1 Angluin-Style learning of NFA

An example of a residual. $\Sigma = \{0, 1\}$, $L = 0^*10^*$. Then $L' = 10^*$ is a residual language of L, because $L' = 0^{-1}L$.

$$L' \text{ residual of } L \iff \exists u \in \Sigma^* : \forall w \in L' : uw \in L$$

The set of all residuales of a language L is $Res(L)$.

A residual FSA (RFSA) is a NFA $\mathcal{R} = (Q, Q_0, F, \delta)$ such that

$$\forall q \in Q : L_q \in Res\left(L\left(\mathcal{R}\right)\right)$$

What does this mean? Evidently, $L_{Q_0} = L(\mathcal{R})$. Then, each state must accept some residual of this language. For example, if $\alpha^*\beta^*\gamma^= L$, the following could be languages of other states:

$$\begin{aligned}
&\beta\gamma^* \\
&\beta^*\gamma \\
&\{\beta\beta\beta\gamma, \beta\gamma\} \\
&\{\epsilon\} \\
&\left\{\gamma, \gamma\gamma\gamma, \ldots, \gamma^{2k+1}\right\}
\end{aligned}$$

In a sense, a RFSA is well-structured in the sense that the language of each state "completes" some portion of the language of the automaton.

A residual L' of L is prime if it is not the union of other residual languages.

The canonical RFSA of a language L is $\mathcal{R}(L) = (Q, Q_0, 1F, \delta)$ with $Q = Primes(L)$,

$$\begin{aligned}
Q_0 &= \{L' \in Q : L' \subseteq L\} \\
F &= \{L' \in Q : \epsilon \in L'\} \\
\delta(L_1, a) &= \left\{L_2 \in Q \mid L_2 \subseteq a^{-1}L_1\right\}
\end{aligned}$$

Unpacking: Given a symbol a, the automaton transitions to any prime residual language that completes the word a into a word in the language.