Planetary Atmospheres - Equation and Value Tables

Group Effort

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1 Gasses and Equation of State

Mole-based Equation	**Mass-based Equation**
Ideal Gas Constant (R)	Specific Gas Constant (R_s)
$pV = nRT \quad (n = \text{mol})$	$pV = mR_sT \quad (m = \text{mass})$

Table 1: Comparison of Mole-based and Mass-based Ideal Gas Equations

Symbol	**Unit**	**Note**
p	Pa	Pressure
V	m^3	Volume
m	kg	Mass
R_s	$\text{J}\cdot\text{kg}^{-1}\cdot\text{K}^{-1}$	Specific Gas Constant
R	$\text{J}\cdot\text{mol}^{-1}\cdot\text{K}^{-1}$	Ideal Gas Constant
T	K	Temperature
M	$\text{kg}\cdot\text{mol}^{-1}$	Molecular Weight (Molar mass)
n_V	m^{-3}	Number of molecules per unit volume
ρ	$\text{kg}\cdot\text{m}^{-3}$	Density
α	$\text{m}^3\cdot\text{kg}^{-1}$	Specific Volume
n	–	Moles
N	$\text{kg}\cdot\text{m}\cdot\text{s}^{-2}$	Newton (force)
Pa	$\text{kg}\cdot\text{m}^{-1}\cdot\text{s}^{-2}$	Pascal (pressure)
J	$\text{kg}\cdot\text{m}^2\cdot\text{s}^{-2}$	Joule (energy)
N_A	mol^{-1}	Avogadro's Number (6.022×10^{23} particles/mol)

Table 2: Physical symbols, units, and associated meanings

2 Wave Symbols and Quantities

Symbol	Name	Meaning (Wave Context)
λ	Lambda	Wavelength – distance between wave crests (m)
ν	Nu	Frequency – cycles per second (Hz = 1/s)
$\bar{\nu}$	Nu-bar	Wave number – cycles per meter (1/m)
k	k	Angular (circular) wave number – $k = 2\pi/\lambda$ (rad/m)
ω	Omega	Angular (circular) frequency – $\omega = 2\pi\nu$ (rad/s)
T	T	Period – time per cycle (s)
v_p	v-sub-p	Phase speed – speed at which wave phase propagates (m/s)

Table 3: Wave Symbols and Their Meanings

	λ	ν	$\bar{\nu}$	k	ω
λ	1	$\frac{c}{\nu}$	$\frac{1}{\bar{\nu}}$	$\frac{2\pi}{k}$	$\frac{2\pi c}{\omega}$
ν	$\frac{c}{\lambda}$	1	$c\bar{\nu}$	$\frac{2\pi k}{c}$	$2\pi\omega$
$\bar{\nu}$	$\frac{1}{\lambda}$	$\frac{\nu}{c}$	1	$\frac{2\pi}{k}$	$\frac{2\pi\omega}{c}$
k	$\frac{2\pi}{\lambda}$	$\frac{\nu c}{2\pi}$	$\frac{\bar{\nu}}{2\pi}$	1	$\frac{1}{c\omega}$
ω	$\frac{2\pi c}{\lambda}$	$\frac{\nu}{2\pi}$	$\frac{2\pi\bar{\nu}}{c}$	ck	1

Table 4: Conversion between wave parameters

3 Radiomentric Quantities

Quantity	Symbol	Units	Physical Meaning	Equation
Radiant Power (Radiative Flux)	Φ, F	W	Total radiant energy emitted, transferred, or received per second.	$\Phi = \frac{dQ}{dt}$
Radiant Energy (Thermal energy)	Q_e, E, W	J	Total electromagnetic energy accumulated over time.	$Q = \int \Phi(t)\, dt$
Radiant Power per Unit Area (Irradiance, Radiative Flux Density, Exitance)	E, I	$\mathrm{W\,m^{-2}}$	Power received per unit surface area (in, through, or out).	$E = \frac{d\Phi}{dA}$
Radiance (Specific Intensity)	L	$\mathrm{W\,m^{-2}\,sr^{-1}}$	Radiant power per unit area per solid angle in a specific direction.	$L = \frac{d^2\Phi}{dA\cos\theta\, d\omega}$

Table 5: Radiometric Quantities: Symbols, Units, and Definitions

Equation	Name of Equation	Units of Result
$L_\star = 4\pi R_\star^2 \sigma T_\star^4$	Stefan–Boltzmann Law (Star Luminosity)	W (watts)
$F = \dfrac{L_\star}{4\pi d^2}$	Solar Constant / Stellar Flux at Planet	$\mathrm{W/m^2}$
$T_p = \left(\dfrac{(1-A)F}{\sigma}\right)^{1/4}$	Effective Temperature of a Planet	K (kelvin)

Table 6: Key equations for planetary energy balance

4 Energy Balance

Equation	Name of Equation	Units of Result
$L_\star = 4\pi R_\star^2 \sigma T_\star^4$	Stefan–Boltzmann Law (Star Luminosity)	W (watts)
$F = \dfrac{L_\star}{4\pi d^2}$	Solar Constant / Stellar Flux at Planet	$\mathrm{W/m^2}$
$T_p = \left(\dfrac{(1-A)F}{\sigma}\right)^{1/4}$	Effective Temperature of a Planet	K (kelvin)

Table 7: Key equations for planetary energy balance

Definition of Values in above Equations:

- $\sigma = 5.67 \times 10^{-8}\,\mathrm{W \cdot m^{-2} \cdot K^{-4}}$ — Stefan–Boltzmann constant
- $R_\star$ = Radius of the star
- d = Distance from star to planet
- $T_\star$ = Effective temperature of the star
- T_p = Effective temperature of the planet
- $L_\star$ = Stellar luminosity
- F = Flux at the planet
- A = Albedo of the planet

Useful Reference Values:

- $R_\odot = 6.96 \times 10^8\,\mathrm{m}$ — Solar radius
- $\mathrm{AU} = 1.496 \times 10^{11}\,\mathrm{m}$ — Astronomical unit

5 Lee

5.1 Beers Law

Gives the change of intensity of light as it passes through a medium;

$$\mathrm{d}\, I_\lambda = I_\lambda(s + \mathrm{d}\, s) - I_\lambda(s) = -I_\lambda(s)\beta_e(s)\,\mathrm{d}\, s$$

can be written as

$$\frac{\mathrm{d}\, I_\lambda}{I_\lambda} = \mathrm{d}\log I_\lambda = -\beta_s\,\mathrm{d}\, s$$

or integrating out

$$I_\lambda(s_2) = I_\lambda(s_1)\exp\left[-\int_{s_1}^{s_2} \beta_e(s)\,\mathrm{d}\, s\right] \tag{1}$$

Optical Depth is defined as:

$$\tau(s_1, s_2) = \int_{s_1}^{s_2} \beta_e(s)\,\mathrm{d}\,s \tag{2}$$

5.2 Scattering

For scattering the dimensionless parameter is defined as:

$$x = \frac{2\pi r}{\lambda} \tag{3}$$

which relates the size of the particle with the wavelength. For $x \in (0.2, 0.002)$, we observe Rayleigh scattering. For $x \in (2000, 0.02)$, we observe Mie scattering.

Rayleigh scattering is driven by the electric dipole moment of the photon induces oscillation of the electrons in the atmospheric gases.

Mie scattering is driven by larger particles which scatter on a homogeneous sphere.

They got different patterns for forward and backward scattering.

5.3 Energy Balance

The Earth is in Energy Balance over centuries. Plot of what happens in the atmosphere. Optical transparent, black body at $300K$.

The four main drivers of energy balance are:

- Radiation
- Latent Heat (Tropics)
- Sensible Heat
- Winds

Global Circulation driven by different energy per area in the tropics.

Three transport cells:

- Hadley Cell
- Ferrel Cell
- Polar Cell

What would happen if the Earth was rotating faster?

Coriolis Force is driving the number of transport cells.

Links with the oceans:

Hayline driven with gradient in salinity

Box Model; if measure form a point; lagrange model; if data form ballon; 3D climate models

Fundamental equations of climate models; radiative transfer; eddy mixing; central chemical equation;