# Categorical Reconstruction Theory

2025-07-04

Tony Zorman

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# The classical story

**Tannaka duality studies algebraic structures through their categories of representations.**

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One solution: Linearise!

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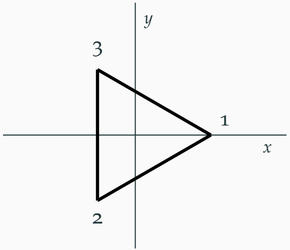

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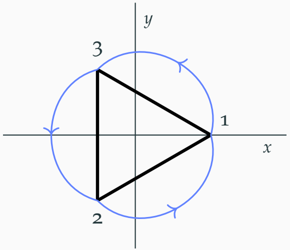

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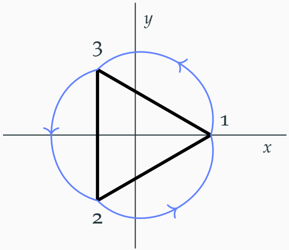

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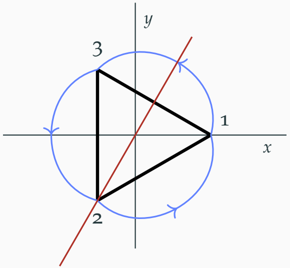


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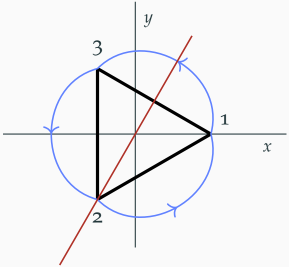

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A **morphism of representations** from $(V, \rho)$ to $(W, \sigma)$ is a linear map $f\colon V \longrightarrow W$ such that $f\rho_g = \sigma_g f$ for all $g \in G$.

# Special building blocks

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$$V \otimes_{\Bbbk} W \cong \Bbbk\{ v_i \otimes w_j \mid v_i \in B_V, w_j \in B_W \}$$

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$$\begin{aligned} \rho^* \colon G &\longrightarrow \mathrm{GL}_n(\mathbb{k}) \\ g &\longmapsto (\rho_{g^{-1}})^{\mathrm{T}}. \end{aligned}$$

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**Theorem (Tannaka reconstruction)**

*The monoidal automorphisms of $U$ are, as a group, isomorphic to $G$.*

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# Categorical generalisations

- [HZ23] with Sebastian Halbig: *Duality in Monoidal Categories*. arXiv: 2301.03545.
- [HZ24a] with Sebastian Halbig: *Diagrammatics for Comodule Monads*. In: Appl. Categ. Struct. 32 (2024).
- [HZ24b] with Sebastian Halbig: *Pivotality, twisted centres, and the anti-double of a Hopf monad*. In: Theory Appl. Categ. 41 (2024).
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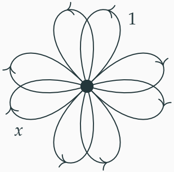

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  - Sets with $\times$, sets with $\sqcup$, groups with $\times$, vector spaces with $\otimes_{\mathbb{k}}$, ...
- A monoidal category is **rigid** if there exists a dual for every object.
  - Finite-dimensional vector spaces, representations, finite-dimensional modules over a finite-dimensional Hopf algebra, ...

**Can we generalise Tannaka duality to other algebraic structures and their categories of representations?**

**Given a (rigid monoidal) category $\mathscr{C}$, can one tell that this category is of the form $\mathsf{rep}(A)$, for some algebraic object $A$?**

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**Theorem (Moerdijk, McCrudden, Bruguières–Virelizier)**

*Given a monad $T$ on a rigid monoidal category $\mathscr{C}$, there exists a bijection*

$$\left\{ \textit{Hopf monad structures on } T \right\} \xleftrightarrow{\cong} \left\{ \begin{array}{l} \textit{Rigid monoidal structure on } \mathsf{rep}(T) \\ \textit{such that } U \textit{ is strong monoidal} \end{array} \right\}$$

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We, for example, require the existence of coherent isomorphisms

$$\alpha_{x,y,m}\colon \rho_{x\otimes y}(m) \xrightarrow{\sim} \rho_x\big(\rho_y(m)\big), \qquad \text{for all } x, y \in \mathscr{C} \text{ and } m \in \mathscr{M}.$$

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In representation categories, we can define **actions** and **coactions** of functors.

# A Kelly-type theorem for representation categories

**Theorem (Halbig–Z)**

*Given*

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*then $G$-coactions on $F$ are in bijection with $V$-actions on $U$.*

# Tannaka duality for representation categories

**Corollary (Halbig–Z)**

*Let $T$ be a Hopf monad on the rigid monoidal category $\mathscr{C}$, and $K$ a monad on a $\mathscr{C}$-representation $\mathscr{M}$. Then there is a bijection:*

$$\left\{ T\text{-Coactions on } K \right\} \xleftrightarrow{\cong} \left\{ \begin{array}{l} \mathsf{rep}(T)\text{-representations on } \mathsf{rep}(K) \\ \text{such that } U^T \text{ acts strongly on } U^K \end{array} \right\}$$

# Reconstruction up to Morita equivalence

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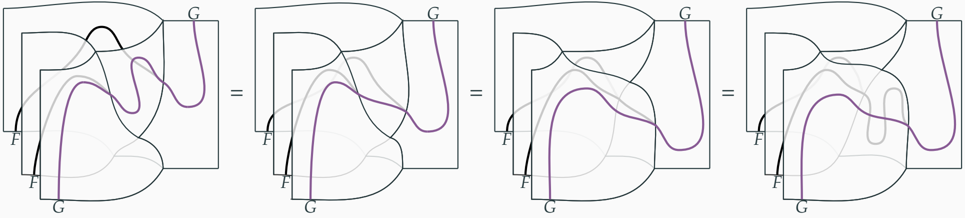
$$\mathscr{M} \simeq \mathsf{rep}(\Gamma_\ell).$$

*Furthermore, there is a bijection*

$$\{(\mathscr{M}, \ell) \text{ as above}\} \big/ \mathscr{M} \simeq \mathscr{N} \xleftrightarrow{\cong} \left\{ \begin{matrix} \textit{Right exact lax } \mathscr{C}\textit{-rep} \\ \textit{monads on } \mathscr{C} \end{matrix} \right\} / \ \mathsf{rep}(T) \simeq \mathsf{rep}(S)$$

# Thanks!



# Results from other articles

**Theorem (Halbig–Z)**

*There exists a tensor representable category that is not rigid.*

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*There are pivotal structures on the centre of a monoidal category that are not induced by the Picard heap of its twisted centre.*

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# References i


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