

a) We want to prove

$$\langle f(y) \rangle_r = \int dy \, r(y) \cdot f(y)$$

$$\begin{aligned} \langle f(y) \rangle &= \frac{1}{N} \sum f(y_i) = \frac{1}{N} \sum f(\Pi^{-1}(x_i)) \quad \left| \begin{array}{l} \text{after what} \\ \text{was given} \end{array} \right. \\ &= \int dx \, f(\Pi^{-1}(x)) \quad \left| \text{for } N \rightarrow \infty \right. \end{aligned}$$

Substitute $\Pi^{-1}(x)$ with y

$$\frac{dy}{dx} = (\Pi^{-1}(x))' \Leftrightarrow \frac{dy}{(\Pi^{-1}(x))'} = dx$$

$$dx = dy \cdot \Pi'(y) \quad \left| \begin{array}{l} \text{inverse function} \\ \text{theorem} \end{array} \right.$$

$$dx = dy \cdot r(y)$$

$$\Pi'(y) = \left(\int_{-\infty}^y r(t) dt \right)' = r(y) \quad \left| \begin{array}{l} \text{fundamental} \\ \text{theorem of} \\ \text{calculus} \end{array} \right.$$

$$= \int dy \cdot r(y) \cdot f(y)$$



Task 1

$$c) \langle \psi_{1s} | \psi_{1s} \rangle = \int_{-\infty}^{\infty} dx dy dz |\psi_{1s}(\vec{r})|^2 \stackrel{!}{=} 1$$

$$\begin{aligned} x &= r \sin \Theta \sin \varphi \\ y &= r \sin \Theta \cos \varphi \\ z &= r \sin \Theta \end{aligned}$$

Substitution

$$= \int dx dy dz |A e^{-|r|/a_0}|^2$$

$$= \int_0^{2\pi} d\varphi \int_0^{\pi} d\Theta \int_0^{\infty} dr A^2 r^2 e^{-\frac{2r}{a_0}}$$

$$= 2\pi \int_0^{\infty} dr A^2 r^2 e^{-\frac{2r}{a_0}} \stackrel{!}{=} 1$$

$$= A^2 2\pi \int_0^{\infty} dr r^2 e^{-\frac{2r}{a_0}}$$

$$\cancel{2\pi \int_0^{\infty} dr r^2 e^{-\frac{2r}{a_0}} = A^2}$$

$$A^2 = \frac{1}{2\pi \int_0^{\infty} dr r^2 e^{-\frac{2r}{a_0}}}$$