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Medical Images Denoising Based on Total Variation Algorithm

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Abstract

Because of the limitation of equipment and the transmission process, the medical images' quality is dropped, which cannot satisfy the medical analysis and the applied requirements. The denoising of medical image plays an important role in the image processing, and it is the basic of further analysis and computation. The image edge are easily interrupted by noise, but the traditional image denoising smooth out the edges of the reconstructed images which caused edge to be blurred, and made the information lost. The denoising algorithms based on partial differential of total variation is proposed by analyzing the requirements of medical image features from the view of image denoising, and use the OpenCV platform to simulate the program. The emphasis is to study the algorithm which applies in the medical image denoising technique. Theoretical analysis and experimental results show that the algorithm is effectiveness and superiority, which can make sure to obtain clear medical image and preserve the edge information integrally at the same time.

Keywords: medical image, image denoising, total variation, edge preserve;

1. Introduction

Nowadays, with the development of the signal processing, the technique of image processing is widely used in medical research. In medical image collection and transmission process will produce some signal noise inevitably, thus it seriously affects the quality of the images, and it brought greatly difficult for the further study. The traditional method of image denoising use smoothing (such as average filters and Gaussian filtering, etc) for sake of achieving good results, but they may be lost edges information and texture[1]. However, medical image consist of many details such as angular and edge, and the lack of image edge information brings difficulties for subsequent analysis. From the above, in the image

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restoration field desperately needs an image restoration process technology which can remove the noise and better keep image edge information.

The development of variational partial differential equation based on image restoration techniques offer a new thought to address the problem about image denoising and image edge preserve. It has become the research hotspot in recent years. The denoising algorithms based on partial differential of total variation is proposed by analyzing requirements of medical image features from the point of image denoising, the emphasis is studying the algorithm to apply in the medical image denoising technology[2]. Theoretical analysis and experimental results show that the algorithm obtain clear medical image while preserving the edge information integrally. It certifies that the total variation algorithm is effectiveness and superiority in medical image denoising.

2. Based on Total Variation Method of Image Denoising Processing

2.1. Algorithm of Total Variational

Total variational algorithm is the hotspot in image restoration field, and it used to deduce the image from the observation to the original image. Inverse problem of Image processing (also called inverse deconvolution) are ill-posed, ill-posed problem which is an inherent characteristics. This essential quality is difficult to overcome without the additional information (for example, monotonicity and boundedness smoothness or data). The traditional method introduced the smooth constraints into it. But there is much edge information and details in the actual image, smoothing the solution means to lost some useful information. Whereas the total variational model (ROF) [3] can effectively solve this contradiction about image smooth and edge details preserve, and it also can reduce the noise while preserving the edge detail extremely well.

Total variational is defined as the integral of gradient magnitude, which is represented as[4]:

$$J_{TV}(f) = \int |\nabla f| \, dxdy = \int \sqrt{f_x^2 + f_y^2} \, dxdy \tag{1}$$

In the formula, $f_x = \partial f / \partial x ; f_y = \partial f / \partial y$. All authors must sign the Transfer of Copyright agreement before the article can be published. This transfer agreement enables Elsevier to protect the copyrighted material for the authors, but does not relinquish the authors' proprietary rights. The copyright transfer covers the exclusive rights to reproduce and distribute the article, including reprints, photographic reproductions, microfilm or any other reproductions of similar nature and translations. Authors are responsible for obtaining from the copyright holder permission to reproduce any figures for which copyright exists.

In practical application, total variation of the image with noise is extremely bigger than total variation of image without noise. Therefore, limit total variation can limit noise; and not produce the smooth effect at the same time, so the image edge information was preserved. Thus, the problem of image restoration become to compute the minimum of

$$J(f) = J_1(f) + \alpha J_{TV}(f) \tag{2}$$

Compute the first derivative of (1)

$$\nabla J(f) = \alpha \nabla \cdot (\nabla f / |\nabla f|) + (K^* K f - K^* g) = 0 \tag{3}$$

Nonlinear equations in (3) are the requirement which is needed to solve the image restoration equations. In the processing to compute the nonlinear equations, however, there are many challenges, which are the total variation is not differentiable at zero, operator is the highly nonlinear and the ill-posed of the operator.

Iterative gradient method is used to address the problems. Because the total variation is non-differentiable at zero, we add a small positive constant value β to $|\nabla f|$; in order to make sure at $\sqrt{|\nabla f|^2+\beta^2}=0$, the total variation is differentiable. At same time, we also obtained a partial differential operator to simplify calculation, express as

$$L(f)=\nabla\cdot(\nabla\Big/\sqrt{|f|^2+\beta^2}) \tag{4}$$

So the (3) can represented as [4]

$$\nabla J(f)=\alpha L(f)+(K^*Kf-K^*g)=0 \tag{5}$$

2.2. Method of Fix-point Iterations

The fix-point method which is introduced by Vogel [5] is employed to address the nonlinear problem in (4). The method is to add a diffusion factor $1/|\nabla f|$ which can translate a non-linear operation to a linear one:

$$[K^*K+\alpha L(f^n)]f^{n+1}=K^*g \tag{6}$$

The fix-point operation starts at f^0 , then at every step all need to calculate the (6). About f^0 and $\nabla J(f)=0$ need to compute equation $K^TK+\alpha(L(f))f=K^Tg$. Operator $L(f)$ is provided in (9).

Taking the 2-D image for example, the penalty function can be defined as:

$$J(f)=\frac{1}{2}\sum_{i=1}^{n_x}\sum_{j=1}^{n_y}\Psi((D_{i,j}^x f)^2+(D_{i,j}^y f)^2) \tag{7}$$

The gradient of $J(f)$:

$$\begin{aligned}\nabla J(f)&=\frac{d}{dt}J(f)\\&=\frac{1}{2}\sum_{i=1}^{n_x}\sum_{j=1}^{n_y}\Psi'_{i,j}((D_{i,j}^x f)^2+(D_{i,j}^y f)^2)\end{aligned} \tag{8}$$

According to the (7), we can get the expression of $L(f)$:

$$L(f)=D_x^T diag(\Psi'(f))D_x+D_y^T diag(\Psi'(f)) \tag{9}$$

Based on the equations when the fix-point iteration is convergence, we can get $f^n=f^{n+1}$. These can stratify the (2). The convergence of fix-point iteration is linear and fast convergence. The experiment shows that this method is not only monotone convergence, but also has good robustness.

2.3. Flow of Algorithm

In the actual operation of the algorithm for solving total variation is used fixed-point iteration method. The fixed-point iteration method of total variation was selected to minimize the non-linear expression [1]:

$$\begin{aligned}f_{n+1}&=[K^TK+\alpha(L(f))f]K^Tg\\&=f_n-[K^TK+\alpha(L(f))f]^{-1}gradT(f_n)\end{aligned} \tag{10}$$

The equation above is using Newton method [6]. First, near the approximate $x^{(k)}$ of the objective function $f(x)$'s minimum point x^*, expanding $f(x)$ into second-order Taylor equation and using the equation to approximate $f(x)$. So we can get a new approximate point $x^{(k+1)}$. Then the objective function $f(x)$'s minimum point x^* is approximated by new point $x^{(k+1)}$. After applying the number of x^0, the Iterative sequence $\{x^k\}$ is computed by (11) [7].

$$x^{(k+1)} = x^{(k)} - \nabla^2 f(x^k)^{-1} \nabla f(x^{(k)}) \tag{11}$$

Equation (11) is called Newton iterative formula. According to the (7), operator $L(f)$ can translate into (12) [1].

$$L(f) = [D_x^T D_y^T] \begin{bmatrix} diag(\Psi'(f)) & 0 \\ 0 & diag(\Psi'(f)) \end{bmatrix} \begin{bmatrix} D_x^T \\ D_y^T \end{bmatrix} \tag{12}$$

An iterative algorithm for the specific process of the experiment is shown in Fig. 1.

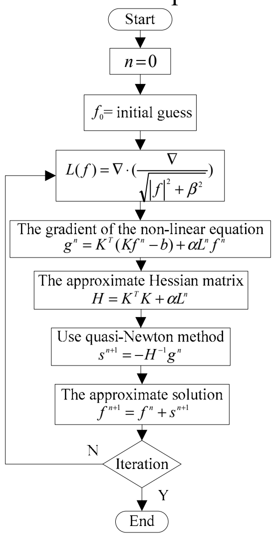


Fig. 1. flow chart of iterative algorithm

Because experimental image is stored in a discrete array, the continuous algorithm of integration is not suitable. Discrete gradient is introduced in experiment.

$$g(f^n) = \mathrm{sgn}(f^n - f^{n-1}) - \mathrm{sgn}(f^{n+1} - f^n) \tag{13}$$

Where $\forall n = 2, \cdots, N-1$. At the start point $g(f^1)$ and end point $g(f^N)$, there is

$$g(f^1) = -\mathrm{sgn}(f^2 - f^1),\ g(f^N) = \mathrm{sgn}(f^N - f^{N-1}) \tag{14}$$

Thus, the total variation algorithm can be simulated in the OpenCV platform.

3. Total Variation Algorithm Achieved in Image Denoising

3.1. Algorithm Simulation and Parameter Selection

All figures should be numbered with Arabic numerals (1,2,...n). All photographs, schemas, graphs and diagrams are to be referred to as figures. Line drawings should be good quality scans or true electronic output. Low-quality scans are not acceptable. Figures must be embedded into the text and not supplied separately. Lettering and symbols should be clearly defined either in the caption or in a legend provided as part of the figure. Figures should be placed at the top or bottom of a page wherever possible, as close as possible to the first reference to them in the paper.

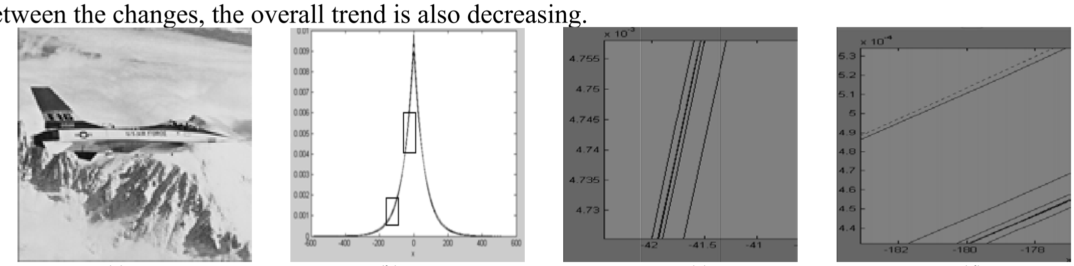
Firstly, a clear image which is 256 by 256 and 8-bit grayscale is chosen for the experiments. Then, using fspecial function in Matlab to create a Gaussian filter which is used to corrupted image with Gaussian noise of zero mean, variance of 0.05 and point spread function of 13×13, so that we can get a blurred and noisy input image. The mean and variance of Gaussian noise also can be changed in order to get better contrast.

Parameter α plays a balancing role between $J_1(f)$ and $J_2(f)$, it also can control the finest scale which is needed to maintain. Appropriate parameter α leads to obtain the most appropriate recovery image, and minimizes the error between recovery image and real images. For the sake of acquiring the appropriate recovery image, PSF, GGM [8] and gradient function are combined with to choose the regulation parameter α . The image in Fig. 2(a) is the clear image named airplane. Table 1 shows the shape parameters and scale parameters which are computed from the clear image with different variances. The Generalized Gaussian distribution curve and its Parts of the Enlargement is shown on the Fig. 2(b), (c) and (d).

Table 1. The comparison of some parameters of the generalized Gaussian distribution

Variances	**0**	**1**	**3**	**5**	**7**	**9**	**11**
Shape parameters	1.95	1.95	1.95	1.95	1.95	1.95	1.95
Scale parameters	9.4987	9.4834	9.1596	9.1596	9.1034	9.0915	9.0942

After analyzing and comparison we can see that despite the image with different variance, their shape parameters are still same, which means the images have same attenuation of the generalized Gaussian function. Although the scale parameter values in several adjacent parameters increase or decrease between the changes, the overall trend is also decreasing.



(a) (b) (c) (d)

Fig. 2. The Generalized Gaussian distribution curve and its part enlargements about the clear image: (a) clear test image-airplane. (b) Generalized Gaussian distribution curve. (c) Parts of the Enlargement about Generalized Gaussian distribution curve. (d) Parts of the Enlargement about Generalized Gaussian distribution curve

According to the literatures, the parameter α values between 0 and 1 in general. We select 10 regularization parameters which have 0.1 intervals to process the image, and then compute their gradients. The results are shown on Fig. 3. In Fig. 3 we can see that from $\alpha = 0.5$, the gradient of image tend to be stable. Combining the General Gaussian and visual experience, we have chosen regulation parameter $\alpha = 0.5$.

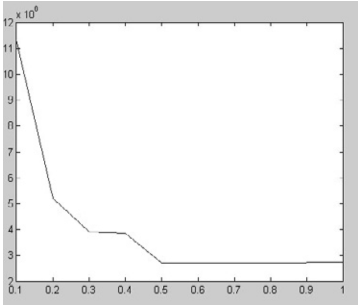
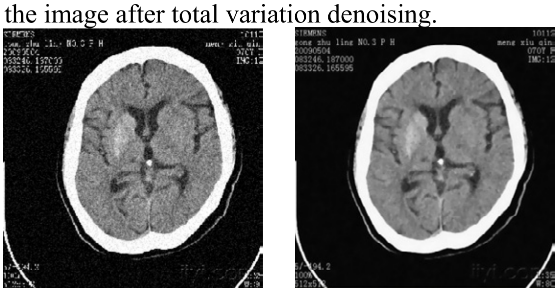


Figure.3. The gradients of image with different regulation parameters

3.2. Analysis and Comparison of the Algorithm Used Medical Image

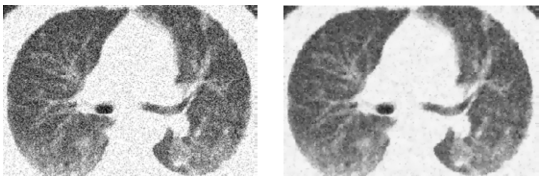
Through the above theoretical analysis, the algorithm is simulated on OpenCV. Experiments use total variation algorithm to remove the noise from BMP format medical image. The result is shown in Fig. 4 and Fig.5.

Taking the figure for example, having a structural information comparison between original image and denoised image, this is shown in Fig. 6. Matlab take the 100th arrange data in images as comparative data, and draw the comparative curves. In the Fig. 6, "…"represents clear image, "-.-"represents image with noise, and "-+-" represents the image after total variation denoising.



(a) (b)

Figure 4. The comparative images: (a) the original image with noise. (b) the image after total variation denoising.



(a) (b)

Figure 5. The comparative images: (a) the original image with noise. (b) the image after total variation denoising.

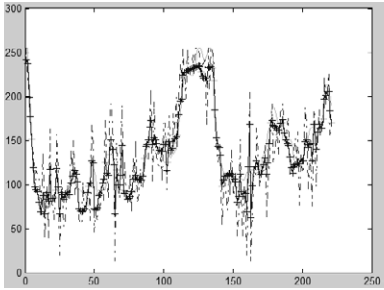


Figure 6. the structure information comparison between clear image, noise image and restoration image

The result shows that the treatment of total variation algorithm, the noise is effectively restrained, and have trend to approach the clear image at transition region.

Ultimately, with the visual and structure information in Fig. 6, we can clearly found that after treatment of total variation algorithm image's effect of image denoising is obvious. Compared with the original noisy image, denoised image produced little fuzzy, better preservation of edge detail, and edge feature has also been enhanced.

The experimental comparison between traditional denoising methods (e.g. mean filtering, median filtering and SNN) and total variation algorithm shows in Fig. 7. From the images we can see obviously that total variation denoising algorithm is superior to the traditional denoising methods.

Matlab take the 70th arrange data in images as comparative data, which is shown in Fig.8, in which the smooth curve is the data of clear image, "--." represents the image after total variation denoising.

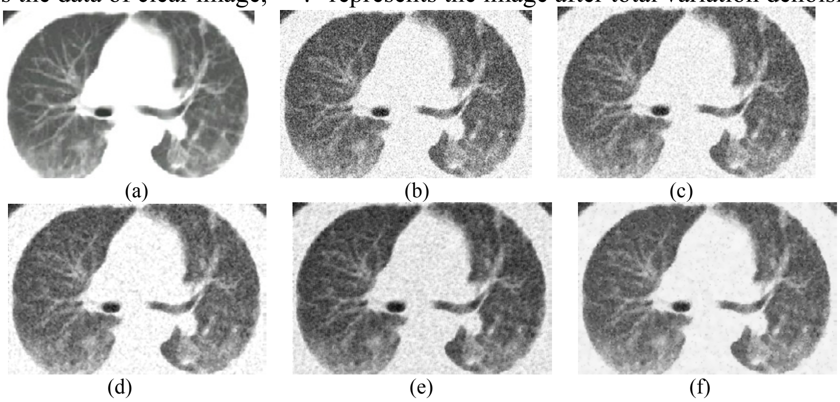


Figure 7. The comparison between clear image and images after denoising processing. (a) Original image with no noise. (b) Original image with noise.(c) Iimage after mean filtering. (d) Image after median filtering. (e) image after SNN. (f) image after total variation algorithm

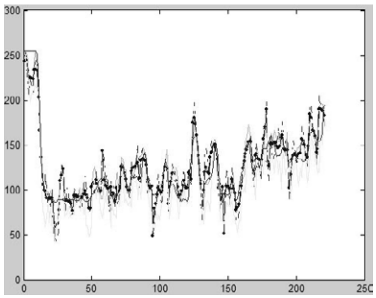


Figure 8. Denoising image structure comparison of the methods

From Fig.8, the total variation denoising curve is closer to the original image than other curves. In the peak or the edge of the image, the traditional methods in different places have appeared different from the original image changes, which mean the edge information at these places, have been changed or lost. However, the image data curve after the total variation algorithm denoising is closer to the original image data curve. Judging from this, image denoising of the total variation algorithm can preserve the edge information effectively.

Briefly, denoising image of total variation technique has been inhibited the image noise and image's edge and detail has preserved intact.

4. Conclusion

This paper introduced the total variation algorithm into medical image denoising, proposed the denoising algorithms based on partial differential of total variation. And paper used fix-point method to simplify the solution of partial differential equations, also combined general Gaussian function and Tenegrad gradient to choose the regulation parameter, and then simulated the algorithm on the OpenCV platform, in order to achieve the best image restoration. Comparing with the traditional methods of image denoising, total variation algorithm remove image noise and blur, and while preserving edge information. Theoretical analysis and experimental results show the total variation algorithm based on the partial differential equations is an effective method of image denoising, which not only can acquire clear medical image, but also can completely maintain the edge information.

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