Assingment 8 - Task 1 - Robin Nehls & Yves Müller

a) We want to prove

$$\langle f(y)\rangle_{\gamma} = \int dy\ \gamma(y)\cdot f(y)$$

$$\langle f(y)\rangle = \frac{1}{N}\sum f(y_i) = \frac{1}{N}\sum f(\Pi^{-1}(x_i)) \quad \Big| \text{after what was given}$$

$$= \int dx\ f(\Pi^{-1}(x)) \quad \Big| \text{for } N\to\infty$$

Substitute $\Pi^{-1}(x)$ with y

$$\frac{dy}{dx} = \left(\Pi^{-1}(x)\right)' \Leftrightarrow \frac{dy}{\left(\Pi^{-1}(x)\right)'} = dx$$

$$dx = dy\cdot\Pi'(y) \quad \Big| \text{inverse function theorem}$$

$$dx = dy\cdot\gamma(y)$$

$$\Pi'(y) = \left(\int_{\infty}^{x}\gamma(t)\,dt\right)' = \gamma(y) \quad \Big| \text{fundamental theorem of calculus}$$

$$= \int dy\cdot\gamma(y)\cdot f(y)$$

$\boxed{}$

Task 1

c) $$\langle \varphi_{1s} | \varphi_{1s} \rangle = \int_{-\infty}^{\infty} dx\, dy\, dz \, \left| \varphi_{1s}\begin{pmatrix} x \\ y \\ z \end{pmatrix} \right|^2 \doteq 1$$

$$x = r \sin \Theta \sin \varphi$$
$$y = r \sin \Theta \cos \varphi$$
$$z = r \sin \Theta$$

Substitution

$$= \int dx\, dy\, dz \, \left| A\, e^{-|r|/a_0} \right|^2$$

$$= \int_0^{2\pi} d\varphi \int_0^{\pi} d\Theta \int_0^{\infty} dr\, A^2 \cdot r^2\, e^{-\frac{2r}{a_0}}$$

$$= 2\pi \int_0^{\infty} dr\, A^2 \cdot r^2\, e^{-\frac{2r}{a_0}} \doteq 1$$

$$= A^2\, 2\pi \int_0^{\infty} dr\, r^2 \cdot e^{-\frac{2r}{a_0}}$$

$$\cancel{2\pi \int_0^{\infty} dr\, r^2 \cdot e^{-\frac{2r}{a_0}} = A^2}$$

$$A^2 = \frac{1}{2\pi \int_0^{\infty} dr\, r^2 \cdot e^{-\frac{2r}{a_0}}}$$

e) The resulting distribution, looks like a gaussian distribution.