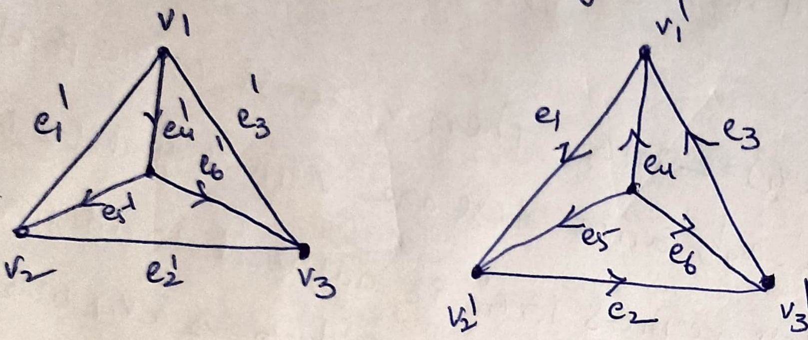
Isomorphic Digraphs:- Two digraphs are said to be isomorphic if (a) their corresponding undirected graphs are isomorphic
(b) Directions of the corresponding edges are also agree.

The following two digraphs are not isomorphic because the directions of the two corresponding edges e_4 and e_4' do not agree (although their corresponding undirected graphs are isomorphic)

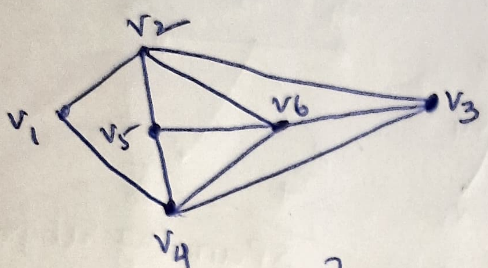


Subgraphs:- Let $G = (V(G), E(G))$ be a graph with vertex set $V(G)$ and edge set $E(G)$ then a graph H $(V(H), E(H))$ is said to be a sub graph of graph G if

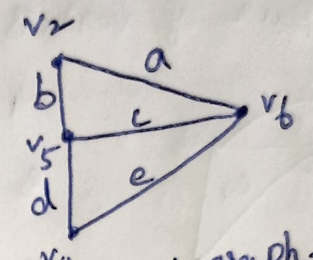
i) All the vertices of H are in G i.e., $V(H) \subseteq V(G)$

ii) All the edges of H are in G i.e. $E(H) \subseteq E(G)$

iii) Each edge of H has the same end vertices in H as in G.



[Graph-G]



(sub graph-H)

NOTE:- ① Every graph is a subgraph of itself

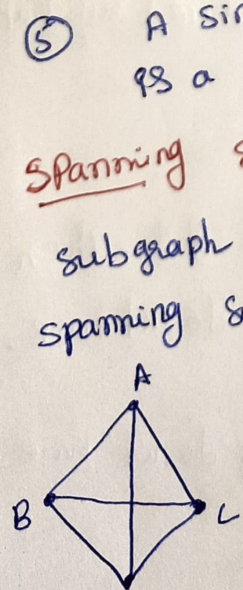
② Every simple graph of n vertices is a subgraph of the complete graph K_n.

③ If G_1 is a subgraph of a graph G_2 and G_2 is a sub graph of a graph G, then G_1 is a subgraph of G.

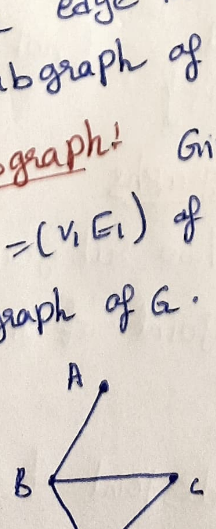
④ A single vertex in a graph G is a subgraph of G.

⑤ A single edge in a graph G, together with its end vertices, is a subgraph of G.

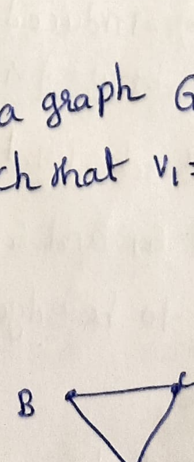
Spanning subgraph: Given a graph $G = (V, E)$ if there is a subgraph $G_1 = (V_1, E_1)$ of G such that $V_1 = V$, then G_1 is called a spanning subgraph of G.



G



G_1

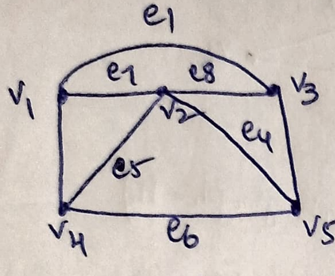


G_2

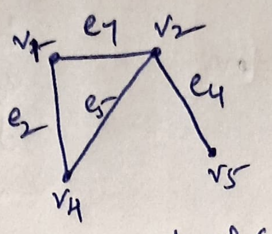
The graph G_1 is a spanning subgraph where as the Graph G_2 is a subgraph but not a spanning subgraph.

Induced subgraph: If W is any subset of vertexset of G, then the subgraph generated or induced by W is the subgraph H of G obtained by taking $V(H) = W$ and $E(H)$ to be those edges of G that joins pair of vertices in W.

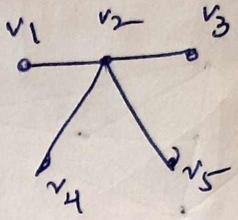
eg



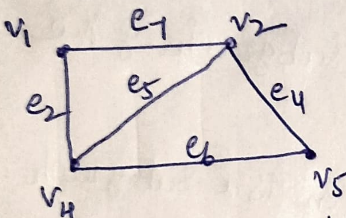
Graph G



subgraph of G

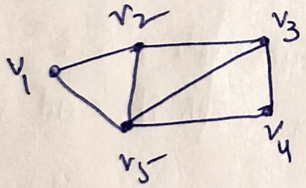


Spanning subgraph of G

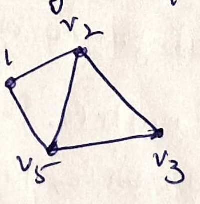


subgraph induced by $W = \{v_1, v_2, v_4, v_5\}$

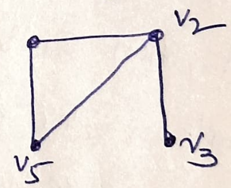
2



Graph



induced subgraph by $W = \{v_1, v_2, v_3, v_5\}$



(not induced subgraph)

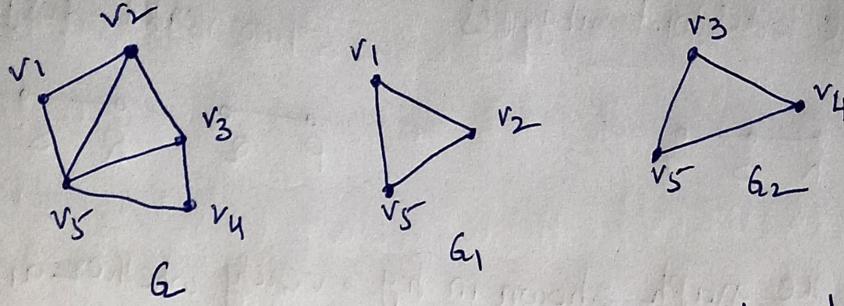
Edge disjoint and vertex-disjoint Subgraphs

Let G be a graph and G_1 and G_2 be two subgraphs of G. Then

(1) G_1 and G_2 are said to be edge-disjoint if they do not have any common edge.

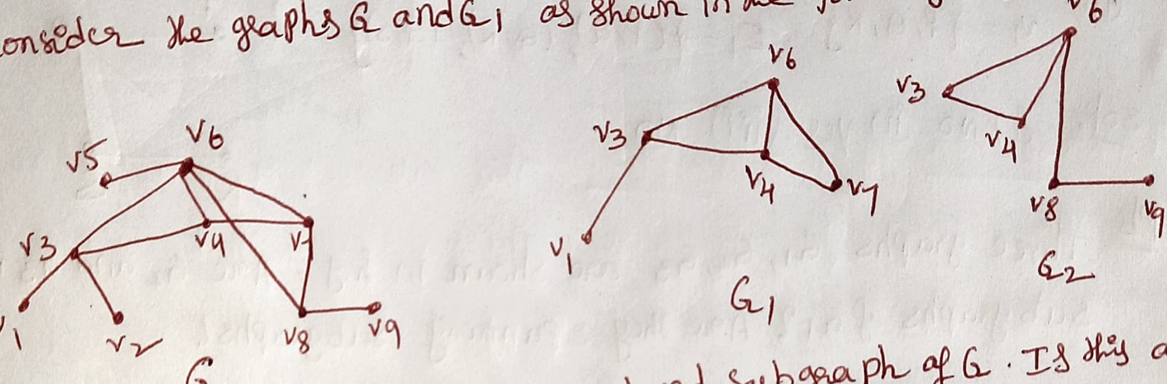
(2) G_1 and G_2 are said to be vertex-disjoint if they do not have any common edge and any common vertex.

Ex's



The graphs G_1 and G_2 are edge-disjoint but not vertex-disjoint subgraphs.

Pb consider the graphs G and G_1 as shown in the following figure



Ⓐ Verify that the graph G_1 is an induced subgraph of G. Is this a spanning subgraph of G?

(b) Draw the subgraph G_2 of G induced by the set $V_2 = \{v_3, v_4, v_6, v_8, v_9\}$

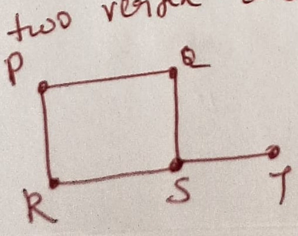
(a) The vertex set of the graph G_1, namely $V_1 = \{v_1, v_3, v_4, v_6, v_7\}$ is a subset of the vertex set $V = \{v_1, v_2, v_3, \dots, v_9\}$ of G. Also, all the edges of G_1 are in G. Further, each edge in G_1 has the same end vertices in G as in G_1. Therefore, G_1 is a subgraph of G.

We further check that every edge $\{v_i, v_j\}$ of G where $v_i, v_j \in V_1$ is an edge of G_1. Therefore, G_1 is an induced subgraph of G.

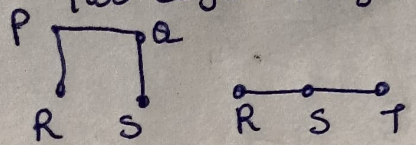
Since $V_1 \neq V$, G_1 is not a spanning subgraph of G.

(b) $V_2 = \{v_3, v_4, v_6, v_8, v_9\}$ is an induced subgraph of G.

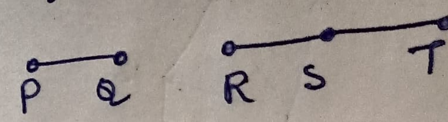
Pb For the graph shown below, find two edge-disjoint subgraphs and two vertex-disjoint subgraphs.



Two edge-disjoint subgraphs



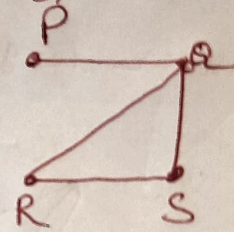
Two vertex-disjoint subgraphs



Pb Let G be the graph shown in fig. verify whether $G_1 = (V_1, E_1)$ is a subgraph of G in the following cases:

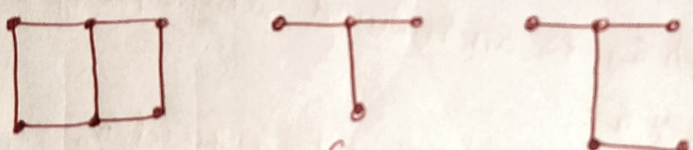
(i) $V_1 = \{P, Q, S\}$ $E_1 = \{\{P,Q\}, \{P,S\}\}$ (ii) $V_1 = \{Q\}$, $E_1 = \phi$, the nullset

(iii) $V_1 = \{P, Q, R\}$ $E_1 = \{(P,Q), (Q,R), (Q,S)\}$



Sol: i) NO (ii) yes (iii) NO

Pb Three graphs G_1, G_2, G_3 are shown in fig. Are G_2 and G_3 induced subgraphs of G_1? Are they spanning subgraphs?



G_1 G_2 G_3

G_2 is an induced subgraph of G_1; it is not a spanning subgraph

G_3 is not an induced subgraph of G_1; it is a spanning subgraph

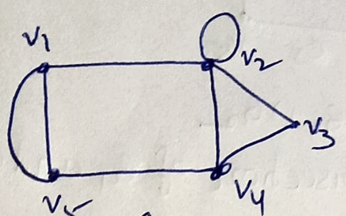
Pb can a finite graph be isomorphic to one of its subgraphs? (other than itself)? NO.

Operations on Graphs:-

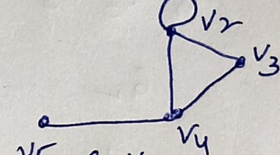
Deleting a vertex:- Let G be a graph with $V(G) = \{v_1, v_2, v_3, \ldots, v_n\}$ then $G - v_i$ is the graph obtained deleting or removing the vertex v_i from G together with all edges incident on v_i.

More generally, we write $G - \{v_1, v_2, \ldots, v_k\}$ for the graph obtained by deleting the vertices $v_1, v_2 \ldots v_k$ and all edges incident on any of them.

Ex:-



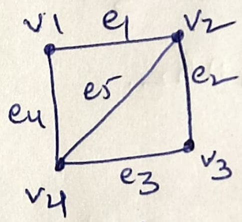
G



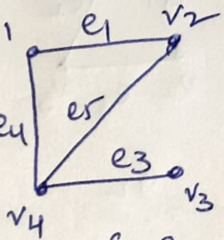
$G - v_1$

Deleting an edge:- Let G be a graph with $E(G) = \{e_1, e_2, \ldots, e_n\}$ then $G - e_i$ is the graph obtained by removing or deleting the edge e_i without deleting the vertices which are incident with e_i.

Ex:-



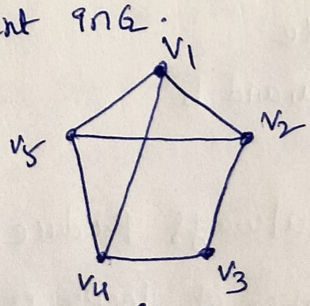
G



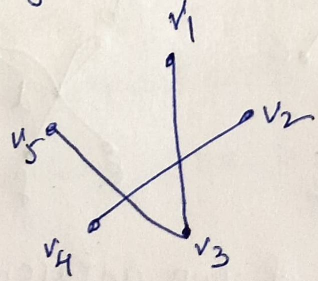
$G - e_2$

Complement of a graph:- The complement of a graph G is the graph $\overline{G}$ with the same vertices as G. An edge exists in $\overline{G} \Leftrightarrow$ it does not exist in G. In other words, two vertices adjacent in $\overline{G} \Leftrightarrow$ they are not adjacent in G.

Ex:-



G

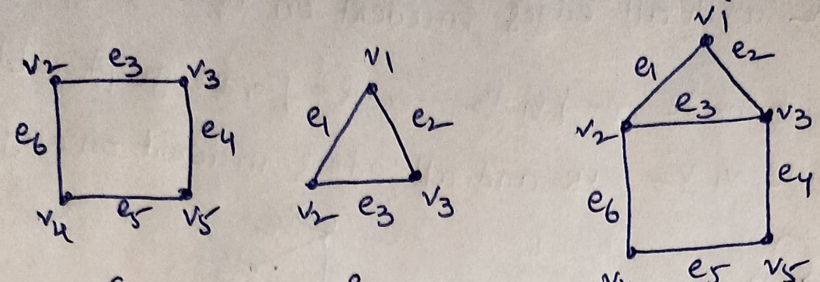


$\overline{G}$

A graph and its complement.

Union of the two graphs:- Let G and G' be two graphs. The union of G and G' is the graph with the vertex set $V(G) \cup V(G')$ and edge set $E(G) \cup E(G')$. Hence $G \cup G' = (V \cup V', E \cup E')$

Ex:-

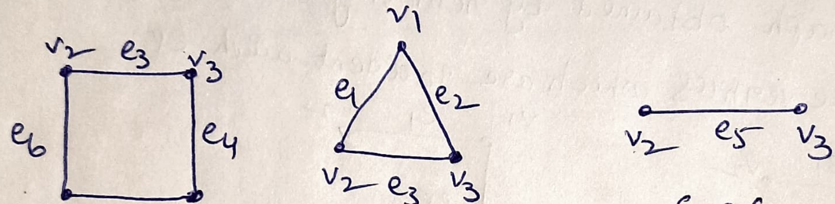


G_1 $\quad$ G_2 $\quad$ $G_1 \cup G_2$

Intersection of two graphs:- The intersection of G_1 and G_2 is the graph consisting only those vertices and edges that are both G_1 & G_2.

$$\Rightarrow G_1 \cap G_2 = (V_1 \cap V_2, E_1 \cap E_2)$$

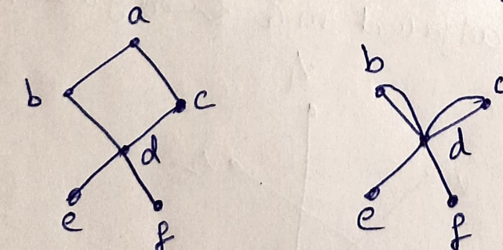
Ex:



G_1 $\quad$ G_2 $\quad$ $G_1 \cap G_2$.

Fusion:- Fusion of two vertices a and b in a graph G is an operation G on which two vertices a and b are fused (merged) together without deletion of any edge of G.

Ex:-



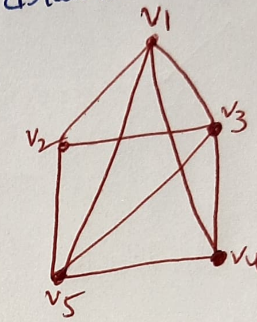
Fusion of the vertices a and b

Note:- Fusion of two adjacent vertices always produce a loop at the point of fusion and the number of loops is equal to the number of edges between the vertices which are fused together.

Planarity: A graph G is said to be planar if it can be drawn in the plane without its edges crossing. otherwise G is nonplanar.

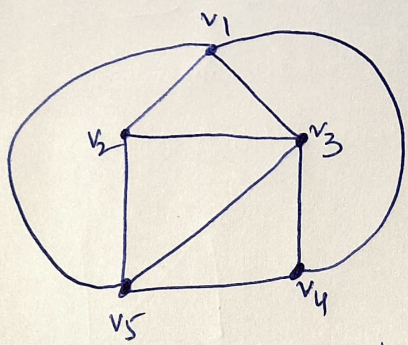
NOTE: A graph may be planar even if it is usually drawn with edge crossings, since it may be possible to draw it in a different way without any edge crossings. we say that a planar graph is a plane graph if it is already drawn in the plane without edge crossings.

Ex: check whether the graph



is planar (or) not.

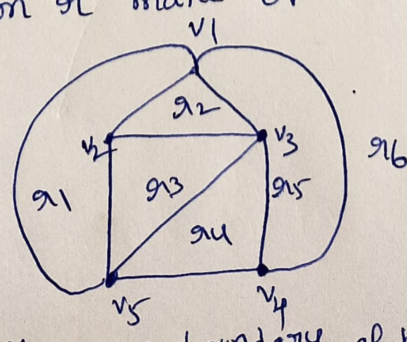
Sol:



There is no edge crossings, so the given graph is a planar graph.

NOTE: ① A plane graph divides the plane into regions. A region is characterized by the cycle that forms its boundary. These regions are connected portions of the plane.

② In each plane, plane graph G determines a region of infinite area called the exterior region of G. In the above example r_6 is the exterior region. The vertices and edges of G incident with a region r make up boundary of the region r.



$v_1-v_2-v_5-v_1$ is the boundary of the region r_1. The degree of the region r_1 is the length of its boundary. $\deg(r_1)=3$