# Gauss' law-II

Phy 108 course
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# Gauss' "Law"

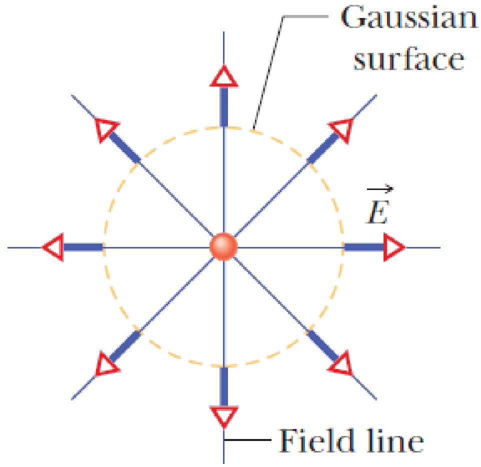



**Figure 23-1** Electric field vectors and field lines pierce an imaginary, spherical Gaussian surface that encloses a particle with charge $+Q$.

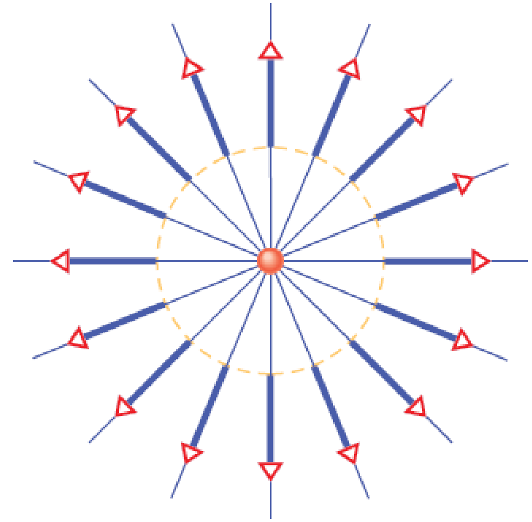
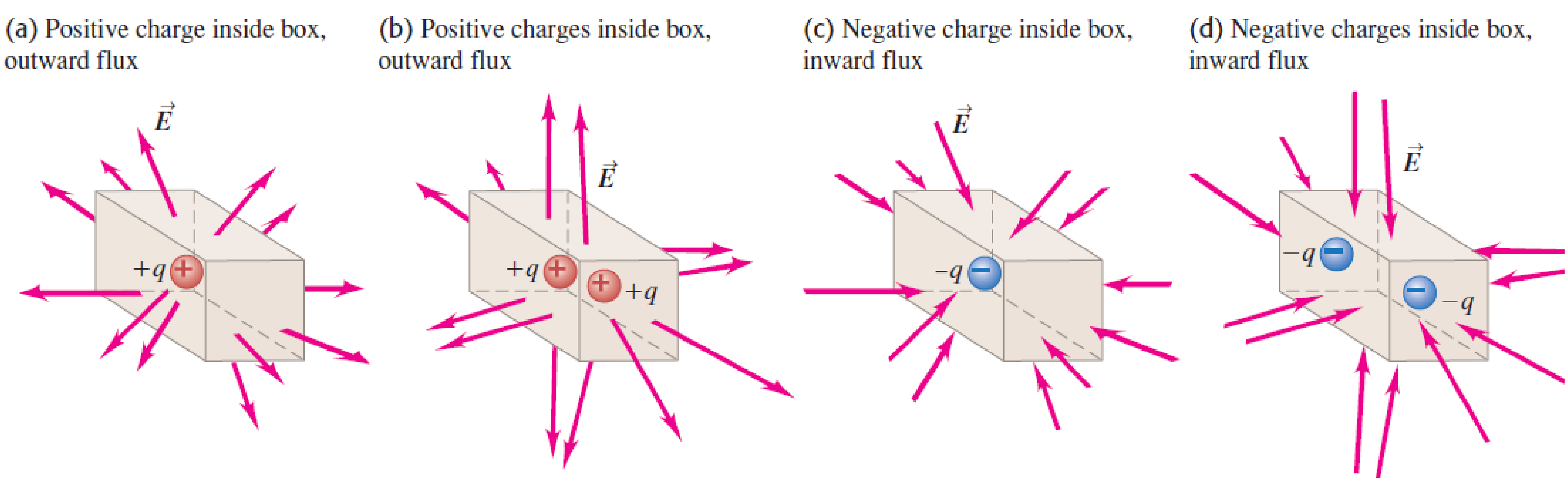


**Figure 23-2** Now the enclosed particle has charge $+2Q$.



# Gauss' "Law"

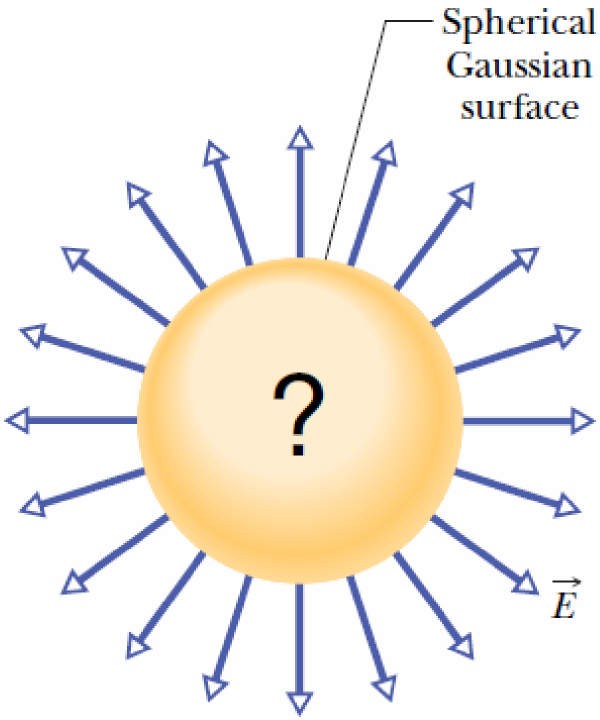



**Fig. 23-1** A spherical Gaussian surface. If the electric field vectors are of uniform magnitude and point radially outward at all surface points, you can conclude that a net positive distribution of charge must lie within the surface and have spherical symmetry.

Gauss' law relates the electric fields at points on a (closed) Gaussian surface to the net charge enclosed by that surface.

# Consider the *flux* (number of electric field lines) passing through a closed surface and the amount of charge inside.

Gauss's law states that the total electric flux through any closed surface (a surface enclosing a definite volume) is proportional to the total (net) electric charge inside the surface.

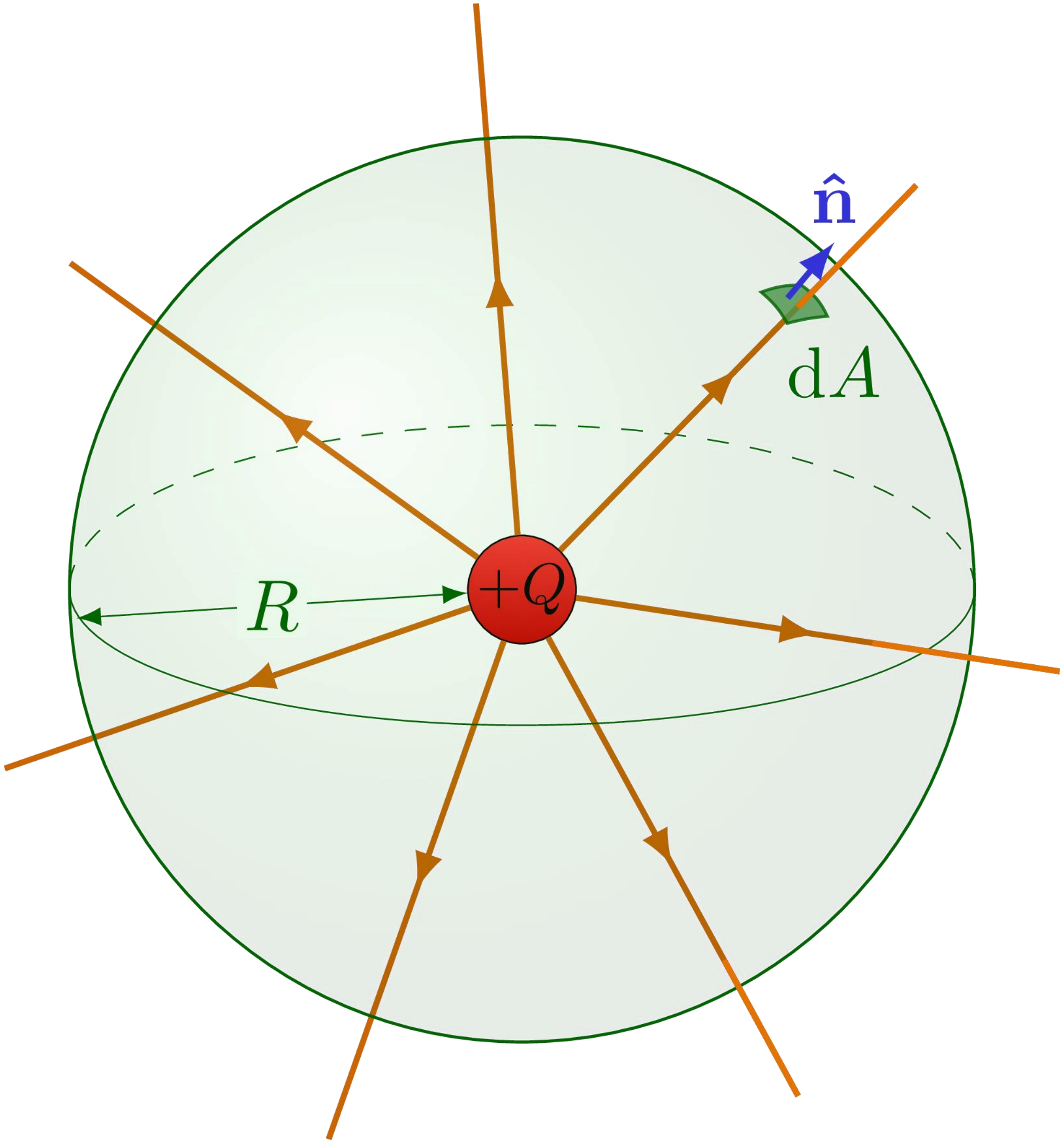
# Gauss' "Law"

Gauss's law generalizes this result to the case of any number of charges and any location of the charges in the space inside the closed surface.

According to Gauss's law, the flux of the electric field through any closed surface, also called a **Gaussian surface**, is equal to the net charge enclosed divided by the permittivity of free space :

$$\Phi_E = \iint \vec{E}.d\vec{A} \qquad \Phi_E = \oint \vec{E} \cdot d\vec{A} = \frac{q_{enclosed}}{\varepsilon_o}$$

The ***Gaussian surface*** does not need to correspond to a real, physical object; indeed, it rarely will. It is a mathematical construct that may be of any shape, provided that it is closed.



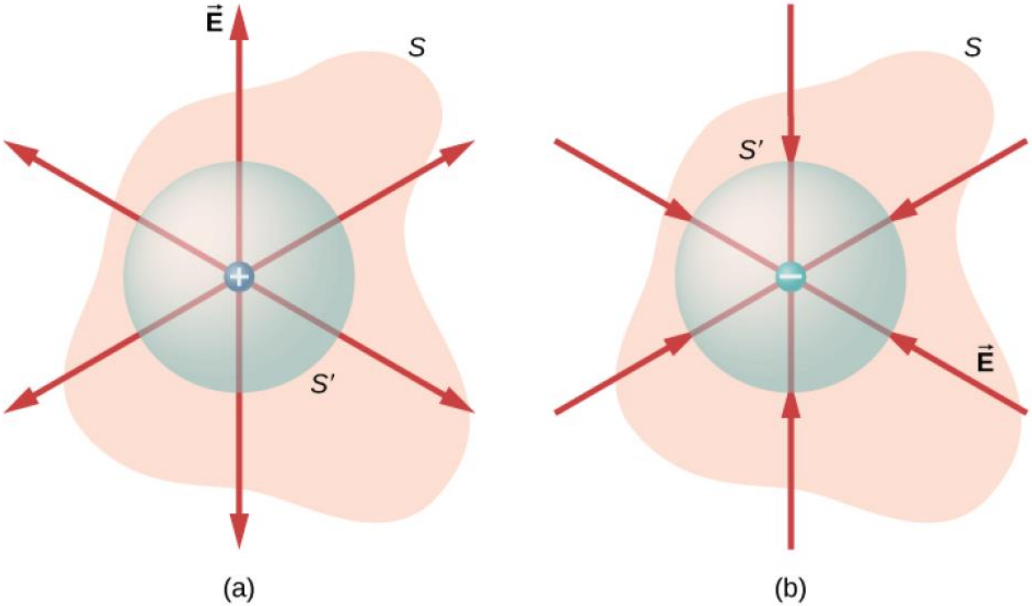
$\hat{\mathbf{n}}$
$\mathrm{d}A$
$R$
$+Q$

# Gauss' "Law"

$$\Phi_E = \oint \vec{E} \cdot d\vec{A} = \frac{q_{enclosed}}{\varepsilon_o}$$

This equation holds for *charges of either sign*, because we define the area vector of a closed surface to point outward.

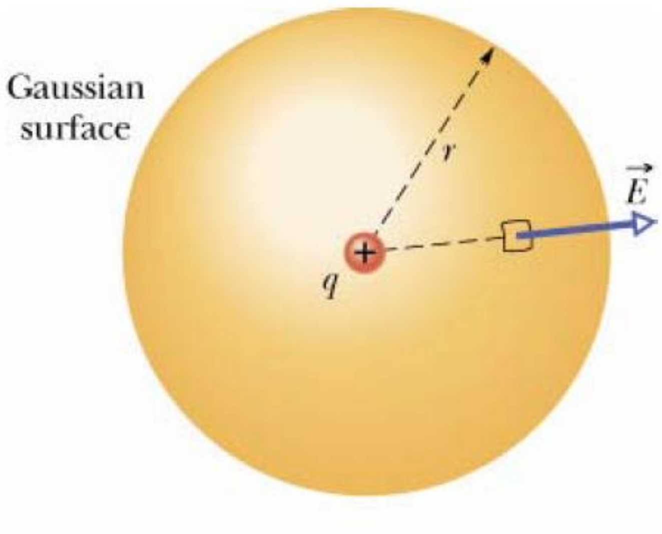
If the enclosed charge is negative (Figure ), then the flux through either or is negative.



# Example: Flux Through Sphere

➔Assume point charge +Q

➔E points radially outward (normal to surface!)

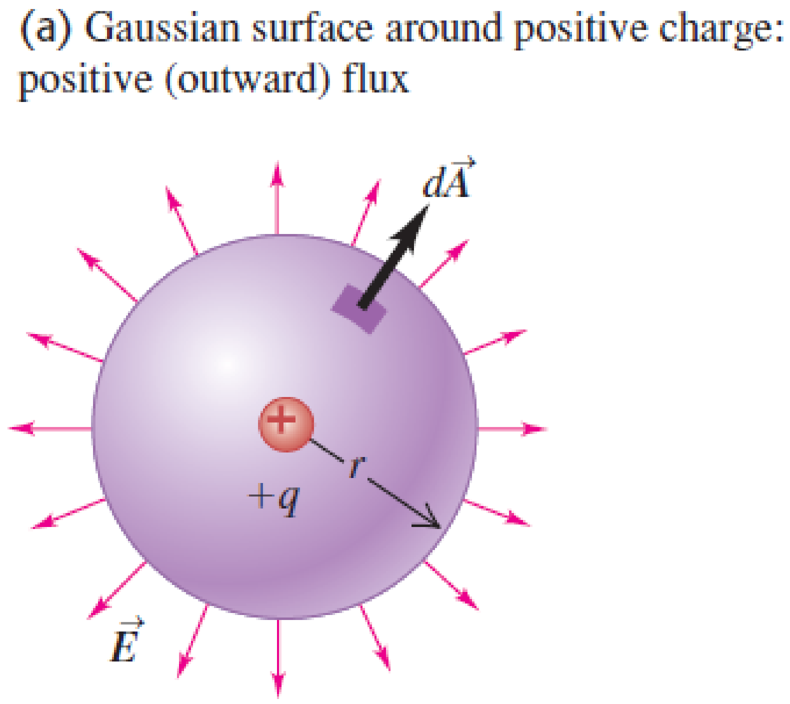


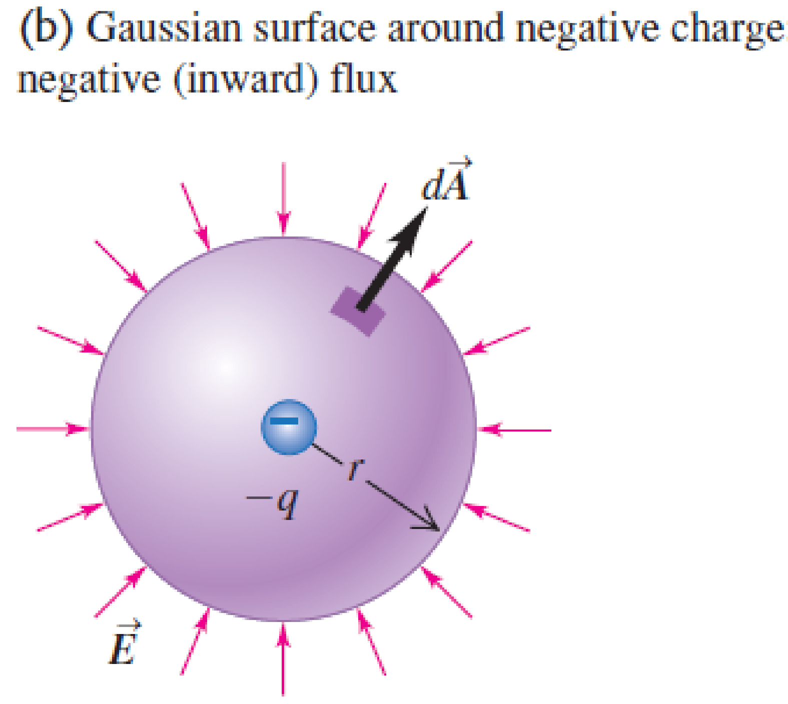

$$\Phi_E = \oint_S \mathbf{E} \cdot d\mathbf{A}$$

$$= \left(\frac{kQ}{r^2}\right)\left(4\pi r^2\right)$$

$$= 4\pi kQ = \frac{Q}{\varepsilon_0}$$

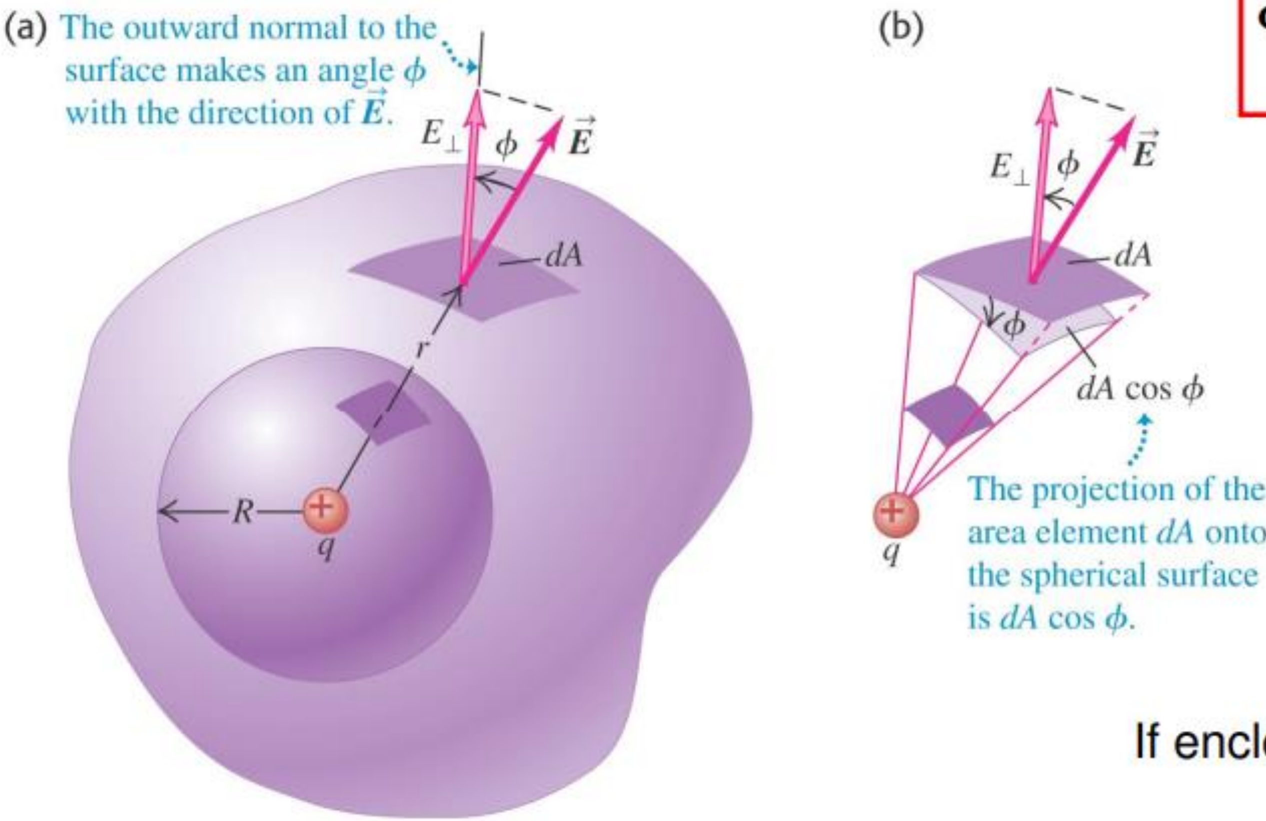
# Gauss' "Law"




$$\Phi_E = \oint E_{\perp}\, dA = \oint \left(\frac{q}{4\pi\epsilon_0 r^2}\right) dA = \frac{q}{4\pi\epsilon_0 r^2} \oint dA = \frac{q}{4\pi\epsilon_0 r^2}\, 4\pi r^2 = \frac{q}{\epsilon_0}$$

$$\Phi_E = \oint E_{\perp}\, dA = \oint \left(\frac{-q}{4\pi\epsilon_0 r^2}\right) dA = \frac{-q}{4\pi\epsilon_0 r^2} \oint dA = \frac{-q}{4\pi\epsilon_0 r^2} 4\pi r^2 = \frac{-q}{\epsilon_0}$$




$$\boxed{\Phi_E = \oint \vec{E} \cdot d\vec{A} = \frac{q}{\varepsilon_0}}$$

Integral through a closed surface

Valid for + / - q

If enclosed q = 0 → $\Phi_E = 0$

# Gauss' "Law"

The ***Gaussian surface*** does not need to correspond to a real, physical object; indeed, it rarely will. It is a mathematical construct that may be of any shape, provided that it is closed. However, since our goal is to integrate the flux over it, we tend to choose shapes that are highly symmetrical.

We will find that Gauss' "Law" gives a simple way to calculate electric fields for charge distributions that exhibit a high degree of symmetry…

# Gauss' "Law"

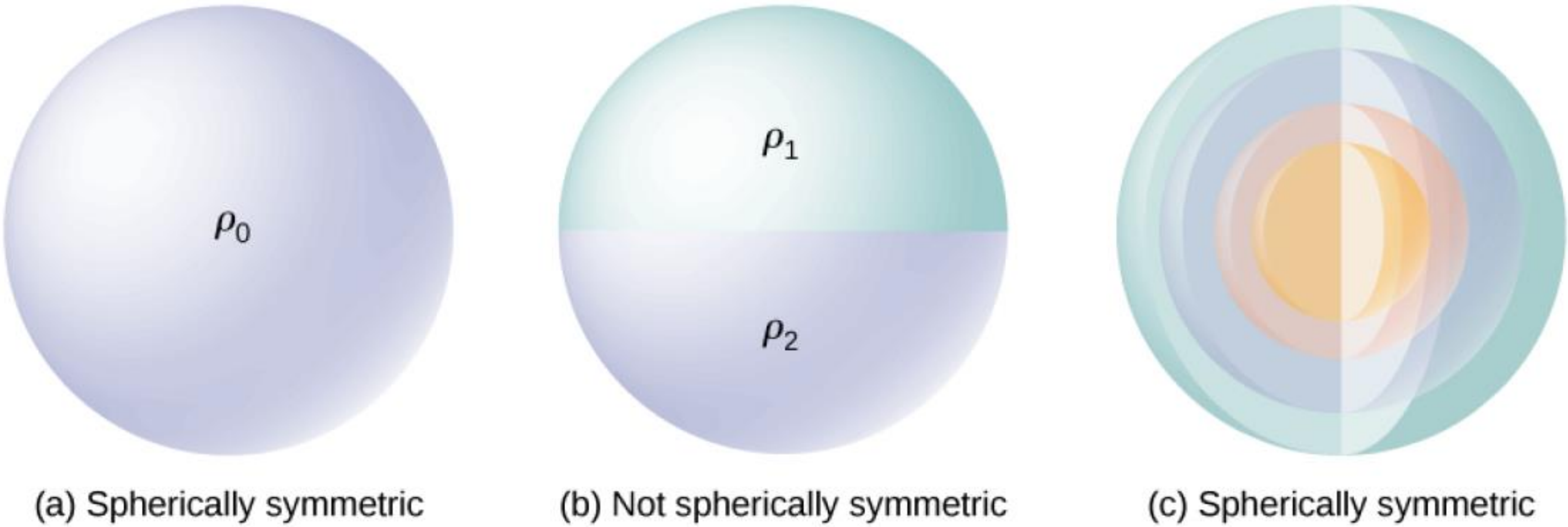



Figure 6.4.1: Illustrations of spherically symmetrical and nonsymmetrical systems. Different shadings indicate different charge densities. Charges on spherically shaped objects do not necessarily mean the charges are distributed with spherical symmetry. The spherical symmetry occurs only when the charge density does not depend on the direction. In (a), charges are distributed uniformly in a sphere. In (b), the upper half of the sphere has a different charge density from the lower half; therefore, (b) does not have spherical symmetry. In (c), the charges are in spherical shells of different charge densities, which means that charge density is only a function of the radial distance from the center; therefore, the system has spherical symmetry.

# Gauss' "Law"

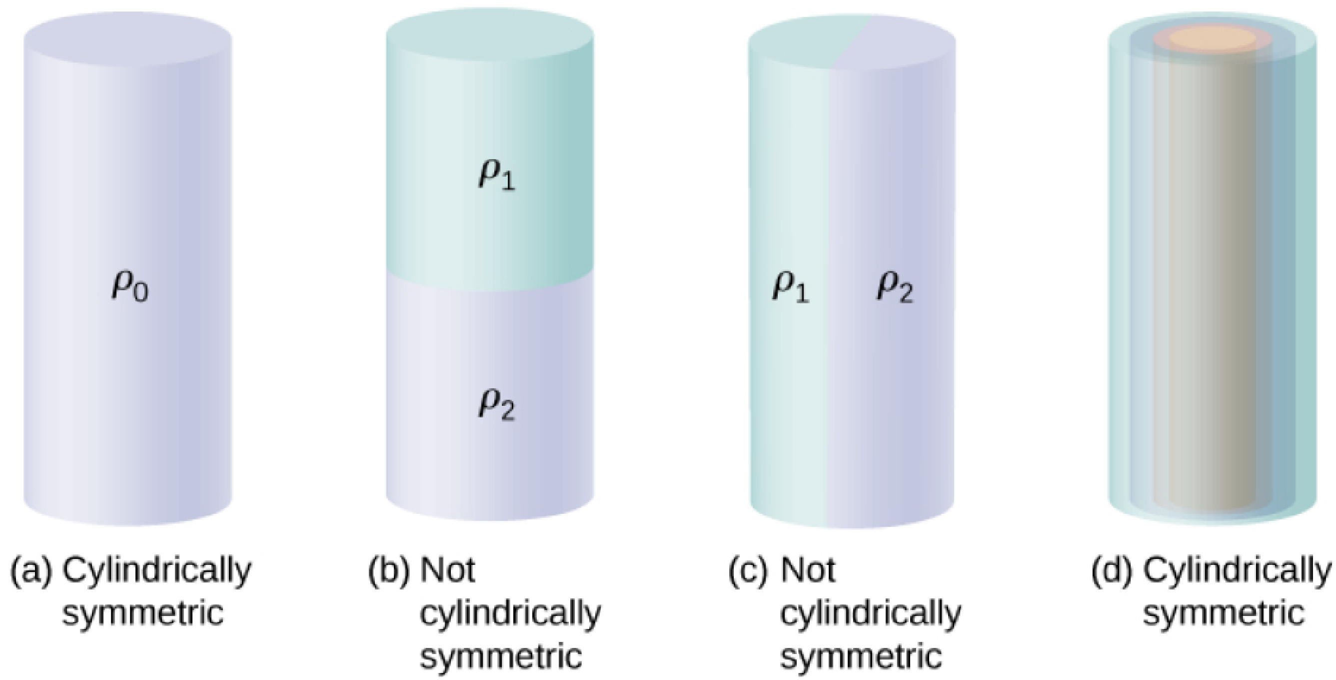
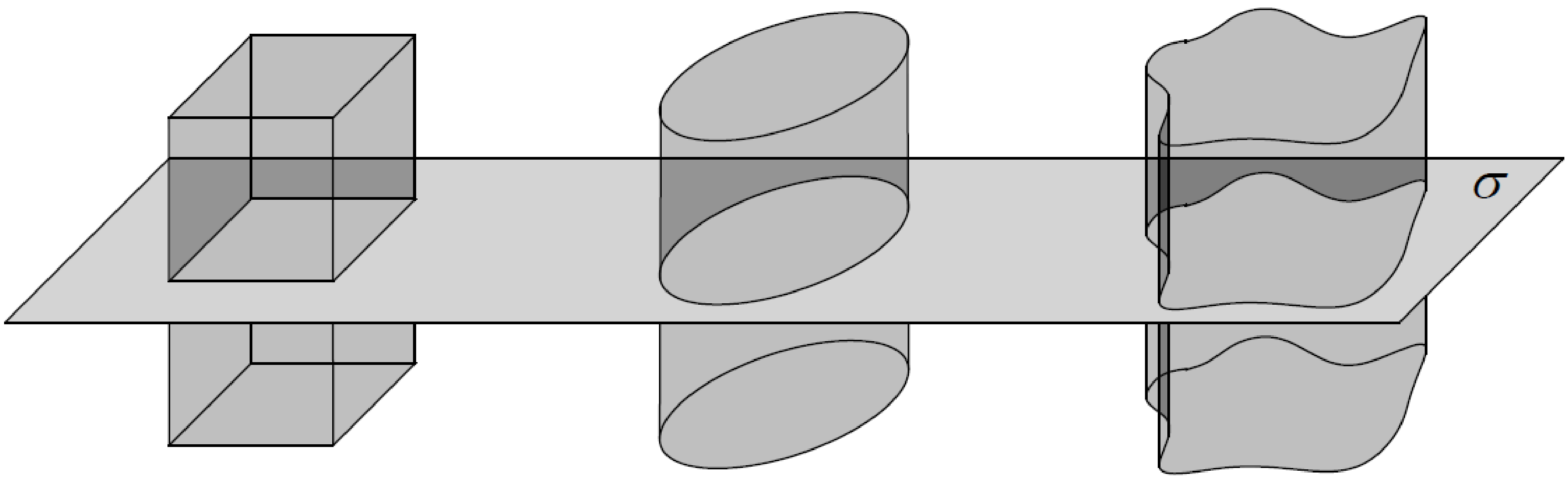


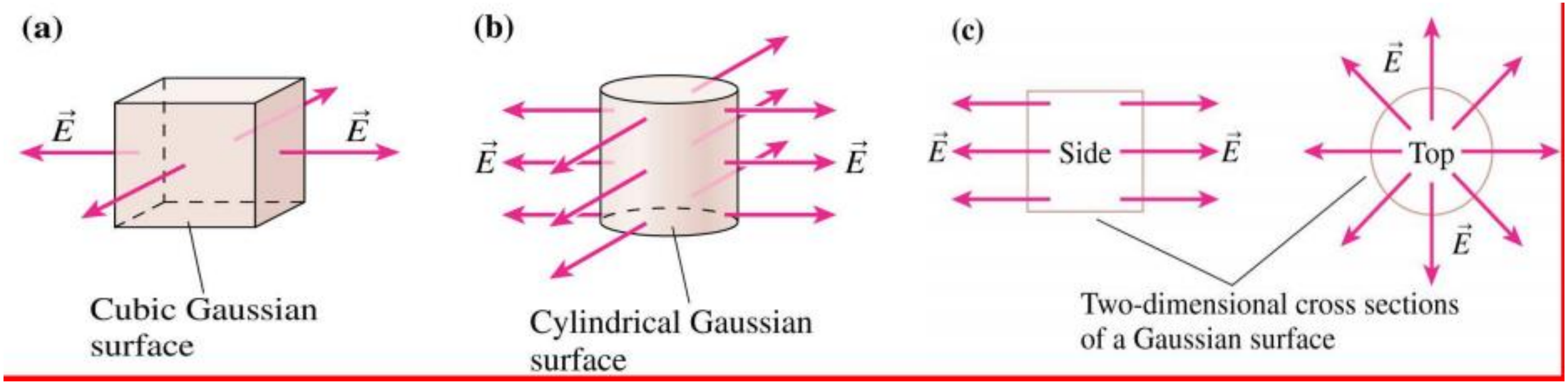

Figure $6.4.7$: To determine whether a given charge distribution has cylindrical symmetry, look at the cross-section of an "infinitely long" cylinder. If the charge density does not depend on the polar angle of the cross-section or along the axis, then you have cylindrical symmetry. (a) Charge density is constant in the cylinder; (b) upper half of the cylinder has a different charge density from the lower half; (c) left half of the cylinder has a different charge density from the right half; (d) charges are constant in different cylindrical rings, but the density does not depend on the polar angle. Cases (a) and (d) have cylindrical symmetry, whereas (b) and (c) do not.

# Gauss' "Law"

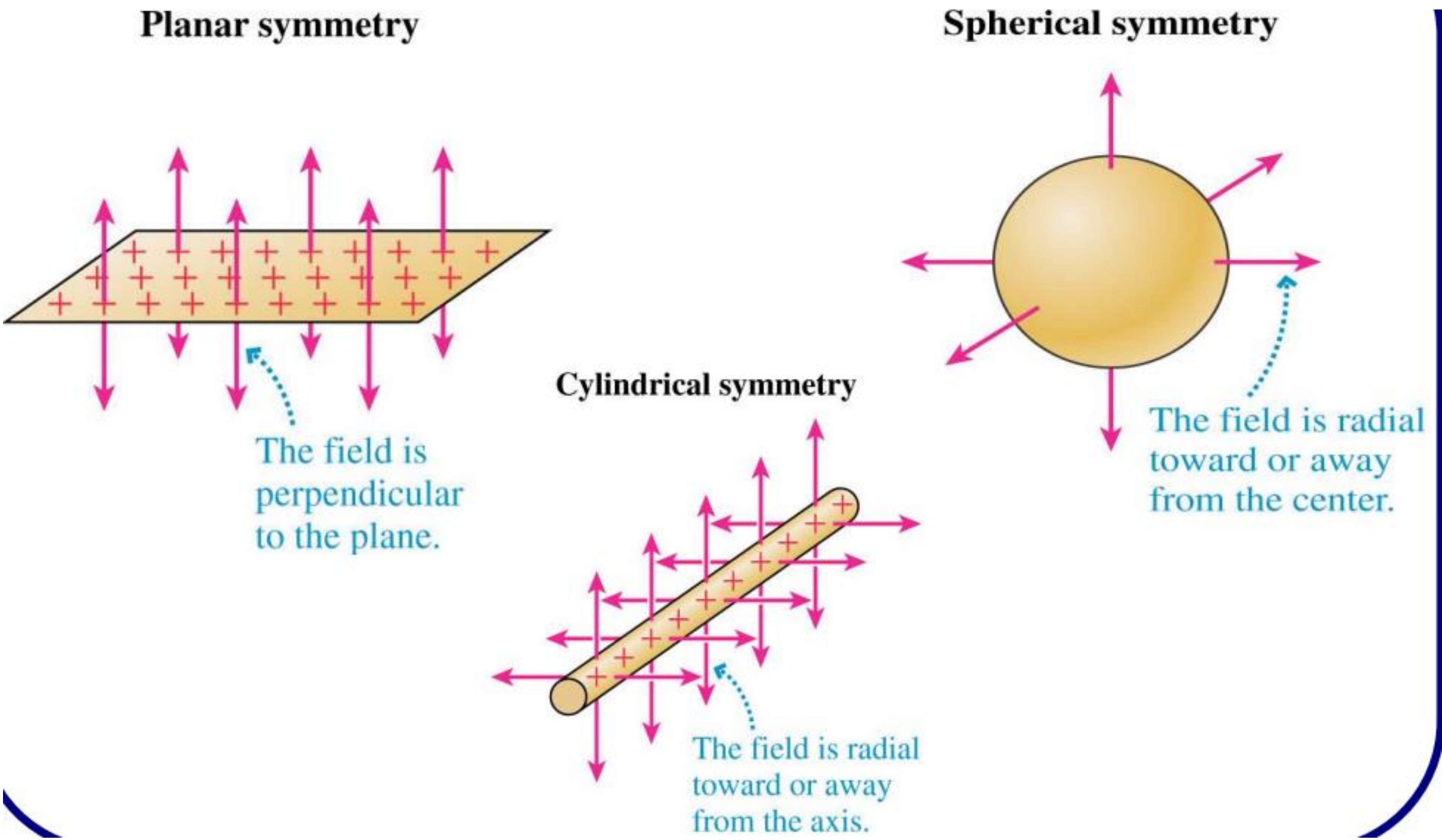
- So the Gaussian surface has to be symmetrical about the plane, and have all sides either parallel or perpendicular to the plane. Any shape with these properties will do.

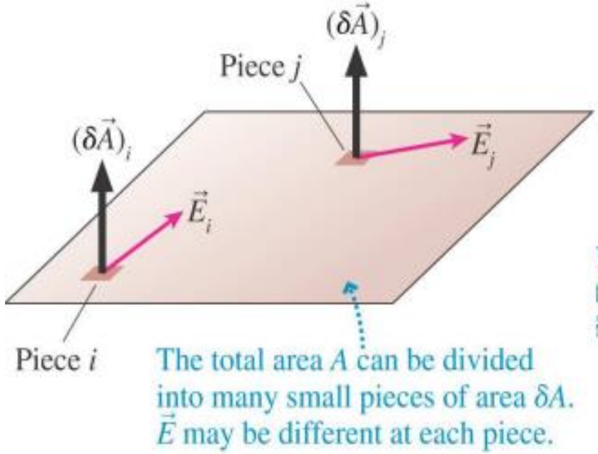


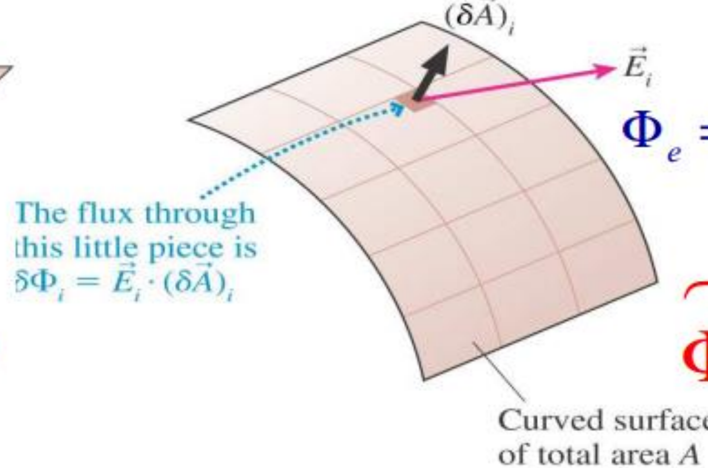
**The electric field pattern through the surface is particularly simple if the closed surface matches the symmetry of the charge distribution inside.**



Gaussian surface which does not match symmetry of charge is not useful!



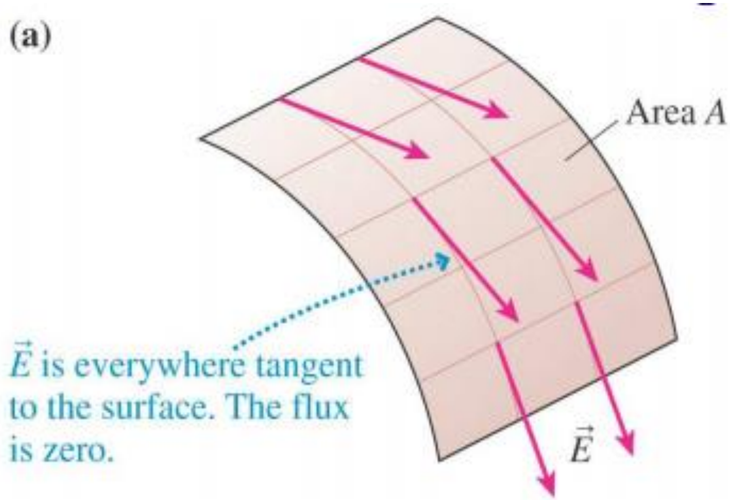


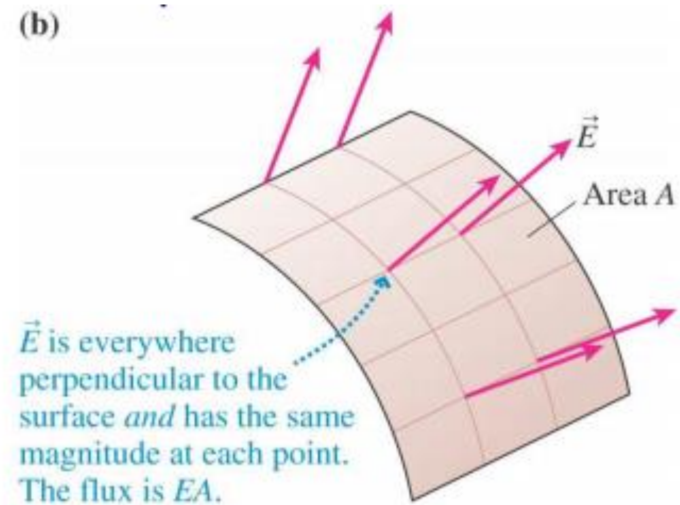




$$\Phi_e = \sum_{i=1}^{N} \delta\Phi_i = \sum_{i=1}^{N} \vec{E} \cdot (\delta\vec{A})_i$$

$$\underbrace{\qquad\qquad\qquad}_{N \to \infty,\ \delta\vec{A} \to d\vec{A}}$$

$$\Phi_e = \int_{surface} \vec{E} \cdot d\vec{A}$$



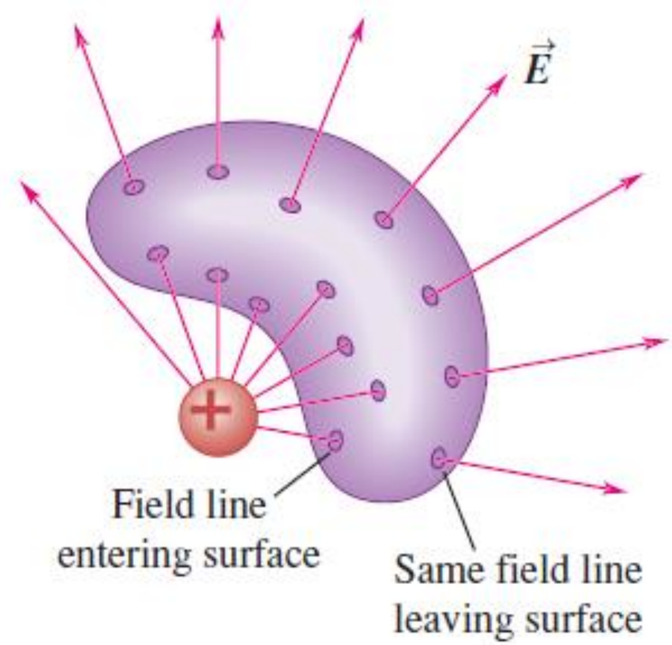
# Gauss' "Law"

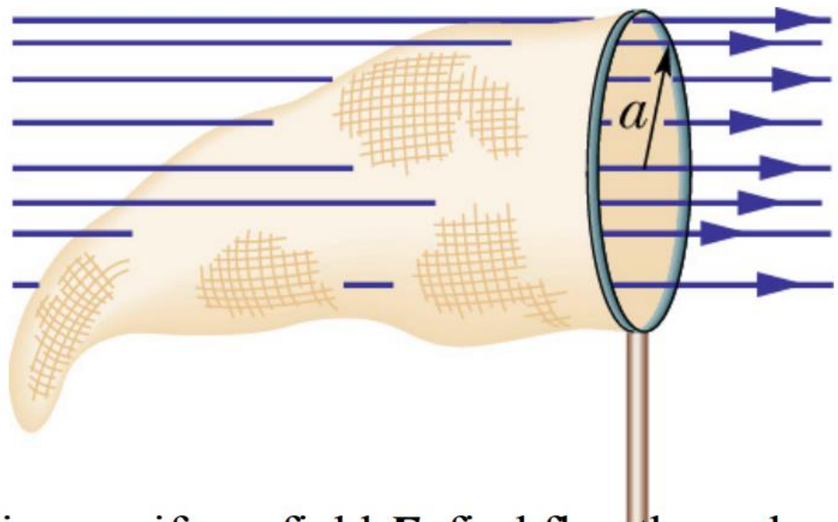
$$\Phi_E = \oint \vec{E} \cdot d\vec{A} = \frac{q_{enclosed}}{\varepsilon_o}$$

The net charge $q_{enc}$ is the algebraic sum of all the enclosed positive and negative charges, and it can be positive, negative, or zero.

The electric field at the surface is due to all the charge distribution, including both that inside and *outside* the surface.

**A point charge outside a closed surface that encloses no charge. If an electric field line from the external charge enters the surface at one point, it must leave at another.**



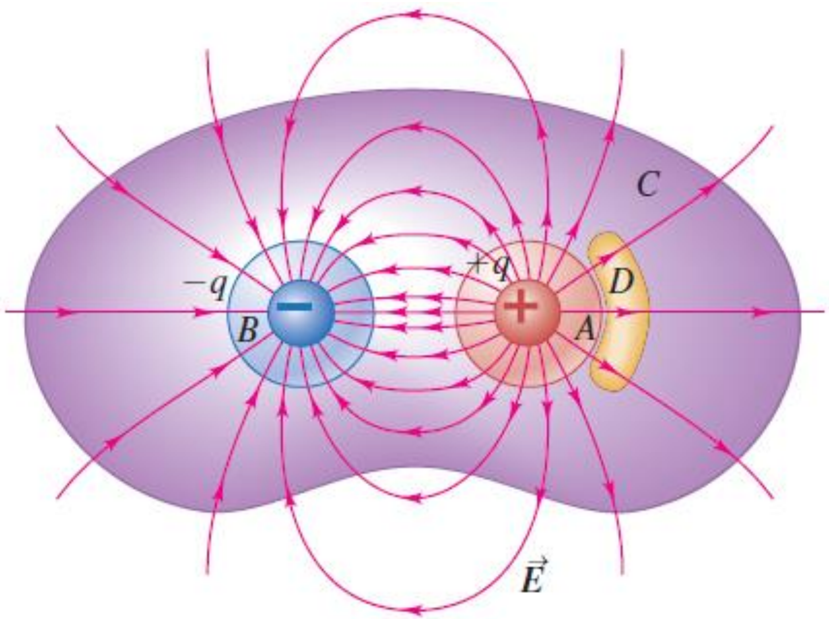



Given uniform field $\boldsymbol{E}$, find flux through net.

$$\Phi_{circle} = \pi a^2 E$$

$$\Phi_{net} = \underline{-\pi a^2 E}$$

# Gauss' "Law"

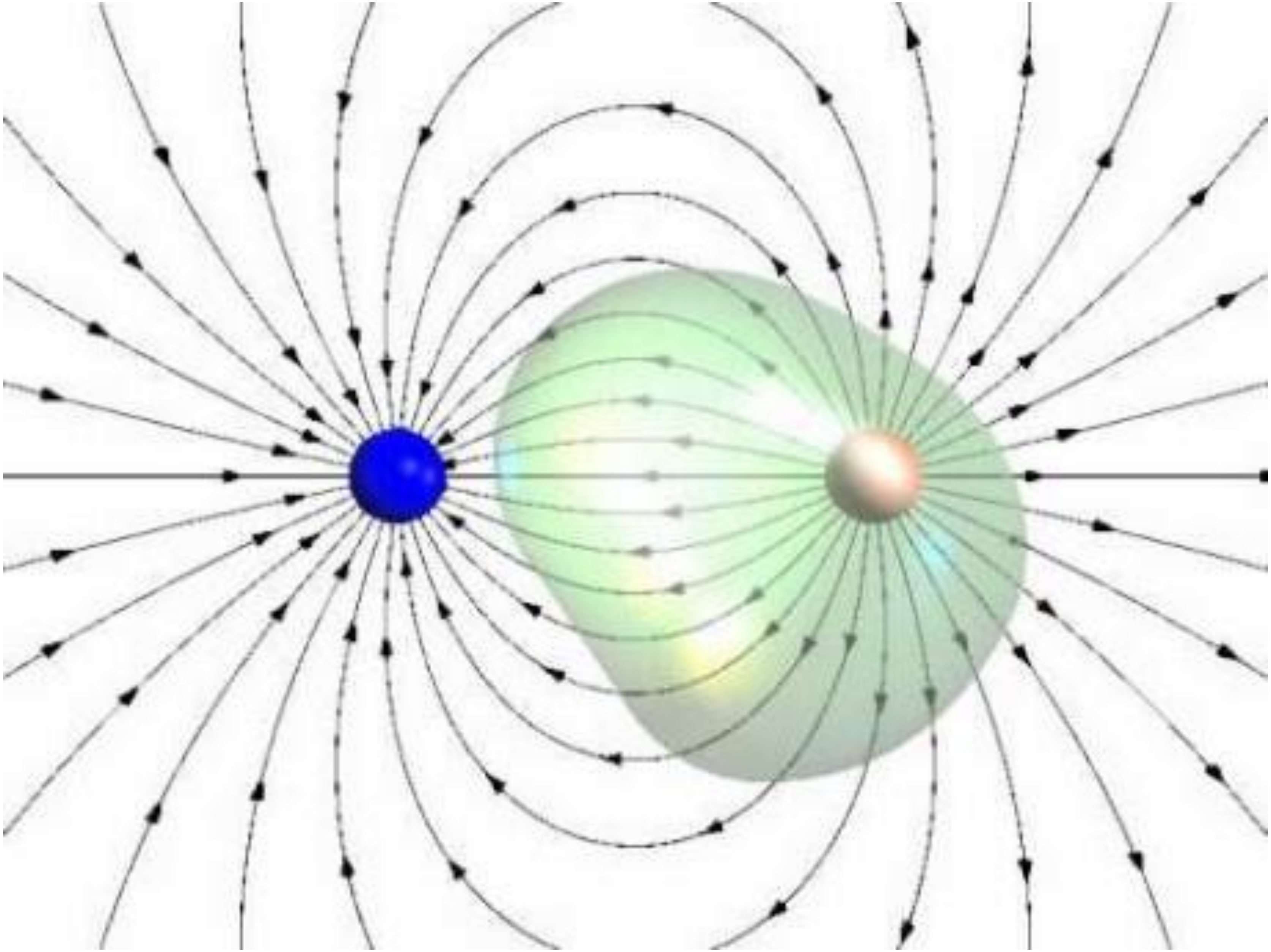



The net number of field lines leaving a closed surface is proportional to the total charge enclosed by that surface.

Two charges, equal in magnitude but opposite in sign, and the field lines that represent their net electric field.

Four Gaussian surfaces are shown in cross section.

Surface *A* encloses the positive charge.
Surface *B* encloses the negative charge.
Surface *D* encloses no charge.
Surface *C* encloses both charges and thus no net charge.



https://www.youtube.com/watch?v=_dzEXr4oprk

# Example, Relating the net enclosed charge and the net flux:

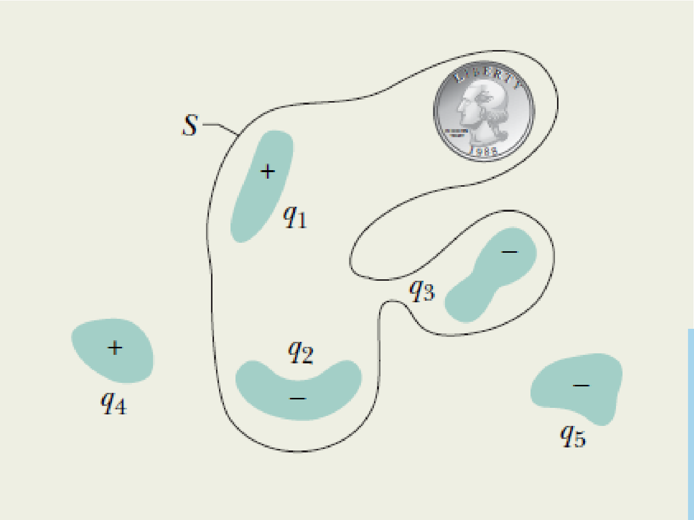



**Fig. 23-7** Five plastic objects, each with an electric charge, and a coin, which has no net charge. A Gaussian surface, shown in cross section, encloses three of the plastic objects and the coin.

Figure 23-7 shows five charged lumps of plastic and an electrically neutral coin. The cross section of a Gaussian surface $S$ is indicated. What is the net electric flux through the surface if $q_1 = q_4 = +3.1$ nC, $q_2 = q_5 = -5.9$ nC, and $q_3 = -3.1$ nC?

**KEY IDEA**

The *net* flux $\Phi$ through the surface depends on the *net* charge $q_{enc}$ enclosed by surface $S$.

***Calculation:*** The coin does not contribute to $\Phi$ because it is neutral and thus contains equal amounts of positive and negative charge. We could include those equal amounts, but they would simply sum to be zero when we calculate the *net* charge enclosed by the surface. So, let's not bother. Charges $q_4$ and $q_5$ do not contribute because they are outside surface $S$. They certainly send electric field lines through the surface, but as much enters as leaves and no net flux is contributed. Thus, $q_{enc}$ is only the sum $q_1 + q_2 + q_3$ and Eq. 23-6 gives us

$$\Phi = \frac{q_{enc}}{\varepsilon_0} = \frac{q_1 + q_2 + q_3}{\varepsilon_0}$$

$$= \frac{+3.1 \times 10^{-9}\,\text{C} - 5.9 \times 10^{-9}\,\text{C} - 3.1 \times 10^{-9}\,\text{C}}{8.85 \times 10^{-12}\,\text{C}^2/\text{N}\cdot\text{m}^2}$$

$$= -670\ \text{N}\cdot\text{m}^2/\text{C}. \qquad \text{(Answer)}$$

The minus sign shows that the net flux through the surface is inward and thus that the net charge within the surface is negative.

[https://www.geogebra.org/m/r7Ue9Nac](https://www.geogebra.org/m/r7Ue9Nac)

## *Gauss' Law and Coulomb's Law:*

$$\varepsilon_0 \oint \vec{E} \cdot d\vec{A} = \varepsilon_0 \oint E\, dA = q_{\text{enc}}.$$

$$\varepsilon_0 E \oint dA = q\,.$$

$$\varepsilon_0 E(4\pi r^2) = q$$

$$E = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}.$$

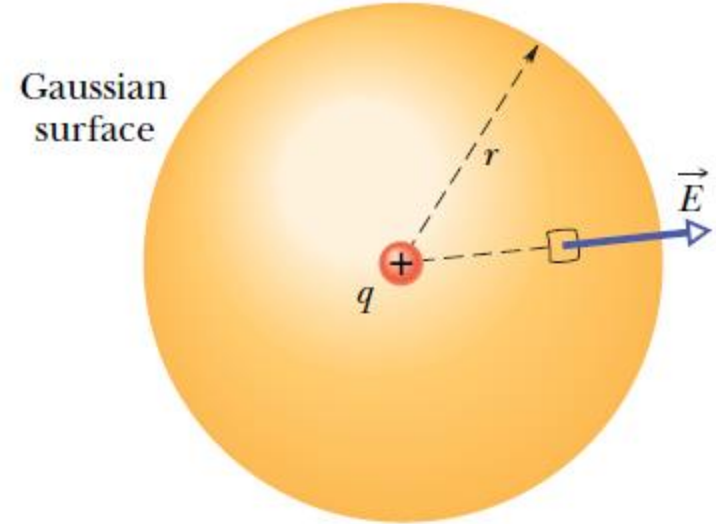
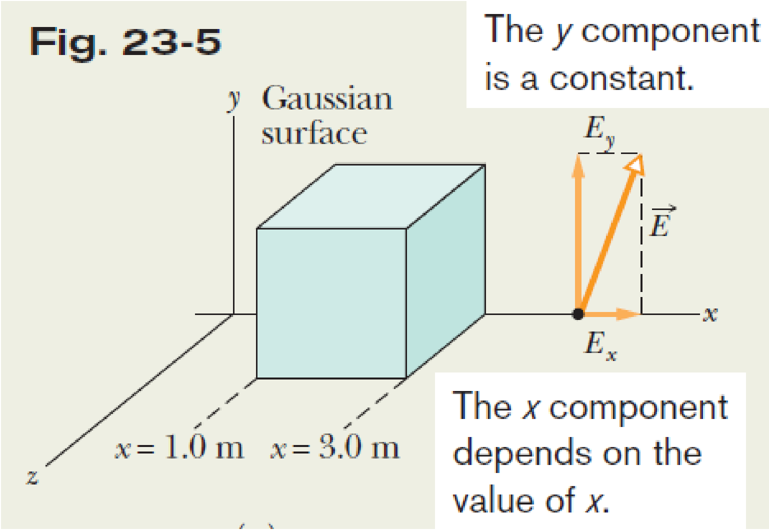



**Fig. 23-8** A spherical Gaussian surface centered on a point charge $q$.

One may also derive Gauss' law from Coulomb's law.
These two laws are equivalent.

# Example, Enclosed charge in a non-uniform field:




What is the net charge enclosed by the Gaussian cube of Fig. 23-5, which lies in the electric field $\vec{E} = 3.0x\hat{\mathrm{i}} + 4.0\hat{\mathrm{j}}$? ($E$ is in newtons per coulomb and $x$ is in meters.)

**KEY IDEA**

The net charge enclosed by a (real or mathematical) closed surface is related to the total electric flux through the surface by Gauss' law as given by Eq. 23-6 ($\varepsilon_0\Phi = q_{enc}$).

***Flux:*** To use Eq. 23-6, we need to know the flux through all six faces of the cube. We already know the flux through the right face ($\Phi_r = 36$ N·m²/C), the left face ($\Phi_l = -12$ N·m²/C), and the top face ($\Phi_t = 16$ N·m²/C).

For the bottom face, our calculation is just like that for the top face *except* that the differential area vector $d\vec{A}$ is now directed downward along the $y$ axis (recall, it must be *outward* from the Gaussian enclosure). Thus, we have $d\vec{A} = -dA\hat{\mathrm{j}}$, and we find

$$\Phi_b = -16\ \mathrm{N\cdot m^2/C}.$$

For the front face we have $d\vec{A} = dA\hat{\mathrm{k}}$, and for the back face, $d\vec{A} = -dA\hat{\mathrm{k}}$. When we take the dot product of the given electric field $\vec{E} = 3.0x\hat{\mathrm{i}} + 4.0\hat{\mathrm{j}}$ with either of these expressions for $d\vec{A}$, we get 0 and thus there is no flux through those faces. We can now find the total flux through the six sides of the cube:

$$\Phi = (36 - 12 + 16 - 16 + 0 + 0)\ \mathrm{N\cdot m^2/C} = 24\ \mathrm{N\cdot m^2/C}.$$

***Enclosed charge:*** Next, we use Gauss' law to find the charge $q_{enc}$ enclosed by the cube:

$$q_{enc} = \varepsilon_0\Phi = (8.85 \times 10^{-12}\ \mathrm{C^2/N\cdot m^2})(24\ \mathrm{N\cdot m^2/C}) = 2.1 \times 10^{-10}\ \mathrm{C}. \quad \text{(Answer)}$$

Thus, the cube encloses a *net* positive charge.

# Gauss' "Law"

$$\oint \vec{V} . d\vec{a} = \iiint \nabla . \vec{V} \, dV$$ (Gauss theorem in vector analysis)

$$\nabla \equiv \hat{x}\frac{\partial}{\partial x} + \hat{y}\frac{\partial}{\partial y} + \hat{z}\frac{\partial}{\partial z}$$ (Differential operator in vector analysis)

$$\Phi_E = \oint \vec{E} \cdot d\vec{A} = \frac{q_{enclosed}}{\varepsilon_o}$$

$$\epsilon_0 \iiint \nabla . \vec{E} \, dV = q_{enc}$$

$$\nabla \bullet \vec{E} = \frac{\rho_e}{\varepsilon_0}$$ (Gauss' law in the differential form)

$$(\nabla \cdot \vec{g} = 4pGr_m)$$ **Gauss's law for gravity!!**

$\rho_e$
Charge density

$\rho_m$
Mass density

# Strategy for Solving Gauss' "Law" Problems

- Select a Gaussian surface with symmetry that matches the charge distribution.

- Draw the Gaussian surface so that the electric field is either constant or zero at all points on the Gaussian surface.

- Use symmetry to determine the direction of $\vec{E}$ on the Gaussian surface.

- Evaluate the surface integral (electric flux).

- Determine the charge inside the Gaussian surface.

- Solve for $\vec{E}$.

# *Applying Gauss' Law and Cylindrical Symmetry:*

We know that the field must point perpendicularly away from the wire and be cylindrically symmetrical.

So if we construct an **imaginary cylinder** of radius *r* and height *h* co-axial with the line, the flux through the circular ends is zero and $|E|$ is uniform on the cylinder walls.

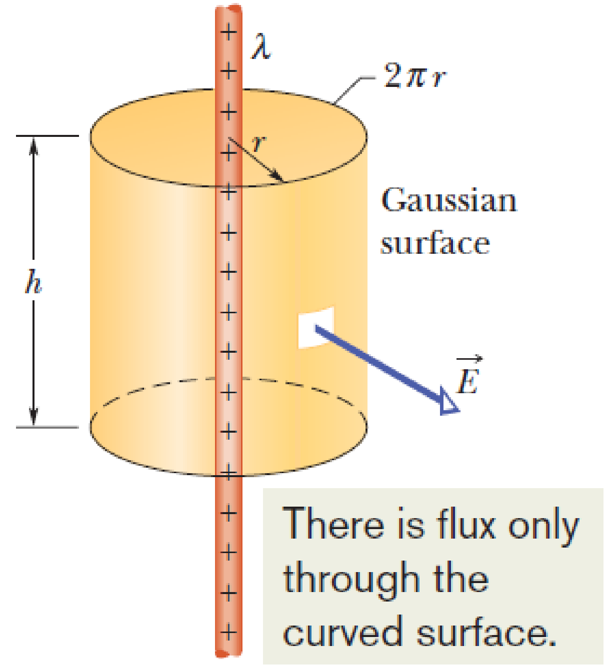
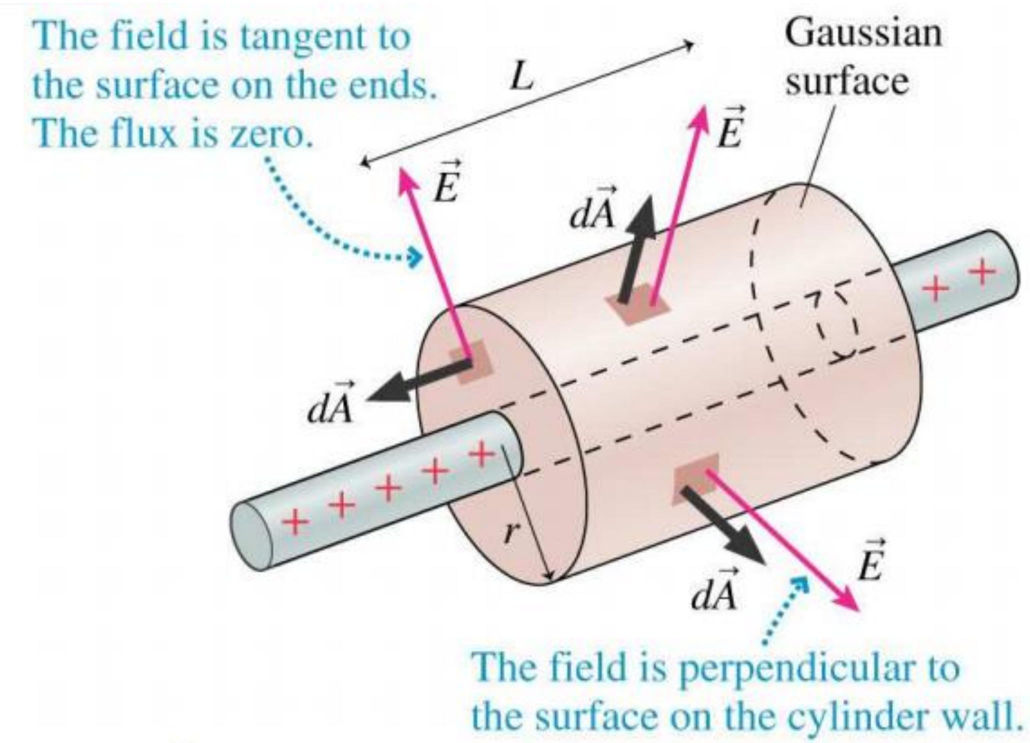
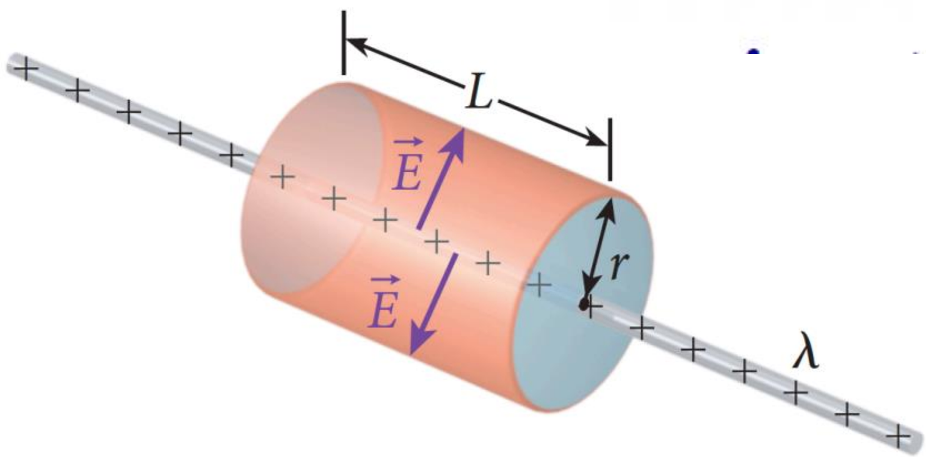



**Fig. 23-12** A Gaussian surface in the form of a closed cylinder surrounds a section of a very long, uniformly charged, cylindrical plastic rod.

# Applying Gauss' Law and Cylindrical Symmetry:



## Applying Gauss' Law and Cylindrical Symmetry:

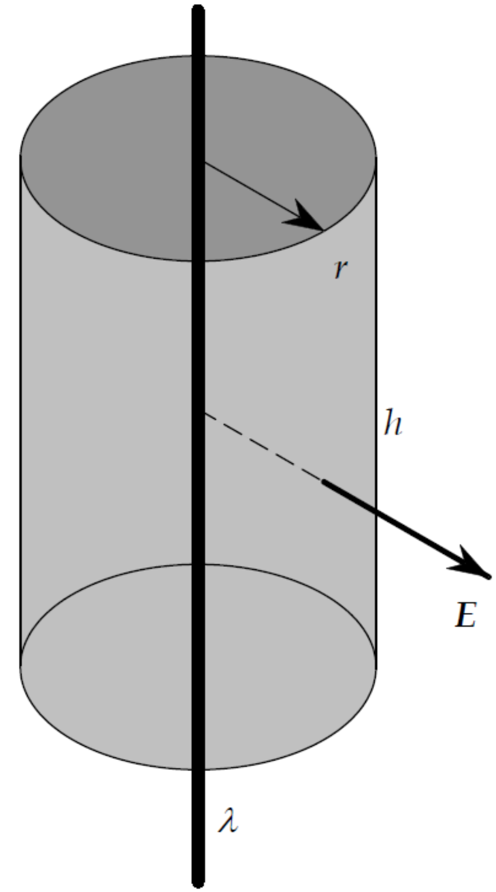
- Furthermore, as with the sphere, $\cos\theta = 1$ everywhere on the cylinder walls, so

$$\oint \boldsymbol{E}\boldsymbol{\cdot}d\boldsymbol{A} = E \int_{\text{cyl. walls}} dA = 2\pi r h E$$

- The cylinder encloses a charge $Q_{\text{enclosed}} = \lambda h$.
- So the $h$s cancel – it doesn't matter how long the cylinder is – and

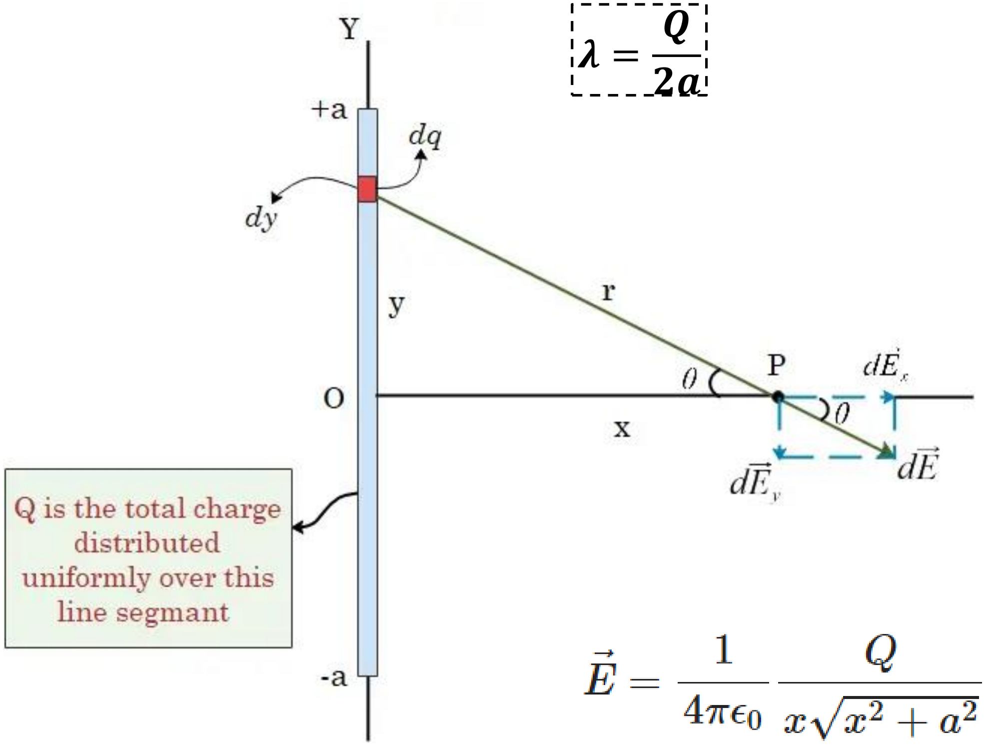
$$2\pi r h E = \frac{\lambda h}{\varepsilon_0} \quad \Rightarrow \quad \boxed{E = \frac{\lambda}{2\pi\varepsilon_0 r}}.$$

Same result as last time, but easier.



# Same calculation we did using Coulomb's law

**Field of a charged line**




$$\lambda = \frac{Q}{2a}$$

$$\vec{E} = \frac{1}{4\pi\epsilon_0}\frac{Q}{x\sqrt{x^2+a^2}}\hat{i}$$

$$dE = \frac{1}{4\pi\epsilon_0}\frac{dQ}{r^2} = \frac{1}{4\pi\epsilon_0}\frac{Q}{2a}\frac{dy}{(x^2+y^2)}$$

$$d\vec{E}_x = \frac{1}{4\pi\epsilon_0}\frac{Q}{2a}\frac{xdy}{(x^2+y^2)^{3/2}}$$

$$d\vec{E}_y = \frac{1}{4\pi\epsilon_0}\frac{Q}{2a}\frac{ydy}{(x^2+y^2)^{3/2}}$$

$$E_x = \frac{1}{4\pi\epsilon_0}\frac{Q}{2a}\int_{-a}^{+a}\frac{xdy}{(x^2+y^2)^{3/2}}$$

$$E_x = \frac{1}{4\pi\epsilon_0}\frac{Q}{2a}\int_{-a}^{+a}\frac{xdy}{(x^2+y^2)^{3/2}}$$

# Applying Gauss' Law, Spherical Symmetry:

To find the constant, consider a point charge at the center of an imaginary sphere.

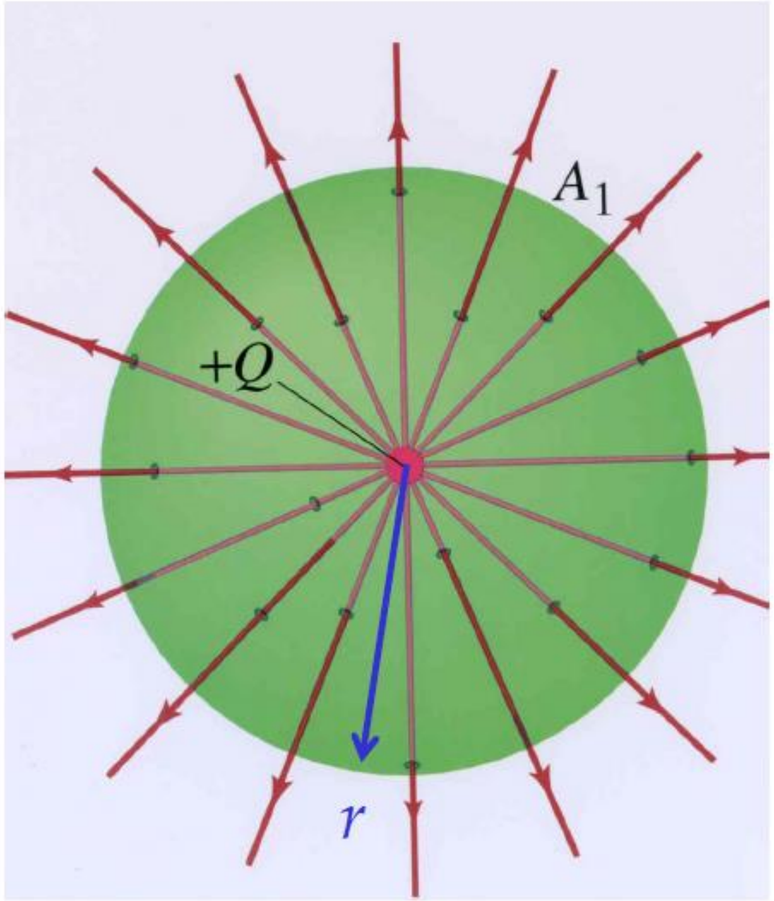
- Since for spheres the surface is perpendicular to the radius, $\cos\theta$ =1 for all elements.

- Since all the points on the sphere lie a distance $r$ away from the charge, $E$ is the same for all elements.

- Thus $\oint \boldsymbol{E} \bullet d\boldsymbol{A} = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}\oint dA$

$$= \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}\left(4\pi r^2\right)$$

$$= Q/\varepsilon_0$$



# Applying Gauss' Law, Spherical Symmetry:

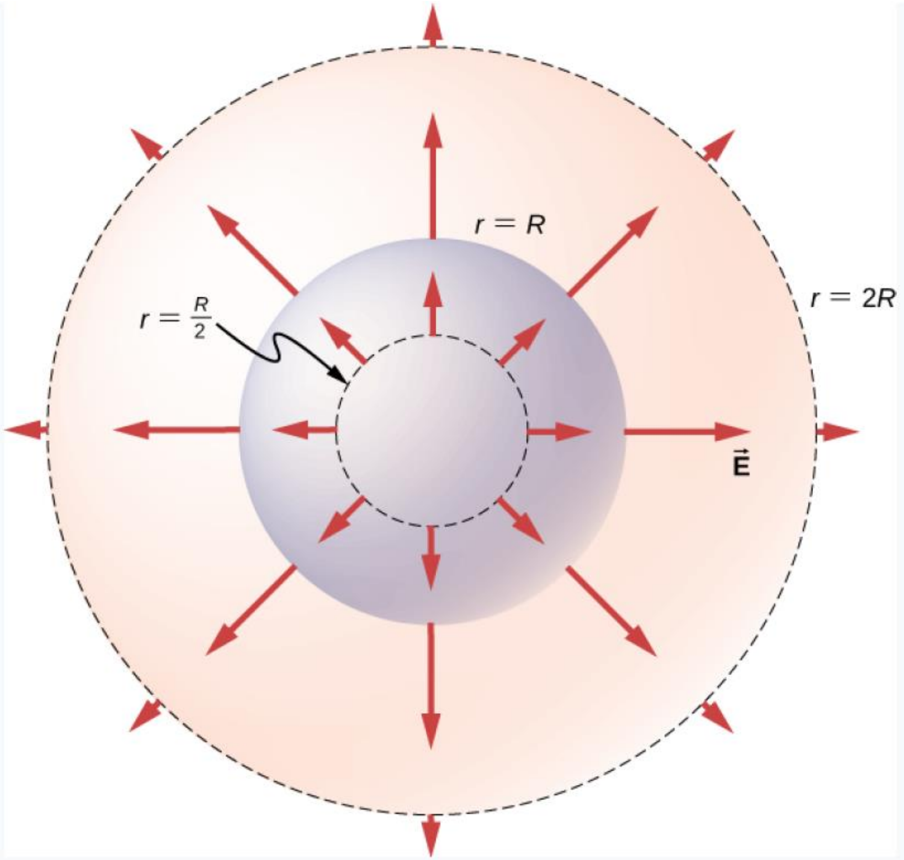
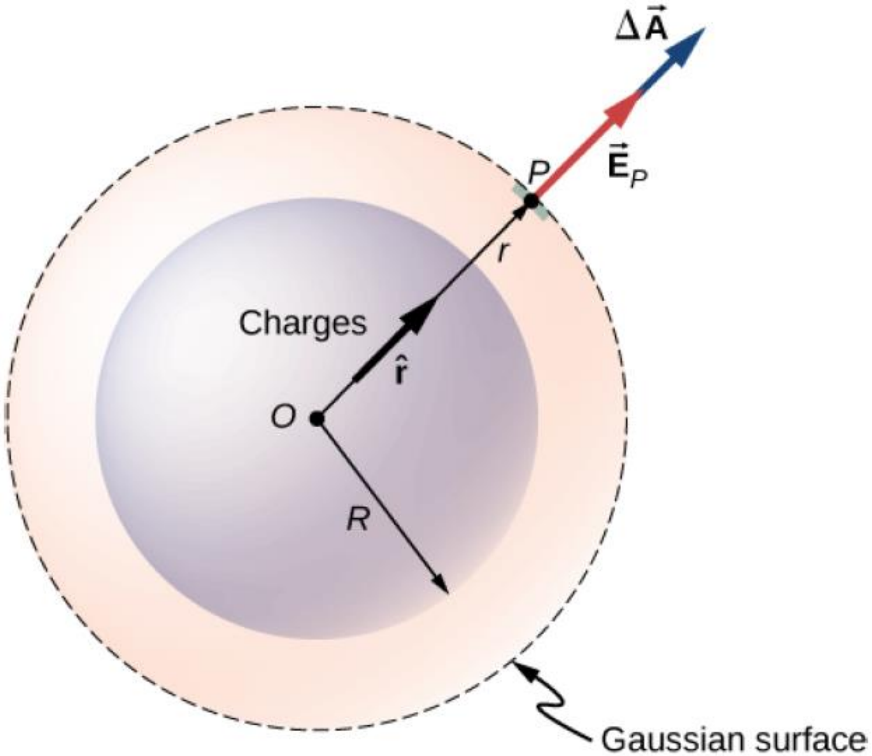



Figure 6.4.5: Electric field vectors inside and outside a uniformly charged sphere.



# Applying Gauss' Law, Spherical Symmetry:

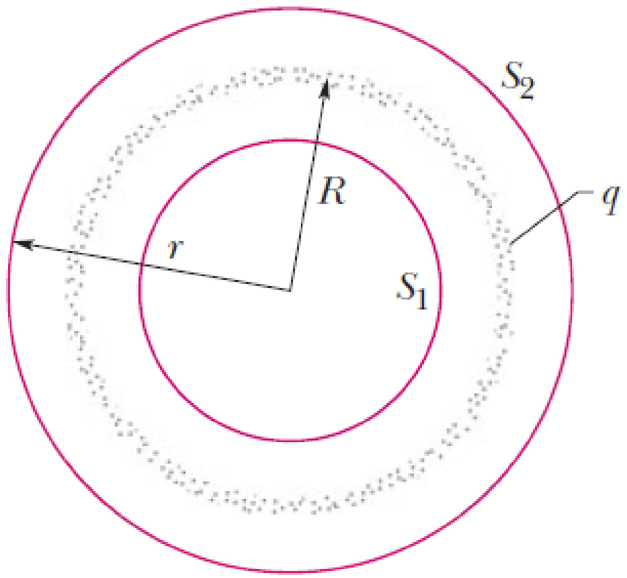
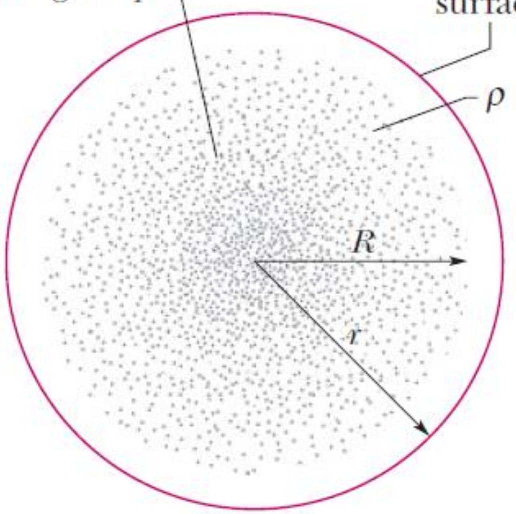
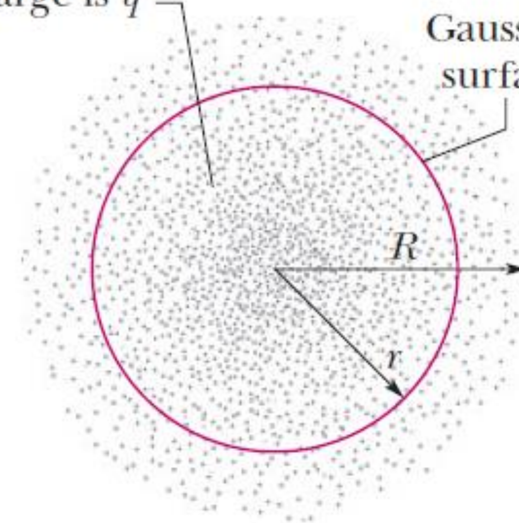



**Figure 23-20** A thin, uniformly charged, spherical shell with total charge $q$, in cross section. Two Gaussian surfaces $S_1$ and $S_2$ are also shown in cross section. Surface $S_2$ encloses the shell, and $S_1$ encloses only the empty interior of the shell.

$$\boxed{E = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2} \qquad \text{(spherical shell, field at } r \geq R).}$$




($a$)




($b$)

The flux through the surface depends on only the *enclosed* charge.

$$E = \frac{1}{4\pi\varepsilon_0}\frac{q'}{r^2} \quad \text{(spherical distribution, field at } r \leq R\text{)}.$$

$$\frac{\begin{pmatrix}\text{charge enclosed by}\\ \text{sphere of radius } r\end{pmatrix}}{\begin{pmatrix}\text{volume enclosed by}\\ \text{sphere of radius } r\end{pmatrix}} = \frac{\text{full charge}}{\text{full volume}}$$

$$\frac{q'}{\frac{4}{3}\pi r^3} = \frac{q}{\frac{4}{3}\pi R^3}.$$

$$q' = q\frac{r^3}{R^3}.$$

$$E = \left(\frac{q}{4\pi\varepsilon_0 R^3}\right) r \quad \text{(uniform charge, field at } r \leq R\text{)}.$$

# Applying Gauss' Law, Spherical Symmetry:

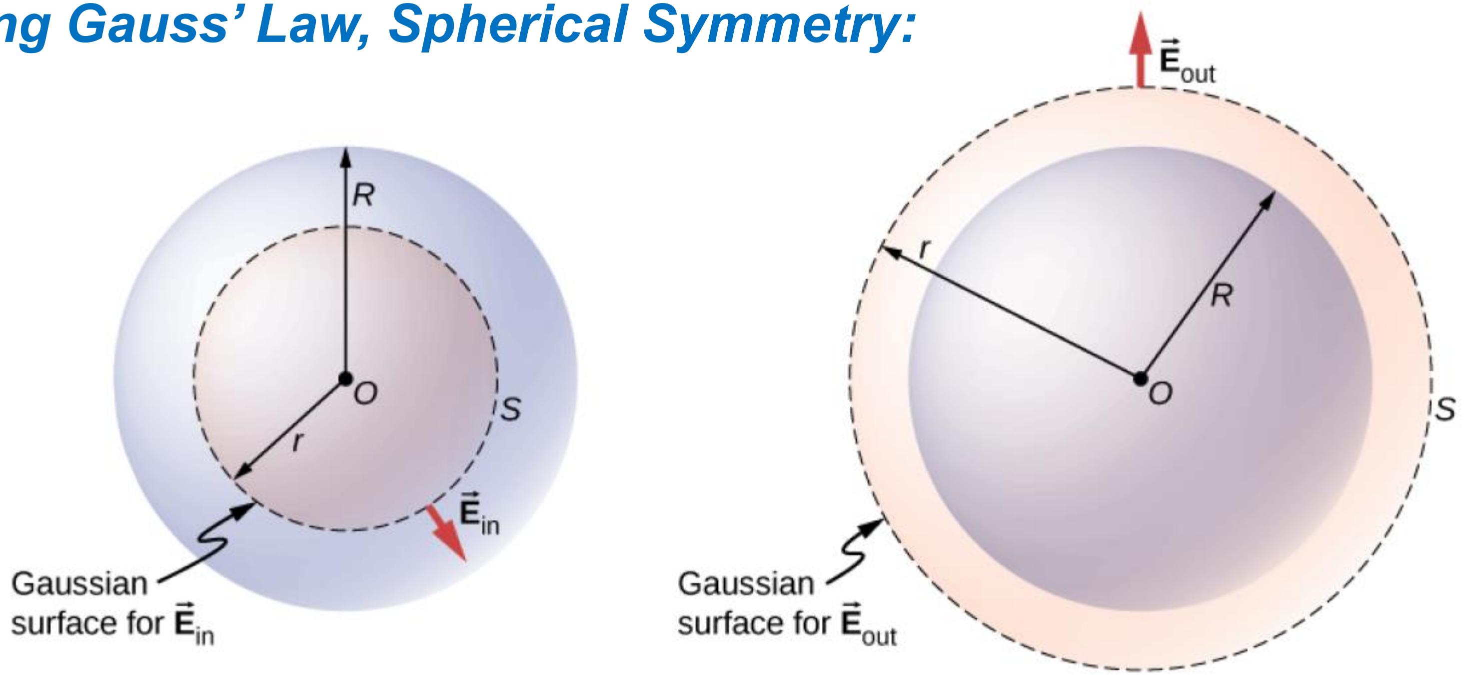
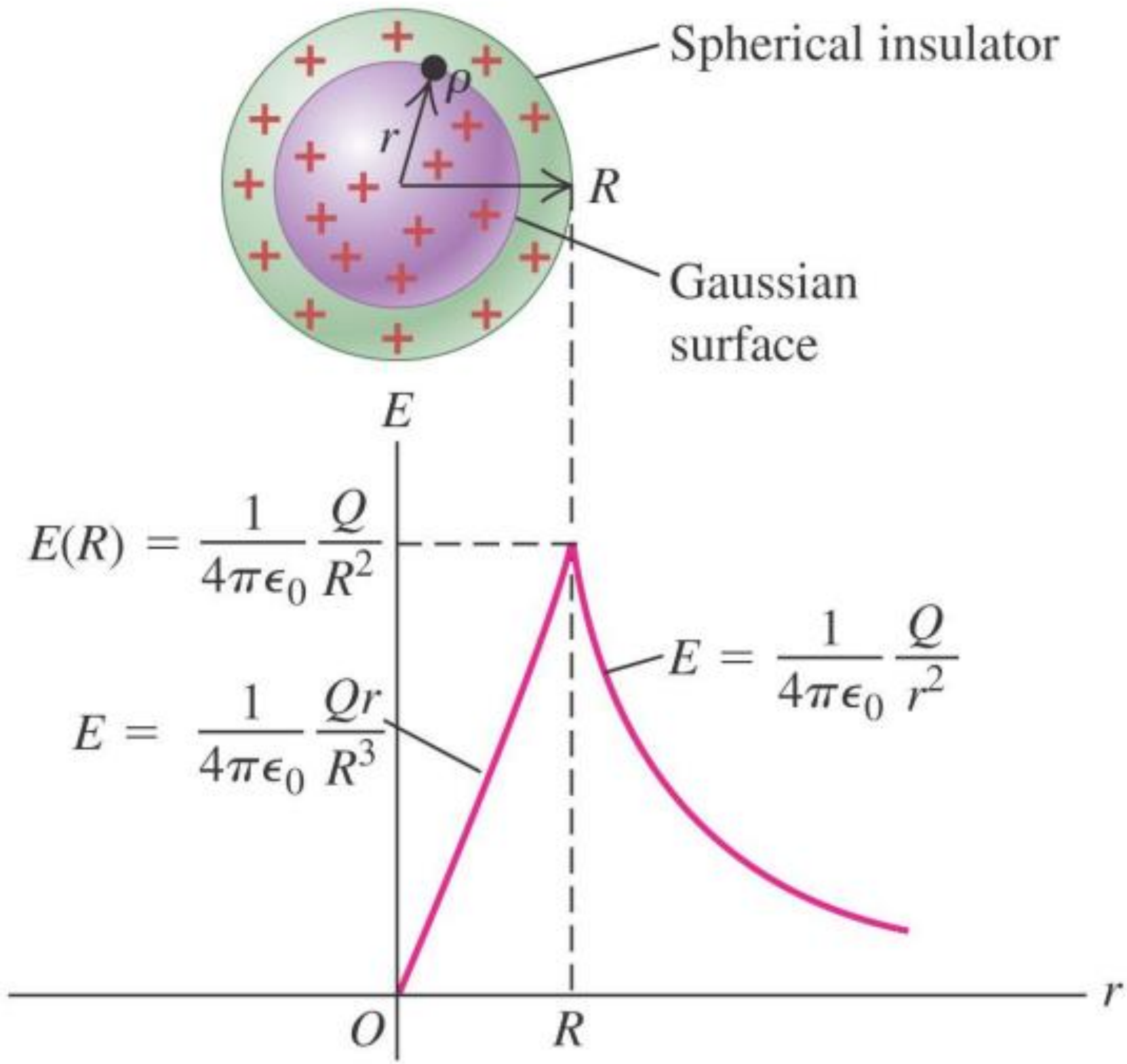



Figure 6.4.3: A spherically symmetrical charge distribution and the Gaussian surface used for finding the field (a) inside and (b) outside the distribution.

$$E = \left(\frac{q}{4\pi\varepsilon_0 R^3}\right) r \quad \text{(uniform charge, field at } r \leq R\text{).}$$

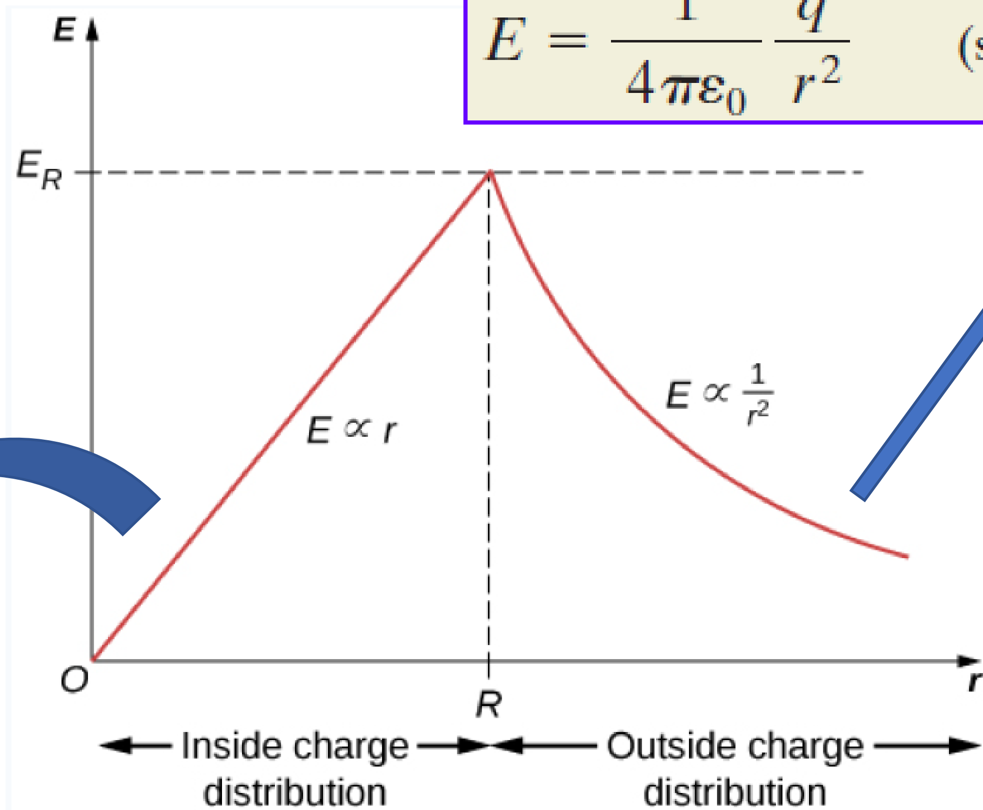
$$\boxed{E = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2} \quad \text{(spherical shell, field at } r \geq R\text{).}}$$



Spherical insulator
Gaussian surface
$\rho$
$r$
$R$
$E$
$E(R) = \frac{1}{4\pi\epsilon_0}\frac{Q}{R^2}$
$E = \frac{1}{4\pi\epsilon_0}\frac{Q}{r^2}$
$E = \frac{1}{4\pi\epsilon_0}\frac{Qr}{R^3}$
$O$
$R$
$r$

## Applying Gauss' Law, Spherical Symmetry:

$$E = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2} \quad \text{(spherical shell, field at } r \geq R\text{).}$$

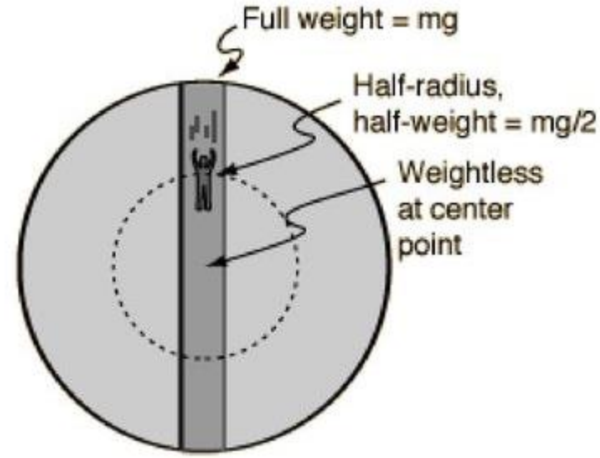


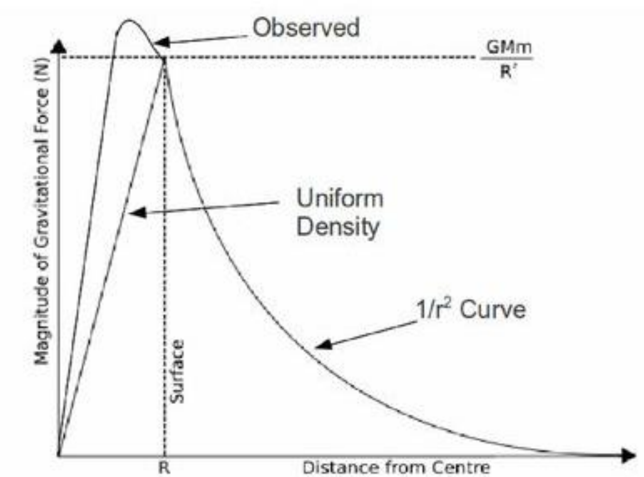

$$E = \left(\frac{q}{4\pi\varepsilon_0 R^3}\right)r \quad \text{(uniform charge, field at } r \leq R\text{).}$$

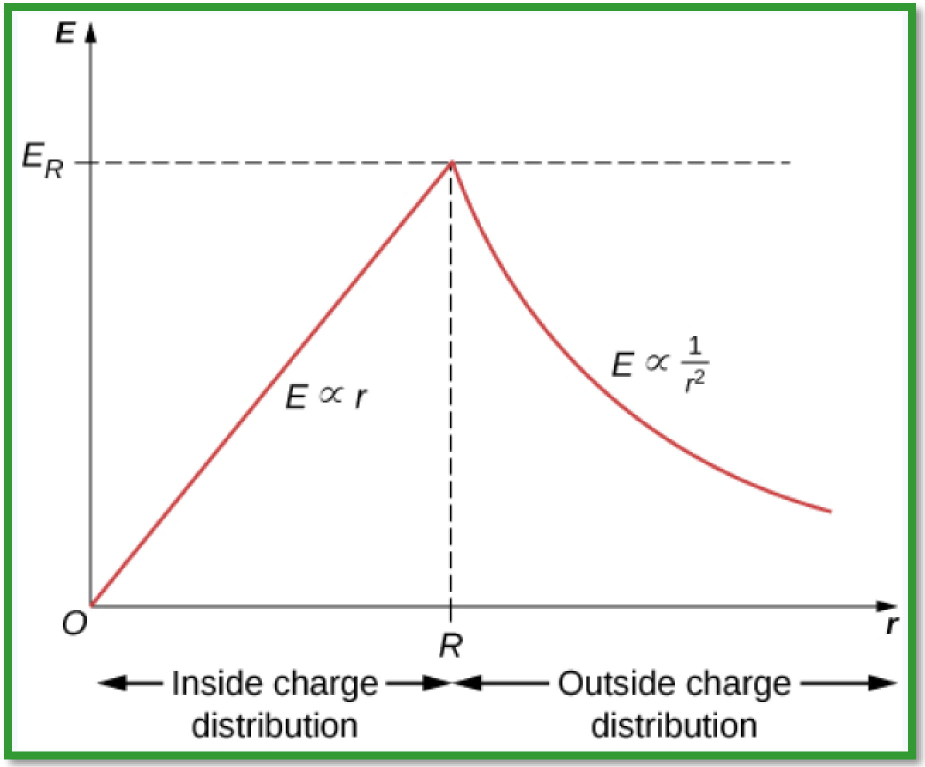
# Recall the case for Gravitation.

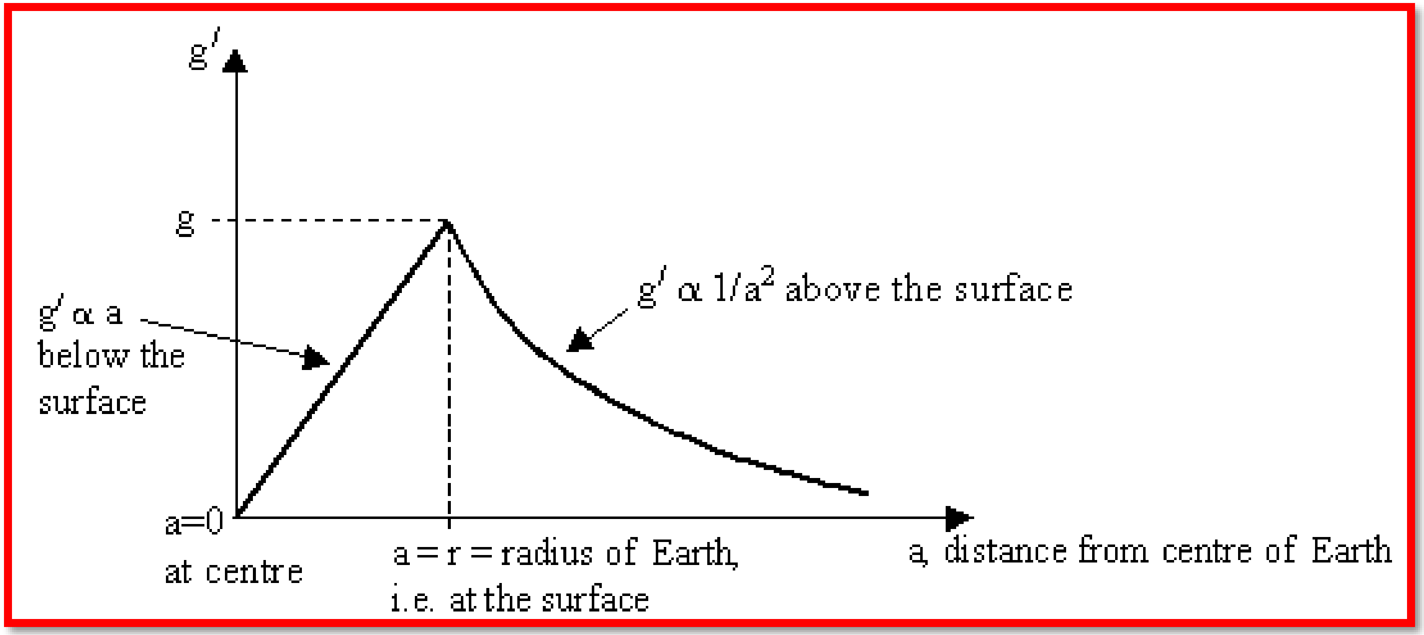
# Gravitational Field of Earth

- Gravitational Field outside Earth (apple graph)
- Gravitational field inside Earth
- Gravitational field greatest at surface of planet
- Gravitational field weakest in center of planet



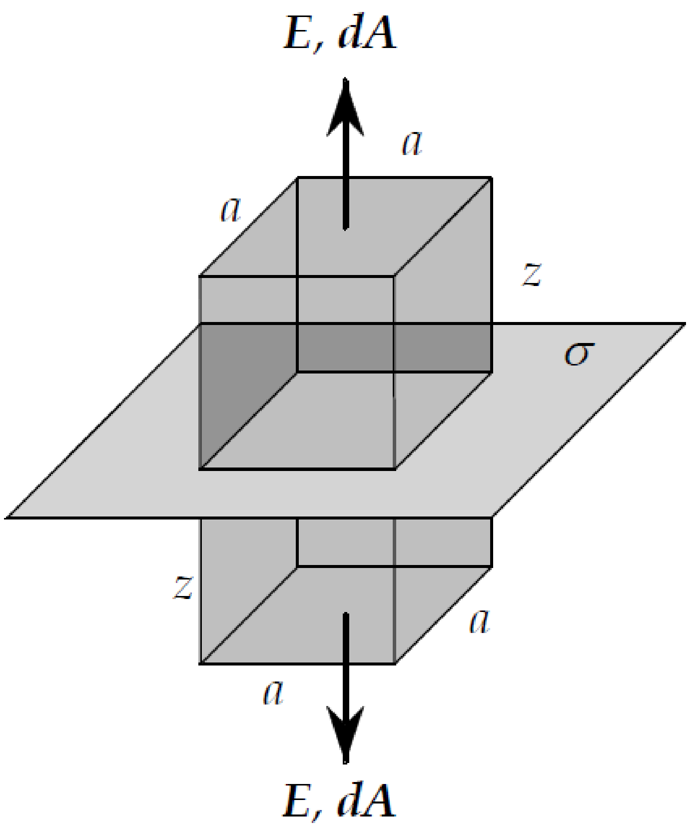




E
$E_R$
$E \propto r$
$E \propto \frac{1}{r^2}$
O
R
r
Inside charge distribution
Outside charge distribution

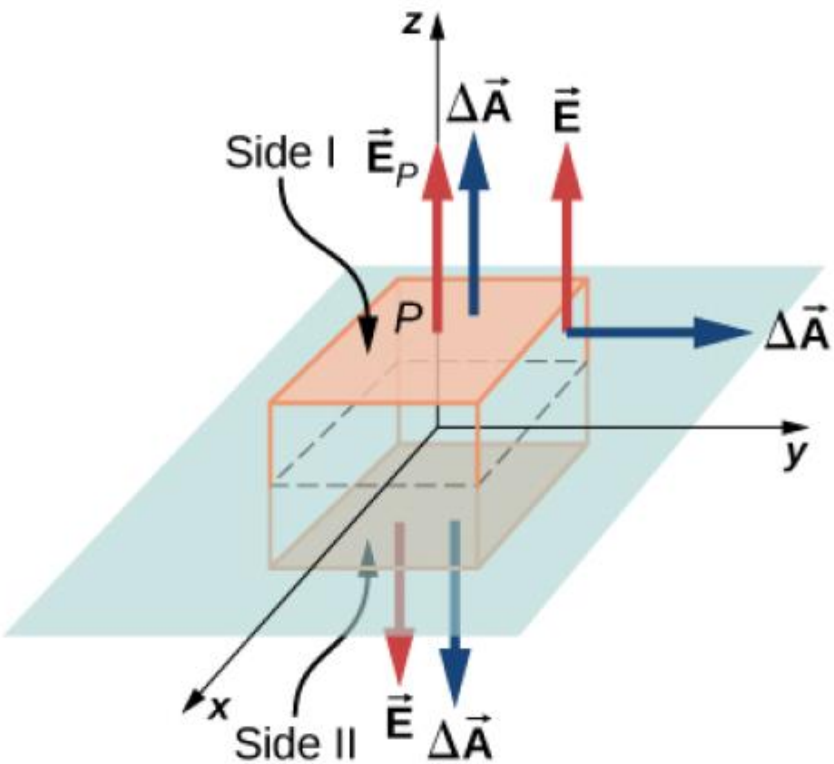

g'
g
g' α a below the surface
g' α 1/a² above the surface
a=0 at centre
a = r = radius of Earth, i.e. at the surface
a, distance from centre of Earth

# Applying Gauss' Law, Planar Symmetry Non-conducting Sheet:

- Let's choose the Gaussian surface with rectangular sides: let its dimensions be $a \times a \times 2z$, and let the charged plane bisect the $a$-$2z$ sides.
- Because $\boldsymbol{E}$ has to be $\perp$ to the plane ($\cos\theta = 0$), the flux is **zero** through all of the vertical, $a \times z$ faces.
- And because $\boldsymbol{E}$ is uniform on the $a \times a$ faces and points the same way as the area vectors (i.e. away from the charged plane; $\cos\theta = 1$), the flux through **each** of these faces is $Ea^2$.



# *Applying Gauss' Law, Planar Symmetry Non-conducting Sheet:*




A thin charged sheet and the Gaussian box for finding the electric field at the field point *P*. The normal to each face of the box is from inside the box to outside.

On two faces of the box, the electric fields are parallel to the area vectors.

On the other four faces, the electric fields are perpendicular to the area vectors.

# Applying Gauss' Law, Planar Symmetry Non-conducting Sheet:

$$\varepsilon_0 \oint \vec{E} \cdot d\vec{A} = q_{\text{enc}},$$

$$\varepsilon_0(EA + EA) = \sigma A,$$

$$E = \frac{\sigma}{2\varepsilon_0} \qquad \text{(sheet of charge).}$$

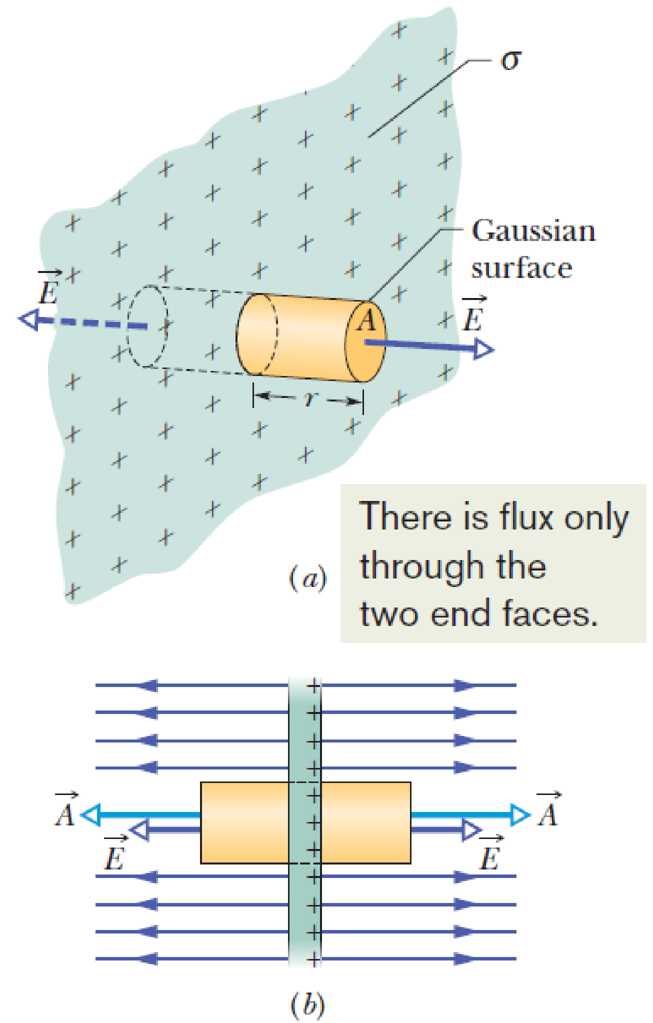
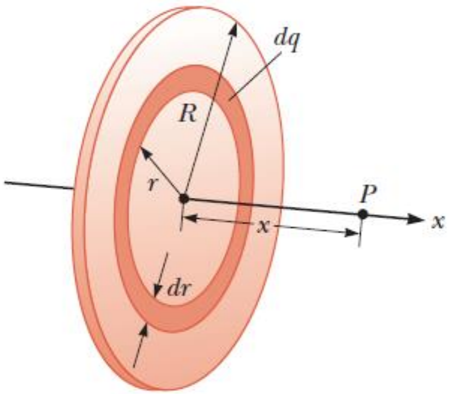



**Fig. 23-15** (*a*) Perspective view and (*b*) side view of a portion of a very large, thin plastic sheet, uniformly charged on one side to surface charge density $\sigma$. A closed cylindrical Gaussian surface passes through the sheet and is perpendicular to it.

# Same calculation we did using Coulomb's law

**Electric Field on the Axis of a Uniformly Charged Disk**




$$dq = \sigma \, dA = \sigma(2\pi r \, dr) = 2\pi\sigma r \, dr$$

$$dE_x = \frac{k_e x}{(r^2 + x^2)^{3/2}} (2\pi\sigma r \, dr)$$

$$E_x = k_e x \pi\sigma \int_0^R \frac{2r \, dr}{(r^2 + x^2)^{3/2}}$$

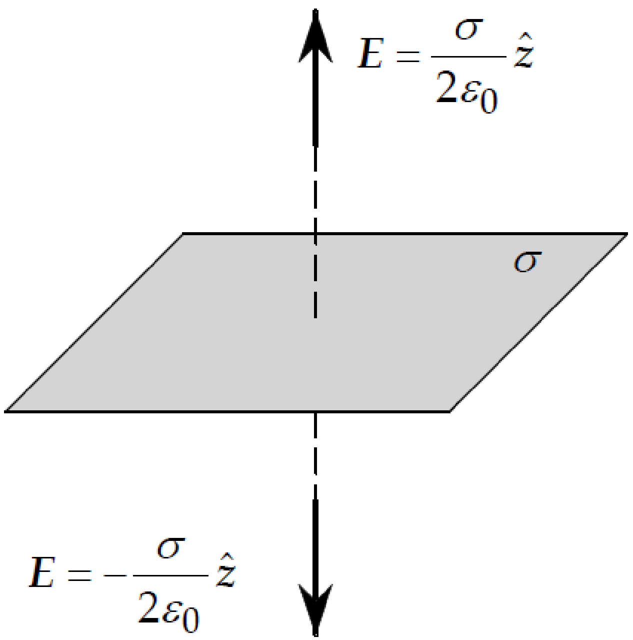
$$= k_e x \pi\sigma \int_0^R (r^2 + x^2)^{-3/2} d(r^2)$$

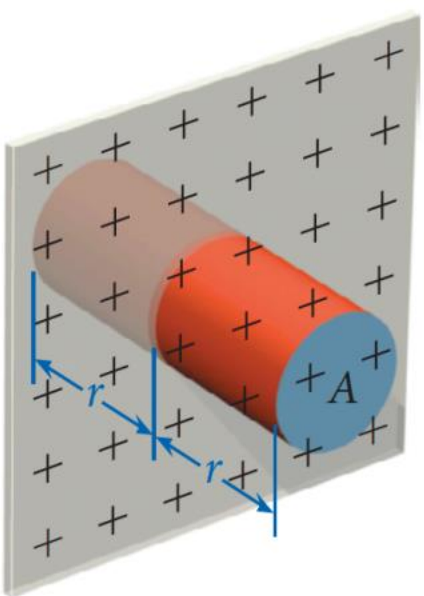
$$= k_e x \pi\sigma \left[ \frac{(r^2 + x^2)^{-1/2}}{-1/2} \right]_0^R = \boxed{2\pi k_e \sigma \left[ 1 - \frac{x}{(R^2 + x^2)^{1/2}} \right]}$$

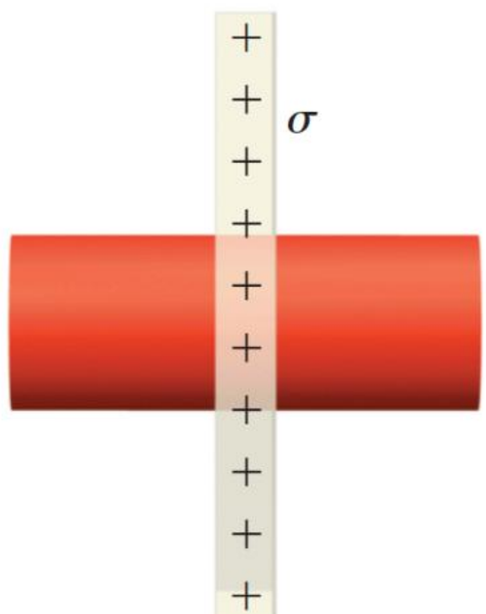
# Applying Gauss' Law, Planar Symmetry Non-conducting Sheet:

- The complete answer is

$$\boxed{E = \begin{cases} \dfrac{\sigma\hat{z}}{2\varepsilon_0}, & z > 0 \\ -\dfrac{\sigma\hat{z}}{2\varepsilon_0}\hat{z}, & z < 0 \end{cases}}$$



# The Electric Field inside a Conductor Vanishes

→E is zero everywhere inside

Why? Conductors are full of mobile charges (e.g., conduction electrons in a background formed by immobile positive ions). If there were E, then the charges must be moving around due to force F=qE. This would contradict "no current."

Note: even if there is an externally imposed E, it cannot go inside

→All excess charge must be on outer surface.

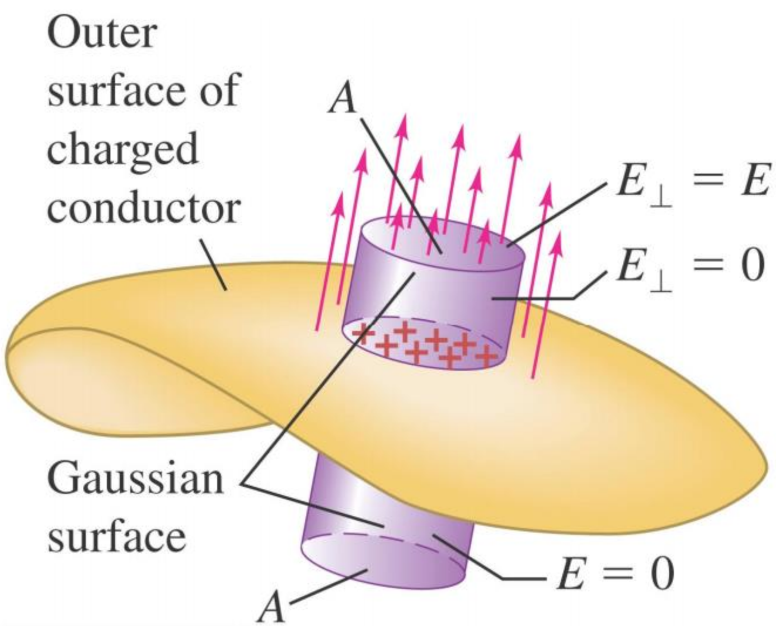
Why? Since E=0 everywhere inside, $q_{enc}$ enclosed by any Gaussian surface is also zero everywhere inside.

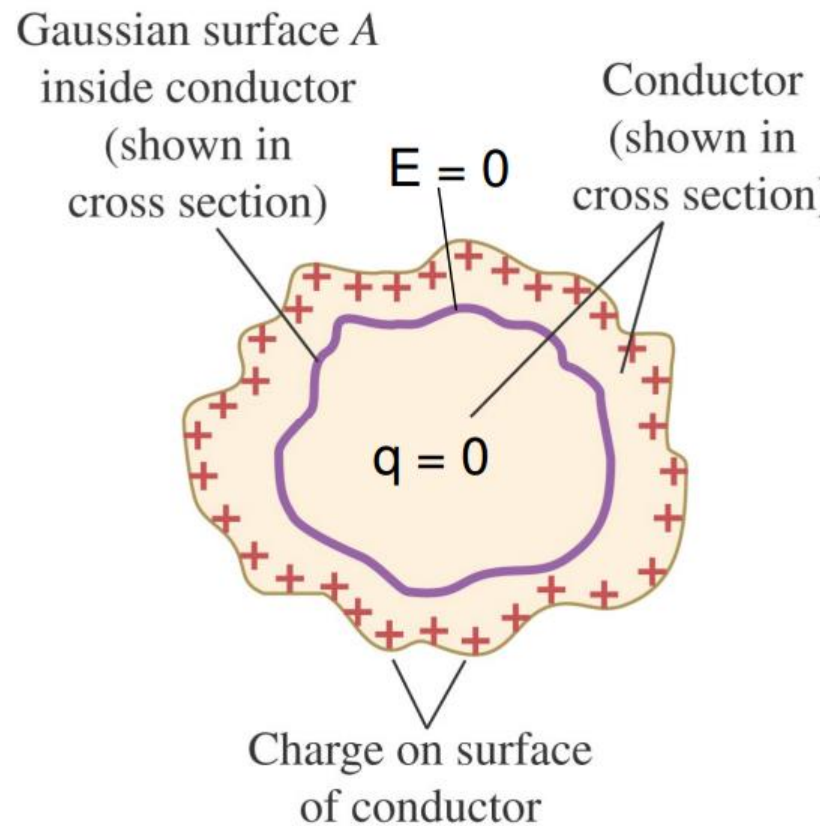
Note: distribution of surface charge must be such to make E=0 everywhere inside

→E is always normal to surface on conductor

Why? E component parallel to surface would cause surface charge to move. This would contradict "no current."

# The Electric Field inside a Conductor Vanishes

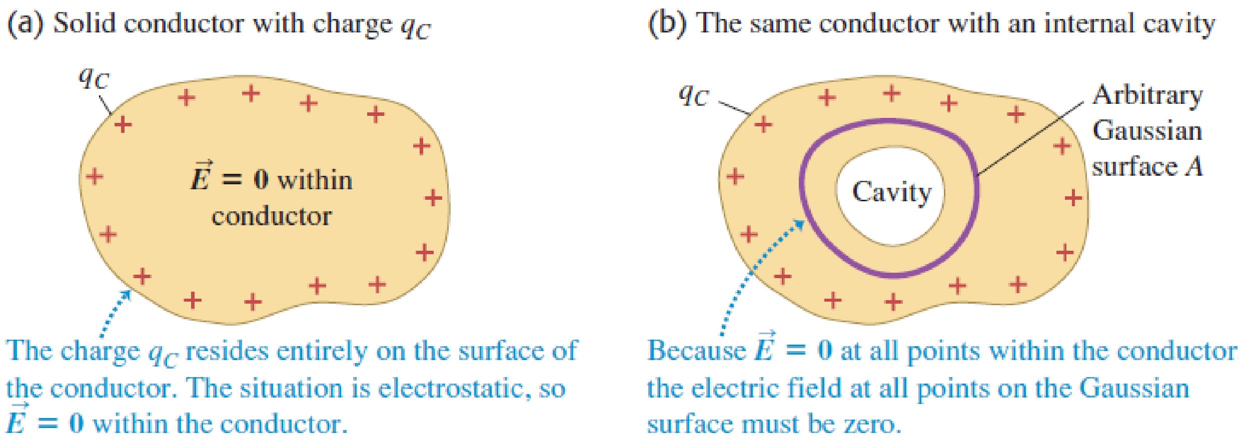


## *A Charged Isolated Conductor:*

If an excess charge is placed on an isolated conductor, that amount of charge will move entirely to the surface of the conductor.

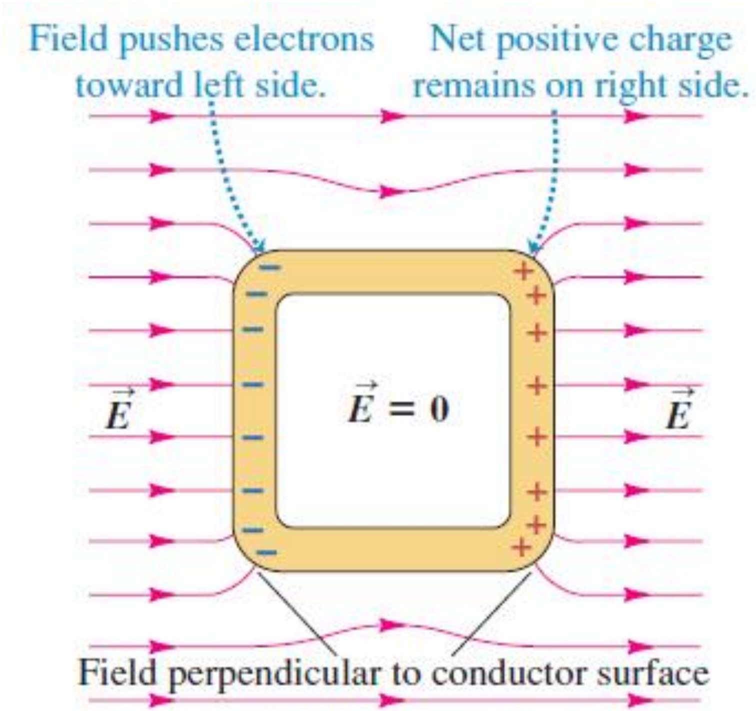
None of the excess charge will be found within the body of the conductor.




The Gaussian surface is placed just inside the actual surface of the conductor. The electric field inside this conductor must be zero. Since the excess charge is not inside the Gaussian surface, it must be outside that surface, which means it must lie on the actual surface of the conductor.

A Gaussian surface is drawn surrounding the cavity, close to its surface but inside the conducting body. Inside the conductor, there can be no flux through this new Gaussian surface. All the excess charge remains on the outer surface of the conductor.

# Faraday's cage




A conductor placed in an *electric field* will be *polarized*.

# Faraday's cage

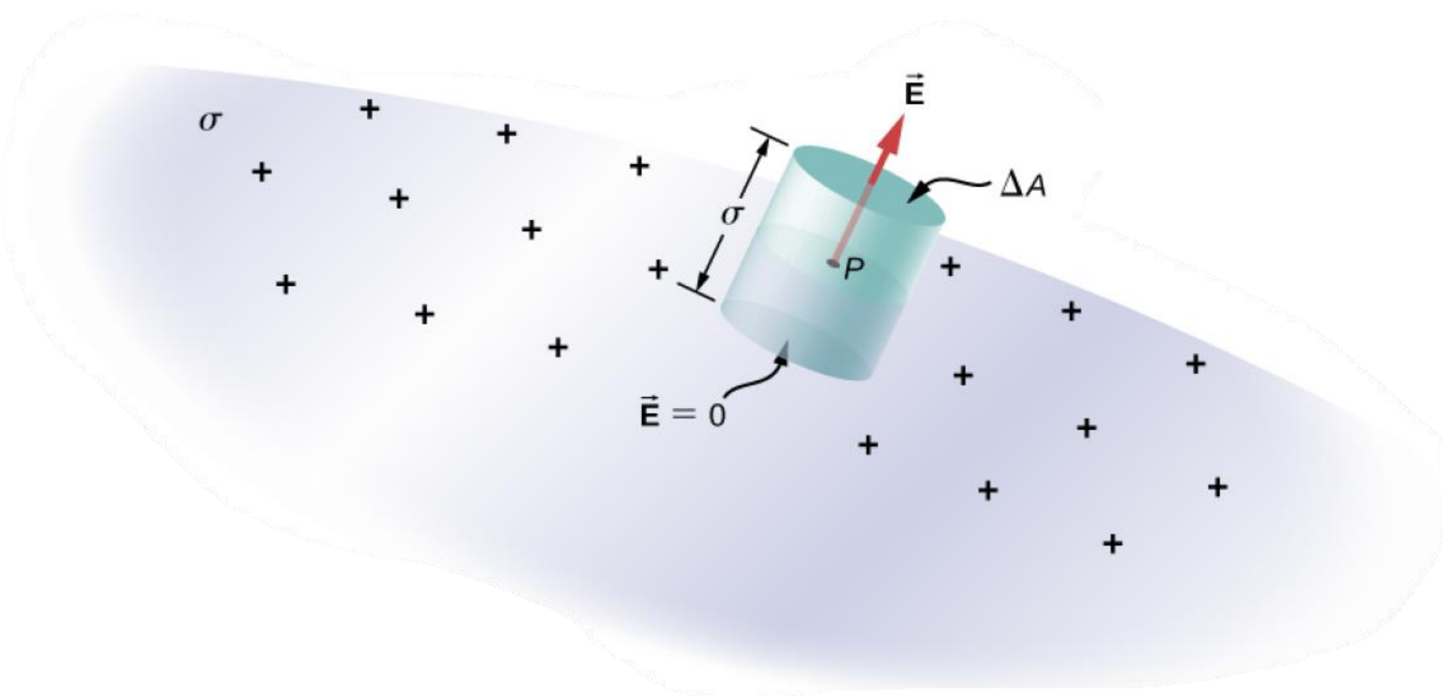
https://www.youtube.com/watch?v=93OhpY65Xo0

# *Applying Gauss' Law Planar Symmetry* *A Charged Isolated Conductor*

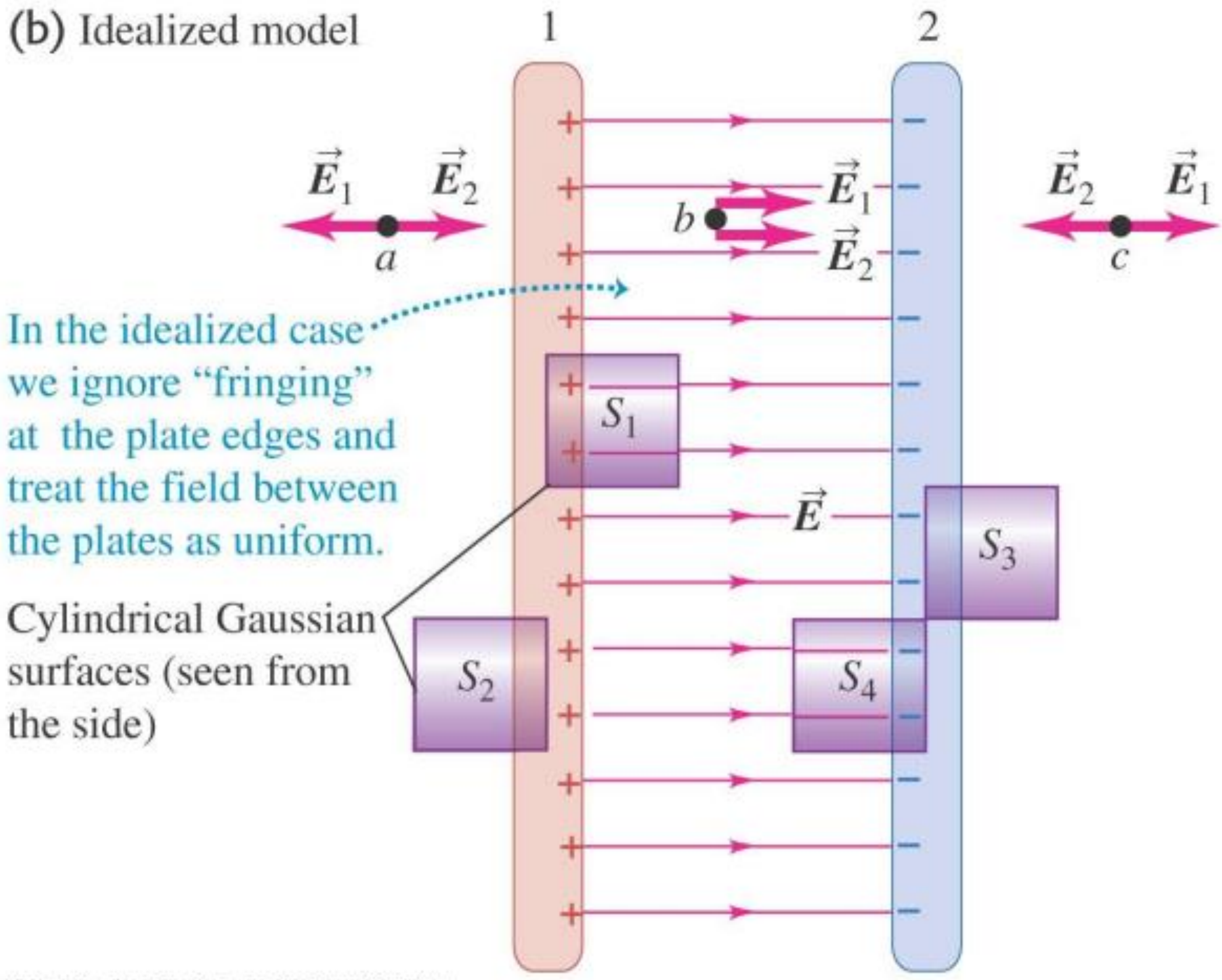
The external electric field near the *surface of a charged conductor* is perpendicular to the surface and has a magnitude that depends on the surface charge density $\sigma$ (charge per unit area)

$$\varepsilon_0 EA = \sigma A,$$

$$E = \frac{\sigma}{\varepsilon_0} \qquad \text{(conducting surface).}$$

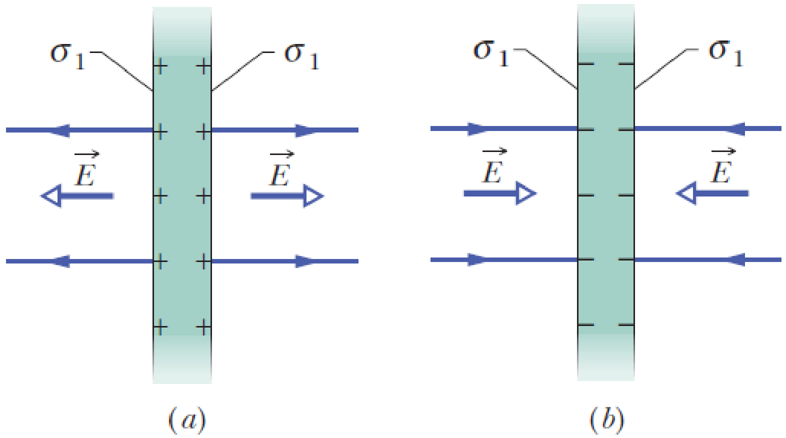


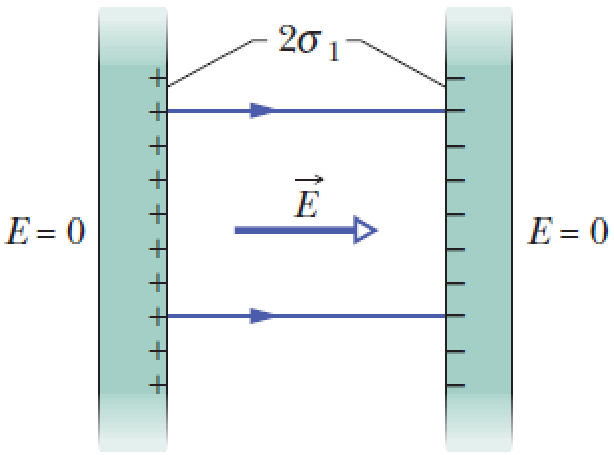
# Same calculation we did using Coulomb's law



# *Applying Gauss' Law Planar Symmetry Two Conducting Plates:*

$$\vec{E} = \frac{\sigma_1}{\varepsilon_0}$$




When the plates bring close to each other and are parallel. The excess charge on one plate attracts the excess charge on the other plate, and all the excess charge moves onto the inner faces of the plates. With twice as much charge now on each inner face, the new surface charge density,

$$\sigma = 2\sigma_1$$

Thus, the electric field at any point between the plates has the magnitude

$$E = \frac{2\sigma_1}{\varepsilon_0} = \frac{\sigma}{\varepsilon_0}.$$

# Applying Gauss' Law Spherical Symmetry A Charged Isolated Conductor

The isolated conducting sphere (Figure ) has a radius *R* and an excess charge *q*. What is the electric field both inside and outside the sphere?

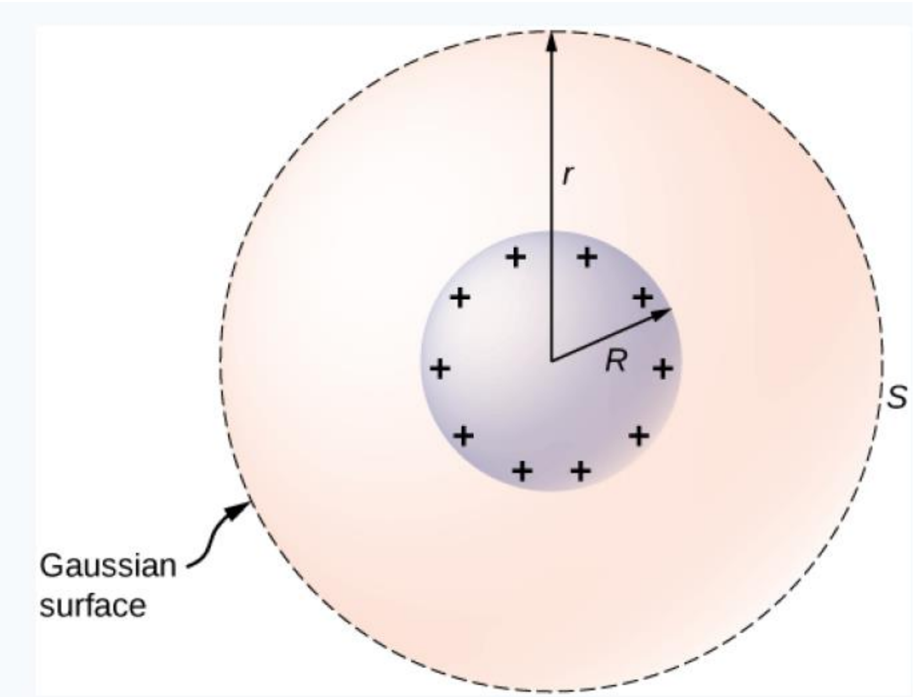
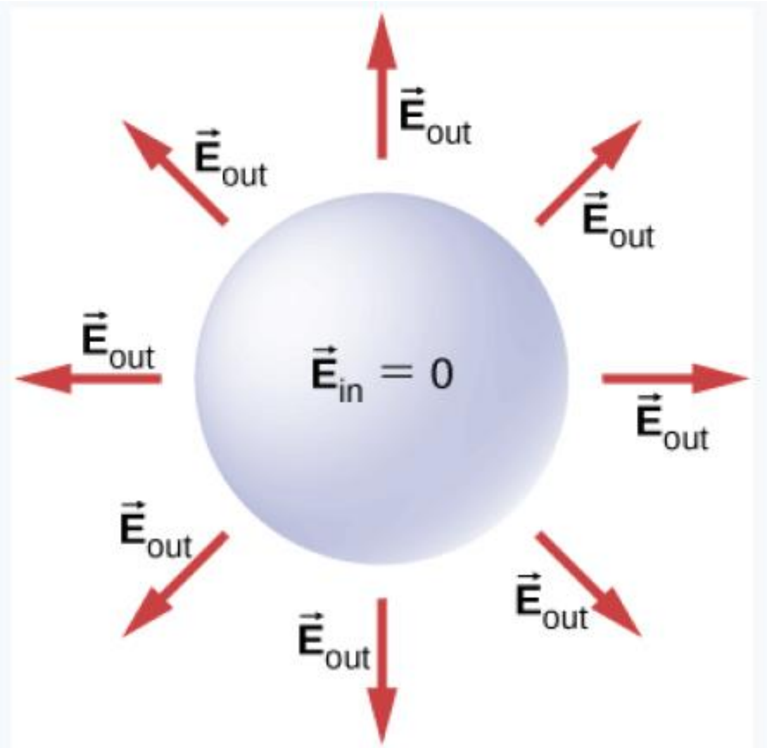



Figure 6.5.9: An isolated conducting sphere.



# Applying Gauss' Law Spherical Symmetry A Charged Isolated Conductor

Since $r$ is constant and $\hat{n} = \hat{r}$ on the sphere,

$$\oint_S \vec{E} \cdot \hat{n} dA = E(r) \oint_S dA = E(r) 4\pi r^2.$$

For $r < R$, $S$ is within the conductor, so $q_{enc} = 0$, and Gauss's law gives

$$E(r) = 0,$$

as expected inside a conductor. If $r > R$, $S$ encloses the conductor so $q_{enc} = q$. From Gauss's law,

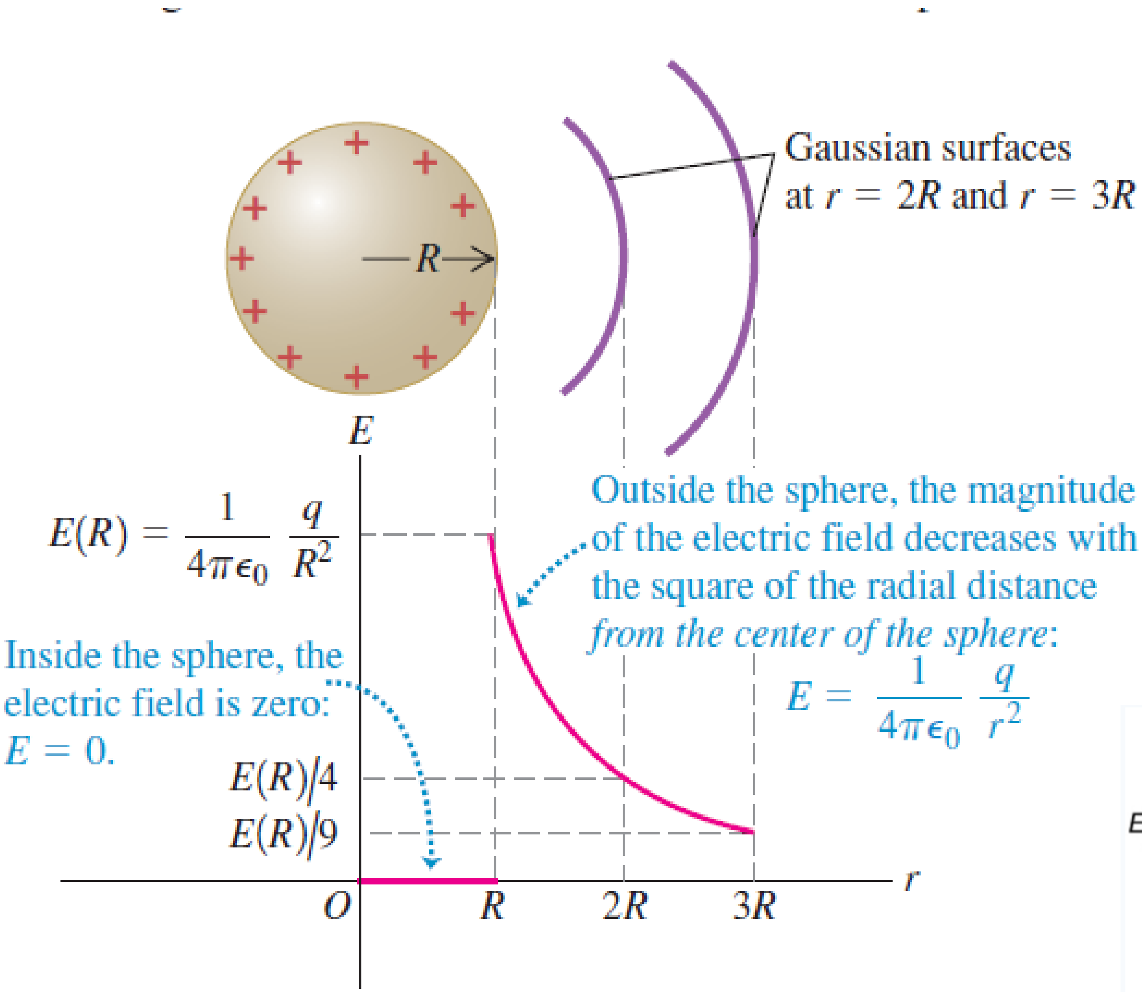
$$E(r) 4\pi r^2 = \frac{q}{\epsilon_0}.$$

The electric field of the sphere may therefore be written as

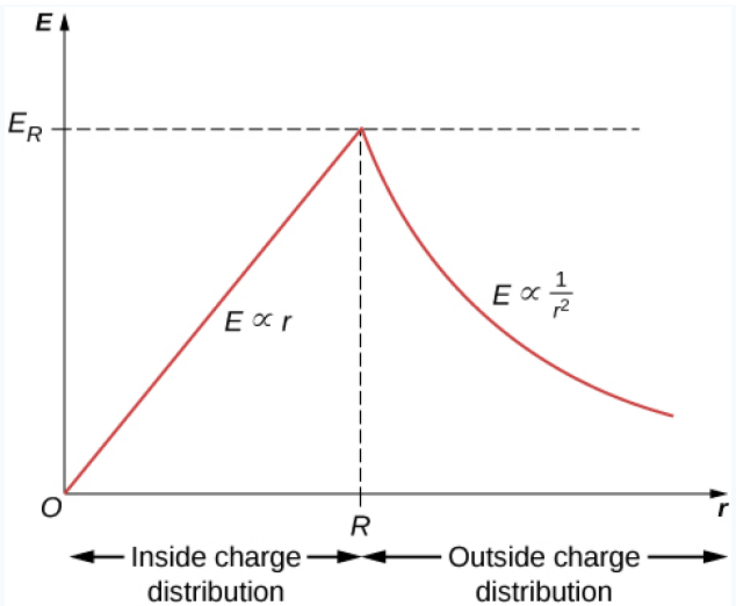
$$\vec{E} = \vec{0} \; (r < R),$$

$$\vec{E} = \frac{1}{4\pi\epsilon_0} \frac{q}{r^2} \hat{r} \; (r \geq R).$$

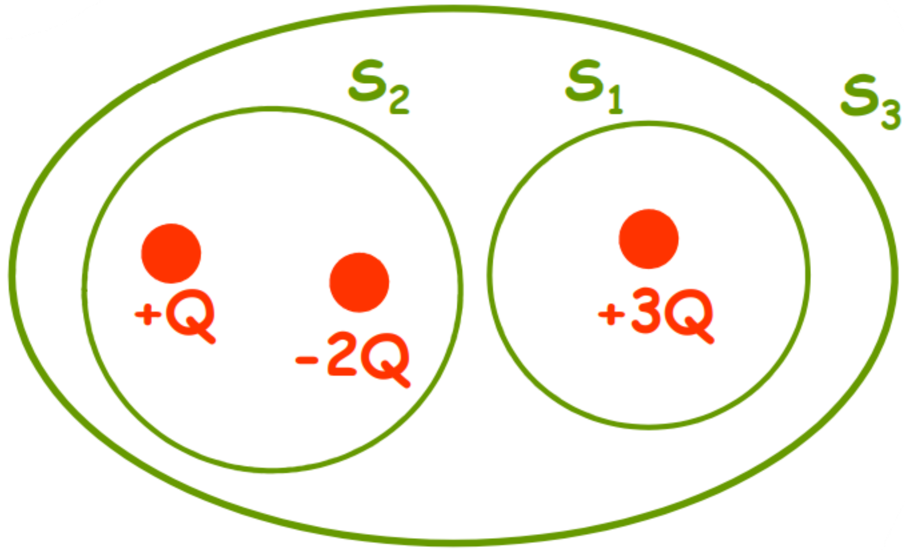
# Applying Gauss' Law Spherical Symmetry A Charged Isolated Conductor



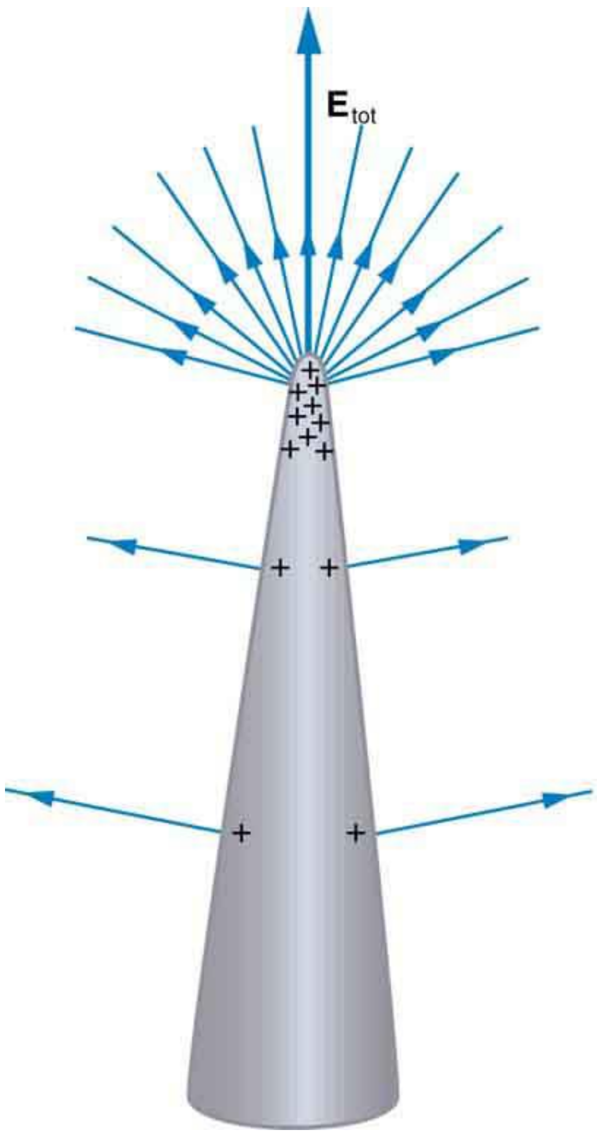

## Case of non-conducting sphere



Let $\Phi_1$, $\Phi_2$, $\Phi_3$, be the fluxes through the 3 closed surfaces. Then,



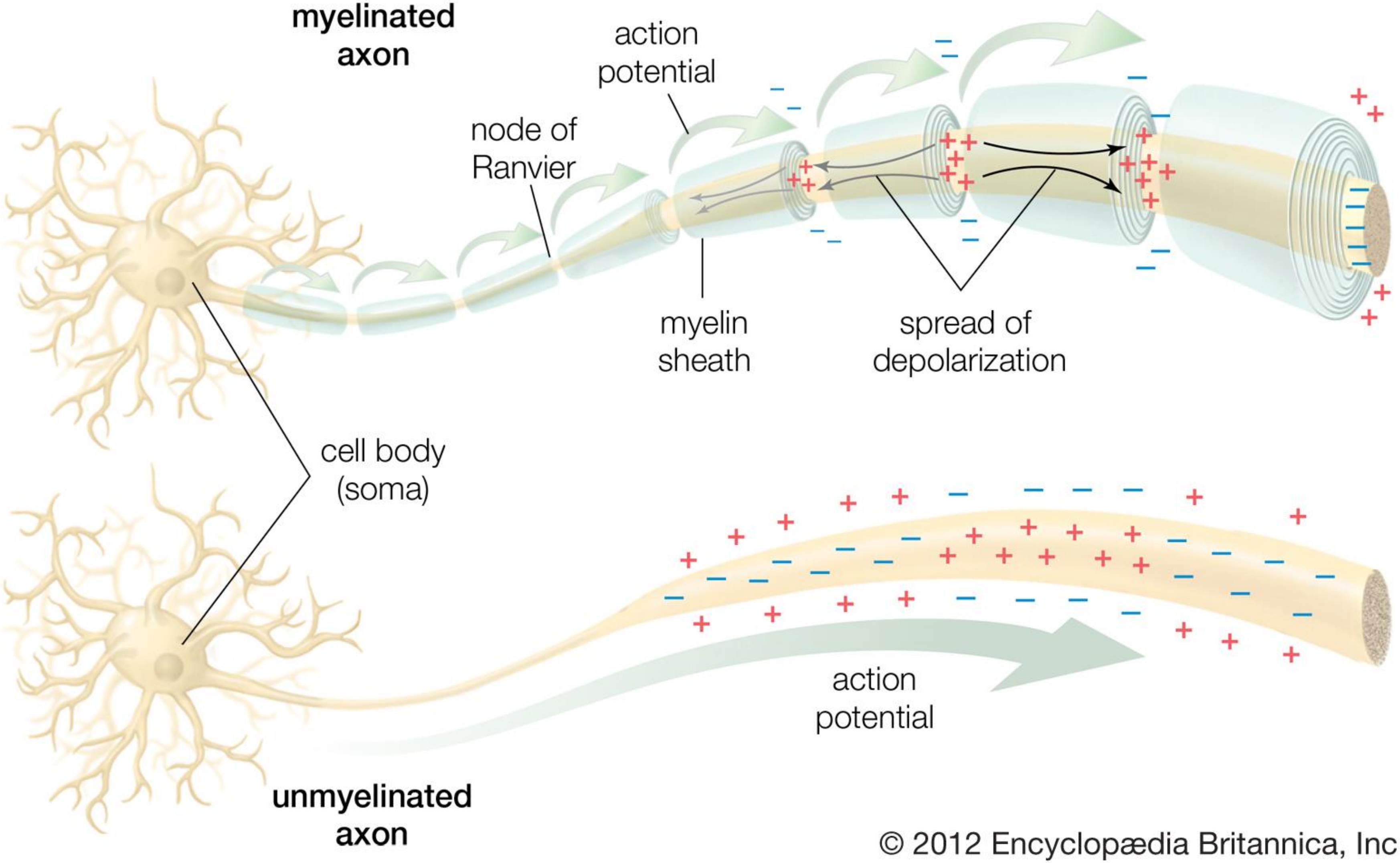

A) $|\Phi_1| > |\Phi_2| > |\Phi_3|$
B) $|\Phi_1| > |\Phi_3| > |\Phi_2|$
C) $|\Phi_3| > |\Phi_2| > |\Phi_1|$
D) $|\Phi_2| > |\Phi_1| > |\Phi_3|$
E) none of the above is true

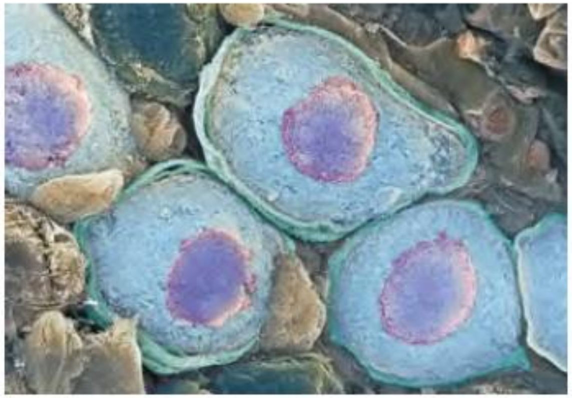


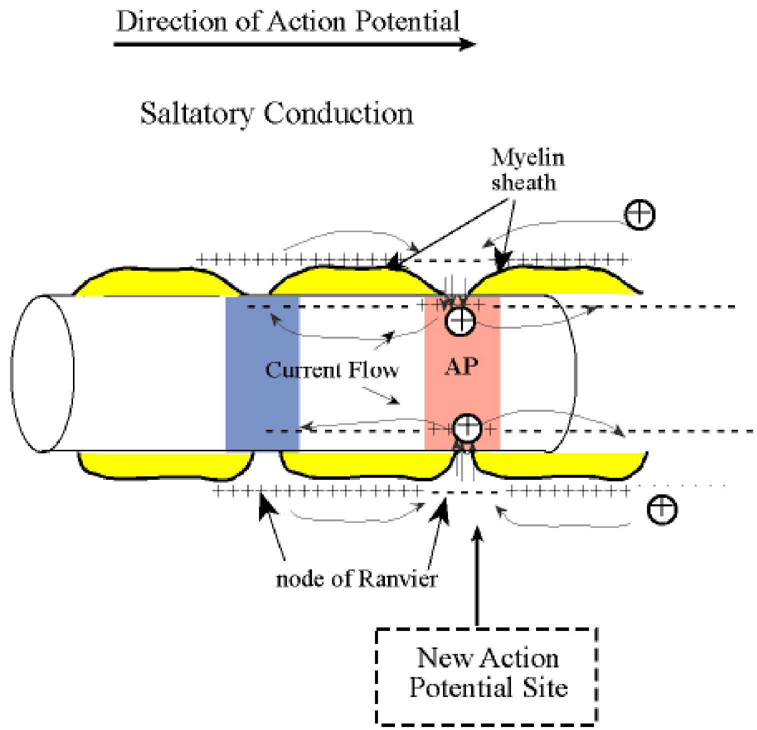

Excess charge on a nonuniform conductor becomes most concentrated at the location of greatest curvature.

A very sharp pointed conductor has a large charge concentration at the tip. The electric field is very strong at the point and can exert a force large enough to transfer charge on or off the conductor. **Lightning rods are used to prevent the buildup of large excess charges on structures and, thus, are pointed.**

# Charge Distribution Inside a Nerve Cell







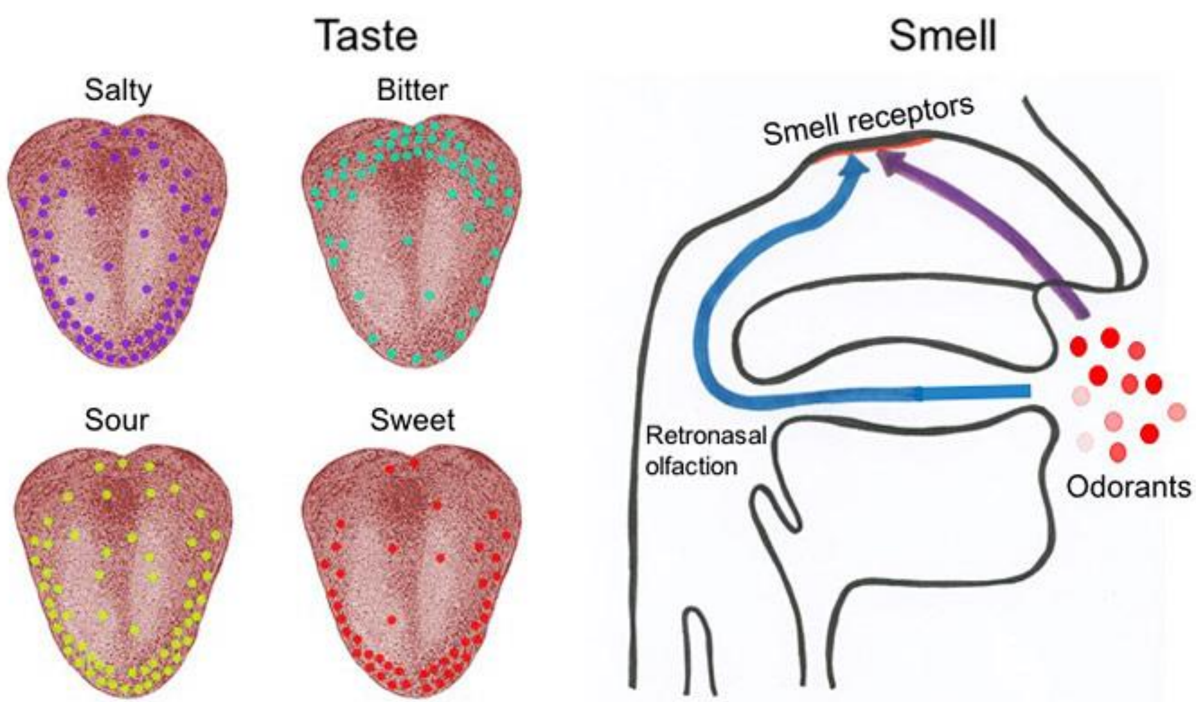

The interior of a human nerve cell contains both positive potassium ions and negatively charged protein molecules. Potassium ions can flow out of the cell through the cell membrane, but the much larger protein molecules cannot.

The result is that the interior of the cell has a net negative charge.

The fluid within the cell is a good conductor, so the molecules distribute themselves on the outer surface of the fluid—that is, on the inner surface of the cell membrane, which is an insulator. This is true no matter what the shape of the cell.

## Charge Distribution spice and hot

The sensation of pain that people who eat peppers feel is also due to the charge distributions in molecules!!



**Electric field of various symmetric charge distributions:** The following table lists electric fields caused by several symmetric charge distributions. In the table, $q$, $Q$, $\lambda$, and $\sigma$ refer to the *magnitudes* of the quantities.

| Charge Distribution | Point in Electric Field | Electric Field Magnitude |
|---|---|---|
| Single point charge $q$ | Distance $r$ from $q$ | $E = \frac{1}{4\pi\epsilon_0}\frac{q}{r^2}$ |
| Charge $q$ on surface of conducting sphere with radius $R$ | Outside sphere, $r > R$ | $E = \frac{1}{4\pi\epsilon_0}\frac{q}{r^2}$ |
| | Inside sphere, $r < R$ | $E = 0$ |
| Infinite wire, charge per unit length $\lambda$ | Distance $r$ from wire | $E = \frac{1}{2\pi\epsilon_0}\frac{\lambda}{r}$ |
| Infinite conducting cylinder with radius $R$, charge per unit length $\lambda$ | Outside cylinder, $r > R$ | $E = \frac{1}{2\pi\epsilon_0}\frac{\lambda}{r}$ |
| | Inside cylinder, $r < R$ | $E = 0$ |
| Solid insulating sphere with radius $R$, charge $Q$ distributed uniformly throughout volume | Outside sphere, $r > R$ | $E = \frac{1}{4\pi\epsilon_0}\frac{Q}{r^2}$ |
| | Inside sphere, $r < R$ | $E = \frac{1}{4\pi\epsilon_0}\frac{Qr}{R^3}$ |
| Infinite sheet of charge with uniform charge per unit area $\sigma$ | Any point | $E = \frac{\sigma}{2\epsilon_0}$ |
| Two oppositely charged conducting plates with surface charge densities $+\sigma$ and $-\sigma$ | Any point between plates | $E = \frac{\sigma}{\epsilon_0}$ |
| Charged conductor | Just outside the conductor | $E = \frac{\sigma}{\epsilon_0}$ |

## Summary Topics

- ❑ Apply Gauss' law to relate the net flux through a closed surface to the net enclosed charge.

- ❑ Derive the expression for the magnitude of the electric field of a charged particle by using Gauss' law.

- ❑ Identify how the algebraic sign of the net enclosed charge corresponds to the direction (inward or outward) of the net flux through a Gaussian surface.

- ❑ Identify that for a charged particle or uniformly charged sphere, Gauss' law is applied with a Gaussian surface that is a concentric sphere.

- ❑ Identify the value of the electric field inside an isolated conductor.

- ❑ Explain how Gauss' law is used to derive the electric field magnitude outside a line of charge or a cylindrical surface (such as a plastic rod) with a uniform linear charge density.

- ❑ Apply Gauss' law to derive the electric field magnitude *E* near a large, flat, nonconducting surface with a uniform surface charge density s.