Categorical Reconstruction Theory

2025-07-04

Tony Zorman

tony.zorman@tu-dresden.de

The classical story

Tannaka duality studies algebraic structures through their categories of representations.

Groups, as people, will be known by their actions

Motto: groups are everywhere.

Groups, as people, will be known by their actions

Motto: G-sets are everywhere.

Groups, as people, will be known by their actions

Motto: G-sets are everywhere.

Problem: Understanding G-sets is difficult.

Groups, as people, will be known by their actions

Motto: G-sets are everywhere.

Problem: Understanding G-sets is difficult.

One solution: Linearise!

Representations of a group

$$\mathrm{Sym}\left(\triangle\right)$$

Representations of a group

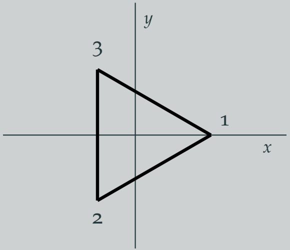
$$\mathrm{Sym}\left(\triangle \right) = S_3$$

Representations of a group

$$\mathrm{Sym}\left(\triangle\right) = S_3 \longhookrightarrow \mathrm{GL}_2(\mathbb{R})$$

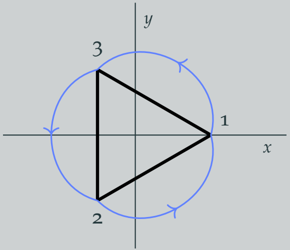
Representations of a group

$$\mathrm{Sym}\left(\triangle\right) = S_3 \hookrightarrow \mathrm{GL}_2(\mathbb{R})$$



Representations of a group

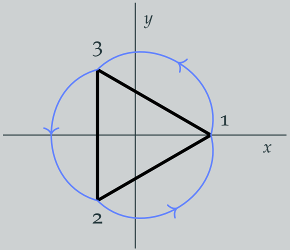
$$\mathrm{Sym}\left(\triangle\right) = S_3 \hookrightarrow \mathrm{GL}_2(\mathbb{R})$$



Representations of a group

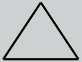
$$\mathrm{Sym}\left(\triangle\right) = S_3 \hookrightarrow \mathrm{GL}_2(\mathbb{R})$$

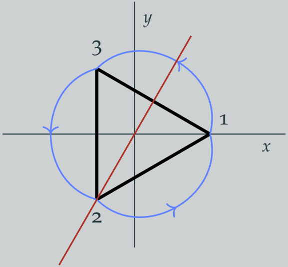
$$\boxed{(1\,2\,3) \rightsquigarrow \begin{pmatrix} \cos\frac{2\pi}{3} & -\sin\frac{2\pi}{3} \\ \sin\frac{2\pi}{3} & \cos\frac{2\pi}{3} \end{pmatrix}}$$



Representations of a group

$$\mathrm{Sym}\Big(\ \ \Big) = S_3 \hookrightarrow \mathrm{GL}_2(\mathbb{R})$$

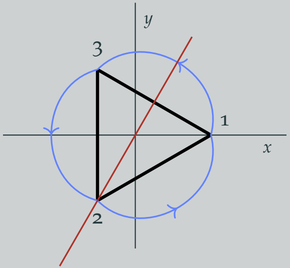




$$(1\,3) \rightsquigarrow \begin{pmatrix} \cos\frac{2\pi}{3} & \sin\frac{2\pi}{3} \\ \sin\frac{2\pi}{3} & -\cos\frac{2\pi}{3} \end{pmatrix}$$

Representations of a group

$$\mathrm{Sym}\left(\triangle \right) = S_3 \hookrightarrow \mathrm{GL}_2(\mathbb{R})$$



Representations of a group

$$\mathrm{Sym}\left(\triangle\right) = S_3 \hookrightarrow \mathrm{GL}_2(\mathbb{R})$$

A **representation** of a finite group G consists of a finite-dimensional $\Bbbk$-vector space V and a group homomorphism, called the **action**:

$$\rho \colon G \longrightarrow \mathrm{GL}(V) := \mathrm{Aut}(V).$$

Representations of a group

$$\mathrm{Sym}\left(\triangle\right) = S_3 \hookrightarrow \mathrm{GL}_2(\mathbb{R})$$

A **representation** of a finite group G consists of a finite-dimensional $\Bbbk$-vector space V and a group homomorphism, called the **action**:

$$\rho\colon G \longrightarrow \mathrm{GL}_{\dim V}(\Bbbk).$$

Representations of a group

$$\mathrm{Sym}\left(\triangle\right) = S_3 \hookrightarrow \mathrm{GL}_2(\mathbb{R})$$

A **representation** of a finite group G consists of a finite-dimensional $\Bbbk$-vector space V and a group homomorphism, called the **action**:

$$\rho \colon G \longrightarrow \mathrm{GL}(V) := \mathrm{Aut}(V).$$

A **morphism of representations** from (V, ρ) to (W, σ) is a linear map $f \colon V \longrightarrow W$ such that $f \rho_g = \sigma_g f$ for all $g \in G$.

Special building blocks

Representations can be **tensored**:

$$(V, \rho) \otimes (W, \sigma) := (V \otimes_{\Bbbk} W, \rho \otimes \sigma),$$

Special building blocks

Representations can be te $V \otimes_{\Bbbk} W \cong \Bbbk\{\, v_i \otimes w_j \mid v_i \in B_V,\, w_j \in B_W \,\}$

$$(V, \rho) \otimes (W, \sigma) := (V \otimes_{\Bbbk} W, \rho \otimes \sigma),$$

Special building blocks

Representations can be tensored:

$$(n, \rho) \otimes (m, \sigma) := (nm, \rho \otimes \sigma)$$

Special building blocks

Representations can be tensored:

$$(n, \rho) \otimes (m, \sigma) := (nm, \rho \otimes \sigma)$$

where

$$\begin{aligned} \rho \otimes \sigma \colon G &\longrightarrow \mathrm{GL}_{nm}(\Bbbk) \\ g &\longmapsto \rho_g \otimes \sigma_g := \begin{pmatrix} (\rho_g)_{1,1}\sigma_g & \cdots & (\rho_g)_{1,n}\sigma_g \\ \vdots & \ddots & \vdots \\ (\rho_g)_{n,1}\sigma_g & \cdots & (\rho_g)_{n,n}\sigma_g \end{pmatrix} \end{aligned}$$

Special building blocks

Representations can be tensored:

where

$$
\begin{aligned}
\rho \otimes \sigma \colon G &\longrightarrow \mathrm{GL}_{nm}(\Bbbk) \\
g &\longmapsto \rho_g \otimes \sigma_g := \begin{pmatrix} (\rho_g)_{1,1}\sigma_g & \cdots & (\rho_g)_{1,n}\sigma_g \\ \vdots & \ddots & \vdots \\ (\rho_g)_{n,1}\sigma_g & \cdots & (\rho_g)_{n,n}\sigma_g \end{pmatrix}
\end{aligned}
$$

$$
\begin{pmatrix} (\rho_g)_{1,1}(\sigma_g)_{1,1} & \cdots & (\rho_g)_{1,1}(\sigma_g)_{1,m} \\ \vdots & \ddots & \vdots \\ (\rho_g)_{1,1}(\sigma_g)_{m,1} & \cdots & (\rho_g)_{1,1}(\sigma_g)_{m,m} \end{pmatrix}
$$

Special building blocks

Representations can be tensored:

$$(n, \rho) \otimes (m, \sigma) := (nm, \rho \otimes \sigma)$$

where

$$\begin{aligned} \rho \otimes \sigma \colon G &\longrightarrow \mathrm{GL}_{nm}(\Bbbk) \\ g &\longmapsto \rho_g \otimes \sigma_g := \begin{pmatrix} (\rho_g)_{1,1}\sigma_g & \cdots & (\rho_g)_{1,n}\sigma_g \\ \vdots & \ddots & \vdots \\ (\rho_g)_{n,1}\sigma_g & \cdots & (\rho_g)_{n,n}\sigma_g \end{pmatrix} \end{aligned}$$

Duals of representations

If (V, ρ) is a representation, then (V^*, ρ^*) is one as well.

Duals of representations

If (V, ρ) is a representation, then (V^*, ρ^*) is one as well; with action

$$\begin{aligned} \rho^*\colon G &\longrightarrow \mathrm{GL}_n(\Bbbk) \\ g &\longmapsto (\rho_{g^{-1}})^{\mathrm{T}}. \end{aligned}$$

Putting it all together

There is a distinguished map U:

$$\begin{aligned} U\colon \mathsf{rep}(G) &\longrightarrow \mathsf{vect} \\ (V,\rho) &\longmapsto V. \end{aligned}$$

Putting it all together

There is a dist Representations of G

$$
\begin{aligned}
U\colon \mathsf{rep}(G) &\longrightarrow \mathsf{vect} \\
(V, \rho) &\longmapsto V.
\end{aligned}
$$

Putting it all together

There is a dist

Representations of G

Finite-dimensional vector spaces

$$
\begin{aligned}
U\colon \mathsf{rep}(G) &\longrightarrow \mathsf{vect}\\
(V,\rho) &\longmapsto V.
\end{aligned}
$$

Putting it all together

There is a distinguished map U:

$$\begin{aligned} U\colon \mathsf{rep}(G) &\longrightarrow \mathsf{vect} \\ (V,\rho) &\longmapsto V \\ \big((V,\rho)\longrightarrow(W,\sigma)\big) &\longmapsto (f\colon V\longrightarrow W). \end{aligned}$$

Putting it all together

There is a distinguished map U:

$$\begin{aligned} U\colon \mathsf{rep}(G) &\longrightarrow \mathsf{vect} \\ (V,\rho) &\longmapsto V \\ \big((V,\rho) \longrightarrow (W,\sigma)\big) &\longmapsto (f\colon V \longrightarrow W). \end{aligned}$$

Theorem (Tannaka reconstruction)

The monoidal automorphisms of U are, as a group, isomorphic to G.

Representations can be defined for any monoid M.

Representations can be defined for any monoid M. We can detect whether M is a group based on whether $\mathsf{rep}(M)$ has duals.

Representations can be defined for any monoid M. We can detect whether M is a group based on whether $\mathsf{rep}(M)$ has duals. This is called Tannaka duality.

Categorical generalisations

- [HZ23] with Sebastian Halbig: *Duality in Monoidal Categories*. arXiv: 2301.03545.
- [HZ24a] with Sebastian Halbig: *Diagrammatics for Comodule Monads*. In: Appl. Categ. Struct. 32 (2024).
- [HZ24b] with Sebastian Halbig: *Pivotality, twisted centres, and the anti-double of a Hopf monad*. In: Theory Appl. Categ. 41 (2024).
- [SZ24] with Mateusz Stroiński: *Reconstruction of module categories in the infinite and non-rigid settings*. arXiv: 2409.00793.
- [CSZ25] with Kevin Coulembier and Mateusz Stroiński: *Simple algebras and exact module categories*. arXiv: 2501.06629.
- [Zor25]: *Duoidal R-Matrices*. arXiv: 2503.03445.

- [HZ23] with Sebastian Halbig: *Duality in Monoidal Categories*. arXiv: 2301.03545.
- [HZ24a] with Sebastian Halbig: *Diagrammatics for Comodule Monads*. In: Appl. Categ. Struct. 32 (2024).
- [HZ24b] with Sebastian Halbig: *Pivotality, twisted centres, and the anti-double of a Hopf monad*. In: Theory Appl. Categ. 41 (2024).
- [SZ24] with Mateusz Stroiński: *Reconstruction of module categories in the infinite and non-rigid settings*. arXiv: 2409.00793.
- [CSZ25] with Kevin Coulembier and Mateusz Stroiński: *Simple algebras and exact module categories*. arXiv: 2501.06629.
- [Zor25]: *Duoidal R-Matrices*. arXiv: 2503.03445.

Categories and their ilk

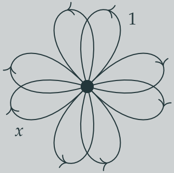
- A **category** is a collection of objects and morphisms between them.

Categories and their ilk

- A **category** is a collection of objects and morphisms between them.
  - Sets, groups, vector spaces, representations, …

Categories and their ilk

- A **category** is a collection of objects and morphisms between them.
  - Sets, groups, vector spaces, representations, . . .
  - A monoid M is a category with a single object:



Categories and their ilk

- A **category** is a collection of objects and morphisms between them.
  - Sets, groups, vector spaces, representations, …
  - A monoid M is a category with a single object.
  - Homomorphisms of categories are called **functors**.

Categories and their ilk

- A **category** is a collection of objects and morphisms between them.
  - Sets, groups, vector spaces, representations, …
  - A monoid M is a category with a single object.
  - Homomorphisms of categories are called **functors**.

- A category is **monoidal** if there exists a way to tensor two objects.

Categories and their ilk

- A **category** is a collection of objects and morphisms between them.
  - Sets, groups, vector spaces, representations, …
  - A monoid M is a category with a single object.
  - Homomorphisms of categories are called **functors**.

- A category is **monoidal** if there exists a way to tensor two objects.
  - Sets with $\times$, sets with $\sqcup$, groups with $\times$, vector spaces with $\otimes_{\Bbbk}$, …

Categories and their ilk

- A **category** is a collection of objects and morphisms between them.
  - Sets, groups, vector spaces, representations, $\ldots$
  - A monoid M is a category with a single object.
  - Homomorphisms of categories are called **functors**.
- A category is **monoidal** if there exists a way to tensor two objects.
  - Sets with $\times$, sets with $\sqcup$, groups with $\times$, vector spaces with $\otimes_{\Bbbk}$, $\ldots$
- A monoidal category is **rigid** if there exists a dual for every object.

Categories and their ilk

- A **category** is a collection of objects and morphisms between them.
  - Sets, groups, vector spaces, representations, …
  - A monoid M is a category with a single object.
  - Homomorphisms of categories are called **functors**.

- A category is **monoidal** if there exists a way to tensor two objects.
  - Sets with $\times$, sets with $\sqcup$, groups with $\times$, vector spaces with $\otimes_{\Bbbk}$, …

- A monoidal category is **rigid** if there exists a dual for every object.
  - Finite-dimensional vector spaces, representations, finite-dimensional modules over a finite-dimensional Hopf algebra, …

Can we generalise Tannaka duality to other algebraic structures and their categories of representations?

Given a (rigid monoidal) category $\mathscr{C}$, can one tell that this category is of the form $\mathsf{rep}(A)$, for some algebraic object A?

Duality theory for monads

Any monoidal category $\mathscr{C}$ admits the notion of a **monoid**.

Duality theory for monads

Any monoidal category $\mathscr{C}$ admits the notion of a **monoid**.

- $(\mathsf{Set}, \times)$: ordinary monoids.
- $(\mathsf{Group}, \times)$: abelian groups.
- $(\mathsf{Vect}, \otimes_{\Bbbk})$: $\Bbbk$-algebras.

Duality theory for monads

Any monoidal category $\mathscr{C}$ admits the notion of a **monoid**.

- $(\mathsf{Set}, \times)$: ordinary monoids.
- $(\mathsf{Vect}, \otimes_{\Bbbk})$: $\Bbbk$-algebras.
- $(\mathsf{Group}, \times)$: abelian groups.
- $(\mathsf{End}(\mathscr{C}), \circ)$: monads.

Duality theory for monads

Any monoidal category $\mathscr{C}$ admits the notion of a **monoid**.

- $(\mathsf{Set}, \times)$: ordinary monoids.
- $(\mathsf{Vect}, \otimes_{\mathbb{k}})$: $\mathbb{k}$-algebras.
- $(\mathsf{Group}, \times)$: abelian groups.
- $(\mathsf{End}(\mathscr{C}), \circ)$: monads.

Theorem (Moerdijk, McCrudden, Bruguières–Virelizier)

Given a monad T on a rigid monoidal category $\mathscr{C}$, there exists a bijection

$$\left\{ \textit{Hopf monad structures on } T \right\} \xleftrightarrow{\cong} \left\{ \begin{matrix} \textit{Rigid monoidal structure on } \mathsf{rep}(T) \\ \textit{such that } U \textit{ is strong monoidal} \end{matrix} \right\}$$

Duality theory for monads

Any monoidal category $\mathscr{C}$ admits the notion of a **monoid**.

- $(\mathsf{Set}, \times)$: ordinary monoids.
- $(\mathsf{Vect}, \otimes_{\mathbb{k}})$: $\mathbb{k}$-algebras.
- $(\mathsf{Group}, \times)$: abelian groups.
- $(\mathsf{End}(\mathscr{C}), \circ)$: monads.

Theorem (Moerdijk, McCrudden, Bruguières–Virelizier)

Given a monad T on a rigid monoidal category $\mathscr{C}$, there exists a bijection

$$\left\{ \textit{Hopf monad structures on } T \right\} \overset{\cong}{\longleftrightarrow} \left\{ \begin{array}{l} \textit{Rigid monoidal structure on } \mathsf{rep}(T) \\ \textit{such that } U \textit{ is strong monoidal} \end{array} \right\}$$

$U : \mathsf{rep}(T) \longrightarrow \mathscr{C}$

Representation categories

Monoidal categories $\mathscr{C}$ are also monoids.

Representation categories

Monoidal categories $\mathscr{C}$ are also monoids—a **$\mathscr{C}$-representation** consists of a category $\mathscr{M}$ and a compatible **action**

$$\rho\colon \mathscr{C} \longrightarrow \mathsf{End}(\mathscr{M}).$$

Representation categories

Monoidal categories $\mathscr{C}$ are also monoids—a **$\mathscr{C}$-representation** consists of a category $\mathscr{M}$ and a compatible **action**

$$\rho\colon \mathscr{C} \longrightarrow \mathsf{End}(\mathscr{M}).$$

We, for example, require the existence of coherent isomorphisms

$$\alpha_{x,y,m}\colon \rho_{x\otimes y}(m) \xrightarrow{\sim} \rho_x\big(\rho_y(m)\big), \qquad \text{for all } x, y \in \mathscr{C} \text{ and } m \in \mathscr{M}.$$

Representation categories

Monoidal categories $\mathscr{C}$ are also monoids—a **$\mathscr{C}$-representation** consists of a category $\mathscr{M}$ and a compatible **action**

$$\rho\colon \mathscr{C} \longrightarrow \mathsf{End}(\mathscr{M}).$$

We, for example, require the existence of coherent isomorphisms

$$\alpha_{x,y}\colon \rho_{x\otimes y} \overset{\sim}{\Longrightarrow} \rho_x\rho_y, \qquad \text{for all } x, y \in \mathscr{C}.$$

Representation categories

Monoidal categories $\mathscr{C}$ are also monoids—a $\mathscr{C}$**-representation** consists of a category $\mathscr{M}$ and a compatible **action**

$$\rho\colon \mathscr{C} \longrightarrow \mathsf{End}(\mathscr{M}).$$

We, for example, require the existence of coherent isomorphisms

$$\alpha_{x,y}\colon \rho_{x\otimes y} \overset{\sim}{\Longrightarrow} \rho_x\rho_y, \qquad \text{for all } x, y \in \mathscr{C}.$$

$\boxed{\rho_{xy} = \rho_x\rho_y}$

Representation categories

Monoidal categories $\mathscr{C}$ are also monoids—a $\mathscr{C}$**-representation** consists of a category $\mathscr{M}$ and a compatible **action**

$$\rho\colon \mathscr{C} \longrightarrow \mathsf{End}(\mathscr{M}).$$

We, for example, require the existence of coherent isomorphisms

$$\alpha_{x,y}\colon \rho_{x\otimes y} \overset{\sim}{\Longrightarrow} \rho_x\rho_y, \qquad \text{for all } x, y \in \mathscr{C}.$$

In representation categories, we can define **actions** and **coactions** of functors.

A Kelly-type theorem for representation categories

Theorem (Halbig–Z)

Given

$$\begin{tikzcd} \mathscr{C} \arrow[r, shift left=2, "F"] \arrow[r, phantom, "\perp"] & \mathscr{D} \arrow[l, shift left=2, "U"] \end{tikzcd}$$

A Kelly-type theorem for representation categories

Theorem (Halbig–Z)

Given

$$\begin{tikzcd} \mathscr{C} \arrow[r, shift left=2, "F"] \arrow[r, phantom, "\perp_{\otimes}"] & \mathscr{D} \arrow[l, shift left=2, "U"] \end{tikzcd}$$

A Kelly-type theorem for representation categories

Theorem (Halbig–Z)

Given

$$\begin{tikzcd} \mathscr{C} \arrow[loop above, "\mathscr{M}"] \arrow[r, shift left=2, "F"] \arrow[r, phantom, "\perp_{\otimes}"] & \mathscr{D} \arrow[loop above, "\mathscr{N}"] \arrow[l, shift left=2, "U"] \end{tikzcd}$$

A Kelly-type theorem for representation categories

Theorem (Halbig–Z)

Given

$$\begin{tikzcd}
\mathscr{M} \arrow[r, shift left=2, "G"] \arrow[r, phantom, "\perp"] \arrow[d, phantom, "\circlearrowright"] & \mathscr{N} \arrow[l, shift left=2, "V"] \arrow[d, phantom, "\circlearrowright"] \\
\mathscr{C} \arrow[r, shift left=2, "F"] \arrow[r, phantom, "\perp_{\otimes}"] & \mathscr{D} \arrow[l, shift left=2, "U"]
\end{tikzcd}$$

A Kelly-type theorem for representation categories

Theorem (Halbig–Z)

Given

$$
\begin{tikzcd}
\mathcal{M} \arrow[r, shift left=2, "G"] \arrow[r, phantom, "\perp"] & \mathcal{N} \arrow[l, shift left=2, "V"] \\
\mathcal{C} \arrow[r, shift left=2, "F"] \arrow[r, phantom, "\perp_{\otimes}"] & \mathcal{D} \arrow[l, shift left=2, "U"]
\end{tikzcd}
$$

then G-coactions on F are in bijection with V-actions on U.

Tannaka duality for representation categories

Corollary (Halbig–Z)

Let T be a Hopf monad on the rigid monoidal category $\mathscr{C}$, and K a monad on a $\mathscr{C}$-representation $\mathscr{M}$. Then there is a bijection:

$$\left\{ T\text{-Coactions on } K \right\} \xleftrightarrow{\cong} \left\{ \begin{array}{l} \mathsf{rep}(T)\text{-representations on } \mathsf{rep}(K) \\ \text{such that } U^T \text{ acts strongly on } U^K \end{array} \right\}$$

Reconstruction up to Morita equivalence

Theorem (Stroiński–Z)

Let $\mathscr{C}$ be an abelian monoidal category and $(\mathscr{M}, \rho)$ an abelian representation of $\mathscr{C}$.

Reconstruction up to Morita equivalence

Theorem (Stroiński–Z)

Let $\mathscr{C}$ be an abelian monoidal category and $(\mathcal{M}, \rho)$ an abelian representation of $\mathscr{C}$. Suppose $\mathcal{M}$ contains a "nice" object ℓ.

Reconstruction up to Morita equivalence

Theorem (Stroiński–Z)

Let $\mathscr{C}$ be an abelian monoidal category and $(\mathscr{M}, \rho)$ an abelian representation of $\mathscr{C}$. Suppose $\mathscr{M}$ contains a "nice" object ℓ. Then there exists a monad Γ_ℓ such that there is an equivalence of representations

$$\mathscr{M} \simeq \mathsf{rep}(\Gamma_\ell).$$

Reconstruction up to Morita equivalence

Theorem (Stroiński–Z)

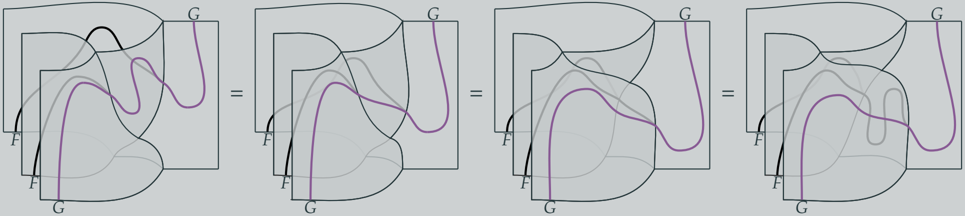
Let $\mathscr{C}$ be an abelian monoidal category and $(\mathscr{M}, \rho)$ an abelian representation of $\mathscr{C}$. Suppose $\mathscr{M}$ contains a "nice" object ℓ. Then there exists a monad Γ_ℓ such that there is an equivalence of representations

$$\mathscr{M} \simeq \mathsf{rep}(\Gamma_\ell).$$

Furthermore, there is a bijection

$$\{(\mathscr{M}, \ell) \text{ as above}\}\big/_{\mathscr{M} \simeq \mathscr{N}} \xleftrightarrow{\cong} \left\{\begin{matrix} \textit{Right exact lax } \mathscr{C}\textit{-rep} \\ \textit{monads on } \mathscr{C} \end{matrix}\right\} / \,\mathsf{rep}(T) \simeq \mathsf{rep}(S)$$

Thanks!



Results from other articles

Theorem (Halbig–Z)

There exists a tensor representable category that is not rigid.

Theorem (Halbig–Z)

There are pivotal structures on the centre of a monoidal category that are not induced by the Picard heap of its twisted centre.

Theorem (Z)

The formalism of R-matrices for bialgebras extends to separately oplax monoidal monads in preduoidal categories.

Results from other articles

Theorem (Halbig–Z)

There exists a tensor representable category that is not rigid.

Theorem (Halbig–Z)

There are pivotal structures on the centre of a monoidal category that are not induced by the Picard heap of its twisted centre.

Theorem (Z)

The formalism of R-matrices for bialgebras extends to separately oplax monoidal monads in preduoidal categories.

Results from other articles

Theorem (Halbig–Z)

There exists a tensor representable category that is not rigid.

Theorem (Halbig–Z)

There are pivotal structures on the centre of a monoidal category that are not induced by the Picard heap of its twisted centre.

Theorem (Z)

The formalism of R-matrices for bialgebras extends to separately oplax monoidal monads in preduoidal categories.

References i

[BV07] Alain Bruguières and Alexis Virelizier. **Hopf monads.** In: **Advances in Mathematics** 215.2 (2007), pp. 679–733. ISSN: 0001-8708. DOI: 10.1016/j.aim.2007.04.011.

[CSZ25] Kevin Coulembier, Mateusz Stroiński, and Tony Zorman. **Simple algebras and exact module categories.** In: **arXiv e-prints** (2025). arXiv: 2501.06629 [math.RT].

[HZ23] Sebastian Halbig and Tony Zorman. **Duality in Monoidal Categories.** In: **arXiv e-prints** (2023). arXiv: 2301.03545 [math.CT].

[HZ24a] Sebastian Halbig and Tony Zorman. **Diagrammatics for Comodule Monads.** In: **Appl. Categ. Struct.** 32 (2024). Id/No 27, p. 17. ISSN: 0927-2852. DOI: 10.1017/CBO9781139542333.

[HZ24b] Sebastian Halbig and Tony Zorman. **Pivotality, twisted centres, and the anti-double of a Hopf monad.** English. In: **Theory Appl. Categ.** 41 (2024), pp. 86–149. ISSN: 1201-561X.

[Kel74] G. M. Kelly. **Doctrinal adjunction.** English. Category Sem., Proc., Sydney 1972/1973, Lect. Notes Math. 420, 257-280 (1974). 1974.

[McC02] Paddy McCrudden. **Opmonoidal monads.** In: **Theory and Applications of Categories** 10 (2002), No. 19, 469–485.

[Moe02] Ieke Moerdijk. **Monads on tensor categories.** In: **J. Pure Appl. Algebra** 168.2-3 (2002). Category theory 1999 (Coimbra), pp. 189–208. ISSN: 0022-4049. DOI: 10.1016/S0022-4049(01)00096-2.

References ii

[SZ24] Mateusz Stroiński and Tony Zorman. **Reconstruction of module categories in the infinite and non-rigid settings.** In: **arXiv e-prints** (2024). arXiv: `2409.00793 [math.QA]`.

[Tan38] Tadao Tannaka. **Über den Dualitätssatz der nichtkommutativen topologischen Gruppen.** German. In: **Tôhoku Math. J.** 45 (1938), pp. 1–12. ISSN: 0040-8735.

[Zor25] Tony Zorman. **Duoidal R-Matrices.** In: **arXiv e-prints** (2025). arXiv: `2503.03445 [math.CT]`.