Current and Resistance-I

Phy 108 course

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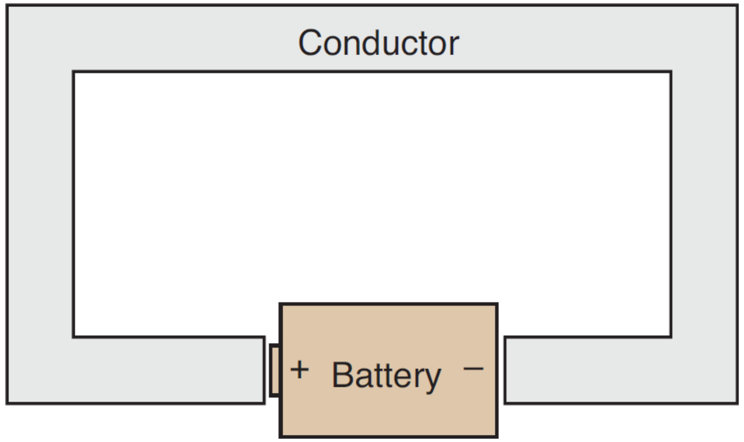
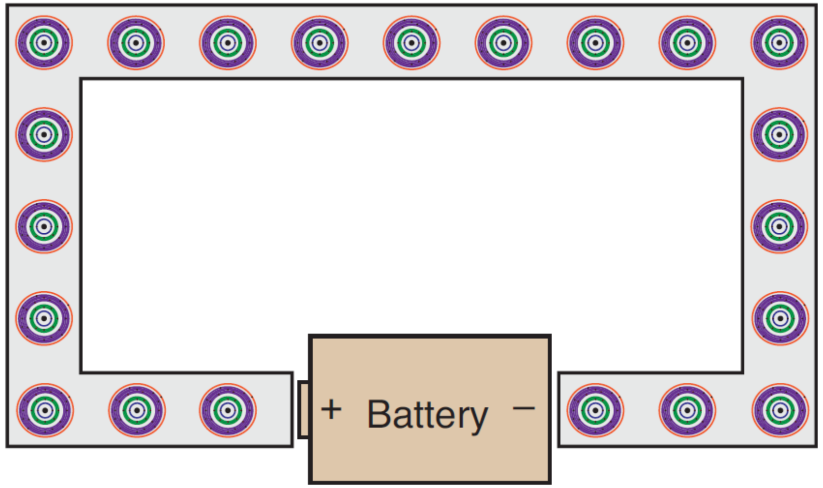
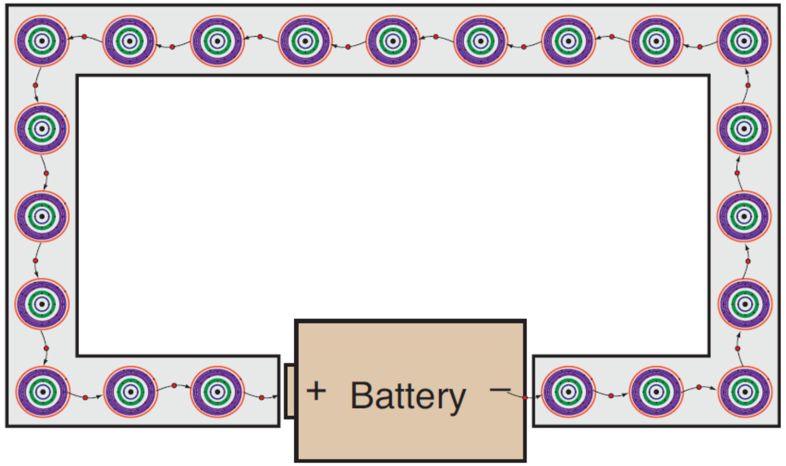
Electrical current in a trivial circuit



Electrical current: atomic model



Electrical Current: electron flow



Electric Current

The free electrons (conduction electrons) in an isolated length of copper wire are in random motion at speeds of the order of 10^6 m/s.

Electrostatics

Any isolated conducting loop—regardless of whether it has an excess charge—is all at the same potential. No electric field can exist within it or along its surface. Although conduction electrons are available, no net electric force acts on them and thus there is no current.

Electrodynamics

By connecting the ends of the wire to a battery electric fields act inside the material making up the loop, exerting forces on the conduction electrons, causing them to move and thus establishing a current. After a very short time, the electron flow reaches a constant value and the current is in its steady state (it does not vary with time).

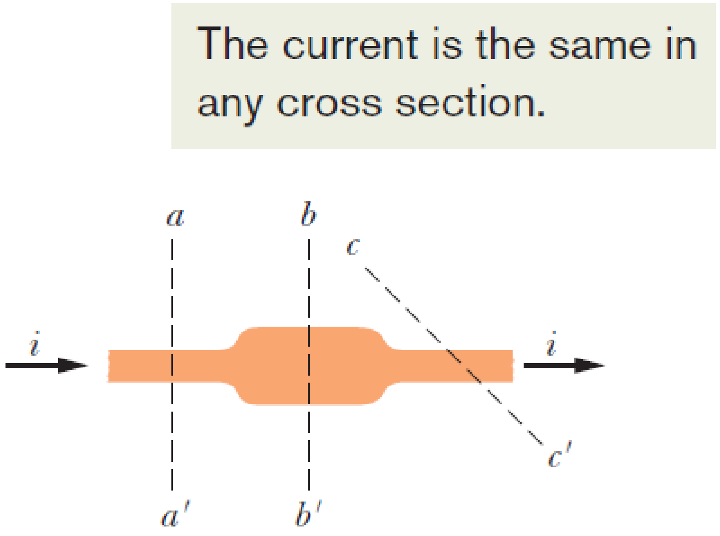
We restrict ourselves largely to the study—within the framework of classical physics—of steady currents of conduction electrons moving through metallic conductors such as copper wires.

Electric Current

If charge dq passes through a hypothetical plane (such as aa) in time dt, then the current i through that plane is defined as, $$i = \frac{dq}{dt}$$

We can find the charge that passes through the plane in a time interval extending from 0 to t by integration,

$$q = \int dq = \int_0^t i\, dt$$

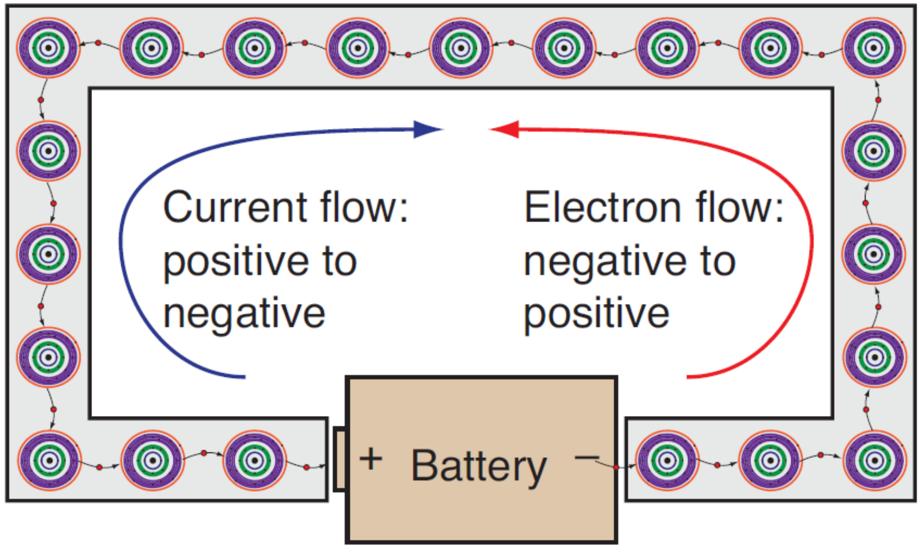


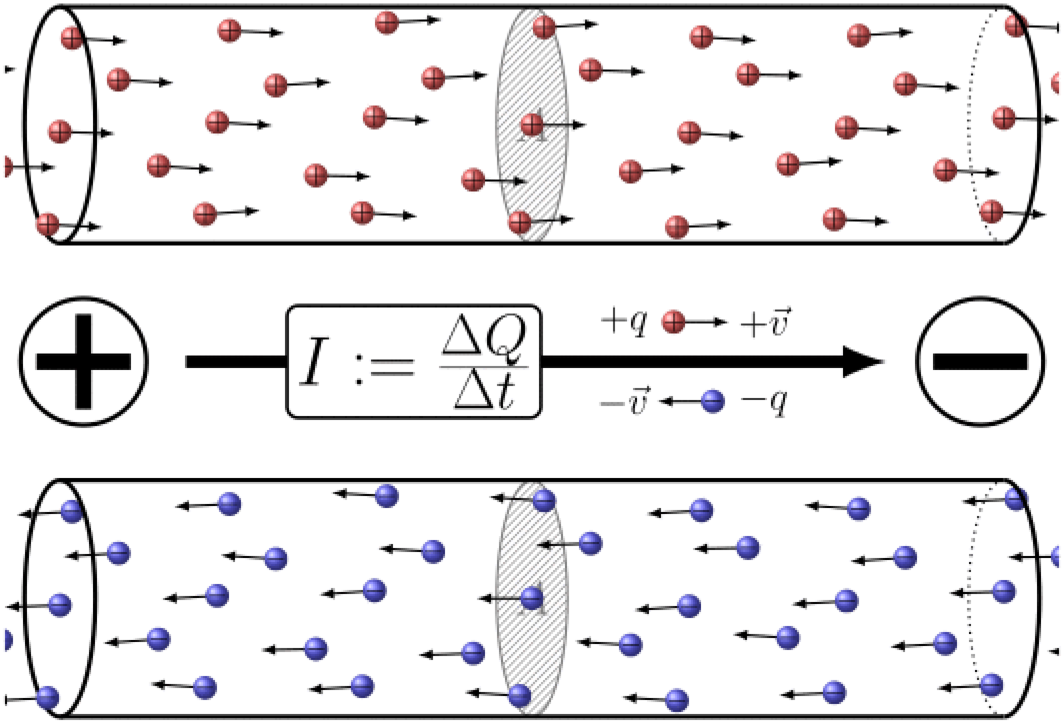
SI unit:
1 ampere = 1 A = 1 coulomb per second = 1 C/s.

The formal definition of the ampere will be discussed later, based on force between two conducting wires

The Directions of Currents

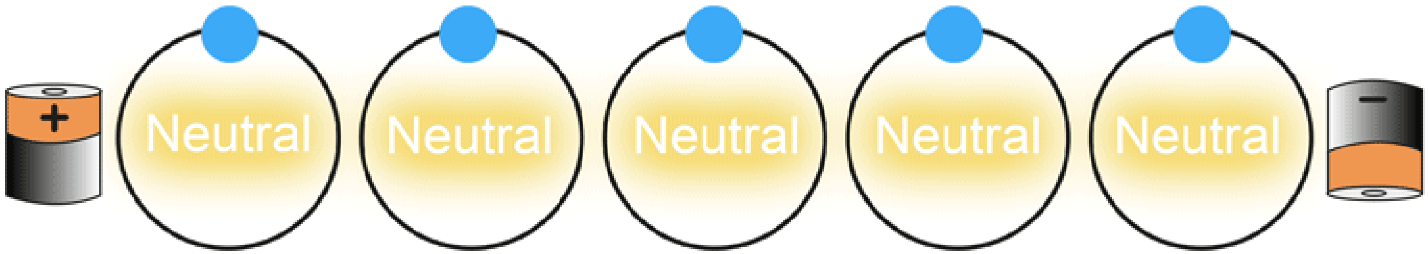
Electrical Current: current convention

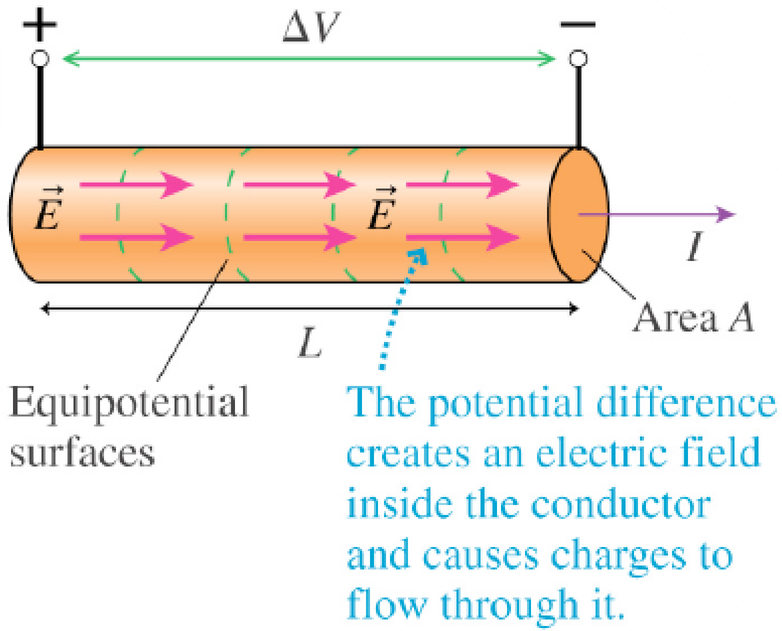


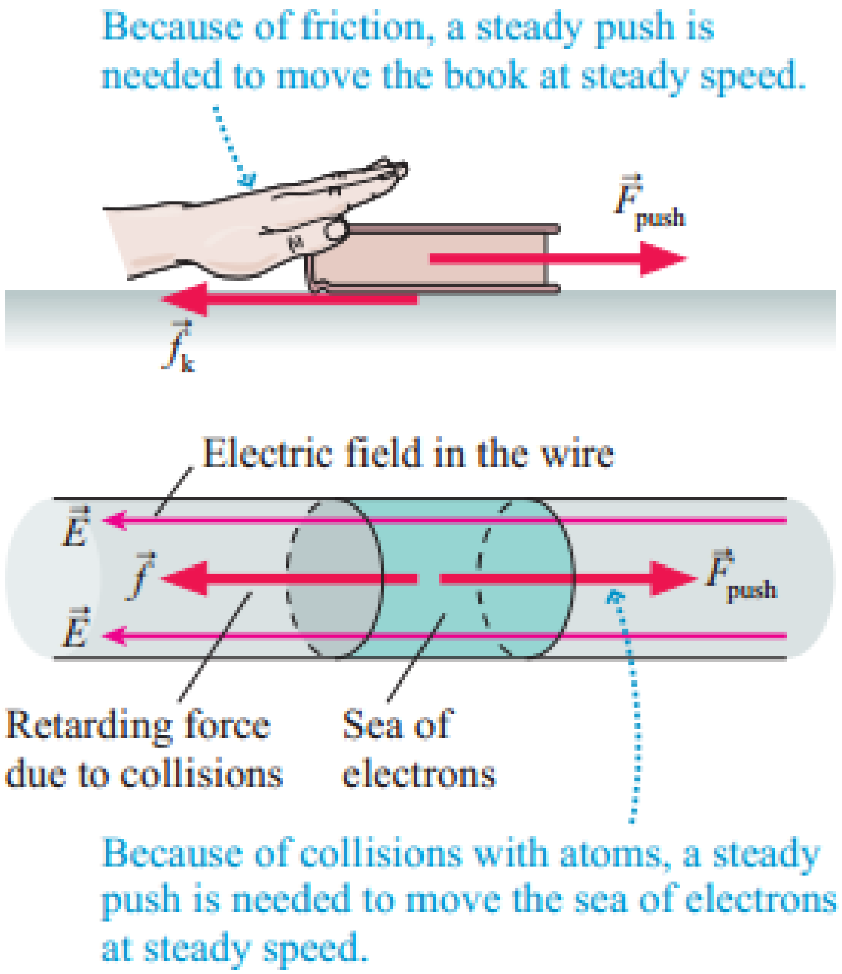


Current due to potential difference

An electron current is a nonequilibrium motion of charges sustained by an internal electric field due to imposed potential difference.

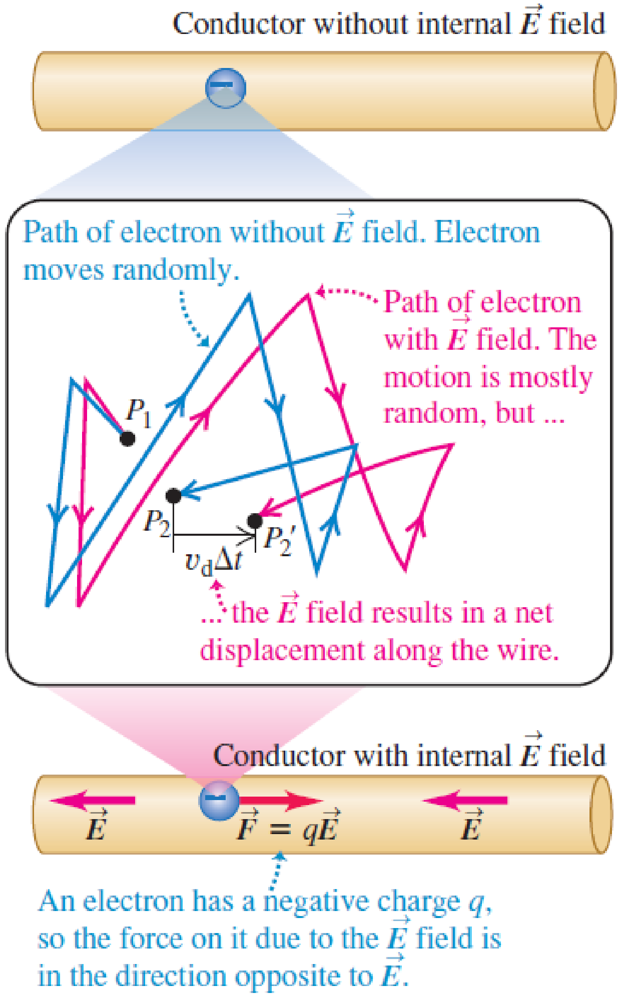


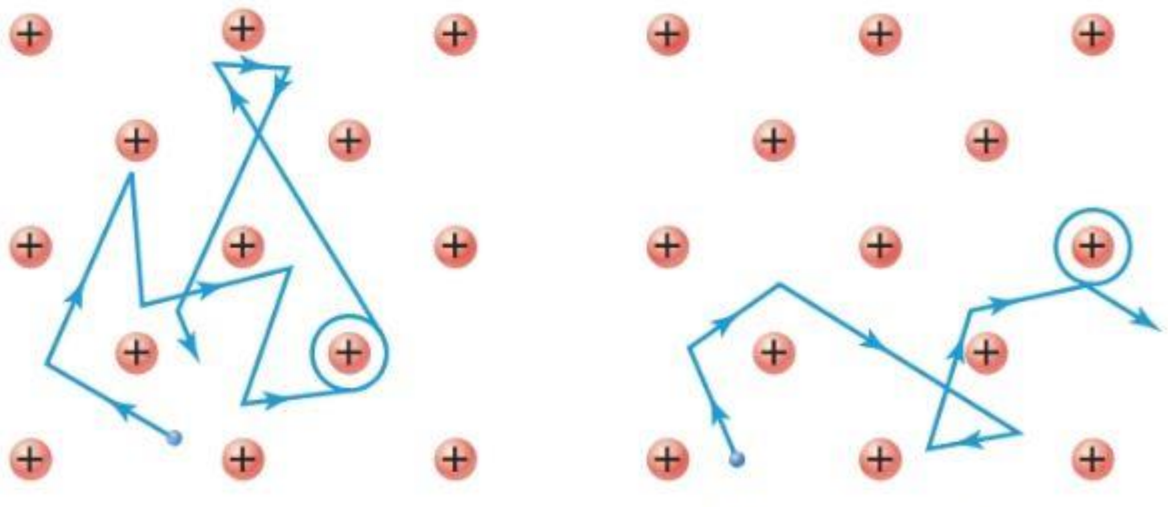




Current, Drift Velocity, and Current Density

If there is no electric field inside a conductor, an electron moves randomly from point P1 to point P2 in a time Δt. If an electric field $\vec{E}$ is present, the electric force $\vec{F} = q\vec{E}$ imposes a small drift (greatly exaggerated here) that takes the electron a distance $v_d t$



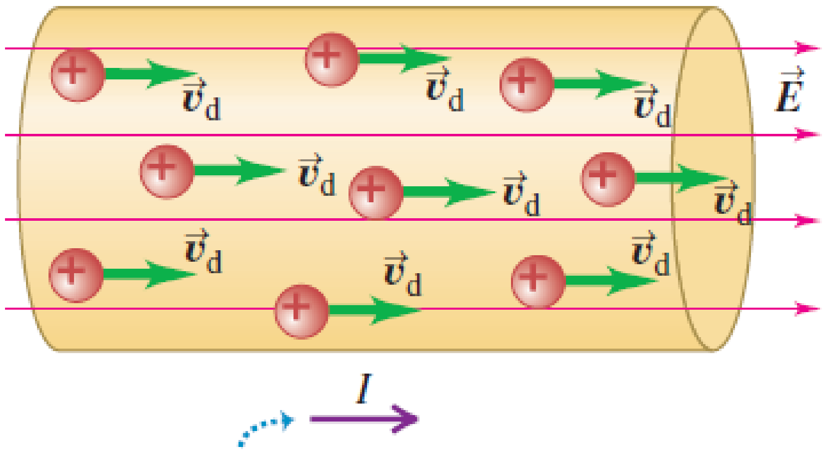


(a) Random motion

(b) Electron drift due to an external energy source

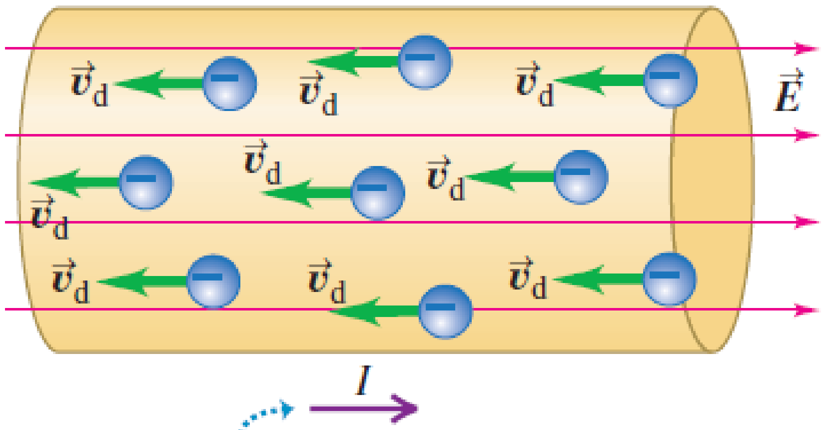
Current, Drift Velocity, and Current Density

The same current can be produced by (a) positive charges moving in the direction of the electric field $\vec{E}$ (b) the same number of negative charges moving at the same speed in the direction opposite to $\boldsymbol{E}$.



A conventional current is treated as a flow of positive charges, regardless of whether the free charges in the conductor are positive, negative, or both.

When an electric field is applied, the field exerts a force on each free electron, giving it a change in velocity in the direction opposite the field.



In a metallic conductor, the moving charges are electrons — but the *current* still points in the direction positive charges would flow.

The net result of this repeated acceleration and dissipation of energy is that the electrons drift along the wire with a small average velocity, called the *drift velocity*. The drift speed is the magnitude of the drift velocity.

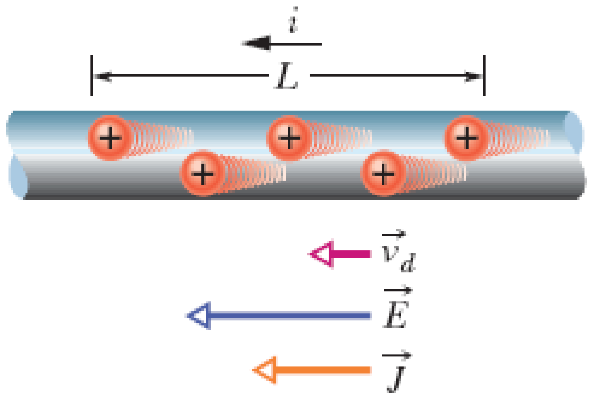
Current, Drift Velocity, and Current Density

The current per unit cross sectional area is called the current density. For each element of the cross section, the magnitude *J* is equal to the current per unit area through that element.

$$i = \int \vec{J} . d\vec{A} = J \int dA = JA$$

$$J = \frac{i}{A}$$

Current is said to be due to positive charges that are propelled by the electric field.



For each element of the cross section, the magnitude *J* is equal to the current per unit area through that element.

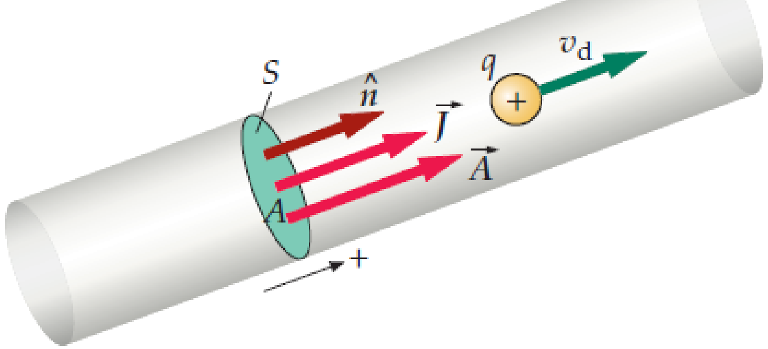
Current is not a vector

Current Density is a vector

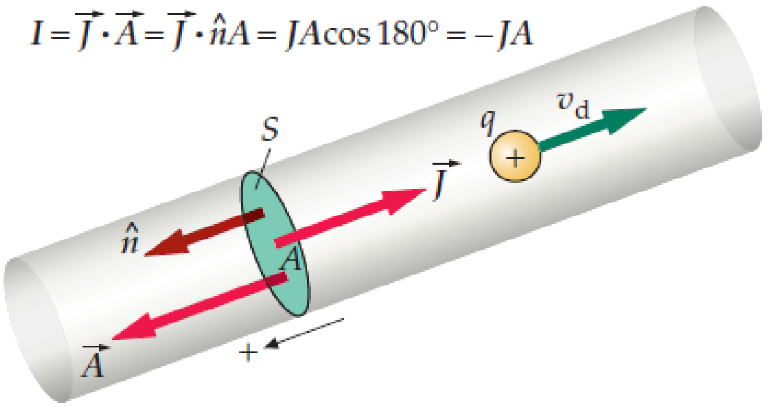
The difference is that the current density describes how charges flow at a certain point, and the vector's direction tells you about the direction of the flow at that point.
By contrast, the current describes how charges flow through an extended object such as a wire.

Current, Drift Velocity, and Current Density

$$I = \vec{J} \cdot \vec{A} = \vec{J} \cdot \hat{n} A = JA\cos(0) = +JA$$

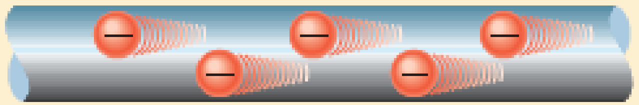


$$I = \vec{J} \cdot \vec{A} = \vec{J} \cdot \hat{n} A = JA\cos 180° = -JA$$



Checkpoint 2

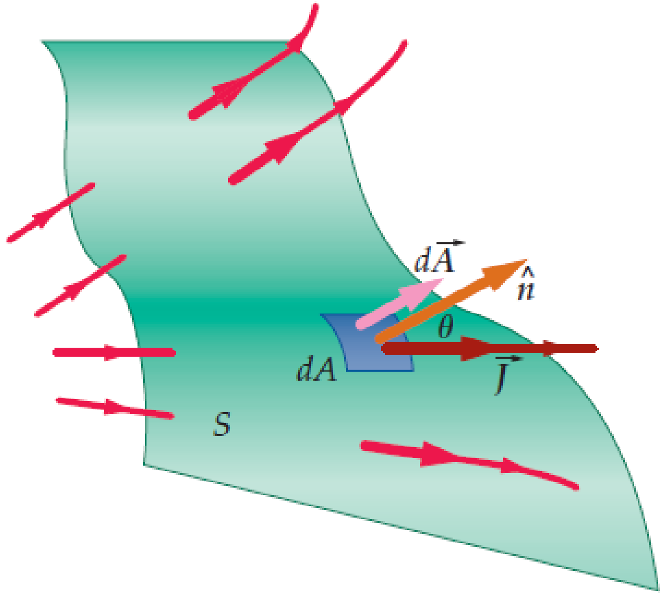
The figure shows conduction electrons moving leftward in a wire. Are the following leftward or rightward: (a) the current i, (b) the current density $\vec{J}$, (c) the electric field $\vec{E}$ in the wire?

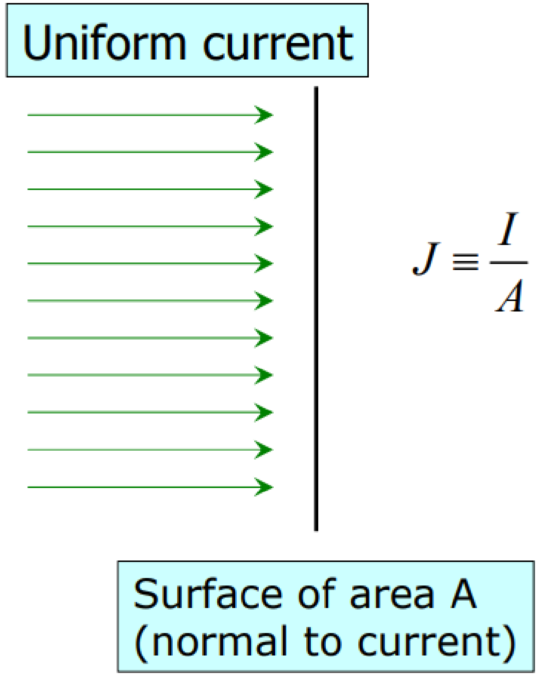


Current, Drift Velocity, and Current Density

$$I = \frac{dQ}{dt} = n|q|v_d A \qquad J = \frac{I}{A} = n|q|v_d$$

$$\vec{J} = nq\vec{v}_d \qquad \text{(vector current density)}$$





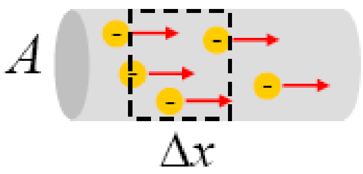
$$J \equiv \frac{I}{A} \qquad J = \text{"current density" (A/m}^2\text{)}$$

For positive carriers, nq is positive and $\vec{J}$ have the same direction as $\vec{v_d}$. For negative carriers, ne is negative and $\vec{J}$ and $\vec{v_d}$ have opposite directions.

Current, Drift Velocity, and Current Density

Current and Electron Drift Speed

- Consider a material where current (electrons) is flowing
  - Let n_e = # free charge carriers / m^3
  - Let q = charge per charge carrier
  - Let A = cross sectional area of material



- Total charge ΔQ in volume element moving past a point

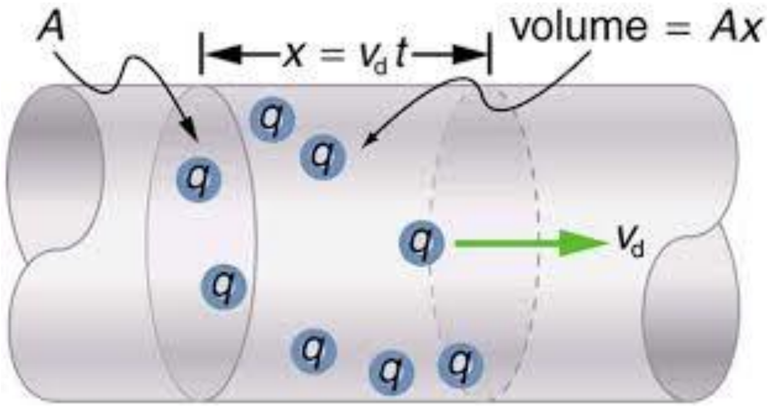
$$\Delta Q = (n_e A \Delta x) q \qquad \text{using } \Delta V = A\Delta x$$

- If charges moving with drift speed v_d, then $\Delta x = v_d \Delta t$

$$\Delta Q = (n_e A v_d \Delta t) q$$

- Thus, current can be written in terms of basic quantities

$$i = \frac{\Delta Q}{\Delta t} = n_e q A v_d$$



Example of Electron Flow

➔Consider a current of 1A. Find the number of electrons flowing past a point per second

$$\frac{\Delta q}{\Delta t} = 1\ \text{A} \Rightarrow 1\ \text{coulomb / sec}$$

➔So, in one second, number of electrons passing a point is

$$N_e = \frac{1\ \text{coulomb}}{1.6\times 10^{-19}} = 6.2\times 10^{18}\ \text{electrons}$$

Example of Drift Speed

- 10A flowing through a copper wire of diameter 2mm
  - Density of Cu = 8.92 g/cm3
  - 1 free electron per Cu atom
  - Atomic mass A_{Cu} = 63.5
- Find drift speed v_d using $i = n_e e A v_d$
  - e is electron charge $e = 1.6 \times 10^{-19}$
  - Find A: $A = \pi r^2 = 3.14 \times \left(10^{-3}\right)^2 = 3.14 \times 10^{-6}\ \mathrm{m}^2$
  - Still need n_e = density of electrons (#/m³)

$$n_e = \frac{\rho_{\mathrm{Cu}}}{m_{\mathrm{Cu}}} \times 1 = \frac{8.92 \times 10^3}{63.5 \times 10^{-3} / 6.02 \times 10^{23}} = 8.5 \times 10^{28} / \mathrm{m}^3$$

Example of Drift Speed (cont.)

- Solve for electron drift speed v_d

$$v_d = \frac{i}{n_e e A} = \frac{10}{\left(8.5\times10^{28}\right)\left(1.6\times10^{-19}\right)\left(3.14\times10^{-6}\right)} = 2.4\times10^{-4}\ \text{m/s}$$

- Thus v_d is 0.24 mm/sec: ~1 hour to move 1 m

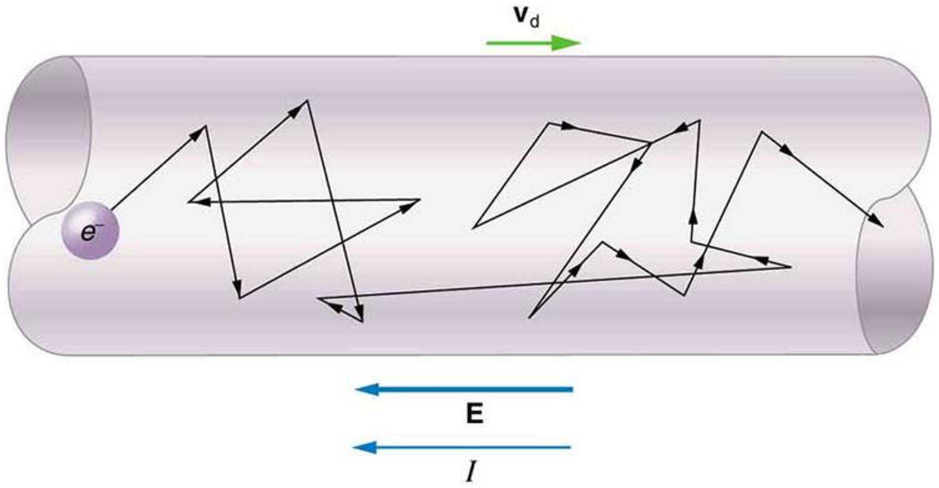
- But electrons actually move ~ 10^6 m/s in material!
  - This is ~ 4×10^9 times larger than drift speed

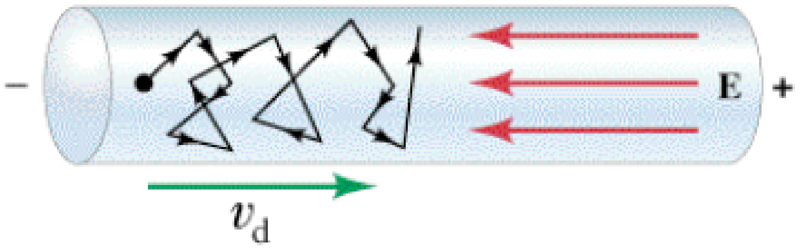
Current, Drift Velocity, and Current Density

Electrons in the Wire

➔ If the electrons move so slowly through the wire, why does the light go on right away when we flip a switch?

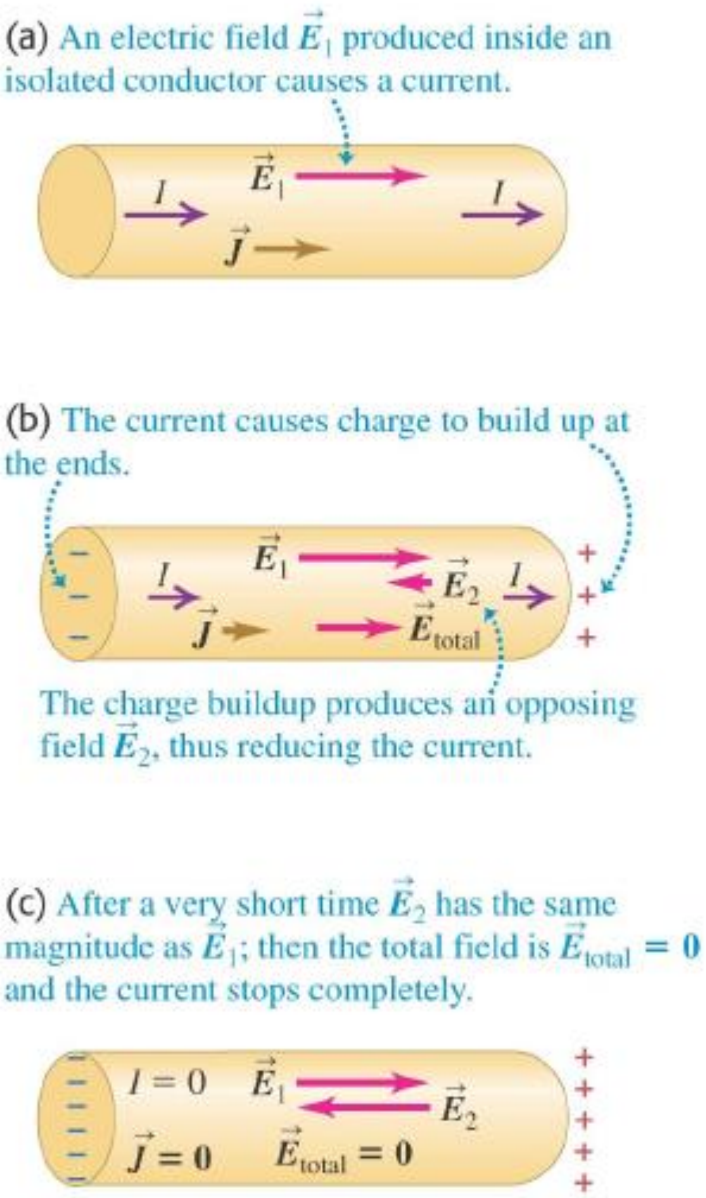
- Household wires have almost no resistance
- The electric field inside the wire travels much faster
- Light switches do not involve currents
- None of the above



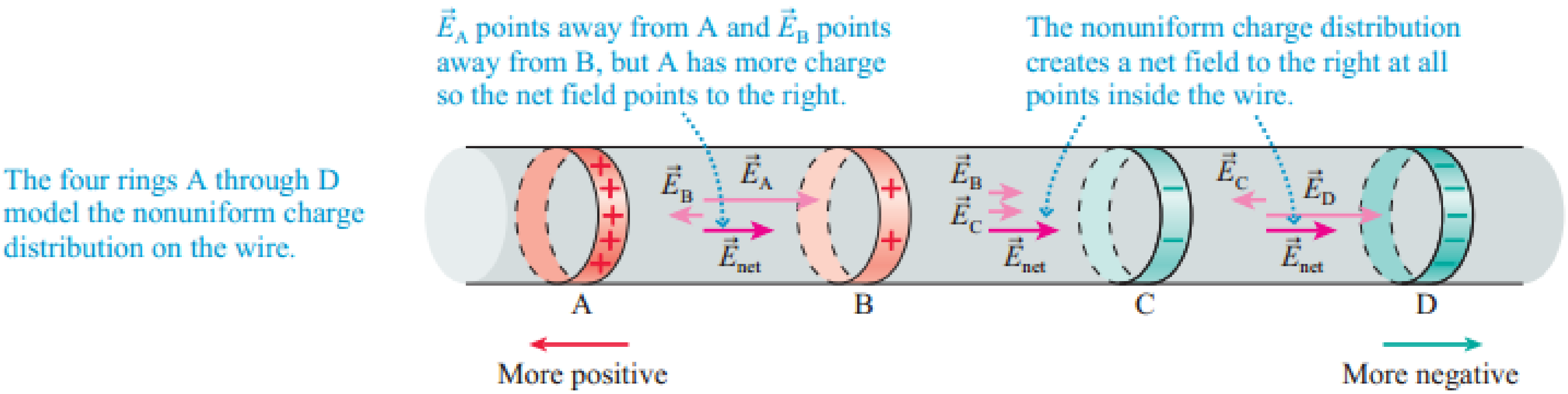


Like a hose full of water when you turn on the faucet

A *nonuniform* distribution of surface charges along a wire creates a net electric field *inside* the wire that points from the more positive end of the wire toward the more negative end of the wire. This is the internal electric field E that pushes the electron current through the wire.



the wire.



Current, Drift Velocity, and Current Density

4 Figure 26-18 shows plots of the current i through a certain cross section of a wire over four different time periods. Rank the periods according to the net charge that passes through the cross section during the period, greatest first.

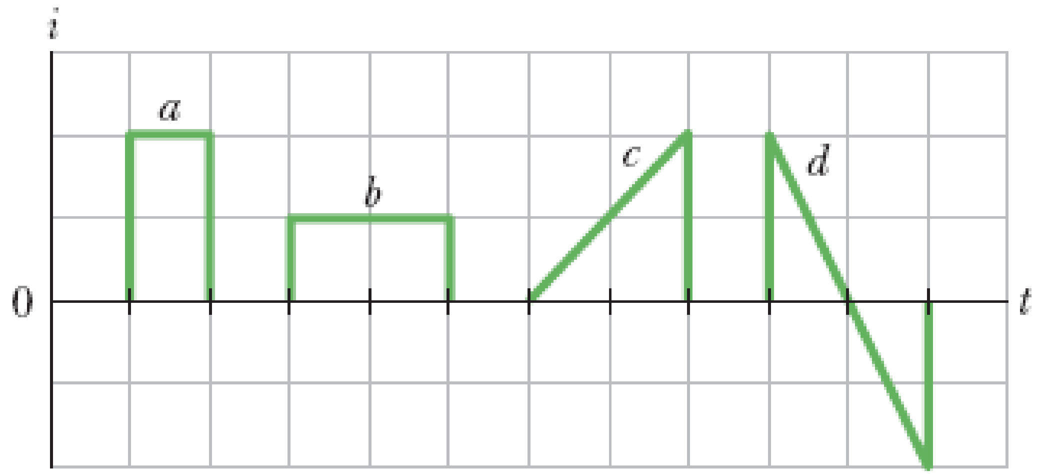
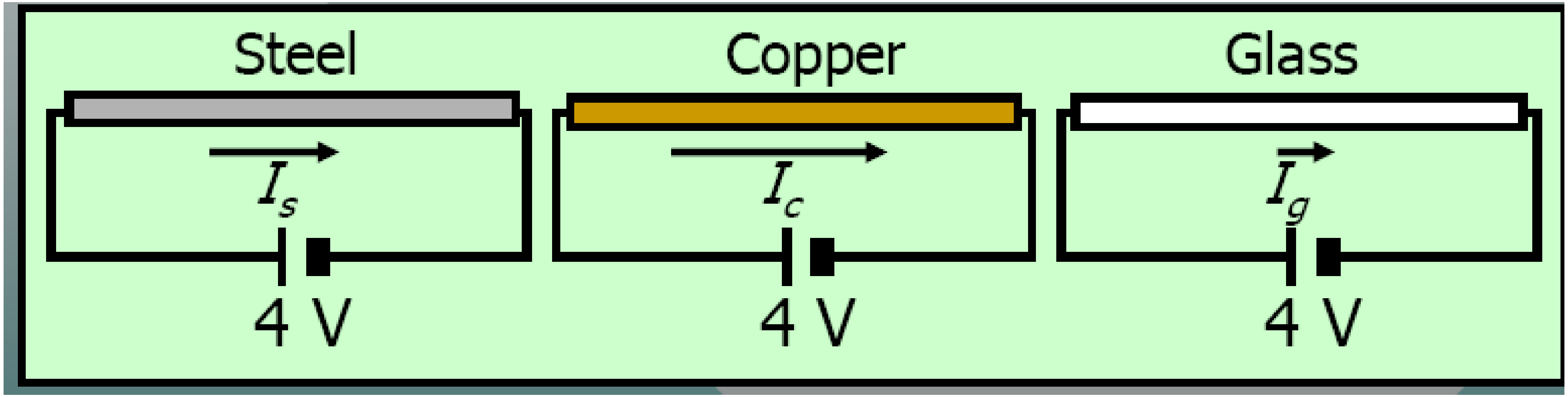


Figure 26-18 Question 4.

Consider a long straight cylindrical wire with radius **R** carrying the ***nonuniform*** current density, $\boldsymbol{J(r)} = \boldsymbol{ar^2}$, where **r** is the distance from the center axis of the wire. How much total current **I** passes through the wire?

Consider a long straight cylindrical wire with radius **R = 0.1 m** is carrying the ***non-uniform*** current density which points along the axis of the wire and has a magnitude, $\boldsymbol{J(r)} = \boldsymbol{a/r}$, where **r** is the distance from the center axis of the wire. If **a = 2 A/m**, how much total current **I** passes through the wire (**in A**)?

Resistance and Resistivity



The current in glass is much less than for steel or iron, suggesting a property of materials called electrical resistance R.

We define the **resistivity** of a material as the ratio of the magnitudes of electric field and current density: for uniform material where E is same

$$\rho = \frac{E}{J} \qquad \text{(definition of resistivity)}$$

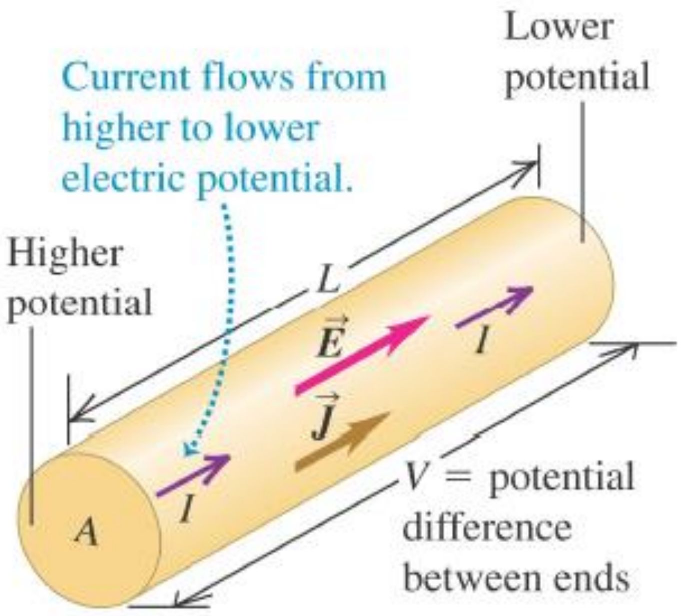
$$\sigma = \frac{\vec{J}}{\vec{E}}$$ Definition of conductivity

Whereas the resistance of an object is the ratio of electric potential and current:

$$R = \frac{V}{I}$$

Resistance is a property of an object.

Resistivity is a property of a material.



Resistance unit: Ohm or Ω

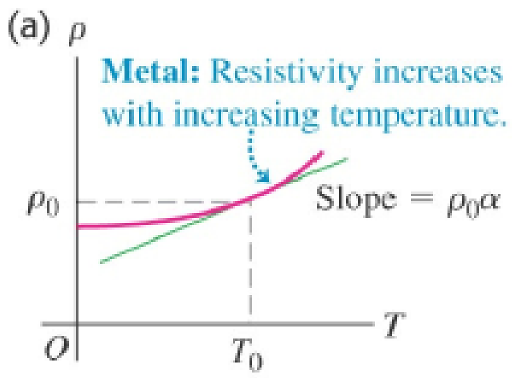
Resistivity unit: Ω-m

Conductivity unit: Ω^{-1}-m^{-1} or mho

Conductivity: $1/\rho$

Metals: good electrical and thermal conductors. Very large difference in conductivity of metals vs. insulators → possible to confine electric currents.

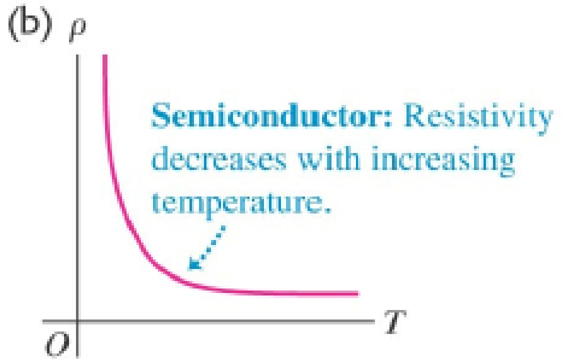
Semiconductors: intermediate resistivity between metal & insulator.



Resistivity and Temperature:

$$\boxed{\rho(T) = \rho_0[1+\alpha(T-T_0)]}$$

α = temperature coefficient of resistivity

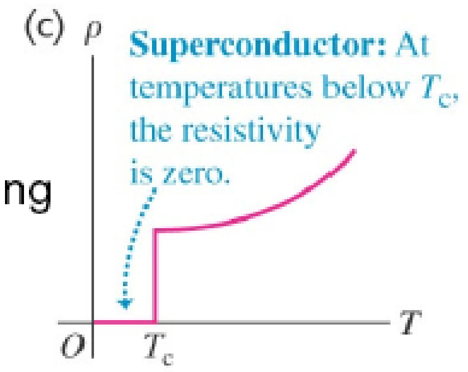


Metal: ρ increases with T

Semiconductor: ρ decreases with T

Superconductor: ρ first decreases smoothly with decreasing T and becomes zero < Tc (critical T)

Highest T_c = 233 K (2009) → $Ta_5Ba_4Ca_2Cu_{10}O_x$



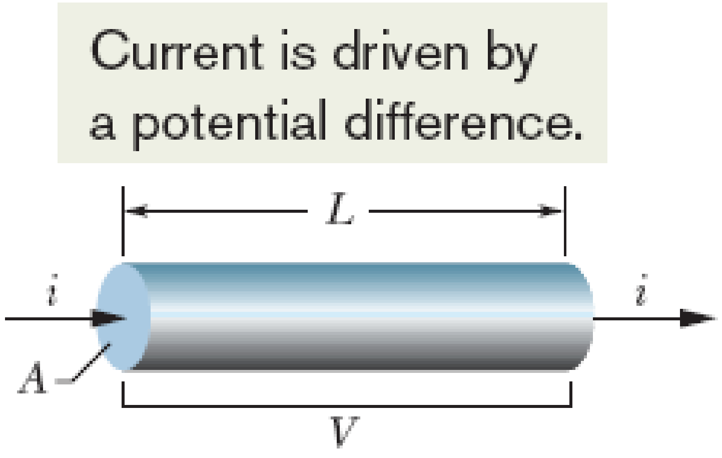


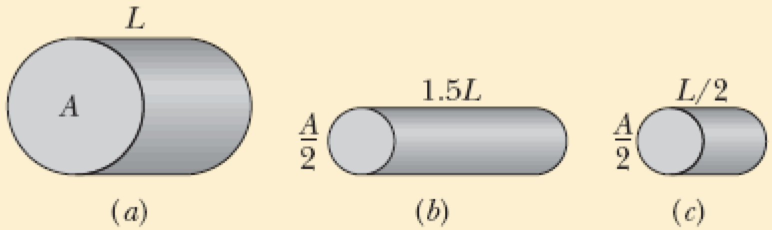
Figure 26-9 A potential difference V is applied between the ends of a wire of length L and cross section A, establishing a current i.

$$E = V/L \quad \text{and} \quad J = i/A.$$

$$\rho = \frac{E}{J} = \frac{V/L}{i/A}.$$

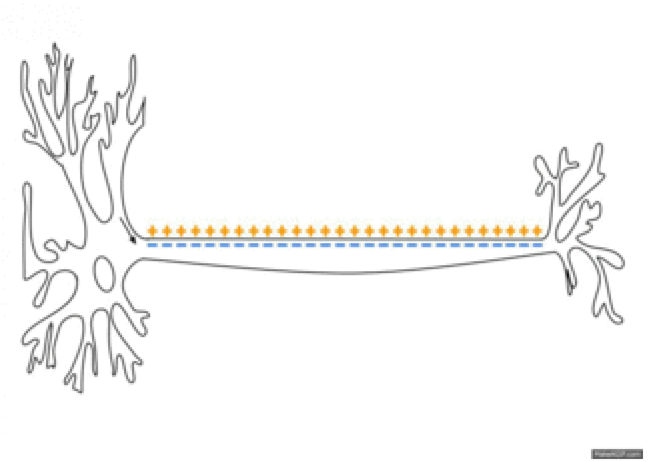
$$\boxed{R = \rho\frac{L}{A}.}$$

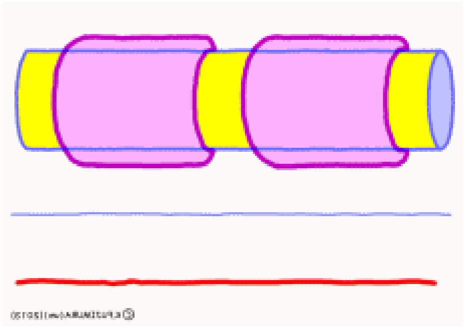
The figure here shows three cylindrical copper conductors along with their face areas and lengths. Rank them according to the current through them, greatest first, when the same potential difference V is placed across their lengths.



Application Resistivity and Nerve Conduction

This false-color image from an electron microscope shows a cross section through a nerve fiber about 1 μm (10^{-6} m) in diameter. A layer of an insulating fatty substance called myelin is wrapped around the conductive material of the axon. The resistivity of myelin is much greater than that of the axon, so an electric signal traveling along the nerve fiber remains confined to the axon. This makes it possible for a signal to travel much more rapidly than if the myelin were absent.

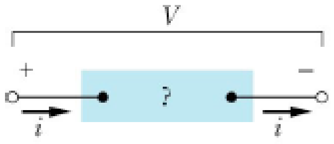




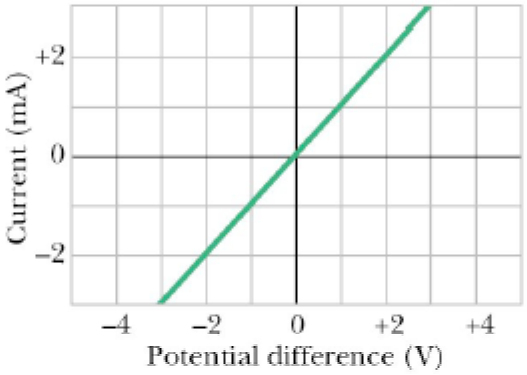
Ohm’s Law

Ohm’s law is an assertion that the current through a device is *always* directly proportional to the potential difference applied to the device.

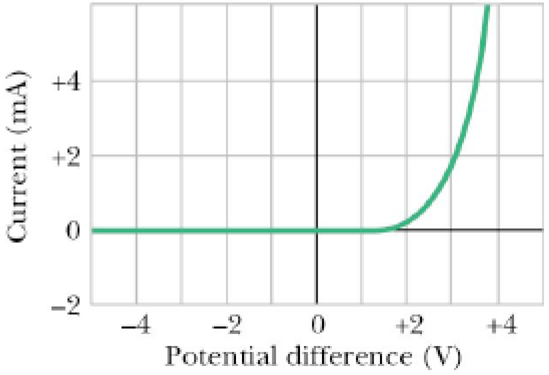
A test of whether or not a material satisfies Ohm’s Law



(a)



(b)



(c)

$$V = IR$$

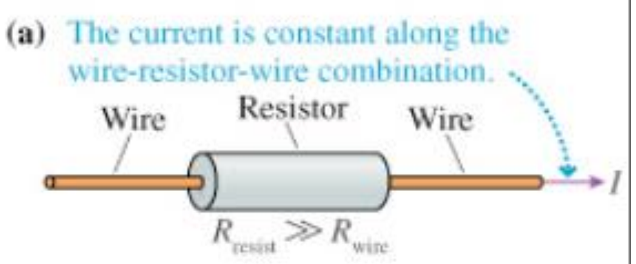
$$I = \frac{V}{R}$$

$$\text{Slope} = \frac{1}{R} = \text{constant}$$

Ohm's law is satisfied

Here the slope depends on the potential difference. Ohm's Law is violated for a pn junction diode.

Ohmic and Nonohmic materials; Ideal Wire Model



(a) Ohmic material

- ideal wires: $R = 0 \Rightarrow$

  $\Delta V = 0$ even $I \neq 0$

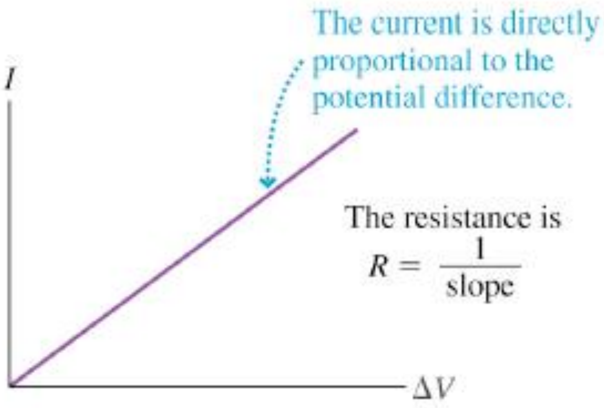
- resistors:

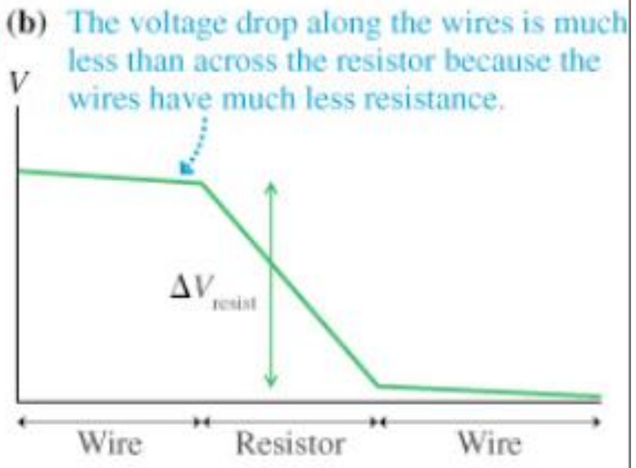
  10 to $10^6 \Omega$

- ideal insulators:

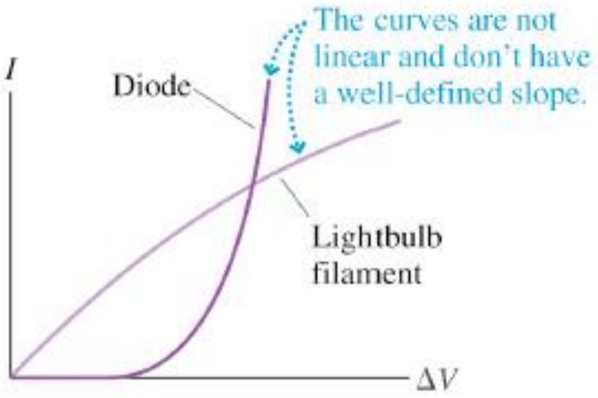
  $R = \infty \Rightarrow$

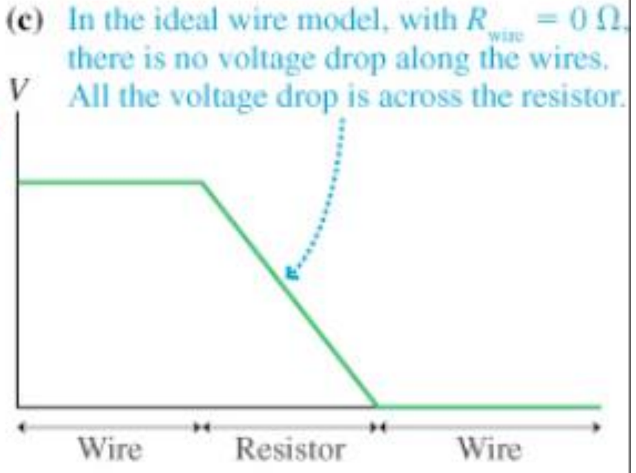
  $I = 0$ even if $\Delta V \neq 0$





(b) Nonohmic materials





A conducting device obeys Ohm’s law when the resistance of the device is independent of the magnitude and polarity of the applied potential difference.

A conducting material obeys Ohm’s law when the resistivity of the material is independent of the magnitude and direction of the applied electric field.

All homogeneous materials, whether they are conductors like copper or semiconductors like pure silicon or silicon containing special impurities, obey Ohm’s law within some range of values of the electric field. If the field is too strong, however, there are departures from Ohm’s law in all cases.

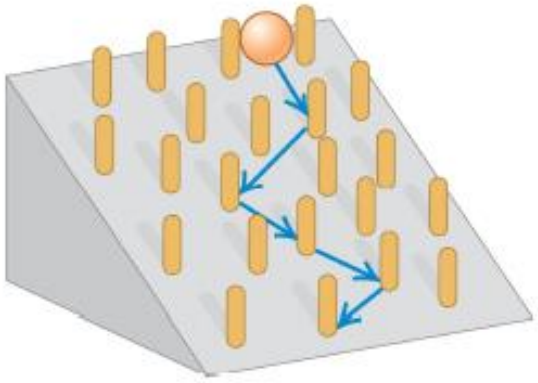
A Microscopic View of Ohm's Law

6. Theory of Metallic Conduction

- If no E → free e- move in straight lines between collisions with + ions
  → random velocities, in average, no net displacement.

- If E → e- path curves due to acceleration caused by F_e → drift speed.

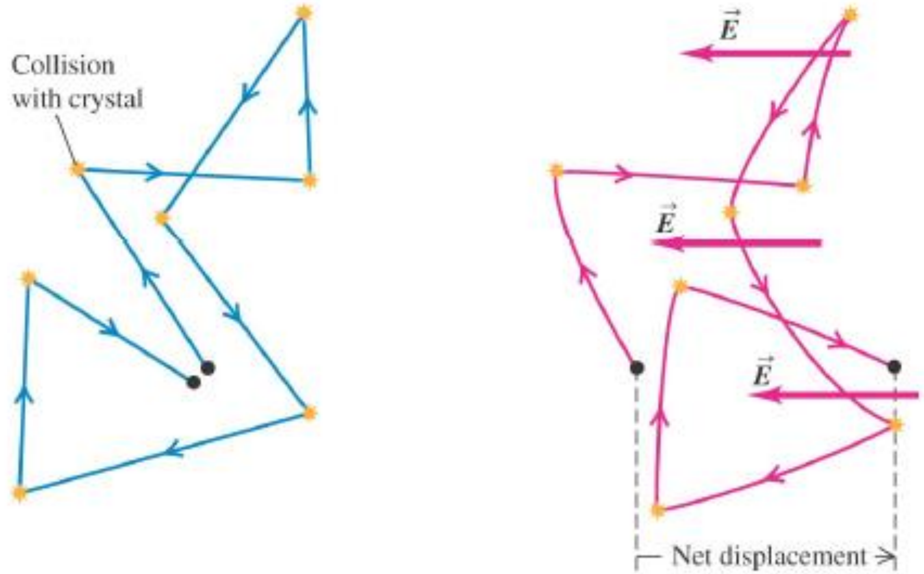
Mean free time (τ): average time between collisions.



Analogy to motion of e- with E.

(a) Typical trajectory for an electron in a metallic crystal *without* an internal $\vec{E}$ field

(b) Typical trajectory for an electron in a metallic crystal *with* an internal $\vec{E}$ field



A Microscopic View of Ohm's Law

$$\rho = \frac{E}{J} \qquad \vec{J} = n \cdot q \cdot \vec{v}_d \qquad \vec{a} = \frac{\vec{F}}{m} = \frac{q\vec{E}}{m}$$

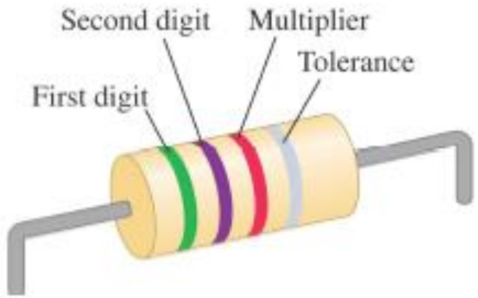
$$\vec{v} = \vec{v}_0 + \vec{a}\,\tau \qquad \vec{v}_{avg} = \vec{a}\,\tau = \frac{q\tau}{m}\vec{E} = \vec{v}_d$$

$$\vec{J} = n \cdot q \cdot \vec{v}_d = \frac{nq^2\tau}{m}\vec{E}$$

$$\boxed{\rho = \frac{E}{J} = \frac{m}{q^2 n\tau} = \frac{m}{e^2 n\tau}}$$

Thus, because the right side of equation is independent of the field magnitude, metals obey Ohm's law.

Resistor: circuit device with a fixed R between its ends.



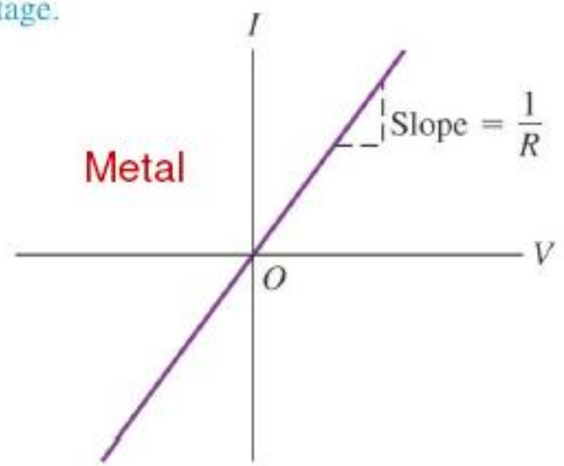
Ex: 5.7 kΩ = green (5) violet (7) red multiplier (100)

Table 25.3 Color Codes for Resistors

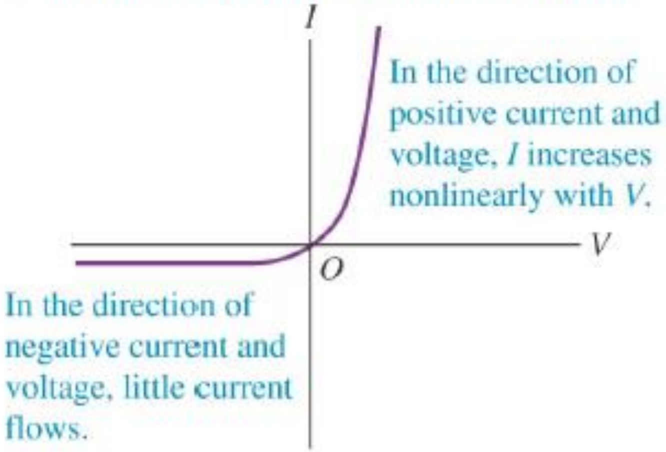
Color	Value as Digit	Value as Multiplier
Black	0	1
Brown	1	10
Red	2	10^2
Orange	3	10^3
Yellow	4	10^4
Green	5	10^5
Blue	6	10^6
Violet	7	10^7
Gray	8	10^8
White	9	10^9

Current-voltage curves

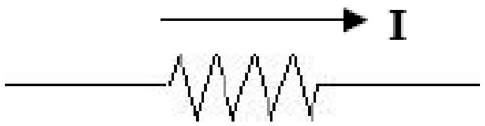
Ohmic resistor (e.g., typical metal wire): At a given temperature, current is proportional to voltage.



Semiconductor diode: a nonohmic resistor



Power dissipation resistors



Potential energy decrease

$$\Delta U = \Delta Q(-V)$$

$$\frac{\Delta U}{\Delta t} = \frac{\Delta Q}{\Delta t}(-V)$$

$$P = IV$$ (drop the minus sign)

Electric power P
is the rate at which electric energy is expended, or work per unit of time.

Rate of potential energy decreases equals rate of thermal energy increases in resistor.

Called Joule heating

- good for stove and electric oven
- nuisance in a PC – need a fan to cool computer

Also since V = IR,

$$P = I^2R \text{ or } \frac{V^2}{R}$$ All are equivalent.

Example: How much power is dissipated when I = 2A flows through the Fe resistor of

R = 10,000 Ω. $P = I^2R = 2^2 x 10^4\ \Omega = 40,000$ Watts

4. Electromotive Force and Circuits

- No steady motion of charge in incomplete circuit.

Electromotive Force (emf)

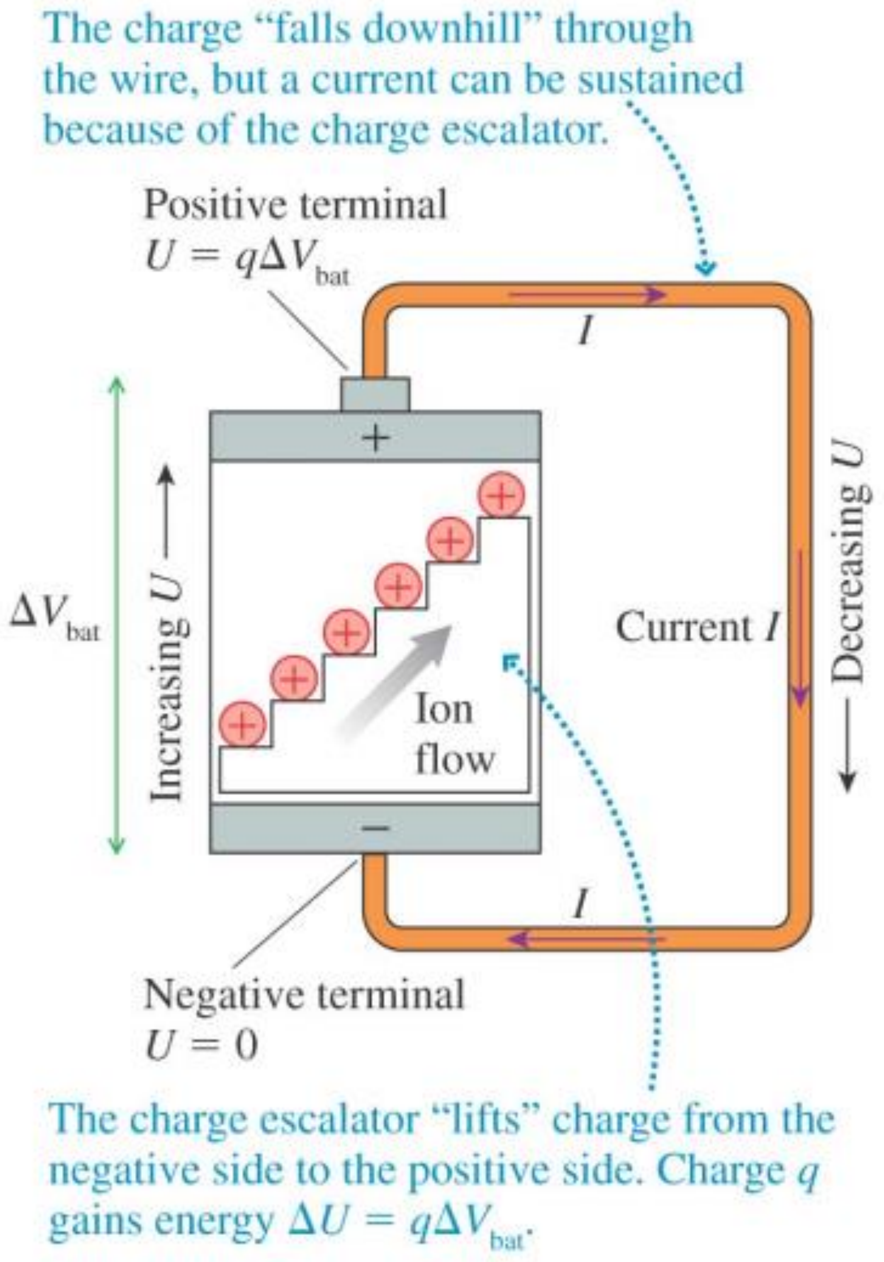
- In an electric circuit there should be a device that acts like the water pump in a fountain = source of emf.

- In this device, the charge travels "uphill" from lower to higher V (opposite to normal conductor) due to the emf force.

- emf is not a force but energy/unit charge

Units: 1 V = 1 J/C

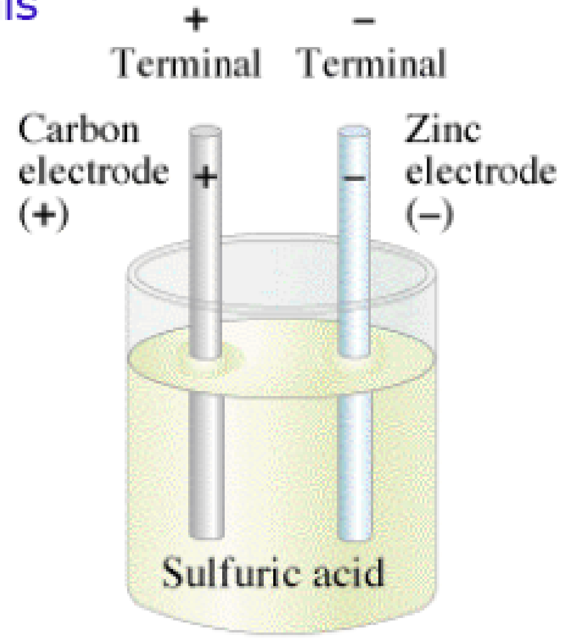
- emf device convert energy (mechanical, chemical, thermal) into electric potential energy and transfer it to circuit.



- EMF device performs work on charge carriers
  - Converts energy to electrical energy
  - Moves carriers from low potential to high potential
  - Maintains potential difference across terminals
- Various types of EMF devices

Battery	Electrolytic reaction
Generator	Magnetic field
Fuel cell	Oxidation of fuel
Solar cell	Electromagnetic energy
Thermopile	Nuclear decay

- Example: battery
  - Two electrodes (different metals)
  - Immersed in electrolyte (dilute acid)
  - One electrode develops + charge, the other – charge



Ideal diagram of “open” circuit

- Ideal emf device maintains a constant potential difference between its terminals, independent of I.

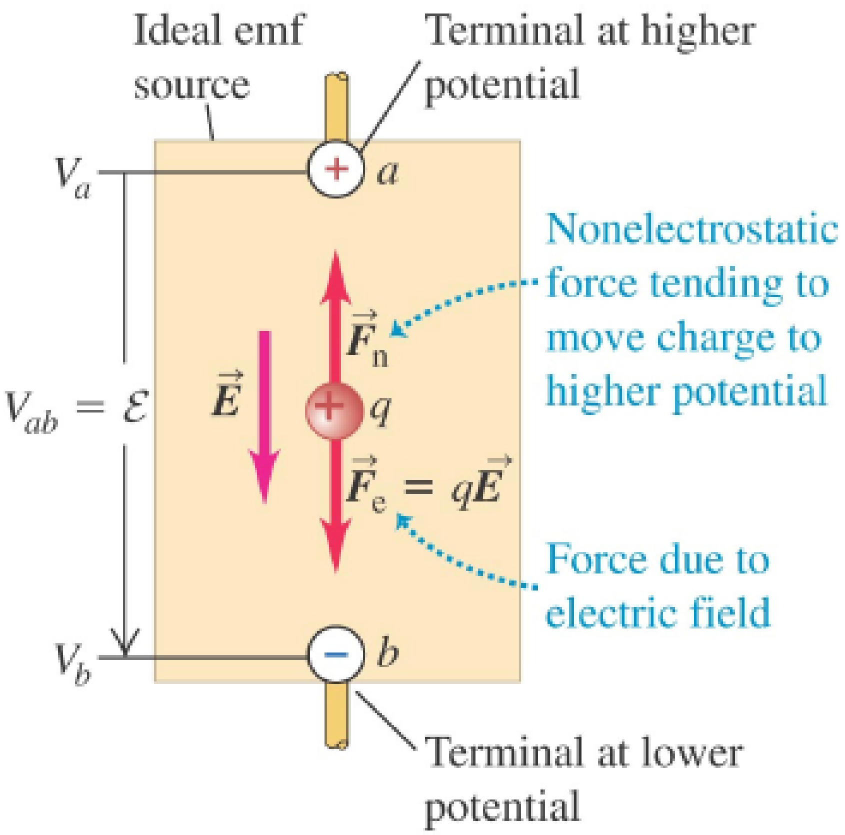
Electric force: $\vec{F}_e = q\vec{E}$

Non electrostatic force: $\vec{F}_n$

maintains potential difference between terminals. If $F_n=0$ → charge will flow between terminals until $V_{ab}=0$

$W_n = q\varepsilon$ displacement opposite to F_e → potential energy Increases by $q \cdot V_{ab}$

$$W_n = \Delta E = q\varepsilon = \Delta K + \Delta U$$
$$= U_a - U_b = q(V_a - V_b)$$



When the emf source is not part of a closed circuit, $F_n = F_e$ and there is no net motion of charge between the terminals.

$$\mathcal{E} = \frac{dW}{dq}$$

$$dW = \mathcal{E}dq$$

$$dW = \mathcal{E}idt$$

$$\mathcal{E}idt = i^2Rdt$$

$$iR = \mathcal{E}$$

$$i = \frac{\mathcal{E}}{R}$$

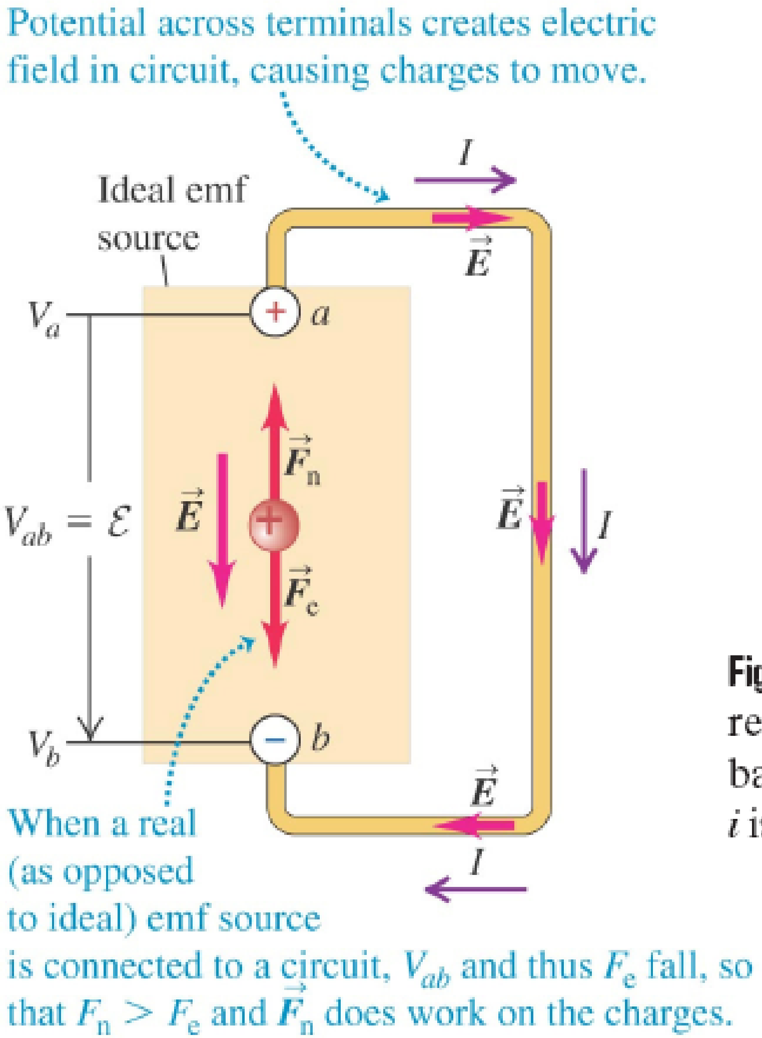
$\boxed{V_{ab} = \varepsilon}$ Ideal source of emf $(F_e = F_n)$ Total work on q = 0

$$\boxed{V_{ab} = \varepsilon = I\,R}$$

- When a positive charge q flows around a circuit, the potential rise ε as it passes through the ideal source is equal to the potential drop V_{ab} as it passes through reminder of circuit.

-The current is same at every point of a circuit, even if wire thickness different at different points of circuit. Charge is conserved and cannot be accumulated in circuit.

Ideal diagram of "closed" circuit



The battery drives current through the resistor, from high potential to low potential.

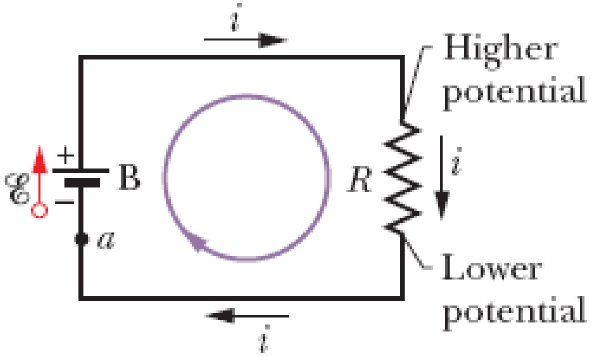


Figure 27-3 A single-loop circuit in which a resistance R is connected across an ideal battery B with emf $\mathscr{E}$. The resulting current i is the same throughout the circuit.

Internal resistance

- In a battery, you only get 12 V when it isn't connected.

- Making connections allows electrons to flow, but internal resistance within battery delivers incrementally less than 12 V.

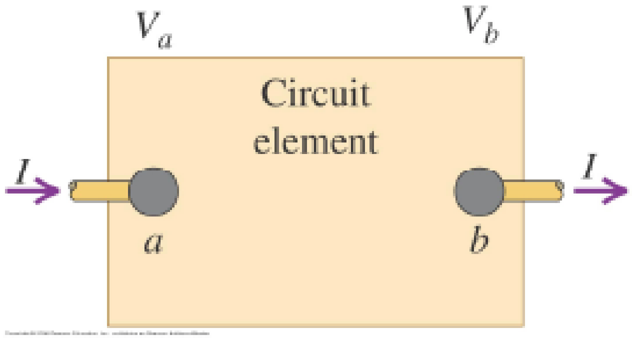
- The potential difference across a real source is not equal to emf. Charge moving through the material of the source encounters internal resistance (r).

Terminal voltage: $\boxed{V_{ab} = \varepsilon - Ir}$ Source with internal resistance

- For a real source, $V_{ab} = \varepsilon$ (emf) only if no current flows through source.

$$I = \frac{\varepsilon}{R + r}$$

5. Energy and Power in Circuits



Power: rate at which energy is delivered to or extracted from a circuit element.

$$\boxed{P = V_{ab}\, I = (V_a - V_b)\, I}$$

Units: 1 Watt = W = V A = (J/C) (C/s) = J/s

Potential Input to a Pure Resistance

$$\boxed{P = V_{ab} I = I^2 R = \frac{V_{ab}^2}{R}}$$

Rate of transfer of electric potential energy into the circuit ($V_a > V_b$) → energy dissipated (heat) in resistor at a rate $I^2 R$.

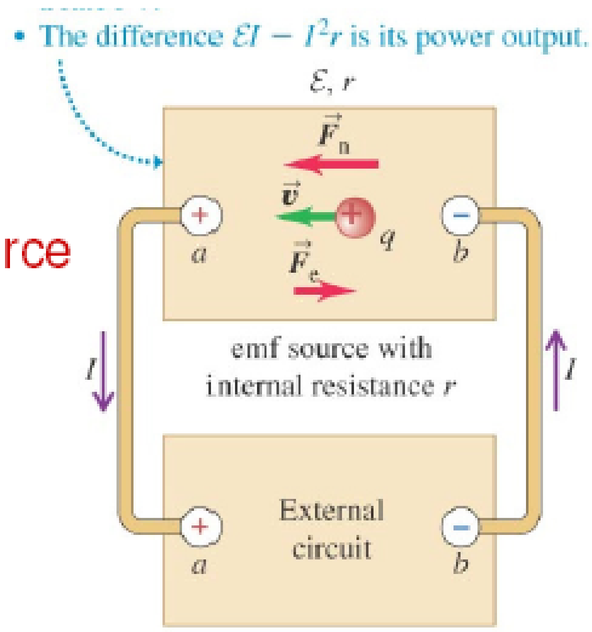
Potential Output of a source

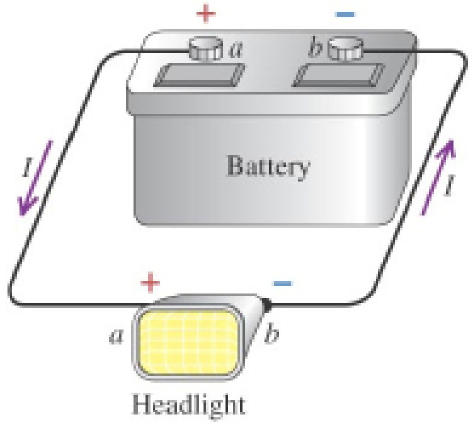
$$\boxed{P = V_{ab} I = (\varepsilon - Ir) I = \varepsilon \cdot I - I^2 r}$$

ε I = rate at which the emf source converts nonelectrical to electrical energy.

I^2 r = rate at which electric energy is dissipated at the internal resistance of source.

Potential Output of a source



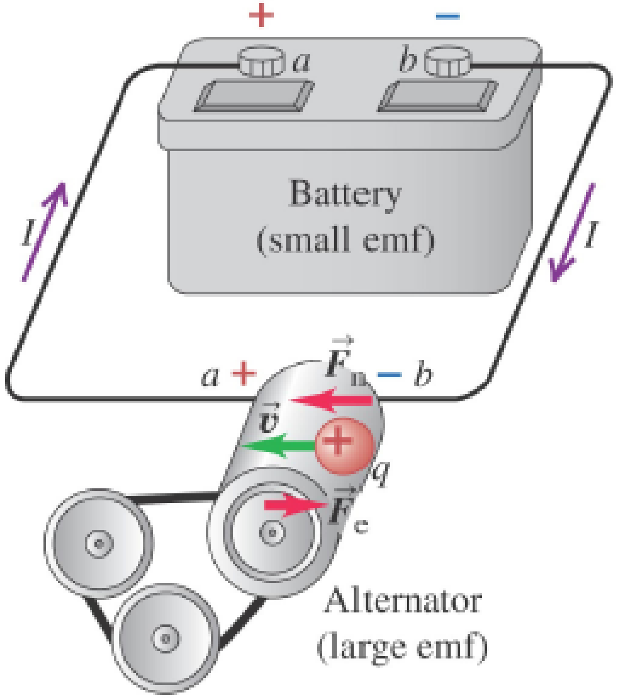


Potential Input to a source

$$\boxed{P = V_{ab} I = (\varepsilon + Ir)I = \varepsilon \cdot I + I^2 r}$$

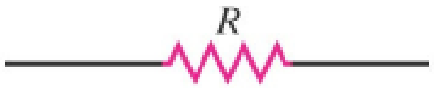
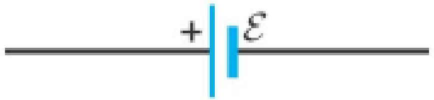
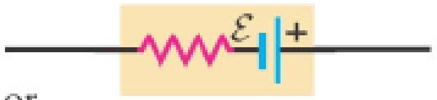
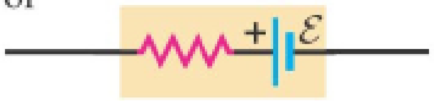
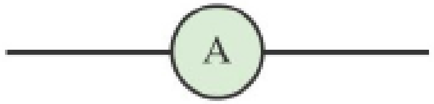
Conversion of electrical energy into non-electrical energy in the upper source at a rate ε I.

$I^2 r$ = rate of dissipation of energy.



Lower source pushing current upward through upper source.

Table 25.4 Symbols for Circuit Diagrams

	Conductor with negligible resistance
	Resistor
	Source of emf (longer vertical line always represents the positive terminal, usually the terminal with higher potential)
 or 	Source of emf with internal resistance r (r can be placed on either side)
	Voltmeter (measures potential difference between its terminals)
	Ammeter (measures current through it)

- The meters do not disturb the circuit in which they are connected.
- Voltmeter → infinite resistance → I = V/ R → I =0 (measures V)
- Ammeter →zero resistance → V = I R = 0 (measures I)