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Mth 256 - HW 10

1.) Suppose the charge on a capacitor in a simple electric circuit is governed by:

$$2y'' + y' + 2y = \delta(t), \quad y(0) = 0, \quad y'(0) = 0$$

Suppose the forcing function $\delta(t)$ represents a spike (impulse) in the voltage at $t_0 = 5$ seconds.

Find $y(t)$.

Given: Impulse at $t_0 = 5$

So, $2y'' + y' + 2y = \delta(t-5)$

$$\mathcal{L}\{2y'' + y' + 2y\} = \mathcal{L}\{\delta(t-5)\}$$

$$2(Ys^2) + sY + 2Y = e^{-5s}$$

$$Y(2s^2 + s + 2) = e^{-5s}$$

$$Y = \frac{e^{-5s}}{(2s^2 + s + 2)}$$

$$Y = \frac{e^{-5s}}{2(s^2 + \frac{1}{2}s + 1)}$$

$$Y = \frac{e^{-5s}}{2\left[(s + \frac{1}{4})^2 + \frac{15}{16}\right]}$$

$$Y = \frac{e^{-5s}}{1} \cdot \frac{1}{2} \cdot \frac{4}{\sqrt{15}} \cdot \frac{\frac{\sqrt{15}}{4}}{(s+\frac{1}{4})^2 + (\frac{\sqrt{15}}{4})^2}$$

$$\frac{2e^{-5s}}{\sqrt{15}} \cdot \frac{\frac{\sqrt{15}}{4}}{(s+\frac{1}{4})^2 + (\frac{\sqrt{15}}{4})^2}$$

using $\mathcal{L}\{e^{at}\sin(bt)\}$

$$\mathcal{L}\{e^{-\frac{1}{4}t}\sin(\tfrac{\sqrt{15}}{4}t)\} = \frac{\frac{\sqrt{15}}{4}}{(s+\frac{1}{4})^2 + (\frac{\sqrt{15}}{4})^2}$$

using 2nd shifting Thm.

$$y(t) = h(t-5)\frac{2e^{-\frac{1}{4}(t-5)}}{\sqrt{15}} \cdot \sin\left[\frac{\sqrt{15}}{4}(t-5)\right]$$