Bennet Sloan
Mth 342
Lab 0

1.) $x = (1, 2i, 1+i)^T$

$y = (i, 2-i, 3)^T$

$(x,y) = y^* x = (-i, 2+i, 3)\begin{pmatrix} 1 \\ 2i \\ 1+i \end{pmatrix}$

$= (-i) + (-2+4i) + (3+3i) = \underline{1+6i}$

$(3x, 2iy) = -6i(x,y) = \underline{36-6i}$

2.) $(x,y) = x_1 y_1 - x_2 y_2$

Consider $(x,x) = x_1 x_1 - x_2 x_2$

If $x_2 > x_1$ then $(x,x) < 0$,

so $(x,y) = x_1 y_1 - x_2 y_2$ is <u>not</u> an inner product.

3.) $\|x+y\|^2 + \|x-y\|^2$

$(x+y, x+y)$	$(x-y, x-y)$
$(x+y, x) + (x+y, y)$	$(x-y, x) + (x-y, -y)$
$(x,x) + (y,x) + (x,y) + (y,y)$	$(x,x) - (y,x) - (x,y) + (y,y)$
$(x,x) + (x,y) + (x,y) + (y,y)$	$(x,x) - (x,y) - (x,y) + (y,y)$
$(x,x) + 2(x,y) + (y,y)$	$(x,x) - 2(x,y) + (y,y)$

3 continued.)

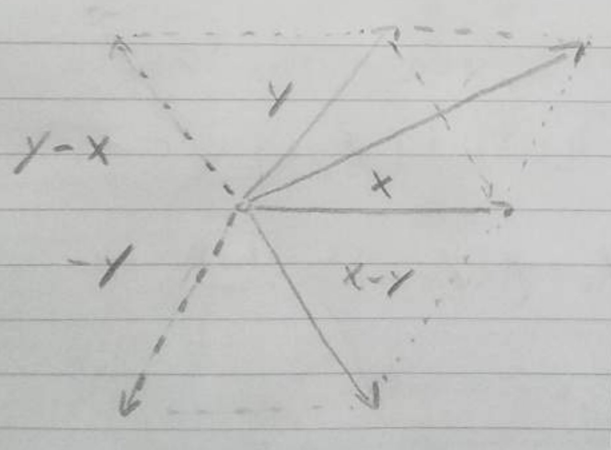
So $\|x+y\|^2 + \|x-y\|^2$

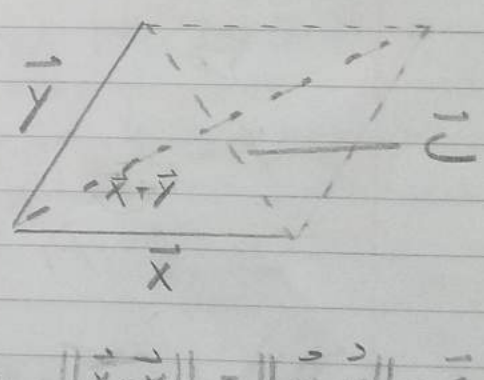
$= [(x,x) + 2(x,y) + (y,y)] + [(x,x) - 2(x,y) + (y,y)]$

$= (x,x) + (y,y) + (x,x) + (y,y)$

$= 2[(x,x) + (y,y)]$

$= 2[\|x\|^2 + \|y\|^2]$ since $(x,x) = \|x\|^2$.

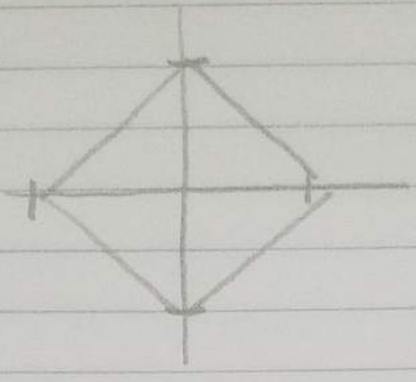




where $\|\vec{x}-\vec{y}\| = \|\vec{y}-\vec{x}\| = \vec{c}$

The sum of the squares of a parallelograms diagonals is equal to the sum of the squares of all 4 sides, in $\mathbb{R}^2$.

4.) For $p = 1$

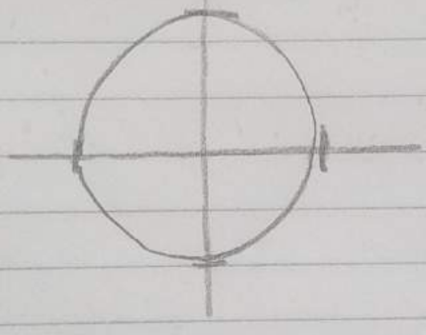


$$\|x\|_p \le 1$$

$$\left[\sum_{i=1}^{n} |x_i|^p\right]^{1/p} \le 1$$

$$\sum_{i=1}^{2} |x_i| \le 1$$

For $p = 2$

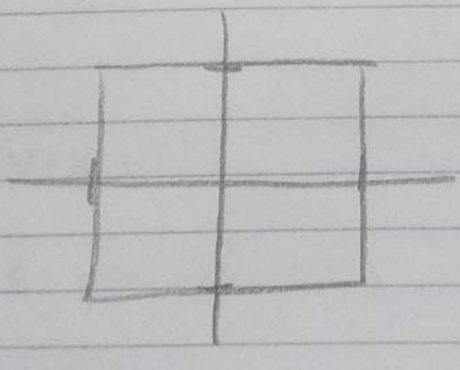


$$\|x\|_p \le 1$$

$$\left[\sum_{i=1}^{n} |x_i|^p\right]^{1/p} \le 1$$

$$\left[\sum_{i=1}^{2} |x_i|^2\right]^{1/2} \le 1$$

For $p = \infty$



$$\|x\|_p \le 1$$

$$\left[\sum_{i=1}^{n} |x_i|^p\right]^{1/p} \le 1$$

$$\max_i |x_i| \le 1$$

For any other p, B_p will be some superellipse since the unit circle must be convex and centrally symmetric.