



# Planning for charters: A stochastic maritime fleet composition and deployment problem<sup>☆</sup>



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## ABSTRACT

This paper introduces a chartering problem that arises in the shipping industry. The chartering decisions determine the time-charter contracts to enter into, in particular, how many ships of each type to charter in, and for how long they are to be hired. We show that this problem can be modeled as a tactical fleet composition problem, with integrated fleet deployment and speed optimization, which also takes into account market uncertainties. We propose a two-stage stochastic programming model, and present a computational study based on the case of Odfjell, a leading chemical shipping company based in Bergen, Norway. We show how the charter plans produced can change depending on different modeling choices. We also show why and how different charter plans affect the company's overall performance, in order to provide guidance in helping the company make its chartering decisions.

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## 1. Introduction

Ever since the 2008–2009 financial crisis, the shipping industry has been under the pressure of low freight rates brought by the global oversupply of ship capacity. This is partly due to the long lead times associated with the delivery of newbuildings. For example, in 2009, the annual growth rate of the world fleet was 7% even though the world seaborne trade fell by 4.5% [29]; and in 2014, the world fleet grew by 3.5% which is still above that of global GDP and trade growth, and even slightly higher than that of the growth of seaborne trade [30]. In 2016, the grim news about the Hanjin Shipping bankruptcy has further put the shipping industry on alert. For maritime transportation providers, it is therefore more critical than ever to ensure efficient management and operation of their fleets in order to survive the crisis.

This study considers a real-life chartering problem faced by Odfjell, a Norwegian public listed company based in Bergen, Norway. As a leading company in the global market for the transportation and storage of bulk liquid chemicals, Odfjell provides services on trading routes all around the world. Each year by the end of October, Odfjell determines the time-charter contracts to enter into as a supplement to the capacity of the fleet they currently own. On a time charter, a daily hire is paid to the ship owner while

the shipping company also bears the sailing costs including fuel and port/canal fees etc. These time charters represent a significant portion of Odfjell's annual expenses, around 24% in 2015 [18], and will decide how many ships of each type to charter in, and for how long they are to be hired.

Several aspects of the future market, such as customer demands, can be highly uncertain. For example, some transport contracts only state percentages of the customers' actual production rather than absolute amounts, which make the committed volumes needing transport uncertain and dependent on the market condition of some specific chemical products. As a result of these uncertainties, the imbalances between supplies and demands for transport capacity in different regions are common in chemical shipping. But this also results in possibilities of picking up optional cargoes from the spot market. Therefore, with the underlying market uncertainties (such as contractual demands and the size of the spot market) affecting the shipping capacity required, the decision making on charters has become rather complicated.

The chartering problem described in this paper can be seen as a *tactical fleet composition problem* with a focus on capacity adjustment given an existing fleet [8]. However, without taking into consideration the operational details to some degree, fleet composition decisions may be based on a too simplified view. Hence, an integration of deployment or routing into the fleet composition decisions is warranted in most cases. In our study, we include fleet deployment decisions to support the capacity evaluation necessary for the making of the charter plan.

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The contribution of this paper is to present a novel stochastic programming model for the chartering problem, taking into account some of the uncertainties affecting the market. These include stochastic demands, fuel prices, charter rates and freight rates. Even though the model is rather general and applicable to many shipping segments and companies, we demonstrate its use to the case of Odfjell, and focus on the decisions for time charters. We show how the charter plans change as we alter some of the modeling: we vary the level of detail in the modeling of fleet deployment; we use the deterministic version of the original stochastic model; we assume uncorrelated random variables; and we treat speed optimization in different ways. We also show how the different chartering plans affect the company's overall performance, in order to provide guidance in helping the company make its chartering decisions.

The remainder of this paper is organized as follows. We first give a brief literature review in Section 2. The chartering problem is described in Section 3, and the stochastic programming model in Section 4. Section 5 introduces the case of Odfjell and we present our computational study in Section 6. We conclude in Section 7.

## 2. Literature review

This section presents a survey of papers that are particularly relevant to our study, regarding maritime fleet composition (Section 2.1), maritime fleet deployment (Section 2.2) and decision making under uncertainty in maritime fleet composition or fleet deployment problems (Section 2.3).

### 2.1. Maritime fleet composition (fleet size and mix)

Fleet composition models help determine the size and mix of the fleet and can be found at all levels of the decision hierarchy. At strategic level, fleet composition decisions usually involve considerable capital investments and deal with such as newbuildings, sale and purchase of second-hand vessels, and demolition of current vessels; at tactical and operational levels, the problem is more related to capacity adjustment given an existing fleet, e.g., the acquisition of additional capacity through time-charters over a relatively short period of time. We refer the readers to the following two surveys and the references therein. A survey on fleet composition problems in both maritime and land-based contexts was presented by Hoff et al. [8], who discussed the industrial aspects of combined fleet composition and routing. Pantuso et al. [19] presented another literature survey on fleet size and mix problems in maritime transportation.

### 2.2. Maritime fleet deployment

Fleet deployment problems normally consist of finding the optimal allocation of the available fleet to services, e.g., trading routes. In the maritime sector, there exists a broad literature on the modeling and solution methods for fleet deployment problems. A survey of fleet operation optimization and fleet deployment was presented by Perakis [22]. Some other examples are: Andersson et al. [2], Fagerholt et al. [5], Gelareh and Meng [6], Gelareh and Pisinger [7], Meng and Wang [16], Wang and Meng [31].

### 2.3. Decision making under uncertainty

In most (if not all) situations, decisions are made under uncertainty. For the problem addressed in this study, some information needed to make the charter plan is highly uncertain, e.g., demand, spot rates and ship operating costs (largely affected by fuel prices), especially when it comes to market conditions further into the future. It is generally understood that in these situations, stochastic models are more appropriate than deterministic ones [13].

There are not many studies on maritime fleet composition or fleet deployment problems in the literature that take uncertainty into account, some exceptions include: Meng and Wang [16] with uncertain demand, using a distribution-based model and chance constraints; Shyshou et al. [27] with uncertain weather conditions and vessel rates using simulation analysis; Alvarez et al. [1] with uncertain second-hand purchase and sale prices, charter rates etc. (but not demand), and using robust optimization; Loxton et al. [15] with uncertain numbers of each type of vehicles needed using a method based on dynamic programming and Golden section search; Wang et al. [32] with uncertain demand and chance constraints using sample average approximation; Fagerholt et al. [4] with uncertain demand quantities and patterns, using simulation and a rolling-horizon framework; Pantuso et al. [20] with uncertain demand, fuel costs and ship values etc., and long-term multi-period considerations in a maritime fleet renewal problem, while Mørch et al. [17] also presented a similar shipping capacity renewal problem with financial factors.

To capture the uncertainty considered in our study, a scenario-based stochastic programming model is proposed, i.e., the uncertainty of the problem is approximated by a set of scenarios, each representing a complete realization of all the stochastic elements. Scenarios used in this context are typically generated based on sampling or on matching statistical properties of the stochastic phenomena [13]. Whichever scenario generation method is used, an important but often overlooked question is how the properties of the uncertain phenomena affect the stochastic program, see discussions in, e.g., Chopra and Ziemba [3], Kallberg and Ziemba [10], Kaut et al. [11], Lium et al. [14], Pantuso et al. [21]. These properties may include, for example, correlations among uncertain elements and shapes of the stochastic distributions (uniform, triangular, normal etc.). In this study, we particularly seek to investigate the impact of correlations on the chartering decisions produced by the stochastic model, and the consequences of failing to take correlations into account when they are sometimes common in reality.

## 3. Problem description

We consider a shipping company seeking to fulfill the contractual demands for the next year by providing services using its own fleet as well as time-charters. To meet demand, in terms of both volume and service frequency required by the transport contracts, the shipping company needs to make a charter plan and deploy its ships to assure timely and cost-effective services. The charter plan is responsible for the additional transport capacity brought in to add to the current fleet for the operations of next year, and the deployment decisions allocate the ships, including both owned and chartered vessels, to different trading routes. Such allocation concerns the number of times each type of ships will operate a given trading route, in order to meet the transport demand of the associated trade.

A somewhat high level abstraction of demand and service is used in this paper, which is based on *trade lanes* between *geographic areas*. Similar abstractions can be found in, e.g., Alvarez et al. [1] and Pantuso et al. [20]. A trade represents a transportation arrangement from one geographic area to another; and a trade lane consists of a number of loading and discharging ports at the origin and destination geographic areas, respectively. In Fig. 1, an example trade lane from Northwest Europe to the US East Coast is shown. This trade lane is serviced when a ship picks up cargoes at each of the four origin ports in Northwest Europe, sails cross the Atlantic Ocean, and unloads the cargoes at their respective destination ports on the US East Coast. Depending on the contractual requirements of the cargoes serviced by the trade, a trade lane may require servicing several times within a specific planning period (e.g., twice a month). Also, some special constraints regarding the

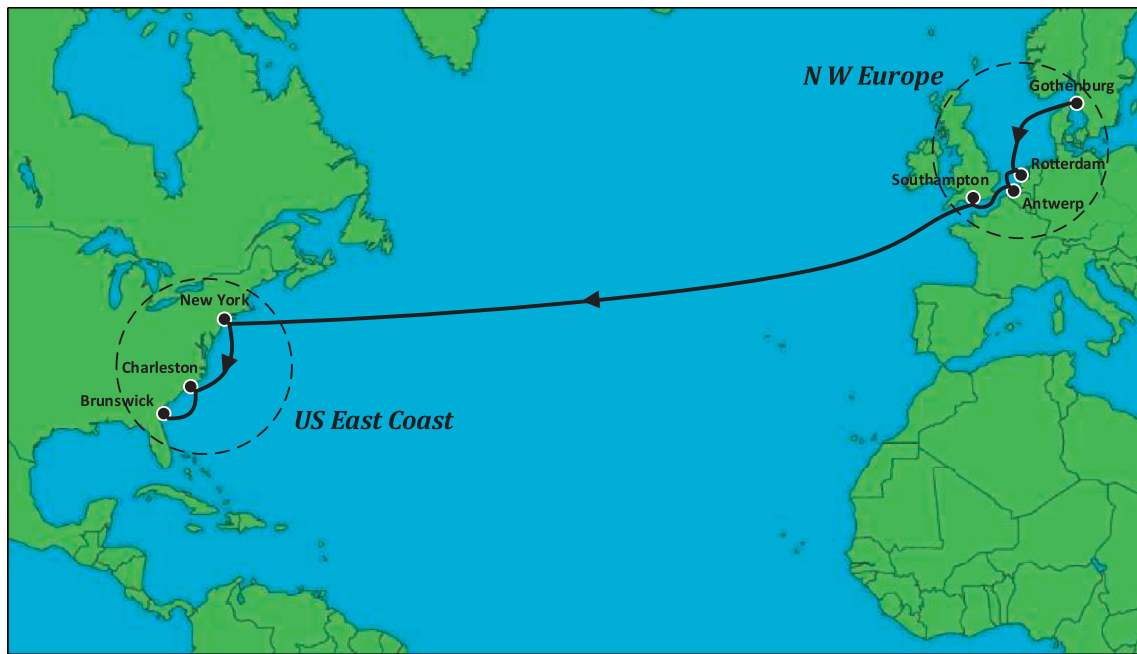


Fig. 1. A trade lane from Northwest Europe to US East Coast.

compatibility between cargo types and ships (or tanks in chemical shipping) may restrict which vessels that can be assigned to a particular trade lane.

The goal of the shipping company is to find the best chartering strategy that provides sufficient transport capacity with minimized total costs, which consist of fixed costs associated with the time charters; and variable costs, of which ship operating costs are an important component. Ship operating costs are affected by the ship deployment decisions as well as decisions with respect to sailing speeds, because of the non-linear relationship between speed and fuel consumption, and of the fact that fuel costs represent a major portion of a ship's operating costs [26,28]. In maritime transportation, many of the OR/MS models found in the literature assume fixed speeds for the ships either explicitly, or implicitly through the calculation of other inputs such as sailing times, due dates and fuel costs [24]. In this study, we consider speed optimization simultaneously with deployment decisions in our chartering problem in order to capture the economic trade-offs between (a) the lower charter costs associated with a higher speed and (b) the higher fuel costs and hence higher operating costs associated with such higher speeds.

For some tonnage not covered by the fleet when there is a surge in demand during operation, the shipping company may choose to acquire (usually more expensive) “on the spot” charters from the spot market instead of planning for the occasional surge in advance with extra ship(s). In the meantime, unused capacity may also be chartered out at a certain rate in the spot market during the operation of the fleet. In addition, apart from contractual cargoes that have to be delivered, picking up optional cargoes from the spot market where possible can also generate revenue. These possibilities associated with the spot market are also taken into account when making the charter plan and deployment decisions. Note that spot markets in the chemical shipping industry are sometimes limited in terms of both demand and supply of shipping capacity, unlike in other shipping sectors such as dry bulk. This is mainly due to the fact that ships used in the chemical shipping industry are specialized tankers and therefore “on the spot” charters are not always available at short notice. Proper

planning for time charters is therefore of great significance for chemical shipping companies.

#### 4. The model

In this section we propose our model for the chartering problem. The modeling approaches and assumptions are introduced in Section 4.1, and the mathematical formulation is presented in Section 4.2.

##### 4.1. Model development

In our chartering problem, the charter plan for the next year is made at the end of the given year. In the model, we further divide the planning period into two periods: the first (**P-1**) is from January to March, and the second (**P-2**) from April to December. Such division in time reflects the difference in the shipping company's confidence in its estimates of market conditions over the two periods: the shipping company is quite sure of the demands (contractual and spot) they are facing and is also confident about its prediction on fuel prices, spot rates etc., for **P-1**; but much less so for **P-2**, due to high market volatility. The charter plan therefore, in accordance with the time division, consists of two sub-decisions: the first determines before **P-1** how many and what types of ships to charter in for the next year; and the second makes further adjustments to the chartered-in ships, between **P-1** and **P-2**, by determining whether or not the shipping company should increase or decrease the charters for **P-2**. These adjustments make up part of the charter plan and are determined simultaneously with the “main” chartering decisions. This is due to the fact that the chemical tanker market is relatively small, and therefore charters are not always available on the spot market.

We illustrate in Fig. 2 the composition of a charter plan and how it affects the fleet over time. The charter plan details the types and numbers of ships to charter in at the start of January with “sub-decision1”, and the increase/decrease in the charters at the end of March with “sub-decision2”. An example is also given in Fig. 2 to show how the fleet in operation is affected by the charter plan. Take “Ship Type 3” for example, the shipping company

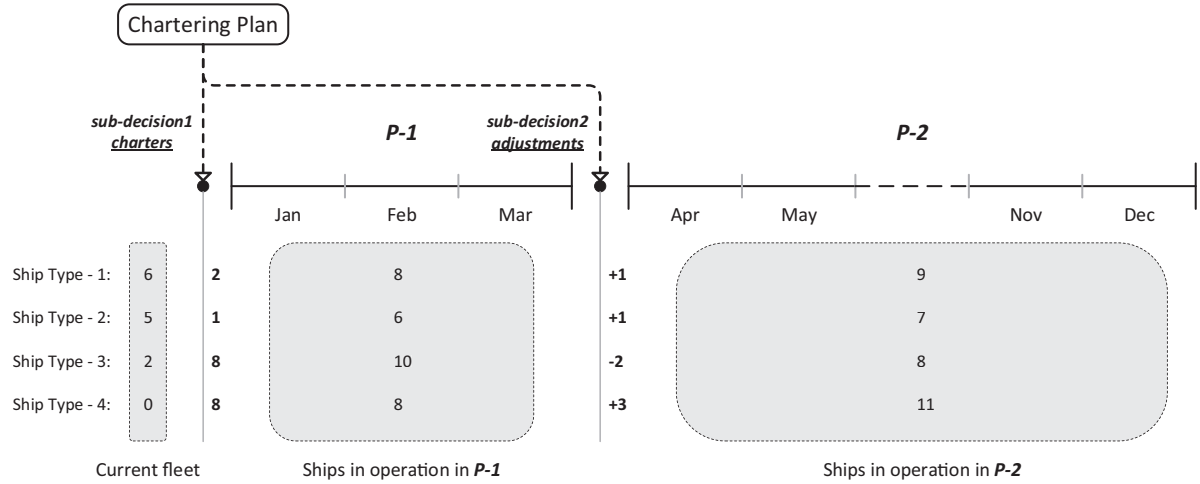


Fig. 2. An example illustrating the decision process of a charter plan, and how it affects the fleet in operation over the planning period.

has two ships of this type in its own fleet. By committing to the charter plan, eight ships of Type-3 are chartered in starting from January, and two of them are then taken out of the chartering contract by the end of March. As a result, the shipping company has ten ships of this type to use during **P-1** (January–March) and eight during **P-2** (April–December).

As mentioned in Section 3, a high level abstraction of demand and service is used in this study, which is based on trade lanes between geographic areas. With every trade lane we associate one or more *contracts*, each representing the aggregated demand on the trade lane that may be carried using the same type of capacity (tanks). Every contract, in this context, can therefore represent transport demands from numerous customers.

To satisfy demand, fleet deployment decisions are made for the available ships, and are also kept at an aggregate level. Like Pantuso et al. [20], we define a *loop* as a round-trip route servicing a number of trade lanes that start and end in the same geographic area. When the destination area of one trade lane is not the same as the origin area of the immediately next trade lane within a loop, ballast (empty) sailing will take place to connect the two consecutive trade lanes. We show in Fig. 3 a small example with three trade lanes and several potential loops that can be derived from these trade lanes. In Fig. 3a, three trade lanes, TR1, TR2 and TR3 are represented by solid arrows. The dashed arrows are the ballast voyages needed in order to form loop L1, involving two trade lanes TR2 and TR3. The graph representation of the loop is further illustrated with bold lines in Fig. 3b, where the circle nodes represent trade lanes and the directed edges represent potential ballast sailings to connect the trade lanes. Therefore, if a ship is assigned to loop L1=TR2→TR3, it sails in ballast to the origin area of TR3 after servicing TR2. And then, subsequent to servicing TR3, the ship sails in ballast back to the origin area of TR2. Loops consisting of three trade lanes can also be constructed based on the graph representation in Fig. 3b, e.g. TR1→TR2→TR3 and TR1→TR3→TR2. Moreover, if a loop contains only one trade lane, it would consist of sailing back to the origin area of the trade lane after servicing it, i.e. the dashed arrow pointing back to itself. After constructing potential loops with the trade lanes that need servicing, the fleet deployment decisions *assign ships to loops* in order to fulfill the demand of each trade lane.

We also include speed optimization in our deployment decisions, since speed is a key determinant of: (a) the time required to sail the loops; and (b) the fuel costs, as fuel consumption per time unit is approximately proportional to the third power of speed [23,25], which gives a quadratic consumption function per

distance unit. However, instead of a function, shipping companies often have fuel consumption data for a number of discrete speeds, this is also the case for Odfjell. We therefore use the fuel consumption data for different speed points and use linear combinations of these points to approximate the fuel consumption rates between these speeds. For instance, if a particular speed  $v^*$  can be written as  $v^* = a \times v_1 + b \times v_2$ ,  $a + b = 1$ ,  $a, b \geq 0$ , where the fuel consumption rates of speed points  $v_1$  and  $v_2$  are known as  $F_1$  and  $F_2$ , respectively; then the fuel consumption rate  $F^*$  at speed  $v^*$  is approximated by  $F^* = a \times F_1 + b \times F_2$ . See Andersson et al. [2] for a more detailed discussion on this approximation approach.

Following from the notions on the division in time of the planning period (into **P-1** and **P-2**), and on the difference in the shipping company's confidence in its estimates on market conditions over the two periods, we assume that at the point when the charter plan has to be made, the market information about **P-1** and **P-2** is deterministic and stochastic, respectively, and that there is no known seasonal or monthly changes in the market during **P-2**. The market information is described with numerical parameters which include contractual and spot demand, ship charter in/out rates and spot freight rates etc. For period **P-2**, these parameters are assumed to be uncertain and are discretized in order to model the random process with scenarios, where each scenario is a complete realization of all of the uncertain parameters over **P-2**.

We model the decision-making in our chartering problem with a two-stage structure. The first stage decisions are made before the realization of the uncertain market parameters of period **P-2**, and comprise the charter plan as well as the deployment of the fleet during period **P-1** where the associated market information is known and deterministic. Note that we assume that acquiring extra charters from the spot market in **P-1** is not possible because the company, as a major capacity provider in the chemical shipping industry, always makes plans to cover all known demands. Then for the second stage, the uncertain market parameters during period **P-2** are realized, and the shipping company needs to make certain recourse decisions to adapt for the observed market parameters. For example, under some realized scenarios the shipping company may need to acquire some extra charters “on the spot”, which are normally more expensive than advance ones, in case of surges in demand; and under other scenarios where demands are lower, the shipping company may pick up more spot cargoes or charter out their ships more often because of the excess capacity. These scenario-dependent recourse decisions, though not considered as important first-stage decisions, serve to send the right

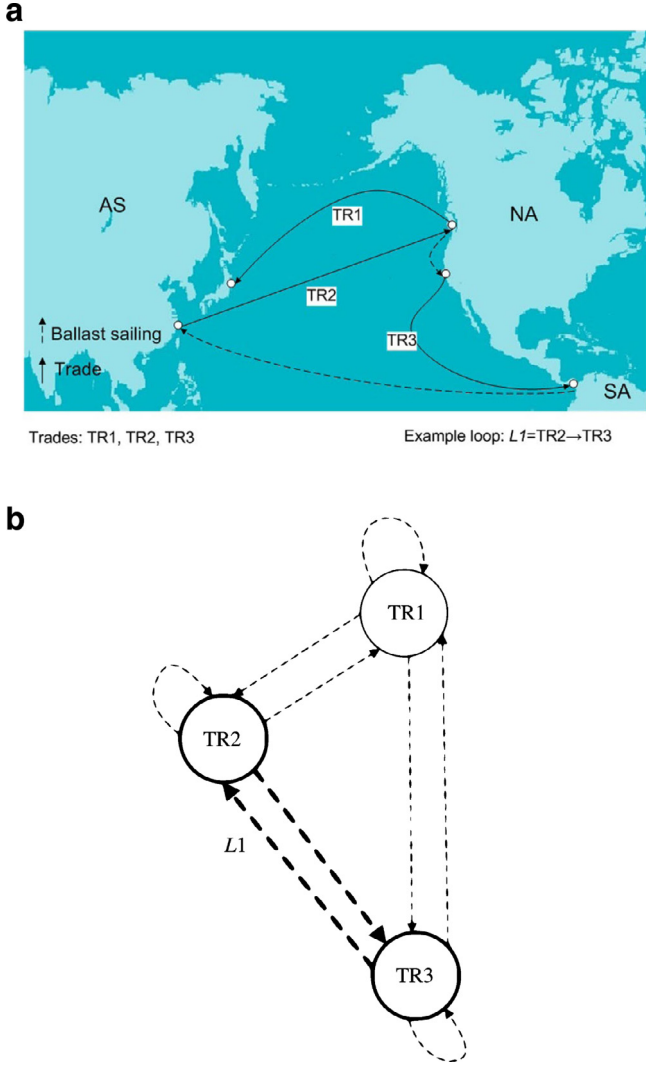


Fig. 3. Example with three trade lanes (TR1, TR2 and TR3) and potential loops [20].

signals back to the first stage where the here-and-now charter plan is made. The charter plan, which determines the total shipping capacity of the fleet, therefore, needs to be carefully contemplated when striking a balance between bringing in too much and too little capacity for the planning period.

#### 4.2. Mathematical formulation

The mathematical formulation of the two-stage stochastic model is as follows, with the notation shown in Table 1.

$$\min \sum_{v \in \mathcal{V}} \left( C_v^I M_v^1 w_v + C_v^I M_v^2 (w_v - w_v^\ominus + w_v^\oplus) + C_v^\ominus M_v^1 w_v^\ominus + C_v^\oplus M_v^2 w_v^\oplus \right) \quad (1a)$$

$$+ \sum_{v \in \mathcal{V}} \sum_{r \in \mathcal{R}_v} \sum_{e \in \mathcal{E}_v} C_{vre}^{RT} x_{vre} - \sum_{v \in \mathcal{V}} R_v^O z_v - \sum_{i \in \mathcal{N}} \sum_{v \in \mathcal{V}} \sum_{k \in \mathcal{K}} R_{ik}^{SP} q_{ivk}^{SP} \quad (1b)$$

$$+ \sum_{s \in \mathcal{S}} p^s \left( \sum_{v \in \mathcal{V}} \sum_{r \in \mathcal{R}_v} \sum_{e \in \mathcal{E}_v} C_{vres}^{RT} x_{vres} + \sum_{v \in \mathcal{V}} C_{vs}^I y_{vs} - \sum_{v \in \mathcal{V}} R_{vs}^O z_{vs} - \sum_{i \in \mathcal{N}} \sum_{v \in \mathcal{V}} \sum_{k \in \mathcal{K}} R_{iks}^{SP} q_{ivks}^{SP} \right) \quad (1c)$$

**Table 1**  
Notation.

Sets	
$\mathcal{V}, \mathcal{K}, \mathcal{C}$	The set of ship types, capacity types and contracts, respectively.
$\mathcal{N}, \mathcal{R}$	The set of trade lanes and loops, respectively.
$\mathcal{E}_v$	The set of speed alternatives for ship type $v$ .
$\mathcal{R}_v \subseteq \mathcal{R}$	The set of loops that can be sailed by ship type $v$ .
$\mathcal{R}_{iv} \subseteq \mathcal{R}$	The set of loops servicing trade lane $i$ that can be sailed by ship type $v$ .
$\mathcal{C}_i^{TR} \subseteq \mathcal{C}$	The set of contracts serviced by trade lane $i$ .
$\mathcal{V}_i \subseteq \mathcal{V}$	The set of ship types that can sail trade lane $i$ .
$\mathcal{C}_k^{CP} \subseteq \mathcal{C}$	The set of contracts compatible with capacity type $k$ .
$\mathcal{K}_c \subseteq \mathcal{K}$	The set of capacity types compatible with contract $c$ .
$\mathcal{S}$	The set of scenarios.
Deterministic parameters	
$N_v$	No. of ships of type $v$ owned by the shipping company.
$M_v^1, M_v^2$	No. of available service days for a ship of type $v$ during <b>P-1</b> and <b>P-2</b> , respectively.
$Q_{ik}$	Volume of capacity type $k$ installed on ship type $v$ .
$T_{vre}$	Total travel time for ship type $v$ to complete a round trip on loop $r$ with speed alternative $e$ , including sailing time and time spent at ports, etc.
$F_c^1, F_c^2$	Frequency requirement of contract $c$ in <b>P-1</b> and <b>P-2</b> , respectively.
$D_c$	Demand of contract $c$ in <b>P-1</b> .
$C_{vre}^{RT}$	Cost for ship type $v$ to complete a round trip on loop $r$ with speed alternative $e$ in <b>P-1</b> , including fuel cost, port fees, canal tolls, etc.
$C_v^I$	Daily charter-in rate for a ship of type $v$ on a “long-term” charter ( <b>P-1</b> plus <b>P-2</b> ).
$C_v^\ominus, C_v^\oplus$	(Both positive values) adjusting factors for “short-term” charters, representing the additional daily charter-in rate for ship type $v$ if hired only for <b>P-1</b> , and only for <b>P-2</b> , respectively.
$R_v^O$	Revenue of chartering out a ship of type $v$ per day in <b>P-1</b> .
$D_{ik}^{SP}$	Volume of spot cargo available on trade lane $i$ that is compatible with capacity type $k$ in <b>P-1</b> .
$R_{ik}^{SP}$	Revenue of delivering one ton of spot cargo with capacity type $k$ on trade lane $i$ in <b>P-1</b> .
Stochastic parameters	
$p^s$	The probability of scenario $s$ taking place in <b>P-2</b> .
$D_{cs}$	Demand of contract $c$ for scenario $s$ in <b>P-2</b> .
$C_{vres}^{RT}$	Cost for ship type $v$ to complete a round trip on loop $r$ with speed alternative $e$ for scenario $s$ in <b>P-2</b> .
$C_{vs}^I$	Cost of chartering in a ship of type $v$ per day for scenario $s$ in <b>P-2</b> (“on the spot” extra time charters).
$R_{vs}^O$	Revenue of chartering out a ship of type $v$ per day for scenario $s$ in <b>P-2</b> .
$D_{iks}^{SP}$	Volume of spot cargo available on trade lane $i$ that is compatible with capacity type $k$ for scenario $s$ in <b>P-2</b> .
$R_{iks}^{SP}$	Revenue of delivering one ton of spot cargo with capacity type $k$ on trade lane $i$ for scenario $s$ in <b>P-2</b> .
Decision variables	
$w_v$	(Charter plan) no. of ships of type $v$ chartered in at the start of <b>P-1</b> .
$w_v^\ominus, w_v^\oplus$	(Charter plan) no. of ships of type $v$ to reduce or add (based on $w_v$ ), respectively, at the end of <b>P-1</b> .
$x_{vre}, x_{vres}$	No. of round trips sailed by a ship of type $v$ on loop $r$ with speed alternative $e$ in <b>P-1</b> , and for scenario $s$ in <b>P-2</b> .
$y_{vs}$	No. of days of extra charter-in for ship type $v$ in scenario $s$ in <b>P-2</b> .
$z_v, z_{vs}$	No. of days of chartering out ship type $v$ in <b>P-1</b> , and for scenario $s$ in <b>P-2</b> .
$q_{ivkc}, q_{ivks}$	Volume of contract $c$ carried by capacity type $k$ installed on ship type $v$ on trade lane $i$ in <b>P-1</b> , and for scenario $s$ in <b>P-2</b> .
$q_{ivkc}^{SP}, q_{ivks}^{SP}$	Volume of spot cargo carried by capacity type $k$ installed on ship type $v$ on trade lane $i$ in <b>P-1</b> , and for scenario $s$ in <b>P-2</b> .

s.t.

$$\sum_{r \in \mathcal{R}_v} \sum_{e \in \mathcal{E}_v} T_{vre} x_{vre} + z_v = M_v^1 (N_v + w_v) \quad v \in \mathcal{V} \quad (1)$$

$$\sum_{v \in \mathcal{V}_i} \sum_{r \in \mathcal{R}_{iv}} \sum_{e \in \mathcal{E}_v} x_{vre} \geq F_c^1 \quad i \in \mathcal{N}, c \in \mathcal{C}_i^{TR} \quad (2)$$

$$\sum_{v \in \mathcal{V}_i} \sum_{k \in \mathcal{K}_c} q_{ivkc} = D_c \quad i \in \mathcal{N}, c \in \mathcal{C}_i^{TR} \quad (3)$$

$$\sum_{r \in \mathcal{R}_{iv}} \sum_{e \in \mathcal{E}_v} Q_{vke} x_{vres} \geq \sum_{c \in \mathcal{C}_i^{TR} \cap \mathcal{C}_k^{CP}} q_{ivkc} + q_{ivk}^{SP} \quad i \in \mathcal{N}, v \in \mathcal{V}_i, k \in \mathcal{K} \quad (4)$$

$$\sum_{v \in \mathcal{V}_i} q_{ivk}^{SP} \leq D_{ik}^{SP} \quad i \in \mathcal{N}, k \in \mathcal{K} \quad (5)$$

and (2nd-stage constraints)

$$\sum_{r \in \mathcal{R}_{vs}} \sum_{e \in \mathcal{E}_v} T_{vres} x_{vres} + z_{vs} = M_v^2 (N_v + w_v + w_v^{\oplus} - w_v^{\ominus}) + y_{vs} \quad v \in \mathcal{V}, s \in \mathcal{S} \quad (6)$$

$$\sum_{v \in \mathcal{V}_i} \sum_{r \in \mathcal{R}_{iv}} \sum_{e \in \mathcal{E}_v} x_{vres} \geq F_c^2 \quad i \in \mathcal{N}, c \in \mathcal{C}_i^{TR}, s \in \mathcal{S} \quad (7)$$

$$\sum_{v \in \mathcal{V}_i} \sum_{k \in \mathcal{K}_c} q_{ivkcs} = D_{cs} \quad i \in \mathcal{N}, c \in \mathcal{C}_i^{TR}, s \in \mathcal{S} \quad (8)$$

$$\sum_{r \in \mathcal{R}_{iv}} \sum_{e \in \mathcal{E}_v} Q_{vke} x_{vres} \geq \sum_{c \in \mathcal{C}_i^{TR} \cap \mathcal{C}_k^{CP}} q_{ivkcs} + q_{ivks}^{SP} \quad i \in \mathcal{N}, v \in \mathcal{V}_i, k \in \mathcal{K}, s \in \mathcal{S} \quad (9)$$

$$\sum_{v \in \mathcal{V}_i} q_{ivks}^{SP} \leq D_{iks}^{SP} \quad i \in \mathcal{N}, k \in \mathcal{K}, s \in \mathcal{S} \quad (10)$$

where (variable domains)

$$w_v, w_v^{\oplus}, w_v^{\ominus} \in \{0\} \cup \mathbb{Z}^+ \quad v \in \mathcal{V} \quad (11)$$

$$w_v \geq w_v^{\ominus} \quad v \in \mathcal{V} \quad (12)$$

$$x_{vr}, x_{vrs}, y_{vs}, z_v, z_{vs} \geq 0 \quad v \in \mathcal{V}, r \in \mathcal{R}_v, s \in \mathcal{S} \quad (13)$$

$$q_{ivkc}, q_{ivkcs} \geq 0 \quad i \in \mathcal{N}, v \in \mathcal{V}_i, k \in \mathcal{K}, c \in \mathcal{C}_i^{TR} \cap \mathcal{C}_k^{CP}, s \in \mathcal{S} \quad (14)$$

$$q_{ivk}^{SP}, q_{ivks}^{SP} \geq 0 \quad i \in \mathcal{N}, v \in \mathcal{V}_i, k \in \mathcal{K}, s \in \mathcal{S} \quad (15)$$

Expression (1a) of the objection function represents the cost of the charters decided by the charter plan. Note that, the charter plan is represented by decision variables  $w_v$ ,  $w_v^{\oplus}$  and  $w_v^{\ominus}$ . The relatively long-term (P-1 plus P-2) charter is effectively  $w_v - w_v^{\ominus}$  for ship type  $v$ ; and the short-term (P-1 only or P-2 only) charters are  $w_v^{\oplus}$  and  $w_v^{\ominus}$  for the first and second periods, respectively. Also note that, Expression (1a) together with Constraints (6) and (11) ensure that for each ship type  $v$ ,  $w_v^{\oplus}$  and  $w_v^{\ominus}$  will never be simultaneously positive in an optimal solution, i.e., at least one will be zero.

Expression (1b) represents the operating costs (including revenues from delivering spot cargoes and chartering out unused capacity) in the first period, while expression (1c) represents the expected operating costs over all scenarios in the second period. Therefore, expressions (1a) and (1b) make up the first-stage costs of the objective function, and expression (1c) the expected second-stage costs.

Constraints (1) state that, in P-1, all transport availability of the fleet (owned and chartered) should be used up, either through the carrier's own operations or chartered out. Constraints (2) ensure the satisfaction of the frequency requirement of every contract and Constraints (3) the demand requirement. Constraints (4) ensure that the total volume of capacity type  $k$  installed on ship type  $v$  sailing on trade lane  $i$  is respected, and may be used to carry either contractual or spot cargoes. Constraints (5) restrict the amount of

spot cargo carried by the shipping company within the size of the spot market for the respective capacity type. Constraints (6)–(10) are the stochastic P-2 versions of constraints (1)–(5) for the second stage.

Constraints (11)–(15) determine the domains of the decision variables. Note that only the chartering decisions  $w_v$ ,  $w_v^{\oplus}$  and  $w_v^{\ominus}$  are required to take integral values. The deployment variables,  $x_{vres}$  and  $x_{vres}$ , are continuous, because in maritime transportation a round trip can take a long time and therefore the fractional part can mean that the final round trip will be finished in the next planning period. In addition, variables  $y_{vs}$ ,  $z_v$ ,  $z_{vs}$  are assumed to be continuous for simplicity as their integrality is less important.

## 5. The case

This section presents our case study. Some of the data input, such as the shipping network and the numbers used for contract volumes and charter rates are not real due to confidentiality reasons, but reflect the real situation in the chemical shipping industry as well as in many other shipping segments. See Table 6 for an overview of the data input.

### 5.1. Network, ships and contracts

We consider the following nine geographic areas in the shipping network: Asia Pacific, Middle East, Northwest Europe, South Africa, South East Asia, US Atlantic Coast, US Gulf, South America West Coast, and South America East Coast. We further include in total 22 trade lanes among these 9 areas for our case study, where each trade lane can be considered as a single origin single destination trading route, e.g. Northwest Europe to US Atlantic Coast. With every trade lane we associate one contract and we therefore have in total 22 contracts.

Each contract is characterized by two types of parameters, demand volume and service frequency, over the planning period. As mentioned in Section 3, the planning period is divided into two periods P-1 and P-2, representing January–March and April–December, respectively. Note that the length of period P-2 is three times as that of period P-1. The volume and frequency of a contract is therefore also distributed over the two periods proportional to the lengths of the periods. However, the demand volume of any contract during period P-2 is considered to be stochastic. Take Contract 1 on trade lane 1 for example, this contract consists of a deterministic volume of 180,000 tons of cargo to be delivered during P-1, and a stochastic demand volume for P-2 with an expected value of 540,000 tons, which is three times that of P-1. Furthermore, Contract 1 requires servicing 24 times during the planning period, i.e., twice a month. Hence, we ensure that the trade lane associated to the contract will be serviced at least 6 and 18 times during P-1 and P-2, respectively.

We consider four different types of chemical tankers operated by Odfjell: Kvaerner, Poland, 19k and 33k. The ships of these four classes have total capacities of 34,782, 51,085, 21,646 and 37,027 deadweight tons, respectively. In practice, the ships that belong to the same ship class usually have slightly different capacities (within 5%), depending on such as the ship's year of making. In our experiments, however, we assume the ships have the same capacity if they are of the same class. Two capacity types are considered, which correspond to two types of tanks installed on the tankers: stainless steel and zinc coated. The compatibility relations in association with tank types are reflected by the setting of sets  $\mathcal{K}_c \subseteq \mathcal{K}$  in our model, which are the set of capacity types compatible with contract  $c$ .

From the fleet data given by the shipping company, two speed points (with corresponding fuel consumption rates), design speed and minimum speed, are defined for each ship class. For our case

study, we only use these two speeds for speed optimization when deploying the fleet, as the two speed points are close to each other (within 15%) in each case.

### 5.2. Construction of feasible loops

In order to satisfy the demand on each trade lane, the deployment decisions assign ships to service routes, which are loops in our model. Therefore, potential loops are to be constructed based on the given trade lanes.

Inspired by Pantuso et al. [20], we start by defining  $C^{max} \geq 1$  as the highest loop cardinality when constructing the set of loops. If  $C^{max} = 3$ , for instance, we may let a loop contain three trade lanes at the most. We also introduce a measure  $\mu$  for any specific loop  $r$ , computed by

$$\mu(r) = \frac{\text{ballast sailing distance of loop } r}{\text{total sailing distance of loop } r}$$

as the ballast ratio of loop  $r$ . We then define  $\mu_k^{max} \in [0.0, 1.0]$ ,  $k \leq C^{max}$  as the maximum acceptable ballast ratio for loops made of  $k$  trade lanes.

We present the method of constructing the set of feasible loops in Algorithm 1. The algorithm takes as input the set of trade lanes

---

#### Algorithm 1 Construction of feasible loops.

---

```

1:  $\mathcal{R} \leftarrow \emptyset$  ▷ initialize the set of feasible loops
2:  $k \leftarrow 1$  ▷ start from loops with one trade lane
3: while  $k \leq C^{max}$  do
4:   for each  $\mathcal{N}^{sub} \subseteq \mathcal{N}$ ,  $\text{card}(\mathcal{N}^{sub}) = k$  do
5:      $\mathcal{L} \leftarrow \emptyset$  ▷ a temporary set of loops
6:     for each circular permutation of  $\mathcal{N}^{sub}$  do
7:       Construct the corresponding loop by connecting the
       trade lanes
8:       Add this loop to  $\mathcal{L}$ 
9:     end for
10:     $r^* \leftarrow$  the shortest loop in  $\mathcal{L}$ 
11:    Calculate  $\mu(r^*)$  ▷ calculate the ballast ratio of loop  $r^*$ 
12:    if  $\mu(r^*) \leq \mu_k^{max}$  then ▷ all  $\mu_k^{max}$ ,  $k \leq C^{max}$  are given
13:      Add  $r^*$  to  $\mathcal{R}$ 
14:    end if
15:  end for
16:   $k \leftarrow k + 1$ 
17: end while
18: return  $\mathcal{R}$ 

```

---

$\mathcal{N}$ , the highest loop cardinality  $C^{max}$  and maximum acceptable ballast ratios  $\mu_k^{max}$  for all  $k \leq C^{max}$ , and returns the set of feasible loops  $\mathcal{R}$  as output. For each  $k \leq C^{max}$ , the algorithm enumerates all possible  $k$ -combinations of  $\mathcal{N}$  (Line 4 of Algorithm 1); and for every such  $k$ -combination, the algorithm constructs all possible loops, by connecting the trade lanes in a sequence using ballast sailings as shown in Section 4.1, and then finds the shortest one (Line 5 to Line 10). This is done because some loops may be completely dominated by others in terms of sailing efficiency. For example, as mentioned in Section 4.1, there may be two loops that involve all three trade lanes in Fig. 3:  $\text{TR1} \rightarrow \text{TR2} \rightarrow \text{TR3}$  and  $\text{TR1} \rightarrow \text{TR3} \rightarrow \text{TR2}$ . One may clearly see that the latter loop makes little sense compared with the former, as it includes sailing across the Pacific Ocean in ballast twice. In such a case, the less efficient loop can be discarded as it is dominated by another loop which services the same trade lanes. In addition, the algorithm only adds to  $\mathcal{R}$  the loops with ballast ratios within certain thresholds described by  $\mu_k^{max}$  (Line 12 of Algorithm 1), in order to eliminate those infeasible loops as they involve too much ballast sailing.

We can therefore, by configuring  $C^{max}$  as well as  $\mu_k^{max}$  when creating the set of feasible loops, determine the level of detail

considered within the deployment of the fleet. If more loops are included, potentially we should expect more efficient and eventually better deployment decisions. In our computational study, we will test the effect of including different sets of loops in our model by altering the configuration of  $C^{max}$  and  $\mu_k^{max}$  values (detailed results reported in Section 6.1), in order to find out what kind of loops are more likely to be used when deploying the ships, and therefore of higher value to the shipping company.

### 5.3. Charter costs

In the chartering problem introduced in this paper, fleet deployment is integrated as a means to give a more accurate estimation of the actual tonnage needed during the planning period, so that the chartering decisions can be properly made. In particular, should the company plan for more capacity in advance before the planning period, or should they rely more on spot capacity “on the fly” after the uncertain parameters have been realized? When striking a balance between the two when planning for charters, the charter costs (in advance and spot) are important factors to consider.

In our model,  $C_v^l$ ,  $v \in \mathcal{V}$  is used to denote the charter-in rate for a ship of type  $v$  on an advance long-term (P-1 plus P-2) charter. Then, for every ship type  $v$ , we set both  $C_v^{\ominus}$  and  $C_v^{\oplus}$  to be equal to 8% of the value of  $C_v^l$  in our case study, which are the additional charter rates if a ship of this type is hired only for P-1 or P-2. In other words, when planning for advance charters, the company pays 8% more on daily charter rates for short-term charters than the relatively long-term ones.

After the realization of the uncertain parameters in period P-2, extra capacity is also available from the spot market in the form of spot charters, in case of insufficient fleet capacity under some scenarios. The price for the extra capacity is determined by  $C_{vs}^l$  under scenario  $s$ , and the mean value for  $C_{vs}^l$  across all scenarios is set to be  $1.5 * C_v^l$  for any ship type  $v$ . Note that, in practice, 50% more for spot charters is sometimes too high an estimate in cases where spot charters are available; on the other hand, there are cases where they are not at all available from the spot market, especially in more specialized shipping segments. We therefore set the spot charters to be 50% more expensive to make a balance between these two situations, so as to avoid relying too much on the spot market and to lower the risk of not being able to find extra capacity. Similarly, the charter-out rates,  $R_v^O$  and the mean value for  $R_{vs}^O$ , are set to be  $0.5 * C_v^l$  to reflect the reality that the company may not always be able to sell the excess capacity in the spot market, and to prevent our solutions from chartering for speculation purposes since this is not the focus of the shipping company.

### 5.4. Scenario generation for random parameters

Recall that the demand of every contract in P-2 ( $D_{cs}$ ) is considered to be stochastic. These stochastic demands represent the most important uncertain phenomena in our problem, as they directly affect the actual capacity needed. All stochastic demands are assumed to follow symmetric triangular distributions with standard deviations equal to around 40% of their corresponding means. Furthermore, the following parameters for period P-2 are also considered to be stochastic:  $D_{iks}^{SP}$ , optional demands from the spot market;  $C_{vrs}^{RT}$ , round trip sailing costs;  $C_{vs}^l$ , extra charter-in rates;  $R_{vs}^O$ , charter-out rates;  $R_{iks}^{SP}$  freight rates for spot cargo delivery. To represent the uncertainty of these random parameters, we assume the following relationship that connects an underlying random variable to every random parameter:  $RP = \xi * E[RP]$ , where  $RP$  is the random parameter,  $E[RP]$  its expected value and  $\xi$  the underlying ran-

dom variable which can also be seen as a scaling factor based on the expected values.

We use in total five underlying random variables. We associate random parameters  $D_{iks}^{SP}$  to variable  $\xi^1$ , which determines the realization of optional demands from the spot market. Note that, by making such association we assume that all random parameters associated to a given random variable are perfectly correlated. For example, for each  $i, i' \in \mathcal{N}, k, k' \in \mathcal{K}$ ,  $D_{iks}^{SP}$  are perfectly correlated to  $D_{i'k's}^{SP}$ . Variable  $\xi^2$  determines the realization of sailing costs which are largely dependent on fuel price, and is associated to random parameters  $C_{irs}^{RT}$ . Similarly, we associate random parameters  $C_{is}^L$ ,  $R_{vs}^0$  and  $R_{iks}^{SP}$  to variables  $\xi^3$ ,  $\xi^4$  and  $\xi^5$ , respectively.

There are, therefore, 27 random variables during period **P-2** to be included in our model: 22 of them are the stochastic demands for each of the 22 contracts; and five underlying random variables  $\xi^1 \sim \xi^5$ . Every underlying random variable is assumed to be subject to the same triangular distribution with lower limit 0, mode 1 and upper limit 2. We also assume a correlation matrix of 27\*27 elements to represent the inter-relationship within the set of random variables, in order to investigate whether or not we should take it into account when making the charter plan. We set all values in the matrix to 0.65, indicating that there is a relatively strong positive correlation between any pair of random variables. We make this assumption due to the fact that, generally speaking, all demands and prices are to some extent dependent on market prosperity, and therefore increase or decrease simultaneously.

In order to discretize the given distributions of the random variables with given correlations, the scenario generation process is performed using the moment-matching method introduced by Høyland et al. [9]. This method takes as input the first four marginal moments and correlations, and generates the desired number of scenarios with equal probabilities, i.e.,  $p^s = 1/|S|$  for all  $s \in S$  in our case.

However, as faced in other stochastic programming problems where stochasticity is approximated with discrete distributions, the quality of such an approximation hinges upon the quality of the scenario-generation method used. Since the chosen scenario generation method is a heuristic approach which also involves random elements, the scenario tree generated on every run is different, though with the same input of marginal distributions and correlations. To make sure the results are not much affected by the particular scenario tree used, we check the *in-sample stability* and *out-of-sample stability* [12] for using 50 scenarios in our stochastic program. We generate 20 scenario trees (each containing 50 scenarios) and solve the stochastic problem with each of them. In the in-sample stability test, we observe a difference between the highest and lowest objective function values, of 1.34%. To test for out-of-sample stability, we sampled a much larger scenario tree with 1000 scenarios to represent the uncertainty of the “true” problem. We calculate the “true” objective function values corresponding to the 20 solutions (charter plan decisions) coming from different (smaller) scenario trees, and observe a difference of 0.46% between the highest and lowest. In addition, the average in- and out-of-sample values are only 2.06% apart. These stability calculations yield acceptable difference values and confirm that our model using 50 scenarios is in- and out-of-sample stable. In this paper, therefore, we generate a scenario tree that contains 50 scenarios with the distributions and correlation matrix given in this section and name it **Stoch-COR**. We use this set of scenarios to represent the uncertain “reality” the company is facing in our case study.

## 6. Computational study

In this section we present a computational study of the case described in Section 5. All programs are implemented in C++ on Microsoft Visual Studio 2010, and solved using CPLEX 12.6.1 on an Intel 3.4 GHz processor with 16 GB memory.

### 6.1. Effect of level of detail in deployment

In our chartering problem, the deployment decisions select loops to operate from a set of feasible loops, and assign shipping capacity to those selected. As explained in Section 5.2, we may increase the level of detail in the fleet deployment by using a larger set of feasible loops, i.e., by increasing  $C^{max}$  and  $\mu_k^{max}$  when creating the loops.

In order to see how varying the level of detail in the deployment modeling influences the results, we obtain several different sets of feasible loops by altering  $C^{max}$  and  $\mu_k^{max}$  values, and solve the stochastic problem with each such set. We report in Table 2 the optimal objective function value and run time for each case. As an example, in Case 3, i.e.,  $C^{max} = 3$ ,  $\mu_1^{max} = 1.0$ ,  $\mu_2^{max} = 1.0$ ,  $\mu_3^{max} = 0.5$ , we accept as feasible loops: (1) all loops made of one trade lane (as they all have a ballast ratio of 0.5); (2) those loops made of two trade lanes that have a ballast ratio less than 1.0; (3) those loops made of three trade lanes that have a ballast ratio less than 0.5. Note that in this table, Case 3 is set as the benchmark case for performance comparison; and “n/a” indicates that the parameter is not applicable in that particular case.

In Case 1, where only the loops made of one or two trade lanes are accepted as feasible loops, we observe a loss in total cost of 6.8%. Then by increasing  $C^{max}$  to 3 and adding some loops made of three trade lanes with ballast ratios under 0.2, the loss is significantly lowered from 6.8% to 2.0% in Case 2. This shows that these loops, which are made of three trade lanes and have low ballast sailing ratios, are potentially of great value to the shipping company. When  $\mu_3^{max}$  is further increased to 0.5 (Case 3), thus allowing more loops with higher ballast ratio in the model, we obtain better performance in terms of relative loss (0.0%). No improvement can be gained by further increasing  $\mu_3^{max}$  to 1.0 (Case 4). Moreover, in Cases 5 and 6, where  $C^{max}$  is 4 and  $\mu_4^{max}$  is set to 0.5 and 1.0, respectively, we obtain even better results. However, the improvements are less significant while the run times grow dramatically. Also, in practice, operating long service loops consisting of four or more trade lanes is not desirable for the shipping company, as they are more affected by possible delays during the voyages and are thus more risky. Therefore, in all of our remaining experiments, we use the feasible set containing 972 loops (Case 3) for fleet deployment in our programs since it represents a good balance between solution quality and run time.

Notice that by comparing Case 1 ( $C^{max} = 2$ ) and Cases 2, 3, and 4 ( $C^{max} = 3$ ), we observe that only considering loops with two trades may lead to some noticeable loss, compared with allowing loops with three trades. The absolute values of these losses are, however, affected by the demand structure of the problem, especially the sizes of the one-way-only trades operated by the company. Under a different circumstance, for example, if for every trade there is usually another trade with a comparable size but in the opposite direction, the simple back-and-forth round trips may already be sufficient and hence including longer loops may not be very beneficial. In Table 2 and this paper, therefore, we do not focus on the absolute values of the losses, and no general statements should be drawn from these values with regards to the absolute quality of a certain parameter setting. Instead, we emphasize the more general insights: when the model is used as a decision support tool, one should always try increasing the level of detail in the fleet deployment to evaluate if the potential gains are large

**Table 2**  
Expected total costs and run times using different loop sets in deployment.

Case	Parameter setting					Number of loops	Opt. obj. func. val.	Loss	Run time (s)
	$C^{\max}$	$\mu_1^{\max}$	$\mu_2^{\max}$	$\mu_3^{\max}$	$\mu_4^{\max}$				
1.	2	1.0	1.0	n/a	n/a	253	351	6.8%	14.0
2.	3	1.0	1.0	0.2	n/a	284	335	2.0%	14.5
3.	<b>3</b>	<b>1.0</b>	<b>1.0</b>	<b>0.5</b>	n/a	<b>972</b>	<b>329</b>	<b>0.0%</b>	<b>40.8</b>
4.	3	1.0	1.0	1.0	n/a	1793	329	0.0%	95.5
5.	4	1.0	1.0	1.0	0.5	6350	327	-0.6%	477.5
6.	4	1.0	1.0	1.0	1.0	9108	327	-0.6%	1056.2

Note: the losses are increases in total cost relative to Case 3 (marked in bold).

**Table 3**  
Charter plans produced with different input.

		Charter plan								
		w	$w^{\ominus}$	$w^{\oplus}$	w	$w^{\ominus}$	$w^{\oplus}$	w	$w^{\ominus}$	$w^{\oplus}$
Ship Type		<b><u>Determ</u></b>			<b><u>Det-65</u></b>			<b><u>Det-75</u></b>		
	Kvaerner	4	0	0	4	0	1	4	0	3
	Poland	3	0	0	3	0	1	3	0	4
	19k	10	0	0	10	0	2	10	0	2
	33k	3	0	0	4	0	0	4	0	1
		<b><u>Det-85</u></b>			<b><u>Stoch-IND</u></b>			<b><u>Stoch-COR</u></b>		
	Kvaerner	4	0	3	4	0	3	4	0	4
	Poland	3	0	5	3	0	2	3	0	3
	19k	10	0	2	10	0	2	10	0	3
	33k	4	0	3	4	0	0	5	0	0

enough to be worthwhile, as there may also be underlying and implicit costs in association with operational changes that are not considered in this model; and the computational efforts required to do so are not likely to become prohibitively expensive quickly as loops that are much too long are generally not desirable.

## 6.2. Effect of using different input

### 6.2.1. Input cases to be tested

The charter plan differs depending on how we describe the market conditions in period **P-2**. We shall view the input labeled **Stoch-COR** (see Section 5.4) as our base case. If we only use one scenario, where every random variable is replaced by its mean value (as many companies do), the charter plan produced may be very different. We denote by **Determ** the input in the latter case. Note that our model can easily be set up with only one scenario. As shown in the literature [e.g., 33,34], using deterministic demands higher than the means sometimes gives better performance, as that results in higher capacities, giving better flexibility in operations. Recall that all stochastic demands are assumed to follow symmetric triangular distributions with standard deviations equal to around 40% of their corresponding means. We therefore also create three other input cases (to be used in the deterministic version of the model), denoted **Det-65**, **Det-75** and **Det-85**, referring to demands set at the 65th, 75th and 85th percentile of their demand distribution. Moreover, in order to establish whether modeling the correlations among random variables matters, we generate another set of scenarios, denoted **Stoch-IND** (to be used in the stochastic model), using the moment-matching heuristic [9], but with all elements in the correlation matrix set to zero, which indicate that the random variable are uncorrelated.

### 6.2.2. Charter plans produced with different input

We show in Table 3 the charter plans produced with the six input cases. For example, the charter plan made with input **Det-75** is to charter in four ships of class Kvaerner, three ships of class Poland, 10 ships of class 19k, and four ships of class 33k for the entire planning period (**P-1** plus **P-2**), and further charter in 3, 4,

2 and 1 ships of the four classes, respectively, for period **P-2**. As expected, when planning with the deterministic model and with higher demand expectation, i.e., **Det-65**, **Det-75** and **Det-85**, the shipping capacity brought in by the charter plan is higher than using **Determ** as input. We may also clearly see that, when planning with the stochastic model, regardless of the correlation matrix used, the capacity is higher than that in the **Determ** case. This is because the optimal solution is using slack capacity to hedge against market uncertainty.

### 6.2.3. Evaluation of different charter plans

Now we evaluate the performance of the charter plans, always comparing to base scenario set **Stoch-COR**. For example, to evaluate the charter plan produced with **Determ**, we fix the advance charter-in variables ( $w_v$ ,  $w_v^{\ominus}$  and  $w_v^{\oplus}$ ) to the corresponding values shown in Table 3, solve the stochastic problem again using the input **Stoch-COR**, and observe the resulting optimal solution.

In Table 4, we report the cost evaluation for the charter plans introduced in Section 6.2.2, in which a detailed breakdown of the total cost is shown in each case (column). In particular, we show the subtotals for the three major cost components: advance charter-in cost (or the cost of the charter plan), total cost for fleet operation in **P-1** and total (expected) cost for fleet operation in **P-2**. These three components, in fact, correspond sequentially to Expression (1a), Expression (1b) and Expression (1c), which make up the objective function of the mathematical formulation presented in Section 4.2.

We see from Table 4 that the charter plan produced with **Determ** gives the largest loss in total cost (12.7%), relative to the optimal charter plan produced with **Stoch-COR**. In the **Determ** case, the company invests around 60 million USD less (compared with the optimal case) in the charter plan, but ends up spending almost 100 million USD more operating the fleet during **P-2**. This is mainly due to the high extra charter-in cost (89 mill USD on average), which can be explained by the advance charter-in capacity being insufficient, and therefore the company has to turn to the more expensive “on the spot” capacity in case of demand surges. Such loss brought by capacity insufficiency can be mitigated by

**Table 4**  
Cost evaluation of implementing different charter plans.

(USD mill)	Using deterministic model				Using stochastic model	
	Determ	Det-65	Det-75	Det-85	Stoch-IND	Stoch-COR
<i>Charter plan</i>						
<b>Adv Chrt-In</b>	94	115	148	166	132	151
<b>Operation P-1:</b>						
Deployment	55	54	54	54	54	53
Charter out	0	0	0	0	0	0
Spot cargoes	−21	−21	−21	−21	−21	−21
<b>Total(P-1)</b>	34	33	33	33	33	32
<b>Operation P-2:</b>						
Deployment	209	207	209	210	207	208
Extra Chrt-In	89	59	22	9	34	17
Charter out	−1	−3	−7	−9	−5	−7
Spot cargoes	−54	−55	−61	−62	−56	−71
<b>Total(P-2)</b>	243	208	163	148	180	146
<b>Grand total</b>	371	355	344	346	345	329
<b>Loss (%)</b>	12.7	8.0	4.5	5.3	4.9	0.0

Note: the losses are increases in total cost relative to Stoch-COR.

planning with higher demand expectation, as suggested by the decreased loss values for cases **Det-65**, **Det-75** and **Det-85**.

If the charter plan is made using the stochastic model, but with zero correlation among all pairs of random variables, i.e., case **Stoch-IND**, the loss also decreases compared to **Determ**. In this case, disregarding the correlation information leads to a loss of 4.9%, which may also be partly explained by the insufficient investment on the charter plan (132 versus the “optimal” 151).

However, from the results and comparison displayed in Table 4, it is not enough to conclude that insufficient advance charter-in capacity has been the only reason for losses. Unlike some other transportation problems with homogeneous fleets or capacity, the shipping capacity in our problem is heterogeneous, there are incompatibility issues between capacity (tank) types and contracts, and also (more importantly) between trade lanes and ship classes. The latter incompatibility relations are represented by the setting of sets  $\mathcal{V}_i \subseteq \mathcal{V}$  in the model, which indicate that some ship types are not allowed to sail certain trade lanes. In chemical shipping, this is usually not due to the physical restrictions of the ships (e.g. draft, tank layout), but is mainly due to the undesired operational complexity caused by large deviations from the company’s old deployment arrangements. For example, Odfjell has always used the large tankers on trade lanes between Northwest Europe and the US Gulf, and is therefore reluctant to use small tankers on these trade lanes, where there are economies of scale, even if this may lead to an overall improvement in its total costs. Such restrictions have complicated the chartering problem, as the market uncertainty cannot be simply dealt with using increased investments in total advance capacity. This can be seen by comparing **Det-75** with **Stoch-COR** in Table 4. The charter plan produced by **Det-75** amounts to a total investment of 148 million USD in advance charters, which is almost the same size as the optimal amount; however, its resulting expected operational cost during **P-2** (163 mill USD) is over 10% higher than that in the optimal case (146 mill USD). Moreover, for the **Det-85** case, where even more investment is made in advance charters, the expected **P-2** operation cost is still slightly higher than that of case **Stoch-COR**. By looking into the results in detail, we have observed that although the charter plans produced with **Det-75** and **Det-85** seem to have prepared for “enough” total capacity in advance, they have both brought in some “wrong” ships (compared to the “correct” and optimal charter plan produced by **Stoch-COR**, see Table 3), which eventually result in losses.

Therefore, to obtain better chartering decisions, the shipping company should, where possible, use the stochastic model and

take both individual distributions and correlation information into account. If the company has to use the deterministic model, results show that planning with higher demand expectation than the means can lead to better decisions. However, due to the incompatibilities between trade lanes and ship types, deterministic models may struggle with providing the “correct” combination (mix) of the different types of ships to charter in.

### 6.3. Effect of including speed optimization

Speed optimization is included in our modeling of fleet deployment, as this may eventually contribute to a better tonnage evaluation in the charter plan. In practical life, speed differentiation is, however, only implemented at the operational stage; tactical plans are usually made assuming one speed for each ship type. This is also the case for the shipping company we are working with, as they normally only use design speeds to make their tactical charter plans.

In all the experiments conducted before Section 6.3 we have included both design and minimum speeds for all ship classes (see Section 5.1) and taken speed optimization into account already at the tactical planning stage. To investigate how much we lose by not taking speed optimization into account in the first stage, we create two additional input cases, both assuming that the charter plan is made with only design speeds for all ship classes, and compare their results with the other input cases introduced earlier. Note that when applying different speed options one only needs to change the input, e.g.,  $\mathcal{E}_v$  (the set of speed alternatives for ship type  $v$ ) and  $T_{vre}$  (total travel time for ship type  $v$  to complete a round trip on loop  $r$  with speed alternative  $e$ ), while the model stays the same. We report the comparison in Table 5. The two additional input cases, which are the one-speed versions of **Determ** and **Stoch-COR**, respectively, are grouped under the “Planning with design speed” title, while the original six input cases are under the “Planning with design & min speeds” title. We report the chartering costs, total costs and relative losses for the two additional input cases in Table 5, evaluated the same way as in Table 4. We also emphasize that for all cases in Table 5, regardless of how the first-stage charter plan is made (planning with design and min speeds or planning with only design speed), the second-stage deployment always performs “full” speed optimization with all speed alternatives considered.

The results show that if planning with design speed only, the company would invest much less in advance charters compared to the corresponding two-speed (design and min) case. The resulting

**Table 5**  
Effect of planning with different speeds.

	Planning with design and min speeds						Planning with design speed	
	Determ	Det-65	Det-75	Det-85	Stoch -IND	Stoch -COR	Determ	Stoch -COR
Adv Chrt-In (USD mill)	94	115	148	166	132	151	71	117
Total cost (USD mill)	371	355	344	346	345	329	397	360
Loss (%)	12.7	8.0	4.5	5.3	4.9	0.0	20.6	9.5
Avg speed P-1 (knots)	13.5	13.2	13.2	13.2	13.2	13.0	14.4	14.4
Avg speed P-2 (knots)	13.6	13.6	13.5	13.4	13.5	13.4	13.7	13.4

**Table 6**

The value/range of the parameters in the computational study. Note: *t* for tons, M for millions, and K for thousands.

Parameters		Value/range
<b>Sets</b>		
$ \mathcal{V} ,  \mathcal{K} ,  \mathcal{C} $	No. of ship types, capacity types and contracts.	4, 2, 22
$ \mathcal{N} $	No. of trade lanes.	22
$ \mathcal{R} $	No. of loops.	253–9108
$ \mathcal{E}_v $	No. of speed alternatives for ship type <i>v</i> .	2
$ \mathcal{S} $	No. of scenarios.	50
<b>Deterministic parameters</b>		
$N_v$	No. of ships of type <i>v</i> owned by the shipping company.	(6, 6, 5, 5)
$M_v^1$	No. of available days for a ship of type <i>v</i> during <b>P-1</b> .	90
$M_v^2$	No. of available days for a ship of type <i>v</i> during <b>P-2</b> .	270
$T_{vre}$	Total travel time for ship type <i>v</i> to complete a round trip on loop <i>r</i> with speed alternative <i>e</i> , including sailing time and time spent at ports, etc.	11–145 [days]
$F_c^1$	Frequency requirement of contract <i>c</i> in <b>P-1</b> .	3 or 6
$F_c^2$	Frequency requirement of contract <i>c</i> in <b>P-2</b> .	9 or 18
$D_c$	Demand of contract <i>c</i> in <b>P-1</b> .	108,000–180,000[t]
$C_{vre}^{RT}$	Cost for ship type <i>v</i> to complete a round trip on loop <i>r</i> with speed alternative <i>e</i> in <b>P-1</b> , including fuel cost, port fees, canal tolls, etc.	\$ 0.14–4.68M
$C_v^l$	Daily charter-in rate for a ship of type <i>v</i> on a “long-term” charter ( <b>P-1</b> plus <b>P-2</b> ).	\$ 12–30K
$C_v^{\ominus}, C_v^{\oplus}$	(Both positive values) adjusting factors for “short-term” charters, representing the additional daily charter-in rate for ship type <i>v</i> if hired only for <b>P-1</b> , and only for <b>P-2</b> , respectively.	\$ 0.96–2.4K (i.e., both $8\% \times C_v^l$ )
$R_v^O$	Revenue of chartering out a ship of type <i>v</i> per day in <b>P-1</b> .	\$ 6–15K (i.e., $0.5 \times C_v^l$ )
$D_{ik}^{SP}$	Volume of spot cargo available on trade lane <i>i</i> that is compatible with capacity type <i>k</i> in <b>P-1</b> .	15,000–30,000[t]
$R_{ik}^{SP}$	Revenue of delivering one ton of spot cargo with capacity type <i>k</i> on trade lane <i>i</i> in <b>P-1</b> .	\$ 16–129
<b>Stochastic parameters (expected values)</b>		
$p^s$	The probability of scenario <i>s</i> taking place in <b>P-2</b> .	0.02
$D_{cs}$	Demand of contract <i>c</i> for scenario <i>s</i> in <b>P-2</b> .	324,000–540,000[t]
$C_{vrs}^{RT}$	Cost for ship type <i>v</i> to complete a round trip on loop <i>r</i> with speed alternative <i>e</i> for scenario <i>s</i> in <b>P-2</b> .	\$ 0.14–4.68M
$C_{vs}^l$	Cost of chartering in a ship of type <i>v</i> per day for scenario <i>s</i> in <b>P-2</b> (“on the spot” extra time charters).	\$ 18–45K (i.e., $1.5 \times C_v^l$ )
$R_{vs}^O$	Revenue of chartering out a ship of type <i>v</i> per day for scenario <i>s</i> in <b>P-2</b> .	\$ 6–15K
$D_{iks}^{SP}$	Volume of spot cargo available on trade lane <i>i</i> that is compatible with capacity type <i>k</i> for scenario <i>s</i> in <b>P-2</b> .	45,000–90,000[t]
$R_{iks}^{SP}$	Revenue of delivering one ton of spot cargo with capacity type <i>k</i> on trade lane <i>i</i> for scenario <i>s</i> in <b>P-2</b> .	\$ 16–129

losses relative to the optimal case, 20.6% and 9.5%, suggest that taking speed optimization into account is beneficial and can lead to better chartering decisions. Technically, this is obvious as speed flexibility adds to the set of feasible solutions to the problem.

Furthermore, to better illustrate how planning with single speeds influences the actual speed decisions during the operations of the fleet, we introduce a simple approach, in the following, to give an approximation of the average speed of the fleet in operation.

Let  $E_{ve}$  be the speed of ship type *v* at speed alternative *e* in knots,  $W_v$  the total capacity of each ship of type *v* in dead weight tons, and  $L_r$  the length of loop *r*. Also let  $x_{vre}^*$ ,  $v \in \mathcal{V}$ ,  $r \in \mathcal{R}_v$ ,  $e \in \mathcal{E}_v$  be the deployment decisions during **P-1** in the optimal solution to a specific chartering problem; and let  $x_{vrs}^*$ ,  $s \in \mathcal{S}$ ,  $v \in \mathcal{V}$ ,  $r \in \mathcal{R}_v$ ,  $e \in \mathcal{E}_v$  be the deployment decisions for **P-2**. We then define  $\bar{E}^{P-1}$  and  $\bar{E}^{P-2}$  as the average ton-mile speed (in knots) of the whole fleet in operation during period **P-1** and **P-2**, respectively, and they can be calculated using the following Eqs. (16) and (17):

$$\hat{E}^{P-1} = \frac{\sum_{v \in V} \sum_{r \in R_v} \sum_{e \in E_v} E_{ve} W_v L_r X_{vre}^*}{\sum_{v \in V} \sum_{r \in R_v} \sum_{e \in E_v} W_v L_r X_{vre}^*} \quad (16)$$

$$\hat{E}^{P-2} = \frac{\sum_{s \in S} \sum_{v \in V} \sum_{r \in R_v} \sum_{e \in E_v} p^s E_{ve} W_v L_r X_{vres}^*}{\sum_{s \in S} \sum_{v \in V} \sum_{r \in R_v} \sum_{e \in E_v} p^s W_v L_r X_{vres}^*} \quad (17)$$

Note that the above approach implies that larger ships carry more weight (than smaller ones) in the calculation of the fleet's average speed. We calculate the average ton-mile speeds for every input case, when evaluated under the same uncertain “reality”, and the results are shown in Table 5.

We observe that the fleet slows down under the optimal charter plan **Stoch-COR**, made with both design and minimum speeds considered. In fact, the average speeds in this case are the lowest among all. We also see that in the one-speed versions of the **Determ** and **Stoch-COR** cases (planning with design speed only), lower investments on advance charters (71 and 117 mill USD) are made compared to their two-speed counterparts (94 and 151 mill USD). This is because in the one-speed cases, speeds are fixed at design speeds (which are higher than minimum speeds) when the charter plans are made, without the option to slow the fleet down. However, during the operational phase (especially period **P-2** during which extra charters are allowed), where the option of slow-steaming is available, the optimal solutions would employ slower speeds in order to improve their overall performances. In contrast, although slow-steaming is also possible in **P-1**, the average speed of the fleet is maintained at a high level to ensure sufficient shipping capability, since the company under-invests in advance charters and lacks access to the spot market for extra capacity. Hence we see the drop in average fleet speed in the last column of Table 5, from 14.4 knots in period **P-1** to 13.4 knots in period **P-2**.

## 7. Conclusion

In this paper, we have introduced and studied a chartering problem in the shipping industry. This problem is modeled as a tactical fleet composition problem, with integrated fleet deployment and speed optimization, which also takes into account market uncertainties. A scenario-based two-stage stochastic programming model is proposed, and is applied in a computational study on the case of Odfjell, a leading chemical shipping company based in Norway. In practice, the model may be used as a decision support tool for making the charter plans and can easily be converted to an adaptive one by rerunning the programs on the fly with updated information.

In the computational study, we have shown that the charter plans and their costs differ when certain aspects of the modeling change. We studied the level of detail in the modeling of fleet deployment (such as speed optimization and loop construction), using the deterministic version of the model instead of its stochastic counterpart, and assuming uncorrelated random variables.

In particular, our results show that better results can be obtained by increasing the level of detail in fleet deployment, but at the expense of higher computational efforts. By evaluating the charter plans produced using the deterministic model, and using the stochastic model but without considering correlations, we show that the shipping company should, where possible, use the stochastic model and take both individual distributions and correlation information into account. If the company has to use the deterministic model, we show that planning with higher demands than the means can lead to better decisions. However, due to the

possible incompatibilities between trade lanes and ship types, deterministic models may struggle with providing the “correct” combination (mix) of the different types of ships to charter in. We have also shown the benefit of integrating speed optimization when making the charter plans.

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