UNIT-IV
ELEMENTARY COMBINATORY

Basic rules of counting (i) Sum Rule (ii) The product Rule

The Sum Rule:- Suppose two tasks T_1 and T_2 are to be performed. If the task T_1 can be performed in 'm' different ways and the task T_2 can be performed in 'n' different ways and if these two tasks can't be performed simultaneously, then one of the two tasks ie T_1 or T_2 can be performed in $m+n$ ways.

In generally, if $T_1, T_2 \ldots T_k$ are k tasks such that no two of these tasks can be performed at the same time and if the task T_i can be performed in n_i different ways, then one of the k tasks ie T_1 or T_2 or ... or T_k can be performed in $n_1+n_2+\cdots+n_k$ ways.

Eg: ① Suppose there are 16 boys and 18 girls in a class and we wish to select one of these students (either a boy or a girl) as the class representative. The no. of ways of selecting a boy is 16 and the no. of ways of selecting a girl is 18.

∴ The no. of ways of selecting a student (boy or girl) is $16+18=34$

② How many ways can we get a sum of 4 or of 8 when two distinguishable dice (say one die is red and the other is white) are rolled? - How many ways can we get an even sum?

Sol: We obtain the sum 4 from the outcomes (1,3), (2,2) (3,1)

Thus there are 3 ways to obtain the sum 4

Likewise, obtain the sum 8 from the outcomes (2,6) (3,5), (4,4), (5,3), (6,2)

Thus there are 5 ways to obtain the sum 8.

∴ $3+5=8$ outcomes whose sum is 4 or 8.

→ The no. of ways to obtain an even sum is the same as the no. of ways to obtain either the sum 2, 4, 6, 8, 10 or 12

Sum 2 → (1,1) , Sum 4 → (2,2), (1,3), (3,1)

Sum 6 → (1,5), (5,1) (2,4), (4,2), (3,3)

Sum 8 → (2,6), (3,5), (4,4) (5,3), (6,2)

Sum 10 → (4,6) (6,4), (5,5)

Sum 12 → (6,6)

∴ There are $1+3+5+5+3+1=18$ ways to obtain an even sum

The product Rule:- Suppose that two tasks T_1 and T_2 are to be performed one after the other. If T_1 can be performed in n_1 different ways and for each of these ways T_2 can be performed in n_2 different ways; Then both of the tasks can be performed in $n_1 n_2$ different ways.

In general, suppose that k tasks $T_1, T_2, \cdots T_k$ are to be performed in a sequence. If T_1 can be performed in n_1 different ways and for each of these ways T_2 can be performed in n_2 different ways, and for each of $n_1 n_2$ different ways of performing T_1 and T_2 in that order, T_3 can be performed in n_3 different ways and so on, then the sequence of tasks $T_1, T_2 \cdots T_k$ can be performed in $n_1 n_2 n_3 \cdots n_k$ different ways.

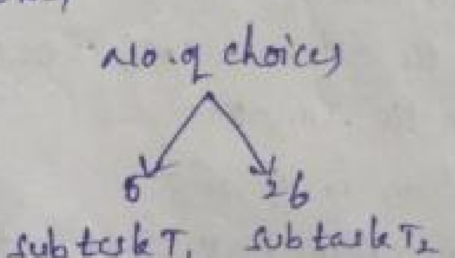
① Find the number of two letter words that begin with a vowel a, e, i, o, u

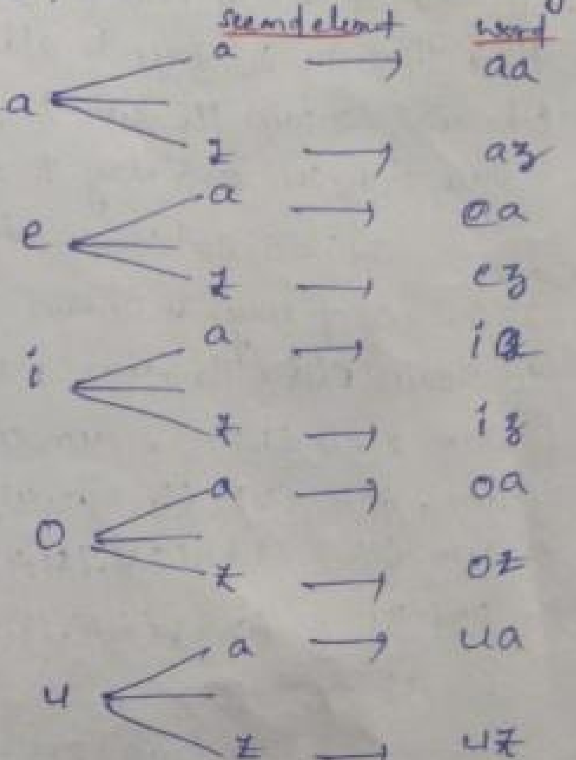
Sol: The task of forming a two letter word consists of two subtasks T_1 and T_2.

T_1 consists of selecting the first letter and T_2 consists of selecting the second letter.

Since each word must begin with "a" vowel T_1 can be accomplished in 5 ways. There are no restrictions on the choice of the second letter so T_2 can be done in 26 ways.

$\therefore$ By Multiplication principle, the task can be performed $5 \times 26 = 130$ different ways. In otherwords, 130 two-letter words begin with a vowel





1. Show that a set 'S' with n elements has 2^n subsets

Soln: Given that a set S with n elements

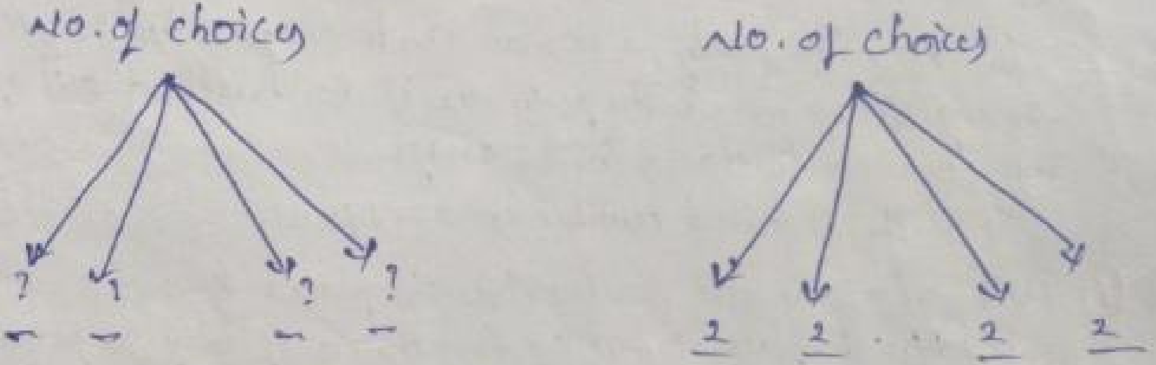
we prove that the set 'S' has 2^n subsets

Every subset of 'S' can be uniquely identified by n-bit word. The task of forming an n-bit word can be broken down to n sub tasks. selecting a bit for each of the n positions

Each position in the word has two choices 0 or 1

so by The multiplication principle, the total number of n-bit words that can be formed $\underbrace{2.2\ldots 2}_{n\text{ times}} = 2^n$

In otherwords, S has 2^n subsets



2. If 2 distinguishable dice are rolled, in how many ways can they fall

3. Suppose that The license plate can be manufactured a repetition of letters of a Certain state require 3 English letters followed by 4 digit.

(a) How many different plates can be manufactured if repetition of letters and digits are allowed

(b) How many plates are possible if only the letters can be repeated

(c) How many plates are possible if only the digit can be repeated.

(d) How many plates are possible if no repetitions are allowed at all

Soln: (a) $26^3 . 10^4$, since there are 26 possibility for each of 3 letters and 10 possibilities for each of 4 digit

(b) $26^3 . 10.9.8.7$

(c) $26.25.24.10^4$

(d) $26.25.24.10.9.8.7$

(P) (a) How many 3-digit numbers can be formed using the digits 1,3,4,5,6,8,9

(b) How many can be formed if no digit can be repeated.

Sol: (a) There are 7^3 such 3-digit numbers since each of the 3 digits can be filled with 7 possibility

(b) There are 7.6.5 such 3-digit numbers

since there are 7 possibilities for the hundreds place but digit is used it is not available for the tens place (since no digit can be repeated in this place). Thus there only one 6 possibility for the tens place and then for the same reason there are only 5 possibility for the units place

(P) There are 20 married couple in a party. find the number of way of choosing one women and one man from the party such that the two are not married to each other

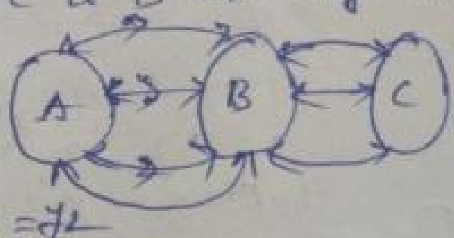
Sol: from the party, a woman can be chosen in 20 ways

Among the 20 men in the party one is her husband out of the 19 other men, one can be chosen in 19 ways

$\therefore$ The required number is $20 \times 19 = 380$

(P) There are four bus routes b/w the places A and B and three bus routes b/w the places B and C. find the no. of ways a person can make a round trip from A to A via B if he does not use a route more than once

Sol: The person can travel from A to B in four ways and from B to C in three ways, but only in two ways from C to B and only in three ways from B to A if he does not use a route more than once.



$\therefore$ The no. of ways he can make the round trip under the given conditions is $4 \times 3 \times 2 \times 3 = 72$

(P) A license plate consists of two English letters followed by four digits. If repetitions are allowed, how many of the plates have only vowels A, E, I, O, U and even digits

Sol: Each of the first two positions in a plate can be filled in 5 ways with vowels

And each of the remaining four places can be filled in 5 ways i.e 0,2,4,6,8.

$\therefore$ The no. of possible license plates of the given type is

$(5 \times 5) \times (5 \times 5 \times 5 \times 5) = 5^6 = 15625$ //

(P) find the number of 3-digit even numbers with no repeated digits

Sol: $(1 \times 9) + (4 \times 8) = 41$.

$\therefore$ Desired number $= 41 \times 8 = 328$

Factorial notation:. The product of first 'n' natural numbers $1, 2, 3 \ldots n$ is denoted by $n!$ and it is read as factorial n (or) n-factorial

i.e $n! = n(n-1)(n-2) \ldots 3.2.1$

NOTE: $0! = 1$

Permutation:- The ordered selection or arrangements of a given set of objects. taken some or all of them at a time is called permutation of the objects.

The total no. of permutations of n objects taken r at a time is denoted by $p(n, r)$ or ^np_r

Results:- 1. $p(n, r) = \frac{n!}{(n-r)!} = {}^np_r$

2. $^np_n = n!$

3. $^np_{n-1} = {}^np_n$

4. The number of permutations of elements of A of n objects taken r at a time, allowing repetion is n^r.

5. The number of permutations of n objects of which p objects are of one type, q are of second-type, r of third-type and remaining objects are different is $\frac{n!}{p! \times q! \times r!}$

(P) Find out how many 5-digit numbers greater than 30,000 can be formed from the digits 1, 2, 3, 4, 5?

Soln: In order to find the 5-digit numbers greater than 30,000 from the digits 1,2,3,4,5

The first digit can be taken as 3, 4 or 5. But, the first digit can be chosen in $3p_1 = 3$ ways.

The remaining 4 digits can be any of the 4 digits taken in $4p_4$ ways.

Hence, The total no. of 5-digit numbers greater than 30,000 will be $3p_1 \times 4p_4 = 3 \times 4!$

(P) Find the no. of permutations of letters of the word 'ENGINEERING'

Soln: The given word consist of 11 letters of which there are 3E's, 3N's, 2G's 2I's and rest all are different.

$\therefore$ The required no. of permutations is $\frac{11!}{3! \times 3! \times 2! \, 2!} = 2,77,200$

(P) Find the no. of 4-digit numbers that can be ~~done~~ formed using the digits 1, 2, 3, 4, 5, 6 that are divisible by 3 when repetition is allowed.

Soln. consider 4 blank places _ _ _ _

To prepare four digit numbers and repetition is allowed, we can fill up each place in 6 ways,

So, we get 6^4 numbers. Among these 6^4 numbers, some of them may not be divisible by 3

By fixing first 3 places with some fixed digits from 1, 2, 3, 4, 5, 6 we get the following six consecutive numbers

$a_1 a_2 a_3 1$

$a_1 a_2 a_3 2$

$a_1 a_2 a_3 3$

$a_1 a_2 a_3 4$

$a_1 a_2 a_3 5$

$a_1 a_2 a_3 6$

w.k.t out of any six consecutive integers exactly two are divisible by 3.

$\therefore$ One third of these 6^4 numbers will be divisible by 3

$\therefore$ The no. of 4 digit numbers which are divisible by 3 $= \frac{1}{3} 6^4 = 432$

(P) In how many ways can 7 women and 3 men be arranged in a row if the 3 men must be always stand next to each other

Soln: No. of men = 3, No. of women = 7

No. of ways of arranging the 3 men = 3!

" " " 7 women = 7!

Since the 3 men always stand next to each other. We treat them as a single entity, which we denote by x

Then if $w_1, w_2 \ldots w_7$ represents the women, we next are interested in the no. of ways of arranging $[x, w_1, w_2 \ldots w_7]$

There are 8! ways.

Hence there are (3!) (8!) permutations altogether.

(P) How many 4 letter words can be formed using the letters of the word "ARTICLE" such that

a) Each word begin with vowel

b) The word contains A but not E

c) Each word must contain atleast one vowel

Soln. Given that, word 'ARTICLE' contains 7 letters

From these 7 letters we have to select 4 and then these 4 letters has to be arranged. We get $^7P_4$ words without any restriction.

a) vowels in the word "ARTICLE" are A, I, E

Since, each 4-letter word must begin with a vowel i.e A or I or E (9)
This can be done in 3 ways
The remaining 3 places can be fill with any one of the remaining six letters in $6P_3$ ways
By fundamental principle, required no. of 4 letter words
$= 3 \times 6P_3$

(b) Let us consider 4 blank places _ _ _ _
Since the ~~letter~~ 4 letter word must contain A, arrange A in any one of the 4 blank places. It can be done 4 ways
Since, the word must not contain E, the ~~rep~~ remaining 3 places can be filled from the remaining 5 letters. This can be done in $5P_3$ ways.
By fundamental principle, the required no. of arrangements
$= 4 \times 5P_3$

(c) The no. of 4 letter words which contain atleast one vowel
= (The no. of 4 letter words without any restriction) − (The number of 4 letter words which contain no vowel)
$= 7P_4 - 4P_4 = 816$

(P) Q find the no. of ways of arranging the letters of the word 'FATHER' so that
(a) The relative positions of vowels and consonents are not disturbed
(b) no vowel occupies even place

Soln. (a) This can be done $4P_4 \cdot 2P_2 = 48$ ways
(b) Let us take 6 blanks $\underset{1}{\_}\ \underset{2}{\_}\ \underset{3}{\_}\ \underset{4}{\_}\ \underset{5}{\_}\ \underset{6}{\_}$
Among these 6 places, 3 are even places. Since no vowel occupies even place, we have to arrange the two vowels in 3 odd places. It can be done in $3P_2$ ways
Now, we shall be left with 4 places (3 even and one odd) in which the consonents ~~can~~ can be arranged. It can be done in $4P_4$ ways.
∴ Required no. of arrangements $= 3P_2 \cdot 4P_4 = 144$

(9). If the letters of the word 'SIPRON' are arranged in all possible ways and the words thus formed are arranged in dictionary order. find the rank of the word "PRISON"

Sol^n. The letters of the word SIPRON in dictionary order is I, N, O, P, R, S

The no. of words which begin with I = $^5P_5 = 120$

" " " " N = $^5P_5 = 120$

" " " " O = $^5P_5 = 120$

The no. of words begin with PI = $^4P_4 = 24$

" " " PN = $^4P_4 = 24$

" " " PO = $^4P_4 = 24$

" " " PRIN = $^2P_2 = 2$

" " " PRIO = $^2P_2 = 2$

" " " PRISNO = $^1P_1 = 1$

The next word is PRISON

∴ The rank of the word = 438

(P) If the letters of the word 'STREAM' are arranged in all possible ways and the words thus formed are arranged as in a dictionary find the word whose rank is 257.

Sol^n. The order of the letters of the word 'STREAM' is A, E, M, R, S, T

We know 257 = 5! + 5! + 3! + 3! + 2! + 2! + 1!

The first 5! words begin with A

Next 5! " " " E

Next 3! " " " MAE

Next 3! " " " MAR

Next 2! " " " MASE

Next 2! " " " MASR

The next word is MASTER

∴ The word with rank 257 is "MASTER"

(P) How many positive integers n can we ...

permutation with repetion

The repetition is allowed then the number of permutations of r objects from a set of 'n' objects is n^r.

(P). Consider the 6 digits numbers 2, 3, 4, 5, 6 and 8 and repetitions of digits are allowed

(a). How many 3 digits can be formed

(b) How many 3 digit numbers must contain the digit 5

Sol^n: (a) For a 3 digit number we have to fill up three places. Since repetitions of the digits is allowed, each of the places can be filled up in 6 ways.

Hence the required 3 digit numbers $= 6\times6\times6 = 6^3 = 216$

(b) Excluding the digit 5, the no. of 3 digit numbers that can be formed from the remaining 5 digits 2, 3, 4, 6 & 8

is $5\times5\times5 = 5^3 = 125$

Hence the number must contain the digit 5

= Total 3 digit numbers − The no. of 3 digit numbers that do not contain '5'

$= 216 - 125 = 91$

(P) There are 25 true or false questions on an examination. How many different ways can a student do the examination if he or she can also choose to leave the answer blank.

Sol^n: Given that total number questions = 25

First question can answers 3 ways (T or F or Blank)

Second question can answers 3 ways (T or F or Blank)

⋮

25^{th} question can answers 3 ways (T or F or Blank)

∴ Different ways can a student do the examination $= 3^{25}$

(P) How many integers b/w 10^5 and 10^6

(a) Have no digit other than 2, 5 or 8

(b) Have no digit other than 0, 2, 5 or 8

Sol^n: (a) The integer b/w 10^5 and 10^6 will contain 6 digits since only 3 digits 2, 5 or 8 are available and the repetition is allowed.

The six places can be filled up in 3^6 ways

Hence the required no. of integer is 3^6

(b) Since only 4 digits 0, 2, 5 or 8 are available and the repetition is allowed.

The first place can be filled up in 3 ways and the remaining places can be filled up with 4^5 ways.

Hence the required no. of integers is 3×4^5 $\underline{3}\ \underline{4}\ \underline{4}\ \underline{4}\ \underline{4}\ \underline{4}$

(1). How many integers b/w 1 and 10^4 contain exactly one 8 and one 9

Soln:- (i) The number of 2-digit numbers contain exactly one 8 and one 9 is 89 and 98

(ii) we arrange the digit 8, 9 in 3 places $^3P_2$ ways i.e 6

(a) We arrange the digits 8, 9 in last two positions in 2 way i.e unit place and tenth position.

In this case we arrange the digits in 100^{th} position in 7 ways i.e we choose digits 1 to 7

$\therefore$ The total no. of ways $7+7=14$ $\_\ \underline{8}\ \underline{9}$, $\_\ \underline{9}\ \underline{8}$

(b) We arrange the digits 8, 9 in 1^{st} and last position in 2 ways i.e units and 10^{th} position

In this we arrange the digit in 10^{th} position in 8 ways i.e we choose digit 0 to 7 $\underline{8}\ \_\ \underline{9}$, $\underline{9}\ \_\ \underline{8}$

$\therefore$ The total number of ways is $8+8=16$

(c) We arrange the digit 8, 9 in 1^{st} two positions in 2 ways i.e 10^{th} and 100^{th} position $\underline{8}\ \underline{9}\ \_$, $\underline{9}\ \underline{8}\ \_$

In this case we arrange the digits in units position in 8 ways i.e (we choose digit 0 to 7)

$\therefore$ The total no. of ways is $8+8=16$

(iii) We arrange the digits 8, 9 in 4 places, $^4P_2$ ways i.e 12

The remaining 2 positions with the numbers 0-7 in 7×8 ways and 8×8 ways

$\therefore$ The total no. of ways is $6 \times 7 \times 8 + 6 \times 8 \times 8 = 720$

∴ The number of integers b/w 1 and 10^4 exactly one 8 and one 9 is $2+14+16+16+720=768$

(B) A ~~pln~~ palindrome is word that reads the same forward or backword. How many 9-letter palindromes are possible using English alphabets?

Sol^n: Since 1^{st} and 9^{th}, 2^{nd} and 8^{th}, 3^{rd} and 7^{th}, 4^{th} and 6^{th} are equal.

We have to select five letters

There are 26 letters, so $26\times26\times26\times26\times26=26^5=11,881,376$

(P) How many positive integers with non-repeated digits are ~~there~~ such that $10\le n\le 9999$

Sol^n: Initially let us consider the number 10

(i) First we find two digit numbers (Non-repeated digits)

$\underset{1}{-}\ \underset{2}{-}$

We choose the 1^{st} digit in 9 ways (from 1 to 9) and the 2 second digit we choose in 9 ways (from 0 to 9) without repetition

∴ No. of two digit numbers are $9\times9=81$

(ii) We find 3 digit numbers (Non-Repeated digits) $\underset{1}{-}\ \underset{2}{-}\ \underset{3}{-}$

We choose the 1^{st} digit in 9 ways (from 1 to 9).

the 2^{nd} digit in 9 ways (from 0 to 9).

the 3^{rd} digit in 8 ways (from 0 to 9) without repetition

∴ The no. of three digit numbers are $9\times9\times8=648$

(iii) We find 4 digit numbers (Non-Repeated digits) $\underset{1}{-}\ \underset{2}{-}\ \underset{3}{-}\ \underset{4}{-}$

We choose the 1^{st} digit in 9 ways (from 1 to 9)

2^{nd} digit in 9 ways (from 0 to 9 without repetition

3^{rd} digit in 8 ways

4^{th} digit in 7 ways

∴ The no. of 4-digit numbers are $9\times9\times8\times7=4536$

∴ The number of +ve integers b/w 10 and 9999 with non-repeated digits is $81+648+4536=5265$

(P). How many 7-digit numbers are there with exactly one 5

Sol^n. We have the no. of digits are 10 i.e 0, 1, 2, --- 9

We arrange the digit 5 in 7 places $^7P_1$ ways i.e 7 ways

(i) If we take the digit 5 in first position, then the remaining 6 positions can filled in 9^6 ways

$\underset{1}{\underline{5}}\ \underset{2}{\_}\ \underset{3}{\_}\ \underset{4}{\_}\ \underset{5}{\_}\ \underset{6}{\_}\ \underset{7}{\_}$

(ii) If we take the digit 5 in 2^{nd} position, the the 1^{st} position choose in 8 ways (using digits 1, 2, --9) and the remaining positition can filled in 9^5 way i.e $8 \cdot 9^5$ ways

$\underset{1}{\_}\ \underset{2}{\underline{5}}\ \underset{3}{\_}\ \underset{4}{\_}\ \underset{5}{\_}\ \underset{6}{\_}\ \underset{7}{\_}$

$l l^y$ we take the digit 5 in 3^{rd}, 4^{th}, 5^{th}, 6^{th} and 7^{th} posititions then the remaining positition filled in $8 \cdot 9^5$ way

$\therefore$ The 7-digit numbere are with exactly one 5 is

$$= 9^6 + 6(8 \cdot 9^5) = 9^5 \cdot 57 //$$

(P). Find the number of 4 letter words that can be formed using the letters of the word 'ARTICLE' in which atleast one letter is repeated

Sol^n. We know that the number 4 letter words that can be formed using 7 letters $= 7^4$ (repetition is allowed)

These 7^4 words can be divided into two exclusive categories.

(i) The 4 letter words without repetition

(ii) The 4 letter words with atleast one repetition of a letter.

The 4 letter words without repetition will be $^7P_4$

$\therefore$ The 4 letter words with atleast one repetition of a letter $= 7^4 - {}^7P_4$ //

(P) Find the number of 5 letter words that can be formed using the letters of the word 'DELHI' that begin and end with vowel when repetitions are allowed (7)

Sol^n: Given that the word 'DELHI'

No. of letters in the word = 5 ; No. of vowel = 2

consider 5 blank places $\underset{1}{-}\ \underset{2}{-}\ \underset{3}{-}\ \underset{4}{-}\ \underset{5}{-}$

(i) Suppose 1^{st} and 5^{th} posititions filled with E and I and the remaining places can be filled in $5 \times 5 \times 5 = 125$ ways

(ii) suppose 1^{st} and 5^{th} posititions filled with I and E and the remaining places can be filled in $5^3 = 125$ ways

(iii) suppose 1^{st} and 5^{th} posititions filled with E and the remaining places can be filled in $5^3 = 125$ ways

(iv) Suppose 1^{st} and 5^{th} positions filled with I and the remaining places can be filled in $5^3 = 125$ ways

$\therefore$ Total no. of 5 letter words $= 125 + 125 + 125 + 125 = \underline{500}$

(P) Find the number of 4-digit numbers that can be formed using the digits 0, 1, 2, 3, 4, 5 which are divisible by 6 when repetition of digits is allowed.

(P) How many positive integers 'n' can we form using the digits 3, 4, 4, 5, 5, 6, 7 if we want n to exceed 5,000,000?

Soln: Let 'n' be must be of the form, $n = x_1 x_2 x_3 x_4 x_5 x_6 x_7$ where $x_1, x_2, x_3 \dots x_7$ are the given digits with $x_1 = 5, 6$ or 7

(i) Suppose we take $x_1 = 5$ then $x_2 x_3 x_4 x_5 x_6 x_7$ is an arrangement of the remaining 6 digits which contains ~~one each~~ two 4's and one each of 3, 5, 6, 7

$\therefore$ The no. of permutation $= \dfrac{6!}{2!\,1!\,1!\,1!\,1!} = 360$

(ii) We take $x_1 = 6$ then $x_2 x_3 x_4 x_5 x_6 x_7$ is an arrangement of the remaining 6 digits which contains two 4's, two 5's and one each of 3, 6, 7

$\therefore$ The no. of arrangement is $\dfrac{6!}{1!\,2!\,2!\,1!} = 180$

(iii) We take $x_1 = 7$,

$\therefore$ The no. of arrangements is $\dfrac{6!}{1!\,2!\,2!\,1!\,1!\,1!} = 180$

By the Sum Rule, The number of n's of the desired type is $360 + 180 + 180 = 720$ //

(P) A store has 25 flags to hang along the front of the store to celebrate a special occasion. If there are 10 red flags, 5 white flags, 4 yellow flags and 6 blue flags, then how many distinguishable ways can the flags be displayed?

Soln: Total no. of flags = 25

Red flags = 10, White flags = 5, Yellow flags = 4, Blue flags = 6

$\therefore$ No. of distinguishable ways can the flags be displayed is

$$\frac{25!}{10!\,5!\,4!\,6!}$$

(P) Find the no. of ways of arranging the letters of the word $a^4b^3c^5$ in expanded form

<u>Soln:</u> Given word is $a^4b^3c^5$

No. of letters in the given word = 12

$\therefore$ The no. of arrangements by taking all at a time = $\frac{12!}{3!4!5!}$

(P) Find the number of ways of arranging the letters of the word 'BRINGING' so that they begin and end with 'I'

<u>Soln:</u> Given word "BRINGING"

No. of letters in the given word = 8.

It contains 2 I's, 2 G's, 2 N's and B, R

Fix I at first and at last, Arranging the remaining 6 letters b/w these 2 I's

Total no. of arrangements = $\frac{6!}{2!2!} = 180$

(P) Find the number of ways of arranging the letters of the word 'SHIPPING' such that (i) 2 P's will come together

(ii) 2 I's do not come together

<u>Soln.</u> Given word 'SHIPPING'

No. of letters in the given word = 8.

It contains 2 P's, 2 I's and S, H, N, G one each

(i) Since P's must come together. Consider 2 P's as a unit

Now it will have 7 letters of which there are 2 I's

$\therefore$ No. of ways of arrangements = $\frac{7!}{2!}$

We need not arrange the 2P's among themselves because they are alike.

$\therefore$ Required no. of arrangements = $\frac{7!}{2!}$

(ii) Since the 2 I's do not come together, arrange the remaining letters at first.

It can be done in $\frac{6!}{2!}$ ways. Because there are 2 P's among the remaining six

_ _ _ _ _ _

Now we get 7 gaps in which 2 I's can be arranged

It can be done = $\frac{7P_2}{2!}$

$\therefore$ Total No. of arrangements = $\frac{6!}{2!} \times \frac{7P_2}{2!} = 7560$

Circular permutation

A circular permutation is an arrangements of objects ~~allowed~~ around a circle or other simple closed curves.

In circular permutation, clockwise and anticlockwise arrangements of objects are possible and both are distinguishable.

If objects are arranged in a circular order, then the circular permutations of 'n' different objects is $(n-1)!$

If anti-clockwise and clockwise order of arrangements are not distinct.

If we have to arrange 'n' different objects around a table such that no two similar objects are neighbour, then the no. of permutations will be $\frac{(n-1)!}{2}$

Ex:- Arrangement of beads in a necklace, arrangement of flowers in a garland. etc

(P) In how many different ways can 5 gentleman and 5 ladies can sit around a table if (i) there is no restriction (ii) no two ladies sit together.

Soln. (i) There are 10 persons

The number of different arrangement is $(10-1)! = 9!$

(ii) The no. of ways 5 gentlemen can sit around a table is $4! = 24$

Between any two men let a woman be seated. Hence all the 5 ladies can be seated in 5 intermediate places in $5!$ ways

$\therefore$ The required no. of ways is $24 \times 5! = 2880$

(P) If 10 persons were invited for a party, in how many ways can they and the host be seated at a circular table? In how many ~~ways~~ of those ways will two particular persons be seated on either side of the host

Soln. (a) There are 11 persons, including the host to be seated ~~be~~ around a circular table i.e $(11-1)! = 10!$ ways

(b) Let P_1, P_2 be two particular persons and H be the host. There two particular persons can be seated on either side of the host in the following two ways

(i) $P_1 H P_2$ (ii) $P_2 H P_1$

Consider the two particular persons and the host as one person, we have 9 persons in all.

These 9 persons can be seated round a circular table in $(9-1)! = 8$ ways. But two particular persons can be seated on either side of the host in 2 ways.

So the no. of ways of seating 11 persons at a ~~particular~~ ~~table~~ circular table with particular persons on either side of the host $= 8! \times 2$

(P) A family consists of father, mother, 2 daughters and 2 sons. In how many different ways can they sit at a round table if 2 daughters wish to sit on either side of the father.

Sol: First arrange father in any place.

Since the 2 daughters wish to sit on either side of the father they can be arranged in 2 places in ${}^2P_2 = 2$ ways

The remaining 3 persons can be arranged in 3! ways

∴ By product rule, the required no. of arrangements

$$= 2 \times 3! = 12$$

(P) Find the no. of ways to arrange 8 persons around circular table if (i) two specified persons wish to sit together (ii) never sit together

Sol: (i) Since, in circular permutations, the person whom we are going to arrange at first and at which place we are arranging him is not important.

Let the special persons be A, B. So, arrange A in any place. Since A, B must sit together. B can be arranged in two ways i.e in the two adjacent places on either side of A

It can be done in 2 ways

The remaining 6 persons can be arranged in 6! ways

∴ By product rule, The no. of arrangements $= 2 \times 6!$

(ii) First arrange A in any place. Since A, B do not come together, deleting the two places on either side of A, now B can be arranged in 5 ways.

The remaining 6 persons can be arranged in 6! ways

$\therefore$ The required number of arrangements $= 5 \times 6!$

(P) Find the no. of ways of arranging 6 boys and 6 girls around a circle so that

(i) all the girls come together (ii) no two girls come together.

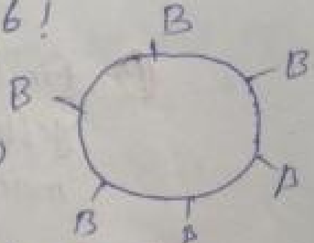
Sol) No. of girls $= 6$; No. of boys $= 6$

(i) Since all the girls come together

Consider '6' girls as a unit. Now we have to arrange 6 boys + 1 unit i.e 7 persons along a round table in 6! ways.

The girls can be arranged among themselves in 6! ways

$\therefore$ The required no. of arrangements $= 6! \times 6!$



(ii) Since no two girls come together.

First arrange 6 boys along circle. It can be done in 5! ways

We get exactly 6 gaps, in which the girls can be arranged. This can be done in 6! ways

$\therefore$ The required no. of arrangements $= 5! \times 6!$

NOTE: (i) The number of circular permutations of n distinct things taken 'r' at a time $= \frac{{}^nP_r}{r}$, $1 \le r \le n$

(ii) The number of permutations of n things taken 'r' at a time in case of hanging type $= \frac{1}{2} \cdot \frac{{}^nP_r}{r}$ ($\because$ we consider only one direction)

(iii) The number of permutations of n different objects taken 'r' at a time in which p particular objects do not occur is ${}^{n-p}P_r$

(iv) The no. of permutations of n different objects taken 'r' at a time in which p particular objects are present is ${}^{n-p}P_{r-p} \times {}^rP_p$

Q. In how many ways 20 different coloured flowers can be arranged into a garland by taking 10 at a time so that 2 Specified colours must occur in the garland but not come together

Soln. Let the two specified colours be A, B
Now fix A at any place of the 10 places along the circle
Since A, B do not come together, we can arrange B only in 7 places. It can be done in 7 ways

Now the remaining 8 places can be filled with remaining 18 flowers by selecting 8 flowers from 18 flowers.

This can be done in $^{18}P_8$ ways

By product rule, The no. of hanging type of garlands

$$= 7 \times \frac{^{18}P_8}{2}$$

Q. Find the number of ways of arranging 6 red roses and 3 yellow roses of different sizes into a garland. In how many of them all the yellow roses come together.

Soln. The nine different roses can be arranged along a circle in 8! ways

Since it is hanging type circular arrangement, we consider only one direction.

$\therefore$ The no. of garlands $= \frac{8!}{2}$

Since all the 3 different yellow roses come together, Consider them as a unit.

The number of roses are 6+1 units. They can be arranged in 6! ways. The yellow roses can be arranged among themselves in 3! ways.

$\therefore$ The no. of garlands $= \frac{6! \times 3!}{2}$

(P) P.T (a) $2n_{P_n} = 2^n[1.3.5...(2n-1)]$

(b) $^nP_r = n \times {}^{(n-1)}P_{(r-1)}$

Soln. (a) $^{2n}P_n = \frac{(2n)!}{n!} = \frac{(2n)(2n-1)(2n-2)\cdots4.3.2.1}{n!}$

$= \frac{[2n(2n-2)\cdots6.4.2][(2n-1)(2n-3)\cdots3.1]}{n!}$

$= \frac{2^n.\cancel{n!}[(2n-1)(2n-3)\cdots5.3.1]}{\cancel{n!}} = 2^n[(2n-1)(2n-3)\cdots5.3.1]$

(b) $R.H.S = n \times {}^{(n-1)}P_{(r-1)} = n \times \frac{(n-1)!}{[(n-1)-(r-1)]!}$

$= n\frac{(n-1)!}{(n-r)!} = \frac{n!}{(n-r)!} = {}^nP_r = L.H.S.$

(P) If $^{(2n+1)}P_{n-1} : {}^{(2n-1)}P_n = 3:5$, find 'n'

(P) find the value of 'n' if the number of permutations of 'n' objects taken 4 at a time is equal to 12 times the no. of permution of n objects taken '2' at a time.

(P) find the value of n, if $^{2n}P_3 = 2 \times {}^nP_4$

COMBINATIONS

Any un ordered selection of $r (\le n)$ objects from a set of n-objects is called an r-combinations of n ~~digits~~ objects and is denoted by $C(n,r)$ or ^nC_r

i.e $C(n,r) = {}^nC_r = \dfrac{n!}{r!(n-r)!}$

Important Results

(i) $C(n,0) = 1$ i.e $^nC_0 = 1$

(ii) $^nC_n = 1$, $^nC_r = {}^nC_{n-r}$

(iii) $C(n+1,r) = C(n,r-1) + C(n,r)$

i.e $^{n+1}C_r = {}^nC_{r-1} + {}^nC_r$

(iv) $^nC_1 = n$, $^nC_2 = \dfrac{n(n-1)}{2!}$, $^nC_3 = \dfrac{n(n-1)(n-2)}{3!}$ and so on

(P) How many committees of five with a given chairperson can be selected from 12 persons?

Soln. The chairperson can be chosen in 12 ways

The other four on the committee can be choosen in $^{11}C_4$ ways

∴ The possible no. of such committee is

$12 \times {}^{11}C_4 = 3960$

(P) out of 5 men and 2 women a committee of 3 is to be formed. In how many ways can it be formed if atleast one women is to be included

Soln:- There are two possible ways

(i) 2 men and 1 women (ii) 1 men and 2 women

(i) The no. of ways of selecting the 2 men and 1 women is

$= {}^5C_2 \cdot {}^2C_1 = 20$

(ii) The no. of ways of selecting the 1 men and 2 women is

$= {}^5C_1 \cdot {}^2C_2 = 5$

∴ The no. of ways of forming the committee $= 20 + 5 = 25$

(P) A Certain question paper contains two parts A and B each containing 4 questions, How many different ways a student can answer 5 questions by selecting atleast 2 questions from each part

Soln: There are two possible ways

(i) $4C_3 \times 4C_2 = 24$ (ii) $4C_2 \times 4C_3 = 24$

$\therefore$ Total no. of ways $= 24+24=48$

(P). In how many ways can four students be selected out of twelve students. If (a) two particular students are not included at all (b) two particular students are included

Soln:. The number ways in which 4 students can be selected out of 12 students are $^{12}C_4 = 495$

(a) When two particular students are not included, 4 are selected out of 10 in $^{10}C_4 = 210$

(b) When two particular students are included, they can be selected only one way. The number of ways of selecting 2 out of 10 students are $^{10}C_2 = 45$

Hence total no. of ways are $= 1 \times {}^{10}C_2 = 45$ //

(P) How many ~~ways can~~ committees of 5 or more can be chosen from 9 peoples

Soln: $^9C_5 + {}^9C_6 + {}^9C_7 + {}^9C_8 + {}^9C_9$

(P). How many 5 card hands consist of cards from a single suit.

Soln. For each of the 4 suits. spades, hearts, diamonds or clubs there are $^{13}C_5$ 5-card hands

Hence, there are a total of $4 \times {}^{13}C_5$ such hands.

(P) How many 5-card hands have 2 cards of one suit and 3 cards of a different suit?

Soln. For a fixed choice of 2 suits there are $2 \times {}^{13}C_2 \times {}^{13}C_3$ ways to choose 2 from one of the suits and 3 from the other. We can choose the 2 suits in $4C_2$ ways

Thus there are $2 \times {}^{13}C_2 \times {}^{13}C_3 \times 4C_2$ such 5-card hands

(13)

Ⓞ. Find the number of committees of 5 that can be selected from 7 men and 5 women if the committee is to consist of atleast 1 man and atleast 1 woman

Soln:- Given that there are 12 persons (5 women and 7 men) from the given 12 persons the no. of committees of 5 that can be formed is $^{12}C_5$

Among these possible committees, there are $^7C_5$ committees consisting of 5 men and $1 = {^5C_5}$ committee consisting of 5 women.

∴ The no. of committees containing atleast one man and one women is

$$^{12}C_5 - {^7C_5} - {^5C_5} = 770$$

Ⓟ. A woman has 11 close relatives and she wishes to invite 5 of them to dinner. In how many ways can she invite them in the following situations

(a) There is no restriction on the choice

(b) Two particular persons will not attend separately

(c) Two particular persons will not attend together

Soln (a) Since there is no restriction on the choice of invitees, five out of 11 can be invited in $^{11}C_5 = 462$ ways

(b) Since two particular persons will not attend separately they should both be invited or not invited.

If both of them are invited, then three more invitees are to be selected from the remaining 9 relatives. This can be done in $^9C_3 = 84$ ways

If both of them are not invited, then five invitees are to be selected from 9 relatives. This can be done in $^9C_5 = 126$ ways.

∴ The total no. of ways in which the invitees can be selected in this case is $84 + 126 = 210$

(c) Since two particular persons will not attend together (say A and B), only one of them can be invited or none of them can be invited.

The number of ways of choosing the invitees with A invited is

$^9C_4 = 126$

Similarly, the number ways of choosing the invitees with B invited is 126

If both A and B are not invited, the number of ways of choosing the invitees is

$^9C_5 = 126$

Thus, the total no. of ways in which the invitees can be selected in this case is $126+126+126 = 378$ /

(P) Find the number of arrangements of the letters in TALLAHASSEE which have no adjacent A's

Soln. No. of letters is 11 of which 3 are A's, 2 each are L's, S's, E's and 1 each are T and H

First, let us disregard the A's. The remaining 8 letters can be arranged in $\frac{8!}{2!2!2!1!1!1!1!} = 5040$ ways

In each of these arrangements, there are 9 possible locations for the three A's. $-T-L-L-H-S-S-E-E-$

These locations can be choosen in $C(9,3)$ or $^9C_3$ ways.

∴ By the product rule, the required number of arrangements is $5040 \times {}^9C_3 = 423,360$

(P) A party is attended by n persons. If each person in the party shake hands with all the others in the party, find the number of handshakes.

Soln. Each handshake is determined by exactly two persons ∴ If each person shakehands with all other persons, the total number of handshakes is equal to the total number of combinations of two persons that can be selected from the n persons.

This number is $^nC_2 = \frac{n!}{(n-2)!\,2!} = \frac{1}{2}n(n-1)$

Combinations with Repetitions

The number of combinations of r objects among 'n' objects if the repetitions are allowed and r is not important is $C(n+r-1, n-1)$ i.e $(n+r-1)_{C_{(n-1)}} = \frac{(n+r-1)!}{r!\,(n-1)!}$

In otherwords $C(n+r-1, r) = C(n+r-1, n-1)$ represents the number of combinations of 'n' distinct objects taken 'r' at a time with repetitions allowed.

NOTE: (i) $C(n+r-1, r) = C(n+r-1, n-1)$ represents the no. of ways 'r' identical objects can be distributed among 'n' distinct containers.

(ii) $C(n+r-1, r) = C(n+r-1, n-1)$ represents the number of non-negative integer solutions of the eqn $x_1+x_2+\cdots+x_n = r$

(iii) The number of ways of distributing 'r' chocklets to 'n' children so that each child get atleast one is $(r-1)_{C_{(r-n)}}$

(iv) The no. of +ve integer solutions of $x_1+x_2+\cdots+x_n = r$ is $C(r-1, r-n)$

(P) The bag contains coins of 7 different denominations with atleast one dozen coins in each denominations. In how many ways can we select a dozen coin from the bag.

Soln: The selection consists in choosing with repetitions $r=12$ coins of $n=7$ distinct denominations

∴ The number of ways of making this selection is

$$C(7+12-1, 12) = C(18, 12) = 18_{C_{12}} = 18,564$$

(P) In how many ways can we distribute 10 identical marbles among 6 distinct containers?

Soln: Here $n=6$, $r=10$ $\quad$ ∴ $C(n+r-1, r) = 15_{C_{10}} = 3003$

(P) Find the number of non-negative integer solutions of the equation $x_1+x_2+x_3 = 17$, where x_1, x_2, x_3 are non-ve integers

Soln: Given that the eqn $x_1+x_2+x_3 = 17$.

Each solution of the given eqn. is equivalent to distribution of 17 identical balls in 3 numbered boxy with repetitions. where x_i represents the number of balls in the i^{th} box.

Here $n=3,\ r=17$

$\therefore$ The required no. of sol^n is $C(n+r-1, r) = {}^{19}C_{17} = 171$

(P) find the number of distinct terms in the expansion of $(x_1+x_2+x_3+x_4+x_5)^{16}$

sol^n. Each term in the expansion is of the form $\binom{16}{n_1\, n_2\, n_3\, n_4\, n_5}$ where each n_i is a non-ve integer and these n_i's ~~are 16~~ sum to 16

$\therefore$ The no. of distinct terms in the expansion is precisely equal to the number non-ve integers solutions of the eqn $n_1+n_2+n_3+n_4+n_5=16$

Here $n=5,\ r=16$

$\therefore$ The number is $C(n+r-1, r) = {}^{20}C_{16} = 4845$

(P) find the no. of non-ve integer solutions of the inequality $x_1+x_2+\cdots+x_6<10$

sol^n: Given that the inequality $x_1+x_2+\cdots+x_6<10$

we have to find the no. of non-negative integer solutions of the eqn $x_1+x_2+\cdots+x_6 = 9-x_7$

where $9-x_7 \le 9$, so that x_7 is non-ve integer.

Thus the required number is the no. of non-ve solutions of the eqn $x_1+x_2+\cdots+x_7=9$

Here $n=7,\ r=9$

$\therefore\ C(n+r-1, r) = {}^{15}C_9 = 5005$

(P). find the number of distinct triply (x_1, x_2, x_3) of non-ve integers satisfying $x_1+x_2+x_3<15$

sol^n. Given that $x_1+x_2+x_3<15$

Since the values are only integers $x_1+x_2+x_3 = 0$ to 14

Here $r=14,\ n=3$

$\therefore$ No. of sol^n. $= C(n+r-1, r) = {}^{16}C_{14} = 120.$

$r=13$, $C(15,13)=105$ $\quad$ $r=12$, $n=3$ $C(14,12)=91$

$r=11$, $n=3$, $C(13,11)=78$ $\quad$ $r=10$, $n=3$, $C(12,10)=66$

$r=9$, $n=3$ $C(11,9)=55$ $\quad$ $r=8$, $n=3$, $C(10,8)=45$

$r=7$, $n=3$, $C(9,7)=36$ $\quad$ $r=6$, $n=3$ $C(8,6)=28$

$r=5$, $n=3$, $C(7,5)=21$ $\quad$ $r=4$ $n=3$ $C(6,4)=15$

$r=3$, $n=3$ $C(5,3)=10$ $\quad$ $r=2$, $n=3$ $C(4,2)=6$

$r=1$, $n=3$ $C(3,1)=3$ $\quad$ $r=0$, $n=3$ $C(2,0)=1$

$\therefore$ The total no. of solⁿ = 680 //

(P) Find the +ve integer solutions of $x+y+z=9$

Solⁿ: Given that $x+y+z=9$.

This is equal to arranging 'n' stones in 'r' boxes such that each box should have atleast one stone.

The no. of ways of arranging 'r' stones in 'n' boxes such that there will be atleast one stone in each box is

$C(r-1, r-n) = {}^{(r-1)}C_{(r-n)}$. Here $r=9$, $n=3$

$\therefore \quad = {}^{8}C_{2} = 28$

The possible 28 solutions are (7,1,1), (1,7,1), (1,1,7), (6,1,2), (6,2,1), (1,6,2), (2,6,1), (1,2,6), (2,1,6), (5,2,2), (2,5,2), (2,2,5), (5,1,3), (5,3,1), (1,5,3), (3,5,1), (1,3,5), (3,1,5), (4,4,1), (1,4,4), (4,1,4), (4,2,3), (4,3,2), (2,4,3), (3,4,2), (2,3,4), (3,2,4), (3,3,3)

(P) How many +ve integer solutions will be there for $x+y+z=100$

Solⁿ: Given $x+y+z=100$

This is equal to assigning 'n' stones in 'r' boxes such that each box contains atleast one stone is ${}^{(r-1)}C_{(r-n)} = 4851$

Here $r=100$, $n=3$ - i.e

(P) Find the no. of +ve integer solutions of $x_1+x_2+x_3+x_4+x_5+x_6 < 10$

Solⁿ: Given that $x_1+x_2+x_3+x_4+x_5+x_6 < 10$

Each x_i value should be ≥ 1. So minimum value is 1

$\therefore x_1+x_2+x_3+x_4+x_5+x_6 = 6$ or 7 or 8 or 9

$r=6, n=6$ The possible solution is $(1,1,1,1,1,1)=1$

$r=7, n=6$ " " are ${}^{(r-1)}C_{(r-n)} = {}^{6}C_{1} = 6$

$r=8, n=6$ " " " " $= {}^{7}C_{2} = 21$

$r=9, n=6$ " " " " $= {}^{8}C_{3} = 56$

$\therefore$ Total no. of +ve integer solutions $= 1+6+21+56 = 84$ //.

(P) Find the number of +ve integer solutions of the equation $x_1+x_2+x_3=17$

Sol: Given that $x_1+x_2+x_3=17$

Here we require $x_1 \geq 1, x_2 \geq 1, x_3 \geq 1$. Here $r=17, n=3$

$\therefore$ The required no. of +ve integer solutions are

$${}^{(r-1)}C_{(r-n)} = {}^{16}C_{14} = 120 \text{ //}$$

(P) Find how many solutions are there to the given equation that satisfies the given condition $x_1+x_2+x_3 = 20$ where each x_i is a +ve integer

Sol: Given that $x_1+x_2+x_3=20 \rightarrow (1)$

We require $x_1 \geq 1, x_2 \geq 1, x_3 \geq 1$

Let us set $y_1 = x_1-1, y_2 = x_2-1, y_3 = x_3-1$

Then y_1, y_2, y_3 are all non -ve integers.

Sub. $x_1 = y_1+1, x_2 = y_2+1, x_3 = y_3+1$ in (1)

we get $y_1+y_2+y_3 = 17 \rightarrow (2)$

The no. of non-negative integer solutions of this eqn is the required number.

Here $n=3, r=17$

$\therefore$ No. of non -ve integer solutions of (3) is

$${}^{(r-1)}C_{(r-n)} = {}^{19}C_{17} \text{ //}$$

(P). In how many ways can five different messages be delivered by three messenger boys if no messenger boy left unemployed.

Soln: Let first messenger delivered x messages

second " " y "

Third " " z "

$\therefore x+y+z=5$ for $x \geq 1, y \geq 1, z \geq 1 \rightarrow$ (1)

Let us set $y_1 = x-1$, $y_2 = y-1$, $y_3 = z-1$

Then y_1, y_2, y_3 are all non -ve integers

Sub. $x = 1+y_1$, $y = 1+y_2$, $z = 1+y_3$ in (1), we get

$\cancel{x+y+z} = y_1+y_2+y_3 = 2 \rightarrow$ (2)

Here $n=3$, $r=2$

$\therefore$ No. of non-negative integer solutions of (2) is

$C(n+r-1, r) = {}^4C_2 = 6$

There are $(1,2,2), (2,1,2), (2,2,1), (1,1,3), (1,3,1), (3,1,1)$ ways delivered by three messengers boys //

(P) How many integers b/w 1 and 1000 have a sum of digits at integer numbers equal to 10.

Soln. Let x_1, x_2, x_3 be the position of the digits b/w integers 1 and 1000

Let $x_1+x_2+x_3 = 10$ for $x_1 \geq 0, x_2 \geq 0, x_3 \geq 0$

Here $r=10$, $n=3$

The number of non-negative integer solutions gives the number of integers b/w 1 and 1000 and sum of digits of integer equal to 10.

$\therefore$ The no. of non-negative integer solutions is

$C(n+r-1, r) = C(12, 10) = 66$

$\therefore$ The required no. of integers is 66 //

(P) Determine the no. of ways possible to wear 5 rings on 4 fingers. (30)

Sol. Let x_1, x_2, x_3, x_4 are fingers and no. of rings are 5

i.e. Let $x_1+x_2+x_3+x_4=5 \rightarrow (1)$

The no. of non-negative integer solutions of above eqn. are gives the required result.

Here $r=5, n=4$

$\therefore$ The no. of non-negative integer solutions of (1) is

$C(n+r-1, r) = C(8,5) = 56$

(P) If $x>2, y>0, z>0$ then find the no. of solutions of $x+y+z+w=21$

Sol. Given that the eqn $x+y+z+w=21 \rightarrow (1)$ and the given constraints $x>2, y>0, z>0$ i.e. $x\geq 3, y\geq 1, z\geq 1$

Let $x_1 = x-3$, $y_1 = y-1$, $z_1 = z-1$

So that x_1, y_1, z_1 are non-negative integers

Sub. $x = x_1+3$, $y = y_1+1$, $z = z_1+1$ in (1), we get

$x_1+y_1+z_1+w = 16 \rightarrow (2)$

Here $n=4$, $r=16$

$\therefore$ The no. of non-negative integer solutions of the eqn (2) is $C(n+r-1, r) = C(19,16) = 969$ //

(P) Find the number of integer solutions of $x_1+x_2+x_3+x_4+x_5 = 30$ where $x_1\geq 2, x_2\geq 3, x_3\geq 4, x_4\geq 2, x_5\geq 0$ [Ans: 8855]

(P) In how many ways can we distribute 12 identical pencils to 5 children so that every child get atleast one pencil [Ans: 330]

(P) Find the number of distinct triples (x_1, x_2, x_3) of non-ve integers satisfying $x_1+x_2+x_3<15$ [Ans: 680] $n=4, r=14$

(P) How many different outcomes are possible by tossing 10 similar coins.

Soln. This is same as the placing 10 similar balls into two boxes labeled 'heads' and 'tails'.

$$x_1 + x_2 = 10 . \text{ Here } r = 10, n = 2$$

$\therefore$ Total no. of outcomes $C(n+r-1, r) = C(11, 10) = 11$

(P) How many different outcomes are possible from tossing 10 similar dice. Ans: 3003

(P) Out of large supply of pennies, nickels, dimes and quarters, in how many ways can 10 coins be selected.
Ans: $C(13, 10)$

(P) Enumerate the number of ways of placing 20 indistinguishable balls into 5 boxes when each box is non-empty.
Ans. ${}^{19}C_4$

(P) How many integral solutions are there to $x_1 + x_2 + x_3 + x_4 + x_5 = 20$ when each $x_i \geq 2$ Ans: $C(14, 10)$.

(1) five balls are to be placed in three boxes. Each box can hold all the five balls. In how many ways can we place the balls in that no box is empty, if

(i) balls and boxes are different
(ii) balls are identical and boxes are different
(iii) balls are different and boxes are identical
(iv) balls and boxes are identical

Soln: (i) If balls and boxes are different, then the five balls can be distributed in the following ways

Distribution of five balls in three boxes

	Box 1	Box 2	Box 3
No. of Balls	3	1	1
	1	3	1
	1	1	3
	2	2	1
	2	1	2
	1	2	2

If boxes I, II and III contain 3, 1 and 1 balls resply. then the no. of ways of distributing the balls

$$\frac{5!}{3!\,1!\,1!} = 20$$

Ily, if boxes I, II, III contain 1, 3 and 1 balls resply then the no. of ways of distributing the balls $= \frac{5!}{3!1!1!} = 20$

If boxes I, II and III contain 1, 1 and 3 balls resply then the no. of way $= \frac{5!}{1!\,1!\,3!} = 20$

Also the boxes I, II and III contain 2, 2 and 1 balls resply Then the no. of ways $= \frac{5!}{2!2!1} = 30$

i.e. 2, 1, 2 $= \frac{5!}{2!\,1!\,2!} = 30$

1, 2, 2, $= \frac{5!}{2!\,2!\,2!} = 30$

Hence the total no. of required ways $= 20+20+20+30+30+30 = 150$

(ii) Balls are identical and boxes are different

Since the balls are identical, repetitions are allowed. We select three balls and put one ball each in each of the three boxes. We must distribute two more identical balls.

Hence the no. of ways distributing five identical balls in three different boxes $= {}^{(3+2-1)}C_2 = {}^4C_2 = 6$

(iii) Balls are different and boxes are identical

Hence, the distributions of balls i.e 1,1,3 ; 1,3,1 and 3,1,1 are identical.

∴ The no. of ways of distributing one ball in each of any two boxes and three balls in the third box

$$= \frac{5!}{3!\,1!\,1!} = 20$$

$||^{ly}$ the number of ways of distributing 2 balls in each of any two boxes and one ball in the third box

$$= \frac{5!}{2!\,2!\,1!} = 30$$

Thus the required no. of ways $= 20 + 30 = 50$

(iv) Balls are identical and boxes are identical

Hence the required no. of ways $= 2$ (i.e 1,1,3 and 1,2,2)

(P) How many words of 4 letters can be formed from the letters of the word INFINITE

Sol^n: Given word INFINITE. There are 8 letters of which 3 are I's, 2 are N's and F, T, E are distinct.

We have to form 4 letters words

Case (i): All distinct selection of 4 distinct letters from 5 letter is ${}^5C_4$ ways

No. of permutations of there are $4!$

Hence the total no. of words $= 4!\,{}^5C_4 = 120$

Case (ii): 3 alike, 1 distinct

No. of words in this case are ${}^1C_1\,{}^4C_1 \frac{4!}{3!} = 16$

Case (iii): 2 alike, 2 alike

No. of words $\;{}^2C_2 \cdot \frac{4!}{2!\,2!} = 6$

(P). How many 4 letter words can be formed using the letters of the word 'PROPOSITION' Ans: 20+18+360+360=758

(P) find the number of 4 letters that can be formed using the letters of the word 'KAMALA' Ans: 24+12+36=72

(P)

The pigeonhole principle

If 'n' pigeons are accomdated in 'm' pigeonholes and $n > m$ then one pigeonhole must atleast $\lceil \frac{n}{m} \rceil$ pigeons' i.e atleast one pigeonhole contains two or more pigeons.

(P). P.T in any set of 29 persons atleast 5 persons must have born on same day of week.

Soln: No. of persons (pigeons) = 29 = n

A week contain 7 days (pigeonholes) i.e $m = 7$

i.e $\text{ceil}\left(\frac{29}{7}\right) = \text{ceil}(4.1) = 5$

(P) Suppose there are 26 students and 7 cars to transport them. Show that atleast one car must have 4 or more passengers

Soln: No. of students (pigeons) = 26 = n

No. of cars (pigeonholes) = 7 = m

$\text{ceil}\left(\frac{26}{7}\right) = \text{ceil}(3.7) = \lceil 3.7 \rceil = 4$

(P) If there are 6 possible grades A, B, C, D, E & F. What is the minimum no. of students required in a class to be sure that atleast 7 will receive the same grade

Soln, No. of students (pigeons) n = ?

$\lceil \frac{n}{m} \rceil = 7 \Rightarrow \frac{n}{6} = 7 \Rightarrow n = 42$

(P) show that if four numbers from 1 to 7 are chosen, then two of them will add up to 8

Soln. Make three sets, each containing two numbers whose sum is 8 i.e $A_1 = \{1, 7\}$ $A_2 = \{2, 6\}$, $A_3 = \{3, 5\}$

The four numbers chosen must belong to one of these sets

As there are only three sets, two of the chosen numbers belong to the same set whose sum is 8.