# Case Solution Group 5

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2025-05-10

## 1. Empirical Specification and Hypotheses

Baseline model:

$$WAGE_i = \beta_1 + \beta_2\, EDUCi + \alpha\, X_i + \mu_i$$

Where Xi includes AGE, RACE, MARRIED, SMSA, and REGION.

### Hypothesis Tests:

**EDUC: Education** H0: B2 $\leq 0 \rightarrow$ Education has no or a negative effect on wages.

H1: B2 $> 0 \rightarrow$ Education has a positive effect on wages.

One-sided test: Economic theory predicts that more education increases wages, so a negative effect would be counterintuitive.

**AGE: Age** H0: A(AGE) $\leq 0 \rightarrow$ Age has no or a negative effect on wages.

H1: A(AGE) $> 0 \rightarrow$ Age has a positive effect on wages.

One-sided test: Age is often associated with experience, which is expected to increase wages.

**MARRIED: Marital Status** H0: A(MARRIED) $\leq 0 \rightarrow$ Being married has no or a negative effect on wages.

H1: A(MARRIED) $> 0 \rightarrow$ Being married has a positive effect on wages.

One-sided test: Being married may indicate stability and a positive impact on productivity, leading to higher wages.

**SMSA: Metropolitan Area** H0: A(SMSA) $\leq 0 \rightarrow$ Living in a metropolitan area has no or a negative effect on wages.

H1: A(SMSA) $> 0 \rightarrow$ Living in a metropolitan area has a positive effect on wages.

One-sided test: Urban areas typically have more job opportunities and higher wages, driving this expectation.

##### **RACE: Race (1 = Black, 0 = White)**

H0: A(RACE) $\geq 0 \rightarrow$ Race has no or a positive effect on wages.

H1: A(RACE) $< 0 \rightarrow$ Race has a negative effect on wages.

One-sided test: Structural inequalities often result in lower wages for minority groups.

**REGION: Geographic Region** H0: A(REGION) $\neq 0 \rightarrow$ (There are no positive wage differences across regions.)

H1: A(REGION) $= 0 \rightarrow$ (At least one region has a positive effect on wages.)

Two-sided test: Region could impact wages in either direction

## 2. Explore the Data

```
head(df)
```

```
##       WAGE EDUC AGE RACE SMSA MARRIED REGION QOB REGION1 REGION2 REGION3
## 1 580.1000    9  45    0    0       1      9   3       0       0       0
## 2 642.2115   17  47    0    0       1      3   4       0       0       1
## 3 577.0192   12  42    0    0       1      7   2       0       0       0
## 4 999.1346   10  43    0    0       1      3   2       0       0       1
## 5 307.7885   12  41    0    0       1      6   3       0       0       0
## 6 280.1000   12  40    1    0       1      5   2       0       0       0
##   REGION4 REGION5 REGION6 REGION7 REGION8 REGION9 QOB1 QOB2 QOB3 QOB4
## 1       0       0       0       0       0       1    0    0    1    0
## 2       0       0       0       0       0       0    0    0    0    1
## 3       0       0       0       1       0       0    0    1    0    0
## 4       0       0       0       0       0       0    0    1    0    0
## 5       0       0       1       0       0       0    0    0    1    0
## 6       0       1       0       0       0       0    0    1    0    0
```

```
summary(df)
```

```
##       WAGE                EDUC            AGE             RACE
##  Min.   :    0.096   Min.   : 0.00   Min.   :40.00   Min.   :0.0000
##  1st Qu.:  278.558   1st Qu.:12.00   1st Qu.:42.00   1st Qu.:0.0000
##  Median :  384.712   Median :12.00   Median :45.00   Median :0.0000
##  Mean   :  436.524   Mean   :12.71   Mean   :44.68   Mean   :0.0832
##  3rd Qu.:  520.100   3rd Qu.:15.00   3rd Qu.:47.00   3rd Qu.:0.0000
##  Max.   :10167.500   Max.   :20.00   Max.   :50.00   Max.   :1.0000
##       SMSA            MARRIED           REGION           QOB
##  Min.   :0.0000   Min.   :0.0000   Min.   :1.000   Min.   :1.000
##  1st Qu.:0.0000   1st Qu.:1.0000   1st Qu.:3.000   1st Qu.:1.000
##  Median :0.0000   Median :1.0000   Median :5.000   Median :3.000
##  Mean   :0.1813   Mean   :0.8609   Mean   :4.767   Mean   :2.502
##  3rd Qu.:0.0000   3rd Qu.:1.0000   3rd Qu.:7.000   3rd Qu.:3.000
##  Max.   :1.0000   Max.   :1.0000   Max.   :9.000   Max.   :4.000
##     REGION1          REGION2          REGION3          REGION4
##  Min.   :0.0000   Min.   :0.0000   Min.   :0.0000   Min.   :0.0000
##  1st Qu.:0.0000   1st Qu.:0.0000   1st Qu.:0.0000   1st Qu.:0.0000
##  Median :0.0000   Median :0.0000   Median :0.0000   Median :0.0000
##  Mean   :0.0549   Mean   :0.1584   Mean   :0.1949   Mean   :0.0732
##  3rd Qu.:0.0000   3rd Qu.:0.0000   3rd Qu.:0.0000   3rd Qu.:0.0000
##  Max.   :1.0000   Max.   :1.0000   Max.   :1.0000   Max.   :1.0000
##     REGION5          REGION6          REGION7          REGION8
##  Min.   :0.0000   Min.   :0.000   Min.   :0.0000   Min.   :0.0000
```

```
##  1st Qu.:0.0000   1st Qu.:0.000   1st Qu.:0.0000   1st Qu.:0.0000
##  Median :0.0000   Median :0.000   Median :0.0000   Median :0.0000
##  Mean   :0.1773   Mean   :0.064   Mean   :0.0995   Mean   :0.0494
##  3rd Qu.:0.0000   3rd Qu.:0.000   3rd Qu.:0.0000   3rd Qu.:0.0000
##  Max.   :1.0000   Max.   :1.000   Max.   :1.0000   Max.   :1.0000
##     REGION9           QOB1            QOB2            QOB3
##  Min.   :0.0000   Min.   :0.0000   Min.   :0.0000   Min.   :0.0000
##  1st Qu.:0.0000   1st Qu.:0.0000   1st Qu.:0.0000   1st Qu.:0.0000
##  Median :0.0000   Median :0.0000   Median :0.0000   Median :0.0000
##  Mean   :0.1284   Mean   :0.2533   Mean   :0.2354   Mean   :0.2674
##  3rd Qu.:0.0000   3rd Qu.:1.0000   3rd Qu.:0.0000   3rd Qu.:1.0000
##  Max.   :1.0000   Max.   :1.0000   Max.   :1.0000   Max.   :1.0000
##       QOB4
##  Min.   :0.0000
##  1st Qu.:0.0000
##  Median :0.0000
##  Mean   :0.2439
##  3rd Qu.:0.0000
##  Max.   :1.0000
```

```
describe(df)
```

```
##          vars     n   mean     sd median trimmed    mad  min     max   range
## WAGE        1 10000 436.52 295.37 384.71  401.88 173.27  0.1 10167.5 10167.4
## EDUC        2 10000  12.71   3.28  12.00   12.72   2.97  0.0    20.0    20.0
## AGE         3 10000  44.68   2.93  45.00   44.67   4.45 40.0    50.0    10.0
## RACE        4 10000   0.08   0.28   0.00    0.00   0.00  0.0     1.0     1.0
## SMSA        5 10000   0.18   0.39   0.00    0.10   0.00  0.0     1.0     1.0
## MARRIED     6 10000   0.86   0.35   1.00    0.95   0.00  0.0     1.0     1.0
## REGION      7 10000   4.77   2.46   5.00    4.65   2.97  1.0     9.0     8.0
## QOB         8 10000   2.50   1.12   3.00    2.50   1.48  1.0     4.0     3.0
## REGION1     9 10000   0.05   0.23   0.00    0.00   0.00  0.0     1.0     1.0
## REGION2    10 10000   0.16   0.37   0.00    0.07   0.00  0.0     1.0     1.0
## REGION3    11 10000   0.19   0.40   0.00    0.12   0.00  0.0     1.0     1.0
## REGION4    12 10000   0.07   0.26   0.00    0.00   0.00  0.0     1.0     1.0
## REGION5    13 10000   0.18   0.38   0.00    0.10   0.00  0.0     1.0     1.0
## REGION6    14 10000   0.06   0.24   0.00    0.00   0.00  0.0     1.0     1.0
## REGION7    15 10000   0.10   0.30   0.00    0.00   0.00  0.0     1.0     1.0
## REGION8    16 10000   0.05   0.22   0.00    0.00   0.00  0.0     1.0     1.0
## REGION9    17 10000   0.13   0.33   0.00    0.04   0.00  0.0     1.0     1.0
## QOB1       18 10000   0.25   0.43   0.00    0.19   0.00  0.0     1.0     1.0
## QOB2       19 10000   0.24   0.42   0.00    0.17   0.00  0.0     1.0     1.0
## QOB3       20 10000   0.27   0.44   0.00    0.21   0.00  0.0     1.0     1.0
## QOB4       21 10000   0.24   0.43   0.00    0.18   0.00  0.0     1.0     1.0
##            skew kurtosis   se
## WAGE       7.39   170.46 2.95
## EDUC      -0.07     0.55 0.03
## AGE        0.05    -1.18 0.03
## RACE       3.02     7.11 0.00
## SMSA       1.65     0.74 0.00
## MARRIED   -2.09     2.35 0.00
## REGION     0.35    -1.06 0.02
## QOB       -0.03    -1.35 0.01
## REGION1    3.91    13.27 0.00
```

```
## REGION2  1.87     1.50 0.00
## REGION3  1.54     0.37 0.00
## REGION4  3.28     8.74 0.00
## REGION5  1.69     0.85 0.00
## REGION6  3.56    10.69 0.00
## REGION7  2.68     5.16 0.00
## REGION8  4.16    15.29 0.00
## REGION9  2.22     2.93 0.00
## QOB1     1.13    -0.71 0.00
## QOB2     1.25    -0.44 0.00
## QOB3     1.05    -0.90 0.00
## QOB4     1.19    -0.58 0.00
```

```
cov(df)
```

```
##                  WAGE          EDUC          AGE          RACE          SMSA
## WAGE    87241.0836001  3.157684e+02  5.256929832 -1.049000e+01 -1.485766e+01
## EDUC      315.7684062  1.074171e+01 -0.667730053 -1.375518e-01 -1.882395e-01
## AGE         5.2569298 -6.677301e-01  8.591725483 -4.168257e-03 -2.355067e-02
## RACE      -10.4899998 -1.375518e-01 -0.004168257  7.628539e-02 -3.784538e-03
## SMSA      -14.8576552 -1.882395e-01 -0.023550665 -3.784538e-03  1.484452e-01
## MARRIED    10.1783750  2.177522e-02  0.021109281 -1.082796e-02  5.419372e-03
## REGION      8.8550412  2.740826e-01 -0.125475448  1.185719e-03  3.264616e-02
## QOB         2.3974020  1.174544e-01 -0.386878218 -3.758456e-03 -4.794949e-03
## REGION1    -0.8536330  1.648109e-02  0.016140984 -2.867967e-03 -1.753545e-03
## REGION2     3.8199259  4.274331e-02  0.009094829  1.421262e-03 -1.461938e-02
## REGION3     3.3789188 -4.004056e-02  0.006221992 -2.615942e-03 -3.735744e-03
## REGION4    -1.1378266 -4.941294e-04 -0.011381978 -3.590599e-03  1.183002e-02
## REGION5    -6.8153854 -5.376850e-02 -0.013634873  9.649605e-03  2.855796e-03
## REGION6    -3.9603260 -6.772837e-02  0.005963796  2.675468e-03  1.069787e-02
## REGION7    -1.9950947 -3.218602e-02  0.002769627  7.216722e-04 -1.239474e-03
## REGION8    -0.2171291  2.021066e-02 -0.016629443 -2.610341e-03  5.944374e-03
## REGION9     7.7805501  1.147825e-01  0.001455066 -2.783158e-03 -9.979918e-03
## QOB1       -0.8194058 -4.086161e-02  0.193104600 -1.745775e-04  2.876998e-03
## QOB2       -0.4775954  1.293153e-02 -0.063772357  2.514971e-03 -8.781078e-04
## QOB3        1.0160062 -2.073263e-02 -0.064890869 -7.477548e-04 -2.079828e-03
## QOB4        0.2809950  4.866271e-02 -0.064441374 -1.592639e-03  8.093809e-05
##                MARRIED       REGION           QOB       REGION1       REGION2
## WAGE    10.1783750456  8.855041195  2.3974019986 -0.8536330190  3.8199258573
## EDUC     0.0217752175  0.274082608  0.1174543854  0.0164810881  0.0427433143
## AGE      0.0211092809 -0.125475448 -0.3868782178  0.0161409841  0.0090948295
## RACE    -0.0108279628  0.001185719 -0.0037584558 -0.0028679668  0.0014212621
## SMSA     0.0054193719  0.032646165 -0.0047949495 -0.0017535454 -0.0146193819
## MARRIED  0.1197631663 -0.022112511 -0.0027859886 -0.0006634763 -0.0004666067
## REGION  -0.0221125113  6.064117412 -0.0024575458 -0.2068289829 -0.4383366337
## QOB     -0.0027859886 -0.002457546  1.2445208421  0.0009457846  0.0047995200
## REGION1 -0.0006634763 -0.206828983  0.0009457846  0.0518911791 -0.0086970297
## REGION2 -0.0004666067 -0.438336634  0.0047995200 -0.0086970297  0.1333227723
## REGION3  0.0019107811 -0.344422742 -0.0027205821 -0.0107010801 -0.0308752475
## REGION4  0.0018823082 -0.056150015 -0.0064397240 -0.0040190819 -0.0115960396
## REGION5  0.0005624862  0.041315032  0.0024133713 -0.0097347435 -0.0280871287
## REGION6  0.0015025503  0.078919892 -0.0026218622 -0.0035139514 -0.0101386139
## REGION7  0.0007405241  0.222205721  0.0044613961 -0.0054630963 -0.0157623762
## REGION8  0.0015716972  0.159726173 -0.0008939494 -0.0027123312 -0.0078257426
```

```
## REGION9 -0.0070402640  0.543571557  0.0000560456 -0.0070498650 -0.0203405941
## QOB1    -0.0005660266  0.004519352 -0.3804693169 -0.0009062606 -0.0028230023
## QOB2     0.0015442944 -0.013753175 -0.1181590759  0.0009766377  0.0022128613
## QOB3     0.0013954795  0.016405841  0.1332052605 -0.0001802780 -0.0007562356
## QOB4    -0.0023737474 -0.007172017  0.3654231323  0.0001099010  0.0013663766
##              REGION3       REGION4       REGION5       REGION6       REGION7
## WAGE     3.3789187829 -1.1378265754 -6.8153853707 -3.9603259637 -1.9950946778
## EDUC    -0.0400405641 -0.0004941294 -0.0537684968 -0.0677283728 -0.0321860186
## AGE      0.0062219922 -0.0113819782 -0.0136348735  0.0059637964  0.0027696270
## RACE    -0.0026159416 -0.0035905991  0.0096496050  0.0026754675  0.0007216722
## SMSA    -0.0037357436  0.0118300230  0.0028557956  0.0106978698 -0.0012394739
## MARRIED  0.0019107811  0.0018823082  0.0005624862  0.0015025503  0.0007405241
## REGION  -0.3444227423 -0.0561500150  0.0413150315  0.0789198920  0.2222057206
## QOB     -0.0027205821 -0.0064397240  0.0024133713 -0.0026218622  0.0044613961
## REGION1 -0.0107010801 -0.0040190819 -0.0097347435 -0.0035139514 -0.0054630963
## REGION2 -0.0308752475 -0.0115960396 -0.0280871287 -0.0101386139 -0.0157623762
## REGION3  0.1569296830 -0.0142681068 -0.0345592259 -0.0124748475 -0.0193944894
## REGION4 -0.0142681068  0.0678485449 -0.0129796580 -0.0046852685 -0.0072841284
## REGION5 -0.0345592259 -0.0129796580  0.1458792979 -0.0113483348 -0.0176431143
## REGION6 -0.0124748475 -0.0046852685 -0.0113483348  0.0599099910 -0.0063686369
## REGION7 -0.0193944894 -0.0072841284 -0.0176431143 -0.0063686369  0.0896087109
## REGION8 -0.0096290229 -0.0036164416 -0.0087594959 -0.0031619162 -0.0049157916
## REGION9 -0.0250276628 -0.0093998200 -0.0227675968 -0.0082184218 -0.0127770777
## QOB1     0.0014319732  0.0015585959  0.0007899890  0.0012889289 -0.0008034303
## QOB2     0.0003205721  0.0007687969 -0.0020366237 -0.0003656366 -0.0030226023
## QOB3    -0.0022164816  0.0002263426 -0.0007100910 -0.0005136514  0.0039940994
## QOB4     0.0004639364 -0.0025537354  0.0019567257 -0.0004096410 -0.0001680668
##              REGION8       REGION9          QOB1          QOB2          QOB3
## WAGE    -2.171291e-01  7.7805501010 -0.8194057861 -4.775954e-01  1.0160061545
## EDUC     2.021066e-02  0.1147825183 -0.0408616062  1.293153e-02 -0.0207326333
## AGE     -1.662944e-02  0.0014550655  0.1931046005 -6.377236e-02 -0.0648908691
## RACE    -2.610341e-03 -0.0027831583 -0.0001745775  2.514971e-03 -0.0007477548
## SMSA     5.944374e-03 -0.0099799180  0.0028769977 -8.781078e-04 -0.0020798280
## MARRIED  1.571697e-03 -0.0070402640 -0.0005660266  1.544294e-03  0.0013954795
## REGION   1.597262e-01  0.5435715572  0.0045193519 -1.375318e-02  0.0164058406
## QOB     -8.939494e-04  0.0000560456 -0.3804693169 -1.181591e-01  0.1332052605
## REGION1 -2.712331e-03 -0.0070498650 -0.0009062606  9.766377e-04 -0.0001802780
## REGION2 -7.825743e-03 -0.0203405941 -0.0028230023  2.212861e-03 -0.0007562356
## REGION3 -9.629023e-03 -0.0250276628  0.0014319732  3.205721e-04 -0.0022164816
## REGION4 -3.616442e-03 -0.0093998200  0.0015585959  7.687969e-04  0.0002263426
## REGION5 -8.759496e-03 -0.0227675968  0.0007899890 -2.036624e-03 -0.0007100910
## REGION6 -3.161916e-03 -0.0082184218  0.0012889289 -3.656366e-04 -0.0005136514
## REGION7 -4.915792e-03 -0.0127770777 -0.0008034303 -3.022602e-03  0.0039940994
## REGION8  4.696434e-02 -0.0063435944  0.0006870487 -2.876288e-05 -0.0011096710
## REGION9 -6.343594e-03  0.1119246325 -0.0012238424  1.174757e-03  0.0012659666
## QOB1     6.870487e-04 -0.0012238424  0.1891580258 -5.963278e-02 -0.0677391939
## QOB2    -2.876288e-05  0.0011747575 -0.0596327833  1.800048e-01 -0.0629522552
## QOB3    -1.109671e-03  0.0012659666 -0.0677391939 -6.295226e-02  0.1959168317
## QOB4     4.513851e-04 -0.0012168817 -0.0617860486 -5.741980e-02 -0.0652253825
##                 QOB4
## WAGE     2.809950e-01
## EDUC     4.866271e-02
## AGE     -6.444137e-02
## RACE    -1.592639e-03
```

```
## SMSA     8.093809e-05
## MARRIED -2.373747e-03
## REGION  -7.172017e-03
## QOB      3.654231e-01
## REGION1  1.099010e-04
## REGION2  1.366377e-03
## REGION3  4.639364e-04
## REGION4 -2.553735e-03
## REGION5  1.956726e-03
## REGION6 -4.096410e-04
## REGION7 -1.680668e-04
## REGION8  4.513851e-04
## REGION9 -1.216882e-03
## QOB1    -6.178605e-02
## QOB2    -5.741980e-02
## QOB3    -6.522538e-02
## QOB4     1.844312e-01
```

### 2.1 Statistics per region

WAGE: Largest wage disparities in regions 2, 6, and 9; high SD, skewness, and kurtosis → outliers with very high incomes.

EDUC: Highest in region 9, lowest in region 6; slightly more variation in regions 5 and 6.

AGE: Almost uniformly ~45 years across regions; low spread, minimal differences.

RACE: Mostly white population (>90%); region 5 relatively more diversity; high kurtosis indicates outliers.

SMSA: Regions 4 & 6 are more urban, region 2 is the most rural; skewed → most live outside urban areas.

MARRIED: High marriage rates across all regions (~85–89%); region 9 slightly lower.

```
## # A tibble: 9 x 11
##   REGION mean_wage mean_educ mean_age sd_wage sd_educ sd_age var_wage var_educ
##    <int>     <dbl>     <dbl>    <dbl>   <dbl>   <dbl>  <dbl>    <dbl>    <dbl>
## 1      1      421.      13.0     45.0    244.    3.10   2.93   59622.     9.62
## 2      2      461.      13.0     44.7    355.    3.17   2.96  126021.    10.0
## 3      3      454.      12.5     44.7    264.    3.07   2.89   69754.     9.40
## 4      4      421.      12.7     44.5    263.    3.15   2.94   69295.     9.90
## 5      5      398.      12.4     44.6    261.    3.48   2.99   68143.    12.1
## 6      6      375.      11.7     44.8    330.    3.74   2.77  108586.    14.0
## 7      7      416.      12.4     44.7    266.    3.57   2.98   70638.    12.7
## 8      8      432.      13.1     44.3    236.    2.95   2.95   55764.     8.73
## 9      9      497.      13.6     44.7    346.    2.92   2.89  119854.     8.54
## # i 2 more variables: var_age <dbl>, count <int>
```

```
##
##  Descriptive statistics by group
## REGION: 1
##         vars   n   mean     sd median trimmed    mad   min    max   range  skew
## REGION     1 549   1.00   0.00   1.00    1.00   0.00  1.00    1.0    0.00   NaN
## WAGE       2 549 420.98 244.18 384.71  389.65 171.07  3.94 1562.5 1558.56  1.74
## EDUC       3 549  13.01   3.10  12.00   12.91   2.97  5.00   20.0   15.00  0.33
## AGE        4 549  44.97   2.93  45.00   45.02   4.45 40.00   50.0   10.00 -0.05
```

```
## RACE       5 549   0.03   0.17   0.00    0.00   0.00  0.00    1.0    1.00  5.40
## SMSA       6 549   0.15   0.36   0.00    0.06   0.00  0.00    1.0    1.00  1.96
## MARRIED    7 549   0.85   0.36   1.00    0.93   0.00  0.00    1.0    1.00 -1.94
##          kurtosis    se
## REGION       NaN  0.00
## WAGE        4.51 10.42
## EDUC       -0.16  0.13
## AGE        -1.22  0.13
## RACE       27.22  0.01
## SMSA        1.85  0.02
## MARRIED     1.78  0.02
## ------------------------------------------------------------
## REGION: 2
##          vars    n   mean     sd median trimmed    mad  min     max   range
## REGION      1 1584   2.00   0.00   2.00    2.00   0.00  2.0     2.0     0.0
## WAGE        2 1584 460.64 354.99 403.94  422.27 171.07  2.4 10167.5 10165.1
## EDUC        3 1584  12.98   3.17  12.00   12.90   2.97  0.0    20.0    20.0
## AGE         4 1584  44.74   2.96  45.00   44.73   4.45 40.0    50.0    10.0
## RACE        5 1584   0.09   0.29   0.00    0.00   0.00  0.0     1.0     1.0
## SMSA        6 1584   0.09   0.28   0.00    0.00   0.00  0.0     1.0     1.0
## MARRIED     7 1584   0.86   0.35   1.00    0.95   0.00  0.0     1.0     1.0
##           skew kurtosis   se
## REGION     NaN      NaN 0.00
## WAGE     13.60   352.35 8.92
## EDUC      0.18     0.42 0.08
## AGE       0.04    -1.22 0.07
## RACE      2.82     5.94 0.01
## SMSA      2.88     6.32 0.01
## MARRIED -2.05     2.20 0.01
## ------------------------------------------------------------
## REGION: 3
##          vars    n   mean     sd median trimmed    mad   min     max   range
## REGION      1 1949   3.00   0.00    3.0    3.00   0.00  3.00    3.00    0.00
## WAGE        2 1949 453.86 264.11  409.2  425.41 164.62  0.15 4634.17 4634.02
## EDUC        3 1949  12.51   3.07   12.0   12.43   1.48  0.00   20.00   20.00
## AGE         4 1949  44.71   2.89   45.0   44.70   2.97 40.00   50.00   10.00
## RACE        5 1949   0.07   0.25    0.0    0.00   0.00  0.00    1.00    1.00
## SMSA        6 1949   0.16   0.37    0.0    0.08   0.00  0.00    1.00    1.00
## MARRIED     7 1949   0.87   0.34    1.0    0.96   0.00  0.00    1.00    1.00
##           skew kurtosis   se
## REGION     NaN      NaN 0.00
## WAGE      4.11    43.18 5.98
## EDUC      0.22     0.56 0.07
## AGE       0.05    -1.12 0.07
## RACE      3.37     9.39 0.01
## SMSA      1.83     1.36 0.01
## MARRIED -2.21     2.88 0.01
## ------------------------------------------------------------
## REGION: 4
##          vars   n   mean     sd median trimmed    mad   min     max range  skew
## REGION      1 732   4.00   0.00   4.00    4.00   0.00  4.00    4.00     0   NaN
## WAGE        2 732 420.98 263.24 384.71  390.86 146.34 10.25 4076.25  4066  4.76
## EDUC        3 732  12.71   3.15  12.00   12.63   1.48  0.00   20.00    20  0.16
## AGE         4 732  44.52   2.94  44.00   44.48   4.45 40.00   50.00    10  0.11
```

```
## RACE       5 732   0.03   0.18   0.00    0.00   0.00  0.00    1.00     1  5.12
## SMSA       6 732   0.34   0.48   0.00    0.30   0.00  0.00    1.00     1  0.66
## MARRIED    7 732   0.89   0.32   1.00    0.98   0.00  0.00    1.00     1 -2.43
##         kurtosis   se
## REGION       NaN 0.00
## WAGE       52.28 9.73
## EDUC        0.58 0.12
## AGE        -1.19 0.11
## RACE       24.24 0.01
## SMSA       -1.57 0.02
## MARRIED     3.93 0.01
## ------------------------------------------------------------
## REGION: 5
##         vars    n   mean     sd median trimmed    mad  min     max   range
## REGION     1 1773   5.00   0.00   5.00    5.00   0.00  5.0    5.00    0.00
## WAGE       2 1773 398.09 261.04 346.25  362.95 189.49  0.1 2501.25 2501.15
## EDUC       3 1773  12.41   3.48  12.00   12.46   2.97  0.0   20.00   20.00
## AGE        4 1773  44.60   2.99  45.00   44.59   4.45 40.0   50.00   10.00
## RACE       5 1773   0.14   0.34   0.00    0.05   0.00  0.0    1.00    1.00
## SMSA       6 1773   0.20   0.40   0.00    0.12   0.00  0.0    1.00    1.00
## MARRIED    7 1773   0.86   0.34   1.00    0.95   0.00  0.0    1.00    1.00
##          skew kurtosis   se
## REGION    NaN      NaN 0.00
## WAGE     2.23     8.76 6.20
## EDUC    -0.19     0.27 0.08
## AGE      0.06    -1.22 0.07
## RACE     2.10     2.42 0.01
## SMSA     1.52     0.31 0.01
## MARRIED -2.12     2.51 0.01
## ------------------------------------------------------------
## REGION: 6
##         vars   n   mean     sd median trimmed    mad   min  max   range  skew
## REGION     1 640   6.00   0.00   6.00    6.00   0.00  6.00    6    0.00   NaN
## WAGE       2 640 374.65 329.52 334.13  335.99 181.61  1.98 6005 6003.02  8.62
## EDUC       3 640  11.66   3.74  12.00   11.65   2.97  0.00   20   20.00 -0.03
## AGE        4 640  44.77   2.77  45.00   44.74   2.97 40.00   50   10.00  0.06
## RACE       5 640   0.12   0.33   0.00    0.03   0.00  0.00    1    1.00  2.26
## SMSA       6 640   0.35   0.48   0.00    0.31   0.00  0.00    1    1.00  0.63
## MARRIED    7 640   0.88   0.32   1.00    0.98   0.00  0.00    1    1.00 -2.40
##         kurtosis    se
## REGION       NaN  0.00
## WAGE      133.54 13.03
## EDUC        0.08  0.15
## AGE        -1.08  0.11
## RACE        3.12  0.01
## SMSA       -1.60  0.02
## MARRIED     3.76  0.01
## ------------------------------------------------------------
## REGION: 7
##         vars   n   mean     sd median trimmed    mad   min     max   range
## REGION     1 995   7.00   0.00   7.00    7.00   0.00  7.00    7.00    0.00
## WAGE       2 995 416.47 265.78 373.17  381.59 191.03  4.69 2501.25 2496.56
## EDUC       3 995  12.39   3.57  12.00   12.56   2.97  0.00   20.00   20.00
## AGE        4 995  44.71   2.98  45.00   44.71   4.45 40.00   50.00   10.00
```

```
## RACE       5 995   0.09   0.29   0.00    0.00   0.00  0.00    1.00    1.00
## SMSA       6 995   0.17   0.37   0.00    0.09   0.00  0.00    1.00    1.00
## MARRIED    7 995   0.87   0.34   1.00    0.96   0.00  0.00    1.00    1.00
##          skew kurtosis   se
## REGION    NaN      NaN 0.00
## WAGE     2.01     7.02 8.43
## EDUC    -0.47     0.56 0.11
## AGE      0.02    -1.19 0.09
## RACE     2.85     6.14 0.01
## SMSA     1.77     1.12 0.01
## MARRIED -2.18     2.74 0.01
## ------------------------------------------------------------
## REGION: 8
##         vars   n   mean     sd median trimmed    mad   min     max   range
## REGION     1 494   8.00   0.00   8.00    8.00   0.00  8.00    8.00    0.00
## WAGE       2 494 432.13 236.14 384.71  401.01 145.41  1.06 1469.49 1468.43
## EDUC       3 494  13.12   2.95  12.00   13.09   2.97  1.00   20.00   19.00
## AGE        4 494  44.34   2.95  44.00   44.27   2.97 40.00   50.00   10.00
## RACE       5 494   0.03   0.17   0.00    0.00   0.00  0.00    1.00    1.00
## SMSA       6 494   0.30   0.46   0.00    0.25   0.00  0.00    1.00    1.00
## MARRIED    7 494   0.89   0.31   1.00    0.99   0.00  0.00    1.00    1.00
##          skew kurtosis    se
## REGION    NaN      NaN  0.00
## WAGE     1.84     4.94 10.62
## EDUC     0.06     0.95  0.13
## AGE      0.17    -1.16  0.13
## RACE     5.46    27.84  0.01
## SMSA     0.86    -1.26  0.02
## MARRIED -2.53     4.41  0.01
## ------------------------------------------------------------
## REGION: 9
##         vars    n   mean     sd median trimmed    mad  min  max  range  skew
## REGION     1 1284   9.00   0.00   9.00    9.00   0.00  9.0    9    0.0   NaN
## WAGE       2 1284 497.11 346.20 440.48  454.64 196.73  7.1 7335 7327.9  7.21
## EDUC       3 1284  13.61   2.92  13.00   13.56   1.48  0.0   20   20.0  0.01
## AGE        4 1284  44.69   2.89  45.00   44.68   2.97 40.0   50   10.0  0.03
## RACE       5 1284   0.06   0.24   0.00    0.00   0.00  0.0    1    1.0  3.65
## SMSA       6 1284   0.10   0.30   0.00    0.00   0.00  0.0    1    1.0  2.60
## MARRIED    7 1284   0.81   0.40   1.00    0.88   0.00  0.0    1    1.0 -1.55
##         kurtosis   se
## REGION       NaN 0.00
## WAGE      121.20 9.66
## EDUC        0.85 0.08
## AGE        -1.18 0.08
## RACE       11.30 0.01
## SMSA        4.76 0.01
## MARRIED     0.39 0.01
```

### 2.2 Statistics per age

AGE GROUPS:

AGE >= 40 & AGE < 42 ~ '40-41 GROUP 1',

AGE >= 42 & AGE < 44 ~ '42-43 GROUP 2',

AGE >= 44 & AGE < 46 ~ '44-45 GROUP 3',

AGE >= 46 & AGE < 48 ~ '46-47 GROUP 4',

AGE >= 48 & AGE <= 50 ~ '48-50 GROUP 5'

```
##
##  Descriptive statistics by group
## AGE_GROUP: 1
##           vars    n   mean     sd median trimmed    mad   min  max   range
## WAGE         1 1835 439.82 340.56 384.71  401.20 171.07  1.06 7335 7333.94
## EDUC         2 1835  12.98   3.11  12.00   12.93   2.97  0.00   20   20.00
## AGE          3 1835  40.56   0.50  41.00   40.58   0.00 40.00   41    1.00
## RACE         4 1835   0.09   0.28   0.00    0.00   0.00  0.00    1    1.00
## SMSA         5 1835   0.18   0.39   0.00    0.10   0.00  0.00    1    1.00
## MARRIED      6 1835   0.85   0.36   1.00    0.94   0.00  0.00    1    1.00
## AGE_GROUP    7 1835   1.00   0.00   1.00    1.00   0.00  1.00    1    0.00
##            skew kurtosis   se
## WAGE       8.61   139.95 7.95
## EDUC       0.07     0.54 0.07
## AGE       -0.25    -1.94 0.01
## RACE       2.95     6.70 0.01
## SMSA       1.65     0.71 0.01
## MARRIED   -1.95     1.82 0.01
## AGE_GROUP   NaN      NaN 0.00
## ------------------------------------------------------------
## AGE_GROUP: 2
##           vars    n   mean     sd median trimmed    mad  min     max   range
## WAGE         1 2029 433.85 281.52 384.71  397.82 178.77  2.4 4634.17 4631.76
## EDUC         2 2029  12.90   3.22  12.00   12.90   2.97  0.0   20.00   20.00
## AGE          3 2029  42.50   0.50  43.00   42.50   0.00 42.0   43.00    1.00
## RACE         4 2029   0.08   0.27   0.00    0.00   0.00  0.0    1.00    1.00
## SMSA         5 2029   0.19   0.39   0.00    0.12   0.00  0.0    1.00    1.00
## MARRIED      6 2029   0.86   0.35   1.00    0.94   0.00  0.0    1.00    1.00
## AGE_GROUP    7 2029   2.00   0.00   2.00    2.00   0.00  2.0    2.00    0.00
##            skew kurtosis   se
## WAGE       3.45    30.53 6.25
## EDUC      -0.05     0.55 0.07
## AGE       -0.01    -2.00 0.01
## RACE       3.07     7.45 0.01
## SMSA       1.55     0.41 0.01
## MARRIED   -2.02     2.07 0.01
## AGE_GROUP   NaN      NaN 0.00
## ------------------------------------------------------------
## AGE_GROUP: 3
##           vars    n   mean     sd median trimmed    mad   min     max   range
## WAGE         1 2035 426.73 245.77 384.71  398.04 171.07  1.98 2884.62 2882.64
## EDUC         2 2035  12.75   3.26  12.00   12.75   2.97  0.00   20.00   20.00
## AGE          3 2035  44.48   0.50  44.00   44.47   0.00 44.00   45.00    1.00
## RACE         4 2035   0.08   0.28   0.00    0.00   0.00  0.00    1.00    1.00
## SMSA         5 2035   0.19   0.40   0.00    0.12   0.00  0.00    1.00    1.00
## MARRIED      6 2035   0.87   0.34   1.00    0.96   0.00  0.00    1.00    1.00
## AGE_GROUP    7 2035   3.00   0.00   3.00    3.00   0.00  3.00    3.00    0.00
##            skew kurtosis   se
## WAGE       2.12     9.53 5.45
```

```
## EDUC      -0.01     0.38 0.07
## AGE        0.09    -1.99 0.01
## RACE       3.03     7.19 0.01
## SMSA       1.55     0.39 0.01
## MARRIED   -2.16     2.66 0.01
## AGE_GROUP   NaN      NaN 0.00
## ------------------------------------------------------------
## AGE_GROUP: 4
##           vars    n   mean     sd median trimmed    mad   min     max    range
## WAGE         1 1876 444.74 341.11 391.25  408.69 191.22  0.15 10167.5 10167.35
## EDUC         2 1876  12.62   3.38  12.00   12.61   2.97  0.00    20.0    20.00
## AGE          3 1876  46.51   0.50  47.00   46.52   0.00 46.00    47.0     1.00
## RACE         4 1876   0.09   0.28   0.00    0.00   0.00  0.00     1.0     1.00
## SMSA         5 1876   0.17   0.38   0.00    0.09   0.00  0.00     1.0     1.00
## MARRIED      6 1876   0.86   0.35   1.00    0.95   0.00  0.00     1.0     1.00
## AGE_GROUP    7 1876   4.00   0.00   4.00    4.00   0.00  4.00     4.0     0.00
##            skew kurtosis   se
## WAGE      13.04   351.30 7.88
## EDUC      -0.06     0.47 0.08
## AGE       -0.05    -2.00 0.01
## RACE       2.96     6.74 0.01
## SMSA       1.74     1.03 0.01
## MARRIED   -2.05     2.19 0.01
## AGE_GROUP   NaN      NaN 0.00
## ------------------------------------------------------------
## AGE_GROUP: 5
##           vars    n   mean     sd median trimmed    mad  min    max  range
## WAGE         1 2225 438.28 266.11 384.71  404.37 173.64  0.1 2667.5 2667.4
## EDUC         2 2225  12.37   3.37  12.00   12.40   2.97  0.0   20.0   20.0
## AGE          3 2225  48.69   0.67  49.00   48.62   1.48 48.0   50.0    2.0
## RACE         4 2225   0.08   0.27   0.00    0.00   0.00  0.0    1.0    1.0
## SMSA         5 2225   0.17   0.37   0.00    0.08   0.00  0.0    1.0    1.0
## MARRIED      6 2225   0.87   0.33   1.00    0.97   0.00  0.0    1.0    1.0
## AGE_GROUP    7 2225   5.00   0.00   5.00    5.00   0.00  5.0    5.0    0.0
##            skew kurtosis   se
## WAGE       2.05     7.54 5.64
## EDUC      -0.18     0.63 0.07
## AGE        0.45    -0.79 0.01
## RACE       3.06     7.37 0.01
## SMSA       1.79     1.21 0.01
## MARRIED   -2.24     3.00 0.01
## AGE_GROUP   NaN      NaN 0.00
```

### 2.3 Covariance matrices per regio

[[1]]

Table 1: Covariance Matrix for REGION: 1

| | WAGE | EDUC | AGE | RACE | SMSA | MARRIED | QOB | QOB1 | QOB2 | QOB3 | QOB4 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| WAGE | 59622.389 | 326.291 | 19.889 | -4.301 | -19.309 | 8.220 | -2.715 | 4.997 | -6.518 | 0.758 | 0.763 |

| | WAGE | EDUC | AGE | RACE | SMSA | MARRIED | QOB | QOB1 | QOB2 | QOB3 | QOB4 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| EDUC | 326.291 | 9.617 | -0.350 | -0.024 | -0.152 | -0.054 | 0.029 | 0.053 | -0.071 | -0.046 | 0.064 |
| AGE | 19.889 | -0.350 | 8.596 | 0.001 | -0.054 | 0.074 | -0.228 | 0.149 | -0.103 | -0.013 | -0.033 |
| RACE | -4.301 | -0.024 | 0.001 | 0.030 | -0.005 | -0.004 | -0.005 | 0.002 | 0.001 | -0.003 | 0.000 |
| SMSA | -19.309 | -0.152 | -0.054 | -0.005 | 0.127 | -0.005 | -0.010 | 0.001 | 0.006 | -0.005 | -0.002 |
| MARRIED | 8.220 | -0.054 | 0.074 | -0.004 | -0.005 | 0.129 | -0.005 | 0.001 | 0.000 | 0.002 | -0.003 |
| QOB | -2.715 | 0.029 | -0.228 | -0.005 | -0.010 | -0.005 | 1.217 | -0.360 | -0.132 | 0.127 | 0.365 |
| QOB1 | 4.997 | 0.053 | 0.149 | 0.002 | 0.001 | 0.001 | -0.360 | 0.181 | -0.060 | -0.063 | -0.058 |
| QOB2 | -6.518 | -0.071 | -0.103 | 0.001 | 0.006 | 0.000 | -0.132 | -0.060 | 0.189 | -0.067 | -0.062 |
| QOB3 | 0.758 | -0.046 | -0.013 | -0.003 | -0.005 | 0.002 | 0.127 | -0.063 | -0.067 | 0.195 | -0.065 |
| QOB4 | 0.763 | 0.064 | -0.033 | 0.000 | -0.002 | -0.003 | 0.365 | -0.058 | -0.062 | -0.065 | 0.186 |

[[2]]

Table 2: Covariance Matrix for REGION: 2

| | WAGE | EDUC | AGE | RACE | SMSA | MARRIED | QOB | QOB1 | QOB2 | QOB3 | QOB4 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| WAGE | 126020.884 | 335.536 | 29.869 | -13.086 | -6.225 | 16.273 | -5.375 | 0.966 | 1.492 | -0.507 | -1.951 |
| EDUC | 335.536 | 10.031 | -0.673 | -0.158 | -0.088 | 0.091 | -0.105 | 0.026 | 0.025 | -0.024 | -0.027 |
| AGE | 29.869 | -0.673 | 8.753 | -0.016 | 0.009 | -0.012 | -0.391 | 0.195 | -0.045 | -0.106 | -0.045 |
| RACE | -13.086 | -0.158 | -0.016 | 0.084 | -0.007 | -0.014 | -0.002 | -0.003 | 0.007 | -0.001 | -0.002 |
| SMSA | -6.225 | -0.088 | 0.009 | -0.007 | 0.081 | 0.001 | -0.003 | 0.003 | -0.002 | -0.002 | 0.001 |
| MARRIED | 16.273 | 0.091 | -0.012 | -0.014 | 0.001 | 0.122 | 0.014 | -0.003 | -0.004 | 0.001 | 0.005 |
| QOB | -5.375 | -0.105 | -0.391 | -0.002 | -0.003 | 0.014 | 1.226 | -0.361 | -0.133 | 0.123 | 0.371 |
| QOB1 | 0.966 | 0.026 | 0.195 | -0.003 | 0.003 | -0.003 | -0.361 | 0.180 | -0.059 | -0.062 | -0.060 |
| QOB2 | 1.492 | 0.025 | -0.045 | 0.007 | -0.002 | -0.004 | -0.133 | -0.059 | 0.187 | -0.066 | -0.063 |
| QOB3 | -0.507 | -0.024 | -0.106 | -0.001 | -0.002 | 0.001 | 0.123 | -0.062 | -0.066 | 0.194 | -0.066 |
| QOB4 | -1.951 | -0.027 | -0.045 | -0.002 | 0.001 | 0.005 | 0.371 | -0.060 | -0.063 | -0.066 | 0.189 |

[[3]]

Table 3: Covariance Matrix for REGION: 3

| | WAGE | EDUC | AGE | RACE | SMSA | MARRIED | QOB | QOB1 | QOB2 | QOB3 | QOB4 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| WAGE | 69754.484 | 247.163 | -9.152 | -6.535 | -13.792 | 10.302 | -13.651 | 4.382 | 1.401 | -2.296 | -3.487 |
| EDUC | 247.163 | 9.398 | -0.690 | -0.083 | -0.106 | -0.007 | 0.111 | -0.007 | -0.029 | -0.031 | 0.067 |
| AGE | -9.152 | -0.690 | 8.344 | -0.021 | -0.035 | -0.018 | -0.386 | 0.188 | -0.069 | -0.040 | -0.079 |
| RACE | -6.535 | -0.083 | -0.021 | 0.065 | -0.011 | -0.006 | -0.004 | 0.002 | -0.001 | 0.000 | -0.001 |
| SMSA | -13.792 | -0.106 | -0.035 | -0.011 | 0.136 | 0.006 | -0.002 | 0.001 | -0.001 | 0.000 | 0.000 |
| MARRIED | 10.302 | -0.007 | -0.018 | -0.006 | 0.006 | 0.113 | -0.022 | 0.002 | 0.008 | 0.000 | -0.010 |
| QOB | -13.651 | 0.111 | -0.386 | -0.004 | -0.002 | -0.022 | 1.264 | -0.388 | -0.116 | 0.131 | 0.373 |
| QOB1 | 4.382 | -0.007 | 0.188 | 0.002 | 0.001 | 0.002 | -0.388 | 0.193 | -0.062 | -0.067 | -0.064 |
| QOB2 | 1.401 | -0.029 | -0.069 | -0.001 | -0.001 | 0.008 | -0.116 | -0.062 | 0.181 | -0.061 | -0.058 |
| QOB3 | -2.296 | -0.031 | -0.040 | 0.000 | 0.000 | 0.000 | 0.131 | -0.067 | -0.061 | 0.191 | -0.063 |
| QOB4 | -3.487 | 0.067 | -0.079 | 0.001 | 0.000 | -0.010 | 0.373 | -0.064 | -0.058 | -0.063 | 0.186 |

[[4]]

Table 4: Covariance Matrix for REGION: 4

| | WAGE | EDUC | AGE | RACE | SMSA | MARRIED | QOB | QOB1 | QOB2 | QOB3 | QOB4 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| WAGE | 69294.730 | 276.252 | -25.048 | -4.650 | -20.462 | 12.774 | 13.151 | -5.239 | -0.306 | 3.178 | 2.367 |
| EDUC | 276.252 | 9.903 | -0.788 | -0.043 | -0.246 | 0.082 | 0.221 | -0.085 | 0.038 | -0.041 | 0.089 |
| AGE | -25.048 | -0.788 | 8.668 | 0.008 | 0.015 | 0.020 | -0.339 | 0.157 | -0.039 | -0.056 | -0.063 |
| RACE | -4.650 | -0.043 | 0.008 | 0.033 | -0.005 | -0.004 | -0.007 | 0.004 | -0.003 | 0.000 | -0.002 |
| SMSA | -20.462 | -0.246 | 0.015 | -0.005 | 0.226 | 0.005 | -0.005 | 0.003 | 0.004 | -0.012 | 0.005 |
| MARRIED | 12.774 | 0.082 | 0.020 | -0.004 | 0.005 | 0.101 | -0.006 | -0.002 | 0.003 | 0.005 | -0.006 |
| QOB | 13.151 | 0.221 | -0.339 | -0.007 | -0.005 | -0.006 | 1.211 | -0.389 | -0.102 | 0.159 | 0.332 |
| QOB1 | -5.239 | -0.085 | 0.157 | 0.004 | 0.003 | -0.002 | -0.389 | 0.199 | -0.068 | -0.074 | -0.057 |
| QOB2 | -0.306 | 0.038 | -0.039 | -0.003 | 0.004 | 0.003 | -0.102 | -0.068 | 0.186 | -0.067 | -0.051 |
| QOB3 | 3.178 | -0.041 | -0.056 | 0.000 | -0.012 | 0.005 | 0.159 | -0.074 | -0.067 | 0.198 | -0.057 |

| | WAGE | EDUC | AGE | RACE | SMSA | MARRIED | QOB | QOB1 | QOB2 | QOB3 | QOB4 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| QOB4 | 2.367 | 0.089 | -0.063 | -0.002 | 0.005 | -0.006 | 0.332 | -0.057 | -0.051 | -0.057 | 0.166 |

[[5]]

Table 5: Covariance Matrix for REGION: 5

| | WAGE | EDUC | AGE | RACE | SMSA | MARRIED | QOB | QOB1 | QOB2 | QOB3 | QOB4 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| WAGE | 68142.878 | 359.308 | -13.139 | -15.912 | -13.042 | 10.883 | 10.164 | -3.215 | -0.293 | 0.067 | 3.441 |
| EDUC | 359.308 | 12.144 | -0.823 | -0.232 | -0.257 | 0.040 | 0.124 | -0.070 | 0.056 | -0.025 | 0.040 |
| AGE | -13.139 | -0.823 | 8.956 | 0.017 | -0.038 | 0.053 | -0.347 | 0.181 | -0.043 | -0.110 | -0.028 |
| RACE | -15.912 | -0.232 | 0.017 | 0.119 | 0.001 | -0.019 | -0.015 | 0.001 | 0.004 | 0.003 | -0.008 |
| SMSA | -13.042 | -0.257 | -0.038 | 0.001 | 0.159 | 0.003 | -0.009 | 0.004 | -0.005 | 0.005 | -0.005 |
| MARRIED | 10.883 | 0.040 | 0.053 | -0.019 | 0.003 | 0.118 | -0.001 | 0.000 | -0.001 | 0.002 | -0.001 |
| QOB | 10.164 | 0.124 | -0.347 | -0.015 | -0.009 | -0.001 | 1.276 | -0.391 | -0.115 | 0.128 | 0.379 |
| QOB1 | -3.215 | -0.070 | 0.181 | 0.001 | 0.004 | 0.000 | -0.391 | 0.191 | -0.058 | -0.068 | -0.066 |
| QOB2 | -0.293 | 0.056 | -0.043 | 0.004 | -0.005 | -0.001 | -0.115 | -0.058 | 0.174 | -0.059 | -0.057 |
| QOB3 | 0.067 | -0.025 | -0.110 | 0.003 | 0.005 | 0.002 | 0.128 | -0.068 | -0.059 | 0.194 | -0.067 |
| QOB4 | 3.441 | 0.040 | -0.028 | -0.008 | -0.005 | -0.001 | 0.379 | -0.066 | -0.057 | -0.067 | 0.190 |

[[6]]

Table 6: Covariance Matrix for REGION: 6

| | WAGE | EDUC | AGE | RACE | SMSA | MARRIED | QOB | QOB1 | QOB2 | QOB3 | QOB4 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| WAGE | 108586.360 | 289.292 | -38.510 | -10.947 | -20.349 | 3.987 | 32.889 | -8.655 | -4.054 | 1.185 | 11.524 |
| EDUC | 289.292 | 14.004 | -1.028 | -0.234 | -0.204 | 0.117 | 0.345 | -0.130 | -0.013 | 0.071 | 0.072 |
| AGE | -38.510 | -1.028 | 7.682 | 0.018 | 0.036 | -0.011 | -0.341 | 0.185 | -0.085 | -0.042 | -0.057 |
| RACE | -10.947 | -0.234 | 0.018 | 0.110 | 0.006 | -0.022 | 0.010 | -0.004 | 0.003 | -0.001 | 0.003 |
| SMSA | -20.349 | -0.204 | 0.036 | 0.006 | 0.227 | 0.011 | -0.028 | 0.005 | 0.007 | -0.001 | -0.011 |
| MARRIED | 3.987 | 0.117 | -0.011 | -0.022 | 0.011 | 0.102 | 0.003 | -0.004 | 0.003 | 0.003 | -0.002 |
| QOB | 32.889 | 0.345 | -0.341 | 0.010 | -0.028 | 0.003 | 1.272 | -0.400 | -0.106 | 0.140 | 0.366 |

| | WAGE | EDUC | AGE | RACE | SMSA | MARRIED | QOB | QOB1 | QOB2 | QOB3 | QOB4 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| QOB1 | -8.655 | -0.130 | 0.185 | -0.004 | 0.005 | -0.004 | -0.400 | 0.199 | -0.063 | -0.071 | -0.065 |
| QOB2 | -4.054 | -0.013 | -0.085 | 0.003 | 0.007 | 0.003 | -0.106 | -0.063 | 0.177 | -0.060 | -0.055 |
| QOB3 | 1.185 | 0.071 | -0.042 | -0.001 | -0.001 | 0.003 | 0.140 | -0.071 | -0.060 | 0.192 | -0.062 |
| QOB4 | 11.524 | 0.072 | -0.057 | 0.003 | -0.011 | -0.002 | 0.366 | -0.065 | -0.055 | -0.062 | 0.181 |

[[7]]

Table 7: Covariance Matrix for REGION: 7

| | WAGE | EDUC | AGE | RACE | SMSA | MARRIED | QOB | QOB1 | QOB2 | QOB3 | QOB4 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| WAGE | 70637.866 | 357.447 | 26.277 | -11.392 | -12.199 | 11.726 | 8.040 | -2.625 | -4.360 | 8.554 | -1.569 |
| EDUC | 357.447 | 12.717 | -0.614 | -0.183 | -0.219 | 0.008 | 0.193 | -0.053 | -0.012 | -0.012 | 0.076 |
| AGE | 26.277 | -0.614 | 8.851 | 0.008 | -0.026 | 0.055 | -0.552 | 0.236 | -0.066 | -0.025 | -0.145 |
| RACE | -11.392 | -0.183 | 0.008 | 0.082 | 0.004 | -0.010 | 0.001 | -0.003 | 0.006 | -0.003 | 0.000 |
| SMSA | -12.199 | -0.219 | -0.026 | 0.004 | 0.140 | 0.004 | 0.016 | -0.001 | -0.005 | -0.002 | 0.008 |
| MARRIED | 11.726 | 0.008 | 0.055 | -0.010 | 0.004 | 0.114 | -0.009 | 0.002 | -0.005 | -0.005 | 0.008 |
| QOB | 8.040 | 0.193 | -0.552 | 0.001 | 0.016 | -0.009 | 1.224 | -0.380 | -0.112 | 0.140 | 0.352 |
| QOB1 | -2.625 | -0.053 | 0.236 | -0.003 | -0.001 | -0.002 | -0.380 | 0.185 | -0.050 | -0.075 | -0.059 |
| QOB2 | -4.360 | -0.012 | -0.066 | 0.006 | -0.005 | 0.005 | -0.112 | -0.050 | 0.163 | -0.063 | -0.050 |
| QOB3 | 8.554 | -0.012 | -0.025 | -0.003 | -0.002 | 0.005 | 0.140 | -0.075 | -0.063 | 0.213 | -0.075 |
| QOB4 | -1.569 | 0.076 | -0.145 | 0.000 | 0.008 | -0.008 | 0.352 | -0.059 | -0.050 | -0.075 | 0.184 |

[[8]]

Table 8: Covariance Matrix for REGION: 8

| | WAGE | EDUC | AGE | RACE | SMSA | MARRIED | QOB | QOB1 | QOB2 | QOB3 | QOB4 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| WAGE | 55763.738 | 204.967 | 8.289 | -5.134 | -19.118 | 0.818 | 7.934 | -4.199 | 3.356 | -2.050 | 2.893 |
| EDUC | 204.967 | 8.725 | -0.586 | -0.044 | -0.214 | 0.034 | 0.208 | -0.084 | 0.024 | -0.004 | 0.064 |
| AGE | 8.289 | -0.586 | 8.700 | -0.033 | -0.049 | 0.027 | -0.462 | 0.259 | -0.089 | -0.139 | -0.032 |
| RACE | -5.134 | -0.044 | -0.033 | 0.030 | -0.005 | -0.001 | 0.014 | -0.008 | 0.003 | 0.005 | 0.000 |

| | WAGE | EDUC | AGE | RACE | SMSA | MARRIED | QOB | QOB1 | QOB2 | QOB3 | QOB4 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| SMSA | -19.118 | -0.214 | -0.049 | -0.005 | 0.211 | 0.008 | 0.024 | -0.008 | 0.000 | -0.001 | 0.009 |
| MARRIED | 0.818 | 0.034 | 0.027 | -0.001 | 0.008 | 0.096 | 0.003 | 0.000 | 0.001 | -0.006 | 0.005 |
| QOB | 7.934 | 0.208 | -0.462 | 0.014 | 0.024 | 0.003 | 1.293 | -0.397 | -0.114 | 0.127 | 0.384 |
| QOB1 | -4.199 | -0.084 | 0.259 | -0.008 | -0.008 | 0.000 | -0.397 | 0.196 | -0.063 | -0.066 | -0.068 |
| QOB2 | 3.356 | 0.024 | -0.089 | 0.003 | 0.000 | 0.001 | -0.114 | -0.063 | 0.180 | -0.058 | -0.060 |
| QOB3 | -2.050 | -0.004 | -0.139 | 0.005 | -0.001 | -0.006 | 0.127 | -0.066 | -0.058 | 0.185 | -0.062 |
| QOB4 | 2.893 | 0.064 | -0.032 | 0.000 | 0.009 | 0.005 | 0.384 | -0.068 | -0.060 | -0.062 | 0.189 |

[[9]]

Table 9: Covariance Matrix for REGION: 9

| | WAGE | EDUC | AGE | RACE | SMSA | MARRIED | QOB | QOB1 | QOB2 | QOB3 | QOB4 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| WAGE | 119854.165 | 270.881 | 33.568 | -8.149 | -7.110 | 10.765 | -1.088 | 0.768 | -2.282 | 3.349 | -1.834 |
| EDUC | 270.881 | 8.544 | -0.359 | -0.046 | 0.113 | -0.022 | 0.145 | -0.052 | 0.022 | -0.034 | 0.064 |
| AGE | 33.568 | -0.359 | 8.370 | -0.012 | -0.016 | 0.051 | -0.413 | 0.207 | -0.087 | -0.035 | -0.086 |
| RACE | -8.149 | -0.046 | -0.012 | 0.058 | -0.005 | -0.009 | -0.008 | 0.002 | 0.004 | -0.005 | 0.000 |
| SMSA | -7.110 | -0.113 | -0.016 | -0.005 | 0.093 | 0.005 | -0.003 | 0.003 | 0.001 | -0.008 | 0.004 |
| MARRIED | 10.765 | -0.022 | 0.051 | -0.009 | 0.005 | 0.156 | 0.008 | -0.002 | -0.002 | 0.001 | 0.003 |
| QOB | -1.088 | 0.145 | -0.413 | -0.008 | -0.003 | 0.008 | 1.207 | -0.367 | -0.123 | 0.138 | 0.351 |
| QOB1 | 0.768 | -0.052 | 0.207 | 0.002 | 0.003 | -0.002 | -0.367 | 0.184 | -0.060 | -0.068 | -0.057 |
| QOB2 | -2.282 | 0.022 | -0.087 | 0.004 | 0.001 | -0.002 | -0.123 | -0.060 | 0.185 | -0.068 | -0.057 |
| QOB3 | 3.349 | -0.034 | -0.035 | -0.005 | -0.008 | 0.001 | 0.138 | -0.068 | -0.068 | 0.201 | -0.065 |
| QOB4 | -1.834 | 0.064 | -0.086 | 0.000 | 0.004 | 0.003 | 0.351 | -0.057 | -0.057 | -0.065 | 0.180 |

### 2.4 Correlation matrices per regio

[[1]]

Table 10: Correlation Matrix for REGION: 1

| | WAGE | EDUC | AGE | RACE | SMSA | MARRIED | QOB | QOB1 | QOB2 | QOB3 | QOB4 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| WAGE | 1.000 | 0.431 | 0.028 | -0.102 | -0.222 | 0.094 | -0.010 | 0.048 | -0.061 | 0.007 | 0.007 |
| EDUC | 0.431 | 1.000 | -0.038 | -0.045 | -0.137 | -0.049 | 0.008 | 0.040 | -0.053 | -0.033 | 0.048 |
| AGE | 0.028 | -0.038 | 1.000 | 0.002 | -0.052 | 0.071 | -0.071 | 0.119 | -0.080 | -0.010 | -0.026 |
| RACE | -0.102 | -0.045 | 0.002 | 1.000 | -0.075 | -0.071 | -0.027 | 0.024 | 0.017 | -0.036 | -0.004 |
| SMSA | -0.222 | -0.137 | -0.052 | -0.075 | 1.000 | -0.037 | -0.026 | 0.007 | 0.038 | -0.031 | -0.014 |
| MARRIED | 0.094 | -0.049 | 0.071 | -0.071 | -0.037 | 1.000 | -0.013 | 0.008 | 0.000 | 0.011 | -0.019 |
| QOB | -0.010 | 0.008 | -0.071 | -0.027 | -0.026 | -0.013 | 1.000 | -0.768 | -0.274 | 0.261 | 0.767 |
| QOB1 | 0.048 | 0.040 | 0.119 | 0.024 | 0.007 | 0.008 | -0.768 | 1.000 | -0.324 | -0.334 | -0.318 |
| QOB2 | -0.061 | -0.053 | -0.080 | 0.017 | 0.038 | 0.000 | -0.274 | -0.324 | 1.000 | -0.349 | -0.332 |
| QOB3 | 0.007 | -0.033 | -0.010 | -0.036 | -0.031 | 0.011 | 0.261 | -0.334 | -0.349 | 1.000 | -0.342 |
| QOB4 | 0.007 | 0.048 | -0.026 | -0.004 | -0.014 | -0.019 | 0.767 | -0.318 | -0.332 | -0.342 | 1.000 |

[[2]]

Table 11: Correlation Matrix for REGION: 2

| | WAGE | EDUC | AGE | RACE | SMSA | MARRIED | QOB | QOB1 | QOB2 | QOB3 | QOB4 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| WAGE | 1.000 | 0.298 | 0.028 | -0.127 | -0.062 | 0.131 | -0.014 | 0.006 | 0.010 | -0.003 | -0.013 |
| EDUC | 0.298 | 1.000 | -0.072 | -0.173 | -0.098 | 0.082 | -0.030 | 0.020 | 0.018 | -0.017 | -0.020 |
| AGE | 0.028 | -0.072 | 1.000 | -0.019 | 0.011 | -0.011 | -0.119 | 0.156 | -0.035 | -0.081 | -0.035 |
| RACE | -0.127 | -0.173 | -0.019 | 1.000 | -0.084 | -0.139 | -0.005 | -0.028 | 0.053 | -0.012 | -0.014 |
| SMSA | -0.062 | -0.098 | 0.011 | -0.084 | 1.000 | 0.007 | -0.010 | 0.025 | -0.016 | -0.015 | 0.007 |
| MARRIED | 0.131 | 0.082 | -0.011 | -0.139 | 0.007 | 1.000 | 0.036 | -0.017 | -0.025 | 0.009 | 0.033 |
| QOB | -0.014 | -0.030 | -0.119 | -0.005 | -0.010 | 0.036 | 1.000 | -0.768 | -0.277 | 0.252 | 0.771 |
| QOB1 | 0.006 | 0.020 | 0.156 | -0.028 | 0.025 | -0.017 | -0.768 | 1.000 | -0.320 | -0.331 | -0.323 |
| QOB2 | 0.010 | 0.018 | -0.035 | 0.053 | -0.016 | -0.025 | -0.277 | -0.320 | 1.000 | -0.344 | -0.335 |
| QOB3 | -0.003 | -0.017 | -0.081 | -0.012 | -0.015 | 0.009 | 0.252 | -0.331 | -0.344 | 1.000 | -0.347 |
| QOB4 | -0.013 | -0.020 | -0.035 | -0.014 | 0.007 | 0.033 | 0.771 | -0.323 | -0.335 | -0.347 | 1.000 |

[[3]]

Table 12: Correlation Matrix for REGION: 3

| | WAGE | EDUC | AGE | RACE | SMSA | MARRIED | QOB | QOB1 | QOB2 | QOB3 | QOB4 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| WAGE | 1.000 | 0.305 | -0.012 | -0.097 | -0.142 | 0.116 | -0.046 | 0.038 | 0.012 | -0.020 | -0.031 |
| EDUC | 0.305 | 1.000 | -0.078 | -0.107 | -0.093 | -0.007 | 0.032 | -0.006 | -0.022 | -0.023 | 0.051 |
| AGE | -0.012 | -0.078 | 1.000 | -0.029 | -0.033 | -0.019 | -0.119 | 0.149 | -0.057 | -0.032 | -0.064 |
| RACE | -0.097 | -0.107 | -0.029 | 1.000 | -0.115 | -0.075 | -0.015 | 0.021 | -0.011 | -0.004 | -0.007 |
| SMSA | -0.142 | -0.093 | -0.033 | -0.115 | 1.000 | 0.049 | -0.005 | 0.008 | -0.006 | 0.000 | -0.003 |
| MARRIED | 0.116 | -0.007 | -0.019 | -0.075 | 0.049 | 1.000 | -0.057 | 0.013 | 0.057 | -0.002 | -0.067 |
| QOB | -0.046 | 0.032 | -0.119 | -0.015 | -0.005 | -0.057 | 1.000 | -0.786 | -0.242 | 0.267 | 0.769 |
| QOB1 | 0.038 | -0.006 | 0.149 | 0.021 | 0.008 | 0.013 | -0.786 | 1.000 | -0.331 | -0.348 | -0.339 |
| QOB2 | 0.012 | -0.022 | -0.057 | -0.011 | -0.006 | 0.057 | -0.242 | -0.331 | 1.000 | -0.327 | -0.319 |
| QOB3 | -0.020 | -0.023 | -0.032 | -0.004 | 0.000 | -0.002 | 0.267 | -0.348 | -0.327 | 1.000 | -0.335 |
| QOB4 | -0.031 | 0.051 | -0.064 | -0.007 | -0.003 | -0.067 | 0.769 | -0.339 | -0.319 | -0.335 | 1.000 |

[[4]]

Table 13: Correlation Matrix for REGION: 4

| | WAGE | EDUC | AGE | RACE | SMSA | MARRIED | QOB | QOB1 | QOB2 | QOB3 | QOB4 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| WAGE | 1.000 | 0.333 | -0.032 | -0.097 | -0.164 | 0.153 | 0.045 | -0.045 | -0.003 | 0.027 | 0.022 |
| EDUC | 0.333 | 1.000 | -0.085 | -0.076 | -0.164 | 0.082 | 0.064 | -0.061 | 0.028 | -0.029 | 0.069 |
| AGE | -0.032 | -0.085 | 1.000 | 0.015 | 0.010 | 0.021 | -0.104 | 0.119 | -0.030 | -0.042 | -0.053 |
| RACE | -0.097 | -0.076 | 0.015 | 1.000 | -0.057 | -0.075 | -0.037 | 0.053 | -0.038 | 0.004 | -0.023 |
| SMSA | -0.164 | -0.164 | 0.010 | -0.057 | 1.000 | 0.031 | -0.010 | 0.013 | 0.022 | -0.058 | 0.025 |
| MARRIED | 0.153 | 0.082 | 0.021 | -0.075 | 0.031 | 1.000 | -0.018 | -0.012 | 0.024 | 0.033 | -0.049 |
| QOB | 0.045 | 0.064 | -0.104 | -0.037 | -0.010 | -0.018 | 1.000 | -0.791 | -0.215 | 0.324 | 0.741 |
| QOB1 | -0.045 | -0.061 | 0.119 | 0.053 | 0.013 | -0.012 | -0.791 | 1.000 | -0.351 | -0.375 | -0.316 |
| QOB2 | -0.003 | 0.028 | -0.030 | -0.038 | 0.022 | 0.024 | -0.215 | -0.351 | 1.000 | -0.348 | -0.294 |
| QOB3 | 0.027 | -0.029 | -0.042 | 0.004 | -0.058 | 0.033 | 0.324 | -0.375 | -0.348 | 1.000 | -0.313 |
| QOB4 | 0.022 | 0.069 | -0.053 | -0.023 | 0.025 | -0.049 | 0.741 | -0.316 | -0.294 | -0.313 | 1.000 |

[[5]]

Table 14: Correlation Matrix for REGION: 5

| | WAGE | EDUC | AGE | RACE | SMSA | MARRIED | QOB | QOB1 | QOB2 | QOB3 | QOB4 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| WAGE | 1.000 | 0.395 | -0.017 | -0.177 | -0.125 | 0.122 | 0.034 | -0.028 | -0.003 | 0.001 | 0.030 |
| EDUC | 0.395 | 1.000 | -0.079 | -0.193 | -0.185 | 0.034 | 0.032 | -0.046 | 0.039 | -0.017 | 0.026 |
| AGE | -0.017 | -0.079 | 1.000 | 0.017 | -0.032 | 0.052 | -0.103 | 0.138 | -0.034 | -0.083 | -0.022 |
| RACE | -0.177 | -0.193 | 0.017 | 1.000 | 0.008 | -0.157 | -0.037 | 0.008 | 0.029 | 0.018 | -0.053 |
| SMSA | -0.125 | -0.185 | -0.032 | 0.008 | 1.000 | 0.023 | -0.019 | 0.025 | -0.028 | 0.028 | -0.027 |
| MARRIED | 0.122 | 0.034 | 0.052 | -0.157 | 0.023 | 1.000 | -0.003 | 0.000 | -0.004 | 0.013 | -0.010 |
| QOB | 0.034 | 0.032 | -0.103 | -0.037 | -0.019 | -0.003 | 1.000 | -0.791 | -0.245 | 0.257 | 0.769 |
| QOB1 | -0.028 | -0.046 | 0.138 | 0.008 | 0.025 | 0.000 | -0.791 | 1.000 | -0.317 | -0.352 | -0.345 |
| QOB2 | -0.003 | 0.039 | -0.034 | 0.029 | -0.028 | -0.004 | -0.245 | -0.317 | 1.000 | -0.321 | -0.314 |
| QOB3 | 0.001 | -0.017 | -0.083 | 0.018 | 0.028 | 0.013 | 0.257 | -0.352 | -0.321 | 1.000 | -0.350 |
| QOB4 | 0.030 | 0.026 | -0.022 | -0.053 | -0.027 | -0.010 | 0.769 | -0.345 | -0.314 | -0.350 | 1.000 |

[[6]]

Table 15: Correlation Matrix for REGION: 6

| | WAGE | EDUC | AGE | RACE | SMSA | MARRIED | QOB | QOB1 | QOB2 | QOB3 | QOB4 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| WAGE | 1.000 | 0.235 | -0.042 | -0.100 | -0.130 | 0.038 | 0.088 | -0.059 | -0.029 | 0.008 | 0.082 |
| EDUC | 0.235 | 1.000 | -0.099 | -0.189 | -0.114 | 0.097 | 0.082 | -0.078 | -0.008 | 0.043 | 0.045 |
| AGE | -0.042 | -0.099 | 1.000 | 0.019 | 0.027 | -0.012 | -0.109 | 0.149 | -0.073 | -0.035 | -0.048 |
| RACE | -0.100 | -0.189 | 0.019 | 1.000 | 0.041 | -0.203 | 0.026 | -0.030 | 0.018 | -0.008 | 0.022 |
| SMSA | -0.130 | -0.114 | 0.027 | 0.041 | 1.000 | 0.070 | -0.052 | 0.022 | 0.037 | -0.006 | -0.054 |
| MARRIED | 0.038 | 0.097 | -0.012 | -0.203 | 0.070 | 1.000 | 0.009 | -0.030 | 0.023 | 0.024 | -0.016 |
| QOB | 0.088 | 0.082 | -0.109 | 0.026 | -0.052 | 0.009 | 1.000 | -0.795 | -0.223 | 0.283 | 0.762 |
| QOB1 | -0.059 | -0.078 | 0.149 | -0.030 | 0.022 | -0.030 | -0.795 | 1.000 | -0.335 | -0.363 | -0.342 |
| QOB2 | -0.029 | -0.008 | -0.073 | 0.018 | 0.037 | 0.023 | -0.223 | -0.335 | 1.000 | -0.323 | -0.305 |
| QOB3 | 0.008 | 0.043 | -0.035 | -0.008 | -0.006 | 0.024 | 0.283 | -0.363 | -0.323 | 1.000 | -0.330 |
| QOB4 | 0.082 | 0.045 | -0.048 | 0.022 | -0.054 | -0.016 | 0.762 | -0.342 | -0.305 | -0.330 | 1.000 |

[[7]]

Table 16: Correlation Matrix for REGION: 7

| | WAGE | EDUC | AGE | RACE | SMSA | MARRIED | QOB | QOB1 | QOB2 | QOB3 | QOB4 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| WAGE | 1.000 | 0.377 | 0.033 | -0.149 | -0.122 | 0.130 | 0.027 | -0.023 | -0.041 | 0.070 | -0.014 |
| EDUC | 0.377 | 1.000 | -0.058 | -0.179 | -0.164 | 0.007 | 0.049 | -0.034 | -0.008 | -0.007 | 0.050 |
| AGE | 0.033 | -0.058 | 1.000 | 0.010 | -0.023 | 0.055 | -0.168 | 0.184 | -0.055 | -0.018 | -0.114 |
| RACE | -0.149 | -0.179 | 0.010 | 1.000 | 0.036 | -0.105 | 0.003 | -0.025 | 0.048 | -0.020 | 0.002 |
| SMSA | -0.122 | -0.164 | -0.023 | 0.036 | 1.000 | 0.033 | 0.039 | -0.007 | -0.036 | -0.010 | 0.052 |
| MARRIED | 0.130 | 0.007 | 0.055 | -0.105 | 0.033 | 1.000 | -0.025 | -0.013 | 0.036 | 0.034 | -0.057 |
| QOB | 0.027 | 0.049 | -0.168 | 0.003 | 0.039 | -0.025 | 1.000 | -0.797 | -0.251 | 0.273 | 0.743 |
| QOB1 | -0.023 | -0.034 | 0.184 | -0.025 | -0.007 | -0.013 | -0.797 | 1.000 | -0.289 | -0.380 | -0.322 |
| QOB2 | -0.041 | -0.008 | -0.055 | 0.048 | -0.036 | 0.036 | -0.251 | -0.289 | 1.000 | -0.338 | -0.287 |
| QOB3 | 0.070 | -0.007 | -0.018 | -0.020 | -0.010 | 0.034 | 0.273 | -0.380 | -0.338 | 1.000 | -0.377 |
| QOB4 | -0.014 | 0.050 | -0.114 | 0.002 | 0.052 | -0.057 | 0.743 | -0.322 | -0.287 | -0.377 | 1.000 |

[[8]]

Table 17: Correlation Matrix for REGION: 8

| | WAGE | EDUC | AGE | RACE | SMSA | MARRIED | QOB | QOB1 | QOB2 | QOB3 | QOB4 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| WAGE | 1.000 | 0.294 | 0.012 | -0.127 | -0.176 | 0.011 | 0.030 | -0.040 | 0.033 | -0.020 | 0.028 |
| EDUC | 0.294 | 1.000 | -0.067 | -0.087 | -0.158 | 0.037 | 0.062 | -0.064 | 0.019 | -0.003 | 0.050 |
| AGE | 0.012 | -0.067 | 1.000 | -0.065 | -0.036 | 0.029 | -0.138 | 0.198 | -0.071 | -0.109 | -0.025 |
| RACE | -0.127 | -0.087 | -0.065 | 1.000 | -0.065 | -0.015 | 0.070 | -0.107 | 0.041 | 0.064 | 0.006 |
| SMSA | -0.176 | -0.158 | -0.036 | -0.065 | 1.000 | 0.057 | 0.046 | -0.038 | 0.000 | -0.005 | 0.044 |
| MARRIED | 0.011 | 0.037 | 0.029 | -0.015 | 0.057 | 1.000 | 0.009 | 0.002 | 0.007 | -0.046 | 0.036 |
| QOB | 0.030 | 0.062 | -0.138 | 0.070 | 0.046 | 0.009 | 1.000 | -0.789 | -0.236 | 0.259 | 0.777 |
| QOB1 | -0.040 | -0.064 | 0.198 | -0.107 | -0.038 | 0.002 | -0.789 | 1.000 | -0.335 | -0.344 | -0.351 |
| QOB2 | 0.033 | 0.019 | -0.071 | 0.041 | 0.000 | 0.007 | -0.236 | -0.335 | 1.000 | -0.316 | -0.322 |
| QOB3 | -0.020 | -0.003 | -0.109 | 0.064 | -0.005 | -0.046 | 0.259 | -0.344 | -0.316 | 1.000 | -0.331 |
| QOB4 | 0.028 | 0.050 | -0.025 | 0.006 | 0.044 | 0.036 | 0.777 | -0.351 | -0.322 | -0.331 | 1.000 |

[[9]]

Table 18: Correlation Matrix for REGION: 9

| | WAGE | EDUC | AGE | RACE | SMSA | MARRIED | QOB | QOB1 | QOB2 | QOB3 | QOB4 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| WAGE | 1.000 | 0.268 | 0.034 | -0.098 | -0.067 | 0.079 | -0.003 | 0.005 | -0.015 | 0.022 | -0.013 |
| EDUC | 0.268 | 1.000 | -0.042 | -0.066 | -0.127 | -0.019 | 0.045 | -0.041 | 0.017 | -0.026 | 0.052 |
| AGE | 0.034 | -0.042 | 1.000 | -0.017 | -0.018 | 0.044 | -0.130 | 0.167 | -0.070 | -0.027 | -0.070 |
| RACE | -0.098 | -0.066 | -0.017 | 1.000 | -0.066 | -0.096 | -0.032 | 0.021 | 0.035 | -0.050 | -0.004 |
| SMSA | -0.067 | -0.127 | -0.018 | -0.066 | 1.000 | 0.044 | -0.009 | 0.021 | 0.009 | -0.056 | 0.029 |
| MARRIED | 0.079 | -0.019 | 0.044 | -0.096 | 0.044 | 1.000 | 0.018 | -0.011 | -0.010 | 0.005 | 0.016 |
| QOB | -0.003 | 0.045 | -0.130 | -0.032 | -0.009 | 0.018 | 1.000 | -0.777 | -0.260 | 0.281 | 0.755 |
| QOB1 | 0.005 | -0.041 | 0.167 | 0.021 | 0.021 | -0.011 | -0.777 | 1.000 | -0.323 | -0.352 | -0.314 |
| QOB2 | -0.015 | 0.017 | -0.070 | 0.035 | 0.009 | -0.010 | -0.260 | -0.323 | 1.000 | -0.352 | -0.315 |
| QOB3 | 0.022 | -0.026 | -0.027 | -0.050 | -0.056 | 0.005 | 0.281 | -0.352 | -0.352 | 1.000 | -0.343 |
| QOB4 | -0.013 | 0.052 | -0.070 | -0.004 | 0.029 | 0.016 | 0.755 | -0.314 | -0.315 | -0.343 | 1.000 |

```
## Warning: Use of ‘df$EDUC‘ is discouraged.
## i Use ‘EDUC‘ instead.
```

```
## Warning: Use of `df$WAGE` is discouraged.
## i Use `WAGE` instead.
```


## 3.Estimate the baseline Specification

```
#linear model
linear_model1=lm(WAGE~ EDUC + AGE + RACE + SMSA + MARRIED + REGION2 + REGION3
                 + REGION4 + REGION5 + REGION6 + REGION7 + REGION8 + REGION9  , data = df)
```

```
##
## ===========================================================
##                              Dependent variable:
##                          -------------------------------
##                                       WAGE
## -----------------------------------------------------------
## EDUC                                 26.696***
##                                       (0.869)
##                                      t = 30.724
##                                      p = 0.000
## AGE                                   2.244**
##                                       (0.942)
##                                      t = 2.382
##                                      p = 0.018
```

```
## RACE                                -77.632***
##                                      (10.225)
##                                     t = -7.592
##                                     p = 0.000
## SMSA                                -63.756***
##                                      (7.393)
##                                     t = -8.624
##                                     p = 0.000
## MARRIED                             78.001***
##                                      (8.026)
##                                     t = 9.719
##                                     p = 0.000
## REGION2                             41.193***
##                                      (13.644)
##                                     t = 3.019
##                                     p = 0.003
## REGION3                             49.088***
##                                      (13.305)
##                                     t = 3.690
##                                     p = 0.0003
## REGION4                              18.845
##                                      (15.600)
##                                     t = 1.208
##                                     p = 0.228
## REGION5                              4.205
##                                      (13.492)
##                                     t = 0.312
##                                     p = 0.756
## REGION6                              7.604
##                                      (16.123)
##                                     t = 0.472
##                                     p = 0.638
## REGION7                              17.081
##                                      (14.646)
##                                     t = 1.166
##                                     p = 0.244
## REGION8                              15.897
##                                      (17.106)
##                                     t = 0.929
##                                     p = 0.353
## REGION9                             63.710***
##                                      (14.046)
##                                     t = 4.536
##                                    p = 0.00001
## Constant                             -81.669*
##                                      (46.614)
##                                     t = -1.752
##                                     p = 0.080
## ------------------------------------------------------------
## Observations                          10,000
## R2                                    0.134
## Adjusted R2                           0.133
## Residual Std. Error          275.029 (df = 9986)
## F Statistic         118.953*** (df = 13; 9986) (p = 0.000)
```

```
## ============================================================
## Note:                          *p<0.1; **p<0.05; ***p<0.01
```

### 3.1 Interpretation of the estimated Coefficient

EDUC (26.696, $p < 0.001$)

A one-year increase in education leads to a \$26.696 increase in weelky wages. This is highly significant ($p = 0.000$).

AGE ( 2.244, $p = 0.020$) A one-year increase in age increases wages by \$2.23. Significant at the 5% level ($p = 0.020$).

RACE (-77.478, $p < 0.001$) Suggests a wage penalty of \$77.48 for certain racial groups (assuming a binary variable where non-white = 1). Highly significant ($p = 0.000$).

SMSA (-63.654, $p < 0.001$) Living in an SMSA (Standard Metropolitan Statistical Area) is associated with a \$63.65 lower wage. Significant at $p = 0.000$.

MARRIED (77.927, $p < 0.001$) Being married increases wages by \$77.93. Highly significant ($p = 0.000$).

Regional Effects on Wages

Significant Regions: REGION2 ($B = 41.204$, $p = 0.003$) REGION3 ($B = 49.071$, $p = 0.0003$) REGION9 ($B = 63.543$, $p = 0.00001$) These regions have higher wages compared to the reference region.

Non-Significant Regions: REGION4, REGION5, REGION6, REGION7, REGION8 ($p > 0.05$) These regions do not significantly differ from the reference region in terms of wages.

```
##
## Linear hypothesis test:
## AGE = 0
## RACE = 0
## MARRIED = 0
## SMSA = 0
##
## Model 1: restricted model
## Model 2: WAGE ~ EDUC + AGE + RACE + SMSA + MARRIED + REGION2 + REGION3 +
##     REGION4 + REGION5 + REGION6 + REGION7 + REGION8 + REGION9
##
##   Res.Df       RSS Df Sum of Sq      F    Pr(>F)
## 1   9990 773577022
## 2   9986 755352847  4  18224175 60.232 < 2.2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

The joint significance test shows that AGE, RACE, MARRIED, and SMSA jointly contribute significantly to explaining WAGE. The F-statistic is 60.23 with a p-value < 2.2e-16, indicating strong statistical significance. We reject the null hypothesis that these four variables have no joint effect on wages.

```
##
## Linear hypothesis test:
## REGION2 = 0
## REGION3 = 0
## REGION4 = 0
## REGION5 = 0
## REGION6 = 0
```

```
## REGION7 = 0
## REGION8 = 0
## REGION9 = 0
##
## Model 1: restricted model
## Model 2: WAGE ~ EDUC + AGE + RACE + SMSA + MARRIED + REGION2 + REGION3 +
##     REGION4 + REGION5 + REGION6 + REGION7 + REGION8 + REGION9
##
##   Res.Df       RSS Df Sum of Sq      F    Pr(>F)
## 1   9994 759957235
## 2   9986 755352847  8   4604388 7.6089 3.389e-10 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

Since the p-value is extremely small (<0.001), we reject the null hypothesis. This means that at least one of the region coefficients is significantly different from zero, implying that region does have a statistically significant effect on wages.

# 4. Evaluating Gauss-Markov Assumptions and Applying Remedial Measures

**Gauss-Markov assumptions:**

Assumption 1: Linearity in the parameters: CHECK

Assumption 2a: The X -values are fixed over repeated sampling (fixed regressor model) FAIL

Correlation between explanatory variables and error terms:

```
residuals <- residuals(linear_model1)
print(mean(residuals))
```

```
## [1] -4.605628e-16
```

```
print(t.test(residuals, mu = 0))
```

```
##
##  One Sample t-test
##
## data:  residuals
## t = -1.6757e-16, df = 9999, p-value = 1
## alternative hypothesis: true mean is not equal to 0
## 95 percent confidence interval:
##  -5.387624  5.387624
## sample estimates:
##     mean of x
## -4.605628e-16
```

```
if (show_interpretation) {
  cat("Assumption 3: The expected value of the error terms is zero( nulhypothese niet verwerpen : CHECK
}
```

```
## Assumption 3: The expected value of the error terms is zero( nulhypothese niet verwerpen : CHECK
```

## 4(a). Stochastic Regressors

All the variables are stochastic.

Strictly exogenous: AGE, RACE, QOB

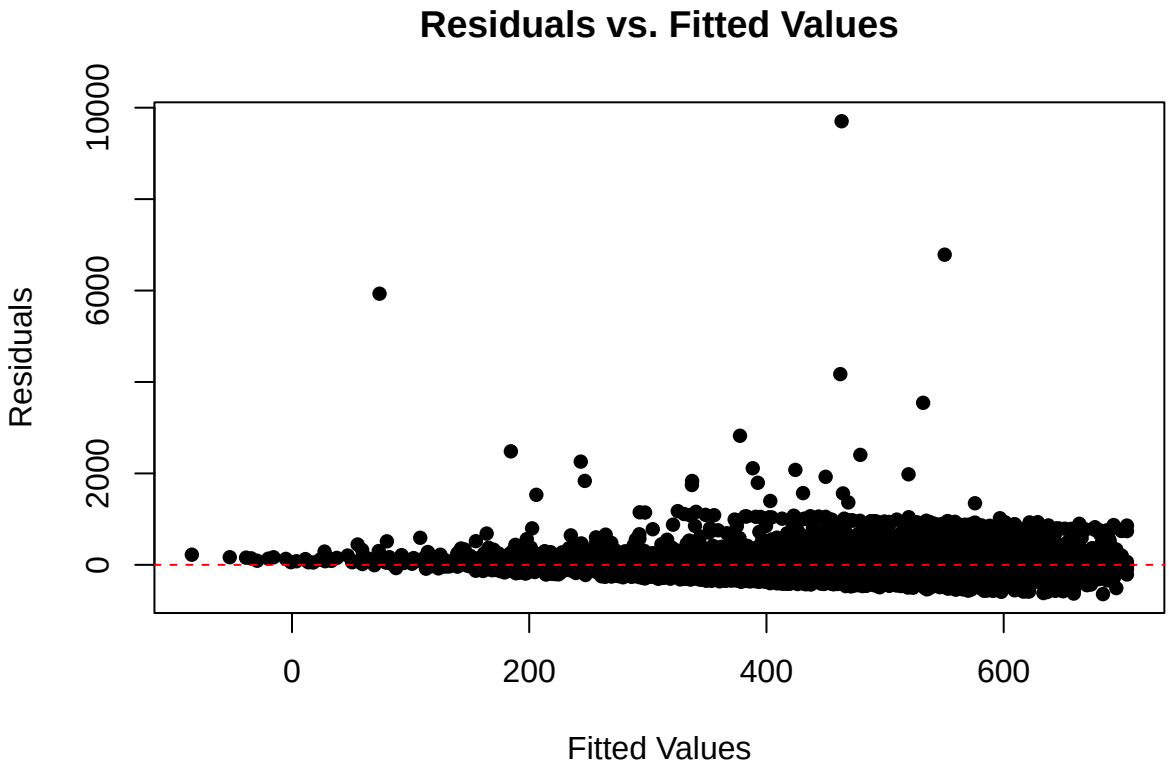
- These variables are fixed and not influenced by wages or the error term.

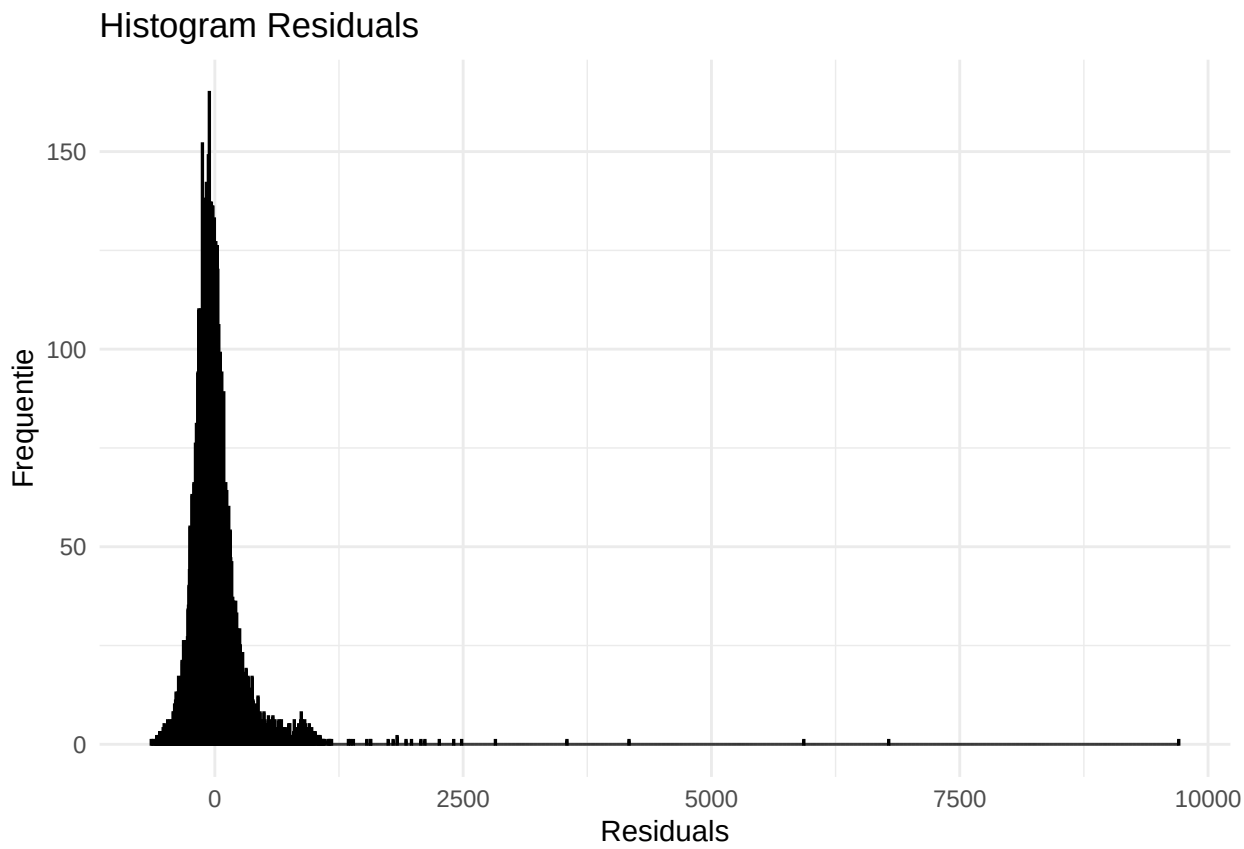
Weakly exogenous: SMSA, REGION, MARRIED

- These variables may be correlated with unobserved factors affecting wages, but are not directly influenced by wages.

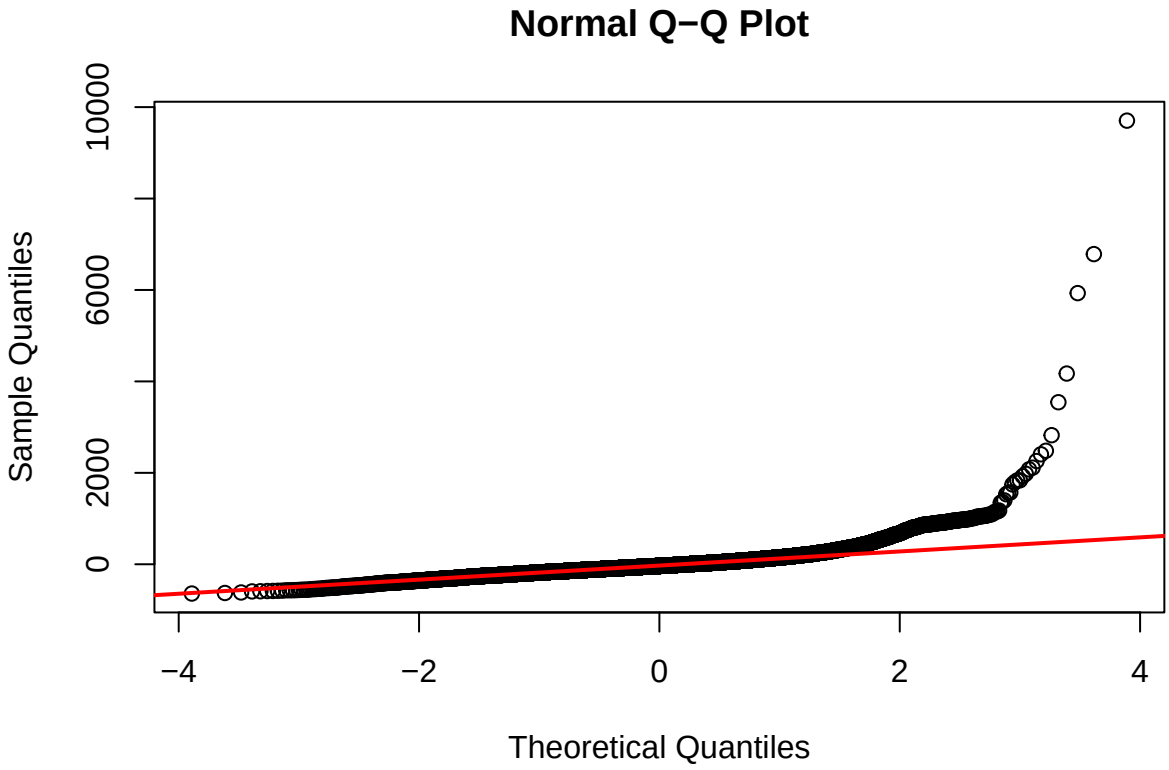
Endogenous: EDUC

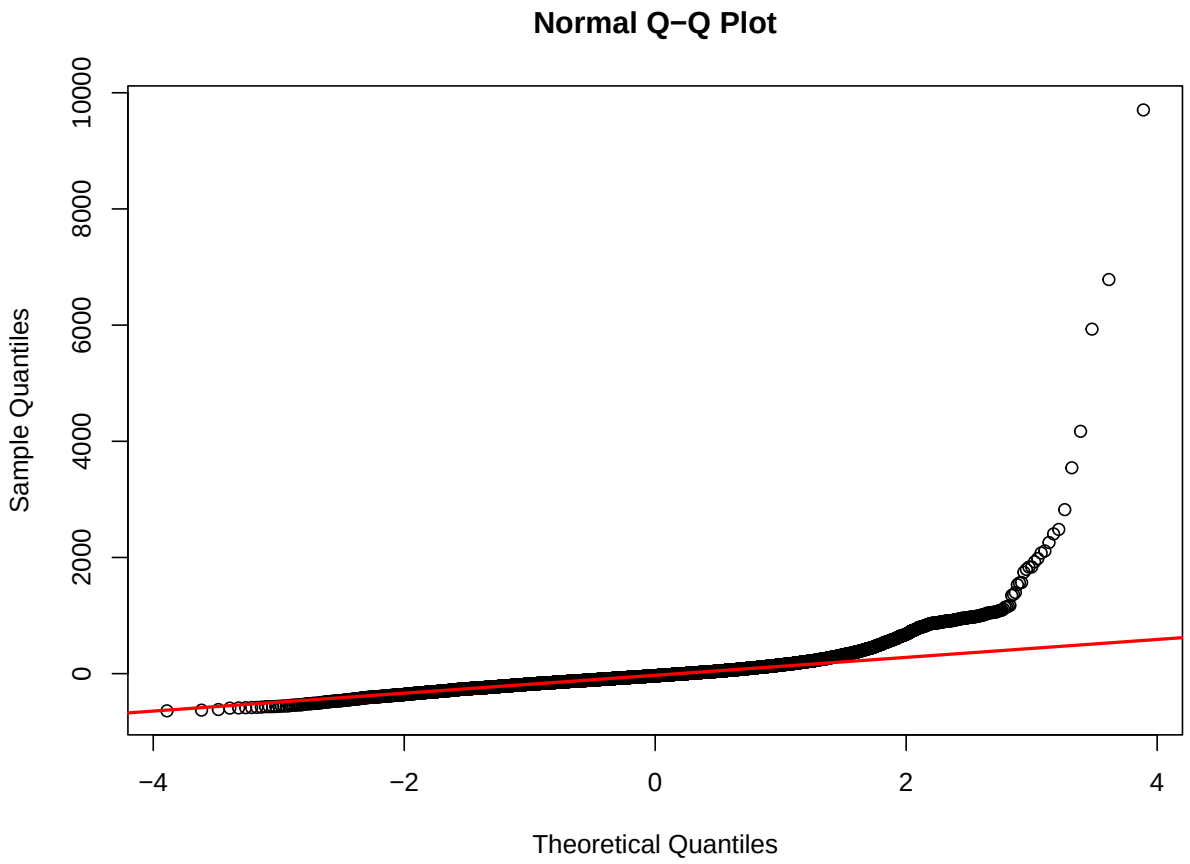
- Potential for reverse causality or correlation with the error term (e.g., higher wages → more education or higher likelihood of marriage).




Histogram Residuals
Frequentie
150
100
50
0
0
2500
5000
7500
10000
Residuals

4(b). Normality Error Terms






```
Jarque Bera Test
```

data: residuals X-squared = 21722226, df = 2, p-value < 2.2e-16

Since the p value is small, we reject the null hypothesis. Therefore the residuals are NOT normally distributed.

Implications for OLS:

1) Unbiasedness: Non-normal residuals do not affect the unbiasedness of OLS estimators as long as key assumptions (like linearity, exogeneity of regressors, and zero-mean error term) are satisfied.

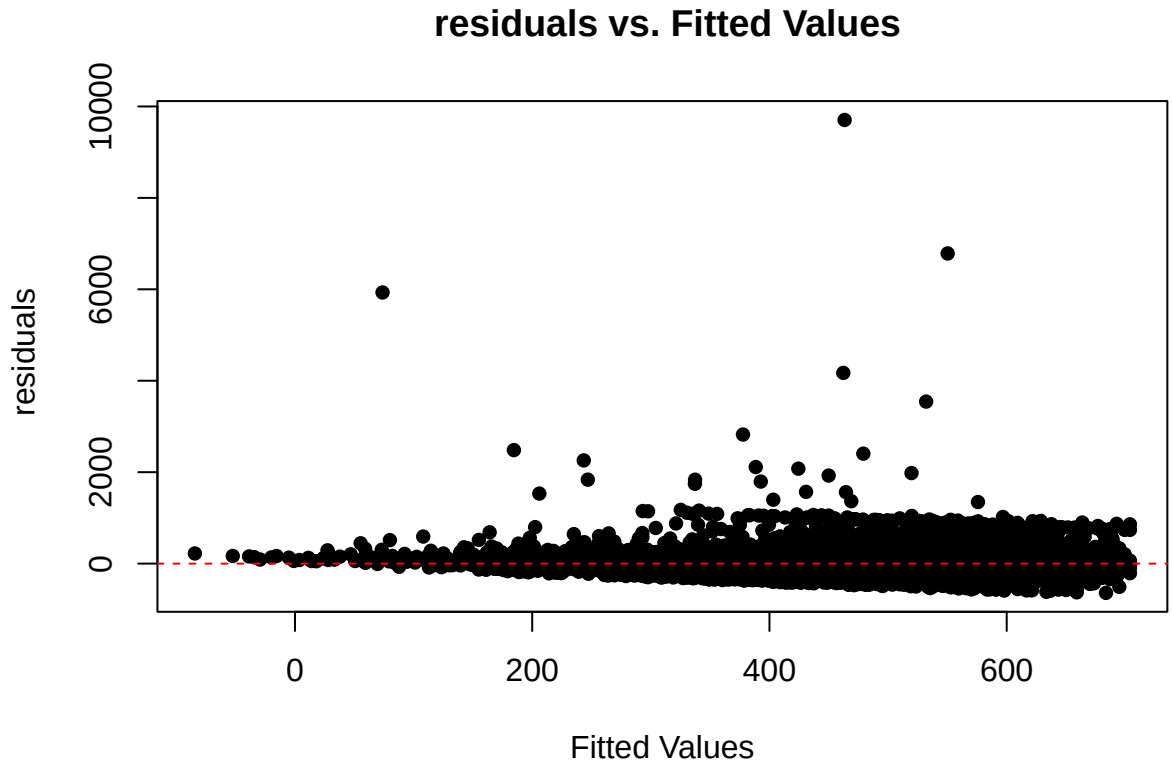
2) Efficiency: OLS estimators may lose their efficiency because normality of residuals is necessary for OLS to be the Best Linear Unbiased Estimator (BLUE) under the Gauss-Markov assumptions.

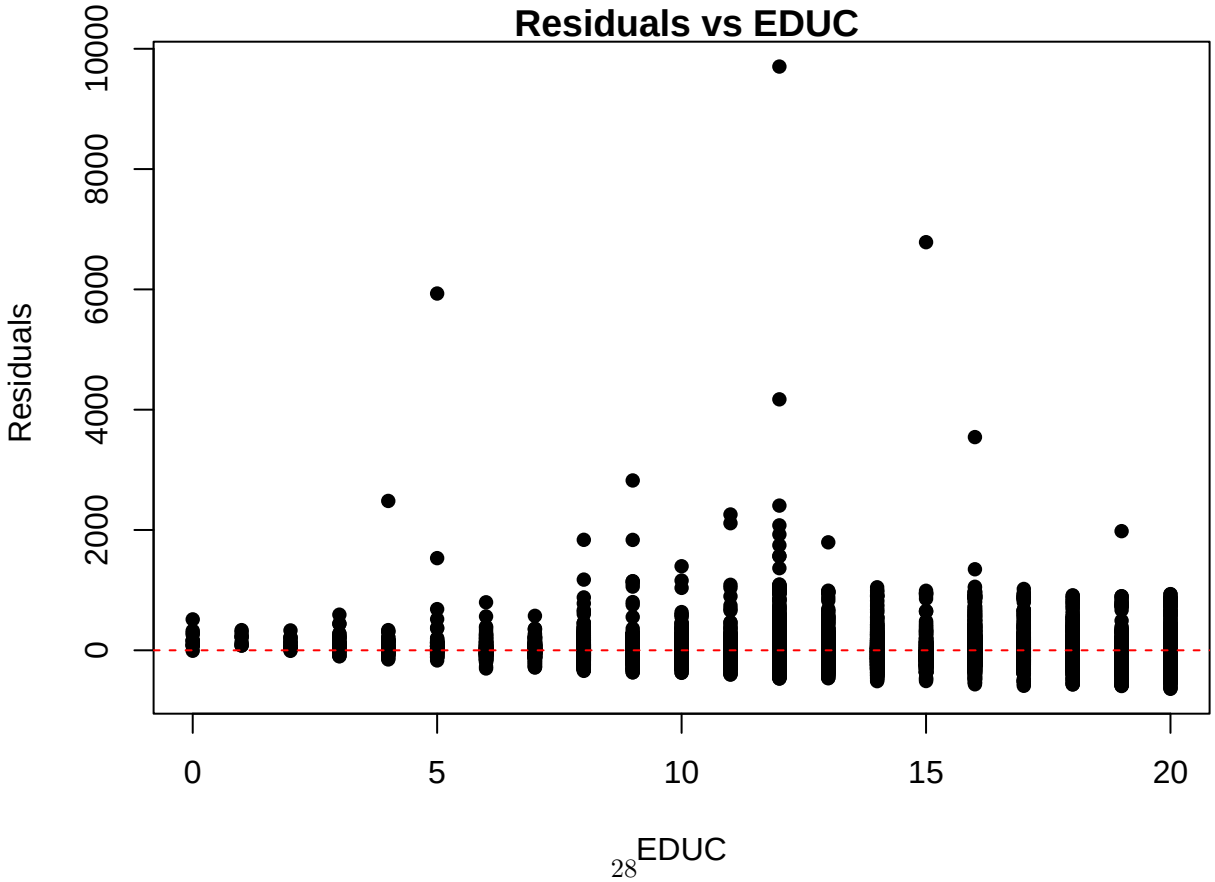
3) Hypothesis Testing: The validity of tests (e.g., t-tests and F-tests) and confidence intervals relies on the normality assumption. Non-normal residuals can lead to incorrect p-values.

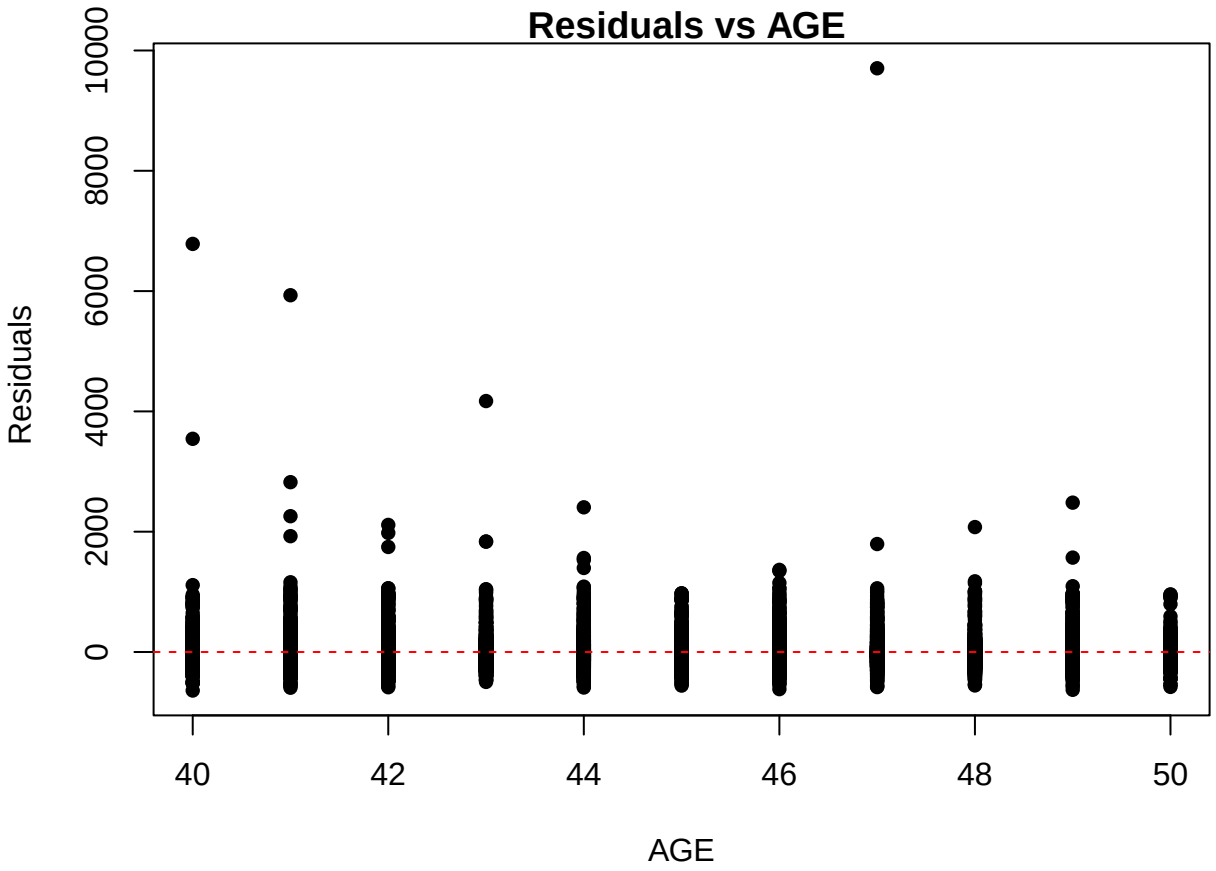
4) Heteroscedasticity or Outliers: Non-normality can signal issues like heteroscedasticity (non-constant error variance), the presence of outliers, or omitted variable bias.

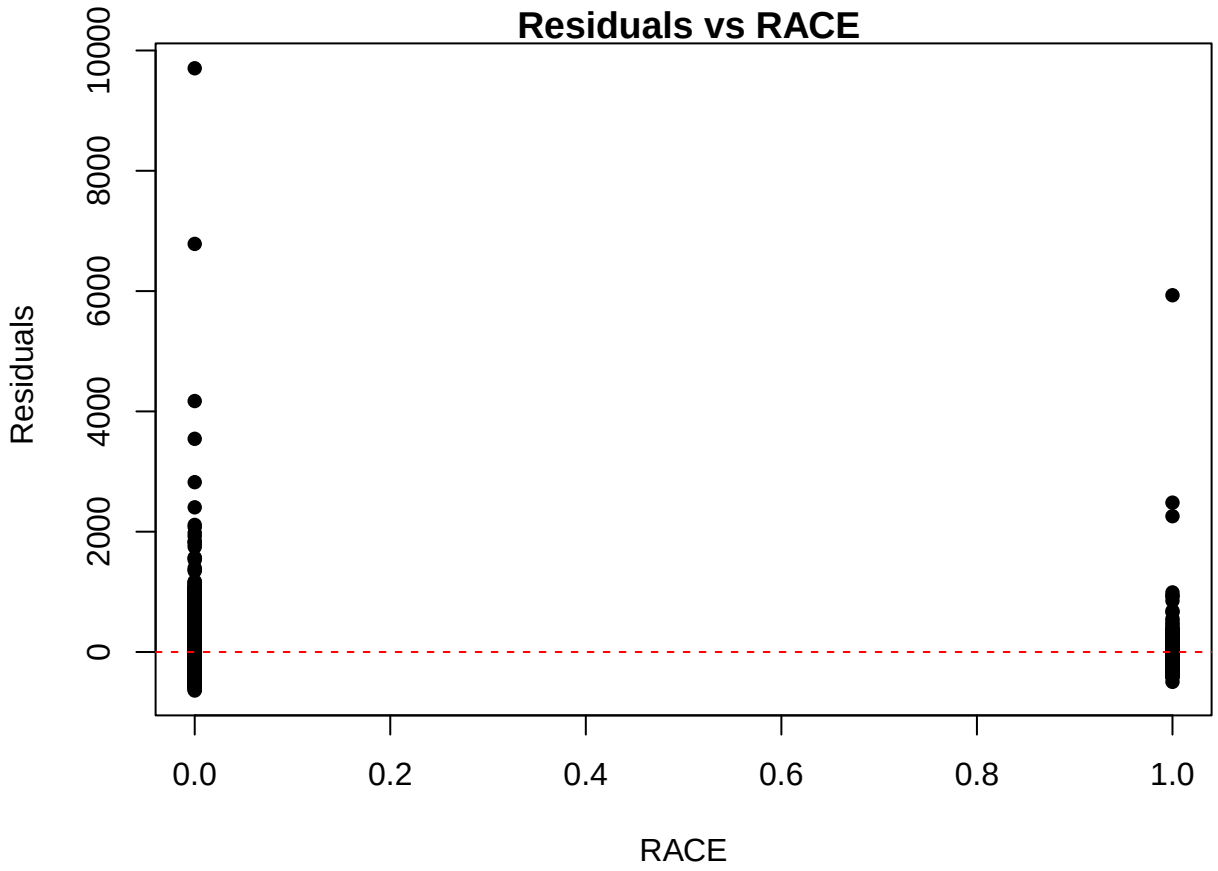
Remediation: Based on the histogram and Q-Q Plot which is slightly right-skewed, we can conclude that the residuals deviate slightly from normality, so OLS may still perform adequately, particularly in our large sample where asymptotic normality applies.

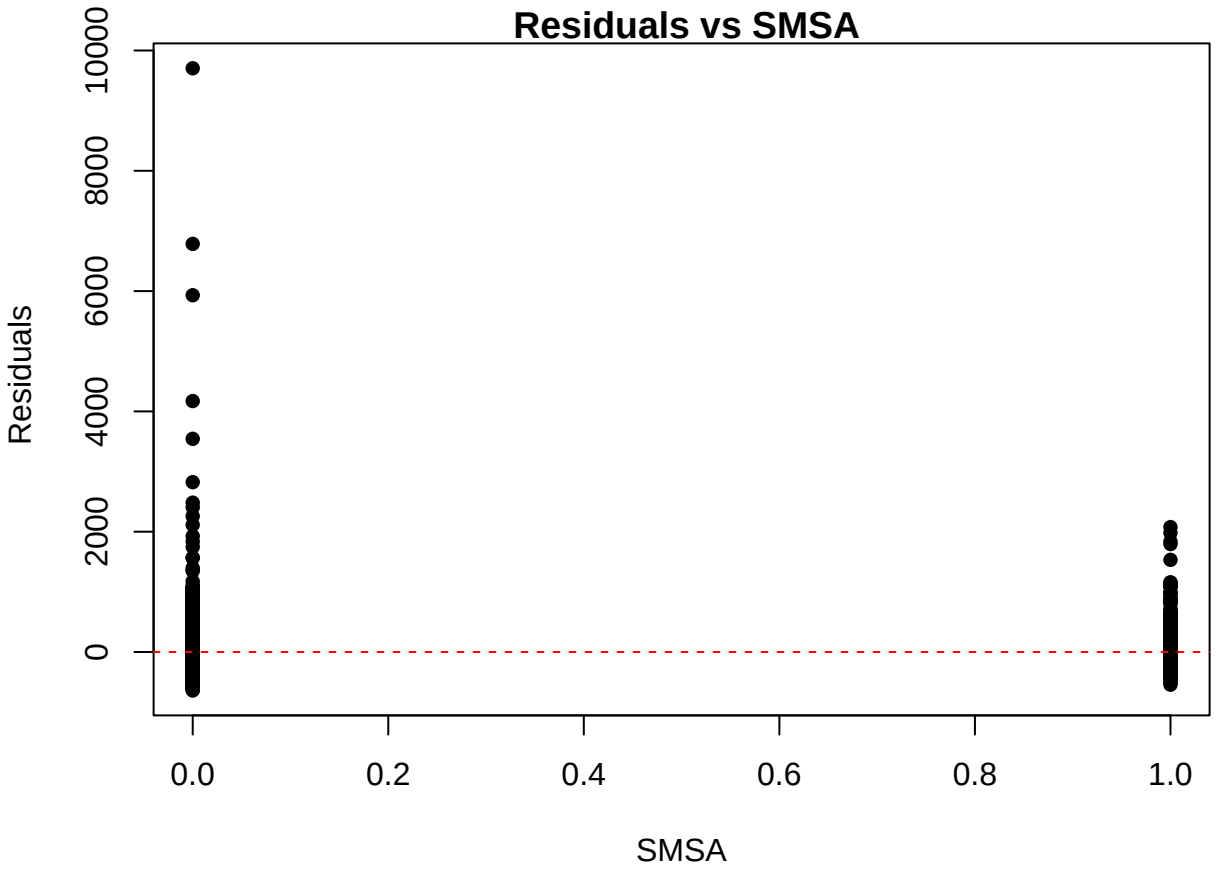
### 4(d) Heteroskedasticity

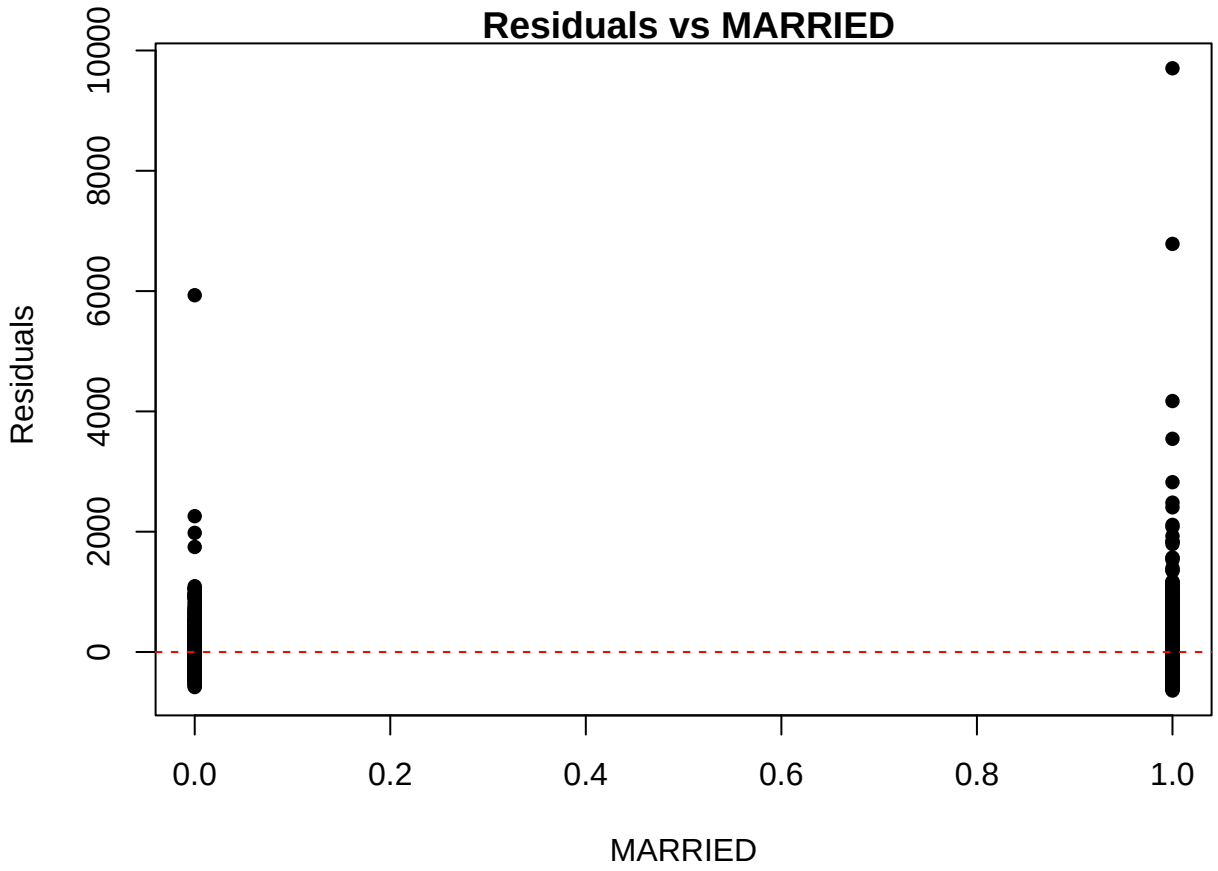


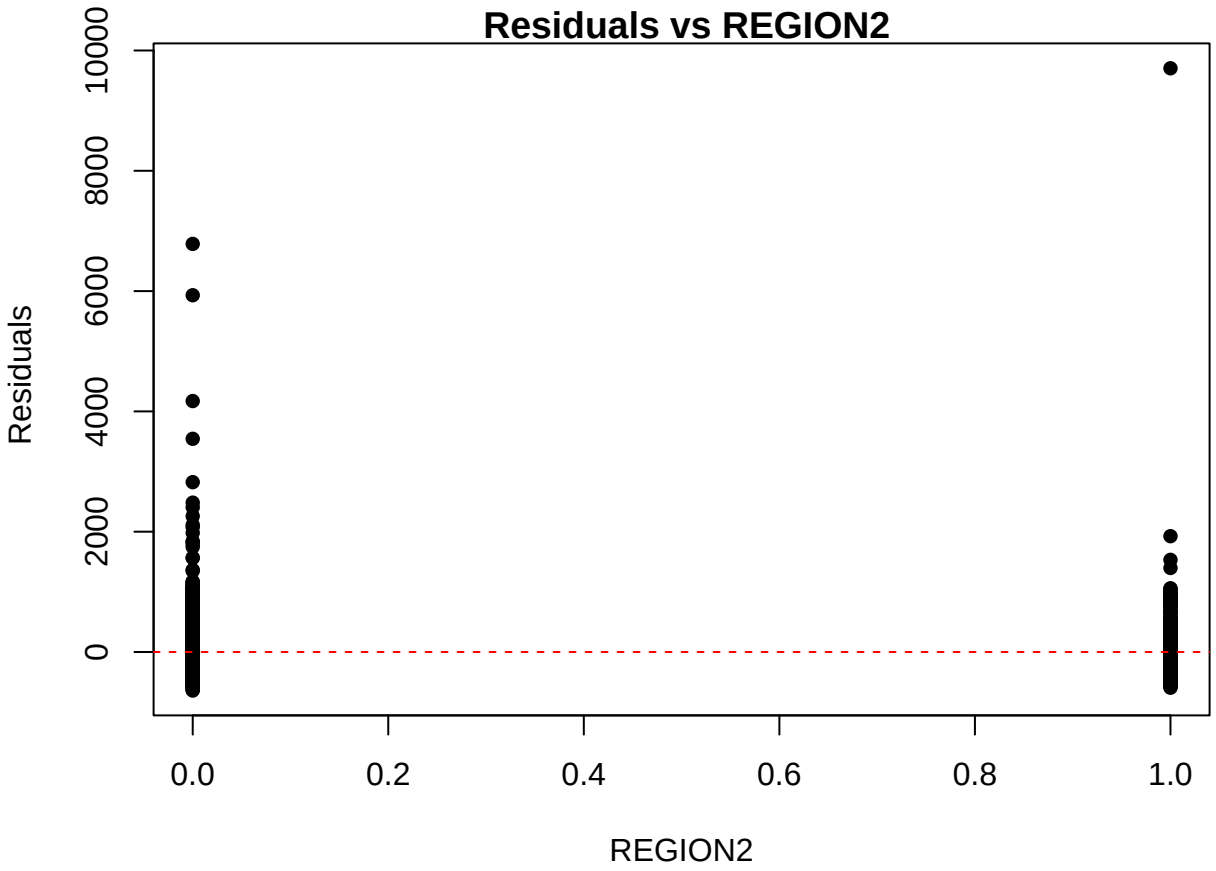



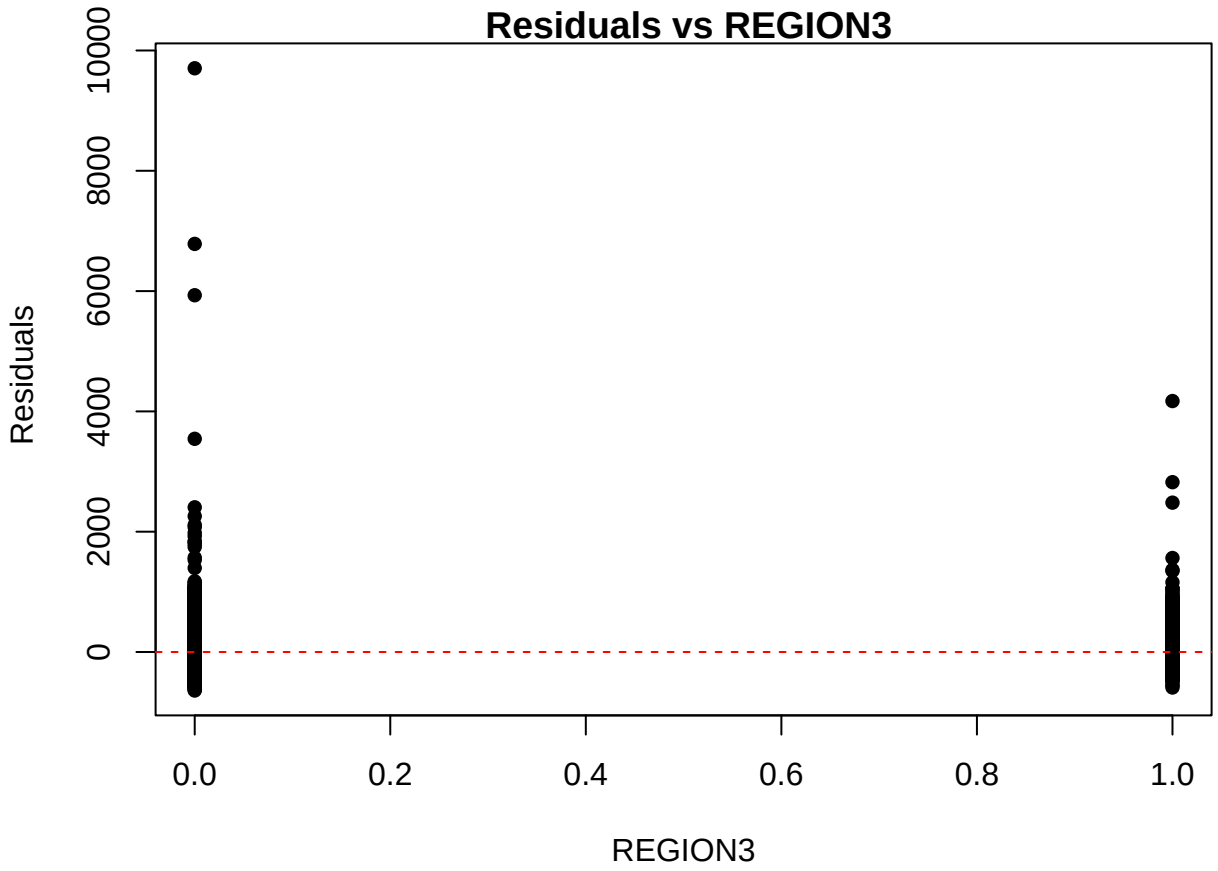

Residuals vs AGE
Residuals
0
2000
4000
6000
8000
10000
40
42
44
46
48
50
AGE

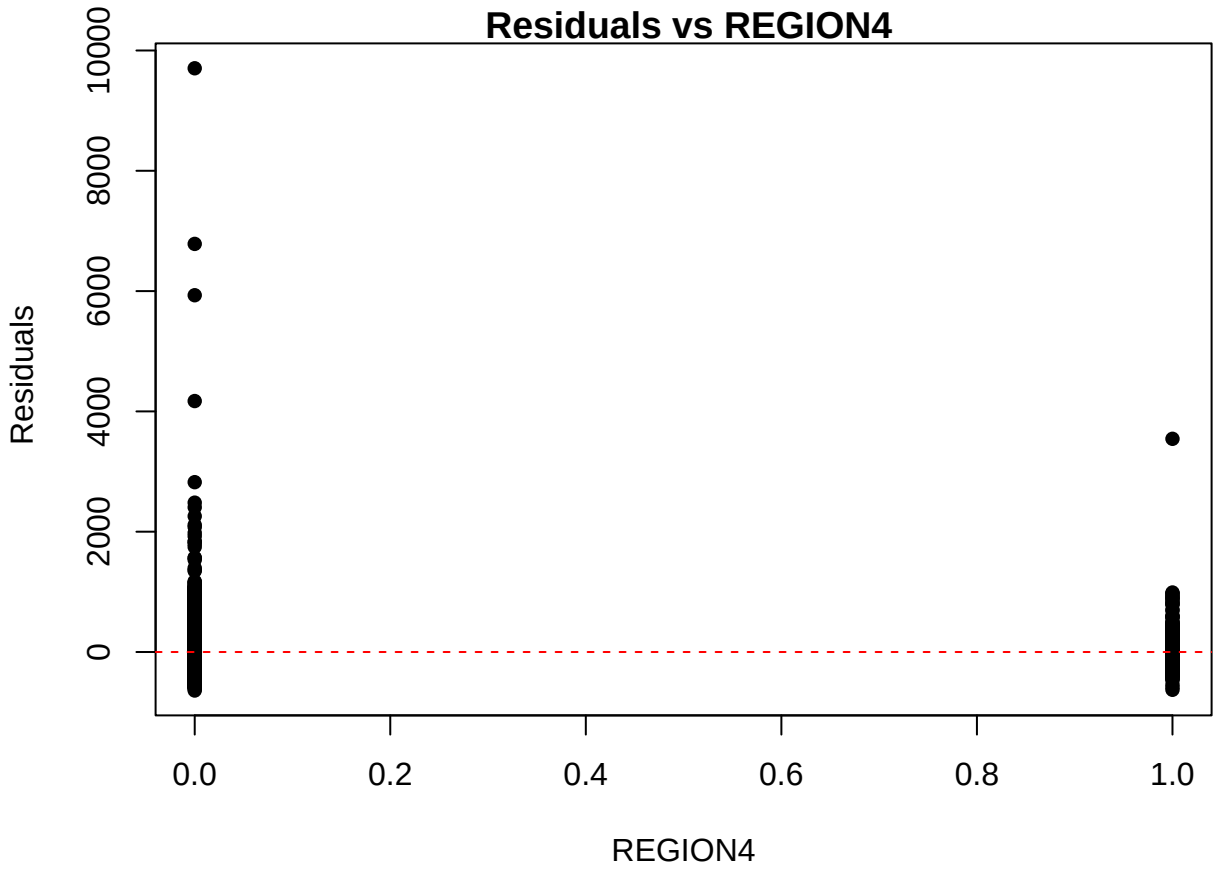

Residuals vs RACE
Residuals
0
2000
4000
6000
8000
10000
0.0
0.2
0.4
0.6
0.8
1.0
RACE

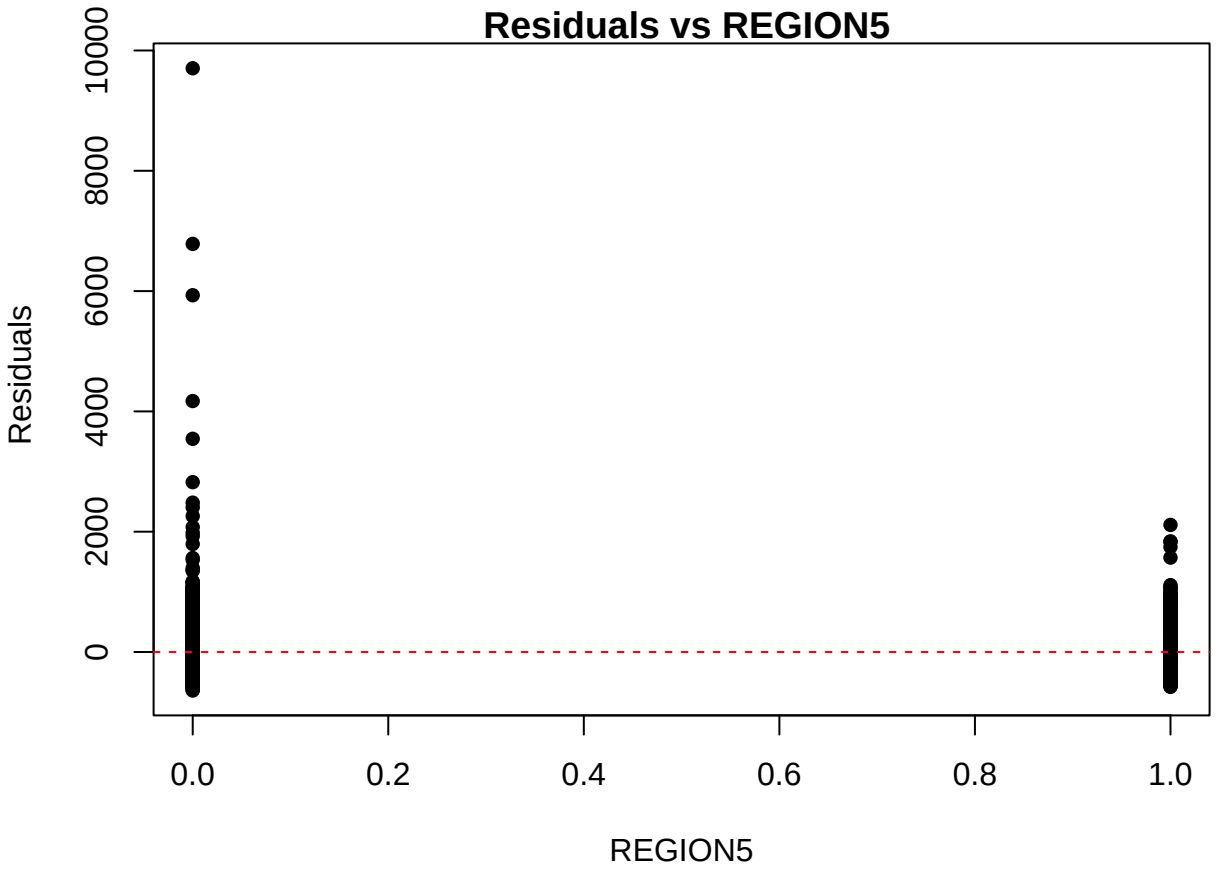

Residuals vs SMSA
Residuals
0
2000
4000
6000
8000
10000
0.0
0.2
0.4
0.6
0.8
1.0
SMSA

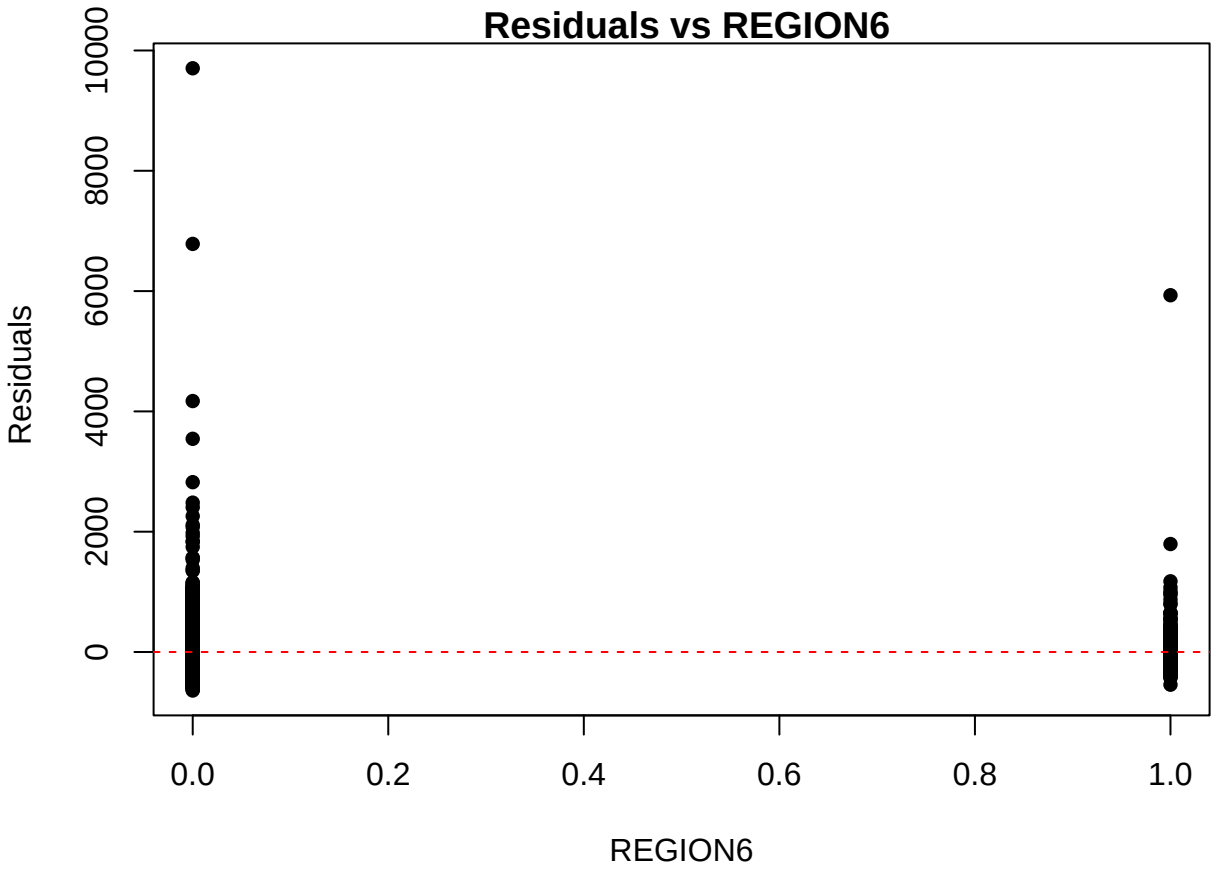

Residuals vs MARRIED
Residuals
0
2000
4000
6000
8000
10000
0.0
0.2
0.4
0.6
0.8
1.0
MARRIED

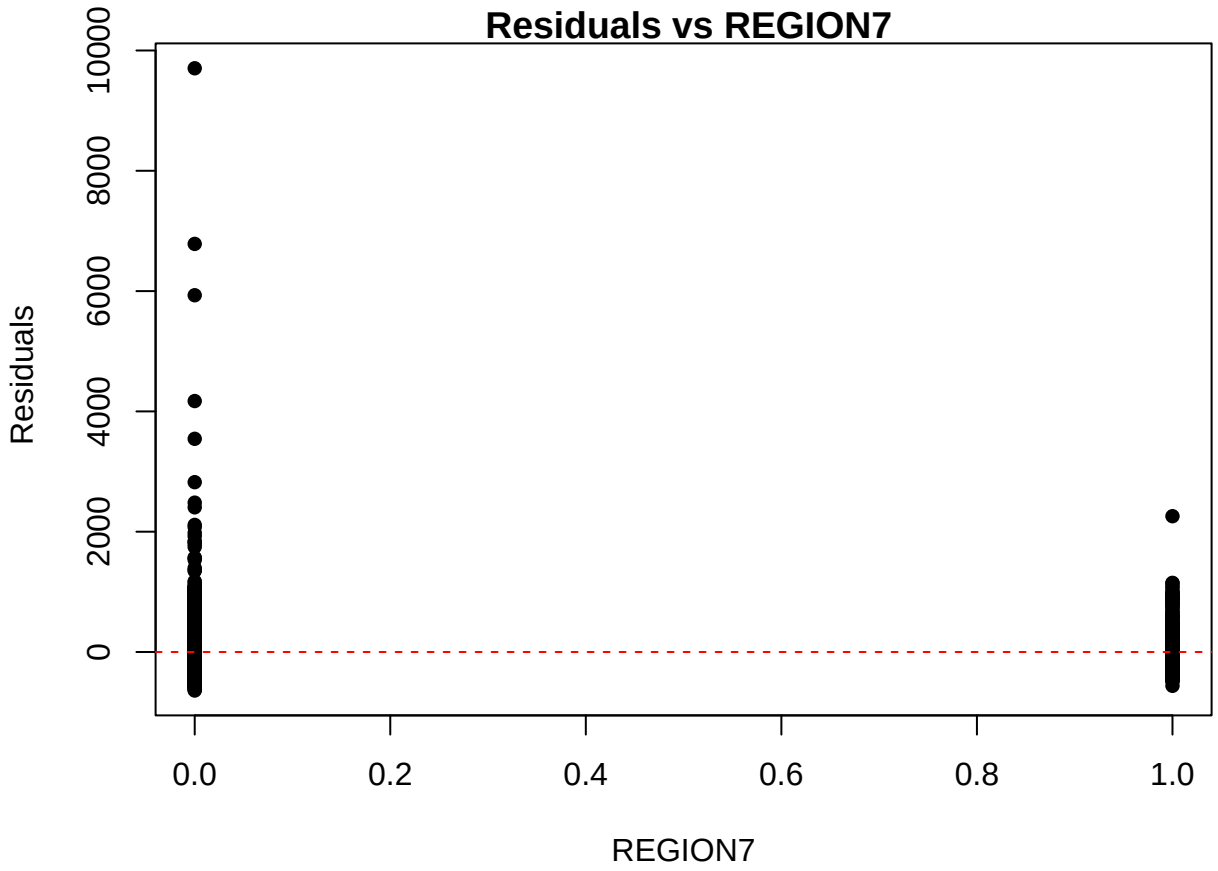

Residuals vs REGION2
Residuals
0
2000
4000
6000
8000
10000
0.0
0.2
0.4
0.6
0.8
1.0
REGION2

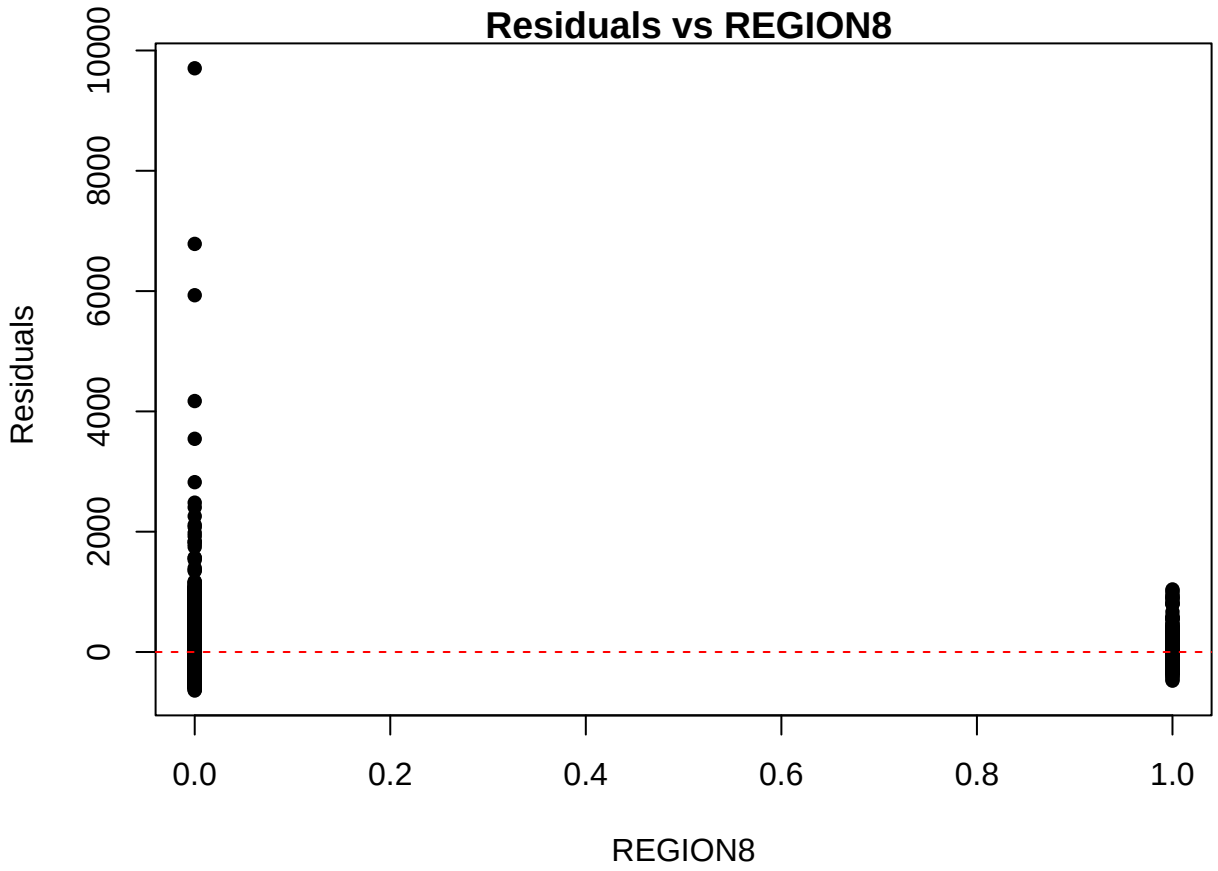

Residuals vs REGION3
Residuals
0
2000
4000
6000
8000
10000
0.0
0.2
0.4
0.6
0.8
1.0
REGION3

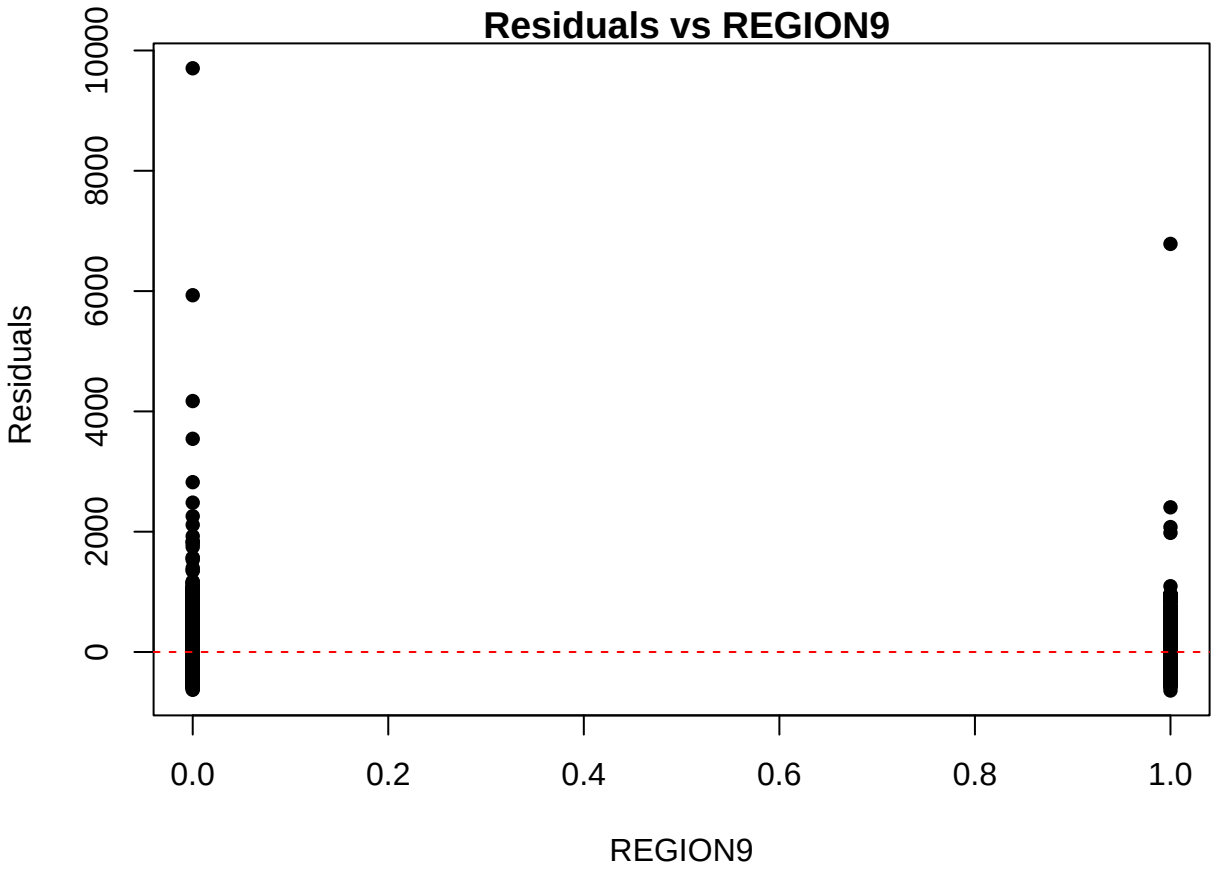

Residuals vs REGION4
Residuals
0
2000
4000
6000
8000
10000
0.0
0.2
0.4
0.6
0.8
1.0
REGION4


Residuals vs REGION5
Residuals
0
2000
4000
6000
8000
10000
0.0
0.2
0.4
0.6
0.8
1.0
REGION5


Residuals vs REGION6
Residuals
0
2000
4000
6000
8000
10000
0.0
0.2
0.4
0.6
0.8
1.0
REGION6


Residuals vs REGION7
Residuals
0
2000
4000
6000
8000
10000
0.0
0.2
0.4
0.6
0.8
1.0
REGION7


Residuals vs REGION8
Residuals
0
2000
4000
6000
8000
10000
0.0
0.2
0.4
0.6
0.8
1.0
REGION8




**Interpretation of Residual Plots:**

1. **Fitted Values**:
   - increase in spread as fitted values increase
   - This pattern indicates potential heteroskedasticity
2. **EDUC**:
   - Initial increase followed by decrease in spread
   - Non-constant variance pattern visible
3. **RACE**:
   - Larger spread observed for white participants
   - Unequal variance between groups ( aigan moer white than black , see SMSA)
4. **SMSA**:
   - Non-metropolitan areas show greater spread
   - Variance differs by urban/rural classification, but this is
     our data is more or less 90 percent non metropolitan (more data , more spread)
5. **MARRIED**:
   - Similar variance pattern as other categorical variables
   - same thing as SMSA , 90 percent of df is married ( so these are natural spreads , and the problem of heteroskedasticity will not be here )
6. **REGION**:

- Different spreads observed across regions (0-1 range)
- Geographic heterogeneity in variance

**Conclusion**:

Clear signs of heteroskedasticity across multiple predictors, suggesting violations of the constant variance assumption.

### 4(d).1Heteroskedasticity tests**

```
white_test <- bptest(linear_model1, ~ fitted(linear_model1) + I(fitted(linear_model1)^2))
print(white_test)
```

**white's general test**

```
##
##  studentized Breusch-Pagan test
##
## data:  linear_model1
## BP = 7.0287, df = 2, p-value = 0.02977
```

the p value of 0.02977 says that we reject the null hyptothis , which implies that we reject homoskedastiscity

**properties**

-for white there are not properties, ( idk zie slide bij verbetring stond in rood , maar ze zei er niks over

**white's general test trimmed dataset**

```
##
##  studentized Breusch-Pagan test
##
## data:  linear_model_trimmed
## BP = 361.55, df = 2, p-value < 2.2e-16
```

even with trimmed date , we have even a smaller p value reject the null hyptothis , which implies that we reject homoskedastiscity

**properties**

-for white there are not properties

```
gqtest_result <- gqtest(linear_model1, order.by =df$EDUC ,fraction = 1000)
print(gqtest_result)
```

**goldfeld-quant test**

```
##
##  Goldfeld-Quandt test
##
## data:  linear_model1
## GQ = 1.8257, df1 = 4486, df2 = 4486, p-value < 2.2e-16
## alternative hypothesis: variance increases from segment 1 to 2
```

very small p value , we order by educ bcs the plot educ vs residuals was a clear sign of heteroskedasticity, t we strongly reject the null hyptothis , which implies that we reject homoskedastiscity

**properties**

-You split the dataset into two non-overlapping groups( splitting by increasin educ) (dropping middle observations).

-Errors are normally distributed., but in our case they are not so be carefull with the interpratation of this test

**Implications for OLS Estimators**

-the coef( the betas ) are not normally distr in small samples

-var of the coef changes , we have a more robust variance formula if we use OLS

-OLS is no longer effecient , we use EGLS wich has a lower variance of the estimator

-the st errors of the coef are underestimated and are not to be trusted, therefore infernce bout the significance is wrong

### 4(d).2 adressing heteroskedasticity

```
##    Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
##  0.9189  8.2973 11.4464 12.6197 13.0128 49.8318
```

```
##
## % Table created by stargazer v.5.2.3 by Marek Hlavac, Social Policy Institute. E-mail: marek.hlavac a
## % Date and time: za, mei 10, 2025 - 22:56:37
## \begin{table}[!htbp] \centering
##   \caption{OLS Regression with Robust Standard Errors}
##   \label{}
## \begin{tabular}{@{\extracolsep{5pt}}lc}
## \\[-1.8ex]\hline
## \hline \\[-1.8ex]
##  & \multicolumn{1}{c}{\textit{Dependent variable:}} \\
## \cline{2-2}
## \\[-1.8ex] & Weekly Wage \\
## \hline \\[-1.8ex]
##  Education & 26.6959$^{***}$ \\
##   & (0.9189) \\
##   & \\
##  Age & 2.2442$^{**}$ \\
##   & (0.9897) \\
##   & \\
##  Race & $-$77.6318$^{***}$ \\
##   & (9.3927) \\
##   & \\
##  SMSA & $-$63.7562$^{***}$ \\
```

```
##   & (6.2889) \\
##   & \\
##  Married & 78.0013$^{***}$ \\
##   & (7.9321) \\
##   & \\
##  Region 2 & 41.1931$^{***}$ \\
##   & (12.5732) \\
##   & \\
##  Region 3 & 49.0879$^{***}$ \\
##   & (10.7566) \\
##   & \\
##  Region 4 & 18.8453 \\
##   & (12.9527) \\
##   & \\
##  Region 5 & 4.2049 \\
##   & (10.8923) \\
##   & \\
##  Region 6 & 7.6044 \\
##   & (15.3882) \\
##   & \\
##  Region 7 & 17.0805 \\
##   & (12.0005) \\
##   & \\
##  Region 8 & 15.8968 \\
##   & (13.7257) \\
##   & \\
##  Region 9 & 63.7105$^{***}$ \\
##   & (13.0328) \\
##   & \\
##  Constant & $-$81.6693 \\
##   & (49.8318) \\
##   & \\
## \hline \\[-1.8ex]
## Observations & 10,000 \\
## R$^{2}$ & 0.1341 \\
## Adjusted R$^{2}$ & 0.1330 \\
## Residual Std. Error & 275.0294 (df = 9986) \\
## F Statistic & 118.9531$^{***}$ (df = 13; 9986) \\
## \hline
## \hline \\[-1.8ex]
## \textit{Note:}  & \multicolumn{1}{r}{$^{*}$p$<$0.1; $^{**}$p$<$0.05; $^{***}$p$<$0.01} \\
##  & \multicolumn{1}{r}{Heteroskedasticity-consistent standard errors in parentheses} \\
## \end{tabular}
## \end{table}
```

**Interpretation:**

- Robust standard errors are larger than conventional OLS standard errors
- This pattern confirms the presence of heteroskedasticity in the data
- The HC1 estimator provides consistent inference under heteroskedasticity
- Coefficient estimates remain unchanged, but significance levels may differ

**EGLS**

% Table created by stargazer v.5.2.3 by Marek Hlavac, Social Policy Institute. E-mail: marek.hlavac at gmail.com % Date and time: za, mei 10, 2025 - 22:56:37

**Comparison with Robust Standard Errors:**

- EGLS provides more efficient estimates than OLS with robust standard errors
- Standard errors are typically larger than OLS with incorrect variance formula
- The efficiency gain comes from properly modeling the heteroskedasticity structure
- Interpretation of coefficients remains the same as OLS

Multicollinearity

```
Gauss-Markov assumption (no perfect multicollinearity) is met.


Analysis of Variance Inflation Factors and correlation:
Low VIFs (<5): Most of the variables have VIF values close to 1, including EDUC (1.07), AGE (1.01), RA


Moderate VIFs (Between 2 and 5): Some regional variables-REGION2 (3.28), REGION3 (3.67), REGION4 (2.18


High VIFs (>5): None of the variables exceed 5, which suggests that severe multicollinearity is not a
```

in the explanatory variables can still lead to precise estimates.

```
Implications for the properties and precision of the OLS estimator:
1) Parameters remain identifiable.

2) Under the CNLRM assumptions, the OLS estimator remains BLUE and normally distributed.

3) OLS estimators exhibit larger variance and covariance, especially the regions.

4) Wider confidence intervals and lower t-statistics
->variables appear less significantly different from zero, higher probability of making a Type II erro

5) The overall model fit (R2) is largely unaffected: even if individual coefficients are insignificant
```

indicate that the coefficients are jointly significant, and can be estimated with high precision.

```
6) Regression coefficients may change substantially

![](Data_exploration_files/figure-latex/multicollin-1.pdf)<!-- -->
```

```
print(correlation_matrix)
```

```
##                EDUC          AGE         RACE        SMSA      MARRIED
## EDUC     1.000000000 -0.069506286 -0.151952924 -0.14907033  0.019198364
## AGE     -0.069506286  1.000000000 -0.005148653 -0.02085355  0.020809977
## RACE    -0.151952924 -0.005148653  1.000000000 -0.03556389 -0.113282921
## SMSA    -0.149070326 -0.020853549 -0.035563888  1.00000000  0.040644736
```

Table 19: EGLS Estimation Results

| | *Dependent variable:* |
|---|---|
| | Transformed WAGE |
| Intercept | 435.7491*** |
| | (51.0221) |
| EDUC | 17.9831*** |
| | (0.7695) |
| AGE | −6.5454*** |
| | (1.0558) |
| RACE | −69.0106*** |
| | (7.8876) |
| SMSA | −78.0229*** |
| | (5.5790) |
| MARRIED | 53.2469*** |
| | (5.7072) |
| REGION2 | 37.1832* |
| | (19.6988) |
| REGION3 | 60.2545*** |
| | (19.3016) |
| REGION4 | 25.2715 |
| | (18.7175) |
| REGION5 | 9.2088 |
| | (18.0923) |
| REGION6 | 12.2196 |
| | (18.5587) |
| REGION7 | 37.1148* |
| | (19.4049) |
| REGION8 | 40.5698* |
| | (24.2491) |
| REGION9 | 65.6674*** |
| | (21.2749) |
| Observations | 10,000 |
| $R^2$ | 0.6063 |
| Adjusted $R^2$ | 0.6058 |
| Residual Std. Error | 1.0188 (df = 9986) |
| F Statistic | 1,098.5050*** (df = 14; 9986) |
| *Note:* | *p<0.1; **p<0.05; ***p<0.01 |
| | Variables transformed by dividing by y hat) |

```
## MARRIED  0.019198364  0.020809977 -0.113282921  0.04064474  1.000000000
## REGION2  0.035717362  0.008497716  0.014092931 -0.10391873 -0.003692641
## REGION3 -0.030839760  0.005358420 -0.023908637 -0.02447606  0.013937881
## REGION4 -0.000578807 -0.014907585 -0.049908666  0.11787788  0.020881363
## REGION5 -0.042953090 -0.012179069  0.091472810  0.01940650  0.004255528
## REGION6 -0.084427585  0.008312525  0.039575771  0.11343959  0.017738535
## REGION7 -0.032806167  0.003156501  0.008728591 -0.01074680  0.007148294
## REGION8  0.028455078 -0.026179022 -0.043610641  0.07119329  0.020956712
## REGION9  0.104683025  0.001483814 -0.030119960 -0.07742498 -0.060808526
##              REGION2     REGION3      REGION4      REGION5      REGION6
## EDUC     0.035717362 -0.03083976 -0.000578807 -0.042953090 -0.084427585
## AGE      0.008497716  0.00535842 -0.014907585 -0.012179069  0.008312525
## RACE     0.014092931 -0.02390864 -0.049908666  0.091472810  0.039575771
## SMSA    -0.103918730 -0.02447606  0.117877877  0.019406502  0.113439595
## MARRIED -0.003692641  0.01393788  0.020881363  0.004255528  0.017738535
## REGION2  1.000000000 -0.21345470 -0.121923453 -0.201399480 -0.113442761
## REGION3 -0.213454705  1.00000000 -0.138274958 -0.228409743 -0.128656896
## REGION4 -0.121923453 -0.13827496  1.000000000 -0.130465639 -0.073487689
## REGION5 -0.201399480 -0.22840974 -0.130465639  1.000000000 -0.121390774
## REGION6 -0.113442761 -0.12865690 -0.073487689 -0.121390774  1.000000000
## REGION7 -0.144209678 -0.16355005 -0.093418354 -0.154313279 -0.086920406
## REGION8 -0.098898407 -0.11216196 -0.064065924 -0.105827415 -0.059609658
## REGION9 -0.166513184 -0.18884475 -0.107866461 -0.178179412 -0.100363538
##              REGION7     REGION8      REGION9
## EDUC    -0.032806167  0.02845508  0.104683025
## AGE      0.003156501 -0.02617902  0.001483814
## RACE     0.008728591 -0.04361064 -0.030119960
## SMSA    -0.010746803  0.07119329 -0.077424983
## MARRIED  0.007148294  0.02095671 -0.060808526
## REGION2 -0.144209678 -0.09889841 -0.166513184
## REGION3 -0.163550053 -0.11216196 -0.188844746
## REGION4 -0.093418354 -0.06406592 -0.107866461
## REGION5 -0.154313279 -0.10582742 -0.178179412
## REGION6 -0.086920406 -0.05960966 -0.100363538
## REGION7  1.000000000 -0.07577645 -0.127583227
## REGION8 -0.075776449  1.00000000 -0.087496055
## REGION9 -0.127583227 -0.08749605  1.000000000
```

```
print(max(abs(correlation_matrix)))
```

```
## [1] 1
```

```
print('VIF values')
```

```
## [1] "VIF values"
```

```
vif_values = vif(linear_model1)
print(vif_values)
```

```
##     EDUC      AGE     RACE     SMSA  MARRIED  REGION2  REGION3  REGION4
## 1.072047 1.008200 1.054291 1.072522 1.019728 3.280936 3.672041 2.182705
##  REGION5  REGION6  REGION7  REGION8  REGION9
## 3.510183 2.058605 2.540830 1.816650 2.919060
```

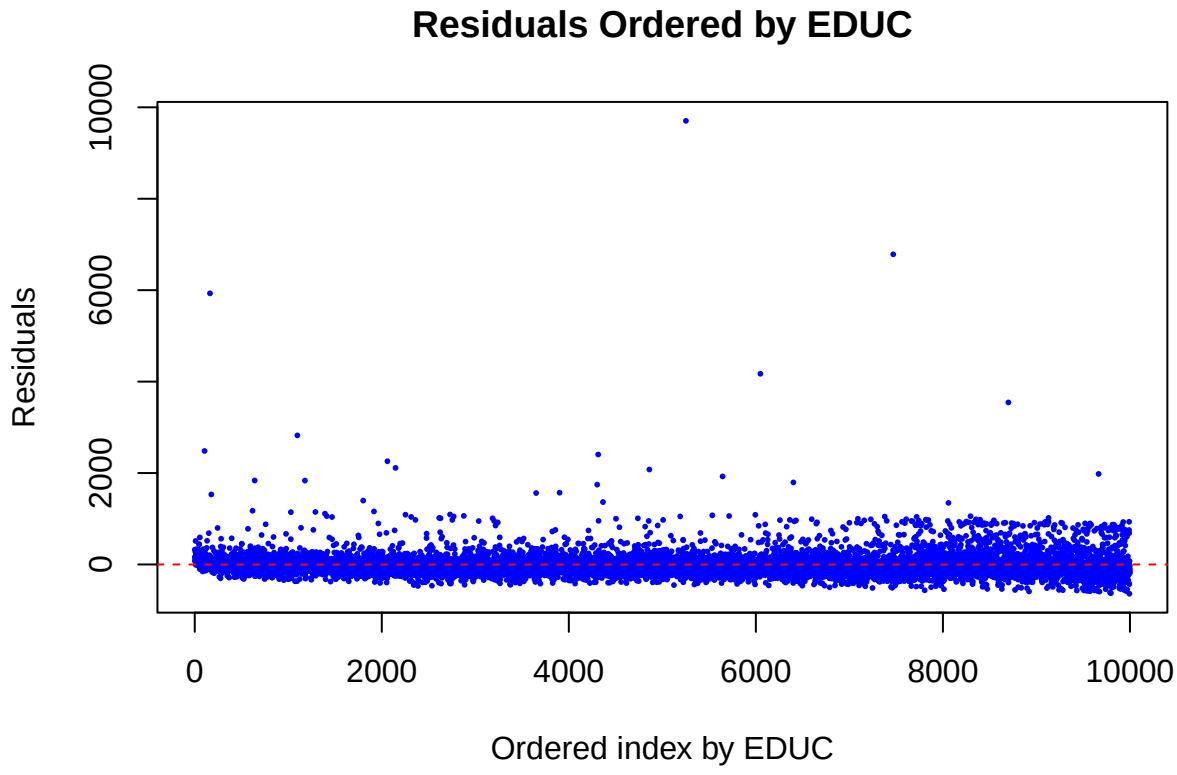
### 4(e) Autocorrelation

```r
df_ordered_educ <- df %>% arrange(EDUC)

# Re-run baseline linear model on ordered data
linear_model_ordered <- lm(
  WAGE ~ EDUC + AGE + RACE + SMSA + MARRIED +
  REGION2 + REGION3 + REGION4 + REGION5 +
  REGION6 + REGION7 + REGION8 + REGION9,
  data = df_ordered_educ
)

# Store residuals
residuals_ordered <- residuals(linear_model_ordered)

plot(residuals_ordered,
    type = "p",  # "p" for points instead of "l" for lines
     pch = 16,    # Solid circle points (other options: 1-25)
     col = "blue", # Color of points
     cex = 0.4,   # Size of points
     main = "Residuals Ordered by EDUC",
     ylab = "Residuals",
     xlab = "Ordered index by EDUC")
abline(h = 0, lty = 2, col = "red")  # Reference line
```



```
Runs Test
```

data: residuals_ordered statistic = -2.6201, runs = 4870, n1 = 5000, n2 = 5000, n = 10000, p-value = 0.00879 alternative hypothesis: nonrandomness

sippe tijden , p value is hoog dus toch corr

========== Test-statistic P-value ——————- 1.957 0.015 ——————- Assumptions: 1) Error terms follow an AR(1) process 2) Error terms are normally distributed 3) The regression model does not include lagged dependent variables hahahah dikke pech , toch corr :)

========== Test-statistic P-value ——————- 13.468 0.036 ——————- Assumptions: 1) Allows for stochastic regressors 2) Tests for higher-order autocorrelation 3) Allows for lagged dependent variables geprankt er is gwn corr , idk

Implications for OLS if autocorrelation is present: 1) Standard variance formula is incorrect 2) OLS estimator no longer efficientInference: Standard errors are incorrect, leading to invalid t-tests, F-tests, and confidence intervalsIt does not make sense to interpret autocorrelation as a structural feature since there is no natural ordering for the dataset.

==========
Dependent variable:
——————— WAGE
———————————————- EDUC -23.274**
(9.053)
t = -2.571
p = 0.011
AGE -2.364*
(1.220)
t = -1.937
p = 0.053
RACE 44.302
(27.709)
t = 1.599
p = 0.110
SMSA 48.280**
(23.258)
t = 2.076
p = 0.038
MARRIED -60.243**
(27.647)
t = -2.179
p = 0.030
I(Yfit2) 0.003***
(0.001)
t = 3.419
p = 0.001
I(Yfit3) -0.00000*
(0.00000)
t = -1.742
p = 0.082
REGION2 -38.952**
(19.736)
t = -1.974
p = 0.049
REGION3 -43.479**

(21.499)
t = -2.022
p = 0.044
REGION4 -17.440
(16.867)
t = -1.034
p = 0.302
REGION5 -9.229
(13.555)
t = -0.681
p = 0.496
REGION6 -16.522
(16.341)
t = -1.011
p = 0.313
REGION7 -20.320
(15.930)
t = -1.276
p = 0.203
REGION8 -13.612
(17.950)
t = -0.758
p = 0.449
REGION9 -61.727**
(26.282)
t = -2.349
p = 0.019
Constant 410.901***
(82.840)
t = 4.960
p = 0.00000
———————————————- Observations 10,000
R2 0.143
Adjusted R2 0.142
Residual Std. Error 273.622 (df = 9984)
F Statistic 111.158*** (df = 15; 9984) (p = 0.000) ==========
Note: *p<0.1;* ***p<0.05;*** p<0.01

========== Test-statistic P-value ————————- 52.514 0
————————-

RESET test

data: linear_model1 RESET = 52.514, df1 = 2, df2 = 9984, p-value < 2.2e-16

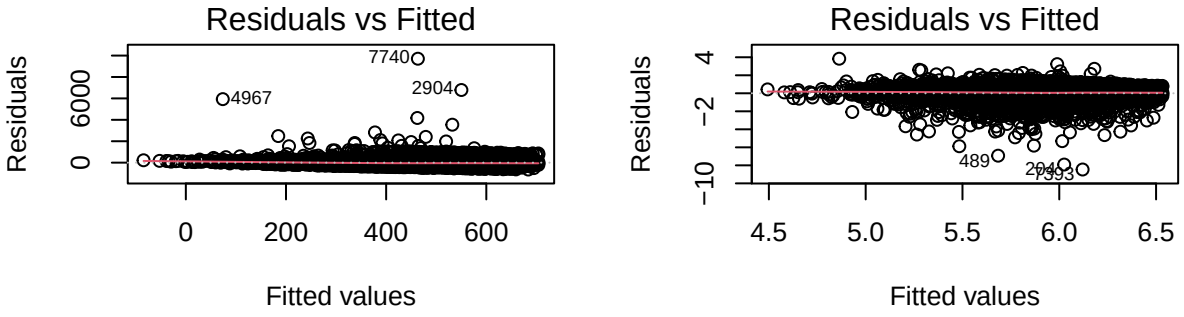
========== Dependent variable:
——————————— res
——————————————— EDUC -77.297***
(12.079)
t = -6.399
p = 0.000
I(EDUC2) 5.621***
(1.081)
t = 5.202

p = 0.00000
I(EDUC3) -0.125***
(0.030)
t = -4.133
p = 0.00004
Constant 321.854***
(43.433)
t = 7.410
p = 0.000
——————————————— Observations 10,000
R2 0.007
Adjusted R2 0.007
Residual Std. Error 273.882 (df = 9996)
F Statistic 24.612*** (df = 3; 9996) (p = 0.000) ==========
Note: *p<0.1;* ***p<0.05;*** p<0.01

========== Test-statistic P-value ——————- 73.323 0
————————-

```
           df       AIC
```

linear_model1 15 140732.32 log_model 15 18897.87 df BIC linear_model1 15 140840.48 log_model 15 19006.02 The log model has a lower AIC and BIC than the linear model, so log model has a better




fit

```
quad_model <- lm(WAGE~ EDUC + I(EDUC^2) + AGE + I(AGE^2) + RACE + SMSA + MARRIED + REGION2 + REGION3
                 + REGION4 + REGION5 + REGION6 + REGION7 + REGION8 + REGION9  , data = df)
linearHypothesis(quad_model, c("I(EDUC^2) = 0", "I(AGE^2) = 0"))
```

```
##
## Linear hypothesis test:
## I(EDUC^2) = 0
## I(AGE^2) = 0
##
## Model 1: restricted model
## Model 2: WAGE ~ EDUC + I(EDUC^2) + AGE + I(AGE^2) + RACE + SMSA + MARRIED +
##     REGION2 + REGION3 + REGION4 + REGION5 + REGION6 + REGION7 +
##     REGION8 + REGION9
##
##   Res.Df       RSS Df Sum of Sq      F    Pr(>F)
## 1   9986 755352847
## 2   9984 750864421  2   4488426 29.841 1.199e-13 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

You reject the null hypothesis that the squared terms are irrelevant. This means:

The relationship between wage and education, and wage and age, is not linear.

Quadratic model fits the data significantly better when it includes these quadratic terms.

So: include I(EDUC^2) and I(AGE^2) in the preferred model going forward

Linear hypothesis test: EDUC:AGE = 0 EDUC:RACE = 0

Model 1: restricted model Model 2: WAGE ~ EDUC + AGE + RACE + EDUC * AGE + EDUC * RACE + I(EDUC^2) + I(AGE^2) + SMSA + MARRIED + REGION2 + REGION3 + REGION4 + REGION5 + REGION6 + REGION7 + REGION8 + REGION9

Res.Df RSS Df Sum of Sq F Pr(>F)
1 9984 750864421
2 9982 749099187 2 1765234 11.761 7.91e-06 *** — Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1 You reject the null hypothesis that the interaction terms are irrelevant.The interaction effects between EDUC and AGE, and between EDUC and RACE, contribute significantly to explaining variation in wages.

========== Test-statistic P-value ----------- 2,176,957.000 0 -----------

========== Test-statistic P-value ----------- 846.434 1.000 -----------

**4g. Endogeneity and Instrumental Variables** We assumed in 4a that Education is endogenous.

- If EDUC is endogenous (i.e., correlated with the error term), the OLS estimator is biased and inconsistent.
- To solve this, we use the instrumental variables (IV) approach, specifically Two-Stage Least Squares (2SLS). This method replaces the endogenous regressor with a predicted version from instruments that are: relevant (correlated with EDUC) and exogenous.
- Proposed instrument and justification: we use Quarter of Birth (QOB) as an instrument for EDUC. Justification of choice: QOB affects the age at which individuals can legally drop out of school due to school-starting age laws. QOB is strictly exogenous: it is randomly assigned at birth and unlikely to be correlated with unobserved factors that affect wages directly.
- Assessing the strength of the instrument: we check the F-statistic of the instrument in the first-stage regression.
- Testing for endogeneity: we perform a Hausman test.

**2SLS estimation**

```
## Loading required package: survival
```

```
##
## Call:
## ivreg(formula = WAGE ~ EDUC + AGE + RACE + MARRIED + SMSA + factor(REGION) |
##     QOB + AGE + RACE + MARRIED + SMSA + factor(REGION), data = df)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -620.56 -132.12  -37.68   73.95 9700.42
##
## Coefficients:
##                 Estimate Std. Error t value Pr(>|t|)
## (Intercept)      -24.421    702.921  -0.035  0.97229
## EDUC              23.287     41.777   0.557  0.57727
## AGE                1.966      3.533   0.556  0.57790
## RACE             -83.598     73.807  -1.133  0.25739
## MARRIED           78.514     10.195   7.701 1.48e-14 ***
## SMSA             -67.962     52.056  -1.306  0.19174
## factor(REGION)2   41.131     13.676   3.007  0.00264 **
## factor(REGION)3   47.565     22.917   2.076  0.03796 *
## factor(REGION)4   18.488     16.215   1.140  0.25426
## factor(REGION)5    2.875     21.159   0.136  0.89191
## factor(REGION)6    4.298     43.605   0.099  0.92149
## factor(REGION)7   15.307     26.206   0.584  0.55916
## factor(REGION)8   16.707     19.790   0.844  0.39856
## factor(REGION)9   65.668     27.795   2.363  0.01817 *
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 275.2 on 9986 degrees of freedom
## Multiple R-Squared: 0.1328,  Adjusted R-squared: 0.1316
## Wald test: 46.29 on 13 and 9986 DF,  p-value: < 2.2e-16
```

In the 2SLS estimation, the coefficient on EDUC is positive (23.29) but not statistically significant ($p = 0.577$), indicating no strong evidence of a causal effect of education on wages in this IV setup. In contrast, being married significantly increases wages (coef = 78.51, $p < 0.001$). Some regional dummies (REGION2, REGION3, REGION9) are also significant, suggesting regional wage differences. Other variables, including AGE and RACE, are not significant.

**First-stage regression (Instrument Relevance)**

```
##
## Call:
## lm(formula = EDUC ~ QOB + AGE + RACE + MARRIED + SMSA + factor(REGION),
##     data = df)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -13.9228  -1.5791  -0.4719   2.1043   9.4704
##
```

```
## Coefficients:
##                  Estimate Std. Error t value Pr(>|t|)
## (Intercept)     16.51952    0.52632  31.387  < 2e-16 ***
## QOB              0.05952    0.02861   2.080 0.037510 *
## AGE             -0.07883    0.01090  -7.235 4.98e-13 ***
## RACE            -1.74628    0.11644 -14.998  < 2e-16 ***
## MARRIED          0.15138    0.09240   1.638 0.101382
## SMSA            -1.23208    0.08423 -14.628  < 2e-16 ***
## factor(REGION)2 -0.01863    0.15710  -0.119 0.905587
## factor(REGION)3 -0.44419    0.15313  -2.901 0.003731 **
## factor(REGION)4 -0.09780    0.17965  -0.544 0.586198
## factor(REGION)5 -0.38929    0.15530  -2.507 0.012202 *
## factor(REGION)6 -0.96658    0.18540  -5.214 1.89e-07 ***
## factor(REGION)7 -0.52131    0.16856  -3.093 0.001988 **
## factor(REGION)8  0.24120    0.19696   1.225 0.220746
## factor(REGION)9  0.57584    0.16163   3.563 0.000369 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 3.167 on 9986 degrees of freedom
## Multiple R-squared:  0.06761,    Adjusted R-squared:  0.0664
## F-statistic:  55.7 on 13 and 9986 DF,  p-value: < 2.2e-16
```

The instrument QOB is statistically significant ($p = 0.038$), with a positive coefficient (0.0595). This confirms that QOB is correlated with EDUC, satisfying the relevance condition for an instrumental variable

**Hausman test**

```
##
## Call:
## lm(formula = WAGE ~ EDUC + AGE + RACE + MARRIED + SMSA + factor(REGION) +
##     first_stage_residuals, data = df)
##
## Residuals:
##    Min     1Q Median     3Q    Max
## -639.9 -131.9  -35.8   76.0 9703.9
##
## Coefficients:
##                       Estimate Std. Error t value Pr(>|t|)
## (Intercept)           -24.421    702.414  -0.035  0.97227
## EDUC                   23.287     41.747   0.558  0.57699
## AGE                     1.966      3.531   0.557  0.57763
## RACE                  -83.598     73.754  -1.133  0.25704
## MARRIED                78.514     10.188   7.707 1.41e-14 ***
## SMSA                  -67.962     52.019  -1.306  0.19142
## factor(REGION)2        41.131     13.666   3.010  0.00262 **
## factor(REGION)3        47.565     22.901   2.077  0.03782 *
## factor(REGION)4        18.488     16.204   1.141  0.25392
## factor(REGION)5         2.875     21.143   0.136  0.89184
## factor(REGION)6         4.298     43.574   0.099  0.92143
## factor(REGION)7        15.307     26.187   0.585  0.55888
## factor(REGION)8        16.707     19.775   0.845  0.39822
## factor(REGION)9        65.668     27.775   2.364  0.01808 *
```

```
## first_stage_residuals    3.411     41.756   0.082  0.93490
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 275 on 9985 degrees of freedom
## Multiple R-squared:  0.1341, Adjusted R-squared:  0.1329
## F-statistic: 110.4 on 14 and 9985 DF,  p-value: < 2.2e-16
```

The coefficient on the first-stage residuals is not statistically significant (p = 0.935). This suggests no evidence of endogeneity in EDUC. OLS is consistent, and IV is not strictly necessary in this case.