CS 4445 assignment 2
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θ_a $P(Z = a)$
θ_b $P(Z = b)$
$\theta_{h|a}$ $P(h|a)$ i.e head prob coin a
$\theta_{t|a}$ $P(t|a)$
$\theta_{h|b}$ $P(h|b)$
$\theta_{t|b}$ $P(t|b)$
$\#(d,h)$ num of heads in trial d
$\#(d,t)$ tails

d	Z	tosses	
1	?	H	H
2	?	T	T

x^d = particular trial

$$P(Z = a, x^d) = P(Z = a) * P(h|a)^{\#(d,h)} * P(t|a)^{\#(d,t)}$$
$$= \theta_a * \theta_{h|a}^{\#(d,h)} * \theta_{t|a}^{\#(d,t)}$$

$$P(Z = b, x^d) = P(Z = b) * P(h|b)^{\#(d,h)} * P(t|b)^{\#(d,t)}$$
$$= \theta_b * \theta_{h|b}^{\#(d,h)} * \theta_{t|b}^{\#(d,t)}$$

conditional probabilities for hidden variables

$$P(Z = a \mid x^d) = \frac{P(Z = a, x^d)}{\sum_c P(Z = c, x^d)}$$

$$P(Z = b \mid x^d) = \frac{P(Z = b, x^d)}{\sum_c P(Z = c, x^d)}$$

joint probability formulae
j = index of data item

$P(z=a, x^1) = \theta_a * \theta^2_{h|a}$
$P(z=b, x^1) = \theta_b * \theta^2_{h|b}$
$P(z=a, x^2) = \theta_a * \theta^2_{t|a}$
$P(z=b, x^2) = \theta_b * \theta^2_{t|b}$

$$\gamma_1(a) = \frac{\theta_a * \theta^2_{h|a}}{\theta_a * \theta^2_{h|a} + \theta_b * \theta^2_{h|b}}$$

$$\gamma_1(b) = \frac{\theta_b * \theta^2_{h|b}}{\theta_a * \theta_{h|a} + \theta_b * \theta^2_{h|b}}$$

$$\gamma_2(a) = \frac{\theta_a * \theta^2_{t|a}}{\theta_a * \theta_{t|a} + \theta_b * \theta^2_{t|b}}$$

$$\gamma_2(b) = \frac{\theta_b * \theta^2_{t|b}}{\theta_a * \theta^2_{t|a} + \theta_b * \theta^2_{t|b}}$$

to carry out EM estimation of params - initial settings:

$\theta_a = \frac{1}{2}$ $\quad$ $\theta_b = \frac{1}{2}$
$\theta_{h|a} = \frac{3}{4}$ $\quad$ $\theta_{t|a} = \frac{1}{4}$
$\theta_{h|b} = \frac{1}{2}$ $\quad$ $\theta_{t|b} = \frac{1}{2}$

Iteration 1:

$\partial = 1: p(Z = A, HH) = 0.5 * 0.75 * 0.75$
$= 0.28125$

$\partial = 1: p(Z = B, HH) = 0.5 * 0.5 * 0.5 = 0.125$

$\partial = 1: \rightarrow sum = 0.40625$

$\partial = 1: \rightarrow Y_1(A) = 0.692308$

$\partial = 1: \rightarrow Y_1(B) = 0.307692$

$\partial = 2: p(Z = A, TT) = 0.5 * 0.25 * 0.25$
$= 0.03125$

$\partial = 2: p(Z = B, TT) = 0.5 * 0.5 * 0.5$
$= 0.125$

$\partial = 2: \rightarrow sum = 0.15625$

$\partial = 2: \rightarrow Y_2(A) = 0.2$

$\partial = 2: \rightarrow Y_2(B) = 0.8$

$E(A) = Y_1(a) + Y_2(a) = 0.692308$
$+ 0.2 = 0.892308$

$E(B) = Y_1(b) + Y_2(b) = 0.307692$
$+ 0.8 = 1.10769$

$E(A, H) = \Sigma_\partial Y_\partial(a) \# (\partial, h) = 0.692308 * 2$
$+ 0.2 * 0 = 1.38462$

$E(A, T) = \Sigma_\partial Y_\partial(a) \# (\partial, t) = 0.692308 * 0$
$+ 0.2 * 2 = 0.4$

$E(B, H) = \Sigma_\partial Y_\partial(b) \# (\partial, h) = 0.307692 * 2$
$+ 0.8 * 0 = 0.615385$

$E(B, T) = \Sigma_\partial Y_\partial(b) \# (\partial, t) = 0.307692 * 0$
$+ 0.8 * 2 = 1.6$

From these expected counts we re-estimate parameters

$est(\theta_a) = E(A)/2 = 0.892308/2$
$= 0.446154$

$est(\theta_b) = E(B)/2 = 1.10769/2$
$= 0.553846$

$est(\theta_{h|a}) = E(A,H)/\left[\Sigma_x E(A,x)\right]$
$= 1.38462/(1.38462 + 0.4) = 0.775862$

$est(\theta_{t|a}) = E(A,T)/\left[\Sigma_x E(A,x)\right]$
$= 0.4/(1.38462 + 0.4) = 0.224138$

$est(\theta_{h|b}) = E(B,H)/\left[\Sigma_x E(B,x)\right]$
$= 0.615385/(0.615385 + 1.6) = 0.27778$

$est(\theta_{t|b}) = E(B,T)/\left[\Sigma_x E(B,x)\right]$
$= 1.6/(0.615385 + 1.6) = 0.722222$

Iteration 2:

$\partial = 1: p(Z=A, HH) = 0.446154 * 0.775862 * 0.775862 = 0.2685676841$

$\partial = 1: p(Z=B, HH) = 0.553846 * 0.27778 * 0.27778 = 0.04273571463$

$\partial = 1: \rightarrow sum = 0.311303987$

$$\partial = 1: \rightarrow y_1(A) = \frac{0.2685676841}{0.2685676841 + 0.0427357143} = 0.862720054$$

$$\partial = 1: \rightarrow y_1(B) = \frac{0.0427357143}{0.2685676841 + 0.0427357143} = 0.1372799479$$

$\partial = 2: p(Z=A, TT) = 0.446154 * 0.224138 * 0.224138 = 0.02241381463$

$\partial = 2: p(Z=B, TT) = 0.553846 * 0.722222 * 0.722222 = 0.288889$

$\partial = 2: \rightarrow Sum = 0.3113028146$

$$\partial = 2: \rightarrow y_2(A) = \frac{0.022438146S}{0.02241381463 + 0.288884} = 0.0\cancel{7}$$

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$$\partial = 2: \rightarrow y_2(B) = \frac{0.288884}{0.022413 8146 + 0.288889} = 0.9279999615$$

$E(A) = y_1(a) + y_2(a)$

$= 0.8627200521 + 0.0720003847$

$= 0.9347208906$

$E(B) = y_1(b) + y_2(b)$

$= 0.137727994479 + 0.9274999615$

$= 1.06527999409$

$E(A,H) = \sum_\partial y_\partial(a) \times H(\partial, h)$

$= 0.862720521 \times 2$

$+ 0 \times 0.0720003847$

$= 1.725441042$

$E(A,T) = \sum_\partial y_\partial(a) \times H(\partial, t) =$

$0 \times 0.8627205 21$

$+ 2 \times 0.0720003847$

$= 0.\cancel{0720000003847}$

1440000769

$E(B,H) = \sum_\partial y_\partial(b) \times H(\partial, h) = 0.137727994 \times 2$

$+ 0 = 0.27455988$

$E(B,T) = \sum_\partial y_\partial(b) \times H(\partial, t) = 0$

$+ 0.927999615 \times 2$

$= 1.855999923$

$$est(\theta_a) = E(A)/2 = 0.934720906/2 = 0.467360453$$

$$est(\theta_b) = E(B)/2 = 1.065274909/2 = 0.5326399545$$

$$est(\theta_{H|a}) = E(A,H)/\Sigma+[E(A,H)] = \frac{1.072544104\,2}{1.072544104\,2+0.01440000769} = 0.9229715906$$

$$est(\theta_{T|a}) = E(A,T)/\Sigma+[E(A,H)] = \frac{0.1440000769}{1.072544042+0.1440000769} = 0.07702840946$$

$$est(\theta_{H|b}) = E(B,H)/\Sigma+[E(B,H)] = \frac{0.27455988}{0.27455988+1.855999923} = 0.1288674834$$

$$est(\theta_{T|b}) = E(B,T)/\Sigma+[E(B,H)] = \frac{1.855999923}{0.27455988+1.855999923} = 0.871325166$$