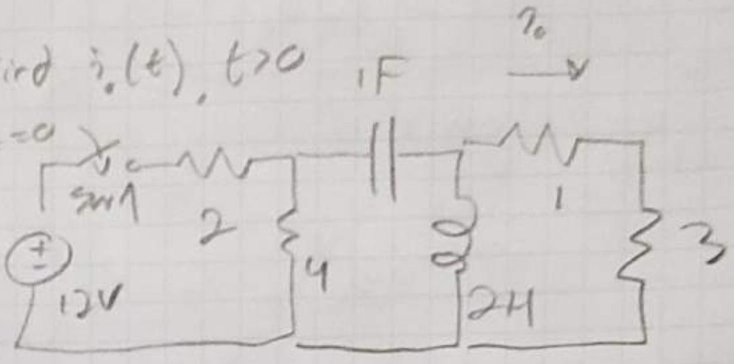
Bennet Sloan

5/15/19

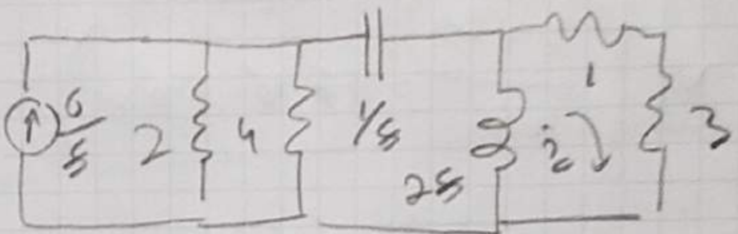
6.1 sw1 closes at $t=0$, Find $i_o(t)$, $t>0$



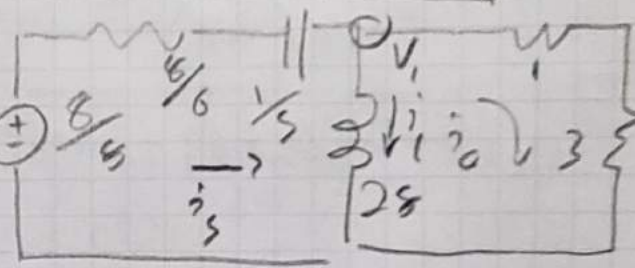
$C = \frac{1}{s}$

$L = 2s$

S.T. $\rightarrow \frac{12}{s} \cdot \frac{1}{2} = \frac{6}{s}$



S.T. $\rightarrow \frac{6}{s} \cdot 2 \| 4 = \frac{8}{s}$



N.V. #1) $i_s = i_1 + i_o$

$$\frac{\frac{8}{s} - V_1}{\frac{8}{6} + \frac{1}{s}} = \frac{V_1}{2s} + \frac{V_1}{4}$$

$$\frac{6s}{6s}\left[\frac{\frac{8}{s} - V_1}{\frac{8}{6} + \frac{1}{s}}\right] \rightarrow \frac{48 - 6s(V_1)}{8s + 6}$$

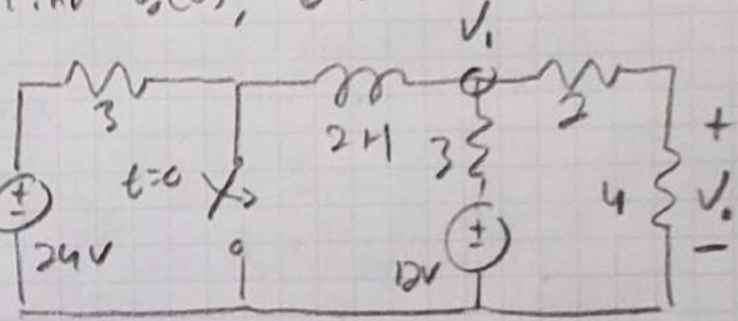
$$\frac{V_1}{2s}\frac{2}{2} + \frac{V_1}{4}\frac{s}{s} \rightarrow \frac{2V_1 + sV_1}{4s}$$

So, $i_o(s) = \dfrac{0.907\angle{-34.71^\circ}}{s+0.344+j0.507} + \dfrac{0.907\angle{34.71^\circ}}{s+0.344-j0.507}$

$i_o(t) = 0.907e^{-j34.71} \cdot e^{-P_1(t)} + 0.907e^{j34.21} \cdot e^{-P_1^*(t)}$

Therefore $\boxed{i_o(t) = 0.907e^{-0.34(t)} \cos(0.507(t) + 34.21^\circ)}$

<u>6.2</u> SW1 closes at t=0. Find $V_o(t)$, $t>0$

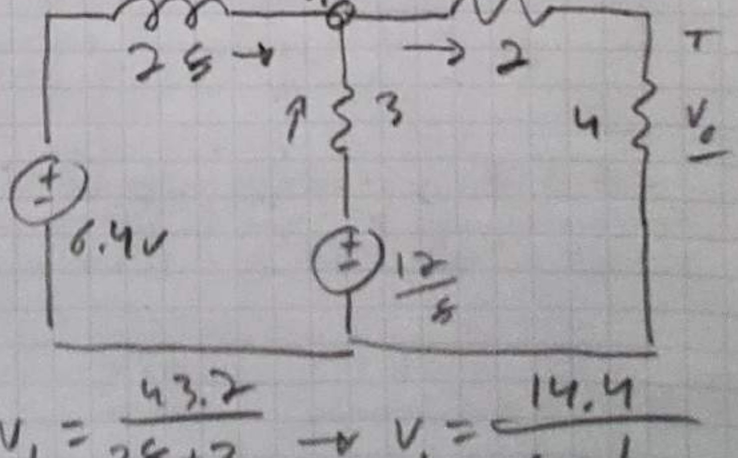


N.V #1) $6\left(\dfrac{24-V_1}{3} + \dfrac{12-V_1}{3} = \dfrac{V_1}{6}\right)$

$48 - 2V_1 + 24 - 2V_1 = V_1$

$72 = 5V_1$

$V_1 = 14.4 \rightarrow i(0^-) = \dfrac{24-14.4}{3} = 3.2A$



For $t>0$

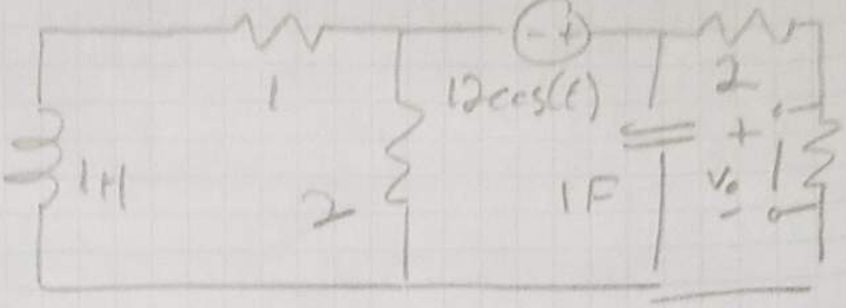
$6s\left(\dfrac{6.4-V_1}{2s} + \dfrac{\frac{12}{s}-V_1}{3} = \dfrac{V_1}{6}\right)$

$= 19.2 - 3V_1 + 24 - 2sV_1 = sV_1$

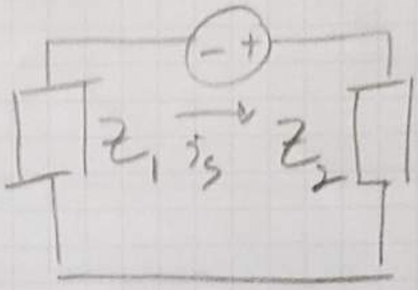
$\rightarrow 43.2 = V_1(3s+3) \rightarrow V_1 = \dfrac{43.2}{3s+3} \rightarrow V_1 = \dfrac{14.4}{s+1}$

But $V_o = \frac{4}{6}V_1 = 9.6\frac{1}{s+1}$, so $\boxed{V_o(t) = 9.6e^{-t}\,v}$

6.3 Find $V_o(t)$



$$Z_1 = \frac{2(s+1)}{s+3}$$



$$Z_2 = \frac{s^{-1}\cdot 3}{3+s^{-1}}\cdot\frac{s}{s} = \frac{3}{3s+1}$$

$$Z_1+Z_2 = \frac{(2s+2)(3s+1)+3(s+3)}{(s+3)(3s+1)} = \frac{6s^2+8s+2+3s+9}{(s+3)(3s+1)}$$

$$= \frac{6s^2+11s+11}{(s+3)(3s+1)}$$

But, $i_s = \frac{V}{Z_1+Z_2}$

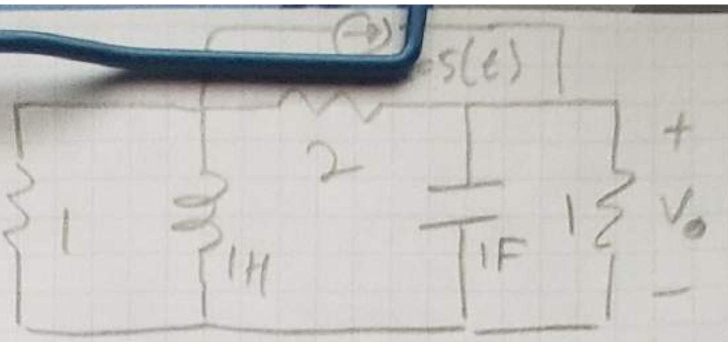
so, $$i_s = \frac{12\frac{s}{s^2+1}}{\frac{6s^2+11s+11}{(s+3)(3s+1)}} = \frac{12s(s+3)(3s+1)}{6s^2+11s+11(s^2+1)}$$

Take $i_o = i_s\frac{s^{-1}}{s^{-1}+3} = \frac{1}{3s+1}i_s$ where $V_o = i_s$

Then $$V_o = \frac{\cancel{(3s+1)}(12s)(s+3)}{\cancel{(3s+1)}(s^2+1)(6s^2+11s+11)} = \frac{12s}{s^2+1}\cdot\frac{s+3}{6s^2+11s+11}$$

H(s) is approx. $\frac{j\omega+3}{6(j\omega)^2+11(j\omega)+11} \xrightarrow{\omega=1} \frac{3+j}{5+j11} \rightarrow 0.26\angle -47.1^\circ$

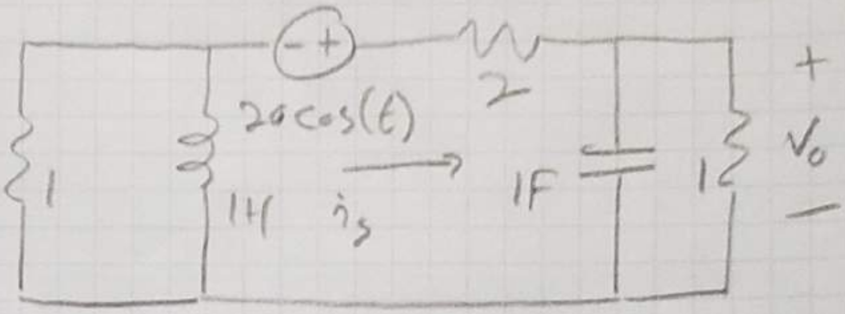
Since $\frac{12s}{s^2+1}$ is source $\rightarrow$ $\boxed{V_o = 12(0.26\cos(t-47.1^\circ))}$



6.4 Find $V_o(t)$

s.t. → $10\cos(t)\cdot 2$

$= 20\cos(t)$ V



Need $$\frac{i_s}{V_s} = \frac{1}{(Z_C\|1)+(Z_L\|1)+2} = \frac{1}{\frac{s^{-1}}{s^{-1}+1}+\frac{s}{s+1}+2}$$

$$= \frac{1}{\frac{1}{s+1}+\frac{s}{s+1}+\frac{(s+1)2}{s+1}} = \frac{1}{3}$$

But $\frac{i_o}{i_s} = \frac{s^{-1}}{s^{-1}+1} = \frac{1}{s+1}$, so $\frac{i_o}{V_s} = \frac{1}{3}\left(\frac{1}{s+1}\right)$

$= \frac{1/3}{s+1}$, And $\frac{V_o}{i_o} = 1$

Therefore $\frac{V_o}{V_s} = \frac{1}{3}\cdot\frac{1}{s+1}$. Now let $s = j\omega$ where $\omega = 1$.

Then $\frac{V_o}{V_s} = \frac{1}{3}\frac{1}{1+j} = \frac{1}{3+3j} = 0.236\angle{-45^\circ}$

Lastly $V_o = 0.236\cdot 20\cos(t-45) \to \boxed{V_o = 4.72\cos(t-45^\circ)}$