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A Stochastic Sailing Speed Optimization and Vessel Deployment Problem in Liner Shipping

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RESEARCH ARTICLE

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Abstract

In this research, we consider the stochastic sailing speed optimization and vessel deployment problem and examine the trade-off between sailing speed and the number of vessels required to provide a certain service in international liner shipping. Solving this kind of stochastic program is a challenging task. To address this issue, we construct a mathematical formulation based on Ng [1] and Ng [2] and enhance the linearization techniques from Wang and Meng [3] so that problem instances of an even larger scale can be solved. The proposed formulation has been numerically applied to solve realistic cases, and the empirical results show that the proposed framework is effective in solving this stochastic programming problem. The managerial insights from our work can help liner shipping companies determine bunker consumption and vessel deployment strategies.

Keywords: Liner shipping, Speed optimization, Vessel deployment, Stochastic optimization

1. Introduction

As maritime international shipping becomes increasingly competitive, many strategies have been proposed to reduce operational costs. The slow steaming strategy, that seeks to lower the operational speed of ships, was first introduced by a pioneering line shipper who sought to improve the financial performance of their carriers by reducing bunker oil consumption [4]. The strategy is especially useful when the bunker price is high and the time window is flexible. However, there is an apparent trade-off between vessel deployment and the slow steaming strategy when a shipping liner company wants to maintain service frequency (i.e., either weekly frequency (Mulder and Dekker [5]), daily (Lin and Tsai [6] or flexible (Ng [7], Ng [8], Ng [9], Ng and Lin [10]). Various past studies have investigated the trade-off between speed optimization and vessel deployment (i.e., Ronen [11], Wang and Meng [12]). However, the trade-off between the number of vessels and speed optimization is not explicitly considered in these studies.

Furthermore, as there can be various uncertainties associated with the vessel speed optimization problem, we incorporate the uncertainty in bunker consumption functions and vessel deployment strategies in the proposed model following the work by Ng [1] and Ng [2]. The resulting formulation is a mixed integer nonlinear optimization problem, which makes it challenging to solve. Therefore, only relatively small problem instances can be solved. One of the typical and most effective solution techniques to address the computational challenge of solving mixed integer nonlinear problems is through linearization (Wang and Meng [3]). Even though their study did not consider the window constraint, preliminary experiment (Ng [2]) shows it is still time-consuming to solve the stochastic model with the linearization techniques reported in Wang and Meng [3]. To fill the gap in the literature, the current research proposes a reformulation so that the problem can be approached effectively. To summarize, the current research constructs the mathematical formulation based on Ng [1] and Ng [2] and enhances the linearization techniques from

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Wang and Meng [3] so that problem instances of an even larger scale can be solved.

The remainder of this paper is structured as follows. Section 2 critically overviews the related work, and Section 3 presents the mathematical formulation for the bunker optimization problem in container shipping with time windows. Section 4 empirically evaluates the formulation and summarizes the results. The final section offers conclusions and suggestions for future research.

2. Literature review

As the competition in international shipping becomes increasingly intense, bunker consumption optimization has attracted increased research attention, as it is one of the constituent industry costs. Bunker consumption optimization also serves as the primary approach in reducing emissions in shipping (i.e., Norlund and Gribkovskaia [13] and Norlund and Gribkovskaia [14]). A detailed review of the mathematical solution methods for bunker optimization problems can be found in Wang, Meng [15]. Interested readers are referred to their work for more details. In this section, we simply provide an updated review of the relevant studies.

Wang and Meng [3] investigated the speed optimization and bunker refueling in liner shipping considering that the actual speed can be different from the planning speed. A mixed-integer nonlinear robust optimization model and a closed-form expression for the worst-base bunker consumption were proposed. The nonlinear model was linearized so that the resulting formulation could be solved as a mixed-integer linear programming formulation. However, our preliminary experiment shows that it is still time-consuming to solve our model with their linearization techniques. Zhang, Teo [16] analyzed the speed optimization problem considering the time window at each port. Optimality properties for the problem and the uniqueness of the solution were established. Their findings provide intuition for the development of algorithms for this problem. Wang and Wang [17] designed a polynomial-time algorithm to solve the speed optimization problem that balances the fuel consumption, level of service and the number of ships required to provide the service.

Aydin, Lee [18] investigated the speed optimization problem in liner shipping considering stochastic port times and time windows. A dynamic programming model was developed by discretizing the port arrival times to provide approximate solutions. The numerical results showed that speed optimizations that consider port time uncertainty can decrease fuel consumption costs. Ng [1] discussed the trade-off between vessel speed and the number of vessels required to maintain a given service frequency and proposed a vessel speed optimization model. It was found that a liner shipping company has a very limited choice in the number of vessels to deploy, and incorporating the findings identified in the research significantly reduced the computational efforts required to solve the speed optimization problem. However, the proposed model is in a nonlinear form and can only be solved by nonlinear optimization packages. Wang, Gao [19] incorporated speed optimization, a bunkering strategy and a shipment strategy and proposed a freight revenue optimization model for a single liner shipping service. A mixed-integer nonlinear programming model and an equivalent mixed-integer linear programming model were proposed and empirically applied to real liner service routes.

After reviewing the relevant studies from recent years, we made a model comparison summary of the most relevant past studies in Table 1 and highlighted the contributions of our work.

As observed from Table 1, our model considers the most related factors in the problem and reflects the actual practice with the greatest fidelity. In summary, the current research constructs the mathematical formulation based on Ng [1] and Ng [2] and enhances the linearization techniques from Wang and Meng [3] so that problem instances of an even larger scale can be solved. Further, we improve the

Table 1. Model comparison.

	Wang and Meng [3]	Ng [1]	Ng [2]	Our model
Model Type	MILP[a]	MINLP[b]	MINLP[b]	MILP[a]
Port arrival time window	✖	✖	○	○
Fuel consumption scenario	✖	○	○	○
Linearization of fuel consumption function	○	✖	✖	○
Discriminating pricing	○	✖	✖	○
Linearization of discriminating pricing function	○	✖	✖	○
Flexible vessel selection	✖	✖	✖	○

[a] MILP: mixed integer linear programming

[b] MINLP: mixed integer nonlinear programming.

linearization of bunker price function and incorporate the method of vessel type selection, which makes the overall framework more applicable to realistic cases. Empirically, the results in a later section show that the proposed reformulation can make the large-scale stochastic vessel optimization problem solvable within a reasonable timeframe.

3. Mathematical formulation

In this section, we first formally state the problem and summarize the assumptions imposed in this research. Then we define the sets, parameters and decision variables that will be used throughout the paper, followed by a detailed mathematical formulation.

In this study, we consider the stochastic sailing speed optimization and vessel deployment problem and examine the trade-off between sailing speed and the number of vessels required to provide a certain service in international liner shipping. A few assumptions are imposed. First of all, we assume that the liner shipping company has only limited number of vessels to deploy. Further, the vessel types of the company are exogenous and cannot change over the planning horizon. However, the number of chartered vessels is unlimited. Further, since we consider a liner shipping problem, the shipping company needs to maintain a periodical schedule. Finally, the liner shipping cannot change the routes in the planning horizon.

Sets	
R	set of routes
K	set of vessel classes
P^{bun}	set of ports that provide bunkering service
R_k	set of routes that deploy vessels of class $k \in K$
J_r	set of legs on route $r \in R$
J_ρ	set of legs whose starting port is $\rho \in P^{bun}$
U	set of transit time ranges
O	set of bunker price types
Ω	set of possible realizations of the bunker consumption function
Parameters	
c_k	cost of deploying a vessel of class $k \in K$ (in \$/week). This cost does not include the bunker and vessel chartering costs (if the vessel is chartered)
d_{jr}	length of leg $j \in J_r$ on route $r \in R$
m_k	number of vessels of class $k \in K$ owned by the liner shipping company
t_r^p	total port time on a roundtrip voyage on route $r \in R$
$t_r^{p_j}$	port time at port $j \in J_r$ on route $r \in R$
π_{jr}	inventory cost per unit time in leg $j \in J_r$ on route $r \in R$ (in \$/hour)
p^ω	probability of scenario $\omega \in \Omega$ occurring
$B_{kjr}(t_{jr})$	bunker consumption (in tons/day) for a vessel of class $k \in K$ when the transit time on leg $j \in J_r$, $r \in R$ is t_{jr}. $B_{kjr}(t_{jr}) = \sum_{\omega \in \Omega} p^\omega B^\omega_{kjr}(t_{jr}) = \sum_{\omega \in \Omega} \alpha^\omega_k (d_{jr}/t_{jr})^{\beta^\omega_k}$.
t_{jr}^{min}	shortest possible transit time on leg $j \in J_r$ of route $r \in R$
t_{jr}^{max}	longest possible transit time on leg $j \in J_r$ of route $r \in R$
r_k	charter rate for a vessel of class $k \in K$ (in \$/week)
$f_{jr}(.)$	cost function per ton of bunker at port $j \in J_r$ on route $r \in R$. The function, for instance, can be in a piecewise linear form (see Wang and Meng [3] for details).
$[\lambda_{jr}, \upsilon_{jr}]$	time window at port $j \in J_r$ on route $r \in R$
C_k	bunker tank capacity of ship of class $k \in K$
BP_o	bunker price in type $o \in O$
δ_o	purchase limit for bunker price type $o \in O$
Decision variables	
t_{jr}	transit time on leg $j \in J_r$ of route $r \in R$.
x_r	total number of vessels to deploy on route $r \in R$.
y_k	number of vessels of class $k \in K$ to charter (assuming a bareboat charter).
z_{jr}^-	the remaining bunker amount in the ship's tank when arriving at the starting port of leg $j \in J_r$ on route $r \in R$.
z_{jr}^+	bunker amount in the ship's tank when departing from the starting port of leg $j \in J_r$ on route $r \in R$.
a_{jr}	arrival time of the first ship in a string deployed on route $r \in R$ at the starting port of leg $j \in J_r$.
b_ρ	total bunker cost of the leg whose starting port is $\rho \in P^{bun}$.
$s_{\rho o}$	total bunker purchased at port $\rho \in P^{bun}$ in price type $o \in O$.
$g_{\rho o}$	binary variables used in Constraints [17,18] to ensure that only when $s_{\rho o}$ approaches its upper limit δ_o can $s_{\rho o+1}$ be greater than 0. Otherwise, $s_{\rho o+1} = 0$.

φ_{ku}^{ω}	intercept of the linearized bunker consumption function of vessels of class $k\in K$ in scenario $\omega\in\Omega$ in range $u\in U$.
θ_{ku}^{ω}	slope of the linearized bunker consumption function of vessels of class $k\in K$ in scenario $\omega\in\Omega$ in range $u\in U$.
h_{rk}	binary variables, where $h_{rk}=1$ indicates that route $r\in R$ is chosen as vessels of class $k\in K$; $h_{rk}=0$ means otherwise.
x_{rk}	total number of vessels of class $k\in K$ to deploy on route $r\in R$.
v_r	total cost of deploying vessels (in \$/week) on route $r\in R$.
i_{jr}	total inventory cost in leg $j\in J_r$ on route $r\in R$.

Based on the notation, the mathematical formulation is as follows:

$$Min \sum_{k,r\in R_k} c_k\cdot x_r+\sum_{k\in K} r_k\cdot y_k+\sum_{j\in J_r,r\in R}\pi_{jr}\cdot t_{jr}+\sum_{\rho\in P^{bun}} b_\rho \tag{1}$$

Subject to

$$z_{jr}^{+}-z_{j+1,r}^{-}\geq\sum_{\omega\in\Omega}p^{\omega}\cdot\left(\varphi_{ku}^{\omega}+\theta_{ku}^{\omega}\cdot t_{jr}\right)$$
$$\forall u\in U,\forall j\in J_r,r\in R,k\in K, If\ j=|J|,\ then\ j+1=1 \tag{2}$$

$$z_{jr}^{+}\geq z_{jr}^{-}\qquad\forall j\in J_r,r\in R \tag{3}$$

$$z_{jr}^{+}\leq C_k\qquad\forall r\in R_k,k\in K \tag{4}$$

$$z_{jr}^{+},\ z_{jr}^{-}\geq 0\qquad\forall j\in J_r,r\in R \tag{5}$$

$$a_{jr}+t_r^{p_j}+t_{jr}=a_{j+1,r}\qquad\forall j\in\{1,2,\ldots,|J_r|-1\},r\in R \tag{6}$$

$$a_{1r}=0\qquad\forall r\in R \tag{7}$$

$$\lambda_{jr}\leq a_{jr}\leq v_{jr}\qquad\forall j\in J_r,r\in R \tag{8}$$

$$\sum_{j\in J_r}t_{jr}+t_r^{p}=168x_r\qquad\forall r\in R \tag{9}$$

$$\sum_{r\in R_k}x_r\leq m_k+y_k\qquad\forall k\in K \tag{10}$$

$$t_{jr}^{min}\leq t_{jr}\leq t_{jr}^{max}\qquad\forall j\in J_r,r\in R \tag{11}$$

$$x_r\in\mathbb{Z}^{+}\qquad\forall r\in R \tag{12}$$

$$y_k\in\mathbb{Z}^{+}\qquad\forall k\in K \tag{13}$$

$$\sum_{o\in O}s_{\rho o}=\sum_{j\in J_\rho}\left(z_{jr}^{+}-z_{jr}^{-}\right)\qquad\forall\rho\in P^{bun} \tag{14}$$

$$b_\rho=\sum_{o\in O}BP_o\cdot s_{\rho o}\qquad\forall\rho\in \mathrm{P}^{bun} \tag{15}$$

$$s_{\rho o}\leq\delta_o\quad\forall\rho\in \mathrm{P}^{bun},\qquad o\in O \tag{16}$$

$$\delta_o-s_{\rho o}\leq M\cdot g_{\rho o}\qquad\forall\rho\in P^{bun},o\in\{1,2,\ldots,|O|-1\} \tag{17}$$

$$s_{\rho o+1}\leq M\left(1-g_{\rho o}\right)\quad\forall\rho\in P^{bun},\ o\in\{1,2,\ldots,|O|-1\} \tag{18}$$

Objective (1) minimizes the total cost, comprising the vessel voyage cost $\left(\sum_{k,r\in R_k}c_k\cdot x_r\right)$, vessel chartering cost $\left(\sum_{k\in K}r_k\cdot y_k\right)$, inventory cost $\left(\sum_{j\in J_r,r\in R}\pi_{jr}\cdot t_{jr}\right)$ and bunker cost $\left(\sum_{\rho\in P^{bun}}b_\rho\right)$.

Constraint (2) calculates the bunker consumption between two subsequent ports. The bunker consumption function is $B_{kjr}(t_{jr})=\alpha_k(d_{jr}/t_{jr})^{\beta_k}$. Since the function is convex, we can use an unbounded approach to perform a piecewise linearization. Assume that the problem is to minimize the convex function with the parameters, $d_{jr}=1000$, $\alpha_k=0.003$ and $\beta_k=2.5$ (An illustration is shown as Fig. 1). Note that α_k and β_k are the parameters of bunker consumption function and their values here are used to demonstrate the piecewise linearization method. Their actual values depend on the vessel type and will be provided in later section (see Fig. 2).

First, we cut the feasible interval ($t_{jr}^{max}-t_{jr}^{min}$) of the transit time into ranges (U set of ranges). With the unbounded approach, the curve (the bunker consumption function) can be approximated with the tangent in each range as illustrated in

Then, as $B_{kjr}(t_{jr})=\alpha_k(d_{jr}/t_{jr})^{\beta_k}$, we can rearrange the equations accordingly:

$$z_{jr}^{+}-z_{j+1,r}^{-}\geq t_{jr}\Big/24\cdot B_{kjr}(t_{jr})$$

$$\rightarrow z_{jr}^{+}-z_{j+1,r}^{-}\geq t_{jr}\Big/24\cdot\alpha_k\left(d_{jr}/t_{jr}\right)^{\beta_k}$$

$$\rightarrow z_{jr}^{+}-z_{j+1,r}^{-}\geq\alpha_k\Big/24\cdot d_{jr}{}^{\beta_k}\Big/t_{jr}{}^{\beta_k-1}$$

$$\rightarrow z_{jr}^{+}-z_{j+1,r}^{-}\geq\left(\alpha_k\cdot d_{jr}{}^{\beta_k}\right)\Big/\left(24\cdot t_{jr}{}^{\beta_k-1}\right)$$

With this derivation, we then can use the constraint (2a) in the rest of the paper.

$$z_{jr}^{+}-z_{j+1,r}^{-}\geq\left(\alpha_k\cdot d_{jr}{}^{\beta_k}\right)\Big/\left(24\cdot t_{jr}{}^{\beta_k-1}\right)$$

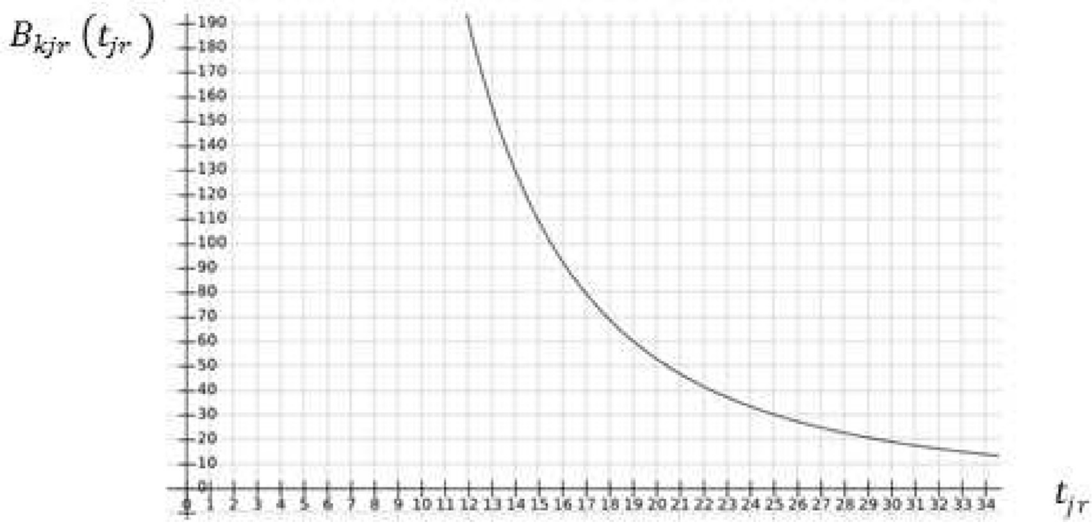

Fig. 1. Bunker consumption function ($d_{jr} = 1000$, $\alpha_k = 0.003$ and $\beta_k = 2.5$).

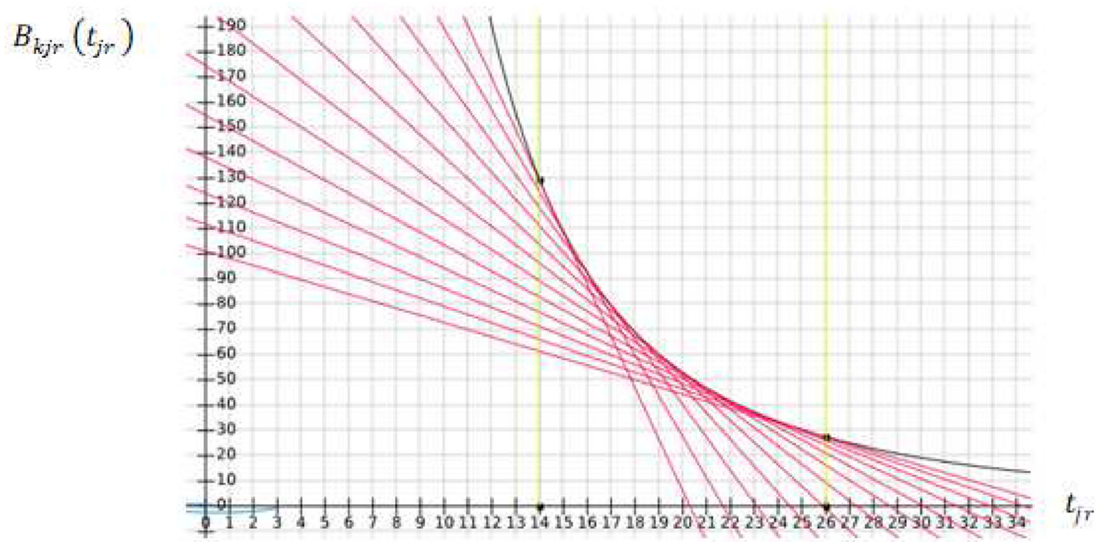

Fig. 2. Tangents in each range.

$$\forall u \in U,\ \forall j \in J_r,\ \forall r \in R,\ \forall k \in K,$$
$$If\ j = |J|, then\ z^{-}_{j+1,r} \to z^{-}_{1,r} \tag{2a}$$

Note that mp_{ku} is the midpoint by range u. In order to find the midpoint of u, we need to know the time length for each range ($\frac{t_{jr}^{max} - t_{jr}^{min}}{|U|}$) and the index of u^{th} interval. To find the mp_{ku} of u^{th} range, we need to minus 0.5 to the index of u. To summarize, mp_{ku} can be calculated as follows:

$$mp_{ku} = time\ length\ for\ each\ range \cdot (serial\ number\ of\ u - 0.5)$$

In other words,

$$mp_{ku} = \frac{t_{jr}^{max} - t_{jr}^{min}}{|U|} \cdot (u - 0.5)$$

Then

$$\theta_{ku} = \frac{\partial\ (\alpha_k \cdot d_{jr}{}^{\beta_k}) / (24 \cdot t_{jr}{}^{\beta_k - 1})}{\partial\ t_{jr}} \cdot mp_{ku}$$

$$\frac{\partial\ (\alpha_k \cdot d_{jr}{}^{\beta_k}) / (24 \cdot t_{jr}{}^{\beta_k - 1})}{\partial\ t_{jr}}$$

$$= \frac{\alpha_k \cdot d_{jr}{}^{\beta_k}}{24} \cdot \frac{\partial\, t_{jr}{}^{1-\beta_k}}{\partial\, t_{jr}}$$

$$= \frac{\left[\alpha_k \cdot d_{jr}{}^{\beta_k} \cdot (1-\beta_k)\right]}{24} \cdot t_{jr}{}^{-\beta_k}$$

$$= \frac{\left[\alpha_k \cdot d_{jr}{}^{\beta_k} \cdot (1-\beta_k)\right]}{24 \cdot t_{jr}{}^{\beta_k}}$$

Therefore, $\varphi_{ku} = (\alpha_k \cdot d_{jr}{}^{\beta_k})/(24 \cdot t_{jr}{}^{\beta_k - 1}) - [\theta_{ku} \cdot m\, p_{ku}]$.

Constraints (3)-(5) are simple bounds on the bunker level. Constraints ((6-8)) are time window constraints to ensure that the ships must arrive at the ports within a designated time window. Constraint (9) is a fundamental relation in liner shipping (see Ng [1] and Ng [2] for more details). In using Constraint (10), it is ensured that the number of vessels deployed does not exceed the ocean carrier's own number of vessels plus the chartered vessels. Limits on the sailing speeds are provided in Constraint (11). Constraint (12) states that x_r are nonnegative integers.

Constraints (14)-(18) calculate the bunkering cost of each port that provides bunkering services. The transform bunker price functions $c_\rho^{bun}(x_\rho)$ are shown below. In these equations, x_ρ is the refueling volume of port ρ, c_ρ is the total fuel cost of port ρ and g_ρ^1 and g_ρ^2 are two binary variables. For details of the transformation, refer to Wang and Meng [3],

$$c_\rho \geq C_\rho^2 + (x_\rho - S_\rho^2) l_2 - (1 - g_\rho^2) \cdot M \quad \forall \rho \in P^{bun} \tag{19}$$

$$c_\rho \geq C_\rho^1 + (x_\rho - S_\rho^1) l_1 - (1 - g_\rho^1) \cdot M - g_\rho^2 \cdot M \quad \forall \rho \in P^{bun} \tag{20}$$

$$c_\rho \geq l_0 \cdot x_\rho - g_\rho^1 \cdot M - g_\rho^2 \cdot M \quad \forall \rho \in P^{bun} \tag{21}$$

$$x_\rho - S_\rho^1 \leq g_\rho^1 \cdot M \quad \forall \rho \in P^{bun} \tag{22}$$

$$x_\rho - S_\rho^2 \leq g_\rho^2 \cdot M \quad \forall \rho \in P^{bun} \tag{23}$$

To incorporate the discriminating pricing scheme, we introduce the following equation when calculating the bunker price.

$$c_\rho^{bun}(x_\rho) = \begin{cases} l_0 \cdot x_\rho, & if\ x_\rho \leq S_j^1 \\ C_j^1 + l_1 \cdot \left(x_\rho - S_j^1\right), & if\ S_j^1 < x_\rho \leq S_j^2 \\ C_j^2 + l_2 \cdot \left(x_\rho - S_j^2\right), & if\ x_\rho > S_j^2 \end{cases}$$

When the amount of bunker purchased is low, the price is high. On the other hand, if the amount of bunker purchased is high, it is possible to have a certain discount, and the resulting price is low.

For clarity, we call the formulation formed by Constraints (1)-(12) and (19)-(23) **P1**, which resembles the formulation by Wang and Meng [3]. However, from our preliminary experiment, solving **P1** is challenging for commercial optimization packages because of the bunker price transformation (Constraints (19)-(23)). Therefore, we enhance the formulation as follows.

We can rewrite the above constraints and define the purchase amount of different bunker prices in each bunker port as an independent variable $s_{\rho o}$ $\forall \rho \in P^{bun}$, $o \in O$. Only when $s_{\rho o}$ approaches its upper limit δ_o, can $s_{\rho o+1}$ be greater than 0, otherwise, $s_{\rho o+1} = 0$.

$$b_\rho^{bun}(s_{\rho o}) = \begin{cases} BP_1 \cdot s_{\rho 1}, & if\ s_{\rho 1} \leq \delta_1 \\ \sum_{o=\{1,2\}} BP_o \cdot s_{\rho o}, & if\ s_{\rho 1} = \delta_1, s_{\rho 2} \leq \delta_2 \\ \sum_{o=\{1,2,3\}} BP_o \cdot s_{\rho o}, & if\ s_{\rho 1} = \delta_1,\ s_{\rho 2} = \delta_2 \end{cases}$$

We call the new formulation Constraints (1)-(18) **P2**.

To enhance the formulation and find the optimal fleet configuration, objective (1) can be rewritten as follows.

$$\min \sum_{r \in R} v_r + \sum_{k \in K} r_k \cdot y_k + \sum_{j \in J_r, r \in R} i_{jr} + \sum_{\rho \in P^{bun}} b_\rho \tag{1f}$$

With the new objective function, we additionally need to include constraints (24)–(26).

$$v_r + (1 - h_{rk}) \cdot M \geq c_k \cdot x_r \quad \forall r \in R\ ,\ k \in K \tag{24}$$

$$i_{jr} + (1 - h_{rk}) \cdot M \geq \pi_{jr} \cdot t_{jr} \quad \forall j \in J_r\ ,\ r \in R\ ,\ k \in K \tag{25}$$

$$\sum_{k \in K} h_{rk} = 1 \quad \forall j \in J_r\ ,\ r \in R \tag{26}$$

with the introduction of these three constraints, the model can determine the appropriate vessels to deploy on each route (instead of the designated vessels that are fixed and cannot be changed). Then, we rewrite constraint (2) as (2f) and constraint (10) as (10f-1) and (10f-2) to complete the new formulation.

$$z_{jr}^{+} - z_{j+1,r}^{-} + (1 - h_{rk}) \cdot M \geq \sum_{\omega \in \Omega} p^{\omega} \cdot (\varphi_{ku}^{\omega} + \theta_{ku}^{\omega} \cdot t_{jr})$$

$$\forall u \in U,\ \forall j \in J_r,\ \forall r \in R,\ \forall k \in K,\ If\ j = |J|, then\ j+1 = 1 \quad (2f)$$

$$x_{rk} + (1 - h_{rk}) \geq x_r \quad \forall r \in R\ ,\ k \in K \quad (10f1)$$

$$\sum_{r \in R} x_{rk} \leq m_k + y_k \quad \forall k \in K \quad (10f2)$$

The new formulation (objective function (1f), constraints (2f), (3)-(9), (10f1), (10f2) and (11)-(26)) can determine the number of vessels to deploy and is called the *flexible vessel selection* model in the empirical study.

4. Case study

To validate the formulation and evaluate the effectiveness of the reformulation, we solve problem instances of different sizes. In the experiments, we employ the commercial optimization package CPLEX 12.7.1 to solve the formulation. The numerical experiments are conducted on a Windows-based machine with an Intel i9-9900K CPU with 3.60 GHz and 64 GB of memory.

The sailing route, vessel information and fuel consumption parameters are summarized in Tables 2–4, respectively. The sailing routes are from Wang and Meng [3]. The information in Tables 3 and 4 is retrieved from https://www.synchronet.eu/news/slow-steaming/the-rise-the-fallen-and-the-possible-solution-about-slow-steaming/?cn-reloaded=1 (accessed 2021). The vessels are the ones that are commonly used in liner shipping.

We first experiment with the time window considerations and summarize the results in Table 5. In the experiment, the minimum speeds of vessel classes 1, 2 and 3 are 13, 12 and 11, respectively. The maximum speeds of vessel classes 1, 2, and 3 are 26, 24 and 22, respectively, while the fuel tank capacities of vessel classes 1, 2 and 3 are 8,333, 11,666 and 15,000, respectively. The arrival TW considered is imposed on each arrival port in each route.

Table 2. Sailing route.

Route	Vessel Type	Port rotation
1	1	Singapore → Brisbane → Sydney → Melbourne → Adelaide → Fremantle → Singapore
2	1	Xiamen → Chiwan → Hong Kong → Singapore → Port Klang → Salalah → Jeddah → Aqabah → Salalah → Singapore → Xiamen
3	2	Yokohama → Tokyo → Nagoya → Kobe → Shanghai → Yokohama
4	2	Ho Chi Minh → Laem Chabang → Singapore → Port Klang → Ho Chi Minh
5	2	Brisbane → Sydney → Melbourne → Adelaide → Fremantle → Jakarta → Singapore → Brisbane
6	2	Manila → Kaohsiung → Xiamen → Hong Kong → Yantian → Chiwan → Hong Kong → Manila
7	2	Dalian → Xingang → Qingdao → Shanghai → Ningbo → Shanghai → Kwangyang → Busan → Dalian
8	2	Chittagong → Chennai → Colombo → Cochin → Nhava Sheva → Cochin → Colombo → Chennai → Chittagong
9	1	Sokhna → Aqabah → Jeddah → Salalah → Karachi → Jebel Ali → Salalah → Sokhna
10	3	Southampton → Thamesport → Hamburg → Bremerhaven → Rotterdam → Antwerp → Zeebrugge → Le Havre → Southampton
11	3	Southampton → Sokhna → Salalah → Colombo → Singapore → Hong Kong → Xiamen → Shanghai → Busan → Dalian → Xingang → Qingdao → Shanghai → Hong Kong → Singapore → Colombo → Salalah → Southampton

Note: the ports highlighted in gray are the ones that provide bunkering service.

Table 3. Vessel information.

Vessel Type	Max Speed (knots)	Min Speed (knots)	Tank Capacity (tons)	Charter Cost (USD)	Deploy Cost (USD)
1	26	13	8,333	280,000	100,000
2	24	12	11,666	420,000	150,000
3	22	11	15,000	560,000	200,000

Table 4. Fuel consumption parameters.

		Vessel Type		
		1	2	3
Scenarios	1	$\alpha = 0.04225; \beta = 2.7$	$\alpha = 0.016284; \beta = 3$	$\alpha = 0.006376; \beta = 3.3$
	2	$\alpha = 0.088965; \beta = 2.7$	$\alpha = 0.03738; \beta = 3$	$\alpha = 0.015706; \beta = 3.3$
	3	$\alpha = 0.041161; \beta = 2.7$	$\alpha = 0.016068; \beta = 3$	$\alpha = 0.006273; \beta = 3.3$

Table 5. The results of adjusting the arrival time window (TW).

	CPU Time (sec)	Objective Value	Vessel Number				Total Transit time	Total Bunker Cost
			Class 1	Class 2	Class 3	Total		
P1 without arrival TW	0.90	52,443,804.97	11	9	11	31	4,015.00	16,135,994.2
P2 without arrival TW	0.28	52,443,804.97	11	9	11	31	4,015.00	16,135,994.9
P1 with 12-h TW	0.42	54,732,481.39	11	10	13	34	4,519.00	13,448,173.8
P2 with 12-h TW	0.19	54,732,481.39	11	10	13	34	4,519.00	13,448,173.9
P1 with 6-h TW	0.43	55,122,763.14	11	10	13	34	4,519.00	13,838,453.8
P2 with 6-h TW	0.18	55,122,763.14	11	10	13	34	4,519.00	13,838,453.9
P1 with 3-h TW	0.18	55,231,182.35	11	10	13	34	4,519.00	13,946,912.8
P2 with 3-h TW	0.17	55,231,182.35	11	10	13	34	4,519.00	13,946,873.0

With linearization, all the cases can be solved within one CPU second, demonstrating the efficiency of the technique. When comparing the cases with time window considerations, the objective values are the lowest when we do not consider the constraint (52,443,804.97). The objective value increases after the time window constraint is imposed. As the time window becomes more restrictive (i.e., from 12 h to 6 h or 3 h), the objective value increases further since it is more difficult to obtain a better solution with tighter constraints. In addition, the total transit time increases and the total bunker cost decreases as the time window becomes tighter. The reason is that a tighter time window constraint prevents the vessels from detouring. It is also observed that some routes require more chartered vessels after imposing more rigid time windows (i.e. routes 5 and 11), indicating that time window does impact the total cost drastically. Finally, in terms of computational improvement, the solution time required to solve **P2** is 47.49% less than that of **P1**, indicating the effectiveness of the reformulation. The computation advantage will become more apparent in later experiments.

The second study experiments with the bunker price (BP) and evaluates the sensitivity of the objective value, total transit time and total bunker cost with BP. With the same parameters (i.e., minimum speed, maximum speed and fuel tank capacity), we summarize the results in Table 6. In this table, the BP is adjusted in all ports with the same percentage. The refilling details are presented in Tables 7 and 8 for cases without and with time window consideration.

When there is no time window consideration, the objective value and total bunker cost increase simultaneously with an increasing BP. Interestingly, the total transit time also increases with an increasing BP. After examining the results, we found that the increase in total transit time is due to the slow steaming strategy, which is a common practice where container ships operate at significantly less than their maximum speed when BP is high. The results show that our model can generate results that are consistent with the actual practice in great fidelity. Again, the computational effort required for **P2** is 36.94% less than that for **P1**.

Table 6. The results of adjusting bunker prices.

	CPU Time (sec)	Objective Value	Vessel Number				Total Transit time	Total Bunker Cost
			Class 1	Class 2	Class 3	Total		
Without Time Window								
P1 with 50%BP	0.48	4,3151,611.44	9	9	10	28	3,511.00	10,551,707.7
P2 with 50% BP	0.16	4,3151,611.44	9	9	10	28	3,511.00	10,551,707.7
P1 with 100% BP	0.90	52,443,804.97	11	9	11	31	4,015.00	16,135,994.2
P1 with 200% BP	0.58	68,579,849.72	11	9	11	31	4,015.00	32,272,109.8
P2 with 100% BP	0.28	52,443,804.97	11	9	11	31	4,015.00	16,135,994.9
P2 with 200% BP	0.15	68,579,849.72	11	9	11	31	4,015.00	32,272,110.0
P1 with 200% BP	0.54	65,864,193.92	13	10	12	35	4,686.99	24,480,209.4
P2 with 200% BP	0.66	65,864,193.92	13	10	12	35	4,687.00	24,480,239.8
With 3-h time window								
P1 with 50% BP	0.17	48,257,747.49	11	10	13	34	4,519.00	6,973,433.8
P2 with 50% BP	0.18	48,257,747.49	11	10	13	34	4,519.00	6,973,433.0
P1 with 100% BP	0.18	55,231,182.35	11	10	13	34	4,519.00	13,946,912.8
P2 with 100% BP	0.17	55,231,182.35	11	10	13	34	4,519.00	13,946,873.0
P1 with 200% BP	0.19	69,178,052.07	11	10	13	34	4,519.00	27,893,705.5
P2 with 200% BP	0.18	69,178,052.07	11	10	13	34	4,519.00	27,893,705.0

*BP: bunker price.

Table 7. Amount of bunker purchased without time window at refill ports.

Port	50% Bunker price		100% Bunker price		200% Bunker price	
	Volume	Cost	Volume	Cost	Volume	Cost
Colombo	2,496.51	579,266.00	2,496.51	1,158,530.00	2,496.51	2,317,070.00
Hamburg	95.49	23,872.60	95.49	47,745.20	95.49	95,490.40
Hong Kong	626.91	156,728.00	626.91	313,455.00	626.91	626,911.00
Rotterdam	317.69	79,422.10	317.69	158,844.00	317.69	317,688.00
Shanghai	2,417.52	562,679.00	2,417.52	1,125,360.00	2,417.52	2,250,720.00
Singapore	37,519.00	7,933,980.00	28,906.10	12,250,500.00	19,630.00	16,709,200.00
Sokhna	5,527.41	1,215,760.00	2,313.24	1,081,560.00	2,313.24	2,163,130.00

Table 8. Amount of bunker purchased with 3-h time window.

Port	50% Bunker price		100% Bunker price		200% Bunker price	
	Volume	Cost	Volume	Cost	Volume	Cost
Colombo	2,527.28	585,728.00	2,527.28	1,171,460.00	2,527.28	2,342,910.00
Hamburg	90.90	22,724.40	90.90	45,448.80	90.90	90,897.50
Hong Kong	651.62	162,906.00	651.62	325,811.00	651.62	651,622.00
Rotterdam	323.17	80,791.40	323.17	161,583.00	323.17	323,166.00
Shanghai	2,417.52	562,679.00	2,417.52	1,125,360.00	2,417.52	2,250,720.00
Singapore	22,146.10	4,705,680.00	23,622.30	10,031,400.00	23,622.30	20,062,700.00
Sokhna	3,799.64	852,925.00	2,323.45	1,085,850.00	2,323.45	2,171,690.00

When a 3-h time window is imposed, the liner shipping company cannot adopt the slow steaming strategy, as there is a time window constraint. Therefore, the total transit time does not vary due to the change in BP. Therefore, the objective value and total bunker cost increase with an increasing BP. In this case, the computational advantage of the reformulation is not apparent, which can be the result of the smaller solution space with a 3-h time window.

Next, we perturb the minimum and maximum speeds of the vessels and observe the impact. Note that the min and max speed can be out of the speed range mentioned earlier in Table 3 in this sensitivity analysis. The purpose is to help the liner shipping company to evaluate the impact of speed when purchasing future vessels with better performance. As shown in Table 9, the decrease in the objective value is negligible with the speed change, indicating that the original operating speed is very close to the optimal speed. In these cases, we still see the computational advantage of using reformulation **P2**, which can save 74.72% of the CPU time on average.

Similarly, as the original operating speed is very close to the optimal speed, imposing the time window does not change the objective value significantly.

In the next experiment, we change the number of ports that can provide refilling services and observe the impact. Table 10 summarizes the results. Note that we limit the computational time allowed to one hour in all cases.

For the problem investigated in this research, the problem complexity grows significantly with the

Table 9. The results of adjusting the vessel's speed interval.

	CPU Time (sec)	Objective Value	Min Speed	Max Speed	Vessel Number				Total Transit time	Total Bunker Cost
					Class 1	Class 2	Class 3	Total		
Without Time Window										
P1	0.90	52,443,804.97	13/12/11	26/24/22	11	9	11	31	4,015.00	1,6135,994.2
P2	0.28	52,443,804.97	13/12/11	26/24/22	11	9	11	31	4,015.00	16,135,994.9
P1	1.22	52,443,577.63	11/10/9	28/26/24	11	9	11	31	4,015.00	16,135,821.0
P2	0.29	52,443,577.63	11/10/9	28/26/24	11	9	11	31	4,015.00	16,135,791.0
P1	1.26	52,442,987.39	9/8/7	30/28/26	11	9	11	31	4,015.00	16,135,258.8
P2	0.26	52,442,987.39	9/8/7	30/28/26	11	9	11	31	4,015.00	16,135,259.0
With 3-h time window										
P1	0.17	55,231,182.35	13/12/11	26/24/22	11	10	13	34	4519.00	13,946,912.8
P2	0.17	55,231,182.35	13/12/11	26/24/22	11	10	13	34	4519.00	13,946,873.0
P1	0.18	55,230,812.02	11/10/9	28/26/24	11	10	13	34	4519.00	13,946,526.9
P2	0.19	55,230,812.02	11/10/9	28/26/24	11	10	13	34	4519.00	13,946,526.9
P1	0.26	54,930,971.49	9/8/7	30/28/26	11	10	13	34	4351.00	15,450,787.8
P2	0.18	54,930,971.49	9/8/7	30/28/26	11	10	13	34	4351.00	15,450,788.0

Table 10. The results of adjusting the number of refill ports.

	Cplex Gap	CPU Time (sec)	Objective Value	Number of Refill Ports
Without Time Window				
P1	0.00%	0.90	52,443,804.97	7
P2	0.00%	0.28	52,443,804.97	7
P1	0.01%	9.03	52,443,804.97	14
P2	0.00%	0.55	52,443,804.68	14
P1	0.00%	477.18	52,443,804.97	28
P2	0.02%	1.81	52,443,804.97	28
P1	1.54%	3,601.48	52,443,804.97	46 (all port)
P2	0.02%	12.47	52,443,804.97	46 (all port)
With 3-h time window				
P1	0.00%	0.18	55,231,182.35	7
P2	0.00%	0.17	55,231,182.35	7
P1	0.13%	1.91	55,231,182.35	14
P2	0.00%	0.30	55,231,182.35	14
P1	1.42%	3,600.79	55,231,182.35	28
P2	0.07%	1.39	55,231,182.35	28
P1	0.87%	3,600.52	55,231,182.35	46 (all port)
P2	0.03%	5.92	55,231,182.35	46 (all port)

number of available refilling ports. Therefore, the computational advantage of using the reformulation becomes apparent, and on average an 80.61% saving of CPU time can be observed. However, with or without a time window, the current 7 refilling ports are sufficient, and therefore. Therefore, the value of objective function does not improve by varying the number of refill ports.

In the previous experiments, we employed the vessels designated in Table 2 and could not change the vessel type for each route. We call this operation mode the *fixed vessel type*. However, a liner shipping company may have the option to deploy an appropriate vessel type for each route based on its own situation. We call this operation mode the *flexible vessel selection*. Therefore, we conduct the test using the proposed framework and enable the model to select the appropriate type among the three types. The results are provided in Table 11.

In the cases where there is no time window constraint, the objective value decreases by 21.24%. Furthermore, all routes adopt class 1 as the vessel type, showing that this class of vessel is advantageous in deployment costs. When a 3-h time window constraint is imposed, the objective value still decreases drastically by 5.53%. However, class 1 vessels are not always the best option for all routes in this case. In some routes (i.e., route 5 and 11), other vessel types are more appealing (i.e., classes 2 and 3). In other words, the time window constraint can have an impact on vessel type selection. Nevertheless, the reformulation still demonstrates computational advantages and saves 29.26% of the CPU seconds.

5. Concluding remarks

In this research, we consider the stochastic sailing speed optimization and vessel deployment problem and propose a mixed integer nonlinear formulation. To address the challenge of solving the complicated mathematical formulation, we derived a new formulation based on Ng [1] and Ng [2]. Furthermore, the linearization techniques by Wang and Meng [3] are enhanced to tackle the problem. In other words, the current study employs the findings from Ng [1], Wang and Meng [3] and Ng [2] so that problem instances of an even larger scale can be solved. The proposed formulation has been numerically applied to solve realistic cases, and the following observations were made as a result of the numerical experiments [1]. The reformulation technique proposed in this research can save computational resources in solving this problem, especially when the number of refilling ports increases [2]. With an increase in bunker price, the slow steaming strategy can be an attractive operational strategy [3]. The slow steaming strategy is not favorable when the time window is considered [4]. The current operating speed is very close to the optimal speed [5]. A flexible vessel type operation can provide more flexibility for a liner shipping company and can result in lower operational costs. Furthermore,

Table 11. Comparison of fixed and flexible vessel type operations.

	CPU Time (sec)	Objective Value	Vessel Number				Total Transit time	Total Bunker Cost
			Class 1	Class 2	Class 3	Total		
Without Time Window								
P1 fixed vessel type	0.90	52,443,804.97	11	9	11	31	4,015.00	16,135,994.2
P2 fixed vessel type	0.28	52,443,804.97	11	9	11	31	4,015.00	16,135,994.9
P1 flexible vessel selection	7.09	41,305,531.76	31	0	0	31	4,015.00	16,022,453
P2 flexible vessel selection	6.69	41,305,531.76	31	0	0	31	4,015.00	16,022,453
With 3-h time window								
P1 fixed vessel type	0.18	55,231,182.35	11	10	13	34	4,519.00	13,946,912.8
P2 fixed vessel type	0.17	55,231,182.35	11	10	13	34	4,519.00	13,946,873.0
P1 flexible vessel selection	4.95	52,176,245.38	19	4	11	34	4,519.00	13,935,000.2
P2 flexible vessel selection	3.77	52,176,245.38	19	4	11	34	4,519.00	13,935,000.2

the time window constraint can have an impact on vessel type selection. The managerial insights from this study can help liner shipping companies determine bunker consumption and vessel deployment strategies.

The current research can be extended in various directions. For instance, we did not consider the loading/unloading of containers or cargo assignments in our formulation. Although incorporating container loading/unload and cargo assignments can significantly complicate the problem, it can also make the formulation more suitable for practical use. Furthermore, as there can be unexpected occurrences during the shipping process, many other parameters (i.e., shipping speed and oil price et al.) can be stochastic. Stochasticity can be good or bad for a transport system Wang and Wu [19]. Therefore, exploring a stochastic extension with the current framework can be an interesting and useful direction for future research. Finally, as major ports obtain the most revenue from bunkering services, the competition between refill ports can be considered in the future research.

Conflicts of interest

The authors declare that they have no conflicts of interest.

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