MTH 342 OSU Winter 2019
Monday, Mar. 4, Lab P, done in class.

1. Let V be an inner product space with an orthonormal basis $\mathcal{B} = \{\mathbf{v}_1, \mathbf{v}_2, \dots, \mathbf{v}_n\}$.

(a) Prove:

$$\text{For any } \mathbf{x} = \sum_{i=1}^{n} \alpha_i \mathbf{v_i} \text{ and } \mathbf{y} = \sum_{i=1}^{n} \beta_i \mathbf{v_i}, \ \ (\mathbf{x}, \mathbf{y}) = \sum_{i=1}^{n} \alpha_i \overline{\beta_i}.$$

(b) Deduce from this *Parseval's identity:*

$$\text{For all } \mathbf{x}, \mathbf{y} \in V, \ (\mathbf{x}, \mathbf{y}) = \sum_{i=1}^{n} (\mathbf{x}, \mathbf{v_i})\overline{(\mathbf{y}, \mathbf{v_i})}.$$

2. Find all vectors in $\mathbb{R}^4$ that are orthogonal to $(1, 1, 1, 1)^T$ and $(1, 2, 3, 4)^T$.

Bennet Sloan

Mth 342 3/7/19

Lab P

1.a.) From conjugate linearity, it follows:

$$(x, y) = \left(\sum_{i=1}^{n} \alpha_i v_i , \sum_{i=1}^{n} \beta_i v_i\right)$$
$$= \sum_{i=1}^{n} \sum_{j=1}^{n} (\alpha_i v_i , \beta_j v_j)$$
$$= \sum_{i=1}^{n} \sum_{j=1}^{n} \alpha_i \bar{\beta}_j (v_i , v_j)$$
$$= \sum_{i=1}^{n} \alpha_i \bar{\beta}_i (v_i , v_i) \quad \text{Orthogonal}$$
$$= \sum_{i=1}^{n} \alpha_i \bar{\beta}_i \quad \text{Orthonormal}$$

Since orthogonality implies $(v_i , v_j) = 0$ for $i \neq j$

And orthonormality implies $(v_i , v_i) = 1$.

1.b.) From linearity and orthonormality it follows:

$$\text{If } (x, v_i) = \left(\sum_{j=1}^{n} \alpha_j v_j , v_i\right)$$
$$= \sum_{j=1}^{n} (\alpha_j v_j , v_i)$$
$$= (\alpha_i v_i , v_i) \quad \text{Orthogonal}$$
$$= \alpha_i (v_i , v_i)$$
$$= \alpha_i \quad \text{Orthonormal}$$

Then similarly $(y, v_i) = \beta_i \rightarrow \bar{\beta}_i = \overline{(y, v_i)}$

Therefore $(x, y) = \sum_{i=1}^{n} \alpha_i \bar{\beta}_i = \sum_{i=1}^{n} (x, v_i)\overline{(y, v_i)}$

2.) Let $x \in \mathbb{R}^4$ be orthogonal to both $v_1 = [1,1,1,1]^T$ & $v_2 = [1,2,3,4]^T$.

Then, $(x, v_1) = (x, v_2) = 0$

So $x_1 + x_2 + x_3 + x_4 = 0$

And $x_1 + 2x_2 + 3x_3 + 4x_4 = 0$

Equivalently $\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 2 & 3 & 4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = 0$

$$\begin{bmatrix} 1 & 0 & -1 & -2 \\ 0 & 1 & 2 & 3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = 0$$

So, $x = x_3 \begin{bmatrix} 1 \\ -2 \\ 1 \\ 0 \end{bmatrix} + x_4 \begin{bmatrix} 2 \\ -3 \\ 0 \\ 1 \end{bmatrix}$

Therefore any vector $a \begin{bmatrix} 1 \\ -2 \\ 1 \\ 0 \end{bmatrix} + b \begin{bmatrix} 2 \\ -3 \\ 0 \\ 1 \end{bmatrix}$, $a, b \in \mathbb{R}$ is orthogonal to v_1 & v_2.