NTNU

TDT4138 - Assignment 2

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1 Models and Entailment in Propositional Logic

1. For each statement below, determine whether the statement is true or false by building the complete model table.

   a) $\neg A \wedge \neg B \models \neg B$

A	B	$\neg A$	$\neg B$	$\neg A \wedge \neg B$	$\neg A \wedge \neg B \Rightarrow \neg B$
false	false	true	true	true	true
false	true	true	false	false	true
true	false	false	true	false	true
true	true	false	false	false	true

   The statement is true, this because in every case where $\neg A \wedge \neg B$ is true, $\neg B$ is also true.

   b) $\neg A \vee \neg B \models \neg B$

A	B	$\neg A$	$\neg B$	$\neg A \vee \neg B$	$\neg A \vee \neg B \Rightarrow \neg B$
false	false	true	true	true	true
false	true	true	false	true	false
true	false	false	true	true	true
true	true	false	false	false	true

   The statement is false, this because not all models are true.

c) $\neg A \wedge B \models A \vee B$

A	B	$\neg A$	$\neg A \wedge B$	$A \vee B$	$\neg A \wedge B \Rightarrow A \vee B$
false	false	true	false	false	true
false	true	true	true	true	true
true	false	false	false	true	true
true	true	false	false	true	true

The statement is true, this because not all models are true.

d) $A \Rightarrow B \models A \Leftrightarrow B$

A	B	$A \Rightarrow B$	$A \Leftrightarrow B$	$\Rightarrow B \Rightarrow A \Leftrightarrow B$
false	false	true	true	true
false	true	true	false	false
true	false	false	false	true
true	true	true	true	true

The statement is false, this because not all models are true.

e) $(A \Rightarrow B) \Leftrightarrow C \models A \vee \neg B \vee C$

A	B	C	$(A \Rightarrow B) \Leftrightarrow C$	$A \vee \neg B \vee C$	$(A \Rightarrow B) \Leftrightarrow C \Rightarrow A \vee \neg B \vee C$
false	false	false	false	true	true
false	false	true	true	true	true
false	true	false	false	false	true
false	true	true	true	true	true
true	false	false	true	true	true
true	false	true	false	true	true
true	true	false	false	true	true
true	true	true	true	true	true

The statement is true, this because all models are true.

f) $(\neg A \Rightarrow \neg B) \wedge (A \wedge \neg B)$ is satisfiable.

A	B	$\neg A$	$\neg B$	$(\neg A \Rightarrow \neg B)$	$(A \wedge \neg B)$	$(\neg A \Rightarrow \neg B) \wedge (A \wedge \neg B)$
false	false	true	true	true	false	false
false	true	true	false	false	false	false
true	false	false	true	true	true	true
true	true	false	false	true	false	false

The statement is satisfiable, this because some models are true.

g) $(\neg A \Leftrightarrow \neg B) \wedge (A \wedge \neg B)$ is satisfiable.

A	B	$\neg A$	$\neg B$	$(\neg A \Rightarrow \neg B)$	$(A \land \neg B)$	$(\neg A \Rightarrow \neg B) \land (A \land \neg B)$
false	false	true	true	true	false	false
false	true	true	false	false	false	false
true	false	false	true	false	true	false
true	true	false	false	true	false	false

The statement is not satisfiable, this because no models are true.

2. Consider a logical knowledge base with 100 variables, $A_1, A_2, ..., A_100$. This will have $Q = 2^{100}$ possible models. For each logical sentence below, give the number of models that satisfy it.

   Feel free to express your answer as a fraction of Q (without writing out the whole number 1267650600228229401496703205376 = 2^{100}) or to use other symbols to represent large numbers.

   Example: The logical sentence A1 will be satisfied by $\frac{1}{2}Q = \frac{1}{2}2^{100} = 2^{99}$ models.

   a) $\neg A_{38} \land \neg A_{49}$

A_{38}	A_{49}	$\neg A_{38} \land \neg A_{49}$
true	true	false
true	false	false
false	true	false
false	false	true

   We notice that in only one case will the model be true. $\frac{1}{4}Q = \frac{1}{4}2^{100} = 2^{-2} \times 2^{100} = 2^{98}$ models.

   b) $\neg A_{27} \land \neg A_{46} \land A_{57}$

A_{27}	A_{46}	A_{57}	$\neg A_{27} \land \neg A_{46} \land A_{57}$
true	true	true	false
true	true	false	false
true	false	true	false
true	false	false	false
false	true	true	false
false	true	false	false
false	false	true	true
false	false	false	false

   We notice that in only one case will the model be true. $\frac{1}{8}Q = \frac{1}{8}2^{100} = 2^{-3} \times 2^{100} = 2^{97}$ models.

   c) $\neg A_{27} \land (A_{46} \land \neg A_{57})$

A_{27}	A_{46}	A_{57}	$A_{27} \wedge (A_{46} \vee \neg A_{57})$
true	true	true	true
true	true	false	true
true	false	true	true
true	false	false	false
false	true	true	false
false	true	false	false
false	false	true	false
false	false	false	false

$\frac{3}{8}Q = \frac{3}{8}2^{100} = 3 \times 2^{-3} \times 2^{100} = 3 \times 2^{97}$ models.

d) $\neg A_{85} \Rightarrow \neg A_{91}$

A_{85}	A_{91}	$\neg A_{85} \Rightarrow A_{91}$
true	true	true
true	false	false
false	true	true
false	false	true

$\frac{3}{4}Q = \frac{3}{4}2^{100} = 3 \times 2^{-2} \times 2^{100} = 3 \times 2^{98}$ models.

e) $(\neg A_{14} \Leftrightarrow \neg A_{19}) \wedge (A_{21} \Rightarrow A_{22})$

A_{14}	A_{19}	A_{21}	A_{22}	$(\neg A_{14} \Leftrightarrow \neg A_{19}) \wedge (A_{21} \Rightarrow A_{22})$
true	true	true	true	true
true	true	true	false	true
true	true	false	true	false
true	true	false	false	true
true	false	true	true	false
true	false	true	false	false
true	false	false	true	false
true	false	false	false	false
false	true	true	true	false
false	true	true	false	false
false	true	false	true	false
false	true	false	false	false
false	false	true	true	true
false	false	true	false	false
false	false	false	true	true
false	false	false	false	true

$\frac{6}{16}Q = \frac{6}{16}2^{100} = 6 \times 2^{-4} \times 2^{100} = 6 \times 2^{96}$ models.

f) $A_{41} \wedge \neg A_{59} \wedge A_{64} \wedge \neg A_{85} \wedge A_{87} \wedge \neg A_{90}$

We notice that all since this models is all *and* statements every variable has to be true. We therefore have $2^6 = 64$ solutions and only $\frac{1}{61}$ will be true. We then get $\frac{1}{64}Q = \frac{1}{64}2^{100} = 2^{-6} \times 2^{100} = 2^{94}$

3. Wompus shit

3.1	3.2	3.3	4.4	$3.1 \wedge \neg 3.2 \wedge (3.3 \vee 4.4)$
true	true	true	true	false
true	true	true	false	false
true	true	false	true	false
true	true	false	false	false
true	false	true	true	true
true	false	true	false	true
true	false	false	true	true
true	false	false	false	false
false	true	true	true	false
false	true	true	false	false
false	true	false	true	false
false	true	false	false	false
false	false	true	true	false
false	false	true	false	false
false	false	false	true	false
false	false	false	false	false

From the model table we get 3 worlds:

w1 = P[3.1, 3.3, 4.4]
w2 = P[3.1, 3.3]
w3 = P[3.1, 4.4]

Sentences to check:
α_1 = "There is no pit in [3.2]"
α_2 = "There is a pit in [4.4]"
α_3 = "There is no pit in [4.4]"
α_4 = "There is a pit in [3.3] or [4.4]"

	α_1	α_2	α_3	α_4
w1	true	true	false	true
w2	true	false	true	true
w3	true	true	false	true

Based on the knowledge base we can no specify that $KB \models \alpha_1$ and $KB \models \alpha_4$.

2 Resolution in Propositional Logic

1. Convert each of the following sentences to Conjunctive Normal Form (CNF).

   a) $A \vee (B \wedge C \wedge \neg D)$
   Distribute:
   $(A \vee B) \wedge (A \vee C) \wedge (A \vee \neg D)$

   b) $\neg(A \Rightarrow \neg B) \wedge \neg(C \Rightarrow \neg D)$
   Eliminate $\Rightarrow$:
   $\neg(\neg A \vee \neg B) \wedge \neg(\neg C \vee \neg D)$
   Move $\neg$ inwards:
   $(A \wedge B) \wedge (C \wedge D)$
   Remove parenthesis:
   $A \wedge B \wedge C \wedge D$

   c) $\neg((A \Rightarrow B) \wedge (C \Rightarrow D))$
   Eliminate $\Rightarrow$:
   $\neg((\neg A \vee B) \wedge (\neg C \vee D))$
   Move $\neg$ inwards:
   $\neg(\neg A \vee B) \vee \neg(\neg C \vee D)$
   Move $\neg$ inwards:
   $(A \wedge \neg B) \vee (C \wedge \neg D)$
   Distribute:
   $((A \wedge \neg B) \vee C) \wedge ((A \wedge \neg B) \vee \neg D)$
   Distribute:
   $((A \vee C) \wedge (\neg B \vee C)) \wedge ((A \vee \neg D) \wedge (\neg B \vee \neg D))$
   Remove paranthesis:
   $(A \vee C) \wedge (\neg B \vee C) \wedge (A \vee \neg D) \wedge (\neg B \vee \neg D)$

   d) $(A \wedge B) \vee (C \Rightarrow D)$
   Eliminate $\Rightarrow$:
   $(A \wedge B) \vee (\neg C \vee D)$
   Distribute:
   $((\neg C \vee D) \vee A) \wedge ((\neg C \vee D) \vee B))$
   Remove parenthesis:
   $(\neg C \vee D \vee A) \wedge (\neg C \vee D \vee B)$

   e) $A \Leftrightarrow (B \Rightarrow \neg C)$
   Eliminate $\Leftrightarrow$:
   $(A \Rightarrow (B \Rightarrow \neg C)) \wedge ((B \Rightarrow \neg C) \Rightarrow A)$
   Eliminate $\Rightarrow$:
   $(\neg A \vee (\neg B \vee \neg C)) \wedge (\neg(\neg B \vee \neg C) \vee A)$
   deMorgan:
   $(\neg A \vee (\neg B \vee \neg C)) \wedge ((B \wedge C) \vee A)$
   Distribute:
   $(\neg A \vee (\neg B \vee \neg C)) \wedge ((B \vee A) \wedge (C \vee A))$

Remove parenthesis:
$(\neg A \lor \neg B \lor \neg C) \land (B \lor A) \land (C \lor A)$

2. Consider the following Knowledge Base (KB):

   a) $(D \land E) \Rightarrow C)$

   b) $\neg A \Rightarrow \neg B$

   c) $\neg C \land E$

   d) $\neg D \Rightarrow B$

   First we convert the KB to CNF:

   a) $C \lor \neg D \lor \neg E$

   b) $A \lor \neg B$

   c) $\neg C \land E$

   d) $B \lor D$

   Then we negate our desired conclusion:
   $KB \models A = KB \models \neg A$

Step	Formula	Derivation
1	$C \lor \neg D \lor \neg E$	Given
2	$A \lor \neg B$	Given
3	$\neg C$	Given
4	E	Given
5	$B \lor D$	Given
6	$\neg A$	Negated conclusion
7	$C \lor \neg D$	Resolution rule: 1,4
8	$\neg D$	Resolution rule: 7,3
9	B	Resolution rule: 5,8
10	A	Resolution rule: 2,9
11	■	Resolution rule: 10,6

3. Exercise 7.18 from the textbook. With the sentence:

   $$(\neg Party \Rightarrow \neg(Food \lor Drinks)) \Rightarrow (Food \Rightarrow Party)$$

   a) Determine, using enumeration, whether this sentence is valid, satisfiable (but not valid), or unsatisfiable.

Party	Food	Drinks	$(\neg Party \Rightarrow \neg(Food \vee Drinks)) \Rightarrow (Food \Rightarrow Party)$
true	true	true	true
true	true	false	true
true	false	true	true
true	false	false	true
false	true	true	true
false	true	false	true
false	false	true	true
false	false	false	true

The sentence is valid because all of the models are true.

b) Convert the left-hand and right-hand sides of the main implication into CNF, showing each step, and explain how the results confirm your answer to (a).

Converting both sides to CNF:

Left:
$(\neg Party \Rightarrow \neg(Food \vee Drinks))$

Eliminate $\Rightarrow$:
$(Party \vee \neg(Food \vee Drinks))$

DeMorgan on $\neg$:
$(Party \vee \neg(\neg Food \wedge \neg Drinks))$

Distribute $\wedge over \vee$:
$(Party \vee \neg Food) \wedge (Party \vee \neg Drinks)$

Right:
$Food \Rightarrow Party$

Eliminate $\Rightarrow$:
$\neg Food \vee Party$

If left side is true, then the right side must also be true. Therefore we can conclude that this sentence is valid.

c) Prove your answer to (a) using resolution.

To prove the resolution we prove by contradiction that: $(Party \lor \neg Food) \land (Party \lor \neg Drinks) \Rightarrow (Food \lor Party)$ is unsatisfible.

We do this by saying that the right side implies negative left side:
$\neg Food \lor Party$ Becomes: $\neg(\neg Food \lor Party)$

DeMorgan:

$Food \land \neg Party$

Step	Formula	Derivation
1	$Party \lor \neg Food$	Given
2	$Party \lor \neg Drinks$	Given
3	$Food$	Negated
4	$\neg Party$	Negated
5	P	Resolution rule: 1,3
6	$\blacksquare$	Resolution rule: 5, 4

3 Representations in First-Order Logic

1. Consider a vocabulary with the following symbols:

   $Occupation(p, o)$: Predicate person p has Occupation o.
   $Customer(p1, p2)$: Predicate. Person $p1$ is a customer of person $p2$.
   $Boss(p1, p2)$: Predicate. Person $p1$ is a boss of person $p2$.
   $Doctor, Surgeon, Lawyer, Actor$: Constants denoting occupations.
   $Emily, Joe$: Constants denoting people.

   Use these symbols to write the following assertions in first-order logic:

   a) Emily is either a surgeon or a lawyer.

      $Occupation(Emily, Surgeon) \lor Occupation(Emily, Lawyer)$

   b) Joe is an actor, but he also holds another job.

      $Occupation(Joe, Actor) \land Occupation(Joe, Other)$

   c) All surgeons are doctors.

      $\forall Surgeon, Occupation(Surgeon, Doctor)$

d) Joe does not have a lawyer (i.e., is not a customer of any lawyer).

$\neg Customer(Joe, Lawyer)$

e) Emily has a boss who is a lawyer.

$Boss(Lawyer, Emily)$

f) There exists a lawyer all of whose customers are doctors.

$\exists Lawyer, \forall Customer(Doctor, Lawyer)$

g) Every surgeon has a lawyer.

$\forall Surgeon, Customer(Surgeon, Lawyer)$

2. Consider a first-order logical knowledge base that describes worlds containing movies, actors, direc- tors and characters. The vocabulary contains the following symbols:

PlayedInMovie(a,m): predicate. Actor/person a played in the movie m
PlayedCharacter(a,c): predicate. Actor/person a played character c
CharacterInMovie(c,m): predicate. Character c is in the movie m.
Directed(p,m): person p directed movie m.
Male(p): p is male
Female(p): p is female
Constants related to the name of the movie, person or character with obvious meaning (to simplify you may use the surname or abbreviation).

Express the following statements in First-Order Logic:

a) The character "Batman" was played by Christian Bale, George Clooney and Val Kilmer.

$PlayedCharacter(ChristianBale, Batman) \wedge PlayedCharacter(GeorgeClooney, Batman)$
$\wedge\ PlayedCharacter(ValKilmer, Batman)$

b) The character "Batman" was played by male actors.

$\forall\ PlayerCharacter(male(p), Batman)$

c) The character "Batwoman" was played by female actresses.

$\forall\ PlayerCharacter(female(p), Batwoman)$

d) Heath Ledger and Christian Bale did not play the same characters.

$PlayerCharacter(HeathLedger, c) \land \neg(PlayedCharacter(ChristianBale, c)$

e) In all "Batman" movies directed by Christopher Nolan, Christian Bale played the character Bat- man (tip: note that in this case Batman is a character of the movie, not the name of the movie).

$\forall\ Directed(ChristopherNolan, Batman), PlayedCharacter(ChristianBale, Batman)$

f) The Joker and Batman are characters that appear together in some movies.

$\exists$ movie m, $CharacterInMovie(Batman, m) \land CharacterInMovie(Joker, m)$

g) Kevin Costner directed and starred in the same movie.

$Directed(KevinCostner, m) \land PlayedInMovie(KevinCostner, m)$

h) George Clooney and Tarantino never played in the same movie and Tarantino never directed a film that George Clooney played.

$\forall\ PlayedInMovie(GeorgeClooney, m),\ \neg PlayedInMovie(Tarantino, m) \land \neg Directed(Tarantino, m)$

i) Uma Thurman is female actress who played a character in some movies directed by Tarantino.

$\exists\, Directed(Tarantino, m), Female(UmaThurman) \land PlayedInMovie(UmaThurman, m)$

3. a) An integer number x is divisible by y if there is some integer z less than x such that $x = z \times y$ (in other words, define the predicate $Divisible(x, y)$).

$\exists x, y, z((z < x) \land (z \in \mathbb{Z}) \land (x = z \times y)) \Leftrightarrow Divisible(x, y)$

b) A number is even if and only if it is divisible by 2 (define the predicate $Even(x)$).

$Divisible(x, 2) \Rightarrow Even(x)$

c) An odd number is not divisible by 2 (define the predicate $Odd(x)$.

$\neg Divisible(x, 2) \Rightarrow Odd(x)$

d) The result of summing an even number with 1 is an odd number (define the predicate $Odd(x)$).

$\forall x Even(x) \Leftrightarrow Odd(x+1)$

e) A prime number is divisible only by itself (define the predicate $Prime(x)$).

$\exists x \forall y\ ((Prime(x) \wedge Even(x)) \wedge (Prime(y) \wedge Even(y))) \Rightarrow (x = y)$

f) There is only one even prime number.

$\exists x \forall y((Prime(x) \wedge Even(x)) \wedge (Prime(y) \wedge Even(y))) \Rightarrow (x = y)$

g) Every integer number is equal to a product of prime numbers. (you can use $\prod_{i=1}^{k} pk$ to express a product of numbers, or use . . . to express a repeating pattern, like $p1, ..., pn$, meaning $p1, p2, p3$ until pn).

$\forall x \exists p_1, ..., p_n\ (Prime(p_1) \wedge ... \wedge Prime(p_n)) \Rightarrow x = \prod_{i=1}^{n} p_i$

4 Resolution in First-Order Logic

1. Find the unifier (θ) – if possible – for each pair of atomic sentences. Here, $Owner(x, y)$, $Horse(x)$ and $Rides(x, y)$ are predicates, while $FastestHorse(x)$ is a function that maps a person to the name of their fastest horse:

   a) Horse(x) ... Horse(Rocky)
   Answer: $\theta = \{x/Rocky\}$

   b) Owner(Leo, Rocky) ... Owner(x, y)
   Answer: $\theta = \{x/Leo, y/Rocky\}$

   c) Owner(Leo, x) ... Owner(y, Rocky)
   Answer: $\theta = \{x/Leo, y/Rocky\}$

   d) Owner(Leo, x) ... Rides(Leo, Rocky)
   Answer: Impossible to unify.

   e) Owner(x, FastestHorse(x)) ... Owner(Leo, Rocky)
   Answer: $\theta = \{x/Leo\}$

f) Owner(Leo, y) ... Owner(x, FastestHorse(x))
Answer: $\theta = \{x/Leo, y/FastestHorse(Leo)\}$

g) Rides(Leo FastestHorse(x)) ... Rides(y, FastestHorse(Marvin))
Answer: $\theta = \{x/Marvin, y/Leo\}$

2. Using the predicates $Philosopher(x)$, $StudentOf(y,x)$, $Write(x,z)$, $Read(y,z)$ and $Book(z)$ perform skolemization with the following expressions:

   a) $\exists x \exists y : Philosopher(x) \wedge StudentOf(y,x)$
   $Philosopher(a) \wedge StudentOf(b,a)$

   b) $\forall y,x : Philosopher(x) \wedge StudentOf(y,x) \to [\exists z : Book(z) \wedge Write(x,z) \wedge Read(y,z)]$

   Eliminate implication:
   $\forall y,x : \neg Philosopher(x) \vee \neg StudentOf(y,x) \vee [\exists z : Book(z) \wedge Write(x,z) \wedge Read(y,z)]$

   Skolemize: substitute z by $h(x,y)$:
   $\forall y,x : \neg Philosopher(x) \vee \neg StudentOf(y,x) \vee [Book(h(x,y)) \wedge Write(x,h(x,y)) \wedge Read(y,h(x,y))]$

3. Use resolution to prove SuperActor(Tarantino) given the information below. You must first convert each sentence into CNF. Feel free to show only the applications of the resolution rule that lead to the desired conclusion. For each application of the resolution rule, show the unification bindings, θ. We are using in this case the same predicates of Exercise 3.1 (movies, actors, etc).

   - $\forall x : SuperActor(x) \Leftrightarrow [\exists m : PlayedInMovie(x,m) \wedge Directed(x,m)]$
   - $\forall m : Directed(Tarantino,m) \Leftrightarrow PlayedInMovie(UmaThurman,m)$
   - $\exists m : PlayedInMovie(UmaThurman,m) \wedge PlayedInMovie(Tarantino,m)$

   a) Show all the steps in the proof (or the diagram).

   •$\forall x : SuperActor(x) \Leftrightarrow [\exists m : PlayedInMovie(x,m) \wedge Directed(x,m)]$
   Eliminate $\Leftrightarrow$:

   $\forall x : (SuperActor(x) \Rightarrow [\exists m : PlayedInMovie(x,m)) \wedge Directed(x,m)]) \wedge ([\exists m : PlayedInMovie(x,m) \wedge Directed(x,m)] \Rightarrow SuperActor(x))$

   Eliminate $\Rightarrow$:
   $\forall x : (\neg SuperActor(x) \vee [\exists m : PlayedInMovie(x,m) \wedge Directed(x,m)]) \wedge$

$(\neg[\exists m : PlayedInMovie(x,m) \wedge Directed(x,m)] \vee SuperActor(x))$

DeMorgan:
$\forall x : (\neg SuperActor(x) \vee [\exists m : PlayedInMovie(x,m) \wedge Directed(x,m)]) \wedge$
$([\exists m : \neg PlayedInMovie(x,m) \vee \neg Directed(x,m)] \vee SuperActor(x))$

Substitute m:
$\forall x : (\neg SuperActor(x) \vee [PlayedInMovie(x, f(m)) \wedge Directed(x, f(m))]) \wedge$
$([\neg PlayedInMovie(x, f(m)) \vee \neg Directed(x, f(m))] \vee SuperActor(x))$

Distribute:
$\forall x : (\neg SuperActor(x) \vee PlayedInMovie(x, f(m)) \wedge (SuperActor(x) \vee Directed(x, f(m))) \wedge$

$([\neg PlayedInMovie(x, f(m)) \vee \neg Directed(x, f(m))] \vee SuperActor(x))$

Remove unified quantifiers:
$(\neg SuperActor(x) \vee PlayedInMovie(x, f(m)) \wedge (SuperActor(x) \vee Directed(x, f(m))) \wedge$

$([\neg PlayedInMovie(x, f(m)) \vee \neg Directed(x, f(m))] \vee SuperActor(x))$

Remove parenthesis:
$(\neg SuperActor(x) \vee PlayedInMovie(x, f(m)) \wedge (SuperActor(x) \vee Directed(x, f(m)) \wedge$

$(\neg PlayedInMovie(x, f(m)) \vee \neg Directed(x, f(m))) \vee SuperActor(x)$

•$\forall m : Directed(Tarantino, m) \Leftrightarrow PlayedInMovie(UmaThurman, m)$

Eliminate $\Leftrightarrow$:
$\forall m : (Directed(Tarantino, m) \Rightarrow PlayedInMovie(UmaThurman, m)) \wedge$
$(PlayedInMovie(UmaThurman, m) \Rightarrow Directed(Tarantino, m))$

Eliminate $\Rightarrow$:
$\forall m : (\neg Directed(Tarantino, m) \vee PlayedInMovie(UmaThurman, m)) \wedge$
$(\neg PlayedInMovie(UmaThurman, m) \vee Directed(Tarantino, m))$

Remove unified quantifiers:
$(\neg Directed(Tarantino, m) \vee PlayedInMovie(UmaThurman, m)) \wedge$
$(\neg PlayedInMovie(UmaThurman, m) \vee Directed(Tarantino, m))$

•$\exists m : PlayedInMovie(UmaThurman, m) \wedge PlayedInMovie(Tarantino, m)$

Substitute m:
$PlayedInMovie(UmaThurman, f(m)) \wedge PlayedInMovie(Tarantino, f(m))$

Step	Formula	Derivation
1	$\neg SuperActor(x) \vee PlayedInMovie(x, f(m))$	Given
2	$SuperActor(x) \vee Directed(x, f(m))$	Given
3	$\neg PlayedInMovie(x, f(m)) \vee \neg Directed(x, f(m)) \vee SuperActor(x)$	Given
4	$\neg Directed(Tarantino, m) \vee PlayedInMovie(UmaThurman, m))$	Given
5	$\neg PlayedInMovie(UmaThurman, m) \vee Directed(Tarantino, m))$	Given
6	$PlayedInMovie(UmaThurman, f(m))$	Given
7	$PlayedInMovie(Tarantino, f(m))$	Given
8	$\neg SuperActor(Tarantino)$	Negated conclusion
9	$Directed(x, f(m))$	Resolution rule: 5,6 $\theta = \{y/UmaThurman\}$
10	$\neg PlayedInMovie(x, f(m)) \vee SuperActor(x)$	Resolution rule: 3,9
11	$SuperActor(x)$	Resolution rule: 7,10
12	$SuperActor(Tarantino)$	$\theta = \{x/Tarantino\}$
13	$\blacksquare$	Resolution rule: 12, 8

4. Translate the information given in FOL into English (or Norwegian) and describe in high level the reasoning you could apply in English to have the same result (in other words, describe a proof of the result in natural language).

   - $\forall x : SuperActor(x) \Leftrightarrow [\exists m : PlayedInMovie(x, m) \wedge Directed(x, m)]$

     A person is a super actor if and only if he or she played in, and directed the same movie.

   - $\forall m : Directed(Tarantino, m) \Leftrightarrow PlayedInMovie(UmaThurman, m)$

     Uma Thurman has played in every movie directed by Tarantino, and Tarantino has directed every movie Uma Thurman has played in.

   - $\exists m : PlayedInMovie(UmaThurman, m) \wedge PlayedInMovie(Tarantino, m)$

     There exists a movie in which bot Uma Thurman and Tarantino playes a role.

     Since there Tarantino has directed every movie Uma Thurman has played in, and there exists a movie where both Tarantino and Uma Thurman plays a role, then Tarantino must be a super actor.