MTH 342 OSU Winter 2019

Friday, Feb. 22, Lab M, done in class.

Complete this and submit it to Canvas by the posted due date.

Here is a way to phrase linear independence of subspaces:
Let V be a vector space. A system of subspaces V_1, V_2, $\dots V_p$ is linearly independent if and only if for any $\vec{v}_1, \vec{v}_2, \dots, \vec{v}_p$ with $\vec{v}_k \in V_k$ (one from each) the vectors $\vec{v}_1, \vec{v}_2, \dots, \vec{v}_p$ are linearly independent.

Theorem 2.2 (p. 108, LADW). Let λ_1, λ_2, $\dots \lambda_r$ be distinct eigenvalues of a matrix A, and let $\vec{v}_1$, $\vec{v}_2$, $\dots \vec{v}_r$ be the corresponding eigenvectors. Then $\vec{v}_1$, $\vec{v}_2$, $\dots \vec{v}_r$ are linearly independent.

1. Explain why this theorem implies the following statement:

Let A be a matrix. Then the system of eigenspaces

$$E_k = \ker(A - \lambda_k I), \quad \lambda_k \in \sigma(A)$$

is linearly independent. Here $\sigma(A)$ denotes the spectrum of A; it is the set of eigenvalues of A.

2. Let A be the following matrix:

$$A = \begin{pmatrix} 4 & -2 \\ 1 & 1 \end{pmatrix}.$$

Find all square roots of A, in other words, find all matrices B such that $B^2 = A$. Start by diagonalizing A. The square-roots of a diagonal matrix are easy to find.

Show how to compute one of the square roots in great detail, and then just indicate how to compute the other square roots. State how many square roots there are.