Problem 7. Monte Carlo VaR for Call Option (20pts)
Recall that price of a Black-Scholes Call option can be written as

$$C^{\mathrm{BS}}(t, S, \sigma, ; K, T) = S_t \Phi(d_+) - K e^{-r(T-t)} \Phi(d_-)$$

where

$$d_\pm = \frac{1}{\sigma\sqrt{T-t}} \left[\ln\left(\frac{S_t}{K}\right) + \left(r \pm \frac{\sigma^2}{2}\right)(T-t) \right]$$

Let us generate scenarios for tomorrows option price using multiplicative changes via

$$S^i_{t+\Delta t} = S_t \cdot \Delta S_i, \qquad \sigma^i_{t+\Delta t} \cdot \Delta\sigma_i$$

with $\Delta S, \Delta\sigma$ are sampled from correlated Gamma distributions

$$\Delta S \sim \Gamma(\alpha, \beta), \quad \Delta\sigma \sim \Gamma(\alpha, \beta)$$

and $corr(\Delta S, \Delta\sigma) = \rho$.

- Generate 2500 ΔS_i and $\Delta\sigma_i$ from bivariate-$\Gamma(1, 0.25)$ and evaluate 2500 new option prices under those scenarios, i.e.

$$C^i_{BS} = C^{\mathrm{BS}}_i(t + \Delta t, S^i_{t+\Delta t}, \sigma^i_{t+\Delta t}; K, T)$$

to calculate the 99% VaR and ES with the following parameters.

- $t = 0$, $T = 1$, $S_0 = K = 100$, $r = 0.025$, $\sigma_0 = 0.25$, $\rho = 0.5$

VaR_{99} = ______________

ES_{99} = ______________