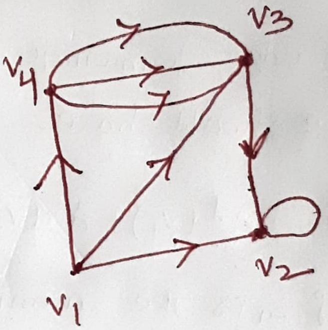
Total degree In a derected graph, the sum of the out degree and the indegree of v is called its total degree.

i.e., total degree of v = (indegree + outdegree) of v.

In case of an underected graph, the total degree or the degree of a node v is equal to the number of edges incident with v. The total degree of an isolated vertex is 0.

Pb Find the indegree, out degree and total degree of each vertex of the graph



vertex	Indegree	out deg	total deg
v_1	0	3	3
v_2	3	1	4
v_3	3+1	1	5
v_4	1	3	4

Th: Show that the degree of a vertex of a simple graph G on n vertices cannot exceed $n-1$.

Proof: Let v be a vertex of G, Since G is simple, no multiple edges or loops are allowed in G. thus v can be adjacent to atmost all the remaining $n-1$ vertices of G.

Hence, v may have maximum degree $n-1$ in G.

then $0 \leq \deg(v) \leq n-1$ for all $v \in V(G)$.

Isomorphic Graph (Isomorphism of Two Graphs)

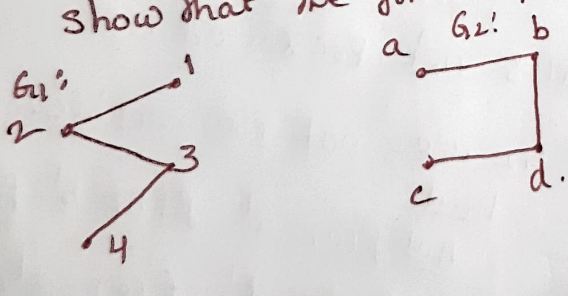
Two graphs $G_1 = (V_1, E_1)$ and $G_2 = (V_2, E_2)$ are said to be isomorphic if there exists a functions $f: V_1 \to V_2$ such that

(i) f is one-to-one and onto i.e., f is bijective.

(ii) $\{a, b\}$ is an edge in E_1, iff $\{f(a), f(b)\}$ is an edge in E_2 for any two elements $a, b \in V_1$. Here the function f is called an isomorphism between G_1 and G_2 and we say that G_1 and G_2 are isomorphic graphs.

In other words, two graphs G_1 and G_2 are said to be isomorphic (to each other) if there is a one-to-one correspondance between their vertices and between their edges such that the adjacency of vertices is preserved (means that if (U, V) are adjacent vertices in G_1, then the corresponding vertices (U_1, V_1) are also adjacent in G_2

Pb1 Show that the given pair of graphs are isomorphic



Sol: Here $V(G_1) = \{1, 2, 3, 4\}$ $V(G_2) = \{a, b, c, d\}$

$E(G_1) = \{\{1,2\} \{2,3\} \{3,4\}\}$ $E(G_2) = \{\{a,b\} \{b,d\} \{c,d\}\}$

$\therefore |V(G_1)| = |V(G_2)|$ & $|E(G_1)| = |E(G_2)|$

The vertices of degree 1 in G_1 are $\{1, 4\}$ and in G_2 are $\{a, c\}$

" " 2 " " " $\{2, 3\}$ " " " " $\{b, d\}$

Define a function $f: V(G_1) \to V(G_2)$ as $f(1) = a$ $f(2) = b$

$f(3) = d$, $f(4) = c$

f is clearly one-one & onto.

Further, $\{1,2\} \in E(G_1)$ and $\{f(1), f(2)\} = \{a, b\} \in E(G_2)$

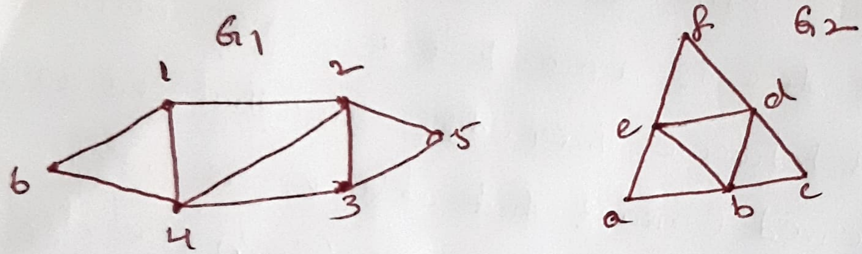
$\{2,3\} \in E(G_1)$ and $\{f(2), f(3)\} = \{b, d\} \in E(G_2)$

$\{3,4\} \in E(G_1)$ and $\{f(3), f(4)\} = \{d, c\} \in E(G_2)$.

Hence f preserves adjacency of the vertices.

$\therefore$ G_1 is isomorphic to G_2. i.e., $G_1 \cong G_2$.

Pb2 check whether the given two graphs G_1 and G_2 are isomorphic or not? Give reasons



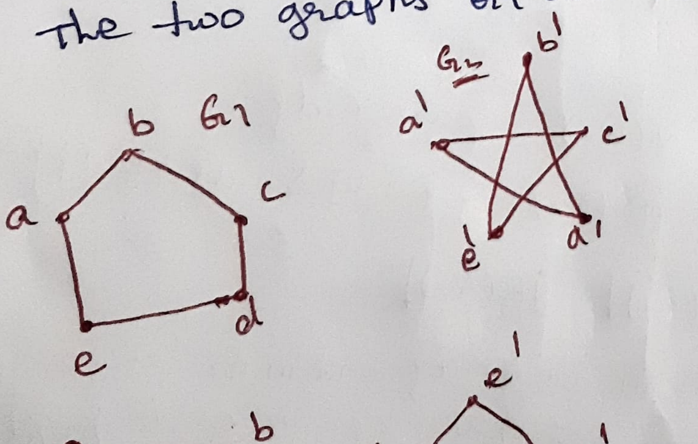
Sol: we observe that $|V(G_1)| = |V(G_2)|$ & $|E(G_1)| = |E(G_2)|$

But G_1 has 2 vertices of degree 4 whereas G_2 has 3 vertices of degree 4.

the adjacency of vertices not preserved.

$\Rightarrow$ The two graphs G_1 and G_2 are not isomorphic

Pb3

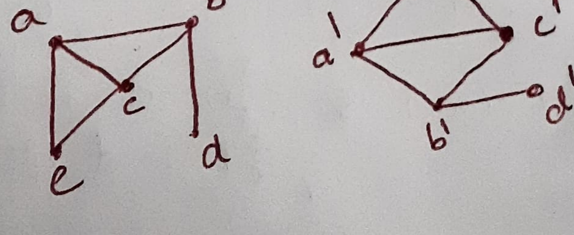


Hint

Ans $f(a) = a'$ $f(b) = b'$ $f(c) = c'$ $f(d) = d'$ $f(e) = e'$.

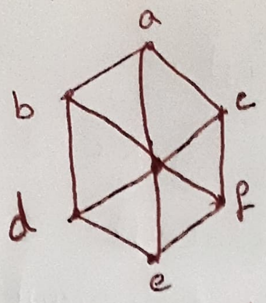
Hence f preserves adjacency of the vertices $G_1 \cong G_2$

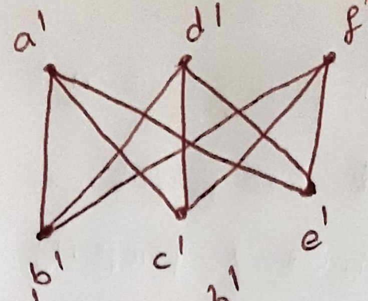
Pb4



Ans Isomorphic.

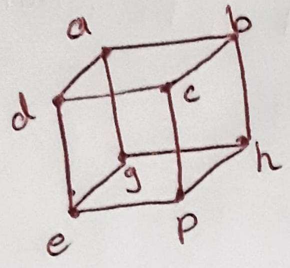
Pb5

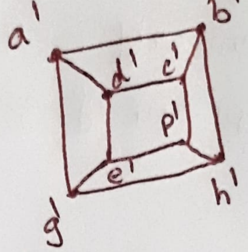




Ans Isomorphic

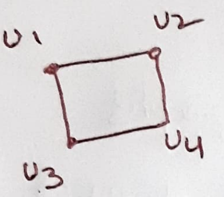
Pb6

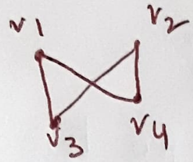




Ans Isomorphic.

Pb7





Ans Isomorphic

Determining when graphs are not isomorphic:

we can Prove that two graphs are not isomorphic by showing that they do not share a property that isomorphic graphs must have, such a property is called an invariant. with respect to the isomorphism of graphs.

The invariants are

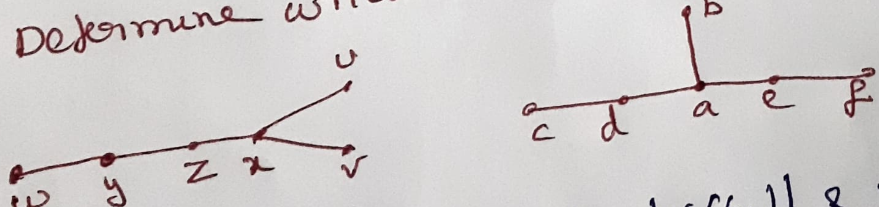
1. The number of vertices
2. the number of edges and
3. The degree sequences of the two graphs

If any of these quantities differ in two graphs, those graphs cannot be isomorphic.

when these invariants are the same it doesn't mean that the two graphs are isomorphic. Apart from these invariants we need a one-to-one and onto function which preserves adjacency of the vertices in simple graph and preserves the direction of edges in digraphs.

Pb Determine whether the following graphs are isomorphic

(a)

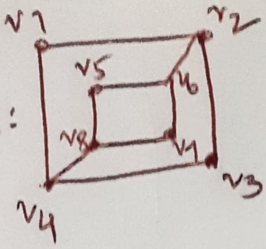


Sol: we observe that $|V(G_1)| = |V(G_2)|$ & $|E(G_1)| = |E(G_2)|$
& $\deg(a) = 3$ in G_2.

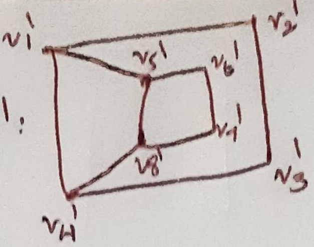
$\deg(x)$ in $G_1 = 3$

G_1 and G_2 are not isomorphic, because the vertex x is adjacent to two pendent vertices, whereas vertex a is adjacent to only one pendent vertex.

(b) G:



G':



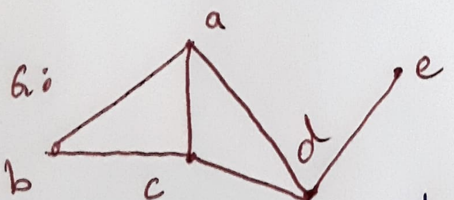
sol:- The graphs G and G' both have 8 vertices and 10 edges.

They both have 4 vertices each of degree 3 and 4 vertices each of degree 2.

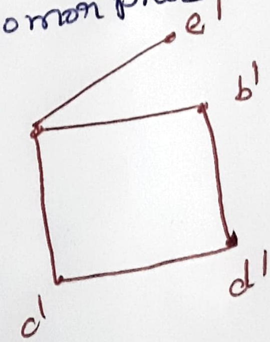
Now consider $\deg(v_1) = 2$ in G. Then v_1 must correspond to either v_2', v_3', v_6' v_7', since these are vertices of deg 2 in G'. However, each of these vertices in G' is adjacent to another vertex of deg 2 in G' but v_1 is adjoint to v_2 & v_4 in G which are of degree 3. Thus the preservation of adjacency of the vertices is not maintained.

$\therefore$ G and G' are not isomorphic.

(c) G:



G':



$|E(G)| \neq |E(G')|$