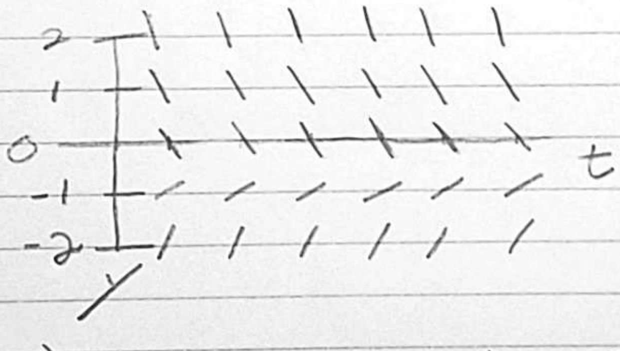
Bennet C. Sloan 9/27/2018
Mth 256 - HW 1

1.) Determine the value(s) of r such that $y(t) = e^{rt}$ is a solution to $y'' + y' - 6y = 0$

Given $y = e^{rt}$
so, $y' = re^{rt}$
And $y'' = r^2 e^{rt}$

Consider
$$y'' + y' - 6y = 0$$
$$r^2 e^{rt} + re^{rt} - 6e^{rt} = 0$$
$$e^{rt}(r^2 + r - 6) = 0$$
$$(r-2)(r+3) = 0$$
Thus $\boxed{r = 2, -3}$

2a.) Sketch a direction field for $\frac{dy}{dt} = -1 - 2y$



y	dy/dt
2	-5
1	-3
0	-1
-1	1
-2	3

2b.) The equation $y' = -1 - 2y$ is an autonomous ODE, invariant under horizontal translation. So, the end behavior is determined by horizontal asymptotes where $\frac{dy}{dt} = 0$.
Graphically, there is one asymptote which is stable.
So, $0 = -1 - 2y$
$y = -\frac{1}{2}$ (H.A. at $y = -\frac{1}{2}$)
Therefore, $\boxed{\lim_{t \to \infty} y(t) = -\frac{1}{2}}$