Problem 8. Zero-Coupon Bond Pricing with JtD Model (20pts)

Recall that in the hazard-rate framework, the zero-coupon bond price is:

$$B_{t,T} = \mathbb{E}^{\mathcal{Q}}\left[N(1-R)e^{-r(\tau_d - t)}\mathbb{1}_{\{\tau_d < T\}} + Ne^{-r(T-t)}\mathbb{1}_{\{\tau_d \geq T\}}\right]$$

where the default-time τ_d is a random variable.

Consider a default time τ_d given by:

$$\tau_d := \inf\{t \in [0,T] : \exp(-\int_0^t \lambda(S_u)du) \leq U\}$$

where U is drawn from a $Unif[0,1]$ distribution and the risk-neutral dynamics of S_t are:

$$dS_t = (r + \lambda(S_t))S_t dt + \sigma S_t dW_t - S_t N_t$$

where the jump-process N_t has jumps of size 1 with cumulative probability $\Lambda_t = \int_0^t \lambda(S_u)du$.

Compute the zero-coupon bond price with the following parameters:

- $S_0 = 100, S_* = 100, \lambda_* = 0.05, p = 1, r = 0.01, \sigma = 0.25,\ T = 5,\ t = 0,$ $N = 100,\ R = 0.5.$

$$B_{0,5} = \underline{\hspace{3cm}}$$