1 New Comments, December 18, 1:05 am

1.1 CFL

Looks better.

1.2 Method of Characteristics: Constant Coefficients

Looks better.

1.3 Fourier Series: Complex

No mastery credit yet.

- The definition of a complex Fourier series at the very beginning of this section is incorrect. What are the appropriate bounds on the summation?
- You are doing just fine with your calculation, though, up until you discuss the roots of your denominator. At that point, you seem to just forget two of the roots. Please put them back in and remember to calculate the relevant c_k values.
- Also, the way that you are using the summations at the end is completely wrong. This may partially, but not entirely, relate to your original mistake. The fact that you are expressing things in terms of k matters! If we write that $c_{-3} = -\frac{i}{8}$, that means that the $k = -3$ term in the complex Fourier series is $-\frac{i}{8}e^{-3ix}$.

1.4 Heat Equation

Much better.

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2 Order of Error

A few comments, mostly typographical:

- You should probably delete (or at least comment out) the blue instructions in this section now that you have written up your own comments. And you should probably comment our (or delete) the other commentary that I have give you as examples, as well.
- When you are moving from prose to displaystyle equations without ending a sentence you should usually transition with a colon, and then your equations should be formatted using eqnarray or eqnarray*, e.g. To calculate the order of the error, we first use a Taylor Series to expand each function used to approximate the derivative except $u(x)$:

$$\begin{aligned} u(x-3h) &= u(x) - u'(x)(3h) + \frac{u''(x)(3h)^2}{2!} - \frac{u'''(x)(3h)^3}{3!} + \ldots \\ u(x-2h) &= u(x) - u'(x)(2h) + \frac{u''(x)(2h)^2}{2!} - \frac{u'''(x)(2h)^3}{3!} + \ldots \end{aligned}$$

- If you want $\ldots$ (or $\cdots$) there are special commands for those. Please check the .tex form of these comments.

- It doesn't follow properly to write "Now ..." with the next sentence beginning "Next, ..." because you didn't actually *do* anything in the 'Now' sentence. I think you should consider replacing 'Next' with 'First' or 'We shall begin by ...' or some such phrasing.

- Why is it that both of your statements $A + B = 0$ and $8A + 4B = 0$ cannot be zero? For that matter, where did the $u''(x)$-associated term come from? What I see is the pair of equations

$$\begin{aligned} A + B &= 0 \\ 9A + 4B &= 0 \end{aligned}$$

  which can certainly be simultaneously true (just not in a useful way for your problem here!). In order to make the system *not* simultaneously solvable you have to include the coefficient for $u'(x)$ with something like

$$u'(x) : -3A - 2B \neq 0.$$

- Please ensure you are writing in complete sentences.

3 Numerical Methods: The CFL Condition

- Headline: No mastery credit for this yet. Please fix the mathematics and resubmit.

- t, x plane rather than t, x plane

- "[...] $u_{j+1,m}$—the time step of the solution we are interested in—as well." rather than "[...] $u_{j+1,m}$ - the time step of the solution we are interested in - as well."

- I love your picture of the region of numerical dependence!

- Don't forget your subscripts, and where did your right-hand inequality come from? You clearly wrote in your figure that the upper bound of your region was just x_m:

$$x_m - 2(j + 1) \leq \xi \leq x_m + (j + m)$$

  should be

$$x_{m-2(j+1)} \leq \xi \leq x_m$$

4 Method of Characteristics: Constant Wave Speed

- Headline: No mastery credit for this yet. Please fix the mathematics and resubmit.

- You want to use $\arctan(x)$ not $arctan(x)$. Also, if you are referring to a variable it should be x not x.

- Don't forget to use complete sentences. e.g. "First, let's consider what we know about $u(t, x)$ and $u(0, x) = f(x)$."

- "First, solve the homogeneous case for $u(t, x)$, that is for $u_t - 5u_x = 0$."

- You want to write this like so:

$$h = u(t, x(t)) = \text{constant}.$$

- It is called a total derivatative.

- Your final answer is incorrect. The function $h = 12t + g(\xi)$ is correct, but the point is that $g(\xi)$ is the function you already know—the homogeneous solution. You must fix this.

5 Method of Characteristics: Polynomial Wave Speed

- Don't forget to use cos() and sin() and tan() and arctan() and ln() and so forth.
- When you want "normal" text in math mode you should use mbox, as in

$$h = u(t, x(t)) = \text{constant}.$$

- Also, please write in complete sentences. The math all looks good, though.

6 Fourier series: Real

- I expect you want $2L$ rather than 2L?
- Don't forget to use cos() and sin() and tan() and arctan() and ln() and so forth.
- You want to use left and right commands to "right-size" the parentheses here:

$$\begin{aligned} a_k &= < f(x), \cos(kx) >= \frac{1}{L}\int_{-L}^{L} f(x)\cos\left(\frac{k\pi x}{L}\right) dx, \qquad k \geq 0 \ \text{ integers} \\ b_k &= < f(x), \sin(kx) >= \frac{1}{L}\int_{-L}^{L} f(x)\sin\left(\frac{k\pi x}{L}\right) dx, \qquad k \geq 1 \ \text{ integers} \end{aligned}$$

- Overall, good introduction for this one. This is the style you would like in general with sufficient detail to help you solve problems of this type.
- Please use complete sentences.
- Did you check what the graph looks like? This is a very good place to support your work with a figure showing the graph of $|x|$ and the graph of the first 50-100 of the terms of the Fourier series you just computed.

7 Fourier series: Complex

- Headline: No mastery credit for this yet. Please fix the mathematics and resubmit.
- Don't forget to use cos() and sin() and tan() and arctan() and ln() and so forth.
- By convention we use c_k rather than C_k for these coefficients.
- You want to use left and right commands to "right-size" the parentheses here:

$$\begin{aligned} c_k &= \frac{1}{2\pi}\int_{-\pi}^{\pi} \left(\sin^3(x) + \cos^2(5x)\right) e^{-ikx} dx \\ c_k &= \left(\frac{1}{2\pi}\right)\left(\frac{ie^{10ix-ikx}}{4(k-10)} + \frac{ie^{-10ix-ikx}}{4(k+10)} + \frac{ie^{-ikx}}{2k} + \frac{e^{i(3-k)x}}{8(3-x)} - \frac{e^{i(-3-k)x}}{8(-k-3)} - \frac{3e^{i(1-k)x}}{8(1-k)} + \frac{3e^{i(-1-k)x}}{8(-k-1)}\right) \end{aligned}$$

- Why is it immediately obvious to you given the presentation of c_k given above that values of c_k with $|k| \neq 0, 1, 3$ or 10 are equal to 0? I do like that presentation to make it clear which values of k need to be computed separately, but perhaps you want to give the expression in terms of a rational function times $\sin(\pi k)$ as well?

- Don't forget your curly braces around superscripts and subscripts so you get c_{10} rather than c_10
- I don't understand your final result at all. It is not tru that

$$f(x) \sim (\frac{1}{2} + \pi) + \sum_{k=1}^{\infty} c_k e^{ikx}$$

with

$$\left(\frac{1}{2\pi}\right)\left(\frac{ie^{10ix-ikx}}{4(k-10)} + \frac{ie^{-10ix-ikx}}{4(k+10)} + \frac{ie^{-ikx}}{2k} + \frac{e^{i(3-k)x}}{8(3-x)} - \frac{e^{i(-3-k)x}}{8(-k-3)} - \frac{3e^{i(1-k)x}}{8(1-k)} + \frac{3e^{i(-1-k)x}}{8(-k-1)}\right).$$

For one thing, what happened to all of the c_{10} and c_{-3} and so forth that you just laboriously computed? And where did that constant term come from? What are the simplified values for c_k?

8 Separation of Variables: Equilibrium behavior

- Perhaps you want something more like:
  Find the equilibrium solution for the heat equation $u_t = .005u_{xx}$ given the initial and boundary conditions:

$$\begin{aligned} u(0,x) &= f(x) \\ u(t,0) &= 6 \\ u(t,5) &= -3 \end{aligned}$$

- Better phrasing:
  Since our heat equation has both u_t and u_{xx}, let's identify the partial differential equation that $u^\star(x)$ satisfies:

$$\frac{\partial u^\star}{\partial t} = 0$$

because $u^\star$ doesn't depend on t at all, and

$$\frac{\partial u^\star}{\partial x} = (u^\star)''$$

because $u^\star$ is a function of x.

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9 Separation of variables: 1D heat equation

- Headline: No mastery credit yet. Please fix the mathematics and the explanations and resubmit.
- $u(t,x) = w(t)v(x)$ not u(t,x) = w(t)v(x)

- Both typographical and mathematical fixes. Note the fact that there is a second derivative with respect to x in the heat equation.

$$\begin{aligned}
\frac{\partial u}{\partial t} &= w'(t)v(x) \\
\frac{\partial^2 u}{\partial x^2} &= \gamma w(t)v''(x) \\
w'(t)v(x) &= \gamma w(t)v''(x) \\
\frac{w'(t)}{w(t)} &= \gamma\frac{v''(x)}{v(x)} = \text{constant} = -\lambda
\end{aligned}$$

- "We know $u(t, x) = w(t)v(x)$ and we assume that $w(t)$ is not always 0, which means we can divide by it. Therefore, v(x)=0 when w(t) $\neq$ 0 " What? We don't want *either* $w(t) = 0$ or $v(x) = 0$ because both of those would generate a trivial solution $u(t, x) = 0$. Also, this shouldn't be a new paragraph and there should be punctuation at the end of your sentence. Keep an eye out for that.

- Don't forget to use cos() and sin() and cosh() and sinh() and so forth.

- You want to use left and right commands to "right-size" the parentheses throughout:

$$u(0, x) = \cos\left(\frac{\pi}{4}x\right)$$

  Also, you shouldn't have two different initial conditions. There should only be one value of $u(0, x)$. Please stick with $u(0, x) = x^2 - 8x$ as that is the one I remember giving you, as well as what you are working with down below.

- "Neumann" not "Nuemann"

- $v'(x)$ not v'(x)

- You don't want to write $v(x) = B + A\cos\left(\frac{k\pi x}{L}\right)$ with $\lambda > 0$. You need to note that you have found a whole family of solutions: $v_0(x) = B$ as well as $v_k(x) = A_k \cos\left(\frac{k\pi x}{L}\right)$. By the principle of superposition, your general solution will be a linear combination of all of these.

- You also need to remember that when you put it back together into $u(t, x) = w(t)v(x)$ you have to keep the λ values straight. In particular, you should not have $a_0 e^{-\frac{k^2\pi^2\gamma}{L^2}t}$ as one of your terms. What should it be?

- When you compute a_0, please be exact. Do not round your solution.

- You can simplify the a_k you found as well. Please do so.