

Homework # 6 - Wed Aug 12 2020

This will be last homework assignment. Please make sure to turn in your work by 4:00PM of Wednesday August 12 so it can be returned before the final exam on Friday August 14.

Problem 1: Recall that given a vector norm $|X|$ the operator norm of a matrix A is given by $|A| = \max_{|X|=1} |AX|$.

Part I: Show that $|I| = 1$ for any the vector norm used. As always, I is the identity matrix.

Part II: Give at least one example to show that if A is invertible, the condition number of A , $\kappa(A) = |A||A^{-1}|$ satisfies

$$\kappa(A) > 1.$$

Problem 2: Do Problem 4 in page 302 of the text.

Problem 3: Let $a = 10^{-10}$ and consider the system

$$\begin{pmatrix} 1 & 1 \\ a & 0 \\ 0 & a \end{pmatrix} X = \begin{pmatrix} -a \\ 1+a \\ 1-a \end{pmatrix}$$

Part I: Set up the normal equations $A^T A X = A^T B$. Verify by hand that $X = [1, -1]^T$ is the solution of the normal equations, corresponding to the least square solution.

Part II: Compute the condition number in the 1 and ∞ norms of $M = A^T A$. Recall that for a 2×2 matrix M the inverse is easy to find, see for example formula (6.20) in the text.

Extra Credit Part I: Find the condition number in the 2 norm of $M = A^T A$. This requires finding eigenvalues of M .

Extra Credit Part II: This requires using MATLAB Compare the solutions obtained using MATLAB when you solve the NORMAL equations $A^T A X = A^T B$ as opposed to use the `qr` command to compute the least square solution by the QR method.
