This writing is entirely based on the work of Fischer et al (2021).

Let $\mathbf{x} = (x_1, \ldots, x_n)$ be some real-valued data sampled from some typical distribution with parameter θ_0. We say $\mathbf{x}$ follows a parametric epidemic changepoint model if there is some function

$$\theta(t) = \begin{cases} \theta_1 & s_1 < t \leq e_1 \\ \vdots & \\ \theta_K & s_K < t \leq e_K \\ \theta_0 & c.c. \end{cases}$$

s.t. x_t has PDF $f(x_t, \theta(t))$. (Informally, there are K anomalous segments during which the data deviates from the typical distribution).

0.1 Inferring K and the segments

Let $\mathcal{C}(x, \theta)$ be some appropriate cost function, β a penalty. Typically, we let $\beta := C \log n$ with C some appropriate constant. $\mathcal{C}$ might be the negative log-likelihood of x under the parametric model using θ.

Our goal then is to choose $\widetilde{K}, (\widetilde{s_i}, \widetilde{e_i})$ (with $1 \leq i \leq \widetilde{K}$), and $\widetilde{\theta_0}$ so as to minimize:

$$\sum_{t \notin \mathcal{A}} \mathcal{C}(x_t, \widetilde{\theta_0}) + \sum_{j=1}^{\widetilde{K}} \left[\min_{\widetilde{\theta_j}} \left(\sum_{t \in \mathcal{A}_j} \mathcal{C}(x_t, \widetilde{\theta_j}) \right) + \beta \right] \tag{1}$$

with

$$\mathcal{A}_j = [\widetilde{s_j} + 1, \widetilde{e_j}], \qquad \mathcal{A} := \bigcup_{j=1}^{\widetilde{K}} \mathcal{A}_j$$

For instance, using negative log-likelihood as cost function and the normal distribution as the typical distribution, the expression which is to be minimized is:

$$\sum_{t \notin \mathcal{A}} \left[\log(\sigma_0^2) + \left(\frac{x_t - \mu_0}{\sigma_0} \right)^2 \right]$$
$$+ \sum_{j=1}^{\tilde{K}} \left[(\tilde{e}_j - \tilde{s}_j) \left(\log \left(\frac{\sum_{t=\tilde{s}_j+1}^{\tilde{e}_j} (x_t - \bar{x}_{(\tilde{s}_j+1):\tilde{e}_j})^2}{(\tilde{e}_j - \tilde{s}_j)} \right) + 1 \right) + \beta \right],$$