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A stochastic programming formulation for strategic fleet renewal in shipping

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ABSTRACT

Shipping companies repeatedly face the problem of adjusting their vessel fleet to meet uncertain future transportation demands and compensating for aging vessels. In this paper, a new multi-stage stochastic programming formulation for strategic fleet renewal in shipping is proposed. The new formulation explicitly handles uncertainty in parameters such as future demand, freight rates and vessel prices. Extensive computational tests are performed, comparing different discretizations of the uncertain variables and different lengths of the planning horizon. It is shown that significantly better results are obtained when considering the uncertainty of future parameters, compared to using expected values.

1. Introduction

Building a new ship can cost more than 200 million USD and capital expenses may account for 80% of the cost of running a shipping company with a fleet of modern vessels (Stopford, 2009, p. 269). For running a ten-year old ship the capital cost may still account for more than 40% of the total running costs (Stopford, 2009, p. 225), see Table 1. Having an unnecessarily large vessel fleet can thus be very expensive. On the other hand, too low capacity may imply that the shipping company cannot meet its obligations or that it has to use expensive charter options. It is therefore an important task for shipping companies to continually adjust their fleet composition to meet future transportation requirements.

Shipping companies also regularly make adjustments to their fleet due to aging vessels. When vessels reach a certain age they will become unprofitable to operate, due to increased risk of break-downs and increased maintenance and operation costs. Furthermore, vessels with new technology that improves fuel and cost efficiency may become available. Decisions about which fleet adjustments to make are usually considered regularly, typically once a year, following a thorough decision process. The decisions are often made with respect to forecasts that estimate future values of the main uncertain parameters, such as demands and prices. However, the shipping industry is among the most volatile businesses, with fluctuations in parameters that are difficult to predict. Even over just a few years, uncertainty in demands, costs and revenues related to the fleet operation is high. As an example, the capital valuation of a vessel may vary by a factor of three over only 2 years (Stopford, 2009).

In general, problems that deal with determining the size and composition of a fleet are often referred to as *fleet size and mix problems* (FSMP). In cases with only one vessel type, the problem becomes a *fleet size problem*. For land-based

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transportation problems there exists a significant amount of published work, see the review by Hoff et al. (2010). For several reasons, FSMPs for the land-based context are not directly transferable to the *maritime FSMP* (MFSMP). First, investments and thus capital costs are higher for maritime problems than for land-based problems. Second, the lifetime of a ship (typically 20–30 years) is much longer than for trucks, and thus uncertainty over the lifetime of ships are greater than for trucks. Third, ships are often more industry specific, implying that investments are sometimes irreversible and that the market for second-hand vessels is not as liquid as the one for trucks. Fourth, the valuation of ships is more complicated than that of trucks. The value of a truck is often assumed to be strictly decreasing with age, while for ships there are additional factors that affect the value. Adland and Koekebakker (2007) use age and the state of the freight market to determine the value of a ship. Fifth, there are different operational challenges between maritime and land-based FSMPs, see for example the discussion by Christiansen et al. (2013).

The literature on the MFSMP is scarce. However, since the first known work within the topic (Dantzig and Fulkerson, 1954), there have been some specific studies from which to draw information, especially during recent years. Even so, more research is still required. First of all, there exists only a limited number of publications on FSMP for any transportation mode that considers uncertainty (Verderame et al., 2010). Furthermore, even within deterministic approaches to the MFSMP there are knowledge gaps. A recent literature study by Pantuso et al. (2014) surveys the literature regarding the MFSMP. One of the conclusions is that, in addition to the lack of studies treating uncertainty, most research studies consider the design of a brand new fleet to transport a given demand, see for example Jaikumar and Solomon (1987) and Zeng and Yang (2007). This is in contrast to the more common case in the industry where an initial fleet must be adjusted over time, which calls for a time-staged modeling approach. Such an approach must take into account possible changes in demand over time and the timing of the fleet alterations, in addition to the uncertainty factors.

A recent contribution to the literature on MFSMP is by Meng and Wang (2011), which proposed a time-staged model with an initial fleet for the MFSMP in liner shipping. A limited number of possible fleet configurations is considered for each time period, and the problem is solved by using dynamic programming. The possible fleet configurations that meet the expected transportation demand should be provided by the shipping company's experts. However, the proposed model does not take into account uncertainty in the demand forecast. Meng et al. (2014) also base their solution method for the fleet planning problem of liner container shipping on a limited number of fleet configurations made by company experts, but include uncertainty in demand as part of the deployment considerations. The demand is assumed to be dependent on that of the previous period. Which ship is deployed at which route is, together with the number of voyages and amount of lay-up, determined before the actual demand is revealed. Then the amount of cargo carried is determined after demand is revealed. Within airline fleet composition, Listes and Dekker (2005) show the importance of accounting for uncertainty when determining a robust airline fleet composition. The airline mode is the mode most like the international shipping mode in terms of investment costs, but meets different operational challenges. Another contribution on the MFSMP is by Alvarez et al. (2011), who proposed a mixed integer programming (MIP) model of the multi-period fleet sizing and deployment problem in bulk shipping. They extend the MIP model into a robust optimization model to account for uncertainty in purchase prices, sale prices, sunset values, and charter rates. Uncertainty in demand is not considered within the model.

A decision support methodology for strategic planning in tramp and industrial shipping was presented by Fagerholt et al. (2010). This proposed methodology combines simulation and optimization by building a Monte Carlo simulation framework around an optimization-based decision support system for short-term routing and scheduling. The simulation proceeds by considering a series of short-term routing and scheduling problems using a rolling horizon principle where information is revealed as time goes by. The uncertainty of the parameters is then treated within the simulation. The approach can easily be configured to provide decision support for a wide range of strategic planning problems, such as fleet size and mix problems, analysis of long-term contracts, and contract terms. However, as with Meng and Wang (2011), the approach is not efficient in cases where there is a large number of alternative fleet configurations to evaluate. It can therefore only be used as an evaluation tool for a limited number of fleet composition scenarios.

For a comprehensive literature review on FSMP in general, we refer to Hoff et al. (2010), while we refer to Pantuso et al. (2014) for a survey on specific maritime cases. For a recent literature review on planning and scheduling under uncertainty across multiple sectors readers are referred to Verderame et al. (2010).

In contrast to the majority of literature on MFSMP, in most practical situations there already exists an initial fleet. The problem then becomes how and when to make long-term alterations to the vessel fleet, rather than determining the optimal fleet given no starting position. To emphasize the difference in these problem structures, we name the problems that require an initial fleet the *strategic fleet renewal problem in shipping* (SFRPS). The objective of the SFRPS is to minimize the expected long-run cost of serving the company's demand, by doing the best adjustments of the vessel fleet.

Table 1
Rough guide to the cost structure of a 10-year-old Capesize bulk carrier (Stopford, 2009).

Operating costs	14%
Periodic maintenance	4%
Voyage costs	40%
Capital costs	42%

The contributions of this paper are (1) the presentation of a new multi-stage stochastic programming formulation for the SFRPS and (2) a computational study that tests how various scenario tree structures affect the solution. The new formulation considers the existence of an initial fleet, the present uncertainty in the future parameters and different ways of changing the vessel fleet. This paper studies the SFRPS for liner shipping, but the same model can easily be adapted to for example industrial bulk shipping problems.

The remainder of this paper is organized as follows. In the next section the problem is thoroughly examined and explained. In Section 3 a stochastic programming model is outlined. Section 4 presents the computational experiments undertaken, while the concluding remarks are contained in Section 5.

2. Problem description

A shipping company's vessel fleet usually consists of different vessel types, that is, the vessel fleet is *heterogeneous*. The shipping company uses this fleet to operate on trade routes and transport cargo between ports along these routes. In liner shipping the shipping company publishes schedules of which ports they will visit, the sequence in which the ports will be visited, and the time at which each visit will take place. A sequence of port calls is called a *route*. Fig. 1 shows two examples of routes for an international shipping company. A vessel may, for example, first sail the route in Fig. 1(a), then go from Bremerhaven to Gothenburg and sail the route in Fig. 1(b).

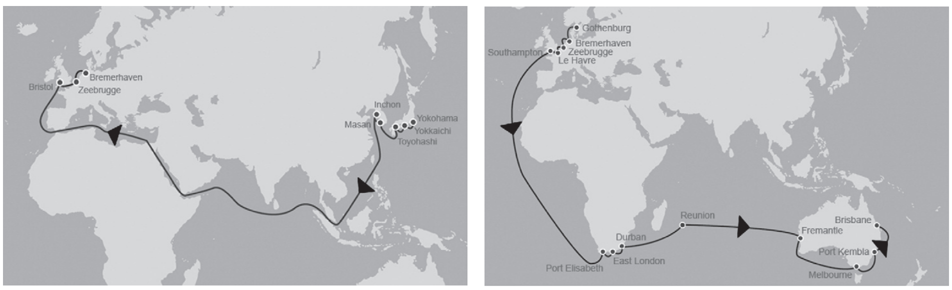
The cargo transported is either *contractual agreed cargo* (CAC) or *spot cargo*. For CAC there exists different contracted terms but all contracts include terms which define how much to transport, when to transport and how often to transport. A spot cargo is a one-time agreement to ship a quantity of cargo from one port to another. In strong market conditions, there is generally a high profit margin for spot cargo. However, in depressed markets spot rates, together with spot demand, can collapse. For example, early in 2009 some container liner shipping companies offered freight rates close to zero on their Asia–Europe trade (UNCTAD, 2010).

The decisions that have to be considered when approaching the SFRPS can be divided into two different types of decisions, strategic decisions and tactical decisions, which can be distinguished by the time horizon for which they affect the company's operation. The strategic decisions include decisions about acquiring vessels and permanently removing vessels from the fleet. These decisions imply changes to the fleet that will persist for a long time, and that will influence later decisions at all levels. The tactical decisions include how many vessels should be chartered in and out, how much spot cargo to carry, as well as how to deploy the ships. These decisions affect the future decisions on a much shorter time horizon.

When the shipping company decides which vessels to acquire and which vessels to remove from the fleet, this is done on the basis of the characteristics and the inherent parameters of the vessels. Characteristics of vessels may vary substantially and define how the vessel can be utilized. For example, different vessels can transport different types of cargo, and a vessel's draft may limit the set of ports which it can visit.

A shipping company can acquire ownership of vessels in two ways: by ordering newbuilds from a shipyard or by acquiring vessels in the secondhand market. Shipyards have limited capacity and can only deliver a limited number of vessels each year, and in the secondhand market there is a limited supply of vessels that can be bought. When ordering from a shipyard the vessels have longer delivery times than if bought in the secondhand market. Capital investments are usually restricted by some sort of budget constraint. When a shipping company wants to remove a vessel, there are also two ways to do this. One way is to sell the vessel in the secondhand market. As with supply in the secondhand market, the demand is limited, and selling a vessel when market expectations are low can be difficult. An option which is always available is to scrap the vessel. When a vessel is scrapped the shipping company gets paid the ships value in steel and the price therefore depends on the steel price.

The SFRPS is to decide how and when to adjust a vessel fleet to efficiently meet transportation demands. The decisions of which ships to buy, build, sell or scrap have to be made with uncertainty in parameters such as future ship prices, demand and charter rates. When solving the SFRPS, we are in principle only interested in the here-and-now decisions, that is, the strategic decisions regarding the adjustments of the fleet that must be made here and now. The tactical decisions, as well



(a) Asia to Europe (b) Europe to Oceania via South Africa

Fig. 1. Example of routes (Wallenius-Wilhelmsen-Logistics, 2010).

as the future strategic decisions, are only relevant for supporting the here-and-now decisions. When the tactical decisions are to be executed some time has elapsed and the company knows more about the relevant parameters. So another difference between the two types of decisions is what information the shipping company possesses when the decisions are made. The strategic decisions are made with greater uncertainty than the tactical ones.

3. A stochastic programming formulation

When modeling a multi-period problem with uncertainty it is important to be aware of when the different decisions are made and when new information arrives. The decision structure that we assume for the SFRPS is illustrated in Fig. 2. The strategic decisions are made without knowing the realization of future parameters, while we assume that the tactical decisions are made when the parameters for that period has become known. Since the SFRPS will normally be repeatedly solved once a year for a shipping company, we are, as mentioned in Section 2, mainly interested in the strategic decisions adjusting the fleet composition that has to be made here and now. However, tactical decisions and future strategic decisions are included to support the here-and-now decisions.

We consider parameters that are linked to the market situation in maritime shipping as uncertain. This includes:

- Newbuild and secondhand prices
- Demand for vessels in the secondhand market
- Supply of vessels in the secondhand market
- Capacity at the wharfs
- Scrap rates
- Demand for charter vessels
- Charter rates
- Amount of CAC to carry (CAC demand)
- Spot cargo demand
- Spot cargo rates
- Operating costs

All of these factors affect the critical decisions regarding which adjustment to make for the current vessel fleet. In the stochastic programming model the uncertain parameters are discretized, by creating a scenario tree. To handle the end of the planning horizon no strategic decisions are made in the last period. Instead a *sunset value* of the vessel fleet is added at the end of the planning horizon. In Alvarez et al. (2011) this value corresponds to the estimated revenue of the remaining lifetime and eventual sale or demolition for the vessel fleet in the last period. We choose to set this equal to the secondhand value of the vessel in the last period of the planning horizon, as it is assumed that the estimated revenue of the remaining lifetime is embedded in the secondhand value.

Since we are not interested in the tactical deployment decisions except for providing support to the strategic here-and-now decisions, the underlying deployment of the vessels is modeled at an aggregated level. Hoff et al. (2010) mentioned that it will typically not make sense to include routing aspects at a very detailed level in strategic fleet planning, unless the transportation demand is highly predictable. We define *trades* as routes that are restricted to only have pick-up ports in one region and delivery ports in another region. Based on this assumption we can model a trade as only having one pickup point, A, and one delivery point, B. The demand on the trade from A to B is then the total demand on all of the pickup ports which are aggregated into A to the delivery ports in B. A trip made on a trade from port A to port B, and then back again to A is defined as a *voyage*. Vessels are divided into different *vessel types*, and a vessel type is defined by its capacity, speed, age and cost structure. The speed used to define a vessel type is its service speed. Using age as a parameter in the definition of vessel type implies that two vessels produced at different years, but otherwise equal, will be classified as two different vessel types. The deployment is solved by determining the number of voyages that is performed by each vessel type for every trade.

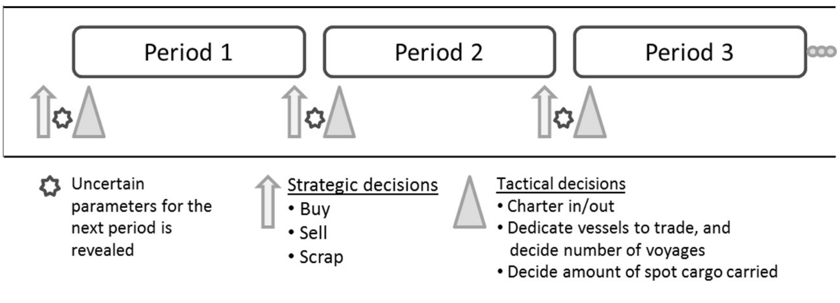


Fig. 2. Decision structure for the SFRPS.

The vessel types have different maintenance needs due to their different age and model characteristics, yielding different costs and times required for maintenance each year. The shipping company has information about this, and in the model it is taken into account by including the maintenance costs in the fixed cost of owning a vessel and restricting the number of days available for operations in any period.

The availability of charter-in vessels are modeled in price steps. At a given price it is possible to charter-in a certain number of vessels, but for a higher price additional vessels may be available. The model formulation facilitates several price steps for each vessel type.

The stochastic programming model for the SFRPS is given as a node formulation. More precisely the presented model is the deterministic equivalent program (DEP) of the problem, which can be solved directly using a standard MIP solver. For an introductory tutorial on stochastic programming we refer to Higle (2005). For more thorough texts we refer to Kall and Wallace (1994) and Birge and Louveaux (2011). In a node formulation every node has an associated time period. In the scenario tree of Fig. 3 there are seven nodes. The time period of node 1 is 0, the time period of nodes 2 and 3 is 1 and the time period of nodes 4 to 7 is 2. This property is used to take into account parameters that change in time or that are scenario specific. For example, the aging of the fleet is modeled by adjusting the related parameters in accordance with the changing physical and monetary properties of an aging ship, such as increased operating costs and more time needed for maintenance.

3.1. Notation

3.1.1. Indices

r	A trade.
k	A vessel type.
e	A node in the scenario tree.
w	A charter-in price level.

3.1.2. Sets

$\mathcal{R}$	Set of all trades.
$\mathcal{K}$	Set of all vessel types.
$\mathcal{E}$	Set of all nodes in the scenario tree.
$\mathcal{E}^1$	Set of all non-leaf nodes in the scenario tree ($\mathcal{E}^1 \subset \mathcal{E}$).
$\mathcal{E}^2$	Set of all nodes in the scenario tree except the root node ($\mathcal{E}^2 \subset \mathcal{E}$).
$\mathcal{H}_{ke}$	Set of all nodes e' of e, such that if a newbuild of vessel type k is ordered in e' it is delivered in e.
$\mathcal{W}$	Set of all charter-in price levels.

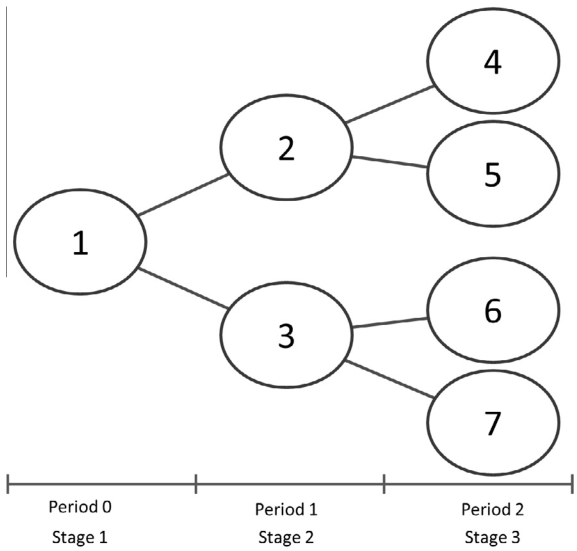


Fig. 3. An example of a scenario tree.

As shown in Fig. 2, not all decisions are made in all periods. The nodes are therefore divided into two sets. Set $\mathcal{E}^1$ contains all nodes corresponding to periods with strategic decisions, and set $\mathcal{E}^2$ contains all nodes corresponding to periods with tactical decisions. These two node sets have a large intersection. Furthermore, the set $\mathcal{H}_{ke}$ has been introduced to be able to describe the number of newbuilds of type k that are to be delivered in node e. Fig. 3 shows an example of a scenario tree with four scenarios and seven nodes. This example has the set of nodes $\mathcal{E} = \{1,2,3,4,5,6,7\}$ and the subsets $\mathcal{E}^1 = \{1,2,3\},\ \mathcal{E}^2 = \{2,3,4,5,6,7\}$.

3.1.3. Parameters

Parameters marked with an asterisk ($*$) are those that are subject to uncertainty.

A_e		Available capital for investment at node e.
B_{ke}^{SH}	$*$	Cost of buying a secondhand vessel of type k at node e.
B_{ke}^{NB}	$*$	Cost of buying a newbuilding of type k at node e.
$B_{ke}^{NB^{IN}}$		Number of vessels of type k delivered at node e from orders placed before period 0.
C_{rke}^{V}	$*$	Cost of one voyage on trade r, with vessel of type k at node e.
C_{ke}^{F}		Fixed cost of one vessel of type k at node e.
D_{re}^{CAC}	$*$	CAC to carry on trade r at node e.
D_{re}^{S}	$*$	Demand in units of spot cargo on trade r at node e.
I_{re}^{S}		Income per unit carried on the spot market on trade r at node e.
K_{ke}		Maximal number of vessel type k available at node e
L_{re}^{V}		The number of voyages that must be sailed on route r at node e.
M_{ke}^{SH}	$*$	Upper bound on the number of secondhand vessels of type k bought at node e.
M_{ke}^{NB}	$*$	Upper bound on the number of new build vessels of type k bought at node e.
M_{ke}^{SOLD}	$*$	Upper bound on the number of vessels of type k sold at node e.
N_{ke}^{O}	$*$	Charter out rate for vessels of type k at node e.
N_{ke}^{I}	$*$	Charter in minimum rate for vessels of type k at node e.
$N_{w}^{I_{exp}}$		Charter in multiplier for price level w. $N_1^{I_{exp}} = 1,\ N_w^{I_{exp}} < N_{w+1}^{I_{exp}}$
N_{ke}^{DEM}	$*$	Upper bound on the number of vessels of type k chartered out at node e.
N_{kew}^{SUPP}	$*$	Upper bound on the number of vessels of type k chartered in at node e at price level w.
P_{rke}		Maximum number of voyages on trade r of one vessel of type k at node e.
Q_k		Capacity in units of vessel type k.
S_{ke}^{SH}	$*$	Selling price for a vessel of type k at node e.
S_{ke}^{SC}	$*$	Scrap rate of a vessel of type k at node e.
V_{k0}^{IN}		Number of vessel type k owned initially.
Z_{e}^{DISC}		Discount factor for node e.
p_e		Probability of reaching node e.
e^p		Preceding node of node e.

3.1.4. Variables

b_{ke}^{SH}	Number of vessels of type k bought secondhand at node e.
b_{ke}^{NB}	Number of vessels of type k ordered at node e.
l_{rke}	Number of voyages on trade r with vessels of type k at node e.
n_{kew}^{I}	Number of vessels of type k chartered in at price level w at node e.
n_{ke}^{O}	Number of vessels of type k chartered out at node e.
s_{ke}^{SH}	Number of vessels of type k sold at node e.
s_{ke}^{SC}	Number of vessels of type k scrapped at node e.
u_{ke}	Number of vessels of type k operated at node e.

(continued on next page)

v_{ke}	Number of vessels of type k owned at node e.
x^S_{re}	Number of units of spot cargo carried on trade r at node e.
y_{rke}	Number of vessels of type k dedicated to trade r at node e.

3.2. Mathematical model

$$\min Z = \sum_{e\in\mathcal{E}^1} \frac{p_e}{Z^{DISC}_e}\left[\sum_{k\in\mathcal{K}}\left(B^{SH}_{ke}b^{SH}_{ke} + B^{NB}_{ke}b^{NB}_{ke} - S^{SH}_{ke}s^{SH}_{ke} - S^{SC}_{ke}s^{SC}_{ke}\right)\right] + \sum_{e\in\mathcal{E}^2}\frac{p_e}{Z^{DISC}_e}\left[\sum_{r\in\mathcal{R}}\sum_{k\in\mathcal{K}} C^V_{rke}l_{rke}\right. \tag{1}$$

$$\left. -\sum_{r\in\mathcal{R}} I^S_{re}x^S_{re} + \sum_{k\in\mathcal{K}}\left(\sum_{w\in\mathcal{W}} N^{I_{exp}}_w N^I_{ke} n^I_{kew} + C^F_{ke}v_{ke} - N^O_{ke}n^O_{ke}\right)\right] - \sum_{e\in\mathcal{E}\setminus\mathcal{E}^1} p_e \frac{S^{SH}_{ke}v_{ke}}{Z^{DISC}_e} \tag{2}$$

The objective function represents the expected net cost, consisting of the cost of vessels bought, the cost of transportation operations on the spot market and operational expenses minus the income of vessels sold and sunset values. It includes every node in the scenario tree, and weights the cost with the probability of ending up going through the node. The first part of term (1) is the net discounted cost of vessels sold, scrapped and bought, that is, the effect of the strategic decisions. The second part is the discounted operational vessel costs. Term (2) adds fixed vessel costs and charter-in costs and subtracts income from carriage of spot cargo and charter out. The last part of (2) is the discounted sunset value of the fleet. Zero freight premium is assumed for newer tonnage, and as all the CAC must be carried in order to produce a feasible solution the CAC is left out of the objective function. The assumption of zero freight premium is consistent with the work of Tamvakis and Thanopoulou (2000), who conclude that in most cases, there has been no statistically significant difference between rates paid to older and newer tonnage.

$$v_{ke} = V^{IN}_{k0} \quad k\in\mathcal{K},\ e\in\mathcal{E}\setminus\mathcal{E}^2 \tag{3}$$

$$v_{ke} = v_{ke^p} + b^{SH}_{ke^p} + \sum_{e'\in\mathcal{H}_{ke}} b^{NB}_{ke'} + B^{NB^{IN}}_{ke} - s^{SH}_{ke^p} - s^{SC}_{ke^p} \quad k\in\mathcal{K},\ e\in\mathcal{E}^2 \tag{4}$$

$$u_{ke} = v_{ke} - n^O_{ke} + \sum_{w\in\mathcal{W}} n^I_{kew} \quad k\in\mathcal{K},\ e\in\mathcal{E}^2 \tag{5}$$

$$x^S_{re} \leqslant D^S_{re} \quad r\in\mathcal{R},\ e\in\mathcal{E}^2 \tag{6}$$

$$\sum_{r\in\mathcal{R}} y_{rke} \leqslant u_{ke} \quad k\in\mathcal{K},\ e\in\mathcal{E}^2 \tag{7}$$

$$l_{rke} \leqslant P_{rke}y_{rke} \quad r\in\mathcal{R},\ k\in\mathcal{K},\ e\in\mathcal{E}^2 \tag{8}$$

$$D^{CAC}_{re} + x^S_{re} \leqslant \sum_{k\in\mathcal{K}} Q_k l_{rke} \quad r\in\mathcal{R},\ e\in\mathcal{E}^2 \tag{9}$$

$$n^I_{kew} \leqslant N^{SUP}_{kew} \quad k\in\mathcal{K},\ e\in\mathcal{E}^2,\ w\in\mathcal{W} \tag{10}$$

$$n^O_{ke} \leqslant N^{DEM}_{ke} \quad k\in\mathcal{K},\ e\in\mathcal{E}^2 \tag{11}$$

$$s^{SH}_{ke} \leqslant M^{SOLD}_{ke} \quad k\in\mathcal{K},\ e\in\mathcal{E}^1 \tag{12}$$

$$b^{SH}_{ke} \leqslant M^{SH}_{ke} \quad k\in\mathcal{K},\ e\in\mathcal{E}^1 \tag{13}$$

$$b^{NB}_{ke} \leqslant M^{NB}_{ke} \quad k\in\mathcal{K},\ e\in\mathcal{E}^1 \tag{14}$$

$$v_{ke} \leqslant K_{ke} \quad k\in\mathcal{K},\ e\in\mathcal{E} \tag{15}$$

$$\sum_{k\in\mathcal{K}} l_{rke} \geqslant L^V_{re} \quad r\in\mathcal{R},\ e\in\mathcal{E}^2 \tag{16}$$

$$\sum_{k\in\mathcal{K}}\left(B^{SH}_{ke}b^{SH}_{ke} + B^{NB}_{ke}b^{NB}_{ke} - S^{SH}_{ke}s^{SH}_{ke} - S^{SC}_{ke}s^{SC}_{ke}\right) \leqslant A_e \quad e\in\mathcal{E}^1 \tag{17}$$

$$x^S_{re} \geqslant 0 \quad r\in\mathcal{R},\ e\in\mathcal{E}^2 \tag{18}$$

$$n^O_{ke}, u_{ke} \geqslant 0 \quad k\in\mathcal{K},\ e\in\mathcal{E}^2 \tag{19}$$

$$n^I_{kew} \geqslant 0 \quad k\in\mathcal{K},\ e\in\mathcal{E}^2,\ w\in\mathcal{W} \tag{20}$$

$$v_{ke} \geqslant 0 \quad k\in\mathcal{K},\ e\in\mathcal{E} \tag{21}$$

$$s^{SH}_{ke},\ s^{SC}_{ke},\ b^{SH}_{ke},\ b^{NB}_{ke} \geqslant 0,\ \text{integer} \quad k\in\mathcal{K},\ e\in\mathcal{E}^1 \tag{22}$$

$$l_{rke},\ y_{rke} \geqslant 0 \quad r\in\mathcal{R},\ k\in\mathcal{K},\ e\in\mathcal{E}^2 \tag{23}$$

Constraints (3) initialize the variables to match the current fleet. Constraints (4) keep track of the adjustments of the vessel fleet from one period to the next. The number of vessels of each type that the company will have available for their own operations is determined by constraints (5). Constraints (6) limit the amount of spot cargo that can be carried.

Constraints (7)–(9) control the use of the vessels to transport cargo. Constraints (7) restrict the number of vessels of each type that can be dedicated to the trades. Based on how many vessels that are dedicated to a trade, constraints (8) limit for each vessel type, the number of voyages that can be performed on a trade. Constraints (9) set a lower limit on the total capacity on each trade. All CAC has to be carried in addition to the chosen spot cargo.

The model considers charters that are less than, or equal to one period. If the periods are set to 1 year, only time-charters with duration of less than 1 year is applicable. The model consider the daily charter cost as fixed over any duration. Constraints (10) limit the number of vessels that can be chartered in for each of the price levels.

Constraints (11)–(14) set an upper limit on the number of vessels chartered out, the number of vessels sold secondhand, the number of vessels bought secondhand and the number of vessels bought from the wharfs, respectively. Constraints (15) limit the total number of vessels of a given type in the fleet. Constraints (15) are also used to model the introduction and retirement of vessels by using a limit of zero both before and after its lifetime. Constraints (16) require the number of voyages on a trade to be higher than a preset number. This is a requirement according to the published schedules of a liner shipping company, based on the number of departures for a given trade. Constraints (17) limit the net investment made in a year to the budget constraint given by a shipping company. These budget constraints represent the amount of capital, both equity and loan, which the shipping company has allocated to finance its vessels. Financing of a ship is not spread over several years since it is assumed that when a vessel is to be bought the capital has to be available. For some fleet renewal problems it would have been reasonable to include funds left from previous periods on the right hand side of constraints (17), see for example, Alvarez et al. (2011).

Constraints (18)–(23) set non-negativity and integer requirements. The strategic decisions have integer requirements while the tactical decisions do not. The modeling of the deployment is aggregated to a level where the position of each individual ship is not accounted for, and the sole purpose is to give feedback to the fleet capacity and composition. Without integer restrictions on l_{rke} (number of voyages) it is possible to sail a fraction of a voyage, and without integer restrictions on y_{rke} it is possible to dedicate a fraction of a vessel to one trade, and the rest to another trade. The sailing between these trades are not explicitly handled in the model. However, a voyage is defined to include both the sailing on the trade and the ballast sailing back to the start, whereas in practice the ballast leg could be taken between different trades instead of back to the start. Whether continuous variables lead to optimistic or pessimistic estimates depends on the lengths of the trades and the distances between the end of one trade and the start of the next trade.

The charter in and charter out variables are not restricted to be integers. This means that it is possible to charter a fraction of a ship. It will for example be possible to charter in 1.5 vessels in a given period. In practice this may correspond to charter in one vessel for the entire period and one vessel for half a period.

4. Computational study

The test case used in this computational study is a hypothetical shipping company, inspired by a roll-on/roll-off liner shipping company, that initially operates a heterogeneous fleet of 44 vessels distributed among five different vessel types. Over the planning horizon, 32 different vessel types are available.

The shipping company has long-term contracts on five trades shown in Table 2. The frequency of the trade gives the minimum number of voyages that has to be performed in one period. The distances have been found using Sea-Rates (Sea-Rates.com, 2011). The modeled demand is both the long-term contracted demand and the demand in the spot market. The values of the model parameters are estimated based on available statistics such as Stopford (2009). Numerical values for some of the parameters are easily calculated based on physical restrictions, for example the upper bound on number of voyages that can be performed by a vessel in one period. Other parameters are more complicated to assess and need a greater degree of estimation, for example the demand for charter vessels. For some parameters, a relation that connects its numerical value to other features or characteristics can be found. The price of a vessel is for example dependent on the vessel's size, age and the market state.

The charter-in market is represented by two levels, that is $|\mathcal{W}| = 2$, with $N_1^{I_{exp}} = 1$ and $N_2^{I_{exp}} = 10$, meaning that vessels chartered in at the second level is ten times as expensive as level one. The values for the second level corresponds to an estimation of the cost of not being able to transport the cargo. The values for N_{ke1}^{SUPP} depend on the market situation while we set

Table 2
Trades served by a hypothetical liner shipping company.

Trade	Distance (Nm)	Frequency
Asia to North America	7753	Weekly
Intra Asia	4934	Biweekly
Asia to Europe	11,599	Weekly
North America to Europe	5183	Monthly
Europe to Australia	12,313	Every 3rd week

$N^{SUPP}_{ke2} = \infty$ for all vessel types k that are within its lifetime which means that there are no limits on chartering in vessels at the second level. This ensures that the model has relatively complete recourse, as long as there is at least one vessel type available at each time period.

The duration of each period is set to 1 year in the following computational study, since the SFRPS is normally solved once a year. The SFRPSs were solved by FICO Xpress Optimization Suite by Dash Optimization on a 64-bit computer running Windows 7 with 16 GB of memory and a quad-core 2.30 GHz processor.

4.1. Modeling uncertainty

The SFRPS model presented in Section 3 requires that the uncertainty is captured in scenarios. That is, we need a discrete representation of the future such that it can be represented by a scenario tree. Here we assume there exists a continuous probability distribution for the future parameters, and we discretize this to create the scenario tree.

In the maritime shipping industry there are close correlations between for example freight rates, demand, spot charter prices and second-hand vessel prices. In the computational study, we assume, as a simplification, that all of our random parameters are perfectly correlated, and that these depend on the general market situation. In other words, the shipping company operates in a market where the uncertainty of the parameters depends only on the market situation. To model this market situation, a stochastic variable `marketStatus` is defined, and once we know its value we also know the uncertain parameters of the model, such as demand and ship prices. Let `marketStatus` vary between 0 and 1, which corresponds to the worst and best possible market situations, respectively. A joint probability distribution between the uncertain parameters can be created so that if $P(\alpha = x, \beta = y) \geqslant 0$ then $P(\alpha = x, \beta = y') = 0$ for all $y' \neq y$ where x and y are realizations of the two different uncertain parameters α and β. Each of the uncertain parameters can be used to determine `marketStatus`. Adland and Koekebakker (2007) use the 1-year time-charter rate as a proxy for the state of the freight market.

Let z be a year in the future, and let $P(m, z)$ be the probability of `marketStatus` having the value m in year z, given that we do not know what the `marketStatus` is today (or that z is sufficiently distant). We assume $P(m, z)$ to have a symmetric, bell shaped, truncated probability distribution with mean $\tilde{\mu} = 0.5$ and lower and upper bounds of 0 and 1, respectively. Furthermore, we assume, for simplicity, that the probability distribution for `marketStatus` for the next period depends only on the `marketStatus` of the current period, that is, it has the Markov property. The `marketStatus` for the next period follows a truncated normal distribution with mean, standard deviation and upper and lower bound determined by the current `marketStatus`. Let $X(m)$ be the `marketStatus` for the next period given the current status m, and let y be the number of years in the next period. Then

$$X(m) \sim N(\mu, \sigma^2), \quad X(m) \in (a, b) \tag{24}$$

$$\mu = \begin{cases} m + \min(\lambda_\mu y(\tilde{\mu} - m), \tilde{\mu} - m) & \text{if } m < \tilde{\mu} \\ m - \min(\lambda_\mu y(m - \tilde{\mu}), m - \tilde{\mu}) & \text{if } m \geqslant \tilde{\mu} \end{cases} \tag{25}$$

$$\sigma = \frac{\underline{\sigma} + \lambda_\sigma (m - \tilde{\mu})^2}{y} \tag{26}$$

$$a = \max(\underline{m}, m - \beta - y/50) \tag{27}$$

$$b = \min(\overline{m}, m + \beta + y/50) \tag{28}$$

where $\lambda_\mu = 0.2$ and $\lambda_\sigma = 0.3$ are multipliers less than one that define how fast the mean approaches the steady state mean, $\tilde{\mu} = 0.5$, and how fast the standard deviation increases with the distance from the steady state mean, respectively. The lowest possible standard deviation for a period of one year is $\underline{\sigma} = 0.1$. The lowest and the highest possible `marketStatus` are represented by $\underline{m} = 0$ and $\overline{m} = 1$, respectively. The truncation of the distribution is adjusted by $\beta = 0.8$.

In the later computational tests, this distribution is taken to represent the true development of the `marketStatus`. However, the model requires a discrete representation of the uncertainty in the form of scenarios. A common method to generate scenarios is to use sampling, as in the sample average approximation method (Verweij et al., 2003), although viable alternatives exists, such as the moment matching heuristic (Høyland et al., 2003). Here, an alternative to sampling is proposed, which is better suited when generating small scenario trees. In the alternative discretization technique used here the possible `marketStatus` values between 0 and 1 are divided into n equally sized subintervals, where the midpoint of the interval is chosen to be the representative point.

In a scenario tree, a scenario is a path through the nodes, from the root node up to a leaf node. In the example in Fig. 4 there are six scenarios. To represent the number of stages and the number of discretization intervals within the stages we introduce the notation $[k_2, k_3, \ldots, k_n]$, where k_i is the number of intervals of `marketStatus` considered in stage i, and n is the number of stages. The first stage is not included in the notation since the first stage corresponds to the current `marketStatus` which is known with certainty. Using this notation the scenario tree of Fig. 4 is $[3, 2]$. To get the numbers of stage 2 the range $(0, 1)$ is divided into 3 equally sized parts: $(0, 0.33)(0.33, 0.67)(0.67.1)$. Each of these intervals are represented by the midpoint of the interval: 0.176, 0.5 and 0.833 respectively.

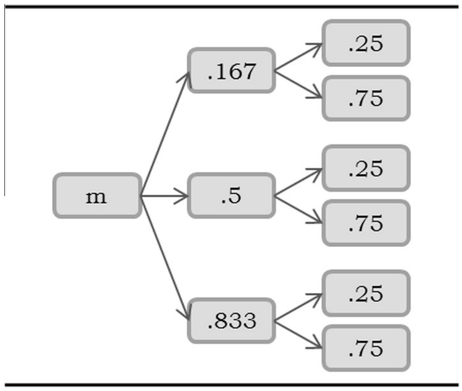


Fig. 4. Scenario tree for a problem with three stages. The first discretization with three intervals and the second with two intervals. The values in the nodes are the corresponding `marketStatus`, and m is the current `marketStatus`.

For every node, except the leaf nodes, a discrete probability distribution for the transition to the next stage is calculated from the presented continuous distribution. The probability of going from a node n_1 to a succeeding node n_2 is calculated using the formula presented in (29). The notation is the same as presented for the continuous distribution, but $\underline{i}$ and $\bar{i}$ is added and is defined to be the lower and upper bound of the interval represented by the node n_2. The `marketStatus` of node n_1 is denoted m, and $\Phi(\bullet)$ is the cumulative distribution function of the standard normal distribution.

$$P(\underline{i} < X(m) < \bar{i}|a < X(m) < b) = P(X(m) < \bar{i}|a < X(m) < b) - P(X(m) < \underline{i}|a < X(m) < b)$$
$$= \frac{\Phi(\frac{\bar{i}-\mu}{\sigma}) - \Phi(\frac{a-\mu}{\sigma})}{\Phi(\frac{b-\mu}{\sigma}) - \Phi(\frac{a-\mu}{\sigma})} - \frac{\Phi(\frac{\underline{i}-\mu}{\sigma}) - \Phi(\frac{a-\mu}{\sigma})}{\Phi(\frac{b-\mu}{\sigma}) - \Phi(\frac{a-\mu}{\sigma})} \tag{29}$$

In the example of Fig. 4 there are three different probability distributions for the transition from the second stage to the third stage. If these are put together they form a *transition matrix* with three states on the vertical axis and two states on the horizontal axis. Table 3 shows this transition matrix with values. There will be as many transition matrices as there are periods. Note that the first stage is not a period and that there is one more stage than there are periods. Based on these transition matrices the probability of visiting a node is calculated.

4.2. Technical analysis of solving the SFRPS

The SFRPS model presented in Section 3 can become hard to solve when there is a large number of variables, which again depends on the size of the scenario tree and the number of vessel types and trades available. The number of integer variables is given by $4 * |\mathcal{E}^1| * |\mathcal{K}|$ and the number of continuous variables is $|\mathcal{E}^2| * |\mathcal{R}| + (2 + |\mathcal{W}| + 2 * |\mathcal{R}|) * |\mathcal{E}^2| * |\mathcal{K}| + |\mathcal{E}| * |\mathcal{K}|$. As an example, for the hypothetical shipping company used in the computational study, the average number of variables for a scenario tree with 2700 scenarios and branching factors $[30, 30, 30]$ is about 1.45 million, of which about 60,000 are integer.

In this section we experimentally investigate the computational complexity of the SFRPS. Seven stages were used in these tests. Fig. 5 shows the computational time for solving the SFRPS to an optimality gap of less than 0.1% as a function of the number of vessel types available. Each line corresponds to a different number of scenarios. The number of available vessel types was varied between 10 and 50.

Fig. 5 shows that with a reasonably small number of scenarios the increase in computational time is only moderate with an increasing number of vessel types. However, with a higher number of scenarios, the computational time more quickly becomes critical when increasing the number of vessel types. Instances with 2250 scenarios and more than 30 vessel types were not solved to optimality within 5 min. Fig. 6 shows the relationship between the time required to reach an optimality gap of less than 0.1% and the number of trades. This figure indicates that the solution time increases approximately linearly with the number of trades.

Table 3
Transition matrix for stage 2 in Fig. 4

From	To	
	0.25	0.75
0.167	0.98	0.02
0.5	0.5	0.5
0.833	0.02	0.98

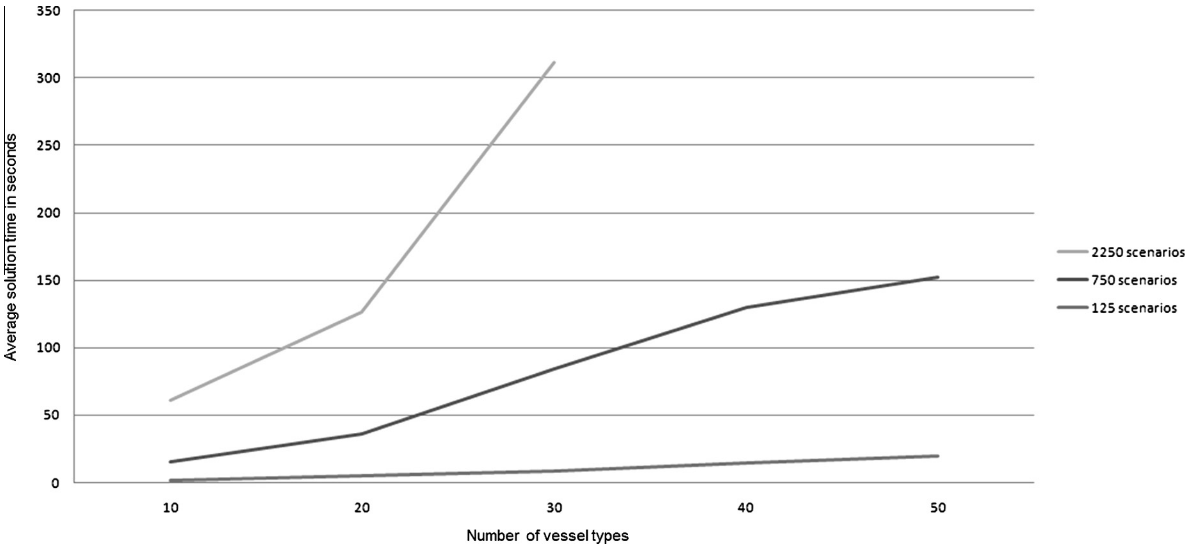


Fig. 5. Solution times for the SFRPS with different number of vessel types and scenarios, considering five trades.

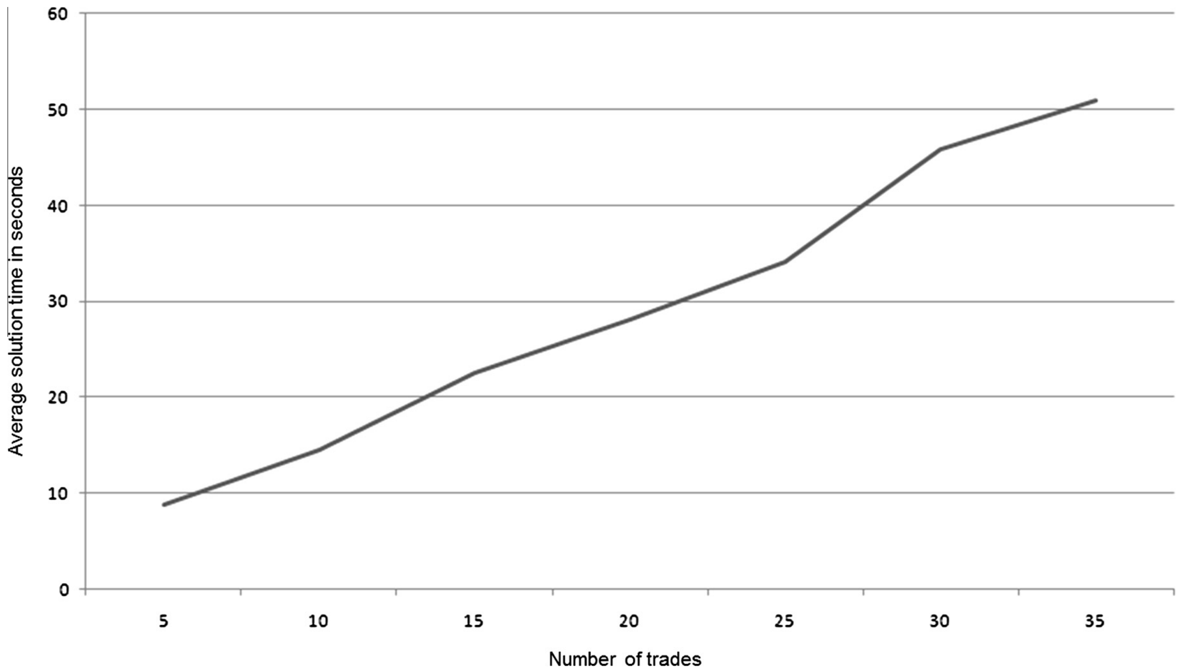


Fig. 6. Solution times for the SFRPS with different number of trades, considering five vessel types and 1500 scenarios.

4.3. Evaluation of scenario tree structures

As mentioned in Section 4.1, the scenario tree for the example in Fig. 4 can be denoted [3,2], indicating that there are three intervals in stage 2 and two in stage 3, and hence also three stages in total. The total number of scenarios is the product of the numbers in the brackets, which is six in the given example. To test how scenario trees with different numbers of stages and scenarios affect the SFRPS decisions, we define and test 24 different *scenario tree structures*, summarized in Table 4.

Scenario tree structures 1–10 consists of only one scenario, that is, they are considering the future as deterministic. Two aspects separate the ten deterministic scenario tree structures from each other: (1) the number of periods in the planning horizon and (2) the assumption about how `marketStatus` might develop. In the first approach regarding how `marketStatus` evolves, we anticipate that the demand observed in the current period will remain the same throughout

Table 4
Overview of scenario tree structures tested.

#	Discretization	Deterministic approach	Planning horizon (Years)
1	[1]	Flat	1
2	[1]	Expected value	1
3	[1,1]	Flat	2
4	[1,1]	Expected value	2
5	[1,1,1]	Flat	3
6	[1,1,1]	Expected value	3
7	[1,1,1,1]	Flat	4
8	[1,1,1,1]	Expected value	4
9	[1,1,1,1,1]	Flat	5
10	[1,1,1,1,1]	Expected value	5
11	[10]		1
12	[20]		1
13	[30]		1
14	[10,10]		2
15	[20,20]		2
16	[30,30]		2
17	[10,10,10]		3
18	[20,20,20]		3
19	[30,30,30]		3
20	[30,20,10]		3
21	[20,10,5,5,5]		5
22	[3,3,3,3,3,3,3,3]		8
23	[5,5,5,10,20]		5
24	[10,20,30]		3

the rest of the planning horizon. This approach is called the *flat approach*. The second deterministic approach anticipates that over time the demand in subsequent periods will revert to the expected value, which we refer to as the *expected value approach*. The remaining scenario tree structures (11–24) consider more than one scenario, combined into different scenario trees and lengths of the planning horizon. The duration of each period is kept equal to 1 year throughout.

The performance from using the different scenario tree structures are evaluated using a simulation framework, elaborated upon in the next paragraph and in Algorithm 1. The performance of each scenario tree structure is evaluated through 67 simulation runs, each simulation spanning a time horizon of 15 years. This means that for each of the 67 simulations the SFRPS must be solved 15 times, once for each year mimicking the repeating decision process of a shipping company. Thus, the process of deciding changes to the vessel fleet is repeated yearly during the simulation by solving the SFRPS model in Section 3, and along the way the accumulated costs and revenues (both from fleet changes and operations) are recorded.

Simulations use a rolling-horizon approach, as in (Mulvey and Vladimirou, 1992), and start by solving a SFRPS using one of the scenario tree structures shown in Table 4. The resulting plan looks n years ahead from the current time and uses m scenarios, where the values of n and m depend on the scenario tree structure. Given the resulting solution for the SFRPS, the here-and-now decisions regarding fleet adjustments are implemented in the simulation. Then the actual value of `marketStatus` for year 1 is revealed before we solve the deployment problem for that year (including decisions about chartering in or out). From the deployment problem we get the cost of operating the fleet in year 1. Then the simulation clock is advanced, and a new SFRPS is solved using the proposed scenario tree structure. Again the planning looks n years ahead and uses m scenarios. A deployment problem for year 2 is then solved after the `marketStatus` for that period has been revealed. This continues until the last year of the simulation.

Algorithm 1. Simulation structure.

Input: t = 0, `Simulation value` = 0, vessel fleet = initial fleet;
while *t* < *simulation horizon* **do**
  Solve the SFRPS for the current period using the model presented in Section 3.2 implemented in a MIP solver;
  Add net investment cost of the here and now decisions to `Simulation Value`;
  Update vessel fleet according to the here and now decisions;
  Let $t = t + 1$;
  Draw realization for the uncertain parameters of period *t*;
  Solve an optimization problem that evaluates how well the company performs in year *t*;
  Add net cost from the evaluation optimization to `Simulation Value`;
  if *t* = *simulation horizon* **then**
  Add value of current fleet to `Simulation Value`;
  end
end

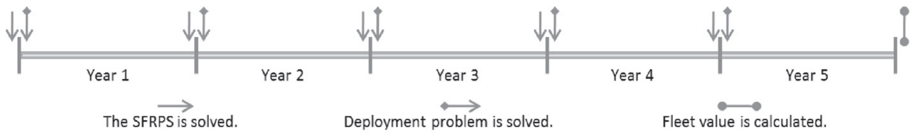


Fig. 7. An illustration of how the simulations are performed. Five planning problems and five deployment problems are solved when the simulation horizon is 5 years. Note that a new `marketStatus` is assumed to be revealed between each period, and that this market status will be constant throughout the year.

The structure of one single simulation is given in Algorithm 1, and Fig. 7 illustrates parts of the structure of one simulation, with focus on where the different optimization problems are solved. The same random numbers were used in the simulations for all scenario tree structures to facilitate a fair comparison between the different scenario tree structures. An acceptance gap of 0.1% and a time limit of 180 s for solving each SFRPS were used: If a feasible solution was less than 0.1% away from the best bound the solver is stopped. If no feasible solution was found during 180 s the solver continues until the first feasible solution is found.

The results from the simulation study with the 24 scenario tree structure are summarized in Table 5. The column "Average Difference" gives the average difference in simulation value between each of the scenario tree structures and the first one. A positive number means that the scenario tree structure on average performs better than the first scenario tree structure. The column "Std Dev Difference" presents the standard deviation of the difference over the 67 simulations, and columns 4 and 5 show the lower and upper bounds on a 95% confidence interval for the actual yearly average difference. The two columns under "Gap" presents the average and the maximum percentage gap experienced when solving the instances of SFPRS in the simulation. The solution time presented is the average time for solving one instance of the SFPRS. All monetary values in the tables of this section are given in thousands of USD.

As seen from the simulated results of the deterministic scenario tree structures (1–10), the lower bound on the confidence interval for the mean is negative, and the average differences in the samples are not high. Thus it cannot be determined which of the deterministic scenario tree structure is best. However, as the scenario trees using longer planning horizons have negative yearly differences, this indicates that when the uncertainty is high, having a longer planning horizon does not improve the solution considerably compared with planning only one year ahead, as long as uncertainty about the future is not taken into account. In fact, scenario tree structures 5, 7 and 9 all have a negative upper bound on the 95% confidence interval for the mean, indicating that they perform worse than scenario tree structure 1. The yearly difference is slightly higher for the expected value approaches than for the flat approaches. All the deterministic scenario tree structures are fast to solve to optimality (within 0.1% gap).

The simulated results of the stochastic scenario tree structures show significant improvement over the deterministic structures. For the best scenario tree structures the results correspond to a 4 % decrease in total costs, after subtracting

Table 5
Results from the simulations, with yearly difference measured in thousands of USD.

#	Average	Std dev	95% confidence		Average	Gap (%)	
	Yearly Difference	Difference	Lower	Upper	Sol time (s)	Average	Max
1	0	0	0	0	0.0	0.04	0.10
2	33	33,085	−8037	8103	0.0	0.04	0.10
3	1602	7333	−186	3391	0.1	0.05	0.10
4	1040	34,519	−7380	9460	0.0	0.05	0.10
5	−7027	16,878	−11,144	−2910	0.1	0.05	0.10
6	−3100	39,863	−12,823	6624	0.1	0.06	0.10
7	−8154	18,000	−12,544	−3763	0.2	0.06	0.10
8	−2790	40,239	−12,606	7025	0.1	0.06	0.10
9	−8166	17,520	−12,440	−3893	0.2	0.06	0.10
10	−3296	40,039	−13,062	6470	0.1	0.06	0.10
11	62,442	46,592	51,077	73,807	0.0	0.04	0.10
12	73,734	53,607	60,659	86,810	0.1	0.03	0.10
13	71,267	51,197	58,779	83,755	0.1	0.03	0.10
14	73,640	52,687	60,788	86,491	0.6	0.06	0.10
15	79,263	57,675	65,195	93,330	2.9	0.06	0.17
16	78,663	55,546	65,114	92,211	6.6	0.06	0.28
17	75,880	52,940	62,967	88,793	18.9	0.07	0.32
18	77,744	60,960	62,875	92,614	95.9	0.14	11.99
19	54,526	72,945	36,733	72,319	342.5	0.89	30.52
20	77,663	57,544	63,627	91,699	101.0	0.14	13.86
21	58,926	67,668	42,420	75,431	411.4	0.70	8.94
22	53,533	50,266	41,272	65,794	136.5	0.21	5.65
23	44,686	77,850	25,697	63,675	211.7	1.30	33.98
24	75,360	56,845	61,495	89,226	53.5	0.07	0.53

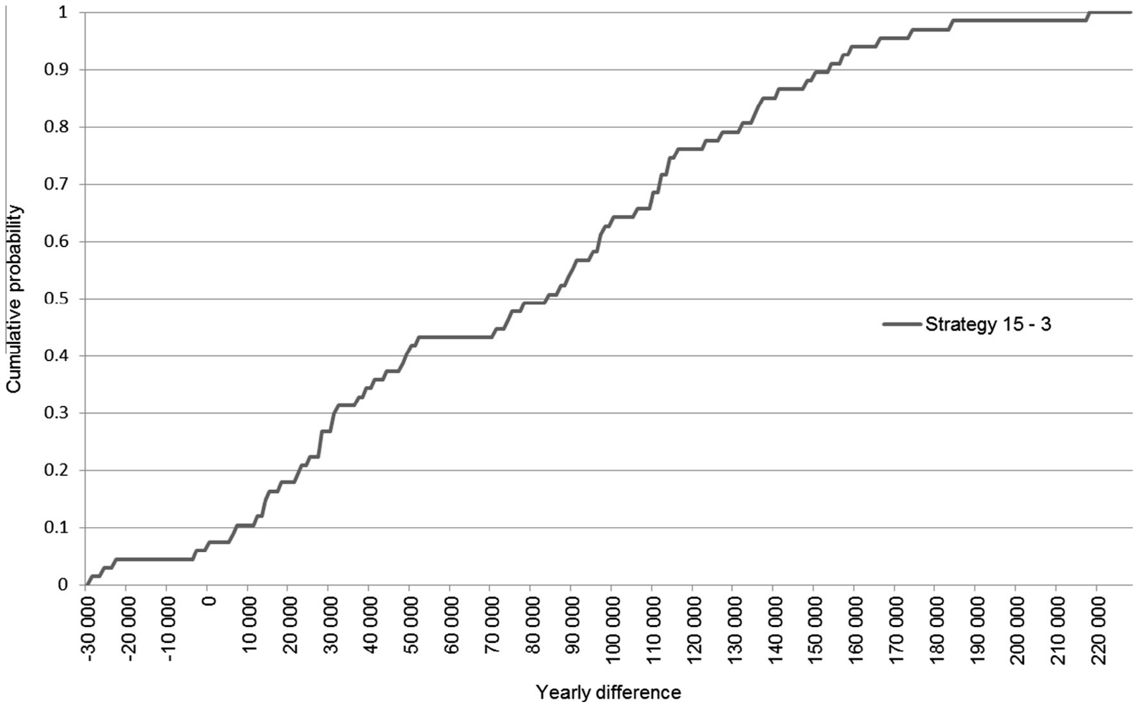


Fig. 8. Difference distribution.

the income from chartering out vessels and transporting spot cargo, compared to using scenario tree structure 1. For structures that involve more difficult instances of the SFRPS, where there is still an occasional optimality gap when the time limit is reached, for example scenario tree structure 19, the results are not as good. Tests show that with increased allowed solution time, these scenario tree structures obtain improve results. If all SFRPSs within the simulation were to be solved to optimality, scenario tree structure 19 is expected to create better results than scenario tree structure 18 in the long run, with the cost of increased computational time. However, with a limited computational budget this does not hold. When comparing the average yearly difference between scenario tree structure 15 and the other scenario tree structures, only structures 16 and 18 have 95% confidence intervals that include 0. Hence, with the limits imposed on the solution times, all scenario tree structures except number 16 and 18 appear to be worse than scenario tree structure 15. When comparing relative to a stochastic scenario tree structure the standard deviations of the yearly difference among the other stochastic scenario tree structures are mainly in the range from 10,000 to 20,000. This means that the stochastic scenario tree structures behave somewhat similar to each other. As do the deterministic ones, but the deterministic and stochastic scenario tree structures behave differently from each other.

Based on average performance, the best deterministic scenario tree structure is number 3 and the best stochastic scenario tree structure is number 15. The results clearly show that considering uncertainty matters. The expected extra yearly profit using the stochastic scenario tree structure instead of the deterministic one is approximately 80 million USD with a lower bound on a 95% confidence interval for the mean at around 65 million USD. Hence the use of a stochastic approach, such as scenario tree structure 15, would be warranted.

The cumulative probability of the difference in simulation values between the best deterministic (i.e. number 3) and the best stochastic scenario tree structure (number 15) are shown in Fig. 8. The difference is calculated taking the simulation value of scenario tree structure 15 subtracted the simulation value of scenario tree structure 3. The figure shows that in our samples the probability of the stochastic approach (scenario tree structure 15) performing worse than the deterministic approach (scenario tree structure 3) over the 15 years is around 5%, while there is a 50% probability that scenario tree structure 15 gives solutions that gives at least 85 million USD in yearly saving (4.5% improvement of the yearly simulation value).

Scenario tree structures 23 and 24 correspond to scenario tree structures 21 and 20, respectively, where the order of the branching factors in the scenario trees is reversed. This means that scenario tree structures 23 and 24 model the uncertainty coarser in the first stage and finer in later stages compared to scenario tree structures 21 and 20. Comparing the results of scenario tree structures 20 and 24 shows that with approximately 90% confidence, scenario tree structure 20 will perform better on average than scenario tree structure 24, and scenario tree structure 20 also has both higher average and maximum gap. Comparing the results of scenario tree structures 21 and 23 shows that scenario tree structure 21 outperforms scenario tree structure 23 with 90% confidence. Thus, the results indicate that it is more important to model the uncertainty of the near future with more details than it is for the later stages.

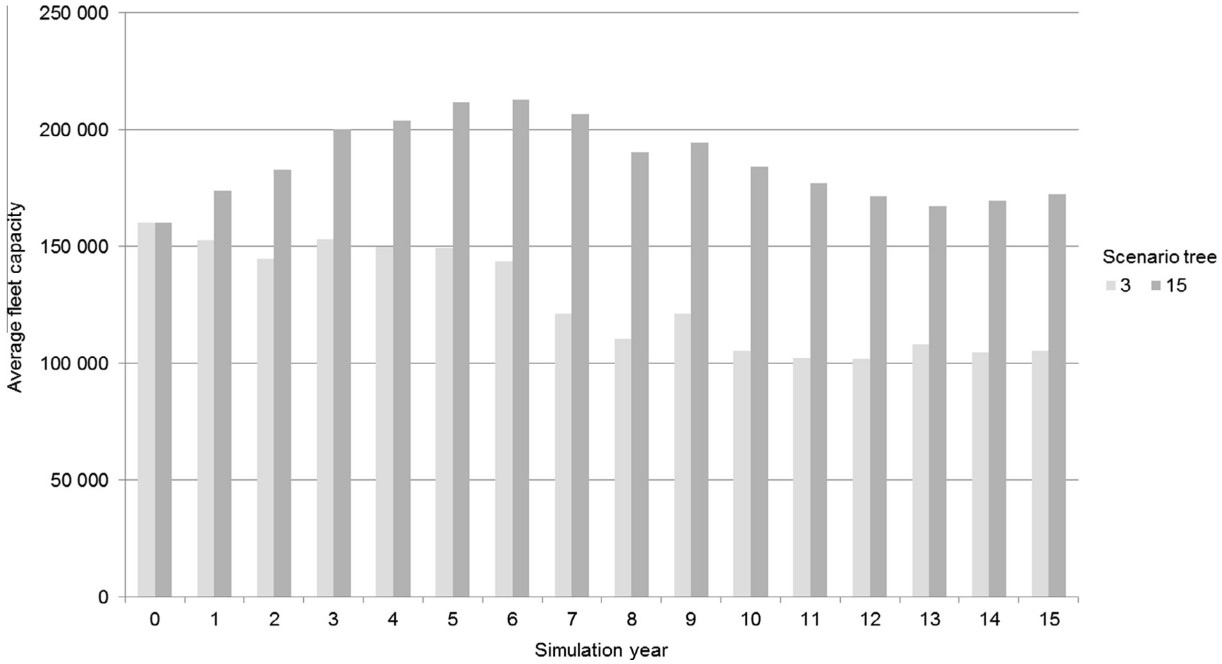


Fig. 9. Fleet development.

Table 6
Average values from the simulations.

Scenario tree structure	Fleet capacity	Vessels chartered		
		Out	In	In, exp
3	125,000	0.7	9.4	1.4
15	188,000	2.0	5.7	0.1

Table 7
Distribution of vessels (in percentage of the number of vessels in fleet).

Vessel size	Approach 3	Approach 15
2600	0.12	0.12
4500	0.45	0.43
8500	0.15	0.15
10,500	0.17	0.17
12,000	0.11	0.13

When taking a closer look at the vessel fleets produced by the best stochastic and the best deterministic scenario tree structure, structural differences are found. Fig. 9 shows the development of the vessel fleet capacity measured by means of the capacity of the vessels owned averaged over the 67 simulations. There is a distinct difference in the fleet capacities suggested by using the two solution scenario tree structures. Table 6 shows the average capacity in units (rounded to nearest thousand) over all the years produced by the best scenario tree structure from each category, together with the average number of charter out, in and in at expensive rate. Scenario tree structure 15 produces an average fleet with approximately 50% more capacity than scenario tree structure 3. Because of this, the stochastic scenario tree structure almost avoids the expensive charter rates. When excess capacity arises it is chartered out to the greatest extent possible. The opposite is true in the deterministic scenario tree structure, as it always tries to meet the expected value, and when the fleet turns out to be too small it is forced to charter in at an expensive rate more frequently. How much of the cost of owning a vessel that can be recovered by chartering out the vessel certainly depends on the demand for charters in the market, so the results presented are data specific.

When investigating the fleet composition further small differences in the choice of vessels are found. The average vessel size was (rounded to nearest hundred) 6900 and 6700 for the best stochastic and the best deterministic scenario tree structure, respectively. Table 7 shows the distribution of the vessel sizes. Scenario tree structure 15 is slightly more inclined to buy larger vessels than scenario tree structure 3. For both scenario tree structures vessels with capacity 4500 has the largest share of the fleet.

5. Conclusion

The contribution of the paper to the literature are (1) the introduction of a new variant of the fleet size and mix problem, called the *strategic fleet renewal problem in shipping* (SFRPS), (2) a new stochastic programming formulation of the SFRPS, and (3) computational experiments showing the importance of modeling uncertainty when solving the SFRPS. The SFRPS is an important problem that ship owners and ship operators regularly face when they are considering changes in their vessel fleets. The SFRPS differ from the usual fleet size and mix problems by explicitly considering an initial fleet and focusing on how to alter this existing fleet. Our stochastic programming formulation models multiple ways of changing capacity and the aging of the vessels, and takes the inherent uncertainty of the problem into account. Specifically it models uncertain demand, ship prices, cargo price, and several other uncertain factors. The model is not designed to replace expert opinions and experience but rather to support the decision process. The formulation is tested in a computational study, using an illustrative test case. Several alternatives for how to model the uncertainty, differing in the number of stages and scenarios as well as the number of planning periods, have been evaluated.

The results from the computational study indicate that taking the inherent uncertainty of the problem into consideration with the use of stochastic programming may significantly improve the fleet adjustment decisions compared to classical deterministic optimization. The results show that alternatives that do not capture the uncertainty of the future (those only considering a single scenario for the future) may make decisions that are too specific and fixed on the expected future. Alternatives that consider the influence of multiple scenarios and account for the consequences of different future realizations are better for determining long term strategic decisions. The benefit given by considering stochastic information is nicely illustrated by the structure of the solutions obtained, where the fleet capacity resulting from considering multiple future scenarios are significantly larger than the fleet capacities suggested by deterministic models. This enables the solutions produced when considering multiple scenarios to satisfy demand at higher levels and make less use of chartering than the solutions obtained by corresponding deterministic approaches.

The results are data specific and thus depend on the particular case studied. However, the results indicate that using an approach that embeds the uncertainty into the model when determining the fleet renewal may be the difference between a profit and a loss for shipping companies. Future research within this area should therefore be fully aware of the potential uncertainties present when designing solution methods for strategic fleet renewal.

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