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Mth 342
Lab L

1.) suppose $A_{n \times n} = SDS^{-1}$

Since A is diagonalizable there exists a basis for $\mathbb{R}^n$ consisting of eigenvalues of A.

Consider: $A = SDS^{-1}$

$$AS = SD$$

Let $S = [\vec{v}_1, \dots, \vec{v}_n]$

And $D = \begin{bmatrix} \lambda_1 & & 0 \\ & \lambda_2 & \\ 0 & & \ddots \; \lambda_n \end{bmatrix}$

Then $SD = [\lambda_1 \vec{v}_1 \dots \lambda_n \vec{v}_n] = AS$

Similarily, $AS = A[\vec{v}_1 \dots \vec{v}_n] = [A\vec{v}_1 \dots A\vec{v}_n]$

Therefore $A\vec{v}_i = \lambda_i \vec{v}_i$ for $i = 1, \dots, n$

Meaning $\vec{v}_i$ is an eigenvector corresponding to λ_i.

2.) Let $P = P^2$, $\lambda \in \sigma(P)$

Let e_λ be an eigenvector of eigenvalue λ.

Then $Pe_\lambda = \lambda e_\lambda$ ☆

Since $P = P^2$,

Then
$$(P^2 - P)e_\lambda = 0$$
$$P^2 e_\lambda - Pe_\lambda = 0$$
$$P^2 e_\lambda = Pe_\lambda$$
$$\underline{P^2 e_\lambda = \lambda e_\lambda} \quad ☆$$

Also,
$$\underline{P^2 e_\lambda} = P(Pe_\lambda)$$
$$= P(\lambda e_\lambda) \quad ☆$$
$$= \lambda(Pe_\lambda)$$
$$= \underline{\lambda^2 e_\lambda} \quad ☆$$

So if
$$\lambda e_\lambda = \lambda^2 e_\lambda$$
$$\lambda = \lambda^2$$
$$\lambda(\lambda - 1) = 0 \rightarrow \therefore \lambda = 1, 0$$