Some observations:

- K_n can span every $G \in \mathcal{G}_n$.
- For any $G \in \mathcal{G}_{n,m}$, there are $M = \binom{n}{2} - m$ edges that must be removed to span it from a K_n.
- The order in which the edges are removed does not matter.
- The space of prunable edges $\mathcal{E}$ is not constant, since an edge may become a bridge and disappear from $\mathcal{E}$.
- Following the previous statement: $\mathcal{E}$ initializes as $\Lambda(n)$ but loses an element per generated bridge.

We know $|\Lambda(n)| = \binom{n}{2}$. There are

$$\binom{\frac{n(n-1)}{2}}{\frac{n(n-1)}{2} - m} = \binom{\frac{n(n-1)}{2}}{m} =: C_{n,m}$$

graphs in $\mathcal{G}_{n,m}$, but some of them are disconnected. The question is: how many of them are disconnected.

Let $\mathcal{A}$ be the class of all graphs. We wish to produce a generating function for $\mathcal{A}$; this is, a series s.t. its kth coefficient is the number of graphs with n vertices, m edges. We know this quantity due to the derivation above, and all that is left is to expand it into a series for each n, m.

The mixed exponential generating function for $\mathcal{A}$ is then

$$\begin{aligned} A(x) &= \sum_{n=0}^{\infty} \left(\sum_{m=0}^{\infty} \binom{\frac{n(n-1)}{2}}{m} y^m \right) \frac{x^n}{n!} \\ &= \sum_{n=0}^{\infty} (1+y)^{\frac{n(n-1)}{2}} \frac{x^n}{n!} \\ &= 1 + \sum_{n=1}^{\infty} (1+y)^{n(n-1)/2} \frac{x^n}{n!} \end{aligned}$$

Now, every graph in $\mathcal{A}$ is a set of connected graphs. In other words, if we define C the class of connected graphs, the relationship between these two clases is the set-of relation. This means

$$A(x) = \exp C(x)$$

But we know $A(x)$, so we can find $C(x)$ by taking $\ln A(x)$:

$$C(x) = \ln \left[1 + \sum_{n=1}^{\infty} (1+y)^{n(n-1)/2} \frac{x^n}{n!} \right]$$

Here, we recall that

$$\log(1+u) = \sum_{k=1}^{\infty} (-1)^{k+1} \frac{u^k}{k}$$

which entails

$$C(x) = \sum_{k=1}^{\infty} \frac{(-1)^{k+1}}{k} \left[\sum_{n=1}^{\infty} (1+y)^{n(n-1)/2} \frac{x^n}{n!} \right]^k$$

Thus, $C(x)$ produces an enumeration of all connected graphs of n vertices, and we can arrive at the expression for all connected graphs of N vertices and M edges:

$$N! y^M x^N \sum_{k=1}^{N} \frac{(-1)^{k+1}}{k} \left[\sum_{n=1}^{N} (1+y)^{n(n-1)/2} \frac{x^n}{n!} \right]^k$$

For example, for $M = N - 1$ across $N = 2, 3, \ldots$, this effectively produces the sequence

$$1, 1, 3, 16, 125, 1296, \ldots$$

which matches the number of trees indicated by the Prufer sequence.