

Case Solution Group 5

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1. Empirical Specification and Hypotheses

Baseline model:

$$WAGE_i = \beta_1 + \beta_2 EDUCi + \alpha X_i + \mu_i$$

Where X_i includes AGE, RACE, MARRIED, SMSA, and REGION.

Hypothesis Tests:

EDUC: Education $H_0: B_2 \leq 0 \rightarrow$ Education has no or a negative effect on wages.

$H_1: B_2 > 0 \rightarrow$ Education has a positive effect on wages.

One-sided test: Economic theory predicts that more education increases wages, so a negative effect would be counterintuitive.

AGE: Age $H_0: A(AGE) \leq 0 \rightarrow$ Age has no or a negative effect on wages.

$H_1: A(AGE) > 0 \rightarrow$ Age has a positive effect on wages.

One-sided test: Age is often associated with experience, which is expected to increase wages.

MARRIED: Marital Status

$H_0: A(MARRIED) \leq 0 \rightarrow$ Being married has no or a negative effect on wages.

$H_1: A(MARRIED) > 0 \rightarrow$ Being married has a positive effect on wages.

One-sided test: Being married may indicate stability and a positive impact on productivity, leading to higher wages.

SMSA: Metropolitan Area

$H_0: A(SMSA) \leq 0 \rightarrow$ Living in a metropolitan area has no or a negative effect on wages.

$H_1: A(SMSA) > 0 \rightarrow$ Living in a metropolitan area has a positive effect on wages.

One-sided test: Urban areas typically have more job opportunities and higher wages, driving this expectation.

RACE: Race (1 = Black, 0 = White)

$H_0: A(RACE) \geq 0 \rightarrow$ Race has no or a positive effect on wages.

$H_1: A(RACE) < 0 \rightarrow$ Race has a negative effect on wages.

One-sided test: Structural inequalities often result in lower wages for minority groups.

REGION: Geographic Region H0: $A(\text{REGION}) \neq 0 \rightarrow$ (There are no positive wage differences across regions.)

H1: $A(\text{REGION}) = 0 \rightarrow$ (At least one region has a positive effect on wages.)

Two-sided test: Region could impact wages in either direction

2. Explore the Data

```
head(df)
```

```
##      WAGE EDUC AGE RACE SMSA MARRIED REGION QOB REGION1 REGION2 REGION3
## 1 580.1000    9  45   0   0     1     9   3     0     0     0
## 2 642.2115   17  47   0   0     1     3   4     0     0     1
## 3 577.0192   12  42   0   0     1     7   2     0     0     0
## 4 999.1346   10  43   0   0     1     3   2     0     0     1
## 5 307.7885   12  41   0   0     1     6   3     0     0     0
## 6 280.1000   12  40   1   0     1     5   2     0     0     0
##      REGION4 REGION5 REGION6 REGION7 REGION8 REGION9 QOB1 QOB2 QOB3 QOB4
## 1          0          0          0          0          0     1     0     0     1     0
## 2          0          0          0          0          0     0     0     0     0     1
## 3          0          0          0          1          0     0     0     1     0     0
## 4          0          0          0          0          0     0     0     1     0     0
## 5          0          0          1          0          0     0     0     0     1     0
## 6          0          1          0          0          0     0     0     1     0     0
```

```
summary(df)
```

```
##      WAGE          EDUC          AGE          RACE
## Min.   : 0.096   Min.   :0.000   Min.   :40.00   Min.   :0.0000
## 1st Qu.: 278.558 1st Qu.:12.00   1st Qu.:42.00   1st Qu.:0.0000
## Median : 384.712 Median :12.00   Median :45.00   Median :0.0000
## Mean   : 436.524 Mean  :12.71   Mean  :44.68   Mean  :0.0832
## 3rd Qu.: 520.100 3rd Qu.:15.00   3rd Qu.:47.00   3rd Qu.:0.0000
## Max.   :10167.500 Max.   :20.00   Max.   :50.00   Max.   :1.0000
##      SMSA          MARRIED         REGION          QOB
## Min.   :0.0000   Min.   :0.0000   Min.   :1.000   Min.   :1.000
## 1st Qu.:0.0000   1st Qu.:1.0000   1st Qu.:3.000   1st Qu.:1.000
## Median :0.0000   Median :1.0000   Median :5.000   Median :3.000
## Mean   :0.1813   Mean   :0.8609   Mean   :4.767   Mean   :2.502
## 3rd Qu.:0.0000   3rd Qu.:1.0000   3rd Qu.:7.000   3rd Qu.:3.000
## Max.   :1.0000   Max.   :1.0000   Max.   :9.000   Max.   :4.000
##      REGION1        REGION2        REGION3        REGION4
## Min.   :0.0000   Min.   :0.0000   Min.   :0.0000   Min.   :0.0000
## 1st Qu.:0.0000   1st Qu.:0.0000   1st Qu.:0.0000   1st Qu.:0.0000
## Median :0.0000   Median :0.0000   Median :0.0000   Median :0.0000
## Mean   :0.0549   Mean   :0.1584   Mean   :0.1949   Mean   :0.0732
## 3rd Qu.:0.0000   3rd Qu.:0.0000   3rd Qu.:0.0000   3rd Qu.:0.0000
## Max.   :1.0000   Max.   :1.0000   Max.   :1.0000   Max.   :1.0000
##      REGION5        REGION6        REGION7        REGION8
## Min.   :0.0000   Min.   :0.0000   Min.   :0.0000   Min.   :0.0000
```

```

## 1st Qu.:0.0000 1st Qu.:0.000 1st Qu.:0.0000 1st Qu.:0.0000
## Median :0.0000 Median :0.000 Median :0.0000 Median :0.0000
## Mean   :0.1773 Mean   :0.064 Mean   :0.0995 Mean   :0.0494
## 3rd Qu.:0.0000 3rd Qu.:0.000 3rd Qu.:0.0000 3rd Qu.:0.0000
## Max.   :1.0000 Max.   :1.000 Max.   :1.0000 Max.   :1.0000
##      REGION9          QOB1          QOB2          QOB3
## Min.   :0.0000  Min.   :0.0000  Min.   :0.0000  Min.   :0.0000
## 1st Qu.:0.0000 1st Qu.:0.0000 1st Qu.:0.0000 1st Qu.:0.0000
## Median :0.0000  Median :0.0000  Median :0.0000  Median :0.0000
## Mean   :0.1284  Mean   :0.2533  Mean   :0.2354  Mean   :0.2674
## 3rd Qu.:0.0000 3rd Qu.:1.0000 3rd Qu.:0.0000 3rd Qu.:1.0000
## Max.   :1.0000  Max.   :1.0000  Max.   :1.0000  Max.   :1.0000
##      QOB4
## Min.   :0.0000
## 1st Qu.:0.0000
## Median :0.0000
## Mean   :0.2439
## 3rd Qu.:0.0000
## Max.   :1.0000

```

```
describe(df)
```

	vars	n	mean	sd	median	trimmed	mad	min	max	range
## WAGE	1	10000	436.52	295.37	384.71	401.88	173.27	0.1	10167.5	10167.4
## EDUC	2	10000	12.71	3.28	12.00	12.72	2.97	0.0	20.0	20.0
## AGE	3	10000	44.68	2.93	45.00	44.67	4.45	40.0	50.0	10.0
## RACE	4	10000	0.08	0.28	0.00	0.00	0.00	0.0	1.0	1.0
## SMSA	5	10000	0.18	0.39	0.00	0.10	0.00	0.0	1.0	1.0
## MARRIED	6	10000	0.86	0.35	1.00	0.95	0.00	0.0	1.0	1.0
## REGION	7	10000	4.77	2.46	5.00	4.65	2.97	1.0	9.0	8.0
## QOB	8	10000	2.50	1.12	3.00	2.50	1.48	1.0	4.0	3.0
## REGION1	9	10000	0.05	0.23	0.00	0.00	0.00	0.0	1.0	1.0
## REGION2	10	10000	0.16	0.37	0.00	0.07	0.00	0.0	1.0	1.0
## REGION3	11	10000	0.19	0.40	0.00	0.12	0.00	0.0	1.0	1.0
## REGION4	12	10000	0.07	0.26	0.00	0.00	0.00	0.0	1.0	1.0
## REGION5	13	10000	0.18	0.38	0.00	0.10	0.00	0.0	1.0	1.0
## REGION6	14	10000	0.06	0.24	0.00	0.00	0.00	0.0	1.0	1.0
## REGION7	15	10000	0.10	0.30	0.00	0.00	0.00	0.0	1.0	1.0
## REGION8	16	10000	0.05	0.22	0.00	0.00	0.00	0.0	1.0	1.0
## REGION9	17	10000	0.13	0.33	0.00	0.04	0.00	0.0	1.0	1.0
## QOB1	18	10000	0.25	0.43	0.00	0.19	0.00	0.0	1.0	1.0
## QOB2	19	10000	0.24	0.42	0.00	0.17	0.00	0.0	1.0	1.0
## QOB3	20	10000	0.27	0.44	0.00	0.21	0.00	0.0	1.0	1.0
## QOB4	21	10000	0.24	0.43	0.00	0.18	0.00	0.0	1.0	1.0
			skew	kurtosis	se					
## WAGE		7.39	170.46	2.95						
## EDUC		-0.07	0.55	0.03						
## AGE		0.05	-1.18	0.03						
## RACE		3.02	7.11	0.00						
## SMSA		1.65	0.74	0.00						
## MARRIED		-2.09	2.35	0.00						
## REGION		0.35	-1.06	0.02						
## QOB		-0.03	-1.35	0.01						
## REGION1		3.91	13.27	0.00						

```

## REGION2 1.87    1.50 0.00
## REGION3 1.54    0.37 0.00
## REGION4 3.28    8.74 0.00
## REGION5 1.69    0.85 0.00
## REGION6 3.56    10.69 0.00
## REGION7 2.68    5.16 0.00
## REGION8 4.16    15.29 0.00
## REGION9 2.22    2.93 0.00
## QOB1    1.13    -0.71 0.00
## QOB2    1.25    -0.44 0.00
## QOB3    1.05    -0.90 0.00
## QOB4    1.19    -0.58 0.00

```

```
cov(df)
```

	WAGE	EDUC	AGE	RACE	SMSA
## WAGE	87241.0836001	3.157684e+02	5.256929832	-1.049000e+01	-1.485766e+01
## EDUC	315.7684062	1.074171e+01	-0.667730053	-1.375518e-01	-1.882395e-01
## AGE	5.2569298	-6.677301e-01	8.591725483	-4.168257e-03	-2.355067e-02
## RACE	-10.4899998	-1.375518e-01	-0.004168257	7.628539e-02	-3.784538e-03
## SMSA	-14.8576552	-1.882395e-01	-0.023550665	-3.784538e-03	1.484452e-01
## MARRIED	10.1783750	2.177522e-02	0.021109281	-1.082796e-02	5.419372e-03
## REGION	8.8550412	2.740826e-01	-0.125475448	1.185719e-03	3.264616e-02
## QOB	2.3974020	1.174544e-01	-0.386878218	-3.758456e-03	-4.794949e-03
## REGION1	-0.8536330	1.648109e-02	0.016140984	-2.867967e-03	-1.753545e-03
## REGION2	3.8199259	4.274331e-02	0.009094829	1.421262e-03	-1.461938e-02
## REGION3	3.3789188	-4.004056e-02	0.006221992	-2.615942e-03	-3.735744e-03
## REGION4	-1.1378266	-4.941294e-04	-0.011381978	-3.590599e-03	1.183002e-02
## REGION5	-6.8153854	-5.376850e-02	-0.013634873	9.649605e-03	2.855796e-03
## REGION6	-3.9603260	-6.772837e-02	0.005963796	2.675468e-03	1.069787e-02
## REGION7	-1.9950947	-3.218602e-02	0.002769627	7.216722e-04	-1.239474e-03
## REGION8	-0.2171291	2.021066e-02	-0.016629443	-2.610341e-03	5.944374e-03
## REGION9	7.7805501	1.147825e-01	0.001455066	-2.783158e-03	-9.979918e-03
## QOB1	-0.8194058	-4.086161e-02	0.193104600	-1.745775e-04	2.876998e-03
## QOB2	-0.4775954	1.293153e-02	-0.063772357	2.514971e-03	-8.781078e-04
## QOB3	1.0160062	-2.073263e-02	-0.064890869	-7.477548e-04	-2.079828e-03
## QOB4	0.2809950	4.866271e-02	-0.064441374	-1.592639e-03	8.093809e-05
	MARRIED	REGION	QOB	REGION1	REGION2
## WAGE	10.1783750456	8.855041195	2.3974019986	-0.8536330190	3.8199258573
## EDUC	0.0217752175	0.274082608	0.1174543854	0.0164810881	0.0427433143
## AGE	0.0211092809	-0.125475448	-0.3868782178	0.0161409841	0.0090948295
## RACE	-0.0108279628	0.001185719	-0.0037584558	-0.0028679668	0.0014212621
## SMSA	0.0054193719	0.032646165	-0.0047949495	-0.0017535454	-0.0146193819
## MARRIED	0.1197631663	-0.022112511	-0.0027859886	-0.0006634763	-0.0004666067
## REGION	-0.0221125113	6.064117412	-0.0024575458	-0.2068289829	-0.4383366337
## QOB	-0.0027859886	-0.002457546	1.2445208421	0.0009457846	0.0047995200
## REGION1	-0.0006634763	-0.206828983	0.0009457846	0.0518911791	-0.0086970297
## REGION2	-0.0004666067	-0.438336634	0.0047995200	-0.0086970297	0.1333227723
## REGION3	0.0019107811	-0.344422742	-0.0027205821	-0.0107010801	-0.0308752475
## REGION4	0.0018823082	-0.056150015	-0.0064397240	-0.0040190819	-0.0115960396
## REGION5	0.0005624862	0.041315032	0.0024133713	-0.0097347435	-0.0280871287
## REGION6	0.0015025503	0.078919892	-0.0026218622	-0.0035139514	-0.0101386139
## REGION7	0.0007405241	0.222205721	0.0044613961	-0.0054630963	-0.0157623762
## REGION8	0.0015716972	0.159726173	-0.0008939494	-0.0027123312	-0.0078257426

```

## REGION9 -0.0070402640 0.543571557 0.0000560456 -0.0070498650 -0.0203405941
## QOB1 -0.0005660266 0.004519352 -0.3804693169 -0.0009062606 -0.0028230023
## QOB2 0.0015442944 -0.013753175 -0.1181590759 0.0009766377 0.0022128613
## QOB3 0.0013954795 0.016405841 0.1332052605 -0.0001802780 -0.0007562356
## QOB4 -0.0023737474 -0.007172017 0.3654231323 0.0001099010 0.0013663766
##          REGION3      REGION4      REGION5      REGION6      REGION7
## WAGE    3.3789187829 -1.1378265754 -6.8153853707 -3.9603259637 -1.9950946778
## EDUC   -0.0400405641 -0.0004941294 -0.0537684968 -0.0677283728 -0.0321860186
## AGE     0.0062219922 -0.0113819782 -0.0136348735 0.0059637964 0.0027696270
## RACE   -0.00261159416 -0.0035905991 0.0096496050 0.0026754675 0.0007216722
## SMSA   -0.0037357436 0.0118300230 0.0028557956 0.0106978698 -0.0012394739
## MARRIED 0.0019107811 0.0018823082 0.0005624862 0.0015025503 0.0007405241
## REGION  -0.3444227423 -0.0561500150 0.0413150315 0.0789198920 0.2222057206
## QOB    -0.0027205821 -0.0064397240 0.0024133713 -0.0026218622 0.0044613961
## REGION1 -0.0107010801 -0.0040190819 -0.0097347435 -0.0035139514 -0.0054630963
## REGION2 -0.0308752475 -0.0115960396 -0.0280871287 -0.0101386139 -0.0157623762
## REGION3 0.1569296830 -0.0142681068 -0.0345592259 -0.0124748475 -0.0193944894
## REGION4 -0.0142681068 0.0678485449 -0.0129796580 -0.0046852685 -0.0072841284
## REGION5 -0.0345592259 -0.0129796580 0.1458792979 -0.0113483348 -0.0176431143
## REGION6 -0.0124748475 -0.0046852685 -0.0113483348 0.0599099910 -0.0063686369
## REGION7 -0.0193944894 -0.0072841284 -0.0176431143 -0.0063686369 0.0896087109
## REGION8 -0.0096290229 -0.0036164416 -0.0087594959 -0.0031619162 -0.0049157916
## REGION9 -0.0250276628 -0.0093998200 -0.0227675968 -0.0082184218 -0.0127770777
## QOB1    0.0014319732 0.0015585959 0.0007899890 0.0012889289 -0.0008034303
## QOB2    0.0003205721 0.0007687969 -0.0020366237 -0.0003656366 -0.0030226023
## QOB3   -0.0022164816 0.0002263426 -0.0007100910 -0.0005136514 0.0039940994
## QOB4    0.0004639364 -0.0025537354 0.0019567257 -0.0004096410 -0.0001680668
##          REGION8      REGION9      QOB1      QOB2      QOB3
## WAGE   -2.171291e-01 7.7805501010 -0.8194057861 -4.775954e-01 1.0160061545
## EDUC   2.021066e-02 0.1147825183 -0.0408616062 1.293153e-02 -0.0207326333
## AGE    -1.662944e-02 0.0014550655 0.1931046005 -6.377236e-02 -0.0648908691
## RACE   -2.610341e-03 -0.0027831583 -0.0001745775 2.514971e-03 -0.0007477548
## SMSA   5.944374e-03 -0.0099799180 0.0028769977 -8.781078e-04 -0.0020798280
## MARRIED 1.571697e-03 -0.0070402640 -0.0005660266 1.544294e-03 0.0013954795
## REGION  1.597262e-01 0.5435715572 0.0045193519 -1.375318e-02 0.0164058406
## QOB    -8.939494e-04 0.0000560456 -0.3804693169 -1.181591e-01 0.1332052605
## REGION1 -2.712331e-03 -0.0070498650 -0.0009062606 9.766377e-04 -0.0001802780
## REGION2 -7.825743e-03 -0.0203405941 -0.0028230023 2.212861e-03 -0.0007562356
## REGION3 -9.629023e-03 -0.0250276628 0.0014319732 3.205721e-04 -0.0022164816
## REGION4 -3.616442e-03 -0.0093998200 0.0015585959 7.687969e-04 0.0002263426
## REGION5 -8.759496e-03 -0.0227675968 0.0007899890 -2.036624e-03 -0.0007100910
## REGION6 -3.161916e-03 -0.0082184218 0.0012889289 -3.656366e-04 -0.0005136514
## REGION7 -4.915792e-03 -0.0127770777 -0.0008034303 -3.022602e-03 0.0039940994
## REGION8  4.696434e-02 -0.0063435944 0.0006870487 -2.876288e-05 -0.0011096710
## REGION9 -6.343594e-03 0.1119246325 -0.0012238424 1.174757e-03 0.0012659666
## QOB1    6.870487e-04 -0.0012238424 0.1891580258 -5.963278e-02 -0.0677391939
## QOB2    -2.876288e-05 0.0011747575 -0.0596327833 1.800048e-01 -0.0629522552
## QOB3   -1.109671e-03 0.0012659666 -0.0677391939 -6.295226e-02 0.1959168317
## QOB4    4.513851e-04 -0.0012168817 -0.0617860486 -5.741980e-02 -0.0652253825
##          QOB4
## WAGE   2.809950e-01
## EDUC   4.866271e-02
## AGE    -6.444137e-02
## RACE   -1.592639e-03

```

```

## SMSA      8.093809e-05
## MARRIED -2.373747e-03
## REGION   -7.172017e-03
## QOB       3.654231e-01
## REGION1  1.099010e-04
## REGION2  1.366377e-03
## REGION3  4.639364e-04
## REGION4 -2.553735e-03
## REGION5  1.956726e-03
## REGION6 -4.096410e-04
## REGION7 -1.680668e-04
## REGION8  4.513851e-04
## REGION9 -1.216882e-03
## QOB1     -6.178605e-02
## QOB2     -5.741980e-02
## QOB3     -6.522538e-02
## QOB4     1.844312e-01

```

2.1 Statistics per region

WAGE: Largest wage disparities in regions 2, 6, and 9; high SD, skewness, and kurtosis → outliers with very high incomes.

EDUC: Highest in region 9, lowest in region 6; slightly more variation in regions 5 and 6.

AGE: Almost uniformly ~45 years across regions; low spread, minimal differences.

RACE: Mostly white population (>90%); region 5 relatively more diversity; high kurtosis indicates outliers.

SMSA: Regions 4 & 6 are more urban, region 2 is the most rural; skewed → most live outside urban areas.

MARRIED: High marriage rates across all regions (~85–89%); region 9 slightly lower.

```

## # A tibble: 9 x 11
##   REGION mean_wage mean_educ mean_age sd_wage sd_educ sd_age var_wage var_educ
##   <int>     <dbl>     <dbl>     <dbl>    <dbl>    <dbl>    <dbl>    <dbl>    <dbl>
## 1     1      421.     13.0      45.0    244.     3.10     2.93    59622.    9.62
## 2     2      461.     13.0      44.7    355.     3.17     2.96   126021.   10.0
## 3     3      454.     12.5      44.7    264.     3.07     2.89    69754.    9.40
## 4     4      421.     12.7      44.5    263.     3.15     2.94    69295.    9.90
## 5     5      398.     12.4      44.6    261.     3.48     2.99    68143.   12.1
## 6     6      375.     11.7      44.8    330.     3.74     2.77   108586.   14.0
## 7     7      416.     12.4      44.7    266.     3.57     2.98    70638.   12.7
## 8     8      432.     13.1      44.3    236.     2.95     2.95    55764.    8.73
## 9     9      497.     13.6      44.7    346.     2.92     2.89   119854.    8.54
## # i 2 more variables: var_age <dbl>, count <int>

##
## Descriptive statistics by group
## REGION: 1
##   vars   n   mean      sd median trimmed    mad   min   max range skew
## REGION   1 549  1.00    0.00   1.00    1.00  0.00  1.00  1.0  0.00  NaN
## WAGE     2 549 420.98 244.18 384.71  389.65 171.07  3.94 1562.5 1558.56 1.74
## EDUC     3 549 13.01   3.10  12.00   12.91  2.97  5.00  20.0 15.00  0.33
## AGE      4 549 44.97   2.93  45.00   45.02  4.45 40.00  50.0 10.00 -0.05

```

```

## RACE      5 549  0.03  0.17  0.00  0.00  0.00  0.00  1.0  1.00  5.40
## SMSA     6 549  0.15  0.36  0.00  0.06  0.00  0.00  1.0  1.00  1.96
## MARRIED  7 549  0.85  0.36  1.00  0.93  0.00  0.00  1.0  1.00 -1.94
##          kurtosis   se
## REGION    NaN  0.00
## WAGE      4.51 10.42
## EDUC     -0.16  0.13
## AGE       -1.22  0.13
## RACE      27.22 0.01
## SMSA      1.85  0.02
## MARRIED   1.78  0.02
## -----
## REGION: 2
##          vars   n   mean     sd median trimmed   mad   min   max range
## REGION   1 1584  2.00  0.00  2.00  2.00  0.00  2.0  2.0  0.0
## WAGE     2 1584 460.64 354.99 403.94 422.27 171.07 2.4 10167.5 10165.1
## EDUC     3 1584 12.98  3.17 12.00 12.90  2.97  0.0 20.0 20.0
## AGE      4 1584 44.74  2.96 45.00 44.73  4.45 40.0 50.0 10.0
## RACE     5 1584  0.09  0.29  0.00  0.00  0.00  0.0  1.0  1.0
## SMSA     6 1584  0.09  0.28  0.00  0.00  0.00  0.0  1.0  1.0
## MARRIED  7 1584  0.86  0.35  1.00  0.95  0.00  0.0  1.0  1.0
##          skew kurtosis   se
## REGION   NaN  NaN  0.00
## WAGE     13.60 352.35 8.92
## EDUC     0.18  0.42 0.08
## AGE      0.04 -1.22 0.07
## RACE     2.82  5.94 0.01
## SMSA     2.88  6.32 0.01
## MARRIED -2.05 2.20 0.01
## -----
## REGION: 3
##          vars   n   mean     sd median trimmed   mad   min   max range
## REGION   1 1949  3.00  0.00  3.0  3.00  0.00  3.00 3.00 0.00
## WAGE     2 1949 453.86 264.11 409.2 425.41 164.62 0.15 4634.17 4634.02
## EDUC     3 1949 12.51  3.07 12.0 12.43  1.48  0.00 20.00 20.00
## AGE      4 1949 44.71  2.89 45.0 44.70  2.97 40.00 50.00 10.00
## RACE     5 1949  0.07  0.25  0.0  0.00  0.00  0.00 1.00 1.00
## SMSA     6 1949  0.16  0.37  0.0  0.08  0.00  0.00 1.00 1.00
## MARRIED  7 1949  0.87  0.34  1.0  0.96  0.00  0.00 1.00 1.00
##          skew kurtosis   se
## REGION   NaN  NaN  0.00
## WAGE     4.11  43.18 5.98
## EDUC     0.22  0.56 0.07
## AGE      0.05 -1.12 0.07
## RACE     3.37  9.39 0.01
## SMSA     1.83  1.36 0.01
## MARRIED -2.21 2.88 0.01
## -----
## REGION: 4
##          vars   n   mean     sd median trimmed   mad   min   max range skew
## REGION   1 732   4.00  0.00  4.00  4.00  0.00  4.00 4.00  0  NaN
## WAGE     2 732 420.98 263.24 384.71 390.86 146.34 10.25 4076.25 4066 4.76
## EDUC     3 732 12.71  3.15 12.00 12.63  1.48  0.00 20.00 20  0.16
## AGE      4 732 44.52  2.94 44.00 44.48  4.45 40.00 50.00 10  0.11

```

```

## RACE      5 732  0.03  0.18  0.00  0.00  0.00  0.00  1.00  1  5.12
## SMSA     6 732  0.34  0.48  0.00  0.30  0.00  0.00  1.00  1  0.66
## MARRIED  7 732  0.89  0.32  1.00  0.98  0.00  0.00  1.00  1 -2.43
##          kurtosis   se
## REGION    NaN 0.00
## WAGE      52.28 9.73
## EDUC      0.58 0.12
## AGE       -1.19 0.11
## RACE      24.24 0.01
## SMSA     -1.57 0.02
## MARRIED   3.93 0.01
## -----
## REGION: 5
##          vars   n   mean     sd median trimmed   mad   min   max range
## REGION   1 1773  5.00  0.00   5.00   5.00  0.00  5.0  5.00  0.00
## WAGE     2 1773 398.09 261.04 346.25 362.95 189.49 0.1 2501.25 2501.15
## EDUC     3 1773 12.41  3.48  12.00  12.46  2.97  0.0  20.00  20.00
## AGE      4 1773 44.60  2.99  45.00  44.59  4.45  40.0  50.00 10.00
## RACE     5 1773  0.14  0.34  0.00  0.05  0.00  0.0  1.00  1.00
## SMSA     6 1773  0.20  0.40  0.00  0.12  0.00  0.0  1.00  1.00
## MARRIED  7 1773  0.86  0.34  1.00  0.95  0.00  0.0  1.00  1.00
##          skew kurtosis   se
## REGION   NaN   NaN 0.00
## WAGE     2.23  8.76 6.20
## EDUC    -0.19  0.27 0.08
## AGE      0.06 -1.22 0.07
## RACE     2.10  2.42 0.01
## SMSA     1.52  0.31 0.01
## MARRIED -2.12  2.51 0.01
## -----
## REGION: 6
##          vars   n   mean     sd median trimmed   mad   min   max range skew
## REGION   1 640   6.00  0.00   6.00   6.00  0.00  6.00  6   0.00  NaN
## WAGE     2 640 374.65 329.52 334.13 335.99 181.61 1.98 6005 6003.02 8.62
## EDUC     3 640 11.66  3.74  12.00  11.65  2.97  0.00  20  20.00 -0.03
## AGE      4 640 44.77  2.77  45.00  44.74  2.97  40.00 50  10.00  0.06
## RACE     5 640  0.12  0.33  0.00  0.03  0.00  0.00  1   1.00  2.26
## SMSA     6 640  0.35  0.48  0.00  0.31  0.00  0.00  1   1.00  0.63
## MARRIED  7 640  0.88  0.32  1.00  0.98  0.00  0.00  1   1.00 -2.40
##          kurtosis   se
## REGION   NaN 0.00
## WAGE     133.54 13.03
## EDUC     0.08 0.15
## AGE      -1.08 0.11
## RACE     3.12 0.01
## SMSA     -1.60 0.02
## MARRIED  3.76 0.01
## -----
## REGION: 7
##          vars   n   mean     sd median trimmed   mad   min   max range
## REGION   1 995   7.00  0.00   7.00   7.00  0.00  7.00  7.00  0.00
## WAGE     2 995 416.47 265.78 373.17 381.59 191.03 4.69 2501.25 2496.56
## EDUC     3 995 12.39  3.57  12.00  12.56  2.97  0.00  20.00 20.00
## AGE      4 995 44.71  2.98  45.00  44.71  4.45  40.00 50.00 10.00

```

```

## RACE      5 995  0.09  0.29  0.00  0.00  0.00  0.00  1.00  1.00
## SMSA     6 995  0.17  0.37  0.00  0.09  0.00  0.00  1.00  1.00
## MARRIED  7 995  0.87  0.34  1.00  0.96  0.00  0.00  1.00  1.00
##          skew kurtosis   se
## REGION   NaN      NaN 0.00
## WAGE     2.01     7.02 8.43
## EDUC    -0.47     0.56 0.11
## AGE      0.02    -1.19 0.09
## RACE     2.85     6.14 0.01
## SMSA    1.77     1.12 0.01
## MARRIED -2.18     2.74 0.01
## -----
## REGION: 8
##      vars   n   mean      sd median trimmed    mad   min   max range
## REGION  1 494  8.00  0.00   8.00   8.00  0.00  8.00  8.00  0.00
## WAGE    2 494 432.13 236.14 384.71 401.01 145.41 1.06 1469.49 1468.43
## EDUC    3 494 13.12  2.95  12.00  13.09  2.97  1.00  20.00 19.00
## AGE     4 494 44.34  2.95  44.00  44.27  2.97  40.00  50.00 10.00
## RACE    5 494  0.03  0.17  0.00  0.00  0.00  0.00  1.00  1.00
## SMSA    6 494  0.30  0.46  0.00  0.25  0.00  0.00  1.00  1.00
## MARRIED 7 494  0.89  0.31  1.00  0.99  0.00  0.00  1.00  1.00
##          skew kurtosis   se
## REGION   NaN      NaN 0.00
## WAGE     1.84     4.94 10.62
## EDUC     0.06     0.95 0.13
## AGE      0.17    -1.16 0.13
## RACE     5.46     27.84 0.01
## SMSA    0.86    -1.26 0.02
## MARRIED -2.53     4.41 0.01
## -----
## REGION: 9
##      vars   n   mean      sd median trimmed    mad   min   max range skew
## REGION  1 1284  9.00  0.00   9.00   9.00  0.00  9.0   9   0.0  NaN
## WAGE    2 1284 497.11 346.20 440.48 454.64 196.73 7.1  7335 7327.9 7.21
## EDUC    3 1284 13.61  2.92  13.00  13.56  1.48  0.0   20  20.0 0.01
## AGE     4 1284 44.69  2.89  45.00  44.68  2.97  40.0  50  10.0 0.03
## RACE    5 1284  0.06  0.24  0.00  0.00  0.00  0.0   1   1.0 3.65
## SMSA    6 1284  0.10  0.30  0.00  0.00  0.00  0.0   1   1.0 2.60
## MARRIED 7 1284  0.81  0.40  1.00  0.88  0.00  0.0   1   1.0 -1.55
##          kurtosis   se
## REGION   NaN 0.00
## WAGE     121.20 9.66
## EDUC     0.85 0.08
## AGE      -1.18 0.08
## RACE     11.30 0.01
## SMSA     4.76 0.01
## MARRIED  0.39 0.01

```

2.2 Statistics per age

AGE GROUPS:

AGE \geq 40 & AGE $<$ 42 ~ '40-41 GROUP 1',

AGE \geq 42 & AGE $<$ 44 ~ '42-43 GROUP 2',

AGE \geq 44 & AGE $<$ 46 ~ '44-45 GROUP 3',
 AGE \geq 46 & AGE $<$ 48 ~ '46-47 GROUP 4',
 AGE \geq 48 & AGE \leq 50 ~ '48-50 GROUP 5'

```

## 
## Descriptive statistics by group
## AGE_GROUP: 1
##      vars   n   mean     sd median trimmed    mad   min   max range
## WAGE      1 1835 439.82 340.56 384.71 401.20 171.07 1.06 7335 7333.94
## EDUC      2 1835 12.98   3.11 12.00 12.93  2.97  0.00  20 20.00
## AGE       3 1835 40.56   0.50 41.00 40.58  0.00 40.00  41 1.00
## RACE      4 1835  0.09   0.28  0.00  0.00  0.00  0.00  1 1.00
## SMSA      5 1835  0.18   0.39  0.00  0.10  0.00  0.00  1 1.00
## MARRIED    6 1835  0.85   0.36  1.00  0.94  0.00  0.00  1 1.00
## AGE_GROUP  7 1835  1.00   0.00  1.00  1.00  0.00  0.00  1 0.00
##          skew kurtosis   se
## WAGE      8.61 139.95 7.95
## EDUC      0.07  0.54 0.07
## AGE      -0.25 -1.94 0.01
## RACE      2.95  6.70 0.01
## SMSA      1.65  0.71 0.01
## MARRIED   -1.95  1.82 0.01
## AGE_GROUP NaN   NaN 0.00
## -----
## AGE_GROUP: 2
##      vars   n   mean     sd median trimmed    mad   min   max range
## WAGE      1 2029 433.85 281.52 384.71 397.82 178.77 2.4 4634.17 4631.76
## EDUC      2 2029 12.90   3.22 12.00 12.90  2.97  0.0 20.00 20.00
## AGE       3 2029 42.50   0.50 43.00 42.50  0.00 42.0 43.00 1.00
## RACE      4 2029  0.08   0.27  0.00  0.00  0.00  0.0  1.00 1.00
## SMSA      5 2029  0.19   0.39  0.00  0.12  0.00  0.0  1.00 1.00
## MARRIED    6 2029  0.86   0.35  1.00  0.94  0.00  0.0  1.00 1.00
## AGE_GROUP  7 2029  2.00   0.00  2.00  2.00  0.00  2.0  2.00 0.00
##          skew kurtosis   se
## WAGE      3.45 30.53 6.25
## EDUC      -0.05 0.55 0.07
## AGE      -0.01 -2.00 0.01
## RACE      3.07  7.45 0.01
## SMSA      1.55  0.41 0.01
## MARRIED   -2.02  2.07 0.01
## AGE_GROUP NaN   NaN 0.00
## -----
## AGE_GROUP: 3
##      vars   n   mean     sd median trimmed    mad   min   max range
## WAGE      1 2035 426.73 245.77 384.71 398.04 171.07 1.98 2884.62 2882.64
## EDUC      2 2035 12.75   3.26 12.00 12.75  2.97  0.00 20.00 20.00
## AGE       3 2035 44.48   0.50 44.00 44.47  0.00 44.00 45.00 1.00
## RACE      4 2035  0.08   0.28  0.00  0.00  0.00  0.00  1.00 1.00
## SMSA      5 2035  0.19   0.40  0.00  0.12  0.00  0.00  1.00 1.00
## MARRIED    6 2035  0.87   0.34  1.00  0.96  0.00  0.00  1.00 1.00
## AGE_GROUP  7 2035  3.00   0.00  3.00  3.00  0.00  3.00  3.00 0.00
##          skew kurtosis   se
## WAGE      2.12  9.53 5.45

```

```

## EDUC      -0.01    0.38 0.07
## AGE       0.09   -1.99 0.01
## RACE      3.03    7.19 0.01
## SMSA      1.55    0.39 0.01
## MARRIED   -2.16    2.66 0.01
## AGE_GROUP  NaN     NaN 0.00
##
## -----
## AGE_GROUP: 4
##          vars   n   mean      sd median trimmed    mad   min   max range
## WAGE      1 1876 444.74 341.11 391.25  408.69 191.22  0.15 10167.5 10167.35
## EDUC      2 1876 12.62   3.38 12.00   12.61   2.97  0.00  20.0  20.0
## AGE       3 1876 46.51   0.50 47.00   46.52   0.00 46.00   47.0  1.00
## RACE      4 1876  0.09   0.28  0.00   0.00   0.00  0.00   0.00  1.00
## SMSA      5 1876  0.17   0.38  0.00   0.09   0.00  0.00   0.00  1.00
## MARRIED   6 1876  0.86   0.35  1.00   0.95   0.00  0.00   0.00  1.00
## AGE_GROUP 7 1876  4.00   0.00  4.00   4.00   0.00  4.00   4.00  0.00
##          skew kurtosis   se
## WAGE     13.04   351.30 7.88
## EDUC     -0.06    0.47 0.08
## AGE      -0.05   -2.00 0.01
## RACE      2.96    6.74 0.01
## SMSA      1.74    1.03 0.01
## MARRIED   -2.05   2.19 0.01
## AGE_GROUP  NaN     NaN 0.00
##
## -----
## AGE_GROUP: 5
##          vars   n   mean      sd median trimmed    mad   min   max range
## WAGE      1 2225 438.28 266.11 384.71  404.37 173.64  0.1 2667.5 2667.4
## EDUC      2 2225 12.37   3.37 12.00   12.40   2.97  0.0  20.0  20.0
## AGE       3 2225 48.69   0.67 49.00   48.62   1.48 48.0  50.0  2.0
## RACE      4 2225  0.08   0.27  0.00   0.00   0.00  0.0   0.0  1.00
## SMSA      5 2225  0.17   0.37  0.00   0.08   0.00  0.0   0.0  1.00
## MARRIED   6 2225  0.87   0.33  1.00   0.97   0.00  0.0   0.0  1.00
## AGE_GROUP 7 2225  5.00   0.00  5.00   5.00   0.00  5.0   5.0  0.0
##          skew kurtosis   se
## WAGE     2.05    7.54 5.64
## EDUC     -0.18    0.63 0.07
## AGE      0.45   -0.79 0.01
## RACE     3.06    7.37 0.01
## SMSA     1.79    1.21 0.01
## MARRIED  -2.24   3.00 0.01
## AGE_GROUP  NaN     NaN 0.00

```

2.3 Covariance matrices per regio

[[1]]

Table 1: Covariance Matrix for REGION: 1

	WAGE	EDUC	AGE	RACE	SMSA	MARRIED	QOB	QOB1	QOB2	QOB3	QOB4
WAGE	59622.389	326.291	19.889	-	-	8.220	-	4.997	-	0.758	0.763
				4.301	19.309		2.715		6.518		

	WAGE	EDUC	AGE	RACE	SMSA	MARRIED	QOB	QOB1	QOB2	QOB3	QOB4
EDUC	326.291	9.617	-	-	-0.152	-0.054	0.029	0.053	-	-	0.064
AGE	19.889	-0.350	8.596	0.001	-0.054	0.074	-	0.149	-	-	-
RACE	-4.301	-0.024	0.001	0.030	-0.005	-0.004	-	0.002	0.001	-	0.000
SMSA	-19.309	-0.152	-	-	0.127	-0.005	-	0.001	0.006	-	-
MARRIED	8.220	-0.054	0.074	-	-0.005	0.129	-	0.001	0.000	0.002	-
QOB	-2.715	0.029	-	-	-0.010	-0.005	1.217	-	-	0.127	0.365
QOB1	4.997	0.053	0.149	0.002	0.001	0.001	-	0.181	-	-	-
QOB2	-6.518	-0.071	-	0.001	0.006	0.000	-	-	0.189	-	-
QOB3	0.758	-0.046	-	-	-0.005	0.002	0.127	-	-	0.195	-
QOB4	0.763	0.064	-	0.000	-0.002	-0.003	0.365	-	-	-	0.186
				0.033				0.058	0.062	0.065	

[[2]]

Table 2: Covariance Matrix for REGION: 2

	WAGE	EDUC	AGE	RACE	SMSA	MARRIED	QOB	QOB1	QOB2	QOB3	QOB4
WAGE	126020.884	335.536	29.869	-	-	16.273	-	0.966	1.492	-	-
EDUC	335.536	10.031	-	-0.158	-	0.091	-	0.026	0.025	-	-
AGE	29.869	-0.673	8.753	-0.016	0.009	-0.012	-	0.195	-	-	-
RACE	-13.086	-0.158	-	0.084	-	-0.014	-	-	0.007	-	-
SMSA	-6.225	-0.088	0.009	-0.007	0.081	0.001	-	0.003	-	-	0.001
MARRIED	16.273	0.091	-	-0.014	0.001	0.122	0.014	-	-	0.001	0.005
QOB	-5.375	-0.105	-	-0.002	-	0.014	1.226	-	-	0.123	0.371
QOB1	0.966	0.026	0.195	-0.003	0.003	-0.003	-	0.180	-	-	-
QOB2	1.492	0.025	-	0.007	-	-0.004	-	-	0.187	-	-
QOB3	-0.507	-0.024	-	-0.001	-	0.001	0.123	-	-	0.194	-
QOB4	-1.951	-0.027	-	-0.002	0.001	0.005	0.371	-	-	-	0.189
				0.045				0.060	0.063	0.066	

[[3]]

Table 3: Covariance Matrix for REGION: 3

	WAGE	EDUC	AGE	RACE	SMSA	MARRIED	QOB	QOB1	QOB2	QOB3	QOB4
WAGE	69754.484	247.163	-	-	-	10.302	-	4.382	1.401	-	-
EDUC	247.163	9.398	9.152	6.535	13.792	-	13.651	-	-	2.296	3.487
AGE	-9.152	-0.690	0.690	0.083	-	-0.035	-0.018	-0.386	0.188	-	-
RACE	-6.535	-0.083	-	0.021	0.065	-0.011	-0.006	-0.004	0.002	-	0.079
SMSA	-13.792	-0.106	-	0.035	0.011	0.136	0.006	-0.002	0.001	-	0.001
MARRIED	10.302	-0.007	-	0.018	0.006	-	0.006	0.113	-0.022	0.002	0.008
QOB	-13.651	0.111	-	0.386	0.004	-	-0.002	-0.022	1.264	-	0.131
QOB1	4.382	-0.007	0.188	0.002	0.001	0.002	-	-0.388	0.193	-	-
QOB2	1.401	-0.029	-	0.069	0.001	-	-0.001	0.008	-0.116	-	-
QOB3	-2.296	-0.031	-	0.040	0.000	0.000	-	0.131	-	-	0.191
QOB4	-3.487	0.067	-	0.079	0.001	0.000	-	-0.010	0.373	-	0.186

[[4]]

Table 4: Covariance Matrix for REGION: 4

	WAGE	EDUC	AGE	RACE	SMSA	MARRIED	QOB	QOB1	QOB2	QOB3	QOB4
WAGE	69294.730	276.252	-	-	-	12.774	13.151	-	-	3.178	2.367
EDUC	276.252	9.903	25.048	4.650	20.462	-	-	5.239	0.306	-	0.089
AGE	-25.048	-0.788	8.668	0.008	0.015	0.020	-	0.157	-	-	-
RACE	-4.650	-0.043	0.008	0.033	-0.005	-0.004	-	0.004	-	0.000	-
SMSA	-20.462	-0.246	0.015	-	0.226	0.005	-	0.003	0.004	-	0.005
MARRIED	12.774	0.082	0.020	-	0.005	0.101	-	-	0.003	0.005	-
QOB	13.151	0.221	-0.339	-	-0.005	-0.006	1.211	-	-	0.159	0.332
QOB1	-5.239	-0.085	0.157	0.004	0.003	-0.002	-	0.389	0.102	-	-
QOB2	-0.306	0.038	-0.039	-	0.004	0.003	-	-	0.186	-	-
QOB3	3.178	-0.041	-0.056	0.000	-0.012	0.005	0.159	-	-	0.198	-

	WAGE	EDUC	AGE	RACE	SMSA	MARRIED	QOB	QOB1	QOB2	QOB3	QOB4
QOB4	2.367	0.089	-0.063	-	0.005	-0.006	0.332	-	-	-	0.166
				0.002				0.057	0.051	0.057	

[[5]]

Table 5: Covariance Matrix for REGION: 5

	WAGE	EDUC	AGE	RACE	SMSA	MARRIED	QOB	QOB1	QOB2	QOB3	QOB4
WAGE	68142.878	359.308	-	-	-	10.883	10.164	-	-	0.067	3.441
EDUC	359.308	12.144	13.139	15.912	13.042	0.040	0.124	3.215	0.293	-	0.040
AGE	-13.139	-0.823	8.956	0.017	-0.038	0.053	-	0.181	-	-	-
RACE	-15.912	-0.232	0.017	0.119	0.001	-0.019	-	0.043	0.110	0.028	0.028
SMSA	-13.042	-0.257	-0.038	0.001	0.159	0.003	-	0.004	-	0.005	-
MARRIED	10.883	0.040	0.053	-0.019	0.003	0.118	-	0.000	-	0.002	-
QOB	10.164	0.124	-0.347	-0.015	-0.009	-0.001	1.276	-	-	0.128	0.379
QOB1	-3.215	-0.070	0.181	0.001	0.004	0.000	-	0.391	0.115	-	-
QOB2	-0.293	0.056	-0.043	0.004	-0.005	-0.001	-	0.191	-	0.058	0.068
QOB3	0.067	-0.025	-0.110	0.003	0.005	0.002	0.128	0.115	0.058	-	0.057
QOB4	3.441	0.040	-0.028	-0.008	-0.005	-0.001	0.379	-	-	0.194	-
								0.066	0.057	0.067	

[[6]]

Table 6: Covariance Matrix for REGION: 6

	WAGE	EDUC	AGE	RACE	SMSA	MARRIED	QOB	QOB1	QOB2	QOB3	QOB4
WAGE	108586.360	289.292	-	-	-	3.987	32.889	-	-	1.185	11.524
EDUC	289.292	14.004	38.510	10.947	20.349	0.117	0.345	8.655	4.054	-	-
AGE	-38.510	-1.028	7.682	0.018	0.036	-0.011	-	0.185	-	-	-
RACE	-10.947	-0.234	0.018	0.110	0.006	-0.022	0.010	-	0.003	-	0.003
SMSA	-20.349	-0.204	0.036	0.006	0.227	0.011	-	0.004	-	0.001	-
MARRIED	3.987	0.117	-0.011	-0.022	0.011	0.102	0.003	-	0.003	0.003	-
QOB	32.889	0.345	-0.341	0.010	-0.028	0.003	1.272	-	-	0.140	0.366
								0.400	0.106	-	

	WAGE	EDUC	AGE	RACE	SMSA	MARRIED	QOB	QOB1	QOB2	QOB3	QOB4
QOB1	-8.655	-0.130	0.185	-0.004	0.005	-0.004	-	0.199	-	-	-
QOB2	-4.054	-0.013	-0.085	0.003	0.007	0.003	-	0.400	0.063	0.071	0.065
QOB3	1.185	0.071	-0.042	-0.001	-0.001	0.003	0.140	-	0.063	0.060	0.055
QOB4	11.524	0.072	-0.057	0.003	-0.011	-0.002	0.366	-	0.071	0.060	0.062
								0.065	0.055	0.062	0.181

[[7]]

Table 7: Covariance Matrix for REGION: 7

	WAGE	EDUC	AGE	RACE	SMSA	MARRIED	QOB	QOB1	QOB2	QOB3	QOB4
WAGE	70637.866	357.447	26.277	-	-	11.726	8.040	-	-	8.554	-
EDUC	357.447	12.717	-	-0.183	-0.219	0.008	0.193	2.625	4.360	-	1.569
AGE	26.277	-0.614	8.851	0.008	-0.026	0.055	-	0.053	0.012	0.012	0.076
RACE	-11.392	-0.183	0.008	0.082	0.004	-0.010	0.001	0.236	-	-	-
SMSA	-12.199	-0.219	-	0.004	0.140	0.004	0.016	-	-	-	0.008
MARRIED	11.726	0.008	0.055	-0.010	0.004	0.114	-	0.001	0.005	0.002	-
QOB	8.040	0.193	-	0.001	0.016	-0.009	1.224	-	-	0.140	0.352
QOB1	-2.625	-0.053	0.236	-0.003	-0.001	-0.002	-	0.380	0.112	-	-
QOB2	-4.360	-0.012	-	0.006	-0.005	0.005	-	-	0.163	-	-
QOB3	8.554	-0.012	-	-0.003	-0.002	0.005	0.140	-	-	0.213	-
QOB4	-1.569	0.076	-	0.000	0.008	-0.008	0.352	-	-	-	0.075
				0.145				0.059	0.050	0.075	

[[8]]

Table 8: Covariance Matrix for REGION: 8

	WAGE	EDUC	AGE	RACE	SMSA	MARRIED	QOB	QOB1	QOB2	QOB3	QOB4
WAGE	55763.738	204.967	8.289	-	-	0.818	7.934	-	3.356	-	2.893
EDUC	204.967	8.725	-	5.134	19.118	-	4.199	-	2.050	-	0.064
AGE	8.289	-0.586	8.700	0.044	-0.049	0.027	-	0.084	-	0.004	-
RACE	-5.134	-0.044	-	0.033	-0.005	-0.001	0.462	-	0.089	0.139	0.032
				0.033			0.014	-	0.003	0.005	0.000
							0.008				

	WAGE	EDUC	AGE	RACE	SMSA	MARRIED	QOB	QOB1	QOB2	QOB3	QOB4
SMSA	-19.118	-0.214	-	-	0.211	0.008	0.024	-	0.000	-	0.009
			0.049	0.005				0.008		0.001	
MARRIED	0.818	0.034	0.027	-	0.008	0.096	0.003	0.000	0.001	-	0.005
				0.001						0.006	
QOB	7.934	0.208	-	0.014	0.024	0.003	1.293	-	-	0.127	0.384
			0.462					0.397	0.114		
QOB1	-4.199	-0.084	0.259	-	-0.008	0.000	-	0.196	-	-	-
				0.008			0.397		0.063	0.066	0.068
QOB2	3.356	0.024	-	0.003	0.000	0.001	-	-	0.180	-	-
			0.089				0.114	0.063		0.058	0.060
QOB3	-2.050	-0.004	-	0.005	-0.001	-0.006	0.127	-	-	0.185	-
			0.139				0.066	0.058			0.062
QOB4	2.893	0.064	-	0.000	0.009	0.005	0.384	-	-	-	0.189
			0.032				0.068	0.060	0.062		

[[9]]

Table 9: Covariance Matrix for REGION: 9

	WAGE	EDUC	AGE	RACE	SMSA	MARRIED	QOB	QOB1	QOB2	QOB3	QOB4
WAGE	119854.165	270.881	33.568	-	-	10.765	-	0.768	-	3.349	-
				8.149	7.110		1.088		2.282		1.834
EDUC	270.881	8.544	-	-	-	-0.022	0.145	-	0.022	-	0.064
			0.359	0.046	0.113			0.052		0.034	
AGE	33.568	-0.359	8.370	-	-	0.051	-	0.207	-	-	-
				0.012	0.016		0.413		0.087	0.035	0.086
RACE	-8.149	-0.046	-	0.058	-	-0.009	-	0.002	0.004	-	0.000
				0.012	0.005		0.008			0.005	
SMSA	-7.110	-0.113	-	-	0.093	0.005	-	0.003	0.001	-	0.004
				0.016	0.005		0.003			0.008	
MARRIED	10.765	-0.022	0.051	-	0.005	0.156	0.008	-	-	0.001	0.003
				0.009				0.002	0.002		
QOB	-1.088	0.145	-	-	-	0.008	1.207	-	-	0.138	0.351
				0.413	0.008	0.003		0.367	0.123		
QOB1	0.768	-0.052	0.207	0.002	0.003	-0.002	-	0.184	-	-	-
							0.367		0.060	0.068	0.057
QOB2	-2.282	0.022	-	0.004	0.001	-0.002	-	-	0.185	-	-
				0.087			0.123	0.060		0.068	0.057
QOB3	3.349	-0.034	-	-	-	0.001	0.138	-	-	0.201	-
				0.035	0.005	0.008		0.068	0.068		0.065
QOB4	-1.834	0.064	-	0.000	0.004	0.003	0.351	-	-	-	0.180
				0.086				0.057	0.057	0.065	

2.4 Correlation matrices per region

[[1]]

Table 10: Correlation Matrix for REGION: 1

	WAGE	EDUC	AGE	RACE	SMSA	MARRIEDQOB	QOB1	QOB2	QOB3	QOB4
WAGE	1.000	0.431	0.028	-0.102	-0.222	0.094	-0.010	0.048	-0.061	0.007
EDUC	0.431	1.000	-0.038	-0.045	-0.137	-0.049	0.008	0.040	-0.053	-0.033
AGE	0.028	-0.038	1.000	0.002	-0.052	0.071	-0.071	0.119	-0.080	-0.010
RACE	-0.102	-0.045	0.002	1.000	-0.075	-0.071	-0.027	0.024	0.017	-0.036
SMSA	-0.222	-0.137	-0.052	-0.075	1.000	-0.037	-0.026	0.007	0.038	-0.031
MARRIED0.094	-0.049	0.071	-0.071	-0.037	1.000	-0.013	0.008	0.000	0.011	-0.019
QOB	-0.010	0.008	-0.071	-0.027	-0.026	-0.013	1.000	-0.768	-0.274	0.261
QOB1	0.048	0.040	0.119	0.024	0.007	0.008	-0.768	1.000	-0.324	-0.334
QOB2	-0.061	-0.053	-0.080	0.017	0.038	0.000	-0.274	-0.324	1.000	-0.349
QOB3	0.007	-0.033	-0.010	-0.036	-0.031	0.011	0.261	-0.334	-0.349	1.000
QOB4	0.007	0.048	-0.026	-0.004	-0.014	-0.019	0.767	-0.318	-0.332	-0.342
										1.000

[[2]]

Table 11: Correlation Matrix for REGION: 2

	WAGE	EDUC	AGE	RACE	SMSA	MARRIEDQOB	QOB1	QOB2	QOB3	QOB4
WAGE	1.000	0.298	0.028	-0.127	-0.062	0.131	-0.014	0.006	0.010	-0.003
EDUC	0.298	1.000	-0.072	-0.173	-0.098	0.082	-0.030	0.020	0.018	-0.017
AGE	0.028	-0.072	1.000	-0.019	0.011	-0.011	-0.119	0.156	-0.035	-0.081
RACE	-0.127	-0.173	-0.019	1.000	-0.084	-0.139	-0.005	-0.028	0.053	-0.012
SMSA	-0.062	-0.098	0.011	-0.084	1.000	0.007	-0.010	0.025	-0.016	-0.015
MARRIED0.131	0.082	-0.011	-0.139	0.007	1.000	0.036	-0.017	-0.025	0.009	0.033
QOB	-0.014	-0.030	-0.119	-0.005	-0.010	0.036	1.000	-0.768	-0.277	0.252
QOB1	0.006	0.020	0.156	-0.028	0.025	-0.017	-0.768	1.000	-0.320	-0.331
QOB2	0.010	0.018	-0.035	0.053	-0.016	-0.025	-0.277	-0.320	1.000	-0.344
QOB3	-0.003	-0.017	-0.081	-0.012	-0.015	0.009	0.252	-0.331	-0.344	1.000
QOB4	-0.013	-0.020	-0.035	-0.014	0.007	0.033	0.771	-0.323	-0.335	-0.347
										1.000

[[3]]

Table 12: Correlation Matrix for REGION: 3

	WAGE	EDUC	AGE	RACE	SMSA	MARRIEDQOB	QOB1	QOB2	QOB3	QOB4
WAGE	1.000	0.305	-0.012	-0.097	-0.142	0.116	-0.046	0.038	0.012	-0.020
EDUC	0.305	1.000	-0.078	-0.107	-0.093	-0.007	0.032	-0.006	-0.022	-0.023
AGE	-0.012	-0.078	1.000	-0.029	-0.033	-0.019	-0.119	0.149	-0.057	-0.032
RACE	-0.097	-0.107	-0.029	1.000	-0.115	-0.075	-0.015	0.021	-0.011	-0.004
SMSA	-0.142	-0.093	-0.033	-0.115	1.000	0.049	-0.005	0.008	-0.006	0.000
MARRIED0.116	-0.007	-0.019	-0.075	0.049	1.000	-0.057	0.013	0.057	-0.002	-0.067
QOB	-0.046	0.032	-0.119	-0.015	-0.005	-0.057	1.000	-0.786	-0.242	0.267
QOB1	0.038	-0.006	0.149	0.021	0.008	0.013	-0.786	1.000	-0.331	-0.348
QOB2	0.012	-0.022	-0.057	-0.011	-0.006	0.057	-0.242	-0.331	1.000	-0.327
QOB3	-0.020	-0.023	-0.032	-0.004	0.000	-0.002	0.267	-0.348	-0.327	1.000
QOB4	-0.031	0.051	-0.064	-0.007	-0.003	-0.067	0.769	-0.339	-0.319	-0.335
										1.000

[[4]]

Table 13: Correlation Matrix for REGION: 4

	WAGE	EDUC	AGE	RACE	SMSA	MARRIEDQOB	QOB1	QOB2	QOB3	QOB4	
WAGE	1.000	0.333	-0.032	-0.097	-0.164	0.153	0.045	-0.045	-0.003	0.027	0.022
EDUC	0.333	1.000	-0.085	-0.076	-0.164	0.082	0.064	-0.061	0.028	-0.029	0.069
AGE	-0.032	-0.085	1.000	0.015	0.010	0.021	-0.104	0.119	-0.030	-0.042	-0.053
RACE	-0.097	-0.076	0.015	1.000	-0.057	-0.075	-0.037	0.053	-0.038	0.004	-0.023
SMSA	-0.164	-0.164	0.010	-0.057	1.000	0.031	-0.010	0.013	0.022	-0.058	0.025
MARRIED0.153	0.153	0.082	0.021	-0.075	0.031	1.000	-0.018	-0.012	0.024	0.033	-0.049
QOB	0.045	0.064	-0.104	-0.037	-0.010	-0.018	1.000	-0.791	-0.215	0.324	0.741
QOB1	-0.045	-0.061	0.119	0.053	0.013	-0.012	-0.791	1.000	-0.351	-0.375	-0.316
QOB2	-0.003	0.028	-0.030	-0.038	0.022	0.024	-0.215	-0.351	1.000	-0.348	-0.294
QOB3	0.027	-0.029	-0.042	0.004	-0.058	0.033	0.324	-0.375	-0.348	1.000	-0.313
QOB4	0.022	0.069	-0.053	-0.023	0.025	-0.049	0.741	-0.316	-0.294	-0.313	1.000

[[5]]

Table 14: Correlation Matrix for REGION: 5

	WAGE	EDUC	AGE	RACE	SMSA	MARRIEDQOB	QOB1	QOB2	QOB3	QOB4	
WAGE	1.000	0.395	-0.017	-0.177	-0.125	0.122	0.034	-0.028	-0.003	0.001	0.030
EDUC	0.395	1.000	-0.079	-0.193	-0.185	0.034	0.032	-0.046	0.039	-0.017	0.026
AGE	-0.017	-0.079	1.000	0.017	-0.032	0.052	-0.103	0.138	-0.034	-0.083	-0.022
RACE	-0.177	-0.193	0.017	1.000	0.008	-0.157	-0.037	0.008	0.029	0.018	-0.053
SMSA	-0.125	-0.185	-0.032	0.008	1.000	0.023	-0.019	0.025	-0.028	0.028	-0.027
MARRIED0.122	0.122	0.034	0.052	-0.157	0.023	1.000	-0.003	0.000	-0.004	0.013	-0.010
QOB	0.034	0.032	-0.103	-0.037	-0.019	-0.003	1.000	-0.791	-0.245	0.257	0.769
QOB1	-0.028	-0.046	0.138	0.008	0.025	0.000	-0.791	1.000	-0.317	-0.352	-0.345
QOB2	-0.003	0.039	-0.034	0.029	-0.028	-0.004	-0.245	-0.317	1.000	-0.321	-0.314
QOB3	0.001	-0.017	-0.083	0.018	0.028	0.013	0.257	-0.352	-0.321	1.000	-0.350
QOB4	0.030	0.026	-0.022	-0.053	-0.027	-0.010	0.769	-0.345	-0.314	-0.350	1.000

[[6]]

Table 15: Correlation Matrix for REGION: 6

	WAGE	EDUC	AGE	RACE	SMSA	MARRIEDQOB	QOB1	QOB2	QOB3	QOB4	
WAGE	1.000	0.235	-0.042	-0.100	-0.130	0.038	0.088	-0.059	-0.029	0.008	0.082
EDUC	0.235	1.000	-0.099	-0.189	-0.114	0.097	0.082	-0.078	-0.008	0.043	0.045
AGE	-0.042	-0.099	1.000	0.019	0.027	-0.012	-0.109	0.149	-0.073	-0.035	-0.048
RACE	-0.100	-0.189	0.019	1.000	0.041	-0.203	0.026	-0.030	0.018	-0.008	0.022
SMSA	-0.130	-0.114	0.027	0.041	1.000	0.070	-0.052	0.022	0.037	-0.006	-0.054
MARRIED0.038	0.038	0.097	-0.012	-0.203	0.070	1.000	0.009	-0.030	0.023	0.024	-0.016
QOB	0.088	0.082	-0.109	0.026	-0.052	0.009	1.000	-0.795	-0.223	0.283	0.762
QOB1	-0.059	-0.078	0.149	-0.030	0.022	-0.030	-0.795	1.000	-0.335	-0.363	-0.342
QOB2	-0.029	-0.008	-0.073	0.018	0.037	0.023	-0.223	-0.335	1.000	-0.323	-0.305
QOB3	0.008	0.043	-0.035	-0.008	-0.006	0.024	0.283	-0.363	-0.323	1.000	-0.330
QOB4	0.082	0.045	-0.048	0.022	-0.054	-0.016	0.762	-0.342	-0.305	-0.330	1.000

[[7]]

Table 16: Correlation Matrix for REGION: 7

	WAGE	EDUC	AGE	RACE	SMSA	MARRIEDQOB	QOB1	QOB2	QOB3	QOB4	
WAGE	1.000	0.377	0.033	-0.149	-0.122	0.130	0.027	-0.023	-0.041	0.070	-0.014
EDUC	0.377	1.000	-0.058	-0.179	-0.164	0.007	0.049	-0.034	-0.008	-0.007	0.050
AGE	0.033	-0.058	1.000	0.010	-0.023	0.055	-0.168	0.184	-0.055	-0.018	-0.114
RACE	-0.149	-0.179	0.010	1.000	0.036	-0.105	0.003	-0.025	0.048	-0.020	0.002
SMSA	-0.122	-0.164	-0.023	0.036	1.000	0.033	0.039	-0.007	-0.036	-0.010	0.052
MARRIED	0.130	0.007	0.055	-0.105	0.033	1.000	-0.025	-0.013	0.036	0.034	-0.057
QOB	0.027	0.049	-0.168	0.003	0.039	-0.025	1.000	-0.797	-0.251	0.273	0.743
QOB1	-0.023	-0.034	0.184	-0.025	-0.007	-0.013	-0.797	1.000	-0.289	-0.380	-0.322
QOB2	-0.041	-0.008	-0.055	0.048	-0.036	0.036	-0.251	-0.289	1.000	-0.338	-0.287
QOB3	0.070	-0.007	-0.018	-0.020	-0.010	0.034	0.273	-0.380	-0.338	1.000	-0.377
QOB4	-0.014	0.050	-0.114	0.002	0.052	-0.057	0.743	-0.322	-0.287	-0.377	1.000

[[8]]

Table 17: Correlation Matrix for REGION: 8

	WAGE	EDUC	AGE	RACE	SMSA	MARRIEDQOB	QOB1	QOB2	QOB3	QOB4	
WAGE	1.000	0.294	0.012	-0.127	-0.176	0.011	0.030	-0.040	0.033	-0.020	0.028
EDUC	0.294	1.000	-0.067	-0.087	-0.158	0.037	0.062	-0.064	0.019	-0.003	0.050
AGE	0.012	-0.067	1.000	-0.065	-0.036	0.029	-0.138	0.198	-0.071	-0.109	-0.025
RACE	-0.127	-0.087	-0.065	1.000	-0.065	-0.015	0.070	-0.107	0.041	0.064	0.006
SMSA	-0.176	-0.158	-0.036	-0.065	1.000	0.057	0.046	-0.038	0.000	-0.005	0.044
MARRIED	0.011	0.037	0.029	-0.015	0.057	1.000	0.009	0.002	0.007	-0.046	0.036
QOB	0.030	0.062	-0.138	0.070	0.046	0.009	1.000	-0.789	-0.236	0.259	0.777
QOB1	-0.040	-0.064	0.198	-0.107	-0.038	0.002	-0.789	1.000	-0.335	-0.344	-0.351
QOB2	0.033	0.019	-0.071	0.041	0.000	0.007	-0.236	-0.335	1.000	-0.316	-0.322
QOB3	-0.020	-0.003	-0.109	0.064	-0.005	-0.046	0.259	-0.344	-0.316	1.000	-0.331
QOB4	0.028	0.050	-0.025	0.006	0.044	0.036	0.777	-0.351	-0.322	-0.331	1.000

[[9]]

Table 18: Correlation Matrix for REGION: 9

	WAGE	EDUC	AGE	RACE	SMSA	MARRIEDQOB	QOB1	QOB2	QOB3	QOB4	
WAGE	1.000	0.268	0.034	-0.098	-0.067	0.079	-0.003	0.005	-0.015	0.022	-0.013
EDUC	0.268	1.000	-0.042	-0.066	-0.127	-0.019	0.045	-0.041	0.017	-0.026	0.052
AGE	0.034	-0.042	1.000	-0.017	-0.018	0.044	-0.130	0.167	-0.070	-0.027	-0.070
RACE	-0.098	-0.066	-0.017	1.000	-0.066	-0.096	-0.032	0.021	0.035	-0.050	-0.004
SMSA	-0.067	-0.127	-0.018	-0.066	1.000	0.044	-0.009	0.021	0.009	-0.056	0.029
MARRIED	0.079	-0.019	0.044	-0.096	0.044	1.000	0.018	-0.011	-0.010	0.005	0.016
QOB	-0.003	0.045	-0.130	-0.032	-0.009	0.018	1.000	-0.777	-0.260	0.281	0.755
QOB1	0.005	-0.041	0.167	0.021	0.021	-0.011	-0.777	1.000	-0.323	-0.352	-0.314
QOB2	-0.015	0.017	-0.070	0.035	0.009	-0.010	-0.260	-0.323	1.000	-0.352	-0.315
QOB3	0.022	-0.026	-0.027	-0.050	-0.056	0.005	0.281	-0.352	-0.352	1.000	-0.343
QOB4	-0.013	0.052	-0.070	-0.004	0.029	0.016	0.755	-0.314	-0.315	-0.343	1.000

Warning: Use of ‘df\$EDUC’ is discouraged.
i Use ‘EDUC’ instead.

```
## Warning: Use of 'df$WAGE' is discouraged.
## i Use 'WAGE' instead.
```



3. Estimate the baseline Specification

```
#linear model
linear_model1=lm(WAGE~ EDUC + AGE + RACE + SMSA + MARRIED + REGION2 + REGION3
+ REGION4 + REGION5 + REGION6 + REGION7 + REGION8 + REGION9 , data = df)

##
## -----
##                               Dependent variable:
## -----
##                               WAGE
## -----
## EDUC                         26.696***  

##                               (0.869)  

##                               t = 30.724  

##                               p = 0.000  

## AGE                          2.244**  

##                               (0.942)  

##                               t = 2.382  

##                               p = 0.018
```

```

## RACE           -77.632***  

##                           (10.225)  

##                           t = -7.592  

##                           p = 0.000  

## SMSA          -63.756***  

##                           (7.393)  

##                           t = -8.624  

##                           p = 0.000  

## MARRIED       78.001***  

##                           (8.026)  

##                           t = 9.719  

##                           p = 0.000  

## REGION2      41.193***  

##                           (13.644)  

##                           t = 3.019  

##                           p = 0.003  

## REGION3      49.088***  

##                           (13.305)  

##                           t = 3.690  

##                           p = 0.0003  

## REGION4      18.845  

##                           (15.600)  

##                           t = 1.208  

##                           p = 0.228  

## REGION5      4.205  

##                           (13.492)  

##                           t = 0.312  

##                           p = 0.756  

## REGION6      7.604  

##                           (16.123)  

##                           t = 0.472  

##                           p = 0.638  

## REGION7      17.081  

##                           (14.646)  

##                           t = 1.166  

##                           p = 0.244  

## REGION8      15.897  

##                           (17.106)  

##                           t = 0.929  

##                           p = 0.353  

## REGION9      63.710***  

##                           (14.046)  

##                           t = 4.536  

##                           p = 0.00001  

## Constant     -81.669*  

##                           (46.614)  

##                           t = -1.752  

##                           p = 0.080  

## -----  

## Observations    10,000  

## R2              0.134  

## Adjusted R2     0.133  

## Residual Std. Error   275.029 (df = 9986)  

## F Statistic     118.953*** (df = 13; 9986) (p = 0.000)

```

```

## =====
## Note: *p<0.1; **p<0.05; ***p<0.01

```

3.1 Interpretation of the estimated Coefficient

EDUC (26.696, p < 0.001)

A one-year increase in education leads to a \$26.696 increase in weekly wages. This is highly significant (p = 0.000).

AGE (2.244, p = 0.020) A one-year increase in age increases wages by \$2.23. Significant at the 5% level (p = 0.020).

RACE (-77.478, p < 0.001) Suggests a wage penalty of \$77.48 for certain racial groups (assuming a binary variable where non-white = 1). Highly significant (p = 0.000).

SMSA (-63.654, p < 0.001) Living in an SMSA (Standard Metropolitan Statistical Area) is associated with a \$63.65 lower wage. Significant at p = 0.000.

MARRIED (77.927, p < 0.001) Being married increases wages by \$77.93. Highly significant (p = 0.000).

Regional Effects on Wages

Significant Regions: REGION2 (B = 41.204, p = 0.003) REGION3 (B = 49.071, p = 0.0003) REGION9 (B = 63.543, p = 0.00001) These regions have higher wages compared to the reference region.

Non-Significant Regions: REGION4, REGION5, REGION6, REGION7, REGION8 (p > 0.05) These regions do not significantly differ from the reference region in terms of wages.

```

##
## Linear hypothesis test:
## AGE = 0
## RACE = 0
## MARRIED = 0
## SMSA = 0
##
## Model 1: restricted model
## Model 2: WAGE ~ EDUC + AGE + RACE + SMSA + MARRIED + REGION2 + REGION3 +
##           REGION4 + REGION5 + REGION6 + REGION7 + REGION8 + REGION9
##
##   Res.Df      RSS Df Sum of Sq      F    Pr(>F)
## 1  9990  773577022
## 2  9986  755352847  4  18224175 60.232 < 2.2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

```

The joint significance test shows that AGE, RACE, MARRIED, and SMSA jointly contribute significantly to explaining WAGE. The F-statistic is 60.23 with a p-value < 2.2e-16, indicating strong statistical significance. We reject the null hypothesis that these four variables have no joint effect on wages.

```

##
## Linear hypothesis test:
## REGION2 = 0
## REGION3 = 0
## REGION4 = 0
## REGION5 = 0
## REGION6 = 0

```

```

## REGION7 = 0
## REGION8 = 0
## REGION9 = 0
##
## Model 1: restricted model
## Model 2: WAGE ~ EDUC + AGE + RACE + SMSA + MARRIED + REGION2 + REGION3 +
##           REGION4 + REGION5 + REGION6 + REGION7 + REGION8 + REGION9
##
##   Res.Df      RSS Df Sum of Sq    F    Pr(>F)
## 1 9994 759957235
## 2 9986 755352847  8   4604388 7.6089 3.389e-10 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

```

Since the p-value is extremely small (<0.001), we reject the null hypothesis. This means that at least one of the region coefficients is significantly different from zero, implying that region does have a statistically significant effect on wages.

4. Evaluating Gauss-Markov Assumptions and Applying Remedial Measures

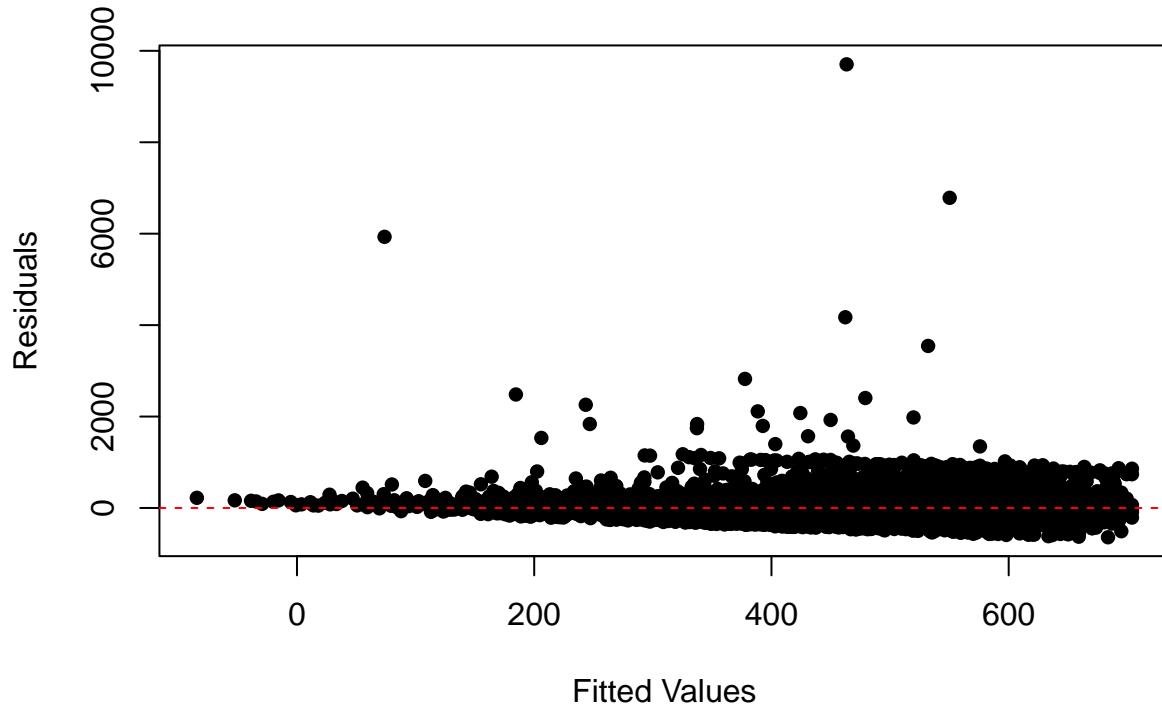
4a. Stochastic Regressors All the variables are stochastic.

Strictly exogenous: AGE, RACE, QOB - These variables are fixed and not influenced by wages or the error term.

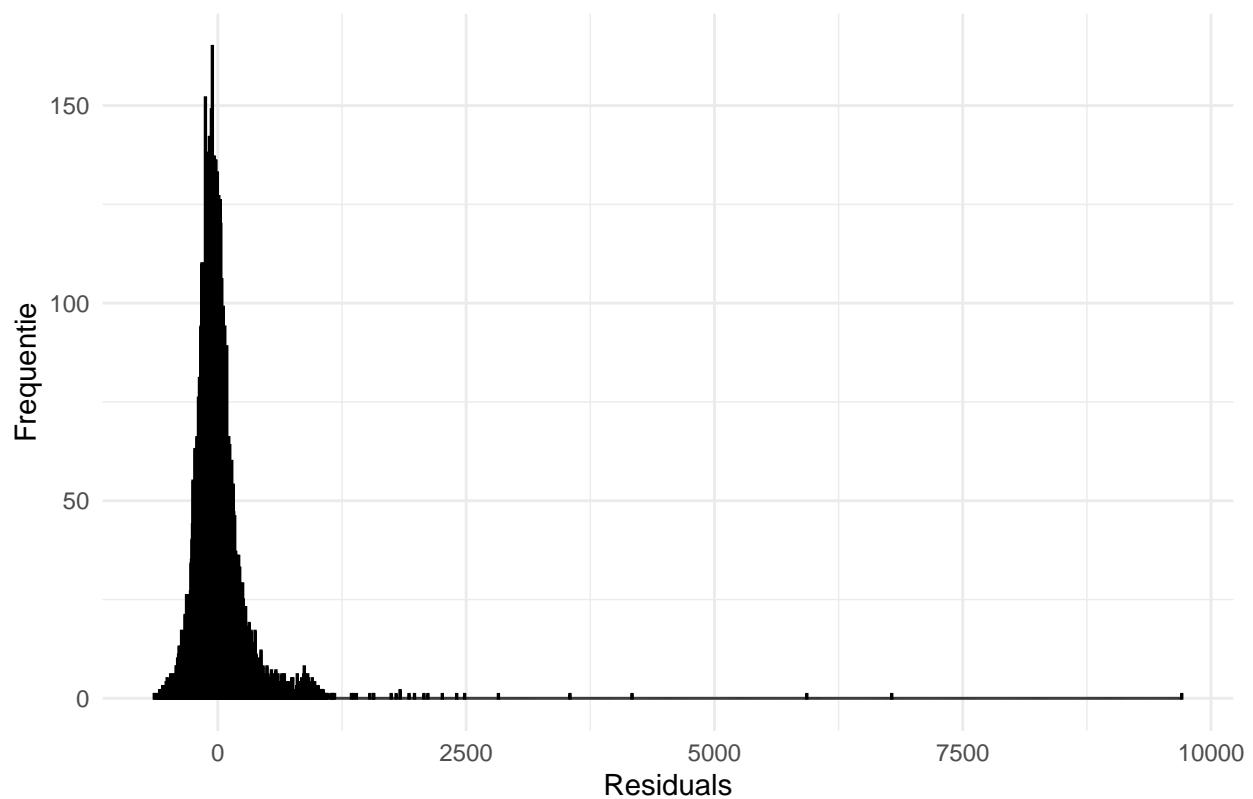
Weakly exogenous: SMSA, REGION, MARRIED - These variables may be correlated with unobserved factors affecting wages, but are not directly influenced by wages.

Endogenous: EDUC, - Potential for reverse causality or correlation with the error term (e.g., higher wages → more education or higher likelihood of marriage).

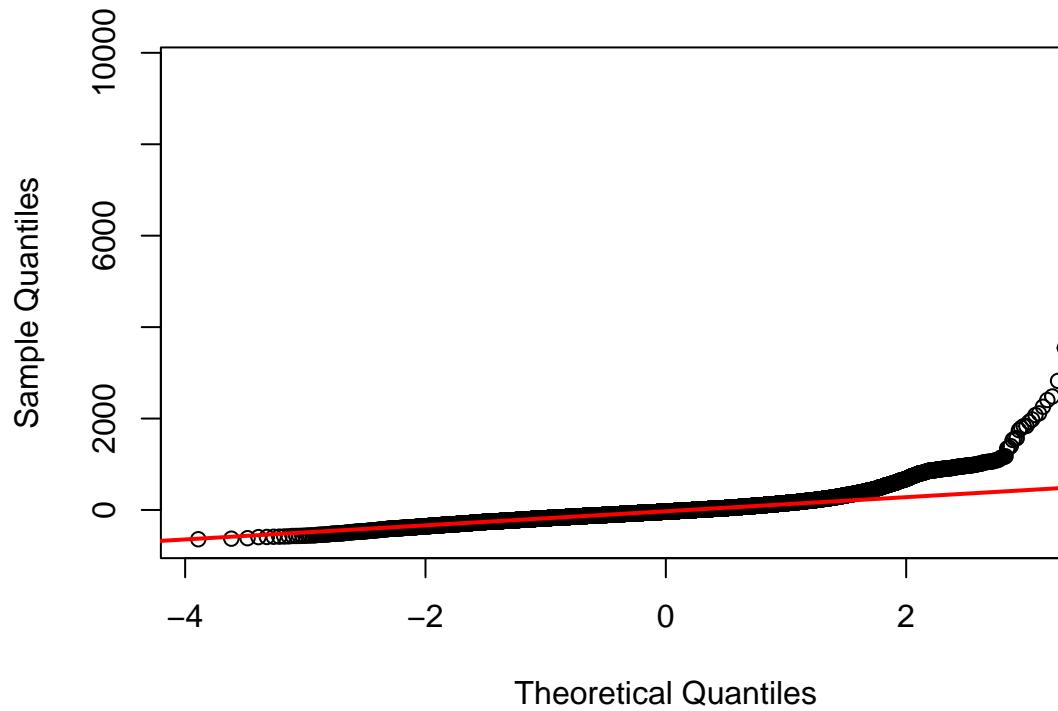
Residuals vs. Fitted Values



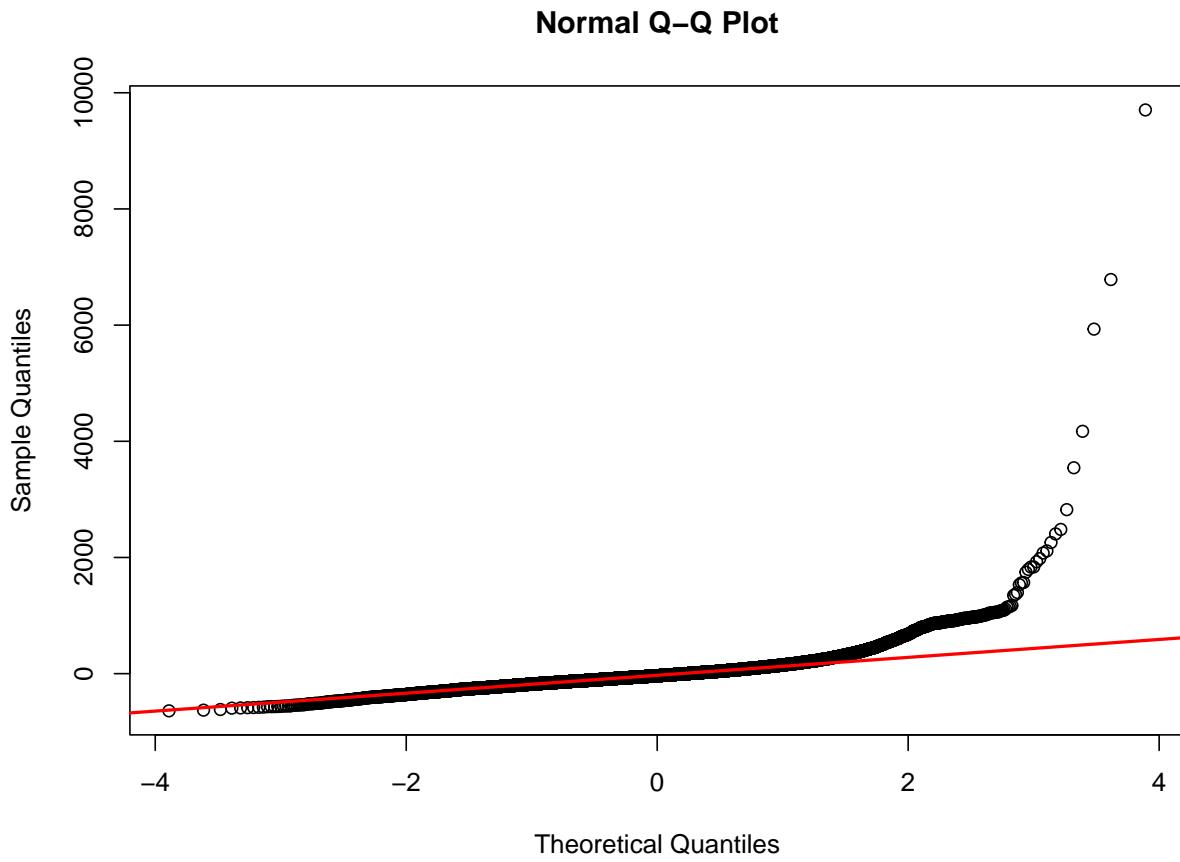
Histogram Residuals



Normal Q–Q Plot



4b. Normality Error Terms



Jarque Bera Test

data: residuals X-squared = 21722226, df = 2, p-value < 2.2e-16

Since the p value is small, we reject the null hypothesis. Therefore the residuals are NOT normally distributed.

Implications for OLS:

- 1) Unbiasedness: Non-normal residuals do not affect the unbiasedness of OLS estimators as long as key assumptions (like linearity, exogeneity of regressors, and zero-mean error term) are satisfied.
- 2) Efficiency: OLS estimators may lose their efficiency because normality of residuals is necessary for OLS to be the Best Linear Unbiased Estimator (BLUE) under the Gauss-Markov assumptions.
- 3) Hypothesis Testing: The validity of tests (e.g., t-tests and F-tests) and confidence intervals relies on the normality assumption. Non-normal residuals can lead to incorrect p-values.
- 4) Heteroscedasticity or Outliers: Non-normality can signal issues like heteroscedasticity (non-constant error variance), the presence of outliers, or omitted variable bias.

Remediation: Based on the histogram and Q-Q Plot which is slightly right-skewed, we can conclude that the residuals deviate slightly from normality, so OLS may still perform adequately, particularly in our large sample where asymptotic normality applies.

```
###Gauss-Markov assumptions:
```

```
Assumption 1: Linearity in the parameters: CHECK
```

```
Assumption 2a: The X -values are fixed over repeated sampling (fixed regressor model) FAIL
```

```
Correlation between explanatory variables and error terms:
```

```
print(mean(residuals))
```

```
## [1] -4.605628e-16
```

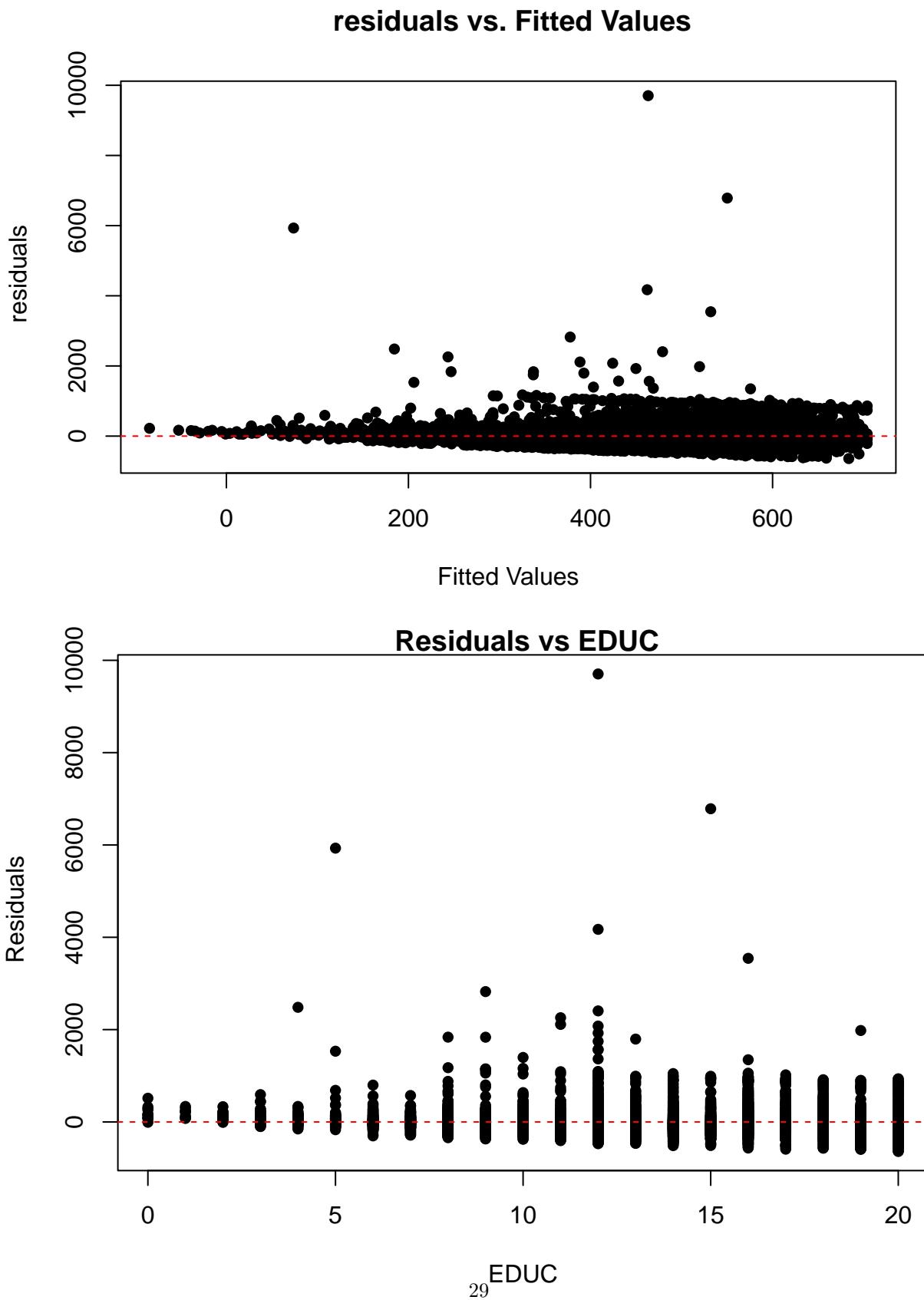
```
print(t.test(residuals, mu = 0))
```

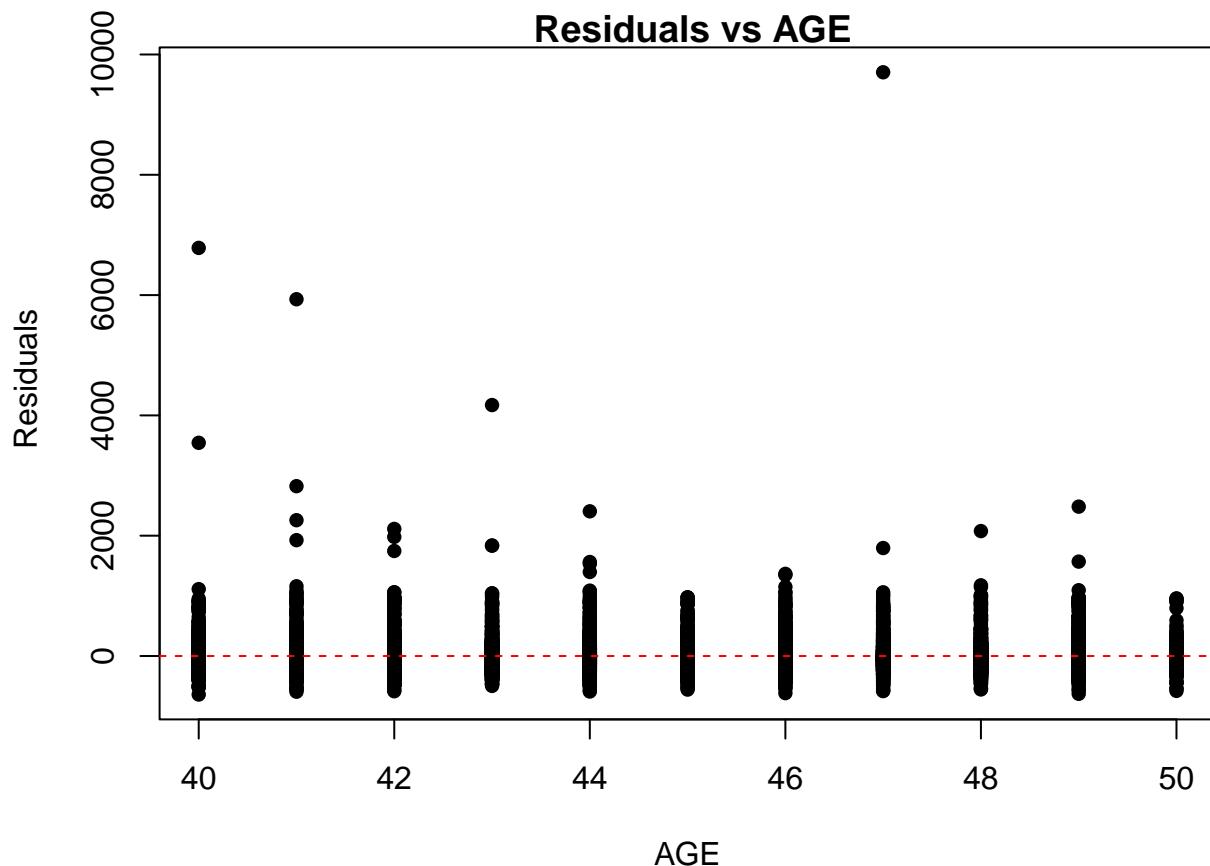
```
##  
##  One Sample t-test  
##  
## data: residuals  
## t = -1.6757e-16, df = 9999, p-value = 1  
## alternative hypothesis: true mean is not equal to 0  
## 95 percent confidence interval:  
## -5.387624 5.387624  
## sample estimates:  
##      mean of x  
## -4.605628e-16
```

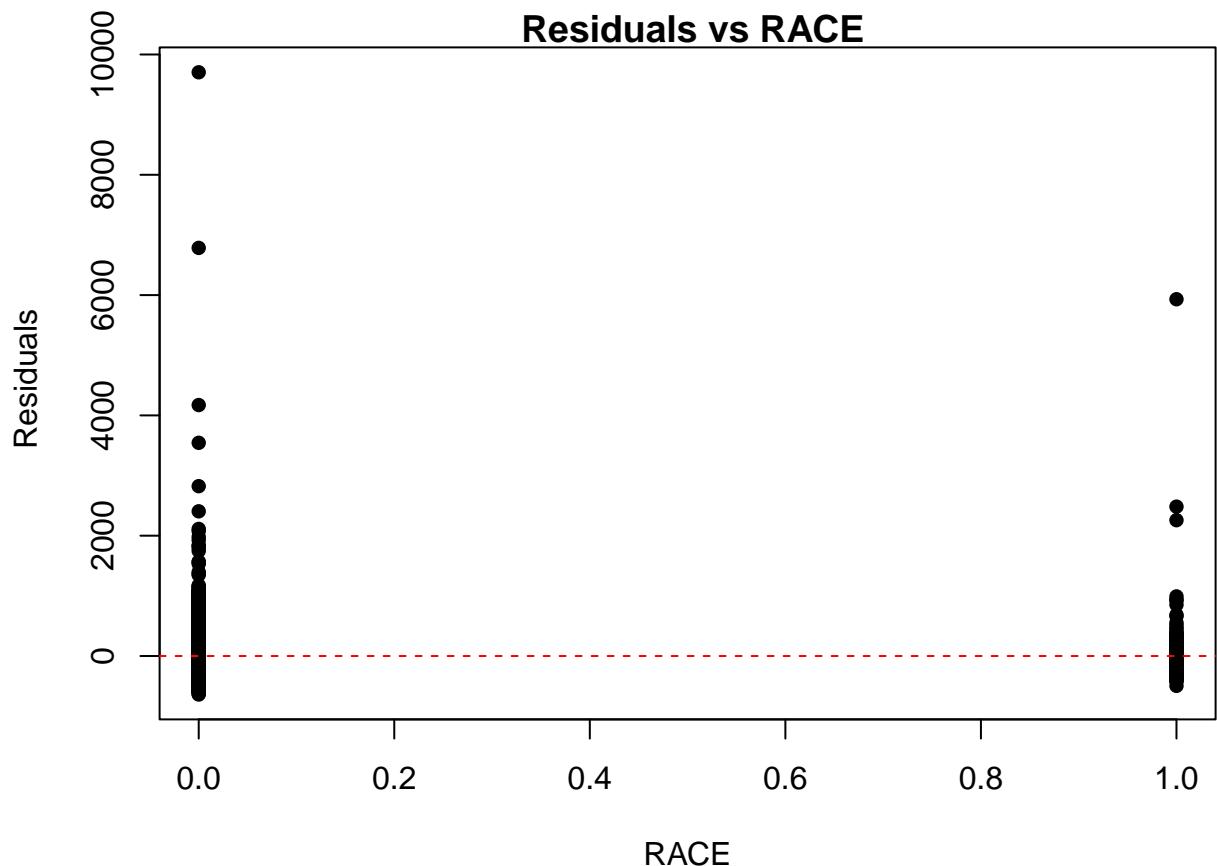
```
if (show_interpretation) {  
  cat("Assumption 3: The expected V value of the error terms is zero: CHECK")  
}
```

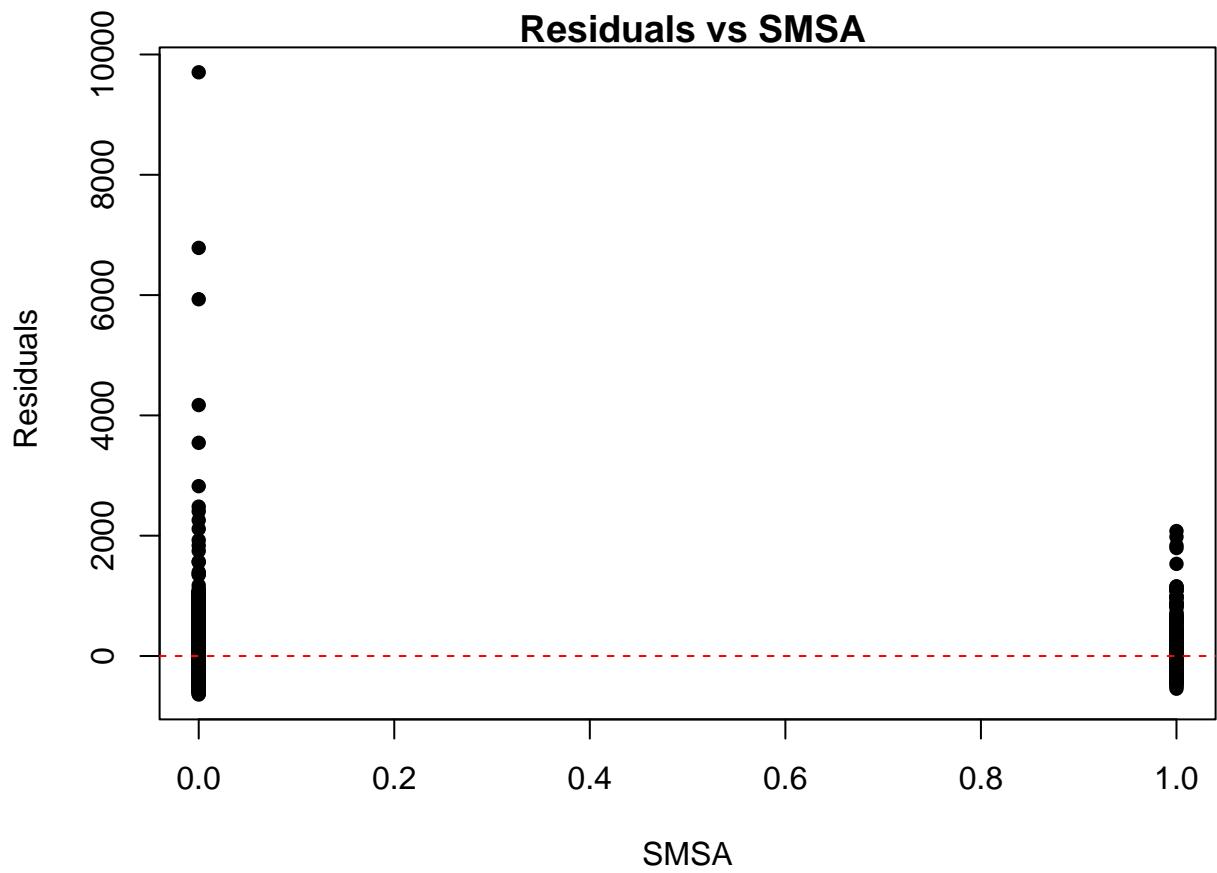
```
## Assumption 3: The expected V value of the error terms is zero: CHECK
```

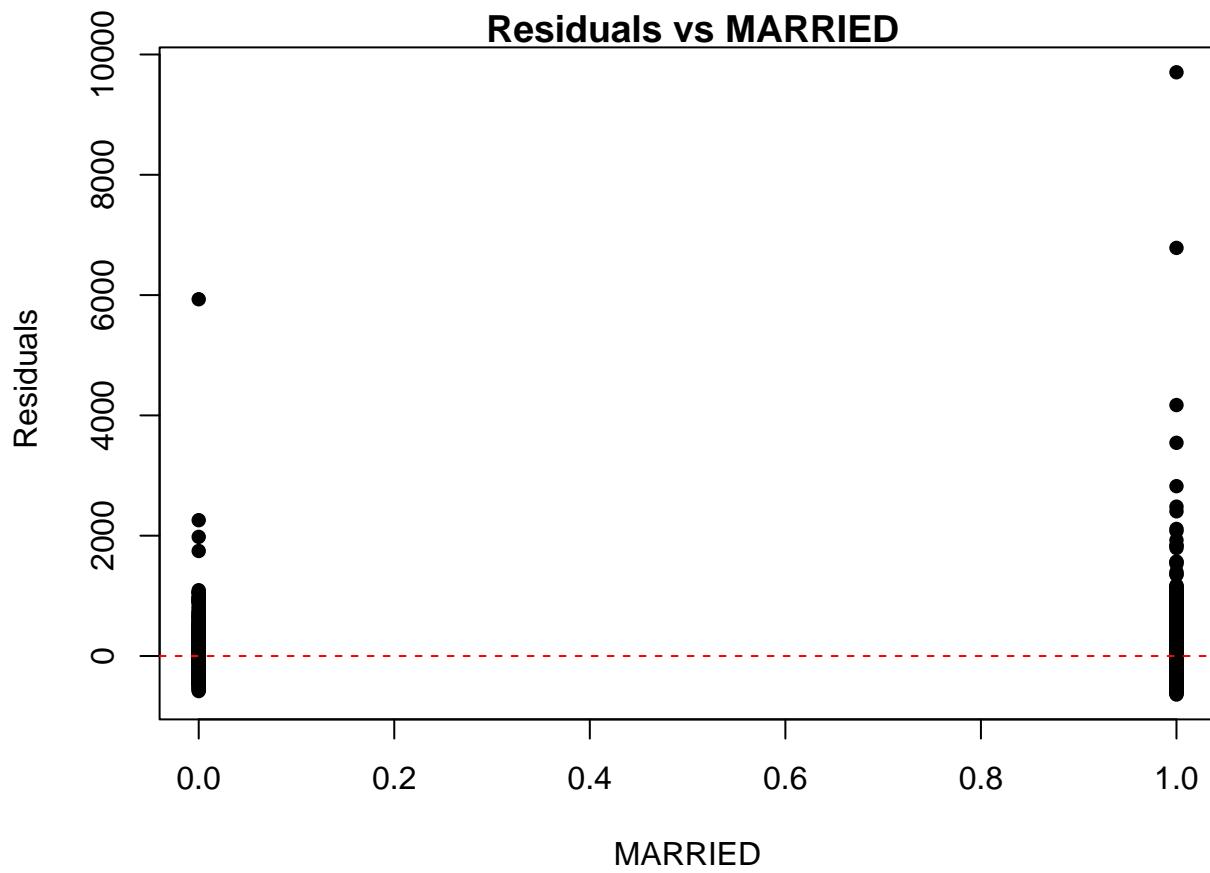
4(d) Heteroskedasticity

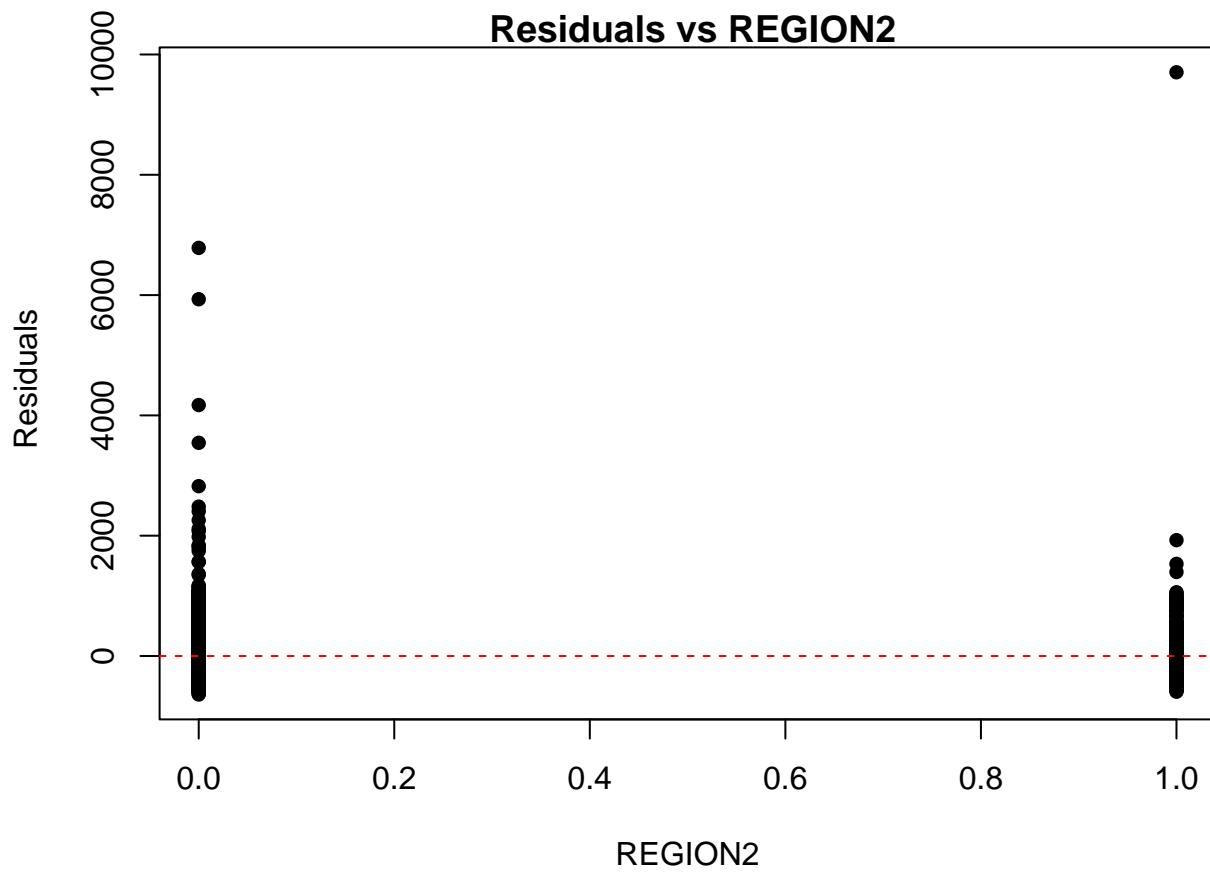


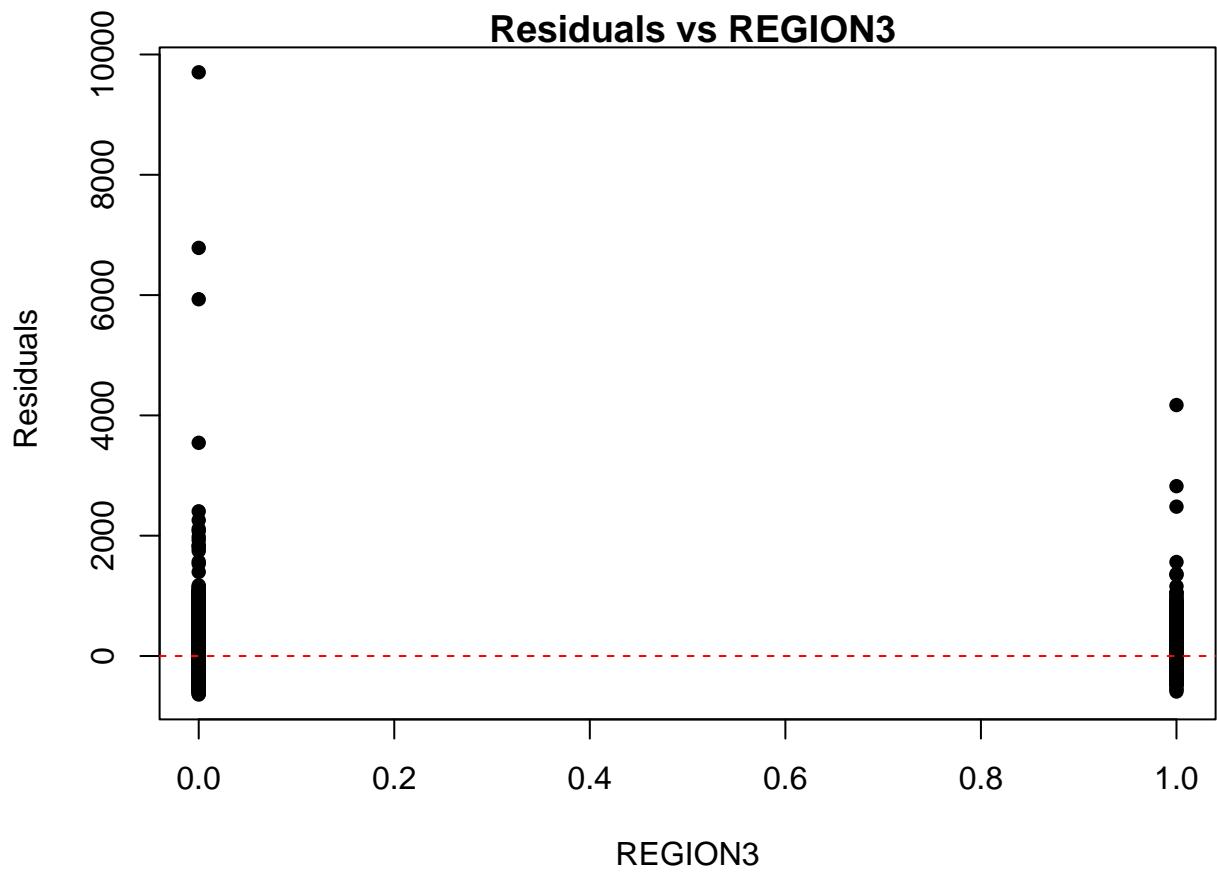


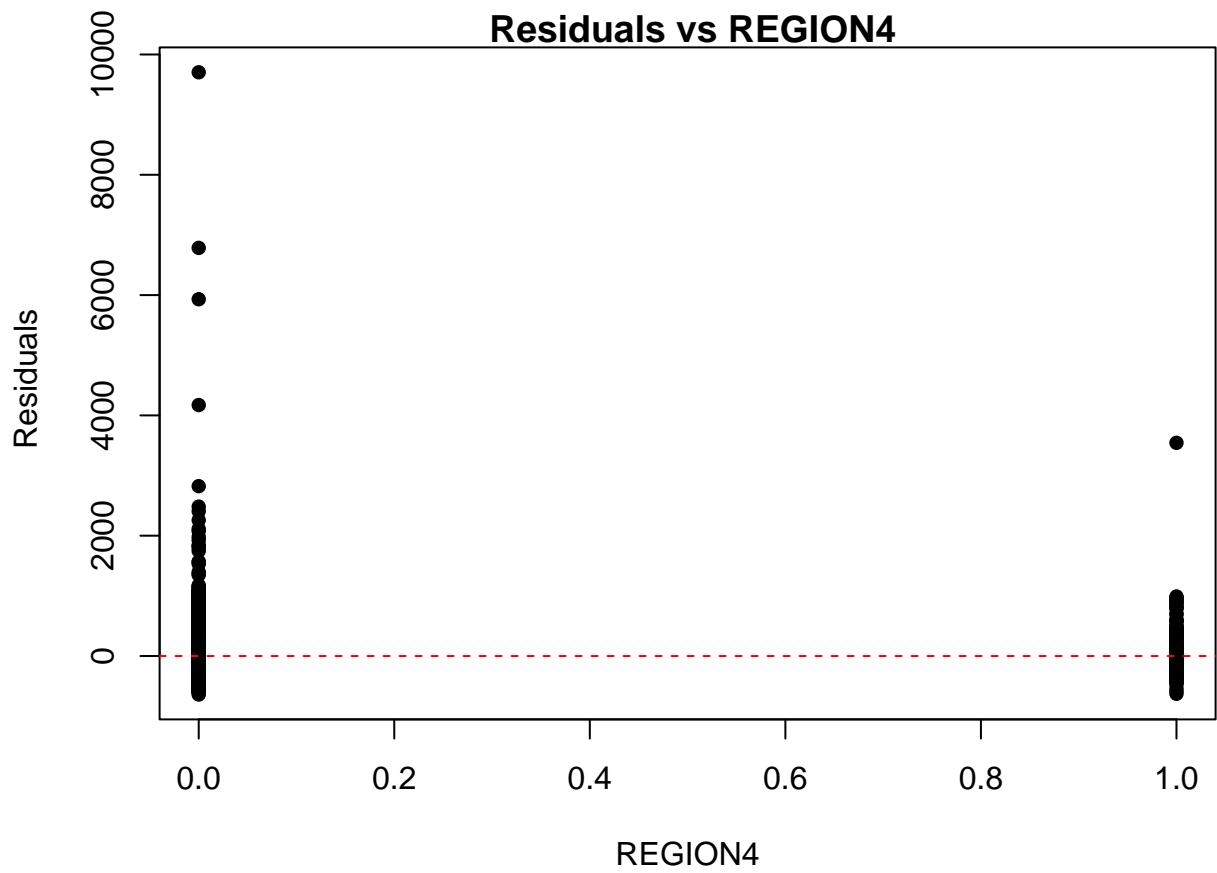


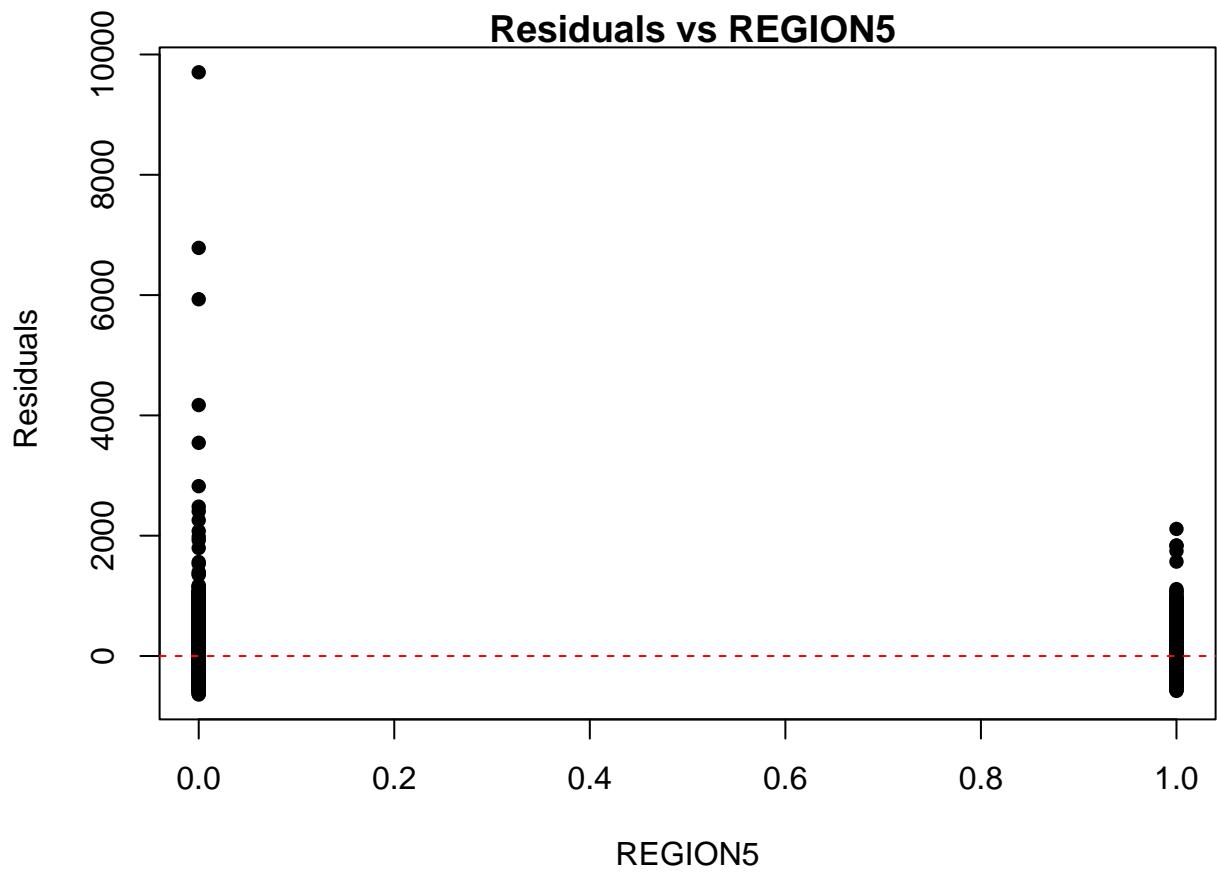


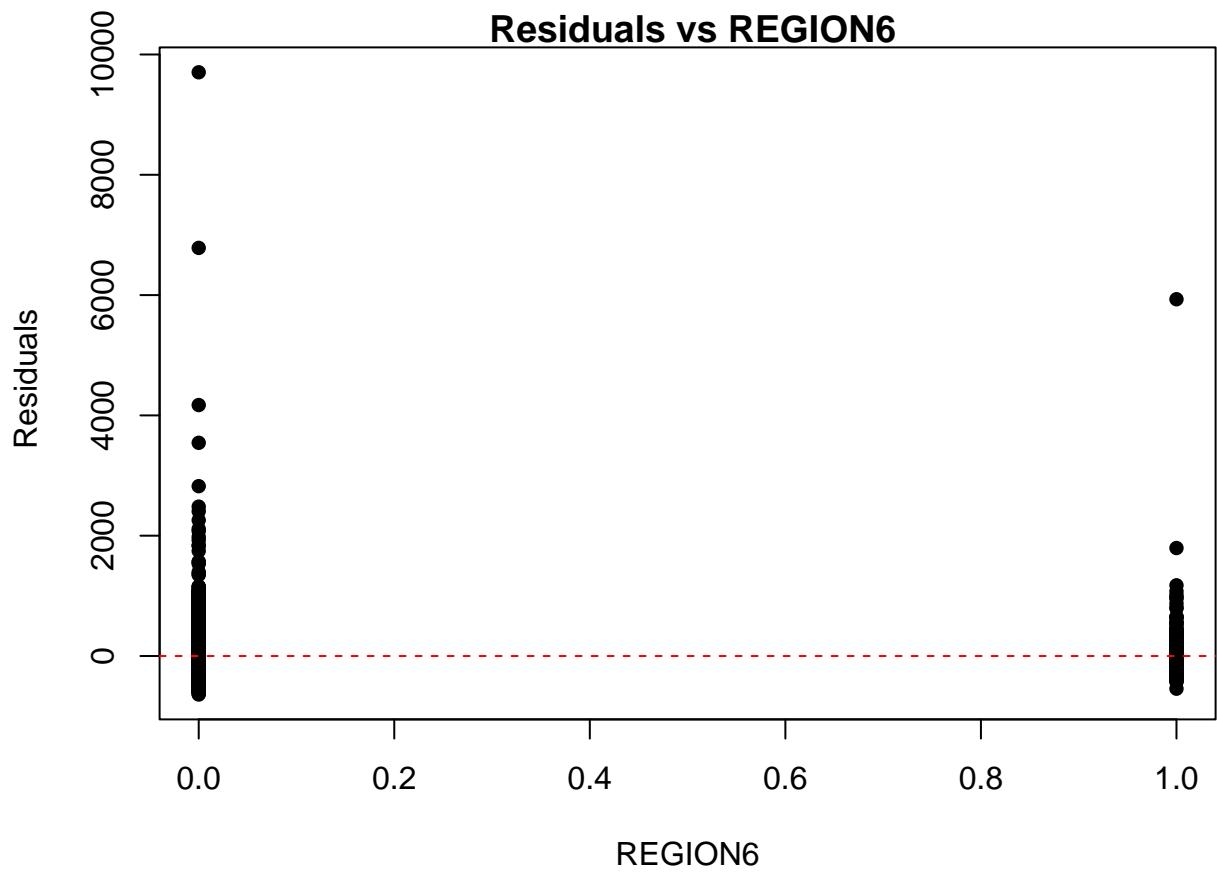


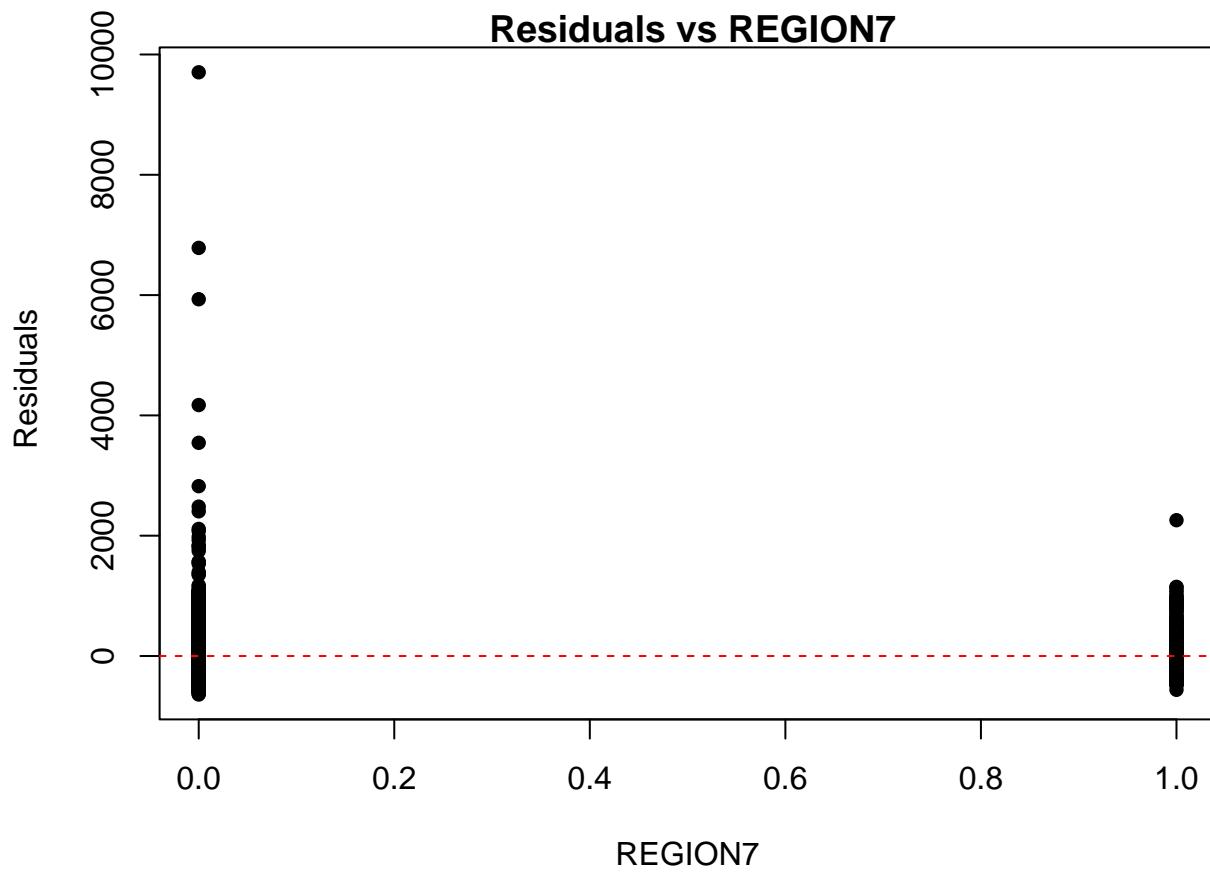


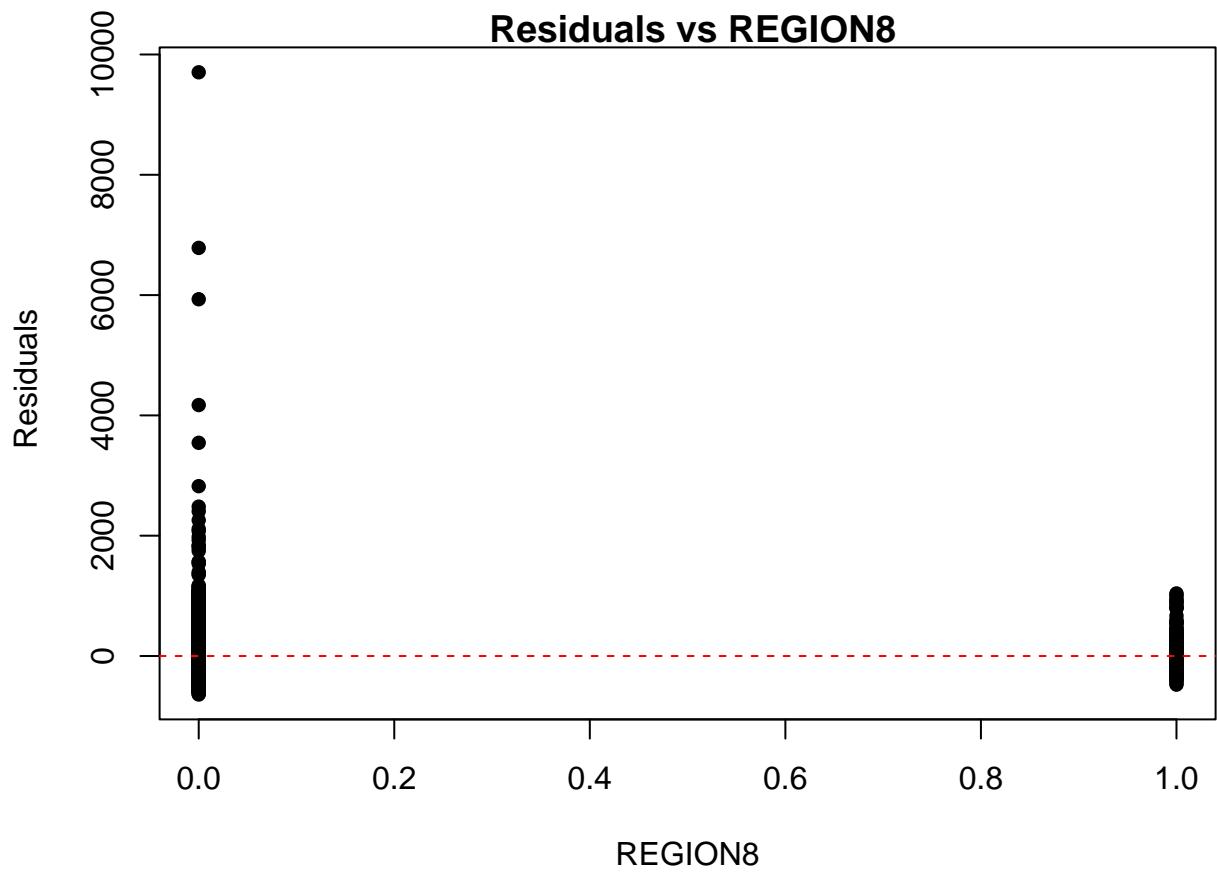


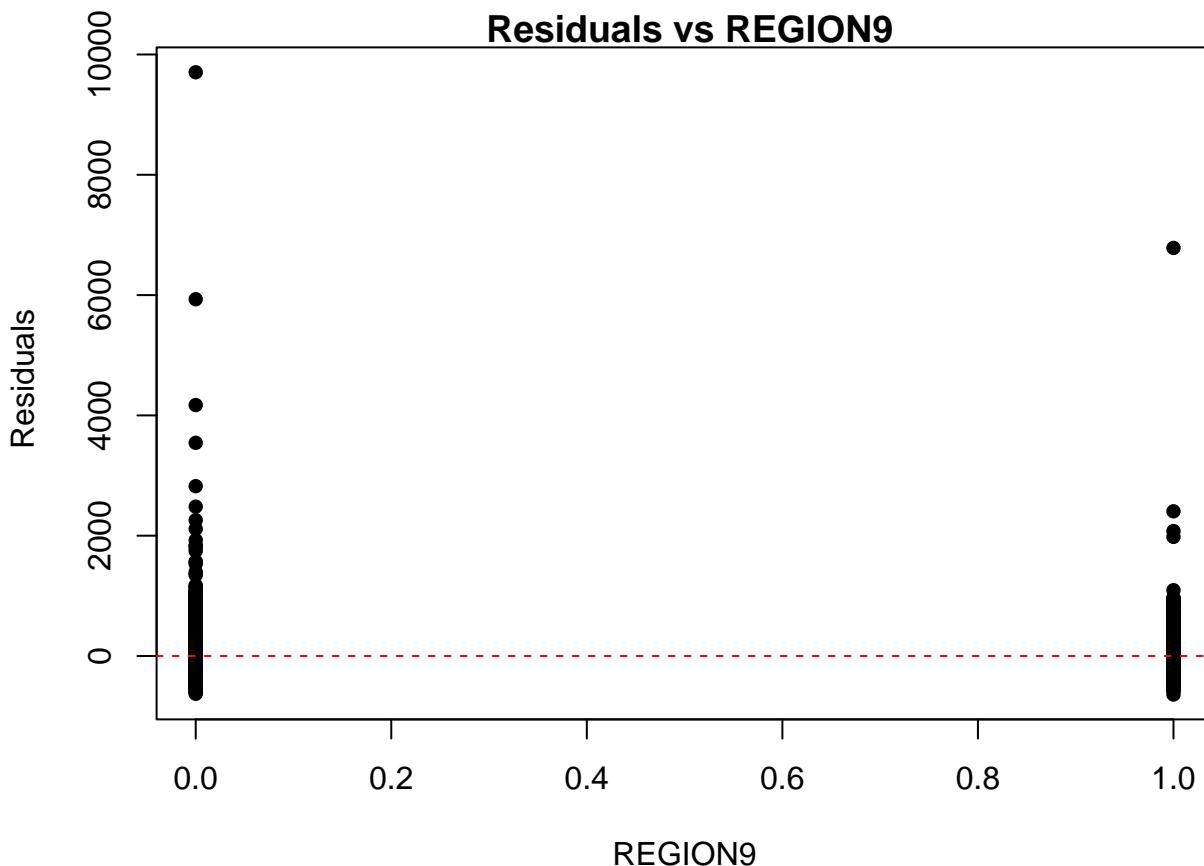












Interpretation of Residual Plots:

1. Fitted Values:

- increase in spread as fitted values increase
- This pattern indicates potential heteroskedasticity

2. EDUC:

- Initial increase followed by decrease in spread
- Non-constant variance pattern visible

3. RACE:

- Larger spread observed for white participants
- Unequal variance between groups (aigan moer white than black , see SMSA)

4. SMSA:

- Non-metropolitan areas show greater spread
- Variance differs by urban/rural classification, but this is our data is more or less 90 percent non metropolitan (more data , more spread)

5. MARRIED:

- Similar variance pattern as other categorical variables
- same thing as SMSA , 90 percent of df is married (so these are natural spreads , and the problem of heteroskedasticity will not be here)

6. REGION:

- Different spreads observed across regions (0-1 range)
- Geographic heterogeneity in variance

Conclusion:

Clear signs of heteroskedasticity across multiple predictors, suggesting violations of the constant variance assumption.

4(d).1 Heteroskedasticity test **

```
white_test <- bptest(linear_model1, ~ fitted(linear_model1) + I(fitted(linear_model1)^2))
print(white_test)
```

white's general test

studentized Breusch-Pagan test

data: linear_model1 BP = 7.0287, df = 2, p-value = 0.02977

the p value of 0.02977 says that we reject the null hypothesis, which implies that we reject homoskedasticity properties

-for white there are not properties, (idk zie slide bij verbetring stond in rood , maar ze zei er niks over

```
# Step 1: Define a trimmed version of your data (removing outliers based on WAGE, for example)
# Adjust variable and trimming logic as needed
trimmed_data <- df %>%
  filter(between(WAGE, quantile(WAGE, 0.01, na.rm = TRUE),
                 quantile(WAGE, 0.99, na.rm = TRUE)))

# Step 2: Refit your linear model on the trimmed dataset
linear_model_trimmed <- lm(WAGE ~ EDUC + AGE + RACE + SMSA + MARRIED + REGION2 + REGION3
                           + REGION4 + REGION5 + REGION6 + REGION7 + REGION8 + REGION9, data = trimmed_data)

# Step 3: White test on the new model
white_test_trimmed <- bptest(linear_model_trimmed,
                               ~ fitted(linear_model_trimmed) + I(fitted(linear_model_trimmed)^2))
print(white_test_trimmed)
```

studentized Breusch-Pagan test

data: linear_model_trimmed BP = 361.55, df = 2, p-value < 2.2e-16 even with trimmed data , we have even a smaller p value reject the null hypothesis , which implies that we reject homoskedasticity

properties

-for white there are not properties

```
gqtest_result <- gqtest(linear_model1, order.by = df$EDUC ,fraction = 1000)
print(gqtest_result)
```

goldfeld-quant test

Goldfeld-Quandt test

data: linear_model1 GQ = 1.8257, df1 = 4486, df2 = 4486, p-value < 2.2e-16 alternative hypothesis: variance increases from segment 1 to 2

very small p value , we order by educ bcs the plot educ vs residuals was a clear sign of heteroskedasticity, t we strongly reject the null hyptothis , which implies that we reject homoskedastiscity

properties

-You split the dataset into two non-overlapping groups(splitting by increasin educ) (dropping middle obser-vations).

-Errors are normally distributed., but in our case they are not so be carefull with the interpratation of this test **Implications for OLS Estimators**

-the coef(the betas) are not normally distr in small samples

-var of the coef changes , we have a more robust variance formula if we use OLS

-OLS is no longer effecient , we use EGLS wich has a lower variance of the estimator

-the st errors of the coef are underestimated and are not to be trusted, therefore infernce bout the significance is wrong **## 4(d).2 adressing heteroskedasticity**

% Table created by stargazer v.5.2.3 by Marek Hlavac, Social Policy Institute. E-mail: marek.hlavac at gmail.com % Date and time: za, mei 03, 2025 - 23:40:10

Interpretation:

- Robust standard errors are larger than conventional OLS standard errors
- This pattern confirms the presence of heteroskedasticity in the data
- The HC1 estimator provides consistent inference under heteroskedasticity
- Coefficient estimates remain unchanged, but significance levels may differ

EGLS

```
# Step 1: Get design matrix and response variable
X <- model.matrix(linear_model1) # Includes intercept & region dummies
Y <- df$WAGE

# Step 2: Get initial residuals from baseline model
resid_init <- residuals(linear_model1)

# Step 3: Create y_bar (mean of Y)
y_bar <- mean(Y)

# Step 4: Transform X and Y
X_star <- X / y_bar
Y_star <- Y / y_bar # Element-wise division
```

Table 19: OLS Regression with Robust Standard Errors

<i>Dependent variable:</i>	
	Weekly Wage
Education	26.6959*** (0.9195)
Age	2.2442** (0.9904)
Race (1=Black)	-77.6318*** (9.3993)
SMSA	-63.7562*** (6.2933)
Married	78.0013*** (7.9377)
Region 2	41.1931*** (12.5820)
Region 3	49.0879*** (10.7642)
Region 4	18.8453 (12.9618)
Region 5	4.2049 (10.9000)
Region 6	7.6044 (15.3990)
Region 7	17.0805 (12.0089)
Region 8	15.8968 (13.7353)
Region 9	63.7105*** (13.0419)
Constant	-81.6693 (49.8668)
Observations	10,000
R ²	0.1341
Adjusted R ²	0.1330
Residual Std. Error	275.0294 (df = 9986)
F Statistic	118.9531*** (df = 13; 9986)

Note:

*p<0.1; **p<0.05; ***p<0.01
 Heteroskedasticity-consistent standard errors in parentheses

```

# Step 5: HGLS estimation (no intercept needed as it's in X_star)
egls_model <- lm(Y_star ~ X_star - 1) # -1 removes additional intercept

# Display results
stargazer(egls_model, type="latex", style="all")

##
## % Table created by stargazer v.5.2.3 by Marek Hlavac, Social Policy Institute. E-mail: marek.hlavac@spio.cz
## % Date and time: za, mei 03, 2025 - 23:40:10
## \begin{table}[!htbp] \centering
##   \caption{}
##   \label{}
## \begin{tabular}{@{\extracolsep{5pt}}lc}
## \hline
## & \textit{Dependent variable:} \\
## \hline
## & Y\_star \\
## X\_star(Intercept) & -$81.669$^{*} \\
## & (46.614) \\
## & t = -$1.752 \\
## & p = 0.080 \\
## X\_starEDUC & 26.696$^{***}$ \\
## & (0.869) \\
## & t = 30.724 \\
## & p = 0.000 \\
## X\_starAGE & 2.244$^{**}$ \\
## & (0.942) \\
## & t = 2.382 \\
## & p = 0.018 \\
## X\_starRACE & -$77.632$^{***}$ \\
## & (10.225) \\
## & t = -$7.592 \\
## & p = 0.000 \\
## X\_starSMSA & -$63.756$^{***}$ \\
## & (7.393) \\
## & t = -$8.624 \\
## & p = 0.000 \\
## X\_starMARRIED & 78.001$^{***}$ \\
## & (8.026) \\
## & t = 9.719 \\
## & p = 0.000 \\
## X\_starREGION2 & 41.193$^{***}$ \\
## & (13.644) \\
## & t = 3.019 \\
## & p = 0.003 \\
## X\_starREGION3 & 49.088$^{***}$ \\
## & (13.305) \\
## & t = 3.690 \\
## & p = 0.0003 \\
## X\_starREGION4 & 18.845 \\
## & (15.600)

```

```

## & t = 1.208 \\
## & p = 0.228 \\
## X_starREGION5 & 4.205 \\
## & (13.492) \\
## & t = 0.312 \\
## & p = 0.756 \\
## X_starREGION6 & 7.604 \\
## & (16.123) \\
## & t = 0.472 \\
## & p = 0.638 \\
## X_starREGION7 & 17.081 \\
## & (14.646) \\
## & t = 1.166 \\
## & p = 0.244 \\
## X_starREGION8 & 15.897 \\
## & (17.106) \\
## & t = 0.929 \\
## & p = 0.353 \\
## X_starREGION9 & 63.710$^{***}$ \\
## & (14.046) \\
## & t = 4.536 \\
## & p = 0.00001 \\
## \hline \\[-1.8ex]
## Observations & 10,000 \\
## R$^2$ & 0.728 \\
## Adjusted R$^2$ & 0.728 \\
## Residual Std. Error & 0.630 (df = 9986) \\
## F Statistic & 1,909.866$^{***}$ (df = 14; 9986) (p = 0.000) \\
## \hline
## \hline \\[-1.8ex]
## \textit{Note:} & \multicolumn{1}{r}{$^{*}p<\$0.1$; $^{**}p<\$0.05$; $^{***}p<\$0.01$} \\
## \end{tabular}
## \end{table}

if (show_interpretation) {
  cat("\n##### **Comparison with Robust Standard Errors:**\n")
  cat("- EGLS provides more efficient estimates than OLS with robust standard errors\n")
  cat("- Standard errors are typically larger than OLS with incorrect variance formula\n")
  cat("- The efficiency gain comes from properly modeling the heteroskedasticity structure\n")
  cat("- Interpretation of coefficients remains the same as OLS\n")
}

##
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## - Interpretation of coefficients remains the same as OLS

```

Multicollinearity

Gauss-Markov assumption (no perfect multicollinearity) is met.

Analysis of Variance Inflation Factors and correlation:

Low VIFs (<5): Most of the variables have VIF values close to 1, including EDUC (1.07), AGE (1.01), R

Moderate VIFs (Between 2 and 5): Some regional variables-REGION2 (3.28), REGION3 (3.67), REGION4 (2.18)

High VIFs (>5): None of the variables exceed 5, which suggests that severe multicollinearity is not a

in the explanatory variables can still lead to precise estimates.

Implications for the properties and precision of the OLS estimator:

1) Parameters remain identifiable.

2) Under the CNLRM assumptions, the OLS estimator remains BLUE and normally distributed.

3) OLS estimators exhibit larger variance and covariance, especially the regions.

4) Wider confidence intervals and lower t-statistics

->variables appear less significantly different from zero, higher probability of making a Type II error

5) The overall model fit (R2) is largely unaffected: even if individual coefficients are insignificant

indicate that the coefficients are jointly significant, and can be estimated with high precision.

6) Regression coefficients may change substantially

```
! [] (Data-exploration_files/figure-latex/multicollin-1.pdf)<!-- -->
```

```
print(correlation_matrix)
```

```
##          EDUC        AGE       RACE      SMSA    MARRIED
## EDUC  1.000000000 -0.069506286 -0.151952924 -0.14907033  0.019198364
## AGE   -0.069506286  1.000000000 -0.005148653 -0.02085355  0.020809977
## RACE  -0.151952924 -0.005148653  1.000000000 -0.035563889 -0.113282921
## SMSA  -0.149070326 -0.020853549 -0.035563888  1.000000000  0.040644736
## MARRIED 0.019198364  0.020809977 -0.113282921  0.04064474  1.000000000
## REGION2 0.035717362  0.008497716  0.014092931 -0.10391873 -0.003692641
## REGION3 -0.030839760  0.005358420 -0.023908637 -0.02447606  0.013937881
## REGION4 -0.000578807 -0.014907585 -0.049908666  0.11787788  0.020881363
## REGION5 -0.042953090 -0.012179069  0.091472810  0.01940650  0.004255528
## REGION6 -0.084427585  0.008312525  0.039575771  0.11343959  0.017738535
## REGION7 -0.032806167  0.003156501  0.008728591 -0.01074680  0.007148294
## REGION8  0.028455078 -0.026179022 -0.043610641  0.07119329  0.020956712
## REGION9  0.104683025  0.001483814 -0.030119960 -0.07742498 -0.060808526
##          REGION2      REGION3      REGION4      REGION5      REGION6
## EDUC    0.035717362 -0.03083976 -0.000578807 -0.042953090 -0.084427585
## AGE     0.008497716  0.00535842 -0.014907585 -0.012179069  0.008312525
## RACE   0.014092931 -0.02390864 -0.049908666  0.091472810  0.039575771
## SMSA   -0.103918730 -0.02447606  0.117877877  0.019406502  0.113439595
## MARRIED -0.003692641  0.01393788  0.020881363  0.004255528  0.017738535
```

```

## REGION2 1.000000000 -0.21345470 -0.121923453 -0.201399480 -0.113442761
## REGION3 -0.213454705 1.000000000 -0.138274958 -0.228409743 -0.128656896
## REGION4 -0.121923453 -0.13827496 1.000000000 -0.130465639 -0.073487689
## REGION5 -0.201399480 -0.22840974 -0.130465639 1.000000000 -0.121390774
## REGION6 -0.113442761 -0.12865690 -0.073487689 -0.121390774 1.000000000
## REGION7 -0.144209678 -0.16355005 -0.093418354 -0.154313279 -0.086920406
## REGION8 -0.098898407 -0.11216196 -0.064065924 -0.105827415 -0.059609658
## REGION9 -0.166513184 -0.18884475 -0.107866461 -0.178179412 -0.100363538
##          REGION7      REGION8      REGION9
## EDUC     -0.032806167  0.02845508  0.104683025
## AGE      0.003156501 -0.02617902  0.001483814
## RACE     0.008728591 -0.04361064 -0.030119960
## SMSA    -0.010746803  0.07119329 -0.077424983
## MARRIED  0.007148294  0.02095671 -0.060808526
## REGION2 -0.144209678 -0.09889841 -0.166513184
## REGION3 -0.163550053 -0.11216196 -0.188844746
## REGION4 -0.093418354 -0.06406592 -0.107866461
## REGION5 -0.154313279 -0.10582742 -0.178179412
## REGION6 -0.086920406 -0.05960966 -0.100363538
## REGION7  1.000000000 -0.07577645 -0.127583227
## REGION8 -0.075776449  1.000000000 -0.087496055
## REGION9 -0.127583227 -0.08749605  1.000000000

```

```
print(max(abs(correlation_matrix)))
```

```
## [1] 1
```

```
print('VIF values')
```

```
## [1] "VIF values"
```

```
vif_values = vif(linear_model1)
print(vif_values)
```

```

##      EDUC      AGE      RACE      SMSA      MARRIED      REGION2      REGION3      REGION4
## 1.072047 1.008200 1.054291 1.072522 1.019728 3.280936 3.672041 2.182705
##      REGION5      REGION6      REGION7      REGION8      REGION9
## 3.510183 2.058605 2.540830 1.816650 2.919060

```

```
df_ordered_educ <- df %>% arrange(EDUC)
```

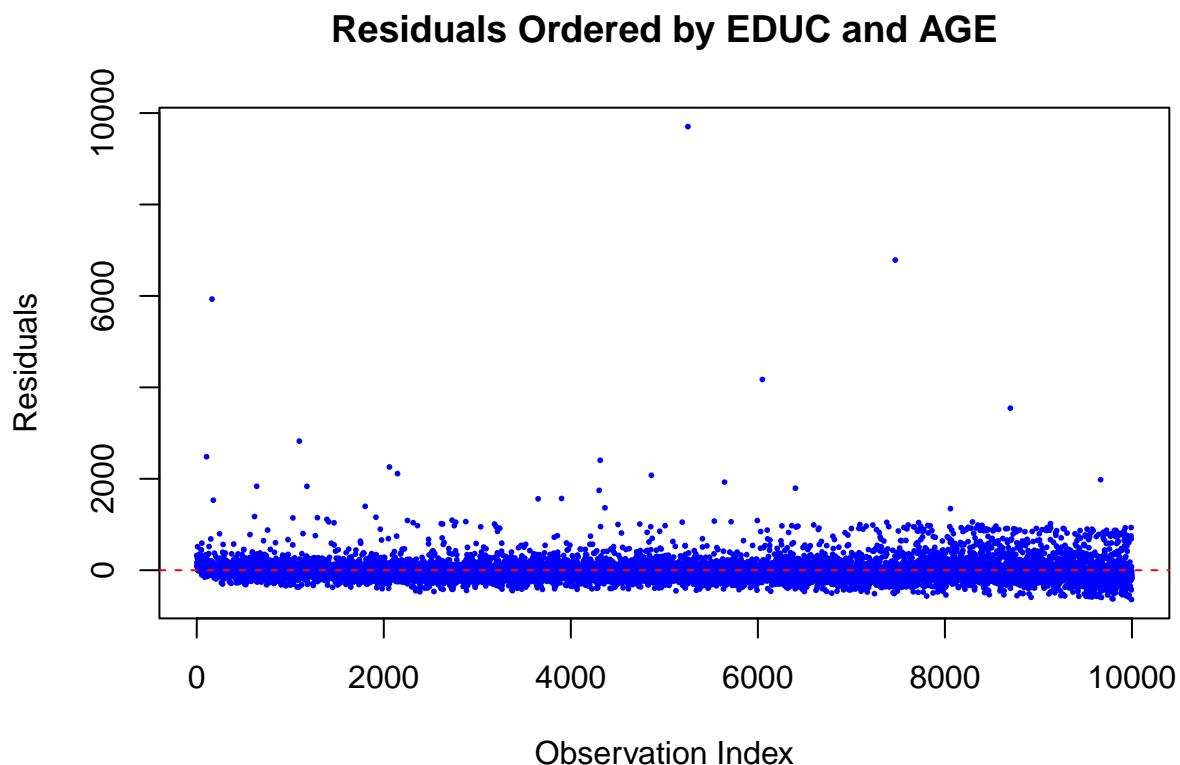
```
# Re-run baseline linear model on ordered data
linear_model_ordered <- lm(
  WAGE ~ EDUC + AGE + RACE + SMSA + MARRIED +
  REGION2 + REGION3 + REGION4 + REGION5 +
  REGION6 + REGION7 + REGION8 + REGION9,
  data = df_ordered_educ
)
```

```
# Store residuals
residuals_ordered <- residuals(linear_model_ordered)
```

```

plot(residuals_ordered,
  type = "p", # "p" for points instead of "l" for lines
  pch = 16,    # Solid circle points (other options: 1-25)
  col = "blue", # Color of points
  cex = 0.4,   # Size of points
  main = "Residuals Ordered by EDUC and AGE",
  ylab = "Residuals",
  xlab = "Observation Index")
abline(h = 0, lty = 2, col = "red") # Reference line

```



```

runs_test_ordered <- runs.test(residuals_ordered)
print(runs_test_ordered)

```

Runs Test

data: residuals_ordered statistic = -2.6201, runs = 4870, n1 = 5000, n2 = 5000, n = 10000, p-value = 0.00879 alternative hypothesis: nonrandomness

```

if (show_interpretation) {
  cat("sippe tijden , p value is hoog dus toch corr ")
}

```

sippe tijden , p value is hoog dus toch corr

```

# Run Durbin-Watson test
dw_test_ordered <- dwtest(linear_model_ordered)

# Create summary properly
DW_summary <- c(dw_test_ordered$statistic, dw_test_ordered$p.value)
names(DW_summary) <- c("Test-statistic", "P-value")
stargazer(DW_summary, type="text")

```

===== Test-statistic P-value ----- 1.957 0.015 -----

```

if (show_interpretation) {
  cat("Assumptions:
  1) Error terms follow an AR(1) process
  2) Error terms are normally distributed
  3) The regression model does not include lagged dependent variables
hahahah dikke pech , toch corr :)")
}

```

Assumptions: 1) Error terms follow an AR(1) process 2) Error terms are normally distributed 3) The regression model does not include lagged dependent variables hahahah dikke pech , toch corr :)

```

bg_test_ordered <- bgtest(linear_model_ordered, order = 6) # Test up to 3 lags
BG_summary=c(bg_test_ordered$statistic,bg_test_ordered$p.value)
names(BG_summary)=c("Test-statistic","P-value")
stargazer(BG_summary, type="text")

```

```

##
## =====
## Test-statistic P-value
## -----
## 13.468      0.036
## -----


if (show_interpretation) {
  cat("Assumptions:
  1) Allows for stochastic regressors
  2) Tests for higher-order autocorrelation
  3) Allows for lagged dependent variables
deze zegt dat er niet autocorr is , idk") }

## Assumptions:
##   1) Allows for stochastic regressors
##   2) Tests for higher-order autocorrelation
##   3) Allows for lagged dependent variables
## deze zegt dat er niet autocorr is , idk

```

```

if (show_interpretation) {
  cat("Implications for OLS if autocorrelation is present:
  1) Standard variance formula is incorrect
  2) OLS estimator no longer efficient")
  cat("Inference: Standard errors are incorrect, leading to invalid t-tests, F-tests, and confidence in")
  cat("It does not make sense to interpret autocorrelation as a structural feature since there is no na")
}

```

```
## Implications for OLS if autocorrelation is present:  
##   1) Standard variance formula is incorrect  
##   2) OLS estimator no longer efficientInference: Standard errors are incorrect, leading to invalid t
```