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Mth 256 - HW6

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1.) Use the method of undetermined coeff. to find the general solution to the DE:

$$\longrightarrow \quad y'' - 3y' + 2y = e^{x} + e^{2x} + e^{-x}$$

For $y_h(x)$: $\quad y'' - 3y' + 2y = 0 \qquad \therefore r = 1, 2$

$$r^2 - 3r + 2 = 0$$

$$(r-1)(r-2) = 0 \qquad y_h(x) = c_1 e^{x} + c_2 e^{2x}$$

For $y_p(x)$: Assume $Ae^{x} + Be^{2x} + Ce^{-x}$ (y_h duplicate)

$$\longrightarrow \quad y_p(x) = Axe^{x} + Bxe^{2x} + Ce^{-x}$$

$$y_p'(x) = A(e^{x} + xe^{x}) + B(e^{2x} + 2xe^{2x}) - Ce^{-x}$$

$$= Ae^{x} + Axe^{x} + Be^{2x} + 2Bxe^{2x} - Ce^{-x}$$

$$y_p''(x) = Ae^{x} + A(e^{x} + xe^{x}) + 2Be^{2x} + 2B(e^{2x} + 2xe^{2x}) + Ce^{-x}$$

$$y_p''(x) = 2Ae^{x} + Axe^{x} + 4Be^{2x} + 4Bxe^{2x} + Ce^{-x}$$

Substitute: $(2Ae^{x} + Axe^{x} + 4Be^{2x} + 4Bxe^{2x} + Ce^{-x})$

$$- 3(Ae^{x} + Axe^{x} + Be^{2x} + 2Bxe^{2x} - Ce^{-x})$$

$$+ 2(Axe^{x} + Bxe^{2x} + Ce^{-x}) = e^{x} + e^{2x} + e^{-x}$$

$$= -Ae^{x} + Be^{2x} + 6Ce^{-x} = e^{x} + e^{2x} + e^{-x}$$

so, $-A = 1$ | $B = 1$ | $6C = 1$

$A = -1$ | | $C = \frac{1}{6}$

1. continued.) $A = -1$, $B = 1$, $C = \frac{1}{6}$

$\rightarrow \quad y_p(x) = -xe^{x} + xe^{2x} + \frac{1}{6}e^{-x}$

Now, $y(x) = y_h(x) + y_p(x)$

$$y(x) = c_1 e^{x} + c_2 e^{2x} - xe^{x} + xe^{2x} + 1/6\, e^{-x}$$

2.) Given that $y = x$ & $y = e^x$ are solutions to the DE:

$$(1-x)y'' + xy' - y = 0$$

on $(1, \infty)$, find the general solution to the DE:

$$(1-x)y'' + xy' - y = (x-1)^2 e^{-x}$$

$\therefore$ $y(x) = y_h(x) + y_p(x)$

IF y_1 & y_2 are Linearly Independent, then $y_h = c_1 y_1 + c_2 y_2$

LI check: $W(y_1, y_2) = \begin{vmatrix} x & e^x \\ 1 & e^x \end{vmatrix} = xe^x - e^x = e^x(x-1) \neq 0$

Since $W \neq 0$, y_1 & y_2 are L.I.

Moreover, $y_h(x) = c_1 x + c_2 e^x$

For $y_p(x)$: Variation of parameters, $f(x) = e^{-x}(1-x)$

$y_p(x) = ux + ve^x \rightarrow u' = -\dfrac{e^{-x}(1-x)\cancel{e^x}}{\cancel{e^x}(x-1)} = -e^{-x}$

$v' = \dfrac{e^{-x}(1-x)\, x}{e^x(1-x)} = -xe^{-2x}$

For $\int \underbrace{-x}_{u}\, \underbrace{e^{-2x}\, dx}_{dv} \rightarrow uv - \int v\, du \rightarrow \frac{x}{2}e^{-2x} - \frac{1}{2}\int e^{-2x}\, dx$

let $u = -x$, $du = -dx$ | $v = -\frac{1}{2}e^{-2x}$, let $v' = e^{-2x}\, dx$

$\therefore$ $y_p(x) = \frac{1}{2}xe^{-2x} + \frac{1}{4}e^{-2x}$

$y(x) = c_1 x + c_2 e^x + \frac{1}{2}xe^{-2x} + \frac{1}{4}e^{-2x}$