Distributions

Descrete distributions:

1) Binomial distribution
2) Poison distribution

Binomial distribution : It was discovered by James Bernoulli in the year 1700 and it is a descrete probability distribution which is used when trial or result of an experiment gives only 2 outcomes success & failure and further result of one trial does not influence the result of another trial & the probability of success at each trial is same from trial to trial .

Note:

Conditions for binomial distribution

1) Trials are independent
2) 1 Trial does not effect outcome of another trial .
3) Result remains same from trial to trial
4) probability of binomial distribution is

$P(X=x) = n_{C_x} p^x q^{n-x}$ (out of n trials getting x successes)

1) A fair coin is tossed 6 times Find probability of getting 4 heads

Let p = probability of Success $= \frac{1}{2}$

q = probability of failure $= 1-p = 1-\frac{1}{2} = \frac{1}{2}$

n = no. of trials = 6

$x = 4$

By binomial distribution $P(X=x) = n_{C_x} p^x q^{n-x}$

$$P(x=4) = 6_{C_4}\left(\frac{1}{2}\right)^4\left(\frac{1}{2}\right)^{6-4}$$

$$= 6_{C_4}\left(\frac{1}{2}\right)^4\left(\frac{1}{2}\right)^2$$

$$= 15\left(\frac{1}{2}\right)^4\left(\frac{1}{2}\right)^2$$

$$= \frac{15}{64}$$

2) Determine th
6 exactly 3
of fair dice
when a pai
no. of outco
The outcome
6 are {(1,5)

$p = \frac{5}{36}$

By binomia

3) A dice i
number i
1) atleast
2) less t
3) 4 Suc

$n = 6$

1) atlea

$P(X \geq$

2) P(

2) Determine the probability of getting the sum 6 exactly 3 times in 7 throws with a pair of fair dice.

When a pair of dice are thrown, the total no. of outcomes = 36

The outcomes which are in favour of getting sum 6 are {(1,5) (5,1) (2,4) (4,2) (3,3)}

$p = \frac{5}{36}$ $\quad q = 1 - \frac{5}{36} = \frac{31}{36}$ $\quad n = 7$ $\quad x = 3$

By binomial distribution $P(x=x) = {}^nC_x p^x q^{n-x}$

$$P(x=3) = {}^7C_3 \left(\frac{5}{36}\right)^3 \left(\frac{31}{36}\right)^{7-3}$$

$$= {}^7C_3 \left(\frac{5}{36}\right)^3 \left(\frac{31}{36}\right)^4$$

$$= 0.051$$

3) A dice is thrown 6 times, if getting an even number is success find the probability of

1) atleast one success
2) less than or equal to 3 success
3) 4 success

$n = 6$, $P = \frac{3}{6} = \frac{1}{2}$, $q = 1 - P = 1 - \frac{1}{2} = \frac{1}{2}$

1) atleast one success:

$$P(x \geq 1) = 1 - P(x < 1)$$

$$= 1 - P(x=0)$$

$$= 1 - {}^nC_0\, p^0 q^{n-0}$$

$$= 1 - {}^6C_0 \left(\frac{1}{2}\right)^0 \left(\frac{1}{2}\right)^{6-0}$$

$$= 1 - \left(\frac{1}{2}\right)^6$$

$$= 1 - \frac{1}{64} = \frac{63}{64}$$

2) $P(x \leq 3) = P(x=0) + P(x=1) + P(x=2) + P(x=3)$

$$= {}^nC_0 p^0 q^{n-0} + {}^nC_1 p^1 q^{n-1} + {}^nC_2 p^2 q^{n-2} + {}^nC_3 p^3 q^{n-3}$$

$$= {}^6C_0 \left(\frac{1}{2}\right)^0 \left(\frac{1}{2}\right)^{6-0} + {}^6C_1 \left(\frac{1}{2}\right)^1 \left(\frac{1}{2}\right)^{6-1} +$$

$${}^6C_2 \left(\frac{1}{2}\right)^2 \left(\frac{1}{2}\right)^{6-2} + {}^6C_3 \left(\frac{1}{2}\right)^3 \left(\frac{1}{2}\right)^{6-3}$$

$= \frac{1}{2^6}\left[6_{C_0} + 6_{C_1} + 6_{C_2} + 6_{C_3}\right]$

$= \frac{1}{64}(42)$

$= \frac{21}{32}$

3) $P(x=4) = n_{C_4} p^4 q^{n-4}$

$= 6_{C_4}\left(\frac{1}{2}\right)^4\left(\frac{1}{2}\right)^{6-4}$

$= 6_{C_4}\left(\frac{1}{2}\right)^6$

$= \frac{15}{64}$.

10 coins are thrown simultaneously. Find the probability of getting 1) atleast 7 heads 2) 6 heads 3) 1 head

$n = 10 \quad p = \frac{1}{2} \quad q = \frac{1}{2}$ (Binomial distribution)

1) atleast 7 heads

$P(x \geq 7) = P(x=7) + P(x=8) + P(x=9) + P(x=10)$

$= n_{C_7} p^7 q^{n-7} + n_{C_8} p^8 q^{n-8} + n_{C_9} p^9 q^{n-9} + n_{C_{10}} p^{10} q^{n-10}$

$= 10_{C_7}\left(\frac{1}{2}\right)^7\left(\frac{1}{2}\right)^3 + 10_{C_8}\left(\frac{1}{2}\right)^8\left(\frac{1}{2}\right)^2 + 10_{C_9}\left(\frac{1}{2}\right)^9\left(\frac{1}{2}\right)^1$

$+ 10_{C_{10}}\left(\frac{1}{2}\right)^{10}\left(\frac{1}{2}\right)^0$

$= \frac{1}{2^{10}}\left[10_{C_7} + 10_{C_8} + 10_{C_9} + 10_{C_{10}}\right]$

$= \frac{1}{1024}(176) = \frac{11}{64} = 0.171$

2) $P(x \geq 6) = n_{C_6} p^6 q^{n-6} + n_{C_7} p^7 q^{n-7} + n_{C_8} p^8 q^{n-8} +$

$= \cancel{10_{C_6}(\tfrac{1}{2}}\; n_{C_9} p^9 q^{n-9} + n_{C_{10}} p^{10} q^{n-10}$

$= 10_{C_6}\left(\frac{1}{2}\right)^6\left(\frac{1}{2}\right)^4 + \frac{176}{1024}$

$= \frac{105}{512} + \frac{176}{1024} = \frac{193}{512} = 0.376$

3) P(

5) Two d
proba
1) atl
n =
p = P
q =
1) at
P
2) P
3) P

3) $P(x \geq 1) = 1 - P(x=0) + P(x=1)$

$= 1 - {}^{n}C_0 \, p^0 q^{n-0} + {}^{n}C_1 \, p^1 q^{n-1}$

$= 1 - {}^{10}C_0 \left(\frac{1}{2}\right)^0 \left(\frac{1}{2}\right)^{10} + {}^{10}C_1 \left(\frac{1}{2}\right)^1 \left(\frac{1}{2}\right)^9$

$= 1 - \frac{1}{1024}$

$= \frac{1023}{1024} = 0.999$

5) Two dice are thrown 5 times find the probability of getting 7 as sum
1) atleast once 2) exactly 2 times 3) $P(1 < x < 5)$

$n = 5$

p = probability of getting 7 as sum $= \frac{6}{36} = \frac{1}{6}$

$q = 1 - p = 1 - \frac{1}{6} = \frac{5}{6}$

1) atleast once

$P(x \geq 1) = 1 - P(x=0)$

$= 1 - {}^{n}C_0 \, p^0 q^{n-0}$

$= 1 - {}^{5}C_0 \left(\frac{1}{6}\right)^0 \left(\frac{5}{6}\right)^{5-0}$

$= 1 - \frac{3125}{7776}$

$= 0.5981$

2) $P(x=2) = {}^{n}C_2 \, p^2 q^{n-2}$

$= {}^{5}C_2 \left(\frac{1}{6}\right)^2 \left(\frac{5}{6}\right)^3 = \frac{625}{3888}$

$= 0.160$

3) $P(1 < x < 5) = P(x=2) + P(x=3) + P(x=4)$

${}^{n}C_2 \, p^2 q^{n-2} + {}^{n}C_3 \, p^3 q^{n-3} + {}^{n}C_4 \, p^4 q^{n-4}$

$= {}^{5}C_2 \left(\frac{1}{6}\right)^2 \left(\frac{5}{6}\right)^3 + {}^{5}C_3 \left(\frac{1}{6}\right)^3 \left(\frac{5}{6}\right)^2 + {}^{5}C_4 \left(\frac{1}{6}\right)^4 \left(\frac{5}{6}\right)^1$

$= 0.16$

6) It has been claimed in 60% of all solar heat installations. The utility bill is reduced by atleast 1/3. Accordingly what are the probabilities that utility will be reduced by 1/3

1) 4 of 5 installations

2) atleast 4 of 5 installations

$$p = 60\% = \frac{60}{100} = 0.6$$

$$q = 1 - p = 1 - 0.6 = 0.4$$

1) $n = 5 \quad x = 4$

$$P(x=4) = n_{C_4} \, p^4 q^{n-4}$$
$$= 5_{C_4} (0.6)^4 (0.4)^1$$
$$= 0.2592$$

2) $P(x \geq 4) = P(x=4) + P(x=5)$

$$= n_{C_4} p^4 q^{n-4} + n_{C_5} p^5 q^{n-5}$$
$$= 5_{C_4} (0.6)^4 (0.4)^1 + 5_{C_5} (0.6)^5 (0.4)^0$$
$$= 0.2592 + 0.0777$$
$$= 0.336$$

7) If 20 tyres are defective & 4 of them are randomly chosen for inspection. What is the probability that only one of defective tyres will be included.

$$p = \frac{1}{20} \qquad q = 1 - p = \frac{19}{20}$$

$n = 4$

$$P(x=1) = n_{C_1} p^1 q^{n-1}$$
$$= 4_{C_1} \left(\frac{1}{20}\right)^1 \left(\frac{19}{20}\right)^3$$
$$= 0.1714$$

8) The incid
industry
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of 6 wo
suffer f

$p = 2$

$n = 6$

$P(x \geq 4$

9) 20% of
defecti
of 5 c

1) None

2) 1 is

$p =$

$n = 5$

1) $P(x$

2) $P($

3) $P(1$

8) The incidence of an occupational disease in an industry is such that the workers have 20% chance of suffering from it. What is probability that out of 6 workers chosen at random. 4 or more will suffer from the disease.

$p = 20\% = \frac{20}{100} = 0.2 \quad q = 0.8$

$n = 6$

$P(X \geq 4) = P(X=4) + P(X=5) + P(X=6)$

$= {}^nC_4 p^4 q^{n-4} + {}^nC_5 p^5 q^{n-5} + {}^nC_6 p^6 q^{n-6}$

$= {}^6C_4 (0.2)^4 (0.8)^2 + {}^6C_5 (0.2)^5 (0.8)^1 + {}^6C_6 (0.2)^6 (0.8)^0$

$= 0.0169$

9) 20% of items produced from a factory are defective. Find the probability that in a sample of 5 choosen at random

1) None is defective 3) $P(1 < x < 4)$

2) 1 is defective

$p = \frac{20}{100} = 0.2 \quad q = 0.8$

$n = 5$

1) $P(X=0) = {}^nC_0 p^0 q^{n-0}$

$= {}^5C_0 (0.2)^0 (0.8)^5$

$= 0.3276$

2) $P(X=1) = {}^nC_1 p^1 q^{n-1}$

$= {}^5C_1 (0.2)^1 (0.8)^4$

$= 0.4096$

3) $P(1 < X < 4) = P(X=2) + P(X=3)$

$= {}^nC_2 p^2 q^{n-2} + {}^nC_3 p^3 q^{n-3}$

$= {}^5C_2 (0.2)^2 (0.8)^3 + {}^5C_3 (0.2)^3 (0.8)^2$

$= 0.256$

10) Assume that 50% of all engineering students are good in mathematics Determine probabilities that among 18 engineering students

1) exactly 10 2) atleast 10 3) atmost 8

4) atleast 2 & atmost 19 are good in mathematics

$p = \frac{1}{2}$ $q = \frac{1}{2}$ $n = 18$

$P(x=10) = n_{C_{10}} p^{10} q^{n-10}$

$= 18_{C_{10}} \left(\frac{1}{2}\right)^{10} \left(\frac{1}{2}\right)^{8}$

$= 0.166$

$P(x \geq 10) = P(x=10) + P(x=11) + P(x=12) + P(x=13) + P(x=14) + P(x=15) - - - - P(x=18)$

$= n_{C_{10}} p^{10} q^{n-10} + n_{C_{11}} p^{11} q^{n-11} - - + n_{C_{18}} p^{18} q^{n-18}$

$= \frac{1}{2^{18}} \left[18_{C_{10}} + 18_{C_{11}} + - - - 18_{C_{18}} \right]$

$= \frac{106762}{262144} = 0.4072$

3) $P(x \leq 8) = P(x=0) + - - - P(x=8)$

$= n_{C_0} p^0 q^{n-0} + - - - n_{C_8} p^8 q^{n-8}$

$= 18_{C_0} \left(\frac{1}{2}\right)^0 \left(\frac{1}{2}\right)^{18} + - - - 18_{C_8} \left(\frac{1}{2}\right)^{18} \left(\frac{1}{2}\right)^{0}$

$= \frac{1}{2^{18}} \left[18_{C_0} + 18_{C_1} + 18_{C_2} + - - - 18_{C_8} \right]$

$= \frac{0.4072}{\cancel{256}}$

4) $P(2 \leq x \leq 9) = P(x=2) + - - - P(x=9)$

$= n_{C_2} p^2 q^{n-2} + - - - + n_{C_9} p^9 q^{n-9}$

$= 18_{C_2} \left(\frac{1}{2}\right)^2 \left(\frac{1}{2}\right)^{16} + - - - 18_{C_9} \left(\frac{1}{2}\right)^9 \left(\frac{1}{2}\right)^9$

$= \frac{1}{2^{18}} \left[18_{C_2} + 18_{C_3} + 18_{C_4} + - - 18_{C_9} \right]$

$= 0.592$

11) The probabil

1) If he fir

of hitting

2) How man

probability

more than

$p = \frac{1}{3}$

1) $P(x \geq 2) =$

2) proba

11) The probability of a man hitting target is $\frac{1}{3}$

1) If he fires 5 times what is the probability of hitting the target atleast twice

2) How many times must he fire so that the probability of hitting the target atleast once is more than 0.9.

$p = \frac{1}{3}$ $q = \frac{2}{3}$ $n = 5$

1) $P(x \geq 2) = P(x=2) + P(x=3) + P(x=4)$

$= 1 - [P(x=0) + P(x=1)]$

$= 1 - [{}^{n}c_0 p^0 q^{n-0} + {}^{n}c_1 p^1 q^{n-1}]$

$= 1 - [{}^{5}c_0 (\frac{1}{3})^0 (\frac{2}{3})^5 + {}^{5}c_1 (\frac{1}{3})^1 (\frac{2}{3})^4]$

$= 1 - 0.460$

$= 0.54$

2) probability $P(x \geq 1) > 90\%$

$1 - P(x < 1) > 0.9$

$1 - P(x=0) > 0.9$

$1 - [{}^{n}c_0 p^0 q^{n-0}] > 0.9$

$1 - [1 (\frac{1}{3})^0 (\frac{2}{3})^n] > 0.9$

$1 - (\frac{2}{3})^n > 0.9$

$1 - 0.9 > (\frac{2}{3})^n$

$(0.1) > (\frac{2}{3})^n$

$\log(0.1) > \log(\frac{2}{3})^n$

$\log(0.1) > n \log(\frac{2}{3})$

$n \log(\frac{2}{3}) < \log(0.1)$

$n < \frac{\log(0.1)}{\log(\frac{2}{3})} = n < 5.6788$

$n \simeq 6$

$\boxed{n = 6}$

12) The probability that John hits a target is $1/2$. He fires 6 times. Find the probability that he hits the target

1) exactly 2 times
2) more than 4 times
3) atleast once

$p = \frac{1}{2} \quad q = \frac{1}{2} \quad n = 6$

1) $P(x=2) = n_{C_2} p^2 q^{n-2}$

$= 6_{C_2} \left(\frac{1}{2}\right)^2 \left(\frac{1}{2}\right)^4 = 0.234$

2) $P(x>4) = P(x=5) + P(x=6)$

$= n_{C_5} p^5 q^{n-5} + n_{C_6} p^6 q^{n-6}$

$= 6_{C_5} \left(\frac{1}{2}\right)^5 \left(\frac{1}{2}\right)^1 + 6_{C_6} \left(\frac{1}{2}\right)^6 \left(\frac{1}{2}\right)^0$

$= \cancel{0.093} = 0.1093$

3) $P(x \geq 1) = 1 - P(x=0) = 1 - n_{C_0} p^0 q^{n-0}$

$= 1 - \left[6_{C_0} \left(\frac{1}{2}\right)^0 \left(\frac{1}{2}\right)^6 \right]$

$= 1 - [0.015] = 0.985$

13) The probability that the life of a bulb of 100 days is 0.05. Find the probability that out of 6 bulls

1) atleast one 2) greater than 4 3) none will be having life of 100 days

$n = 6 \quad p = 0.05 \quad q = 1 - 0.05 = 0.95$

1) $P(x \geq 1) = 1 - P(x=0) = 1 - [n_{C_0} p^0 q^{n-0}]$

$= 1 - [6_{C_0} (0.05)^0 (0.95)^6] = 1 - 0.735$

$= 0.265$

2) $P(x>4) = P(x=5) + P(x=6)$

$= n_{C_5} p^5 q^{n-5} + n_{C_6} p^6 q^{n-6}$

$= 6_{C_5} (0.05)^5 (0.95)^1 + 6_{C_6} (0.05)^6$

$= 1.7968 \times 10^{-6}$

3) $P(x=0) = n_{C_0} p^0 q^{n-0}$

$= 6_{C_0} (0.05)^0 (0.95)^6$

$= 0.735$

If the chance that any of the 10 telephone lines is busy at an instant is 0.2. What is the most probable number of busy lines and what is the probability of this number

$p = 0.2 \quad q = 0.8$

Note:
Mean of a binomial distribution $= n \times p$
and variance $= npq$

$$\text{Mean } (\mu) = \sum p_i x_i$$

$$= \sum_{x=0}^{n} x\, p(x)$$

$$= \sum_{x=0}^{n} x \left[n_{C_x} p^x q^{n-x} \right]$$

$$= 0 + 1 n_{C_1} p^1 q^{n-1} + 2 n_{C_2} p^2 q^{n-2} + 3 n_{C_3} p^3 q^{n-3}$$

$$+ \cdots + n \cdot n_{C_n} p^n q^{n-n}$$

$$= npq^{n-1} + \frac{2n(n-1)}{2!} p^2 q^{n-2} + \frac{3n(n-1)(n-2)}{3 \times 2 \times 1} p^3 q^{n-3} + \cdots + np^n \cdot 1$$

$$= npq^{n-1} + n(n-1)p^2q^{n-2} + \frac{\cancel{3}n(n-1)(n-2)}{\cancel{3}\times 2\times 1}p^3q^{n-3} + \cdots + np^n$$

$$= np\left[q^{n-1} + (n-1)pq^{n-2} + \frac{(n-1)(n-2)}{2!}p^2q^{n-3} + \cdots + p^{n-1}\right]$$

$$= np\left[(n-1)_{C_0}p^0q^{(n-1)-0} + (n-1)_{C_1}p^1q^{(n-1)-1} + (n-1)_{C_2}p^2q^{(n-1)-2} + \cdots + (n-1)_{C_{n-1}}p^{n-1}q^{(n-1)-(n-1)}\right]$$

$$= np\left[(p+q)^{n-1}\right]$$

$$= np\left[(1)^{n-1}\right]$$

$$\boxed{u = np}$$

$$\text{Variance } (\sigma^2) = \sum_{x=0}^{n} x^2p(x) - u^2$$

$$= \sum\left[x^2 - x + x\right]p(x) - (np)^2$$

$$= \sum(x^2 - x)p(x) + \sum xp(x) - (np)^2$$

$$= \sum_{x=0}^{n} x(x-1)\,n_{C_x}p^xq^{n-x} + np - n^2p^2$$

$$= 0 + 1(1-1)n_{C_1}p^1q^{n-1} + 2(2-1)n_{C_2}p^2q^{n-2} + 3(3-1)n_{C_3}p^3q^{n-3} + \cdots + n(n-1)n_{C_n}p^nq^{n-n} + np - n^2p^2$$

$$= 2\,\frac{n(n-1)}{2!}p^2q^{n-2} + 3\cdot 2\cdot\frac{n(n-1)(n-2)}{3\times 2!}\cdot p^3q^{n-3} + \cdots + n(n-1)n_{C_n}p^nq^{n-n} + np - n^2p^2$$

$$= n(n-1)p^2\left[q^{n-2} + (n-2)p^1q^{n-3} + \cdots + n_{C_n}p^{n-2}q^{n-n}\right] + np - n^2p^2$$

$$= n(n-1)p^2\left[(n-2)_{C_0}p^0q^{(n-2)-0} + (n-2)_{C_1}p^1q^{(n-2)-1} + \cdots + (n-2)_{C_{(n-2)}}p^{n-2}q^{(n-2)-(n-2)}\right] + np - n^2p^2$$

$$= n(n-1)p^2\left[(p+q)^{n-2}\right] + np - n^2p^2$$

$$= n(n-1)p'(1) + np - n^2p^2$$

$$= n^2p^2 - np^2 + np - n^2p^2 = np(1-p)$$

$$\boxed{\sigma^2 = npq}$$

13/10

1) If the probability of defective bolt is $\frac{1}{8}$. Find 1) mean 2) variance for the distribution of defective bolts of 640

$p = \frac{1}{8}$ $\quad q = \frac{7}{8}$ $\quad n = 640$

$\text{mean} = np = 640 \times \frac{1}{80} = 80$

$\text{variance} = npq = 640 \times \frac{1}{8} \times \frac{7}{8} = 70$

2) If the probability of defective bolt is 0.2 Find mean, standard deviation for distribution of bolts in a total of 400

$p = 0.2 \quad q = 0.8 \quad n = 400$

$\text{mean} = np = (400)(0.2) = 80$

$\text{variance} = npq = (400)(0.2)(0.8) = 64$

$\text{Standard deviation} = \sqrt{64} = 8$

3) The mean & variance of a binomial distribution are 4 and $\frac{4}{3}$ respectively. Find $P(x \geq 1)$

$\text{mean} = np = 4 \quad \text{—} \; ①$

$\text{variance} = npq = \frac{4}{3} \quad \text{—} \; ②$

$\frac{②}{①} = \frac{\cancel{np}q}{\cancel{np}} = \frac{\frac{4}{3}}{4} = q = \frac{1}{3}$

$p = 1 - \frac{1}{3} = \frac{2}{3}$

$np = 4$

$n = \frac{4}{p} = \frac{4}{\frac{2}{3}} = \frac{12}{2} = 6$

$P(x \geq 1) = 1 - P(x < 1)$

$= 1 - P(x = 0)$

$= 1 - nc_0 \, p^0 q^{n-0}$

$= 1 - 6c_0 \left(\frac{2}{3}\right)^0 \left(\frac{1}{3}\right)^{6-0}$

$= 1 - \frac{1}{3^6}$

$= \frac{728}{729}$

4) A descrete random variable x has the mean 6 and variance 2. If it is assumed that distribution is a binomial distribution. Find probability $P(5 \le x \le 7)$

Given mean $= np = 6$ — (1)

Variance $= npq = 2$ — (2)

$$\frac{(2)}{(1)} = \frac{npq}{np} = \frac{2}{6} = \frac{1}{3}$$

$$p = 1 - q = 1 - \frac{1}{3} = \frac{2}{3} \qquad np = 6$$

$$n = \frac{6}{\frac{2}{3}} = 9$$

$$\left.npq = 6 \times \frac{1}{3} = 2\right\}$$

$$P(5 \le x \le 7) = P(x=5) + P(x=6) + P(x=7)$$

$$= {}^{n}C_5\, p^5 q^{n-5} + {}^{n}C_6\, p^6 q^{n-6} + {}^{n}C_7\, p^7 q^{n-7}$$

$$= {}^{9}C_5 \left(\tfrac{2}{3}\right)^5 \left(\tfrac{1}{3}\right)^4 + {}^{9}C_6 \left(\tfrac{2}{3}\right)^6 \left(\tfrac{1}{3}\right)^3 + {}^{9}C_7 \left(\tfrac{2}{3}\right)^7 \left(\tfrac{1}{3}\right)^2$$

$$= \left\{\frac{1}{3^9}\left[{}^{9}C_5 + {}^{9}C_6 + {}^{9}C_7\right]\right\}^{\times}$$

$$= 0.712$$

5) The mean & variance of a binomial distribution are 2 and $\frac{8}{5}$. Find n

Given mean $= np = 2$

variance $= npq = \frac{8}{5}$

$$\frac{npq}{np} = \frac{\frac{8}{5}}{2} = \frac{8}{5} \times \frac{1}{2} = \frac{4}{5} \qquad p = \frac{1}{5}$$

$$np = 2$$

$$n \times \frac{1}{5} = 2 \Rightarrow n = 10.$$

6) The mean & variance of a binomial variable x with parameters n and p are 16 and 8 find $P(x \ge 1)$ and $P(x > 2)$

mean $= np = 16$

variance $= npq = 8$

$$\frac{npq}{np} = \frac{8}{16} \Rightarrow q = \frac{1}{2} \qquad p = \frac{1}{2}$$

Given $np = 16 \Rightarrow n \times \frac{1}{2} = 16$ $\boxed{n = 32}$

$P(x \ge 1) =$

$=$

$=$

$=$

$P(x > 2) =$

$=$

$=$

$=$

$=$

7) In 8 thro
as succes
Success

$n = 8$

mean $=$

$S.D = \sqrt{}$

8) Find the
in tossi
than 0.1
probabilit
$p =$ prob

$P(x \geq 1) = 1 - P(x=0)$

$= 1 - n_{C_0} p^0 q^{n-0}$

$= 1 - 32_{C_0} \left(\frac{1}{2}\right)^0 \left(\frac{1}{2}\right)^{32}$

$= 1 - \frac{1}{2^{32}}$

$= 0.999$

$P(x > 2) = 1 - [P(x=0) + P(x=1)]$

$= 1 - [n_{C_0} p^0 q^{n-0} + n_{C_1} p^1 q^{n-1}]$

$= 1 - \left[32_{C_0} \left(\frac{1}{2}\right)^0 \left(\frac{1}{2}\right)^{32} + 32_{C_1} \left(\frac{1}{2}\right)^1 \left(\frac{1}{2}\right)^{31}\right]$

$= 1 - \left[\frac{1}{2^{32}} (32_{C_0} + 32_{C_1})\right]$

$= 1 - \left[\frac{1}{2^{32}} (33)\right]$

$= 0.99$

7) In 8 throws of a dice. 5 or 6 is considered as succes find the mean and S.D of the no. of success

$n = 8 \quad p = \frac{2}{6} = \frac{1}{3} \quad q = 1 - \frac{1}{3} = \frac{2}{3}$

mean $= np = 8 \times \frac{1}{3} = \frac{8}{3}$

$S.D = \sqrt{\text{variance}} = \sqrt{npq} = \sqrt{8 \times \frac{1}{3} \times \frac{2}{3}} = \sqrt{\frac{16}{9}} = \frac{4}{3}$

8) Find the maximum n such that getting no head in tossing a fair coin n times is greater than 0.1. Find the maximum no. n such that probability of getting no head

p = probability of getting head $= \frac{1}{2} \quad q = \frac{1}{2}$

$P(x=0) > 0.1$

$n_{C_0} p^0 q^{n-0} > 0.1$

$1 \left(\frac{1}{2}\right)^0 \left(\frac{1}{2}\right)^n > 0.1$

$\frac{1}{2^n} > 0.1$

$\frac{1}{2^n} > \frac{1}{10}$

$10 > 2^n$

$\boxed{2^n < 10}$

$2^1 < 10$

$2^2 < 10$

$2^3 < 10$ (maximum)

2^4 not less than 10

$\boxed{n = 3}$ maximum value

9) In 256 sets of 12 tosses of a coin. In how many cases one can expect 8 heads & 4 tails.

$n = 12$

Let p = probability of getting a head $= \frac{1}{2}$

$q = 1 - p = 1 - \frac{1}{2} = \frac{1}{2}$

$P(x=8) = {}^{n}C_{8}\, p^{8} q^{n-8}$

$= {}^{12}C_{8} \left(\frac{1}{2}\right)^{8} \left(\frac{1}{2}\right)^{4}$

$= 0.1208$

expected number in 256 such cases

$N\,(P(x=8)) = 256 \times 0.1208$

$= 30.9375 \simeq 31$

When 6 dice are thrown 729 times. How many times do you expect atleast 3 dice to show 5 or 6.

$n = 6 \quad p = \frac{2}{6} = \frac{1}{3} \quad q = \frac{2}{3}$

$P(x \geq 3) = 1 - \left[P(x=0) + P(x=1) + P(x=2)\right]$

$= 1 - \left[{}^{n}C_{0}\, p^{0} q^{n-0} + {}^{n}C_{1}\, p^{1} q^{n-1} + {}^{n}C_{2}\, p^{2} q^{n-2}\right]$

$= 1 - \left[{}^{6}C_{0} \left(\frac{1}{3}\right)^{0} \left(\frac{2}{3}\right)^{6} + {}^{6}C_{1} \left(\frac{1}{3}\right)^{1} \left(\frac{2}{3}\right)^{5} + {}^{6}C_{2} \left(\frac{1}{3}\right)^{2} \left(\frac{2}{3}\right)^{4}\right]$

$= 1 - [0.680]$

$= 0.32$

$N\,P(x \geq 3) = 0.32 \times 729 = 233.28 = 233$

11) Two dice are thrown 120 times. Find the average no. of times the no. on first dice exceeds the no on second dice

$p = \frac{15}{36} = \frac{5}{12}$

mean $= np = 120 \times \frac{5}{12} = 50$

12) Out of 800 families with 5 children each. How many would you expect to have 3 boys b) 5 girls c) no girl d) atmost 2 girls. Assume that probabilities for boys & girls are same

Let p = probability of having a boy $= \frac{1}{2}$

$q = \frac{1}{2}$, $n = 5$, $N = 800$

a) $P(x=3)$

∴ No. of ... are ...

b) 5 gir...

N (...

c) No g...

N F...

4) atm...

atleas... boy...

13) In a ... indepe... are o... parame...

a) $P(x=3) = 3 \text{ boys} = {}^{n}c_0 p^0 q^{n-0}$ ×
$= {}^{5}c_0 \left(\frac{1}{2}\right)^0 \left(\frac{1}{2}\right)^5$
$= \frac{1}{32}$

$= {}^{n}c_3 p^3 q^{n-3}$
$= {}^{5}c_3 \left(\frac{1}{2}\right)^3 \left(\frac{1}{2}\right)^2$
$= 0.3125$ (for each family)

∴ No. of families having 5 children i.e 3 boys are $N P(x=3) = 800 \times 0.3125$
$= 250$

b) 5 girls = 0 boys = $P(x=0)$
$= {}^{n}c_0 p^0 q^{n-0}$
$= {}^{5}c_0 \left(\frac{1}{2}\right)^0 \left(\frac{1}{2}\right)^5 = \frac{1}{32}$
$= 0.03125$

$N(P(x=0)) = 800 \times 0.03125 = 800 \times 0.031$
$= 25$

c) No girl = 5 boys = $P(x=5)$
$= {}^{n}c_5 p^5 q^{n-5}$
$= {}^{5}c_5 \left(\frac{1}{2}\right)^5 \left(\frac{1}{2}\right)^0$
$= \frac{1}{32} = 0.031$

$N P(x=5) = 800 \times 0.031 = 25$

4) atmost 2 girls (or) atleast 3 boys $= P(x=3) + P(x=4) + P(x=5)$
$= {}^{n}c_3 p^3 q^{n-3} + {}^{n}c_4 p^4 q^{n-4} + {}^{n}c_5 p^5 q^{n-5}$
$= {}^{5}c_3 \left(\frac{1}{2}\right)^3 \left(\frac{1}{2}\right)^2 + {}^{5}c_4 \left(\frac{1}{2}\right)^4 \left(\frac{1}{2}\right)^1 + {}^{5}c_5 \left(\frac{1}{2}\right)^5 \left(\frac{1}{2}\right)^0$
$= \frac{1}{2^5} \left[{}^{5}c_3 + {}^{5}c_4 + {}^{5}c_5 \right]$
$= \frac{1}{32} (16) = \frac{1}{2}$
$= \cancel{1.5625} = 0.5$

$N P(x \geq 3) = 800 \times \cancel{1.5625}\ 0.5 = 400$

13) In a binomial distribution consisting of 5 independent trials, probability of 1 & 2 success are 0.4096 & 0.2048 respectively. Find the parameter p of the distribution.

$P(x=1) = 0.4096$

$n_{c_1} p^1 q^{n-1} = 0.4096$

$P(x=2) = 0.2048$

$n_{c_2} p^2 q^{n-2} = 0.2048$

$$\frac{P(x=1)}{P(x=2)} = \frac{0.4096}{0.2048} = \frac{n_{c_1} p^1 q^{n-1}}{n_{c_2} p^2 q^{n-2}} = 2$$

$$\frac{\cancel{n} \cancel{p^1} \cancel{q^n} q^{-1}}{\cancel{n}\frac{(n-1)}{2} p^{\cancel{2}} \cancel{q^n} q^{-2}} = 2$$

$$\frac{\cancel{2} q^{-1} q^2}{p(n-1)} = \cancel{2}$$

$q = p(n-1) = p(5-1) = 4p$

$q = 4p$

$\therefore p + q = 1$

$p + 4p = 1$

$5p = 1$

$$\boxed{p = \frac{1}{5}}$$

14/10

Poisson Distribution

A random variable X is said to follow a poisson distribution if it assumes only non -ve values & its probability density function is given by

$$P(X=x)\begin{cases}\dfrac{e^{-\lambda}\lambda^{x}}{x!} & \text{where } x=0,1,2\ldots \\ 0 & \end{cases}$$

here $\lambda = np > 0$ is called parameter of distribution poisson distribution is suitable for rare events for which propability occurence p is very small and the no. of trials is very large where np is finite.

* Note:

mean $= \lambda = np$

variance $= \lambda$

1) If the probability that an individual suffers a bad reaction from a certain injection is 0.001. Determine the probability that out of 2000 individuals 1) exactly 3 2) more than 2 3) None 4) more than one individual suffer a bad reaction.

$p = 0.001$, $n = 2000$

$\lambda = np = 2000 \times 0.001 = 2$

1) P (exactly 3) $= P(X=3)$

$$= \frac{e^{-\lambda}\lambda^{3}}{3!} = \frac{e^{-2}2^{3}}{6} = 0.180$$

2) more than 2 $P(X>2) = 1 - P(X\le 2)$

$$= 1 - [P(X=0) + P(X=1) + P(X=2)]$$

$$= 1 - \left[\frac{e^{-\lambda}\lambda^{0}}{0!} + \frac{e^{-\lambda}\lambda^{1}}{1!} + \frac{e^{-\lambda}\lambda^{2}}{2!}\right]$$

$$= 1 - \left[\frac{e^{-2}2^{0}}{0!} + \frac{e^{-2}2^{1}}{1!} + \frac{e^{-2}2^{2}}{2!}\right]$$

$$= 1 - e^{-2}[1+2+2]$$

$$= 1 - 5e^{-2}$$

$$= 0.323$$

3) $P(\text{none}) = P(x=0)$

$$= \frac{e^{-\lambda}\lambda^0}{0!}$$

$$= \frac{e^{-2}2^0}{1} = e^{-2} = 0.135$$

4) more than $1 = P(x>1)$

$$= 1 - P(x \le 1)$$

$$= 1 - [P(x=0) + P(x=1)]$$

$$= 1 - \left[\frac{e^{-\lambda}\lambda^0}{0!} + \frac{e^{-\lambda}\lambda}{1!}\right]$$

$$= 1 - \left[e^{-2} + e^{-2}(2)\right]$$

$$= 1 - [3e^{-2}]$$

$$= 0.593$$

2) A car hire firm has 2 cars which it hires out day by day. The no. of demands for each day is distributed as poisson distribution with mean 1.5. Calculate the proportion of days

1) on which there is no demand
2) on which demand is refused

λ = mean = 1.5 (poisson distribution)

$$P(x=x) = \frac{e^{-\lambda}\lambda^x}{x!}$$

1) $P(x=0) = \dfrac{e^{-1.5}(1.5)^0}{0!} = 0.2231$

2) $P(x>2) = 1 - [P(x=0) + P(x=1) + P(x=2)]$

$$= 1 - \left[\frac{e^{-1.5}(1.5)^0}{0!} + \frac{e^{-1.5}(1.5)^1}{1!} + \frac{e^{-1.5}(1.5)^2}{2!}\right]$$

$$= 1 - [0.8088]$$

$$= 0.1912$$

3) A hospital switch board receives an average of 4 emergency calls in a 10 min interval. What is the probability that

1) There are atmost 2 emergency calls in a 10 min
2) There are exactly 3 emergency calls in a 10 min interval

mean = np = λ = 4 (poisson distribution)

$$P(x=x) = \frac{e^{-\lambda}\lambda^{x}}{x!}$$

1) $P(x \le 2) = P(x=0) + P(x=1) + P(x=2) + P(x=3)$

$$= \frac{e^{-4}4^{0}}{0!} + \frac{e^{-4}4^{1}}{1!} + \frac{e^{-4}4^{2}}{2!} + \frac{e^{-4}4^{3}}{3!}$$

$$= 0.238$$

2) $P(x=3) = P(x=3) = \frac{e^{-4}4^{3}}{3!}$

$$= 0.195$$

4) A manufacturer knows that the condensors he make contains on an avg 1% defective. He packs them in boxes of 100. What is probability that a box picked at random will contain 3 or more faulty condensor.

n is very large & p is very small.

$\therefore$ poisson distribution $\quad n = 100 \quad p = \frac{1}{100}$

$$\lambda = np = \frac{1}{100} \times 100 = 1$$

$P(x \ge 3) = 1 - [P(x=0) + P(x=1) + P(x=2)]$ P(

$$= 1 - \left[\frac{e^{-1}1^{0}}{0!} + \frac{e^{-1}1^{1}}{1!} + \frac{e^{-1}1^{2}}{2!}\right]$$

$$= 1 - [0.919]$$

$$= 0.081$$

5) A bank receives on the avg 6 bad checks per day. Find probability that it will receive 4 bad checks on any given day.

$\lambda = 6$

$$P(x=4) = \frac{e^{-6}6^{4}}{4!} = 0.1338$$

6) A manufacturer of cotton pins knows that 5% of his product is defective. Pins are sold in boxes of 100. He guarantees that not more than 10 pins will be defective. What is probability a box will fail to meet guarantee quality

$p = 5\% = \frac{5}{100}$ $n = 100$ (poisson distribution)

$\lambda = \text{mean} = np = 100 \times \frac{5}{100} = 5$

$P(x>10) = 1 - [P(x=0) + P(x=1) + P(x=2) - - + P(x=10)]$

$= 1 - \left[\frac{e^{-5} 5^0}{0!} + \frac{e^{-5} 5^1}{1!} + - - - \frac{e^{-5} 5^{10}}{10!} \right]$

$= \{ 1 - [e^{-5} (1 + 2 + 6 + 24 + 120 + 720 + 5040) \times$
$+ 40320 + 362880 + \}$

$= 1 - [e^{-5} (1 + 5 + 5^2 + - - - + 5^{10})]$

$= 1 - 82250.32$

$= -82249.32$

$= 0.013$

7) If the probability is 0.05 that a certain wide flange column will fill under actual load what are probabilities that among 16 columns
1) at ~~mo~~most 2 will fail
2) atleast 4 will fail

$\lambda = \text{mean} = 0.05 \times 16 = \frac{5}{100} \times 16 = 0.8$

1) $P(x \leq 2) = \cancel{1-}\ P(x=0) + P(x=1) + P(x=2)$

$= 1 - \left[\frac{e^{-0.8}(0.8)^0}{0!} + \frac{e^{-0.8}(0.8)^1}{1!} \right] + \frac{e^{-0.8}(0.8)^2}{2!}$

$= \{ 1 - [0.8087] = 0.1913 \} = 0.8088$

2) $P(x \geq 4) = 1 - [P(x=0) + P(x=1) + P(x=2) + P(x=3)]$

$= 1 - \left[\frac{e^{-0.8}(0.8)^0}{0!} + \frac{e^{-0.8}(0.8)^1}{1!} + \frac{e^{-0.8}(0.8)^2}{2!} + \frac{e^{-0.8}(0.8)^3}{3!} \right]$

$= 1 - [0.8472]$

$= 0.1528$

8) It has b
produced
what is
such tool
1) 3% or
$p = 2\%$.
$\lambda = np$
1) $P(x \geq$
$= 1 - [$
$= 1 - [$
$= 1 -$
$= 1 - [$
$= 1 - ($
$= 1 - 0$
$= 0.11$
2) $P($
$= \{1 -$
$= P(x$
$= e^{-8}$
0!
$= e^{-}$
$= e^{-}$
$= 0.$

8) It has been found that 2% of the tools produced by a Certain machine are defective. What is probability that in a shipment of 400 such tools

1) 3% or more 2) 2% or less will prove defective

$p = 2\% = \frac{2}{100}$ $n = 400$ (Poisson distribution)

$\lambda = np = 400 \times \frac{2}{100} = 8$

1) $P(x \geq 3\%) = P(x \geq 3\% \text{ of } 400)$

$= P(x \geq \frac{3}{100} \cdot 400)$

$= P(x \geq 12)$

$= 1 - [P(x < 12)]$

$= 1 - [P(x=0) + \text{----} + P(x=11)]$

$= 1 - \left[\frac{e^{-8} 8^0}{0!} + \text{-----} + \frac{e^{-8} 8^{11}}{11!}\right]$

$= 1 - \left[e^{-8}\left(\frac{8^0}{0!} + \frac{8^1}{1!} + \text{---} + \frac{8^{11}}{11!}\right)\right]$

$= 1 - (e^{-8}(2647.317))$

$= 1 - 0.8880$

$= 0.112$

2) $P(x \leq 2\%) = P(x \leq 2\% \text{ of } 400)$

$= P(x \leq \frac{2}{100} 400$

$= P(x \leq 8)$

$= 1 - [P$

$= P(x=0) + P(x=1) + \text{----} + P(x=8)$

$= \frac{e^{-8} 8^0}{0!} + \frac{e^{-8} 8^1}{1!} + \text{----} + \frac{e^{-8} 8^8}{8!}$

$= e^{-8}\left(\frac{8^0}{0!} + \frac{8^1}{1!} + \text{---} + \frac{8^8}{8!}\right)$

$= e^{-8}(1766.35)$

$= 0.5925$

9) If a random variable has poisson distribution such that $P(1) = P(2)$. Find 1) mean of distribution 2) $P(4)$ 3) $P(x \geq 1)$ 4) $P(1 < x < 4)$

$P(1) = P(2)$

$$\frac{\cancel{e^{-\lambda}} \cancel{\lambda}}{1!} = \frac{\cancel{e^{-\lambda}} \lambda^{\cancel{2}}}{2!}$$

$$\boxed{\lambda = 2}$$

2) $P(4) = P(x=4)$

$$= \frac{e^{-\lambda}\lambda^4}{4!} = \frac{e^{-2}2^4}{4!} = 0.090$$

3) $P(x \geq 1)$

$= 1 - (P(x=0))$

$= 1 - \left[\frac{e^{-2}2^0}{0!}\right]$

$= 1 - 0.135$

$= 0.865$

4) $P(1 < x < 4)$

$= P(x=2) + P(x=3)$

$$= \frac{e^{-2}2^2}{2!} + \frac{e^{-2}2^3}{3!} = 0.45111$$

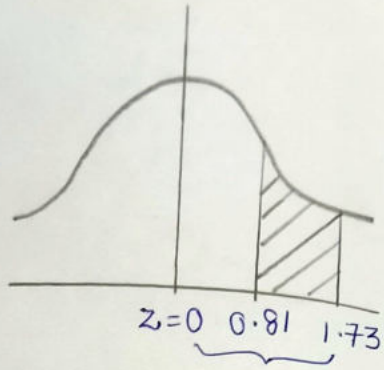
1) For a normally distributed variable with mean 1 and standard deviation 3. Find the probabilities that $(3.43 \le x \le 6.19)$

$\mu = 1$, $\sigma = 3$

Change of Scale is $z = \dfrac{x-\mu}{\sigma}$

when $x = 3.43 \Rightarrow z = \dfrac{x-\mu}{\sigma}$

$= \dfrac{3.43-1}{3} = 0.81$



when $x = 6.19 \Rightarrow \dfrac{6.19-1}{3} = 1.73$

$P(3.43 \le x \le 6.19) = P(0.81 \le x \le 1.73)$

$= \int_{0.81}^{1.73} \phi(z)dz = \int_{0}^{1.73} \phi(z)dz - \int_{0}^{0.81} \phi(z)dz$

$= 0.4582 - 0.2910$

$= 0.1672$

ii) $(-1.43 \le x \le 6.19)$

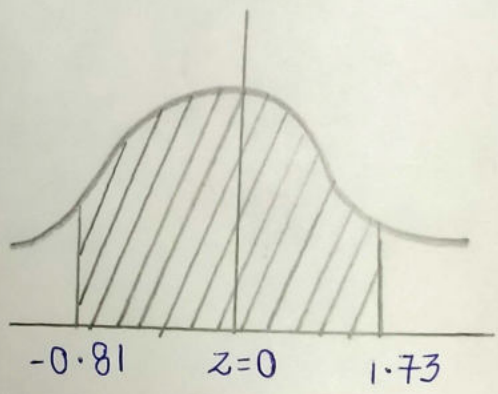
when $x = -1.43 \Rightarrow z = \dfrac{-1.43-1}{3} = -0.81$

$P(-1.43 \le x \le 6.19) = P(-0.81 \le x \le 1.73)$

$= \int_{-0.81}^{1.73} \phi(z)dz$

$= \int_{-0.81}^{0} \phi(z)dz + \int_{0}^{1.73} \phi(z)dz$

$= \int_{0}^{0.81} \phi(z)dz + \int_{0}^{1.73} \phi(z)dz$



$= 0.2916 + 0.4582$

$= 0.7498$

2) If x is normal variable with mean 30 and standard deviation 5. Find i) $P(26 \le x \le 40)$ ii) $P(x \ge 45)$

$\mu = 30 \quad \sigma = 5$

$P(26 \le x \le 40)$

when $x = 26 \Rightarrow z = \dfrac{26-30}{5} = \dfrac{-4}{5} = -0.8$

when $x = 40 \Rightarrow z = \frac{40-30}{5} = \frac{10}{5} = 2$

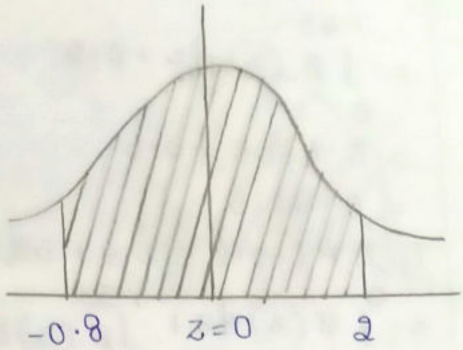
$P(26 \leq x \leq 40) = P(-0.8 \leq x \leq 2)$

$= \int_{-0.8}^{2} \phi(z)dz$

$= \int_{-0.8}^{\infty} \phi(z)dz + \int_{0}^{2} \phi(z)dz$

$= \int_{0}^{0.8} \phi(z)dz + \int_{0}^{2} \phi(z)dz$

$= 0.2881 = 0.7653$



(ii) $P(x \geq 45)$

when $x = 45 \Rightarrow z = \frac{45-30}{5} = \frac{15}{5} = 3$

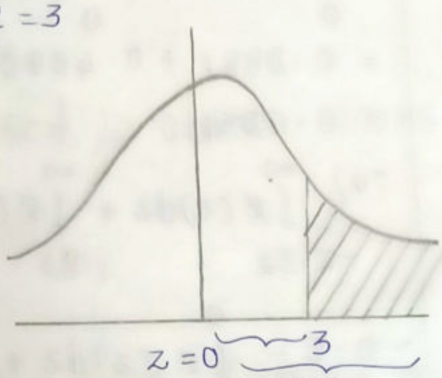
$P(x \geq 45) = P(z \geq 3)$

$= \int_{3}^{\infty} \phi(z)dz$

$= 0.5 - \int_{0}^{3} \phi(z)dz$

$= 0.5 - 0.4987$

$= 0.0013$



3) If x is a normal variable. Find the area

i) to the left of $z = -1.78$

ii) to the right of $z = -1.45$

iii) Corresponding to $(-0.8 \leq z \leq 1.53)$

(iv) to the left of $z = -2.52$ & to the right of $z = 1.83$

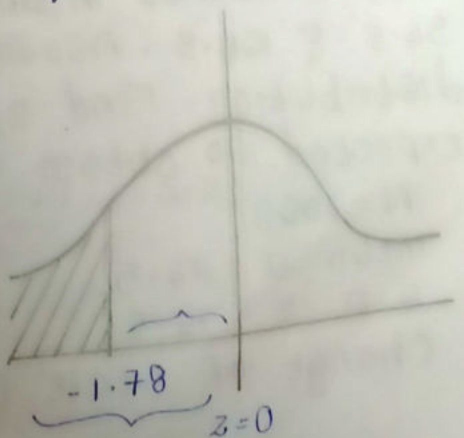
i) $P(z \leq z_1) = P(z \leq -1.78)$

$= 0.5 - \int_{-1.78}^{0} \phi(z)dz$

$= 0.5 - \int_{0}^{1.78} \phi(z)dz$

$= 0.5 - 0.4616$

$= 0.0384$



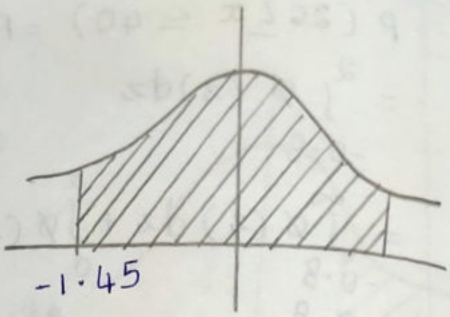
ii) $P(z \geq -1.45)$

$= \int_{-1.45}^{0} \phi(z)\,dz + 0.5$

$= \int_{0}^{1.45} \phi(z)\,dz + 0.5$

$= 0.4255 + 0.5$

$= 0.9265$



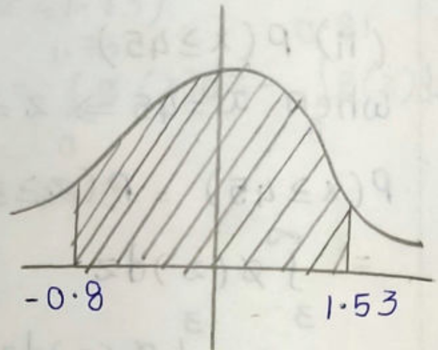
iii) $P(-0.8 \leq z \leq 1.53)$

$= \int_{-0.8}^{0} \phi(z)\,dz + \int_{0}^{1.53} \phi(z)\,dz$

$= \int_{0}^{0.8} \phi(z)\,dz + \int_{0}^{1.53} \phi(z)\,dz$

$= 0.2881 + 0.4370$

$= 0.7251$



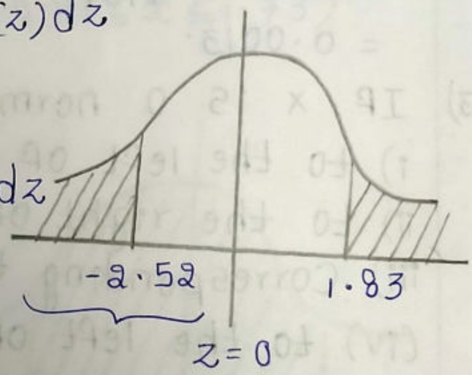
iv) $\int_{-2.52}^{\infty} \phi(z)\,dz + \int_{1.83}^{\infty} \phi(z)\,dz$

$0.5 - \int_{-2.52}^{0} \phi(z)\,dz + 0.5 - \int_{0}^{1.83} \phi(z)\,dz$

$= 1 - \int_{0}^{2.52} \phi(z)\,dz - \int_{0}^{1.83} \phi(z)\,dz$

$= 1 - 0.4941 - 0.4664$

$= 0.0395$



3/10 The mean & S.D of the marks obtained by 1000 students in an examination are respectively 34.5 & 160.5. Assuming the normality of the distribution. Find approximate no. of students expected to obtain marks b/w 30 & 60

$N = 1000$

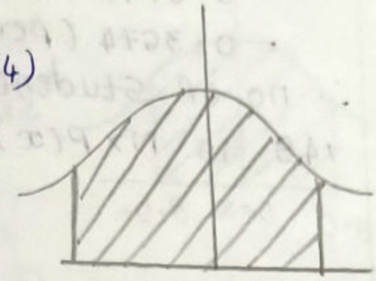
mean $\mu = 34.5$

S.D $\sigma = 16.5$

Change of Scale $(z) = \frac{x - \mu}{\sigma}$

when $x = 30 \Rightarrow z = \frac{30 - 34.5}{16.5} = -0.27$

when $x = 60 \Rightarrow z = \frac{60 - 34.5}{16.5} = 1.54$



$P(30 \leq x \leq 60) = P(-0.27 \leq z \leq 1.54)$

$= \int_{-0.27}^{1.54} \phi(z)\,dz$

$= \int_{-0.27}^{0} \phi(z)\,dz + \int_{z=0}^{1.54} \phi(z)\,dz$

$= \int_{0}^{0.27} \phi(z)\,dz + \int_{0}^{1.54} \phi(z)\,dz$

$= 0.1084 + 0.4382$

$= 0.5466$

$\therefore$ No. of students expected to obtain marks b/w 30 & 60 $= P(x) \times N$

$= 0.5466 \times 1000$

$= 546.6$

$= 547$

2) Suppose the weight of 800 male students are normally distributed with mean $(\mu) = 140$ pounds and S.D $= 10$ pounds find the no. of students whose weights are 1) b/w 138 & 148 2) more than 152 pounds.

$N = 800$

$\mu = 140$

$\sigma = 10$

Change of scale $z = \frac{x - \mu}{\sigma}$

1) $P(138 \leq x \leq 148)$

when $x = 138 \Rightarrow z = \frac{138 - 140}{10} = -0.2$

when $x = 148 \Rightarrow z = \frac{148 - 140}{10} = 0.8$

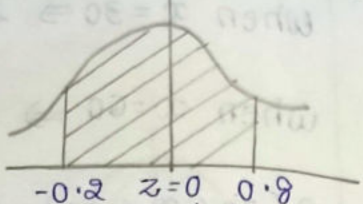
$P(138 \leq x \leq 148) = P(-0.2 \leq z \leq 0.8)$

$= \int_{-0.2}^{0} \phi(z)\,dz + \int_{0}^{0.8} \phi(z)\,dz$

$$= \int_0^{0.2} \phi(z)dz + \int_0^{0.8} \phi(z)dz$$

$$= 0.0793 + 0.2881$$

$$= 0.3674 \text{ (per student)}$$



$\therefore$ no. of students whose weights are b/w 138 & 148 is $N \times P(x) = 800 \times 0.3674$

$$= 293.92$$

$$= 294.$$

2) $P(x > 152)$

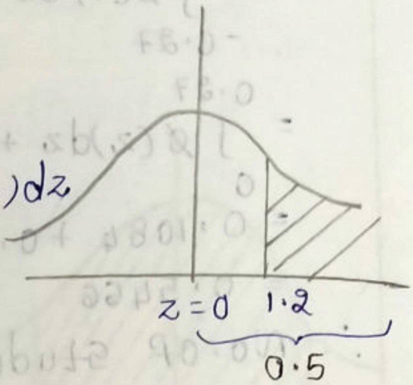
when $x = 152 \Rightarrow z = \frac{152-140}{10} = 1.2$

$P(x > 152) = P(x > 1.2) = \int_{1.2}^{\infty} \phi(z)dz$

$$= 0.5 - \int_0^{1.2} \phi(z)dz$$

$$= 0.5 - 0.3849$$

$$= 0.1151 \text{ (per student)}$$



$\therefore$ no. of students whose weight are more than 152 is $N \times P(x) = 800 \times 0.1151$

$$= 92.08$$

$$= 92$$

3) In a sample of 1000 cases the mean of a certain test is 14 and S.D is 2.5. Assuming the distribution to be normal. Find 1) How many score b/w 12 & 15 2) How many score above 18 3) How many scroe below 18

$N = 1000$

$\mu = 14$

$\sigma = 2.5$

Change of scale $z = \frac{x-\mu}{\sigma}$

1) $P(12 \le x \le 15)$

when $x = 12 \Rightarrow z = \frac{12-14}{2.5} = -0.8$

when $x = 15 \Rightarrow z = \frac{15-14}{2.5} = 0.4$

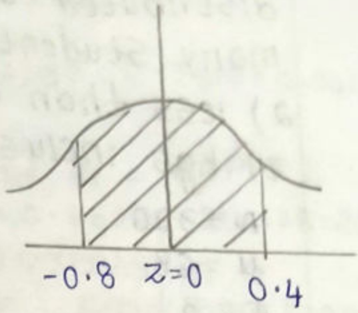
$P(12 \le x \le 15) = P(-0.8 \le x \le 0.4)$

$$= \int_{-0.8}^{0} \phi(z)\,dz + \int_{0}^{0.4} \phi(z)\,dz$$

$$= \int_{0}^{0.8} \phi(z)\,dz + \int_{0}^{0.4} \phi(z)\,dz$$

$$= 0.2881 + 0.1554$$

$$= 0.4435$$



Students Scroed b/w 12 & 15 $= N \times P(x)$

$= 1000 \times 0.4435$

$= 443.5$

$= 444$

2) $P(x > 18)$

when $x = 18 \Rightarrow z = \frac{18-14}{2.5} = 1.6$

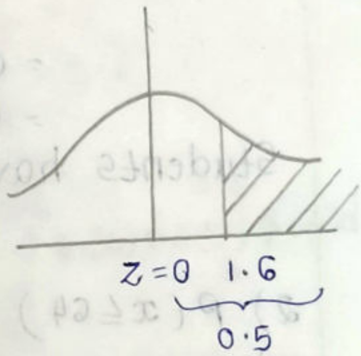
$P(x > 18) = P(x > 1.6)$

$$= \int_{1.6}^{\infty} \phi(z)\,dz$$

$$= 0.5 - \int_{0}^{1.6} \phi(z)\,dz$$

$$= 0.5 - 0.4452$$

$$= 0.0548$$



Students Scroed above 18 $= N \times P(x)$

$= 1000 \times 0.0548$

$= 54.8$

$= 55$

3) $P(x < 18)$

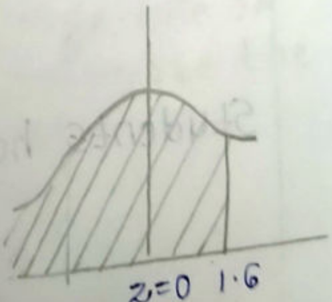
when $x = 18 \Rightarrow z = \frac{18-14}{2.5} = 1.6$

$P(x < 18) = P(x < 1.6)$

$$= 0.5 + \int_{0}^{1.6} \phi(z)\,dz$$

$$= 0.5 + 0.4452$$

$$= 0.9452$$

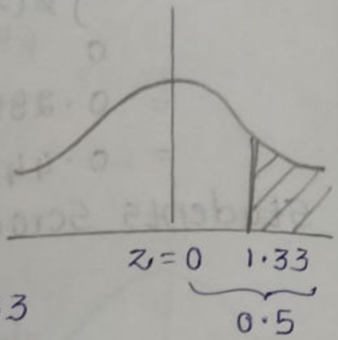


Students scored below 18 $= N \times P(x)$

$= 1000 \times 0.9452$

$= 945.2 = 945$

4) If the masses of 300 students are normally distributed with mean 68 & S.D 3 kgs. How many students have masses 1) greater than 72 kgs 2) less than or equal to 64 kgs 3) b/w 65 and 71 kgs inclusive



$N = 300$
$\mu = 68$
$\sigma = 3$

Change of scale $z = \frac{x-\mu}{\sigma}$

1) $P(x > 72)$

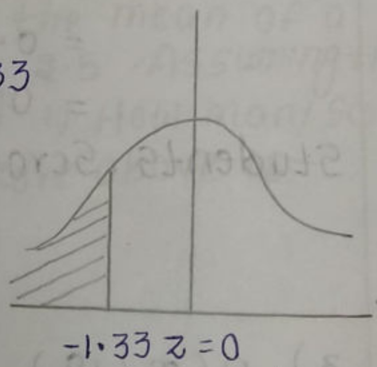
when $x = 72 \Rightarrow z = \frac{72-68}{3} = 1.33$

$$P(x>72) = P(x > 1.33)$$
$$= 0.5 - \int_0^{1.33} \phi(z)\,dz$$
$$= 0.5 - 0.4082$$
$$= 0.0918$$

Students having mass greater than $72 = N \times P(x)$
$$= 300 \times 0.0918$$
$$= 27.54 = 28$$

2) $P(x \leq 64)$

when $x = 64 \Rightarrow z = \frac{64-68}{3} = -1.33$



$$P(x \leq 64) = P(x \leq -1.33)$$
$$= 0.5 - \int_0^{1.33} \phi(z)\,dz$$
$$= 0.5 - 0.4082$$
$$= \cancel{0.9082}\ 0.0918$$

Students having mass less than $64 = N \times P(x)$
$$= 300 \times \cancel{0.9082}\ 0.0918$$
$$= \cancel{272.46} = 27.54$$
$$= \cancel{272} = 28$$

3) $P(65 \leq x \leq 71)$

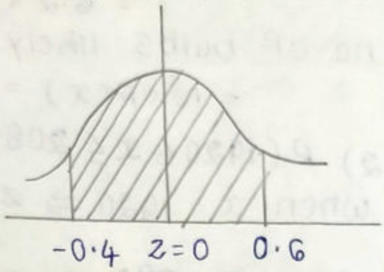
when $x = 65 \Rightarrow \cancel{\left\{ z = \frac{72-65}{3} = 2.33 \right\}}\ \frac{65-68}{3} = \frac{-3}{3} = -1$

when $x = 71 \Rightarrow z = \frac{71-68}{3} = \frac{3}{3} = 1$

$$P(65 \le x \le 71) = P(-1 \le x \le 1)$$
$$= \int_{-1}^{0} \phi(z)\,dz + \int_{0}^{1} \phi(z)\,dz$$
$$= \int_{0}^{1} \phi(z)\,dz + \int_{0}^{1} \phi(z)\,dz = 0.3413 + 0.3413$$
$$= 0.6826$$

Students having mass b/w 65 and 71 $= N \times P(x)$
$= 300 \times 0.6826 = 204.78 = 205$

5) Given that mean height of students in a class is 158 cm with S.D 20 cm. Find how many students heights lie b/w 150 & 170 if there are 100 students in class



$N = 100$
$\mu = 158$
$\sigma = 20$

1) $P(150 \le x \le 170)$

when $x = 150 \Rightarrow z = \frac{150-158}{20} = \frac{-8}{20} = -0.4$

when $x = 170 \Rightarrow z = \frac{170-158}{20} = \frac{12}{20} = 0.6$

$$P(150 \le x \le 170) = P(-0.4 \le x \le 0.6)$$
$$= \int_{-0.4}^{0} \phi(z)\,dz + \int_{0}^{0.6} \phi(z)\,dz = \int_{0}^{0.4} \phi(z)\,dz + \int_{0}^{0.6} \phi(z)\,dz$$
$$= 0.1554 + 0.2258 = 0.3812$$

Students height b/w 150 & 170 $= N \times P(x)$
$= 100 \times 0.3812$
$= 38.12 = 38$

6) In a test on 2000 electric bulbs it was found that the life of a particular make was normally distributed with an average life of 2040 hours and S.D of 40 hrs. Estimate the no. of bulbs lightly to burn for

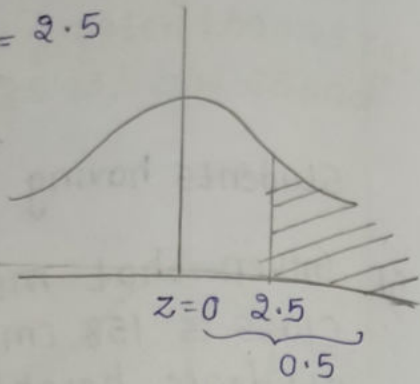
1) more than 2140 hrs
2) b/w 1920 & 2080 hrs
3) less than 1960 hrs

$N = 2000$
$\mu = 2040$
$\sigma = 40$

Change of scale $z = \frac{x-\mu}{\sigma}$

1) $P(x > 2140)$

when $x = 2140 \Rightarrow z = \frac{2140 - 2040}{40} = 2.5$



$$P(x > 2140) = P(x > 2.5)$$
$$= \int_{2.5}^{\infty} \phi(z)\,dz$$
$$= 0.5 - \int_{0}^{2.5} \phi(z)\,dz$$
$$= 0.5 - 0.4938$$
$$= 6.2 \times 10^{-3}$$

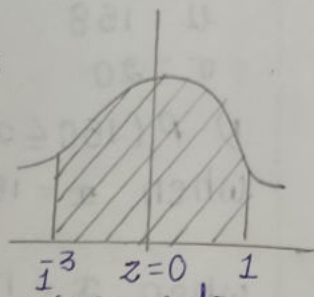
no. of bulbs likely to burn more than 2140 hrs

$= N \times P(x) = 2000 \times 6.2 \times 10^{-3} = 12.4 = 12$

2) $P(1920 \leq x \leq 2080)$

when $x = 1920 \Rightarrow z = \frac{1920 - 2040}{40} = -3$

when $x = 2080 \Rightarrow z = \frac{2080 - 2040}{40} = 1$



$P(1920 \leq x \leq 2080) = P(-3 \leq x \leq 1)$

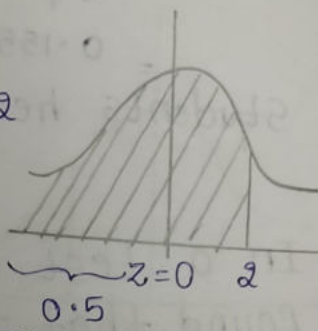
$$\int_{-3}^{0} \phi(z)\,dz + \int_{0}^{1} \phi(z)\,dz = \int_{0}^{3} \phi(z)\,dz + \int_{0} \phi(z)\,dz$$
$$= 0.4987 + 0.3413 = 0.84$$

no. of bulbs likely to burn b/w 1920 & 2080 hrs

$= N \times P(x) = 2000 \times 0.84 = 1680$

3) $P(x < 1960)$

when $x = 1960 \Rightarrow z = \frac{1960 - 2040}{40} = 2$



$$P(x < 1960) = P(x < 2)$$
$$= 0.5 + \int_{0}^{2} \phi(z)\,dz$$
$$= 0.5 + 0.4772 = 0.9772$$

no. of bulbs likely to burn less than 1960 hrs

$= N \times P(x) = 2000 \times 0.9772$

$= 1954.4 = 1954$

7) 1000 Students had written an examination. The mean of test is 35 & S.D is 5. Assuming the distribution to be normal

1) How many Students marks lie b/w 25 & 40

2) How many Students got more than 40

3) below 20

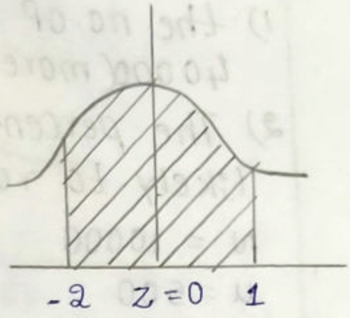
$N = 1000$
$\mu = 35$
$\sigma = 5$

Change of Scale $z = \frac{x-\mu}{\sigma}$

1) $P(25 \le x \le 40)$

when $x = 25 \Rightarrow \frac{25-35}{5} = \frac{-10}{5} = -2$

when $x = 40 \Rightarrow \frac{40-35}{5} = 1$



$P(25 \le x \le 40) = P(-2 \le x \le 1)$

$\int_{-2}^{0} \phi(z)\,dz + \int_{0}^{1} \phi(z)\,dz$

$= \int_{0}^{2} \phi(z)\,dz + \int_{0}^{1} \phi(z)\,dz$

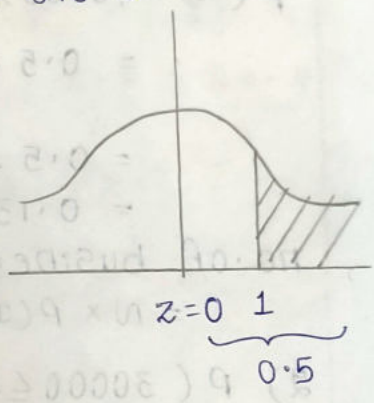
$= 0.4772 + 0.3413 = 0.8185$

Student marks lying b/w 25 & 40 $= N \times P(x)$

$= 1000 \times 0.8185 = 818.5$

2) $P(x > 40)$

when $x = 40 \Rightarrow \frac{40-35}{5} = 1$



$P(x > 40) = P(x > 1)$

$= 0.5 - \int_{0}^{1} \phi(z)\,dz$

$= 0.5 - 0.3413$

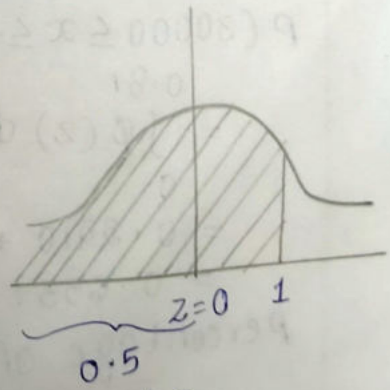
$= 0.1587$

Student marks more than 40 $= N \times P(x)$

$= 1000 \times 0.1587 = 158.7 = 159$

3) $P(x < 40)$

when $x = 40 \Rightarrow \frac{40-35}{5} = 1$



$P(x < 40) = P(x < 1)$

$= 0.5 + \int_{0}^{1} \phi(z)\,dz$

$= 0.5 + 0.3413$

$= 0.8413$

Students who got marks less than 40

$= N \times P(x)$

$= 1000 \times 0.8413 = 841.3 = 841$

8) A sales tax officer has reported that the avg sales of 500 business men that he has to deal with during a year is 36000 with a S.D of 10000. Assuming that the sales in this business are normally distributed. Find

1) the no.of business as the sales of which are 40000 (more than)

2) The percentage of business sales of which are likely to range b/w 30000 & 40000

$N = 10000$

$\mu = 500$

$\sigma = 36000$

1) $P(x > 40000)$

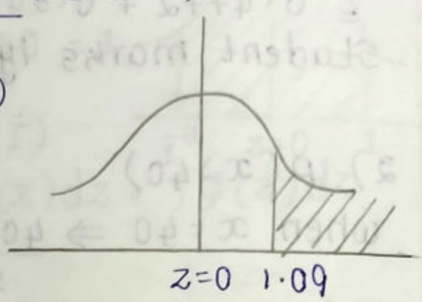
when $x = 40000 \Rightarrow z = \dfrac{40000 - 500}{36000} = 1.097$

$P(x > 40000) = P(x > 1.09)$

$= 0.5 - \int_0^{1.09} \phi(z)\, dz$

$= 0.5 - 0.3621$

$= 0.1379$



no.of business sales more than 40000

$= N \times P(x) = 10000 \times 0.1379 = 1379$

2) $P(30000 \le x \le 40000)$

when $x = 30000 \Rightarrow z = \dfrac{30000 - 500}{36000} = 0.81$

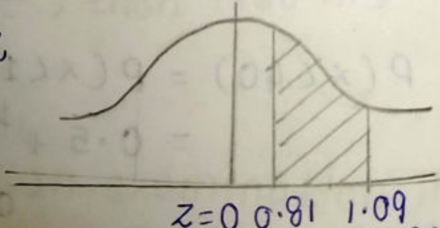
when $x = 40000 \Rightarrow z = \dfrac{40000 - 500}{36000} = 1.09$

$P(30000 \le x \le 40000) = P(0.81 \le x \le 1.09)$

$= \int_0^{0.81} \phi(z)\, dz + \int_0^{1.09} \phi(z)\, dz$

$= 0.2910 + 0.3621$

$= 0.6531$



percentage of business sales b/w 30000 & 40000

$= N \times P(x) = 10000 \times 0.6531$

$= 6531$

9) In a nor
under 35
mean & v

From fig

$\int_0^{z_1} \phi(z) d$

$z_1 =$

$z_1 = \frac{x -}{\sigma}$

$-1.48 =$

-1.48σ

$\mu - 1.$

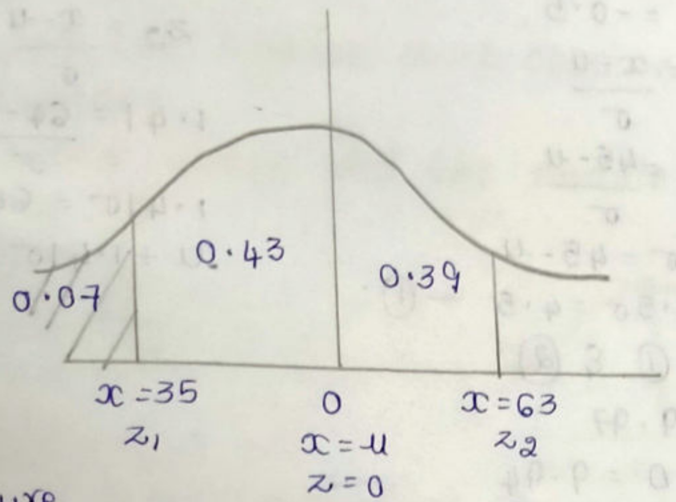
From ①

$\mu =$

Variance

10) In a no
45, 8%
ce of th

9) In a normal distribution 7% of the items are under 35 and 89% are under 63. Determine the mean & variance of the distribution.



From figure

$\int_0^{z_1} \phi(z)dz = 0.43$

$z_1 = -1.48$

$z_1 = \frac{x-\mu}{\sigma}$

$-1.48 = \frac{35-\mu}{\sigma}$

$-1.48\sigma = 35-\mu$

$\mu - 1.48\sigma = 35$ — (1)

From figure

$\int_0^{z_2} \phi(z)dz = 0.39$

$z_2 = 1.23$

$z_2 = \frac{x-\mu}{\sigma} = \frac{63-\mu}{\sigma}$

$1.23 = \frac{63-\mu}{\sigma}$

$1.23\sigma = 63-\mu$

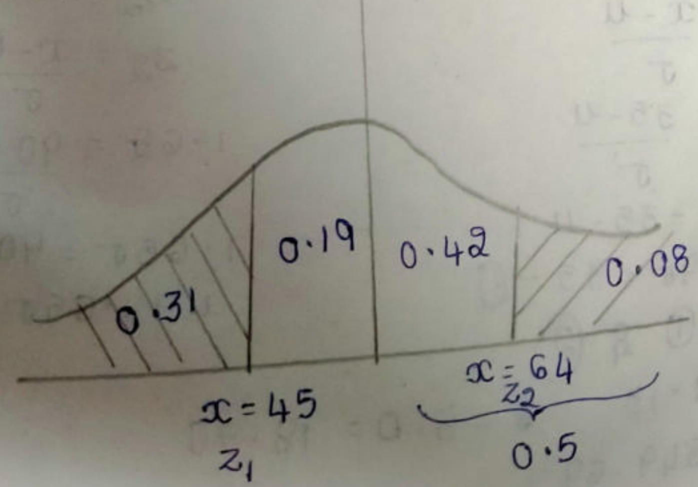
$\mu + 1.23\sigma = 63$ — (2)

From (1) & (2)

$\mu = 50.25$ $\sigma = S.D = 10.33$

Variance $= \sigma^2 = 106.708$

10) In a normal distribution 31% of items are under 45, 8% of items are over 64. Find mean & variance of the distribution.



From figure

$\int_0^{z_1} \phi(z)\,dz = 0.19$

$z_1 = -0.5$

$z_1 = \frac{x-\mu}{\sigma}$

$-0.5 = \frac{45-\mu}{\sigma}$

$-0.5\sigma = 45-\mu$

$\mu - 0.5\sigma = 4.5$ — (1)

From figure

$\int_0^{z_2} \phi(z)\,dz = 0.42$

$z_2 = 1.41$

$z_2 = \frac{x-\mu}{\sigma}$

$1.41 = \frac{64-\mu}{\sigma}$

$1.41\sigma = 64-\mu$

$\mu + 1.41\sigma = 64$ — (2)

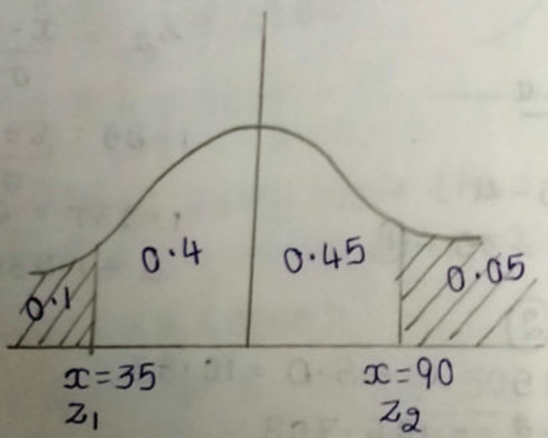
From (1) & (2)

$\mu = 49.97$

$\sigma = S.D = 9.94$

Variance $= \sigma^2 = 98.80$

11) Suppose 10% of the probability for a normal distribution $N(\mu, \sigma^2)$ is below 35 and 5% are above 90. What are the values of μ and σ



From figure

$\int_0^{z_1} \phi(z)\,dz = 0.4$

$z_1 = -1.29$

$z_1 = \frac{x-\mu}{\sigma}$

$-1.29 = \frac{35-\mu}{\sigma}$

$-1.29\sigma = 35-\mu$

$\mu - 1.29\sigma = 35$ — (1)

From figure

$\int_0^{z_2} \phi(z)\,dz = 0.45$

$z_2 = 1.65$

$z_2 = \frac{x-\mu}{\sigma}$

$1.65 = \frac{90-\mu}{\sigma}$

$1.65\sigma = 90-\mu$

$\mu + 1.65\sigma = 90$ — (2)

From (1) & (2)

$\mu = 59.13$ $\sigma = S.D = 18.70$

$\sigma^2 = 349.69$

01/11 1) The marks ... students ... and S.D ... marks abo...

2) What ... 10% of st...

3) Within ... of studen...

$N = 1000$

$\mu = 78\%$

$\sigma = 11\%$

1) $P(x > 90$ ...

when $x =$ ...

$P(x > 0.9$ ...

$=$

$=$

$\therefore$ The n...

$= N \times P$ ...

2)

From f...

$\int_0^{z_1} \phi(z$ ...

z ...

$\frac{x-\mu}{\sigma}$

$\frac{x-0.}{0.1}$ ...

01/11 1) The marks obtained in mathematics by 1000 students is normally distributed with mean 78% and S.D 11%. Determine how many students got marks above 90%.

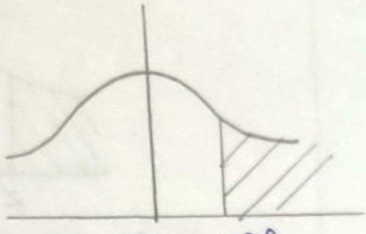
2) What was the highest mark obtained by lowest 10% of students

3) Within what limits did the middle of 90% of students lie.

$N = 1000$

$\mu = 78\% = 0.78$

$\sigma = 11\% = 0.11$



1) $P(x > 90\%) = P(x > 0.9)$

when $x = 0.9 \Rightarrow z = \frac{0.9 - 0.78}{0.11} = 1.09$

$P(x > 0.9) = P(x > 1.09)$

$= 0.5 - \int_0^{1.09} \phi(z)\,dz$

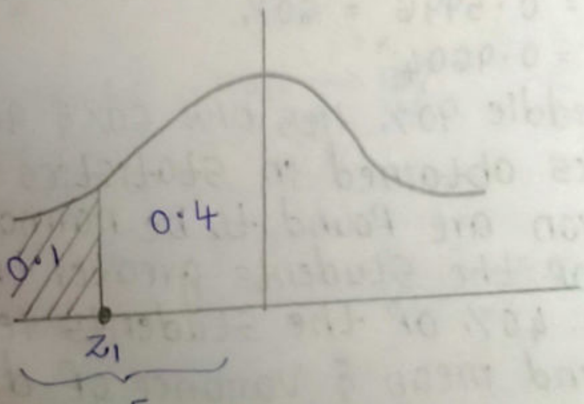
$= 0.5 - 0.3621$

$= 0.138$

$\therefore$ The no. of students with marks more than 90%

$= N \times P(x) = 1000 \times 0.138 = 138$

2)



From figure

$\int_0^{z_1} \phi(z)\,dz = 0.4$

$z_1 = -1.29$

$\frac{x - \mu}{\sigma} = -1.29$

$\frac{x - 0.78}{0.11} = -1.29$

$x - 0.78 = -1.29 \times 0.11$

$x - 0.78 = -0.1419$

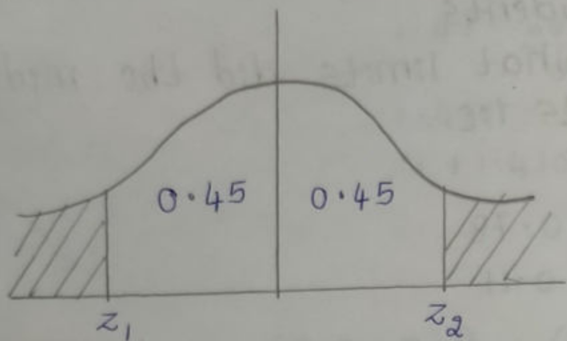
$x = 0.78 - 0.1419$

$x = 0.6381$

$\therefore$ The highest mark obtained by the lowest 10%

is $0.6381 \times 1000 = 638.1\%$

3)



From figure

$\int_0^{z_1} \phi(z)\,dz = 0.45$

$z_1 = -1.64$

$\frac{x-u}{\sigma} = -1.64$

$\frac{x-0.78}{0.11} = -1.64$

$x - 0.78 = -1.64 \times 0.11$

$x - 0.78 = -0.1804$

$x = 0.78 - 0.1804$

$x = 0.5996 = 60\%$

$\{x = 0.9604\}$

From figure

$\int_0^{z_2} \phi(z)\,dz = 0.45$

$z_2 = 1.64$

$\frac{x-u}{\sigma} = 1.64$

$\frac{x-0.78}{0.11} = 1.64$

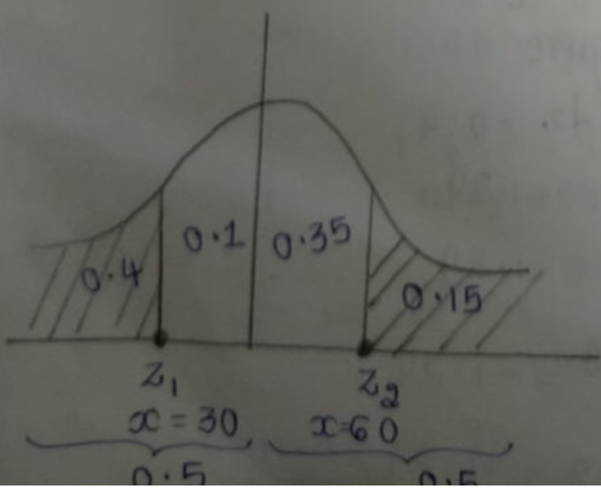
$x - 0.78 = 0.1804$

$x = 0.78 + 0.1804$

$x = 0.9604 = 90\%$

$\therefore$ The middle 90% lies b/w 60% & 96%.

2) The marks obtained in statistics in a certain examination are found to be normally distribute. If 15% of the students greater than or equal to 60 marks. 40% of the students less than 30 marks. Find mean & variance of distribution.



From fig

$\int_0^{z_1} \phi(z)\,dz$

$z_1 = -0.2$

$-0.25 = x$

$\frac{30-u}{\sigma} =$

$30 - u =$

$u - 0.8$

$u = 35$

$\sigma = 23$

$\sigma^2 = V$

3) In an e
student
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60% ma
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of the
5% of
percent
division

less t

60%.

50 -

40

>

From figure

$\int_0^{z_1} \phi(z)\,dz = 0.1$

$z_1 = -0.25$

$-0.25 = \frac{x-\mu}{\sigma}$

$\frac{30-\mu}{\sigma} = -0.25$

$30 - \mu = -0.25\sigma$

$\mu - 0.25\sigma = 30$ — (1)

$\mu = 35.81$

$\sigma = 23.25$

σ^2 = Variance = 540.56

From figure

$\int_0^{z_2} \phi(z)\,dz = 0.35$

$z_2 = 1.04$

$1.04 = \frac{60-\mu}{\sigma}$

$60 - \mu = 1.04\sigma$

$\mu + 1.04\sigma = 60$ — (2)

3) In an examination it is laied down that a student passes if he secures 40% or more he is placed in first, second & third division according as he secures 60% or more marks, b/w 50% & 60% marks & marks b/w 40% & 50% respectively. He gets a distinction in case he secures 75% or more. It is noticed from the result that 10% of the students failed in examination whereas 5% of them obtained distinction. Calculate the percentage of students placed in the second division.

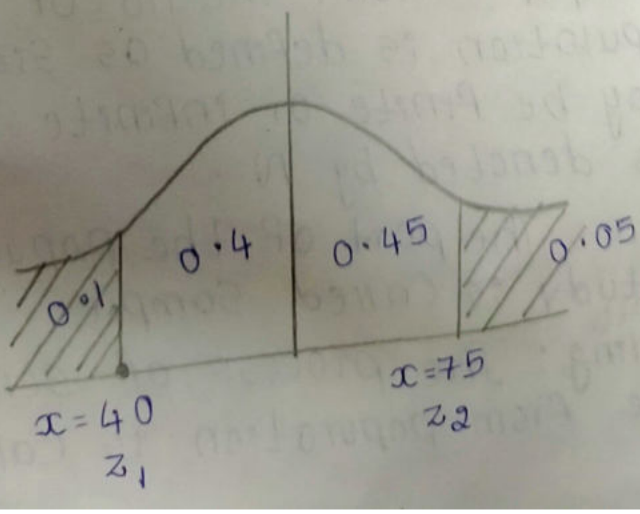
less than 40 - fail → 10%.

60% or more – first

50 - 60% – second

40 - 50 % – third

> 75% – distinction – 5%.



From figure

$$\int_0^{z_1} \phi(z)\,dz = 0.4$$

$z_1 = -1.29$

$-1.29 = \dfrac{40-\mu}{\sigma}$

$40-\mu = -1.29\sigma$

$\mu - 1.29\sigma = 40$ — (1)

From figure

$$\int_0^{z_2} \phi(z)\,dz = 0.45$$

$z_2 = 1.65$

$1.65 = \dfrac{75-\mu}{\sigma}$

$75-\mu = 1.65\sigma$

$\mu + 1.65\sigma = 75$ — (2)

From (1) & (2)

$\mu = 55.35$

$\sigma = 11.90$

$P(50 < x < 60)$

when $x = 50 \Rightarrow z = \dfrac{50-55.35}{11.90} = -0.44$

when $x = 60 \Rightarrow z = \dfrac{60-55.35}{11.90} = 0.39$

$$P(50 < x < 60) = P(-0.44 < z < 0.39)$$

$$= \int_0^{0.44} \phi(z)\,dz + \int_0^{0.39} \phi(z)\,dz$$

$$= 0.1700 + 0.1517$$

$$= 0.3217$$

Sampling Distributions

Population: It is aggregate or totality of statistical data forming a subject of investigation

Size of population: The no. of observations in population is defined as size of population. It may be finite or infinite. It is denoted by N.

Sample: The part of the population selected for study is called sample.

Sampling: The process of selection of sample from population is called sampling

Classific
Samples
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2) Small
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Sample

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1) What
$N = 200$
Correc

2) Find
factor

C.

3) How
Chosen

The n

Classification of Samples:
Samples are classified into 2 types
1) large Sample: If the size of Sample is greater than 30 i.e $n > 30$ then that Sample is called large Sample.
2) Small Sample: If the size of Sample is less than or equal to 30 i.e $n \leq 30$ then the Sample is called Small/ exact/ simple Sample.

Parameters: The Statistical Constants of population namely mean (μ), Variance (σ^2) are called Parameters.

Statistics: The Constants (Stastical) of Sample namely Sample mean ($\bar{x}$), Sample Variance (s^2) are called Statistics.

Sample mean: If $x_1, x_2 --- x_n$ represent a random Sample of size n. Then Sample mean is defined as $\bar{x} = \frac{\sum x_i}{n}$ where $i = 1, 2 --- n$

Sample Variance: $\frac{\sum x_i - (\bar{x})^2}{n-1}$

Correction factor: $\frac{N-n}{N-1}$

1) What is the value of Correction factor if $n=5$, $N = 200$

Correction Factor $= \frac{200-5}{200-1} = \frac{195}{199} = 0.97$

2) Find the value of finite population Correction factor for $n=10$, $N=1000$

$C.F = \frac{1000-10}{1000-1} = 0.99$

3) How many different samples of size 2 can be chosen from a finite population of size 25

$N = 25$, $n = 2$

The no. of different Samples that can be choosen are $N_{C_n} = 25_{C_2} = 300$

4) A population consist of 5 numbers {2, 3, 6, 8, 10} consider all possible samples of size 2 which can be drawn with replacement from this population. 1) Find mean of population 2) S.D of population 3) mean of sampling distribution of means 4) S.D of sampling distribution of means. 2, 3, 6, 8, 11

1) mean of population

$$u = \frac{2+3+6+8+11}{5} = 6$$

2) S.D of population

$$\sigma^2 = \sum \frac{(x_i - u)^2}{n}$$

$$= \frac{(2-6)^2 + (3-6)^2 + (6-6)^2 + (8-6)^2 + (11-6)^2}{5}$$

Variance = 10.8

$S.D = \sigma = \sqrt{10.8} = 3.28$

Samples of size 2 with replacement are

(2,2) (2,3) (2,6) (2,8) (2,11)
(3,2) (3,3) (3,6) (3,8) (3,11)
(6,2) (6,3) (6,6) (6,8) (6,11)
(8,2) (8,3) (8,6) (8,8) (8,11)
(11,2) (11,3) (11,6) (11,8) (11,11)

Sampling distribution of means

$\frac{2+2}{2} = 2$, 2.5, 4, 5, 6.5, 2.5, 3, 4.5, 5.5, 7,
4, 4.5, 6, 7, 8.5
5, 5.5, 7, 8, 9.5
6.5, 7, 8.5, 9.5, 11

mean of sampling distribution of means

$$u(\bar{x}) = \frac{2 + 2.5 + \cdots + 9.5 + 11}{25} = 6$$

4) S.D of sampling distribution of means

$$\text{Variance } (\sigma^2) = \frac{(2-6)^2 + (2.5-6)^2 + (4-6)^2 + \cdots + (9.5-6)^2 + (11-6)^2}{25}$$

$= 5.4$

S.D $\sigma = \sqrt{5.4} = 2.32$

Without replacement

1) mean of population

$$\mu = \frac{2+3+6+8+11}{5} = 6$$

2) S.D of population

$$\sigma^2 = \frac{\sum (x_i-\mu)^2}{n}$$

$$= \frac{(2-6)^2+(3-6)^2+(6-6)^2+(8-6)^2+(11-6)^2}{5}$$

Variance $(\sigma^2) = 10.8$

S.D $= \sigma = \sqrt{10.8} = 3.28$

Samples of size 2 without replacement are

(2,3) (2,6) (2,8) (2,11)

(3,6) (3,8) (3,11)

(6,8) (6,11)

(8,11)

Sampling distribution of means

2.5, 4, 5, 6.5

4.5, 5.5, 7

7, 8.5

9.5

mean of sampling distribution of means

$$\mu(\bar{x}) = \frac{2.5+4+ - - - +9.5}{10} = 6$$

4) S.D of sampling distribution of means

$$\text{Variance}(\sigma^2) = \frac{(2.5-6)^2 + - - - + (9.5-6)^2}{10}$$

$= 4.05$

S.D $\sigma = \sqrt{4.05} = 2.01$

4/11 1) A population consist of 5 10 14 18 13 24. Consider all possible samples of size 2 which can be drawn without replacement from the population. Find 1) mean of population 2) S.D of population 3) mean of sampling distribution of means 4) S.D of sampling distribution of means

5, 10, 14, 18, 13, 24

1) mean of population

$$= \frac{5+10+14+18+13+24}{6}$$

$= 14$

2) S.D of population

$$\sigma^2 = \sum \frac{(x_i - u)^2}{n}$$

$$= \frac{(5-14)^2 + (10-14)^2 + \text{---} + (24-14)^2}{6}$$

$= 35.66$

$S.D = \sqrt{35.66} = 5.97$

Samples of size 2 without replacement

(5,10) (5,14) (5,18) (5,13) (5,24)

(10,14) (10,18) (10,13) (10,24)

(14,18) (14,13) (14,24)

(18,13) (18,24)

(13,24)

Sampling distribution of means

= 7.5, 9.5, 11.5, 9, 14.5

12, 14, 11.5, 17

16, 13.5, 19

15.5, 21

18.5

mean of sampling distribution of means

$$u(\bar{x}) = \frac{7.5 + 9.5 + \text{---} + 18.5}{15}$$

$= 14$

4) S.D of
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S.D = √1

2) A popula
16, 20, 24
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Sampling
4, 8, 12
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Samples

(4,8)

(8,12

(12,16

(16,20

(20,24

Samplin

= 6,

10,

14,

18,

22

4) S.D of sampling distribution of means

$$\text{Variance } (\sigma^2) = \frac{(7.5-14)^2 + --- + (18.5-14)^2}{15}$$

$$= 14.26$$

$S.D = \sqrt{14.26} = 3.77$

2) A population consist of 6 numbers 4, 8, 12, 16, 20, 24. Consider all possible samples of size 2 that can be drawn without replacement. Find population mean, population S.D, mean of sampling distribution of means, S.D of sampling distribution of means.

4, 8, 12, 16, 20, 24

1) mean of population

$$\frac{4+8+12+16+20+24}{6} = 14$$

2) S.D of population

$$\sigma^2 = \sum \frac{(x_i-u)^2}{n}$$

$$= \frac{(4-14)^2 + (8-14)^2 + --- + (24-14)^2}{6}$$

$$= 46.66$$

$S.D = \sqrt{46.66} = 6.83$

Samples of size 2 without replacement

(4,8) (4,12) (4,16) (4,20) (4,24)
(8,12) (8,16) (8,20) (8,24)
(12,16) (12,20) (12,24)
(16,20) (16,24)
(20,24)

Sampling distribution of means

= 6, 8, 10, 12, 14
10, 12, 14, 16
14, 16, 18
18, 20
22

mean of sampling distribution of means

$$u = \frac{6 + 8 + \,---\, + 22}{15}$$

$= 14$

S.D of sampling distribution of means

$$\text{Variance } (\sigma^2) = \frac{(6-14)^2 + \,---\, + (22-14)^2}{15}$$

$= \cancel{14.66}\ 18.66$

$S.D = (\sqrt{14.66} = 3.82)\times \quad \sqrt{18.4} = 4.32$

3) Samples of size 2 are taken from population 1 2 3 4 5 6 1) with replacement 2) without replacement

1 2 3 4 5 6

1) mean of population

$$\frac{1+2+3+4+5+6}{6} = 3.5$$

2) S.D of population

$$\sigma^2 = \sum \frac{(x_i - u)^2}{n}$$

$$= \frac{(1-3.5)^2 + \,---\, + (6-3.5)^2}{6}$$

$= \cancel{2.875}\ 2.916$

$S.D = \sqrt{2.916} = 1.707$

Sampling distribution of means with replacement

(1,1) (1,2) (1,3) (1,4) (1,5) (1,6)
(2,1) (2,2) (2,3) (2,4) (2,5) (2,6)
(3,1) (3,2) (3,4) (3,3) (3,5) (3,6)
(4,1) (4,2) (4,3) (4,4) (4,5) (4,6)
(5,1) (5,2) (5,3) (5,4) (5,5) (5,6)
(6,1) (6,2) (6,3) (6,4) (6,5) (6,6)

mean of s

1 , 1.5

1.5, 2

2, 2.

2.5, 3

3, 3.5

3.5, 4

mean of S

$u = 1$

$= 3$

Variance

$\sigma^2 =$

$S.D = \sqrt{}$

2) Witho

Samplin

(1,2)

(2,3)

(3,4)

(4,5)

(5,6

mean of

1.5

2.5

3.5

4.5

5.5

mean of

mean of Sample distributions

1, 1.5, 2, 2.5, 3, 3.5
1.5, 2, 2.5, 3, 3.5, 4
2, 2.5, 3, 3.5, 4, 4.5
2.5, 3, 3.5, 4, 4.5, 5
3, 3.5, 4, 4.5, 5, 5.5
3.5, 4, 4.5, 5, 5.5, 6

mean of Sampling distribution of means

$$\mu = \frac{1 + 1.5 + \cdots + 6}{36}$$

$$= 3.5$$

variance of sampling distribution of means

$$\sigma^2 = \frac{(1-3.5)^2 + \cdots + (6-3.5)^2}{36}$$

$$= 1.45$$

$$S.D = \sqrt{1.45} = 1.20$$

2) Without replacement

Sampling distribution of means

(1,2) (1,3) (1,4) (1,5) (1,6)
(2,3) (2,4) (2,5) (2,6)
(3,4) (3,5) (3,6)
(4,5) (4,6)
(5,6)

mean of Sample distributions

1.5 = 2.25
0.5 = 0.25
2.5 = 6.25

1.5 2 2.5 3 3.5
2.5 3 3.5 4
3.5 4 4.5
4.5 5
5.5

mean of Sampling distribution of means

$$\mu = \frac{1.5 + 2 + \cdots + 5.5}{15}$$

$$= 3.5$$

Variance of Sampling distribution

$$(\sigma^2) = \sum \frac{(x_i - u)^2}{n}$$

$$= \frac{(1.5-3.5)^2 + --- + (5.5-3.5)^2}{15}$$

$= 1.16 \quad S.D = \sqrt{1.16} = 1.07$

4) Samples of size 2 are taken from the population 3 6 9 15 27 with replacement
1) Find mean of population, S.D of population mean of sampling distribution of mean, S.D
3) mean of sampling distribution of means of sampling distribution of means

3 6 9 15 27

1) mean of population

$$\frac{3+6+9+15+27}{5} = 12$$

2) S.D of population

$$\sigma^2 = \frac{\sum (x_i - u)^2}{n}$$

$$= \frac{(3-12)^2 + (6-12)^2 + --- (27-12)^2}{5}$$

$= 63.33 \quad = 72$

$S.D = \sqrt{63.33} = 7.95 \quad = \sqrt{72} = 8.48$

Sampling distribution of means with replacement

(3,3) (3,6) (3,9) (3,15) (3,27)
(6,3) (6,6) (6,9) (6,15) (6,27)
(9,3) (9,6) (9,9) (9,15) (9,27)
(15,3) (15,6) (15,9) (15,15) (15,27)
(27,3) (27,6) (27,9) (27,15) (27,27)

mean of sampling distributions

= 3	4.5	6	9	15
4.5	6	7.5	10.5	16.5
6	7.5	9	12	18
9	10.5	12	15	21
15	16.5	18	21	27

mean of Sampling distribution of means

$$\mu = \frac{3 + 4.5 + \cdots 27}{25}$$

$= 12$

Variance of Sampling distribution of means

$$\sigma^2 = \frac{(3-12)^2 + \cdots (27-12)^2}{25}$$

$= 36.36$

$S.D = \sqrt{36.36}$

$= 6.02$

5) A population consist of 5 numbers 3, 6, 9, 15, 27. List all possible samples of size 3 that can be drawn without replacement 1) population mean, population S.D

All possible samples of size 3 are

(3, 6, 9) (3, 6, 15) (3, 6, 27)

(3, 9, 15) (3, 9, 27) (3, 15, 27)

(6, 9, 15) (6, 9, 27) (6, 15, 27)

(9, 15, 27)

1) mean of population

$$= \frac{3+6+9+15+27}{5} = 12$$

2) S.D of population

$$\sigma^2 = \sum \frac{(x_i - \mu)^2}{n}$$

$$= \frac{(3-12)^2 + \cdots + (27-12)^2}{5}$$

$= 63.33$ $= 72$

$S.D = \sqrt{63.33} = 7.95$ $= 8.48$

mean of Sampling distributions

6	8	12	
9	13	15	
10	14	16	17

mean of Sampling distribution of means

$$\mu = \frac{6+8+ - - - +17}{10}$$

$$= 12$$

S.D of Sampling distribution of means

$$\sigma^2 = \frac{(6-12)^2 + - - - - + (17-12)^2}{10}$$

$$= 12$$

$$S.D = \sqrt{12}$$

$$= 3.46$$

6) Population Contains 1, 5, 6, 8 Consider all possible Samples of Size 2 with replacement

1) mean of Sample population

$$\mu = \frac{1+5+6+8}{4} = 5$$

2) S.D of population

$$\sigma^2 = \sum \frac{(x_i-\mu)^2}{n}$$

$$= \frac{(1-5)^2 + - - - +(8-5)^2}{4}$$

$$= 6.5$$

$$S.D = \sqrt{6.5} = 2.54$$

Sample distribution of Size 2 with replacement

(1,5) (1,1) (1,6) (1,8)
(5,1) (5,5) (5,6) (5,8)
(6,1) (6,5) (6,6) (6,8)
(8,1) (8,5) (8,6) (8,8)

mean of Sampling distribution of means

3	1	3.5	4.5
3	5	5.5	6.5
3.5	5.5	6	7
4.5	6.5	7	8

mean of Sa
$\mu = 3$
$= 5$
S.D of Sa
$\sigma^2 = ($
$=$
$S.D = \sqrt{2.}$

7/11 Normal
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The nor
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$z_2 = (x$

1) Find th
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$n = 10$
$\mu = np$
$\sigma^2 = np$
$P($
when
when

mean of sampling distribution of means

$$\mu = \frac{3+1+ - - - - +8}{16}$$

$= 5$

S.D of sampling distribution of means

$$\sigma^2 = \frac{(3-5)^2 + - - - + (8-5)^2}{16}$$

$= 2.90$

$S.D = \sqrt{2.90} = 1.70$

7/11 Normal Approximation to binomial distribution

The normal distribution can be used to approximate the binomial distribution. Suppose the no. of success 'x' ranges from x_1 to x_2 then the probability of getting x_1 to x_2 success is given by $\sum_{r=x_1}^{x_2} n_{C_x} p^x q^{n-x}$, for large n the calculation of binomial distribution is very difficult hence binomial curve can be replaced by the normal curve and the required probabilities can be computed where $z_1 = \frac{(x_1 - \frac{1}{2}) - \mu}{\sigma}$

$z_2 = \frac{(x_2 + \frac{1}{2}) - \mu}{\sigma}$ where mean $\mu = np$

1) Find the probability of 100 patients b/w 84 and 95 inclusive can survive a heart operation. Given that the chances of survival is 0.9

$n = 100$, $p = 0.9$ $q = 1 - 0.9 = 0.1$

$\mu = np = 100 \times 0.9 = 90$

$\sigma^2 = npq = 100 \times 0.9 \times 0.1 = 9 \Rightarrow \sigma = \sqrt{npq} = \sqrt{9} = 3$

$P(84 \le x \le 95)$

when $x_1 = 84 \Rightarrow z_1 = \frac{(84 - \frac{1}{2}) - 90}{3} = -2.16$

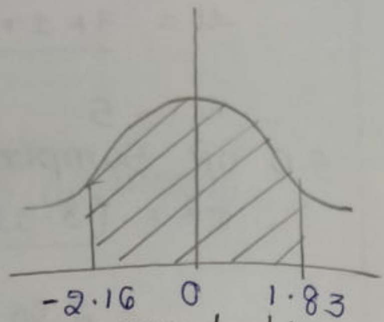
when $x_2 = 95 \Rightarrow z_2 = \frac{(95 + \frac{1}{2}) - 90}{3} = 1.83$

$$= \int_{-2.16}^{0} \phi(z)\,dz + \int_{0}^{1.83} \phi(z)\,dz$$

$$= \int_{0}^{2.16} \phi(z) + 0.4664$$

$$= 0.4846 + 0.4664$$

$$= 0.9514$$



2) 8 Coins are tossed together. Find the probability of getting 1 to 4 heads in single toss

$n = 8$, $p = \frac{1}{2} \Rightarrow np = 8 \times \frac{1}{2} = 4$ $\sqrt{npq} = \sqrt{8 \times \frac{1}{2} \times \frac{1}{2}} = \sqrt{2}$

$p(1 < x < 4)$

when $x_1 = 1 \Rightarrow z_1 = \frac{\left(1 - \frac{1}{2}\right) - 4}{\sqrt{2}} = -2.47$

$x_2 = 4 \Rightarrow z_2 = \frac{\left(4 + \frac{1}{2}\right) - 4}{\sqrt{2}} = 0.35$

$$= \int_{-2.47}^{0} \phi(z)\,dz + \int_{0}^{0.35} \phi(z)\,dz$$

$$= \int_{0}^{2.47} \phi(z)\,dz + \int_{0}^{0.35} \phi(z)\,dz$$

$$= 0.4932 + 0.1368$$

$$= 0.63$$

Find the probability that guess ane a student can correctly answer 25 to 30 questions in a multiple choice quiz consisting of 80 questions assume that in each question 4 choices only 1 choice is correct and student has no knowledge of the subject.

$n = 80$, $p = \frac{1}{4}$, $q = \frac{3}{4}$

$\mu = np = 80 \times \frac{1}{4} = 20$ $\sigma = \sqrt{npq} = \sqrt{80 \times \frac{1}{4} \times \frac{3}{4}} = \sqrt{15}$

$P(25 < x < 30)$

when $x_1 = 25 \Rightarrow z_1 = \frac{\left(25 - \frac{1}{2}\right) - 20}{\sqrt{15}} = 1.16$

$x_2 = 30 \Rightarrow z_2 = \left(30 + \frac{1}{2}\right) - 20 / \sqrt{15} = 2.71$

$$= -\int_0^{1.16} \phi(z) + \int_0^{2.71} \phi(z)$$

$= -0.3770 + 0.4966$

$= \cancel{0.873}$ 0.1196

4) Find the probability of getting an even number on face 3 to 5 times in throwing 10 dice together

$n = 10$, $p = \frac{3}{6} = \frac{1}{2}$ $q = \frac{1}{2} \Rightarrow np = 5$ $\sigma = \sqrt{npq} = \sqrt{10 \times \frac{1}{2} \times \frac{1}{2}}$
$= \frac{\sqrt{10}}{2}$

$P(3 < x < 5)$

when $x_1 = 3 \Rightarrow z_1 = \frac{\left(3 - \frac{1}{2}\right) - 5}{\sqrt{10}/2} = -1.11$

when $x_2 = 5 \Rightarrow z_2 = \frac{\left(5 + \frac{1}{2}\right) - 5}{\sqrt{10}/2} = 0.22$

$$= \int_{-1.11}^{0} \phi(z)\,dz + \int_0^{0.22} \phi(z)\,dz \Rightarrow \int_0^{1.11} \phi(z)\,dz + \int_0^{0.22} \phi(z)\,dz$$

$= 0.3665 + 0.0871$

$= 0.4536$

Chebychevs Inequality

The role of S.D as a parameter, the to characterize variance is precisely interpreted by Chebychevs inequality

Chebychevs Theorem:

If x is a random variable with mean (μ) & Variance (σ^2) then for any positive number k we have probability 1) $P(|x-\mu| \geq k\sigma) \leq \frac{1}{k^2}$

2) $P(|x-\mu| < k\sigma) \geq 1 - \frac{1}{k^2}$

Since x is a random variable, by definition

Variance $(\sigma^2) = E[(x-\mu)^2]$

Proof :

$$\sigma^2 = E[(x-u)^2]$$

$$= \int_{-\infty}^{\infty} (x-u)^2 f(x)\,dx$$

$$= \int_{-\infty}^{u-k\sigma} (x-u)^2 f(x)\,dx + \int_{u-k\sigma}^{u+k\sigma} (x-u)^2 f(x)\,dx + \int_{u+k\sigma}^{\infty} (x-u)^2 f(x)\,dx$$

$$\sigma^2 \geq \int_{-\infty}^{u-k\sigma} (x-u)^2 f(x)\,dx + \int_{u+k\sigma}^{\infty} (x-u)^2 f(x)\,dx \quad - (1)$$

Consider $|x-u| \geq k\sigma$

$$\pm(x-u) \geq k\sigma$$

$$(\pm(x-u))^2 \geq (k\sigma)^2$$

$$(x-u)^2 \geq k^2\sigma^2 \quad - (2)$$

Sub (2) in (1)

$$\sigma^2 \geq \int_{-\infty}^{u-k\sigma} (x-u)^2 f(x)\,dx + \int_{u+k\sigma}^{\infty} (x-u)^2 f(x)\,dx$$

$$\geq \int_{-\infty}^{u-k\sigma} k^2\sigma^2 f(x)\,dx + \int_{u+k\sigma}^{\infty} k^2\sigma^2 f(x)\,dx$$

$$= k^2\sigma^2 \left[\int_{-\infty}^{u-k\sigma} f(x)\,dx + \int_{u+k\sigma}^{\infty} f(x)\,dx \right]$$

$$= k^2\sigma^2 \left[P(x \leq u-k\sigma) - P(x \geq u+k\sigma) \right]$$

$$= k^2\sigma^2 \left[P(x-u \leq -k\sigma) + P(x-u \geq k\sigma) \right]$$

$$= k^2\sigma^2 P(|x-u| \geq k\sigma)$$

$$\therefore \sigma^2 \geq k^2\sigma^2 P(|x-u| \geq k\sigma)$$

$$\frac{\cancel{\sigma^2}}{k^2\cancel{\sigma^2}} \geq P(|x-u| \geq k\sigma)$$

$$\boxed{P(|x-u| \geq k\sigma) \leq \frac{1}{k^2}} \qquad -P(|x-u| \geq k\sigma) \geq \frac{-1}{k^2}$$

w.k.t $P(|x-u| \geq k\sigma) + P(|x-u| < k\sigma) = 1$

$$P(|x-u| < k\sigma) = 1 - P(|x-u| \geq k\sigma$$

$$P(|x-u| < k\sigma) \geq 1 - \frac{1}{k^2}$$

8/11 ASymmetri

lower boun

80 to 120

$n = 600$

$p =$ proba

$u = np =$

$\sigma = \sqrt{npq}$

By Cheb

$P(|x-u$

$P(|x-$

$P(-k$

$P(80$

80

80

$80\times$

$\therefore$ lowe

8/11 A Symmetric die is thrown 600 times Find the lower bound for the probability of getting 80 to 120 Sixes

$n = 600$

p = probability of getting six $= \frac{1}{6}$ $q = \frac{5}{6}$

$\mu = np = \frac{1}{6} \times 600 = 100$

$\sigma = \sqrt{npq} = \sqrt{600 \times \frac{1}{6} \times \frac{5}{6}} = \frac{5\sqrt{30}}{3} = 9.12$

By Chebychevs inequality

$P(|x-\mu| < k\sigma) \geq 1 - \frac{1}{k^2}$

$P\left(|x-100| < k\sqrt{\frac{500}{6}}\right) \geq 1 - \frac{1}{k^2}$

$P\left(-k\sqrt{\frac{500}{6}} < x - 100 < +k\sqrt{\frac{500}{6}}\right) \geq 1 - \frac{1}{k^2}$

$P(80 < x - 100 < 120) \geq 1 - \frac{1}{k^2}$

$80 = -k\sqrt{\frac{500}{6}}$

$80^2 = k^2 \frac{500}{6}$

$\frac{80 \times 80 \times 6}{500} = k^2$

$k^2 = \frac{384}{5}$

$\therefore$ lower boundary is $1 - \frac{1}{k^2}$

$1 - \frac{1}{\frac{384}{5}} = 1 - \frac{5}{384} = \frac{379}{384}$