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LAB T

1.) Let $A_{n\times n} = A^*_{n\times n}$ with eigenvectors $\vec{v}, \vec{w}$.

Suppose $A\vec{v} = \lambda_1 \vec{v}$, $A\vec{w} = \lambda_2 \vec{w}$, $\lambda_1 \neq \lambda_2 \in \mathbb{R}$

Consider, $(A\vec{v}, \vec{w}) = (\lambda_1 \vec{v}, \vec{w}) = \lambda_1 \vec{w}^* \vec{v}$

$= \lambda_1 (\vec{v}, \vec{w})$ $\quad$ Real λ_1

Also, $(A\vec{v}, \vec{w}) = \vec{w}^* A \vec{v} = (A^* \vec{w})^* \vec{v} = (\vec{v}, A^* \vec{w})$ $\quad$ Real A

$= (\vec{v}, A\vec{w})$ $\quad$ $A = A^*$

$= (\vec{v}, \lambda_2 \vec{w})$ $\quad$ $A\vec{w} = \lambda_2 \vec{w}$

$= \lambda_2 (\vec{v}, \vec{w})$ $\quad$ Real λ_2

Therefore $\lambda_1(\vec{v}, \vec{w}) = \lambda_2(\vec{v}, \vec{w})$ $\quad$ Transitive

$\lambda_1(\vec{v}, \vec{w}) - \lambda_2(\vec{v}, w) = 0$

$(\lambda_1 - \lambda_2)(\vec{v}, \vec{w}) = 0$

Since $\lambda_1 \neq \lambda_2$, $(\lambda_1 - \lambda_2) \neq 0$

$\therefore (\vec{v}, \vec{w}) = 0$, $\vec{v}$ & $\vec{w}$ are orthogonal

2.a) $A = \begin{bmatrix} 7 & 2 \\ 2 & 4 \end{bmatrix}$

$$\det(A - \lambda I) = (7-\lambda)(4-\lambda) - 4$$
$$= \lambda^2 - 11\lambda + 28$$
$$= (\lambda - 3)(\lambda - 8) \rightarrow \sigma = \{3, 8\}$$

$$\vec{v}_{\lambda_1} = \ker(A - \lambda_1 I) = \ker \begin{bmatrix} 7-3 & 2 \\ 2 & 4-3 \end{bmatrix} = \ker \begin{bmatrix} 2 & 1 \\ 0 & 0 \end{bmatrix} \rightarrow \begin{bmatrix} -1 \\ 2 \end{bmatrix}$$

$$\vec{v}_{\lambda_2} = \ker(A - \lambda_2 I) = \ker \begin{bmatrix} 7-8 & 2 \\ 2 & 4-8 \end{bmatrix} = \ker \begin{bmatrix} 1 & -2 \\ 0 & 0 \end{bmatrix} \rightarrow \begin{bmatrix} 2 \\ 1 \end{bmatrix}$$

$$\vec{u}_{\lambda_1} = \frac{\vec{v}_{\lambda_1}}{\|\vec{v}_{\lambda_1}\|} = \frac{1}{\sqrt{5}} \begin{bmatrix} -1 \\ 2 \end{bmatrix}$$ (unit eigenvector for $\lambda = 3$)

$$\vec{u}_{\lambda_2} = \frac{\vec{v}_{\lambda_2}}{\|\vec{v}_{\lambda_2}\|} = \frac{1}{\sqrt{5}} \begin{bmatrix} 2 \\ 1 \end{bmatrix}$$ (unit eigenvector for $\lambda = 8$)

$$D = \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix} = \begin{bmatrix} 3 & 0 \\ 0 & 8 \end{bmatrix}, \quad P = \begin{bmatrix} \vec{u}_{\lambda_1} & \vec{u}_{\lambda_2} \end{bmatrix} = \begin{bmatrix} -1 & 2 \\ 2 & 1 \end{bmatrix} \frac{1}{\sqrt{5}}$$

$$A = \begin{bmatrix} -1 & 2 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} 3 & 0 \\ 0 & 8 \end{bmatrix} \begin{bmatrix} -1 & 2 \\ 2 & 1 \end{bmatrix} \frac{1}{5}$$

2.b) $A = \frac{3}{5} \begin{bmatrix} -1 \\ 2 \end{bmatrix} \begin{bmatrix} -1 & 2 \end{bmatrix} + \frac{8}{5} \begin{bmatrix} 2 \\ 1 \end{bmatrix} \begin{bmatrix} 2 & 1 \end{bmatrix}$