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Fleet sizing with time and voyage-chartered vessels for an oil shipping company under demand uncertainty

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ABSTRACT

The fleet sizing problem is one of the main concerns for an industry shipping company under various uncertainty of market conditions. This paper addresses the problem in a single period with uncertain shipment demand for an oil shipping company, who transports the refined petroleum products among multiple origin–destination (OD) pairs. Besides the self-owned vessels, the time-chartered and voyage-chartered vessels are considered by the company as two types of rental vessels from the potential outsourcing market. The company aims to determine the numbers of vessels with different capacity levels and different rental types, deploy them among all OD pairs to minimize the total expected cost including the rent costs of time-chartered and voyage-chartered vessels and the holding cost of time-chartered vessels. Several stochastic programming models are proposed to investigate three scenarios: the aggregative shipment demand and an available fleet with continuously separable vessel capacity in the outsourcing market, the aggregative shipment demand and an available fleet with inseparable vessel capacity, and the OD shipment demand matrix and an available fleet with inseparable vessel capacity. Assuming economies of scale in rental duration, vessel capacity, and voyage distance, we conduct theoretical analysis to deduce the necessary conditions and solution properties for optimal resolutions to the fleet sizing problems. The real-world numerical examples based on China National Petroleum Corporation are adopted to illustrate the proposed models and theoretical results.

1. Introduction

Currently, over 80% of the energy consumption by mankind still comes from fossil fuels including coal, oil and natural gas. Notably, the tanker trade including oil, gas and chemicals occupied about one-third of the international maritime trade and is up to 18,000 billion ton-miles (UNCTD, 2022). According to the statistics of ships of 100 gross tons and above, the total capacity of tankers is 762,446 thousand dead-weight tons, which accounts for 34.67% of the total capacity (Clarksons Research, 2022). The transportation branch company of China National Petroleum Corporation provides over 200 shipping lines from 14 ports in North China to 30 coastal ports in South and East China for five refined petroleum products, such as, octane-92, octane-93, octane-95, octane-98 gasoline and diesel. The annual shipment demand exceeds 200 million tons occupying about 20% of China's domestic refined oil shipping market. Besides the self-owned tankers, the company must rent vessels by time charters and voyage charters

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according to the annual shipment demand forecast of refined petroleum products. The annual rent cost is up to 0.26 billion US dollars.

The fleet sizing or composition is a critical decision problem of an oil shipping company facing various uncertainty of shipment demand, charter rates and voyage conditions (Pantuso et al., 2014). Beside of the self-owned vessels, the chartering of vessels is necessary for the companies to guarantee all cargo is carried. Traditionally, there are two basic types of charter service: the time charter, and the voyage charter (Arslan and Papageorgiou, 2017). Under a time charter, the companies can control the vessel during a contracted time period and pay for the cost due to utilization including bunkers, pilots, tugs, wharfage, and other port charges. A voyage charter is a contract for the carriage of cargo by water from one designated place to another. Under a voyage charter, the shipowner retains full control of the vessel and charges a fee to the companies. Generally, the time-chartered vessels provide the higher flexibility and produce the lower marginal cost or charter rate than the voyage-chartered vessels (Branch, 2007). Nevertheless, excessive time-chartered vessels lead to significant holding costs due to inefficient utilization. The fleet configuration problem is to determine the vessel types and numbers for the time and voyage-chartered service. The time charter is a long-term contract and belongs to the planning problem, while the voyage charter is a temporary contract and is a scheduling problem. These problems stem from the practical needs of a leading oil transportation company.

This paper considers the fleet configuration of an industrial carrier in a planning period (typically, a year) with uncertain shipment demand. Besides the self-owned vessels, the carrier selects the time-chartered vessels from the potential outsourcing market to adjust its fleet according to its origin–destination (OD) shipment demand. Different from the stochastic dynamic scheduling problem, the annual period planning is necessary task for an industry carrier since the time-chartered vessels must be signed for a fixed contract period, typically, a half of year. The rent charges for the time-chartered and voyage-chartered vessels are also affected by the vessel size and sailing distance. The controlled vessels including the self-owned and time-chartered vessels can be assigned to different shipping lines to carry different types of cargo. Furthermore, the realized capacity of the self-owned vessels is also uncertain since the practical schedule of those vessels is impossible ex-ante determined. Therefore, both shipment demand and realized capacity of the self-owned vessels are stochastic. It is challenging for the carrier to determine the numbers and sizes of the time-chartered vessels with uncertain OD shipment demand and capacity realization of the self-owned vessels by incorporating the cost characteristics to minimize the total expected rent cost.

We propose stochastic programming models to capture the optimal fleet sizing problem under three scenarios: (I) the aggregative shipment demand and continuously separable vessel capacity in the outsourcing market, (II) the aggregative shipment demand and an available fleet with inseparable vessel capacity, and (III) the OD shipment demand matrix and an available fleet with inseparable vessel capacity. Scenario (I) is the continuous approximation of the other two scenarios. The total cost is the sum of the rent costs of time and voyage-chartered vessels and the holding cost of time-chartered vessels. For each scenario, the theoretical analysis is conducted to derive the necessary conditions and solution properties to obtain the optimal solutions under mild assumptions. The main contributions of this paper can be summarized as follows. We build the stochastic programming models of the fleet sizing problem under several scenarios to incorporate the shipment demand uncertainty and the cost characteristics of vessels and shipping lines. For the scenario with aggregative shipment demand and continuously separable vessel capacity, the optimal capacity of time-chartered vessels can be exactly captured by the marginal costs of rental types and the cumulative distributions of demand and realized capacity of the self-owned vessels. For the scenarios (II) and (III), we examine the necessary conditions of the optimal fleet sizing and propose the heuristic algorithms to determine the optimal solution of the fleet sizing problem.

The rest of the paper is organized as follows. A literature review on the related studies is given in Section 2. Section 3 describes fleet sizing problem of the aggregative shipment demand with continuously separable vessel capacity in the outsourcing market. The corresponding stochastic programming model and the optimal solution are discussed in the section. Sections Section 4 extends the benchmark case to scenarios (II) and (III). A case study using the real data of the transportation branch company of China National Petroleum Corporation is conducted to further elaborate the developed models and results in Section 5. Section 6 concludes this study and presents future research directions.

2. Literature review

The fleet sizing or composition problem is frequently concerned in transportation area, such as, public transit (Gertsbach and Gurevich, 1977; Beaujon and Turnquist, 1991), taxi (Yang and Wong, 1998; Yang et al., 2002), freight transportation (Dejax and Crainic, 1987; Du and Hall, 1997), rail transportation (Bojovic, 2002; Hoff et al., 2010), and airline industry (Berge and Hopperstad, 1993; Listes and Dekker, 2005). It is also an challenging issue in maritime transportation since the fleet composition can affect the future scheduling and routing for their operational decisions and fleet renewal plan for their strategic decisions (Christiansen et al., 2007). Furthermore, The fleet sizing problem is quite challenging with significant uncertainty regarding the future demand, charter rate and voyage time (Fagerholt et al., 2010; Vagen and Nikolaisen, 2019). Wu et al. (2021) classified the fleet sizing problem into the maritime fleet sizing and mix problem, and the maritime fleet renewal problem. The maritime fleet sizing and mix problem aims to determine the vessel number for each type and/or schedule those vessels to cope with the market change in a future single period. The maritime fleet renewal problem focuses on the dynamic adjustment of fleet composition by adding new or second-hand ships, and selling or scrapping ships in a multiple periods.

Ksciuk et al. (2023) carried out a thorough review on the uncertainty in ship routing and scheduling. They summarized the literature on fleet sizing problem according to the uncertain sources (demand, revenue, charter price, freight rate, etc.), model/solution method (simulation, one-stage or multi-stage stochastic programming), shipping modes (liner, tramp or industry) and considered problems (fleet sizing and mix, fleet renewal). The fleet sizing problem is well investigated in liner shipping under various uncertain sources. With the chance-constrained mixed-integer programming, Meng and Wang (2010) studied joint ship fleet design and deployment plan under demand uncertainty in a single period (typically, 36 months). Meng et al. (2012, 2015) extended their previous work to incorporate the multi-period demand uncertainty. For more details, refer to Ng (2015).

The fleet sizing problem in tramp and industry shipping always enlightened from real industry shipping problems. Schwartz (1968) addressed the fleet composition and scheduling problem for a barge service company. Liu and Sherali (2000) proposed a mixed-integer 0–1 programming model to model the coal shipping and blending problem for the Taiwan Power Company. The fleet composition is modeled the selection from the Panamax-size ship and the Cape-size ship according to practical constraints and coal demand. Fagerholt and Rygh (2002) addressed a real problem to transport fresh water from Turkey to Jordan. The simulation model was adopted to examine the fleet sizing and capacity design problem of other facilities. Persson and Gothe-Lundgren (2005) studied a shipment planning problem by simultaneous planning ship route and allocating products from three refineries of the Nynas Group to a number of depots. The company deployed the self-owned ships and hired from the spot market. Shyshou et al. (2010) studied the fleet configuration by hiring ships with corresponding capacity either on the long-term basis or on the spot market to anchor handling operations for the largest Norwegian offshore oil and gas operator.

In recent years, the fleet sizing problem has attracted more attention of researchers, who mainly focused on the methodologies of the problem and solution algorithms. The stochastic programming with one-stage or multi-stage decision is one of commonly-used mathematical technologies. Tremendous efforts have been made by Pantuso and his co-authors on the fleet renewal problem (Pantuso et al., 2014, 2015a,b). They considered the problem with multi-stage stochastic programming model and the ships in the fleet can be bought, sold and scrapped according to various uncertain market conditions. Bakkehaug et al. (2014) investigated the similar problem with a multi-stage stochastic programming formulation and the scenario tree was adopted to capture the uncertainties. The robust optimization is also adopted to deal with the market uncertainties (Alvarez et al., 2011; Bertsimas et al., 2018). Wu et al. (2021) jointly considered the fleet adjustment problem, the cargo selection problem, and the ship routing problem in tramp shipping with a robust optimization model. Over a given planning horizon, the paper can simultaneously determine the strategic, tactical and operational decisions. Stochastic programming models are also adopted to investigate the investment issues, such as, to maximize the rate of return on the ship investments (Morch et al., 2017), to limit the risk of insolvency due to negative cash flows (Skalnes et al., 2020), to incorporate the effect of ship scrapping subsidies (Yang et al., 2019).

Because of the natural market uncertainties and complexity of the fleet sizing problem in tramp or industry shipping, the main objective of research is to capture the real problem with standard optimization programming models and design efficient solution algorithms. In particular, the time charter and voyage charter are not distinguished since the longer hiring period of a ship would result in less marginal cost and higher flexibility in scheduling for a shipping company. In reality, the time-chartered vessels have the lower marginal cost than the voyage-chartered vessels (Branch, 2007). In particular, the number of time-chartered vessels must be determined before the realization of the demand. To our knowledge, two papers distinguished the time-chartered and voyage-chartered service in the fleet sizing problem. Arslan and Papageorgiou (2017) first introduced the charter durations as decision variable to consider the fleet renewal problem with a multi-stage stochastic programming model. Wang et al. (2018) addressed the chartering problem for a shipping industry by incorporating the fleet deployment and speed optimization to deal with the market uncertainties. The duration length of a chartering vessel is also considered in their model. They mainly developed the simulation-based optimization model and designed an efficient solution algorithm. In this paper, we aim to derive the properties of the optimal solution for the fleet sizing problem under some commonsense assumptions, such as, the economic scale on duration, capacity and distance. Generally, the rent charge of the time-chartered vessels is always lower than that of the voyage-chartered vessels for same type of ships, the marginal rent charge of vessel decreases with respect to the vessels size, and the marginal shipping cost per unit cargo-distance decreases when the shipping distance increases. Under some economical assumptions, we can derive the optimal solution or the optimality conditions for the fleet sizing problem.

3. Benchmark case

3.1. Model formulations

In this section, we first focus on the benchmark case with aggregative shipment demand and continuously separable vessel capacity. Consider a shipping service between an origin port and a destination port for an oil shipping company with a given planning period, typically, a month or a year. The shipping company has a given transportation capacity with self-owned vessels, Y_0 (Tons). In practice, the company provides a network shipping service among several shipping lines connecting multiple origin and destination ports, the capacity of the self-owned vessels is shared by other shipping lines with random and independent shipment demand levels. To consider the influence of shipping network, we introduce a random variable V (Tons) with a cumulative distribution function (CDF) $F(V)$ and a support set $\Theta \subset [0, Y_0]$ to capture the portion allocated to the interested origin and destination pair to process the shipping service. The share V of the capacity cannot be changed once it is predetermined before the planning period to guarantee the equilibrium among all shipping lines. This leads to the supply uncertainty, which has been studied in supply chain Hsieh and Wu (2008) and Chen and Xiao (2015).

Our model treats self-owned shipping capacity Y_0 as a random variable, acknowledging that the shipping company's available capacity for specific origin–destination (OD) pairs is not fixed at the start of the planning period. Instead, it fluctuates based on operational demands across various routes. For instance, a shipping company's fleet may need to be redeployed to meet unexpected demand surges on alternative routes, thereby altering the capacity available for the originally planned OD pairs. We illustrate this situation with data provided by COSCO Maritime Petroleum. The company has a total capacity of 400,000 tons across 14 product tankers. Throughout 2023, 7 of these tankers served eight different routes for other companies, with a total sailing time exceeding 146 days. During this period, COSCO Maritime Petroleum transported 598,807.4 tons of cargo not owned by the company and earned a net profit of 20.4 million RMB. Changes in capacity on one route can directly influence capacities on other routes. This stochastic nature of shipping capacity necessitates a probabilistic modeling framework to capture the dynamics of real-world shipping operations accurately.

The shipment demand of single cargo for the shipping company is naturally uncertain and denoted by Q. We assume that the random variable Q follows a continuous cumulative distribution function (CDF) $G(Q)$ in a sample domain $\Omega \subset R^+$. The outsourcing strategy is typically adopted by a shipping company to deal with the uncertain shipment demand (Cariou and Wolff, 2011). The time-chartered and voyage-chartered vessels are two types of rental service in shipping market with the outsourcing strategy. For the time-chartered vessels, the rent charge for each vessel is contracted by the company and the vessel owner. The rent fee must be paid by the company to the vessel owner whatever the vessels are used and will not change during the period regardless of the freight rate of the shipping market. For the voyage-chartered vessels, a temporary rent charge is determined by the current freight rate of the shipping market.

Let Y (Tons) and $\hat{Y}$ (Tons) denote the capacity levels of time-chartered and voyage-chartered vessels of the company. The sequential decision process for the company can be described as follow: the company first chooses the capacity of time-chartered vessels Y facing the random shipment demand Q with a known CDF $G(Q)$ and random capacity allocation of self-owned vessels V with a known CDF $F(V)$; then the capacity of the voyage-chartered vessels $\hat{Y}$ is finally derived after the capacity allocation V and shipment demand Q are successively realized. Since the capacity of the voyage-chartered vessels $\hat{Y}$ is determined after the realization of both V and Q, the shipment demand must be fulfilled during the planning period. The capacities V and Y of the self-owned and time-chartered vessels must be utilized in priority since their cost is paid in advance. Therefore, $\hat{Y}$ is the conditional expectation when the realized shipment demand Q is excessive to the realized supply $V + Y$, which can be expressed by

$$\hat{Y} = E\left(\max\left\{Q - m\eta\left(V + Y\right), 0\right\}\right) \tag{1}$$

where m is the number of round-trips for a vessel traversing between the given port pair, η is the average loading factor of vessels. The corresponding excessive capacity is also a conditional expectation when the realized supply $V + Y$ is excessive to the realized shipment demand Q, namely,

$$\tilde{Y} = E\left(\max\left\{m\eta\left(V + Y\right) - Q, 0\right\}\right). \tag{2}$$

For convenience of presentation, $m\eta$ is normalized to 1. In fact, we can view the shipment demand Q as an equivalent variable $\frac{Q}{m\eta}$. With those settings, the conditional expectations, $\hat{Y}$ and $\tilde{Y}$, can be rewritten as

$$\hat{Y} = \iint_{\Theta\times\Omega} \max\left\{Q - V - Y, 0\right\} \mathrm{d}F(V)\mathrm{d}G(Q) \tag{3}$$

and

$$\tilde{Y} = \iint_{\Theta\times\Omega} \max\left\{V + Y - Q, 0\right\} \mathrm{d}F(V)\mathrm{d}G(Q) \tag{4}$$

The operational cost for each allocation realization of the self-owned vessels V is also a random variable, denoted by γ(US dollars) including the bunker consumption, labor, and port utilization charge. It is clear to see that, γ and V have the same probability distribution. The rent charges per unit capacity for the time-chartered and voyage-chartered vessels are denoted by c_1 and c_2 (US dollars), respectively. c_1 and c_2 are assumed to be fixed during the whole planning period. Note that, in practice, c_2 is also uncertain which depends on the freight rate of the shipping market. For the planning problem, the company can determine c_2 via statistic estimation.

The fleet sizing problem of the shipping company is to determine the capacity of the time-chartered vessels, Y, and thus, the capacity of the voyage-chartered vessels, $\hat{Y}$, facing the uncertainty caused by both supply and demand sides to minimize the expected cost for a given planning period, which includes the expected operational cost of self-owned vessels, the rent cost of time-chartered vessels, the expected rent cost of voyage-chartered vessels, the holding cost of the excessive capacity. Mathematically,

$$\min_{Y\geq 0}\; C(Y) = \mu_\gamma + c_1 Y + c_2\hat{Y} + c_3\tilde{Y} \tag{5}$$

where μ_γ is the mean of γ, c_3 is the holding cost per unit excessive capacity. To derive the analytical result for other partitions, adopting $\max\{A, 0\} = \frac{|A|+A}{2}$ in $\hat{Y}$ and $\tilde{Y}$ given by Eqs. (3) and (4), the objective function $C(Y)$ defined by Eq. (5) can be rewritten as

$$\begin{aligned} C(Y) =& \mu_\gamma + c_1 Y + \tfrac{c_2-c_3}{2} \iint_{\Theta\times\Omega} (Q - V - Y)\,\mathrm{d}F(V)\mathrm{d}G(Q) \\ &+ \tfrac{c_2+c_3}{2} \iint_{\Theta\times\Omega} |Q - V - Y|\mathrm{d}F(V)\mathrm{d}G(Q) \\ =& \left(\mu_\gamma - \tfrac{1}{2}\left(c_2 - c_3\right)\mu_V\right) + \tfrac{1}{2}\left(c_2 - c_3\right)\mu_Q + \Psi(Y) \end{aligned} \tag{6}$$

where μ_Q is the means of Q, μ_V is the mean of V, and

$$\Psi(Y) = \left(c_1 - \tfrac{c_2-c_3}{2}\right)Y + \tfrac{c_2+c_3}{2} \iint_{\Theta\times\Omega} |Q - V - Y|\mathrm{d}F(V)\mathrm{d}G(Q). \tag{7}$$

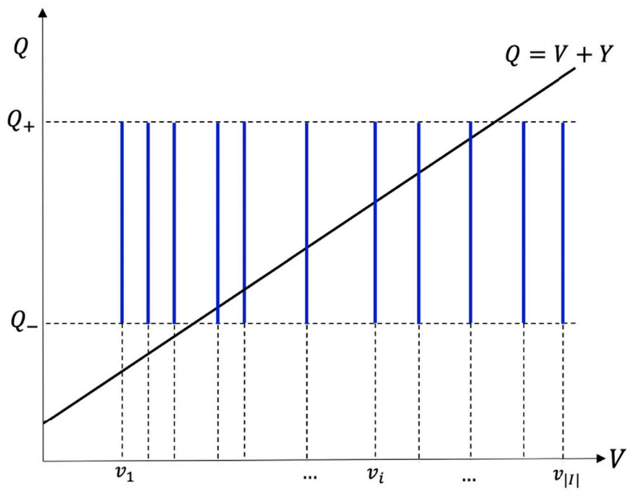

Fig. 1. The graphical explanation partitions of Y with uniform distributions.

It is clear to see that the optimization problem (5) is equivalent to minimization $\Psi(Y)$ with $Y \geq 0$. Note that the company would consider the budget constraint, feasibility of vessel fleet and other practical situations when determining the capacity of the time-chartered vessels Y. Furthermore, the capacity of vessels should be discrete values in practice. And then we will extend the benchmark problem by incorporating more practical considerations.

3.2. Solution properties of optimal fleeting sizing

We now move to examine the properties of the optimal capacity of time-chartered vessels. For the continuous CDF $G(Q)$, we consider two scenarios according to the geometric property of the support set: (1) $\Omega = (0, +\infty)$; and (2) $\Omega = [Q_-, Q_+] \subset (0, +\infty)$. Scenario (1) includes many commonly-used distributions as a special case, for example, the Gamma, Lognormal, truncated normal and exponential distributions. The uniform and Beta distributions with bounded support set is a typical example of Scenario (2).

Scenario (1), for any non-negative realization V and non-negative capacity Y, $\Psi(Y)$, given by (7), can be simplified as

$$
\begin{aligned}
\Psi(Y) = & \left(c_1 - \tfrac{c_2 - c_3}{2}\right) Y + \tfrac{c_2 - c_3}{2} \int_\Theta \mathrm{d}F(V) \int_0^{V+Y} (V + Y - Q)\,\mathrm{d}G(Q) \\
& + \tfrac{c_2 - c_3}{2} \int_\Theta \mathrm{d}F(V) \int_{V+Y}^{+\infty} (Q - V - Y)\,\mathrm{d}G(Q),
\end{aligned}
\tag{8}
$$

which is continuously differentiable with respect to Y. Taking the derivative of $\Psi(Y)$ given by Eq. (8) in Y gives rise to

$$
\frac{\mathrm{d}\Psi(Y)}{\mathrm{d}Y} = \left(c_2 + c_3\right) \left[\int_\Theta (G(V + Y))\,\mathrm{d}F(V) - \frac{c_2 - c_1}{c_2 + c_3} \right].
\tag{9}
$$

Note that, $G(\dots)$ and $F(\dots)$ are the cumulative distribution functions, which are increasing with a range $[0, 1]$. $G(\dots)$ is continuously differentiable and strictly increasing in $(0, +\infty)$ from 0 to 1. We have the following straightforward result, which is also true for discrete random variable V.

Lemma 1. *Suppose the support sets $\Theta \subset [0, Y_0]$ and $\Omega = (0, +\infty)$, the following function*

$$
H(Y) = \int_\Theta G(V + Y)\,dF(V)
\tag{10}
$$

strictly increases from $H(0)$ to 1 as Y increases from 0 to $+\infty$, where $0 < H(0) < 1$.

We now move to investigate scenario (2) with $\Omega = [Q_-, Q_+] \subset (0, +\infty)$. In this case, $\hat{Y}$ and $\tilde{Y}$ are not continuously differentiable in the whole feasible domain of Y. However, we can divide the feasible domain of Y into several portions according to the value Y_0, on which both conditional expectations $\hat{Y}$ and $\tilde{Y}$ are still continuously differentiable with respect to Y. For natural discretization of capacity of the self-owned vessels, we also assume that V follows a discrete distribution with $\Theta = \{v_i, i \in I\}$ and $0 \leq v_1 < v_2 < \cdots < v_{|I|} \leq Y_0$. The known probability $Pr\{V = v_i\} = p_i$, $i \in I$. Fig. 1 provides an explanatory example with discrete distribution of V and bounded support set for any given Y.

Introduce the following partition of support set Θ

$$
\begin{aligned}
\Theta_1(Y) &= \left\{ v_i \in \Theta : v_i \leq Q_- - Y \right\} \\
\Theta_2(Y) &= \left\{ v_i \in \Theta : Q_- - Y < v_i \leq Q_+ - Y \right\} \\
\Theta_3(Y) &= \left\{ v_i \in \Theta : Q_+ - Y < v_i \right\}
\end{aligned}
\tag{11}
$$

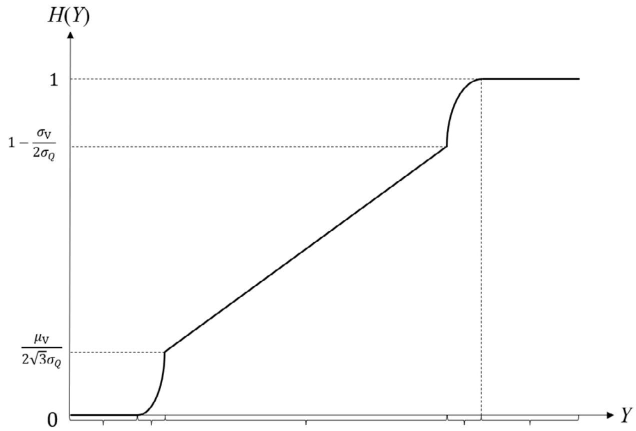

Fig. 2. An example of $H(Y)$ with uniformly distributed V and Q.

We have

$$\begin{aligned}
&\iint_{\Theta\times\Omega} |Q-V-Y| \mathrm{d}F(V)\mathrm{d}G(Q) \\
&= \sum_{v_i\in\Theta_1(Y)} p_i \int_{Q_-}^{Q_+} \left(Q-v_i-Y\right) \mathrm{d}G(Q) \\
&+ \sum_{v_i\in\Theta_2(Y)} p_i \left(\int_{Q_-}^{v_i+Y} \left(v_i+Y-Q\right) + \int_{v_i+Y}^{Q_+} \left(Q-v_i-Y\right)\right) \mathrm{d}G(Q) \\
&+ \sum_{v_i\in\Theta_3(Y)} p_i \int_{Q_-}^{Q_+} \left(v_i+Y-Q\right) \mathrm{d}G(Q)
\end{aligned} \tag{12}$$

Similarly, taking the derivative of $\Psi(Y)$, given by (7), we have

$$\frac{\mathrm{d}\Psi(Y)}{\mathrm{d}Y} = \left(c_2+c_3\right)\left[\sum_{v_i\in\Theta_2(Y)} p_i G\left(v_i+Y\right) + Pr\{\Theta_3(Y)\} - \tfrac{c_2-c_1}{c_2+c_3}\right], \tag{13}$$

where $Pr\{\Theta_3(Y)\}$ is the probability with realized capacity of self-owned vessels being larger than $Q_+ - Y$. Similar to Lemma 1, we have the following conclusion.

Lemma 2. *Suppose the support sets $Pr\{V=v_i\}=p_i$, $i\in I$, and $\Omega=[Q_-,Q_+]\subset(0,+\infty)$, the following function*

$$H(Y) = \sum_{v_i\in\Theta_2(Y)} p_i G\left(v_i+Y\right) + Pr\{\Theta_3(Y)\} \tag{14}$$

strictly increases from $H(0)$ to 1 as Y increases from 0 to $+\infty$ where $0<H(0)<1$.

In comparison with Lemma 1, the main difference of Lemma 2 is that the support set of CDF $G()$ is bounded close set. And thus, the objective function of problem (5) is not smooth. In fact, $H(Y)$ defined in Lemma 1 is a special case of $H(Y)$ defined in Lemma 2 since $\Theta_2(Y)=\Theta$ for all $Y\geq 0$. As a result, the right-hand-side term of Eq. (14) is reduced to that of Eq. (10). Hereinafter, $H(Y)$ refers to Eq. (14). Fig. 2 provides an explanatory example when both V and Q follow the uniform distributions. In this case, by direct calibration, we know that $H(Y)$ is piece-wise continuous and strictly increasing function. It is valuable to point out that, when the capacity of the self-owned vessels V is deterministic for the company, or, $Pr\{V=V_0\}=1$, then, $H(Y)$ defined in Lemmas 1 and 2 can be reduced to $H(Y)=G(V_0+Y)$.

According to Lemmas 1–2, we immediately know that the optimal solution of problem (5).

Proposition 1. *The optimal capacity of the time-chartered vessels of problem (5) must have $Y^*=0$ when the following condition holds*

$$c_2 \leq \frac{c_1+H(0)c_3}{1-H(0)}; \tag{15}$$

otherwise, the optimal capacity of the time-chartered vessels can be determined by the following condition

$$H\left(Y^*\right) = \frac{c_2-c_1}{c_2+c_3}. \tag{16}$$

Proof. According to Lemmas 1 and 2, $H(Y)$ is strictly increasing from $H(0)$ to 1. When $H(0) > \frac{c_2-c_1}{c_2+c_3}$, the terms in square brackets of Eqs. (9) and (13) are always positive. That is to say $\mathrm{d}\Psi(Y)/\mathrm{d}Y > 0$, which means the optimal capacity of the time-chartered vessels of problem (5) must be zero or $Y^* = 0$. When condition (15) does not hold, then $\Psi(Y)$ is first increasing and then decreasing as Y increases from 0 to $+\infty$. The optimal solution of problem (5) exists and is unique. At optimum, condition (16) holds. The proof is completed. □

As we have pointed out that, the time-chartered vessels should not be considered when its rent charges is higher than that of the voyage-chartered vessels. Proposition 1 provides a tighter condition for the shipping company to consider the time-chartered vessels when the objective function incorporates the freight rate of the shipping market and the holding cost into the consideration, as shown by inequality (15). Note that CDF $G()$ is also monotonically increasing. Equality (16) implies the following properties of the optimal outsourcing capacity of the time-chartered vessels. The result is straightforward.

Corollary 1. *The optimal capacity of the time-chartered vessels, Y^* determined by (16) is strictly decreasing with respect to c_1 and c_3, increasing with respect to c_2.*

In practice, the random variables V and Q would be correlated since the company tends to set a higher portion to the shipping line when its shipment demand is expected to be higher. In this case, we can assume that V and Q follow a joint distribution with marginal distribution $F(V)$ and $G(Q)$. Following similar discussion, we can also obtain the same conclusions as the uncorrelated case.

4. Extensions of the fleet sizing problem

In this section, we extend the benchmark case of fleet sizing problem to the more practical situations. In the previous section, the capacity of time-chartered or voyage-chartered vessels is continuously separable and the cost per unit capacity is given and fixed. In reality, the vessels in the outsourcing market are quite different in vessel sizes. And the rent charge per unit capacity is also related to the vessel size, besides of the rental types. Furthermore, the shipping company typically operates a shipping network with multiple origin and destination pairs. The rent charge per unit capacity also affected by the voyage distance even for a same vessel. Therefore, the company must also consider the deployment problem of vessels among the shipping network.

4.1. Available vessels with discrete capacities and limit numbers

For a practical consideration, we assume that the capacity allocation of the self-owned vessels follows a discrete distribution since the capacity and number of the self-owned vessels are given. Namely $Pr\{V = v_i\} = p_i$, $\{i \in I\}$. Each realization of V produces operational cost γ_i, $i \in I$. Suppose an available vessel fleet in the outsourcing market is denoted by $\{y_s, n_s, \tau_s^k\}$, $s \in S$, where S is the set of vessels types, y_s and n_s are the capacity and number of vessel type s, and τ_s^k, $k = 1, 2$ are the rent charges per trip for the time- and voyage-chartered vessels, $s \in S$. $k = 1$ and $k = 2$ represent the time-chartered and voyage-chartered service, respectively. For τ_s^1, we calculate the average cost per trip by using historical data. This involves considering the total time-chartered cost over the entire period and dividing it by the expected number of trips. Introduce the decision variables x_s^k to represent the number of trips for service k with vessel type s, which is an integer variable with $x_s^1 + x_s^2 \le n_s m$, m is the maximal trip number for ship s sailing between the origin and destination. With the above notations, the fleet sizing problem of the company with fleet deployment can be captured by the following stochastic integer programming problem

$$\min C(\mathbf{x}) = \mu_\gamma + \sum_{k\in\{1,2\}} \sum_{s\in S} \tau_s^k x_s^k + c_3 E\left(\max\{V + Y - Q, 0\}\right) \tag{17}$$

subject to

$$E\left(\max\{Q - V - Y, 0\}\right) \le \hat{Y} \tag{18}$$

$$\sum_{k\in\{1,2\}} x_s^k \le n_s m, \forall s \in S \tag{19}$$

$$Y = \sum_{s\in S} x_s^1 y_s, \ \ \hat{Y} = \sum_{s\in S} x_s^2 y_s, \tag{20}$$

$$E\left(\max\{Q - V - Y, 0\}\right) = \sum_{i\in I} p_i E\left(\max\{Q - v_i - Y, 0\}\right) \tag{21}$$

$$E\left(\max\{V + Y - Q, 0\}\right) = \sum_{i\in I} p_i E\left(\max\{v_i + Y - Q, 0\}\right) \tag{22}$$

$$x_s^1 \in \{0, m, 2\,m, \dots, n_s m\}, \quad x_s^2 \in \{0, 1, 2, \dots, n_s m\}, \quad \forall s \in S. \tag{23}$$

In problem (17)–(23), $\mathbf{x}$ is the vector of decision variables, $\mathbf{x} = \{x_s^k, s \in S, k = 1, 2\}$; In our model, we assume a risk-neutral decision-maker who optimizes capacity decisions based on expected values. While it is true that time charter capacity is determined before observing random shipping demand and voyage charter capacity is determined afterward, our model optimizes the allocation for both by focusing on expected values prior to demand realization. This approach allows for simultaneous decision-making for both time and voyage charters based on the information available before actual demand is known. As a result, there may be differences between the model's decisions and those made in real-world scenarios where post-demand information is available. However, from an expected value perspective, our decision-making approach ensures that the allocation remains optimal. V is discrete random variable and Q is continuous random variable. Eq. (20) are the capacities of the time-chartered vessels Y and voyage-chartered vessels $\hat{Y}$. the conditional expectations in objective (17) and constraint (18) are calculated by Eqs. (21) and (21), respectively. Furthermore, the holding cost is still considered in objective function of problem (17)–(23) to avoid the waste of the transportation capacity.

For voyage charter, the charterer primarily focuses on the lump-sum charter rate paid to the ship owner, with the total charter cost determined by multiplying the number of voyages completed by the charter rate. In contrast, time charter involves two types of costs for the charterer: a fixed charter rate paid to the ship owner (regardless of the number of voyages completed) and variable costs (such as fuel, cargo handling, and port charges) that are dependent on the number of voyages completed. For a given round trip leg, assuming a constant sailing speed, the maximum number of round trips that can be completed within the charter period is a fixed value. Rationally, the time-chartered vessel should aim to complete the maximum number of round trips within the charter period. To reflect this in our model, we restricted the decision variable x_s^1 for time-chartered vessels, as shown in Eq. (23). This constraint ensures that the number of round trips for a time-chartered vessel must be an integer multiple of the maximum possible round trips, providing a logical and practical basis for allocating the fixed charter cost across each round trip. Violating the x_s^1 constraint would lead to underutilization of the vessel's capacity within the charter period. Consequently, the fixed cost of the time charter can be distributed across the completed round trips within the charter period, resulting in a fixed cost per round trip. This, when combined with the variable cost for each round trip, constitutes τ_s^1.

Generally, the traditional branch-and-cut algorithm can be obtained to calculate the optimal solution of the stochastic integer programming problem (17)–(23). To derive the exact solution of the problem theoretically, we introduce the following assumption.

Assumption 1. (1) The marginal effect on rental duration $\tau_s^1 < \tau_s^2$ for all $s \in S$;

(2) The marginal rental cost per unit capacity strictly decreases with the increase of the vessel size, namely, $\frac{\tau_s^k}{y_s} < \frac{\tau_{s'}^k}{y_{s'}}$ provided that $y_s > y_{s'}$, for all $s, s' \in S$, $k = 1, 2$;

(3) The marginal rental cost saving by the time-chartered service is non-decreasing in the vessel size, $\frac{\tau_s^2 - \tau_s^1}{y_s} < \frac{\tau_{s'}^2 - \tau_{s'}^1}{y_{s'}}$ provided that $y_s > y_{s'}$, for all $s, s' \in S$.

The first part of the assumption captures the **marginal effect on rental duration**, namely, the rent charge of the time-chartered vessels is always lower than that of the voyage-chartered vessels for same type of ships. It is commonly seen in rental market that the marginal rental cost decreases when the rental duration increases since the vessel owner tends to earn the certain payoff. Analyzing historical vessel leasing data from an oil transportation company revealed an interesting trend: Assumption 1 holds when the lease duration of the vessels extends beyond three months. This indicates that for lease periods shorter than three months, Assumption 1 may not apply or the observed trends differ. Unless otherwise specified, the charter period for the time-chartered vessels in this paper is assumed to exceed three months. The last two parts of the assumption capture the **marginal effect on vessel size**, namely, the marginal rent charge of vessel decreases with respect to the vessels size. The larger of the vessels size, the lower of the rent charge per unit capacity. Furthermore, the vessel size would increase the marginal rental cost saving by the time-chartered service. The larger of the vessel size, the larger of the marginal cost saving by the time-chartered service. The assumption is practically reasonable. The shipping company is willing to rent a huge vessel since the shipping market has significant scale economies.

We first consider the simplest case with $|S| = 1$, namely, only one type of vessels in the fleet. The optimal solution of problem (17)–(23) with $|S| = 1$ can be described in the following proposition.

Proposition 2. *Suppose Assumption 1 holds and $|S| = 1$, the optimal number of the time-chartered vessels of problem (17)–(23) must have $x_s^1 = 0$ when the following condition holds*

$$\tau_s^2 \leq \frac{\tau_s^1 + c_3 H(0) y_s}{1 - H(0)}; \tag{24}$$

where $H(Y)$ is given by Eq. (14); otherwise, $x_s^1 = n_s m$ if $n_s \leq n^* = \lfloor \frac{Y^*}{y_s} \rfloor$; and $x_s^1 = n^* m$ or $n^* + 1$ if $n_s > n^*$.

Proof. Let $c_k = \frac{\tau_s^k}{y_s}$, $k = 1, 2$. It is clear to see that $c_1 < c_2$ according to Assumption 1 (1). For problem (5) with continuous separable capacity Y, according to Proposition 1, we immediately know that, when condition (15) or, correspondingly, condition (24) holds, the company should not consider the time-chartered vessels, and all the shipment demand will be fulfilled with the self-owned vessels and voyage-chartered vessels. When condition (15) or condition (24) is violated, then there exists a unique Y^* to minimize the total cost. Denote $n^* = \lfloor \frac{Y^*}{y_s} \rfloor$, where $\lfloor A \rfloor$ is the maximal integer not larger than A. Since $C(Y)$ strictly decreases in domain $[0, Y^*)$ and then strictly increases in domain $(Y^*, +\infty)$. Therefore, when $n_s \leq n^* + 1$, the number of time-chartered vessels with a fleet must be n_s; when $n_s \geq n^* + 1$, the company can choose n^* or $n^* + 1$ by comparing the corresponding cost. The proof is completed. □

Note that, the number of voyage-chartered vessels x_s^2 can be determined by the constraint (18) with given x_s^1, namely,

$$x_s^2 = \left\lceil \frac{E\left(max\{Q - V - x_s^1 y_s, 0\}\right)}{y_s} \right\rceil. \tag{25}$$

where $\lceil A \rceil$ is the minimal integer not less than A.

It is useful to consider the simplest case described in Proposition 2. In fact, when the optimal solution of problem (5) with continuous separable capacity Y^* exists and for any integer n with $ny_s \leq Y^*$, introduce the random variable $\bar{V} = V + ny_s$ with operational cost $\bar{\gamma} = \gamma + nt_s^1$. Then we can obtain a variant of fleet sizing problem (17)–(23) with an available fleet $\bar{n}_s = n_s - n$. It is easy to see that, the variant problem is completely equivalent to the original problem (17)–(23) since they have the same optimal solution. More generally, we move to prove the following result.

Proposition 3. *$\left(\bar{x}_s^1, \bar{x}_s^2\right)$, $s \in S$, is the optimal solution of problem (17)–(23) if and only if $\left(\bar{x}_s^1 - \hat{x}_s^1, \bar{x}_s^2\right)$ is the following stochastic programming problem provided that $\hat{x}_s^1 \leq \bar{x}_s^1$ with at least one strict inequality*

$$\min C(\mathbf{x}) = \mu_{\bar{\gamma}} + \sum_{k\in\{1,2\}} \sum_{s\in S} \tau_s^k x_s^k + c_3 E\left(\max\{\bar{V} + Y - Q, 0\}\right) \tag{26}$$

subject to

$$E\left(\max\{Q - \bar{V} - Y, 0\}\right) \leq \hat{Y} \tag{27}$$

$$\sum_{k\in\{1,2\}} x_s^k \leq \bar{n}_s m, \forall s \in S \tag{28}$$

$$x_s^1 \in \{0, m, 2\ m, \ldots, \bar{n}_s m\}, \quad x_s^2 \in \{0, 1, 2, \ldots, n_s m\}, \quad \forall s \in S, k = 1, 2 \tag{29}$$

where $\bar{V} = V + \sum_{s\in S} \hat{x}_s^1 y_s$, $\bar{\gamma} = \gamma + \sum_{s\in S} \hat{x}_s^1 \tau_s^1$, $\bar{n}_s = n_s - \hat{x}_s^1/m$.

Proof. For any feasible solution of problem (17)–(23) can be expressed as $\left(x_s^1, x_s^2\right) = \left(x_s^1 - \hat{x}_s^1, x_s^2\right)$, $s \in S$, provided that $\hat{x}_s^1 \leq x_s^1$. The objective function (17) can be expressed as

$$\begin{aligned}\min C(\mathbf{x}) = &\left(\mu_{\bar{\gamma}} - \sum_{s\in S} \tau_s^1 \hat{x}_s^1\right) \\ &+ \sum_{s\in S} \tau_s^1 \left(x_s^1 - \hat{x}_s^1\right) + \sum_{s\in S} \tau_s^1 \hat{x}_s^1 + \sum_{s\in S} \tau_s^2 x_s^1 \\ &+ c_3 E\left(\max\{\left(\bar{V} + \sum_{s\in S} \hat{x}_s^1\right) + \left(Y - \sum_{s\in S} \hat{x}_s^1\right) - Q, 0\}\right)\end{aligned} \tag{30}$$

With direct arrangement, the constraint conditions (27)–(29) correspond to (18)–(23). Therefore, $\left(\bar{x}_s^1, \bar{x}_s^2\right)$, $s \in S$, is the optimal solution of problem (17)–(23), then $\left(\bar{x}_s^1 - \hat{x}_s^1, \bar{x}_s^2\right)$, $s \in S$, must be the optimal solution of problem (26)–(29) provided that $\hat{x}_s^1 \leq \bar{x}_s^1$; and vice versa. The proof is completed. □

Suppose $\left(\bar{x}_s^1, \bar{x}_s^2\right)$, $s \in S$, is the optimal solution of problem (17)–(23), we can reduce problem (17)–(23) to the equivalent problem (26)–(29) when $\bar{x}_s^1 > 0$, according to Proposition 3. The original problem (17)–(23) with $|S| > 1$ can be finally reduced to the problem with $|S| = 1$. From Proposition 2, we further have the following result.

Proposition 4. *Under Assumption 1, suppose $\left(\bar{x}_s^1, \bar{x}_s^2\right)$, $s \in S$, is the optimal solution of problem (17)–(23), then $\bar{x}_s^1 = 0$ when condition (24) holds; furthermore, if $n_{s_1} > \bar{x}_{s_1}^1$, then $\sum_{y_s < y_{s_1}, s\in S} y_s \bar{x}_s^1 < y_{s_1}$.*

According to Proposition 4, we can determine the candidates of time-chartered vessels provided that condition (24) is violated. For the candidates of time-chartered vessels, it is natural for the shipping company to allocate the capacity to the ship with larger size since the marginal rental cost per unit capacity strictly decreases with the increase of the vessel size, according to (2) and (3) in Assumption 1. When we consider one of the candidates, we can reduce the original problem (17)–(23) to problem (26)–(29) by subtracting the allocating capacity according to Proposition 3 . The optimal number of the current vessel can be determined by Proposition 2 with $|S| = 1$. $\bar{x}_s^1$ is obtained by iteratively implementing Propositions 2 and 3 for all candidate vessels. $\bar{x}_s^2$ is easily determined by considered the following linear programming problem

$$\min \sum_{s\in S} \tau_s^2 x_s^2 \tag{31}$$

subject to

$$\sum_{s\in S} x_s^2 y_s \geq E\left(\max\{Q - V - \sum_{s\in S} \bar{x}_s^1 y_s, 0\}\right) \tag{32}$$

$$x_s^2 \in \{0, 1, 2, \ldots, n_s m - \bar{x}_s^1\}, \forall s \in S. \tag{33}$$

According to the above discussion, we propose the constructive heuristic algorithm to determine the optimal solution of problem (17)–(23), which follows the idea of steepest descent method. According to Assumption 1, we sort all vessels in descent order of the vessel sizes, then set the vessels as the time-chartered service one by one, and others with less sizes are considered as the voyage-chartered service till the expected shipment demand is served. The algorithm can be described in Algorithm 1. In practical applications, Algorithm 1 serves as a heuristic method, aiming to provide high-quality solutions efficiently. While it may not guarantee global optimality in every instance, it is designed to approach near-optimal solutions by leveraging the properties provided by Proposition 2.

Algorithm 1 Algorithm of problem (17)-(23)

1: Selecting candidate time-chartered vessels

$$\bar{S} = \left\{ s \in S : \tau_s^2 > \frac{\tau_s^2 + H(0)c_3 y_s}{1 - H(0)} \right\};$$

2: Ranking the marginal rent charge, $\frac{\tau_{s_1}^1}{y_{s_1}} < \frac{\tau_{s_2}^1}{y_{s_2}} < \cdots,\ s_j \in \bar{S}$;

3: Determining the optimal number of time-chartered vessels, set $c_k = \frac{\tau_{s_1}^1}{y_{s_1}}$, $k = 1, 2$, and determine $\bar{x}_{s_1}^1$ according to **Proposition 2**;

4: Moving to Step 3 for type s_2 if $\bar{x}_{s_1}^1 = n_{s_1} m$;

5: Stopping when all types in $\bar{S}$ are allocated;

6: Finding $\bar{x}_s^2$ with given $\bar{x}_s^1$ by solving problem (31)-(33).

4.2. Fleet sizing problem with a shipping network

A shipping company typically provides transhipment service between multiple OD pairs in practice. We now move to extend the fleet sizing problem into a shipping network. The transportation branch company of China National Petroleum Corporation provides about 15 shipping lines with 20 self-owned vessels. For the company, all original ports are located on Bohai Bay, and the destination ports are distributed in southeast coast. Therefore, we consider a shipping network with one origin and multiple destinations. A vessel only serves an origin and destination for each journey. It is reasonable for the refined oil shipping service, different from the container shipping service.

Denote the set of the shipping lines as L with an index l. Let m_l represent the maximal round-trip numbers of shipping line l by all vessels sailing with same average speed. Let V_l represent the allocation of the capacity of the self-owned vessels on shipping line l. V_l, $l \in L$, is a correlated random vector and can be determined by the statistics on the historical data of self-owned vessel allocation. We assume that V_l is exogenously given and focus our discussion on the outsourcing capacity determination of the time-chartered and voyage-chartered vessels. The capacity and operational cost on a specified shipping line are still represented by γ_l, $l \in L$. Furthermore, since the joint distribution of V_l, $l \in L$, is exogenously given, we can easily determine the marginal distribution of V_l, still denoted by $F_l(V_l)$. The function in the square bracketed term of (34) is the same as that of problem (17)–(23).

Note that different shipping lines can share the same vessel. However, a vessel cannot provide different service types, namely, a vessel cannot rent as a time-chartered and voyage-chartered types simultaneously. Therefore, unlike the previous subsection, it is convenient to consider each vessel, separately. Denote the set of vessels by S. Each vessel is associated with a capacity y_s and rent charges τ_{sl}^k for service type k, $s \in S$, $l \in L$, $k = 1, 2$. Introduce decision variable x_{sl}^k to represent the number of round-trips for vessel s on shipping line l for service type k. The problem of the company is to select the time-chartered vessels among all shipping lines facing the uncertain shipment demand Q_l of each shipping line to minimize the total expected cost. In the same spirit as the previous subsection, the fleet sizing problem with a shipping network can be captured as the following stochastic integer programming model.

$$\min C(\mathbf{x}) = \sum_{l\in L} \left[\mu_{\gamma_l} + \sum_{k\in\{1,2\}} \sum_{s\in S} \tau_{sl}^k x_{sl}^k + c_3 E\left(\max\{V_l + Y_l - Q_l, 0\} \right) \right] \tag{34}$$

subject to

$$E\left(\max\{Q_l - V_l - Y_l, 0\} \right) \le \hat{Y}_l, l \in L \tag{35}$$

$$x_{sl'}^1 x_{sl''}^2 = 0, s \in S, l' \in L, l'' \in L \tag{36}$$

$$\sum_{l\in L} \frac{x_{sl}^k}{m_l} \le 1, s \in S, k = 1, 2 \tag{37}$$

$$Y_l = \sum_{s\in S} x_{sl}^1 y_s,\ \ \hat{Y}_l = \sum_{s\in S} x_{sl}^2 y_s \tag{38}$$

$$x_{sl}^k \in \mathbb{N}. \tag{39}$$

where $\mathbf{x}$ is the vector of decision variables, $\mathbf{x} = \{x_{sl}^k, s \in S, l \in L, k = 1, 2\}$; $\mathbb{N}$ is the set of the non-negative integers. Eq. (38) are the definitions of the capacities of the time-chartered vessels and voyage-chartered vessels. Constraint (36) implies that a vessel either is rent as time-chartered service or voyage-chartered service. Constraint (37) means that when a vessel provides transportation service for several shipping lines, the feasible combinations of the round-trip numbers must satisfy the constraint during the planned period. Constraint (37) implicitly expresses a limitation on the service frequency, or total sailing time, for different lines served by the same vessel.

To incorporate the effect of the voyage distances of different shipping lines, we make the following assumption.

Assumption 2.
(1) The marginal effect on rental duration, $\tau^1_{sl} < \tau^2_{sl}$ for all s and l;
(2) The marginal effect on vessel size, $\frac{\tau^k_{sl}}{y_s} < \frac{\tau^k_{s'l}}{y_{s'}}$ and $\frac{\tau^2_{sl}-\tau^1_{sl}}{y_s} < \frac{\tau^2_{s'l}-\tau^1_{s'l}}{y_{s'}}$ provided that $y_s > y_{s'}$ for all s, s' and k;
(3) The marginal effect on voyage distance, $m_l \tau^k_{sl} < m_{l'} \tau^k_{sl'}$ provided that $m_l < m_{l'}$ for all s and k.

(1) and (2) in Assumption 2 are discussed in the previous sections. The last assumptions implies that, the marginal shipping cost per unit cargo-distance is strictly decreasing when the shipping distance increases. In the shipping area, the transportation cost would have the economic scale on distance. Suppose the planning period T and average sailing speed u, the marginal shipping cost per unit cargo-distance can be expressed as $\frac{m_l \tau^k_{sl}}{uT}$. Therefore, $m_l < m_{l'}$ means that the shipping distance along shipping line l is larger than that along l'. Therefore, Assumption 2(3) represents the economic scale on voyage distance. In reality, the shipping company has an incentive to rent a vessel with a larger size for the long-hull shipping service, which is captured by the marginal effects described in Assumption 2.

Under Assumption 2, following the method described in Section 4, we can derive $H_l(Y)$ with given CDFs $F_l(V_l)$ and $G_l(Q_l)$. Therefore, the following conclusion can be easily proved.

Proposition 5. *Under Assumption 2, suppose $(\bar{x}^1_s, \bar{x}^2_s)$, $s \in S$, is the optimal solution of problem (34)–(39), then*
(1) $\bar{x}^1_s = 0$ when $\tau^2_{sl} \le \frac{\tau^1_{sl} + H_l(0)c_3 y_s}{1-H_l(0)}$ for all s and l;
(2) $\bar{x}^k_{sl} = 0$ implies $\bar{x}^k_{sl'} = 0$ when $m_l \tau^k_{sl} < m_{l'} \tau^k_{sl'}$ for all s and k;
(3) $\bar{x}^k_{sl} = 0$ implies $\bar{x}^k_{s'l} = 0$ when $\frac{\tau^k_{sl}}{y_s} < \frac{\tau^k_{s'l}}{y_{s'}}$ for all l and k.

Proof. When $\tau^2_{sl} \le \frac{\tau^1_{sl} + H_l(0)c_3 y_s}{1-H_l(0)}$, we can change the time-chartered service for vessel s to the voyage-chartered service provided that $\bar{x}^1_{sl} > 0$. By doing this, the cost of shipping line l and the thus the total cost of the company decreases. We obtain the result (1). According to the marginal effect of the transportation distance, the total cost must decrease when the company assigns vessel s to shipping line l provided that $\bar{x}^k_{sl'} > 0$ and $\bar{x}^k_{sl} = 0$ since $m_l \tau^k_{sl} < m_{l'} \tau^k_{sl'}$. Result (2) is obtained. Similarly, the marginal effect of the vessel size guarantees result (3) of the proposition. The proof is completed. □

It is straightforward to prove the results described in Proposition 5 according to Assumption 2. Proposition 5 provides the rule of selecting the time-chartered vessels: the vessels with larger capacity should be allocated to shipping line with long distance since the marginal shipping cost is lower than the others. We can revise Algorithm 1 to determine the optimal solution of problem (34)–(39). In this case, we should rank the shipping lines according to the voyage distance or sort m_l, $l \in L$. For shipping line $l = 1$, the candidate time-chartered vessels can be selected according to Proposition 5(1). The optimal time-chartered vessels $\bar{x}^1_{s,l}$ is determined according to Algorithm 1 for all candidates. Then, following the same method, we can consider the next shipping line till $\bar{x}^1_{s,l}$ are determined for all shipping lines. Note that, the feasible set of vessel numbers are different for different shipping lines. The voyage-chartered vessels $\bar{x}^2_{sl}$ again can be determined by the following deterministic integer programming problem

$$\min \sum_{l\in L} \sum_{s\in S} \tau^2_{sl} x^2_{sl} \tag{40}$$

subject to constraints (37), (39) and

$$\sum_{s\in S} x^2_{sl} y_s \ge E\left(\max\left\{Q_l - \bar{V}_l - \sum_{s\in S} \bar{x}^1_{sl} y_s, 0\right\}\right), \forall l \in L. \tag{41}$$

Note that, in reality, there are several types of refined-oil products including octane-92, octane-93, octane-98 gasoline and diesel. The vessels can transport different products. However, different products cannot be transported simultaneously by a single vessel since there is typically only one tank for each vessel. Furthermore, different refined-oil products have different density. And thus, due to the limitation of the tank volume, each vessel has different levels of capacity in weight for different oil products. In this case, we can view the shipping line with different oil products into different shipping lines according to the product type, and convert different oil products into the gasoline because the gasoline has a lower density. By doing so, the results described in Proposition 5 and the above algorithm are also valid.

The model incorporates inventory holding costs and year-end excess inventory loss costs by assuming that the vessel charter costs can be evenly distributed across each unit of inventory. Similar to the newsvendor problem, the model derives optimal inventory-related conclusions, which provide useful guidance for chartering decisions at a macro level. However, this model diverges from the traditional newsvendor problem in several ways: (1) Diverse Vessel Types: The model incorporates various vessel types, each with distinct economic and industry characteristics. This includes considerations for diminishing marginal returns in rental costs both by duration and vessel size, reflecting preferences for time-chartered vessels and cost efficiencies of larger vessels. (2) Complex Shipping

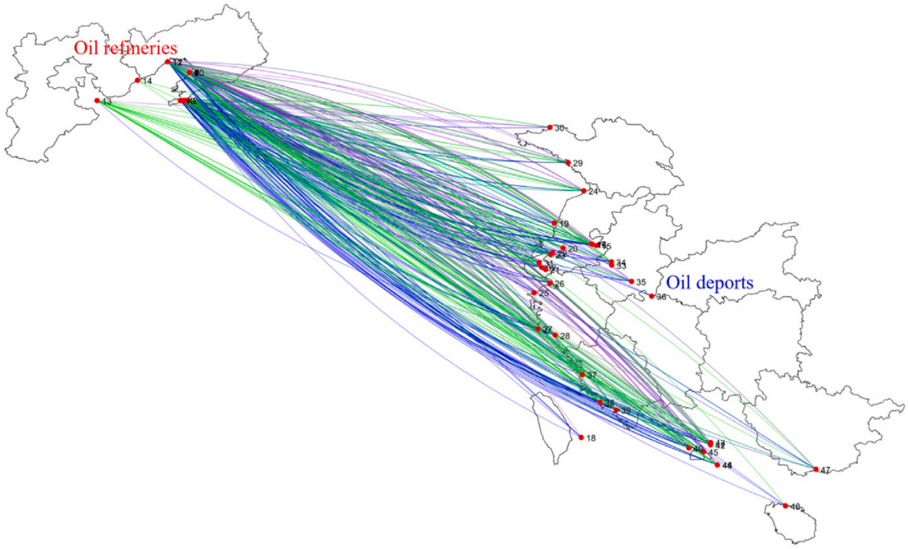

Fig. 3. The network of the refined-oil shipping company.

Networks: The model extends from a point-to-point market to a complex shipping network, highlighting the economic characteristic of diminishing marginal returns on distance and the ability of vessels to serve multiple segments simultaneously. These unique aspects and generalizations address the specificities of the chartering industry, offering a more comprehensive framework than the classic newsvendor problem.

5. Case study

In this case study, we consider the real fleet sizing problem of time-chartered and voyage-chartered vessels for the COSCO Shipping Energy Transportation Co., Ltd. The planning period is assumed to be one year. The company occupies about 40 percentage of the total refined-oil products in China. One of the maximal subsidiary companies, the north company provides the refined-oil shipping service between the 14 ports in the Liaoning Province and the 36 deports distributed along the coastal area of China, shown in Fig. 3. Each year, the company received over 200 million tons covering five types of refined-oil products, such as, octane-92, octane-95, octane-98, octane-E92 gasoline and diesel. Since all oil refineries locate in Liaoning Province, we can simplify the shipping network as one refineries and multiple deports. Distinguishing different oil types, there are 538 shipping lines provided by the company, shown in Fig. 3.

5.1. Benchmark case with a single shipping line

Note that, the shipment demand between the refined ports in Dalian and the deports in Jiangsu Province occupies about 15% of the total quantity of the company. The distance between the ports is quite short both for the refined ports and deports. To examine the benchmark case, we bundle all the shipping lines and consider one kind of oil product, typically, the octane-92 gasoline. The voyage distance is assumed to be the average of all those shipping lines, about $l = 725.5$ nautical miles. The shipment demand in 2022 is about $Q_0 = 254,315$ tons. We assume that the annual shipment demand follows a uniform distribution with 40% fluctuation of the quantity. That is to say, $Q \backsim [(1-0.4)\times Q_0, (1+0.4)\times Q_0]$. Even thought the company has a self-owned vessel fleet, it is uncertain to determine the realized capacity of the those self-owned vessels for the future planning period.

Suppose the average sailing speed for all vessels is 10 knots. The round-trip time for a vessel along the shipping line includes the loading time at origin and destination ports, typically, two days, sailing time back and force between the two ports. We assume that the idle time for maintenance or dealing with other tasks occupies 20% of the whole round-trip time. Given the annual working time 300 days, the feasible number of round-trip for a vessel is estimated to be 38, which is identical for all vessels. According to the practical data of the refined-oil shipping company, both the rent charges of time-chartered and voyage-chartered vessels vary with vessel size, as described in Assumptions 1 and 2. For a time-chartered vessel, dividing the annual rent cost of a specific vessel by its vessel size, we obtain the annual rent charge c_1 Yuan per year when the vessel is allocated as the time-chartered service. For a time-chartered vessel, it is simple to calculate the corresponding annual rent charge c_2 by multiplying the one-time rent charge and the round-trip number. Then we can estimate the rent charges for vessels with different sizes and service types in the outsourcing market with an interpolation method. Fig. 4 shows the time and voyage-chartered vessels with different sizes. The rent cost for voyage-chartered vessels c_2 is directly estimated by the statistics from the outsourcing market. The blue circles represent the real rent charge for time-chartered vessels used in the company. A negative exponential function is adopted to fit the relationship

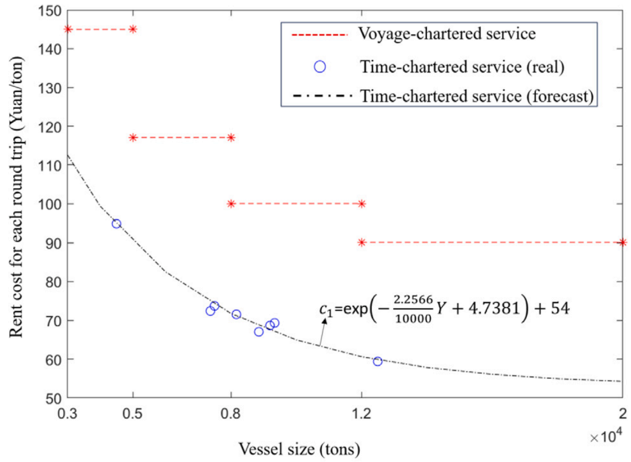

Fig. 4. Rent charges for time and voyage-chartered vessels with different sizes.

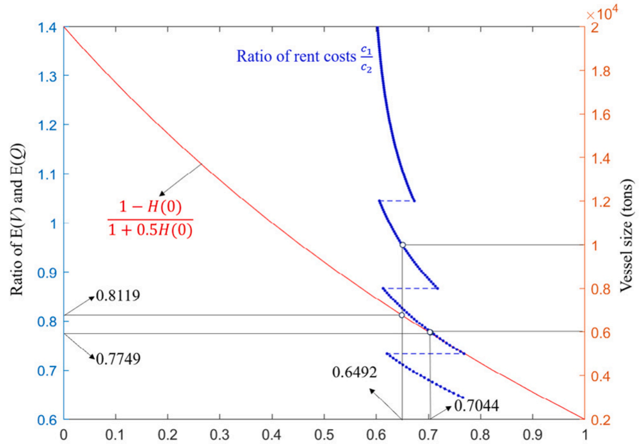

Fig. 5. Rent cost for time and voyage-chartered vessels with different sizes.

between the marginal rent charge c_1 and the vessel size, as shown in Fig. 4. In addition, we assume that holding cost c_3 is a half of the rent charge of the time-chartered service.

With above assumption, we know that condition (15) in Proposition 1 can be rearranged to

$$\frac{c_1}{c_2} \geq \frac{1-H(0)}{1+\frac{1}{2}H(0)} = \begin{cases} 1 & \frac{E(V)}{E(Q)} < 0.6 \\ \frac{24}{5}\frac{1}{1+E(V)/E(Q)} - 2 & 0.6 \leq \frac{E(V)}{E(Q)} \leq 1.4 \\ 0 & \frac{E(V)}{E(Q)} > 1.4 \end{cases} \tag{42}$$

where $E(V)$ is the average capacity of self-owned vessels assigned to the shipping lines between Dalian and the deports in Jiangsu Province. Fig. 5 plots the function $\frac{1-H(0)}{1+H(0)/2}$ and the ratio of c_1 and c_2 used the data in Fig. 4. The figure is useful to tell the time-chartered service with a specific vessel is beneficial for the company when given the capacity realization of the self-owned vessels. For example, consider a 10,000-ton vessel, $\frac{c_1}{c_2} = 0.6492$. When $E(V) > 0.8119E(Q)$, the company should not consider the time-chartered service. Furthermore, the optimal capacity of time-chartered service with 10,000-ton vessels can be determined by Eq. (16) as $Y^* = 0.8119E(Q) - E(V)$ when $E(V) < 0.8119E(Q)$. Similarly, when $E(V) > 0.7749E(Q)$, the company should not consider the time-chartered service with 6000-ton vessels. And, the optimal capacity of time-chartered service with 6000-ton vessels is $Y^* = 0.7749E(Q) - E(V)$ when $E(V) < 0.7749E(Q)$. Specially, condition (24) always holds since $H(0) = 0$ when $Pr\{V = 0\} = 0$. And thus, all vessels can be considered as time-chartered service.

We now move to consider the fleet sizing problem for the company facing a potential vessel fleet with 247 vessels. The capacity of the vessels varies from 2100 to 19 800 tons. We consider the fleet sizing problem with $|S| = 1$. To do so, we view the vessels with same capacity level as a fleet and determine the optimal solution of problem (17)–(23) for each fleet. According to constraint (19), there are 102 feasible vessel fleets as the time-chartered service. With direct calibration, the optimal capacity

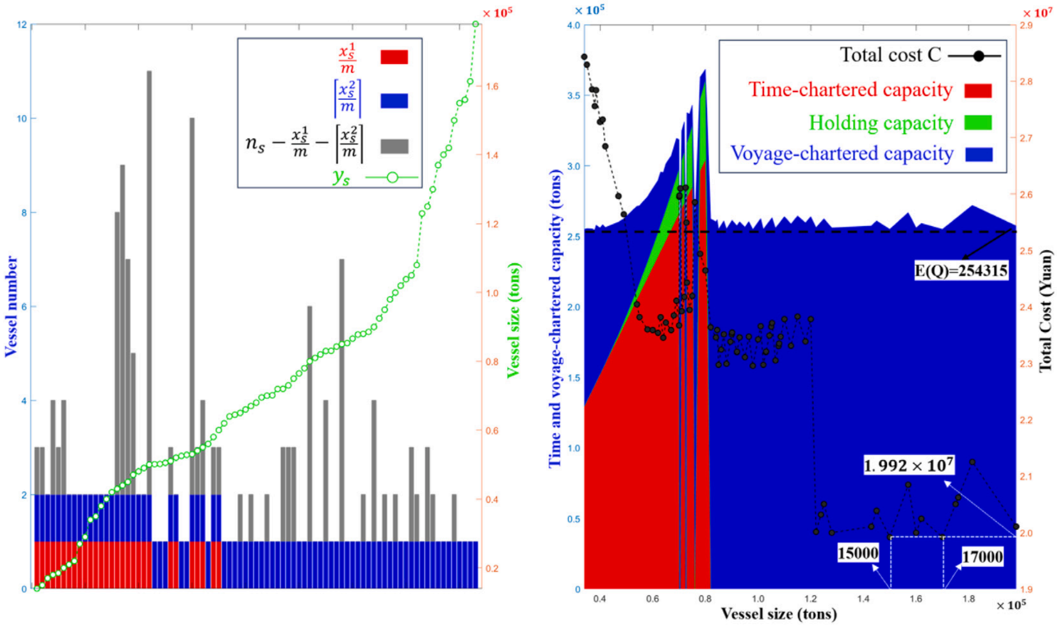

Fig. 6. Optimal numbers of time-chartered and voyage-chartered vessels with $|S| = 1$. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

$Y^* = \max\{0, \left(\frac{24}{5(2+c_1/c_2)} - 1\right) E(Q) - E(V)\}$, then $n^* = \lfloor \frac{Y^*}{y_s} \rfloor$ defined in Proposition 2. Finally, the optimal numbers of time-chartered and voyage-chartered vessels can be determined by comparing the levels of the total objective when $x_s^1 = n^* m$ and $x_s^1 = (n^*+1)m$, where the number of round-trips $m = 38$. To be convenient, we assume $Pr\{V = 0\} = 1$. With the uniform distribution of the shipment demand and x_s^1, the holding capacity and residual capacity can be calculated by

$$E(\max\{Y - Q, 0\}) = \int_{0.6E(Q)}^{A} \frac{x_s^1 y_s - Q}{0.8E(Q)} \mathrm{d}Q = \frac{(A - 0.6E(Q))\left(2x_s^1 y_s - A - 0.6E(Q)\right)}{1.6E(Q)} \tag{43}$$

and

$$E(\max\{Q - Y, 0\}) == \frac{(1.4E(Q) - A)\left(1.4E(Q) + A - 2x_s^1 y_s\right)}{1.6E(Q)}. \tag{44}$$

where $A = \min\{\max\{0.6E(Q), x_s^1 y_s\}, 1.4E(Q)\}$. Fig. 6 shows the optimal solution for each given vessel fleet. Given each vessel type, there are 83 vessel fleets satisfying constraint (19) at the corresponding optimal solution. The left-hand-side sub-figure shows that, when the capacity of vessel y_s is lower than or equal to 8000 tons, at the optimum, the company should rent 1 vessel as the time-chartered service, dotted by the red color; when the capacity of vessel y_s is higher than 8200 tons, at the optimum, the company should not consider the time-chartered service. Specially, several fleets with capacity level between 7000 and 8000 tons are not considered as time-chartered service, as shown in Fig. 6, because there is only one vessel for each vessel type and it is better to rent the vessel as voyage-chartered service. The right-hand-side sub-figure of Fig. 6 shows the time-chartered capacity, holding capacity and voyage-chartered capacity for each given vessel fleet. Observing from Fig. 6, we can see that, when the capacity of vessel is 15,000 or 17,000 tons, the company only considers the voyage-chartered service since the shipment demand is quite low. The total rent cost achieves minimum $C = 1.992 \times 10^7$ RMB Yuan.

To examine the validity of Algorithm 1 for a fleet with multiple vessel types, we randomly select several vessels from the given outsourcing market to construct a fleet with multiple vessel types. The holding cost per unit capacity is assumed to be average of the time-chartered rent charge of the vessels in the fleet. For the uniform distribution and $Pr\{V = 0\} = 1$, we know $H(0) = 0$ and condition (24) always holds for all vessels. According to Algorithm 1, we sort all vessels according to their sizes and consider the vessels as time-chartered service. Then the holding capacity and residual capacity can be calculated by Eqs. (43) and (44). The cumulative transhipment capacity by the voyage-chartered vessels is the summation of sizes of other vessels according to the cumulative round-trip numbers. Fig. 7 shows the optimal solutions for two scenarios: (I)$10Q_0$ and 30 vessels; (II) $5Q_0$ and 20 vessels. For both scenarios, with the increase of the time-chartered capacity, the total cost first decreases and then increases. For scenario (I),

Table 1
Self-owned vessel fleet.

Capacity for octane-92 (tons)	5400	9800	10 300	24 500
Number of vessels	1	2	4	5

the minimal cost is achieved when the largest three vessels are rent by the company as the time-chartered service. For scenario (II), the minimal cost is achieved when the largest four vessels are rent as the time-chartered service. The optimal solutions are shown in upper sub-figures of Fig. 7 (I) and (II). The lower sub-figures of Fig. 7 (I) and (II) shows the corresponding holding capacity and residual capacity for each feasible solution. It must be pointed out that, the vessel sizes of the fleet for scenario (II) are quite small in comparison with those of the fleet for scenario (I). As a result, even though the average shipment demand of scenario (II) is quite lower than that of scenario (I), the total cost at optimum of scenario (II), 2.1236×10^8 RMBYuan, is higher than that at optimum of scenario (I), 2.1080×10^8 RMBYuan. That is because, the rent charge for both time and voyage-chartered vessels are very high for scenario (II).

5.2. General case with a shipping network

In this subsection, we move to consider the fleet sizing problem with a shipping network and multiple types of products. There are 538 shipping lines by distinguishing different OD pairs and product types, shown in Fig. 3. Viewing octane-92 gasoline as reference, the converting factors of the shipment demands for other oil products are $1.014 \cong \frac{0.735}{0.725}$, $1.039 \cong \frac{0.753}{0.725}$ and $1.172 \cong \frac{0.850}{0.725}$, respectively, where the densities for octane-92, octane-93, octane-98 gasoline and diesel are 0.725, 0.735, 0.753 and 0.850 ton/m^3. The density of octane-E92 gasoline mixed with ethanol is almost the same with that of octane-92 gasoline. We can unify all oil types to octane-92 gasoline by dividing those converting factors. Similarly, the shipment demand for each shipping line is assumed to follow a uniform distribution with 40% fluctuation of the quantity collected in 2022. That is to say, $Q_l \backsim [(1-0.4) \times E(Q_l), (1+0.4) \times E(Q_l)]$, where $E(Q_l)$ is the shipment demand in 2022 for shipping line l.

The company has a self-owned fleet with 16 vessels. The capacities for octane-92 gasoline and the corresponding number of vessels are listed in Table 1. It is difficult to assign the self-owned fleet for the company facing the uncertain shipment demand. In the numerical example, we can first determine the assignment the self-owned vessels among all shipping lines by assuming that the probability of the shipment demand fulfilled by the self-owned vessels along a specific shipping line must exceed $P_0 = 100\%$ once the shipping line is selected to be served by the self-owned vessels, mathematically,

$$\Pr\left\{\sum_{s \in S_0} x_{sl}^0 y_s^0 - Q_l z_l\right\} \geq P_0, \forall l \in L, \tag{45}$$

where $z_l \in \{0,1\}$, $l \in L$; integer x_{sl}^0 is the number of the round-trip of vessel s along shipping line l, $s \in S_0$, and S_0 is the set of the self-owned vessels. x_{sl}^0 must satisfy the following constraint to guarantee the available of the vessel number

$$\sum_{l \in L} \frac{x_{sl}^0}{m_l} \leq n_s, \forall s \in S_0. \tag{46}$$

Note that, the service guaranteed constraint (45) can be expressed as

$$\sum_{s \in S_0} x_{sl}^0 y_s^0 \geq G_l^{-1}(P_0) z_l = 1.4 E(Q_l) z_l, \forall l \in L. \tag{47}$$

The objective of the company is to maximize the total fulfilled shipment demand, namely $\max \sum_{l \in L} 1.4 E(Q_l) z_l$. By adopting the linear integer programming solver of Matlab, we can obtain the feasible solution for the problem. As a result, 164 shipping lines can be served by the self-owned vessels with 100% service guaranteed constraint. We only consider the other 374 shipping lines for the time and voyage-chartered service.

Consider 237 vessels without distinguishing the vessel size in the outsourcing market. Therefore, n_s for all vessels can be set to 1. According to Proposition 5, we sort the shipping lines in the ascending order of the round-trip numbers and sort the vessels in the descending order of the vessel sizes. Furthermore, to simply calibrate and obtain some feasible solutions, we assume that the maximal trip number of ship s allocated to shipping line l, x_{sl}^1 does not exceed $\left\lfloor \frac{E(Q_l)}{0.8 y_s} \right\rfloor$ to avoid huge holding cost by the unreasonable assignment. We also assume that $\sum_{l \in L} \frac{x_{sl}^0}{m_l} \geq 0.9$ to guarantee the full utilization of the time-chartered vessels. According to those rules, we can select the first feasible time-chartered vessel and determine the corresponding feasible assignment x_{sl}^1 on the shipping lines. Then, after selecting the first time-chartered vessel, we add one more time-chartered vessel from the remaining vessels. According the same rules, we determine the feasible joint assignment x_{sl}^1 for the two time-chartered vessels. We obtain one feasible x_{sl}^1 corresponding to a specified number of the time-chartered vessels. For each feasible x_{sl}^1, we can calculate the residual capacity for all shipping lines, $\hat{Y}_l$, similar to Section 5.1. Obtain the optimal solution x_{sl}^2 by the integer programming problem (40)–(41). By comprising all those feasible solutions, we can find the approximate optimal solution.

Fig. 8 shows the rent costs of time and voyage-chartered vessels, the holding cost and the total rent cost vary with the number of the time-chartered vessels. As the number of the time-chartered vessels increases, both rent cost and holding cost of the time-chartered vessels strictly increase, shown as Fig. 8(a) and (b). Because of the demand uncertainty and the network effect, the rent

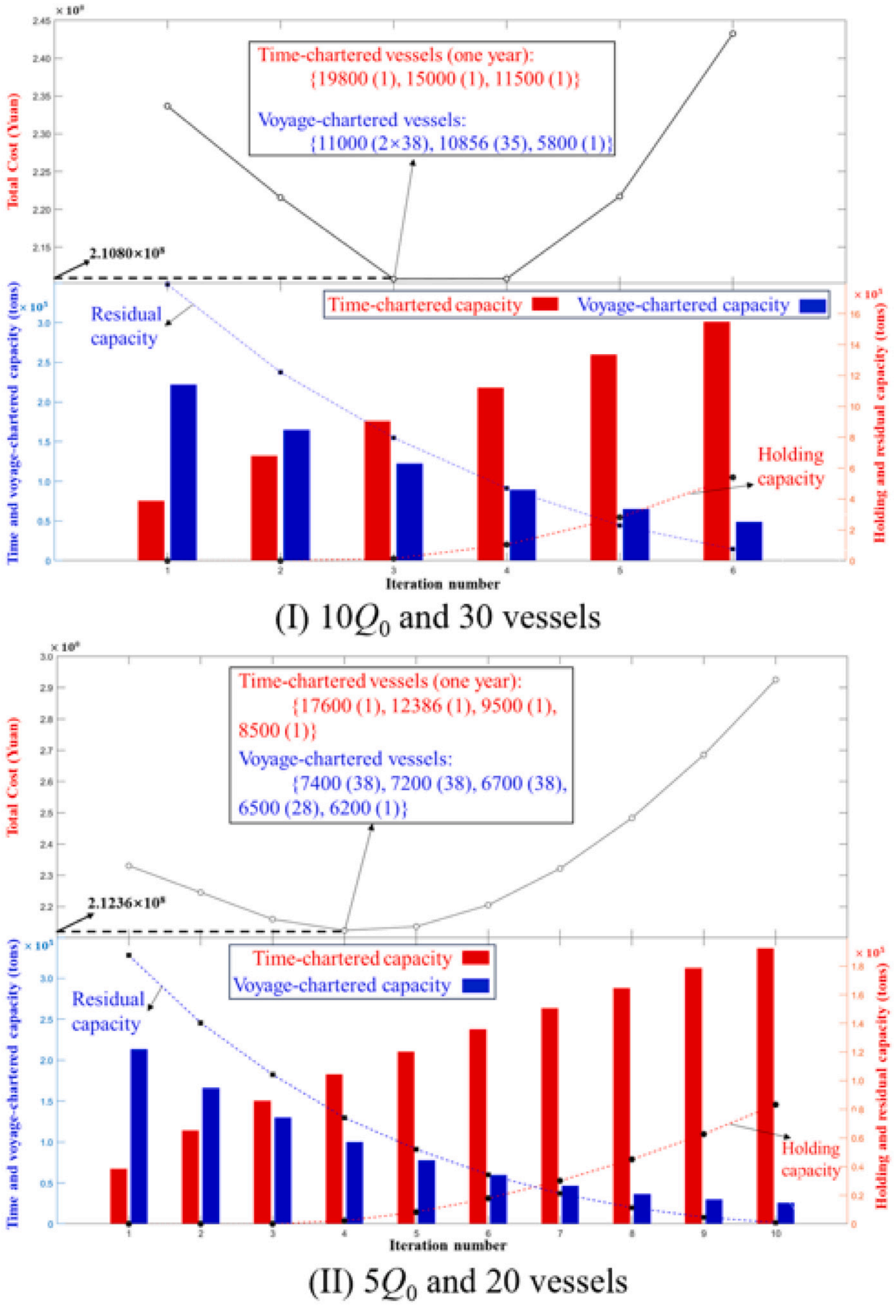

Fig. 7. Optimal time and voyage-chartered vessels for scenarios (I) and (II).

cost of the voyage-chartered vessels, and thus the total rent cost, fluctuates with the increase of the time-chartered vessels since the voyage-chartered vessels must accomplish the residual shipment demand or satisfy constraint (41). However, observed from Fig. 8(d), at optimum, the number of the time-chartered vessels achieves 10 and the minimum of the total rent cost is about 2.7002 billion RMBYuan. In this case, the number of the voyage-chartered vessels is 78 with size from 17 500 tons to 2000 tons, which can be determined by linear programming problem (40) with constraints (37), (39) and (41). And, the minimal rent cost of the voyage-chartered vessels is achieved, about, 2.4399 billion RMBYuan, at the number of the time-chartered vessels is up to 19. In comparison with the case without considering time-chartered service, the total rent cost can be reduced by about 15%.

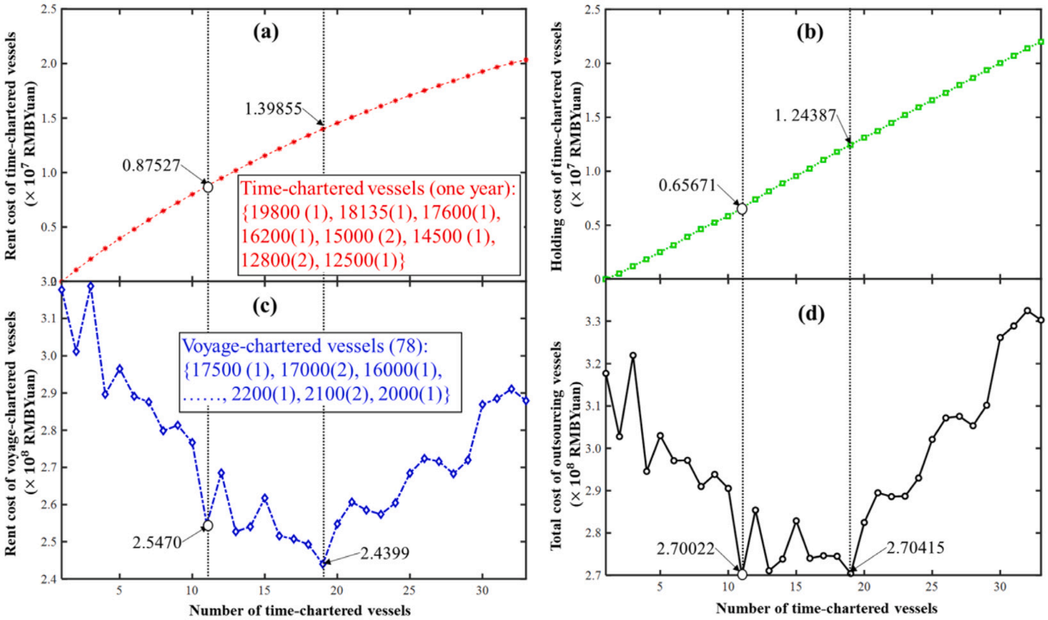

Fig. 8. Fleet sizing with time and voyage-chartered vessels with a shipping network.

6. Conclusions

The fleet sizing is a key decision problem for the industry and tramp shipping, which is challenging to the researchers due to various uncertainty of market conditions. This paper considers the fleet sizing problem of an industry carrier in a planning period with uncertain shipment demand. Besides the self-owned vessels, the carrier selects the time-chartered and voyage-chartered vessels from the potential outsourcing market according to her uncertain demand. We first assumed that there exists the economic scale on rent duration, vessels capacity and voyage distance. That is to say, the rent charge of the time-chartered vessels is always lower than that of the voyage-chartered vessels for same type of ships, the marginal rent charge of vessel decreases with respect to the vessels size, and the marginal shipping cost per unit cargo-distance decreases when the voyage distance increases. Under some economical assumptions, we considered three scenarios: (I)the aggregative shipment demand and an available fleet with continuously separable vessel capacity in the outsourcing market, (II)the aggregative shipment demand and an available fleet with inseparable vessel capacity, and (III)the origin–destination (OD) shipment demand matrix and an available fleet with inseparable vessel capacity. For each scenario, the corresponding stochastic programming model of fleet sizing problem was proposed to minimize the total expected cost including the rent costs of time-chartered and voyage-chartered vessels and the holding cost of time-chartered vessels. The theoretical analysis was conducted to derive the necessary conditions and solution properties. For the scenario with the aggregative shipment demand and an available fleet with continuously separable vessel capacity, the optimal capacity of time-chartered vessels can be exactly captured by the marginal costs of service types and the cumulative distributions of demand and realized capacity of the self-owned vessels. For the scenarios with an available fleet with inseparable vessel capacity in the outsourcing market, we examine the necessary conditions of the optimal fleet sizing and propose heuristic algorithms to determine the fleets.

There are several directions to extend the current research. One is to consider various uncertainties including demand, charter rates and voyage time. Charter rates have significant effect on the decision behavior of carriers since the rent cost is directly related to the charter rate. How to capture the stochastic properties on the rent chargers of time and voyage-chartered vessels is quite challenging. The voyage time would affect the efficiency of the time-chartered vessels in the deployment and the total cost. The second direction is to adopt the multi-stage stochastic programming model to capture the scheduling problem of voyage-chartered vessels once the market conditions are realized. More practically, the AIS (Automatic identification System)-based data can be used to estimate the real-time location of vessels and the dynamic vessel-cargo matching method can be developed. Last but not least, the fleet renewal problem is not considered in the current paper. It is important extension to incorporate the renewal problem of the self-owned vessels in a long-term decision for the industry shipping company. The current model uses aggregated annual demand data, which fails to capture the impact of daily or weekly demand fluctuations. Future research should consider refining time granularity by incorporating daily, weekly, or monthly demand data to simulate the effect of short-term demand fluctuations on fleet size decisions. Additionally, analyzing the seasonal and cyclical variations in demand will provide a more detailed demand forecasting model. Operational details play a crucial role in determining fleet size, and future research can integrate these by considering the layout of ports, route design, and transshipment points in the shipping network to optimize vessel scheduling and route selection. A detailed analysis of vessel usage plans, including loading and unloading times, sailing times, and maintenance schedules, is necessary to assess the effective capacity of the vessels more accurately. Furthermore, introducing order management and fulfillment strategies will help simulate the impact of different strategies on fleet size and operational efficiency.

CRediT authorship contribution statement

Zhijia Tan: Writing – review & editing, Supervision, Resources, Funding acquisition. **Liwei Du:** Writing – original draft, Investigation, Formal analysis. **Ming Wang:** Writing – original draft, Validation. **Hai Yang:** Writing – review & editing, Supervision. **Lingxiao Wu:** Writing – review & editing, Validation.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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References

Alvarez, J.F., Tsilingiris, P., Engebrethsen, E.S., Kakalis, N.M., 2011. Robust fleet sizing and deployment for industrial and independent bulk ocean shipping companies. INFOR Inf. Syst. Oper. Res. 49 (2), 93–107.
Arslan, A.N., Papageorgiou, D.J., 2017. Bulk ship fleet renewal and deployment under uncertainty: A multi-stage stochastic programming approach. Transp. Res. E: Logist. Transp. Rev. 97, 69–96.
Bakkehaug, R., Eidem, E.S., Fagerholt, K., Hvattum, L.M., 2014. A stochastic programming formulation for strategic fleet renewal in shipping. Transp. Res. E: Logist. Transp. Rev. 72, 60–76.
Beaujon, G.J., Turnquist, M.A., 1991. A model for fleet sizing and vehicle allocation. Transp. Sci. 25 (1), 19–45.
Berge, M.E., Hopperstad, C.A., 1993. Demand driven dispatch: A method for dynamic aircraft capacity assignment, models and algorithms. Oper. Res. 41 (1), 153–168.
Bertsimas, D., Gupta, V., Kallus, N., 2018. Data-driven robust optimization. Math. Program. 167 (2), 235–292.
Bojovic, N.J., 2002. A general system theory approach to rail freight car fleet sizing. European J. Oper. Res. 136 (1), 136–172.
Branch, A.E., 2007. Elements of Shipping. Routledge.
Cariou, P., Wolff, F.C., 2011. Ship-owners' decisions to outsource vessel management. Transp. Rev. 31 (6), 709–724.
Chen, K., Xiao, T., 2015. Outsourcing strategy and production disruption of supply chain with demand and capacity allocation uncertainties. Int. J. Prod. Econ. 170, 243–257.
Christiansen, M., Fagerholt, K., Nygreen, B., Ronen, D., 2007. Maritime transportation. In: Barnhart, C., Laporte, G. (Eds.), Transportation. Handbooks in Operations Research and Management Science. Vol. 14, pp. 189–284.
Clarksons Research, 2022. Shipping Review and Outlook. Spring.
Dejax, P.J., Crainic, T.G., 1987. A review of empty flows and fleet management models in freight transportation. Transp. Sci. 21 (4), 227–248.
Du, Y., Hall, R., 1997. Fleet sizing and empty equipment redistribution for center-terminal transportation networks. Manage. Sci. 43 (2), 145–157.
Fagerholt, K., Christiansen, M., Hvattum, L.M., Johnsen, T.A., Vabo, T.J., 2010. A decision support methodology for strategic planning in maritime transportation. Omega 38 (6), 465–474.
Fagerholt, K., Rygh, B., 2002. Design of a sea-borne system for fresh water transport-a simulation analysis. Belg. J. Oper. Res. Stat. Comput. Sci. 40, 137–146.
Gertsbach, I., Gurevich, Y., 1977. Constructing an optimal fleet for a transportation schedule. Transp. Sci. 11 (1), 20–36.
Hoff, A., Andersson, H., Christiansen, M., Hasle, G., Lokketangen, A., 2010. Industrial aspects and literature survey: fleet composition and routing. Comput. Oper. Res. 37 (12), 2041–2061.
Hsieh, C.C., Wu, C.H., 2008. Capacity allocation, ordering, and pricing decisions in a supply chain with demand and supply uncertainties. European J. Oper. Res. 184 (2), 667–684.
Ksciuk, J., Kuhlemann, S., Tierney, K., Koberstein, A., 2023. Uncertainty in maritime ship routing and scheduling: a literature review. European J. Oper. Res. 308 (2), 499–524.
Listes, O., Dekker, M., 2005. A scenario aggregation–based approach for determining a robust airline fleet composition for dynamic capacity allocation. Transp. Sci. 39 (3), 367–382.
Liu, C.M., Sherali, H.D., 2000. A coal shipping and blending problem for an electric utility company. Omega 28 (4), 433–444.
Meng, Q., Wang, T., 2010. A chance constrained programming model for short-term liner ship fleet planning problems. Mar. Policy Manag. 37 (4), 329–346.
Meng, Q., Wang, T., Wang, S., 2012. Short-term liner ship fleet planning with container transshipment and uncertain container shipment demand. European J. Oper. Res. 223 (1), 96–105.
Meng, Q., Wang, T., Wang, S., 2015. Multi-period liner ship fleet planning with dependent uncertain container shipment demand. Mar. Policy Manag. 42 (1), 43–67.
Morch, O., Fagerholt, K., Pantuso, G., Rakke, J., 2017. Maximizing the rate of return on the capital employed in shipping capacity renewal. Omega 67, 42–53.
Ng, M., 2015. Container vessel fleet deployment for liner shipping with stochastic dependencies in shipping demand. Transp. Res. B 74, 79–87.
Pantuso, G., Fagerholt, K., Hvattum, L.M., 2014. A survey on maritime fleet size and mix problems. European J. Oper. Res. 235 (2), 341–349.
Pantuso, G., Fagerholt, K., Wallace, S.W., 2015a. Solving hierarchical stochastic programs: Application to the maritime fleet renewal problem. INFORMS J. Comput. 27, 89–102.
Pantuso, G., Fagerholt, K., Wallace, S.W., 2015b. Uncertainty in fleet renewal: A case from maritime transportation. Transp. Sci. 50 (2), 390–407.
Persson, J.A., Gothe-Lundgren, M., 2005. Shipment planning at oil refineries using column generation and valid inequalities. European J. Oper. Res. 163 (3), 631–652.
Schwartz, N., 1968. Discrete programs for moving known cargos from origins to destinations on time at minimum bargeline fleet cost. Transp. Sci. 2 (2), 134–145.
Shyshou, A., Gribkovskaia, I., Barcelo, J., 2010. A simulation study of the fleet sizing problem arising in offshore anchor handling operations. European J. Oper. Res. 203 (1), 230–240.
Skalnes, J., Fagerholt, K., Pantuso, G., Wang, X., 2020. Risk control in maritime shipping investments. Omega 96, 102087.
United Nations Conference on Trade and Development (UNCTD), 2022. Review on Maritime Transport 2022. United Nations Publications, New York.
Vagen, S.S., Nikolaisen, J.B., 2019. A Combined Optimization-Simulation Framework for a Maritime Inventory Routing Problem under Uncertainty Master thesis. Norwegian University of Science and Technology.

Wang, X., Fagerholt, K., Wallace, S.W., 2018. Planning for charters: A stochastic maritime fleet composition and deployment problem. Omega 79, 54–66.
Wu, L., Wang, S., Laporte, G., 2021. The robust bulk ship routing problem with batched cargo selection. Transp. Res. B 143, 124–159.
Yang, Z., Jiang, Z., Notteboom, T., Haralambides, H., 2019. The impact of ship scrapping subsidies on fleet renewal decisions in dry bulk shipping. Transp. Res. E: Logist. Transp. Rev. 126, 177–189.
Yang, H., Wong, S.C., 1998. A network model of urban taxi services. Transp. Res. B 32 (4), 235–246.
Yang, H., Wong, S.C., Wong, K.I., 2002. Demand–supply equilibrium of taxi services in a network under competition and regulation. Transp. Res. B 36 (9), 799–819.