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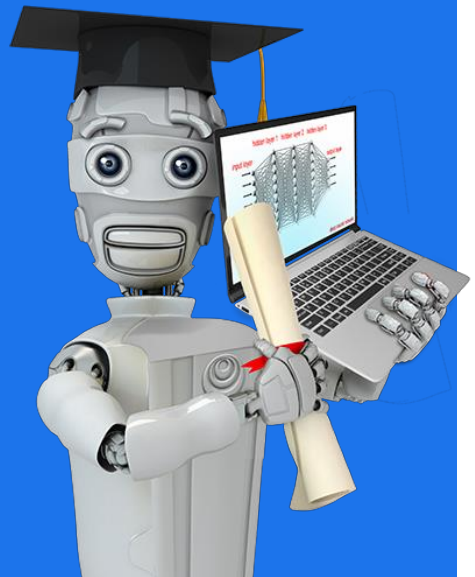

Linear Regression with Multiple Variables

Multiple Features

Multiple features (variables)

one feature →

Size in feet^2 (x)	Price ($) in 1000's (y) ←
2104	400
1416	232
1534	315
852	178
…	…

$$f_{w,b}(x) = wx + b$$

Multiple features (variables)

	Size in feet²	Number of bedrooms	Number of floors	Age of home in years	Price ($) in $1000's
	x_1	x_2	x_3	x_4	
	2104	5	1	45	460
$i=2$	1416	3	2	40	232
	1534	3	2	30	315
	852	2	1	36	178
	...	...	...	...	...

$j = 1 \ldots 4$

$n = 4$

x_j = j^{th} feature

n = number of features

$\vec{x}^{(i)}$ = features of i^{th} training example

$x_j^{(i)}$ = value of feature j in i^{th} training example

$\vec{x}^{(2)} = [1416 \quad 3 \quad 2 \quad 40]$

$x_3^{(2)} = 2$

Model:

Previously: $f_{w,b}(x) = wx + b$

$$f_{w,b}(X) = w_1 X_1 + w_2 X_2 + w_3 X_3 + w_4 X_4 + b$$

example

$$f_{w,b}(X) = 0.1 X_1 + 4 X_2 + 10 X_3 + -2 X_4 + 80$$

X_1: size; X_2: #bedrooms; X_3: #floors; X_4: years; 80: base price

$$f_{w,b}(x) = w_1 x_1 + w_2 x_2 + \cdots + w_n x_n + b$$

$$f_{\vec{w},b}(\vec{x}) = w_1 x_1 + w_2 x_2 + \cdots + w_n x_n + b$$

$\vec{w} = [w_1 \; w_2 \; w_3 \ldots w_n]$ parameters of the model

b is a number

vector $\vec{x} = [x_1 \; x_2 \; x_3 \ldots x_n]$

$$f_{\vec{w},b}(\vec{x}) = \vec{w} \cdot \vec{x} + b = w_1 x_1 + w_2 x_2 + w_3 x_3 + \cdots + w_n x_n + b$$

dot product

multiple linear regression

(not multivariate regression)

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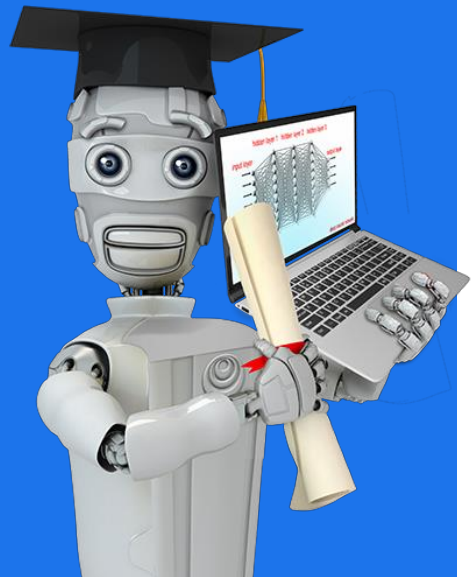

Linear Regression with Multiple Variables

Vectorization Part 1

Parameters and features

$\vec{w} = [w_1 \quad w_2 \quad w_3]$ $n=3$

b is a number

$\vec{x} = [x_1 \quad x_2 \quad x_3]$

linear algebra: count from 1

NumPy

w[0] w[1] w[2]

```
w = np.array([1.0,2.5,-3.3])
b = 4
x = np.array([10,20,30])
```

x[0] x[1] x[2]

code: count from 0

Without vectorization

$n = 100{,}000$

$f_{\vec{w},b}(\vec{x}) = w_1 x_1 + w_2 x_2 + w_3 x_3 + b$

```
f = w[0] * x[0] +
    w[1] * x[1] +
    w[2] * x[2] + b
```

Without vectorization

$f_{\vec{w},b}(\vec{x}) = \left(\sum_{j=1}^{n} w_j x_j\right) + b$

$\sum_{j=1}^{n} \rightarrow j = 1 \ldots n$ 1, 2, 3

range(0, n) → j = 0 ... n-1

```
f = 0      range(n)
for j in range(0,n):
    f = f + w[j] * x[j]
f = f + b
```

Vectorization

$f_{\vec{w},b}(\vec{x}) = \vec{w} \cdot \vec{x} + b$

```
f = np.dot(w,x) + b
```

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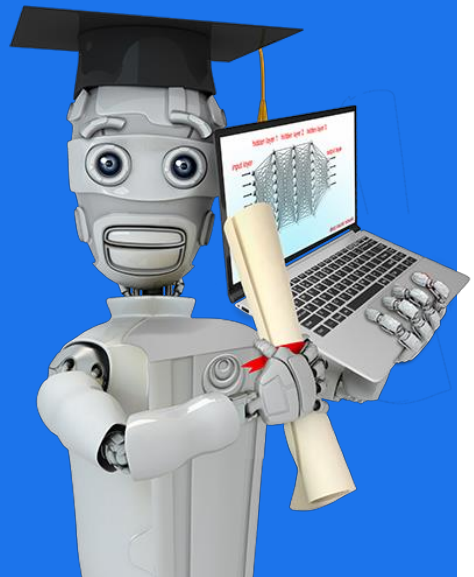


Linear Regression with Multiple Variables

Vectorization Part 2

Without vectorization

```
for j in range(0,16):
    f = f + w[j] * x[j]
```

t_0
```
f + w[0] * x[0]
```

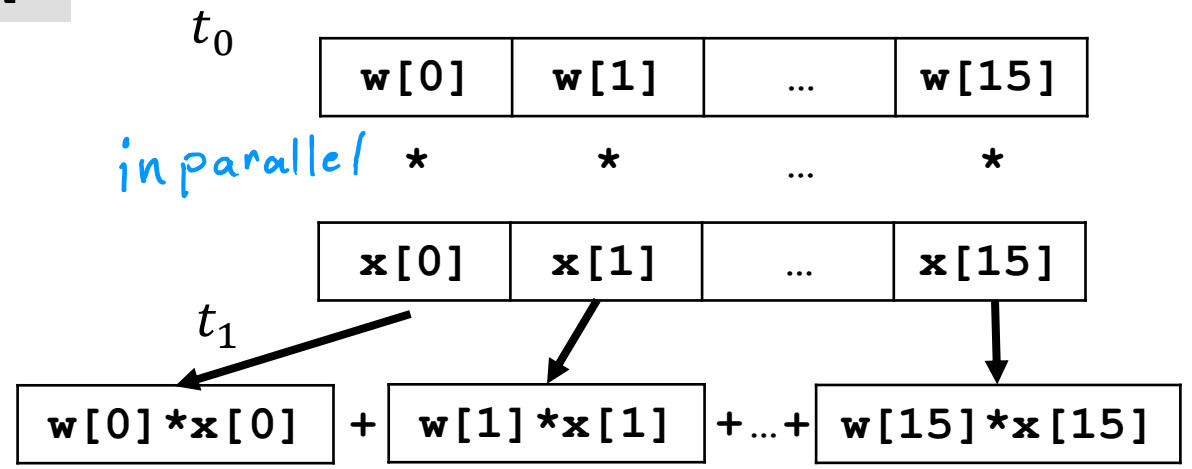
t_1
```
f + w[1] * x[1]
```

…

t_{15}
```
f + w[15] * x[15]
```

Vectorization

```
np.dot(w,x)
```



efficient → scale to large datasets

Gradient descent

$\vec{w} = (w_1 \quad w_2 \quad \cdots \quad w_{16})$ ~~b~~ parameters

derivatives $\vec{d} = (d_1 \quad d_2 \quad \cdots \quad d_{16})$

```
w = np.array([0.5, 1.3, … 3.4])
d = np.array([0.3, 0.2, … 0.4])
```

learning rate α

compute $w_j = w_j - 0.1 d_j$ for $j = 1 \ldots 16$

Without vectorization

$w_1 = w_1 - 0.1 d_1$

$w_2 = w_2 - 0.1 d_2$

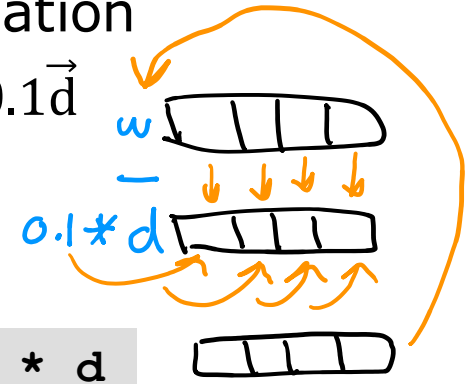
$\vdots$

$w_{16} = w_{16} - 0.1 d_{16}$

```
for j in range(0,16):
    w[j] = w[j] - 0.1 * d[j]
```

With vectorization

$\vec{w} = \vec{w} - 0.1\vec{d}$




```
w = w - 0.1 * d
```

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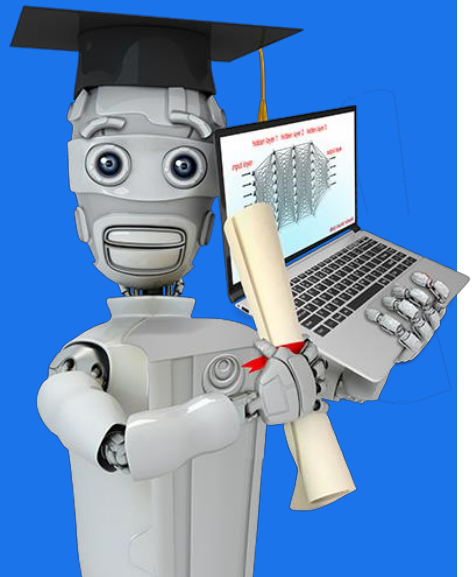

Linear Regression with Multiple Variables

Gradient Descent for Multiple Regression

Previous notation

Parameters $w_1, \cdots, w_n$

b

Model $f_{\vec{w},b}(\vec{x}) = w_1 x_1 + \cdots + w_n x_n + b$

Cost function $J(w_1, \cdots, w_n, b)$

Vector notation

vector of length n

$\vec{w} = [w_1 \quad \cdots \quad w_n]$

b still a number

$f_{\vec{w},b}(\vec{x}) = \vec{w} \cdot \vec{x} + b$

dot product

$J(\vec{w}, b)$

Gradient descent

repeat {

$w_j = w_j - \alpha \frac{\partial}{\partial w_j} J(w_1, \cdots, w_n, b)$

$b = b - \alpha \frac{\partial}{\partial b} J(w_1, \cdots, w_n, b)$

}

repeat {

$w_j = w_j - \alpha \frac{\partial}{\partial w_j} J(\vec{w}, b)$

$b = b - \alpha \frac{\partial}{\partial b} J(\vec{w}, b)$

}

Gradient descent

One feature

repeat {

$$w = w - \alpha \frac{1}{m}\sum_{i=1}^{m}\left(f_{w,b}\left(x^{(i)}\right) - y^{(i)}\right)x^{(i)}$$

$$\frac{\partial}{\partial w}J(w,b)$$

$$b = b - \alpha \frac{1}{m}\sum_{i=1}^{m}\left(f_{w,b}\left(x^{(i)}\right) - y^{(i)}\right)$$

simultaneously update w, b

}

n features ($n \geq 2$)

repeat {

$j = 1$

$$w_1 = w_1 - \alpha \frac{1}{m}\sum_{i=1}^{m}\left(f_{\vec{w},b}\left(\vec{x}^{(i)}\right) - y^{(i)}\right)x_1^{(i)}$$

$$\frac{\partial}{\partial w_1}J(\vec{w},b)$$

$\vdots$

$j = n$

$$w_n = w_n - \alpha \frac{1}{m}\sum_{i=1}^{m}\left(f_{\vec{w},b}\left(\vec{x}^{(i)}\right) - y^{(i)}\right)x_n^{(i)}$$

$$b = b - \alpha \frac{1}{m}\sum_{i=1}^{m}\left(f_{\vec{w},b}\left(\vec{x}^{(i)}\right) - y^{(i)}\right)$$

simultaneously update

w_j (for $j = 1, \cdots, n$) and b

}

An alternative to gradient descent

Normal equation
- Only for linear regression
- Solve for w, b without iterations

Disadvantages
- Doesn't generalize to other learning algorithms.
- Slow when number of features is large (> 10,000)

What you need to know
- Normal equation method may be used in machine learning libraries that implement linear regression.
- Gradient descent is the recommended method for finding parameters w,b

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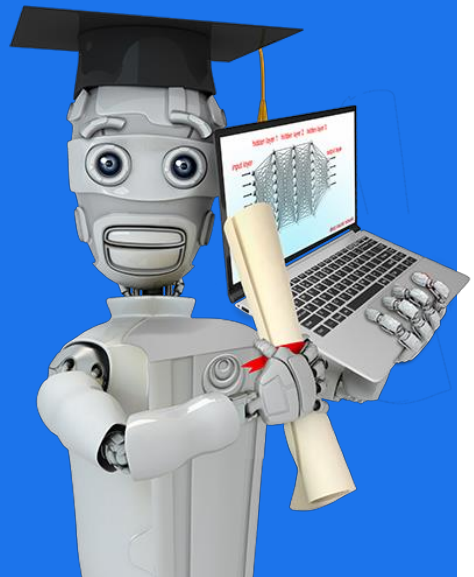


Practical Tips for Linear Regression

Feature Scaling Part 1

Feature and parameter values

$\widehat{price} = w_1 x_1 + w_2 x_2 + b$

size # bedrooms

x_1: size (feet^2)
range: $300 - 2{,}000$

large

x_2: # bedrooms
range: $0 - 5$

small

House: $x_1 = 2000$, $x_2 = 5$, $price = \$500\text{k}$

one training example

size of the parameters w_1, w_2?

$w_1 = 50, \quad w_2 = 0.1, \quad b = 50$

$\widehat{price} = \underbrace{50 * 2000}_{100{,}000\text{k}} + \underbrace{0.1 * 5}_{0.5\text{k}} + \underbrace{50}_{50\text{k}}$

$\widehat{price} = \$100{,}050.5\text{k} = \$100{,}050{,}500$

$w_1 = 0.1, \quad w_2 = 50, \quad b = 50$

small large

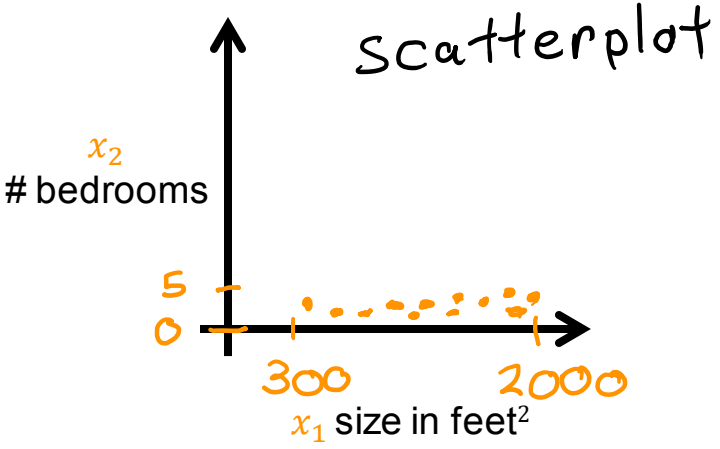
$\widehat{price} = \underbrace{0.1 * 2000k}_{200\text{k}} + \underbrace{50 * 5}_{250\text{k}} + \underbrace{50}_{50\text{k}}$

$\widehat{price} = \$500\text{k}$ more reasonable

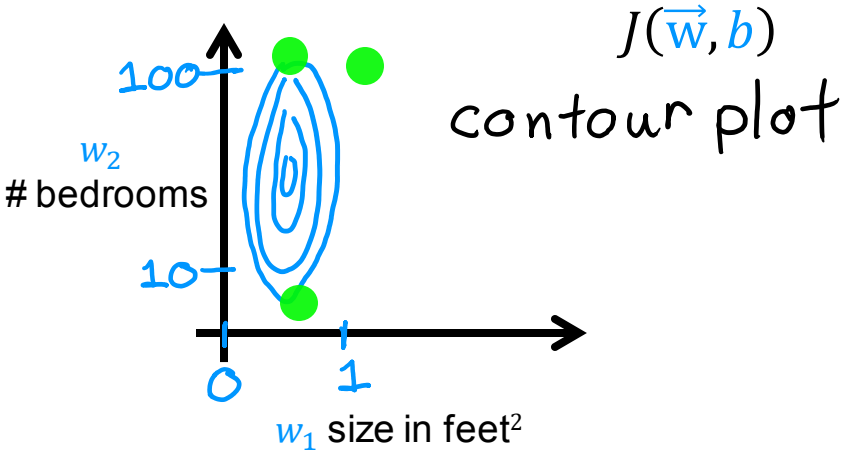
Feature size and parameter size

	size of feature x_j	size of parameter w_j
size in feet^2	⟷	⟷
#bedrooms	⟷	⟷

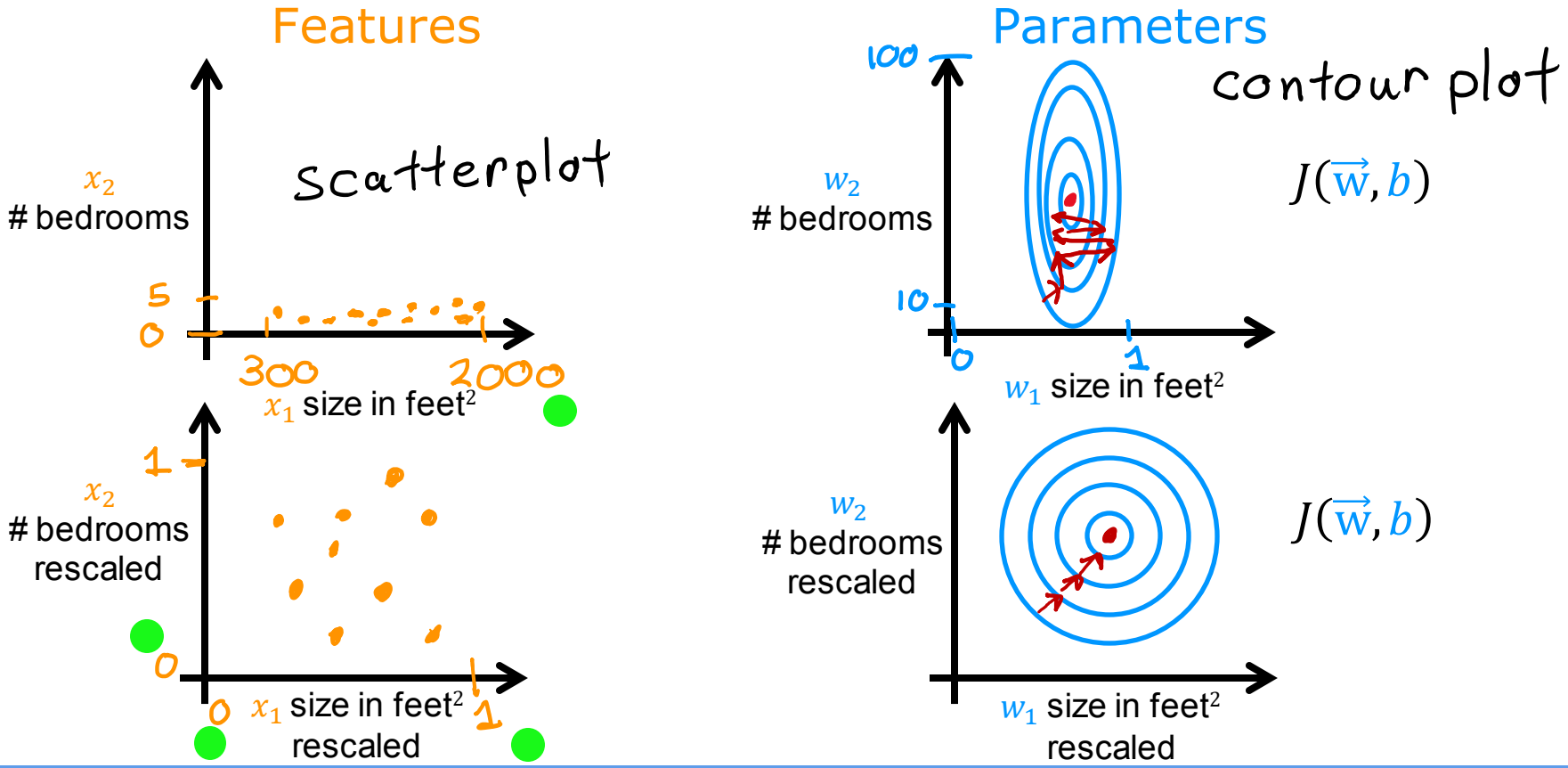
Features



Parameters



Feature size and gradient descent



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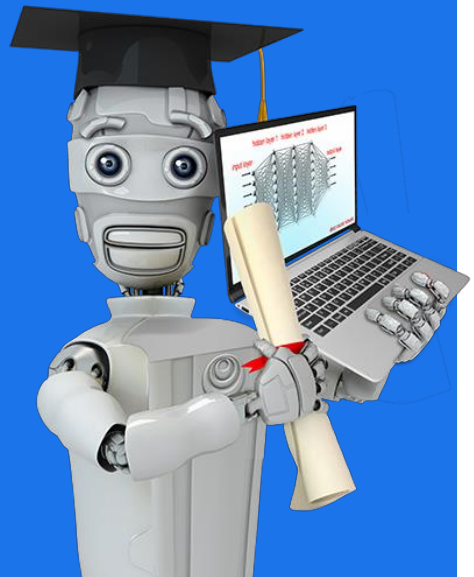
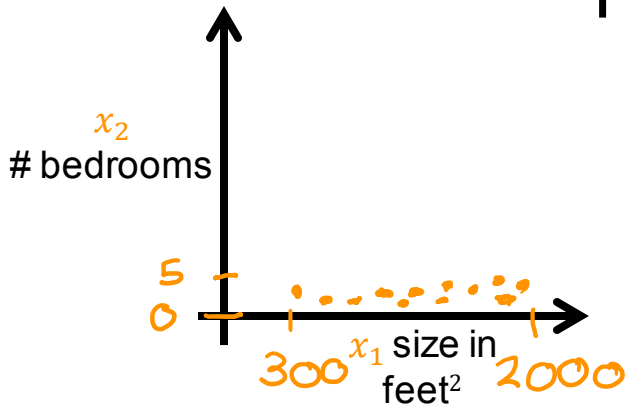
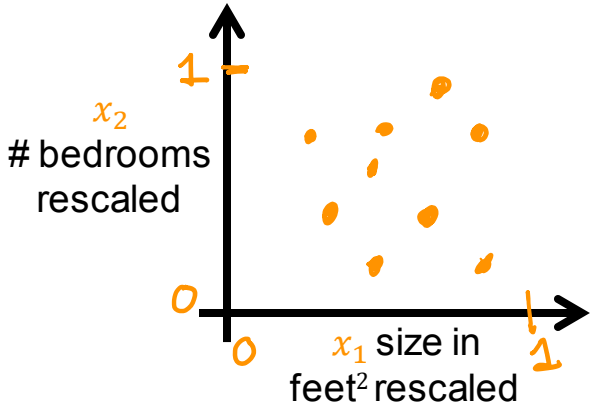


Practical Tips for Linear Regression

Feature Scaling Part 2

Feature scaling





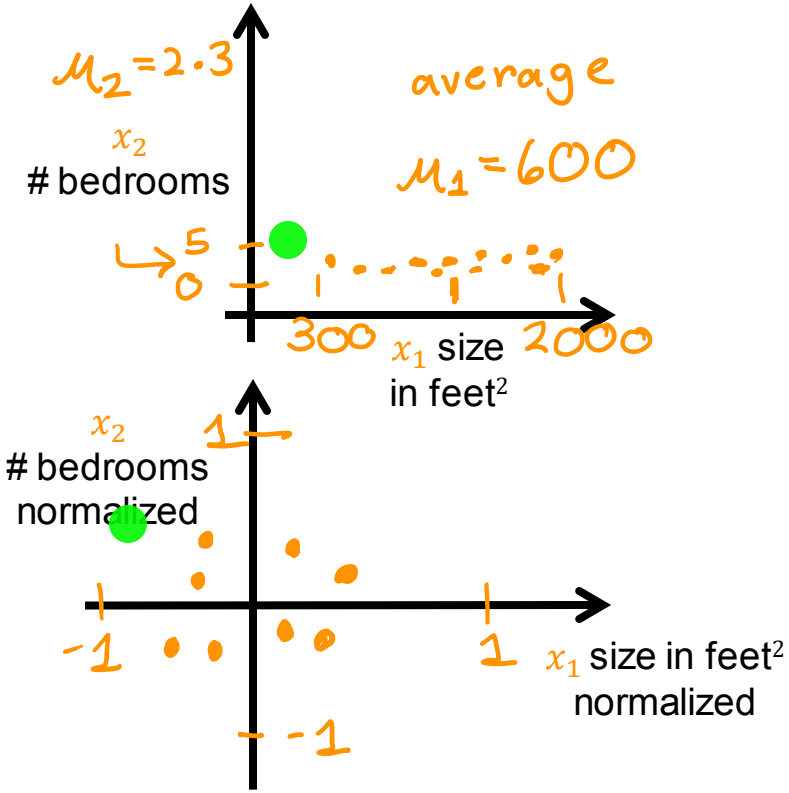
$$300 \le x_1 \le 2000 \qquad 0 \le x_2 \le 5$$

$$x_{1,scaled} = \frac{x_1}{2000} \qquad x_{2,scaled} = \frac{x_2}{5}$$

max max

$$0.15 \le x_{1,scaled} \le 1 \qquad 0 \le x_{2,scaled} \le 1$$

Mean normalization



$$300 \le x_1 \le 2000$$

$$x_1 = \frac{x_1 - \mu_1}{2000 - 300}$$

max-min

$$-0.18 \le x_1 \le 0.82$$

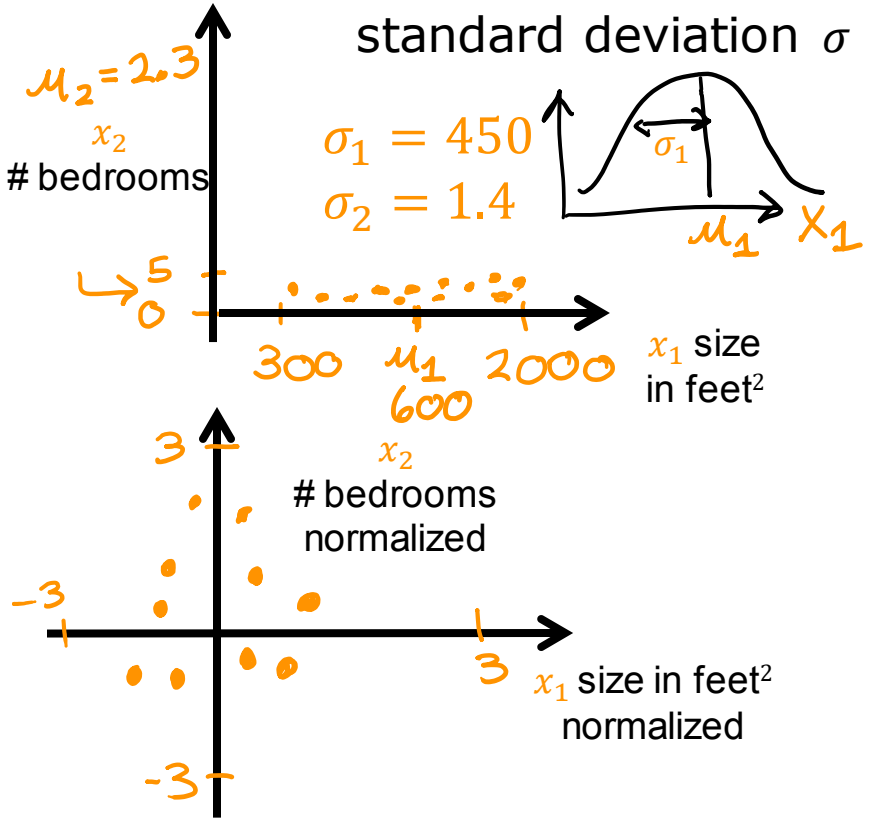
$$0 \le x_2 \le 5$$

$$x_2 = \frac{x_2 - \mu_2}{5 - 0}$$

max-min

$$-0.46 \le x_2 \le 0.54$$

Z-score normalization



$300 \leq x_1 \leq 2000$

$$x_1 = \frac{x_1 - \mu_1}{\sigma_1}$$

$-0.67 \leq x_1 \leq 3.1$

$0 \leq x_2 \leq 5$

$$x_2 = \frac{x_2 - \mu_2}{\sigma_2}$$

$-1.6 \leq x_2 \leq 1.9$

Feature scaling

aim for about $-1 \leq x_j \leq 1$ for each feature x_j

$-3 \leq x_j \leq 3$

$-0.3 \leq x_j \leq 0.3$

} acceptable ranges

$0 \leq x_1 \leq 3$ okay, no rescaling

$-2 \leq x_2 \leq 0.5$ okay, no rescaling

$-100 \leq x_3 \leq 100$ too large → rescale

$-0.001 \leq x_4 \leq 0.001$ too small → rescale

$98.6 \leq x_5 \leq 105$ too large → rescale

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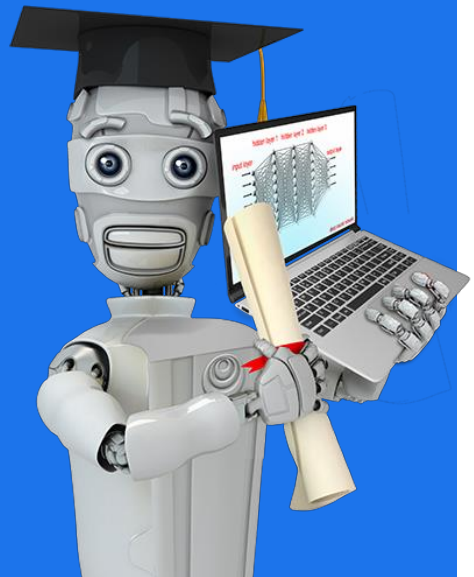


Practical Tips for Linear Regression

Checking Gradient Descent for Convergence

Gradient descent

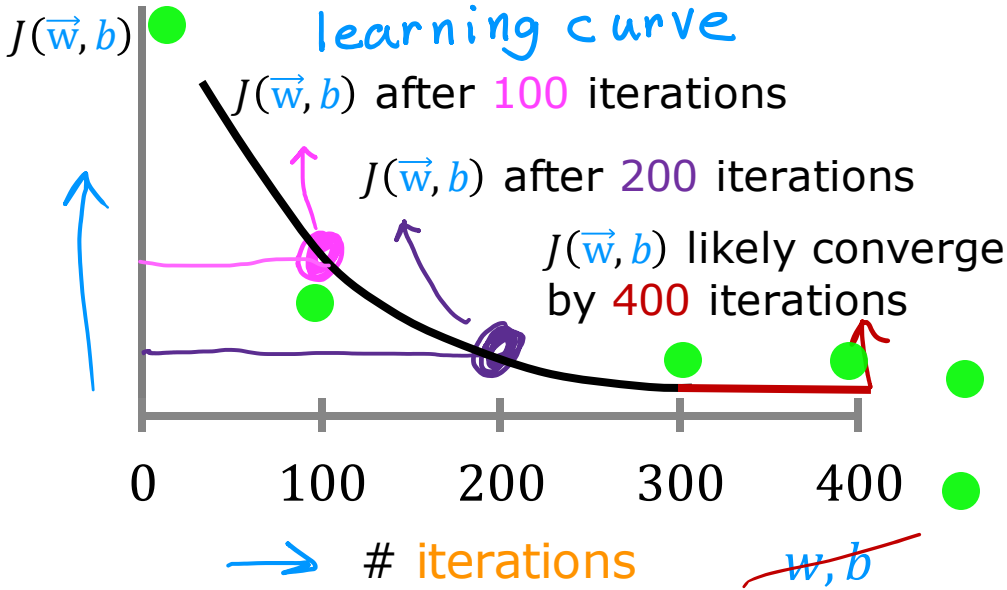
$$w_j = w_j - \alpha \frac{\partial}{\partial w_j} J(\vec{w}, b)$$

$$b = b - \alpha \frac{\partial}{\partial b} J(\vec{w}, b)$$

Make sure gradient descent is working correctly

objective: $\min_{\vec{w},b} J(\vec{w}, b)$

$J(\vec{w}, b)$ should decrease after every iteration



iterations needed varies 30 1,000 100,000

Automatic convergence test

Let ε "epsilon" be 10^{-3}. 0.001

If $J(\vec{w}, b)$ decreases by $\leq \varepsilon$ in one iteration, declare convergence.

(found parameters $\vec{w}, b$ to get close to global minimum)

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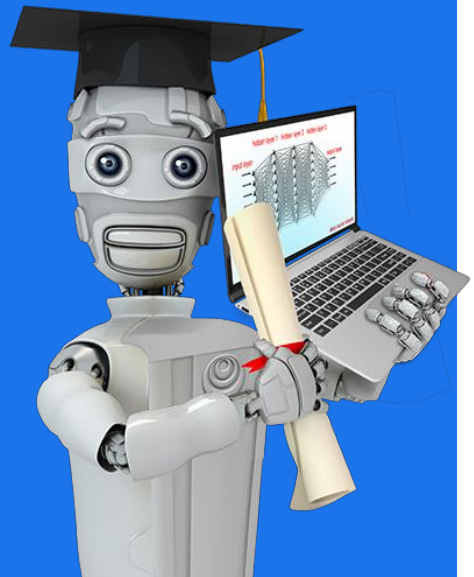
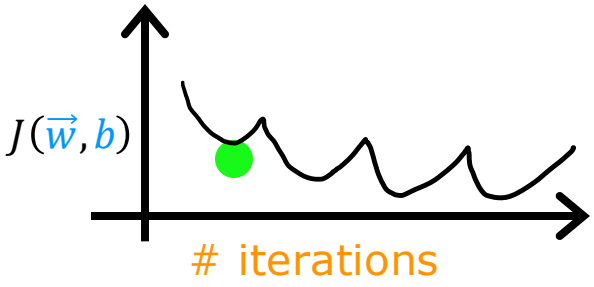
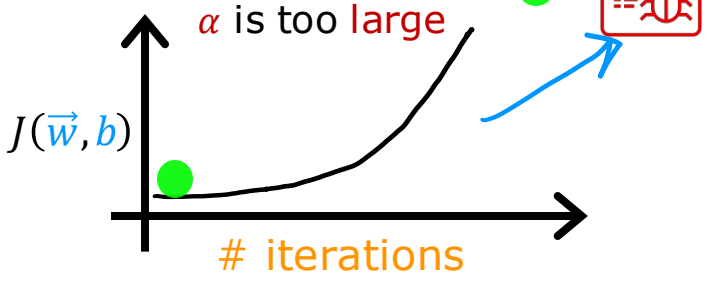


Practical Tips for Linear Regression

Choosing the Learning Rate

Identify problem with gradient descent





or learning rate is too large

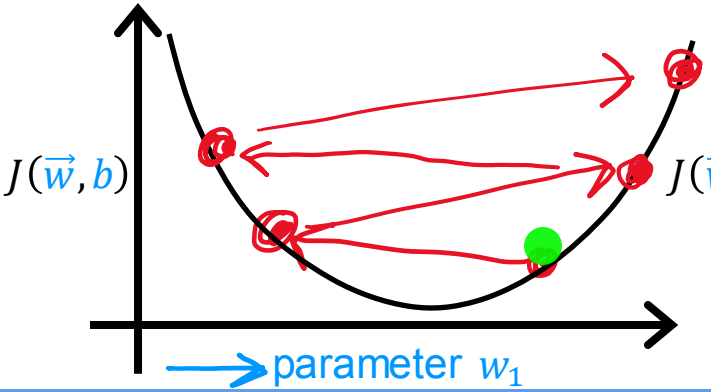
$w_1 = w_1 + \alpha d_1$

use a minus sign

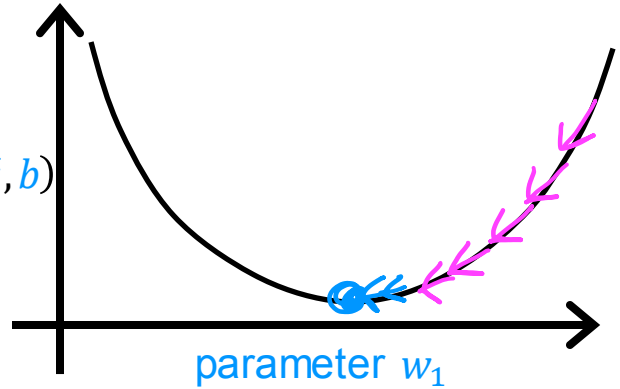
$w_1 = w_1 - \alpha d_1$

Adjust learning rate

α is too big



Use smaller α



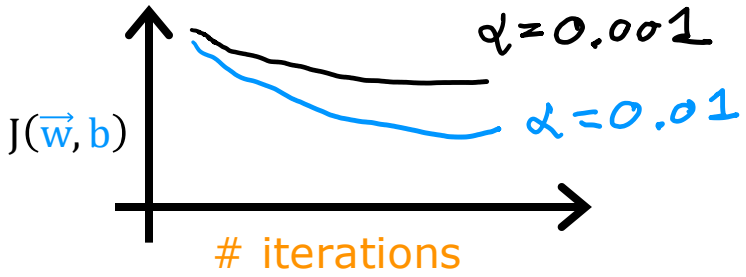
With a small enough α, $J(\vec{w}, b)$ should decrease on every iteration

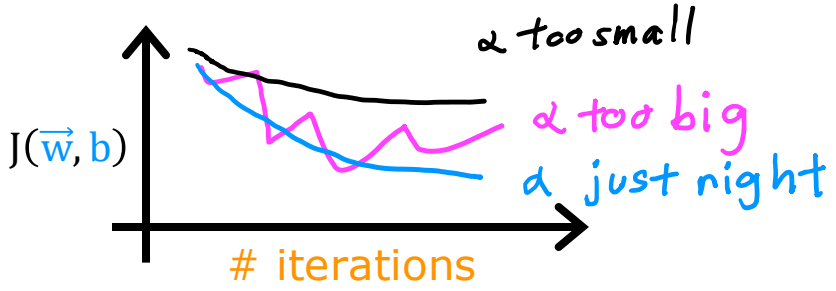
If α is too small, gradient descent takes a lot more iterations to converge

Values of α to try:

... 0.001 0.003 0.01 0.03 0.1 0.3 1 ...

3x ≈3x 3x ≈3x 3x ≈3x





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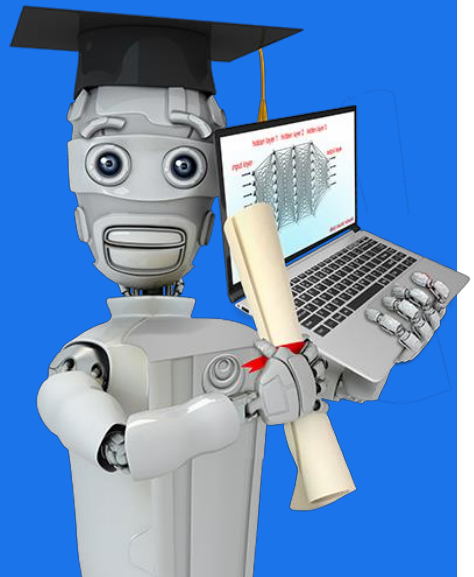


Practical Tips for Linear Regression

Feature Engineering

Feature engineering

$$f_{\vec{w},b}(\vec{x}) = w_1 x_1 + w_2 x_2 + b$$

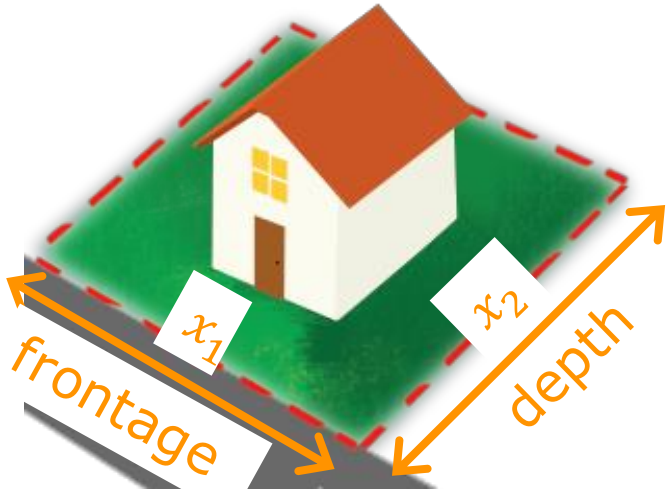
frontage depth

$$area = frontage \times depth$$

$x_3 = x_1 x_2$

new feature

$$f_{\vec{w},b}(\vec{x}) = w_1 x_1 + w_2 x_2 + w_3 x_3 + b$$



Feature engineering:
Using intuition to design new features, by transforming or combining original features.

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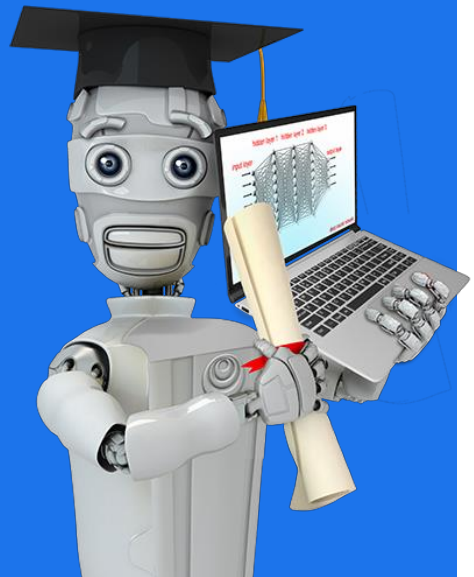
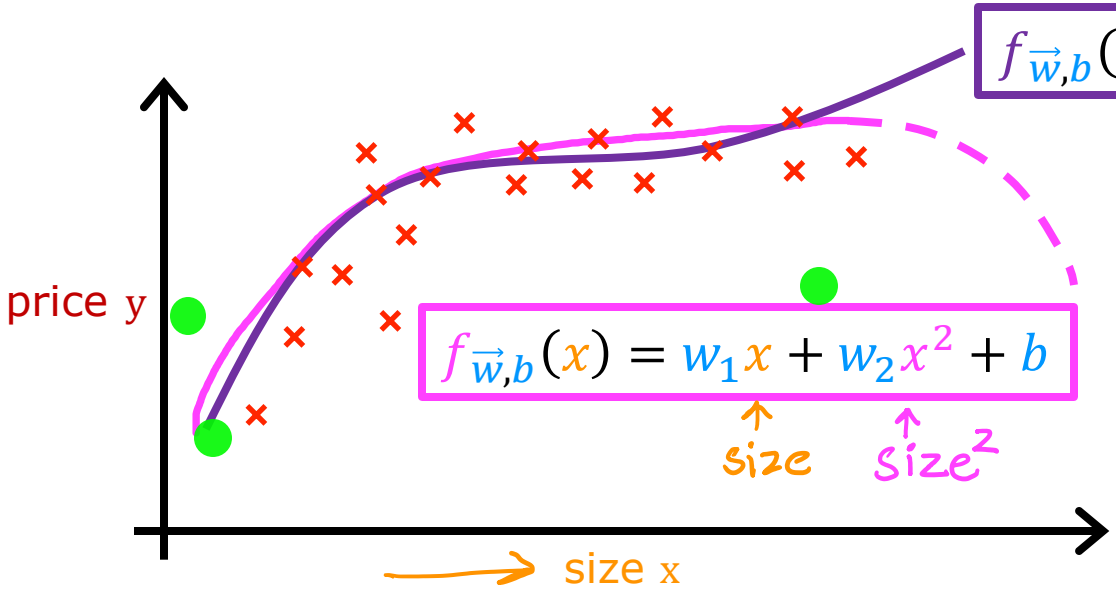


Practical Tips for Linear Regression

Polynomial Regression

Polynomial regression



$$f_{\vec{w},b}(x) = w_1 x + w_2 x^2 + b$$

size x x^2

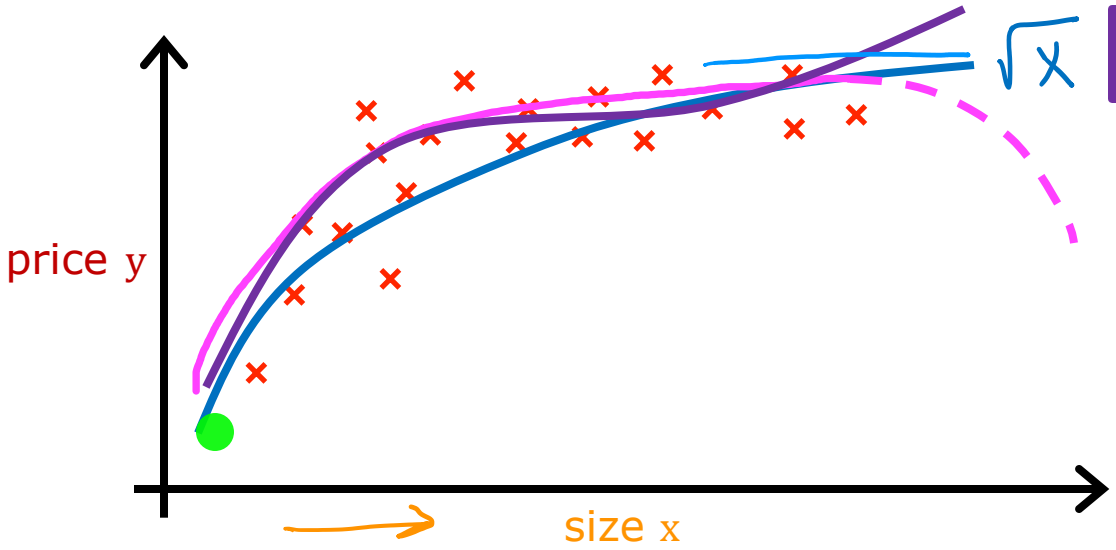
$$f_{\vec{w},b}(x) = w_1 x + w_2 x^2 + w_3 x^3 + b$$

size x x^2 x^3

size $1 - 10^3$ $1 - 10^6$ $1 - 10^9$

feature scaling

Choice of features



$$f_{\vec{w},b}(x) = w_1 x + w_2 \sqrt{x} + b$$

size

$\sqrt{\text{size}}$

what features to use?
↳ course 2