Lab Section 1: First Order Circuits

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ECE 203-01

April 16, 2019

Abstract

In the case of constant resistance loads, capacitive components charge and discharge at a rate which is proportional to the instantaneous charge. Therefore, voltage behaves similarly as q=Cv, where C is fixed. For a circuit with only one energy storing element, the transient voltage response will be an integral curve of some first order ODE. However, considering that the exact solution is asymptotic, it is practical to truncate the solution after enough time has passed to consider the response negligible. As for the response duration itself, an acceptable approximation will be 5 time-coefficients as elaborated further on.

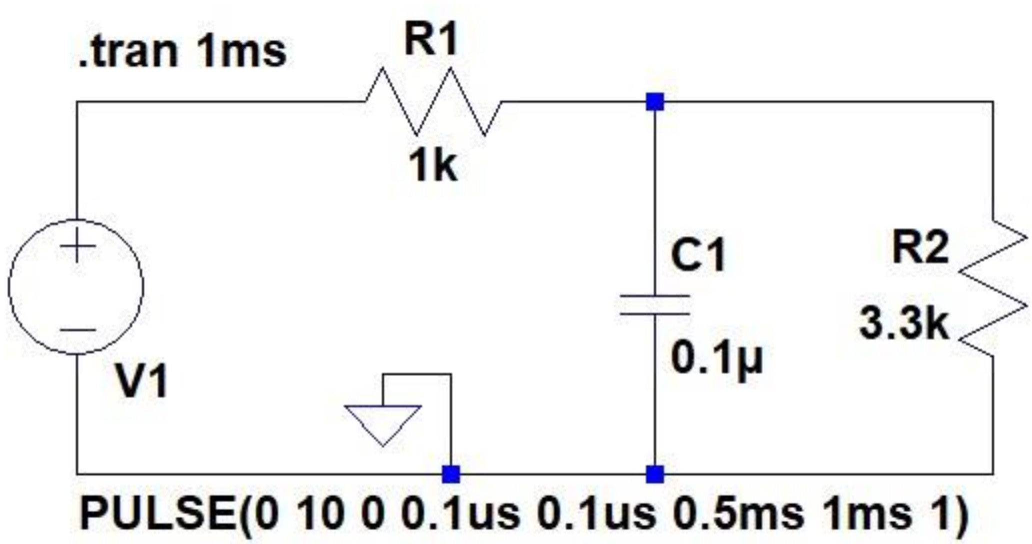
Equipment

- LTspiceXVII
- MATLAB/Excel

Procedure

1. Simulate circuit 1.1
2. Export trace data to Excel and/or MATLAB
3. Isolate the natural response data (moment of pulse low as initial condition)
4. Graph the natural response using semi-log for voltage
5. Compute the time constant from data and compare with theory.

Circuits



Circuit 1.1 - RC circuit with Square wave input

Calculations

Theory

For the theoretical response of the capacitor upon discharge,

Consider: $v_T = v_{ss} + (v_0 - v_{ss})e^{(\tau^{-1})t}$

And: $\tau = [R_{TH}C]^{-1}(s^{-1})$

Where:
$$R_{TH} = R_1||R_2 = 767.4\Omega$$
$$v_0 = 10\left(\frac{R_2}{R_1+R_2}\right) = 7.6744v$$
$$v_{ss} = 0v$$
$$\boldsymbol{\tau = 0.000076744s}$$

So: $v_T = 7.6744e^{(13{,}030.\overline{30})t}$

Simulated

Next, the trace data will be checked by calculating tau from two voltage moments.

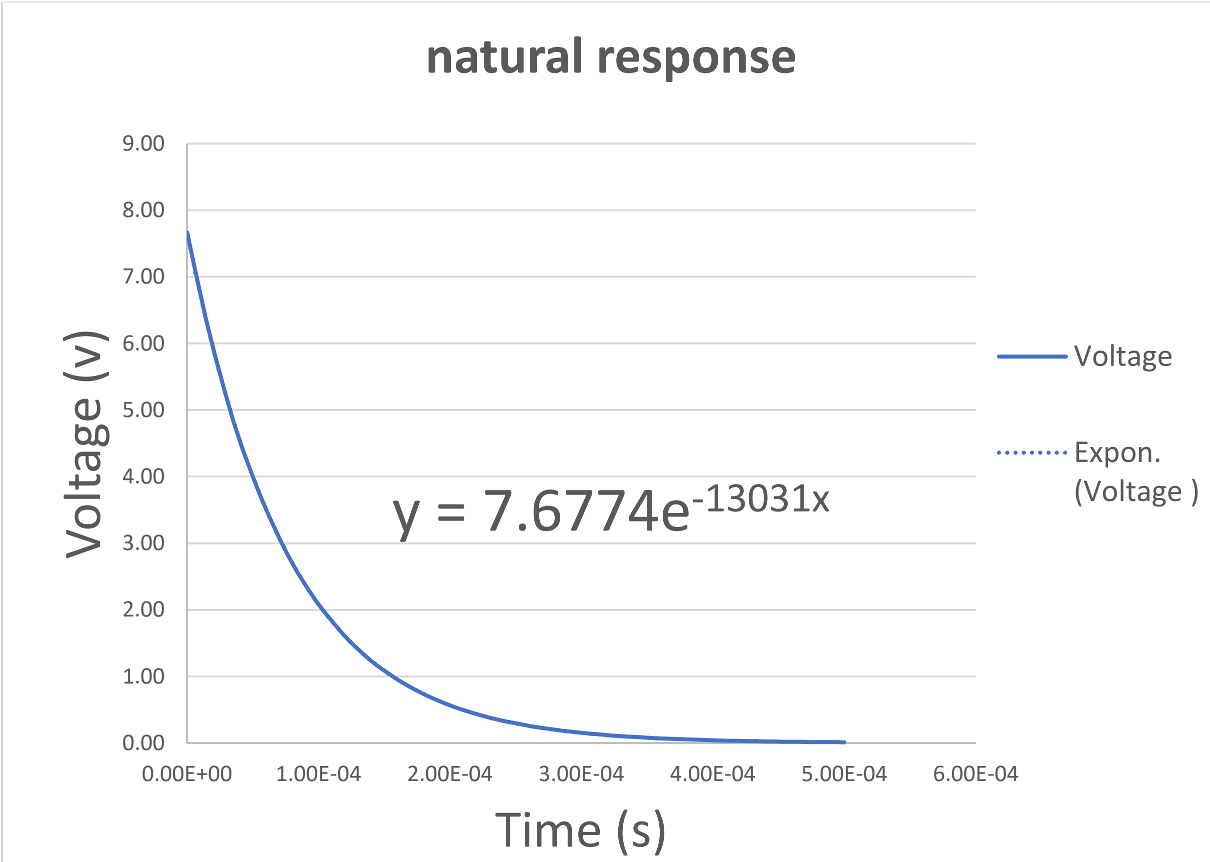
We know: $\frac{\ln[v_0(t_1)-v_0(\infty)]-\ln[(t_2)-v_0(\infty)]}{(t_2-t_1)} = \tau$

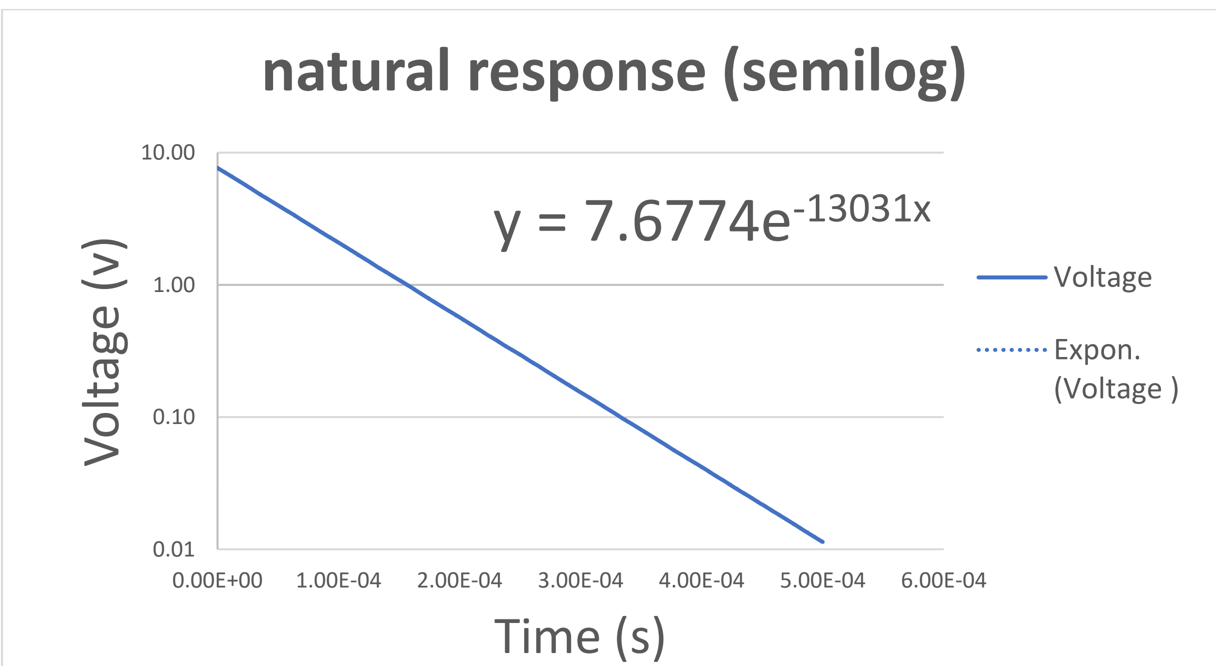
Using:
$$v(0.00016) = 0.9477542$$
$$v(0.00030) = 0.1538742$$
$$v(\infty) = 0$$

Then: $\frac{\ln[0.9477542]-\ln[0.1538742]}{0.00014} = 13{,}031.135 = \tau^{-1}$

Moreover: $\boldsymbol{\tau = 0.000076739s}$

Finally, using the simulated trace data, excel fits the equation shown below to the transient period of the capacitor's discharge. The dataset has been shifted by -5ms.





Conclusions

The time constant was calculated in three ways with the following results:

Theory: $0.000076744s$

Differential: 0.000076739s

Excel: 0.000076739s

From this it can be concluded that the simulation was accurate, as well as the calculation of tau from two voltage moments. The minor deviation may be caused by the fall time of the square wave source, which was 0.1 micro seconds and not instantaneous like theory assumes. The difference could also simply be in the density of the slew data, having 7 significant figures in this case. Regarding response time, the capacitor should be discharged after $5\tau = 0.0003837$ seconds of transience. In general, I would expect these results to be very similar to a real-world analysis, though deviation would likely occur. A possible algorithm of deriving this curve from real oscilloscope data would be the linear least squares method; wherein a matrix equation $A\vec{x} = \vec{b}$ is constructed using transient voltages $\vec{b}$ and augmented time coordinates A. Here, the solution vector $\vec{x}$ is approximated by taking the projection of $\vec{b}$ onto the column space of A which is equivalent to left multiplying the matrix equation by the adjoint of A. That would give a curve of best fit to a sampling of voltage data. In summary, there are many ways of obtaining the time constant for an energy storing device which all work well. It is important to consider the source voltage rise and fall times in real world applications as it can be significant, especially with smaller faster capacitors such as those in timing circuits.