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1 Index set theorems

A set $A \subseteq \omega$ is an index set if for all $x, y \in \omega$,

$$x \in A \wedge \varphi_x = \varphi_y \Rightarrow y \in A$$

In other words, a set is an index set if all elements in the set index the same partial computable function.

Would it not be better to set $\varphi_x \simeq \varphi_y$? Think about this.

Trivially, ω and $\emptyset$ are index sets.

Theorem 1 (Index set theorem)**.** If A is a non-trivial index set, then either $K \leq_1 A$ or $K \leq_1 \overline{A}$. Furthermore, choose e_0 s.t. $\varphi_{e_0}(y)$ is undefined for all y. If $e_0 \in \overline{A}$, then $K \leq_1 A$.

Proof. Take $x \in A$ (the case for $x \in \overline{A}$ is analogous).