Turn this in by making a pdf scan of your work and submitting it to Canvas by 11:59 pm on the due date.

How to write up homework. Please put your name and assignment number on the first page. You may use this page as a cover sheet. Show your work and explain it. In general, just writing down an answer without showing the steps to arriving at the answer is not sufficient for credit.

Current reading is Chapter 2 of Linear Algebra Done Wrong (LADW).

1. This is a proof writing exercise. Prove the following statements about matrices A and B. Proofs need words that connect mathematical statements together. A proof should explain how to get from the hypothesis to the conclusion.

   (a) If $AB = \mathbf{0}$, then either $B = \mathbf{0}$ or A is not invertible.

   (b) If A is symmetric and invertible, then A^{-1} is symmetric.

   For the second statement you can use without proof $(AB)^T = B^T A^T$.

2. Recall that $C[0,1]$ denotes the vector space of all continuous real valued functions defined on $[0,1]$ (with addition and scalar multiplication defined appropriately). For each $a \in [0,1]$ the evaluation map

$$E_a : C[0,1] \to \mathbb{R}, \qquad E_a(f) = f(a),$$

   is a linear transformation. Describe $\ker E_a$, the kernel of E_a.

3. Let V and W be vector spaces and let $T : V \to W$ be a linear transformation. Let $\mathbf{v}_1, \mathbf{v}_2, \ldots, \mathbf{v}_k \in V$.

   (a) Show that if the vectors $T(\mathbf{v}_1), T(\mathbf{v}_2), \ldots, T(\mathbf{v}_k)$ are linearly independent then the vectors $\mathbf{v}_1, \mathbf{v}_2, \ldots, \mathbf{v}_k$ are linearly independent.

   (b) Show by counterexample that the converse of (a) is false.

   (c) A function f is called *injective* if $f(x) = f(y)$ implies $x = y$. Show the converse of (a) holds if we assume that T is injective.

4. Let U and V be subspaces of a vector space W.

   (a) The *subspace sum* of U and V is

   $$U + V = \{\mathbf{x} + \mathbf{y} \mid \mathbf{x} \in U, \mathbf{y} \in V\}.$$

   Show that $U + V$ is also a subspace.

   (b) Let $\mathbf{w} \in W$, but $\mathbf{w} \notin V$. Show that for all $\mathbf{v} \in V$, $\mathbf{v} + \mathbf{w} \notin V$.

   (c) Show that if $U \not\subseteq V$ and $V \not\subseteq U$ then $U \cup V$ is not a subspace.