Sea B instancia de 3-sat. Construyamos el siguiente 3-hiper grafo. Let $i \in \{1, \ldots, n\}$, $j \in \{1, \ldots, m\}$, $r \in \{1, 2, 3\}$, $k \in \{1 \ldots m(n-1)\}$.

Vertices. Define

$$
\begin{aligned}
X &= \{a_{ij}\} \cup \{s_j\} \cup \{h_k\} \\
Y &= \{b_{ij}\} \cup \{t_j\} \cup \{g_k\} \\
Z &= \{u_{ij}\} \cup \{w_{ij}\}
\end{aligned}
$$

Define

$$
v_{jr} = \begin{cases} u_{ij} & \exists i : l_{ir} = x_k \\ w_{ij} & \exists i : l_{ir} = \overline{x_k} \end{cases}
$$

Let E be the union of

$$
\begin{aligned}
E_0 &= \{a_{ij}, b_{ij}, u_{ij}\} \\
E_1 &= \{a_{(i+1)j}, b_{ij}, w_{ij}\} \\
E_2 &= \{h_k, g_k, u_{ij}\} \cup \{h_k, g_k w_{ij}\} \\
E_3 &= \{s_j, t_j, v_{jr}\}
\end{aligned}
$$

Assume there is a perfect matching. Take an arbitrary i. If there is some j s.t. a_{ij} belongs to the matching, then $a_{ij} \in E_0$ or $a_{ij} \in E_1$. If $a_{ij} \in E_0$, this is if $\{a_{ij}, b_{ij}, u_{ij}\}$ is in the matching, it cannot be the case that $a_{(i+1)j, b_{ij}, w_{ij}}$ is in the matching. This inductively implies that now member of E_1 is in the matching and all members of E_0 are in the matching. Call this CASE 0.

A similar reasoning reveals that if any side of E_1 belongs to the matching, all sides of E_1 belong to the matching and no side of E_0 belong to the matching. Call this CASE 1.

Define

$$
\overrightarrow{b_i} = \begin{cases} 1 & \textbf{CASE 1 for } i \\ 0 & \textbf{CASE 0 for } i \end{cases}
$$

We must only prove there is some l_{ir} s.t. $l_{ir}(\overrightarrow{b}) = 1$ for any i. There are two cases.

If $l_{ir} = x_k$, then $v_{jr} = u_{ij}$. Then $\left\{s_j, t_j, u_{ij}\right\}$ belongs to the matching. But then $\left\{a_{ij}, b_{ij}, u_{ij}\right\}$ cannot belong to the matching. Then Case 1 holds and $l_{ir}(\overrightarrow{b}) = 1$.

If $l_{ir} = \overline{x_k}$, $v_{jr} = w_{ij}$. Then $\left\{s_j, t_j, w_{ij}\right\}$ belongs to the matching. But then $\left\{a_{(i+1)j}, b_{ij}, w_{ij}\right\}$ cannot belong to the matching and we are in case 0. Then $l_{ir}(\overrightarrow{b}) = 1$.

In both cases, $l_{ir}(\overrightarrow{b}) = 1$.

($\Leftarrow$) Assume there is some $\overrightarrow{b}$ s.t. $B(\overrightarrow{b}) = 1$. We will build a matching with edges $F_0 \cup \ldots \cup F_3$ such that $F_i \subseteq E_i$.

In particular, we let

$$F_0 = \left\{\left\{a_{ij}, b_{ij}, u_{ij}\right\} : b_i = 0\right\}$$
$$F_1 = \left\{a_{(i+1)j, b_{ij}, w_{ij}} : b_i = 1\right\}$$

Now, for any i there is some l_{ir} s.t. $l_{ir}(\overrightarrow{b}) = 1$. I make

$$F_3 = \left\{s_j, t_j, v_{jr}\right\}$$

If an edge belongs to F_0 and F_3, we must have $v_{jr} = u_{ij}$. But this implies both $l_{jr} = x_i$ and $b_i = 0$, which is absurd.

If an edge belongs to F_1 and F_3 then we must have $v_{jr} = w_{ij}$.But this implies $l_{jr} = \overline{x_k}$ and $b_i = 1$, which is absurd.

So, we have a matching. But we must see that it is perfect. For this purpose, we define:

$$N = \{z \in Z : z \text{ not covered by } F_0, F_1, F_2\}$$

Clearly, $|F_3| = m$. Let p be the amount of is s.t. $b_i = 0$ and q the amount of is s.t. $b_i = 1$. Clearly, $n = p + q$, $|F_0| = mp$, $|F_1| = mq$. Then

$$|Z - N| = |F_0| + |F_1| + |F_3| = mp + mq + m = m(n + 1)$$

Then $|N| = |Z| - |Z - N| = 2mn - m(n+1) = m(n-1)$

Then there is a bijection $f : \{1, \dots, m(n-1)\} : N$. We define

$$\{g_k, h_k, f(k)\}$$

which has disjoint edges because it is injective and covers every uncovered z because it is surjective.

All sides of Z are now covered and since $|X| = |Y| = |Z|$ the matching is perfect.