Homework # 3 - Due Tuesday July 13 2020

Problem 1: Use Newton's method to find $\alpha = 3^{1/6}$ (i.e. α is the sixth root of the number 3.) For this, let $f(x) = x^6 - 3$. Then α is the root of the equation $f(x) = 0$.

Starting with $x_0 = 2$, write a program (Matlab if possible) to compute 7 iterates of Newton's method. Organize your calculations in a manner similar to the files posted in Canvas as part of the lectures of June 29 and July 7. In particular your solution should include the program that you have written, and a table showing the iterates obtained using Newton's method.

Describe how would you use the calculated values to check that the iterates are converging at a 'quadratic rate'.

Problem 2: Use the Secant method to find $\alpha = 3^{1/6}$. As initial guesses, use $x_0 = 1$ and $x_1 = 2$. Compute 10 iterates. As in problem 1, describe how would you check that the iterates are converging at a faster than linear but not quadratic rate.

Problem 3: When Newton's method is used to find the root of $f(x) = x^6 - 3$, the iterates can be written as

$$x_{n+1} = px_n + (1-p)\frac{3}{x_n^5}$$

where $p = 5/6$. [Check this!.]

By using different values of p, $0 < p < 1$, one obtains iterates that, if convergent, they also would give the desired value of α. [Check this!.]

Part I: Do 20 iterates using $p = 3/4$. Do the iterates converge? Is the rate quadratic or linear? How can you tell the difference?

Part II: Do 20 iterates using $p = 1/2$. The iterates should not converge. What explanation can you give for this fact?
