Relations and Functions

Ordered pair: An ordered pair is a pair of objects whose components occur in a special order. It is written by listing the components in the specified order, separating them by a comma and enclosing the pair in parenthesis. In the ordered pair (a, b), a is called the first component and b, the second component.

Cartesian product of sets: Let A and B be sets. Cartesian product of A and B, denoted by $A \times B$ and is defined as $A \times B = \{(a,b) : a \in A \& b \in B\}$

i.e., $A \times B$ is the set of all possible ordered pairs whose first component comes from A and whose second component comes from B.

For example: if $A = \{a, b\}$ and $B = \{1\}$ then $A \times B = \{(a,1), (b,1)\}$

$B \times A = \{(1,a), (1,b)\}$ $A \times A = \{(a,a)(a,b)(b,b)(b,a)\}$ $B \times B = \{(1,1)\}$

NOTE 1: ① $A \times B \neq B \times A$

② if a set A has "m elements" and B has n elements then $|A \times B| = mn$

i.e., $|A \times B| = |A||B|$.

→ The idea of cartesian product of sets can be extended to any finite no of sets. For any non-empty sets $A_1, A_2 \cdots A_k$, the k-fold product $A_1 \times A_2 \times \cdots \times A_k$ is defined as the set of all ordered k-tuples $(a_1, a_2, \cdots, a_k)$, where $a_i \in A_i$, $i = 1, 2, \cdots, k$. That is

$$A_1 \times A_2 \times \cdots \times A_n = \{(a_1, a_2, \ldots a_k) \mid a_i \in A_i \ i = 1, 2, \cdots, k\}$$

and $|A_1 \times A_2 \times \cdots \times A_n| = |A_1| |A_2| \cdots |A_k|$

For example, if $A = \{1, 0\}$ $B = \{2, -2\}$ $C = \{0, -1\}$, then

$A \times B \times C = \{(1,2,0)(1,-2,0)(1,2,-1)(1,-2,-1)(0,2,0)(0,2,-1)$
$(0,-2,0)(0,-2,-1)\}$

Pb 1 Find x and y in each of the following cases.

① $(2x-3, 3y+1) = (5, 7)$ ② $(x+2, 4) = (5, 2x+y)$

③ $(x, y) = (x^2, y^2)$ ④ $(x, y) = (y^2, x^2)$

Ans
① $(4, 2)$
② $(3, -2)$
③ $x = 0, 1$ & $y = 0, 1$
④ $x = y = 0$ (or) $x = y^2$ where $x^3 = 1$

(5) $\underset{}{\overset{(3,-2)}{(2x, x+y)}} = (6,1)$ (6) $(y-2, 2x+1) = (x-1, y+2)$ Ans $(2,3)$

(4) $x = y^2$ & $y = x^2$ sub(2) in(1) $x = x^4 \Rightarrow x^3 = 1 \Rightarrow x = 1;\ y = 1$

$x(1-x^3) = 0 \Rightarrow x = 0;\ y = 0$

Pb2 Let $A = \{1,3,5\}$ $B = \{2,3\}$ and $C = \{4,6\}$. write the following

(1) $A \times B = \{(1,2)(1,3)(3,2)(3,3)(5,2)(5,3)\}$

(2) $B \times A =$

(3) $B \times C =$

(4) $A \times C =$

(5) $(A \cup B) \times C = \{1,2,3,5\} \times \{4,6\} = \{(1,4)(1,6),(2,4)(2,6),(3,4)(3,6)(5,4)(5,6)\}$

(6) $A \cup (B \times C) = \{1,3,5\} \cup \{(2,4)(2,6)(3,4)(3,6)\} = \{1,3,5,(2,4)(2,6)(3,4)(3,6)\}$

(7) $(A \times B) \cup C = \{(1,2)(1,3)(3,2)(3,3)(5,2)(5,3), 4, 6\}$

(8) $A \cap (B \times C) = \phi$

(9) $(A \times B) \cup (B \times C) = \{(1,2),(1,3)(3,2)(3,3)(5,2)(5,3),(2,4)(2,6)(3,4)(3,6)\}$

(10) $(A \times B) \cap (B \times A) = \{(3,3)\}$

(11) $(A \times B) \cap (B \times C) = \phi$

Pb3 if $A = \{1,2,3\}$ $B = \{4,5\}$ $C = \{1,2,3,4,5\}$ find (i) $A \times B$ (ii) $C \times B$ (iii) $B \times B$

Pb4 Given $A = \{a,b\}$ and $B = \{1,2,3\}$ find $A \times B$, $B \times A$, $A \times A$, & $B \times B$

Pb5 Given $A = \{1,2\}$, $B = \{a,b,c\}$ and $C = \{3,4\}$ find $A \times B \times C$ and $B \times C \times A$

Pb6 Let $A = \{1,2,3,4\}$ $B = \{2,5\}$ $C = \{3,4,7\}$ write down the following $A \times B$, $B \times A$, $A \cup (B \times C)$, $(A \cup B) \times C$, $(A \times C) \cup (B \times C)$

(7) if $A = \{2,3\}$ $B = \{-1,2\}$ and $C = \{a,b\}$, verify that $A \times (B \cup C) = (A \times B) \cup (A \times C)$ and $A \times (B \cap C) = (A \times B) \cap (A \times C)$

Properties of Cartesian Product:

For the four sets A, B, C and D

1. $(A \cap B) \times (C \cap D) = (A \times C) \cap (B \times D)$
2. $(A - B) \times C = (A \times C) - (B \times C)$
3. $(A \cup B) \times C = (A \times C) \cup (B \times C)$
4. $A \times (B \cap C) = (A \times B) \cap (A \times C)$
5. $(A \cap B) \times C = (A \times C) \cap (B \times C)$
6. $A \times (B - C) = (A \times B) - (A \times C)$

(1) Prove that $A \times (B \cap C) = (A \times B) \cap (A \times C)$

__Proof:__ Let (x, y) be any element of $A \times (B \cap C)$, Then $x \in A$ and $y \in (B \cap C)$

$\Rightarrow x \in A$ and $(y \in B$ and $y \in C)$ $\Rightarrow (x \in A$ and $y \in B)$ and $(x \in A$ and $y \in C)$

$\Rightarrow (x, y) \in (A \times B)$ and $(x, y) \in (A \times C)$

$\Rightarrow (x, y) \in (A \times B) \cap (A \times C)$

$\therefore A \times (B \cap C) \subseteq (A \times B) \cap (A \times C) \rightarrow (1)$

similarly we can prove $(A \times B) \cap (A \times C) \subseteq A \times (B \cap C) \rightarrow (2)$

from (1) & (2) $A \times (B \cap C) = (A \times B) \cap (A \times C)$

(2) If A, B, C are three sets such that $A \subseteq B$ show that $(A \times C) \subseteq (B \times C)$

__Sol:__ Let $(x, y) \in A \times C \Rightarrow x \in A$ and $y \in C$

Since $A \subseteq B \Rightarrow x \in B$ and $y \in C \Rightarrow (x, y) \in (B \times C)$

$\therefore A \times C \subseteq (B \times C)$

Relations:- The word relation is used to indicate a relationship between two objects. There are many kinds of relationship in the world. we deal with relationship between student and teachers, an employee and his salary, and so on. The often used relations in mathematics are less than (<) greater than (>), "subset of" and so on. A relation between two objects can be defined by listing the two objects as an ordered pair. A set of all such ordered pairs, in each of which the first member has some definite relationship to the second, describes a particular relation. This method of specifying a relation does not require any special symbol or description and so is suitable for any relation between any two sets. In this topic, we discuss the mathematics of relations discuss the mathematics of relations defined on sets, various ways of representing relations and explore various properties they may have

Def:- Let A and B be two sets. Then a subset of $A \times B$ is called a relation from A to B. Thus, if R is a relation from A to B then R is a set of ordered pairs (a,b) where $a \in A$ and $b \in B$.

→ If $(a,b) \in R$, we say that "a is related to b by R"; this is denoted by aRb.

→ If R is a relation from A to A, that is, if R is subset of $A \times A$, we say that R is a binary relation on A.

→ If $(a,b) \notin R$, then a is not related to b by R and is written as $a \not R b$.

Domain and Range of a relation: The set of first coordinates of every ordered pair (element) of R is called the domain of R and the set of second coordinates of every ordered pair (element) of R is called Range of R. symbolically we can write:

If R is a relation from A to B, then domain of $R = d(R) = \{x : x \in A$ and $(x,y) \in R\}$

and range of $R = r(R) = \{y : y \in B, (x,y) \in R\}$.

Inverse Relation: If R is relation from a set A to a set B, then the inverse of R, denoted by R^{-1} is the relation from the set B to set A which contains all the ordered pairs (elements) of R in which first and second coordinates of each ordered pair are inter-changed. It means that R^{-1} shall consists of all those ordered pairs which if reversed shall belong to R. That is

$$R^{-1} = \{(y,x) : (x,y) \in R\}.$$

Identity Relation: A relation R in a set A is said to identity relation generally denoted by I_A, if $I_A = \{(x,x) : x \in A\}$

Ex: Let $A = \{1,2,3\}$ then $I_A = \{(1,1)(2,2)(3,3)\}$ is an identity relation in A.

n-ary Relation: Let $\{A_1, A_2, \cdots, A_n\}$ be a finite collection of sets. A subset R of $A_1 \times A_2 \times \cdots \times A_n$ is called n-ary relation on $A_1, A_2 \cdots A_n$.

Representation of a relations:

There are five main methods to represent a relation R from a set A to set B.

1. **Roaster method:** In this method all the elements (ordered pairs) of the relation are enclosed within braces.

Ex: Let $A = \{1,2,3\}$ and $B = \{x,y,z\}$ then $R = \{(1,y)(1,z)(3,y)\}$

(2) **matrix method or matrix of a Relation:** consider the sets $A = \{a_1, a_2 \cdots a_n\}$ and $B = \{b_1, b_2, \cdots b_n\}$ of orders m and n respectively. Then $A \times B$ consists of all ordered pairs of the form (a_i, b_j) $1 \le i \le m$, $1 \le j \le n$ which are mn in number.

Let R be a relation from A to B so that R is subset of $A \times B$

now, let us put $m_{ij} = (a_i, b_j)$ and assign the values 1 or 0 to m_{ij} as follows $m_{ij} = \begin{cases} 1 & \text{if } (a_i, b_j) \in R \\ 0 & \text{if } (a_i, b_j) \notin R \end{cases}$

The $m \times n$ matrix formed by these m_{ij}'s is called the adjacency matrix or the matrix of the relation R or the relation matrix for R, and is denoted by M_R (or) $M(R)$.

It is to be noted that the rows of M_R correspond to the elements of A and the columns to those of B.

when $B = A$, the matrix M_R is a square matrix with 'n elements'.

Ex: 1 The relation $R = \{(1,y), (1,z), (3,y)\}$ from $A = \{1,2,3\}$ to $B = \{x,y,z\}$ can be represented in matrix form as follows

$$M_R = \begin{array}{c} \\ 1 \\ 2 \\ 3 \end{array} \begin{array}{c} \begin{array}{ccc} x & y & z \end{array} \\ \begin{bmatrix} 0 & 1 & 1 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \end{array}$$

(2) Define the relation R for the adjacency matrix given by

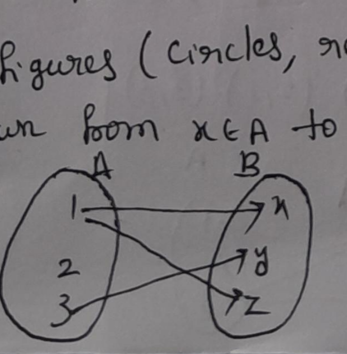
$$M_R = \begin{array}{c} \\ a \\ b \\ c \end{array} \begin{array}{c} \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 1 & 0 & 1 & 0 \end{bmatrix}_{3 \times 4} \end{array}$$

sol:- Let R be a relation from set A to set B. As M_R is 3×4, the A and B have 3 & 4 elements

Let sets $A = \{a, b, c\}$ $B = \{1, 2, 3, 4\}$

$\therefore R = \{(a,1), (a,4), (b,2), (b,3), (c,1), (c,3)\}$

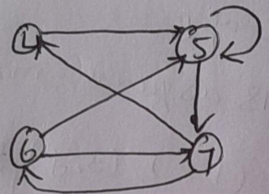
(3) Arrow diagram:- The elements of sets A and B are written in two disjoint plane figures (circles, rectangles, discs etc.) and then arrows are drawn from $x \in A$ to $y \in B$ if xRy.



The diagram corresponding to the relation $R = \{(1,x)(1,z)(3,y)\}$ is

Digraph of a relation on sets: when a relation is from a finite set A to itself can be represented by a digraph also. First the elements of A are written down. Then arrows are drawn from each element x to each element y $(x, y \in A)$ whenever x is related to y i.e., xRy.

Ex: Let the set $A = \{4, 5, 6, 7\}$ and the relation R on A $= \{(4,5)(5,5)(5,7)(6,5)(6,7)(7,4)(7,6)\}$ this relation can be represented by a digraph as shown below



Set Builder form: In this method the rule that associates the first and second coordinates of each ordered Pair is given.

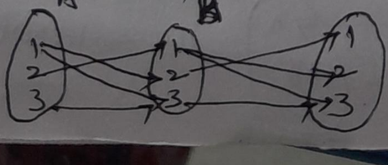
Ex: Relation from A to B given by $R = \{(x,y) : x \in A \text{ and } y \in B, x < y\}$ where $A = \{1, 2, 4\}$, $B = \{2, 3, 5\}$ $R = \{(1,2)(1,3)(1,5), (2,3), (2,5), (4,5)\}$

Composition of Relations: If A, B and C are three non empty sets and R and S are the relations from A to B and B to C respectively then $R \subseteq A \times B$ and $S \subseteq B \times C$. Then we define a relation from A to C denoted by $R \circ S$ given by $R \circ S = \{(x, z) : \exists \text{ some } y \in B \ni (x,y) \in R \text{ \& } (y,z) \in S\}$ This relation is called a composition of R and S or a composite relation of R and S.

Ex: R and S are relations on $A = \{1, 2, 3\}$ $R = \{(1,1)(1,2)(2,3)(3,1)(3,3)\}$ $S = \{(1,2)(1,3), (2,1), (3,3)\}$ then find $R \circ S$ and $S^2 = S \circ S$.

(a) To find $R \circ S = \{(1,2)(1,1)(2,3)(3,2)(3,3)(1,3)\}$

$S^2 = S \circ S = \{(1,1)(1,3)(2,2)(3,3)(2,3)\}$



Universal Relation: A relation R in a set A is called universal relation if $R = A \times A$.

For example if $A = \{1,2,3\}$ then $R = \{(1,1)(1,2)(1,3)(2,1)(2,2)(2,3)(3,1)(3,2)(3,3)\}$

Operations on Relations: Since a relation is a subset of the cartesian product of two sets, the set-theoretic operations may be used to construct new relations from given relations.

Union Given the relations R_1 and R_2 from a set A to set B, the union of R_1 and R_2, denoted by $R_1 \cup R_2$ is defined as a relation from A to B with the property that $(a,b) \in R_1 \cup R_2$ iff $(a,b) \in R_1$ or $(a,b) \in R_2$

Intersection: The intersection of R_1 and R_2, denoted by $R_1 \cap R_2$, is defined as a relation from A to B with the property that $(a,b) \in R_1 \cap R_2$ iff $(a,b) \in R_1$ and $(a,b) \in R_2$.

→ Evidently, $R_1 \cup R_2$ is the union of the sets R_1 and R_2 and $R_1 \cap R_2$ is the intersection of the sets R_1 and R_2 in the universal set $A \times B$.

Complement of a relation: Given a relation R from a set A to a set B the complement of R, denoted by $\bar{R}$ (or) R', is defined as a relation from A to B with the property that $(a,b) \in \bar{R}$ iff $(a,b) \notin R$. In other words $\bar{R}$ is the complement of the set R in the universal set $A \times B$.

Difference of R and S: The difference of R and S is denoted by $R-S$ and is defined as a relation from A to B with the property that $(a,b) \in R-S$ iff $(a,b) \in R$ and $(a,b) \notin S$.

→ **Converse of a relation:** Given a relation R from a set A to set B the converse of R is defined as a relation from B to A with the property that $(a,b) \in R$ iff $(b,a) \in$ converse of R. and is denoted by R^C.

NOTE: If M_R is the matrix of R, then $(M_R)^T$ is the matrix of converse of R. and $(R^C)^C = R$

Ex) Consider the sets $A = \{a, b, c\}$ and $B = \{1,2,3\}$ and the relations $R = \{(a,1)\ (b,1)\ (c,2)\ (c,3)\}$ and $S = \{(a,1)\ (a,2)\ (b,1)\ (b,2)\}$ from A to B. Determine $\bar{R}$, $\bar{S}$, $R \cup S$, $R \cap S$, $R - S$, converse of R, S.

Sol:- $A \times B - \{(a,1)\ (a,2)\ (a,3)\ (b,1)\ (b,2)\ (b,3)\ (c,1)\ (c,2)\ (c,3)\}$

$\bar{R} = \{(a,2)(a,3)\ (b,2)(b,3)\ (c,1)\}$

$\bar{S} = \{(a,3)\ (b,3)\ (c,1)(c,2)\ (c,3)\}$

$R \cup S = \{(a,1)\ (a,2)\ (b,1)\ (b,2)\ (c,2)\ (c,3)\}$

$R \cap S = \{(a,1)\ (b,1)\}$

$R - S = \{(a,2)\ (b,2)\}$

converse of $R = \{(1,a)\ (1,b)\ (2,c)\ (3,c)\}$

$S = \{(1,a)\ (2,a)\ (1,b)\ (2,b)\}$

Pb2 Let $A = \{1,2,3\}$ and $B = \{1,2,3,4\}$. The relations R and S from A to B are represented by the matrices $m_R = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{bmatrix}$ $m_S = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \end{bmatrix}$

Find the relations $\bar{R}$, $R \cup S$, $R \cap S$ and converse of S and their matrix representations.

$R = \{(1,1)(1,3)\ (2,4)(3,1)\ (3,2)(3,3)\}$

(i) $\bar{R} = \{(1,2)(1,4)\ (2,1)\ (2,2)(2,3)(3,4)\}$; $m_{\bar{R}} = \begin{array}{c|cccc} & 1 & 2 & 3 & 4 \\ \hline 1 & 0 & 1 & 0 & 1 \\ 2 & 1 & 1 & 1 & 0 \\ 3 & 0 & 0 & 0 & 1 \end{array}$

$S = \{(1,1)\ (1,2)\ (1,3)\ (1,4)(2,4)\ (3,2)(3,4)\}$

(ii) $R \cup S = \{(1,1)\ (1,3)\ (2,4)\ (3,1)\ (3,2)\ (3,3)\ (1,2)\ (1,4)\ (3,4)\}$

$m_{R \cup S}$ (from A) (To B)

$$\begin{array}{c|cccc} & 1 & 2 & 3 & 4 \\ \hline 1 & 1 & 1 & 1 & 1 \\ 2 & 0 & 0 & 0 & 1 \\ 3 & 1 & 1 & 1 & 1 \end{array}$$

(iii) $R \cap S = \{(1,1)\ (1,3)\ (2,4)\ (3,2)\}$

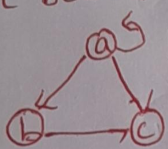
$m_{R \cap S} = \begin{array}{c|cccc} & 1 & 2 & 3 & 4 \\ \hline 1 & 1 & 0 & 1 & 0 \\ 2 & 0 & 0 & 0 & 1 \\ 3 & 0 & 1 & 0 & 0 \end{array}$

(iv) converse of $S = \{(1,1)\ (2,1)\ (3,1)\ (4,1)\ (4,2)\ (2,3)\ (4,3)\}$

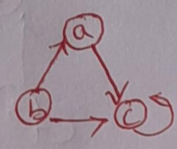
which is a relation from B to A.

$$m = \begin{array}{c|ccc} & 1 & 2 & 3 \\ \hline 1 & 1 & 0 & 0 \\ 2 & 1 & 0 & 1 \\ 3 & 1 & 0 & 0 \\ 4 & 1 & 1 & 1 \end{array}$$

Pb The degraphs of two relations R and S on the set $A = \{a, b, c\}$ are given below. Draw the degraphs of $\bar{R}$, $R \cup S$, $R \cap S$ and converse of R.



(i) R



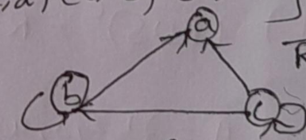
(ii) : S

By examining the given digraphs, we note that

$R = \{(a,a)\ (a,b)\ (b,c)\ (a,c)\}$ & $S = \{(b,a)\ (b,c)\ (a,c)\ (c,c)\}$

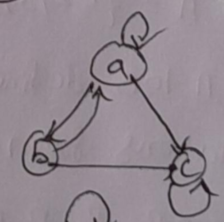
$A \times A = \{(a,a) (a,b) (a,c) (b,a) (b,b) (b,c) (c,a) (c,b) (c,c)\}$

$\bar{R} = \{(c,a)\ (b,a)\ (b,b)\ (c,b)\ (c,c)\}$

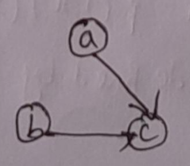


$\bar{R}$

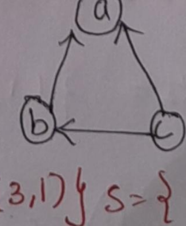
$R \cup S = \{(a,a)\ (a,b)\ (a,c)\ (b,c)\ (b,a)\ (c,c)\}$



$R \cap S = \{(a,c)\ (b,c)\}$



converse of $R = \{(a,a)\ (b,a)\ (c,b)\ (c,a)\}$

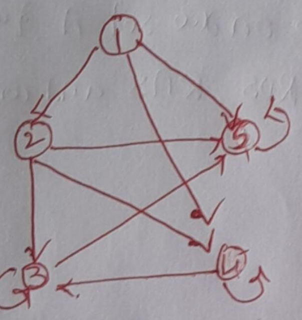


Pb ① Let $A = B = \{1,2,3\}$ and $R = \{(1,1)\ (1,2)\ (2,3)\ (3,1)\}$ $S = \{(2,1) (3,1) (3,2) (3,3)\}$ compute $\bar{R}$, $R \cap S$, $R \cup S$ and converse of S, $R - S$, $S - R$.

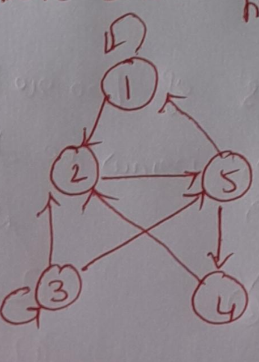
② Let $A = B = \{a, b, c, d\}$ $R = \{(a,a) (a,c) (b,c) (c,a) (d,b) (d,d)\}$ and $S = \{(a,b) (b,c) (c,a) (c,b) (d,c)\}$. compute $M(R \cap S)$, $M(R \cup S)$, M(converse of R) and $M(S)$

Pb Let $A = \{1, 2, 3\}$ and R and S be relations on A whose matrices are as given below. Find the matrices of $\bar{R}$, converse of R, $R \cap S$ and $R \cup S$ and $R - S$. $m_R = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix}$ $m_S = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 1 & 0 \\ 0 & 1 & 0 \end{bmatrix}$

Pb Let $A = \{1, 2, 3, 4, 5\}$ and R and S be relations on A whose corresponding digraphs are as given below. Find $\bar{R}$, $R \cup S$, $R \cap S$



(i): R



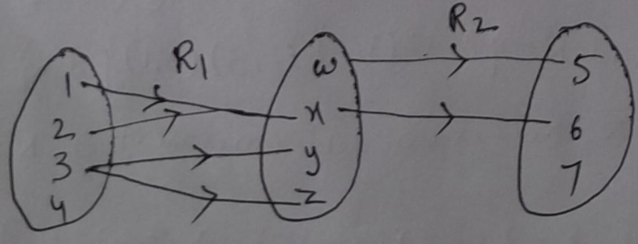
(ii): S

Pb Let R and S be relations from a set A to set B. Prove the following: (i) If $R \subseteq S$, then $R^c \subseteq S^c$ (ii) $(R \cap S)^c = R^c \cap S^c$ (iii) $(R \cup S)^c = R^c \cup S^c$.

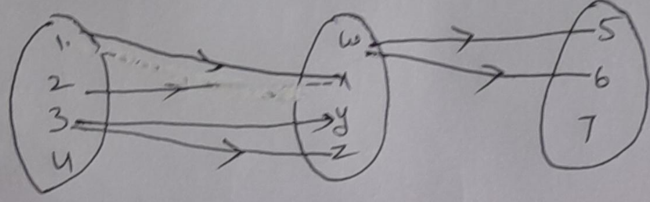
Pb Let $A = \{1, 2, 3, 4\}$ $B = \{w, x, y, z\}$ and $C = \{5, 6, 7\}$. Let R_1 be a relation from A to B defined by $R_1 = \{(1, x)(2, x)(3, y)(3, z)\}$ and R_2, R_3 be the relations from B to C, defined by $R_2 = \{(w, 5)(x, 6)\}$; $R_3 = \{(w, 5)(w, 6)\}$

Find $R_1 \circ R_2$ & $R_1 \circ R_3$

Sol: $R_1 \circ R_2 = \{(1, 6)(2, 6)\}$ $R_1 \circ R_3 = \{\} = \phi$.



$R_1 \circ R_2$



$R_1 \circ R_3$

For the relations R_1 and R_2 in the above example, find $m(R_1)$, $m(R_2)$ and $m(R_1 \circ R_2)$. verify that $m(R_1 \circ R_2) = m(R_1) \cdot m(R_2)$

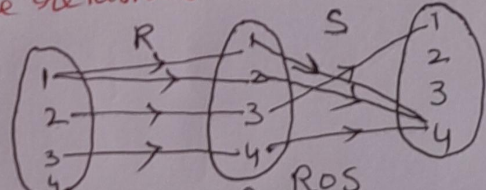
Sol:

$$m_{R_1} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}; \quad m_{R_2} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad m_{(R_1 \circ R_2)} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

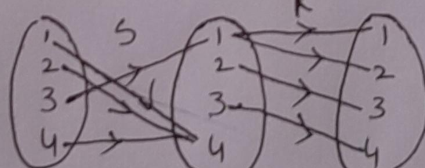
$$m_{R_1} \cdot m_{R_2} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} = m_{(R_1 \circ R_2)}.$$

Pb Let $A = \{1,2,3,4\}$ and $R = \{(1,1)(1,2),(2,3),(3,4)\}$ $S = \{(3,1),(4,4),(2,4),(1,4)\}$ be relations on A. Find the relations $R \circ S$, $S \circ R$, R^2 and S^2.

$R \circ S = \{(1,4)(2,1)(3,4)\}$



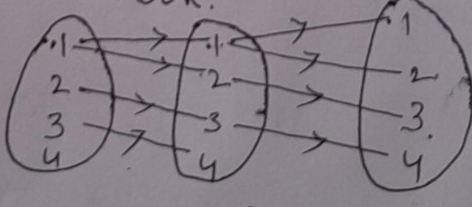
$R \circ S$



$S \circ R = \{(3,1)(3,2)\}$

$S \circ R$.

$R^2 = \{(1,1)(1,)(1,3)(2,4)\}$



R^2

$S^2 = \{(1,4)(2,4)(3,4)(4,4)\}$