1) What is the probability that for a leap year to have 52 Mondays 53 Sundays

In leap year there are 366 days out of which 52 weeks were present. The remaining 2 days can be {(Sat, Sun), (Sun, Mon), (mon, Tue), (Tue, wed) (wed, Thur), (Thur, Fri), (Fri, Sat)}

$$P(E) = \frac{\text{Favourable no. of outcomes}}{\text{Total no. of outcomes}}$$

$$P(E) = \frac{1}{7}$$

2) In a class there are 10 boys and 5 girls. A Committe of 4 students is to be selected from the class. Find the probability for the committee to contain atleast 3 girls.

No. of students in a class = 15

Out of 15 students a committee of 4 members can be formed in 15_{C_4} ways

Favourable no. of outcomes for forming a committee of 4 having atleast 3 girls

= (3 girls & 1 boys) (or) (4 Girls)

$= (5_{C_3} \times 10_{C_1}) + 5_{C_4}$

$$P(E) = \frac{\text{Favourable no. of outcomes}}{\text{Total no. of outcomes}}$$

$$= \frac{(5C_3 \times 10_{C_1}) + 5_{C_4}}{15_{C_4}}$$

3) A class consists of 6 girls and 10 boys. If a Committee of 3 is chosen at random find the probability that

(i) 3 boys are selected

(ii) exactly 2 girls are selected

A Commitee of 3 can be Selected from a class containing of 6 girls & 10 boys in 16_{C_3} ways

(i) 3 boys are selected in 10_{C_3} ways

$$P(E) = \frac{10_{C_3}}{16_{C_3}}$$

(ii) Exactly 2 girls are selected in $10_{C_1} \times 6_{C_2}$

$= 6_{C_2} \times 10_{C_1}$ (2 girls & 1 boy)

$$P(E) = \frac{6_{C_2} \times 10_{C_1}}{16_{C_3}}$$

* Axioms of Probability:

1) If E is any event in a Sample space (S) then $0 \le P(E) \le 1$ (axiom of positivity)

2) $P(S) = 1$ where S is the sample space.

* Axiom of Union:

- If E_1, E_2 are disjoint subsets of S then $P(E_1 \cup E_2) = P(E_1) + P(E_2)$

Numbers A 2 3 4 5 6 7 8 9 10 J
1 11

52

Red (26) Black (26)

what is the probability that a card is drawn at random from pack of cards

(i) either a queen (or) a king

Total no. of outcomes for drawing a card out of 52 cards is 52_{C_1} ways.

Probability of getting the card as a king (or) queen

$= 4_{C_1} + 4_{C_1}$ ways

$$P(E) = \frac{4_{C_1} + 4_{C_1}}{52_{C_1}}$$

Probability

Set: A Collection of well defined objects is Called a Set. The objects Comprising the set are Called elements

Subset: Suppose A is a Set, B is such that every element of B, belonging to the set A then B is a Subset of A. $B \subseteq A$

Union: Let A and B be two Sets. Union of A and B is the set of all those elements which belong to either A or B or both

$A \cup B = \{ x / x \in A$ or $x \in B$ or $x \in$ both $\}$

Let A and B be two sets. Interse-
intersection: ction of A and B is the set of all those elements which are common to A and B.

$A \cap B = \{ x / x \in A$ and $x \in B \}$

Null set or empty Set $\emptyset$: $\emptyset$ is the set which contains no elements.

Disjoint Sets: If two sets have no element in common, they are Called disjoint Sets.

$A \cap B = \emptyset$

Universal Set: All Sets are assumed to be Subsets of some fixed set Called universal set.

Complement of A: The set of elements which do not belong to A.

$A^c = \{ x / x \in U, x \notin A \}$

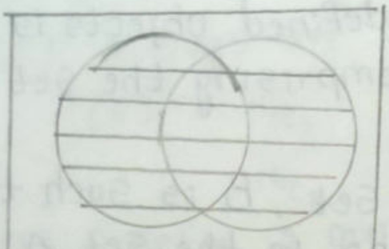
difference of Sets: A-B is the Sets of elements which belongs to A but not to B

$A - B = \{ x / x \in A$ and $x \notin B \}$

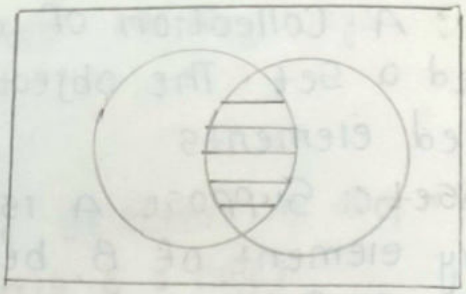
Note:

1) A-B, B are disjoint Sets
2) A' or $A^c = U - A$ where U is universal set

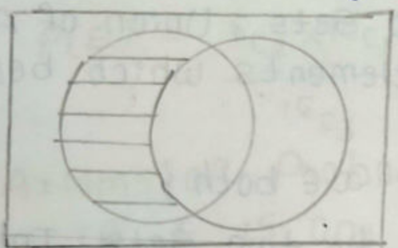
Venn diagrams



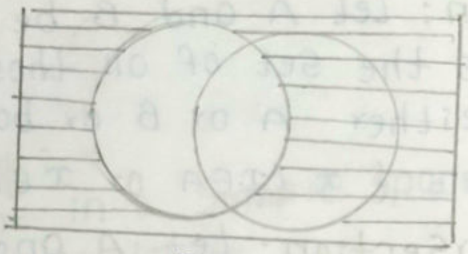
$A \cup B$ = Shaded region



$A \cap B$



$A - B$



$A^c = A'$

Note:

1) $A \cup (B \cup C) = (A \cup B) \cup C$ (Associative law)
2) $A \cup A^c = U$
3) $A \cap A^c = \emptyset$
4) $(A^c)^c = A$ or $(A')' = A$
5) $U^c = \emptyset$, $\emptyset^c = U$

Demorgans law:

1) $(A \cup B)' = A' \cap B'$
2) $(A \cap B)' = A' \cup B'$

Distributive laws:

1) $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$
2) $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

Note: If $A_1, A_2 - - - A_n$ are sets then

1) $A_1 \cup A_2 - - - \cup A_n = \bigcup_{i=1}^{n} A_i$
2) $A_1 \cap A_2 - - - \cap A_n = \bigcap_{i=1}^{n} A_i$
3) $A \subseteq B \Rightarrow A \cap B = A$
4) $A \subseteq B \Rightarrow A \cup B = B$

Multiplication of Choices: If $A_1, A_2 --- A_n$ are sets contain $I_1, I_2 --- I_n$ elements respectively then there are $I_1, I_2 --- I_n$, ways of choosing first element of A_1 then element of $A_2 --- a_n$ element of A_n.

Ex: In how many different ways can a student answer 5 questions which are having four options each.

$4 \cdot 4 \cdot 4 \cdot 4 \cdot 4 = 4^5 = 1024$

Permutation: If r objects are chosen from a set of n distinct objects, any particular arrangement or order of these objects is called a permutation. The no. of permutations of r objects selected from a set of n objects is $^nP_r = \frac{n!}{(n-r)!}$

Combinations: To find the number of ways in which r objects can be selected from a set of n distinct objects is called the number of combinations. The number of ways in which r objects can be selected from a set of n objects is

$$^nC_r = \frac{n!}{r!\,(n-r)!}$$

1) In how many ways 3 students are selected from 15 students

$$= {}^{15}C_3 = \frac{15!}{3!\,(15-3)!} = \frac{15 \times 14 \times 13}{3 \times 2 \times 1}$$

2) A student is to answer 5 out of 8 questions in an examination. In how many ways he/she can choose

1) If any five
2) If first question is compulsory
3) 2 out of first 3.

1) $^8C_5 = \frac{8!}{5!\,(8-5)!} = \frac{8 \times 7 \times 6 \times 5 \times 4}{5 \times 4 \times 3 \times 2 \times 1} = 56$

2) First question is Compulsory so he has to choose remaining 4 out of 7

$^7C_4 = \frac{7!}{4!(7-4)!} = \frac{7\times6\times5\times4\times3\times2\times1}{4\times3\times2\times1\times3\times2\times1} = 35$

3) 2 out of first 3 means 2 out of first 3 and 3 out of remaining 5

$^3C_2 \times {}^5C_3 = \frac{3!}{2!(3-2)!} \times \frac{5!}{3!(5-3)!} = 3\times10 = 30$

3) Three light bulbs are chosen at random from 12 bulbs of which 5 are defective. Find the probability that 1) All are defective 2) One is defective 3) two are defective

Total no. of outcomes out of 12 bulbs 3 are Selected $= {}^{12}C_3$ ways

1) All are defective

out of 5 defective 3 should be selected no of ways in $^5C_3$

Probability is $\frac{^5C_3}{^{12}C_3} = \frac{10}{220} = \frac{1}{22}$

2) One is defective

out of 5D, one is selected and out of 7ND two should be selected

∴ No of possible ways of Selection is $^5C_1 \times {}^7C_2$

Probability $= \frac{^5C_1 \times {}^7C_2}{^{12}C_3} = \frac{5\times35}{220} = \frac{21}{44}$

3) two are defective

$P(E) = \frac{^5C_2 \times {}^7C_1}{^{12}C_3}$

4) A bag Contains 5 red balls, 8 blue balls and 11 white balls. Three balls are drawn together from the box. Find the probability that 1) one is red, one is blue and one is white

2) Two white, one red

3) Three white

Total no. of balls in a bag = $5+8+11=24$

Out of 24 balls three balls are drawn in 24_{C_3} ways

$$= \frac{24!}{3!\,(24-3)!} = \frac{24\times23\times22\times21!}{3\times2\times1\times(21)!} = 2024$$

P(E) = favourable - 78

1) One Red, one blue, one white

$5_{C_1} \times 8_{C_1} \times 11_{C_1} = 5\times8\times11$

$\therefore$ probability $= \dfrac{\text{favourable no. of outcomes}}{\text{Total no. of outcomes}}$

$$= \frac{5\times8\times11}{2024} = \frac{55}{253}$$

2) Two white and one red

$5_{C_1} \times 11_{C_2}$

$$P(E) = \frac{5_{C_1}\times 11_{C_2}}{24_{C_3}} = \frac{5\times55}{2024} = \frac{25}{184}$$

3) Three white

no. of ways is $11_{C_3} = \dfrac{11!}{3!\,(11-3)!} = \dfrac{11\times10\times9}{3\times2}$

$$P(E) = \frac{165}{2024}$$

Cards : 52 cards

♠ → Spades (13)

♡ → Heart (13)

◇ → diamond (13)

♣ → Clubs (13)

Colours → Red, black

Red ↓ 26

black ↓ 26

no's - 13 — Ace (1) 2 - - - 9 10 J Q K

(J = 11, Q = 12, K = 13)

1) Find the probability of getting 2 diamonds if we draw 2 cards at random from a pack of 52 cards

Out of 52 cards 2 cards are drawn

no. of ways is $52_{C_2} = \dfrac{52!}{2!\,(52-2)!} = \dfrac{52\times51}{2} = 1326$

Out of 13 diamonds 2 diamonds are drawn

favourable no. of outcomes $13_{C_2} = \dfrac{13!}{2!\,(13-2)!} = \dfrac{13\times12}{2} = 78$

$P(E) = \frac{\text{Favourable}}{\text{Total}} = \frac{78}{1326} = \frac{1}{17}$

2. Three Cards are drawn from a pack of 52 Cards. Find the probability that

1) 3 are spades
2) 2 Spades and one diamond
3) 1 spade, 1 diamond, 1 heart

Out of 52 Cards 3 Cards are drawn in 52_{C_3} ways

$= \frac{52!}{3!\,(52-3)!} = \frac{52 \times 51 \times 50}{3 \times 2}$

Total no. of ways $= 52_{C_3} = 22100$

no. of possible (favourable) cases for drawing all 3 spades is $13_{C_3} = \frac{13!}{3!\,(13-3)!} = \frac{13 \times 12 \times 11}{6} = 286$

$P(E) = \frac{\text{Favourable}}{\text{Total}} = \frac{286}{22100} = \frac{11}{850}$

2) 2 Spades and one diamond

$\frac{13_{C_2} \times 13_{C_1}}{22100} = \frac{39}{850}$

3) 1 Spade, 1 diamond, 1 heart

$\frac{13_{C_1} \times 13_{C_1} \times 13_{C_1}}{22100} = \frac{169}{1700}$

3) Two Cards are selected at random from 10 Cards numbered 1 to 10. Find the probability that the sum is even if 1) two no's are drawn together 2) two cards are drawn one after another with replacement

Out of 10 Cards two are selected

Total no. of outcomes $= 10_{C_2} = \frac{10!}{2!\,(10-2)!} = 45$

Sum is even if both Cards are even (or) both are odd

Favourable no. of outcomes

$5_{C_2} + 5_{C_2} = 2 \times \frac{5!}{2!\,(5-2)!} = 20$

$P(E) = \frac{20}{45} = \frac{4}{9}$

2) First card can be drawn in $10c_1$ ways and second card can also be drawn in $10c_1$ ways (with replacement)
Total no. of ways $= 10 \times 10 = 100$ $(10c_1 \times 10c_1)$
Sum is even if both are even or both or odd
favourable no. of outcomes $5\times5 + 5\times5 = 25+25 = 50$

$$P(E) = \frac{50}{100}$$

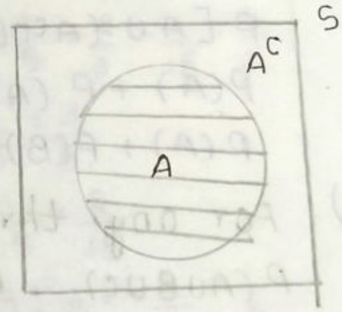
4) What is the probability of drawing an ace from a well shuffled deck of 52 playing cards
drawing one card out of $52 = 52c_1$
out of 4 aces one card $= 4c_1$

$$P(E) = \frac{4c_1}{52c_1} = \frac{4}{52} = \frac{1}{13}$$

Axioms of probability

1) $0 \leq P(A) \leq 1$
2) $P(S) = 1$
3) If A and B are any two mutually exclusive events then $P(A \cup B) = P(A) + P(B)$

Theorems

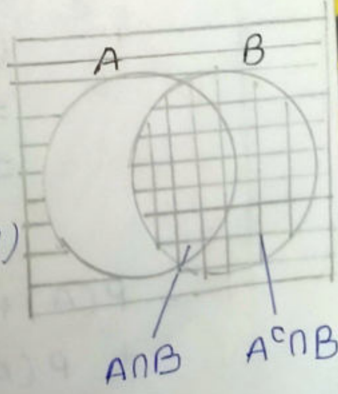


1) $P(A^c) = 1 - P(A)$

proof:
Let S be the universal set

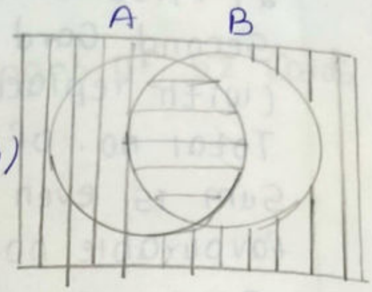
$S = A \cup A^c$
$P(S) = P(A \cup A^c)$
$1 = P(A) + P(A^c)$
$P(A^c) = 1 - P(A)$

2) For any events A and B $P(A^c \cap B) = P(B) - P(A \cap B)$



proof:
From the figure
$(A^c \cap B)$ and $A \cap B$ are distinct sets
$P(A^c \cap B) \cup (A \cap B) = P(A^c \cap B) + P(A \cap B)$
$P(B) = P(A^c \cap B) + P(A \cap B)$
$\therefore P(A^c \cap B) = P(B) - P(A \cap B)$

3) $P(A \cap B^c) = P(A) - P(A \cap B)$



proof:

$A \cap B^c$ and $A \cap B$ are disjoint sets

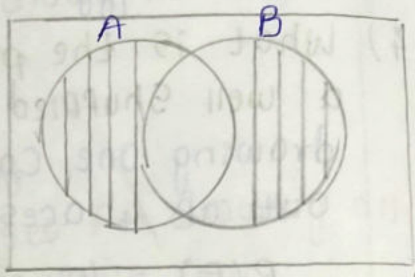
$P\,(A \cap B^c) \cup (A \cap B) = P(A \cap B^c) + P(A \cap B)$

$P(A) = P(A \cap B^c) + P(A \cap B)$

$\therefore P(A \cap B^c) = P(A) - P(A \cap B)$

4) For any two events A and B

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$



proof:

from 2 3 theorems

$P(A^c \cap B) = P(B) - P(A \cap B)$ — 1

$P(A \cap B^c) = P(A) - P(A \cap B)$ — 2

1 + 2

$P(A^c \cap B) + P(A \cap B^c) = P(A) + P(B) - 2P(A \cap B)$

$P[(A^c \cap B) \cup (A \cap B^c)] = P(A) + P(B) - 2P(A \cap B)$

(or)

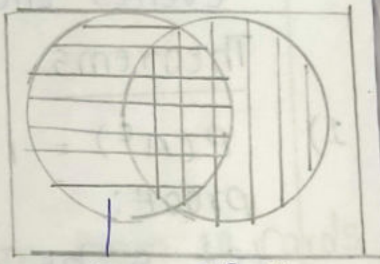
A and $A^c \cap B$ are mutually exclusive disjoint sets

$A \cup (A^c \cap B) = A \cup B$

$P[A \cup (A^c \cap B)] = P(A \cup B)$

$P(A) + P(A^c \cap B) = P(A \cup B)$

$P(A) + P(B) - P(A \cap B) = P(A \cup B)$



5) For any three events A, B and C

$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C) - P(C \cap A) + P(A \cap B \cap C)$

proof:

$P(A \cup B \cup C) = P[(A \cup B) \cup C]$

$= P(A \cup B) + P(C) - P[(A \cup B) \cap C]$

$= P(A) + P(B) - P(A \cap B) + P(C) - P[(A \cap C) \cup (B \cap C)]$

$= P(A) + P(B) - P(A \cap B) + P(C) - P(A \cap C) - P(B \cap C) + P[(A \cap C) \cap (B \cap C)]$

$= P(A) + P(B) + P(C) - P(A \cap B) - P(A \cap C) - P(B \cap C) + P(A \cap B \cap C)$

6) If $A \subseteq B$ then prove that

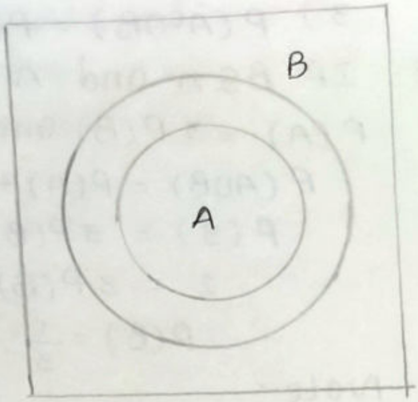
1) $P(A^c \cap B) = P(B) - P(A)$

proof:

By theorem (2)

$P(A^c \cap B) = P(B) - P(A \cap B)$

$P(A^c \cap B) = P(B) - P(A)$



2) $P(A^c \cap B) \geq 0$

$P(B) - P(A) \geq 0$

$P(B) \geq P(A)$

7) If A and B are mutually exclusive then $P(A) \leq P(B^c)$

proof:

$P(A \cup B) = P(A) + P(B)$

$P(A \cup B) = P(A) + P(B) \leq 1$

$P(A) \leq 1 - P(B)$

$P(A) \leq P(B^c)$

1) $P(A) = \frac{1}{5}$, $P(B) = \frac{2}{3}$, $P(A \cap B) = \frac{1}{15}$

Find 1) $P(A \cup B)$ 2) $P(A^c \cap B)$ 3) $P(A \cap B^c)$ 4) $P(A^c \cap B^c)$

1) $P(A \cup B) = P(A) + P(B) - P(A \cap B) = \frac{1}{5} + \frac{2}{3} - \frac{1}{15} = \frac{4}{5}$

2) $P(A^c \cap B) = P(B) - P(A \cap B) = \frac{2}{3} - \frac{1}{15} = \frac{3}{5}$

3) $P(A \cap B^c) = P(A) - P(A \cap B) = \frac{1}{5} - \frac{1}{15} = \frac{2}{15}$

4) $P(A^c \cap B^c) = P((A \cup B)^c) = 1 - P(A \cup B) = 1 - \frac{4}{5} = \frac{1}{5}$

5) $P(A^c \cup B^c) = P((A \cap B)^c) = 1 - P(A \cap B) = 1 - \frac{1}{15} = \frac{14}{15}$

2. If $P(A \cup B) = \frac{4}{5}$, $P(B^c) = \frac{1}{3}$, $P(A \cap B) = \frac{1}{5}$

Find 1) $P(B)$ 2) $P(A)$ 3) $P(A^c \cap B)$

1) $P(B) = 1 - P(B^c) = 1 - \frac{1}{3} = \frac{2}{3}$

2) $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$\frac{4}{5} = P(A) + \frac{2}{3} - \frac{1}{5}$

$P(A) = \frac{1}{3}$

3) $P(A^c \cap B) = P(B) - P(A \cap B) = \frac{2}{3} - \frac{1}{5} = \frac{7}{15}$

3. If $B \subseteq A$ and A and B are two events such that $P(A) = 3P(B)$ and $A \cup B = S$ find $P(B)$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B) \qquad \left(\because \begin{array}{l} B \subseteq A \\ A \cap B = B \end{array} \right)$$

$$P(S) = 3P(B) + P(B) - P(B)$$

$$1 = 3P(B)$$

$$P(B) = \frac{1}{3}$$

* Note:

4) Two dice are thrown let A be the envent that sum is of the points on the faces is 9. Let B be the event that atleast one number is 6. Find the probability of the following events

1) $A \cap B$ 2) $A \cup B$ 3) $A \cap B^c$ 4) $A^c \cap B$ 5) $A^c \cap B^c$

6) $A^c \cup B^c$

$$S = \{ (1,1) - - - (1,6)$$
$$(2,1) - - - (2,6)$$
$$\vdots$$
$$(6,1) - - - (6,6) \}$$

$P(A) = \frac{4}{36} = \frac{1}{9}$ $\quad P(B) = \frac{11}{36}$ $\quad P(A \cap B) = \frac{2}{36} = \frac{1}{18}$

$P(A \cap B) = \frac{1}{18}$

2) $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$$= \frac{1}{9} + \frac{11}{36} - \frac{1}{18} = \frac{13}{36}$$

3) $P(A \cap B^c) = P(A) - P(A \cap B) = \frac{1}{9} - \frac{1}{18} = \frac{1}{18}$

4) $P(A^c \cap B) = P(B) - P(A \cap B) = \frac{11}{36} - \frac{1}{18} = \frac{9}{36} = \frac{1}{4}$

5) $P(A^c \cup B^c) = P((A \cap B)^c) = 1 - P(A \cap B) = 1 - \frac{1}{18} = \frac{17}{18}$

6) $P(A^c \cap B^c) = P((A \cup B)^c) = 1 - P(A \cup B) = 1 - \frac{13}{36} = \frac{23}{36}$

5. If A and B are two events and $P(A) = \frac{3}{5}$, $P(B) = \frac{1}{2}$. Then 1) P.T $P(A \cup B) \geq \frac{3}{5}$

2) $\frac{1}{10} \leq P(A \cap B) \leq \frac{1}{2}$

$A \subseteq A \cup B \Rightarrow P(A) \leq P(A \cup B)$

$$\frac{3}{5} \leq P(A \cup B)$$

2) $P(A\cup B) = P(A)+P(B)-P(A\cap B) \le 1$

$$\frac{3}{5}+\frac{1}{2}-P(A\cap B) \le 1$$

$$\frac{3}{5}+\frac{1}{2}-1 \le P(A\cap B)$$

$$\frac{1}{10} \le P(A\cap B) \quad \text{— } ①$$

$A\cap B \subseteq B \Rightarrow P(A\cap B) \le P(B)$

$$P(A\cap B) \le \frac{1}{2} \quad \text{— } ②$$

Combining ① and ②

$$\frac{1}{10} \le P(A\cap B) \le \frac{1}{2}$$

6) If $P(A)=a$, $P(B)=b$, $P(A\cap B)=c$, express the following probabilites interms of a, b, c

1) $P(A^c\cup B^c) = P((A\cap B)^c) = 1-P(A\cap B) = 1-c$

2) $P(A\cap B^c) = P(A)-P(A\cap B) = a-c$

3) $P(A^c\cap B) = P(B)-P(A\cap B) = b-c$

4) $P(A^c\cap B^c) = P((A\cup B)^c) = 1-P(A\cup B)$

$$= 1-[P(A)+P(B)-P(A\cap B)]$$

$$= 1-a-b+c$$

5) $P(A^c\cup B) = P(A^c)+P(B)-P(A^c\cap B)$

$$= 1-P(A)+P(B)-[P(B)-P(A\cap B)]$$

$$= 1-a+c$$

6) $P(A^c\cap(A\cup B)) = P[(A^c\cap A)\cup(A^c\cup B)) = 1$

$\Rightarrow P(A^c\cup A\cup B) = P(S) = 1$

$$P(A^c)+P(A\cup B)-P(A^c\cap(A\cup B)) = 1$$

$$P(A^c)+P(A)+P(B)-P(A\cap B)-1 = P(A^c\cap(A\cup B))$$

$$1-P(A)+P(A)+P(B)-P(A\cap B)-1 = P(A^c\cap(A\cup B))$$

$$1-a+a+b-c-1 = P(A^c\cap(A\cup B))$$

$$b-c = P(A^c\cap(A\cup B))$$

7) $P(A\cup(A^c\cap B)) = P(A)+P(A^c\cap B)-P(A\cap(A^c\cap B))$

$$= a+P(B)-P(A\cap B)-0$$

$$= a+b-c$$

8) $P(A\cap B)^c) = 1-P(A\cap B) = 1-c$

9) $P((A\cup B)^c) = 1-P(A\cup B)$

$$= 1-[P(A)+P(B)-P(A\cap B)]$$

$$= 1-(a+b-c) = 1-a-b+c$$

7) Among 150 students 80 are studying maths 40 are studying only physics and 30 are studying maths and physics. If a student is chosen at random, find the probability that the student

1) Studying maths or physics
2) Student studying neither maths nor physics

Let A be the event of student studying maths
B = Student studying physics

$P(A) = \frac{80}{150}$ $P(B) = \frac{40}{150}$ $P(A \cap B) = \frac{30}{150}$

1) maths or physics $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$$= \frac{80}{150} + \frac{40}{150} - \frac{30}{150} = \frac{90}{150} = \frac{3}{5}$$

2) neither maths nor physics

$$P(A^c \cap B^c) = P((A \cup B)^c) = 1 - P(A \cup B) = 1 - \frac{3}{5} = \frac{2}{5}$$

8. The probabilites that students A, B, C, D solve a problem are $\frac{1}{3}, \frac{2}{5}, \frac{1}{5}$ and $\frac{1}{4}$ respectively. If all of them try to solve the problem what is the probability that problem is solved

P(A) = probability that A solves the problem

$P(A) = \frac{1}{3} \Rightarrow P(A^c) = \frac{2}{3}$

$P(B) = \frac{2}{5}$ $P(C) = \frac{1}{5}$ $P(D) = \frac{1}{4}$

$P(B^c) = \frac{3}{5}$ $P(C^c) = \frac{4}{5}$ $P(D^c) = \frac{3}{4}$

The probability that the problem is solved is

$$P(A \cup B \cup C \cup D) = 1 - P(A \cup B \cup C \cup D'))$$
$$= 1 - P(A^c \cap B^c \cap C^c \cap D^c)$$
$$= 1 - [P(A^c) . P(B^c) . P(C^c) . P(D^c)]$$
$$= 1 - \left[\frac{2}{3} \times \frac{3}{5} \times \frac{4}{5} \times \frac{3}{4}\right]$$
$$= 1 - \frac{6}{25} = \frac{19}{25}$$

9) Two bolts are drawn from a box containing 4 good and 6 bad bolts. Find the probability that second bolt is good if the first one is found to be bad

Probability that first bad and second good is

$$\frac{6}{10} \times \frac{4}{9} = \frac{4}{15}$$

10) A Class has 10 boys and 5 girls. Three Students are Selected at random one after the other. Find the probability that

1) First two are boys and third is a girl
2) First and third of Same Sex and Second is opposite Sex

First two are boys and third is a girl

$$= \frac{10}{15} \cdot \frac{9}{14} \cdot \frac{5}{13} = \frac{15}{91}$$

2) There are two possibilities first, third are boys and Second is a girl

another possibility is first, third are girls second is a boy, required probability is

Sum of these two $= \frac{10}{15} \times \frac{5}{14} \times \frac{9}{13} + \frac{5}{15} \times \frac{10}{14} \times \frac{4}{13} = \frac{5}{7}$

11) A problem of Statistics is given to three Students A, B, C whose Chances of Solving it are $\frac{1}{2}$, $\frac{3}{4}$ and $\frac{1}{4}$ respectively. What is the probability that the problem is Solved

$P(A) = \frac{1}{2}$, $P(B) = \frac{3}{4}$, $P(C) = \frac{1}{4}$

$$P(A \cup B \cup C) = 1 - (P(A \cup B \cup C)^c)$$
$$= 1 - P(A^c \cap (B^c \cap C^c)$$
$$= 1 - [P(A^c) \cdot P(B^c) \cdot P(C^c)]$$
$$= 1 - \left(\frac{1}{2} \times \frac{1}{4} \times \frac{3}{4}\right) = 1 - \frac{3}{32} = \frac{29}{32}$$

12) Out of 10 girls in a class, 3 have blue eyes of 2 of the girls are chosen at random what is the probability that

1) Both have blue eyes
2) atleast one have blue eyes

Total $= {}^{10}C_2$

favourable $= {}^{3}C_2$

$P(E) = \frac{^{3}C_2}{^{10}C_2} = \frac{3\times2}{10\times9} = \frac{1}{15}$

2) atleast one have blue eyes

$P(x \geq 1) = 1 - P(x < 1)$

$= 1 - P(x=0)$ (no blue eyes)

$= 1 - \frac{^{7}C_2}{^{10}C_2} = 1 - \frac{21}{45} = \frac{8}{15}$

Conditional probability

If E_1 & E_2 are two events in sample space S and $P(E_1) \neq 0$ then the probability of happening of E_2 after the happinening of E_1 is called Conditional probability of event E_2 given E_1

$\therefore P\left(\frac{E_2}{E_1}\right) = \frac{P(E_2 \cap E_1)}{P(E_1)}$ (or) $\frac{P(E_1 \cap E_2)}{P(E_1)}$

Multiplication theorem of probability

In a random experiment, if E_1 & E_2 are two events such that $P(E_1) \neq 0$, $P(E_2) \neq 0$ then

$P(E_1 \cap E_2) = P(E_1) \cdot P\left(\frac{E_2}{E_1}\right)$

$P(E_2 \cap E_1) = P(E_2) \cdot P\left(\frac{E_1}{E_2}\right)$

Note:

1) If E_1 & E_2 are any two independent event then $P\left(\frac{E_1}{E_2}\right) = P(E_1)$

2) If E_1 & E_2 are pairwise independent events then $P(E_1 \cap E_2) = P(E_1) \cdot P(E_2)$

Imp Baye's theorem

If $E_1, E_2 --- E_n$ are mutually exclusive and exhaustive events such that $P(E_i) > 0$ for $i = 1, 2 -- n$ in a sample space S and A is another event in S which is intersecting with every E_i i.e A can only occur in combination with events $E_1, E_2 --- E_n$

If E_i is any of the events of $E_1, E_2 ---- E_n$ where $P(E_1) + P(E_2) --- P(E_n)$ and $P\left(\frac{A}{E_1}\right), P\left(\frac{A}{E_2}\right) --- P\left(\frac{A}{E_n}\right)$ are known then

$$P\left(\frac{E_k}{A}\right) = \frac{P(E_k) \cdot P\left(\frac{A}{E_k}\right)}{P(E_1) P\left(\frac{A}{E_1}\right) + P(E_2) P\left(\frac{A}{E_2}\right) + -- P(E_n) P\left(\frac{A}{E_n}\right)}$$

proof:

$E_1, E_2 --- E_n$ are mutually exclusive events

$E_i \cap E_j = \emptyset$

$S = E_1 \cup E_2 ----\cup E_n$

$A \cap S = A$

$A = A \cap S = A \cap [E_1 \cup E_2 --- E_n]$.

$A = (A \cap E_1) \cup (A \cap E_2) --- \cup (A \cap E_n)$

$P(A) = P[(A \cap E_1) \cup (A \cap E_2) --- \cup (A \cap E_n)]$

$= P(A \cap E_1) + P(A \cap E_2) + ---+ P(A \cap E_n)$

$P(A) = P(E_1 \cap A) + P(E_2 \cap A) + ---+ P(E_n \cap A)$

$P(A) = P(E_1) \cdot P\left(\frac{A}{E_1}\right) + P(E_2) \cdot P\left(\frac{A}{E_2}\right) + --+ P(E_n) \cdot P\left(\frac{A}{E_n}\right)$ - (1)

(By multiplication theorem)

By conditional probability definition

$$P\left(\frac{E_k}{A}\right) = \frac{P(E_k \cap A)}{P(A)}$$

$$= \frac{P(E_k) \cdot P\left(\frac{A}{E_k}\right)}{P(A)}$$ (By using multiplication theorem)

$$P\left(\frac{E_k}{A}\right) = \frac{P(E_k) \cdot P\left(\frac{A}{E_k}\right)}{P(E_1) P\left(\frac{A}{E_1}\right) + P(E_2) P\left(\frac{A}{E_2}\right) + --+ P(E_n) P\left(\frac{A}{E_n}\right)}$$

(From (1))

Note:
If A and B are any two independent events then $P\left(\frac{A}{B}\right) = P(A)$

1) If A and B are any two events such that $P(A) \neq 0$ $P(B) \neq 0$. If A is independent of B then B is also independent of A

A is independent of B

$P\left(\frac{A}{B}\right) = P(A)$ (By above note)

$\frac{P(A \cap B)}{P(B)} = P(A)$ (By conditional probability)

$\frac{P(A \cap B)}{P(A)} = P(B)$

$P\left(\frac{B}{A}\right) = P(B)$

B is independent of A

2) If A and B are independent events then A^c and B^c are also independent events

Given A and B are independent events

$\therefore$ A & B are independent events

$P(A \cap B) = P(A) \cdot P(B)$ — (1)

$$\begin{aligned}
P(A^c \cap B^c) &= P((A \cup B)^c) \\
&= 1 - P(A \cup B) \\
&= 1 - [P(A) + P(B) - P(A \cap B)] \\
&= 1 - P(A) - P(B) + P(A \cap B) \\
&= 1 - P(A) - P(B) + P(A)P(B) \\
&= 1 - P(A) - P(B)[1 - P(A)] \\
&= [1 - P(A)][1 - P(B)]
\end{aligned}$$

$P(A^c \cap B^c) = P(A^c) \cdot P(B^c)$

$\therefore$ A^c and B^c are independent events

3) If A and B are any two events then A and B^c are independent

$\because$ A and B are independent events

$P(A \cap B) = P(A) \cdot P(B)$ — (1)

$P[(A\cap B^c)\cup(A\cap B)] = P(A)$

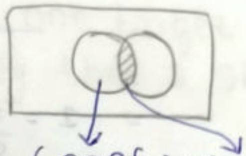
$P(A\cap B^c) + P(A\cap B) = P(A)$

$P(A\cap B^c) = P(A) - P(A\cap B)$

$= P(A) - P(A)P(B)$

$= P(A)[1-P(B)]$

$= P(A)\cdot P(B^c)$



$(A\cap B^c)\cup(A\cap B) = A$

$P(A\cap B^c) = P(A)\cdot P(B^c)$

$\therefore$ A and B^c are independent events

4) If A, B, C are mutually independent events then $A\cup B$ and C are also independent

A, B, C are mutually independent events

$P(A\cap B\cap C) = P(A)P(B)P(C)$

Consider $P((A\cup B)\cap C) = P((A\cap C)\cup(B\cap C))$

$= P(A\cap C) + P(B\cap C) - P[(A\cap C)\cap(B\cap C)]$

$= P(A)P(C) + P(B)P(C) - P(A\cap B\cap C)$

$= P(A)P(C) + P(B)P(C) - P(A)P(B)P(C)$

$= P(C)[P(A)+P(B)-P(A)P(B)]$

$= P(C)[P(A)+P(B)-P(A\cap B)]$

$P[(A\cup B)\cap C] = P(C)\cdot P(A\cup B)$

1) If $P(A^c)=\frac{3}{8}$, $P(B^c)=\frac{1}{2}$, $P(A\cap B)=\frac{1}{4}$ then find

1) $P\left(\frac{A}{B}\right)$ 2) $P\left(\frac{B}{A}\right)$ 3) $P\left(\frac{A^c}{B^c}\right)$ 4) $P\left(\frac{B^c}{A^c}\right)$

$P(A^c) = \frac{3}{8} \Rightarrow P(A) = 1-\frac{3}{8} = \frac{5}{8}$

$P(B^c) = \frac{1}{2} \Rightarrow P(B) = 1-\frac{1}{2} = \frac{1}{2}$

1) $P\left(\frac{A}{B}\right) = \frac{P(A\cap B)}{P(B)} = \frac{\frac{1}{4}}{\frac{1}{2}} = \frac{1}{2}$

2) $P\left(\frac{B}{A}\right) = \frac{P(B\cap A)}{P(A)} = \frac{\frac{1}{4}}{\frac{5}{8}} = \frac{1}{4}\times\frac{8}{5} = \frac{2}{5}$

3) $P\left(\frac{A^c}{B^c}\right) = \frac{P(A^c\cap B^c)}{P(B^c)} = \frac{P((A\cup B)^c)}{P(B^c)} = \frac{1-P(A\cup B)}{P(B^c)}$

$= \frac{1-[P(A)+P(B)-P(A\cap B)]}{P(B^c)}$

$$= \frac{1 - \left[\frac{5}{8} + \frac{1}{2} - \frac{1}{4}\right]}{\frac{1}{2}} = \frac{1 - \left[\frac{5+4-2}{8}\right]}{\frac{1}{2}}$$

$$= \frac{1 - \frac{7}{8}}{\frac{1}{2}} = \frac{1}{8} \times \frac{2}{1} = \frac{1}{4}$$

4) $P\left(\frac{B^c}{A^c}\right) = \frac{P(B^c \cap A^c)}{P(A^c)} = \frac{P((B \cup A)^c)}{P(A^c)} = \frac{1 - P((B \cup A)^c)}{P(A^c)}$

$$= \frac{\frac{1}{8}}{\frac{3}{8}} = \frac{1}{3}$$

2) Among the workers in a factory 50% receied bonus. Among those receiving bonus 30% are Skilled what is the probability that a randomly Selected worker who is Skilled and received bonus

Let A be the event of receiving bonus

Let B be the event, they are Skilled

$P(A) = 50\% = \frac{50}{100} = \frac{1}{2} = 0.5$

$P\left(\frac{B}{A}\right) = 30\% = \frac{30}{100} = 0.3$

$P(B \cap A) = P(A \cap B) = P(A) \times P\left(\frac{B}{A}\right)$

$= 0.5 \times 0.3$

$= 0.15$

3) If the probability that a communication System will have high fiedelity is 0.51 and the probability that it have high fidelity and Selectivity is 0.18 what is the probability that System with high fidelity will also have high Selectivity

Let A be the event of happening of high fidelity

Let B be the event of having high Selectivity

$P(A) = 0.51$

$P(A \cap B) = 0.18$

$P\left(\frac{B}{A}\right) = \frac{P(B \cap A)}{P(A)} = \frac{0.18}{0.51} = 0.35$

4) A can hit a target once in 5 shots. B can hit 2 targets in 3 shots. C can hit one target in 4 shots. What is the probability that 2 shots hit the target

$P(A) = \frac{1}{5}$ $\quad P(B) = \frac{2}{3}$ $\quad P(C) = \frac{1}{4}$

$P(A^c) = \frac{4}{5}$ $\quad P(B^c) = \frac{1}{3}$ $\quad P(C^c) = \frac{3}{4}$

Two shots hits the target is possible only when A,B hits the target C will not hit target (or) A, C hit the target B will not hit target (or) B, C hit the target A will not hit target

i.e $P[(A\cap B\cap C^c) \cup (A\cap B^c\cap C) \cup (A^c\cap B\cap C)]$

$= P(A\cap B\cap C^c) + P(A\cap B^c\cap C) + P(A^c\cap B\cap C)$

$= P(A)P(B)P(C^c) + P(A)P(B^c)P(C) + P(A^c)P(B)P(C)$

$= \left(\frac{1}{5}\right)\left(\frac{2}{3}\right)\left(\frac{3}{4}\right) + \left(\frac{1}{5}\right)\left(\frac{1}{3}\right)\left(\frac{1}{4}\right) + \left(\frac{4}{5}\right)\left(\frac{2}{3}\right)\left(\frac{1}{4}\right)$

$= \frac{6}{60} + \frac{1}{60} + \frac{8}{60} = \frac{15}{60} = \frac{1}{4}$

5) The probabilities of passing in subject A, B, C, D are $\frac{3}{4}, \frac{2}{3}, \frac{4}{5}, \frac{1}{2}$ respectively. To qualify in examination A student should pass in 'A' & 2 subjects among 3. What is the probability of qualifying the examination?

$P(A) = \frac{3}{4}$ $\quad P(B) = \frac{2}{3}$ $\quad P(C) = \frac{4}{5}$ $\quad P(D) = \frac{1}{2}$

$P(A^c) = \frac{1}{4}$ $\quad P(B^c) = \frac{1}{3}$ $\quad P(C^c) = \frac{1}{5}$ $\quad P(D^c) = \frac{1}{2}$

To qualify in the examination, he can qualify in A, B, C, D (all subjects) or A, B, C but not D or A, B, D but not C or A, C, D but not B

$P[(A\cap B\cap C\cap D) \cup (A\cap B\cap C\cap D^c) \cup (A\cap B\cap C^c\cap D) \cup (A\cap B^c\cap C\cap D)]$

$= P(A\cap B\cap C\cap D) + P(A\cap B\cap C\cap D^c) + P(A\cap B\cap C^c\cap D) + P(A\cap B^c\cap C\cap D)$

$= P(A)P(B)P(C)P(D) + P(A)P(B)P(C)P(D^c) + P(A)P(B)P(C^c)$
$P(D) + P(A)P(B^c)P(C)P(D)$

$$= \left(\frac{3}{4}\right)\left(\frac{2}{3}\right)\left(\frac{4}{5}\right)\left(\frac{1}{2}\right) + \left(\frac{3}{4}\right)\left(\frac{2}{3}\right)\left(\frac{4}{5}\right)\left(\frac{1}{2}\right) + \left(\frac{3}{4}\right)\left(\frac{2}{3}\right)\left(\frac{1}{5}\right)\left(\frac{1}{2}\right)$$
$$+ \left(\frac{3}{4}\right)\left(\frac{1}{3}\right)\left(\frac{4}{5}\right)\left(\frac{1}{2}\right)$$
$$= \frac{24}{120} + \frac{24}{120} + \frac{6}{120} + \frac{12}{120} = \frac{66}{120} = \frac{33}{60} = \frac{11}{20}$$

6) If the probability of event A is 0.2 and event B is 0.3 and probability $A \cap B = 0.08$ are 3 events independent

$P(A) = 0.2 \quad P(B) = 0.3 \quad P(A \cap B) = 0.08$

$P(A)\,P(B) = 0.2 \times 0.3 = 0.06$

$P(A)\,P(B) \neq P(A \cap B)$

$\therefore$ A & B are not independent events

7) A class has 10 boys & 5 girls, 3 students are selected at random one after another

7) 2 marbles are drawn in succession from a box containing 10 red, 30 white, 20 blue, 15 orange with replacement being made after each draw. Find probability that

1) both are white
2) first is red & second is white

Total no. of marbles in box are $10+20+30+15 = 75$

1) Both are white (with replacement)

probability of getting both white marbles is

$$\frac{30C_1}{75C_1} \times \frac{30C_1}{75C_1}$$

2) First is red & second is white

$$\frac{10C_1}{75C_1} \times \frac{30C_1}{75C_1}$$

Problems on Baye's theorem

8) In a certain college 25% of boys & 10% of girls are studying mathematics, the girls constitute 60% of student body

1) what is probability that mathematics being studied
2) If a student is selected at random and is found to be studying mathematics.

Find the probability that Student is a girl
Student is a boy

1) Boys $P(B) = 40\% = \frac{40}{100} = 0.4 \xrightarrow{\text{maths}} P\left(\frac{m}{B}\right) = 25\% = \frac{25}{100} = 0.25$

Girls $P(G) = 60\% = \frac{60}{100} = 0.6 \xrightarrow{\text{maths}} P\left(\frac{m}{G}\right) = 10\% = \frac{10}{100} = 0.1$

$$P(m) = P(B) \cdot P\left(\frac{m}{B}\right) + P(G) \cdot P\left(\frac{m}{G}\right)$$

$$= 0.4 \times 0.25 + 0.6 \times 0.1$$

$$= 0.16 \quad 0.19$$

probability that mathematics student is a girl

$$P\left(\frac{G}{m}\right) = \frac{P(G) \cdot P\left(\frac{m}{G}\right)}{P(B) P\left(\frac{m}{B}\right) + P(G) P\left(\frac{m}{G}\right)}$$

$$= \frac{0.6 \times 0.1}{0.16} = \frac{0.06}{0.16} = 0.375$$

probability that mathematics student is a boy

$$P\left(\frac{B}{m}\right) = \frac{P(B) \cdot P\left(\frac{m}{B}\right)}{P(B) P\left(\frac{m}{B}\right) + P(G) P\left(\frac{m}{G}\right)}$$

$$= \frac{0.4 \times 0.25}{0.16} = \frac{0.1}{0.16} = 0.625$$

2) 'A' bag contains 2 white & 3 red ballas & bag 'B' contains 4 white & 5 red balls. One ball is drawn at random from one of the bags and it is found to be red. Find the probability that the red ball drawn is from bag 'B'.

A bag $P(A) = \frac{1}{2}$: 2 W $P\left(\frac{W}{A}\right) = \frac{2}{5}$; 3 R $P\left(\frac{R}{A}\right) = \frac{3}{5}$

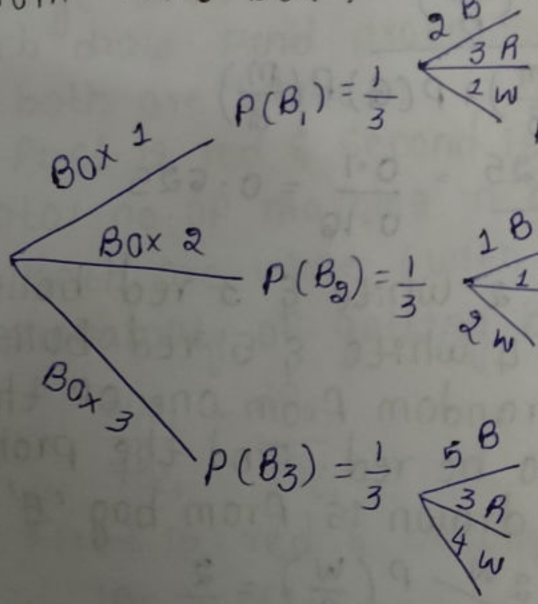
B bag $P(B) = \frac{1}{2}$: 4 W $P\left(\frac{W}{B}\right) = \frac{4}{9}$; 5 R $P\left(\frac{R}{B}\right) = \frac{5}{9}$

Probability that the red ball is drawn is from bag 'B'

$$P\left(\frac{B}{R}\right) = \frac{P(B)\times P\left(\frac{R}{B}\right)}{P(A)\times P\left(\frac{R}{A}\right) + P(B)P\left(\frac{R}{B}\right)}$$

$$= \frac{\frac{1}{2}\times\frac{5}{9}}{\frac{1}{2}\times\frac{3}{5}+\frac{1}{2}\times\frac{5}{9}} = \frac{\frac{5}{18}}{\frac{3}{10}+\frac{5}{18}} = \frac{25}{52}$$

3) First box contains 2 black, 3 red, 1 white, second box contains 1 black, 1 red, 2 white. Third box contains 5 black, 3 red, 4 white balls of these a box is selected at random from it a red ball is randomly drawn 1) If the ball is red find probability that is from second box
2) If the ball is white, find probability that is from first box.
3) If the ball is black, find probability that it is from third box.



$P\left(\frac{B}{B_1}\right) = \frac{2}{6} = \frac{1}{3}$

$P\left(\frac{R}{B_1}\right) = \frac{3}{6} = \frac{1}{2}$

$P\left(\frac{W}{B_1}\right) = \frac{1}{6}$

$P\left(\frac{B}{B_2}\right) = \frac{1}{4}$

$P\left(\frac{R}{B_2}\right) = \frac{1}{4}$

$P\left(\frac{W}{B_2}\right) = \frac{2}{4} = \frac{1}{2}$

$P\left(\frac{B}{B_3}\right) = \frac{5}{12}$

$P\left(\frac{R}{B_3}\right) = \frac{3}{12} = \frac{1}{4}$

$P\left(\frac{W}{B_3}\right) = \frac{4}{12} = \frac{1}{3}$

1) Probability the ball is red from second box

$$P\left(\frac{B_2}{R}\right) = \frac{P(B_2)\,P\left(\frac{R}{B_2}\right)}{P(B_1)\,P\left(\frac{R}{B_1}\right) + P(B_2)\,P\left(\frac{R}{B_2}\right) + P(B_3)\,P\left(\frac{R}{B_3}\right)}$$

$$P\left(\frac{B_2}{B_1}\right) = \frac{\frac{1}{3}\times\frac{1}{4}}{\frac{1}{3}\times\frac{1}{2}+\frac{1}{3}\times\frac{1}{4}+\frac{1}{3}\times\frac{1}{4}} = \frac{1}{4}$$

2) probability that ball is white from first box

$$P\left(\frac{B_1}{W}\right) = \frac{P(B_1)\,P\left(\frac{W}{B_1}\right)}{P(B_1)\,P\left(\frac{W}{B_1}\right)+P(B_2)\,P\left(\frac{W}{B_2}\right)+P(B_3)\,P\left(\frac{W}{B_3}\right)}$$

$$= \frac{\frac{1}{3}\times\frac{1}{6}}{\frac{1}{3}\times\frac{1}{6}+\frac{1}{3}\times\frac{1}{2}+\frac{1}{3}\times\frac{1}{3}} = \frac{1}{6}$$

3) probability that ball is black from third box

$$P\left(\frac{B_3}{B}\right) = \frac{P(B_3)\,P\left(\frac{B}{B_3}\right)}{P(B_1)\,P\left(\frac{B}{B_1}\right)+P(B_2)\,P\left(\frac{B}{B_2}\right)+P(B_3)\,P\left(\frac{B}{B_3}\right)}$$

$$= \frac{\frac{1}{3}\times\frac{5}{12}}{\frac{1}{3}\times\frac{1}{3}+\frac{1}{3}\times\frac{1}{4}+\frac{1}{3}\times\frac{5}{12}} = \frac{5}{12}$$

4) Suppose 25 women out 10000 and 5 men out of 100 are colour blind. A colour blind person is selected at random. what is probability of the person being male. Assume that male & female to be in equal numbers.

men — $P(m) = \frac{1}{2} \longrightarrow P\left(\frac{CB}{m}\right) = \frac{5}{100}$

women — $P(W) = \frac{1}{2} \longrightarrow P\left(\frac{CB}{W}\right) = \frac{25}{10{,}000}$

$$P\left(\frac{m}{CB}\right) = \frac{P(m)\times P\left(\frac{CB}{m}\right)}{P(m)\times P\left(\frac{CB}{m}\right)+P(W)\times P\left(\frac{CB}{W}\right)}$$

$$= \frac{\frac{1}{2}\times\frac{5}{100}}{\frac{1}{2}\times\frac{5}{100}+\frac{1}{2}\times\frac{25}{10{,}000}} = \frac{20}{21}$$

5) In a bolt factory machines A, B, C manufacture 20%, 30% & 50% of their total output and 6%, 3% & 2% are defective. A boat is drawn at random and is found to be defective. Find probability that it is manufacture from 1) machine A 2) machine B, 3) machine C.

$P(A) = \frac{20}{100} = \frac{1}{5} \longrightarrow P\left(\frac{D}{A}\right) = \frac{6}{100} = 6\%$

$P(B) = 30\% = \frac{30}{100} = \frac{3}{10} \longrightarrow P\left(\frac{D}{B}\right) = \frac{3}{100} = 3\%$

$P(C) = 50\% = \frac{50}{100} = \frac{1}{2} \longrightarrow P\left(\frac{D}{C}\right) = \frac{2}{100} = 2\%$

$$P\left(\frac{A}{D}\right) = \frac{P(A)\,P\left(\frac{D}{A}\right)}{P(A)\,P\left(\frac{D}{A}\right) + P(B)\,P\left(\frac{D}{B}\right) + P(C)\,P\left(\frac{D}{C}\right)}$$

$$= \frac{\frac{1}{5} \times \frac{6}{100}}{\frac{1}{5} \times \frac{6}{100} + \frac{3}{10} \times \frac{3}{100} + \frac{1}{2} \times \frac{2}{100}} = \frac{12}{31}$$

$$P\left(\frac{B}{D}\right) = \frac{P(B)\,P\left(\frac{D}{B}\right)}{P(A)\,P\left(\frac{D}{A}\right) + P(B)\,P\left(\frac{D}{B}\right) + P(C)\,P\left(\frac{D}{C}\right)}$$

$$= \frac{\frac{3}{10} \times \frac{3}{100}}{\frac{1}{5} \times \frac{6}{100} + \frac{3}{10} \times \frac{3}{100} + \frac{1}{2} \times \frac{2}{100}} = \frac{9}{31}$$

$$P\left(\frac{C}{D}\right) = \frac{\frac{1}{2} \times \frac{2}{100}}{\frac{1}{5} \times \frac{6}{100} + \frac{3}{10} \times \frac{3}{100} + \frac{1}{2} \times \frac{2}{100}} = \frac{10}{31}$$

6) A business man goes to hotels x, y, z, 20%, 50%, 30% of time respectively. It is known that 5%, 4%, 8% of the rooms in x, y, z hotels have faulty plumbings. What is probability that business man room having faulty plumbing is assigned

to hotel z

x, y, z

$P\left(\frac{z}{FP}\right)$

7) In a f

& machi

in 1000

in 250

drawn a

what is

to hotel z

x: $P(x) = \frac{20}{100} = \frac{1}{5} \longrightarrow P\left(\frac{FP}{x}\right) = \frac{5}{100} = \frac{1}{20}$

y: $P(y) = \frac{50}{100} = \frac{1}{2} \longrightarrow P\left(\frac{FP}{y}\right) = \frac{4}{100} = \frac{1}{25}$

z: $P(z) = \frac{30}{100} = \frac{3}{10} \longrightarrow P\left(\frac{FP}{z}\right) = \frac{8}{100} = \frac{2}{25}$

$$P\left(\frac{z}{FP}\right) = \frac{P(z)\,P\left(\frac{FP}{z}\right)}{P(x)\,P\left(\frac{FP}{x}\right) + P(y)\,P\left(\frac{FP}{y}\right) + P(z)\,P\left(\frac{FP}{z}\right)}$$

$$= \frac{\frac{3}{10} \times \frac{2}{25}}{\frac{1}{5} \times \frac{1}{20} + \frac{1}{2} \times \frac{1}{25} + \frac{3}{10} \times \frac{2}{25}} = \frac{4}{9}$$

7) In a factory machine A produces 40% of output & machine B produces 60%. on the average 9 items in 1000 produced by A are defective and 1 item in 250 produced by B is defective. An item is drawn at random from a days output is defective what is probability that is produced by A or B.

$P(A) = 40\% = \frac{40}{100} = 0.4 \longrightarrow P\left(\frac{D}{A}\right) = \frac{9}{1000}$

$P(B) = 60\% = \frac{60}{100} = 0.6 \longrightarrow P\left(\frac{D}{B}\right) = \frac{1}{250}$

$$P\left(\frac{A}{D}\right) = \frac{P(A)\,P\left(\frac{D}{A}\right)}{P(A)\,P\left(\frac{D}{A}\right) + P(B)\,P\left(\frac{D}{B}\right)}$$

$$= \frac{0.4 \times \frac{9}{1000}}{0.4 \times \frac{9}{1000} + 0.6 \times \frac{1}{250}} = 0.6$$

$$= \frac{24}{16} = 0.4$$

$$P\left(\frac{B}{D}\right) = \frac{P(B)\,P\left(\frac{D}{B}\right)}{P(A)\,P\left(\frac{D}{A}\right) + P(B)\,P\left(\frac{D}{B}\right)}$$

$\therefore$ probability that it is defective but produced by A or B $= P\left(\frac{A}{D}\right) \cup P\left(\frac{B}{D}\right)$

$$= P\left(\frac{A}{D} \cup \frac{B}{D}\right)$$

$$= P\left(\frac{A}{D}\right) + P\left(\frac{B}{D}\right)$$

$$= 0 \cdot 6 + 0 \cdot 4$$

$$= 1$$

8) Companies B_1, B_2, B_3 produce 30%, 45% and 25% of the cars respectively. It is known that 2%, 3% and 2% of the cars produced from B_1, B_2, B_3 are defective 1) What is probability that car is defective

2) If a car purchased is found to be defected what is the probability that this car is produced by Company B_3.

$$P(B_1) = \frac{30}{100} \longrightarrow P\left(\frac{D}{B_1}\right) = \frac{2}{100} = \frac{2}{100} = 0.02$$

$$P(B_2) = \frac{45}{100} \longrightarrow P\left(\frac{D}{B_2}\right) = \frac{3}{100} = 0.03$$

$$P(B_3) = \frac{25}{100} \longrightarrow P\left(\frac{D}{B_3}\right) = \frac{2}{100} = 0.02$$

1) $P(D) = P(B_1) P\left(\frac{D}{B_1}\right) + P(B_2) P\left(\frac{D}{B_2}\right) + P(B_3) P\left(\frac{D}{B_3}\right)$

$$= \frac{30}{100} \times \frac{2}{100} + \frac{45}{100} \times \frac{3}{100} + \frac{25}{100} \times \frac{2}{100}$$

$$= 0 \cdot 0245$$

2) $P\left(\frac{B_3}{D}\right) = \dfrac{P(B_3) \cdot P\left(\frac{D}{B_3}\right)}{P(B_1) P\left(\frac{D}{B_1}\right) + P(B_2) P\left(\frac{D}{B_2}\right) + P(B_3) P\left(\frac{D}{B_3}\right)}$

$$= \frac{\frac{25}{100} \times \frac{2}{100}}{\frac{30}{100} \times \frac{2}{100} + \frac{45}{100} \times \frac{3}{100} + \frac{25}{100} \times \frac{2}{100}}$$

$$= \frac{0 \cdot 005}{0 \cdot 0245} = 0 \cdot 20$$

9) Suppose x, y, z produce TV. x produce twice as many as y while y and z produce the same number. It is known that 2% of x, 2% of y & 4% of z are defective. All TV's are produced and kept in one shop and then one TV is chosen at random

1) What is probability that TV is defective

2) Suppose a TV chosen is defective. What is the probability that this TV is produced by company x.

Let the total production be 100%.

Let total production by y & z Companies be B & C

$\therefore B = C$

Let Company x be A

$\therefore A = 2B$

$\therefore 2B + B + B = 100\%$

$4B = 100$

$B = 25\%$

$x = 50\%, \; y = 25\%, \; 25\%$

$$P(x) = \frac{50}{100} = \frac{1}{2} \longrightarrow P\left(\frac{D}{x}\right) = \frac{2}{100} = 0.02$$

$$P(y) = \frac{25}{100} = \frac{1}{4} \longrightarrow P\left(\frac{D}{y}\right) = \frac{2}{100} = 0.02$$

$$P(z) = \frac{25}{100} = \frac{1}{4} \longrightarrow P\left(\frac{D}{z}\right) = \frac{4}{100} = 0.04$$

1) $$P(x)P\left(\frac{D}{x}\right) + P(y)P\left(\frac{D}{y}\right) + P(z)P\left(\frac{D}{z}\right) = P(D)$$

$$\frac{50}{100} \times \frac{2}{100} + \frac{25}{100} \times \frac{2}{100} + \frac{25}{100} \times \frac{4}{100}$$

$$= \frac{1}{40} = 0.025$$

2) $$P\left(\frac{x}{D}\right) = \frac{P(x)P\left(\frac{D}{x}\right)}{P(x)P\left(\frac{D}{x}\right) + P(y)P\left(\frac{D}{y}\right) + P(z)P\left(\frac{D}{z}\right)}$$

$$= \frac{\frac{50}{100} \times 0.02}{\frac{1}{2} \times 0.02 + \frac{1}{4} \times 0.02 + \frac{1}{4} \times 0.04}$$

$$= \frac{0.01}{0.025} = 0.4$$

10) In a bolt factory machine A, B & C manufacturing 25%, 35% & 45% out of their total output. 5%, 4% & 2% are defective bolts. A bolt is drawn from product & found to be defective. What are the probabilities that it is manufactured by machine A, B, C.

$$P(A) = \frac{25}{100} \longrightarrow P\left(\frac{D}{A}\right) = \frac{5}{100}$$

$$P(B) = \frac{35}{100} \longrightarrow P\left(\frac{D}{B}\right) = \frac{4}{100}$$

$$P(C) = \frac{45}{100} \longrightarrow P\left(\frac{D}{C}\right) = \frac{2}{100}$$

1) $$P\left(\frac{A}{D}\right) = \frac{P(A)P\left(\frac{D}{A}\right)}{P(A)P\left(\frac{D}{A}\right)+P(B)P\left(\frac{D}{B}\right)+P(C)P\left(\frac{D}{C}\right)}$$

$$= \frac{\frac{25}{100}\times\frac{5}{100}}{\frac{25}{100}\times\frac{5}{100}+\frac{35}{100}\times\frac{4}{100}+\frac{45}{100}\times\frac{2}{100}} \qquad \cancel{= \frac{25}{69}}$$

$$= \frac{25}{71}$$

2) $$P\left(\frac{B}{D}\right) = \frac{P(B)P\left(\frac{D}{B}\right)}{P(A)P\left(\frac{D}{A}\right)+P(B)P\left(\frac{D}{B}\right)+P(C)P\left(\frac{D}{C}\right)}$$

$$= \frac{\frac{35}{100}\times\frac{4}{100}}{\frac{25}{100}\times\frac{5}{100}+\frac{35}{100}\times\frac{4}{100}+\frac{45}{100}\times\frac{2}{100}}$$

$$= \frac{28}{71}$$

3) $$P\left(\frac{C}{D}\right) = \frac{P(C)P\left(\frac{D}{C}\right)}{P(A)P\left(\frac{D}{A}\right)+P(B)P\left(\frac{D}{B}\right)+P(C)P\left(\frac{D}{C}\right)}$$

$$= \frac{\frac{45}{100}\times\frac{2}{100}}{\frac{25}{100}\times\frac{5}{100}+\frac{35}{100}\times\frac{4}{100}+\frac{45}{100}\times\frac{2}{100}} = \frac{18}{71}$$

11) A dogs
Box 1 C
Contains
3 white
a box
to be
ty the

1) P(

2) P

3)

11) ~~A dogs van conta~~

Box 1 Contains 1 white, 2 red, 3 green balls, Box 2 Contains 2 white, 3 red, 1 green balls. Box 3 Contains 3 white, 1 red, 2 green balls. A ball is drawn from a box contains chosen at random. These are found to be 1 white and 1 red. Determine the probability that the balls are drawn from Box 2.

$P(B_1) = \frac{1}{3}$: $P\left(\frac{W}{B_1}\right) = \frac{1}{6}$, $P\left(\frac{R}{B_1}\right) = \frac{2}{6}$, $P\left(\frac{G}{B_1}\right) = \frac{3}{6}$

$P(B_2) = \frac{1}{3}$: $P\left(\frac{W}{B_2}\right) = \frac{2}{6}$, $P\left(\frac{R}{B_2}\right) = \frac{3}{6}$, $P\left(\frac{G}{B_2}\right) = \frac{1}{6}$

$P(B_3) = \frac{1}{3}$: $P\left(\frac{W}{B_3}\right) = \frac{3}{6}$, $P\left(\frac{R}{B_3}\right) = \frac{1}{6}$, $P\left(\frac{G}{B_3}\right) = \frac{2}{6}$

1) $$P\left(\frac{B_2}{W}\right) = \frac{P(B_2)\,P\left(\frac{W}{B_2}\right)}{P(B_1)\,P\left(\frac{W}{B_1}\right) + P(B_2)\,P\left(\frac{W}{B_2}\right) + P(B_3)\,P\left(\frac{W}{B_3}\right)}$$

$$= \frac{\frac{1}{3} \times \frac{2}{6}}{\frac{1}{3} \times \frac{1}{6} + \frac{1}{3} \times \frac{1}{3} + \frac{1}{3} \times \frac{1}{2}} = \frac{1}{3}$$

2) $$P\left(\frac{B_2}{R}\right) = \frac{P(B_2)\,P\left(\frac{R}{B_2}\right)}{P(B_1)\,P\left(\frac{R}{B_1}\right) + P(B_2)\,P\left(\frac{R}{B_2}\right) + P(B_3)\,P\left(\frac{R}{B_3}\right)}$$

$$= \frac{\frac{1}{3} \times \frac{1}{2}}{\frac{1}{3} \times \frac{1}{3} + \frac{1}{3} \times \frac{1}{2} + \frac{1}{3} \times \frac{1}{6}}$$

$$= \frac{1}{2} \quad \frac{1}{3}$$

3) probability that ball drawn is 1 white and 1 red $= \frac{1}{3} \times \frac{1}{2} = \frac{1}{6}$

A manufacturer produces steel pipes in 3 plants with daily production volume of 500, 1000, 2000 units respectively. According to past experience it is known that the fraction of defective outputs produced by these plants are 0·005, 0·008 & 0·01. If a pipe is selected from this total production and found to be defective

1) From which plant the pipe came
2) What is probability that it came from first plant

$$P(P_1) = \frac{500}{3500} \longrightarrow P\left(\frac{D}{P_1}\right) = 0\cdot005$$

$$P(P_2) = \frac{1000}{3500} \longrightarrow P\left(\frac{D}{P_2}\right) = 0\cdot008$$

$$P(P_3) = \frac{2000}{3500} \longrightarrow P\left(\frac{D}{P_3}\right) = 0\cdot010$$

1) $$P(D) = P(P_1)\,P\left(\frac{D}{P_1}\right) + P(P_2)\,P\left(\frac{D}{P_2}\right) + P(P_3)\,P\left(\frac{D}{P_3}\right)$$

$$= \frac{1}{7}\times 0\cdot005 + \frac{2}{7}\times 0\cdot008 + \frac{4}{7}\times 0\cdot010$$

$$= \frac{61}{7000} = 8\cdot714\times 10^{-3}$$

2) $$P\left(\frac{P_1}{D}\right) = \frac{P(P_1)\times P\left(\frac{D}{P_1}\right)}{P(P_1)\,P\left(\frac{D}{P_1}\right) + P(P_2)\,P\left(\frac{D}{P_2}\right) + P(P_3)\,P\left(\frac{D}{P_3}\right)}$$

$$= \frac{7\cdot14\times 10^{-4}}{8\cdot714\times 10^{-3}} = 0\cdot8193\times 10^{-1}$$

The chance that a doctor 'A' when diagonised a disease 'x' correctly is 60%. The chance that a patient will die by his treatment after correct diagnosis is 40% and the chance of death by wrong diagnosis is 70%. A patient of doctor A who had disease 'x' die. What is the chance that his disease was diagnoised correctly.

$$P(A) = \frac{60}{100} \longrightarrow P\left(\frac{D}{A}\right) = \frac{40}{100}$$

$$P(W) = \frac{40}{100} \longrightarrow P\left(\frac{D}{W}\right) = \frac{70}{100}$$

probability that the person die that after his disease was diagnoised correctly

$$P\left(\frac{A}{D}\right) = \frac{P(A)\,P\left(\frac{D}{A}\right)}{P(A)\,P\left(\frac{D}{A}\right) + P(W)\,P\left(\frac{D}{W}\right)}$$

$$= \frac{\frac{60}{100} \times \frac{40}{100}}{\frac{60}{100} \times \frac{40}{100} + \frac{40}{100} \times \frac{70}{100}}$$

$$= \frac{6}{13} = 0.46$$

Of the 3 men, the chances that a politician business man or academitian will be appointed of a vice chancellor of university is 0·5, 0·3, 0·2. probability of research which promoted by these persons if they are appointed as VC are 0·3, 0·7, 0·8 respectively.

1) Determine the probability that research is promoted.

2) If research is promoted. What is the probability that vc is an academitian.

$$P(P) = 0.5 \longrightarrow P\left(\frac{R}{P}\right) = 0.3$$

$$P(B) = 0.3 \longrightarrow P\left(\frac{R}{B}\right) = 0.7$$

$$P(A) = 0.2 \longrightarrow P\left(\frac{R}{A}\right) = 0.8$$

$$1)\ P(R) = P(P) \times P\left(\frac{R}{P}\right) + P(B) \times P\left(\frac{R}{B}\right) + P(A) \times P\left(\frac{R}{A}\right)$$

$$= 0.5 \times 0.3 + 0.3 \times 0.7 + 0.2 \times 0.8$$

$$= \frac{2}{5} = 0.4 \,\}\times\ 0.52$$

$$2)\ P\left(\frac{A}{R}\right) = \frac{P(A) \times P\left(\frac{R}{A}\right)}{P(P) \times P\left(\frac{R}{P}\right) + P(B) \times P\left(\frac{R}{B}\right) + P(A) \times P\left(\frac{R}{A}\right)}$$

$$= \frac{0.2 \times 0.8}{0.5 \times 0.3 + 0.3 \times 0.7 + 0.2 \times 0.2}$$

$$= \left\{\frac{0.04}{0.52} = 0.1\right\}^{\times} \qquad = \frac{0.16}{0.52} = \frac{4}{13}$$

Random Variable

A real variable 'x' is a function, a random variable 'x' whose value is determined outcome of a random experiment is called random variable. There are 2 types

1) Descrete random variable
2) Continuous " "

Descrete random variable: which contain only finite no. of descrete values in an interval of domain in other words if the random variable takes the value only on the set $\{0, 2, 4 - - - n\}$ is called a descrete random variable

Continuous random variable: A random variable which can take values continuously i.e which takes all possible values in a given interval is called continuous random variable

probabilities of descrete R.V

For Descrete R V 'x', the real valued function $P(x)$ is such that $P(X=x) = P(x)$ then $P(x)$ is called probability function

properties of probability function

1) $P(x) \geq 0$

2) $\sum P(x) = 1$

3) $P(x)$ can never be negative

probability distribution function

Let x be a random variable then the probability distribution function is defined by $F(x) = P(X \leq x)$

properties of distribution fun

1) $P(a < x \leq b) = F(b) - F(a)$

2) $P(a \leq x \leq b) = P(x=a) + P(a < x \leq b)$
$= P(x=a) + F(b) - F(a)$

3) $P(a < x < b) = P(a < x \leq b) - P(x=b)$
$= [F(b) - F(a)] - P(x=b)$

4) $P(a \leq x < b) = P(x=a) + F(b) - F(a) - P(x=b)$

Expectation / mean

Suppose a random variable 'x' assumes the values $x_1, x_2 - - - x_n$ with respective probabilities $P_1, P_2 - - P_n$ then the mathematical expectation

$$\mu = E(x) = \sum_{i=1}^{n} P_i x_i = P_1 x_1 + P_2 x_2 + - - - + P_n x_n$$

It is denoted with 'μ'

Variance :

Variance is mathematical expecatation of $(x-\mu)^2$

$$\text{variance } (\sigma^2) = E((x-\mu)^2)$$
$$= E(x^2) - [E(x)]^2$$

Alternate Formula $= \sigma^2 = \sum P_i x_i^2 - \mu^2$

Standard Deviation: The square root of variance is called standard deviation

Let x denotes the

1) Two dice are thrown. Let x assign to each point A, B in S the max of its numbers. Find probability distribution also find mean, variance & SD

$$S = \{ (1,1) \; - - - - \; (1,6)$$
$$(2,1) \; - - - - \; (2,6)$$
$$\vdots$$
$$(6,1) \; - - - - \; (6,6) \}$$

when 2 dice are thrown the total no. of outcomes are $6^2 = 36$ $\therefore$ Sample space $S = \{(1,1) - - - - (1,6)$
$\vdots$
$(6,1) - - - - (6,6)\}$

Given x = max of 2 numbers

$\therefore x(1,1) = \max \text{ of } (1,1) = 1$

$x(1,2) = \max \text{ of } (1,2) = 2$

$x(1,3) = \max (1,3) = 3$

$x(1,4) = \max (1,4) = 4$

$x(1,5) = \max (1,5) = 5$

$x(1,6) = \max (1,6) = 6$

$\therefore x = \{1, 2, 3, 4, 5, 6\}$

$P(x=1)$ = probability of getting '1' as max $= \frac{1}{36}$

$P(x=2) = \frac{3}{36}$ $\quad$ $P(x=5) = \frac{9}{36}$

$P(x=3) = \frac{5}{36}$ $\quad$ $P(x=6) = \frac{11}{36}$

$P(x=4) = \frac{7}{36}$

Probability distribution table

X	1	2	3	4	5	6
$P(X=x)$	$\frac{1}{36}$	$\frac{3}{36}$	$\frac{5}{36}$	$\frac{7}{36}$	$\frac{9}{36}$	$\frac{11}{36}$

1) mean $= \sum P_i x_i$

$$u = \left(\frac{1}{36}\times 1\right) + \left(\frac{3}{36}\times 2\right) + \left(\frac{5}{36}\times 3\right) + \left(\frac{7}{36}\times 4\right) + \left(\frac{9}{36}\times 5\right) + \left(\frac{11}{36}\times 6\right)$$

$$u = 4.47$$

2) Variance $(\sigma^2) = \sum P_i x_i^2 - u^2$

$$(\sigma^2) = \left(\frac{1}{36}\times 1\right) + \left(\frac{3}{36}\times 4\right) + \left(\frac{5}{36}\times 9\right) + \left(\frac{7}{36}\times 16\right) + \left(\frac{9}{36}\times 25\right) + \left(\frac{11}{36}\times 36\right) - (4.47)^2$$

$$= 21.972 - 19.980$$

$$= 1.992$$

3) Standard deviation $(\sigma) = \sqrt{1.992} = 1.411$

2) Given X = min of 2 numbers

X (1,1) = main of (1,1) = 6
X (2,2) = min of (2,2) = 1
X (6,1) = min of (1,1) = 1
X (6,2) = min (6,2) = 2
X (6,3) = min (6,3) = 3
X (6,4) = min (6,4) = 4
X (6,5) = min (6,5) = 5
X (6,6) = min (6,6) = 6

X = {1, 2, 3, 4, 5, 6}

$P(X=1)$ = probability of getting '1' as min $= \frac{11}{36}$

$P(X=2) = \frac{9}{36}$

$P(X=3) = \frac{7}{36}$

$P(X=4) = \frac{5}{36}$

$P(X=5) = \frac{3}{36}$

$P(X=6) = \frac{1}{36}$

probability distribution table

x	1	2	3	4	5	6
$P(x=x)$	$\frac{11}{36}$	$\frac{9}{36}$	$\frac{7}{36}$	$\frac{5}{36}$	$\frac{3}{36}$	$\frac{1}{36}$

1) mean $= \sum P_i x_i$

$$= \left(1 \times \frac{11}{36}\right) + \left(2 \times \frac{9}{36}\right) + 3\left(\frac{7}{36}\right) + \left(4 \times \frac{5}{36}\right) + \left(5 \times \frac{3}{36}\right)$$

$$+ \left(6 \times \frac{1}{36}\right)$$

$$= 2.52$$

2) Variance $(\sigma^2) = \sum P_i x_i^2 - u^2$

$$= \left(1 \times \frac{11}{36}\right) + \left(4 \times \frac{9}{36}\right) + \left(9 \times \frac{7}{36}\right) + \left(16 \times \frac{5}{36}\right) + \left(25 \times \frac{3}{36}\right)$$

$$+ \left(36 \times \frac{1}{36}\right) - (2.52)^2$$

$$= 8.361 - 6.350$$

$$= 2.011$$

3) Standard deviation $(\sigma) = \sqrt{2.011} = 1.418$

3) Given x = Sum of 2 numbers on dice

$x(1,1) = \text{Sum}(1,1) = 2$

$x(1,2) = \text{Sum}(1,2) = 3$

$x(1,3) = \text{Sum}(1,3) = 4$

$x(1,4) = \text{Sum}(1,4) = 5$

$x(1,5) = \text{Sum}(1,5) = 6$

$x(1,6) = \text{Sum}(1,6) = 7$

$x(4,4) = \text{Sum}(4,4) = 8$

$x(4,5) = \text{Sum}(4,5) = 9$

$x(5,5) = \text{Sum}(5,5) = 10$

$x(5,6) = \text{Sum}(5,6) = 11$

$x(6,6) = \text{Sum}(6,6) = 12$

probability distribution table

x	2	3	4	5	6	7	8	9	10	11	12
$P(x=x)$	$\frac{1}{36}$	$\frac{2}{36}$	$\frac{3}{36}$	$\frac{4}{36}$	$\frac{5}{36}$	$\frac{6}{36}$	$\frac{5}{36}$	$\frac{4}{36}$	$\frac{3}{36}$	$\frac{2}{36}$	$\frac{1}{36}$

1) mean $= \sum p_i x_i$

$(\mu) = \left(2\times\frac{1}{36}\right) + \left(3\times\frac{2}{36}\right) + \left(4\times\frac{3}{36}\right) + \left(5\times\frac{4}{36}\right) + \left(6\times\frac{5}{36}\right) +$
$\left(7\times\frac{6}{36}\right) + \left(8\times\frac{5}{36}\right) + \left(9\times\frac{4}{36}\right) + \left(10\times\frac{3}{36}\right) + \left(11\times\frac{2}{36}\right) +$
$\left(12\times\frac{1}{36}\right)$

$\mu = 7$

2) Variance $(\sigma^2) = \sum p_i x_i^2 - \mu^2$

$= \left(4\times\frac{1}{36}\right) + \left(9\times\frac{2}{36}\right) + \left(16\times\frac{3}{36}\right) + \left(25\times\frac{4}{36}\right) + \left(36\times\frac{5}{36}\right)$
$+ \left(49\times\frac{6}{36}\right) + \left(64\times\frac{5}{36}\right) + \left(81\times\frac{4}{36}\right) + \left(100\times\frac{3}{36}\right) +$
$\left(121\times\frac{2}{36}\right) + \left(144\times\frac{1}{36}\right) - \mu^2$

$= 54.83 - 49$

$= 5.83$

3) Standard deviation $(\sigma) = \sqrt{5.83} = 2.414$

1) Let x denote the no. of heads in a single toss of 4 fair coins. Determine

1) probability $(x < 2)$

2) $P(1 < x \leq 3)$

When 4 fair coins are tossed total no. of outcomes $= 4^2 = 16$

Sample space (S) = {H H H H, HHHT, HHTH, HTHH, TTHH, THHH, HHTT, HTTH, THHT, HTTT, THTH, HTHT, TTTH, TTTT, THTT, TTHT}

Given x = no. of heads

x (HHHH) = 4

x (HHHT) = 3 = x (HHTH) = x (HTHH) = x (THHH)

x (HHTT) = x (HTTH) = x (TTHH) = x (THTH) = 2

x (HTTT) = x (THTT) = x (TTHT) = x (TTTH) = 1

x (TTTT) = 0

Range of x = {0, 1, 2, 3, 4}

probability at $x = 0 \Rightarrow P(x=0) = \frac{1}{16}$ = probability of getting zero heads

$P(x=1) = \frac{4}{16}$ $\quad P(x=3) = \frac{4}{16}$

$P(x=2) = 6/16$ $\quad P(x=4) = 1/16$

probability distribution table

x	0	1	2	3	4
$P(x=x)$	$1/16$	$\frac{4}{16}$	$\frac{6}{16}$	$\frac{4}{16}$	$\frac{1}{16}$

1) $P(x<2) = P(x=0)+P(x=1)$

$= \frac{1}{16} + \frac{4}{16} = \frac{5}{16}$

2) $P(1<x<3) = P(x=2)+P(x=3)$

$= \frac{6}{16} + \frac{4}{16} = \frac{10}{16} = \frac{5}{8}$

2) A random variable x has the following probability distribution.

x	1	2	3	4	5	6	7	8
$P(x=x)$	k	2k	3k	4k	5k	6k	7k	8k

Find value of 1) k 2) $P(x\le 2)$ 3) $P(2\le x\le 5)$

we know that $\Sigma P_i = 1$

1) $k+2k+3k+4k+5k+6k+7k+8k=1$

$36k=1$

$\boxed{k=\frac{1}{36}}$

2) $P(x\le 2) = P(x=1)+P(x=2)$

$= k+2k$

$= 3k = 3\left(\frac{1}{36}\right) = \frac{1}{12}$

3) $P(2\le x\le 5) = P(x=2)+P(x=3)+P(x=4)+P(x=5)$

$= 2k+3k+4k+5k$

$= 14k = 14\left(\frac{1}{36}\right) = \frac{7}{18}$

3) A random variable x has following probability distribution

Values of x	0	1	2	3	4	5	6	7	8
$P(X=x)$	a	3a	5a	7a	9a	11a	13a	15a	17a

1) Determine
2) Find P(
3) Find the
we know
$a+3a$

2) $P(x<$

$P(x\ge$

$P(0<$

3) Cumul
F(

x
0
1
2
3
4
5
6
7
8

1) Determine the value of a
2) Find $P(X<3)$, $P(X\geq 3)$ & $P(0<X<5)$
3) Find the distribution function $F(x)$

1) we know that sum of all probabilities = 1

$$a+3a+5a+7a+9a+11a+13a+15a+17a=1$$
$$81a=1$$
$$a=\frac{1}{81}$$

2) $P(X<3) = P(X=0)+P(X=1)+P(X=2)$

$$= a+3a+5a = 9a = 9\left(\frac{1}{81}\right) = \frac{1}{9}$$

$P(X\geq 3) = P(X=3)+P(X=4)+P(X=5)+P(X=6)+P(X=7)+P(X=8)$

$$= 7a+9a+11a+13a+15a+17a = 72a$$
$$= \frac{72}{81} = \frac{8}{9}$$

$P(0<X<5) = P(X=1)+P(X=2)+P(X=3)+P(X=4)$

$$= 3a+5a+7a+9a$$
$$= 24a = 24\left(\frac{1}{81}\right) = \frac{8}{27}$$

3) Cumulative distribution function

$$F(x) = P(X\leq x)$$

x	$F(x) = P(X\leq x)$
0	$F(0) = P(X\leq 0) = a = \frac{1}{81}$
1	$F(1) = P(X\leq 1) = P(X=0)+P(X=1) = 4a = \frac{4}{81}$
2	$F(2) = 9a = \frac{9}{81} = \frac{1}{9}$
3	$F(3) = 16a = \frac{16}{81}$
4	$F(4) = 25a = 25/81$
5	$F(5) = 36a = \frac{36}{81}$
6	$F(6) = 49a = \frac{49}{81}$
7	$F(7) = 64a = \frac{64}{81}$
8	$F(8) = 81a = \frac{81}{81}$

4) The probability distribution function of a variate X

X	0	1	2	3	4	5	6
$P(X=x)$	k	$3k$	$5k$	$7k$	$9k$	$11k$	$13k$

1) Find k 2) Find $P(X<4)$, $P(X\geq 5)$, $P(3<X\leq 6)$
3) What will be the min value of k so that $P(X\leq 2)\geq 0.3$

1) we know that sum of all probabilities = 1

$$k+3k+5k+7k+9k+11k+13k=1$$
$$49k=1$$
$$k=\frac{1}{49}$$

2) $P(X<4) = P(X=0)+P(X=1)+P(X=2)+P(X=3)$
$= k+3k+5k+7k = 16k = 16\left(\frac{1}{49}\right) = \frac{16}{49}$

$P(X\geq 5) = P(X=6)+P(X=5)$
$= 13k+11k = 24k = 24\left(\frac{1}{49}\right) = \frac{24}{49}$

$P(3<X\leq 6) = P(X=4)+P(X=5)+P(X=6)$
$= 9k+11k+13k$
$= 33k$
$= 33\left(\frac{1}{49}\right) = \frac{33}{49}$

3) $P(X\leq 2) > 0.3$

$$P(X=0)+P(X=1)+P(X=2) > 0.3$$
$$k+3k+5k > 0.3$$
$$9k > 0.3$$
$$9k > \frac{3}{10}$$
$$k > \frac{3}{90}$$
$$\boxed{k > \frac{1}{30}}$$ min value of k

5) A random variable X has following probability function

X	-3	-2	-1	0	1	2	3
$P(X=x)$	k	0.1	k	0.2	$2k$	0.4	$2k$

1) Find k
deviation
1) we know th
k + 0

2) mean (

3) varianc

4) Stan

6) A desc
probabi

X
$P(X=x)$

1) Find
distrib
1) we

2) P

6)

1) Find k 2) mean 3) Variance 4) Standard deviation

1) We know that Sum of all probabilities = 1

$$k + 0.1 + k + 0.2 + 2k + 0.4 + 2k = 1$$

$$6k + 0.7 = 1$$

$$6k = 1 - 0.7 = 0.3$$

$$k = \frac{3}{60}$$

$$\boxed{k = \frac{1}{20}}$$

2) mean (μ) $= \left(\frac{1}{20} \times -3\right) + (0.1 \times -2) + \left(\frac{1}{20} \times -1\right) + (0 \times 0.2) + \left(1 \times \frac{2k}{20}\right) + (2 \times 0.4) + 3 \times \frac{2}{20}$

$$= \left(\frac{-1}{50} = -0.02\right) \times \frac{4}{5} = 0.8$$

3) variance (σ) $= \left(9 \times \frac{1}{20}\right) + (4 \times 0.1) + \left(1 \times \frac{1}{20}\right) + (0 \times 0.2) + \left(1 \times \frac{2}{20}\right) + (4 \times 0.4) + \left(9 \times \frac{2}{20}\right) - \mu^2$

$$= \frac{7}{2} = 3.5 - (0.8)^2 = 3.5 - 0.64$$

$$= 2.86$$

4) Standard deviation $= \sqrt{2.86}$

$$= 1.69$$

6) A descrete random variable x has following probability distribution.

x	1	2	3	4	5	6	7	8
$P(x=x)$	$2k$	$4k$	$6k$	$8k$	$10k$	$12k$	$14k$	$4k$

1) Find value of k 2) $P(x<3)$ & $P(x \geq 5)$ 3) Find the distribution function of x

1) we know sum of all probabilities = 1

$$2k + 4k + 6k + 8k + 10k + 12k + 14k + 4k = 1$$

$$74k = 1$$

$$k = \frac{1}{74}$$

2) $P(x<3) = P(x=1) + P(x=2) = 2k + 4k = 6k = 6\left(\frac{1}{74}\right) = \frac{3}{37} = \frac{1}{9}$

$P(X \geq 5) = P(X=5) + P(X=6) + P(X=7) + P(X=8)$

$= 10k + 12k + 14k + 4k = 40k = 40\left(\frac{1}{74}\right)$

$= \frac{20}{37}$

Cumulative distribution function

$F(x) = P(X \leq x)$

x	$F(x) = P(X \leq x)$
1	$F(1) = P(X \leq 1) = 2k = \frac{2}{74} = \frac{1}{37}$
2	$F(2) = P(X \leq 2) = 6k = \frac{6}{74} = \frac{3}{37}$
3	$F(3) = P(X \leq 3) = 12k = \frac{12}{74} = \frac{6}{37}$
4	$F(4) = P(X \leq 4) = 20k = \frac{20}{74} = \frac{10}{37}$
5	$F(5) = P(X \leq 5) = 30k = \frac{30}{74} = \frac{15}{37}$
6	$F(6) = P(X \leq 6) = 42k = \frac{42}{74} = \frac{21}{37}$
7	$F(7) = P(X \leq 7) = 56k = \frac{56}{74} = \frac{28}{37}$
8	$F(8) = P(X \leq 8) = 60k = \frac{60}{74} = \frac{30}{37}$

7) A random variable X has following probability function

x	1	2	3	4	5	6
$P(X=x)$	k	$3k$	$5k$	$7k$	$9k$	$11k$

Determine 1) k 2) Expectation 3) Variance

1) we know that sum of all probabilities = 1

$k + 3k + 5k + 7k + 9k + 11k = 1$

$36k = 1$

$k = \frac{1}{36}$

mean (μ) = (

(

=

Variance (σ^2)

8) A random
bility func

x	0
$P(X=x)$	$\frac{k}{45}$

Determin
deviation
we know

mean (

Varian

Star

$\text{mean}(\mu) = \left(1 \times \frac{1}{36}\right) + \left(2 \times \frac{3}{36}\right) + \left(3 \times \frac{5}{36}\right) + \left(4 \times \frac{7}{36}\right) + \left(5 \times \frac{9}{36}\right) + \left(6 \times \frac{11}{36}\right)$

$= 4.47$

$\text{Variance}(\sigma^2) = \left(1 \times \frac{1}{36}\right) + \left(4 \times \frac{3}{36}\right) + \left(9 \times \frac{5}{36}\right) + \left(16 \times \frac{7}{36}\right) + \left(25 \times \frac{9}{36}\right) + \left(36 \times \frac{11}{36}\right) - (4.47)^2$

$= 21.97 - 19.98$

$= 1.99$

8) A random variable x has the following probability function

x	0	1	2	3	4	5	6	7	8
$P(x=x)$	$\frac{k}{45}$	$\frac{k}{15}$	$\frac{k}{9}$	$\frac{k}{5}$	$\frac{2k}{45}$	$\frac{6k}{45}$	$\frac{7k}{45}$	$\frac{8k}{45}$	$\frac{4k}{45}$

Determine 1) mean 2) variance 3) standard deviation.

We know that sum of all probabilities = 1

$\frac{k}{45} + \frac{k}{15} + \frac{k}{9} + \frac{k}{5} + \frac{2k}{45} + \frac{6k}{45} + \frac{7k}{45} + \frac{8k}{45} + \frac{4k}{45} = 1$

$45\frac{k}{45} = 1$

$\boxed{k = 1}$

$\text{mean}(\mu) = \left(0 \times \frac{1}{45}\right) + \left(1 \times \frac{1}{15}\right) + \left(2 \times \frac{1}{9}\right) + \left(3 \times \frac{1}{5}\right) + \left(4 \times \frac{2}{45}\right) + \left(5 \times \frac{6}{45}\right) + \left(6 \times \frac{7}{45}\right) + \left(7 \times \frac{8}{45}\right) + \left(8 \times \frac{4}{45}\right)$

$= 4.62$

$\text{variance}(\sigma^2) = \left(0 \times \frac{1}{45}\right) + \left(1 \times \frac{1}{15}\right) + \left(4 \times \frac{1}{9}\right) + \left(9 \times \frac{1}{5}\right) + \left(16 \times \frac{2}{45}\right) + \left(25 \times \frac{6}{45}\right) + \left(36 \times \frac{7}{45}\right) + \left(49 \times \frac{8}{45}\right) + \left(64 \times \frac{4}{45}\right) - (4.62)^2$

$= 26.35 - 21.34 = 5.01$

Standard deviation $= \sqrt{5.01} = 2.23$

9) A random variable x has following probability function

x	0	1	2	3	4	5	6	7
$P(x=x)$	0	k	$2k$	$2k$	$3k$	k^2	$2k^2$	$7k^2+k$

Determine k

Evaluate $P(x<6)$, $P(x\geq 6)$, $P(0<x<5)$ & $P(0\leq x\leq 4)$

Determine distribution function of x

mean, variance, S.D

$P(x\leq k) > \frac{1}{2}$ Find the min value of k

1) we know that sum of all probabilities = 1

$$0+k+2k+2k+3k+k^2+2k^2+7k^2+k=1$$

$$9k+10k^2=1$$

$$10k^2+9k-1=0$$

$$k=\frac{1}{10}, -1$$

$$\boxed{k=\frac{1}{10}}$$

2) $P(x<6) = P(x=0)+P(x=1)+P(x=2)+P(x=3)+P(x=4)+P(x=5)$

$$= 0+k+2k+2k+3k+k^2$$

$$= 8k+k^2 = 8\left(\frac{1}{10}\right)+\left(\frac{1}{100}\right) = \frac{8}{10}+\frac{1}{100}$$

$$= 0.81 = \frac{81}{100}$$

3) $P(x\geq 6) = P(x=6)+P(x=7)$

$$= 2k^2+7k^2+k$$

$$= 9k^2+k = 9\left(\frac{1}{100}\right)+\frac{1}{10} = \frac{9}{100}+\frac{1}{10}$$

$$= \frac{19}{100}$$

4) $P(0<x<5) = P(x=1)+P(x=2)+P(x=3)+P(x=4)$

$$= k+2k+2k+3k = 8k$$

$$= 8\left(\frac{1}{10}\right) = \frac{8}{10}$$

5) Cumulative distribution function
$F(x) = P(X \le x)$

x	$F(x) = P(X \le x)$
0	$P(X \le 0) = F(0) = 0$
1	$F(1) = P(X \le 1) = \frac{1}{10} = k$
2	$F(2) = P(X \le 2) = 3k = \frac{3}{10}$
3	$F(3) = P(X \le 3) = 5k = \frac{5}{10}$
4	$F(4) = P(X \le 4) = 8k = \frac{8}{10}$
5	$F(5) = P(X \le 5) = 8k + k^2 = \frac{8}{10} + \frac{1}{100} = \frac{81}{100}$
6	$F(6) = P(X \le 6) = 8k + 3k^2 = \frac{8}{100} + \frac{3}{100} = \frac{83}{100}$
7	$F(7) = P(X \le 7) = 9k + 10k^2 = \frac{9}{10} + \frac{10}{100} = 1$

6) $\text{mean} = (0 \times 0) + \left(1 \times \frac{1}{10}\right) + \left(2 \times \frac{2}{10}\right) + \left(3 \times \frac{2}{10}\right) + \left(4 \times \frac{3}{10}\right) + \left(5 \times \frac{1}{100}\right) + \left(6 \times \frac{2}{100}\right) + \left(7 \times \frac{17}{100}\right)$

$= 3.66$

7) $\text{Variance} = (0 \times 0) + \left(1 \times \frac{1}{10}\right) + \left(4 \times \frac{2}{10}\right) + \left(9 \times \frac{2}{10}\right) + \left(16 \times \frac{3}{10}\right) + \left(25 \times \frac{1}{100}\right) + \left(36 \times \frac{2}{100}\right) + \left(49 \times \frac{17}{100}\right) - (3.66)^2$

$= 16.8 - 13.39 = 3.41$

8) Standard deviation $= 1.84$

9) $P(X \le k) > \frac{1}{2}$

$P(X \le 0) = P(X = 0) = 0 > \frac{1}{2}$ ×

$P(X \le 1) = P(X=0) + P(X=1) = k = \frac{1}{10} > \frac{1}{2}$ ×

$P(X \le 2) = 3k = \frac{3}{10} > \frac{1}{2}$ ×

$P(X \le 3) = 5k = \frac{5}{10} > \frac{1}{2}$ ×

$P(X \le 4) = 8k = \frac{8}{10} > \frac{1}{2} = 0.8 > 0.5$ ✓

1) From a lot of 10 items containing 3 defectives a sample of 4 items is drawn at random. Let the random variable x denote the no. of defective items in the sample. Find the probability distribution of x when sample is drawn without replacement

Let x denotes the no. of defective items

$\therefore x = \{0, 1, 2, 3\}$

$P(x=0)$ = probability of getting zero defective i.e all 4 are N.D

$= \frac{{}^{7}C_4}{{}^{10}C_4} = \frac{1}{6}$

$P(x=1)$ = probability of getting 1 D and 3 ND

$= \frac{{}^{3}C_1 \times {}^{7}C_3}{{}^{10}C_4} = \cancel{\frac{3}{10}}\ \frac{1}{2}$

$P(x=2)$ = probability of getting 2 D and 2 ND

$= \frac{{}^{3}C_2 \times {}^{7}C_2}{{}^{10}C_4} = \frac{3}{10}$

$P(x=3)$ = probability of getting 3 D and 1 ND

$= \frac{{}^{3}C_3 \times {}^{7}C_1}{{}^{10}C_4} = \frac{1}{10}$

2) A player wins if he gets 5 on a single throw of a dice. He loses if he gets 2 or 4. If he wins he gets ₹50. If he loses he gets ₹10. Otherwise he has to pay ₹15. Find the value of the game to the player. Is it favourable

The random variable x contains $x = \{-15, 10, 50\}$

$P(x=-15) = \frac{3}{6} = \frac{1}{2}$

$P(x=10) = \frac{2}{6} = \frac{1}{3}$

$P(x=150) = \frac{1}{6}$

probability distribution table

x	-15	10	50
$P(x=x)$	$\frac{1}{2}$	$\frac{1}{3}$	$\frac{1}{6}$

$$\text{mean} = \Sigma P_i x_i$$

$$= \left(-15 \times \frac{1}{2}\right) + \left(10 \times \frac{1}{3}\right) + \left(50 \times \frac{1}{6}\right)$$

$$= \frac{-15}{2} + \frac{10}{3} + \frac{50}{6}$$

$$= 4.16$$

Since mean is greater than zero, the game is favourable to the player.

3) A player tosses 3 fair coins. He wins ₹500 if 3 heads appear. ₹300 if 2 heads appear. ₹100 if 1 head appears on the other hand he loses ₹1500 if three tails occur. Find the expected gain to the pair

When 3 coins are tossed total no. of outcomes $= 2^3 = 8$ = {HHH, TTT, HTT, THT, TTH, HHT, HTH, THH}

Range of x = {-1500, 100, 300, 500}

$P(x=-1500) = \frac{1}{8}$

$P(x=100) = \frac{3}{8}$

$P(x=300) = \frac{3}{8}$

$P(x=500) = \frac{1}{8}$

probability distribution table

x	-1500	100	300	500
$P(x=x)$	$\frac{1}{8}$	$\frac{3}{8}$	$\frac{3}{8}$	$\frac{1}{8}$

$$\text{mean} = \Sigma P_i x_i$$

$$= \left(-1500 \times \frac{1}{8}\right) + \left(100 \times \frac{3}{8}\right) + \left(300 \times \frac{3}{8}\right) + \left(500 \times \frac{1}{8}\right)$$

$$= 25$$

4) A player tosses 2 fair coins he wins ₹100 if head appears, ₹200 if 2 heads appears on the other hand he loses ₹500 if no head appears. Determine the expected value E of the gain. Is the gain is favourable

When 2 fair coins are tossed, the total no. of outcomes $= 2^2 = 4$

$S = \{HH, HT, HT, TT\}$

The random variable x contains $x = \{-500, 100, 200\}$

$P(x=-500) = \frac{1}{4}$

$P(x=100) = \frac{2}{4} = \frac{1}{2}$

$P(x=200) = \frac{1}{4}$

probability distribution table

x	-500	100	200
$P(x=x)$	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{1}{4}$

mean $= \sum p_i x_i$

$= \left(-500 \times \frac{1}{4}\right) + \left(100 \times \frac{1}{2}\right) + \left(200 \times \frac{1}{4}\right)$

$= -25$

$\therefore$ mean is negative i.e less than zero

$\therefore$ The gain is not favourable

5) A fair coin is tossed untill a head or 5 tails occur. Find the expected number E of tosses of the coin.

Given or Let x denotes the number of coins. Coin is tossed until a head or five tails occur. $\therefore$ it is clear that if on $x=1$, head comes then the process will be stopped and if tail comes then coin will be tossed again second time.

Clearly it will be repeated again and again till 5 tails come maximum

Then the value of x will be 1, 2, 3, 4, 5

$S = \{H, TH, TTH, TTTH$ or $TTTTH, TTTTT\}$

probability that head comes in first throw

$P(x=1) = \frac{1}{2}$

$P(x=2) =$

$P(x=3) =$

$P(x=4) =$

$P(x=5)$

probabi

x
P(

mean =

1) Find
lity d

probab

x

P(x=x

mean

Varia

$P(x=2) = P(TH) = P(T)P(H) = \frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$

$P(x=3) = P(TTH) = P(T)P(T)P(H) = \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \frac{1}{8}$

$P(x=4) = P(TTTH) = P(T)P(T)P(T)P(H)$

$= \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \frac{1}{16}$

$P(x=5) = P(TTTTH \cup TTTTT)$

$= P(T)P(T)P(T)P(T)P(H) + P(T)P(T)P(T)P(T)P(T)$

$= \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} + \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \frac{1}{32} + \frac{1}{32}$

$= \frac{1}{16}$

probability distribution table

x	1	2	3	4	5
$P(x)$	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{8}$	$\frac{1}{16}$	$\frac{1}{16}$

mean $= \sum P_i x_i$

$= \left(\frac{1}{2} \times 1\right) + \left(2 \times \frac{1}{4}\right) + \left(3 \times \frac{1}{8}\right) + \left(4 \times \frac{1}{16}\right) + \left(5 \times \frac{1}{16}\right)$

$= \frac{31}{16} = 1.93$

1) Find the mean & variance of uniform probability distribution given by $f(x) = \frac{1}{n}$ for $x = 1, 2 --- n$

probability distribution table

x	1	2	3	4	---	n
$P(x=x)$	$\frac{1}{n}$	$\frac{1}{n}$	$\frac{1}{n}$	$\frac{1}{n}$	----	$\frac{1}{n}$

mean $(\mu) = \left(\frac{1}{n} \times 1\right) + \left(\frac{1}{n} \times 2\right) + \left(\frac{1}{n} \times 3\right) + --- + \left(\frac{1}{n} \times n\right)$

$= \frac{1}{n}(1 + 2 + 3 + --- + n)$

$= \frac{1}{\cancel{n}}\left(\frac{\cancel{n}(n+1)}{2}\right) = \frac{n+1}{2}$

Variance $= \sum P_i x_i^2 - \mu^2$

$= \left(\frac{1}{n} \times 1\right) + \left(\frac{1}{n} \times 4\right) + \left(\frac{1}{n} \times 9\right) + --- + \left(\frac{1}{n} \times n^2\right)$

$- \left(\frac{n+1}{2}\right)^2$

$$= \frac{1}{n}\left[1^2+2^2+\dots+n^2\right) - \frac{(n^2+2n+1)}{4}$$

$$= \frac{1}{\cancel{n}}\, \frac{\cancel{n}(n+1)(2n+1)}{6} - \frac{(n+1)^2}{4}$$

$$= \frac{(n+1)}{2}\left[\frac{2n+1}{3} - \frac{(n+1)}{2}\right]$$

$$= \frac{(n+1)}{2}\left[\frac{4n+2-3n-3}{6}\right]$$

$$= \frac{(n+1)(n-1)}{12} = \frac{n^2-1}{12}$$

Continuous Random Variable

A random variable x is said to have Continuous random variable if ~~he~~ it takes all continuous values in a range

Note :

1) Sum of all probabilities = 1

i.e $\int_{-\infty}^{\infty} f(x)\,dx = 1$

2) mean $(\mu) = \int_{-\infty}^{\infty} x f(x)\,dx$

3) Variance $= \int_{-\infty}^{\infty} x^2 f(x)\,dx - \mu^2$

4) Standard deviation $= \sqrt{\text{variance}}$

5) median : median is the point which divides the entire distribution into 2 equal parts

i.e if x is defined for $a \to b$ & m is the median then $\int_a^m f(x)\,dx = \int_m^b f(x)\,dx = \frac{1}{2}$

6) Mode : mode is the value of x for which f(x) is maximum.

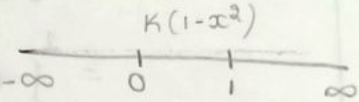
Mode is given by $f'(x)=0$ & $f''(x)<0$

1) If the probability density of a random variable is given by $f(x) = \begin{cases} k(1-x^2) & \text{for } 0<x<1 \\ 0 & \text{otherwise} \end{cases}$ Find the value of k & probabilities that a random variable having this probability density will take on a value b/w 1) 0.1 & 0.2 2) greater than 0.5

$$f(x) = \begin{cases} k(1-x^2) & \text{for } 0<x<1 \\ 0 & \text{otherwise} \end{cases}$$

We know that

$$\int_{-\infty}^{\infty} f(x)\,dx = 1$$



$$\int_{-\infty}^{0} f(x)\,dx + \int_{0}^{1} f(x)\,dx + \int_{1}^{\infty} f(x)\,dx = 1$$

$$\int_{-\infty}^{0} 0\,\cancel{dx} + \int_{0}^{1} k(1-x^2)\,dx + \int_{1}^{\infty} 0\,\cancel{dx} = 1$$

$$k\int_{0}^{1} (1-x^2)\,dx = 1$$

$$k\left[x - \frac{x^3}{3}\right]_0^1 = 1$$

$$k\left[\left(1-\frac{1}{3}\right) - (0-0)\right] = 1$$

$$k\left(\frac{2}{3}\right) = 1$$

$$k = \frac{3}{2}$$

1) P (b/w 0.1 and 0.2)

$$= P(0.1 \le x \le 0.2)$$

$$= \int_{0.1}^{0.2} f(x)\,dx$$

$$= \int_{0.1}^{0.2} k(1-x^2)\,dx$$

$$= k\int_{0.1}^{0.2} (1-x^2)\,dx$$

$$= \frac{3}{2}\left[x - \frac{x^3}{3}\right]_{0.1}^{0.2}$$

$$= \frac{3}{2}\left[(0.2) - \frac{(0.2)^3}{3} - \left(0.1 - \frac{(0.1)^3}{3}\right)\right]$$

$$= \frac{3}{2}\left[0.19 - 0.09\right] = \frac{3}{2}(0.1) = 0.15$$

2) P (greater than 0.5)

$$P(x>0.5) = \int_{0.5}^{\infty} f(x)\,dx$$

$$= \int_{0.5}^{1} f(x)\,dx + \int_{1}^{\infty} f(x)\,dx$$

$$= \int_{0.5}^{1} k(1-x^2)\,dx + \int_{1}^{\infty} 0\,dx$$

$$= k\int_{0.5}^{1}(1-x^2)\,dx$$

$$= \frac{3}{2}\left[x - \frac{x^3}{3}\right]_{0.5}^{1}$$

$$= \frac{3}{2}\left[\left(1-\frac{1}{3}\right) - \left(0.5 - \frac{(0.5)^3}{3}\right)\right]$$

$$= \frac{3}{2}(0.66 - 0.45) = \frac{3}{2}(0.21) = 0.315$$

2) If probability density function $f(x) = \begin{cases} kx^3 & 0 \le x \le 3 \\ 0 & \text{elsewhere} \end{cases}$. Find the value of k and find the probability b/w $x = \frac{1}{2}$ and $x = \frac{3}{2}$

$$f(x) = \begin{cases} kx^3 & 0 \le x \le 3 \\ 0 & \text{elsewhere} \end{cases}$$

we know that

$$\int_{-\infty}^{\infty} f(x)\,dx = 1$$

$$\int_{-\infty}^{0} f(x)\,dx + \int_{0}^{3} f(x)\,dx + \int_{3}^{\infty} f(x)\,dx = 1$$

$$\int_{-\infty}^{0} \cancel{0\,dx} + \int_{0}^{3} kx^3 + \int_{3}^{\infty} 0\,\cancel{dx} = 1$$

$$k\int_{0}^{3} x^3\,dx = 1$$

$$k\left[\frac{x^4}{4}\right]_{0}^{3} =$$

$$\boxed{k = \frac{4}{81}}$$

1) P (b/w $\frac{1}{2}$

$= P(0.5 \le x \le$

3) Find the

Find P (

$f(x)$

we know

$$\int_{-\infty}^{0} f($$

$$\int_{-\infty}^{0}$$

$$k \int_0^3 x^3\, dx = 1$$

$$k\left[\frac{x^4}{4}\right]_0^3 = 1 \Rightarrow \frac{k}{4}\left[3^4 - 0\right] = 1 \Rightarrow \frac{k}{4}(81) = 1$$

$$\boxed{k = \frac{4}{81}}$$

1) $P\left(\text{b/w } \frac{1}{2} \text{ and } \frac{3}{2}\right) = P(\text{b/w } 0.5 \text{ and } 1.5)$

$$= P(0.5 \leq x \leq 1.5) = \int_{0.5}^{1.5} f(x)\, dx$$

$$= \int_{0.5}^{1.5} kx^3\, dx$$

$$= k \int_{0.5}^{1.5} x^3\, dx$$

$$= \frac{4}{81}\left[\frac{x^4}{4}\right]_{0.5}^{1.5}$$

$$= \frac{1}{81}\left[(1.5)^4 - (0.5)^4\right]$$

$$= \frac{1}{81}(5.0625 - 0.0625)$$

$$= \frac{1}{81}\left[\cancel{4.062}5\right] = \frac{5}{81} = 0.61$$

3) Find the Constant k such that $\begin{cases} kx^2 & 0 < x < 1 \\ 0 & \text{otherwise} \end{cases}$

Find $P(0.5 < x \leq 2)$

$$f(x) = \begin{cases} kx^2 & 0 < x < 1 \\ 0 & \text{otherwise} \end{cases}$$

we know that

$$\int_{-\infty}^{\infty} f(x)\, dx = 1$$

$$\int_{-\infty}^{0} f(x)\, dx + \int_{0}^{1} f(x)\, dx + \int_{1}^{\infty} f(x)\, dx = 1$$

$$\int_{-\infty}^{0} \cancel{0dx} + \int_{0}^{1} kx^2\, dx + \int_{1}^{\infty} \cancel{0dx} = 1$$

$k \int_0^1 x^2 dx = 1$

$k \left[\frac{x^3}{3} \right]_0^1 = 1$

$\frac{k}{3} [1 - 0] = 1$

$\boxed{k = 3}$

$P(0.5 < x \leq 2) = \int_{0.5}^{2} f(x) dx$

$= \int_{0.5}^{1} f(x) dx + \int_{1}^{2} f(x) dx$

$= \int_{0.5}^{1} kx^2 dx + \int_{1}^{2} \cancel{0} dx$

$= k \int_{0.5}^{1} x^2 dx$

$= k \left[\frac{x^3}{3} \right]_{0.5}^{1}$

$= \frac{k}{3} \left[1^3 - (0.5)^3 \right]$

$= \frac{3}{3} [1 - 0.125]$

$= 0.875$

4) The probability density function $y = \begin{cases} k(3x^2 - 1) & -1 \leq x \leq 2 \\ 0 & \text{elsewhere} \end{cases}$

Find 1) k 2) $-1 \leq x \leq 0$

$y = f(x) = \begin{cases} k(3x^2 - 1) & -1 \leq x \leq 2 \\ 0 & \text{elsewhere} \end{cases}$

we know that

$\int_{-\infty}^{\infty} f(x) dx = 1$

$\int_{-\infty}^{-1} f(x) dx + \int_{-1}^{2} f(x) dx + \int_{2}^{\infty} f(x) dx = 1$

$\int_{-\infty}^{1} \cancel{0 dx} + \int_{-1}^{2} k(3x^2 - 1) dx + \int_{2}^{\infty} \cancel{0 dx} = 1$

$= k \int_{-1}^{2} (3x^2 - 1$

$= k \left[\frac{3x^3}{3} - \right.$

$= k [x^3 -$

$k [6] =$

$\boxed{k = \frac{1}{6}}$

$P(-1 \leq x \leq 0$

5) A Contin
function

Determin

$f(x)$

We kno

$\int_{-\infty}^{\infty} f($

$\int_{-\infty}^{0} f(x$

$\int_{-\infty}^{0} \cancel{0 dx}$

$$= k\int_{-1}^{2} (3x^2-1)\,dx = 1$$

$$= k\left[\frac{3x^3}{3} - x\right]_{-1}^{2} = 1$$

$$= k\left[x^3 - x\right]_{-1}^{2} = 1 \Rightarrow k\left[2^3 - 2 - ((-1)^3 - (-1))\right] = 1$$

$$k[6] = 1$$

$$\boxed{k = \frac{1}{6}}$$

$$P(-1 \le x \le 0) = \int_{-1}^{0} f(x)\,dx$$

$$= \int_{-1}^{0} k(3x^2-1)\,dx$$

$$= k\int_{-1}^{0}(3x^2-1)\,dx$$

$$= k\left[3\frac{x^3}{3} - x\right]_{-1}^{0} = k\left[x^3 - x\right]_{-1}^{0}$$

$$= k\left[-(-1+1)\right]$$

$$= 0$$

-1 ≤ x ≤ 2
elsewhere

5) A continuous random variable has continuous function $f(x) = \begin{cases} kxe^{-\lambda x} & \text{for } x \ge 0,\ \lambda > 0 \\ 0 & \text{elsewhere} \end{cases}$

Determine 1) k 2) mean 3) Variance

$$f(x) = \begin{cases} kxe^{-\lambda x} & \text{for } x \ge 0,\ \lambda > 0 \\ 0 & \text{elsewhere} \end{cases}$$

We know that

$$\int_{-\infty}^{\infty} f(x)\,dx = 1$$

$$\int_{-\infty}^{0} f(x)\,dx + \int_{0}^{\infty} f(x)\,dx = 1$$

$$\cancel{\int_{-\infty}^{0} 0\,dx} + \int_{0}^{\infty} kxe^{-\lambda x}\,dx = 1$$

$$k \int_0^\infty x e^{-\lambda x} dx = 1$$

$$k \left[\frac{x e^{-\lambda x}}{-\lambda} - \frac{e^{-\lambda x}}{\lambda^2} \right]_0^\infty = 1$$

$$k \left[(0-0) - \left(0 - \frac{1}{\lambda^2}\right) \right] = 1$$

$$\frac{k}{\lambda^2} = 1$$

$$\boxed{k = \lambda^2}$$

2) $\text{mean}(\mu) = \int_{-\infty}^{\infty} f(x) \cdot x \, dx$

$$\mu = \int_{-\infty}^{0} f(x) \cdot x \, dx + \int_0^\infty f(x) \cdot x \, dx$$

$$= \int_{-\infty}^{\infty} \cancel{0 \cdot x} \, dx + \int_0^\infty k x e^{-\lambda x} \cdot x \, dx$$

$$= k \int_0^\infty x^2 e^{-\lambda x} dx$$

$$= k \left[\frac{x^2 e^{-\lambda x}}{-\lambda} - \frac{2x e^{-\lambda x}}{\lambda^2} + \frac{2 e^{-\lambda x}}{-\lambda^3} \right]_0^\infty$$

$$= \lambda^2 \left[(0-0+0) - \left(0 - 0 + \frac{2(1)}{-\lambda^3} \right) \right]$$

$$\mu = \lambda^2 \left(\frac{2}{\lambda^3} \right) = \frac{2}{\lambda}$$

3) Variance

$$\sigma^2 = \int_{-\infty}^{\infty} f(x) x^2 \, dx - \mu^2$$

$$= \int_{-\infty}^{0} f(x) x^2 dx + \int_0^\infty f(x) x^2 dx - \mu^2$$

$$= \int_{-\infty}^{0} \cancel{0 \cdot x^2} dx + \int_0^\infty k x e^{-\lambda x} (x^2 dx) - \left(\frac{2}{\lambda} \right)^2$$

$$= k \int_0^\infty x^3 e$$

$$= \lambda^2 \left[x \right.$$

$$= \lambda^2 \left[\right.$$

$$= \lambda^2 \left(\right.$$

$$\sigma^2 = \frac{6}{\lambda^2}$$

6) $f(x)$

$f(x)$

we kn

$$\int_{-\infty}^{0} f$$

$$\int_{-\infty}^{0} \cancel{0}$$

$$= k\int_0^\infty x^3 e^{-\lambda x}\,dx - \frac{4}{\lambda^2}$$

$$= \lambda^2\left[\frac{x^3e^{-\lambda x}}{-\lambda} - \frac{3x^2e^{-\lambda x}}{\lambda^2} + \frac{6xe^{-\lambda x}}{\lambda^4}\right]_0^\infty - \frac{4}{\lambda^2}$$

$$= \lambda^2\left[0 - \left(0 - 0 + 0 - \frac{6}{\lambda^4}\right)\right] - \frac{4}{\lambda^2}$$

$$= \lambda^2\left(\frac{6}{\lambda^4}\right) - \frac{4}{\lambda^2}$$

$$\sigma^2 = \frac{6}{\lambda^2} - \frac{4}{\lambda^2} = \frac{2}{\lambda^2}$$

6) $$f(x) \begin{cases} cx(2-x) & \text{if } 0 \le x \le 2 \\ 0 & \text{otherwise} \end{cases}$$

$$f(x) = \begin{cases} c(2x - x^2) & \text{if } 0 \le x \le 2 \\ 0 & \text{otherwise} \end{cases}$$

we know that

$$\int_{-\infty}^{0} f(x)\,dx + \int_0^2 f(x)\,dx + \int_2^\infty f(x)\,dx = 1$$

$$\cancel{\int_{-\infty}^{0} 0\,dx} + \int_0^2 c(2x - x^2)\,dx + \cancel{\int_2^\infty 0\,dx} = 1$$

$$c\int_0^2 (2x - x^2)\,dx = 1$$

$$c\left[\frac{2x^2}{2} - \frac{x^3}{3}\right]_0^2 = 1$$

$$c\left[x^2 - \frac{x^3}{3}\right] = 1$$

$$c\left[2^2 - \frac{2^3}{3}\right] = 1$$

$$c\left[4 - \frac{8}{3}\right] = 1$$

$$c\left[\frac{4}{3}\right] = 1$$

$$\boxed{c = \frac{3}{4}}$$

7) $f(x) = \begin{cases} kx^2e^{-x} & \text{when } x \geq 0 \\ 0 & \text{elsewhere} \end{cases}$

Find k, mean, variance, SD

$f(x) = \begin{cases} kx^2e^{-x} & \text{when } x \geq 0 \\ 0 & \text{elsewhere} \end{cases}$

$\boxed{\begin{array}{c} e^{-\infty} = 0 \\ e^{\infty} = \infty \\ e^{-0} = 1 \end{array}}$

we know that

$$\int_{-\infty}^{0} f(x)dx + \int_{0}^{\infty} f(x)dx = 1$$

$$\cancel{\int_{-\infty}^{0} 0 dx} + \int_{0}^{\infty} kx^2e^{-x}dx = 1$$

$$k\int_{0}^{\infty} x^2e^{-x}dx = 1$$

x^2	e^{-x}
$2x$	$+\ \frac{e^{-x}}{-1}$
2	$-\ e^{-x}$
0	$+\ \frac{e^{-x}}{-1}$

$$k\left[-x^2e^{-x} - 2xe^{-x} - 2e^{-x}\right]_0^{\infty} = 1$$

$$k\left[-(-2)\right] = 1$$

$$\boxed{k = \frac{1}{2}}$$

$$\text{mean} = u = \int_{-\infty}^{\infty} f(x).x\,dx$$

$$u = \int_{-\infty}^{0} f(x).x\,dx + \int_{0}^{\infty} f(x).x\,dx$$

$$= \int_{-\infty}^{0} 0.x\,dx + \int_{0}^{\infty} kx^2e^{-x}x\,dx$$

x^3	e^{-x}
$3x^2$	$+\ \frac{e^{-x}}{-1}$
$6x$	$-\ e^{-x}$
6	$+\ \frac{e^{-x}}{-1}$
0	$-\ e^{-x}$

$$= k\int_{0}^{\infty} x^3e^{-x}dx$$

$$= k\left[-x^3e^{-x} - 3x^2e^{-x} + -6xe^{-x} - 6e^{-x}\right]_0^{\infty}$$

$$= k[6] = \frac{1}{2}(6) = 3$$

$$\text{variance} = \int_{-\infty}^{\infty} f(x)x^2dx - u^2$$

$$= \int_{-\infty}^{0} 0\,\cancel{x^2}dx + \int_{0}^{\infty} ke^{-x}x^2x^2dx - u^2$$

$$= k\int_{0}^{\infty} x^4e^{-x} \ldots$$

$$= k\left[-x^4e^{-x} - 4x^3 \ldots\right. - u^2$$

$$= k[24] - \ldots$$

$$= \frac{1}{2}(24) - \ldots$$

Standard de...

1) If a rando...
function f(...

Find the pr...
1) b/w P(1...

1) P(1 < x ...

$$= \int_{1}^{3} f(x)d\ldots$$

$$= \int_{1}^{3} 2e^{-2x}\ldots$$

$$= 2\left[\frac{e^{-\ldots}}{-2}\ldots\right.$$

$$= -1[e^{-}\ldots$$

2) P(x > ...

$$= \int_{0.5}^{\infty} f(x\ldots$$

$$= \int_{0.5}^{\infty} 2e^{-\ldots}$$

$$= 2[\ldots$$

$$= 2[\ldots$$

$$= 2\ldots$$

$$= 1\ldots$$

$$= k\int_0^\infty x^4 e^{-x}\,dx - u^2$$

$$= k\left[-x^4e^{-x} - 4x^3e^{-x} - 12x^2e^{-x} - 24xe^{-x} - 24e^{-x}\right]_0^\infty - u^2$$

$x^4 \quad e^{-x}$

$4x^3 \quad \frac{e^{-x}}{-1}$

$12x^2 \quad e^{-x}$

$24x \quad \frac{e^{-x}}{-1}$

$24 \quad e^{-x}$

$0 \quad \frac{e^{-x}}{-1}$

$$= k[24] - u^2$$

$$= \frac{1}{2}(24) - 9 = 12 - 9 = 3$$

Standard deviation $= \sqrt{3} = 1.732$

1) If a random variable has probability density function $f(x) = \begin{cases} 2e^{-2x} & \text{for } x > 0 \\ 0 & \text{otherwise} \end{cases}$

Find the probabilities that it will take on a value

1) b/w $P(1 \le x \le 3)$ 2) greater than 0.5

1) $P(1 < x < 3)$

$$= \int_1^3 f(x)\,dx$$

$$= \int_1^3 2e^{-2x}\,dx$$

$$= 2\left[\frac{e^{-2x}}{-2}\right]_1^3$$

$$= -1\left[e^{-6} - e^{-2}\right]$$

2) $P(x > 0.5)$

$$= \int_{0.5}^\infty f(x)\,dx$$

$$= \int_{0.5}^\infty 2e^{-2x}\,dx$$

$$= 2\left[\frac{e^{-2x}}{-2}\right]_{0.5}^\infty$$

$$= 2\left[0 - \frac{e^{-1}}{-2}\right]$$

$$= \frac{2e^{-1}}{-2}$$

$$= \frac{1}{e}$$

2) The probability density function of a random variable is defined by
$$f(x) = \begin{cases} \frac{1}{16}(3+x)^2 & -3 \le x < -1 \\ \frac{1}{16}(6-2x^2) & -1 \le x < 1 \\ \frac{1}{16}(3-x)^2 & 1 \le x \le 3 \\ 0 & \text{elsewhere} \end{cases}$$

Verify $f(x)$ is a density function and also find mean

$$\int_{-\infty}^{-3} f(x)dx + \int_{-3}^{-1} f(x)dx + \int_{-1}^{1} f(x)dx + \int_{1}^{3} f(x)dx + \int_{3}^{\infty} f(x)dx$$

$$= \int_{-\infty}^{-3} \cancel{0\,dx} + \int_{-3}^{-1} \frac{1}{16}(3+x)^2 + \int_{-1}^{1} \frac{1}{16}(6-2x^2) + \int_{1}^{3} \frac{1}{16}(3-x)^2 + \int_{3}^{\infty} \cancel{0\,dx}$$

$$= \frac{1}{16}\left[\frac{(3+x)^3}{3}\right]_{-3}^{-1} + \frac{1}{16}\left[6x - \frac{2x^3}{3}\right]_{-1}^{1} + \frac{1}{16}\left[\frac{(3-x)^3}{-3}\right]_{1}^{3}$$

$$= \frac{1}{48}[\overset{8}{\cancel{64}}] + \frac{1}{16}\left[6 - \frac{2}{3} - \left(-6 + \frac{2}{3}\right)\right] + \frac{1}{-48}[0-8]$$

$$= \frac{\cancel{64}8}{48} + \frac{1}{16}\left(12 - \frac{4}{3}\right) + \frac{8}{48}$$

$$= \frac{\cancel{64}8}{48} + \frac{1}{16}\left(\frac{32}{3}\right) + \frac{8}{48}$$

$$= 1$$

$$\therefore \int_{-\infty}^{\infty} f(x)\,dx = 1$$

density function $= 1$

$$\text{mean} = \int_{-\infty}^{\infty} f(x) \cdot x\,dx$$

$$= \int_{-\infty}^{-3} x f(x)dx + \int_{-3}^{-1} f(x)x\,dx + \int_{-1}^{1} f(x)x\,dx + \int_{1}^{3} f(x)x\,dx + \int_{3}^{\infty} f(x)x\,dx$$

$$= \int_{-\infty}^{-3} \cancel{0 \cdot x\,dx} + \int_{-3}^{-1} \ldots$$

$$\int_{1}^{3} \frac{1}{16}(3-x) \ldots$$

$$= \frac{1}{16}\int_{-3}^{-1}[(9+x^2 \ldots$$

$$\frac{1}{16}\int_{1}^{3}(9+x \ldots$$

$$= \frac{1}{16}\int_{-3}^{-1}(9x+ \ldots$$

$$= \frac{1}{16}\left\{\left[\frac{9x^2}{2} + \ldots\right.\right.$$

$$= \frac{1}{16}\left\{\left[\left(\frac{9}{2}+\frac{1}{4} \ldots\right.\right.\right.$$

$$= \frac{1}{16}[-4+4]$$

3) A R.V X ...
with a pr...

1) Find P(x ...
2) Find a n...
P (x ≤ 1/2 ...

2) P(...

$$= \int_{-\infty}^{-3} \cancel{0 \cdot x\,dx} + \int_{-3}^{-1} \frac{1}{16}(3+x)^2 x\,dx + \int_{-1}^{1} \frac{1}{16}(6-2x^2)x\,dx +$$

$$\int_{1}^{3} \frac{1}{16}(3-x)^2 x\,dx + \int_{3}^{\infty} \cancel{0 \cdot x\,dx}$$

$$= \frac{1}{16}\int_{-3}^{-1}\left[(9+x^2+6x)x\right]dx + \frac{1}{16}\int_{-1}^{1}(6x-2x^3)\,dx +$$

$$\frac{1}{16}\int_{1}^{3}(9+x^2-6x)x\,dx$$

$$= \frac{1}{16}\int_{-3}^{-1}(9x+x^3+6x^2)\,dx + \frac{1}{16}\int_{-1}^{1}(6x-2x^3)\,dx + \frac{1}{16}\int_{1}^{3}(9x+x^3-6x^2)\,dx$$

$$= \frac{1}{16}\left\{\left[\frac{9x^2}{2}+\frac{x^4}{4}+\frac{6x^3}{3}\right]_{-3}^{-1} + \left[\frac{6x^2}{2}-\frac{2x^4}{4}\right]_{-1}^{1} + \left[\frac{9x^2}{2}+\frac{x^4}{4}-\frac{6x^3}{3}\right]_{1}^{3}\right\}$$

$$= \frac{1}{16}\left\{\left[\left(\frac{9}{2}+\frac{1}{4}-\frac{6}{3}\right)-\left(\frac{81}{2}+\frac{81}{4}-\frac{162}{3}\right)\right]+\left[\left(3-\frac{1}{2}\right)-\left(3-\frac{1}{2}\right)\right]+\left[\left(\frac{81}{2}+\frac{81}{4}-\frac{162}{3}\right)-\left(\frac{9}{2}+\frac{1}{4}-\frac{6}{3}\right)\right]\right.$$

$$= \frac{1}{16}\left[-4+4\right] = 0$$

3) A R.V x gives measurement of x b/w 0 & 1 with a probability function $f(x) = \begin{cases} 12x^3 - 21x^2 + 10x, & 0 \le x \le 1 \\ 0 & \text{else where} \end{cases}$

1) Find $P(x \le \frac{1}{2})$ 2) $P(x > \frac{1}{2})$

2) Find a number 'k' such that $P(x \le k) = \frac{1}{2}$

$$P\left(x \le \frac{1}{2}\right) = \int_{-\infty}^{1/2} f(x)\,dx$$

$$= \int_{-\infty}^{0} f(x)\,dx + \int_{0}^{1/2} f(x)\,dx$$

$$= \int_{-\infty}^{0} \cancel{0}\,dx + \int_{0}^{1/2}(12x^3 - 21x^2 + 10x)\,dx$$

$$= 12\left[\frac{x^4}{4}\right]_0^{1/2} - 21\left[\frac{x^3}{3}\right]_0^{1/2} + 10\left[\frac{x^2}{2}\right]_0^{1/2}$$

$$= 3\left[\frac{1}{16}\right] - 7\left[\frac{1}{8}\right] + 5\left[\frac{1}{4}\right]$$

$$= \frac{9}{16}$$

2) $P\left(x > \frac{1}{2}\right) = 1 - P\left(x \le \frac{1}{2}\right)$

$$= 1 - \frac{9}{16}$$

$$= \frac{7}{16}$$

$P(x \leq k) = \frac{1}{2}$

$= \int_{-\infty}^{k} f(x)\,dx = \frac{1}{2}$ ($\because k$ is +ve)

$= \int_{-\infty}^{0} f(x)\,dx + \int_{0}^{k} f(x)\,dx = \frac{1}{2}$

$= \int_{-\infty}^{0} 0 + \int_{0}^{k} (12x^3 - 21x^2 + 10x)\,dx = \frac{1}{2}$

$= 12\left[\frac{x^4}{4}\right]_0^k - 21\left[\frac{x^3}{3}\right]_0^k + 10\left[\frac{x^2}{2}\right]_0^k = \frac{1}{2}$

$= 3[k^4] - 7(k^3) + 5(k^2) = \frac{1}{2}$

$2(3k^4 - 7k^3 + 5k^2) = 1$

$6k^4 - 14k^3 + 10k^2 - 1 = 0$

$\boxed{k = 0.452}$

4) A Continuous random variable x has the distribution function
$$F(x) = \begin{cases} 0 & \text{if } x \leq 1 \\ k(x-1)^4 & \text{if } 1 < x \leq 3 \\ 1 & \text{if } x > 3 \end{cases}$$

Determine 1) $f(x)$ 2) k 3) Mean

distribution function

$$F(x) = \begin{cases} 0 & \text{if } x \leq 1 \\ k(x-1)^4 & \text{if } 1 < x \leq 3 \\ 1 & \text{if } x > 3 \end{cases}$$

1) $f(x) = \frac{d}{dx} F(x) = \begin{cases} 0 & \text{if } x \leq 1 \\ k4(x-1)^3 & \text{if } 1 < x \leq 3 \\ 0 & \text{if } x > 3 \end{cases}$

2) $\int_{-\infty}^{\infty} f(x)\,dx = 1$

$\int_{-\infty}^{1} f(x)\,dx + \int_{1}^{3} f(x)\,dx + \int_{3}^{\infty} f(x)\,dx = 1$

$\int_{-\infty}^{1} 0\,dx + \int_{1}^{3} 4k(x-1)^3\,dx + \int_{0}^{\infty} 0\,dx = 1$

$$4k\left(\frac{(x-1)^4}{4}\right)_1^3 = 1$$

$$k(16-0) = 1$$

$$16k = 1$$

$$\boxed{k = \frac{1}{16}}$$

3) Mean $\mu = \int_{-\infty}^{\infty} f(x).x\,dx$

$$= \int_{-\infty}^{1} f(x)x\,dx + \int_{1}^{3} f(x)x\,dx + \int_{3}^{\infty} f(x)x\,dx$$

$$= \cancel{\int_{-\infty}^{1} 0.x\,dx} + \int_{1}^{3} 4k(x-1)^3 x\,dx + \cancel{\int_{3}^{\infty} 0.x\,dx}$$

$$= 4k\int_{1}^{3}(x^3-3x^2+3x-1)x\,dx$$

$$= 4\times\frac{1}{16}\int_{1}^{3}(x^4-3x^3+3x^2-x)\,dx$$

$$= \frac{1}{4}\left[\frac{x^5}{5}-\frac{3x^4}{4}+\frac{3x^3}{3}-\frac{x^2}{2}\right]_1^3$$

$$= \frac{1}{4}\left[\frac{1}{5}(243-1)-\frac{3}{4}(81-1)-\frac{1}{2}(9-1)\right]$$

$$= \frac{1}{4}\left[48.4-60-4\right] = \frac{1}{4}(-15.6) = -3.9$$

5) The trouble shooting capacity of IC Chip in a circuit is a R.V is x whose distribution function is given by $F(x) = \begin{cases} 0 & x \le 3 \\ 1-\frac{9}{x^2} & x > 3 \end{cases}$ Find the

where x denotes the no. of years. probability that the IC Chip will work properly
1) less than 8 years 2) beyond 8 years
3) b/w 5 to 7 years 4) anywhere from 2 to 5

we know that

$$f(x) = \int_{-\infty}^{3} 0\,dx + \int_{3}^{\infty} 1-\frac{9}{x^2}\,dx$$

1) $P(x<8) = \int_{-\infty}^{3} f(x)\,dx + \int_{3}^{8} f(x)\,dx$

$= \cancel{\int_{-\infty}^{3} 0\,dx} + \int_{3}^{8} 1 - \frac{9}{x^2}\,dx$

$= \left[x + \frac{9}{x}\right]_3^8$

$= \left[8 + \frac{9}{8} - \left(3 + \frac{9}{3}\right)\right]$

$= 3.125$

$f(x) = \begin{cases} 0 & x \leq 3 \\ \frac{18}{x^3} & x > 3 \end{cases}$

1) $P(x<8) = \int_{-\infty}^{3} f(x)\,dx + \int_{3}^{8} f(x)\,dx$

$= \cancel{\int_{-\infty}^{3} 0\,dx} + \int_{3}^{8} \frac{18}{x^3}\,dx$

$= 18\left(\frac{1}{-2x^2}\right)_3^8 = -9\left[\frac{1}{64} - \frac{1}{9}\right] = \frac{55}{64}$

2) $P(x>8) = \int_{8}^{\infty} f(x)\,dx$

$= \int_{8}^{\infty} \frac{18}{x^3}\,dx = 18\left(\frac{1}{-2x^2}\right)_8^{\infty}$

$= -9\left[\frac{1}{\infty} - \frac{1}{64}\right]$

$= \frac{9}{64}$

3) $P(5<x<7) = \int_{-\infty}^{3} f(x)\,dx + \int_{3}^{\infty} f(x)\,dx$

$= \int_{5}^{7} \frac{18}{x^3}\,dx$

$= 18\left(\frac{1}{-2x^2}\right)_5^7 = -9\left[\frac{1}{49} - \frac{1}{25}\right]$

$= 0.176$

4) $P(2<x<5) = \int_{-\infty}^{3} f(x)\,dx + \int_{3}^{5} f(x)\,dx$

$= \cancel{\int_{-\infty}^{3} 0\,dx} + \int_{3}^{5} \frac{18}{x^3}\,dx = 18\left(\frac{1}{-2x^2}\right)_3^5$

$= -9\left[\frac{1}{25} - \frac{1}{9}\right] = \frac{16}{25}$

The Cumulat
Continuous

Find 1) f(
distribution

$F(x) = \{$

1) $f(x) =$

2) mean $= \int_{-\infty}^{\infty}$

$u = \int_{-\infty}^{0} x f$

Variance (

$u = \int_{-\infty}^{0} x$

$u = 2$

$= 2$

$= 2$

Variance

The cumulative distribution function for a continuous random variable $F(x) \begin{cases} 1-e^{-2x} & x \geq 0 \\ 0 & x < 0 \end{cases}$

Find 1) $f(x)$ 2) mean 3) Variance 4) S.D

distribution function

$$F(x) = \begin{cases} 1-e^{-2x} & x \geq 0 \\ 0 & x < 0 \end{cases}$$

$$1)\ f(x) = \frac{d}{dx} F(x) = \begin{cases} 0 - \frac{e^{-2x}}{\cancel{-2}} \quad 2e^{-2x} & x \geq 0 \\ 0 & x < 0 \end{cases}$$

$$2)\ \text{mean} = \int_{-\infty}^{\infty} f(x) \cdot x\, dx$$

$$u = \int_{-\infty}^{0} x f(x)\, dx + \int_{0}^{\infty} x f(x)\, dx$$

$$\text{Variance } (\sigma^2) = \sum_{x=0}^{n} x^2 p(x) - u^2$$

$$= \sum [x^2 - x + x]\, p(x) - (np)^2$$

$$= \sum (x^2 - x)\, p(x) + \sum x\, p(x) - (np)^2$$

$$= \sum_{x=0}^{n} x(x-1)\, n_{C_x} p^x q^{n-x} + np - n^2 p^2$$

$$u = \int_{-\infty}^{0} \cancel{x \cdot 0\, dx} + \int_{0}^{\infty} 2e^{-2x} x\, dx$$

$$\begin{array}{ll} x & e^{-2x} \\ 1 & +\frac{e^{-2x}}{-2} \\ 0 & -\frac{e^{-2x}}{4} \end{array}$$

$$u = 2 \int_{0}^{\infty} e^{-2x} \cdot x\, dx$$

$$= 2 \left[-\frac{x e^{-2x}}{2} - \frac{e^{-2x}}{4} \right]_0^{\infty}$$

$$= 2 \left[-\left(-\frac{1}{4}\right) \right] = \frac{2}{4} = \frac{1}{2}$$

$$\text{Variance} = \int_{-\infty}^{\infty} f(x)\, x^2\, dx - u^2$$

$$= \int_{-\infty}^{0} f(x)\, x^2\, dx + \int_{0}^{\infty} f(x)\, x^2\, dx - u^2$$

$$= \int_{-\infty}^{0} \cancel{0 \cdot x^2\, dx} + \int_{0}^{\infty} 2e^{-2x} x^2\, dx - u^2$$

Variance $= 2\int_0^{\infty} x^2 e^{-2x} dx - \mu^2$

$= 2\left[\frac{-x^2 e^{-2x}}{2} - \frac{x e^{-2x}}{2} - \frac{e^{-2x}}{4}\right]_0^{\infty} - \mu^2$

$= 2\left[0 - \left(-\frac{1}{4}\right)\right] = \frac{2}{4} = \frac{1}{2} - \mu^2 = \frac{1}{2} - \frac{1}{4} = \frac{1}{4}$ 0

Standard deviation $= \sqrt{0.25} = \cancel{0.707}$ 0.5

x^2	e^{-2x}
$2x$ (+)	$\frac{e^{-2x}}{-2}$
2 (−)	$\frac{e^{-2x}}{4}$
0 (+)	$\frac{e^{-2x}}{-8}$