Math 351 **Introduction to Numerical Analysis** **Summer 2020**

Midterm - July 17, 2020.

Problem 1: Let

$$I(x) = \int_0^x \frac{1-e^{-t^2}}{t} \mathrm{d}t.$$

Recall that the Taylor series centered at 0 for e^u is

$$e^u = 1 + u + \frac{u^2}{2!} + \frac{u^3}{3!} + \frac{u^4}{4!} + ... + \frac{u^k}{k!} + .. = \sum_{k=0}^{\infty} \frac{u^k}{k!}.$$

Part I: Find the Taylor polynomial of degree 8, $P_8(x)$ that approximates $I(x)$.

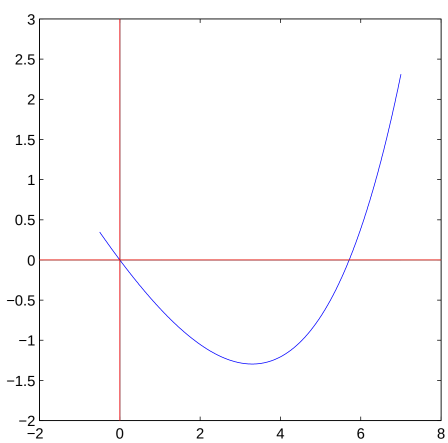
Part II: Bound the error in the approximation of Part I for x such that $-1 \leq x \leq 1$. Justify your answer.

Part III: Determine the degree of the polynomial that should be used so that, for all $-1 \leq x \leq 1$,

$$|I(x) - P_n(x)| < 5 * 10^{-10}.$$

Problem 2: The graph of the function $f(x) = e^{x/3} - 1 - x$ is shown below. From the graph, it is clear that besides $\alpha_1 = 0$ there is another solution α_2 to the equation $f(x) = 0$.

Part I: Determine graphically the value X^* so that if Newton's methods is used with an initial guess $x_0 < X^*$ the iterates converge to the root α_1, but if $x_0 > X^*$, then the iterates converge to α_2.



Part II: Find and solve the equation determining X^*.

Problem 3: Three different fixed point iterations of the form $x_{n+1} = g_j(x_n),\ j = 1, 2, 3$ have been used to find the solution of a nonlinear equation $f(x) = 0$. The graphs below show the line $y = x$ and the graph of each of the functions $y = g_j(x), j = 1, 2, 3$.

Identify which fixed point method will converge the fastest to the root, which one will converge slower and which one will not converge at all. Give a brief justification for your answer.