1 Finance with certainty

A financial operation may be understood as a loan. A lender gives a borrower a certain amount of money, under the condition that the borrower shall return it with interest. We assume the interest is previously agreed upon, which means we are dealing with financial *certainty*.

The initial capital es the amount of money being borrowed. The final capital is the amount that is returned. The *plazo* (term) is the time the operation lasts. The interest is the difference between the initial and the final capital. In general, the interest is proportional to the initial capital.

Since the interest depends on the initial capital, generally what is done is setting an *interest rate*. The interest rate is the interest corresponding to each unit of capital per each unit of time.

Example. For example, 5% to 30 days, or 10% monthly, or daily 0.001 (0.1%), or annual 25%.

Example. An initial capital of 10.000 with an interest rate of 5% every 30 days, we have an interest in 30 days of $10.000 \times 0.05 = 500$.

Example. If an interest of 4000 is paid for a loan of 200.000 for 45 days, the interest rate is to 45 days is 2%:

$$4000/200.000 = 0.02$$

If the interest rate is set to an amount of time t less than the term of the loan, we have different systems.

1. *Simple capitalization system*: In the simple capitalization system, the interest is directly proportional to the term:

   $$F = I(1 + it)$$

   where F, I are the final and initial capitals, i is the interest rate, and t is the time expressed in the time unit of the interest.

2. *Composite capitalization system*: Once we reach time t, we operate as if we had removed the money from the loan and re-inserted the amount obtained.

   $$F = C(1 + i)^t, \qquad i = C\left((1 + i)^t - 1\right)$$

   where C is the capital we have at the moment (with accumulated interests) and t is the number of terms that passed.

Example of simple capitalization. An interest rate of 5% to 30 days and an initial capital of 10.000 gives an interest of: 500 in 30 days, 1000 in 60 days, 1500 in 90 days, etc. Notice that each time 30 days pass, we increment our money by $500 = 10.000 \times 0.05$.

Notice that after 30 days we would have 10.500, and after 60 we would have 11.000. But there's a problem: 11.000 is less than what would be obtained with an interest rate of 5% over the 10.500 we had after the first 30 days.

In other words, if the 5% of 10.000 is 500, the 5% of 10.500 is more than 500. So we see that as time goes by, with the simple capitalization system, the growth becomes less and less relative to the capital at hand.

Example of composite capitalization. An interest rate of 5% every 30 days with initial capital of 10.000. Then the cumulative interest will be:

1. 500 in 30 days.
2. 1025 in 60 days $(500 + 0.05 \times 10.500)$
3. 1576 in 90 days $(1025 + 0.05 \times 11.025)$

Problem. Fixed term to 35 days with $I = 100.000$. An automatic renovation is done in the following two vencimientos. The interest on each term are 2.4%, 2.4%, 2.8%. Compute the final capital after 105 days if: (a) total renovation is done, (b) partial renovation is done.

Solution. The first term goes from day 0 to 35, the second from day 35 to 70, and the third from day 70 to 105. The interest rate of each of these terms was specified above.

With total renovation, this would work as follows. On day 35, after the first term, we would have 100.000×1.025. On day 70 we would apply the interest of the term to the total capital so far accrued, so we would have $100.000(1.025)(1.024)$. Lastly, on day 105, we would apply the interest again to the total capital accrued, resulting in $100.000(1.025)(1.024)(1.028) = 107898.88 = F$. with an interest of $F - I = 7898.88$.

With partial renovation, on day 35 we would again have 100.000×1.025. On day 70, we would add to our accrued capital the interest of the term applied to the initial capital: $100.000(1.025 + 0.024)$. Then, on day 105, we would have $100.000(1.025 + 0.024 + 0.028)$. (If this is unclear, apply the distributive law and it will be.)

1.1 Nominal and effective rates

The TNA (Tasa Nominal Anual) is an annual rate that serves as reference to compute rates in different terms through direct proportionality. For instance, if to 30 days we have a TNA of

25%, this means the that the rate to 30 days is

$$I_{30\text{ days}} = TNA \times \frac{\text{rate in days}}{365} = 0.25 \times \frac{30}{365} = 0.0205 = 2.05\%$$

Notice that $30/365$ is the "how much are 30 days to a year".

The TEA (Tasa Efectiva Anual) is a rate equivalent to the effective rate being applied, but corresponding to an annual term. For instance, if for a fixed term to 30 days we have a rate of 2.05%, then the composite capitalization is

$$1 \times (1.0205)^{\frac{365}{30}} = 1.2800$$

which means the TEA is 28%.

1.2 Continuous capitalization

If the TNA is r, the final capital in a year over an initial capital C, with continuous capitalization, is

$$C \times \lim_{m\to\infty} \left(1 + \frac{r}{m}\right)^m = C \times e^r$$

Then

$$TEA = e^t - 1$$

meaning that the final capital es

$$C(t) = C \times e^{rt}$$

When modeling, we need to take r as $r(t)$, i.e. as a rate that may continuously vary in time, which adds extra complexity.

1.3 Discounted value

When comparing two quantities in different time instances, we determine the *discounted value* or *actual value* of a capital. Let C_1 the capital at time t_1, C_2 the capital at time t_2, with $t_1 < t_2$. Then via composite capitalization with rate i

$$C_1 = \frac{C_2}{(1+i)^{t_2-t_1}}$$

is the discounted value from C_2 at time t_1.

1.4 Excercises

Ejercicio 2. An individual borrows 50.000 with a term of 30 days. Simple capitalization was agreed upon with an annual nominal interest rate (TNA) of 33%.

(a) Determine the amount the individual will have to pay in interest.

(b) Determine the interest rate of the operation with a term of 30 days.

(c) Compute the interest rate of the operation if instead of 30 days the term was 180, 90, or 60.

Recordemos que en el sistema de capitalización simple, el interés es directamente proporcional al plazo y no varía con el capital acumulado, sino que depende sólo del capital inicial. En general,

$$F = I\,(1 + it) \tag{1}$$

Como la tasa de interés nominal anual es 33%, la tasa cada 30 días es

$$\frac{30}{365} \times 0.33 \approx 0.0271 = 2.71\%$$

Por lo tanto, al cabo de los 30 días, el interés será $0.0271 \times 50.000 = 1356.1648$. La tasa de interés en 180, 90 y 60 días es:

$$\frac{60}{365} \times 0.33 = 0.0542, \qquad \frac{90}{365} \times 0.33 = 0.0813, \qquad \frac{180}{365} \times 0.33 = 0.1627$$

or equivalently 5.42%, 8.13% and 16.27%.

Ejercicio 2. Una persona depositó el 1 de marzo de 2025 300.000 en una caja de ahorro en la que se aplica una tasa de interés constante con capitalización simple. Se sabe que el 1 de agosto de 2025 el capital disponible es de 314.229.

(a) ¿Cuál es la tasa de interés diaria y cuál es la tasa anual (365 días) que se aplica en esta cuenta? (b) ¿Cuánto podría retirarse si el capital se deja depositado hasta el 1 de noviembre de 2025 y se mantiene la misma tasa? (c) ¿En qué fecha el monto ascenderá a 317.949?

Del 1 de marzo al 1 de agosto hay 153 días. Notemos que

$$300.000 \times (1 + I_{153}) = 314.299 \Rightarrow I_{153} = 0.0476$$

Es decir que el interés fue del 4.76%. Entonces,

$$I_{153} = TNA \times \frac{153}{365} \Rightarrow TNA = \frac{365}{153} \times 0.0476 = 0.1135$$

Es decir que la tasa nominal anual es del 11.35%. La tasa de interés diaria es

$$0.1135 \times \frac{1}{365} = 0.00031 = 0.0031\%$$

Si el capital se deja hasta el 1 de noviembre, el tiempo total será 245 días, resultando en un interés de

$$0.1135 \times \frac{245}{365} = 0.0761$$

y por lo tanto un capital acumulado de 322.830.

El monto ascenderá a 317.949 si el interés acumulado es 17.949, es decir si aumentamos nuestro capital un 5.983%. Esto sucederá después de k días, donde

$$0.1135 \times \frac{k}{365} = 0.05983 \iff k = 0.05983 \times \frac{365}{0.1135} \iff k = 192.40$$

Es decir, sucederá después de 193 días.

2 Finance

A financial market is a market where financial instruments are exchanged, such as basic and derivative assets. Most financial markets are regulated so that no fraud occurs. However, there are off-the-counter markets (mercado extrabursátil) where instruments are negotiated directly between two parts and are regulated differently than standard financial markets.

In financial markets, one exchanges financial instruments such as:

- Actions (shares) y CEDEAR.
- Divisas
- Commodities (Bienes de consumo, e.g. trigo, petróleo, etc)
- Bonos, obligaciones negociables
- ETF (Exchange Traded Funds).

One might also buy *productos derivados*:

- Forwards
- Futuros
- Opciones
- Derivados sobre tasas

A share (acción) is a financial instrument that represents ownership over some of the equal fractions in which the social capital of a corporation is divided. In other words, it represents ownership over a part of a company. Some shares are quoted (cotizadas) publicly in the stock exchange (la bolsa). This means t hey can be negotiated in the financial market. A share can provide income by periodically giving the shareholders part of the company's money. The valuo of a share varies according to the law of demand and supply.

With regards to currencies, what is quoted is the *tipo de cambio* (exchange rate), i.e. the cost of a currency with regards to another (generally the local one).

Commodities comprise consumer goods, raw materials, and services which are quoted (cotizados) in the market. In general, whatever depends on a future and is not a value is understood to be a commodity. Grain, cotton, energy materials such as gas and oil, metals, and meat, are some examples of commodities.

A bond (bono) is a debt issued by private entities or the state. By debt we mean that it is a promise to pay a certain amount in the future. There is a certain terminology here:

- Amortizar: To return the capital borrowed, in part or in total.

- Bonos con cupón: Periodic coupons are paid according to some coupon rate.
- Bonos cupón cero o bonos a descuento: No coupon is paid: the capital is amortizado upon expiration to a nominal price. The price of emission is inferior to said nominal price.

Bonds have associated some *tasa de retorno*.

2.1 Financial derivative

A financial contract is a financial derivative if its value depends on another instrument S, called the *underlying*. The payoff of a derivative at the moment of its expieration is a function of the value of its underlying,

$$\text{Payoff} = f\left(S(t_{\text{end}})\right) \tag{1}$$

where $S(t)$ is the price of the underlying at the expiration time t_{end}.

A *forward contract* is an agreement to sell or buy an underlying in some future time T to some price K. The contract specifies what the underlying is, the time T and the price K. No matter the market price of the underlying at time T, it will be sold/bought at the agreed-upon-price K. They are negotiated off-the-counter.

In a *forward*, depending on whether the investor is buying or selling the underlying, we say he's *long* or *short*. Long is the one buying the underlying, short is the one who sells it.

The payoff of a forward is the value of the contract at the time of its expiration, and there's an opposition between the long and short positions.

$$\text{Payoff} = \begin{cases} S(t_{\text{end}}) - K & \text{long position} \\ K - S(t_{\text{end}}) & \text{short position} \end{cases} \tag{2}$$

Ejercicio 2: Un inversor entra short en un contrato forward por 100000 libras esterlinas (GBP) por dólares estadounidenses a un tipo de cambio de 1,3000 dólares por libra. ¿ Cuánto gana o pierde el inversor si la tasa de cambio al finalizar el contrato es: a) 1 GBP = 1,2900 UD? b) 1 GBP = 1,3200 UD?

The investor commits himself to *sell* 100.000 GBP with an underlying value of 1,3 dolars per pound upon contract maturity. (a) If, when the contract matures, the price $S(t_{\text{end}})$ is 1,29, then the investor will win since he will sell to a price higher than the market price. In particular, he will win $0.01 \times 100000 = 1000$.

Ejercicio 3. Un inversor entra en una posición long en un contrato futuro sobre algodón cuando el precio de futuros es de 3,50 dólares por kilogramo. El contrato es para la entrega de 50000 kilogramos. ¿Cuánto gana o pierde el comerciante si el precio del algodón al final del contrato es a) 4,82 dólares por kilogramo y b) 3,15 dólares por kilogramo? Interpretar el resultado en el diagrama de payoff correspondiente.

(a) En la posición long, el inversor se compromete a comprar 50.000kg de algodón a 3,50 la unidad. Claramente, si en la maduración del contrato el precio es 4.82, el inversor estará comprando algodón a un precio menor al valor del mercado, lo cual significa que está ahorrando dinero. En particular, estará ahorrando $4,82 - 3,50 = 1,32$ dólares por unidad, es decir un total de $1,32 \times 50.000 = 66.000$ dólares.

(b) Mismo razonamiento, pero en este caso pierde $(3,50 - 3,15)50.000 = 17.500$ dólares.

Ejercicio 4: El 1 de julio de 2024 una compañía entró en un contrato forward para comprar 10 millones de yenes japoneses el 1 de enero de 2025. El 1 de setiembre de 2024 entró en un contrato forward para vender 10 millones de yenes japoneses el 1 de enero de 2025. Describir cuál es el payoff de esta estrategia.

El primero de enero de 2025, la compañía deberá comprar 10 millones de yenes al valor del 1 de Julio de 2024, que denotamos como p_1, y vender 10 millones de yenes al valor del 1 de septiembre de 2024, que denotamos como p_2. Sea p_3 el valor del yen al 1 de enero de 2025.

El payoff será claramente lo ganado/perdido en una operación más lo ganado/perdido en la otra, es decir la suma de los payoffs. Teniendo en cuenta que, por separado,

$$\mathcal{P}_{\text{long}} = (p_3 - p_1)k, \qquad \mathcal{P}_{\text{short}} = (p_2 - p_3)k$$

son los payoffs de las dos operaciones,

$$\mathcal{P} = \mathcal{P}_{\text{long}} + \mathcal{P}_{\text{short}} \tag{3}$$

es el payoff final. La ecuación (3) se simplifica a

$$\mathcal{P} = k\big((p_3 - p_1) + (p_2 - p_3)\big) = k(p_2 - p_1) \tag{4}$$

Es decir que el payoff es directamente proporcional a la diferencia del valor del yen en los dos momentos en que se firmaron los contratos, el 1 de julio y el 1 de septiembre. Lo interesante es que el payoff se vuelve independiente del valor del yen al momento del vencimiento del contrato.

Ejercicio 5. La empresa argentina ExportCo está actualmente exportando mercadería a EE.UU. y sabe que dentro de 8 meses recibirá un pago de 2 millones de dólares. ¿Cómo puede dicha empresa realizar una estrategia de cobertura ante posibles fluctuaciones en el tipo de cambio con contratos forward?

Puede entrar *short* para vender 2 millones de dólares al precio de la fecha para dentro de 8 meses. De ese modo, incluso si el precio del dólar baja en los 8 meses que pasan, seguirá vendiéndolos al precio de ahora.

Ejercicio 6. Un comerciante ingresa en posición short en contratos futuros a julio sobre concentrado de jugo de naranja congelado. Cada contrato es para la entrega de 15.000 libras del producto subyacente. El precio de futuros actual es de 160 centavos por libra, el margen inicial es de 6000 por contrato y el margen de mantenimiento es de 4500 por contrato. ¿Qué cambio de precio daría lugar a una margin call? ¿En qué circunstancias se podrían retirar 2000 de la cuenta?

(a) Sea m_0 la cantidad inicial de dinero en la cuenta marginal *en dólares*; i.e. $m_0 = 6000$. Sean $m_1, m_2, \ldots$ los subsiguientes valores en la cuenta marginal, ajustados de acuerdo con la lógica de este tipo de contratos. Sabemos que

$$m_{k+1} = m_k + (p_k - p_{k+1})C \tag{5}$$

donde p_i es el precio del subyacente en el i-écimo momento y C la cantidad de subyacente involucrado. Notemos que los precios están en *centavos de dólar por libra*, mientras m_{k+1}, m_k están en dólares.

Dicho esto, es fácil ver que por recurrencia la ecuación (5) se traduce en:

$$m_k = m_0 + (p_0 - p_k)C \tag{6}$$

Se nos pregunta qué cambio de precio daría lugar a una margin call. Es decir, se nos pide la solución a la ecuación

$$m_k \leq 4500$$

que por la recurrencia del (6) resulta

$$m_0 + (p_0 - p_k)C \leq 4500\text{USD} \iff 600.000 + (160 - p_k)15.000 \leq 450.000$$

donde ponemos 450.000 y 600.000 para hablar de todas las unidades en centavos de dólar. (Porque los precios p_k están en centavos de dólar por libra.) Vemos que la inecuación se cumple si y solo si $p_k \geq 170$. Es decir que un aumento de 10 centavos de dólar por libra es suficiente para conducir a una margin call.

(b) Se retiran 2000USD si y solo si al tiempo de finalizar el contrato el valor final en la cuenta marginal es $m_{\text{final}} \geq m_0 + 2000 \times 100\text{cents}$. Pero $m_{\text{final}} = m_0 + (p_0 - p_{\text{final}})15.000$, con lo cual necesitamos resolver la inecuación

$$
\begin{aligned}
& m_0 + 2000 \times 100 \geq m_0 + (p_0 - p_{\text{final}})15.000 \\
\iff & \frac{200.000}{15.000} \geq p_0 - p_{\text{final}} \\
\iff & p_{\text{final}} \geq p_0 - 13.33 \\
\iff & p_{\text{final}} \geq 146.67
\end{aligned}
$$

Es decir que una ganancia de al menos 2000USD se consigue si y solo si el precio baja 13.33 centavos de dólar, o bien si al tiempo de maduración estamos vendiendo el subyacente a un valor 13.33 centavos por encima del valor en dicho tiempo.