1.) Find a line $y = mx + b$ to best fit the data:

$$\{(1,2),(3,5),(4,5),(6,8),(6,9),(7,10)\}$$

a.) This data corresponds to $y = f(x) = mx + b$.

① $y(1) = m + b = 2$

② $y(3) = 3m + b = 5$

③ $y(4) = 4m + b = 5$

④ $y(6) = 6m + b = 8$

⑤ $y(6) = 6m + b = 9$

⑥ $y(7) = 7m + b = 10$

b.) This system can be written in matrix form:

$$A \vec{x} = \vec{b}$$

$$\begin{bmatrix} 1 & 1 \\ 3 & 1 \\ 4 & 1 \\ 6 & 1 \\ 6 & 1 \\ 7 & 1 \end{bmatrix} \begin{bmatrix} m \\ b \end{bmatrix} = \begin{bmatrix} 2 \\ 5 \\ 5 \\ 8 \\ 9 \\ 10 \end{bmatrix}$$

c.) The least squares solution will be

$$A\vec{x}^* = \text{proj}_{\text{Ran}(A)} \vec{b}, \quad x^* \text{ approx. } \vec{x}$$

or $A^* A \vec{x}^* = A^* \vec{b}$ which is simpler to compute.

so, $A^* A \vec{x}^* = A^* \vec{b}$ is equivalently:

$$\begin{bmatrix} 1 & 3 & 4 & 6 & 6 & 7 \\ -1 & -1 & -1 & -1 & -1 & -1 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 3 & 1 \\ 4 & 1 \\ 6 & 1 \\ 6 & 1 \\ 7 & 1 \end{bmatrix} \begin{bmatrix} m \\ b \end{bmatrix} = \begin{bmatrix} 1 & 3 & 4 & 6 & 6 & 7 \\ -1 & -1 & -1 & -1 & -1 & -1 \end{bmatrix} \begin{bmatrix} 2 \\ 5 \\ 5 \\ 8 \\ 9 \\ 10 \end{bmatrix}$$

$$\begin{bmatrix} 147 & 27 \\ -27 & -6 \end{bmatrix} \begin{bmatrix} m \\ b \end{bmatrix} = \begin{bmatrix} 209 \\ -39 \end{bmatrix}$$

$$\left[\begin{array}{cc|c} 147 & 27 & 209 \\ -27 & -6 & -39 \end{array}\right] \xrightarrow{\text{RREF}} \left[\begin{array}{cc|c} 1 & 0 & 1.3137 \\ 0 & 1 & 0.5882 \end{array}\right]$$

$$\therefore \vec{x}^* = \begin{bmatrix} m \\ b \end{bmatrix}^* = \begin{bmatrix} 1.3137 \\ 0.5882 \end{bmatrix}$$