## Unit - III

# Set theory, Relations and Functions:

## Set theory :-

**Set defination:** A collection of well defined objects is a set. The objects comprising the set are called elements. We use capital letters to represent sets and small letters to represents elements.

Ex(1): set of 74 students in a class

Ex(2): Set of 10 Natural numbers

**Set union:** Suppose A and B are two sets. The union of A and B is the set of all those elements which belongs to either A or B. we write $A \cup B$.

$$A \cup B = \{x / x \in A \text{ or } x \in B\}$$

**Set intersection:** Suppose A and B are two sets. Intersection of A and B is the set of all those elements which belongs to A and B. we write $A \cap B$

$$A \cap B = \{x / x \in A \text{ and } x \in B\}$$

**Note:** $n(A \cup B) = n(A) + n(B) - n(A \cap B)$ ($\because n(A)$ = no. of elements in A)

**Finite set:** A set contains a finite no. of elements.

Ex(1): $S = \{2, 4, 6, 8, 10\}$

**Infinite set:** A set contains infinite no. of elements (or) un countable no. of elements

Ex(2): $S = \{2, 4, 6, 8, \ldots\}$

**Singleton set:** A set containing a single element.

Ex: $S = \{2\}$

**Null Set:** A set consist of no elements is called a null set

$\phi = \{ \}$

**Set Representations (or) Set Notations:**

The set can be represented in two forms

1. Roster form/Tabular form: ex: set of odd numbers i.e $S = \{1, 3, 5, 7, \ldots\}$
2. Set-Builder form

ex: Set of odd numbers

$S = \{x / p(x)\}$

$= \{x / x \text{ is a odd number}\}$

**Cardinality of a Set:**

The cardinality of a set is the total no. of unique elements in a set.

Ex(1): $S = \{1, 2, 3, 4, 5\}$

$|S| = 4$

Ex(2): $S = \{1, \{2, 3\}, \{3, 4, 5\}\}$

$|S| = 3$

**Subset (or) Set inclusion:**

Suppose A and B are sets such that every element of A belongs to the set B, then A is called a Subset of B and we write $A \subseteq B$.

Ex: $A = \{1, 2, 3\}$, $B = \{1, 2, 3, 4, 5\}$

**Equality of Sets:** Two sets A and B if and only if the elements of A are same as the elements of B.

(OR) Two sets are equal if $A \subseteq B$ and $B \subseteq A$

Ex: $A = \{1, 2, 3\}$, $B = \{1, 2, 3\} \Rightarrow A = B$

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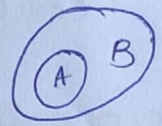
**Proper subset:** Set A is considered to be a proper subset of set B if set B contains at least one element that is not present in set A. (OR) $A \subset B$ if $A \subseteq B$ and $A \neq B$

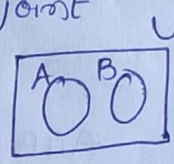
Ex: If $A = \{12, 24\}$ $B = \{12, 24, 36\}$

Then set A is the proper subset of B because 36 is not present in set A.




**Disjoint sets:** Two sets A and B are said to be disjoint if there is no common element for A and B.

we write $A \cap B = \phi$

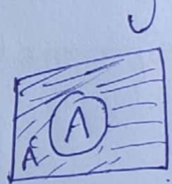



**Note.** Null set is a subset of every set ex: $\phi \subseteq A$

**Universal set** (U or E): Union of all sets

ex: $A = \{1, 2\}$ $B = \{3, 4\}$ $C = \{a, b\}$

$U = \{1, 2, 3, 4, a, b\}$




**Complement of a set:** The set of elements which do not belong to A and denote by $A^c$ (or) $A'$ (or) $\bar{A} = \{x / x \in U$ and $x \notin A\}$ (or) $\sim A$ (or) $\neg A$

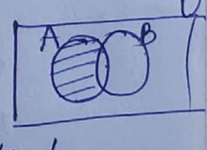
**Difference of two sets:** The difference of two sets A and B is A−B (The set of all elements, which belongs to A and does not belong to B)

ex: $A = \{1, 2, 3, 4, 5\}$, $B = \{3, 5, 6, 7\}$

$A - B = \{x / x \in A$ and $x \notin B\}$.

$A - B = \{1, 2, 3\}$

$B - A = \{6, 7\}$




**power set:** The set of all subsets of a set A is called the power set of A and is written as P(A).

ex: $A = \{a, b\}$

$P(A) = \{\phi, \{a\}, \{b\}, \{a, b\}\}$

here $\phi$ is the minimum subset

$\therefore P(A) = 2^4$

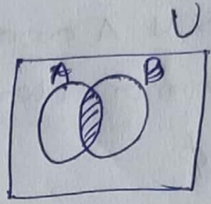
Ex(2): $A = \{1, 2, 3\}$

$P(A) = \{\phi, \{1\}, \{2\}, \{3\}, \{1,2\}, \{1,3\}, \{2,3\}, \{1,2,3\}\}$

$= 9$

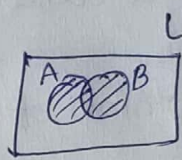
Note: If the set A contains 'n' elements then power set contains $2^n$ elements.

Set operations:



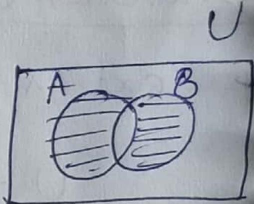

1. Intersection ($\cap$ (or) $\wedge$)

$A \cap B = \{x / x \in A \text{ and } x \in B\}$




2. Union ($\cup$ or $\vee$)

$A \cup B = \{x / x \in A \text{ or } x \in B\}$




3. symmetric difference: $A \Delta B = (A - B) \cup (B - A)$

$A = \{1, 2, 3, 4, 5\}$, $B = \{3, 4, 5\}$ $\therefore A - B = \{1, 2\}$

$A \Delta B = \{1, 2\}$ $\therefore B - A = \phi$

Cartesian product: collection of ordered pairs

$A \times B = \{(x, y) / (x \in A) \wedge (y \in B)\}$

$A = \{1, 2\}$ $B = \{a, b, c\}$

$A \times B = \{(1,a) (1,b), (1,c), (2,a), (2,b) (2,c)\}$

$B \times A = \{(a,1) (a,2), (b,1), (b,2), (c,1) (c,2)\}$

Note: $A \times B \neq B \times A$

① prove that $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$.

Sol Let $x \in A \cup (B \cap C)$

$\Rightarrow x \in A$ or $x \in (B \cap C)$

$\Rightarrow x \in A$ or $x \in B$ and $x \in C$

$\Rightarrow x \in A$ or $x \in B$ and $x \in A$ or $x \in C$

$\Rightarrow x \in (A \cup B)$ and $x \in (A \cup C)$

$\Rightarrow x \in (A \cup B) \cap (A \cup C)$

So $A \cup (B \cap C) \subseteq (A \cup B) \cap (A \cup C)$ — (1)

Let $x \in (A \cup B) \cap (A \cup C)$

$\Rightarrow x \in (A \cup B)$ and $x \in (A \cup C)$

$\Rightarrow x \in A$ or $x \in B$ and $x \in A$ or $x \in C$

$\Rightarrow x \in A$ or $x \in B$ and $x \in C$

$\Rightarrow x \in A$ or $x \in (B \cap C)$

$\Rightarrow x \in A \cup (B \cap C)$

So $(A \cup B) \cap (A \cup C) \subseteq A \cup (B \cap C)$ — (2)

From (1) and (2)

$\therefore A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$.

② prove that $A - B = A \cap B^c$

Sol Let $x \in A - B \Rightarrow x \in A$ and $x \notin B$

$\Rightarrow x \in A$ and $x \in B^c$

$\Rightarrow x \in (A \cap B^c)$

So $A - B \subseteq (A \cap B^c)$ — (1)

Let $x \in (A \cap B^c) = x \in A$ and $x \in B^c$

$= x \in A$ and $x \notin B$

$= x \in (A - B)$

So $(A \cap B^c) \subseteq A - B$ — (2)

From (1) and (2)

$\therefore A - B = A \cap B^c$.

③ $(A \cup B) \times C = (A \times C) \cup (B \times C)$

~~Proof~~: Let $x \in (A \cup B) \times C$

$\Rightarrow x = (a, b)$, where $a \in A \cup B$ and $b \in C$

$\Rightarrow x \in A$ or $B$ and $b \in C$

$\Rightarrow a \in A$ and $b \in C$ or $a \in B$ and $b \in C$

$\Rightarrow (a, b) \in (A \times C)$ or $(a, b) \in (B \times C)$

$\Rightarrow (a, b) \in (A \times C) \cup (B \times C)$

$\Rightarrow x \in (A \times C) \cup (B \times C)$

So $(A \cup B) \times C \subseteq (A \times C) \cup (B \times C)$ —— ①

Let $x \in (A \times C) \cup (B \times C) \Rightarrow x \in (A \times C)$ and/or $x \in (B \times C)$

$\Rightarrow x = (a, b)$, where $a \in A$ and $b \in C$ or $a \in B$ and $b \in C$

$\Rightarrow x \in (A \cup B)$ and $b \in C$

$\Rightarrow (a, b) \in (A \cup B) \times C$

$\Rightarrow x \in (A \cup B) \times C$

So $(A \times C) \cup (B \times C) \subseteq (A \cup B) \times C$ —— ②

From ① and ② $(A \cup B) \times C = (A \times C) \cup (B \times C)$

④ $(A \cap B) \times C = (A \times C) \cap (B \times C)$

Proof: Let $x \in (A \cap B) \times C$

$\Rightarrow x = (a, b)$, where $a \in A \cap B$ and $b \in C$

$\Rightarrow a \in A$ and $a \in B$ and $b \in C$

$\Rightarrow a \in A$ and $b \in C$ and $a \in B$ and $b \in C$

$\Rightarrow (a, b) \in (A \times C)$ and $(a, b) \in (B \times C)$

$\Rightarrow (a, b) \in (A \times C) \cap (B \times C)$

$\Rightarrow x \in (A \times C) \cap (B \times C)$

So $(A \cap B) \times C \subseteq (A \times C) \cap (B \cap C)$ —— ①

Let $x \in (A \times C) \cap (B \times C)$

$\Rightarrow x \in (A \times C)$ and $x \in (B \times C)$

$\Rightarrow x = (a,b)$ where $a \in A$ and $b \in C$ and $a \in B$ and $b \in C$

$\Rightarrow a \in (A \cap B)$ and $b \in C$

$\Rightarrow (a,b) \in (A \cap B) \times C$

$\Rightarrow x \in (A \cap B) \times C$

So $(A \times C) \cap (B \times C) \subseteq (A \cap B) \times C$ —— (2)

From (1) and (2)

$(A \cap B) \times C = (A \times C) \cap (B \times C)$.

(5) $A \times (B - C) = A \times B - A \times C$

Proof: Let $x \in A \times (B - C)$

$\Rightarrow x = (a,b)$, where $a \in A$ and $b \in (B - C)$

$\Rightarrow a \in A$ and $b \in B$ and $b \notin C$

$\Rightarrow (a,b) \in A \times B$ and $(a,b) \notin A \times C$

$\Rightarrow x \in (A \times B)$ and $(a,b) \notin A \times C$

$\Rightarrow x \in ((A \times B) - (A \times C))$

$\Rightarrow$

$A \times (B - C) \subseteq (A \times B) - (A \times C)$ —— (1)

Let $x \in ((A \times B) - (A \times C)) \Rightarrow$

$x \in A \times B$ and $x \notin A \times C$

$\Rightarrow x = (a,b)$ where $a \in A$ and $b \in B$ and $b \notin C$

$\Rightarrow a \in A$ and $b \in (B - C)$

$\Rightarrow (a,b) \in A \times (B - C)$

$\Rightarrow x \in A \times (B - C)$ —— (2)

From (1) & (2).

$(A \times B) - (A \times C) \subseteq A \times (B - C)$

(6) Let $U=R$, $A=\{x \in R; x>0\}$, $B=\{x \in R: x>1\}$ and $C=\{x \in R: x<2\}$

Find the following sets

(a) $A \cup B$

(b) $A \cup C$

(c) $B \cup C$

(d) $A \cap B$

(e) $A \cap C$

(f) $B \cap C$

(g) $A^c$

(h) $A-B$

Sol (a) $A=\{x \in R; x>0\}$

i.e $A$ = The set of real numbers which are greater than 0

$B=\{x \in R: x>1\}$

i.e $B$ = The set of real numbers which are greater than 1

So $B \subseteq A \Rightarrow A \cup B = A$

(b) $A=\{x \in R: x>0\}$, $C=\{x \in R: x<2\}$

$-\infty, \dots -1, 0, 1, 2, \dots \infty$

$C=(-\infty, 2)$, $A=(0, \infty)$

$A \cup C = R$ (The set of real numbers

Ily

(c) $B \cup C = R$

(d) $A \cap B = B$

(e) $A \cap C = (0, 2)$

(f) $B \cap C = (1, 2)$

(g) $A = (-\infty, 0)$

(h) $A-B = (0, 1)$

(7) If A = (1, 2, 3, 4, 5) and B = (2, 5, 6, 8, 10, 12, 14) then find

(a) $A \cup B$ (b) $A \cap B$ (c) $A - B$

Sol (a) $A \cup B = (1, 2, 3, 4, 5, 6, 8, 10, 12, 14)$

(b) $A \cap B = (2, 5)$

(c) $A - B = (1, 3, 4)$.

(8) If $A = \{1, 2, 3\}$ and $B = \{2, 3, 4\}$; how many elements will be there in $A \cup B$ and the power set of $A \cup B$? write all the subsets of $A \cup B$.

Sol $A \cup B = \{1, 2, 3, 4\}$

$\therefore$ no. of elements in $A \cup B$ i.e $n(A \cup B) = 4$

$P(A \cup B) = 2^4 = 16$

The subset of $A \cup B$ or the elements of $A \cup B$:

(i) $\phi$

(ii) $\{1\}$

(iii) $\{2\}$

(iv) $\{3\}$

iv) $\{4\}$

v) $\{1, 2\}$

vi) $\{1, 3\}$

viii) $\{1, 4\}$

ix) $\{2, 3\}$

x) $\{2, 4\}$

xi) $\{3, 4\}$

xii) $\{1, 2, 3\}$

xiii) $\{2, 3, 4\}$

xiv) $\{1, 3, 4\}$

xv) $\{1, 2, 4\}$

xvi) $\{1, 2, 3, 4\}$.

(9) Let $A, B, C \subseteq R^2$, where $A = \{(x,y)/y = 2x+1\}$, $B = \{(x,y)/y = 3x\}$ and $C = \{(x,y)/x-y=7\}$. Determine each of the following.

(i) $A \cap B$

ii) $B \cap C$

iii) $\bar{B} \cup \bar{C}$

iv) $\overline{\bar{A} \cup \bar{B}}$

$A = \{(x,y)/y = 2x+1\}$ $B = \{(x,y)/y = 3x\}$

Sol (i) $A \cap B = \{(x,y)/3x = 2x+1\}$ ~~x=1, when x=1~~ and y=3

$$3x = 2x+1$$

$$x = 1$$

then $y = 3$

$\therefore A \cap B = \{(1,3)\}$

ii) $B \cap C = \{(x,y)/x - 3x = 7\}$ $\because B = \{(x,y)/y = 3x\}$

$C = \{(x,y)/x-y=7\}$

$$x - 3x = 7$$

$$-2x = 7$$

$x = \frac{-7}{2}$ then $y = \frac{-21}{2}$

$\therefore B \cap C = \{(-\frac{7}{2}, -\frac{21}{2})\}$

iii) $\bar{B} \cup \bar{C} = \overline{B \cap C} = R^2 - \{(-\frac{7}{2}, -\frac{21}{2})\}$

iv) $\overline{\bar{A} \cup \bar{B}} = \overline{\overline{A \cap B}} = A \cap B$

$= \{(1,3)\}$

(10) If $A = \{1,2,3,4,5,6\}$ $B = \{2,3,4,5\}$ and $C = \{3,4,5,6,7\}$ then find (i) $A \cup (B \cap C)$ ii) $(A \cup B) \cap (A \cup C)$, and verify that (i) and (ii) are equal.

Sol (i) $B \cap C = \{3,4,5\}$

$A \cup (B \cap C) = \{1,2,3,4,5,6\}$

(ii) $A \cup B = \{1,2,3,4,5,6\}$

$A \cup C = \{1,2,3,4,5,6\}$

$(A \cup B) \cap (A \cup C) = \{1,2,3,4,5,6\}$ $\therefore$ (i) and (ii) are equal.

(11) If $A=\{1,3,5,6\}$, $B=\{3,2,5,8,9\}$ then find (i) $A\times B$ ii) $B\times A$

Sol The set A consists of four elements and B consists of five elements

$\therefore$ $A\times B$ consists of 20 elements.

$A\times B=\{(1,3),(1,2)(1,5),(1,8),(1,9),(3,3),(3,2),(3,5),(3,8),(3,9),$
$(5,3),(5,2),(5,5),(5,8),(5,9),(6,3),(6,2),(6,5),(6,8),(6,9)\}$

$B\times A=\{(3,1),(3,3),(3,5),(3,6),(2,3),(2,1),(2,5),(2,6),(5,1),(5,3),(5,5),$
$(5,6),(8,1),(8,3),(8,5),(8,6),(9,1),(9,3),(9,5),(9,6)\}$

(12) If $A=(a,b,c,d)$, $B=(1,3,5,2,7,8)$ and $C=(3,7,9,11,5,4)$

Find (i) $A\times B$ (iii) $(A\cup B)\times C$

(ii) $A\times(B\cap C)$ iv) $C\times A$

Sol (iii) $A\cup B=\{a,b,c,d,1,3,5,2,7,8\}$, $C=\{3,7,9,11,5,4\}$

$(A\cup B)\times C=\{(a,3),(a,7),(a,9),(a,11),(a,5),(a,4),(b,3),(b,7),(b,9),(b,11)$
$(b,5),(b,4),(c,3),(c,7),(c,9),(c,11),(c,5),(c,4),(d,3),(d,7),(d,9)$
$(d,11),(d,5),(d,4),(1,3)(1,7),(1,9),(1,11),(1,5),(1,4),(3,3),(3,7),$
$(3,9),(3,11),(3,5),(3,4)\ldots\ldots(8,4)\}$

(13) If $A=\{1,3,2\}$, $B=\{2,5,6\}$, $C=\{6,8\}$, find

(i) $A\times B\times C$ ii) $(A\cap B)\times C$ iii) $(A\cup B)\times C$ iv) $(A\times C)\cap(B\times C)$

v) $(A\times C)\cup(B\times C)$

Sol

(i) $A\times B=\{(1,2),(1,5),(1,6),(3,2),(3,5),(3,6),(2,2),(2,5),(2,6)\}$

$A\times B\times C=\{(1,2,6),(1,2,8),(1,5,6),(1,5,8),(1,6,6),(1,6,8),$
$(3,2,6),(3,2,8),(3,5,6),(3,5,8),(3,6,6),(3,6,8),$
$(2,2,6),(2,2,8),(2,5,6),(2,5,8),(2,6,6),(2,6,8)\}$

(14) If $A = (2,4,7)$, $B = (3,4,5)$, $C = (1,2,5)$, find
(i) $A \times (B - C)$ (ii) $(A \times B) - (A \times C)$ verify that (i) = (ii).

(15) If $(x+2, y-7) = (6,4)$ then find the values of $x$ and $y$.

Sol $x + 2 = 6$, $y - 7 = 4$
$x = 8$, $y = 11$

(16) If $(x+2y, 3x-y) = (0, 7)$ then find the values of $x$ and $y$.

Sol $x + 2y = 0$, $3x - y = 7$
$x = -2y$ $3(-2y) - y = 7$
$-7y = 7$
$y = -1$ then $x = +2$

# The laws of Set theory:

The operations on sets Satisfy certain laws. The following are a few of these laws wherein A, B, C are Subsets of a Universal set U.

→ Commutative Laws: (i) $A \cup B = B \cup A$
(ii) $A \cap B = B \cap A$

→ Associative laws: (i) $A \cup (B \cup C) = (A \cup B) \cup C$
(ii) $A \cap (B \cap C) = (A \cap B) \cap C$

→ Distributive Laws: (i) $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$
(ii) $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

→ Idempotent Laws: (i) $A \cup A = A$
(ii) $A \cap A = A$

→ Identity Laws: (i) $A \cup \phi = A$
(ii) $A \cap U = A$ U → Universal set

→ Law of Double complement: $\overline{\overline{A}} = A$

→ Inverse Laws: (i) $A \cup \overline{A} = U$
(ii) $A \cap \overline{A} = \phi$

→ DeMorgan Laws: (i) $\overline{A \cup B} = \overline{A} \cap \overline{B}$
(ii) $\overline{A \cap B} = \overline{A} \cup \overline{B}$

→ Domination Laws: (i) $A \cup U = U$
(ii) $A \cap \phi = \phi$

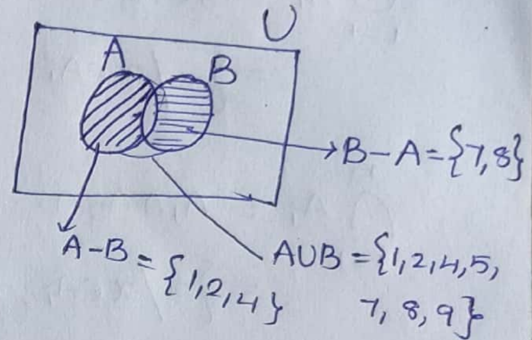
→ Absorption Laws: (i) $A \cup (A \cap B) = A$
(ii) $A \cap (A \cup B) = A$

1) Determine the sets A and B, given that
$A-B=\{1,2,4\}$, $B-A=\{7,8\}$ and $A\cup B=\{1,2,4,5,7,8,9\}$

Sol

$A=(A\cup B)-(B-A)$

$=\{1,2,4,5,9\}$




$B=(A\cup B)-(A-B)$

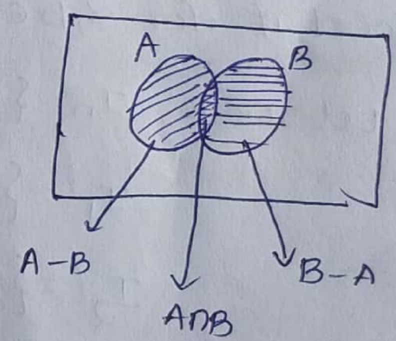
$B=\{5,7,8,9\}$

2) Determine the sets A and B, given that
$A-B=\{1,3,7,11\}$, $B-A=\{2,6,8\}$ and $A\cap B=\{4,9\}$

Sol

$A=(A-B)\cup(A\cap B)$

$=\{1,3,4,7,9,11\}$




$B=(B-A)\cup(A\cap B)$

$=\{2,4,6,8,9\}.$

**Index set:** Let us consider finite collection $n$ sets,

$A_1, A_2, \ldots, A_n$ then

$$\bigcup_{i=1}^{n} A_i = A_1 \cup A_2 \cup \cdots \cup A_n$$

$$= \{x / x \in A_i, \text{ for some } i \text{ such that } 1 \leq i \leq n\}$$

$$\bigcap_{i=1}^{n} A_i = A_1 \cap A_2 \cap \cdots \cap A_n$$

$$= \{x / x \in A_i, \text{ for all } i \text{ such that } 1 \leq i \leq n\}$$

A set $I$ is said to be index set for a family $A$ of sets, if for any $\alpha \in I$, $\exists$ a set $A_\alpha \in A$ and

$$A = \{A_\alpha / \alpha \in I\}$$

The sets $A_\alpha$ are said to be mutually or pairwise disjoint if for $\alpha, \beta \in I$, $\alpha \neq \beta$ implies $A_\alpha \cap A_\beta = \phi$.

ex: Let $n \in N$, $I_n = \{x \in R / -\frac{1}{n} < x < \frac{1}{n}\}$

$$I_1 = \{x \in R / -1 < x < 1\}$$

$$I_2 = \{x \in R / -\frac{1}{2} < x < \frac{1}{2}\}$$

$$\vdots$$

$$\bigcup_{n \in N} I_n = \{x \in R / -1 < x < 1\}$$

$$\bigcap_{n \in N} I_n = \{0\}.$$

## principle of Inclusion - Exclusion (Subtraction principle):

**Sum rule:** $A_i$, $i = 1, 2, 3, \dots n$ are the finite number of sets, then

$$\left| \bigcup_{i=1}^{n} A_i \right| = \sum_{i=1}^{n} |A_i| \text{, when } \boxed{A_i \cap A_j = \phi},\ i \neq j$$

for all i and j
disjoint sets

If $A_i \cap A_j \neq \phi \ \forall i, j$ then

$$\left| \bigcup_{i=1}^{n} A_i \right| = \sum_{i=1}^{} |A_i| - \sum_{\substack{i,j=1 \\ i \neq j}}^{n} |A_i \cap A_j| + \sum_{\substack{i,j,k=1 \\ i \neq j \neq k}}^{n} |A_i \cap A_j \cap A_k| - \dots + (-1)^{n} \left| \bigcap_{i=1}^{n} A_i \right|$$

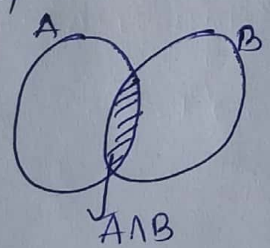
***

Note (i) For two sets

$$|A_1 \cup A_2| = |A_1| + |A_2| - |A_1 \cap A_2|$$

(OR)

$$|A \cup B| = |A| + |B| - |A \cap B|$$




∵ order, size or cardinality of A and is denoted by ~~O(A) = n(A)~~ ~~= |B|~~

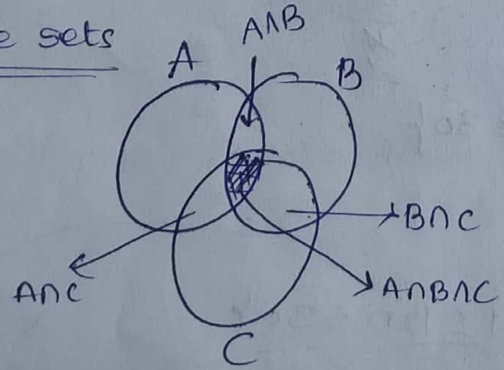
$O(A)$ (or) $|A|$

or $n(A)$

here $|A|$ is read as "order of A" (or) 'size of A' (or) 'cardinality of A'

***

ii) For three sets




$$|A \cup B \cup C| = |A| + |B| + |C| - |A \cap B| - |A \cap C| - |B \cap C| + |A \cap B \cap C|$$

Find $|A \cup B|$ when

Ex (1): $A$ = set of positive integers that are $\leq 30$ and multiple of 4.

$B$ = set of positive integers that are $\leq 30$ and multiple of 6.

Sol $A = \{4, 8, 12, 16, 20, 24, 28\}$

$B = \{6, 12, 18, 24, 30\}$ and $A \cap B = \{12, 24\}$

$|A \cap B| = 2$

$|A| = 7$ $|B| = 5$

$\therefore |A \cup B| = 7 + 5 - 2 = 10$

(OR) $A \cup B = \{4, 6, 8, 12, 16, 18, 20, 24, 28, 30\}$

$|A \cup B| = 10$

Another way:

$A = \{4n / n \in \mathbb{Z},\ 1 \leq 4n \leq 30\}$

$= \{4n / n \in \mathbb{Z},\ 1 \leq n \leq \frac{30}{4}\}$

$|A| = \left[\frac{30}{4}\right] = 7$ $\therefore \frac{30}{4} = 7.5$

$B = \{6n / n \in \mathbb{Z},\ 1 \leq 6n \leq 30\}$

$|B| = \left[\frac{30}{6}\right] = 5$

$\because$ lcm of 6, 4

$2 \,|\, 6, 4$
$\quad 3, 2$

$2 \times 3 \times 2 = 12$

$A \cap B = \{12n / n \in \mathbb{Z},\ 1 \leq 12n \leq 30\}$

$|A \cap B| = \left[\frac{30}{12}\right] = 2$

$\therefore |A \cup B| = 7 + 5 - 2$

$= 10$

③ Determine all the +ve integers less than 2102 and are divisible by atleast one of the primes 2, 3 and 5.

Sol

$A = \{2n : / n \in \mathbb{Z},\ 1 \le 2n \le 2102\}$

$|A| = \lfloor \frac{2102}{2} \rfloor = 1051$

$B = \{3n : / n \in \mathbb{Z},\ 1 \le 3n \le 2102\}$

$|B| = \lfloor \frac{2102}{3} \rfloor = 700$

$C = \{5n : / n \in \mathbb{Z},\ 1 \le 5n \le 2102\}$

$|C| = \lfloor \frac{2102}{5} \rfloor = 420$

$A \cap B = \{6n : / n \in \mathbb{Z}, 1 \le 6n \le 2102\}$

$|A \cap B| = 350$

$A \cap C = \{10n / n \in \mathbb{Z}, 1 \le 10n \le 2102\} \Rightarrow |A \cap B| = 210$

$B \cap C = \{15n / n \in \mathbb{Z}, 1 \le 15n \le 2102\} \Rightarrow |B \cap C| = 140$

$A \cap B \cap C = \{30n / n \in \mathbb{Z}, 1 \le 30n \le 2102\} \Rightarrow |A \cap B \cap C| = 70$

$|A \cup B \cup C| = 1051 + 700 + 420 - (350) - 210 - 140 + 70$

$= 1541$

(4) A computer company require 30 programmers to handle systems programming jobs and 40 programmers for applications programing. If the company appoints 55 programmers to carry out these jobs, how many of these perform jobs of both types? How many handle only system programming jobs? How many handle only applications programming?

Sol Let A denote the set of programmers who handle systems programing job then $|A| = 30$

Let B is the set of programmers who handle applications programming. then $|B| = 40$

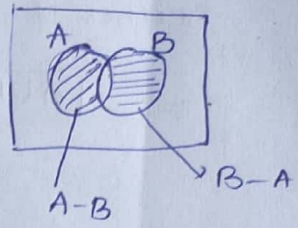
Then $A \cup B$ is the set of programmers appointed to carry out these jobs. $\Rightarrow |A \cup B| = 55$

By addition rule $|A \cup B| = |A| + |B| - |A \cap B|$

$55 = 30 + 40 - |A \cap B|$

$|A \cap B| = 15$. This means that 15 programmers perform both types of jobs.

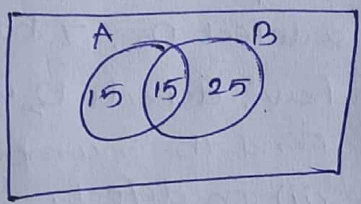
we note that the set of programmers who handle only systems programming is $A-B$




$$|A-B| = |A| - |A\cap B|$$
$$= 30-15$$
$$= 15$$

ly the number of programmers who handle only applications programming $|B-A| = |B| - |A\cap B|$
$$= 40-15$$
$$= 25$$

These results are illustrated in the following venn diagram.




(5) In class of 52 students, 30 are studying C++, 28 are studying pascal and 13 are studying both languages. How many in this class are studying at least one of these languages? How many are studying neither of these languages!

Sol Let A denote the set of students in the class who are studying C++ $\Rightarrow \boxed{|A| = 30}$

Let U denote the set of all students in the class $\Rightarrow \boxed{|U| = 52}$

and B denote the set of students in the class who are studying pascal $\Rightarrow \boxed{|B| = 28}$, $\boxed{|A\cap B| = 13}$

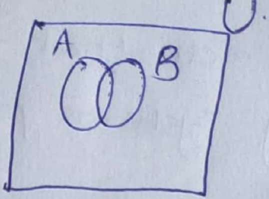
The set of students in the class who are studying both languages

The set of students who are studying atleast one of the two languages is $A\cup B$

$$\Rightarrow |A\cup B| = |A| + |B| - |A\cap B|$$
$$= 30+28-13$$
$$= 45$$

Scanned with CamScanner

The set of Students who are studying neither of these languages is $\overline{A \cup B}$



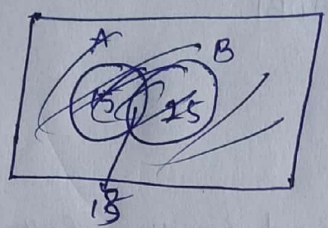

$\Rightarrow |\overline{A \cup B}| = |U| - |A \cup B|$

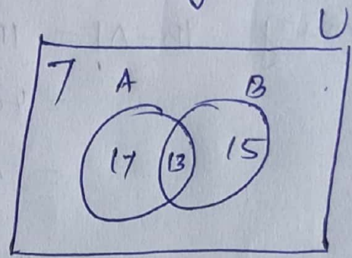
$= 52 - 45$

$= 7$

$|U| = 52$
$|A| = 30$
$|B| = 28$
$|A \cap B| = 13$

The results are illustrated in the following venn diagram.




6) In a sample of 100 Logic chips, 23 have a defect $D_1$, 26 have a defect $D_2$, 30 have a defect $D_3$, 7 have defects $D_1$ and $D_2$, 8 have defects $D_1$ and $D_3$, 10 have defects $D_2$ and $D_3$ and 3 have all the three defects. Find the numbers of chips having (i) at least one defect, (ii) no defect.

Sol Let U denote the set of all chips $\Rightarrow |U| = 100$

$|D_1| = 23$ $\quad |D_1 \cap D_2| = 7$

$|D_2| = 26$ $\quad |D_1 \cap D_3| = 8$

$|D_3| = 30$ $\quad |D_2 \cap D_3| = 10$

$|D_1 \cap D_2 \cap D_3| = 3$

(i) At least one defect

$$|D_1 \cup D_2 \cup D_3| = |D_1| + |D_2| + |D_3| - |D_1 \cap D_2| - |D_2 \cap D_3| - |D_1 \cap D_3| + |D_1 \cap D_2 \cap D_3|$$

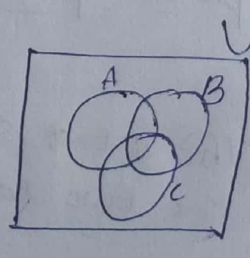
$= 23 + 26 + 30 - 7 - 10 - 8 + 3$

$= 57$

(ii) $|\overline{A \cup B \cup C}| = |U| - |A \cup B \cup C|$

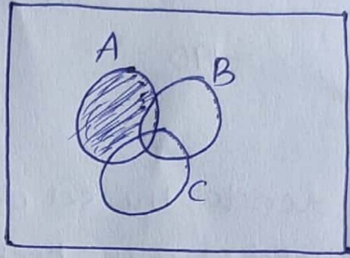
$= 100 - 57$

$= 43$



(7) If A, B, C are finite sets, prove that
$|A-B-C| = |A| - |A\cap B| - |A\cap C| + |A\cap B\cap C|$

proof: We note that $A-B-C$ is the set of elements that belong to A but not B or C




$\therefore |A-B-C| = |A\cup B\cup C| - |B\cup C|$

$= |A| + \cancel{|B|} + \cancel{|C|} - |A\cap B| - |A\cap C|$
$- \cancel{|B\cap C|} + |A\cap B\cap C|$
$- (\cancel{|B|} + \cancel{|C|} - \cancel{|B\cap C|})$

$= |A| - |A\cap B| - |A\cap C| + |A\cap B\cap C|$

(8) A survey of 500 television viewers of a sports channel produced the following information: 285 watch cricket, 195 watch hockey, 115 watch football, 45 watch cricket and football, 70 watch cricket and hockey, 50 watch hockey and football and 50 do not watch any of the three kinds of games.

(a) How many viewers in the survey watch all three kinds of games

(b) How many viewers watch exactly one of them.

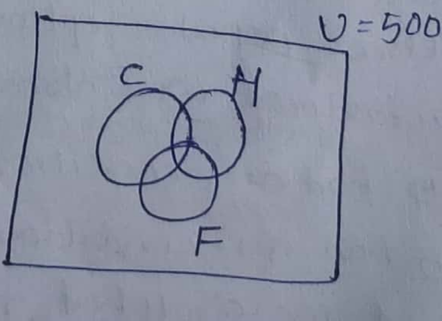
Sol Let U denotes the set of all viewers included in the survey.

$\therefore |U| = 500$

$|C| = 285, \quad |H| = 195, \quad |F| = 115.$

$|C\cap F| = 45, \quad |C\cap H| = 70, \quad |H\cap F| = 50 \qquad |\overline{C\cup H\cup F}| = 50$

$|C\cup H\cup F| = 500 - 50$
$= 450$




(a)
$|C\cup H\cup F| = |C| + |H| + |F| - |C\cap H| - |C\cap F| - |H\cap F|$
$+ |C\cap H\cap F|$

$450 = 285 + 195 + 115 - 45 - 70 - 50 + |C\cap H\cap F|$

$\therefore |C\cap H\cap F| = 20.$

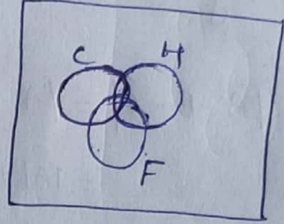
(b)

Let $C_1$ denote the set of viewers who watched only cricket

$$|C_1| = |C| - |C\cap H| - |C\cap F| + |C\cap H\cap F|$$

$$= 285 - 70 - 45 + 20$$

$$= 190$$




Let $H_1$ denote the set of viewers who watched only hockey.

$$|H_1| = |H| - |H\cap F| - |H\cap C| + |C\cap H\cap F|$$

$$= 195 - 50 - 70 + 20$$

$$= 95.$$

and the no. of viewers who watch only football is

$$|F_1| = |F| - |F\cap H| - |F\cap C| + |C\cap H\cap F|$$

$$= 115 - 50 - 45 + 20$$

$$= 40$$

From these, we find that the number of viewers who watch exactly one of the sports is

$$|C_1| + |H_1| + |F_1| = 190 + 95 + 40$$

$$= 325.$$

(9) A survey of a sample of 25 new cars being sold by an auto-dealer was conducted to see which of the three popular options: air-conditioning, radio and power windows, were already installed. The survey found:

15 had air-conditioning, 12 had radio, 11 had power windows, 5 had air-conditioning and power windows, 9 had air-conditioning and radio, 4 had radio and power windows, and 3 had all three options. Find the number of cars that had.

(i) only one power windows

ii) only air-conditioning

iii) only radio

iv) only one of the options

v) at least one option

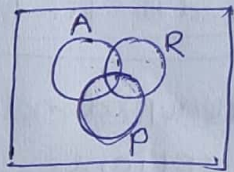
vi) none of the options.

Sol Let $U$ be the set of all cars in the sample $\Rightarrow |U| = 25$

given $|A| = 15$ $\quad |R| = 12$ $\quad |P| = 11$

$|A \cap P| = 5$ $\quad |A \cap R| = 9$ $\quad |R \cap P| = 4$

and $|A \cap R \cap P| = 3$




(i) Only power windows

$|P_1| = |P - A - R|$

$= |P| - |P \cap R| - |P \cap A| + |P \cap A \cap R|$

$P - |P \cap R| - |P \cap A| + |A \cap P \cap R|$

$= 11 - 4 - 5 + 3$

$= 5$

ii) $|A_1| = |A - R - P|$

$= |A| - |A \cap R| - |A \cap P| + |A \cap R \cap P|$

$= 15 - 9 - 5 + 3$

$= 4$

iii) $|R_1| = |R - A - P|$

$= |R| - |R \cap P| - |R \cap P| + |R \cap A \cap P|$

$= 12 - 9 - 4 + 3$

$= 2$

iv) The no. of cars that had only one of the options

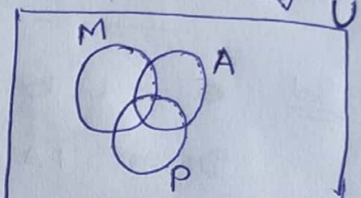
$= |P_1| + |A_1| + |R_1| = 5 + 4 + 2 = 11$

v) $|P \cup A \cup R| = |P| + |A| + |R| - |P \cap A| - |P \cap R| - |A \cap R| + |A \cap P \cap R|$

$= 11 + 15 + 12 - 5 - 4 - 9 + 3 = 23$

vi) $|U| - |P \cup A \cup R| = 25 - 23 = 2$

Q10. 30 cars are assembled in factory. The options available are a music system, an air conditioner and power windows. It is known that 15 of the cars have music systems, 8 have air conditioners and 6 have power windows. Further, 3 have all options. Determine at least how many cars do not have any option at all.




Sol  Let $|U| = 30$

$|M| = 15, \ |A| = 8, \ |P| = 6, \ |M \cap A \cap P| = 3$

need to find $|\overline{M \cup A \cup P}| = ?$

→ $M \cup A \cup P$ denotes the set of cars that have at least one of the options.

→ Then $\overline{M \cup A \cup P}$ is the set of cars that do not have any option.

By addition rule,

$$|M \cup A \cup P| = |M| + |A| + |P| - |M \cap A| - |M \cap P| - |A \cap P| + |M \cap A \cap P| \quad \text{—— (1)}$$

since $M \cap A \cap P$ is a subset of $M \cap A$, $A \cap P$ and $M \cap P$

i.e $|M \cap A \cap P| \leq |M \cap A|$

$|M \cap A \cap P| \leq |A \cap P|$

$|M \cap A \cap P| \leq |M \cap P|$

From (1)

$$|M \cup A \cup P| \geq |M| + |A| + |P| - |M \cap A \cap P| - |M \cap A \cap P| - |M \cap A \cap P| + |M \cap A \cap P|$$

$$= |M| + |A| + |P| - 2|M \cap A \cap P|$$

$$= 15 + 8 + 6 - 2(3)$$

$$= 23$$

$$|\overline{M \cup A \cup P}| = |U| - |M \cup A \cup P|$$

$$\geq |U| - 23$$

$$= 30 - 23$$

$= 7$  This shows that at least 7 cars do not have any of the options.

(11) A student visits a sports club everyday from Monday to Friday after school hours and plays one of the three games: Cricket, Tennis, Football. In how many ways can he play each of the three games at least once during a week (from Monday to Friday)

Sol On each day, the student has three choices of games

Total no. of choices of games in a 5-day period $= 3^5$

$\therefore \boxed{|U| = 3^5}$

Let C denotes the set of all choices of games which excludes cricket.

No. of choices of games in a 5-day period which excludes cricket is $\boxed{|C| = 2^5}$

No. of choices of games " " " Tennis is $\boxed{|T| = 2^5}$

No. of choices of games " " " Football is $\boxed{|F| = 2^5}$

No. of choices of games " " " Cricket and Tennis $\boxed{|C \cap T| = 1^5}$

No. of choices of games " " " cricket and Football $\boxed{|C \cap F| = 1^5}$

No. of choices of games " " " Tennies and Football $\boxed{|T \cap F| = 1^5}$ and $|C \cap T \cap F| = 0$

$$|C \cup T \cup F| = |C| + |T| + |F| - |C \cap T| - |C \cap F| - |T \cap F| + |C \cap T \cap F|$$

no. of choices games which excludes atleast one of the three games in the 5-day

$$= 2^5 + 2^5 + 2^5 - 1^5 - 1^5 - 1^5$$

$$= 93$$

The no. of choices of games in the 5-day period which does not exclude any game is

$|\overline{C \cup T \cup F}| = |U| - |C \cup T \cup F|$

(or) $= 3^5 - 93$

$|\bar{C} \cap \bar{T} \cap \bar{F}| = 150$

$\therefore$ There are 150 ways for the student to select his daily games so that he plays everygame atleast one during a week (from monday to Friday).

(12) Consider a set of integers from 1 to 250 Find out

a) How many numbers are divisible by 3 or 5

b) " " " 5 or 7

c) " " " 3 or 5 but not by 7

d) " " " 3 or 5 or 7

e) " " not divisible by 3, 5, 7. ie 3 or 5 or 7

Sol Let A = Integers divisible by 3

B = " 5

C = " 7

$n(A)$ = no. of integers divisible by 3 $= \left\lfloor \frac{250}{3} \right\rfloor = 83$

$n(B) =$ " 5 $= \left\lfloor \frac{250}{5} \right\rfloor = 50$

$n(C) =$ " 7 $= \left\lfloor \frac{250}{7} \right\rfloor = 35$

$n(A \cap B) =$ " " 3 and 5 $= \left\lfloor \frac{250}{3 \times 5} \right\rfloor = 16$

$n(B \cap C) = \left\lfloor \frac{250}{5 \times 7} \right\rfloor = 7$

lcm of 5, 7

$n(A \cap C) = \left\lfloor \frac{250}{3 \times 7} \right\rfloor = 11$

$n(A \cap B \cap C) = \left\lfloor \frac{250}{3 \times 5 \times 7} \right\rfloor = 2$

$n(A \cup B)$ = no. of integers divisible A or B

$= n(A) + n(B) - n(A \cap B)$

(a) $n(A \cup B) = 83 + 50 - 16$

(b) $n(B \cup C) = n(B) + n(C) - n(B \cap C)$

$= 50 + 35 - 7$

(c) $n(A \cup B \cup C) - n(C)$

$= n(A) + n(B) + n(C) - n(A \cap B) - n(A \cap C) - n(B \cap C)$

$+ n(A \cap B \cap C) - n(C)$

(d) $n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(A \cap C) - n(B \cap C)$

$+ n(A \cap B \cap C)$

(e) $n(\overline{A \cup B \cup C}) = 250 - n(A \cup B \cup C)$

$= 250 - (\ )$

(12) Find the number of integers from 1 to 1000 which are divisible by at least 2, 3, or 5

Sol

$n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(B \cap C) -$

$n(A \cap C) + n(A \cap B \cap C)$ —— (1)

$n(A) = \left\lfloor \frac{1000}{2} \right\rfloor = 500$ $\quad n(A \cap B) = \left\lfloor \frac{1000}{2 \times 3} \right\rfloor =$

$n(B) = \left\lfloor \frac{1000}{3} \right\rfloor = 333$ $\quad n(B \cap C) = \left\lfloor \frac{1000}{3 \times 5} \right\rfloor =$

$n(C) = \left\lfloor \frac{1000}{5} \right\rfloor = 200$ $\quad n(A \cap C) = \left\lfloor \frac{1000}{2 \times 5} \right\rfloor =$

$n(A \cap B \cap C) = \left\lfloor \frac{1000}{2 \times 3 \times 5} \right\rfloor =$

above values subs in (1) we get

(14) How many integers between 50 and 100, inclusive are divisible by 2 or 3 ?

Sol $100-50 = 50+1 = 51$

$n(A) = \lfloor \frac{51}{2} \rfloor = 25$

$n(B) = \lfloor \frac{51}{3} \rfloor = 17$

$n(A \cap B) = \lfloor \frac{\cancel{51}^{\cancel{17}^{8}}}{\cancel{2 \times 3}} \rfloor = 8$

$n(A \cup B) = n(A) + n(B) - n(A \cap B)$

$= 25 + 17 - 8$

$= 34$

(15) How many integers between 1 and 300 (inclusive) are (i) divisible by at least one of 5, 6, 8 ?

ii) divisible by none of 5, 6, 8 ?

Sol $U = \{1, 2, 3, ..., 300\}$

$|U| = 300$

$n(A) = \lfloor \frac{300}{5} \rfloor = 60$

$n(B) = \lfloor \frac{300}{6} \rfloor = 50$

$n(C) = \lfloor \frac{300}{8} \rfloor = 37$

$n(A \cap B) = \lfloor \frac{300}{\text{lcm}(5,6)} \rfloor = \lfloor \frac{300}{5 \times 6} \rfloor = 10$

$n(A \cap C) = \lfloor \frac{300}{\text{lcm}(5,8)} \rfloor = \lfloor \frac{300}{5 \times 8} \rfloor = 7$

$n(B \cap C) = \lfloor \frac{300}{\text{lcm}(6,8)} \rfloor = \lfloor \frac{300}{24} \rfloor = 12$

$n(A \cap B \cap C) = \lfloor \frac{300}{\text{lcm}(5,6,8)} \rfloor = \lfloor \frac{300}{120} \rfloor = 2$

$\frac{17}{\frac{3}{1}2}$

$\therefore 2 \underline{| 6, 8}$
$3, 4$
$2 \times 3 \times 4 = 24$

$\therefore 2 \underline{| 5, 6, 8}$
$5, 3, 4$
$2 \times 5 \times 3 \times 4 =$

(i) $|A\cup B\cup C| = |A|+|B|+|C|-|A\cap B|-|B\cap C|-|A\cap C|+|A\cap B\cap C|$

$= 60+50+37-10-7-12+2$

$= \underline{\underline{120}}$

Thus, 120 elements of U are divisible by atleast one of 5, 6, 8.

(ii) The no. of elements of U that are divisible by none of 5, 6, 8 is

$|\bar{A}\cap\bar{B}\cap\bar{C}| = |U| - |A\cup B\cup C|$

(or)

$|\overline{A\cup B\cup C}|$

$= 300 - 120$

$= \underline{\underline{180}}$

(16) Let X be the set of all three-digit integers; that is $X = \{x \text{ is an integer} / 100 \le x \le 999\}$

If $A_i$ is the set of numbers in X whose $i^{th}$ digit is i, compute the cardinality of the set $A_1 \cup A_2 \cup A_3$.

Sol $A_1 = \{100, 102, 103, 104, \dots, 199\}$, so $|A_1| = 100$

$A_2 = \{120, 121, 122, 123, \dots, 129, 220, 221, 222, \dots, 229, 320, 321, \dots, 329, 420, 421, 422, \dots, 429, \dots, 920, 921, \dots 929\}$

$\therefore \boxed{|A_2| = 90}$

$A_3 = \{103, 113, 123, 133, 143, 153, 163, 173, 183, 193, 203, 213 \dots 293, 303, 313, \dots, 393, \dots, 903, 913, 923, \dots 993\}$

$\therefore \boxed{|A_3| = 90}$

~~$|A_1 \cap A_2| = \{120, 121, 122, \dots 120$~~

$A_1 \cap A_2 = \{120, 121, 122, 123, \dots, 129\} \Rightarrow |A_1 \cap A_2| = 10$

$A_2 \cap A_3 = \{123, 223, 323, \dots, 923\} \Rightarrow |A_2 \cap A_3| = 9$

$A_1 \cap A_3 = \{103, 113, 123, \dots, 193\} \Rightarrow |A_1 \cap A_3| = 10$

$A_1 \cap A_2 \cap A_3 = \{123\} \Rightarrow |A_1 \cap A_2 \cap A_3| = 1$

$\therefore |A_1 \cup A_2 \cup A_3| = |A_1| + |A_2| + |A_3| - |A_1 \cap A_2| - |A_2 \cap A_3| - |A_1 \cap A_3| + |A_1 \cap A_2 \cap A_3|$

$= 100 + 90 + 90 - 10 - 10 - 9 + 1$

$= 252.$