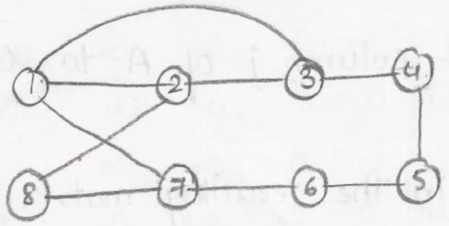
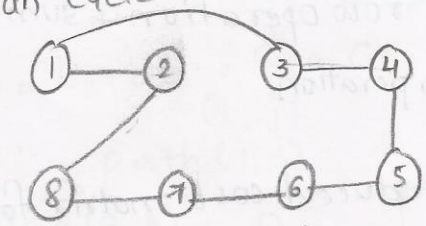


Find the hamiltonian cycle?

Hamiltonian Cycle:



Branch And Bound :

The term Branch & Bound refers to all state space
search methods in which all childrens of Each node are gene-
rated before any other live node can become the E-node. In
Branch & Bound, BFS, state space search are called FIFO
technique and DFS state space search are called LIFO. In
back tracking, we generate solution state space tree using only
DFS, whereas in branch & Bound we use both BFS & DFS techniques.
In backtracking bounding functions are used to avoid generation of
subtrees that do not contain an answer node. Backtracking is more
efficient for decision problems not for optimization problems. The main
difference between branch & bound and backtracking is that if we
get the solution, then we will terminate the search procedure in
backtracking, whereas BB we will continue the process until we
get a optimal solution. BB is applicable only for minimum values.

Travelling Sales Person Problem:-

Let $G=(V,E)$ be a directed graph. Let $(C(i,j))$ C_{ij}
$=\infty$ if $i,j \notin E$. TSP can be calculated based upon the least cost
Branch And Bound (LCBB).

Procedure :-

Let A be reduced cost matrix for root noode R.
Let S be a child of R such that the tree edge R,S corresponds
to include edge i, j in the tool.

3. Change all n-trees in row i & column j of A to ∞.

4. Set $A(j, 1)$ to ∞

5. Reduce all rows & columns in the resulting matrix.

6. Cost of R $\to$ $\hat{c}ost(R)$ = sum of row operation + sum of column operation

reduced matrix.

7. If s is not a leaf then the reduced cost matrix for s can be obtained as

$$\hat{C}(S) = \hat{C}(R) + A(i,j) + r$$

r $\to$ reduced cost matrix.

$\hat{C}(R)$: cost of reduced matrix.

i: ith row of reduced cost matrix

j $\to$ jth column of reduced cost matrix

*.

$$\begin{array}{l} \begin{bmatrix} \infty & 20 & 30 & 10 & 11 \\ 15 & \infty & 16 & 4 & 2 \\ 3 & 5 & \infty & 2 & 4 \\ 19 & 6 & 18 & \infty & 3 \\ 16 & 4 & 7 & 16 & \infty \end{bmatrix} \begin{array}{l} r_1 = 10 \\ r_2 = 2 \\ r_3 = 2 \\ r_4 = 3 \\ r_5 = 4 \end{array} \\ c_1 = \quad c_2 = \quad c_3 = \quad c_4 = \quad c_5 = \end{array}$$

$$\begin{bmatrix} \infty & 10 & 20 & 0 & 1 \\ 13 & \infty & 14 & 2 & 0 \\ 1 & 3 & \infty & 0 & 2 \\ 16 & 3 & 15 & \infty & 0 \\ 12 & 0 & 3 & 12 & \infty \end{bmatrix} \Rightarrow \begin{bmatrix} \infty & 10 & 17 & 0 & 1 \\ 12 & \infty & 11 & 2 & 0 \\ 0 & 3 & \infty & 0 & 2 \\ 15 & 3 & 12 & \infty & 0 \\ 11 & 0 & 0 & 12 & \infty \end{bmatrix} = A$$

$c_1 = 0 \quad c_2 = 0 \quad c_3 = 3 \quad c_4 = 0 \quad c_5 = 0$

$$\hat{C}(R) = 10 + 2 + 2 + 3 + 4 + 1 + 3$$

$$\hat{C}(R) = \underline{25}$$

Step 1:- path$(1, 2)$ (i = 1, j = 2)

1) $(i, j) = \infty$; ith row & jth row

2) $A(j, 1) = \infty$; $A(2, 1) = \infty$

$$\begin{array}{c} \begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ \infty & \infty & 11 & 2 & 0 \\ 0 & \infty & \infty & 0 & 2 \\ 15 & \infty & 12 & \infty & 0 \\ 11 & \infty & 0 & 12 & \infty \end{bmatrix} \end{array} \begin{array}{l} r_1 = 0 \\ r_2 = 0 \\ r_3 = 0 \\ r_4 = 0 \\ r_5 = 0 \end{array}$$

$c_1 = 0 \quad c_2 = 0 \quad c_3 = 0 \quad c_4 = 0 \quad c_5 = 0$

$$\hat{C}(S) = \hat{C}(R) + A(i,j) + r$$
$$= 25 + A(1,2) + r$$
$$= 25 + 10 + 0 = 35 //$$

Step 2:- path $\overset{(i,\ j)}{(1,3)}$

1) $A(3,1) = \infty$

2) $(1,3) = \infty$

$$\begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ 12 & \infty & \infty & 2 & 0 \\ 0 & 3 & \infty & 0 & 2 \\ 15 & 3 & \infty & \infty & 0 \\ 11 & 0 & \infty & 12 & \infty \end{bmatrix} \begin{array}{l} r_1 = 0 \\ r_2 = 0 \\ r_3 = 0 \\ r_4 = 0 \\ r_5 = 0 \end{array} \Rightarrow$$

$$\begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ 1 & \infty & \infty & 2 & 0 \\ \infty & 3 & \infty & 0 & 2 \\ 4 & 3 & \infty & \infty & 0 \\ 0 & 0 & \infty & 12 & \infty \end{bmatrix}$$

$c_1 = 11 \quad c_2 = 0 \quad c_3 = \infty \quad c_4 = \infty \quad c_5 = \infty$

$$\hat{C}(S) = \hat{C}(R) + A(i,j) + r$$
$$= 25 + 17 + 11$$
$$= 53$$

Step 3: path $(1,4)$

1) $1,4 = \infty$

2) $A(4,1) = \infty$

$$\begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ 12 & \infty & 11 & \infty & 0 \\ 0 & 3 & \infty & \infty & 2 \\ 15 & 3 & 12 & \infty & 0 \\ 11 & 0 & 0 & \infty & \infty \end{bmatrix} \begin{array}{l} r_1 = \infty \\ r_2 = 0 \\ r_3 = 0 \\ r_4 = 0 \\ r_5 = 0 \end{array}$$

$c_1 = 0 \quad c_2 = 0 \quad c_3 = 0 \quad c_4 = 0 \quad c_5 = 0$

$$\hat{C}(S) = \hat{C}(R) + A(i,j) + r$$
$$= 25 + A(1,4) + 0$$
$$= 25 + 0 + 0 = 25$$

Step 4 :- path $\overset{i\ \ j}{(1,5)}$

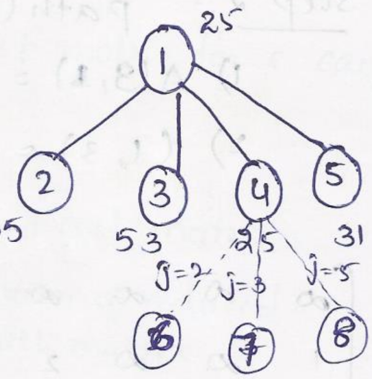
$1,5 = \infty$

$A(5,1) = \infty$

$$\begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ 12 & \infty & 11 & 2 & \infty \\ 0 & 3 & \infty & 0 & \infty \\ 15 & 3 & 12 & \infty & \infty \\ 11 & 0 & 0 & 12 & \infty \end{bmatrix} \begin{array}{l} r_1 = \infty \\ r_2 = 2 \\ r_3 = 0 \\ r_4 = 3 \\ r_5 = 0 \end{array}$$

$$\begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ 10 & \infty & 9 & 0 & \infty \\ 0 & 3 & \infty & 0 & \infty \\ 12 & 0 & 9 & \infty & \infty \\ 11 & 0 & 0 & 12 & \infty \end{bmatrix} = \begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ 10 & \infty & 9 & 0 & \infty \\ 0 & 3 & \infty & 0 & \infty \\ 12 & 0 & 9 & \infty & \infty \\ 11 & 0 & 0 & 12 & \infty \end{bmatrix}$$

$c_1 = 0 \quad c_2 = 0 \quad c_3 = 0 \quad c_4 = 0 \quad c_5 = 0$



$$\hat{C}(S) = \hat{C}(R) + A(1,5) + r$$

$$= 25 + 1 + 5$$

$$\hat{C}(S) = 31$$

Now

$$\begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ 12 & \infty & 11 & \infty & 0 \\ 0 & 3 & \infty & \infty & 2 \\ 3 & 3 & 12 & \infty & 0 \\ 11 & 0 & 0 & \infty & \infty \end{bmatrix} = \hat{A}$$

$\hat{C}(R) = \underline{\underline{25}}$

Step 5:- path$(\overset{i,j}{4,2})$

$4,2 = \infty$

$A(2,1) = \infty$

$$\begin{array}{c} \begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ \infty & \infty & 11 & \infty & 0 \\ 0 & \infty & \infty & \infty & 2 \\ \infty & \infty & \infty & \infty & \infty \\ 11 & 0 & 0 & \infty & \infty \end{bmatrix} \end{array} \begin{array}{l} r_1 = 0 \\ r_2 = 0 \\ r_3 = 0 \\ r_4 = 0 \\ r_5 = 0 \end{array}$$

$c_1 = 0 \quad c_2 = 0 \quad c_3 = 0 \quad c_4 = 0 \quad c_5 = 0$

$$\hat{C}(S) = \hat{C}(R) + A(i,j) + r$$

$$= 25 + 3 + 0 = 28$$

Step 6:- path$(\overset{i,j}{4,3})$

1) $4,3 = \infty$

2) $A(3,1) = \infty$

$$\begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ 12 & \infty & \infty & \infty & 0 \\ \infty & 3 & \infty & \infty & 2 \\ \infty & \infty & \infty & \infty & \infty \\ 11 & 0 & \infty & \infty & \infty \end{bmatrix} \begin{array}{l} r_1 = \infty \\ r_2 = 0 \\ r_3 = 2 \\ r_4 = \infty \\ r_5 = 0 \end{array}$$

$$\begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ 12 & \infty & \infty & \infty & 0 \\ \infty & 3 & \infty & \infty & 0 \\ \infty & \infty & \infty & \infty & \infty \\ 11 & 0 & \infty & \infty & \infty \end{bmatrix} = \begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ 1 & \infty & \infty & \infty & 0 \\ \infty & 1 & \infty & \infty & 0 \\ \infty & \infty & \infty & \infty & \infty \\ 0 & 0 & \infty & \infty & \infty \end{bmatrix}$$

$c_1 = 11 \quad c_2 = 0 \quad c_3 = \infty \quad c_4 = \infty \quad c_5 = 0$

$$\hat{C}(S) = \hat{C}(R) + A(i,j) + r$$
$$= 25 + 12 + 13$$
$$\hat{C}(7) = 50$$

Step 7 :- $\overset{i,\,j}{(4,5)}$

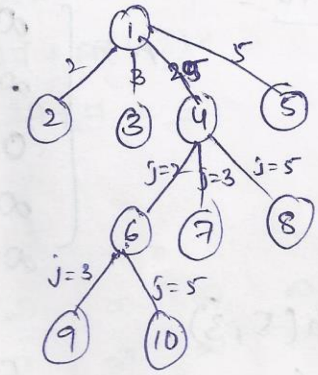
$4,5 = \infty$

$A(5,1) = \infty$

$$\begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ 12 & \infty & 11 & \infty & \infty \\ 0 & 3 & \infty & \infty & \infty \\ \infty & \infty & \infty & \infty & \infty \\ 11 & 0 & 0 & \infty & \infty \end{bmatrix} \begin{matrix} r_1 = \infty \\ r_2 = 11 \\ r_3 = 0 \\ r_4 = 0 \\ r_5 = 0 \end{matrix} \Rightarrow \begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ 1 & \infty & 0 & \infty & \infty \\ 0 & 3 & \infty & \infty & \infty \\ \infty & \infty & \infty & \infty & \infty \\ 11 & 0 & 0 & \infty & \infty \end{bmatrix}$$

$c_1 = 0 \quad c_2 = 0 \quad c_3 = 0 \quad c_4 = 0 \quad c_5 = \infty$

$$\hat{C}(S) = \hat{C}(R) + A(i,j) + r$$
$$= 25 + 0 + 11$$
$$= 36$$



~~Ste~~

$$A = \begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ \infty & \infty & 11 & \infty & 0 \\ 0 & \infty & \infty & \infty & 2 \\ \infty & \infty & \infty & \infty & \infty \\ 11 & \infty & 0 & \infty & \infty \end{bmatrix}$$

$$\hat{C}(R) = 28$$

Step 8 :- path $\overset{i,\,j}{(2,3)}$

$2,3 = \infty$

$A(3,1) = \infty$

$$\begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ \infty & \infty & \infty & \infty & \infty \\ \infty & \infty & \infty & \infty & 2 \\ \infty & \infty & \infty & \infty & \infty \\ 11 & 0 & \infty & \infty & \infty \end{bmatrix} \begin{matrix} r_1 = \infty \\ r_2 = \infty \\ r_3 = 2 \\ r_4 = \infty \\ r_5 = 0 \end{matrix} \Rightarrow \begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ \infty & \infty & \infty & \infty & \infty \\ \infty & \infty & \infty & \infty & 0 \\ \infty & \infty & \infty & \infty & \infty \\ 0 & 0 & \infty & \infty & \infty \end{bmatrix}$$

$c_1 = \overset{11}{\cancel{0}} \quad c_2 = 0 \quad c_3 = \infty \quad c_4 = 0 \quad c_5 = \overset{0}{\cancel{2}}$

$c_1 = 11$

$$\hat{C}(S) = \hat{C}(R) + A(i,j) + r$$
$$= 28 + 11 + 13 = 52$$

$28 + 13 + 11 = 52$

Step 9:- $\overset{i,\ j}{(2, 5)}$

$2,5 = \infty$

$A(5,1) = \infty$

$$\begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ \infty & \infty & \infty & \infty & \infty \\ 0 & \infty & \infty & \infty & \infty \\ \infty & \infty & \infty & \infty & \infty \\ \infty & \infty & 0 & \infty & \infty \end{bmatrix}$$

$C_1 = 11 \quad C_2 = 0 \quad C_3 = 0 \quad C_4 = 0 \quad C_5 = 0$

$$\begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ \infty & \infty & \infty & \infty & \infty \\ 0 & \infty & \infty & \infty & \infty \\ \infty & \infty & \infty & \infty & \infty \\ \infty & \infty & 0 & \infty & \infty \end{bmatrix}$$

$\hat{C}(S) = \hat{C}(R) + A(i,j) + r$

$= 28 + 0 + \infty$

$= 28$

Step 10:-

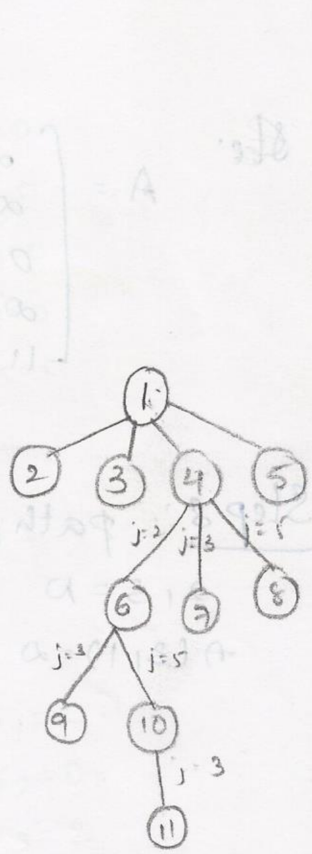
$$A = \begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ \infty & \infty & \infty & \infty & \infty \\ 0 & \infty & \infty & \infty & \infty \\ \infty & \infty & \infty & \infty & \infty \\ \infty & \infty & 0 & \infty & \infty \end{bmatrix}$$

Path(5, 3)

$(5, 3) = \infty$

$(3, 1) = \infty$

$$\begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ \infty & \infty & \infty & \infty & \infty \\ \infty & \infty & \infty & \infty & \infty \\ \infty & \infty & \infty & \infty & \infty \\ \infty & \infty & \infty & \infty & \infty \end{bmatrix}$$



$\hat{C}(S) = \hat{C}(R) + A(i,j) + r$

$= 28 + 0 + 0 = 28$

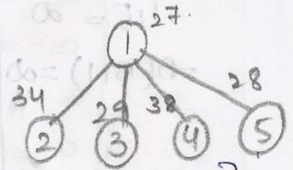
$1 \to 4 \to 2 \to 5 \to 3 = 28$

$$\begin{bmatrix} \infty & 7 & 3 & 12 & 8 \\ 3 & \infty & 6 & 14 & 9 \\ 5 & 8 & \infty & 6 & 18 \\ 9 & 3 & 5 & \infty & 11 \\ 18 & 14 & 9 & 8 & \infty \end{bmatrix} \begin{array}{l} r_1=3 \\ r_2=3 \\ r_3=5 \\ r_4=3 \\ r_5=8 \end{array} \Rightarrow \begin{bmatrix} \infty & 4 & 0 & 9 & 5 \\ 0 & \infty & 3 & 11 & 6 \\ 0 & 3 & \infty & 1 & 13 \\ 6 & 0 & 2 & \infty & 8 \\ 10 & 6 & 1 & 0 & \infty \end{bmatrix} \Rightarrow \begin{bmatrix} \infty & 4 & 0 & 9 & 0 \\ 0 & \infty & 3 & 11 & 1 \\ 0 & 3 & \infty & 1 & 8 \\ 6 & 0 & 2 & \infty & 3 \\ 10 & 6 & 1 & 0 & \infty \end{bmatrix}$$

$c_1=0 \quad c_2=0 \quad c_3=0 \quad c_4=0 \quad c_5=5 \qquad \Downarrow A'$

$\hat{C}(R) = \hat{3}+3+5+3+8+0+0+0+0+5$

$\hat{C}(R) = 27.$



Step 1:- $\overset{(i,j)}{(1,2)}$

$1,2 = \infty$

$A(2,1) = \infty$

$$\begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ 0 & \infty & 6 & 14 & 9 \\ 5 & \infty & \infty & 6 & 18 \\ 9 & \infty & 5 & \infty & 11 \\ 18 & \infty & 9 & 8 & \infty \end{bmatrix} \begin{array}{l} r_1=\infty \\ r_2= \\ \\ \\ \\ \end{array} \begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ \infty & \infty & 3 & 11 & 1 \\ 0 & \infty & \infty & 1 & 8 \\ 6 & \infty & 2 & \infty & 3 \\ 10 & \infty & 1 & 0 & \infty \end{bmatrix} \begin{array}{l} r_1=0 \\ r_2=1 \\ r_3=0 \\ r_4=2 \\ r_5=0 \end{array}$$

$$\begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ \infty & \infty & 2 & 10 & 0 \\ 0 & \infty & \infty & 1 & 8 \\ 4 & \infty & 0 & \infty & 1 \\ 10 & \infty & 1 & 0 & \infty \end{bmatrix}$$

$\hat{C}(S) = \hat{C}(R) + A(i,j) + r$

$= 27 + A(1,2) + 3$

$\hat{C}(S) = 27 + 7 = 34$

$c_1=0 \quad c_2=\infty, \quad c_3=0 \quad c_4=0 \quad c_5=0$

Step 2:- $\overset{i,j}{(1,3)}$

$1,3 = \infty$

$A(3,1) = \infty$

$$\begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ 0 & \infty & \infty & 11 & 1 \\ \infty & 3 & \infty & 1 & 8 \\ 6 & 0 & \infty & \infty & 3 \\ 10 & 6 & \infty & 0 & \infty \end{bmatrix} \begin{array}{l} r_1=\infty \\ r_2=0 \\ r_3=1 \\ r_4=0 \\ r_5=0 \end{array} \begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ 0 & \infty & \infty & 11 & 1 \\ 0 & 2 & \infty & 0 & 7 \\ 6 & 0 & \infty & \infty & 3 \\ 10 & 6 & \infty & 0 & \infty \end{bmatrix}$$

$c_1=0 \quad c_2=0 \quad c_3=\infty \quad c_4=0 \quad c_5=1$

$$\begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ 0 & \infty & \infty & 11 & 0 \\ 0 & 2 & \infty & 0 & 6 \\ 6 & 0 & \infty & \infty & 2 \\ 10 & 6 & \infty & 0 & \infty \end{bmatrix}$$

$\hat{C}(S) = \hat{C}(R) + A(i,j) + r$

$= 27 + 1 + 1 = 29$

$c_1=0 \quad c_2=0 \quad c_3=0 \quad c_4=0 \quad c_5=1$

Step 3: (1,4)

$1,4 = \infty$

$A(4,1) = \infty$

$$\begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ 0 & \infty & 3 & \infty & 1 \\ 0 & 3 & \infty & \infty & 8 \\ \infty & 0 & 2 & \infty & 3 \\ 10 & 6 & 1 & \infty & \infty \end{bmatrix} \begin{array}{l} r_1=0 \\ r_2=0 \\ r_3=0 \\ r_4=0 \\ r_5=1 \end{array} \begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ 0 & \infty & 3 & \infty & 1 \\ 0 & 3 & \infty & \infty & 8 \\ \infty & 0 & 2 & \infty & 3 \\ 9 & 5 & 0 & \infty & \infty \end{bmatrix}$$

$$\begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ 0 & \infty & 3 & \infty & 0 \\ 0 & 3 & \infty & \infty & 7 \\ \infty & 0 & 2 & \infty & 2 \\ 9 & 5 & 0 & \infty & \infty \end{bmatrix}$$

$\hat{c}(S) = \hat{c}(R) + A(4,1;4) + r$

$\hat{c}(4) = 27 + 9 + 2$

$= 27 + 11 = 38.$

Step 4 :- $(1,5) = \infty$

$1,5 = \infty$

$A(5,1) = \infty$

$$\begin{array}{c} \begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ 0 & \infty & 3 & 11 & \infty \\ \infty & 3 & \infty & 1 & \infty \\ 6 & 0 & 2 & \infty & \infty \\ 10 & 6 & 1 & 0 & \infty \end{bmatrix} \begin{matrix} r_1=0 \\ r_2=0 \\ r_3=0 \\ r_4=0 \\ r_5=0 \end{matrix} \\ c_1=0 \quad c_2=0 \quad c_3=1 \quad c_4=0 \quad c_5=\infty \end{array} \begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ 0 & \infty & 2 & 11 & \infty \\ 0 & 3 & \infty & 1 & \infty \\ 6 & 0 & 1 & \infty & \infty \\ 10 & 6 & 0 & 0 & \infty \end{bmatrix}$$

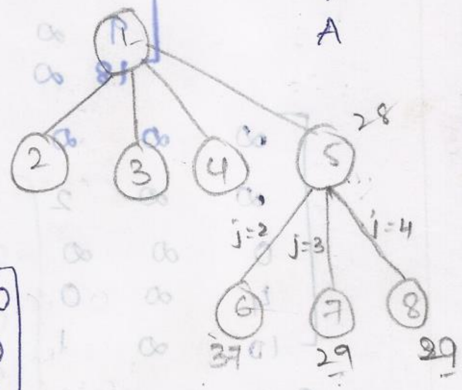
$\Downarrow$

A

$\hat{c}(S) = \hat{c}(R) + A(i,j) + r$

$= 27 + A(1,5) + 1$

$= 27 + 1 = 28$



Step 5 :- $(5,2)$ (i, j)

$(5,2) = \infty$

$A(2,1) = \infty$

$$\begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ \infty & \infty & \infty & 11 & \infty \\ 0 & \infty & \infty & 1 & \infty \\ 6 & \infty & 1 & \infty & \infty \\ \infty & \infty & \infty & \infty & \infty \end{bmatrix}$$

$$A = \begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ \infty & \infty & 2 & 11 & \infty \\ 0 & \infty & \infty & 1 & \infty \\ 6 & \infty & 1 & \infty & \infty \\ \infty & \infty & \infty & \infty & \infty \end{bmatrix}$$

$$\begin{bmatrix} \infty & \infty & 0 & 9 & 0 \\ 0 & \infty & 3 & 11 & 1 \\ 0 & \infty & \infty & 1 & \infty \\ 4 & \infty & 0 & \infty & 1 \\ \infty & \infty & \infty & \infty & \infty \end{bmatrix}$$

$$\begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ \infty & \infty & 2 & 11 & \infty \\ 0 & \infty & \infty & 1 & \infty \\ 6 & \infty & 1 & \infty & \infty \\ \infty & \infty & \infty & \infty & \infty \end{bmatrix} \begin{matrix} \\ r_2=2 \\ \\ r_4=1 \\ \\ \end{matrix}$$

$$\begin{array}{c} \begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ \infty & \infty & 0 & 9 & \infty \\ 0 & \infty & \infty & 1 & \infty \\ 5 & \infty & 0 & \infty & \infty \\ \infty & \infty & \infty & \infty & \infty \end{bmatrix} \\ c_4=1 \end{array} \quad \begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ \infty & \infty & 0 & 8 & \infty \\ 0 & \infty & \infty & 0 & \infty \\ 5 & \infty & 0 & \infty & \infty \\ \infty & \infty & \infty & \infty & \infty \end{bmatrix}$$

$\hat{c}(S) = \hat{c}(R) + A(i,j) + r$

$= 28 + 6 + 4 = 38$

Step 6 :- path $(5,3)$

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$$\begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ 0 & \infty & \infty & 11 & \infty \\ \infty & 3 & \infty & 1 & \infty \\ 6 & 0 & \infty & \infty & \infty \\ \infty & \infty & \infty & \infty & \infty \end{bmatrix} \begin{matrix} r_1=\infty \\ 0 \\ 1 \\ 0 \\ 0 \end{matrix} \Rightarrow \begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ 0 & \infty & \infty & 11 & \infty \\ \infty & 2 & \infty & 0 & \infty \\ 6 & 0 & \infty & \infty & \infty \\ \infty & \infty & \infty & \infty & \infty \end{bmatrix}$$
$$\begin{matrix} 0 & 0 & 0 & 0 & 0 \end{matrix}$$

$\hat{C}(S) = \hat{C}(R) + A(i,j) + r$
$= 28 + 0 + 1 = \underline{29}$

Step 7:- (5,4) path

$5,4 = \infty$
$A(4,1) = \infty$

$$\begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ 0 & \infty & 2 & \infty & \infty \\ 0 & 3 & \infty & \infty & \infty \\ \infty & 0 & 1 & \infty & \infty \\ \infty & \infty & \infty & \infty & \infty \end{bmatrix} \begin{matrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{matrix} \Rightarrow \begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ 0 & \infty & 1 & \infty & \infty \\ 0 & 3 & \infty & \infty & \infty \\ \infty & 0 & 0 & \infty & \infty \\ \infty & \infty & \infty & \infty & \infty \end{bmatrix}$$
$$\begin{matrix} 0 & 0 & 1 & \infty & 0 \end{matrix}$$

$\hat{C}(S) = \hat{C}(R) + A(i,j) + r$
$= 28 + 0 + 1 = 29.$

Step 8:- $A = \begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ 0 & \infty & \infty & 11 & \infty \\ 0 & 2 & \infty & 0 & \infty \\ 6 & 0 & \infty & \infty & \infty \\ \infty & \infty & \infty & \infty & \infty \end{bmatrix}$

(3,2) path
$3,2 = \infty$
$A(2,1) = \infty$

$$\begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ \infty & \infty & \infty & 11 & \infty \\ \infty & \infty & \infty & \infty & \infty \\ 6 & \infty & \infty & \infty & \infty \\ \infty & \infty & \infty & \infty & \infty \end{bmatrix} \begin{matrix} \\ r_2 = 11 \\ \\ r_4 = 6 \\ \\ \end{matrix} \Rightarrow \begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ \infty & \infty & \infty & 0 & \infty \\ \infty & \infty & \infty & \infty & \infty \\ 0 & \infty & \infty & \infty & \infty \\ \infty & \infty & \infty & \infty & \infty \end{bmatrix}$$

$\hat{C}(S) = \hat{C}(R) + A(i,j) + r = 29 + 17 = 46$

Step 9:- (3,4) path

$3,4 = \infty$
$A(4,1) = \infty$

$$\begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ 0 & \infty & \infty & \infty & \infty \\ \infty & \infty & \infty & \infty & \infty \\ \infty & 0 & \infty & \infty & \infty \\ \infty & \infty & \infty & \infty & \infty \end{bmatrix}$$

$\hat{C}(S) = A(3,4) + r + \hat{C}(P) = \underline{29}$

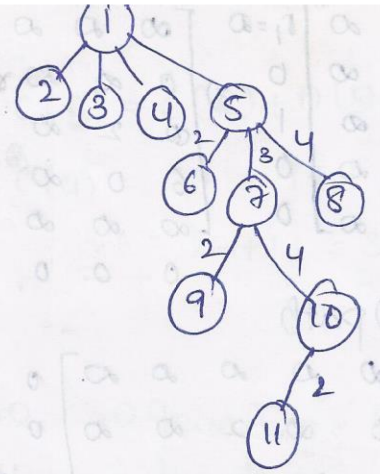
Step 10:- $(4,2) = \infty$

$4,2, = \infty$
$A(2,1) = \infty$

$$\begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ \infty & \infty & \infty & \infty & \infty \\ \infty & \infty & \infty & \infty & 0 \\ \infty & \infty & \infty & \infty & \infty \\ \infty & \infty & \infty & \infty & \infty \end{bmatrix}$$

$\hat{c}(S) = 29$

$1 \rightarrow 5 \rightarrow 3 \rightarrow 4 \rightarrow 2$



1,5,3,4,2

$\overset{11}{\Rightarrow} 26$

$$\begin{bmatrix} \infty & 11 & 0 & 9 & 6 \\ 8 & \infty & 7 & 3 & 4 \\ 8 & 4 & \infty & 4 & 8 \\ 11 & 10 & 5 & \infty & 5 \\ 6 & 9 & 5 & 5 & \infty \end{bmatrix} \begin{matrix} r_1 = 0 \\ r_2 = 3 \\ r_3 = 4 \\ r_4 = 5 \\ r_5 = 5 \end{matrix} \quad \begin{bmatrix} \infty & 11 & 0 & 9 & 6 \\ 5 & \infty & 4 & 0 & 1 \\ 4 & 0 & \infty & 0 & 4 \\ 6 & 5 & 0 & \infty & 0 \\ 1 & 4 & 0 & 0 & \infty \end{bmatrix}$$

$c_1 = 1 \quad c_2 = 0 \quad c_3 = 0 \quad c_4 = 0 \quad c_5 = 0$

$$\begin{bmatrix} \infty & 11 & 0 & 9 & 6 \\ 4 & \infty & 4 & 0 & 1 \\ 3 & 0 & \infty & 0 & 4 \\ 5 & 5 & 0 & \infty & 0 \\ 0 & 4 & 0 & 0 & \infty \end{bmatrix} \Rightarrow A$$

$\hat{c}(R) = 3+4+5+5+1+0+0+0+0$

$\hat{c}(R) = 18$

Step 1:- path(1,2)
(i, j)

$1,2 = \infty$

$A(2,1) = \infty$

$$\begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ \infty & \infty & 4 & 0 & 1 \\ 3 & \infty & \infty & 0 & 4 \\ 5 & \infty & 0 & \infty & 0 \\ 0 & \infty & 0 & 0 & \infty \end{bmatrix} \begin{matrix} r_1 = \infty \\ r_2 = 0 \\ r_3 = 0 \\ r_4 = 0 \\ r_5 = 0 \end{matrix}$$

$c_1 = 0 \quad c_2 = 0 \quad c_3 = 0 \quad c_4 = 0 \quad c_5 = 0$

$\hat{c}(B) = \hat{c}(R) + A(i,j) + r$

$= 18 + A(1,2) + 0$

$= 18 + 11 = 29$

$\hat{c}(S) = 29$

$\hat{c}(2) = 29$

Step 2: path$\overset{i,j}{(1,3)}$

$1,3 = \infty \quad A(3,1) = \infty$

$$\begin{array}{cl}\begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ 4 & \infty & \infty & 0 & 1 \\ \infty & 0 & \infty & 0 & 4 \\ 5 & 5 & \infty & \infty & 0 \\ 0 & 4 & \infty & 0 & \infty \end{bmatrix} & \begin{array}{l} r_1=\infty \\ r_2=0 \\ r_3=0 \\ r_4=0 \\ r_5=0 \end{array}\end{array}$$

$c_1=0 \quad c_2=0 \quad c_3=0 \quad c_4=0 \quad c_5=0$

$\hat{c}(S) = \hat{c}(R) + A(i,j) + r \qquad \hat{c}(3) = 18$

$= 18 + 0 + 0 = 18$

Step 3: path(1,4) (i, j)

$1,4 = \infty$

$A(4,1) = \infty$

$$\begin{array}{cl}\begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ 4 & \infty & 4 & \infty & 1 \\ 3 & 0 & \infty & \infty & 4 \\ \infty & 5 & 0 & \infty & 0 \\ 0 & 4 & 0 & \infty & \infty \end{bmatrix} & \begin{array}{l} r_1=\infty \\ r_2=1 \\ r_3=0 \\ r_4=0 \\ r_5=0 \end{array}\end{array} \qquad \begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ 3 & \infty & 3 & \infty & 0 \\ 3 & 0 & \infty & \infty & 4 \\ \infty & 5 & 0 & \infty & 0 \\ 0 & 4 & 0 & \infty & \infty \end{bmatrix}$$

$c_1=0 \quad c_2=0 \quad c_3=0 \quad c_4=0 \quad c_5=0$

$\hat{c}(S) = \hat{c}(R) + A(i,j) + r$

$\hat{c}(4) = 18 + A(1,4) + 1 = 18 + 9 + 1 = 28$.

Step 4:- path(1,5)

$1,5 = \infty$

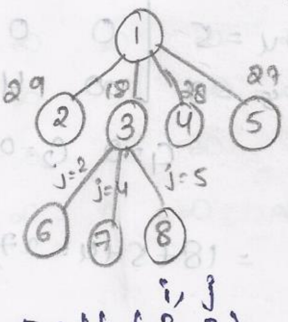
$A(5,1) = \infty$

$$\begin{array}{cl}\begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ 4 & \infty & 4 & 0 & \infty \\ 3 & 0 & \infty & 0 & \infty \\ 5 & 5 & 0 & \infty & \infty \\ \infty & 4 & 0 & 0 & \infty \end{bmatrix} & \begin{array}{l} r_1=\infty \\ r_2=0 \\ r_3=0 \\ r_4=0 \\ r_5=0 \end{array}\end{array}$$

$c_1=3 \quad c_2=0 \quad c_3=0 \quad c_4=0 \quad c_5=0$

$\hat{c}(S) = \hat{c}(R) + A(i,j) + r$

$= 18 + 6 + 3 = \cancel{18}\ 24 + 3 = 27$.



Step 5:- path(3,2) (i, j)

$3,2 = \infty$

$A(2,1) = \infty$

$$A = \begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ 4 & \infty & \infty & 0 & 1 \\ \infty & 0 & \infty & 0 & 4 \\ 5 & 5 & \infty & \infty & 0 \\ 0 & 4 & \infty & 0 & \infty \end{bmatrix}$$

$$\begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ \infty & \infty & \infty & 0 & 1 \\ \infty & \infty & \infty & \infty & \infty \\ 5 & \infty & \infty & \infty & 0 \\ 0 & \infty & \infty & 0 & \infty \end{bmatrix} \begin{array}{l} r_1 = \infty \\ r_2 = 0 \\ r_3 = \infty \\ r_4 = 0 \\ r_5 = 0 \end{array}$$

$c_1 = 0 \quad c_2 = \infty \quad c_3 = 0 \quad c_4 = 0 \quad c_5 = 0$

$\hat{C}(S) = \hat{C}(R) + A(i,j) + r = 18 + A(3,2) + 0$

$\hat{C}(S) = 18 + 0 = 18$

Step 6:- path$\overset{(i,\ j)}{(3,4)}$

$3,4 = \infty$

$A(4,1) = \infty$

$$\begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ 4 & \infty & \infty & \infty & 1 \\ \infty & \infty & \infty & \infty & \infty \\ \infty & 5 & \infty & \infty & 0 \\ 0 & 4 & \infty & \infty & \infty \end{bmatrix} \begin{array}{l} r_1 = \infty \\ r_2 = 1 \\ r_3 = \infty \\ r_4 = 0 \\ r_5 = 0 \end{array}$$

$$\begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ 3 & \infty & \infty & \infty & 0 \\ \infty & \infty & \infty & \infty & \infty \\ \infty & 5 & \infty & \infty & 0 \\ 0 & 4 & \infty & \infty & \infty \end{bmatrix}$$

$c_2 = 4$

$\hat{C}(S) = \hat{C}(R) + A(i,j) + r$

$= 18 + A(3,4) + 1$

$= 18 + 0 + 4 = \underline{\underline{23}}$

Step 7:- path$\overset{i,\ j}{(3,5)}$

$3,5 = \infty$

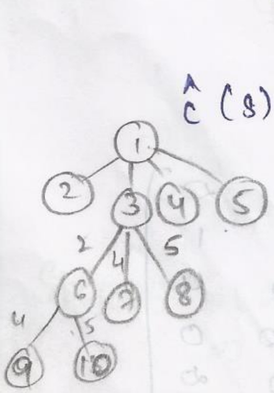
$A(5,1) = \infty$

$$\begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ 4 & \infty & \infty & 0 & \infty \\ \infty & \infty & \infty & \infty & \infty \\ 5 & 5 & \infty & \infty & \infty \\ \infty & 4 & \infty & 0 & \infty \end{bmatrix} \begin{array}{l} r_1 = \infty \\ r_2 = 0 \\ r_3 = \infty \\ r_4 = 5 \\ r_5 = 0 \end{array} \begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ 4 & \infty & \infty & 0 & \infty \\ \infty & \infty & \infty & \infty & \infty \\ 0 & 0 & \infty & \infty & \infty \\ \infty & 4 & \infty & 0 & \infty \end{bmatrix}$$

$c_1 = 0 \quad c_2 = 0 \quad c_3 = 0 \quad c_4 = 0 \quad c_5 = 0$

$\hat{c}(S) = \hat{C}(R) + A(i,j) + r$

$= 18 + 5 + A(3,5) = 18 + 5 + 4 = 27$



Step 8 :- path $\overset{i,j}{(2,4)}$

$$A = \begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ \infty & \infty & \infty & 0 & 1 \\ \infty & \infty & \infty & \infty & \infty \\ 5 & \infty & \infty & \infty & 0 \\ 0 & \infty & \infty & 0 & \infty \end{bmatrix}$$

$2,4 = \infty$

$A(4,1) = \infty$

$$\begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ \infty & \infty & \infty & \infty & \infty \\ \infty & \infty & \infty & & \infty \\ \infty & \infty & \infty & \infty & \infty \\ 0 & \infty & \infty & \infty & \infty \end{bmatrix}$$

$\hat{c}(S) = \hat{c}(R) + A(i,j) + r$

$= 18 + A(2,4)$

$= 18 + 0 = 18.$

Step 9 :- path $\overset{i,j}{(2,5)}$

$2,5 = \infty$

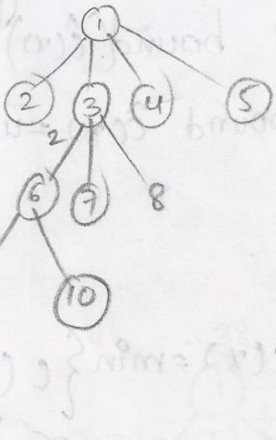
$A(5,1) = \infty$

$$\begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ \infty & \infty & \infty & \infty & \infty \\ \infty & \infty & \infty & \infty & \infty \\ 5 & \infty & \infty & \infty & \infty \\ \infty & \infty & 0 & 0 & \infty \end{bmatrix}$$

$$\begin{array}{c} \\ \\ \\ 5 \\ 0 \end{array} \begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ \infty & \infty & \infty & \infty & \infty \\ \infty & \infty & \infty & \infty & \infty \\ 0 & \infty & \infty & \infty & \infty \\ \infty & \infty & 0 & 0 & \infty \end{bmatrix}$$

$\hat{c}(S) = \hat{c}(R) + A(2,5) + r$

$= 18 + 1 + 5 = 24$



Step 10 :- path $\overset{i,j}{(4,5)}$

$$A = \begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ \infty & \infty & \infty & \infty & \infty \\ \infty & \infty & \infty & \infty & \infty \\ \infty & \infty & \infty & \infty & 0 \\ 0 & \infty & 0 & \infty & \infty \end{bmatrix}$$

$4,5 = \infty$

$A(5,1) = \infty$

$$\begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ \infty & \infty & \infty & \infty & \infty \\ \infty & \infty & \infty & \infty & \infty \\ \infty & \infty & \infty & \infty & \infty \\ \infty & \infty & \infty & \infty & \infty \end{bmatrix}$$

$\hat{c}(S) = \hat{c}(R) + A(i,j) + r$

$= 18 + 0 = 18$ //.

$1 \rightarrow 3 \rightarrow 2 \rightarrow 4 \rightarrow 5$

$= 18$

0/1 Knapsack Problem:-

→ A maximization problem can be easily converted into the minimization problem by changing the sign of the objective function

→ for 0/1 knapsack problem, 2 techniques are used.

(1) FIFOBB expanding the nodes using BFS technique.

(2) LIFOBB expanding the nodes using DFS technique

We expanding the node by taking the $x_i = 1$ for left child and $x_i = 0$ for right child.

1) LIFOBB :- / LCBB (Least Cost Branch & Bound)

Inorder to find the solution for a given problem using LIFO BB for 0/1 knapsack problem.

The following 2 methods are used.

① Lower bound $\hat{c}(x)$ ② Upper bound $(\mu(x))$

Lower bound $\hat{c}(x)$ = upperbound − $\dfrac{\text{remaining weight in knapsack } \sim\sim\text{ bag}}{\text{remaining weight which is not considered}} \times \text{remaining profit}$

$$\hat{C}(x) = \min\{c(lchild(x)), c(rchild(x))\}$$

Upper bond $\mu(x) = -\sum_{i=1}^{n} P_i$

→ Inorder to expand the next level of nodes, we consider ~~lower bound values~~. the lower bound values.

→ If both the ~~bor~~ nodes consists of same lower bound values then select the minimum upper bound values.

1. $n = 4$ $m = 15$ $P = 10, 10, 12, 18$ $w = 2, 4, 6, 9$.

For node(1)

$w_1+w_2+w_3+w_4 \le m$

$2 \le 15$ consider only w_1, w_2, w_3 weights and their profits.

$2+4 \le 15$

$6 \le 15$ ✓

$6+6 \le 15$

$12 \le 15$ ✓

$12+9 \le 15$ ✗

$w_1+w_2+w_3 = 12$

$\hat{c}(1) = ?$

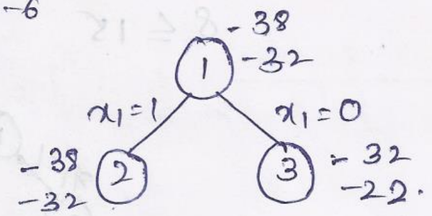
$\mu(1) = -\sum_{i=1}^{4} P_i = (P_1+P_2+P_3)(P_4)$

$= -(10+10+12)$

$= -32$

$\hat{c}(1) = -32 - \frac{3}{9} \times 18 = -32-6$

$= -38$



For node(2)

$w_1+w_2+w_3+w_4 \le m$

$2 \le 15$

$2+4 \le 15$

$2+4+6 \le 15$

$w_1+w_2+w_3 = 12$

$\mu(2) = -(10+10+12) = -32$

$\hat{c}(2) = -32 - \frac{3}{9} \times 18 = -38$

For node(3)

$w_1+w_2+w_3+w_4 \le m$

$\downarrow$
0

$4 \le 15$

$4+6 \le 15$

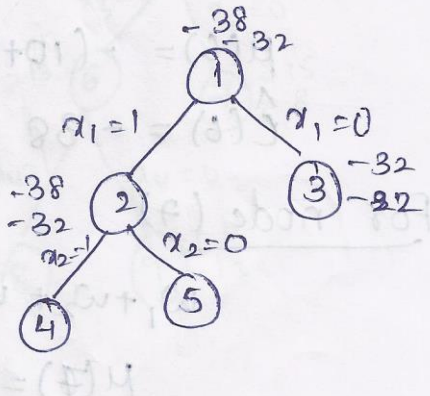
$10+9 \not\le 15$

$w_2+w_3 = 10$

$\mu(3) = -(10+12) = -22$

$\hat{c}(3) = -22 - \frac{5}{9} \times 18$

$= -32$



For node(4)

$w_1+w_2+w_3+w_4 \le m$

$2 \le 15$ $2+4+6 \le 15$

$2+4 \le 15$.

$w_1+w_2+w_3=12$

$\mu(4) = -(10+10+12) = -32$

$\hat{c}(4) = -32 - \frac{3}{9}\times 18 = -38$

for node (5)

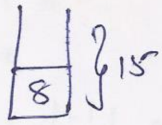
$w_1 = 1 \quad w_2 = 0$

$2 \le 15$

$w_1 \; w_3$

$2+6 \le 15$

$8 \le 15$

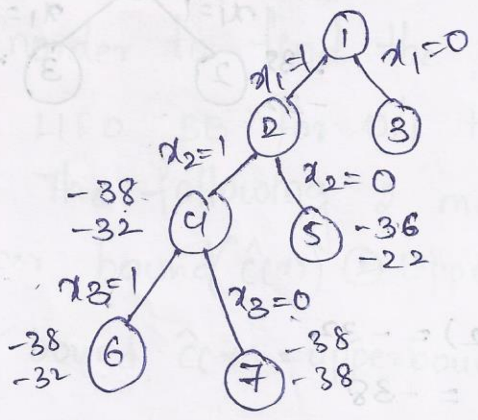


$w_1+w_3 = 8$

$\mu(5) = -(10+12) = -22$

$\hat{c}(5) = -22 - \frac{7}{9}\times 18 = -22-14$

$= -36$



For node (6)

$w_1+w_2+w_3=12$

$\mu(6) = -(10+10+12) = -32$

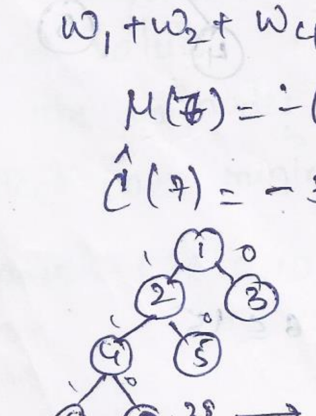
$\hat{c}(6) = -38$

For node (7)

$w_1+w_2+w_4 \le 15$

$\mu(7) = -(10+10+18) = -38$

$\hat{c}(7) = -38 - \frac{0}{9}\times 18 = -38$



L.B & U.B are equal. So kill that node.

For node (8)

$\omega_1 + \omega_2 + \omega_4 \le m$

$2 + 4 + 9 \le 15$

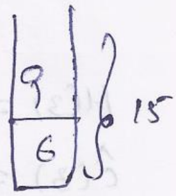
$\mu(8) = -38$

$\hat{c}(8) = -38 - \frac{0}{9} \times 18 = -38$

For node(9)

$\omega_1 + \omega_2 \le m$

$6 \le 15$



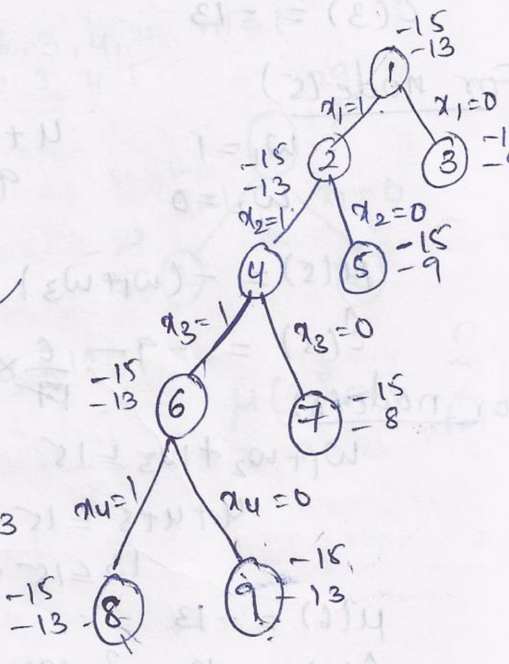
$\mu(9) = -20$

$\hat{c}(8) = -20 - \frac{0}{9} \times \cancel{18}\,2$

$= -20$

The solution is $\underline{\underline{1101}}$

2. $n = 5$, $m = 15$, $w = 4, 4, 5, 8, 9$. $P = 4, 4, 5, 8, 9$



For node(1)

$w_1 + w_2 + w_3 + w_4 + w_5 \le m$

$4 \le 15$

$4 + 4 \le 15$

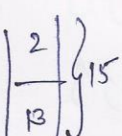
$4 + 4 + 5 \le 15 \Rightarrow 13 \le 15$ ✓

$13 + 8 \le 15$ ✗

$w_1 + w_2 + w_3 \le 15$

$13 \le 15$

$\mu(1) = -\sum_{i=1}^{n} P_i = -(4+4+5) = -13$



$\hat{c}(1) = -13 - \frac{2}{\cancel{9}} \times \cancel{9}$

$\hat{c}(1) = -15$

For node(2)

$w_1 + w_2 + w_3 \le 15$

$4 + 4 + 5 \le 15$ $13 \le 15$ ✓

$\mu(2) = -\sum_{i=1}^{n} P_i = -13$

$\hat{c}(2) = -13 - \frac{2}{\cancel{9}} \times \cancel{9} = -15$

For node(3)

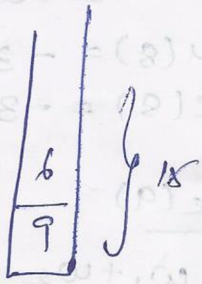
$w_1 = 0$

$4 \le 15$

$4+5 \le 15$

$9 \le 15 \qquad 9+8 \le 18$

$17 \le 15 \times$



$w_2 + w_3 \le 15$

$9 \le 15$

$\mu(3) = -(4+5) = -9$

$\hat{c}(3) = -9 - \frac{6}{\cancel{9}} \times \cancel{9} = -15$

For node(4)

$w_1 = 1 \quad w_2 = 1$

$4+4+5 \le 15$

$13 \le 15$

$\mu(3) = -15$

$\hat{c}(3) = -13$

For node(5)

$w_1 = 1 \qquad 4+5 \le 15$

$w_2 = 0 \qquad 9 \le 15$

$\mu(5) = -(w_1 + w_3) = -(9)$

$\hat{c}(5) = -9 - \frac{6}{\cancel{17}} \times \cancel{17} = -15$

For node(6)

$w_1 + w_2 + w_3 \le 15$

$4+4+5 \le 15$

$13 \le 15$ ✓

$\mu(6) = -13$

$\hat{c}(6) = -13 - \frac{2}{\cancel{18}} \times \cancel{18} = -15$

For node(7)

$w_1 = 1 \quad w_2 = 1 \quad w_3 = 0$

$4+4 \le 15$

$8 \le 15$ ✓ $\qquad 8+9 \le 15$

$\mu(7) = -(4+4) = -8$

$$\hat{c}(7) = -8 - \frac{7}{\cancel{9}} \times \cancel{9} = -15$$

For node(8)

$$w_1 + w_2 + w_3 + w_4 \le 15$$

$$w_1 + w_2 + w_3 \le 15$$

$$\mu(8) = -13$$

$$\hat{c}(8) = -13 - \frac{2}{\cancel{18}} \times \cancel{18} = -15$$

For node(9)

$$w_4 = 0$$

$$w_1 + w_2 + w_3 + w_4 \le 15$$

$$4 \le 15$$

$$8 \le 15$$

$$8+5 \le 15$$

$$13 \le 15$$

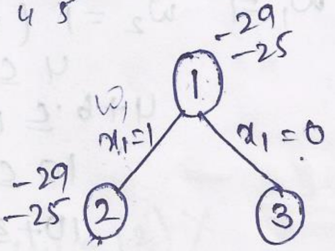
$$w_1 + w_2 + w_3 \le 15$$

$$\mu(9) = -(4+4+5) = -13$$

$$\hat{c}(9) = -15$$

3. $n=5$ $\quad P = 10, 15, 6, 8, 4$ $\quad w = 4, 6, 3, 4, 2$ $\quad m = 12$

For node(1)



$$\cancel{w_5} + w_1 + w_2 + w_3 + w_4 \le 12$$

$$4 \le 12$$

$$4+6 \le 12$$

$$4+6+3 \le 12 \times$$

$$w_1 + w_2 = 12$$

$$\mu(x) = -\sum_{i=1}^{n} P_i$$

$$\mu(x) = (10+15) = -25$$

$$\hat{c}(x) = -25 - \frac{\cancel{2}}{\cancel{9}} \times \overset{2}{\cancel{18}}$$

$$= -25 - 4 = -29$$

For node(2)

$$w_5 + w_1 + w_2 + w_3 + w_4 \le 12$$

$$4 \le 12$$

$$4+6 \le 12$$

$$\mu(x) = -25 \quad \hat{c}(x) = -29$$

For node(3)

$$w_5+w_1+w_2+w_3+w_4 \le 12$$

$x_1=0 \Rightarrow w_1=0$ (we should not consider w_1 weight)

$$w_2+w_3+w_4 \le 12$$

$6 \le 12$

$6+3 \le 12$

$9 \le 12$

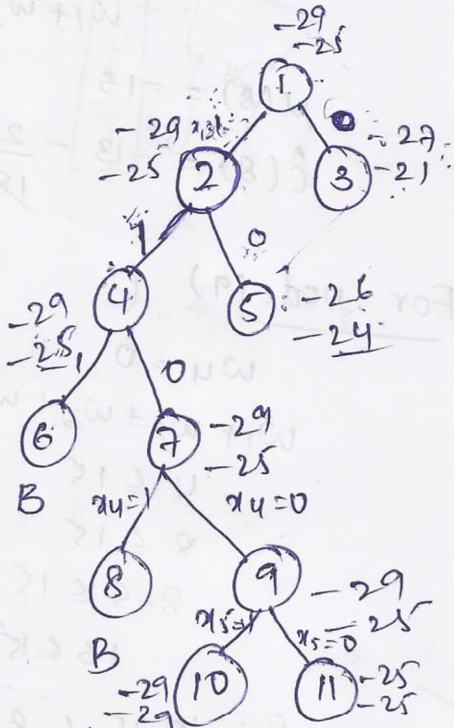
$9+4 \le 12$ ×

$w_2+w_3=12$

6 3

$\mu(x) = -(15+6) = -21$

$\hat{c}(x) = -21 - \frac{3}{\cancel{6}} \times \overset{2}{\cancel{12}} = -27$



For node(4)

$$w_5+w_1+w_2+w_3+w_4 \le 12$$

$w_1=1$ $w_2=1$ (we have to consider w_1 & w_2 wts)

$4 \le 12$ $\qquad w_1+w_2 \le 12$

$4+6 \le 12$

$10 \le 12$

$10+3 \le 12$ ×

$\mu(x) = -(10+15) = -25$

$\hat{c}(x) = -25 - \frac{2}{\cancel{9}} \times \overset{2}{\cancel{18}} = -29$

For node(5)

$$w_1+w_2+w_3+\overset{w_5}{\cancel{w_4}} \le 12$$

$4 \le 12$ $\qquad 11+2 \le 12$ ×

$4+3 \le 12$ $\qquad w_1+w_3+w_4 \le 15$

$7 \le 12$ $\qquad$ B $\mu(x) = -(10+6+8) = -24$

$7+4 \le 12$

$11 \le 12$ ✓ $\qquad \hat{c}(x) = -24 - \frac{1}{\cancel{2}} \times \overset{2}{\cancel{4}}$

$= -26$

For node(6)

$w_1 + w_2 + w_3 + w_4 + w_5 \le 12$

$4 \le 12$

$4+6 \le 12$

$10+3 \le 12$

$13 \le 12$ ✗

Kill the node. Because weight > m.

For node(7)

$w_1 + w_2 + w_3 + w_4 + w_5 \le 12$

$4+6 \le 12$

$10 \le 12$

$2\,\boxed{10}\,\}12$

$\mu(x) = -(25) = -25$

$\hat{c}(x) = -25 - \frac{2}{\cancel{6}} \times \cancel{12}^{2}$

$= -25-4 = -29$

For node(8)

$w_1 + w_2 + w_3 + w_4 + w_5 \le 12$

$4+6+\cancel{4} \le 12$

$10 \le 12$

$10+4 \le 12$ ✗ $(14 \le 12)$ ✗

Kill the node 8 weight > m.

For node(9)

$w_1 + w_2 + w_3 + w_4 + w_5 \le m$

$4+6 \le 12$ $\quad 12 \le 12$

$10 \le 12$

$\mu(x) = -25-4 = 29$

$\hat{c}(x) = -25 - \frac{2}{\cancel{2}} \times 4$

$= -25-4 = -29$

For node (10)

$$w_1 + w_2 + w_5 \le 12$$

$$4 + 6 + 2 \le 12$$

$$12 \le 12$$

$$\hat{\mu}(x) = -29$$

$$\hat{c}(x) = -29$$

For node (11)

$$w_1 + w_2 \le m$$

$$4 + 6 \le 12$$

$$\mu(x) = -25$$

$$\hat{c}(x) = -25 - \frac{2}{0}$$

$$\hat{c}(x) = -25$$

The solution is 11001

Path generated is $1 \to 2 \to 4 \to 7 \to 9 \to 10$

upperbond = −29

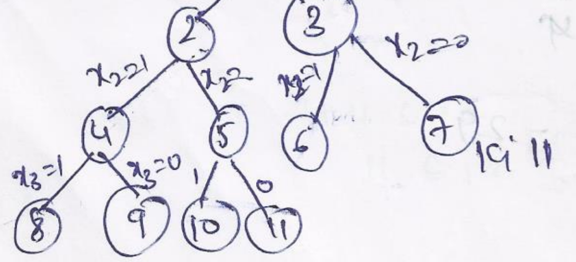
FIFOBB — 0/1 Knapsack Problem.

(1) $\hat{c}(x) \geq U(x)$ kill the node

BFS

$n = 4$, $m = 15$, $P = 10, 10, 12, 18$

$w = 2, 4, 6, 9$



$x_4 = 1$, $x_4 = 0$

For node(1)

$w_1+w_2+w_3+w_4 \le m$

$2 \le 15$

$2+4 \le 15$

$6 \le 15$

$6+6 \le 15$

$12 \le 15$

$12+9 \le 15$ ×

$w_1+w_2+w_3 \le m$

$12 \le 15$

$\mu(1) = -(10+10+12) = -32$

$\hat{c}(1) = -32 - \frac{3}{9} \times 18 = -38$

For node(2)

$w_1+w_2+w_3+w_4 \le m$

$w_2 = 0$

$2 \le 15$

$2+4 \le 15$

$6 \le 15$

$6+6 \le 15$

$12 \le 15$

$12+9 \le 15$ ×

$\mu(2) = -(10+10+12) = -32$

$\hat{c}(2) = -32 - \frac{3}{9} \times 18 = -38$

For node(3)

$w_1+w_2+w_3+w_4 \le m$

$4+6 \le 15$

$10 \le 15$

$\mu(3) = -(10+12) = -22$

$\hat{c}(3) = -22 - \frac{5}{9} \times 18 = -32$

For node(4)

$w_1+w_2+w_3+w_4 \le m$

$2 \le 15$

$2+4 \le 15$

$6 \le 15$

$6+6 \le 15$

$12 \le 15$

$12+9 \le 15$ ×

$\mu(4) = -32$

$\hat{c}(4) = -38.$

For node(5)

$w_1+w_2+w_3+w_4+w_5 \le m$

$\mu(5) = -22$

$\hat{c}(5) = -36$

For node(7)

$w_1+w_2+w_3+w_4 \le m$

$6 \le 15$

$6+9 \le 15$

$15 \le 15$

$\mu(7) = -30$

$\hat{c}(7) = -30 - \frac{0}{0}$

$\hat{c}(7) = -30$

$\hat{c}(x) = \mu(x)$ ∴ kill the node.

For node(6)

$w_1+w_2+w_3+w_4 \le m$

$4 \le 15$

$4+6 \le 15$

$10 \le 15$

$\mu(6) = -22$

$\hat{c}(6) = -22 - \frac{5}{9} \times 18 = -32$

For node(8)

$2+4+6 \le 15$

$12 \le 15$

$\mu(8) = -32$

$\hat{c}(8) = -38$

For node(9)

$2 \le 15$

$2+4 \le 15$

$6 \le 15$

$\mu(9) = -(10+10+18)$

$= -38$

$\hat{c}(9) = -38-0 = -38$

kill node

For node(11)

$2 \le 15$

$2+9 \le 15$

$11 \le 15$

$\mu(11) = -(+28) = -28$

$\hat{c}(11) = -28 - \underline{4}$

$\hat{c}(11) = -28$

For node(13)

010

$4+9 \le 15$

$13 \le 15$

$\mu(13) = -28$

$\hat{c}(13) = -28$

kill node

For node(16)

1011

$2+6 \le 15$

$8 \le 15$

$8+9 \le 15$ X

$\mu(16) = -22$

$\hat{c}(16) = -22-0 = -22$

Bound ~~kill node~~

For node(10)

$2+6 \le 15$

$8 \le 15$

$8+9 \le 15$ X

$\mu(10) = -(+22) = -22$

$\hat{c}(10) = -22 - \frac{7}{9} \times 18^{2}$

$= -22 - 14$

$= -36$

For node(12)

011

$4+6 \le 15$

$10 \le 15$

$\mu(12) = -22$

$\hat{c}(12) = -32$

For node(14)

$2+4+6+9 \le 15$ X

Bound.

For node 15

$12 \le 15$

$\mu(15) = -32$

$\hat{c}(15) = -32 - \frac{3}{\infty}$

$\hat{c}(15) = -32$

kill node.

For node(17)

1010

$2+6 \le 15$

$8 \le 15$

$\mu(17) = -22$

$\hat{c}(17) = -22$

kill node

For node (18)

0111

$4+6+9 \le 15$

$19 \le 15$

Bound.

For node (19)

0110

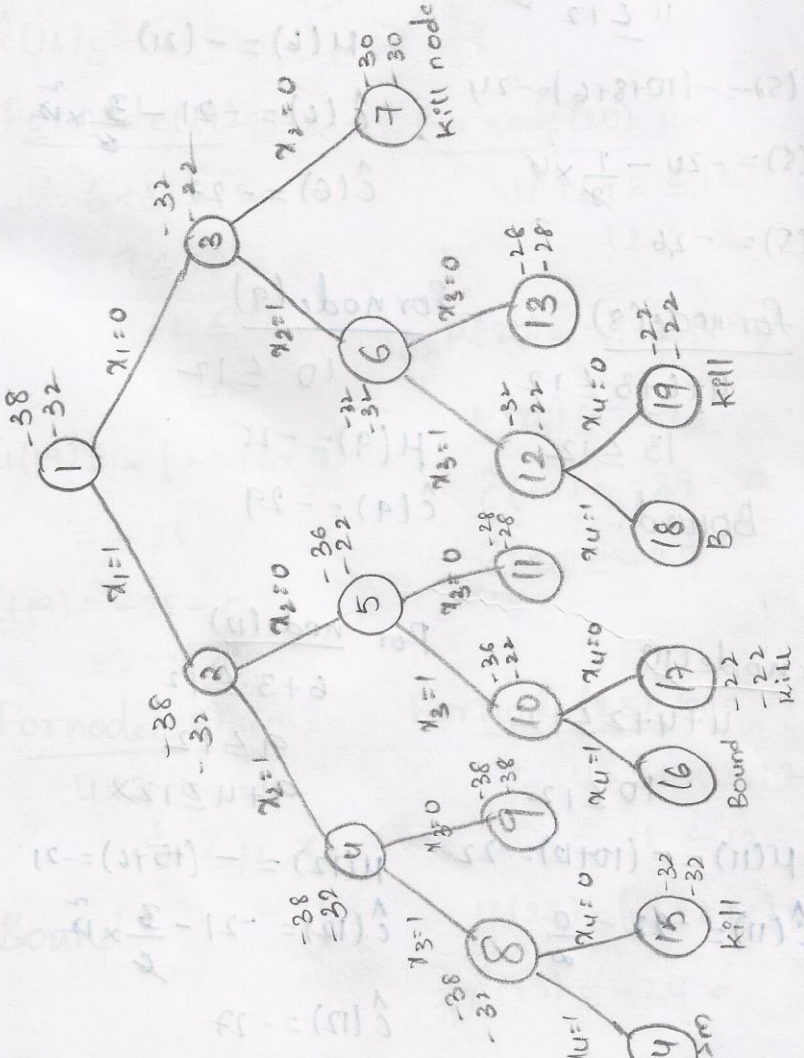
$4+6 \le 15$

$10 \le 15$

$\mu(19) = -22$

$\hat{c}(19) = -22$

Kill node.



2. $n=5$ $P = 10, 15, 6, 8, 4$ (1 2 3 4 5) $w = 4, 6, 3, 4, 2$ (1 2 3 4 5) $m = 12$

For node (1)

$4 \le 12$

$4+6 \le 12$

$4+6+3 \le 12$ ×

$\mu(1) = -25$

$\hat{c}(1) = -25 - \frac{2}{\cancel{9}} \times \cancel{18}^{2}$

$\hat{c}(1) = -29$

For node (2)

$4 \le 12$

$4+6 \le 12$

$10 \le 12$

$\mu(2) = -25$

$\hat{c}(2) = -25 - \frac{2}{\cancel{9}} \times \cancel{18}^{2}$

$\hat{c}(2) = -29$

For node (3)

$6 \le 12$

$9 \le 12$ ✓

$13 \le 12$ ×

$\mu(3) = -(15+6) = -21$

$\hat{c}(3) = -21 - \frac{3}{\cancel{9}} \times \cancel{18}^{2}$

$\hat{c}(3) = -27$.

For node (4)

$4+6 \le 12$

$10 \le 12$

$\mu(4) = -(10+15) = -25$

$\hat{c}(4) = -25 - \frac{2}{\cancel{9}} \times \cancel{18}^{2}$

$\hat{c}(4) = -29$

For node (5)

$4+3+4 \le 12$

$11 \le 12$ ✓

$\mu(5) = -(10+8+6) = -24$

$\hat{c}(5) = -24 - \frac{1}{\cancel{2}} \times \cancel{4}^{2}$

$\hat{c}(5) = -26$

For node (6)

$6+3 \le 12$

$9 \le 12$

$\mu(6) = -(21)$

$\hat{c}(6) = -21 - \frac{3}{\cancel{6}} \times \cancel{12}^{2}$

$\hat{c}(6) = -27$

For node (7)

$3+4+2 \le 12$

$9 \le 12$

$\mu(7) = -(6+8+4) = -18$

$\hat{c}(7) = -18$.

Kill the node.

For node (8)

$4+6+3 \le 12$

$13 \le 12$

Bound.

For node (9)

$10 \le 12$

$\mu(9) = -25$

$\hat{c}(9) = -29$

For node (10)

$4+3+4 \le 12$

$11 \le 12$

$\mu(10) = -(10+6+8) = -24$

$\hat{c}(10) = -24 - \frac{1}{\cancel{2}} \times \cancel{4}^{2}$

$\hat{c}(10) = -26$

For node (11)

$4+4+2 \le 12$

$10 \le 12$

$\mu(11) = -(10+12) = -22$

$\hat{c}(11) = -22 - \frac{0}{\infty}$

For node (12)

$6+3 \le 12$

$9 \le 12$

$9+4 \le 12$ ×

$\mu(12) = -(15+6) = -21$

$\hat{c}(12) = -21 - \frac{3}{\cancel{6}} \times \cancel{12}^{2}$

$\hat{c}(12) = -27$

For node(13) 010

$6+4+2 \le 12$

$12 \le 12$

$\mu(13) = -(15+8+4) = -27$

$\hat{c}(13) = -27 - 0$

$\hat{c}(13) = -27$

For node(14) 1101

$4+6+4 \le 12$ ×

Bound

For node(15) 1000

$4+6 \le 12$

$10 \le 12$

$\mu(15) = -25$

$\hat{c}(15) = -25 - 0 = -25$

For node(16) 1011

$4+3+4 \le 12$

$11 \le 12$

$\mu(16) = -(10+6+8) = -24$

$\hat{c}(16) = -24 - \frac{1}{2} \times 4^2$

$= -26$

$\hat{c}(16) = -26$

For node(17) 1010

$4+3 \le 12$

$7+\overset{2=9}{2} \le 12 \Rightarrow 9 \le 12$

$\mu(17) = -(10+6) = -20$

$\hat{c}(17) = -20 - 0$

$\hat{c}(17) = -20$

For node(18) 0111

$6+3+4 \le 12$

$13 \le 12$ ×

$\mu(18) = -($

$\hat{c}(18) =$

For node(19) 0110

$6+3 \le 12$

$9 \le 12$

$9+2 \le 12$

$11 \le 12$

$\mu(19) = -(15+6+4)$

$= -25$

$\hat{c}(19) = -25 - 0$

$= -25$

For node(20) 11001

$4+6+2 \le 12$

$12 \le 12$

$\mu(20) = -(10+15+4)$

$\mu(20) = -29$

$\hat{c}(20) = -29 - 0$

$\hat{c}(20) = -29$

For node(21) 11000

$4+6 \le 12$

$10 \le 12$

$\mu(21) = -25$

$\hat{c}(21) = -25 - 2$

$\hat{c}(21) = -25$

For node

For node(22) 10111

$4+3+4+2 \le 12$

$13 \le 12$ ×

Bound.

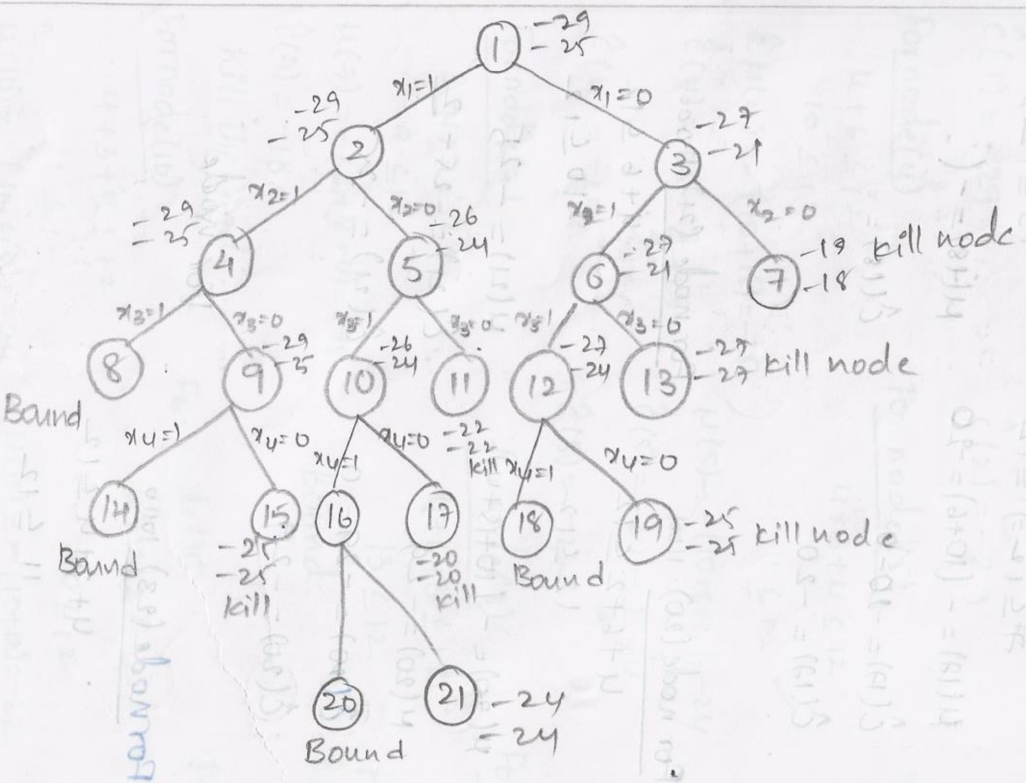
For node(23) 10110

$4+3+4 \le 12$

$11 \le 12$

$\mu(23) = -(10+6+8) = -24$

$\hat{c}(23) = -24$ 0



1 −29 −25
x1=1
x1=0
2 −29 −25
3 −27 −21
x2=1
x2=0
x2=1
x2=0
4 −29 −25
5 −26 −24
6 −27 −21
7 −19 −18 kill node
x3=1
x3=0
x3=1
x3=0
x3=1
x3=0
8
Bound
9 −29 −25
10 −26 −24
11
12 −27 −24
13 −27 −27 kill node
x4=1
x4=0
x4=1
x4=0 −22 −22 kill
x4=1
x4=0
14
Bound
15 −25 −25 kill
16
17 −20 −20 kill
18
Bound
19 −25 −25 kill node
20
Bound
21 −24 −24

3. $n=5$ $m=15$ $P=W=4,4,5,8,9.$

For node (1)

$4 \le 15$

$4+4+5 \le 15$

$13 \le 15$ ✓

$\mu(1) = -(4+4+5) = -13$

$\hat{c}(1) = -13 - \frac{2}{\cancel{17}} \times \cancel{17} = -15$

For node (2)

$4+4+5 \le 15$

$\mu(2) = -13$

$\hat{c}(2) = -15$

For node (3)

$4+5 \le 15$

$9 \le 15$

$8+9 \le 15$ ✗

$\mu(3) = -(4+5) = -9$

$\hat{c}(3) = -9 - \frac{6}{\cancel{17}} \times \cancel{17}$

$\hat{c}(3) = -15$

For node (4)

$4+4 \le 15$

$8 \le 15$

$8+5 \le 15$

$13 \le 15$

$\mu(4) = -(4+4+5) = -13$

$\hat{c}(4) = -13 - \frac{2}{\cancel{17}} \times \cancel{17}$

$\hat{c}(4) = -15$

For node (5)

$4+5 \le 15$

$9 \le 15$

$9+8 \le 15$ ✗

$\mu(5) = -(4+5) = -9$

$\hat{c}(5) = -9 - \frac{6}{\cancel{17}} \times \cancel{17}$

$\hat{c}(5) = -15$

For node (6)

$4+5 \le 15$

$9 \le 15$

$9+8 \le 15$ ✗

$\mu(6) = -(4+5) = 9$

$\hat{c}(6) = -9 - \frac{6}{\cancel{17}} \times \cancel{17}$

$\hat{c}(6) = -15$

For node (7)

$5+8 \le 15$

$13 \le 15$

$\mu(7) = -(5+8) = -13$

$\hat{c}(7) = -13 - \frac{2}{\cancel{9}} \times \cancel{9}$

$= -15$

For node (8) 111

$4+4+5 \le 15$

$13 \le 15$

$\mu(8) = -(4+4+5) = -13$

$\hat{c}(8) = -13 - \frac{2}{\cancel{17}} \times \cancel{17}$

$\hat{c}(8) = -15$

For node (9) (110)

$4+4 \le 15$

$8 \le 15$

$8+8 \le 15$ ✗

$\mu(9) = -(8)$

$\hat{c}(9) = -8 - \frac{7}{\cancel{17}} \times \cancel{17}$

$\hat{c}(9) = -15$

For node (10) (101)

$4+5 \le 15$

$9 \le 15$

$9+8 \le 15$ ✗

$\mu(10) = -(9)$

$\hat{c}(10) = -9 - \frac{6}{\cancel{17}} \times \cancel{17} = -15$

For node (11)

$4+8 \le 15$

$12 \le 15$

$\mu(11) = -12$

$\hat{c}(11) = -12 - \frac{3}{\cancel{9}} \times \cancel{9}$

$= -15$

For node (12) 011

$4+5 \le 15$

$9 \le 15$

$\mu(12) = -9$

$\hat{c}(12) = -9 - \frac{6}{\cancel{17}} \times \cancel{17}$

$\hat{c}(12) = -15$

```
Algorithm UBound(cp, cw, k, m)
{
    b:=cp, c:=cw;
    for i:=k+1 to n do
    {
        if (c+ w[i] ≤m) then
        {
            c: =c+w[i];
        } b: = b-p[i];
    }   return b;
}        }
```