# UNIT - I : MATHEMATICAL LOGIC.

Introduction, Statements and Notation, Connectives, Well formed formulas, tautologies, equivalence of formulas, duality law, functionally complete set of connectives, other connectives.

* **Mathematical Logic:** Logic is the science dealing with the methods of reasoning. Logic which uses a symbolic language to express its principles in precise and unambiguous terms is known as Mathematical Logic (or) Symbolic Logic.
It provides rules and techniques for determining whether a given argument (or) mathematical proof (or) conclusion in a scientific theory is valid (or) not.
Logic is not only used in mathematics to draw conclusions but also used in our daily life to solve many types of problems. One component of logic is proposition calculus which deals with statements with values true (or) false and is concerned with analysis of propositions, the other part is predicate calculus which deals with the predicates which proposition containing variables.

* **Proposition (or) Statement:** A proposition (or) statement is a declarative sentence i.e. either true or false but not both. For example, $3+3=6$ and $3+3=7$ are both statements, the 1st is true, 2nd is false. Similarly $x+y>4$ is not a statement because for some values of $x$ and $y$ the sentence is true $(x=1, y=5)$ and for some values of $x$ and $y$ the sentence is false $(x=1, y=2)$.
Propositions are usually represented by english ... (uppercase also).

The truth (or) falsity of a statement is called its true value. If a proposition is true, we will indicate the truth value by T (or) 1 and if it is false by the symbol 0 (or) F.

Examples: (1) p: $2+3=5$ T

(2) q: every rectangle is square F

(3) r: $x$ is an integer, it is not a proposition as we don't know the value of $x$.

(4) s: take a triangle ABC; it is not at all a declaration so it is not a proposition.

*Logical connectives: The words like NOT, OR, AND, IF THEN, IF AND ONLY IF are called connectives (or) logical operators. A proposition consisting of only a single propositional variable is called an atomic (or) primary (or) primitive (or) simple proposition i.e., they cannot be further sub-divided.

A proposition obtained from the combinations of two (or) more propositions by means of logical operators (or) connectives, to form a single proposition is referred to be molecular (or) composite (or) compound proposition.

*Truth Tables: A truth table is a table that shows the true value of a compound proposition for all possible cases. A truth table consists of columns and rows. The number of columns depends upon the no. of simple propositions and connectives used to form a compound proposition. The no. of rows in a truth table are found on the basis of simple propositions. In general, for 'n' simple propositions, the total no. of rows will be $2^n$.

It is useful in (1) finding out the validity of equivalence relation between the functions.

(2) designing and testing the electronic circuits to perform a given operation based on a certain relationship.

Connectives:

→ Negation: A proposition obtained by inserting the word NOT at an appropriate place in a given proposition is called the negation of the given proposition.

If p is any proposition, negation of p is denoted by ~p (or) ¬p and is read as NOT p.

For example, p: 3 is prime no.

~p: 3 is NOT a prime no.

p: Paris is in France.

~p: Paris is NOT in France.

Truth Table:

| p | ~p |
|---|---|
| T | F |
| F | T |

→ Conjunction: A compound proposition obtained by using AND between two given propositions is called conjunction.

The conjunction of 2 propositions p and q is denoted by $p \wedge q$ and read as p and q.

Truth Table:

| p | q | $p \wedge q$ |
|---|---|---|
| T | T | T |
| T | F | F |
| F | T | F |
| F | F | F |

Examples: Form conjunctions for each of the foll:

(1) p: Ram is healthy  q: He has blue eyes.

$p \wedge q$: Ram is healthy AND Ram has blue eyes.

(2) p: It is cold  q: It is raining.

$p \wedge q$: It is cold AND raining

(3) $p: 5x+6=26$ $\quad q: x>3$

$p \wedge q: 5x+6=26$ AND $x>3$.

Q2) Using the statements p: He is intelligent.
q: He is hardworking, write the foll. statements in symbolic form.

(i) He is intelligent and hardworking.

(A) $p \wedge q$

(ii) He is neither intelligent nor hardworking.

(A) $\sim(p \wedge q)$

(iii) He is intelligent but not hardworking.

(A) $p \wedge \sim q$

(iv) It is false that He is dull (or) hardworking.

(A) $\sim(\sim p \wedge q)$

→ Disjunction: A compound proposition obtained by combining 2 simple propositions by inserting the word OR in between them is called the disjunction.

Disjunction of 2 propositions p and q is denoted by $p \vee q$.

The disjunction $p \vee q$ is false only if both p and q are false. Otherwise, it is true.

Truth Table:

| p | q | $p \vee q$ |
|---|---|---|
| T | T | T |
| T | F | T |
| F | T | T |
| F | F | F |

Examples: Assign a truth value to each of the foll. statements.

(i) $(5<5) \vee (5<6) \Rightarrow F \vee T \Rightarrow$ True

(ii) $(5 \times 4 = 21) \vee (9+7=17) \Rightarrow F \vee F \Rightarrow$ False

(iii) $(6+4=10) \wedge (0>2) \Rightarrow T \wedge F \Rightarrow$ False

(iv) (Hyderabad is capital of Telangana) $\vee$ (Hyderabad is capital of AP) $\Rightarrow T \vee F \Rightarrow$ True

Q2) p: It is cold. q
Write simple ver
each of the follo
(i) $\neg p$ (ii) $p \wedge q$
(i) It is not cold
(ii) It is cold
(iii) It is cold
(iv) It is cold

3Q) Write the
symbolic form
(i) John will t
graduate sch
$\sim(p \vee q)$
James will

Ram will t
to the high

4Q) Form the
p: I will
Ans) I will w

→ Conditiona
combining
'if and th
conditiona
If p and
conditiona
The cond
the cond
The cond
In the
anteced
consequ

2Q) p: It is cold. q: It is raining.
Write simple verbal sentence which describes each of the following statements.
(i) $\neg p$ (ii) $p \wedge q$ (iii) $p \vee q$ (iv) $p \vee \sim q$.
(i) It is not cold.
(ii) It is cold and raining.
(iii) It is cold or raining.
(iv) It is cold or not raining.

3Q) Write the negation of the foll. statements in symbolic form.
(John will take a job in industry or go to graduate school.
$\sim(p \vee q)$
James will bicycle or run tomorrow.

Ram will take a job in software company or go to the higher studies.

4Q) Form the disjunction of
p: I will write homework q: I will go to college.
Ans) I will write homework or I will go to college.

→ Conditional: A compound proposition obtained by combining 2 simple propositions by using the words 'if and 'then at appropriate places is called a conditional.
If p and q are 2 propositions, we can form the conditionals: "if p then q", "if q then p".
The conditional if p then q is denoted by $p \Rightarrow q$,
the conditional if q then p is denoted by $q \rightarrow p$.
The conditionals $p \rightarrow q$ and $q \rightarrow p$ are different.
In the conditional $p \rightarrow q$, the proposition p is called antecedent (or) hypothesis, and q is called the consequent (or) conclusion.

Truth Table:

| $p$ | $q$ | $p \to q$ |
|---|---|---|
| T | T | T |
| T | F | F |
| F | T | T |
| F | F | T |

Examples: Which of the following propositions are true and which are false.

(i) If the earth is round, then the earth travels around the sun. $\Rightarrow$ T $\to$ T $\Rightarrow$ T.

(ii) If Alexander Grahambell invented telephone then tiger has wings. ; T $\Rightarrow$ F ; F.

(iii) If Tigers have wings then RDX is dangerous.

F $\to$ T ; T

→ Biconditional: If $p$ and $q$ are 2 propositions then the compound statement $p$ if and only if $q$ is called a biconditional statement and the connective if and only if is a biconditional connective and it is denoted by $p \Leftrightarrow q$.

The biconditional statement $p \Leftrightarrow q$ can also be stated as $p$ is a necessary and sufficient condition for $q$.

Ex Truth Table:

| $p$ | $q$ | $p \Leftrightarrow q$ |
|---|---|---|
| T | T | T |
| T | F | F |
| F | T | F |
| F | F | T |

Examples: He swims if and only if the water is warm.

Sales of houses ~~is~~ falls if and only if interest rate rises.

Q1) Construct a T...
proposition:

(i) $p \wedge (\sim q \vee p)$

(iii) $(p \vee q) \wedge q$...

(v) $\neg(\neg p \vee \neg q)$...

Sol: (i)

| $p$ | $q$ |
|---|---|
| T | T |
| T | F |
| F | T |
| F | F |

(ii)

| $p$ | $q$ | $p \vee$... |
|---|---|---|
| T | T | T |
| T | F | T |
| F | T | T |
| F | F | |

(iii)

| $p$ | $q$ |
|---|---|
| T | T |
| T | F |
| F | T |
| F | F |

(iv)

| $p$ | $q$ |
|---|---|
| T | |
| T | |
| F | |
| F | |

(v)

| $p$ | $q$ |
|---|---|
| T | |
| T | |
| F | |
| F | |

Q1) Construct a Truth Table for each compound proposition.

(i) $p \wedge (\sim q \vee p)$ (ii) $\sim(p \vee q) \vee (\sim p \wedge \sim q)$

(iii) $(p \vee q) \wedge q$ (iv) $(P \vee Q) \vee \sim P$

(v) $\neg(\neg P \vee \neg Q)$ (vi) $P \to (\neg P \vee R)$

Sol: (i)

| $p$ | $q$ | $\sim q$ | $(\sim q \vee p)$ | $p \wedge (\sim q \vee p)$ |
|---|---|---|---|---|
| T | T | F | T | T |
| T | F | T | T | T |
| F | T | F | F | F |
| F | F | T | T | F |

(ii)

| $p$ | $q$ | $p \vee q$ | $\sim(p \vee q)$ | $\sim p$ | $\sim q$ | $\sim p \wedge \sim q$ | $\sim(p \vee q) \vee (\sim p \wedge \sim q)$ |
|---|---|---|---|---|---|---|---|
| T | T | T | F | F | F | F | F |
| T | F | T | F | F | T | F | F |
| F | T | T | F | T | F | F | F |
| F | F | F | T | T | T | T | T |

(iii)

| $p$ | $q$ | $p \vee q$ | $(p \vee q) \wedge q$ |
|---|---|---|---|
| T | T | T | T |
| T | F | T | F |
| F | T | T | True |
| F | F | F | F |

(iv)

| $P$ | $Q$ | $\sim P$ | $P \vee Q$ | $(P \vee Q) \vee \sim P$ |
|---|---|---|---|---|
| T | T | F | T | T |
| T | F | F | T | T |
| F | T | T | T | T |
| F | F | T | F | T |

(v)

| $P$ | $Q$ | $\neg P$ | $\neg Q$ | $\neg(\neg P \vee \neg Q)$ | $\neg(\neg P \wedge Q)$ |
|---|---|---|---|---|---|
| T | T | F | F | F | T |
| T | F | F | T | T | F |
| F | T | T | F | T | F |
| F | F | T | T | T | F |

(vi)

| p | r | ¬p | ¬p ∨ r | p → (¬p ∨ r) |
|---|---|---|---|---|
| T | T | F | T | T |
| T | F | F | F | F |
| F | T | T | T | T |
| F | F | T | T | T |

(vii) $p \vee (\sim q \rightarrow p)$

(viii) $(\sim(p \wedge q) \vee r) \rightarrow \sim p$.

(viii)

| p | q | r | p∧q | ~(p∧q) | ~(p∧q) ∨ r | ~p | (~(p∧q)∨r) → ~p |
|---|---|---|---|---|---|---|---|
| T | T | T | T | F | T | F | F |
| T | T | F | T | F | F | F | T |
| T | F | T | F | T | T | F | F |
| T | F | F | F | T | T | F | F |
| F | T | T | F | T | T | T | T |
| F | T | F | F | T | T | T | T |
| F | F | T | F | T | T | T | T |
| F | F | F | F | T | T | T | T |

*Logical Equivalence: If 2 propositions $P(p, q, r, \ldots)$ and $Q(p, q, r, \ldots)$ where $p, q, r, \ldots$ are propositional variables, have the same truth values in every possible case (or) $P \leftrightarrow Q$ is a tautology then the propositions are called logically equivalent (or) simply equivalent and denoted by $P \equiv Q$.

*Tautology, Contradiction and Contigency:
A compound proposition that is always true for all possible truth values of its variables (or) in other words, contained only T in the last column of Truth Table is called Tautology.
NOTE: The conjunction of 2 tautologies is also a tautology i.e. if A & B are 2 tautologies, then $A \wedge B$ is also a tautology.

Tautology is ...
from the given ...
*Contradiction ...
always false f...
variables or ...
in the last ...
called contra...
*Contigency ...
tautology nor ...
NOTE: Negation ...
A statement ...
identically ...
is identicall...
*Procedure to ...
tautology ...
(i) By Tr...
Q1) P.T. the ...
(i) $p \vee \sim p$ ...
(v) $(\sim p \wedge (p \rightarrow$ ...
(viii) $((p \rightarrow$ ...
sol: (viii)

| p | q |
|---|---|
| T | T |
| T | T |
| T | F |
| T | F |
| F | T |
| F | |
| F | |
| F | |

I...

Tautology is useful to conclude some statements from the given statements.

*Contradiction : A compound proposition that is always false for all possible values of its variables or in other words, contains only F in the last column of the Truth Table is called contradiction.

*Contigency : A compound proposition that is neither a tautology nor a contradiction is called a contigency.

NOTE: Negation of contradition is tautology.

A statement formula which is a tautology is identically true and a formula which is a contradiction is identically false.

*Procedure to determine whether a given formula is a tautology has 2 methods:

(1) By Truth Table (2) By Substitution method.

Q1) P.T. the foll. propositions are tautology.

(i) $p \vee \sim p$ (ii) $\sim(p \wedge q) \vee q$ (iii) $p \to (p \vee q)$ (iv) $(p \vee q) \to p$

(v) $(\sim P \wedge (P \to Q)) \to (\sim Q)$ (vi) $(P \vee Q) \vee \neg P$ (vii) $(P \wedge (P \Leftrightarrow Q)) \to Q$

(viii) $((p \to q) \wedge (q \to r)) \to (p \to r)$

sol: (viii)

| $p$ | $q$ | $r$ | $p \to q$ | $q \to r$ | $(p \to q) \wedge (q \to r)$ | $p \to r$ | final |
|---|---|---|---|---|---|---|---|
| T | T | T | T | T | T | T | T |
| T | T | F | T | F | F | F | T |
| T | F | T | F | T | F | T | T |
| T | F | F | F | T | F | F | T |
| F | T | T | T | T | T | T | T |
| F | T | F | T | F | F | T | T |
| F | F | T | T | T | T | T | T |
| F | F | F | T | T | T | T | T |

It is a tautology.

(vii)

| $p$ | $q$ | $P \leftrightarrow Q$ | $P \wedge (P \leftrightarrow Q)$ | $(P \wedge (P \leftrightarrow Q)) \rightarrow Q$ |
|---|---|---|---|---|
| T | T | T | T | T |
| T | F | F | F | T |
| F | T | F | F | T |
| F | F | T | F | T |

It is a tautology.

**2Q)** S.T $p \rightarrow (q \wedge r)$ & $\neg p \vee (q \wedge r)$ are logically equivalent

| $p$ | $q$ | $r$ | $q \wedge r$ | $p \rightarrow (q \wedge r)$ |
|---|---|---|---|---|
| T | T | T | T | T |
| T | T | F | F | F |
| T | F | T | F | F |
| T | F | F | F | F |
| F | T | T | T | T |
| F | T | F | F | T |
| F | F | T | F | T |
| F | F | F | F | T |

| $p$ | $q$ | $r$ | $q \wedge r$ | $\neg p$ | $\neg p \vee (q \wedge r)$ |
|---|---|---|---|---|---|
| T | T | T | T | F | T |
| T | T | F | F | F | F |
| T | F | T | F | F | F |
| T | F | F | F | F | F |
| F | T | T | T | T | T |
| F | T | F | F | T | T |
| F | F | T | F | T | T |
| F | F | F | F | T | T |

∴ They are logically equivalent.

$A = p \rightarrow (q \wedge r)$ $B = \neg p \vee (q \wedge r)$

Last column of A & B are same. So they are logically equivalent.

**3Q)** S.T. $\underbrace{\neg(P \leftrightarrow Q)}_{A} \Leftrightarrow \underbrace{(P \wedge \neg Q) \vee (\neg P \vee Q)}_{B}$

Sol:

| $P$ | $Q$ | $P \leftrightarrow Q$ | $\neg(P \leftrightarrow Q)$ |
|---|---|---|---|
| T | T | T | F |
| T | F | F | T |
| F | T | F | T |
| F | F | T | F |

They are not equivalent.

| $P$ | $Q$ | $\neg Q$ | $\neg P$ | $P \wedge \neg Q$ | $\neg P \vee Q$ | $(P \wedge \neg Q) \vee (\neg P \vee Q)$ |
|---|---|---|---|---|---|---|
| T | T | F | F | F | T | T |
| T | F | T | F | T | F | T |
| F | T | F | T | F | T | T |
| F | F | T | T | F | T | [illegible] |

4Q) $\neg(p \leftrightarrow q) \Leftrightarrow (p \wedge \neg q) \vee (\neg p \wedge q)$

Sol:

| p | q | $p \leftrightarrow q$ | $\neg(p \leftrightarrow q)$ |
|---|---|---|---|
| T | T | T | F |
| T | F | F | T |
| F | T | F | T |
| F | F | T | F |

| p | q | $\neg p$ | $\neg q$ | $p \wedge \neg q$ | $\neg p \wedge q$ | $(p \wedge \neg q) \vee (\neg p \wedge q)$ |
|---|---|---|---|---|---|---|
| T | T | F | F | F | F | F |
| T | F | F | T | T | F | T |
| F | T | T | F | F | T | T |
| F | F | T | T | F | F | F |

they are logically equivalent (since their last columns are same).

5Q) S.T. $p \wedge (\neg q \vee r)$ and $p \vee (q \wedge \neg r)$ are logically not equivalent

6Q) P.T. the proposition $(p \to q) \to (p \wedge q)$ is a contigency

7Q) verify the proposition $p \wedge (q \wedge \sim p)$ is a contradiction.

* Tautological Implication: A statement P is said to be tautologically implies a statement Q if and only if $P \to Q$ is a tautology. We shall denote this idea by $P \Rightarrow Q$ which is read as P implies Q. In a similar manner $P \Leftrightarrow Q$ states that P and Q are equivalent.

1Q) S.T. the foll. implications

(i) $(P \wedge Q) \Rightarrow (P \to Q)$ (ii) $P \Rightarrow (Q \to P)$

(iii) $(P \to (Q \to R)) \Rightarrow (P \to Q) \to (P \to R)$

Sol: (i)

| P | Q | $P \wedge Q$ | $P \to Q$ | $(P \wedge Q) \to (P \to Q)$ |
|---|---|---|---|---|
| T | T | T | T | T |
| T | F | F | F | T |
| F | T | F | T | T |
| F | F | F | T | T |

$\therefore$ (i) is an implication since it is a tautology.

(ii) $P \Rightarrow (Q \to P)$

| P | Q | $Q \to P$ | $P \to (Q \to P)$ |
|---|---|---|---|
| T | T | T | T |
| T | F | T | T |
| F | T | F | T |
| F | F | T | T |

∴ It is an implication.

(iii) $(P \to (Q \to R)) \Rightarrow (P \to Q) \to (P \to R)$

| P | Q | R | $Q \to R$ | $P \to Q$ | $P \to R$ | $(P \to Q) \to (P \to R)$ | $P \to (Q \to R)$ |
|---|---|---|---|---|---|---|---|
| T | T | T | T | T | T | T | T |
| T | T | F | F | T | F | F | T |
| T | F | T | ~~F~~ T | T | T | T | F |
| T | F | F | T | T | F | F | ~~F~~ T |
| F | T | T | T | F | T | T | T |
| F | T | F | False | F | T | T | T |
| F | F | T | True | T | T | T | T |
| F | F | F | T | T | T | T | T |

$(P \to (Q \to R)) \to (P \to Q) \to (P \to R)$

T

F

(iii)

| P | Q | R | $Q \to R$ | $A = P \to (Q \to R)$ | $P \to Q$ | $P \to R$ | $B = (P \to Q) \to (P \to R)$ | $A \to B$ |
|---|---|---|---|---|---|---|---|---|
| T | T | T | T | T | T | T | T | T |
| T | T | F | F | F | T | F | F | T |
| T | F | T | T | T | F | T | T | T |
| T | F | F | T | T | F | F | T | T |
| F | T | T | T | T | T | T | T | T |
| F | T | F | F | T | T | T | T | T |
| F | F | T | T | T | T | T | T | T |
| F | F | F | T | T | T | T | T | T |

2Q) show th
(i) $P \to (Q \to$
(ii) $P \to (Q \vee R)$
(iii) $(P \to Q) \wedge$
(iv) $\neg(P \Leftrightarrow Q)$

* Well form
A statem
a string
with or
connect
expressi
which
pronou
. A WF
(1) A st
(2) All
(3) If
(4) If
$(P \Leftrightarrow$
Acc
WFF
$(\neg$
$\neg$

1Q)
(i) (
(ii)
(iii)
(iv)
(v)

2Q) Show that foll. equivalences:

(i) $P \to (Q \to P) \Leftrightarrow \neg P \to (P \to Q)$

(ii) $P \to (Q \vee R) \Leftrightarrow (P \to Q) \vee (P \to R)$

(iii) $(P \to Q) \wedge (R \to Q) \Leftrightarrow (P \vee R) \to Q$

(iv) $\neg(P \Leftrightarrow Q) \Leftrightarrow (P \vee Q) \wedge \neg(P \wedge Q)$

* Well formed formulas:

A statement formula is an expression which is a string consisting of variables (alphabets with or without subscripts, parenthesis and connective symbols). A grammatically correct expression is called a well formed formula which is abbreviated as WFF and can be pronounced as "wuff".

. A WFF can be generated by the foll. rules

(1) A statement variable standing alone is a WFF.

(2) All variables and constants are WFF.

(3) If P is WFF the $\neg P$ is also WFF

(4) If P and q are WFF then $(P \wedge q)$, $(P \wedge q)$, $(P \to q)$ $(P \Leftrightarrow q)$ are also WFF.

According to this definition, the foll. are also WFF:

$\neg(P \wedge Q)$, $\neg(P \vee Q)$, $(P \to (P \vee Q))$, $((P \to Q) \to (Q \to R))$, $(\neg Q \to R)$,

$\neg Q \wedge$ is not a WFF

$(\neg P \vee Q) \to (R \vee P))$ is not a WFF [∵ ( is missing].

1Q) Verify which of the foll. are WFF:

(i) $(P \to (P \vee Q))$ $\Rightarrow$ WFF

(ii) $(P \to \neg P) \to \neg P$ $\Rightarrow$ Not WFF

(iii) $((\neg Q \wedge P) \wedge Q)$ $\Rightarrow$

(iv) $((P \to (Q \to R)) \to (P \to Q) \to (P \to R)]$ $\Rightarrow$

(v) $(\neg P \to Q) \to Q \to P$ $\Rightarrow$

## * Converse, Contrapositive and Inverse.

If $p$ and $q$ are any two propositions then some other conditional propositions related to $p \to q$ are :

(i) Converse : The converse of $p \to q$ is $q \to p$

(ii) Inverse : The inverse of $p \to q$ is $\neg p \to \neg q$

(iii) Contrapositive : The contrapositive of $p \to q$ is $\sim q \to \sim p$

The Truth Table of the above propositions is as follows:

| | | | | Converse | Inverse | Contrapositive | |
|---|---|---|---|---|---|---|---|
| $p$ | $q$ | $\sim p$ | $\sim q$ | $q \to p$ | $\sim p \to \sim q$ | $\sim q \to \sim p$ | $p \to q$ |
| T | T | F | F | T | T | T | T |
| T | F | F | T | T | T | F | F |
| F | T | T | F | F | F | T | T |
| F | F | T | T | T | T | T | T |

## * Laws of Logics:

The foll. results known as the laws of logic which are from the definition of logical equivalence. In these laws $T_0$ denotes a tautology, $F_0$ denotes a contradiction.

(i) Law of Double negation: For any proposition $p$, $\sim(\sim p) \Leftrightarrow p$

(ii) Idempotent laws: For any proposition $p$,
$(p \vee p) \Leftrightarrow p$ , $(p \wedge p) \Leftrightarrow p$

(iii) Identity law: $(p \vee F_0) \Leftrightarrow p$ , $(p \wedge T_0) \Leftrightarrow p$

(iv) Inverse law: $(p \vee \neg p) \Leftrightarrow T_0$ , $(p \wedge \neg p) \Leftrightarrow F_0$

(v) Domination law: $(p \vee T_0) \Leftrightarrow T_0$ , $(p \wedge F_0) \Leftrightarrow F_0$

(vi) Commutative law:

$$(p \vee q) \Leftrightarrow (q \vee p) , \quad (p \wedge q) \Leftrightarrow (q \wedge p)$$

(vii) Associative law:

$$(p \vee (q \vee r)) \Leftrightarrow ((p \vee q) \vee r) , \quad (p \wedge (q \wedge r)) \Leftrightarrow ((p \wedge q) \wedge r)$$

(viii) Distributive law:

$(p \lor (q \land r)) \Leftrightarrow ((p \lor q) \land (p \lor r))$

$(p \land (q \lor r)) \Leftrightarrow ((p \land q) \lor (p \land r))$

(ix) Absorption law: $(p \lor (p \land q)) \Leftrightarrow p$

$(p \land (p \lor q)) \Leftrightarrow p$

(x) De morgan law:

$\neg (p \lor q) \Leftrightarrow (\neg p \land \neg q)$

$\neg (p \land q) \Leftrightarrow (\neg p \lor \neg q)$

Law for negation of conditional:

$\neg (p \to q) \Leftrightarrow (p \land \neg q)$

★★ $p \to q \Leftrightarrow \neg p \lor q$

1Q) S.T. $p \to (q \to r) \Leftrightarrow p \to (\neg q \lor r) \Leftrightarrow (p \land q) \to r$ without constructing a Truth Table.

Sol: WKT $p \to q \Leftrightarrow \neg p \lor q$

consider $p \to (q \to r)$

$\Leftrightarrow (\neg p \lor (q \to r))$

$\Leftrightarrow (\neg p \lor (\neg q \lor r)) \quad \Leftrightarrow p \to (\neg q \lor r)$

$\Leftrightarrow (\neg p \lor \neg q) \lor r$ (associative)

$\Leftrightarrow (\neg p \land q) \lor r$ (Demorgan's)

$\Leftrightarrow p \land q \to r$

2Q) Without constructing the Truth Table P.T.

$p \to (q \to p) \Leftrightarrow \neg p \to (p \to q) \land$

Sol: consider $p \to (q \to p)$

$\equiv p \to (\neg q \lor p)$

$\equiv \neg p \lor (\neg q \lor p)$

$\equiv (\neg p \lor \neg q) \lor p$

$\equiv (\neg p \lor p) \lor \neg q$

$\equiv T \lor \neg q \equiv T$.

consider: $\neg p \to (p \to q)$

$\equiv \neg p \to (\neg p \vee q)$

$\equiv \neg(\neg p) \vee (\neg p \vee q)$

$\equiv p \vee (\neg p \vee q)$

$\equiv (p \vee \neg p) \vee q$

$\equiv T \vee q$

$\equiv T$

$\therefore p \to (q \to p) \Leftrightarrow \neg p \to (p \to q)$

3Q) P.T. the foll. are logically equivalent without using truth tables

(i) $(p \to Q) \wedge (R \to Q) \Leftrightarrow (P \vee R) \to Q$

Sol: consider $(p \to Q) \wedge (R \to Q)$

$\equiv (\neg p \vee Q) \wedge (\neg R \vee Q)$

$\equiv (\neg p \wedge \neg R) \vee Q$ (by distributive law)

$\equiv \neg(p \vee R) \vee Q$

$\equiv (p \vee R) \to Q$

[$\because A \to B \Leftrightarrow \neg A \vee B$ ; $\neg(A \vee B) = \neg A \wedge \neg B$ ; $\neg(A \wedge B) = \neg A \vee \neg B$

(ii) $(P \to R) \wedge (Q \to R) \Leftrightarrow (P \vee Q) \to R$

(iii) $(p \to q) \to q \Rightarrow p \vee q$

Sol: consider $(p \to q) \to q$

$\equiv (\neg p \vee q) \to q$

$\equiv \neg(\neg p \vee q) \vee q$

$\equiv (\neg(\neg p) \wedge \neg q) \vee q$

$\equiv (p \wedge \neg q) \vee q$

$\equiv (p \vee q) \wedge (\neg q \vee q)$ (distributive law)

$\equiv (p \vee q) \wedge T$

$\equiv p \vee q$ (identity law)

(iv) $(p \to q) \wedge (r \to q) \Leftrightarrow (p \vee r) \to q$

(v) $\{\neg P \wedge (\neg Q \wedge R)\} \vee (Q \wedge R) \vee (P \wedge R) \Leftrightarrow R$

Sol: Consider, $\{\neg P \wedge (\neg Q \wedge R)\} \vee (Q \wedge R) \vee (P \wedge R)$

$\equiv \{(\neg P \wedge \neg Q) \wedge R\} \vee (Q \vee P) \wedge R$

(by associative and distributive law)

$\equiv \{\neg (P \vee Q) \wedge R\} \vee (Q \vee P) \wedge R$

$\equiv \{\neg (P \vee Q) \vee (P \vee Q)\} \wedge R \quad [P \vee \neg P = T]$

$\equiv T \wedge R$

$\equiv R$

(vi) $(P \vee Q) \wedge \neg\{\neg P \wedge (\neg Q \vee \neg R)\} \vee (\neg P \wedge \neg Q) \vee (\neg P \wedge \neg R)$ (PT it is a T)

Sol:

$\equiv (P \vee Q) \wedge \neg\{\neg P \wedge \neg(Q \wedge R)\} \vee \neg P \wedge (\neg Q \vee \neg R)$

$\equiv (P \vee Q) \wedge \neg\{\neg (P \vee (Q \wedge R)\} \vee \neg P \wedge \neg(Q \vee R)$

$\equiv (P \vee Q) \wedge \underbrace{(P \vee (Q \wedge R))}_{u} \vee \underbrace{\neg (P \vee (Q \wedge R))}_{\neg u}$

$\equiv (P \vee Q) \vee T \equiv T$

15) Simplify the foll compound propositions using the laws of logics

(i) $(p \vee q) \wedge \neg\{(\neg p \vee p)\}$

$\equiv (p \vee q) \wedge \{\neg(\neg p) \wedge \neg p\}$

$\equiv (p \vee q) \wedge \{p \wedge \neg p\} \quad (R \wedge (S \wedge T) = (R \wedge S) \wedge T)$

$\equiv \{(p \vee q) \wedge p\} \wedge \neg p$ (associative law)

$\equiv \{(p \wedge p) \vee (q \wedge p)\} \wedge \neg p$ (distributive law)

$\equiv \{p \vee (p \wedge q)\} \wedge \neg p$ (idempotent law)

$\equiv (p \wedge \neg p) \vee (p \wedge q \wedge \neg p)$

$\equiv F \vee (q \wedge F)$

$\equiv F \vee F$

$\equiv F$

(ii) $\neg[\neg\{(p\vee q)\wedge r\}\vee\neg q]$   $q\wedge r$

(iii) $p\vee[p\wedge(p\vee q)] \Leftrightarrow p$

(iv) $[p\vee q\vee(\neg p\wedge\neg q\wedge r)] \Leftrightarrow (p\vee q\vee r)$

Sol (iii) $p\vee[p\wedge(p\vee q)]$

(ii) $\neg[\neg\{(p\vee q)\wedge r\}\vee\neg q]$

$\equiv[\neg\neg\{(p\vee q)\wedge r\}\wedge\neg\neg q]$

$\equiv[(p\vee q)\wedge r]\wedge q$

$\equiv (p\vee q)\wedge(q\wedge r)$

$\equiv ((p\vee q)\wedge q)\wedge r$

$\equiv [(p\wedge q)\vee(q\wedge q)]\wedge r$

$\equiv [(p\wedge q)\vee q]\wedge r$

$\equiv q\wedge r$ (by absorbtion law)

(iv) $[p\vee q\vee(\neg p\wedge\neg q\wedge r)]$

$\equiv (p\vee q\vee[\neg(p\vee q)\wedge r])$

$\equiv[(p\vee q)\vee(\neg(p\vee q))]\wedge[(p\vee q)\vee r]$ (distributive law)

$\equiv T\wedge[p\vee q\vee r]$

$\equiv p\vee q\vee r$ (RHS)

$\therefore [p\vee q\vee(\neg p\wedge\neg q\wedge r)] \Leftrightarrow (p\vee q\vee r)$

(iii) $p\vee[p\wedge(p\vee q)]$

$\equiv p\vee p$   [absorbtion law].

$\equiv p$ ($\equiv$ RHS) (idempotent law).

*Duality law: Two formulas A and A* are said to be duals of each other if one can be applied from the other by replacing conjunction by disjunction and disjunction by conjunction. These connectives ∧ and join are also called duals of each other. Also if the formula A has T (or) F then A*, its

dual is obtai…
In addition t…

1Q) Write the…
(i) $(p\wedge\neg q)\vee$…
(iii) $(p\wedge q)\vee T$
(v) $\neg(p\vee q)$…

Sol:
(i) A : $(p\wedge\neg q$…
(ii) A : $(p\vee q$…
(iii) A : $(p\wedge q)$…
(iv) A : $\neg(p\vee q$…
A* : $\neg($…
(v) A : $\neg(p\vee$…
A* : $\neg($…

NOTE (1) If…
then their…
each oth…
as the p…
(2) $(A^*)^*$…

2Q) Writ…
(i) $P\rightarrow Q$…
Sol : (i)…
$\equiv$
$\equiv$

(ii) $(P$…
$\equiv\neg($…
$\equiv$

(iii) $P$…
$\equiv\neg P$…

dual is obtained by replacing T by F & F by T, In addition to the above mentioned interchanges.

1Q) Write the duals of the foll:

(i) $(p \wedge \neg q) \vee (r \wedge T)$ (ii) $(p \vee q) \wedge r$

(iii) $(p \wedge q) \vee T$ (iv) $\neg(p \vee q) \wedge (p \wedge \neg(q \wedge \neg s))$

(v) $\neg(p \vee q) \wedge (p \vee \neg(q \wedge \neg r))$

Sol:

(i) $A : (p \wedge \neg q) \vee (r \wedge T)$ ; $A^* = (p \vee \neg q) \wedge (r \vee F)$

(ii) $A : (p \vee q) \vee r$ ; $A^* = (p \wedge q) \vee r$

(iii) $A : (p \wedge q) \vee T$ ; $A^* = (p \vee q) \wedge F$

(iv) $A : \neg(p \vee q) \wedge (p \wedge \neg(q \wedge \neg s))$

$A^* : \neg(p \wedge q) \vee (p \vee \neg(q \vee \neg s))$

(V) $A : \neg(p \vee q) \wedge (p \vee \neg(q \wedge \neg r))$

$A^* : \neg(p \wedge q) \vee (p \wedge \neg(q \vee \neg r))$

NOTE (1) If any two formulas A & B are equivalent, then their duals $A^*$ & $B^*$ are also equivalent to each other. Thus $A \Leftrightarrow B \Rightarrow A^* \Leftrightarrow B^*$. This is known as the principle of duality.

(2) $(A^*)^* \Leftrightarrow A$

2Q) Write the duals of the foll:

(i) $P \to Q$ (ii) $(p \to q) \to r$ (iii) $P \to (Q \to R)$

Sol: (i) $P \to Q$

$\equiv \neg P \vee Q$

$\equiv \neg P \wedge Q$

(ii) $(p \to q) \to r$

$\equiv \neg(p \to q) \vee r \equiv \neg(\neg p \vee q) \vee r$

$\equiv (\neg(\neg p) \wedge \neg q) \vee r \equiv (p \wedge \neg q) \vee r$

$A^* = (p \vee \neg q) \wedge r$

(iii) $P \to (Q \to R)$

$\equiv \neg P \vee (Q \to R) \equiv \neg P \vee (\neg Q \vee R)$

$A^* = \neg P \wedge (\neg Q \wedge R)$

3Q) P.T. $\neg(p \wedge q) \to (\neg p \vee (\neg p \vee q)) \Leftrightarrow (\neg p \vee q)$. Hence deduce that $(p \vee q) \wedge (\neg p \wedge (\neg p \wedge q)) \Leftrightarrow (\neg p \wedge q)$

Sol: consider $\neg(p \wedge q) \to (\neg p \vee (\neg p \vee q))$ $[\because A \to B = \neg A \vee B]$

$= \neg(\neg(p \wedge q)) \vee (\neg p \vee (\neg p \vee q))$ — ① $[\because A \vee (B \vee C) = (A \vee B) \vee C]$

$\equiv (p \wedge q) \vee (\neg p \vee (\neg p \vee q))$

$\equiv (p \wedge q) \vee ((\neg p \vee \neg p) \vee q)$

$\equiv (\underbrace{p \wedge q}_{R}) \vee (\underbrace{\neg p}_{S} \vee \underbrace{q}_{T})$

$\equiv [(p \wedge q) \vee \neg p] \vee q$

$\equiv [(p \vee \neg p) \wedge (q \vee \neg p)] \vee q$

$\equiv [T \wedge (q \vee \neg p)] \vee q$

$\equiv (q \vee \neg p) \vee q$

$\equiv (q \vee q) \vee \neg p$

$\equiv q \vee \neg p$

$\equiv \neg p \vee q$ (RHS).

Hence, proved.

$(p \wedge q) \vee (\neg p \vee (\neg p \vee q)) \Leftrightarrow (\neg p \vee q)$

from ①: $A : (p \wedge q) \vee (\neg p \vee (\neg p \vee q)) \Leftrightarrow (\neg p \vee q)$

$A^* : (p \vee q) \wedge (\neg p \wedge (\neg p \wedge q)) \Leftrightarrow (\neg p \wedge q)$

4Q) Verify the principle of duality for the foll. equivalents.

(i) $\neg(p \wedge q) \to (\neg p \vee (\neg p \vee q)) \Leftrightarrow (\neg p \vee q)$.

Sol: WKT the principle of duality is

$A \Leftrightarrow B$ then $A^* \Leftrightarrow B^*$.

$\neg(p \wedge q) \to (\neg p \vee (\neg p \vee q))$

$\neg(\neg(p \wedge q)) \vee (\neg p \vee (\neg p \vee q))$

$(p \wedge q) \vee (\neg p \vee (\neg p \vee q)) \Leftrightarrow \neg p \vee q$. — ①

$(p \wedge q) \vee ((\neg p \vee \neg p) \vee q) \times$

$(p \wedge q) \vee (\neg p \vee q) \times$

dual of ① is : $(p \vee q) \wedge (\neg p \wedge (\neg p \wedge q)) \Leftrightarrow \neg p \wedge$

consider $(p \vee q) \wedge (\neg p \wedge (\neg p \wedge q))$

$\equiv (p \vee q) \dots$

$\equiv (\underbrace{p \vee q}_{R}) \dots$

~~$\equiv (p \vee \dots$~~

$\equiv (p \vee \dots$

$\equiv ((p \vee \dots$

$\equiv q \dots$

LHS = …

* Functiona…
Any set of … can be en… containin… called a … It is as… complete… connecti… expresse…
~~Ex~~: p… connec… express…
Ex: p… p… p… p…

Conclu… bicon… $\wedge$, $\vee$… and… Demo… first… fina… obta…

$\equiv (P \vee Q) \wedge (\neg P \wedge \neg P) \wedge Q)$ (idempotent law).

$\equiv \underbrace{(P \vee Q)}_{R} \wedge (\underbrace{\neg P}_{S} \wedge \underbrace{Q}_{T})$

$\cancel{\equiv (P \vee Q \wedge \neg P) \wedge Q}$

$\equiv \underbrace{(P \vee Q)}_{R} \wedge (\underbrace{Q}_{S} \wedge \underbrace{\neg P}_{T})$ (commutative law)

$\equiv ((P \vee Q) \wedge Q) \wedge \neg P$ (associative law)

$\equiv Q \wedge \neg P$ (absorption law).

LHS = RHS. Hence, proved.

* Functionally complete set of connectives:

Any set of connectives in which every formula can be expressed in terms of an equivalent formula containing the connectives from the set is called a functionally complete set of connectives. It is assumed that such a functionally complete set does not contain any redundant connectives i.e. a connective which can be expressed in terms of the other connectives.

~~Ex:~~ $p \vee q$ Functionally complete set of connectives is nothing but we have to express in terms of $\{\neg, \vee\}$ and $\{\neg, \wedge\}$.

Ex: $p \vee q \equiv \neg(p \wedge q)$

$p \wedge q \equiv \neg(p \vee q)$

$p \to q \equiv \neg p \vee q$

$p \Leftrightarrow q \equiv [(p \to q) \wedge (q \to p)]$

$\equiv [(\sim p \vee q) \wedge (\sim q \vee p)]$

Conclusion: Thus all the conditional and biconditional can be replaced by the 3 connectives $\wedge, \vee, \sim$. Similarly we can eliminate conjunction and disjunction from the given formula by using Demorgan's laws. Thus in any formula we can replace first the biconditional, then conditionals and finally all the conjunctions (or) disjunctions to obtain an equivalence formula. this formula contains either $\{\neg, \vee\}$ or $\{\neg, \wedge\}$. Therefore, the set of connectives $\{\neg, \vee\}$ and $\{\neg, \wedge\}$ are functionally complete set.

NOTE:

(i) $\{\neg\}, \{\vee\}, \{\wedge\}, \{\vee, \wedge\}$ are not functionally complete.

(ii) From the five connectives $\sim, \vee, \wedge, \rightarrow, \leftrightarrow$, we have to obtain atleast 2 sets of functionally complete connectives.

1Q) Write an equivalence expression for $[(p \rightarrow q) \wedge r] \vee (r \leftrightarrow s)$ which contains neither '$\rightarrow$' nor '$\leftrightarrow$'.

Sol: $[(\neg p \vee q) \wedge r] \vee [(r \rightarrow s) \wedge (s \rightarrow r)]$

$[(\neg p \vee q) \wedge r] \vee [(\neg r \vee s) \wedge (\neg s \vee r)]$

* Other connectives: (NAND and NOR).

These connectives have useful applications in the design of computer particularly in the design of electrical circuits.

(1) NAND: It is a combination of NOT and AND where NOT stands for $\sim$ and AND stands for the conjunction. The connective NAND is denoted by the symbol '$\uparrow$'. For any 2 propositions $p$ and $q$, $p \uparrow q \Leftrightarrow \neg(p \wedge q)$.

(2) NOR: The word NOR is combination of NOT and OR where NOT stands for $\sim$ and OR for the disjunction. The connective NOR is denoted by '$\downarrow$'. If $p$ and $q$ are any 2 propositions, then $p \downarrow q \Leftrightarrow \neg(p \vee q)$.

To prove that each of the connectives $\uparrow$ & $\downarrow$ is functionally complete:

It will be proved by showing that the set of connectives {¬, ∨} & {¬, ∨} can be expressed either in terms of NAND alone or in terms of NOR alone.

* Expression of ¬, ∨, ∧ in terms of ↑.

- consider: $P \uparrow P = \neg(P \wedge P) \equiv \neg P \vee \neg P \equiv \neg P$

  ¬ expressed in terms of ↑.

- consider: $\underbrace{(P \uparrow Q)}_{A} \uparrow \underbrace{(P \uparrow Q)}_{A} \equiv \underbrace{\neg(P \uparrow Q)}_{\neg A} \equiv \neg(\neg(P \wedge Q))$

  $\equiv P \wedge Q$.

  ∧ expressed in terms of ↑.

- consider: $(P \uparrow P) \uparrow (Q \uparrow Q) \equiv (\neg P \uparrow \neg Q) \equiv \neg(\neg P \wedge \neg Q)$

  $\equiv \neg(\neg(P \vee Q)) \equiv P \vee Q$.

  ∨ expressed in terms of ↑.

* Expression of ¬, ∨, ∧ in terms of ↓.

- consider: $P \downarrow P \equiv \neg(P \vee P) \equiv \neg P \wedge \neg P \equiv \neg P$

  ¬ expressed in terms of ↓.

- consider $(P \downarrow Q) \downarrow (P \downarrow Q) \equiv \neg(P \downarrow Q) \equiv \neg(\neg(P \vee Q)) \equiv P \vee Q$

  ∨ expressed in terms of ↓.

- consider $(P \downarrow P) \downarrow (Q \downarrow Q) \equiv (\neg P \downarrow \neg Q) \equiv \neg(\neg P \vee \neg Q)$

  $\equiv \neg(\neg(P \wedge Q)) \equiv (P \wedge Q)$.

  ∧ expressed in terms of ↓.

Hence a single operator NAND (or) NOR is functionally complete set. Each of ~~sin~~ {↑} (or) {↓} is called a minimal functionally complete set (or) minimal set.

* Properties of NAND and NOR:

- $p \uparrow q \equiv q \uparrow p$ & $p \downarrow q \equiv q \downarrow p$ (commutative law)

  &

- $(p \uparrow q) \uparrow r \not\equiv p \uparrow (q \uparrow r)$

  $(p \downarrow q) \downarrow r \not\equiv p \downarrow (q \downarrow r)$ (not associative)

1Q) P.T. for any 2 propositions

(i) $\neg(p\downarrow q) \Leftrightarrow \neg p \uparrow \neg q$ (ii) $\neg(p\uparrow q) \Leftrightarrow \neg p \downarrow \neg q$

~~(iii) $p\uparrow(Q\uparrow R) \Leftrightarrow (\neg p \vee ($~~ (iii) $p\uparrow q \Leftrightarrow (\neg p \wedge q)\vee(p\wedge \neg q)\vee(\neg p \wedge$

Sol: (i) consider $\neg(p\downarrow q)$

$\equiv \neg(\neg(p\vee q))$

$\equiv \neg(\neg p \wedge \neg q)$

$\equiv \neg p \uparrow \neg q$ (RHS) hence, proved.

(ii) $\neg(p\uparrow q) \equiv \neg(\neg(p\wedge q))$

$\equiv \neg(\neg p \vee \neg q)$

$= \neg p \downarrow \neg q$ (RHS) hence, proved.

(iii) $p\uparrow q \equiv \neg(p\wedge q)$

$\equiv \neg p \vee \neg q$ (Demorgans)

$\equiv (\neg p \wedge T) \vee (\neg q \wedge T)$ (identity law)

$\equiv (\neg p \wedge (q \vee \neg q)) \vee (\neg q \wedge (p \vee \neg p))$ (inverse law)

$\equiv \{(\neg p \wedge q) \vee (\neg p \wedge \neg q)\} \vee \{(\neg q \wedge p) \vee (\neg q \wedge \neg p)\}$ (distributive law)

$\equiv (\neg p \wedge q) \vee (\neg q \wedge p) \vee (\neg p \wedge \neg q)$ (associative law)

2Q) For any 3 propositions P, Q, R, P.T.

(i) $P\uparrow(Q\uparrow R) \Leftrightarrow \neg P \vee (Q \wedge R)$

(ii) $(p\uparrow q)\uparrow r \Leftrightarrow (p\wedge q)\vee \neg r$

(iii) $p\downarrow(q\downarrow r) \Leftrightarrow \neg p \wedge (q \vee r)$

(iv) $(p\downarrow q)\downarrow r \Leftrightarrow (p\vee q)\wedge \neg r$.

Sol: (i) $P\uparrow(Q\uparrow R)$

$\equiv \neg(P\wedge(Q\uparrow R))$

$\equiv \neg(P \wedge \neg(Q\wedge R))$

$\equiv \neg(P \wedge (\neg Q \vee \neg R))$

$\equiv \neg P \vee (\neg(\neg Q \vee \neg R))$

$\equiv \neg P \vee (\neg(\neg(Q\wedge R)))$

$\equiv \neg P \vee (Q \wedge R)$ (RHS) hence, proved

(ii) $(p\wedge q)\wedge r$

~~$\equiv \neg(p\wedge q)\wedge$~~

~~$\equiv \neg(\neg(p\wedge q)$~~

(iii) $P\downarrow(q\downarrow$

(iv) $(p\downarrow q)\downarrow$

3Q) Expre

NAND o

(i) $\neg p$

Sol: (i) $\neg(p$

(ii) p

$p\wedge$

(iii) p

(ii) $(p \uparrow q) \uparrow r \equiv \neg[(p \uparrow q) \wedge r] \equiv \neg[\neg(p \wedge q) \wedge r]$

$\cancel{\equiv \neg(p \wedge q) \uparrow r} \equiv \neg(\neg(p \wedge q)) \vee \neg r$

$\cancel{\equiv \neg(\neg(p \wedge q) \wedge r)} \equiv (p \wedge q) \vee \neg r$ (RHS).

(iii) $p \downarrow (q \downarrow r) \equiv p \downarrow (\neg(q \vee r))$

$\equiv \neg(p \vee (\neg(q \vee r)))$

$\equiv \neg p \wedge \neg(\neg(q \vee r))$

$\equiv \neg p \wedge (q \vee r)$ (RHS).

(iv) $(p \downarrow q) \downarrow r \equiv \neg((p \downarrow q) \vee r)$

$\equiv \neg[\neg(p \vee q) \vee r]$

$\equiv \neg(\neg(p \vee q)) \wedge \neg r$

$\equiv (p \vee q) \wedge \neg r$ (RHS).

3Q) Express the foll. propositions interms of only NAND and only NOR connectives.

(i) $\neg p$ (ii) $p \wedge q$ (iii) $p \vee q$ (iv) $p \to q$ (v) $p \leftrightarrow q$.

Sol: (i) $\neg(p \wedge p) \equiv p \uparrow p$ (NAND)

$\neg p \equiv \neg p \wedge \neg p \equiv \neg(p \vee p) \equiv p \downarrow p$ (NOR).

(ii) $p \wedge q \equiv \neg(\neg p \wedge q) \equiv \neg(\neg p \vee \neg q)$

$\equiv \neg p \downarrow \neg q$

$\equiv (p \downarrow p) \downarrow (q \downarrow q)$ (NOR)

$p \wedge q \equiv (p \wedge q) \vee (p \wedge q) \equiv \neg\neg(p \wedge q) \vee \neg\neg(p \wedge q)$

$\equiv \neg\{\neg(p \wedge q) \wedge \neg(p \wedge q)\}$

$\equiv \neg\{(p \uparrow q) \wedge (p \uparrow q)\}$

$\equiv (p \uparrow q) \uparrow (p \uparrow q)$ (NAND).

(iii) $p \vee q \equiv \neg\{\neg(p \vee q)\} \equiv \neg(\neg p \wedge \neg q) \equiv \neg p \uparrow \neg q$

$\equiv (p \uparrow p) \uparrow (q \uparrow q)$

(NAND).

1Q) $p \vee q \equiv (p \vee q) \wedge (p \vee q) \equiv \{\neg(\neg(p \vee q)) \wedge \neg(\neg(p \vee q))\}$

(i) $\equiv \neg\{\neg(p \vee q) \vee \neg(p \vee q)\}$

~~(iii)~~ $\equiv \neg(p \vee q) \downarrow \neg(p \vee q)$

Sol:( $\equiv (p \downarrow q) \downarrow \neg(p \downarrow q)$ (NOR)

(iv) $p \to q \equiv \neg p \vee q \equiv (\neg p \downarrow q) \downarrow (\neg p \downarrow q)$ - from

$\equiv ((\neg p \downarrow \neg p) \downarrow q) \downarrow ((\neg p \downarrow \neg p) \downarrow q)$ .. from

(ii) $p \to q \equiv \neg p \vee q \equiv (\neg p \uparrow \neg p) \uparrow (q \uparrow q)$ from ③

$\equiv \{(p \uparrow p) \uparrow (p \uparrow p)\} \uparrow (q \uparrow q)$ from ①

(iii) (v) $p \Leftrightarrow q \equiv \underbrace{(p \to q)}_{r} \wedge \underbrace{(q \to p)}_{s}$

$\equiv (r \uparrow s) \uparrow (r \uparrow s)$ from ②

$r \equiv p \to q = ((p \uparrow p) \uparrow (p \uparrow p)) \uparrow (q \uparrow q)$ from ④

$s \equiv q \to p \equiv ((q \uparrow q) \wedge (q \uparrow q)) \uparrow (p \uparrow p)$

2Q)

(i)

(ii) (

(iii)

(iv) (f

Sol

UNIT - II :

* Normal forms :

* Introduction : By comparing truth tables one can determine whether two logical expressions P and Q are equivalent. But the process is very tidious when the no. of variables increases. A better method is to transform the expressions P and Q to some standard forms of expressions P' and Q' such that a simple comparison of P' & Q' shows whether $P \equiv Q$. the standard forms are called normal forms or canonical forms. There are 2 types of normal forms. (1) disjunctive NF (DNF) (2) Conjunctive NF (CNF)

The logical expressions, a product of the variables and their negations is called an elementary products. For example,

$p, \neg p, p \wedge q, \neg p \wedge q, p \wedge \neg q, \neg p \wedge \neg q$. are some elementary products in 2 variables.

similarly a sum of the variables and their negations is called elementary sums in 2 variables. Examples : $p, \neg p, p \vee q, \neg p \vee q, p \vee \neg q, \neg p \vee \neg q$] are some elementary sums.

NOTE: (1) It will be convinient to use the words products and sum in place of the logical connectives conjunction and disjunction.

(2) An elemantary product is identically false if and only if it contains atleast one pair of its factors in which one is the negation of the other.

(3) An elemantary sum is identically true if and only if it contains atleast one pair of factors in which one is the negation of the other.

* DNF : A Logical expression is said to be in DNF if it is the sum of elementary products.

Ex (1): $(p \wedge q) \vee (\neg p \wedge q) \vee (p \wedge \neg q)$; (2) $(p \wedge q) \vee (p \wedge r)$;

(3): $(p \wedge q) \vee (\neg p \wedge \neg q)$

Procedure to obtain a DNF of a given logical e

Step 1: Remove all '$\to$' and '$\leftrightarrow$' by an equival expression containing the connectives $\wedge, \vee, \neg$

For example: ① $p\vee(q\wedge r)\equiv(p\vee q)\wedge(p\vee r)$

$p\wedge(q\vee r)\equiv(p\wedge q)\vee(p\wedge r)$

② $p\to q\equiv\neg p\vee q$

③ $p\leftrightarrow q\equiv(p\to q)\wedge(q\to p)$

$\equiv(\neg p\vee q)\wedge(\neg q\vee p)$

$\equiv((\neg p\vee q)\wedge\neg q)\vee(\neg p\vee q\wedge p)$

$\equiv((\neg p\wedge\neg q)\vee(q\wedge\neg q)\wedge(\neg p\wedge p)\vee(q\wedge p)$

$\equiv((\neg p\wedge\neg q)\vee F)\vee(F\vee(q\wedge p))$

$\equiv(\neg p\wedge\neg q)\vee(q\wedge p)$

Step 2: Eliminate ~ before sum and product b double negation. (or)

① $\neg(\neg p)\equiv p$

② $\neg(p\vee q)\equiv(\neg p\wedge\neg q)$

$\neg(p\wedge q)\equiv(\neg p\vee\neg q)$

Step 3: Apply the distributive law until sum of elementary product is obtained.

* CNF: A logical exp. is said to be in CNF if it consists of a product of elementary sums. The

NOTE: CNF (or) DNF is not unique.

Examples: ① $(p\vee q)\wedge(\neg p\vee q)\wedge(\neg p\vee\neg q)$

② $(\neg p\vee q)\wedge(\neg q\vee p)$

③ $(p\vee q)\wedge(p\vee r)$

Q1) Obtain the DNF of the foll:

(i) $p\wedge(p\to q)$ (ii) $p\vee(\neg p\to(q\vee(q\to\neg r)))$ (iii) $p\to(p\to$

Sol: (iv) $\neg(p\vee q)\leftrightarrow(p\wedge q)$ (v) $\neg[p\to(q\wedge r))$

Sol: (i) $p \wedge (p \to q) \equiv p \wedge (\neg p \vee q)$

$\equiv (p \wedge \neg p) \vee (p \wedge q)$

which is the required DNF.

(ii) $p \vee (\neg p \to (q \vee (q \to \neg r)))$

$\equiv p \vee (\neg p \to (q \vee (\neg q \vee \neg r)))$

$\equiv p \vee (\neg(\neg p) \vee (q \vee (\neg q \vee \neg r)))$

$\equiv p \vee (p \vee (q \vee (\neg q \vee \neg r)))$

$\equiv p \vee (p \vee (q \vee \neg (q \wedge r)))$

$\equiv (p \vee (p \vee (\neg\neg q \vee \neg (q \wedge r))) \equiv p \vee q \vee \neg q \vee \neg r$

$\equiv \cancel{p \vee (p \vee \neg(\neg q \wedge (q \wedge r)))}$ × $\equiv p \vee T \vee \neg r$

$\equiv \cancel{(p \vee p) \vee \neg(\neg q \wedge (q \wedge r)))}$ × $\equiv p \vee \neg r$.

$\equiv \cancel{p \vee \neg(\neg q \wedge (q \wedge r)))}$ ×

$\equiv \cancel{(p \wedge p) \vee \neg(\neg q \wedge q \wedge r))}$ ×

(iii) $p \to (p \to q) \equiv p \to (\neg p \vee q)$

$\equiv \neg p \vee (\neg p \vee q) \equiv \neg p \vee (\neg p \vee \neg\neg q)$

~~$\equiv (\neg p \wedge \neg p) \vee \neg(p \wedge \neg q)$~~ ×

(iv) $\neg \underbrace{(p \vee q)}_{R} \leftrightarrow \underbrace{(p \wedge q)}_{S}$

$\equiv [\neg(p \vee q) \wedge (p \wedge q)] \vee [\neg(\neg(p \vee q)) \wedge \neg(p \wedge q)]$

$\equiv (\neg p \wedge \neg q \wedge p \wedge q) \vee [(p \vee q) \wedge (\neg p \vee \neg q)]$

$\equiv [\neg p \wedge \neg q \wedge p \wedge q] \vee [((p \vee q) \wedge \neg p] \vee [(p \vee q) \wedge \neg q]]$.

$\equiv [\neg p \wedge \neg q \wedge p \wedge q] \vee (p \wedge \neg p) \vee (q \wedge \neg q) \vee (p \wedge \neg q) \vee (q \wedge \neg q)$

(v) $\neg [p \to (q \wedge r)] \equiv \neg [\neg p \vee (q \wedge r)]$

$\equiv \neg\neg p \wedge \neg (q \wedge r)$

$\equiv p \wedge \neg (q \wedge r)$

$\equiv p \wedge (\neg q \vee \neg r)$

$\equiv (p \wedge \neg q) \vee (p \wedge \neg r)$.

2Q) Find DNF of $p\wedge(p\to q)$ using truth tables.

Sol:

| $p$ | $q$ | $\neg p\vee q$ | $p\wedge(\neg p\vee q)$ |
|---|---|---|---|
| T | T | T | T |
| T | F | F | F |
| F | T | T | F |
| F | F | T | F |

In this table, we have truth value T only in the first row and has F in all the other rows. The corresponding values for $p\wedge(\neg p\vee q)$ are Ts.

∴ The required DNF is $p\wedge(\neg p\vee q) \equiv (p\wedge\neg p)\vee(p\wedge$

3Q) obtain the DNF of the foll:

(i) $[p\to(p\to q)]\wedge\neg(\neg p\vee\neg q)$ $\quad (p\to q\equiv\neg p\vee q)$

Sol: $[p\to(\neg p\vee q)]\wedge\neg(\neg(p\wedge q))$ $\quad$ (Demorgans law)

$[\neg p\vee(\neg p\vee q)]\wedge(p\wedge q)$ $\quad$ (double negation)

$[(\neg p\vee\neg p)\vee q]\wedge(p\wedge q)$

$\underbrace{(\neg p\vee q)}_{A\vee B}\wedge\underbrace{(p\wedge q)}_{\equiv C\wedge D}$

$\cancel{(\neg p\wedge p)\vee(p\wedge q)}\vee(\neg p\wedge q)\vee(q\wedge q)$

$(\neg p\wedge p\wedge q)\vee(p\wedge q)$

(ii) $p\wedge\{\neg p\to(q\vee(q\to\neg r))\}$

(iii) $(p\vee q\vee\neg q)\wedge(p\vee\neg p)$

Sol: $[p\vee(q\vee\neg q)]\wedge[p\vee\neg p]$

$[p\vee T]\wedge[T]$

$\equiv[(p\vee q\vee\neg q)\wedge p]\vee[(p\vee q\vee\neg q)\wedge\neg p]$

$\equiv(p\wedge p)\vee(q\wedge p)\vee(\neg q\wedge p)\vee(p\wedge\neg p)\vee(q\wedge\neg p)\vee(\neg q$

$\equiv p\vee(p\wedge q)\vee(\neg q\wedge p)\vee(p\wedge\neg p)\vee(q\wedge\neg p)\vee(\neg q\wedge\neg p)$

(iv) $(p\to q)\Leftrightarrow(q\to r)$

(v) $\neg(p\to(q\wedge r))$

(vi) $(\neg P\vee\neg Q)\to(P\Leftrightarrow Q)$

48) Find the CNF of the foll:

(i) $p \wedge (p \to q)$

$\equiv p \wedge (\neg p \vee q)$ which is required product of sum terms.

(ii) $[q \vee (p \wedge r)] \wedge \neg[(p \vee r) \wedge q]$

$\equiv [(q \vee p) \wedge (q \vee r)] \wedge [\neg(p \vee r) \vee \neg q]$

$\equiv [(q \vee p) \wedge (q \vee r)] \wedge [(\neg p \wedge \neg r) \vee \neg q]$

$\equiv [(q \vee p) \wedge (q \vee r)] \wedge [(\neg p \vee \neg q) \wedge (\neg q \vee \neg r)]$

product is required product of sum terms.

(iii) $\neg(p \vee q) \Leftrightarrow (p \wedge q)$

$\equiv \{\neg[\neg(p \vee q)] \vee (p \wedge q)\} \wedge \{\neg(p \wedge q) \vee \neg(p \vee q)\}$

$A \Leftrightarrow B$
$\equiv (A \to B) \wedge (B \to A)$
$\equiv (\neg A \vee B) \wedge (\neg B \vee A)$

$\equiv \{(\underbrace{p \vee q}_{A}) \vee (\underbrace{p}_{B} \wedge \underbrace{q}_{C})\} \wedge \{(\underbrace{\neg p \vee \neg q}_{A}) \vee (\underbrace{\neg p}_{B} \wedge \underbrace{\neg q}_{C})\}$

$\equiv \{(p \vee q \vee p) \wedge (p \vee q \vee q)\} \wedge \{(\neg p \vee \neg q \vee \neg p) \wedge (\neg p \vee \neg q \vee \neg q)\}$

$\equiv (p \vee q) \wedge (p \vee q) \wedge (\neg p \vee \neg q) \wedge (\neg p \vee \neg q)$

$\equiv (p \vee q) \wedge (\neg p \vee \neg q)$

(iv) $p \to \{(p \to q) \wedge \neg(\neg p \vee \neg q)\}$

$\equiv p \to \{(\neg p \vee q) \wedge \neg(\neg(p \wedge q))\}$

$\equiv \underbrace{\neg p}_{A} \vee \{\underbrace{(\neg p \vee q)}_{B} \wedge \underbrace{(p \wedge q)}_{C}\}$

~~$\neg p \vee \{(\neg p \vee q$~~

$\equiv \{\neg p \vee (\neg p \vee q)\} \wedge \{\neg p \vee (p \wedge q)\}$

$\equiv \{\neg p \vee q\} \wedge \{(\neg p \vee p) \wedge (\neg p \vee q)\}$

$\equiv \{\neg p \vee q\} \wedge \{\neg p \vee p\} \wedge \{\neg p \vee q\}$

(v) $\{q \vee (p \wedge r)\} \wedge \neg\{(p \vee r) \wedge q\}$

(vi) $q \vee (p \wedge \neg q) \vee (\neg p \wedge \neg q)$ } same question

(vii) $q \vee \{(p \wedge \neg q) \vee (\neg p \wedge \neg q)\}$.

Q 5Q) Obtain CNF, DNF of the foll:

(i) $(\neg p \to r) \wedge (p \to q)$

(ii) $[q \vee (p \wedge r)] \wedge \neg[(p \vee r) \wedge q]$

(iii) $\{p \wedge \neg(q \vee r)\} \vee \{[(p \wedge q) \vee \neg r] \wedge p\}$.

**Sol:** (i) Take $P \equiv \neg p \to r$

$Q \equiv p \to q$

$\epsilon = P \wedge Q$.

| $p$ | $q$ | $r$ | $\neg p$ | $\neg p \to r$ ($P$) | $p \to q$ ($Q$) | $P \wedge Q$ |
|---|---|---|---|---|---|---|
| T | T | T | F | T | T | T |
| T | T | F | F | T | T | T |
| T | F | T | F | T | F | F |
| T | F | F | F | T | F | F |
| F | T | T | T | T | T | T |
| F | T | F | T | F | T | F |
| F | F | T | T | T | T | T |
| F | F | F | T | F | T | F |

Observe the last column of the above truth table. It has T in 1st, 2nd, 5th and 7th rows. Then the product terms corresponding to these rows are:

$(p \wedge q \wedge r), (p \wedge q \wedge \neg r), (\neg p \wedge q \wedge r), (\neg p \wedge \neg q \wedge r)$

$\therefore$ required DNF:

$(p \wedge q \wedge r) \vee (p \wedge q \wedge \neg r) \vee (\neg p \wedge q \wedge r) \vee (\neg p \wedge \neg q \wedge r)$.

CNF: Observe the last column from the truth Table we have 'F' in 3rd, 4th, 6th and 8th rows. and the corresponding sum terms are:

$(\neg p \vee q \vee \neg r), (\neg p \vee q \vee r), (p \vee \neg q \vee r), (\cancel{\neg} p \vee q \vee r)$.

$\therefore$ The required CNF is:

$(\neg p \vee q \vee \neg r) \wedge (\neg p \vee q \vee r) \wedge (p \vee \neg q \vee r) \wedge (p \vee q \vee r)$.

* Principal Disjunctive Normal Form:
We know that the DNF (or) CNF is not unique. In order to arrive at a unique NF of a given formula, we ~~studio~~ studied PDNF and PCNF.

* Minterm: A minterm consists of conjunction in which each statement variable (or) its negation, but not both appears only once. For example, for 2 variables p and q, the min terms are $p \wedge q$, $p \wedge \neg q$, $\neg p \wedge q$, $\neg p \wedge \neg q$. For three variables p, q and r, the min terms are $p \wedge q \wedge r$, $p \wedge q \wedge \neg r$, $p \wedge \neg q \wedge r$, $\neg p \wedge q \wedge r$, $\neg p \wedge \neg q \wedge r$, $\neg p \wedge q \wedge \neg r$, $p \wedge \neg q \wedge \neg r$, $\neg q \wedge \neg p \wedge \neg r$.

NOTE: No 2 minterms are equivalent.

* PDNF: PDNF of a given formula is an equivalent formula consisting of disjunctions of minterms only i.e., sum of minterms only. This is also known as the SOP canonical form or simply PDNF.

* Methods to obtain PDNF of a given formula.

(1) By Truth Table:

Step 1: Construct a Truth table of a given compound proposition.

Step 2: For every Truth value T of the given formula, select the min-term which also has the value T for the same combination of the truth value of the statement variables.

Step 3: The sum of the min-terms selected in Step 2 is the required PDNF.

(2) Without Truth Table (Laws of Logics):
To obtain PDNF through algebraic manipulations, we can use the foll. steps

Step 1: Firstly, replace the conditional ($\rightarrow$) and biconditional ($\leftrightarrow$) by their equivalent formulas containing only $\wedge$, $\vee$, and $\neg$.

Step 2: The negations are applied to the variables by using Demorgan's laws followed by the application of distributive laws (as that is applied CNF (or DNF).

Step 3: Drop elementary products which are contradictions such as $p \wedge \neg p$, $q \wedge \neg q$, ...

Step 4: If $p$ and $\neg p$ are missing in an elementary product $\alpha$, then replace $\alpha$ by $(\alpha \wedge p) \vee (\alpha \wedge \neg p)$.

Step 5: Repeat the above step until all elementary products are reduced to sum of minterms.

Step 6: Identical min-terms appearing in the disjunctions are deleted.

1Q) Obtain the PDNF of the foll. by using truth table and without using truth table

(i) $p \to q$

Sol (1)

| $p$ | $q$ | $p \to q$ |
|---|---|---|
| T | T | T |
| T | F | F |
| F | T | T |
| F | F | T |

consider last column. select rows having T.

$p \wedge q$, $\neg p \wedge q$, $\neg p \wedge \neg q$.

$\equiv (p \wedge q) \vee (\neg p \wedge q) \vee (\neg p \wedge \neg q)$

Sol (2) $p \to q$

$\equiv \neg p \vee q$

$\equiv (\neg p \wedge T) \vee (q \wedge T)$

$\equiv [\neg p \wedge (q \vee \neg q)] \vee [q \wedge (p \vee \neg p)]$

$\equiv (\neg p \wedge q) \vee (\neg p \wedge \neg q) \vee (q \wedge p) \vee (q \wedge \neg p)$

$\equiv (q \wedge p) \vee (\neg p \wedge q) \vee (\neg p \wedge \neg q)$

(ii) $p \leftrightarrow q$

| | $p$ | $q$ | $q \leftrightarrow q$ |
|---|---|---|---|
| (1) | T | T | T |
| | T | F | F |
| | F | T | F |
| | F | F | T |

$(p \wedge q), (\neg p \wedge \neg q)$

$(p \wedge q) \vee (\neg p \wedge \neg q)$.

(2) $p \leftrightarrow q \equiv (p \to q) \wedge (q \to p)$

$\equiv (\underbrace{\neg p \vee q}_{A}) \wedge (\underbrace{\neg q}_{B} \vee \underbrace{p}_{C})$

$\equiv [(\neg p \vee q) \wedge \neg q] \vee [(\neg p \vee q) \wedge p]$

(eliminate contradictions) $\equiv (\neg p \wedge \neg q) \vee (\underbrace{\cancel{q \wedge \neg q}}_{F}) \vee (\underbrace{\cancel{\neg p \wedge p}}_{F}) \vee (q \wedge p)$

$\equiv (\neg p \wedge \neg q) \vee (p \wedge q)$ //

(iii) $p \to \{(p \to q) \wedge \neg(\neg q \vee \neg p)\}$

$\equiv p \to \{(\neg p \vee q) \wedge \neg(\neg q \vee \neg p)\}$

$\equiv \neg p \vee \{(\neg p \vee q) \wedge \neg(\neg q \vee \neg p)\}$

$\equiv \neg p \vee \{(\underbrace{\neg p}_{A} \vee \underbrace{q}_{B}) \wedge (\underbrace{q \wedge p}_{C})\}$

$(A \vee B) \wedge C$

$\equiv (A \wedge C) \vee (B \wedge C)$.

$\equiv \neg p \vee \{(\neg p \wedge q \wedge p) \vee (q \wedge q \wedge p)\}$

$\equiv \neg p \vee \{(F \wedge q) \vee (p \wedge q)\}$

$\equiv \neg p \vee (p \wedge q)$

$\equiv (\neg p \wedge T) \vee (p \wedge q)$

$\equiv (\neg p \wedge (q \vee \neg q)) \vee (p \wedge q)$

$\equiv (\neg p \wedge q) \vee (\neg p \wedge \neg q) \vee (p \wedge q)$

(iv) $(p \wedge \neg q \wedge \neg r) \vee (q \wedge r)$

$\equiv (p \wedge \neg q \wedge \neg r) \vee (q \wedge r \wedge T)$.

$\equiv (p \wedge \neg q \wedge \neg r) \vee (\underbrace{q \wedge r}_{A} \wedge (\underbrace{p}_{B} \vee \underbrace{\neg p}_{C}))$

$\equiv (p \wedge \neg q \wedge \neg r) \vee (q \wedge r \wedge p) \vee (q \wedge r \wedge \neg p)$.

(v) $p \vee (p \wedge q)$

So. $\equiv (p \wedge T) \vee (p \wedge q)$

$\equiv (p \wedge (q \vee \neg q)) \vee (p \wedge q)$

$\equiv (p \wedge q) \vee (p \wedge \neg q) \vee (p \wedge q)$

$\equiv (p \wedge q) \vee (p \wedge \neg q)$

(vi) $(P \wedge Q) \vee (\neg P \wedge R) \vee (Q \wedge R)$

$(P \wedge Q) \vee (\neg P \wedge R \wedge T) \vee (Q \wedge R \wedge T)$

$\equiv (P \wedge Q \wedge T) \vee (\neg P \wedge R \wedge (Q \vee \neg Q)) \vee (Q \wedge R \wedge (P \vee \neg P$

$\equiv (P \wedge Q \wedge (R \vee \neg R)) \vee (\neg P \wedge R \wedge Q) \vee (\neg P \wedge R \wedge \neg Q) \vee$

$\equiv (P \wedge Q \wedge R) \vee (P \wedge Q \wedge \neg R) \vee (\neg P \wedge R \wedge Q) \vee (\neg P \wedge R \wedge \neg Q) \vee$

$(Q \wedge R \wedge P) \vee (Q \wedge R \wedge \neg P)$.

36 (i) $\equiv (P \wedge Q \wedge R) \vee (\neg P \vee Q \vee R) \vee (P \wedge Q \wedge \neg R) \vee (\neg P \wedge R \wedge \neg Q$

So

| $P$ | $Q$ | $R$ | $\neg P$ | $P \wedge Q$ | $\neg P \wedge R$ | $Q \wedge R$ | $(P \wedge Q) \vee (\neg P \wedge R)$ | Sol |
|---|---|---|---|---|---|---|---|---|
| T | T | T | F | T | F | T | T | T |
| T | T | F | F | T | F | F | T | T |
| T | F | T | F | F | F | F | F | F |
| T | F | F | F | F | F | F | F | F |
| F | T | T | T | F | T | T | T | T |
| F | ~~F~~T | F | T | F | F | F | F | F |
| F | F | T | T | F | T | F | T | T |
| F | F | F | T | F | F | F | F | F |

(ii)

(iii)

So

$(P \wedge Q \wedge R), (P \wedge Q \wedge \neg R), (\neg P \wedge Q \wedge R), (\neg P \wedge \neg Q \wedge R)$.

$\equiv (P \wedge Q \wedge R) \vee (P \wedge Q \wedge \neg R) \vee (\neg P \wedge Q \wedge R) \vee (\neg P \wedge \neg Q \wedge R)$.

* Advantages of PDNF are:

- PDNF of a given formula is unique.
- 2 formulas are equivalent if and only if the PDNF's coincide.
- If the given compound proposition is a tautology then its PDNF will contain all

possible n

*PCNF:
Max terms
consists of
negation b
max term
for 2 v
$p \vee q$, p
for 3 v
are $p \vee$
$\neg p \vee q \vee$
NOTE: T
Each of
exactly
of the
Princip
PCNF
an eq
of ma
only.
cano
PCNF
NOTE:
-ing
will
min
obta
Dem
(2) If
var
com
don
one
of

possible minterms.

*PCNF:

Max terms: For a given number of variables the maxterms consists of disjunctions in which each variable or its negation but not both appears only once, their the max terms of duals of min terms.

For 2 variables $p, q$, the max terms are $p \vee q$, $p \vee \neg q$, $\neg p \vee q$, $\neg p \vee \neg q$.

For 3 variables; $p, q, r$, the max terms are $p \vee q \vee r$, $p \vee q \vee \neg r$, $p \vee \neg q \vee r$, $p \vee \neg q \vee \neg r$, $\neg p \vee q \vee r$, $\neg p \vee q \vee \neg r$, $\neg p \vee \neg q \vee r$, $\neg p \vee \neg q \vee \neg r$.

NOTE: The no. of maxterms in $n$ variables is $2^n$. Each of the max terms has truth value F for exactly one combination of the truth values of the variables.

Principle Conjunctive Normal Form (PCNF): PCNF of a given formula can be defined as an equivalent formula consisting of conjunctive of max terms only (or) product of max terms only. This is also called the product of sums canonical form. The process of obtaining PCNF is the one followed for PDNF.

NOTE: (1) If PDNF (PCNF) of a given formula A containing 'n' variables then, PDNF (PCNF) of negation A will consist of sum of (products of) remaining min terms (maxterms) ~~which~~ do not appear in the PDNF of A. From $A \equiv \neg(\neg A)$ one can obtain PCNF of A by repeated applications of Demorgan laws to the PDNF of negation A.

(2) If the PCNF of a given formula A containing 'n' variables is known, then PCNF of negation A will consist of product of remaining maxterms which donot appear in the PCNF of A. From $A \equiv \neg(\neg A)$ one can obtain PDNF of A by repeated applications of Demorgan laws to the PCNF of A.

1Q) Obtain PCNF of the following:

2 So (i) $(\neg p \to q) \vee (q \leftrightarrow p)$.

$\equiv (\neg(\neg p) \vee q) \vee [(q \to p) \wedge (p \to q)]$

$\equiv (p \vee q) \vee [(\neg q \vee p) \wedge (\neg p \vee q)]$

$\equiv (p \vee q \vee \neg q \vee p) \wedge (p \vee q \vee \neg p \vee q)$ $\left[\because \begin{array}{l} p \vee p \equiv p \\ q \vee \neg q \equiv T \\ p \vee \neg p \equiv T \end{array}\right.$

$\equiv [p \vee T] \wedge [q \vee T]$

$\equiv p \wedge q$

$\equiv (p \vee F) \wedge (q \vee F)$

$\equiv (p \vee (q \wedge \neg q)) \wedge (q \vee (p \wedge \neg p))$

$\equiv (p \vee q) \wedge (p \vee \neg q) \wedge (p \vee q) \wedge (\neg p \vee q)$

3Q (i) $\equiv (p \vee q) \wedge (\neg p \vee q) \wedge (p \vee \neg q)$.

So (ii) $(\neg p \to q) \wedge (q \leftrightarrow p)$

$\equiv (p \vee q) \wedge (\neg q \vee p) \wedge (\neg p \vee q)$

(iii) $(p \wedge q) \vee (\neg p \wedge q)$

~~$\equiv (p \vee \neg p) \wedge (p \vee$~~

$\equiv [p \vee (\neg p \wedge q)] \wedge [q \vee (\neg p \wedge q)]$

$\equiv [(p \vee \neg p) \wedge (p \vee q)] \wedge [(q \vee \neg p) \wedge (q \vee q)]$

$\equiv T \wedge (p \vee q) \wedge (\neg p \vee q) \wedge q$.

(ii) $\equiv (p \vee q) \wedge (\neg p \vee q) \wedge (q \vee (p \wedge \neg p))$

(iii) $\equiv (p \vee q) \wedge (\neg p \vee q) \wedge (q \vee p) \wedge (q \vee \neg p)$

So $\equiv (p \vee q) \wedge (\neg p \vee q)$

(iv) $p \wedge (p \vee q)$

(v) $\neg p \wedge q$

(vi) $\neg(p \vee q)$

(vii) $(\neg p \to R) \wedge (q \leftrightarrow p)$.

(iv) $p \wedge (p \vee q)$

(iv) $\equiv [p \vee (q \wedge \neg q)] \wedge (p \vee q)$

(v) $\equiv (p \vee q) \wedge (p \vee \neg q) \wedge (p \vee q)$

(vi) $\equiv (p \vee q) \wedge (p \vee \neg q)$

(v) $\neg p \wedge q$.

$\equiv [\neg p \vee (q \wedge \neg q)] \wedge$

$\equiv (\neg p \vee q) \wedge (\neg$

$\equiv (p \vee q) \wedge (\neg$

(vi) $\neg(p \vee q)$.

$\equiv \neg p \wedge \neg q$

$\equiv [\neg p \vee (q$

$\equiv (\neg p \vee q)$

$\equiv (\neg p \vee q) \wedge$

(vii) $(\neg p \to R) \wedge$

$\equiv [\neg(\neg p)$

$\equiv (p \vee R$

$\equiv (p \vee$

$\equiv [p \vee R$

$\equiv (p \vee R$

$\equiv (p \vee R$

(viii) $p \wedge q$

(ix) $(\neg p \vee \neg$

(x) $p \to ($

(xi) $p \Leftrightarrow$

(viii) $p \wedge$

$\equiv [p \vee$

$\equiv (p \vee$

$\equiv (p$

(v) $\neg p \wedge q$

$\equiv [\neg p \vee (q \wedge \neg q)] \wedge [q \vee (p \wedge \neg p)]$

$\equiv (\neg p \vee q) \wedge (\neg p \vee \neg q) \wedge (q \vee p) \wedge (q \wedge \neg p)$

$\equiv (p \vee q) \wedge (\neg p \vee \neg q) \wedge (\neg p \vee q)$.

(vi) $\neg (p \vee q)$

$\equiv \neg p \wedge \neg q$

$\equiv [\neg p \vee (q \wedge \neg q)] \wedge [\neg q \vee (p \wedge \neg p)]$

$\equiv (\neg p \vee q) \wedge (\neg p \vee \neg q) \wedge (p \vee \neg q) \wedge (\neg p \vee \neg q)$

$\equiv (\neg p \vee q) \wedge (p \vee \neg q) \wedge (\neg p \vee \neg q)$

(vii) $(\neg P \rightarrow R) \wedge (Q \leftrightarrow P)$

$\equiv [\neg(\neg P) \vee R] \wedge [(Q \rightarrow P) \wedge (P \rightarrow Q)]$

$\equiv (p \vee R) \wedge [(\neg Q \vee P) \wedge (\neg P \vee Q)]$

$\equiv (P \vee R) \wedge [(\neg Q \vee P) \wedge (\neg P \vee Q)]$

$\equiv [P \vee R \vee (Q \wedge \neg Q)] \wedge [\neg Q \vee P \vee (R \wedge \neg R)] \wedge [\neg P \vee Q \vee (R \wedge \neg R)]$

$\equiv (P \vee R \vee Q) \wedge (P \vee R \vee \neg Q) \wedge (\neg Q \vee P \vee R) \wedge (\neg Q \vee P \vee \neg R) \wedge (\neg P \vee Q \vee R) \wedge (\neg P \vee Q \vee \neg R)$.

$\equiv (P \vee R \vee Q) \wedge (P \vee R \vee \neg Q) \wedge (R \vee \neg Q \vee \neg P) \wedge (P \vee \neg Q \vee R) \wedge (\neg P \vee Q \vee R)$.

(viii) $p \wedge q$

(ix) $(\neg p \vee \neg q) \rightarrow (p \rightarrow \neg q)$

(x) $p \rightarrow (p \wedge (q \rightarrow p))$

(xi) $p \leftrightarrow q$

(viii) $p \wedge q$

$\equiv [p \vee (q \wedge \neg q)] \wedge [q \vee (p \wedge \neg p)]$

$\equiv (p \vee q) \wedge (p \wedge \neg q) \wedge (q \vee p) \wedge (q \wedge \neg p)$

$\equiv (p \vee q) \wedge (\neg p \wedge q) \wedge (p \wedge \neg q)$

(ix) $(\neg p \vee \neg q) \rightarrow (p \rightarrow \neg q)$

$\equiv (\neg p \vee \neg q) \rightarrow (\neg p \vee \neg q)$

$\equiv \neg(\neg p \vee \neg q) \rightarrow (\neg p \vee \neg q)$

$\equiv (\neg\neg p \wedge \neg\neg q) \vee (\neg p \vee \neg q)$

$\equiv (\underset{A}{p} \wedge \underset{B}{q}) \vee (\underbrace{\neg p \vee \neg q}_{C})$

$\equiv (p \vee \neg p \vee \neg q) \wedge (q \vee \neg p \vee \neg q)$

$\equiv (T \vee \neg q) \wedge (\neg p \vee T)$

$\equiv \neg q \wedge \neg p$

$\equiv [\neg q \vee (p \wedge \neg p)] \wedge [\neg p \vee (q \wedge \neg q)]$

$\equiv (\neg q \vee p) \wedge (\neg q \vee \neg p) \wedge (\neg p \vee q) \wedge (\neg p \vee \neg q)$

$\equiv (p \vee \neg q) \wedge (\neg p \vee q) \wedge (\neg p \vee \neg q)$.

Sol (xi) $p \leftrightarrow q$

$\equiv (p \rightarrow q) \wedge (q \rightarrow p)$

$\equiv (\neg p \vee q) \wedge (\neg q \vee p)$

2Q) Obtain PDNF, PCNF by (i) using truth table (ii) without constructing truth table for $(\neg p \rightarrow q) \wedge (p \leftrightarrow q)$.

Sol: (i) truth table:

~~PDNF~~ :

| $p$ | $q$ | $\neg p$ | $\neg p \rightarrow q$ | $p \leftrightarrow q$ | $(\neg p \rightarrow q) \wedge (p \leftrightarrow q)$ |
|---|---|---|---|---|---|
| T | T | F | T | T | T |
| T | F | F | T | F | F |
| F | T | T | T | F | F |
| F | F | T | F | T | F |

$(p \wedge q) = \text{PDNF}$.

PCNF : $(\neg p \vee q)$, $(p \vee \neg q)$, $(p \vee q)$.

$\equiv (\neg p \vee q) \wedge (p \vee \neg q) \wedge (p \vee q)$.

(ii) PCNF :

$\equiv (\neg p \rightarrow q) \wedge (p$

$\equiv (\neg\neg p \vee q) \wedge [($

$\equiv (p \vee q) \wedge (\neg p$

~~$\equiv p \vee q$~~ is

PDNF :

~~$\neg p$~~

PCNF of $\neg A$

now PDNF $\equiv \neg$

$\equiv$

$\equiv$

$\equiv$

3Q) S.T. the

(i) $p \vee (q \wedge$

(ii) $(p \vee q) \wedge$

Sol: (i) PCN

$\equiv (p \vee$

$\equiv ($

$\equiv ($

$\equiv ($

$\equiv [p$

(ii) $(p \vee q)$

$\equiv ($

$\equiv$

$\equiv$

$\equiv$

$\therefore$

4Q) P.T.

(iii) PCNF :

$\equiv (\neg p \to q) \wedge (p \leftrightarrow q)$

$\equiv (\neg\neg p \vee q) \wedge [(p \to q) \wedge (q \to p)]$

$\equiv (p \vee q) \wedge (\neg p \vee q) \wedge (\neg q \vee p)$

$\cancel{\equiv p \vee q}$ is in PCNF.

PDNF :

[ $A \to$ PCNF
find PCNF $\to \neg A$
PDNF $\equiv \neg\neg A$ ]

$\cancel{\equiv \neg p}$

PCNF of $\neg A \equiv (\neg p \vee \neg q)$

now PDNF $\equiv \neg\neg A$

$\equiv \neg(\neg p \vee \neg q)$

$\equiv \neg\neg p \wedge \neg\neg q$

$\equiv (p \wedge q)$ //.

3Q) S.T. the formulas are logically equivalent using PCNF

(i) $p \vee (q \wedge r)$

(ii) $(p \vee q) \wedge (p \vee r)$

Sol: (i) PCNF : $p \vee (q \wedge r)$

$\equiv (p \vee q) \wedge (p \vee r)$.

$\equiv (p \vee q \vee F) \wedge (p \vee r \vee F)$

$\equiv (p \vee q \vee (r \wedge \neg r)) \wedge (p \vee r \vee (q \wedge \neg q))$

$\equiv (p \vee q \vee r) \wedge (p \vee q \vee \neg r) \wedge (p \vee r \vee q) \wedge (p \vee r \vee \neg q)$.

$\equiv (p \vee q \vee r) \wedge (p \vee q \vee \neg r) \wedge (p \vee \neg q \vee r)$.

(ii) $(p \vee q) \wedge (p \vee r)$.

$\equiv (p \vee q \vee (F)) \wedge (p \vee r \vee F)$

$\equiv (p \vee q \vee (r \wedge \neg r)) \wedge (p \vee r \vee (q \wedge \neg q))$

$\equiv (p \vee q \vee r) \wedge (p \vee q \vee \neg r) \wedge (p \vee r \vee q) \wedge (p \vee r \vee \neg q)$.

$\equiv (p \vee q \vee r) \wedge (p \vee q \vee \neg r) \wedge (p \vee \neg q \vee r)$.

$\therefore$ (i) and (ii) are logically equivalent.

4Q) P.T. (i) & (ii) are equivalent using PDNF.

(i) $p \vee (q \wedge r)$

$\equiv (p \wedge T \wedge T) \vee (q \wedge r \wedge T)$

$\equiv [p \wedge (q \vee \neg q) \wedge (r \vee \neg r)] \vee [q \wedge r \wedge (p \vee \neg p)]$

$\equiv [p \wedge q \wedge (r \vee \neg r)] \vee [p \wedge \neg q \wedge (r \vee \neg r)] \vee [q \wedge r \wedge p] \vee [q \wedge r \wedge \neg p]$

$\equiv (p \wedge q \wedge r) \vee (p \wedge q \wedge \neg r) \vee (p \wedge \neg q \wedge r) \vee (p \wedge \neg q \wedge \neg r) \vee (q \wedge r \wedge p) \vee (q \wedge r \wedge \neg p)$

$\equiv (p \wedge q \wedge r) \vee (p \wedge q \wedge \neg r) \vee (p \wedge \neg q \wedge r) \vee (p \wedge \neg q \wedge \neg r) \vee (\neg p \wedge q \wedge r)$

(ii) $(p \vee q) \wedge (p \vee r)$

$\equiv [p \wedge (p \vee r)] \vee [q \wedge (p \vee r)]$

$\equiv (p \wedge p) \vee (p \wedge r) \vee (q \wedge p) \vee (q \wedge r)$.

$\equiv (T \wedge T \wedge p) \vee (p \wedge r \wedge T) \vee (q \wedge p \wedge T) \vee (q \wedge r \wedge T)$

$\equiv [p \wedge (q \vee \neg q) \wedge (r \vee \neg r)] \vee (p \wedge r \wedge (q \vee \neg q)) \vee (q \wedge p \wedge (r \vee \neg r)) \vee (q \wedge r \wedge (p \vee \neg p))$

$\equiv [p \wedge q \wedge (r \vee \neg r)] \vee (p \wedge \neg q \wedge (r \vee \neg r)) \vee (p \wedge r \wedge q) \vee (p \wedge r \wedge \neg q) \vee (q \wedge p \wedge r) \vee (q \wedge p \wedge \neg r) \vee (q \wedge r \wedge p) \vee (q \wedge r \wedge \neg p)$

$\equiv (p \wedge q \wedge r) \vee (p \wedge q \wedge \neg r) \vee (p \wedge \neg q \wedge r) \vee (p \wedge \neg q \wedge \neg r) \vee (p \wedge q \wedge r) \vee (\neg p \wedge q \wedge r) \vee (p \wedge \neg q \wedge r) \vee (p \wedge q \wedge \neg r)$

$\equiv (p \wedge q \wedge r) \vee (\neg p \wedge q \wedge r) \vee (p \wedge \neg q \wedge r) \vee (p \wedge q \wedge \neg r) \vee (p \wedge \neg q \wedge \neg r)$

∴ (i) and (ii) are logically equivalent.

5Q) ~~Given~~ Obtain the PCNF and PDNF of the foll. formula

(i) $P \vee [\neg P \to (Q \vee (\neg Q \to R))]$

~~≡ pv[¬p → Q (~~

$\equiv P \vee [\neg P \to (Q \vee (\neg\neg Q \vee R))]$

$\equiv P \vee [\neg P \to (Q \vee Q \vee R)]$

$\equiv P \vee [\neg P \to (Q \vee R)]$

$\equiv P \vee [\neg\neg P \vee (Q \vee R)]$

$\equiv P \vee [P \vee Q \vee R]$

$\equiv (P \vee P) \vee (Q \vee R)$

$\equiv P \vee Q \vee R$

which is in PCNF. of assume A.

PCNF of $\neg A$ : $(P \vee Q \vee \neg R) \wedge (P \vee \neg Q \vee R) \wedge (\neg P \vee Q \vee R) \wedge (\neg P \vee \neg Q \vee R) \wedge (P \vee \neg Q \vee \neg R) \wedge (\neg P \vee Q \vee \neg R) \wedge (\neg P \vee \neg Q \vee \neg R)$

PDNF $\equiv \neg\neg A$.

$\equiv \neg\{(P \vee Q \vee \neg R) \wedge (P \vee \neg Q \vee R) \wedge (\neg P \vee Q \vee R) \wedge (\neg P \vee \neg Q \vee R) \wedge (P \vee \neg Q \vee \neg R) \wedge (\neg P \vee Q \vee \neg R) \wedge (\neg P \vee \neg Q \vee \neg R)\}$

$\equiv \neg(P \vee Q \vee \neg R) \vee \neg(P \vee \neg Q \vee R) \vee \neg(\neg P \vee Q \vee R) \vee \neg(\neg P \vee \neg Q \vee R) \vee \neg(P \vee \neg Q \vee \neg R) \vee \neg(\neg P \vee Q \vee \neg R) \vee \neg(\neg P \vee \neg Q \vee \neg R)$.

$\equiv (\neg P \wedge \neg Q \wedge R) \vee (\neg P \wedge Q \wedge \neg R) \vee (P \wedge \neg Q \wedge \neg R) \vee (P \wedge Q \wedge \neg R) \vee (\neg P \wedge Q \wedge R) \vee (P \wedge \neg Q \wedge R) \vee (P \wedge Q \wedge R)$.

68) Given, PDNF of A is $(P \wedge Q \wedge R) \vee (P \wedge Q \wedge \neg R) \vee (\neg P \wedge Q \wedge R) \vee (\neg P \wedge Q \wedge \neg R)$.

Find PCNF.

Sol: PDNF of $\neg A$:

$(P \wedge \neg Q \wedge R) \vee (\neg P \wedge \neg Q \wedge R) \vee (P \wedge \neg Q \wedge \neg R) \vee (\neg P \wedge \neg Q \wedge \neg R)$.

PCNF is $\neg\neg A$ :

$\equiv \neg\{(P \wedge \neg Q \wedge R) \vee (\neg P \wedge \neg Q \wedge R) \vee (P \wedge \neg Q \wedge \neg R) \vee (\neg P \wedge \neg Q \wedge \neg R)\}$

$\equiv \neg(P \wedge \neg Q \wedge R) \wedge \neg(\neg P \wedge \neg Q \wedge R) \wedge \neg(P \wedge \neg Q \wedge \neg R) \wedge \neg(\neg P \wedge \neg Q \wedge \neg R)$

$\equiv (\neg P \vee Q \vee \neg R) \wedge (P \vee Q \vee \neg R) \wedge (\neg P \vee Q \vee R) \wedge (P \vee Q \vee R)$.

* The theory of inference for statement calculus:
The main function of logic is to provide rules of inference (or) principles of reasoning. The theory associated with such rules is known as inference theory because it is concerned with the inferring of a conclusion from certain premises.
If a conclusion is derived from a set of

premises by using the accepted set of reasoning then such a process of derivation is called a deduction or a formal proof and the argument is called a valid argument and the conclusion which is arrived by these rules is called a valid conclusion

* Argument : A sequence of statements that ends with a conclusion is known as an argument.
For example, consider a set of proposition $P_1, P_2, \ldots, P_n$ and a proposition $C$, then a Compound proposition $(P_1 \land P_2 \land \ldots \land P_n) \to C$ is called and argument where $P_1, P_2, \ldots P_n$ are called premises of the argument and $C$ is called the conclusion of the argument.
Sometimes an argument of the above form is written in the foll. form:

$$\begin{array}{l} P_1 \\ P_2 \\ P_3 \\ \vdots \\ P_n \\ \hline \therefore C \\ \hline \end{array}$$

* Consistency of Premises : The premises $P_1, P_2, \ldots, P_n$ of an argument are said to be consistent if their conjunction $(P_1 \land P_2 \land \ldots \land P_n)$ is true in atleast one possible situation.

* The premises $P_1, P_2, \ldots, P_n$ of an argument are said to be inconsistent if their conjunction $(P_1 \land P_2 \land \ldots \land P_n)$ is false in every possible situation.

Q1) S.T. the premises $p \to q$, $p \to r$, $q \to \neg r$, $r$ are consistent.

Sol: let $P_1 = p \to q$ ; $P_2 = p \to r$ ; $P_3 = q \to \neg r$ ; $P_4 = r$.

| p | q | r |
|---|---|---|
| T | T | T |
| T | T | F |
| T | F | T |
| T | F | F |
| F | T | T |
| F | T | F |
| F | F | T |
| F | F | F |

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th
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| p | q | r | $P_1$ | $P_2$ | $\neg r$ | $P_3$ | $P_1 \wedge P_2$ | $P_3 \wedge P_4$ | $P_1 \wedge P_2 \wedge P_3 \wedge P_4$ |
|---|---|---|---|---|---|---|---|---|---|
| T | T | T | T | T | F | F | T | F | F |
| T | T | F | T | F | T | T | F | F | F |
| T | F | T | F | T | F | T | F | T | F |
| T | F | F | F | F | T | T | F | F | F |
| F | T | T | T | T | F | F | T | F | F |
| F | T | F | T | T | T | T | T | F | F |
| F | F | T | T | T | F | T | T | T | T |
| F | F | F | T | T | T | T | T | F | F |

since there exists atleast one True in their conjunction, They are said to be consistent.

*Validity using truth tables:

Let A and B be two statement formulas. We say that B logically follows from A (or) B is a valid conclusion of the premise A if and only if $A \rightarrow B$ is a totaulogy.

Similarly the argument which yields a conclusion C from the premises $P_1, P_2, \ldots, P_n$ is valid if and only if the statement $(P_1 \wedge P_2 \wedge \ldots \wedge P_n) \rightarrow C$ is a totaulogy.

*Procedure for testing the validity of an argument using truth table.

Step 1: Identify the premises and conclusion of the argument.

Step 2: Construct a truth table showing the truth values of all premises and the conclusion

Step 3: Find the rows (called critical rows) in which all the premises are true.

Step 4: In each critical row determine whether the conclusion of argument is also true.

a) If in each critical row ~~determine~~ ~~whether~~ the conclusion is also true then the argument is a valid argument

b) If there is atleast one critical row in which the conclusion is false, the argument is invalid.

1Q) Determine whether the conclusion followed logically from the given premises is valid or not. $P_1 : P \to Q$ ; $P_2 : \neg(P \wedge Q)$ ; $C : \neg P$

Sol:

| $P$ | $Q$ | $P \to Q$ ($P_1$) | $P \wedge Q$ | $\neg(P \wedge Q)$ ($P_2$) | $\neg P$ |
|---|---|---|---|---|---|
| T | T | T | T | F | F |
| T | F | F | F | T | F |
| F | T | T | F | T | T |
| F | F | T | F | T | T |

We observe that in the conclusion, 3rd and 4th rows have T and the corresponding premises also have T (in the same rows)
∴ The given argument is a valid argument
wrong

2Q) $P_1 : P \to Q$ ; $P_2 \to \neg Q$ ; $C \to Q$

Sol: wrong

| $P$ | $Q$ | $P \to Q$ | $\neg Q$ | $Q \equiv C$ |
|---|---|---|---|---|
| T | T | T | $\boxed{F}$ | T |
| T | F | F | T | F |
| F | T | T | $\boxed{F}$ | T |
| F | F | T | T | F |

Since the rows holding true in the conclusion do not hold the same true value in their corresponding rows of the premises (1st and 3rd rows), the argument is not said to be valid.

3Q) $P_1 : P \to Q$ ; $P_2 : P$ ; $C : Q$.

4Q) $P_1 : \neg P$ ; $P_2 : P \leftrightarrow Q$ ; $C : \neg(P \wedge Q)$

5Q) $P_1 : P \to Q$ ; $P_2 : Q$ ; $C : P$

6Q) $P_1 : P \to Q$ ; $P_2 : \neg P$ ; $C : P$.

7Q) $P_1 : P \vee Q$ ; $P_2 : P \to R$ ; $C : R$ ; $P_3 : Q \to R$

8Q) $P_1 : P \to (Q \to R)$ ; $P_2 : P \wedge Q$ ; ~~[illegible]~~ ; $C : R$.

9Q) $H_1 : P \to (Q \to R)$ ; $H_2 : R$ ; $C : P$.

Sol: (8) wrong

| P | Q | R | $Q \to R$ | $P \to Q \to R$ | $P \wedge Q$ | $R \equiv C$ |
|---|---|---|---|---|---|---|
| T | T | T | T | T | T | T |
| T | T | F | F | F | T | F |
| T | F | T | T | T | $\boxed{F}$ | T |
| T | F | F | T | T | F | F |
| F | T | T | T | T | $\boxed{F}$ | T |
| F | ~~T~~ | F | F | T | F | F |
| F | F | T | T | T | $\boxed{F}$ | T |
| F | F | F | T | T | F | F |

Since the rows holding T in conclusion do not hold T in all the corresponding premises ∴ The argument is invalid.

(7) wrong

| P | Q | R | $P \vee Q$ | $P \to R$ | $Q \to R$ | R |
|---|---|---|---|---|---|---|
| T | T | T | ~~F~~ T | T | T | T |
| T | T | F | ~~F~~ T | F | F | F |
| T | F | T | T | T | T | T |
| T | F | F | T | F | T | F |
| F | T | T | T | T | T | T |
| F | T | F | T | T | F | F |
| F | F | T | $\boxed{F}$ | T | T | T |
| F | F | F | F | T | T | F |

Since not all rows having T in conclusion have T's in the corresponding premises, it is an invalid argument.

(9) wrong

| P | Q | R | $Q \to R$ | $P \to (Q \to R)$ | R | P |
|---|---|---|---|---|---|---|
| T | T | T | T | T | T | T |
| T | T | F | F | F | F | T |
| T | F | T | T | T | T | T |
| T | F | F | T | T | F | T |
| F | T | T | T | T | T | F |
| F | T | F | F | T | F | F |
| F | F | T | T | T | T | F |
| F | F | F | T | T | F | F |

Since not all rows in the conclusion having T have T in their corresponding premises. The given argument is not a valid argument.

10) S.T. the conclusion C follows from the given premises (i) $P_1 : \neg Q$ ; $P_2 : P \to Q$ ; $C : \neg P$

(ii) $P_1 : P \to Q$ ; $P_2 : \neg P$ ; $C : Q$.

(iii) $P_1 : \neg P \vee Q$ ; $P_2 : \neg(Q \wedge \neg R)$ ; $P_3 : \neg R$ ; $C : \neg P$

Sol: (i)

| P | Q | $\neg Q$ ($P_1$) | $P \to Q$ ($P_2$) | $\neg P$ (C) |
|---|---|---|---|---|
| T | T | F | T | F |
| T | F | T | F | F |
| F | T | F | T | T |
| F | F | T | T | T |

Since, in 4th row the two premises are true Therefore, 4th row is a critical row & Also the conclusion C is true. Hence, the given argument is a valid argument.

(ii)

| P | Q | $P \to Q$ | $\neg P$ | Q | |
|---|---|---|---|---|---|
| T | T | T | F | T | × |
| T | F | F | F | F | × |
| F | T | T | T | T | ✓ critical row (if all P's are T) |
| F | F | T | T | $\boxed{F}$ | ✓ critical row |

Rows 3 and 4 are critical rows, but in row 3, conclusion has T and row 4 has conclusion F which doesn't satisfy the condition of valid argument. Therefore the given argument is an invalid argument.

(iii)

| P | Q | R | $\neg P$ | $\neg P \vee Q$ ($P_1$) | $\neg R$ ($P_3$) | $Q \wedge \neg R$ | $\neg(Q \wedge \neg R)$ ($P_2$) | $\neg P$ | |
|---|---|---|---|---|---|---|---|---|---|
| T | T | T | F | T | F | F | T | F | NCR |
| T | T | F | F | T | T | T | F | F | NCR |
| T | F | T | F | F | F | F | T | F | NCR |
| T | F | F | F | F | T | F | T | F | NCR |
| F | T | T | T | T | F | F | T | T | NCR |
| F | T | F | T | T | T | T | F | T | NCR |
| F | F | T | T | T | F | F | T | T | NCR |
| F | F | F | T | T | T | F | T | T | CR. |

In the 8th row all premises are true so it is ~~the~~ only critical row. Since in the critical row the conclusion is true, the given argument is a valid argument and ⁝ C follows from the given premises.

(iv) $P_1 : P \to Q$ ; $C : P \to (P \wedge Q)$.

(v) $P_1 : \neg P$ ; $P_2 : P \vee Q$ ; $C : Q$.

(vi) $P_1 : P \vee Q$ ; $P_2 : P \to R$ ; $P_3 : Q \to R$ ; $C : R$.

(vii) $P_1 : P \to (Q \to R)$ ; $P_2 : P \wedge Q$ ; $C : R$.

(viii) $P_1 : P \to (Q \to R)$ $P_2 : R$ ; $C : P$.

N.

Sol: (iv)

| P | Q | P → Q | P ∧ Q | P → (P ∧ Q) |
|---|---|---|---|---|
| T | T | T | T | T |
| T | F | F | F | F |
| F | T | T | F | T |
| F | F | T | F | T |

Since premises have 3 critical rows (1st, 3rd and 4th) and their conclusion is also true, the given argument is a valid argument

(v)

| P | Q | $P_1$ ¬P | $P_2$ P ∨ Q | C Q |
|---|---|---|---|---|
| T | T | F | T | T |
| T | F | F | T | F |
| F | T | T | T | T |
| F | F | T | F | F |

Since premises have 3rd row as only critical row and its conclusion is also true, the given argument is valid.

(vi)

| P | Q | R | $P_1$ P ∨ Q | $P_2$ P → R | $P_3$ Q → R | R |
|---|---|---|---|---|---|---|
| T | T | T | T | T | T | T |
| T | T | F | T | F | F | F |
| T | F | T | T | T | T | T |
| T | F | F | T | F | T | F |
| F | T | T | T | T | T | T |
| F | T | F | T | T | F | F |
| F | F | T | F | T | T | ~~F~~ T |
| F | F | F | F | T | T | F |

The premises have 3 critical rows (1, 3, 5) and their conclusion is also True. So the given argument is a valid argument.

(vii)

| P | Q | R |
|---|---|---|
| T | T | T |
| T | T | F |
| T | F | T |
| T | F | F |
| F | T | T |
| F | T | F |
| F | F | T |
| F | F | F |

Since the ... and thei... argumen...

(viii)

| P | Q |
|---|---|
| T | T |
| T | T |
| T | F |
| T | F |
| F | |
| F | T |
| F | |
| F | |

~~Since~~ rows ... rows ... and ... argu...

(vii)

| P | Q | R | $Q \to R$ | $P_1$: $P \to (Q \to R)$ | $P_2$: $P \wedge Q$ | C: R |
|---|---|---|---|---|---|---|
| T | T | T | T | T | T | T |
| T | T | F | F | F | T | F |
| T | F | T | T | T | F | T |
| T | F | F | T | T | F | F |
| F | T | T | T | T | F | T |
| F | T | F | F | T | F | F |
| F | F | T | T | T | F | T |
| F | F | F | T | T | F | F |

Since the premises have 1 critical row (1) and their conclusion is true, the given argument is a valid argument.

(viii)

| P | Q | R | $Q \to R$ | $P_1$: $P \to (Q \to R)$ | $P_2$: R | C: P |
|---|---|---|---|---|---|---|
| T | T | T | T | T | T | T |
| T | T | F | F | F | F | T |
| T | F | T | T | T | T | T |
| T | F | F | T | T | F | T |
| F | T | T | T | T | T | $\boxed{F}$ |
| F | T | F | F | T | F | F |
| F | F | T | T | T | T | $\boxed{F}$ |
| F | F | F | T | T | F | F |

~~Since~~ The given argument has 3 critical rows (1, 3, 7). But only 2 of the critical rows (1, 3) have true in their conclusion and the other has false. So the given argument is not a valid argument.

* Without using truth table: The truth table technique become tedi when the no. of atomic variables present in all the formulas representing the premises and conclusion is large. To overcome this disadvantage, we using the foll. techniques.

Now we discuss the process of derivation by which one demonstrates that a particular formula is a valid consequence (or) conclusion of a given set of premises.

Before we do this, we need 2 rules of inference which are called Rule P and Rule T.

- Rule P: A premise maybe introduced at any point in the derivation.
- Rule T: A formula S maybe introduced in a derivation if it is a tautologically implied by any one or more of the preceding formulas in the derivation.

* Rule of Inference: [ Implications / Equivalences.

(1) Addition:

$p \Rightarrow p \vee q$ $\qquad \dfrac{p}{p \vee q}$

$q \Rightarrow p \vee q$

(2) Simplification:

$p \wedge q \Rightarrow p$ $\qquad \dfrac{p \wedge q}{\therefore p}$

$p \wedge q \Rightarrow q$ $\qquad \dfrac{p \wedge q}{\therefore q}$

(3) conjunction:

$p$

$q$

$\therefore p \wedge q$

(4) Modus Ponens

$p \to q$

$\dfrac{p}{\therefore q}$

(5) Modus Tolle

$p \to q$

$\dfrac{\neg q}{\therefore \neg p}$

(6) hypothetic

$p \to q$

$q \to r$

$\therefore p \to r$

(7) Disjunct

$p \vee$

$\neg p$

(8) Contr

$(p \to$

(9) Dest

$(P$

* Equ

(1) $P \to$

(2) $P \Leftrightarrow$

(3) $\neg($

(4) Modus Ponens:

$$\begin{array}{l} p \to q \\ p \\ \hline \therefore q \end{array}$$

(5) Modus Tollens:

$$\begin{array}{l} p \to q \\ \neg q \\ \hline \therefore \neg p \end{array}$$

(6) hypothetical syllogism:

$$\begin{array}{l} p \to q \\ q \to r \\ \hline \therefore p \to r \end{array}$$

(7) Disjunctive syllogism:

$$\begin{array}{l} p \vee q \\ \neg p \\ \hline q \end{array}$$

(8) Constructive Dilemma:

$$\begin{array}{l} (p \to q) \wedge (r \to s) \\ p \vee r \\ \hline \therefore q \vee s \end{array}$$

(9) Destructive Dilemma:

$$\begin{array}{l} (p \to q) \wedge (r \to s) \\ \neg q \vee \neg s \\ \hline \neg p \vee \neg r \end{array}$$

*Equivalences:

(1) $P \to Q \equiv \neg P \vee Q$

(2) $P \leftrightarrow Q \equiv (P \to Q) \wedge (Q \to P)$

(3) $\neg(P \leftrightarrow Q) \equiv P \leftrightarrow \neg Q$

(4) $\neg(P \to Q) \equiv P \wedge \neg Q$

(5) $P \to (Q \to R) \equiv (P \wedge Q) \to R$.

Problems:

1Q) S.T. $S \vee R$ is tautologically implied by $(P \vee Q) \wedge (P \to R) \wedge (Q \to S)$

2Q) Demonstrate that $R$ is a valid inference from the premises $P \to Q$, $Q \to R$, $P$.

Sol: Given Premises are

$P_1 : P \to Q$ ; $P_2 : Q \to R$ ; $P_3 = P$.

① $P \to Q$ {1}

② $Q \to R$ {2}

③ $P \to R$ {1,2} (hypothetical syllogism)

④ $P$ {4}

⑤ $R$ {1, 2, 4} (modus ponens)

$\therefore$ $R$ is a valid inference.

3Q) S.T. $R \vee S$ follows logically from the premises $C \vee D$, $(C \vee D) \to \neg H$, $\neg H \to (A \wedge \neg B)$, $(A \wedge \neg B) \to R \vee S$.

Sol: Given premises are

$P_1 : C \vee D$ ; $P_2 : (C \vee D) \to \neg H$ ; $P_3 : \neg H \to (A \wedge \neg B)$;

$P_4 : (A \wedge \neg B) \to R \vee S$.

① $(C \vee D) \to \neg H$ {1}

② $C \vee D$ {2}

③ $\neg H$ {1, 2} (by modus ponens)

④ $\neg H \to (A \wedge \neg B)$ {4}

⑤ $(A \wedge \neg B)$ {1, 2, 4} (by modus ponens)

⑥ $(A \wedge \neg B) \to R \vee S$ {6}

⑦ $R \vee S$ {1, 2, 4, 6}

(or)

① $(C \vee D) \to \neg$

② $\neg H \to (A$

③ $(C \vee D) \to ($

④ $(A \wedge \neg B) \to$

⑤ ~~$(A \wedge \neg B) \to$~~

⑤ $C \vee D \to R$

⑥ $C \vee D$ {6}

⑦ $R \vee S$ {

Sol (ii): ① P

② ¬

③

④

⑤

⑥

⑦

⑧

⑨

⑩

4Q) S.T

the p

Sol: Gi

$P_1$ : P

①

②

③

④

⑤

(or)

① $(C \vee D) \rightarrow \neg H$ $\{1\}$

② $\neg H \rightarrow (A \wedge \neg B)$ $\{2\}$

③ $(C \vee D) \rightarrow (A \wedge \neg B)$ $\{1, 2\}$ by hypothetical syllogism

④ $(A \wedge \neg B) \rightarrow R \vee S$ $\{4\}$

~~⑤ $(A \wedge \neg B) \rightarrow R \vee S$ $\{1, 2, 4\}$ (by hypothetical syllogism)~~

⑥ $C \vee D \rightarrow R \vee S$ $\{1, 2, 4\}$ by hypothetical syllogism.

⑥ $C \vee D$ $\{6\}$

⑦ $R \vee S$ $\{1, 2, 4, 6\}$ modus ponens.

Sol (ii): ① $P \vee Q$ $\{1\}$ $[P \rightarrow Q = \neg Q \rightarrow \neg P]$

② $\neg P \rightarrow Q$ $\{1\}$

③ $Q \rightarrow S$ $\{3\}$

④ $\neg P \rightarrow S$ $\{1, 3\}$ (hypothetical syllogism).

⑤ $\neg S \rightarrow \neg(\neg P)$ $\{1, 3\}$

⑥ $\neg S \rightarrow P$ $\{1, 3\}$.

⑦ $P \rightarrow R$ $\{7\}$.

⑧ $\neg S \rightarrow R$ $\{1, 3, 7\}$ (hypothetical syllogism).

⑨ $\neg(\neg S) \vee R$ $\{1, 3, 7\}$.

⑩ $S \vee R$ $\{1, 3, 7\}$.

48) S.T $R \wedge (P \vee Q)$ is a valid conclusion from the premises $P \vee Q$, $Q \rightarrow R$, $P \rightarrow M$, $\neg M$.

Sol: Given premises are:

$P_1 : P \vee Q$ ; $P_2 : Q \rightarrow R$ ; $P_3 : P \rightarrow M$ ; $P_4 : \neg M$.

① $P \rightarrow M$ $\{1\}$

② $\neg M$ $\{2\}$

③ $\neg P$ $\{1, 2\}$ modus tollens.

④ $P \vee Q$ $\{4\}$

⑤ $Q$ $\{1, 2, 4\}$ disjunctive syllogism.

⑥ $Q \to R$ {6}

⑦ $R$ {1, 2, 4, 6} modus ponens.

⑧ $R \wedge (P \vee Q)$ {1, 2, 4, 6} by conjunction

if $P, Q \Rightarrow P \wedge Q$

$\therefore R \wedge (P \vee Q)$ is a valid conclusion.

5Q) S.T. $t$ is a valid conclusion from the following premises $P_1 : p \to q$ ; $P_2 : q \to r$ ; $P_3 : r \to s$ ; $P_4 : \neg s$ ; $P_5 : p \vee t$.

Sol: ① $p \to q$ {1}

② $q \to r$ {2}

③ $p \to r$ {1, 2} hypothetical syllogism.

④ $r \to s$ {4}

⑤ $p \to s$ {1, 2, 4} hypothetical syllogism

⑥ $\neg s$ {6}

⑦ $\neg p$ {1, 2, 4, 6} modus tollens

⑧ $p \vee t$ {7}.

⑨ $t$ {1, 2, 4, 6, 7} by disjunctive syllogism

$\therefore t$ is a valid conclusion.

6Q) Prove the validity of the following arguments:

- If I get the job and work hard then I will get promoted.
- If I get promoted then I will be happy.

Conclusion:

- I will not be happy therefore either I will not get the job or I will not work hard.

Sol: $p$ : I get the job.

$q$ : I work hard

$r$ : I get promoted

$s$ : I will be happy.

The premises are $P_1 : (p \wedge q) \rightarrow r$ ; $P_2 : r \rightarrow s$ .
$P_3 : \neg s$ ; $P_4 : \neg p \vee \neg q$ .

① $(p \wedge q) \rightarrow r$ {1}
② $r \rightarrow s$ {2}
③ $(p \wedge q) \rightarrow s$ {1,2} Hypothetical syllogism.
④ $\neg s$ {4}
⑤ $\neg(p \wedge q)$ {1,2,4} modus tollens.
⑥ $\neg p \vee \neg q$ {1,2,3} demorgan's law.

∴ The given info is valid.

7Q) S.T. $r \wedge s$ is derived from the premises:
~~A~~ $p, q, p \rightarrow r, q \rightarrow s$.

Sol: Given $P_1 : p$ ; $P_2 : q$ ; $P_3 : p \rightarrow r$ ; $P_4 : q \rightarrow s$.

① $p \rightarrow r$ {1}
② $p$ {2}
③ $r$ {1,2} modus ponens.
④ $q \rightarrow s$ {4}
⑤ $q$ {5}
⑥ $s$ {4,5}
⑦ $r \wedge s$ {3,6} conjunction

8Q) consider the foll. statements:

- If my checkbook is on my office table then I paid my phone bill.
- I was ~~checking~~ looking at my phonebill for payment on breakfast table (or) I was looking at my phone bill for payment at my office table.
- If I was looking at the phonebill at breakfast then the checkbook is on the breakfast table.

- I did not pay my phone bill.
- If I was looking at the phonebill in my office, then the check-book is on my office table.

conc: Where was my checkbook?

sol: p: My checkbook is on my office table.
q: I paid my phone bill.
r: I was looking at the phonebill for payment at breakfast.
s: I was looking at the phonebill for payment in my office.
t: The checkbook is on breakfast table.

$P_1: p \to q$
$P_2: r \vee s$
$P_3: r \to t$
$P_4: \neg q$
$P_5: s \to p$

① $s \to p$ $\{1\}$
② $p \to q$ $\{2\}$
③ $s \to q$ $\{1,2\}$ hypothetical syllogism.
④ $\neg q$ $\{4\}$
⑤ $\neg s$ $\{1,2,4\}$ modus tollens.
⑥ $r \vee s$ $\{6\}$
⑦ $r$ $\{1,2,4,6\}$ disjunctive syllogism
⑧ $r \to t$ $\{8\}$
⑨ $t$ $\{1,2,4,6,8\}$.

∴ The checkbook was on the breakfast table.

* Mathematic
The word
inferring
validity of
by which
statement
A formal
mathemat
follows:
let S(n)
positive
true for
(i) S(1)
(ii) assu
(iii)
so the
princip
step 1:
It me
step 2:
S(k) is
step 3:
true
1Q) P.T
all +ve
sol:
Step 2
n = k

* Mathematical Induction:
The word induction means the method of inferring a general statement from the validity of a particular case. It is a technique by which one can prove mathematical statements involving positive integers.
A formal statement of principle of mathematical induction can be stated as follows:
Let $s(n)$ be a statement that involves positive integers $n = 1, 2, 3, \ldots$ then $s(n)$ is true for all +ve integers '$n$' provided that
(i) $s(1)$ is true
(ii) assume $s(k)$ is true.
(iii) ~~####~~ $s(k+1)$ is true whenever $s(k)$ is true.
So there are 3 steps of proof using the principle of mathematical induction.

Step 1: Inductive base: verify that $s(1)$ is true. It means $s(n)$ is true for $n=1$.
Step 2: Inductive hypothesis: assume that $s(k)$ is true for an arbitary value of $k$.
Step 3: Inductive step: Verify that $s(k+1)$ is true on the basis of the inductive hypothesis

1Q) P.T. by mathematical induction, for all +ve integer $n \geq 1$, $1+2+3+\cdots+n = \frac{n(n+1)}{2}$ 3

Sol: Step 1: when $n=1$

LHS: $1 = \frac{2}{2} = 1 \cdot \frac{2}{2} = \frac{1(1+1)}{2}$ (RHS).

hence, the result is true for $n=1$.

Step 2: Assume that the result is true for $n=k$. i.e., $1+2+3+\cdots+k = \frac{k(k+1)}{2}$

Step 3: Let $n = k+1$.

consider: $1+2+3+\cdots+k+k+1 = \frac{k(k+1)}{2} + k+1$

$= \frac{k(k+1)+2(k+1)}{2}$

$= \frac{(k+1)(k+2)}{2}$

$= \frac{(k+1)(k+1+1)}{2}$

$\therefore$ The result is true for $n = k+1$.

2Q) P.T. by mathematical induction,

$1^2+2^2+3^2+\cdots+n^2 = \frac{n(n+1)(2n+1)}{6}$ for $n \geq 1$.

Sol: Step 1: let $n = 1$.

LHS: $1 = \frac{2}{2}\cdot\frac{3}{3} = 1\cdot\frac{2}{2}\cdot\frac{3}{3} = \frac{1(1+1)(2(1)+1)}{2\cdot 3}$

$= \frac{1(1+1)(2(1)+1)}{6}$ (RHS)

Step 2: assume $S(k)$ is true. (i.e. $n=k$).

$1^2+2^2+3^2+\cdots+k^2 = \frac{k(k+1)(2k+1)}{6}$.

Step 3: Let $n = k+1$.

consider: $1^2+2^2+3^2+\cdots+k^2+(k+1)^2 = \frac{k(k+1)(2k+1)}{6}+(k+1)^2$

$= \frac{k(k+1)(2k+1)+6(k+1)^2}{6}$

$= \frac{(k+1)}{6}\left[k(2k+1)+6k+6\right]$

$= \frac{(k+1)}{6}(2k^2+k+6k+6)$

$= \frac{(k+1)}{6}(2k^2+7k+6)$

$= \frac{(k+1)}{6}(2k(k+2)+3(k+2))$

= (k+

= (k+

∴ The result

By mathem

true for

3Q) Using th

induction,

Sol: Step 1:

RHS: $\frac{1}{27}$

= $\frac{1}{\ldots}$

Step 2: ass

3 + 33 + 333

Step 3: Cons

3+33+

= $\frac{1}{27}$ (

$$= \frac{(k+1)(2k+3)(k+2)}{6}$$

$$= \frac{(k+1)((k+1)+1)(2(k+1)+1)}{6}.$$

∴ The result is true for ~~k~~ $n = k+1$.

By mathematical induction, the result is true for $n \geq 1$.

3Q) Using the principle of mathematical induction, $3 + 33 + 333 + \cdots + \underbrace{3\ldots3}_{n \text{ digits}} = \frac{1}{27}(10^{n+1} - 9n - 10)$

Sol: Step 1: let $n = 1$.

RHS: $\frac{1}{27}(10^{n+1} - 9n - 10) = \frac{1}{27}(10^{1+1} - 9(1) - 10)$

$= \frac{1}{27}(100 - 19) = \frac{81}{27} = 3 \Rightarrow$ LHS.

Step 2: assume it is true for $n = k$.

$3 + 33 + 333 + \cdots + \underbrace{3\ldots3}_{k \text{ digits}} = \frac{1}{27}(10^{k+1} - 9k - 10).$

Step 3: Consider $n = k+1$

$3 + 33 + 333 + \cdots + \underbrace{3\ldots3}_{k+1 \text{ digits}}$

$= \frac{1}{27}(10^{k+1} - 9k - 10) + \frac{3^{k+1}(999\ldots9)}{9^{k+1}}$

48) P.T. 2 divides $n^2+n$ where $n$ is a positive integer using mathematical induction.

Sol: Step 1: let $n=1$.

RHS : $1^2+1 = 2$. 2 divides 2.

$\therefore$ It is true for $n=1$.

Step 2: assume $n=k$ is true.

$k^2+k$ is divided by 2.

i.e., $k^2+k = 2m$

Step 3: let $n=k+1$.

$(k+1)^2 + (k+1)$

$= k^2+2k+1+k+1$

$= k^2+3k+2$

$= k^2+k+2k+2$

$= 2m+2(k+1)$

$= 2[m+k+1]$

$= 2n$.

50) S.T. $n^3+2n$ is divisible by 3 for all $n \geq 1$

Q) P.T. $\underbrace{7+77+777+7\cdots7}_{n\,times} = \frac{7}{81}\left(10^{n+1} - 9n - 10\right) \ \forall\ n \in N$

Q1) P.T. $\sqrt{2\sqrt{2+\cdots+\sqrt{2}}} = 2\cos\left(\frac{\pi}{2^{n+1}}\right) \ \forall\ n \in N$.

Sol: step 1: for $n = 1$;

consider RHS: $2\cos\left(\frac{\pi}{2^{1+1}}\right)$

$$= 2\cos\left(\frac{\pi}{4}\right)$$

$$= \sqrt{2}\cancel{2}\cos\frac{1}{\sqrt{\cancel{2}}}$$

$$= \sqrt{2}.$$

$\therefore$ Result is true for $n=1$.

step 2: assume $p(n)$ is true for $n=k$.

$$\underbrace{\sqrt{2+\sqrt{2+\sqrt{2+\cdots\sqrt{2}}}}}_{k\,times} = 2\cos\left(\frac{\pi}{2^{k+1}}\right)$$

step 3: P.T. $p(n)$ is true for $n = k+1$.

$$\sqrt{2+\underbrace{\sqrt{2+\sqrt{2+\cdots+\sqrt{2}}}}_{k\,times}} = \sqrt{2+2\cos\left(\frac{\pi}{2^{k+1}}\right)}$$

$$= \sqrt{2\left(1+\cos\left(\frac{\pi}{2^{k+1}}\right)\right)}$$

$$= \sqrt{2\left(2\cos^2\left(\frac{\pi}{2^{(k+1)+1}}\right)\right)} \quad \left[1+\cos\theta = 2\cos^2\frac{\theta}{2}\right]$$

all $n \geq 1$

$$= \sqrt{\left[2\cos^{\cancel{2}}\left(\frac{\pi}{2^{(k+1)+1}}\right)\right]^{\cancel{2}}}$$

$$= 2\cos\left(\frac{\pi}{2^{(k+1)+1}}\right)$$

$\therefore$ The result is true for $n = k+1$.

Hence, by the principle of mathematical induction, the result is true for all +ve integers [$Z^+$].

2Q) P.T. $1\cdot2 + 2\cdot3 + \cdots + n\cdot(n+1) = \frac{n(n+1)(n+2)}{3}$ for all $n \in N$.

3Q) PT. $a + ar + ar^2 + \cdots + ar^{n-1} = \frac{(ar^n - 1)}{r-1} \ \forall\ n \in N$

4Q) P.T. $\frac{1}{1\cdot2} + \frac{1}{2\cdot3} + \cdots + \frac{1}{n(n+1)} = \frac{n}{n+1}$ for all $n \in N$.

5Q) P.T. $1 + 2 + 2^2 + \cdots + 2^n = 2^{n+1} - 1 \ \forall\ n \in N$.

4 Sol: Step 1: when $n=1$;

Consider RHS: $2^{1+1}-1 = 2^2-1 = 4-1 = 3$ (LHS)

Step 2: assume, result is true for $n=k$.

$$1+2+2^2+\dots+2^k = 2^{k+1}-1$$

Step 3: P.T. the result is true for $n=k+1$.

$$1+2+2^2+\dots+2^k+2^{k+1} \Rightarrow 2^{k+1}-1+2^{k+1}$$

$$= 2^{(k+1)+1}-1 \quad \text{(RHS)}$$

Hence, the result is true for $n=k+1$.

$\therefore$ The result is true $\forall\, n \in N$ by the principle of mathematical induction.

2Q) 3 Sol: $1.2+2.3+\dots+n(n+1) = \dfrac{n(n+1)\cancel{(2n+1)}(n+2)}{\cancel{6}\,3}$

Step 1: when $n=1$;

consider RHS: $\cancel{\dfrac{1(1+1)(1+2)}{6}} = \cancel{\dfrac{1.2.3}{6}} = \cancel{1}$

$\cancel{= 1.2.}$

$$= \frac{1(1+1)(1+2)}{3} = \frac{1.2.3}{3} = \frac{\cancel{6}}{\cancel{3}} = 2.\ \text{(LHS)}$$

Step 2: assume the result is true for $n=k$.

$$1.2+2.3+\dots+(k+1)k = \frac{k(k+1)(k+2)}{3}$$

Step 3: P.T. the result is true for $n=k+1$.

$$1 \cdot 2+2.3+\dots+k(k+1)+(k+1)(k+2)$$

$$= \frac{k(k+1)(k+2)}{3} + (k+1)(k+2)$$

$$= \frac{k(k+1)(k+2)+3(k+1)(k+2)}{3}$$

$$= \frac{(k+1)(k+2)(k+3)}{3}$$

$$= \frac{(k+1)(k+1+1)(k+1+2)}{3} \quad \text{(RHS)}.$$

$\therefore$ The result is true $\forall\, n \in N$.

Type 2: related to divisions

1Q) P.T. $n^3 - n$ is divisible by 3. n is a +ve

2Q) P.T. $n^5 - n$ is divisible by 5. n is a +ve.

3Q) P.T. $6^{n+2} + 7^{2n+1}$ is divisible by 43.

4Q) P.T. $n(n+1)(2n+1)$ is divisible by 6.

5Q) P.T. $x^n - y^n$ is divisible by $x-y$ $\forall$ $n \in N$. where $x$ and $y$ are any 2 different integers.

6Q) P.T. $5^{2n+1} + 3^{(n+2)} \cdot 2^{(n-1)}$ is divisible by 19.

2 Sol: $p(n)$: $n^5 - n$ is divisible by 5.

Step 1: $n = 1$. consider $1^5 - 1 = 0$

0 is divisible by 5. so $p(n)$ is true for $n=1$.

Step 2: assume that the result is true for $n = k$.

$k^5 - k$ is divisible by 5.

Step 3: Consider $(k+1)^5 - (k+1)$

$$= {}^5C_0 k^5 + {}^5C_1 k^4 + {}^5C_2 k^3 + \cdots + {}^5C_5 - k - 1$$

$$= k^5 + 5k^4 + 10k^3 + 10k^2 + 5k + \cancel{1} - k - \cancel{1}$$

$$= k^5 + 5k^4 + 10k^3 + 10k^2 + 5k - k$$

$$= \underbrace{(k^5 - k)}_{\text{Term-I}} + \underbrace{5(k^4 + 2k^3 + 2k^2 + k)}_{\text{Term-II}} \quad - \text{eq } ②$$

Both, terms I & II are divisible by 5.

$\therefore$ eq ② is divisible by 5 and the result is true for $n = k+1$.

The result is true for all $n \in N$.

4' ④ Sol: $p(n) = 6^{n+2} + 7^{2n+1}$ divisible by 43.

Step 1: for $n = 1$:

$6^{1+2} + 7^{1+2} = 6^3 + 7^3 = 216 + 343 = 559$

$= 13 \times 43$.

∴ The result is true for $n = 1$.

Step 2: assume the result is true for $n = k$.

$6^{k+2} + 7^{2k+1}$ is divisible by 43.

Step 3: P.T $p(n)$ is true for $n = k+1$.

$= 6^{(k+1)+2} + 7^{2(k+1)+1}$

$= 6.6^{k+2} + 7^2.7^{2k+1}$

$= 6.6^{k+2} + 49.7^{2k+1}$

$= 6.6^{k+2} + 6.7^{2k+1} + 43.7^{2k+1}$

$= \underbrace{6\{6^{k+2} + 7^{2k+1}\}}_{I} + \underbrace{43.7^{2k+1}}_{II}$

Both terms I & II are divisible by 43.

Hence, proved.

⑤ Sol: $p(n) = x^n - y^n$ is divisible by $x-y$.

Step 1: $n = 1$;

$x^1 - y^1 = x - y$ is divisible by $x-y$.

Step 2: assume $p(n)$ is true for $n = k$.

$x^k - y^k$ is divisible by $x-y$.

Step 3: P.T. $p(n)$ is true for $x = k+1$.

$= x^{k+1} - y^{k+1}$

$= x.x^k - y.y^k - x^k y + x^k y$

$= \underbrace{x^k(x-y)}_{I} + \underbrace{y(x^k - y^k)}_{II}$

Terms I & II are divisible by $x-y$.

Hence, proved.

19) P.T. $3^{2n}$ wh
1 as remainder

Sol: Dividend =
$p(n)$ be the s
divided by
remainder.

Step 1: $n =$
$3^{2(1)}$
∴ resul

Step 2: n
when $3^{2k}$ is
$3^{2k} =$

Step 3:

∴ $3^{2k+}$
rem
Hence

93.

18) P.T. $3^{2n}$ when divided by 8 always leaves a 1 as remainder for all $n \in N$.

Sol: Dividend = Divisor x Quotient + Remainder

P(n) be the statement given by $3^{2n}$ when divided by 8 always leaves 1 as the remainder.

Step 1: $n=1$.

$$3^{2(1)} = 3^2 = 9 = 8+1.$$

$\therefore$ result is true for $n=1$.

Step 2: $n=k$. assume result is true for $n=k$, i.e when $3^{2k}$ is divided by 8 leaves remainder as 1

$$3^{2k} = 8 \times \lambda + 1 \quad - \text{eq } ①.$$

Step 3: Consider $3^{2(k+1)} = 3^{2k+2}$

$$= 9 . 3^{2k}$$

$$= 9(8\lambda+1)$$

$$= 8.9\lambda + 9$$

$$\equiv 8.9\lambda + 8 + 1$$

$$3^{2(k+1)} = 8(9\lambda+1)+1$$

$\therefore 3^{2(k+1)}$, when divided by 8, leaves a remainder 1.

Hence, the result is true for all $n \in N$.

UNIT – III :

* set : Any well defined collection or class or list of different objects is called a set.
(OR) A set is any collection of distinct and distinguishable objects. The objects in a set can be numbers, points in geometry, letters, people, rivers, countries, etc.
The objects in a set are called elements or numbers of the set.

Examples : (1) The set of students of 2nd CSE-B.
(2) set of students of CMRIT.
(3) set of all rivers in India.
(4) set of vowels.
(5) set of districts in Telangana.

• Notation : sets are usually denoted by uppercase letters (A, B, C, ---, X, Y, Z) and the elements in a set are denoted by lower case letters (a, b, ---, y, z)

* Cardinality : The no. of elements in a set A is known as cardinality of the set A and is denoted as n(A) (or) |A|

NOTE : If n(A) is a finite no., then the set is known as finite set. For example, $A = \{a, e, i, o, u\}$
$n(A) = 5$. ∴ A is finite.
$A = \{x / x \in N\} = \{1, 2, 3, ---, N\}$ A is infinite set

* Representation of a set : There are 2 ways :
(1) Roaster method (or) Tabular form.
(2) Rule method (or) set builder form.

1) Roaster (or) Tabular form : In this form, all the elements of the set are listed and the elements being separated by commas and closed within braces. Ex : (1) $A = \{0, 1\}$ ; $A = \{a, b, c\}$ ; $A = \{5, 4, 6\}$.

NOTE(1): The order in which the elements of a set are listed is not important. Thus

$\{1,2,3\} = \{3,2,1\} = \{2,1,3\}$ ....

NOTE(2): Repeated elements in the listing of the elements of a set can be ignored.

Example: Represent the foll. sets in tabular form

(1) $A = \{x : x^2 - 3x + 2 = 0\}$

$A = \{1, 2\}$

(2) $B = \{x : x$ is an integer and $1 < x < 7\}$.

$B = \{2, 3, 4, 5, 6\}$

(2) Rule Method (or) set builder form: In this method, a set is defined by specifying a property that elements of a set have in common. The set is then described as follows:

$A = \{x : p(x)\}$.

A vertical bar can also be placed in place of colon. $A = \{x \mid P(x)\}$.

Given below are some sets in set Builder form

$A = \{x : x = n^2, n \in N\}$.

$A = \{x : x$ is a vowel$\}$

$A = \{x : x$ is even no btw 1 & 8$\}$.

Find the foll. sets in set builder form.

(1) $A = \{3, 6, 9, 12, 15\}$.

$A = \{x : x = 3n, 1 \leq n \leq 5\}$

(2) $B = \{-4, -3, -2, -1, 0, 1, 2, 3\}$.

$B = \{x : x$ is an integer btw $-5 < x < 4\}$.

*Null set (Empty set or void set): Set which contains no elements in it. It is denoted by the symbol $\phi$. i.e., $\phi = \{ \}$.

* singleton set: A set which has only one element is called a singleton set. For example, $\{1\}$, $\{\phi\}$.

* subset: If A & B are sets such that every element of A is also an element of B, then A is said to be a subset of B (or) A is containing B and is denoted by $A \subseteq B$. Here B is called super set of A.

NOTE: (1) If A is not subset of B, i.e., atleast one element of A does not belong to B, we write $A \nsubseteq B$.

(2) Every set A is subset of itself i.e., $A \subseteq A$. (reflexive property).

(3) The null set $\phi$ is subset of any set A. $\phi \subseteq A$

(4) If A is subset of B and B is subset of C, then A is subset of C.

$A \subseteq B$, $B \subseteq C$, then $A \subseteq C$ (transitive)

1Q) Determine whether each of the foll. statements is true or false.

(i) $x \in \{x\}$ True

(ii) $\{x\} \subseteq \{x\}$ True

(iii) $\{x\} \in \{x\}$ False

(iv) $\{x\} \in \{\{x\}\}$ True.

(v) $\phi \subseteq \{x\}$ True.

* No. of subsets (or) Power set: For a set A, the collection of all subsets of A is called the power set of A and is denoted by P(A). If a set contains n elements, then P(A) contains $2^n$ elements. For example, $A=\{1,2,3\}$.

$P(A) = \{\phi, \{1\}, \{2\}, \{3\}, A, \{1,2\}, \{2,3\}, \{3,1\}\}$.

*Proper subset: Any set A is said to be proper subset of another set B if A is subset ... atleast one element of B that does not belong to A. It is ~~written~~ written as $A \subset B$. It is also called a proper inclusion.

* Equal sets: 2 sets A & B are said to be equal if and only if every element of A is and element of B and consequently every element of B is an element of A.
i.e. $A \subseteq B$ & $B \subseteq A \Rightarrow A = B$.

* Universal set: In any application of set theory, all the sets under investigation are likely to be considered as subsets of a particular set. This set is called the universal set (or) universe of dispose set and is denoted by 'U'.

· A set is called a universal set if it includes every set under discussion.

For example In a study of human population, all people in the world maybe assumed to form the universal set. The people of any continent, country, religion is a subset of this universal set.

(2) The set of letters in alphabets is the universal from which the letters of any word maybe chosen to form a set.

1Q) which of the foll. sets are equal.

(i) $A = \{x \mid x$ is a letter in the word follow$\} = \{f, o, l, w\}$.

$B = \{x \mid x$ is a letter in the word $\{$wolf$\} = \{f, o, l, w\}$.

$C = \{x \mid x$ is a letter in the word flow$\} = \{f, l, o, w\}$.

$\therefore A = B = C$.

(ii) $A = \{1, 2, 3, 4\}$

$B = \{2, 1, 4, 3\}$

$C = \{1, 2, 4, 1, 3, 2, 4\}$

$D = \{x \mid x$ is +ve integer less than 5 $\}$.

E = {x | x is an integer less than or equal to 4}

$\therefore A = B = C = D$.

2Q) Which of the foll. sets are different. $\phi, \{\phi\}, \{0\}$

Sol: All 3 sets are different.

3Q) Let $A = \{1, \{1\}, \{2\}, 2\}$ Which of the foll. are true

(i) $1 \in A$ (ii) $1 \subset A$ (iii) $\{1\} \subset A$ (iv) $\{1,2\} \in A$

(v) $\{\{1\}, \{2\}\} \subset A$.

(i) T
(ii) F
(iii) T
(iv) F
(v) T

4Q) Describe the foll sets using the tabulation method.

(i) $A = \{1 + (-1)^n \mid n$ is +ve integer$\}$

Sol: $\Rightarrow \{0, 2\}$.

(ii) $A = \{(n + \frac{1}{n}) \mid n \in \{1,2,3,5,7\}\}$.

Sol: $\{2, 2+\frac{1}{2}, 3+\frac{1}{3}, 5+\frac{1}{5}, 7+\frac{1}{7}\}$

$= \{2, \frac{5}{2}, \frac{10}{3}, \frac{26}{5}, \frac{50}{7}\}$

5Q) Which of the foll. sets are equal to null sets.

(i) $A = \{x \mid x$ is an integer and $x^2 + 4 = 6\}$

$x = \pm\sqrt{2}$ (to obey the condition).

but $\sqrt{2}$ is not an integer.

$\therefore$ ~~Not~~ a It is a null set.

(ii) $B = \{x \mid x$ is an integer and $3x + 5 = 9\}$.

$3x + 5 = 9$

$x = \frac{4}{3}$. (not an integer).

$\therefore$ Null set.

(iii) $C = \{x \mid x$ is a real no & $x^3 = -1\}$

$\{-1\}$ $\therefore$ Not null set.

(iv) D{x | x is a real no and x = x+1}.

{ } null set.

(v) E{x | x is a real no and x+8 = 8}.

{0} Not null set

6Q) Find all subsets of

(i) A = {0, 1}. B = {0, {1, 2}} − {B, ϕ, {0}, {1,2}}

C = {3, 7} D = {2, 3, 7} E = {a, b, c, d}.

7Q) If A, B, C are 3 sets, P.T foll:

(i) if $A \subset B$ & $B \subseteq C$ then $A \subset C$

(ii) if $A \subseteq B$ & $B \subset C$ then $A \subset C$

(iii) if $A \subset B$ & $B \subset C$ then $A \subset C$

(iv) if $A \subseteq B$ & $B \subseteq C$ then $A \subseteq C$.

Sol: (i) Let $x \in A$ and since $A \subset B \Rightarrow x \in B$

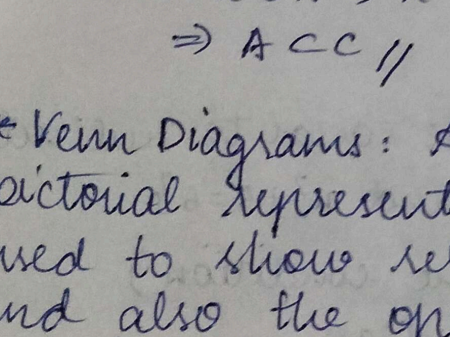
given $B \subseteq C \Rightarrow x \in C$.

$\therefore x \in A \Rightarrow x \in C$

$\Rightarrow A \subset C$ //

* Venn Diagrams: A venn diagram is a pictorial representation of sets which are used to show relationship between sets and also the operations on them. The universal set is represented by the interior of a rectangle and its subsets are represented by circular areas drawn within the rectangle.

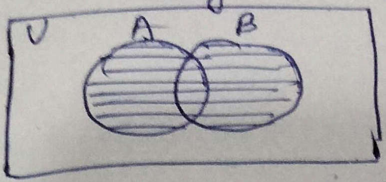
− Venn diagram of $A \subseteq B \subseteq C$



* Operations on sets: In this section, we shall discuss several operations that will combine to yield new sets;

(1) Union: The union of 2 sets A and B is denoted by $A \cup B$ pronounced as A union B. is the set of all elements which belong to A (or) B (or) both i.e., $A \cup B = \{x \mid x \in A \text{ (or) } x \in B\}$.

The venn diagram of $A \cup B$ is as follows:



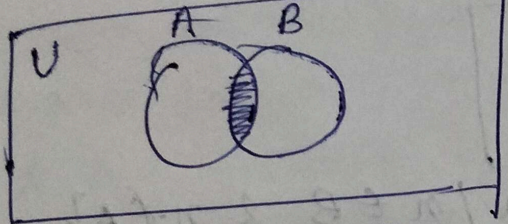

$A \cup B$ (shaded area)

where U is some universal set which contains A & B as subsets.

(2) Intersection: The intersection of 2 sets A & B is denoted by $A \cap B$; pronounced as A intersection B, is the set of all elements which belong to both A and B i.e.

$A \cap B = \{x \mid x \in A \ \& \ x \in B\}$.

The venn diagram of $A \cap B$ is:

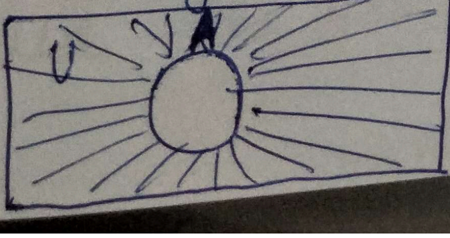



$A \cap B$ (shaded area).

- $A \cap B \subseteq A$, $A \cap B \subseteq B$.
- $A \subseteq A \cup B$, $B \subseteq A \cup B$.

(3) Complement of a set: Let U be the universal set and A be any subset of U, the set of all elements which belong to U but not to A is called complement of A and is denoted by $\bar{A}$, $A^c$, $A'$. Thus $A' = \{x \mid x \notin A \ \& \ x \in U\}$.

The venn diagram for $A'$ is:



For example, if the set of all integers is taken as the universal set and E is the set of all even integers then E' is the set of all odd integers.

NOTE: $U' = \phi$

$\phi' = U$

If $A \subseteq U$ then $\bar{A} \subseteq U$

* $A$ & $\bar{A}$ are disjoint

$A \cup \bar{A} = U$

$A \cap \bar{A} = \phi$

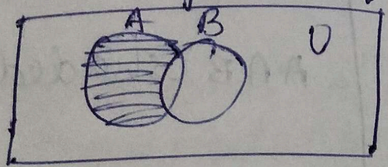
$\overline{(\bar{A})} = A$

$A - B = A \cap \bar{B}$

$B - A = B \cap \bar{A}$

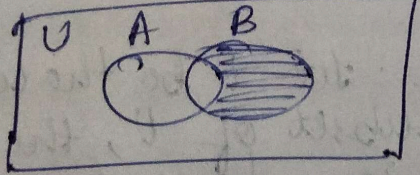
*Relatively complement: If A and B are 2 sets then set of all elements which belong to A but not to B is called relatively complement of B w.r.t. A. and is denoted by A−B i.e., $A - B = \{x / x \in A \ \& \ x \notin B\}$.

The venn diagram of A−B:




Similarly $B - A = \{x / x \in B \ \& \ x \notin A\}$

The venn diagram is:




• Example: Let $A = \{a, b, c\}$ $B = \{b, c, d, e\}$.

then $A - B = \{a\}$

$B - A = \{d, e\}$

NOTE: For any sets A & B, A−B, B−A are disjoint sets.

If A and B a...

$A = (A \cup B) - (B...$

$B = (A \cup B) - (A...$

$A = (A \cap B) \cup ...$

$B = (A \cap B) \cup ...$

* symmetric ... of 2 sets A ... is the set of ... but not bo... of 2 sets.

$A \Delta B = \{x...$

the venn d...

NOTE: A ... A ... A ...

1) Example ... find A−...

Sol: A−B ... B−A ... A ... A ... A ...

* Min ... 2 subs... $A \cap B$, ... w.r.t ... are co...

If A and B are disjoint then $A-B=A$
$B-A=B$.

$A=(A\cup B)-(B-A)$

$B=(A\cup B)-(A-B)$

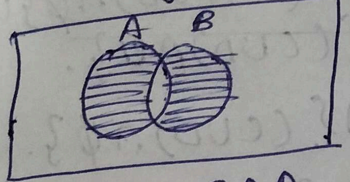
$A=(A\cap B)\cup(A-B)$

$B=(A\cap B)\cup(B-A)$

* Symmetric difference: The symmetric difference of 2 sets A & B is denoted by $A\Delta B$ (or) $A\oplus B$ is the set of all elements that belong to A or B but not both. It is also called the boolean sum of 2 sets.

$A\Delta B=\{x/x$ belongs to exactly any of A & B$\}$.

The venn diagram for $A\Delta B$ is-




NOTE: $A\Delta B=B\Delta A$

$A\Delta B=(A\cup B)-(A\cap B)$

$A\Delta B=(A-B)\cup(B-A)$

1) Example: If $A=\{-3,0,1,2\}$; $B=\{1,2,3,4\}$ then find $A-B$, $B-A$, $A\Delta B$, $A\cup B$, $A\cap B$, ~~$\bar{A}$~~, ~~$\bar{B}$~~.

Sol: $A-B=\{-3,0\}$.

$B-A=\{3,4\}$.

$A\Delta B=\{-3,0,3,4\}$.

$A\cup B=\{-3,0,1,2,3,4\}$.

$A\cap B=\{1,2\}$.

* Min set and Max set: Let A and B be 2 subsets of a universal queue, then the sets $A\cap B$, $\bar{A}\cap B$, $A\cap\bar{B}$, $\bar{A}\cap\bar{B}$ are called min sets w.r.t A & B; the sets $A\cup B$, $\bar{A}\cup B$, $A\cup\bar{B}$, $\bar{A}\cup\bar{B}$ are called max sets w.r.t. A & B.

1Q) If $U=\{1,2,3,4,5,6,7,8,9,10\}$, $A=\{1,3,5,7$
$B=\{4,6,8,10\}$.

**Duality**: If $E$ is any identity of algebraic, then the dual $E^*$ of $E$ is the expression obtained by replacing

(i) Each union operation by the intersection operation

(ii) Each intersection operation by the union operation

(iii) $U$ by $\phi$ and $\phi$ by $U$.

It is known as the principle of duality if $E$ is true then $E^*$ is also true.

1Q) Find the dual of $E=\{(A\cup B)\cap\phi\}\cup\{(C\cap D)\cup$ ~~$\cup\phi\}\cap\{(C\cup D)\cap U\}$.~~

Sol: ~~$E^*=\{(A\cap B)\cup\phi\}$~~

$E^*=\{(A\cap B)\cup U\}\cap\{(C\cup D)\cap\phi\}$.

2Q) If $U=\{1,2,3,4,5,6,7,8,9\}$, $A=\{1,2,4,6,8\}$.
$B=\{2,4,5,9\}$.

(i) $\bar{A}$ (ii) $\bar{B}$ (iii) $\bar{A}\cup\bar{B}$ (iv) $\overline{A\cup B}$ (v) $\bar{A}\cap\bar{B}$ (vi) $\overline{A\cap B}$.
(vii) $A-B$ (viii) $B-A$ (ix) $A\Delta B$ (x) $A\cup B$ (xi) $A\cap B$.

2Q) Determine the sets $A$ & $B$.

(i) G.T. $A-B=\{1,2,4\}$; $B-A=\{7,8\}$ and
$A\cup B=\{1,2,4,5,7,8,9\}$.

(ii) $A-B=\{1,3,7,11\}$; $B-A=\{2,6,8\}$; $A\cap B=\{4,9\}$.

Sol (i) $A=(A\cup B)-(B-A)=\{1,2,4,5,9\}$.

$B=(A\cup B)-(A-B)=\{5,7,8,9\}$.

(ii) $A=(A-B)\cup(A\cap B)=\{1,3,4,7,9,11\}$.

$B=(B-A)\cup(A\cap B)=\{2,4,6,8,9\}$.

3Q) Write the duals statements for each of the foll

(i) $A=A\cap(A\cup B)$ (ii) $A=(A\cup B)\cap(A\cup\phi)$

(iii) $A\cup B=(A\cap B)\cup(A\cap\bar{B})\cup(\bar{A}\cap B)$

(iv) $U=(A\cap B)\cup($

4Q) If $A=\{4,5$
then verify

5Q) If $A=\{1,2,3$
then verify t

(a) $(\overline{A\cup B})=$

(or) $(A\cup B)'=$

The law
the opera
The foll
where

(1) Idemp

(2) Associa

(3) Comm

(4) Dis

(5) De

(6)

(7)

(iv) $U = (A \cap B) \cup (A \cap \overline{B}) \cup (\overline{A} \cap B) \cup (\overline{A} \cap \overline{B})$.

48) If $A = \{4, 5, 7, 8, 10\}$ $B = \{4, 5, 9\}$, $C = \{1, 4, 6, 9\}$. then verify that $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$.

58) If $A = \{1, 2, 3\}$ ; $B = \{2, 3, 4\}$ and $U = \{1, 2, 3, 4, 5, 6\}$ then verify that $(A \cup B)^c = A^c \cap B^c$ & $(A \cap B)^c = A^c \cup B^c$.

(or) $(\overline{A \cup B}) = \overline{A} \cap \overline{B}$ & $(\overline{A \cap B}) = \overline{A} \cup \overline{B}$

(or) $(A \cup B)' = A' \cap B'$ & $(A \cap B)' = A' \cup B'$.

## The laws of set theory (Algebra sets).

The operations on sets satisfy certain laws. The following are a few of these laws, where A, B, C are subsets of U.

(1) Idempotent law : $A \cup A = A$ ; $A \cap A = A$.

(2) Associative law : $A \cup (B \cup C) = (A \cup B) \cup C$.

$A \cap (B \cap C) = (A \cap B) \cap C$

(3) Commutative law : $A \cup B = B \cup A$

$A \cap B = B \cap A$.

(4) Distributive law : $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$.

(5) Demorgan's law : $(A \cup B)' = A' \cap B'$

$(A \cap B)' = A' \cup B'$

(6) Identity law : $A \cup \phi = A$

$A \cap U = A$

(7) Domination law : $A \cup U = U$

$A \cap \phi = \phi$

(8) Involution law : $(A')' = A$.

(9) Absorption law : $A \cup (A \cap B) = A$

$A \cap (A \cup B) = A$.

① P.T. for any 3 sets $A, B, C$, $A \cup (B \cup C) = (A \cup B) \cup C$ and $A \cap (B \cap C) = (A \cap B) \cap C$

Sol: Let $x \in A \cup (B \cup C)$

Part I: $\Rightarrow x \in A$ (or) $x \in (B \cup C)$

$\Rightarrow x \in A$ (or) $\{x \in B$ (or) $x \in C\}$.

$\Rightarrow x \in A$ (or) $x \in B$ (or) $x \in C$

$\Rightarrow x \in A$ (or) $x \in B$ (or) $x \in C$

$\Rightarrow x \in (A \cup B)$ or $x \in C$.

$\Rightarrow x \in (A \cup B) \cup C$

$\therefore A \cup (B \cup C) \subseteq (A \cup B) \cup C$ — ①

Part II: Let $x \in (A \cup B) \cup C$

$\Rightarrow x \in (A \cup B)$ (or) $x \in C$

$\Rightarrow (x \in A)$ (or) $(x \in B)$ (or) $x \in C$

$\Rightarrow x \in A$ (or) $x \in (B \cup C)$

$\Rightarrow x \in A \cup (B \cup C)$

$(A \cup B) \cup C \subseteq A \cup (B \cup C)$ — ②

from ① and ② $(A \cup B) \cup C = A \cup (B \cup C)$.

② P.T. for any 2 sets $A$ and $B$. $A \cap B = B \cap A$
$A \cup B = B \cup A$.

Sol: Let $x \in A \cap B$

$\Rightarrow x \in A$ and $x \in B$

$\Rightarrow x \in B$ and $x \in A$

$\Rightarrow x \in (B \cap A)$

$A \cap B \subseteq B \cap A$ — ①.

Let $x \in B \cap A$

$\Rightarrow x \in B$ and $x \in A$

$\Rightarrow x \in A$ and $x \in B$

$\Rightarrow x \in (A \cap B)$

$B \cap A \subseteq B \cap A$ — ②

from ① & ② $A \cap B = B \cap A$.

③ P.T. for any 3 sets
$A \cup (B \cap C) = (A \cup B) \cap$

Sol: Let $x \in A \cap ($

$\Rightarrow x \in A$ and

$\Rightarrow x \in A$ and

$\Rightarrow \{x \in A$ and

$\Rightarrow \{x \in (A \cap B)$

$\Rightarrow x \in (A \cap$

$A \cap (B \cup C) =$

Let $x \in (A \cap$

$\Rightarrow x \in (A \cap B$

$\Rightarrow (x \in A$ a

$\Rightarrow x \in A$

$\Rightarrow x \in A$

$\Rightarrow x \in A$

$\therefore (A \cap B) \cup$

from ① and

④ P.T. for any

Sol: Let $x \in$

$\Rightarrow x \in$

$\Rightarrow x \notin$

$\Rightarrow x$

$\Rightarrow x$

$\therefore \overline{A}$

Let $x$

$\Rightarrow x$

$\Rightarrow x$

$\Rightarrow$

③ P.T. for any 3 sets $A, B, C$, $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$
$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$.

Sol: Let $x \in A \cap (B \cup C)$

$\Rightarrow x \in A$ and $x \in (B \cup C)$

$\Rightarrow x \in A$ and $\{x \in B$ (or) $x \in C\}$.

$\Rightarrow \{x \in A$ and $x \in B\}$ (or) $\{x \in A$ and $x \in C\}$

$\Rightarrow \{x \in (A \cap B)\}$ (or) $x \in (A \cap C)$.

$\Rightarrow x \in (A \cap B) \cup (A \cap C)$.

$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$ — ①.

Let $x \in (A \cap B) \cup (A \cap C)$

$\Rightarrow x \in (A \cap B)$ (or) $x \in (A \cap C)$

$\Rightarrow (x \in A$ and $x \in B)$ or $(x \in A$ and $x \in C)$

$\Rightarrow x \in A$ and $\{x \in B$ and $x \in C\}$.

$\Rightarrow x \in A$ and $x \in (B \cup C)$

$\Rightarrow x \in A \cap (B \cup C)$.

$\therefore (A \cap B) \cup (A \cap C) \subseteq A \cap (B \cup C)$ — ②.

from ① and ② : $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$.

④ P.T. for any 2 sets $A$ & $B$, $\overline{(A \cup B)} = \bar{A} \cap \bar{B}$
$\overline{A \cap B} = \bar{A} \cup \bar{B}$.

Sol: Let $x \in (\overline{A \cup B})$

$\Rightarrow x \notin (A \cup B)$

$\Rightarrow x \notin A$ (or) $x \notin B$.

$\Rightarrow x \in \bar{A}$ ~~(or)~~ and $x \in \bar{B}$.

$\Rightarrow x \in \bar{A} \cap \bar{B}$.

$\overline{A \cup B} \subseteq \bar{A} \cap \bar{B}$ — ①.

Let $x \in (\bar{A} \cap \bar{B})$.

$\Rightarrow x \in \bar{A}$ and $x \in \bar{B}$

$\Rightarrow x \notin A$ (or) $x \notin B$

$\Rightarrow x \notin (A \cup B)$.

$\Rightarrow x \in (A \cup B)'$

$\bar{A} \cap \bar{B} \subseteq \overline{A \cup B}$ — ②

from ① and ②: $\overline{A \cup B} = \bar{A} \cap \bar{B}$

* The principle of inclusion-enclusion:

Let A and B be two finite sets. We wish to find the no. of elements in A union B. WKT, $A \cup B$ consists of all elements of A (or) B (or) both. So the no. of elements in $A \cup B$ is equal to the no. of elements in A + no. of elements in B − the no. of elements common to both A & B. i.e.,

$$|A \cup B| = |A| + |B| - |A \cap B|$$

$$n(A \cup B) = n(A) + n(B) - n(A \cap B).$$

which is known as the addition rule (or) the principle of inclusion-exclusion for 2 sets.

• The principle of inclusion-enclusion for 3 sets A, B, C is

$$n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(B \cap C) - n(C \cap A) + n(A \cap B \cap C)$$

1Q) A computer company must hire 20 programmers to handle system programming jobs and 30 programmers for applications programming. Of these hired 5 are expected to perform jobs of both types. How many programmers must be hired.

Sol: Let A be the set of system prog. hired.

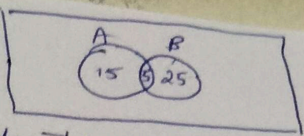
$n(A) = 20$

Let B be the set of app. prog. hired

$n(B) = 30$.

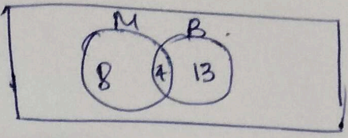
$n(A \cap B) = 5$

$n(A \cup B) = n(A) + n(B) - n(A \cap B)$
$= 20 + 30 - 5$
$= 45$.




28) In a class of 25 students, 12 have taken math, 8 have taken math but not biology. Find the no. of students who have taken math and biology, those who have taken biology but not math.

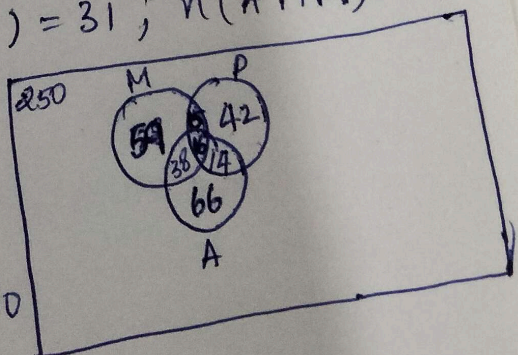
Sol: $n(M \cup B) = 25$.
$n(M) = 12$
$n(M - B) = 8$




(i) $n(M \cap B) = 4$
(ii) $n(B - M) = 13$.

30) Out of 250 candidates who failed in an exam it was revealed that 128 failed in math, 87 in physics and 134 in aggregate. 31 failed in math and phy, 54 failed in aggregate and math, 30 failed in aggregate and phy. Find how many candidates failed (i) in all 3 subjects
(ii) in math but not in phy.
(iii) in aggregate but not in math.
(iv) in phy but not in aggregate or math.
(v) in aggregate or math but not in phy.

Sol: $n(M \cup P \cup A) = 250$; $n(M) = 128$; $n(P) = 87$.
$n(A) = 134$; $n(M \cap P) = 31$; $n(A \cap M) = 54$.
$n(A \cap P) = 30$.




by principle of inclusion-exclusion, WKT (formula)

(i) $n(M \cap P \cap A) = 250 - 128 - 87 - 134 + 31 + 54 + 30$
$= 16$

(ii) $59 + 38 = 97$

(iii) $66 + 14 = 80$

(iv) 42

(v) $59 + 38 + 66 = 163$