$P_0(x) = 1$ $\qquad\qquad\qquad\qquad \omega(x) = 1$

$P_1(x) = x$

$P_2(x) = \frac{1}{2}(3x^2 - 1)$

$P_3(x) = \frac{1}{2}(5x^3 - 3x)$

$0 = P_3(x)$

$= \frac{1}{2}(5x^3 - 3x)$

$= 5x^3 - 3x \quad \Rightarrow \quad x_1^{(3)} = 0$

$= 5x^2 - 3 \qquad\qquad x_{2,3}^{(3)} = \pm\sqrt{\frac{3}{5}}$

$\frac{3}{5} = x^2$

$j = 1, \ldots, 3$ $\qquad$ ~~SLE~~ SLE

I) $p_0(x_1^{(3)})\omega_1^{(3)} + p_0(x_2^{(3)})\omega_2^{(3)} + p_0(x_3^{(3)}) \cdot \omega_3^{(3)} = \int_{-1}^{1} p_0^2(x)$

II) $p_1(x_1^{(3)})\omega_1^{(3)} + p_1 \quad \ldots \quad = 0$

III) $p_2(x_1^{(3)})\omega_1^{(3)} + p_2 \quad \ldots \quad = 0$

for simplification: $\omega_i^{(3)} = \omega_i$ and $x_i^{(3)} = x_i$

I) $\omega_1 + \omega_2 + \omega_3 = [x]_{-1}^{1}$

II) $0\,\omega_1 + \sqrt{\frac{3}{5}}\,\omega_2 + \sqrt{\frac{3}{5}}\,\omega_3 = 0 \quad \Rightarrow \quad \underline{\omega_2 = \omega_3}$

III) $\frac{1}{2}(0-1)\omega_1 + \frac{1}{2}\left(\frac{9}{5}-1\right)\omega_2 + \frac{1}{2}\left(\frac{9}{5}-1\right)\omega_3 = 0$

$-\frac{1}{2}\omega_1 + \frac{2}{5}\omega_2 + \frac{2}{5}\omega_2 = 0$

$-\frac{1}{2}\omega_1 + \frac{4}{5}\omega_2 = 0$

$$\text{I)}\quad w_1 + 2w_2 = 1-(-1)$$

$$w_1 + 2w_2 = 2$$

$$\text{I}+2\cdot\text{III)}\quad +\frac{4}{5}\cdot 2w_2 + 2w_2 = 2$$

$$+\frac{4}{5}w_2 + w_2 = 1$$

$$w_2\frac{9}{5} = 1$$

$$\underline{\underline{w_2 = \frac{5}{9} \approx 0{,}\overline{5} = w_3}}$$

$$\text{in I)}\quad w_1 + 2\cdot\frac{5}{9} = 2$$

$$\underline{\underline{w_1 = 0{,}\overline{8} = \frac{8}{9}}}$$

~~in II)~~

$$\int_{-1}^{1} x^4 \, dx = \sum_{i=1}^{3} (x_i^{(3)})^4 \cdot w_i^{(3)}$$

$$\left[\frac{1}{5}x^5\right]_{-1}^{1} = \frac{8}{9}\cdot 0 + 2\left(\frac{5}{9}\cdot\sqrt{\frac{3}{5}}^{\,4}\right)$$

$$\frac{1}{5}-\left(-\frac{1}{5}\right) = \frac{2}{5} = \frac{10^2}{9}\cdot\frac{9}{25_5} = \frac{2}{5}$$

$$\int_{-1}^{1} x^5 \, dx = \sum_{i=1}^{3} (x_i^{(3)})^5 \cdot w_i^{(3)}$$

$$\left[\frac{1}{6}x^6\right]_{-1}^{1} = \frac{8}{9}\cdot 0 + \frac{5}{9}\cdot\sqrt{\frac{3}{5}}^{\,5} - \frac{5}{9}\sqrt{\frac{3}{5}}^{\,5}$$

$$\frac{1}{6}-\frac{1}{6} = 0$$

$$\underline{\underline{0 = 0}}$$

we expext x_a to be root of $p_n(x)$ and $p_{n+1}(x)$ then:

$$p_n(x_a) = p_{n+1}(x_a)$$

$$(x_a - \lambda_n)\, p_{n-1}(x_a) - \gamma_n^2 \cdot p_{n-2}(x_a) = (x_a - \lambda_{n+1}) \underbrace{p_n(x_a)}_{0} - \gamma_{n+1}^2\, p_{n-1}(x_a)$$

$$= -\gamma_{n+1}^2\, p_{n-1}(x_a)$$

$$0 = \underbrace{\left((x_a - \lambda_n)\, p_{n-1}(x_a) - \gamma_n^2 \cdot p_{n-2}(x_a)\right)}_{=0} + \underbrace{\gamma_{n+1}^2\, p_{n-1}(x_a)}_{=0}$$

$\Rightarrow$

$$0 = \int_{-1}^{1} \frac{\langle p_n | p_n \rangle}{\langle p_{n-1} | p_{n-1} \rangle} \cdot p_{n-1}(x_a)$$

(I think that means $p_{n-1}(x_a)$ is also 0, so that every $p_i(x_a)$ would be zero, which would be in contradiction to $p_0(x) = 1$)