ECE 351 HW3 Bennet Sloan

$f(x) = x^6 - 3$, $x_0 = 2$, 7 iterates

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

$$= x_n - \frac{x_n^6 - 3}{6x_n^5}$$

$$= \frac{(x_n)(6x_n^5) - (x_n^6 - 3)}{6x_n^5}$$

$$= \frac{5x_n^6 + 3}{6x_n^5}$$

$$\lim_{n\to\infty} \frac{|\alpha - x_{n+1}|}{|\alpha - x_n|^p} = \lambda \neq 0$$

Converges with order p, $p \geq 1$.

$$= \left(\frac{5}{6}\right)\left(x_n + \frac{3/5}{x_n^5}\right)$$

→ By using diff(x) in MATLAB, an array of values shows $(n+1) - (n)$ decreasing at a quadratic rate, converging faster as $x \to \alpha$.

P_1.m | P_2.m | P_3.m

```matlab
clear all;
clc;

x(1)=2;                                         %Given x_0 = 2

for k=1:7                                       %For 7 iterates
    x(k+1)=(5/6)*(x(k)+((3/5)/(x(k)^5)));       %Calculated via Newtons Method
end

format longe
x'


xr(1)=2;
diffx=diff(x');

for k=1:7                                       %For 7 iterates
    xr(k+1)=(5/6)*(diffx(k)+((3/5)/(diffx(k)^5)));    %Calculated via Newtons Method
end

format longe
xr'

```

Name	Value
ans	[2;-154.7286;-587.001...
diffx	[-0.3177;-0.2433;-0.15...
k	7
x	[2,1.6823,1.4390,1.280...
xr	[2,-154.7286,-587.001...

Command Window

```
ans =

     2.000000000000000e+00
     1.682291666666667e+00
     1.439017379444474e+00
     1.280210024488632e+00
     1.212241513001959e+00
     1.201197248087276e+00
     1.200937096145414e+00
     1.200936955176044e+00


ans =

     2.000000000000000e+00
    -1.547286487014126e+02
    -5.870011818839581e+02
    -4.950266193585587e+03
    -3.446920610958366e+05
    -3.042887720150964e+09
    -4.195991932388753e+17
    -8.981444987844134e+33

>>
```

2.) $\therefore\ f(x) = x^6 - 3$, $x_0 = 1$, $x_1 = 2$, 10 iterates

$$x_{n+1} = x_n - (x^6 - 3) \cdot \left[\frac{x_n - x_{n-1}}{(x_n^6 - 3) - (x_{n-1}^6 - 3)} \right], \quad n \geq 1$$

3.) $\therefore\ f(x) = x^6 - 3$, $x_{n+1} = p x_n + (1-p)\frac{3}{x_n^5}$

From 1) $\rightarrow \left(\frac{5}{6}\right)\left(x_n + \frac{3/5}{x_n^5}\right) = p x_n + (1-P)\frac{3}{x_n^5}$

For $P = 5/6$ $\rightarrow$

$$= \left(\frac{5}{6}\right) x_n + \left(1 - \frac{5}{6}\right)\frac{3}{x_n^5}$$

$$= \frac{5}{6} x_n + \frac{3}{x_n^5} - \frac{3(5)}{6 x_n^5}$$

$$= \frac{(5x_n)(x_n^5) - 3(5)}{6 x_n^5} + \frac{3}{x_n^5}$$

$$= \frac{5x_n^6 - 15 + 18}{6 x_n^5}$$

$$= \frac{5}{6}\left(x_n + \frac{3/5}{x_n^5}\right)$$

P_1.m | P_2.m | P_3.m

```matlab
clear all;
clc;


x(1)=1;                                        %Given x_0 = 1
x(2)=2;                                        %Given x_1 = 2

for k=3:10                                     %For 10 iterates
    x(k)=x(k-1) - ((x(k-1)^6)-3)*((x(k-1)-x(k-2))/(((x(k-1)^6)-3)-((x(k-2)^6)-3)));
end

format longe
x'

xr(1)=1;                                        %Given x_0 = 1
xr(2)=2;                                        %Given x_1 = 2
diffx=diff(x');
for k=3:10                                     %For 10 iterates
xr(k)=diffx(k-1)-((diffx(k-1)^6)-3)*((diffx(k-1)-diffx(k-2))/...
    (((diffx(k-1)^6)-3)-((diffx(k-2)^6)-3)));
end
xr'
```

Name	Value
ans	[1;2;23.3686;-3.5982;6...
diffx	[1;-0.9683;0.0277;0.21...
k	10
x	[1,2,1.0317,1.0594,1.2...
xr	[1,2,23.3686,-3.5982,6...

Command Window

```
ans =

     1.000000000000000e+00
     2.000000000000000e+00
     1.031746031746032e+00
     1.059404943945980e+00
     1.270846719573732e+00
     1.179236113506492e+00
     1.197930692416822e+00
     1.201076070361364e+00
     1.200936082143545e+00
     1.200936954923209e+00


ans =

     1.000000000000000e+00
     2.000000000000000e+00
     2.336857246993967e+01
    -3.598166519653340e+00
     6.170006306276545e+03
     1.024171305916692e+04
    -5.598529781453075e+05
     1.092811730269000e+09
     1.109698036843875e+13
                       Inf

fx >>
```

Ln 1 Col 1

P_1.m | P_2.m | P_3.m

```matlab
clear all;
clc;

x(1)=2;                                    %Let x_0 = 2
p=1/2;

for k=1:20
    x(k+1)=p*x(k)+(1-p)*(3/(x(k)^5));     %Calculated via Newtons Method
end

format longe
x'
diff(x)'

%{
Part 1.)
    Yes it converges to alpha. With 3 iterates per decade convergence
    appears to be constant/linear.
Part 2.)
    In the case of P=1/2, the sequence oscillates due to the
    derivative of iteration at alpha being negative. This leads to
    the error (x_n - alpha) changing in sign as n varies.
%}
```

Command Window

```
     2.077880595321859e+00
     1.077665020870714e+00
     1.570814994037035e+00
     9.422508320832296e-01
     2.490694021353438e+00
     1.260996110745061e+00
     1.100957859680733e+00
     1.477816335256439e+00
     9.517166551621943e-01


ans =

    -9.531250000000000e-01
     6.694984195303078e-01
    -7.574860402931115e-01
     1.370903591874298e+00
    -1.143042716818646e+00
     4.385886270138628e-02
    -8.381472200294260e-02
     1.828570243173104e-01
    -3.039092229106752e-01
     8.081346687414903e-01
    -8.446200384791468e-01
     1.088625768661589e+00
    -1.000215574451146e+00
     4.931499731663216e-01
    -6.285641619538057e-01
     1.548443189270209e+00
    -1.229697910608377e+00
    -1.600382510643283e-01
     3.768584755757063e-01
    -5.260996800942446e-01

>>
```

Name	Value
ans	20x1 double
k	20
p	0.5000
x	1x21 double