Electric Potential-I

Phy 108 course
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Processes occurring during thunderstorms cause large differences in electric potential between a thundercloud and the ground.

The result of this potential difference is an electrical discharge that we call lightning, such as this display.

Experimentally, physicists and engineers discovered that the electric force is conservative and thus has an associated electric potential energy.

If the charge is free to move, it will do so in response to the electric force.

Therefore, the electric field will be doing work on the charge. This work is *internal* to the system.

This situation is similar to that in a gravitational system: When an object is released near the surface of the Earth, the gravitational force does work on the object.

Experimentally, physicists and engineers discovered that the electric force is conservative and thus has an associated electric potential energy.

If we release a particle at P within an electric field, it will begins to move and thus has kinetic energy. Energy cannot appear by magic, so from where does it come?

It comes from the electric potential energy U associated with the force between the two particles in the arrangement

Electric Potential Trailer

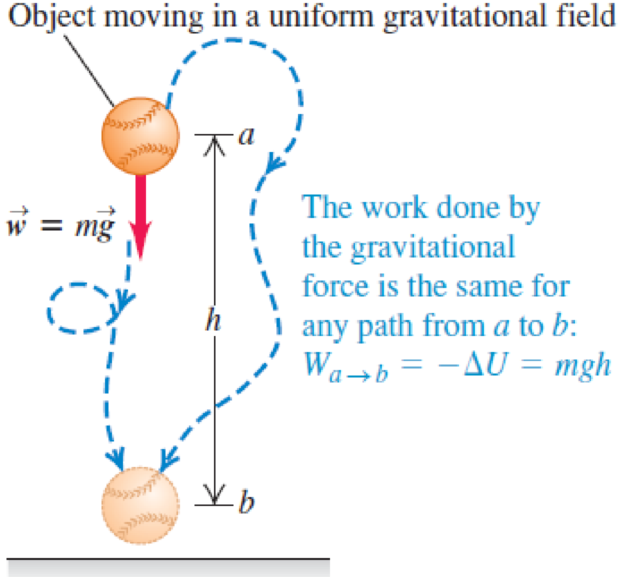
https://www.youtube.com/watch?v=ZZJ5znGvYAU

Potential Energy and Work done

$$W_{a \to b} = \int_a^b \vec{F} \cdot d\vec{l} = \int_a^b F \cos\phi \, dl \quad \text{(work done by a force)}$$

When the particle moves from a point where the potential energy is U_a to a point where it is U_b, the change in potential energy is $\Delta U = U_b - U_a$ and the work $W_{a \to b}$ done by the force is

$$W_{a \to b} = U_a - U_b = -(U_b - U_a) = -\Delta U \quad \text{(work done by a conservative force)}$$



When $W_{a \to b}$ is positive, U_a is greater than U_b, ΔU is negative, and the potential energy *decreases*. That's what happens when a baseball falls from a high point (a) to a lower point (b) under the influence of the earth's gravity; the force of gravity does positive work, and the gravitational potential energy decreases (Fig. 23.1). When a tossed ball is moving upward, the gravitational force does negative work during the ascent, and the potential energy increases.

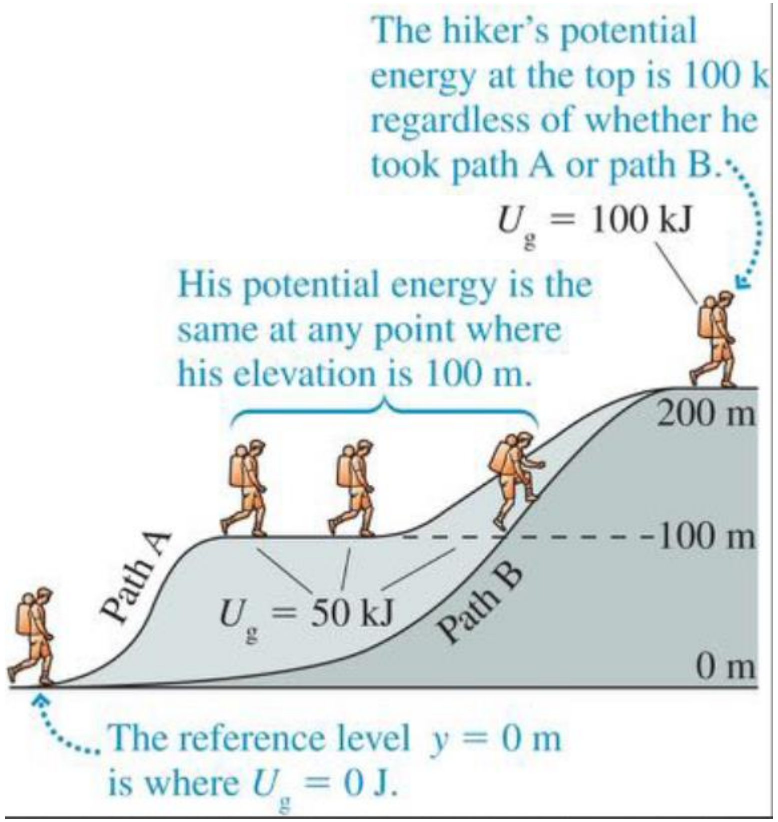
The negative sign tells us that positive work by the conservative force leads to a decrease in potential energy.

Conservative Force

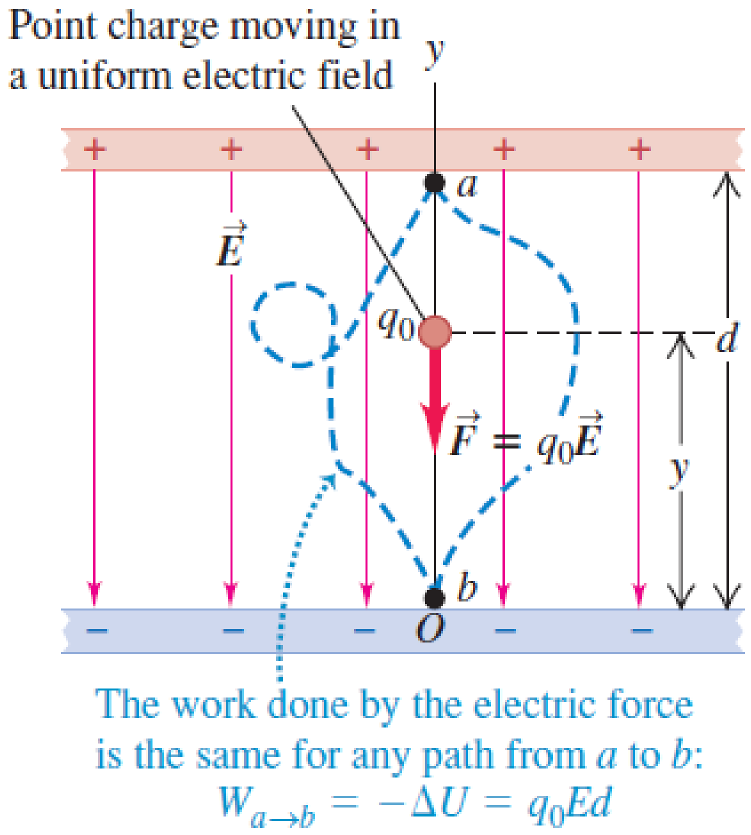
- Work done is Path independent.
- Closed path work done is zero.

The negative sign tells us that positive work by the conservative force leads to a decrease in potential energy.

The gravitational field vector points towards the center of earth.



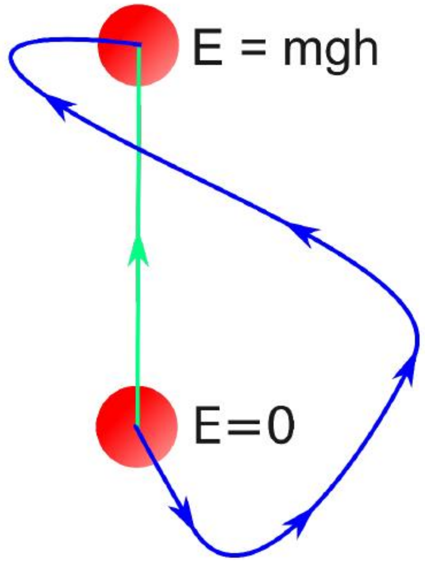
Gravitational Potential Energy change



The electric field vector points from higher potential toward lower potential.

Electric Potential Energy change

- Potential energy difference doesn't depend on the path – only on the two points A and B



Conservative force, where the work done/change in potential energy is independent of path followed.

Conservation of Energy.

If a charged particle moves through an electric field with no force acting on it other than the electric force due to the field, then the mechanical energy is conserved.

$$U_i + K_i = U_f + K_f$$

or, $\Delta K = -\Delta U$

$$\Delta K = -q\Delta V = -q(V_f - V_i)$$

Work by an Applied Force.

If some force in addition to the electric force acts on the particle, we say that the additional force is an applied force or external force, which is often attributed to an external agent,

(initial energy) + (work by applied force) = (final energy)

$$U_i + K_i + W_{app} = U_f + K_f$$

$$\Delta K = -\Delta U + W_{app}$$

The work by the applied force can be positive, negative, or zero, and thus the energy of the system can increase, decrease, or remain the same.

In the special case where the particle is stationary before and after the move, $K_i = K_f$

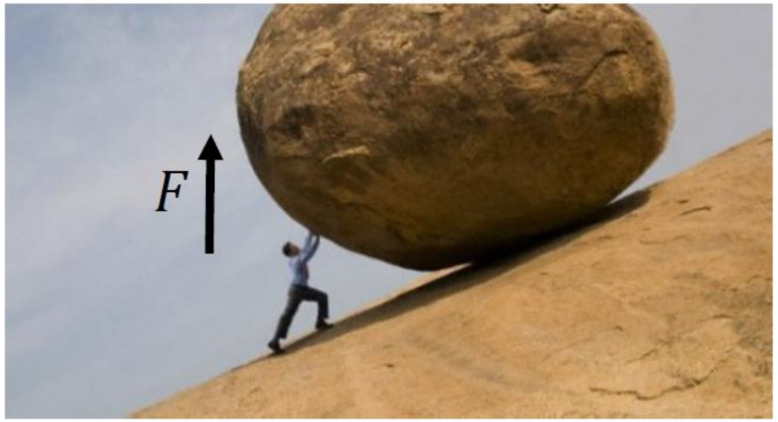
Then, $W_{app} = \Delta U = q\Delta V$

The negative sign tells us that positive work by the conservative force leads to a decrease in potential energy.

- Potential energy U is the energy stored in a system (when work is done against a force)

- Potential energy may be released and converted into other forms (such as kinetic energy)

- e.g. force of gravity …



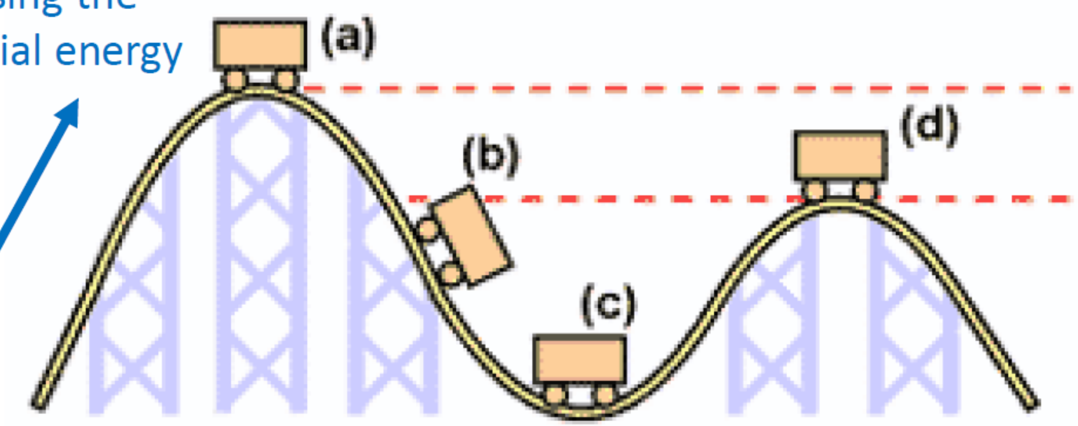
$$F = mg$$

Work = Force x Distance

$$W = F \times h$$
$$= mgh$$
$$\rightarrow U = mgh$$

h

Work is done, increasing the potential energy



- Potential energy difference is the only thing that matters – not the reference (or zero) level

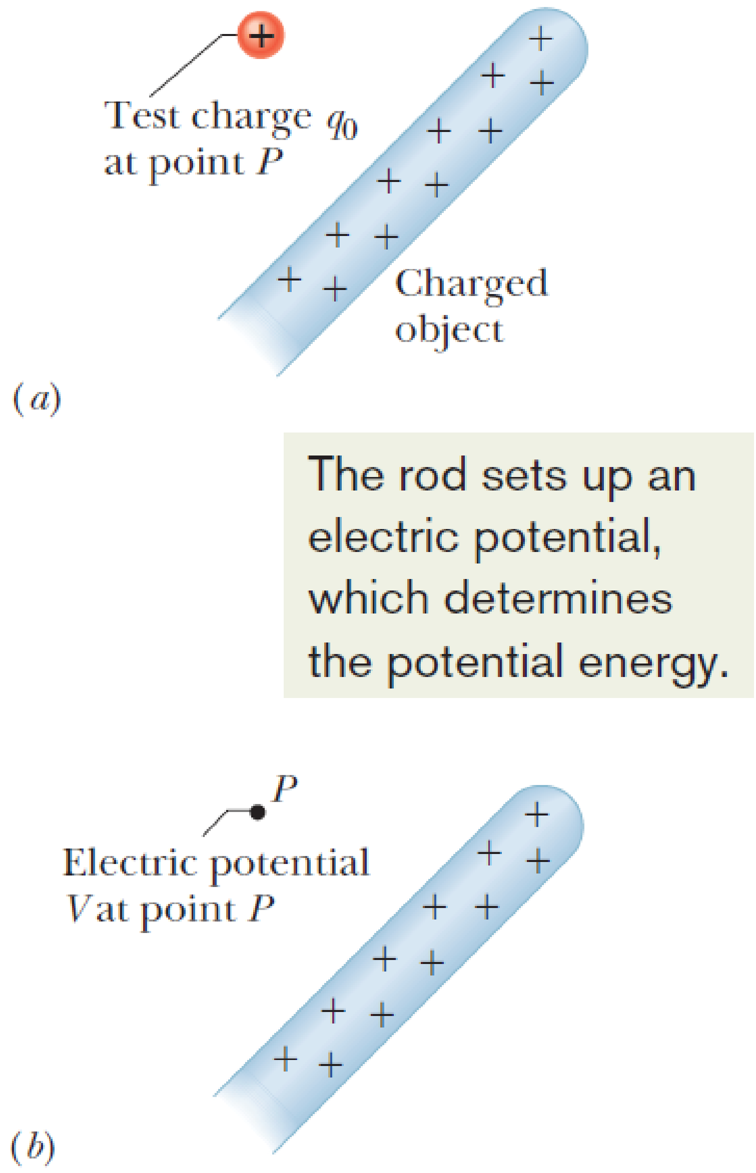
- For example, applying conservation of energy to a mechanics problem:

Final energy = Initial energy

$$KE_{final} + PE_{final} = KE_{initial} + PE_{initial}$$

$$KE_{final} = KE_{initial} + (PE_{initial} - PE_{final})$$

Electric Potential Energy



Let's follow the same procedure with our new conservative force, the electric force. we want to find the potential energy U associated with a positive test charge q_0 located at point P in the electric field of a charged rod.

First, we need a reference configuration for which $U= 0$. A reasonable choice is for the test charge to be infinitely far from the rod, because then there is no interaction with the rod.

Next, we bring the test charge in from infinity to point P to form the configuration of Fig.

Along the way, we calculate the work done by the electric force on the test charge.

Let's use the notation W_∞ to emphasize that the test charge is brought in from infinity.

The work and thus the potential energy can be positive or negative depending on the sign of the rod's charge.

Electric Potential

The electric potential V at P in terms of the work done by the electric force and the resulting potential energy,

$$V = \frac{-W_\infty}{q} = \frac{U}{q} \text{ (since } W = -U)$$

The electric potential, like the electric field, is a property of the source charges.

That is, the electric potential is the amount of electric potential energy per unit charge when a positive test charge is brought in from infinity

$$(\text{electric potential energy}) = (\text{particle's charge})\left(\frac{\text{electric potential energy}}{\text{unit charge}}\right),$$

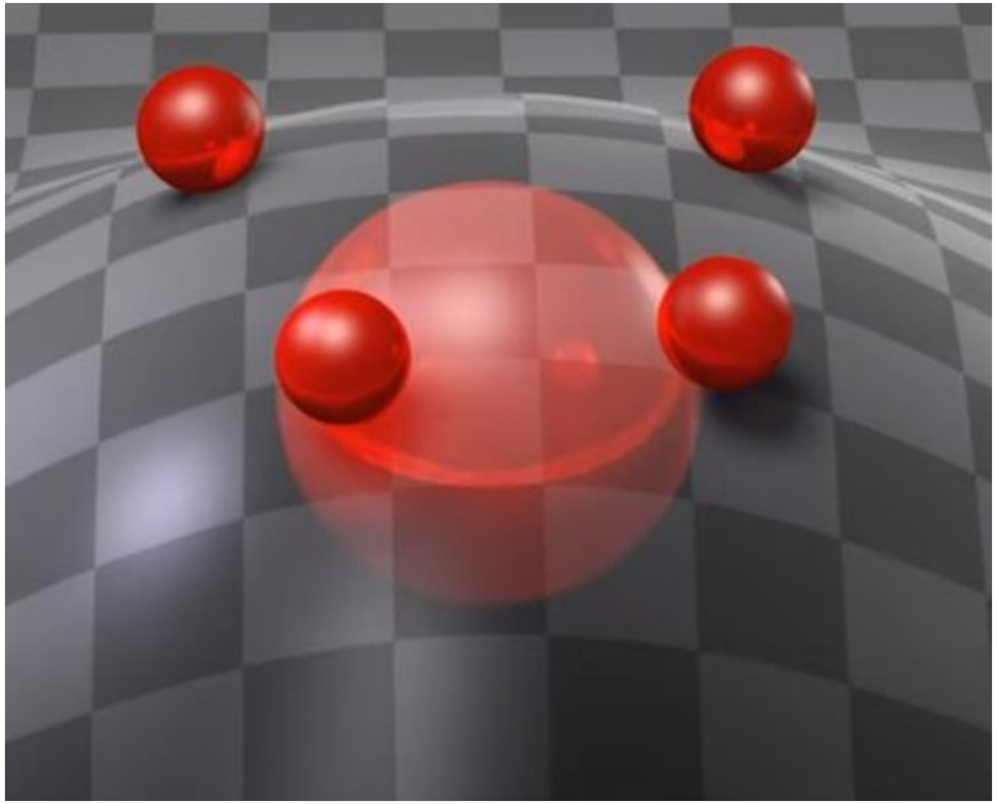
or $$U = qV, \tag{24-3}$$

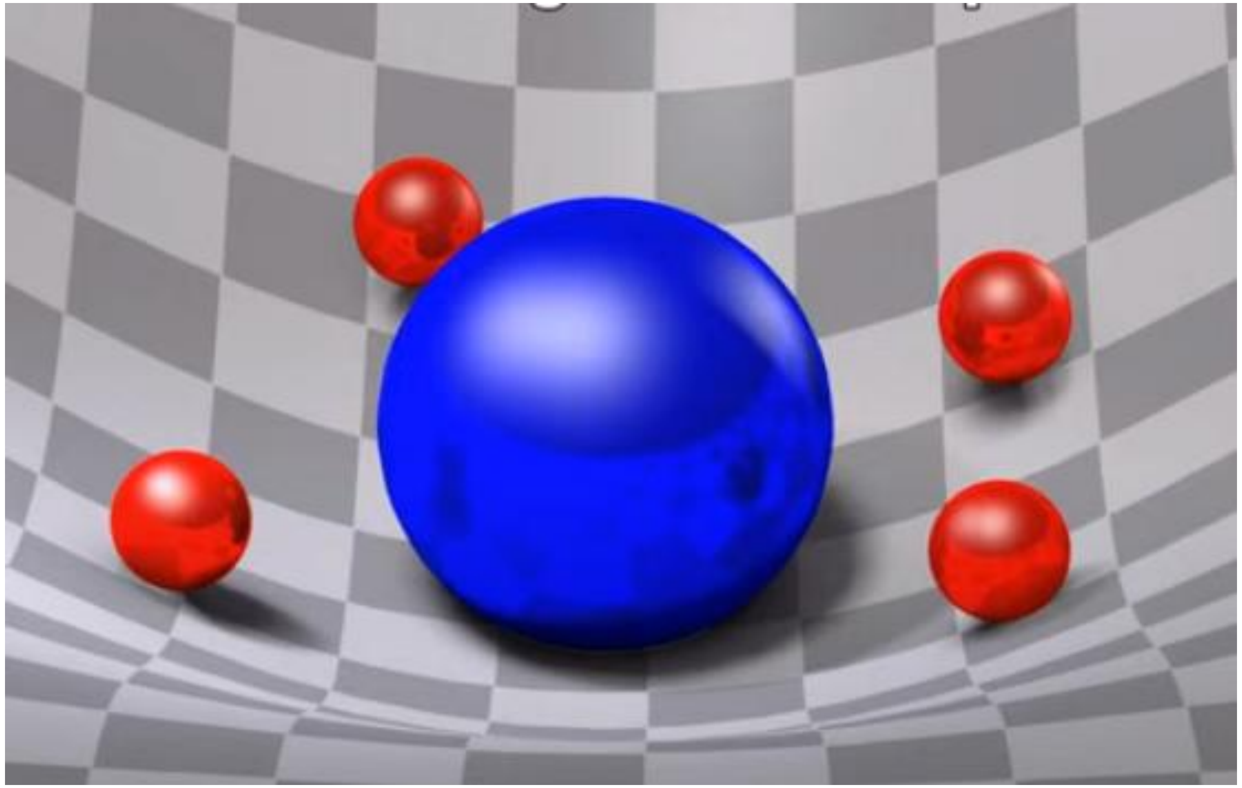
Units.

The SI unit for potential is the joule per coulomb.

Special unit, the *volt* (abbreviated V), is used to represent it.

Thus, $1\ volt\ (V) = 1\ joule\ per\ coulomb$





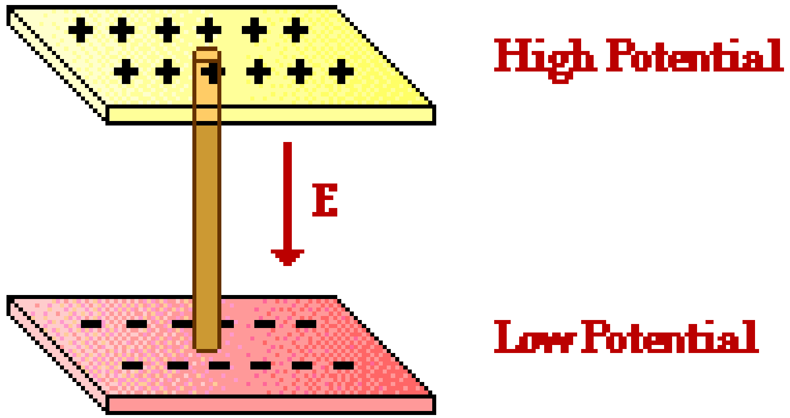
Electric Potential

Whether the test charge is positive or negative, the following general rules apply:

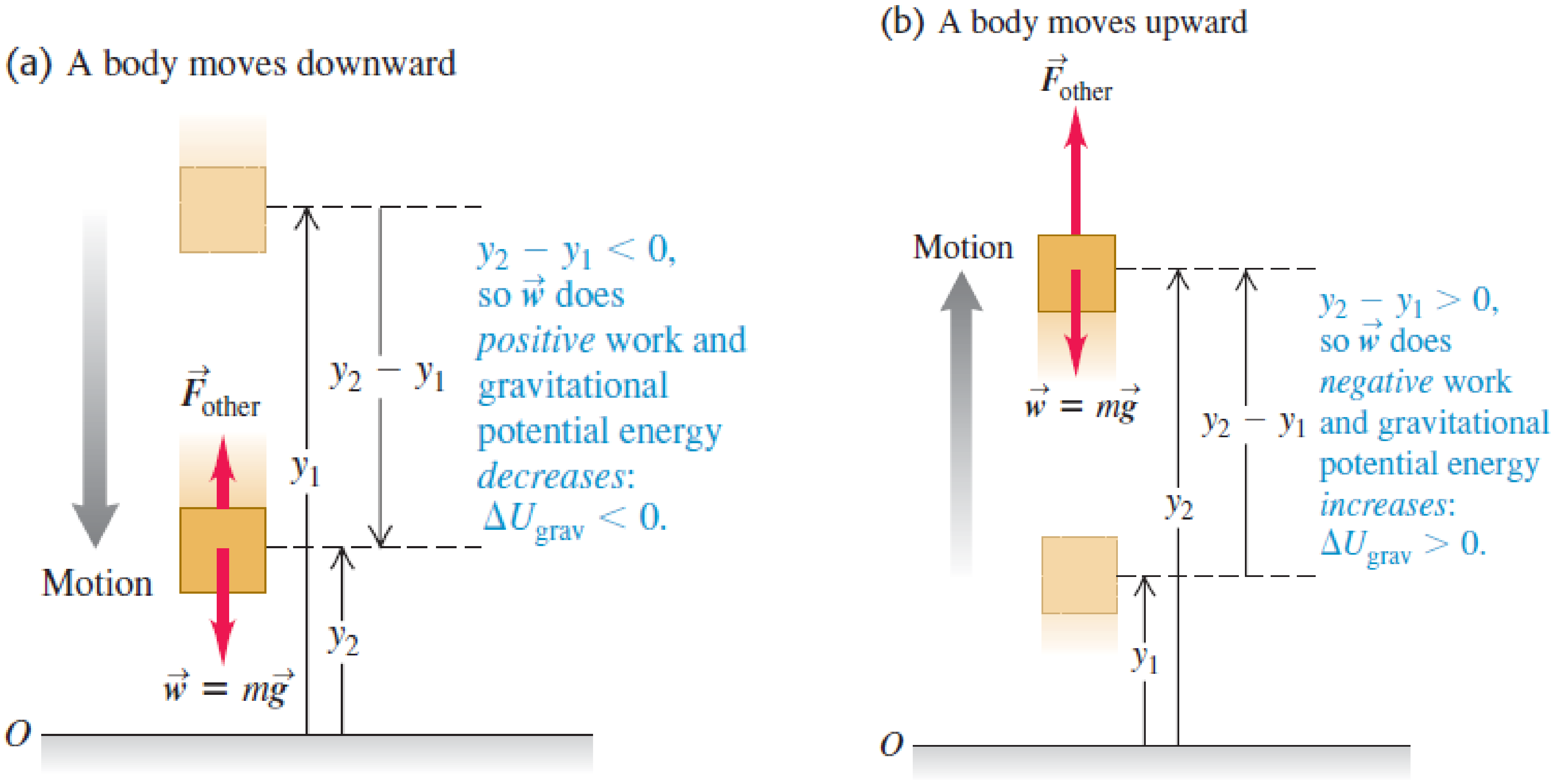
U increases **if the test charge moves in the direction** ***opposite*** **the electric force,** $\boldsymbol{F = q_0 E}$

U decreases **if moves in the** ***same*** **direction as** $\boldsymbol{F = q_0 E}$

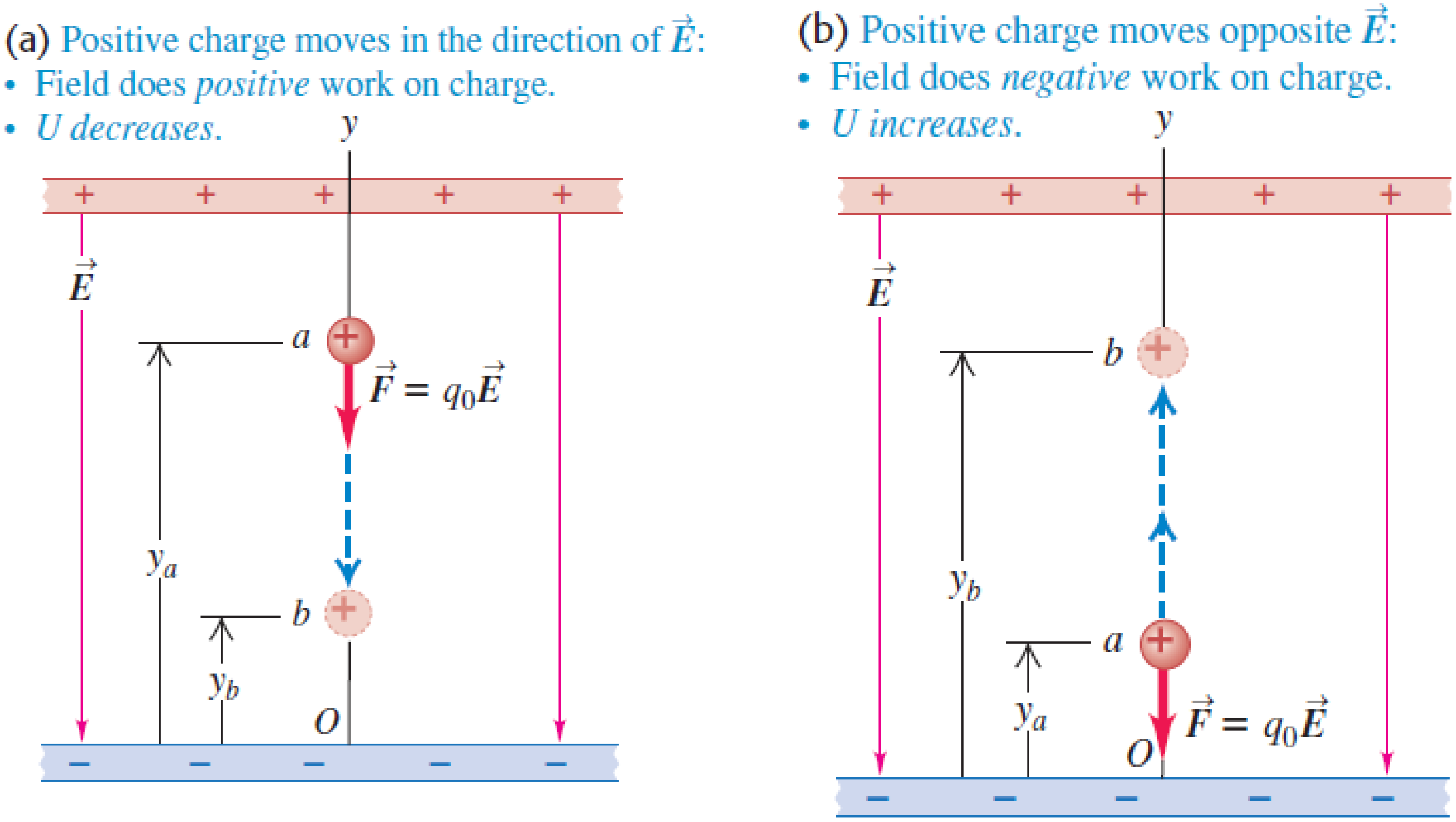
This is the same behavior as for gravitational potential energy, which increases if a mass *m* moves upward (opposite the direction of the gravitational force) and decreases if *m* moves downward (in the same direction as the gravitational force).



Motion Through Gravitational Field



Motion Through an Electric Field

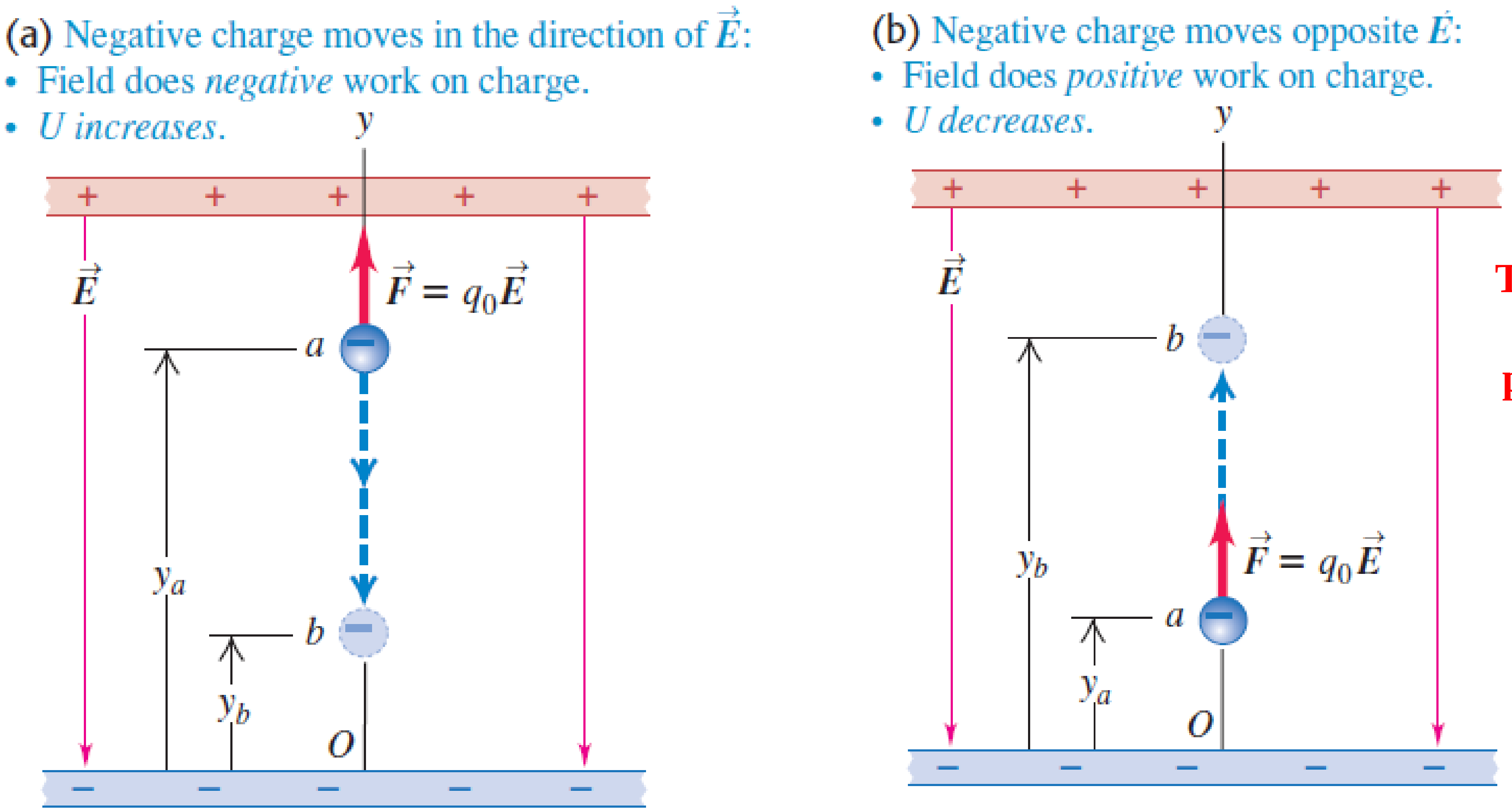


The electric field vector points from higher potential toward lower potential.

When the test charge moves from height y_a to height y_b, the work done on the charge by the field is given by

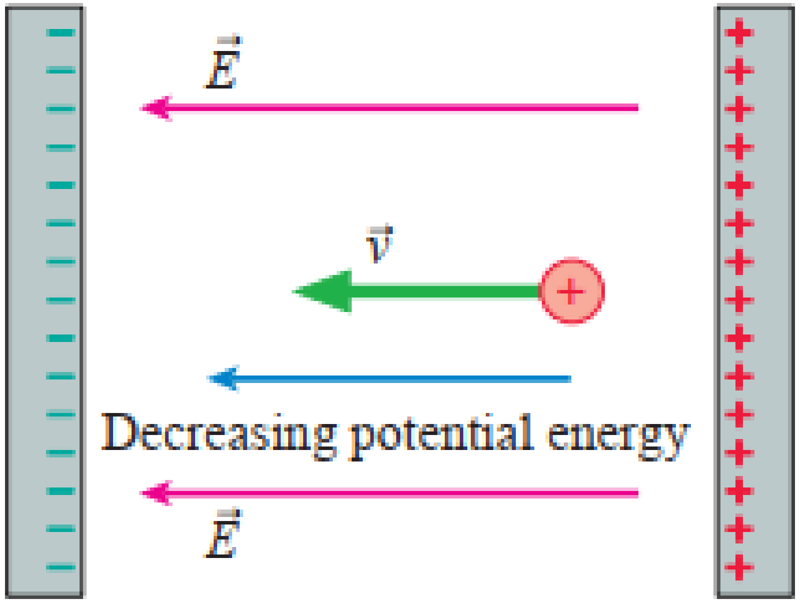
$$W_{a\to b} = -\Delta U = -(U_b - U_a) = -(q_0Ey_b - q_0Ey_a) = q_0E(y_a - y_b) \quad (23.6)$$

Motion Through an Electric Field

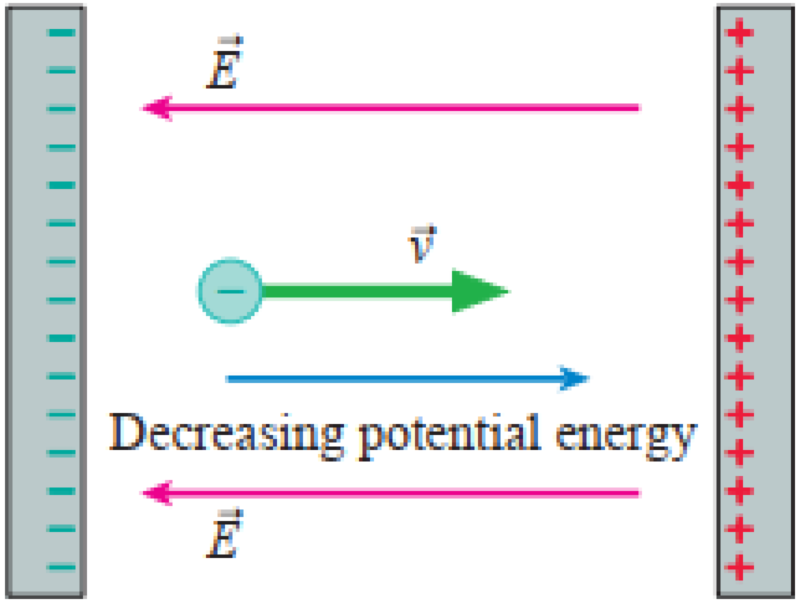


The electric field vector points from higher potential toward lower potential.

A charged particle of either sign gains kinetic energy as it moves in the direction of decreasing potential energy.



The potential energy of a positive charge decreases in the direction of $\vec{E}$. The charge gains kinetic energy as it moves toward the negative plate.



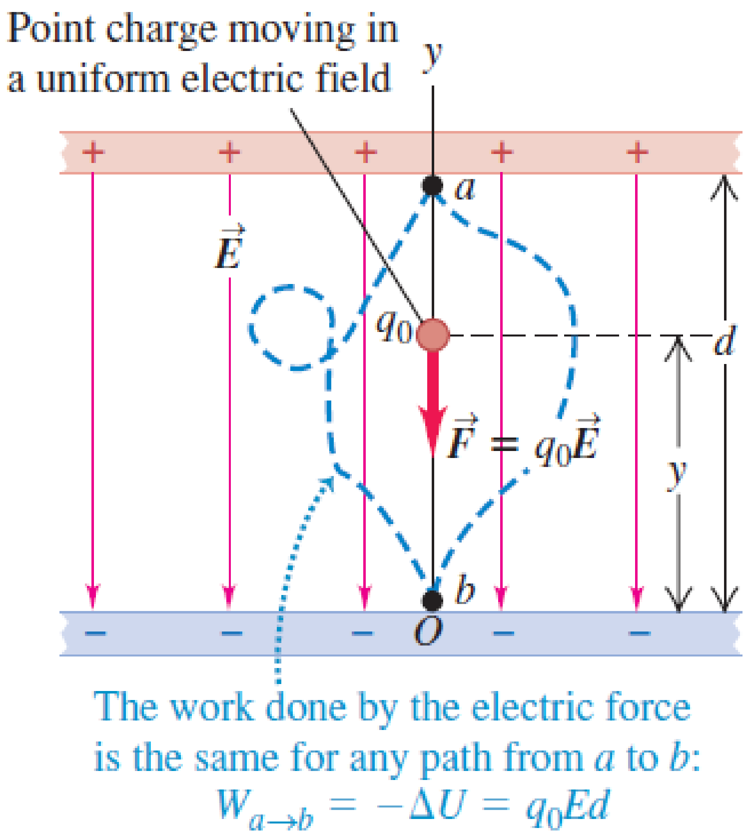
The potential energy of a negative charge decreases in the direction opposite to $\vec{E}$. The charge gains kinetic energy as it moves away from the negative plate.

Motion Through an Electric Field

Let's look at an electrical example of these basic concepts. In Fig. 23.2 a pair of charged parallel metal plates sets up a uniform, downward electric field with magnitude E. The field exerts a downward force with magnitude $F = q_0E$ on a positive test charge q_0. As the charge moves downward a distance d from point a to point b, the force on the test charge is constant and independent of its location. So the work done by the electric field is the product of the force magnitude and the component of displacement in the (downward) direction of the force:

$$W_{a\to b} = Fd = q_0Ed \tag{23.4}$$

This work is positive, since the force is in the same direction as the net displacement of the test charge.

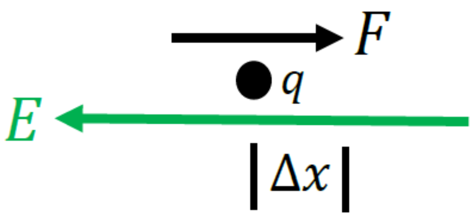


The y-component of the electric force, $F_y = -q_0E$ is constant

Then the corresponding potential energy, $U = q_0Ed$

The electric field vector points from higher potential toward lower potential.

- e.g. moving a charge through an electric field…



$$F = -qE$$

(minus sign because the force is opposite to E)

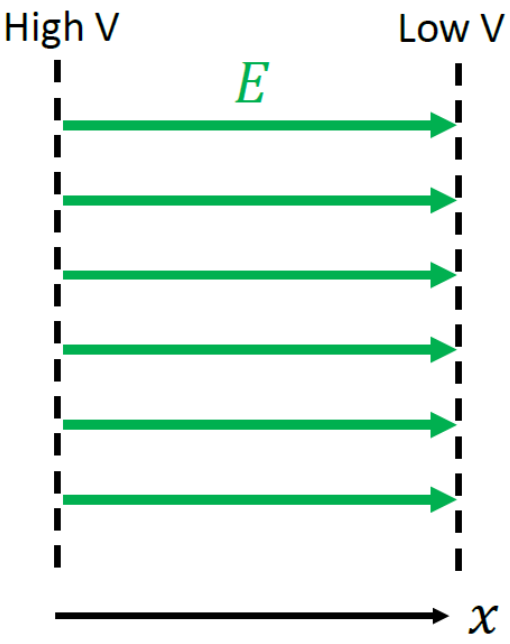
Work = Force x Distance $\quad W = F\,\Delta x \quad = -qE\,\Delta x$

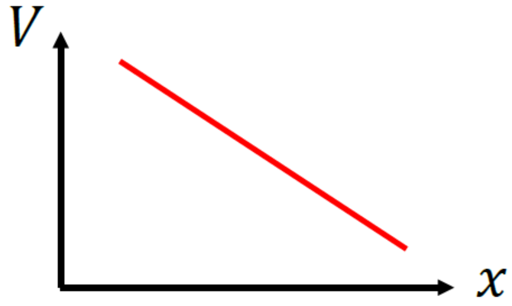
- Potential difference ΔV is work needed to move 1C of charge: $W = q\,\Delta V$

- Equate: $q\,\Delta V = -qE\,\Delta x$

$$\boxed{E = -\frac{\Delta V}{\Delta x}}$$

- Electric field is the gradient of potential $E = -\frac{\Delta V}{\Delta x}$



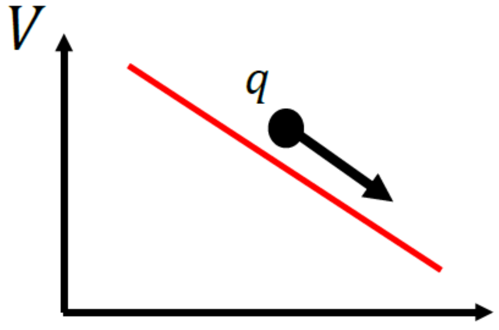


- Positive charges feel a force from high to low potential
- Negative charges feel a force from low to high potential

- Analogy with gravitational potential

Gravitational potential difference exerts force on mass

Electric potential difference exerts force on charge



Electric potential

230 V

1.5 V

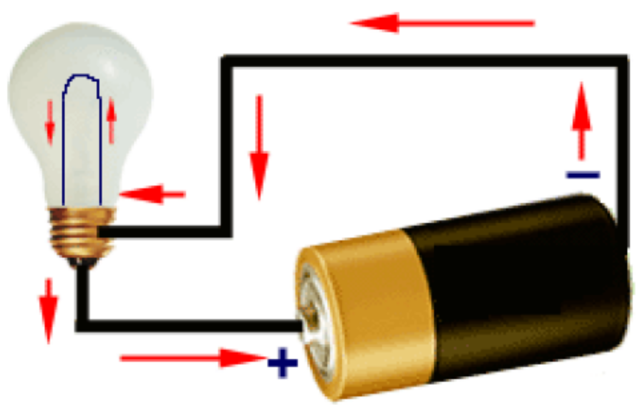
100,000 V

So what is a volt?

- What does it mean when it says “1.5 Volts” on the battery?
- The electric potential difference between the ends is 1.5 Volts

- The electric potential difference ΔV in volts between two points is the work in Joules needed to move 1 C of charge between those points

$$\boxed{W = q \times \Delta V}$$



Simple circuit with light

The 1.5 V battery does 1.5 J of work for every 1 C of charge flowing round the circuit

The electric potential difference between the ground and a cloud in a particular thunderstorm is 1.2×10^9 V. What is the magnitude of the change in energy (in multiples of the electron-volt) of an electron that moves between the ground and the cloud?

Electron-volts.

the *electron-volt* (eV), which is defined to be equal to the work required to move a single elementary charge *e* (such as that of an electron or proton) through a potential difference *V* of exactly one volt,

$$1\ eV = e(1\ V) = 1.602 \times 10^{-19} J$$

According to Einstein famous E=mc^2, find the energy in terms of eV for an electron of rest mass 9.1x10^{-31} kg, where the speed of light is 3x10^8 m/s.

$E=9.1\text{x}10^{-31}\text{x}(3\text{x}10^{8})^2/1.6\text{x}10^{-19}=0.511$ MeV

In the figure, we move a proton from point *i* to point *f* in a uniform electric field.

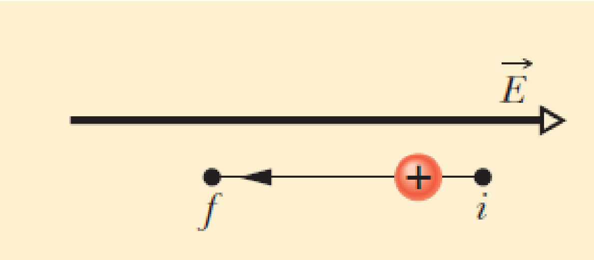
Is positive or negative work done by

(a) the electric field?

(b) applied force?

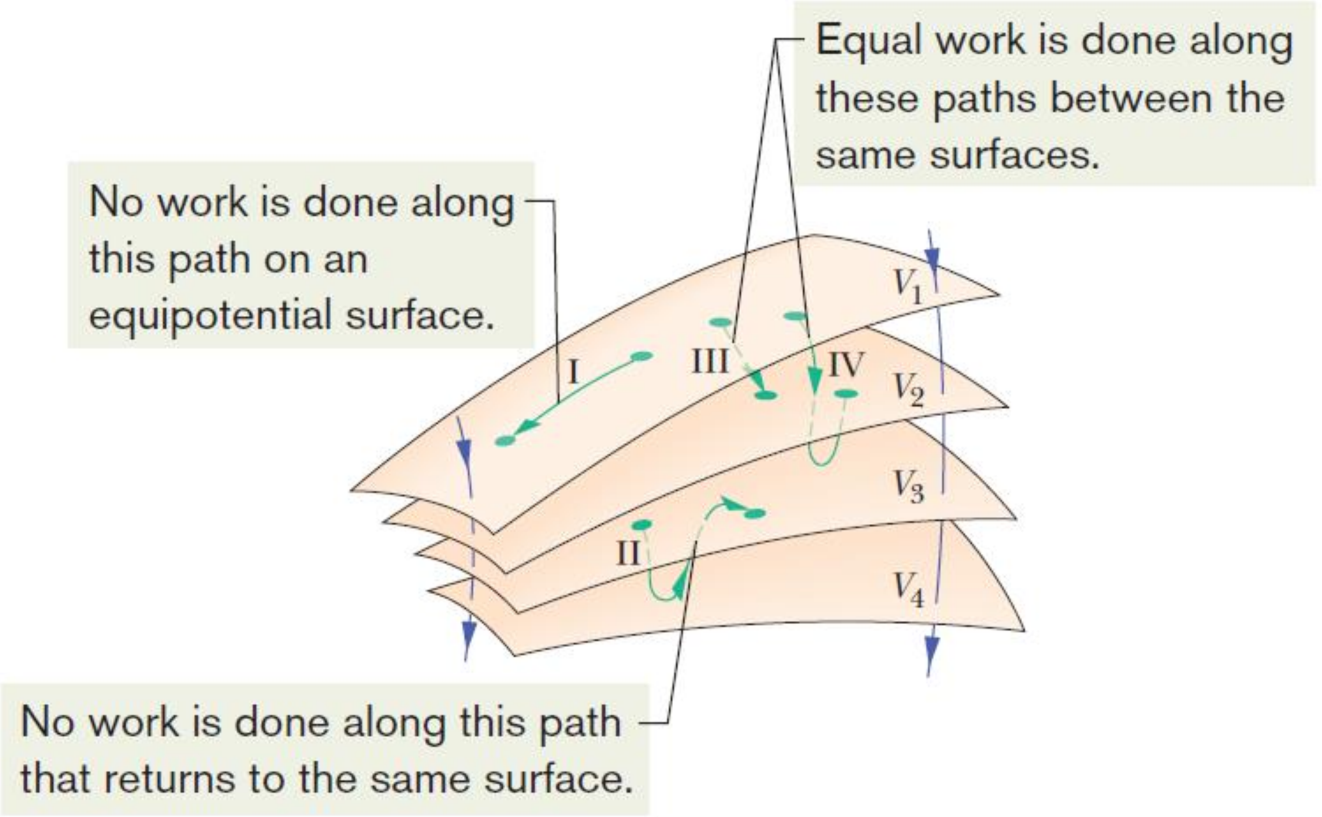
(c) Does the electric potential energy increase or decrease?

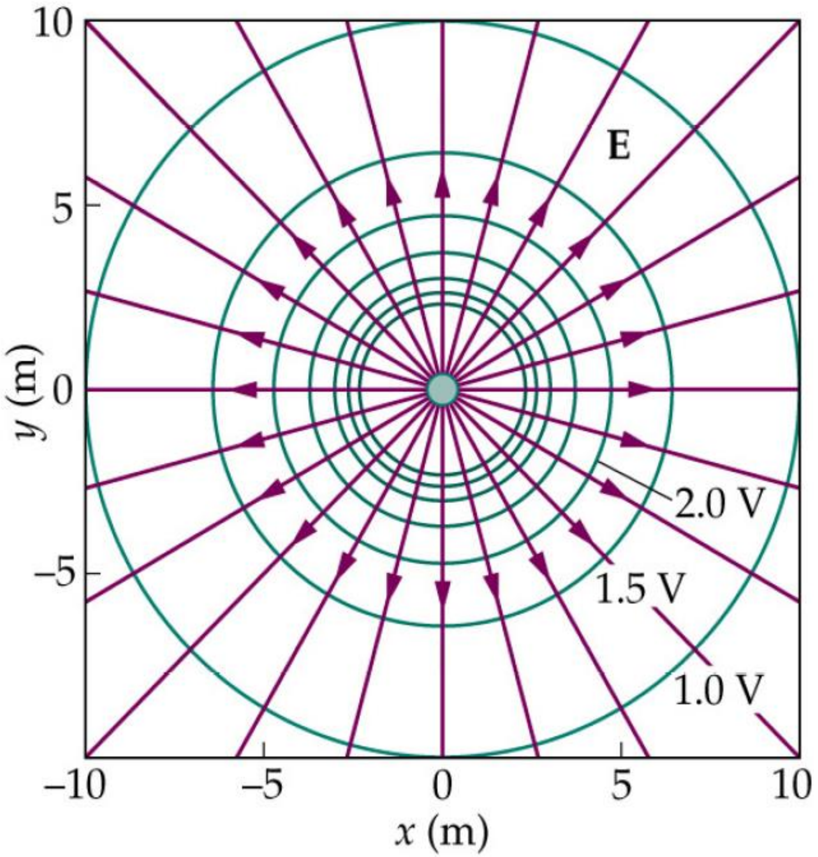
(d) Does the proton move to a point of higher or lower electric potential?

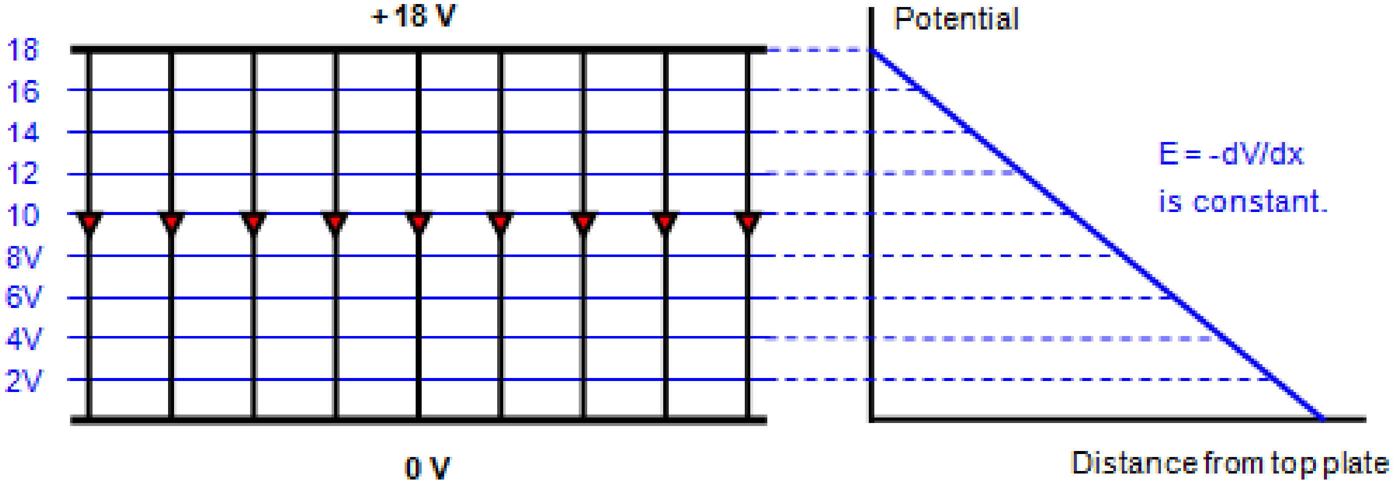


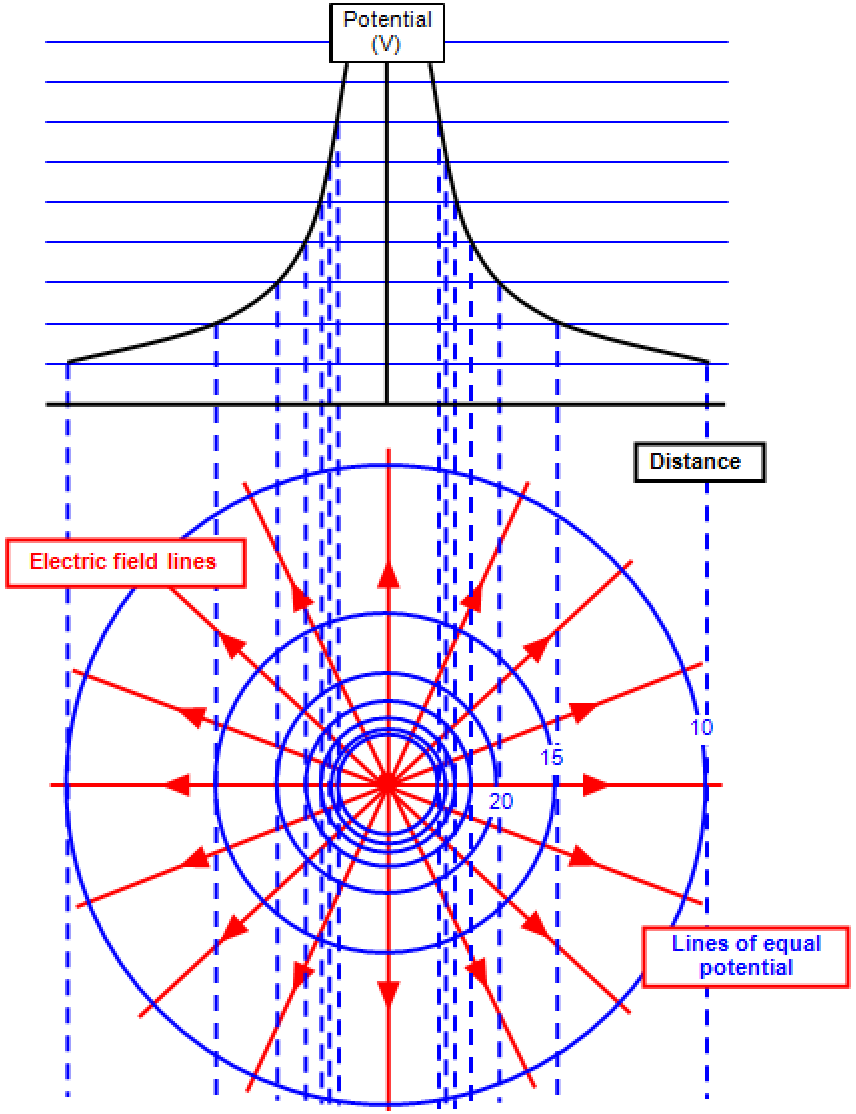
Equipotential Surfaces

Adjacent points that have the same electric potential form an **equipotential surface,** which can be either an imaginary surface or a real, physical surface. No net work *W* is done on a charged particle by an electric field when the particle moves between two points *i* and *f* on the same equipotential surface.






+18 V
18
16
14
12
10
8V
6V
4V
2V
0 V
Potential
E = -dV/dx
is constant.
Distance from top plate


Potential
(V)
Distance
Electric field lines
10
15
20
Lines of equal
potential

Work: positive or negative?

The right figure shows a family of equipotential surfaces associated with the electric field due to some distribution of charges. V1=100 V, V2=80 V, V3=60 V, V4=40 V.

W_I, W_{II}, W_{III} and W_{IV} are the works done by the electric field on a charged particle q as the particle moves from one end to the other. Which statement of the following is not true?

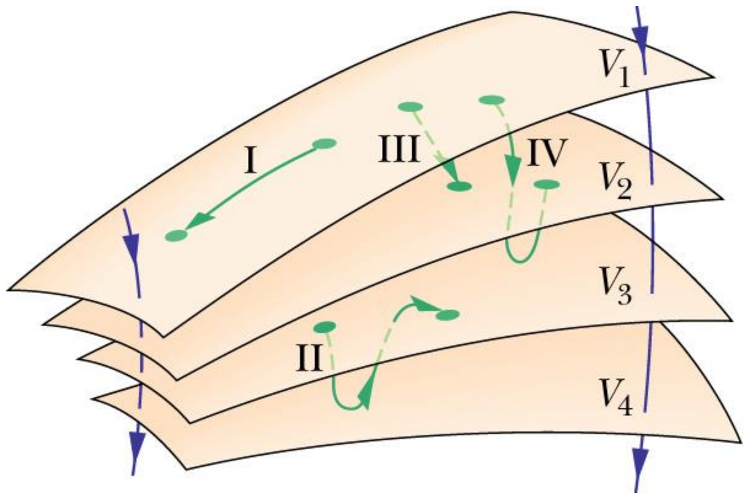
A. $W_I = W_{II}$

B. W_{III} is not equal to zero

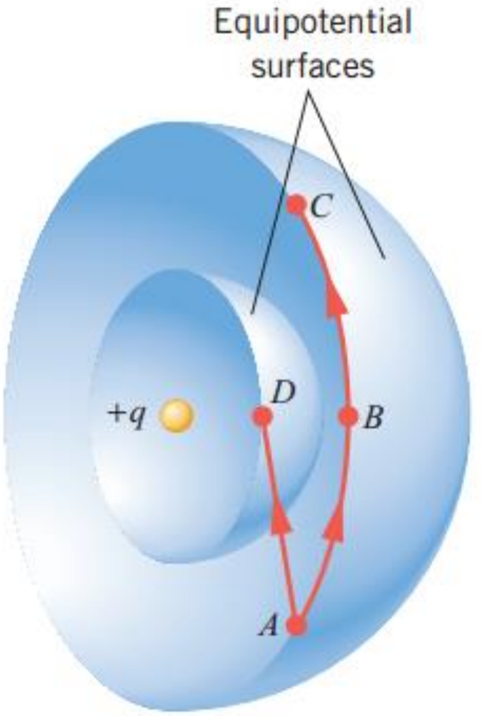
C. W_{II} equals to zero

D. $W_{III} = W_{IV}$

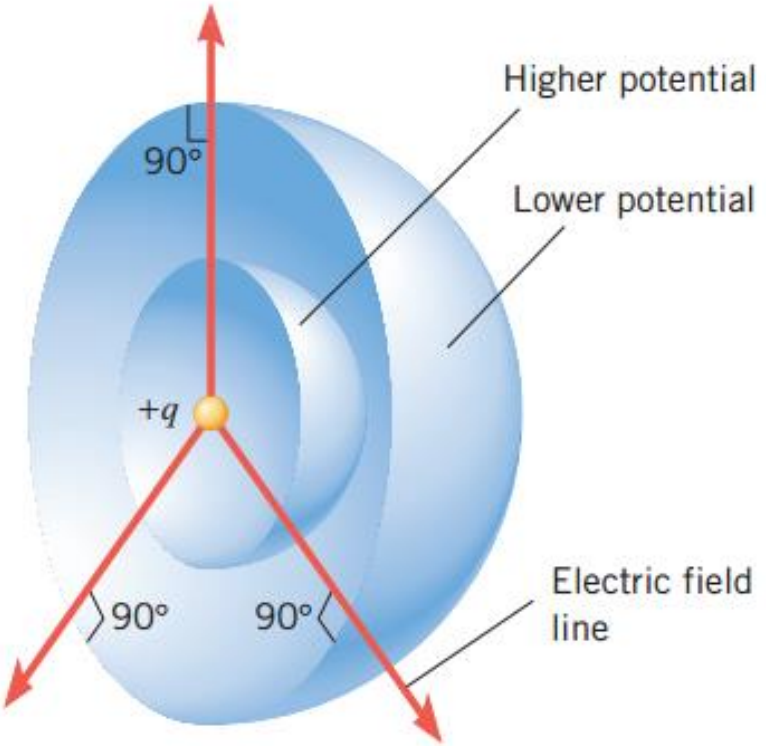
E. W_{IV} is positive



Equipotential Surfaces

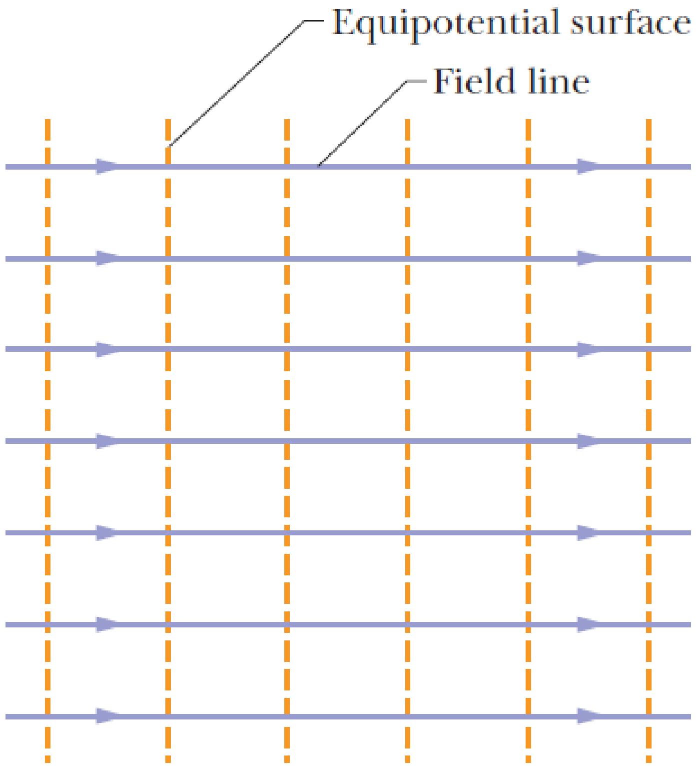


The equipotential surfaces that surround the point charge *+q* are spherical. The electric force does no work as a charge moves on the path *ABC.*
However, work is done by the electric force when a charge moves between two equipotential surfaces, as along the path *AD.*

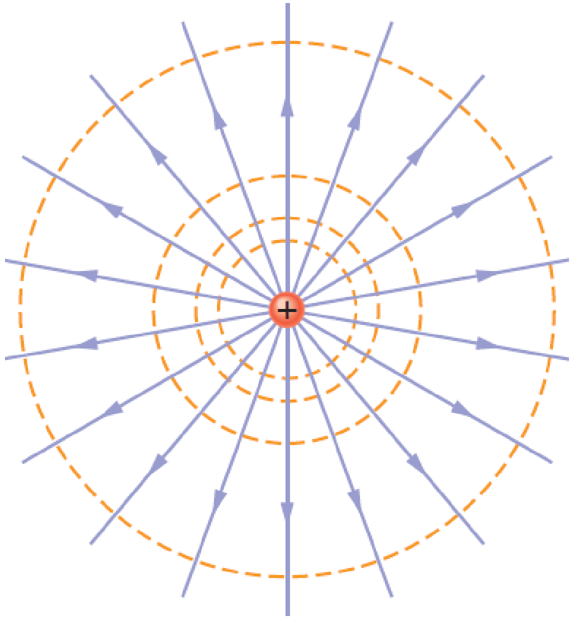


The electric field points in the direction of *decreasing* potential.

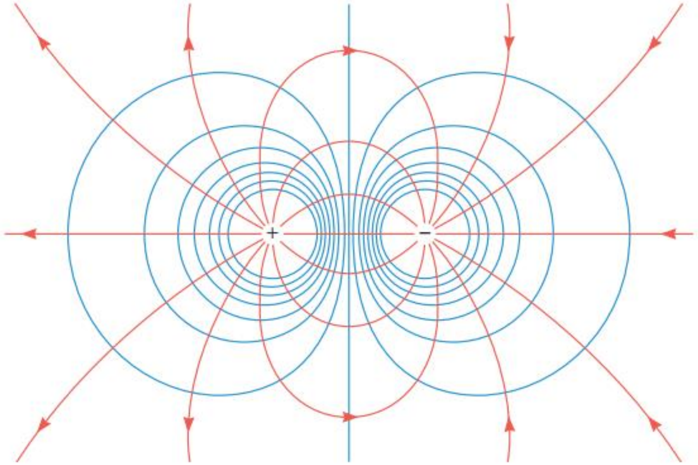
Equipotential Surfaces

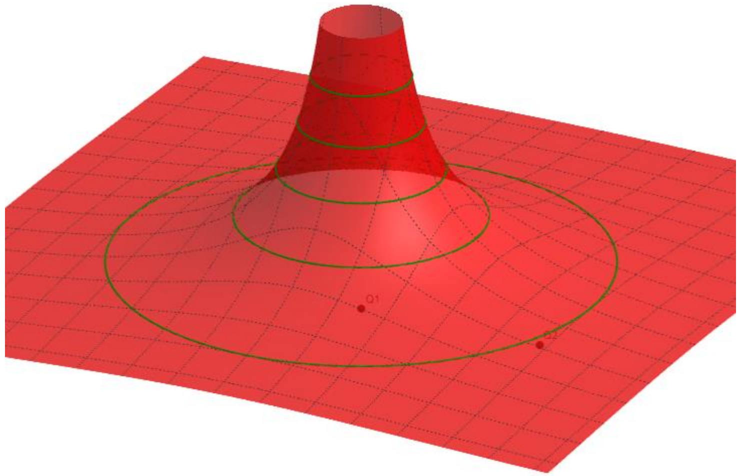


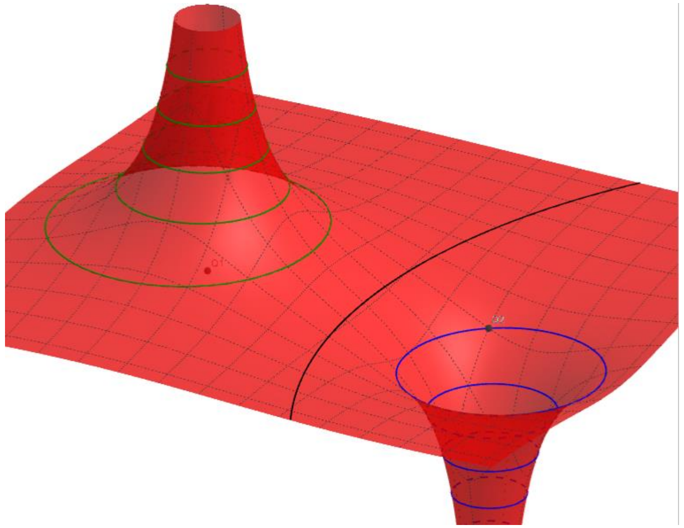
(a)



(b)







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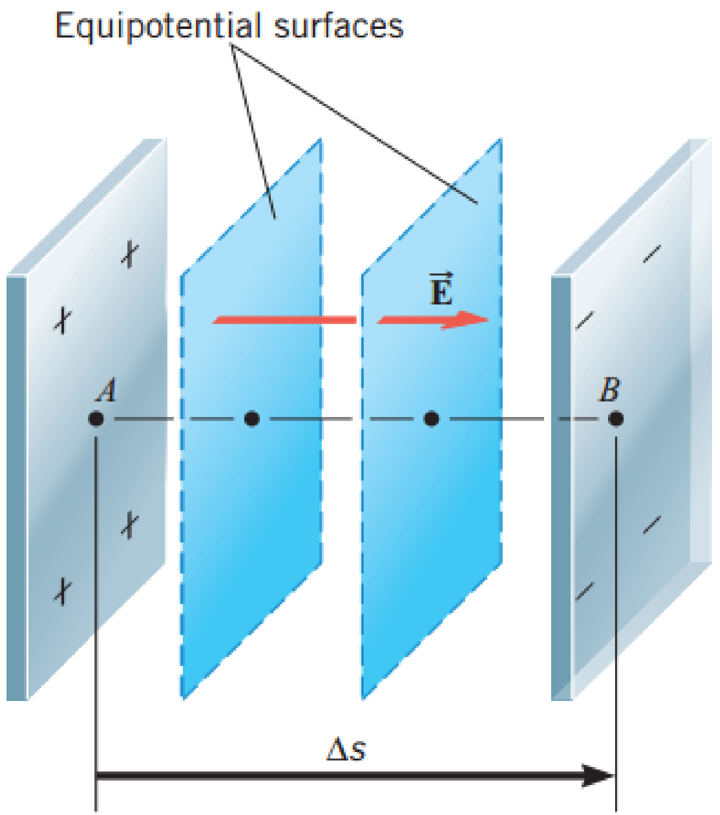
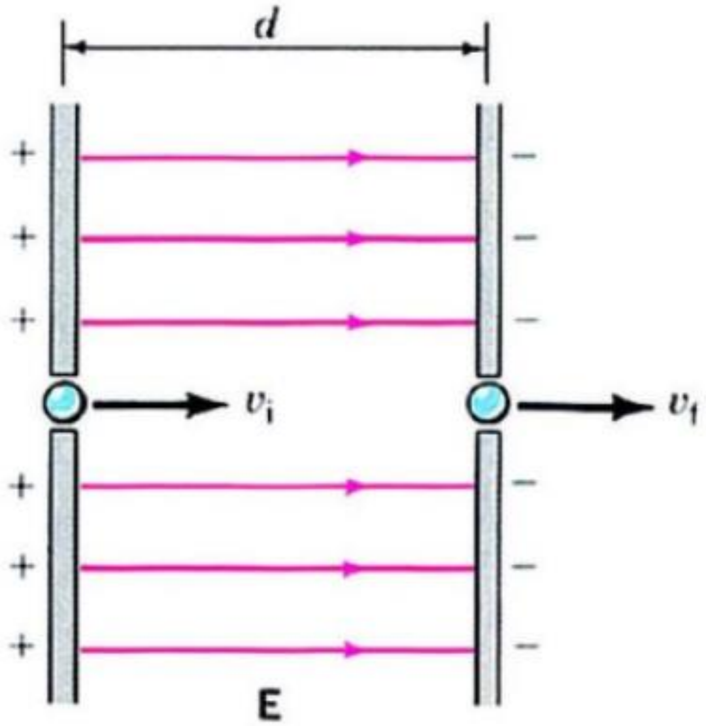


Figure 19.16 The metal plates of a parallel plate capacitor are equipotential surfaces. Two additional equipotential surfaces are shown between the plates. These two equipotential surfaces are parallel to the plates and are perpendicular to the electric field $\vec{E}$ between the plates.



Properties of Equipotential Surface

1.The electric field is always perpendicular to an equipotential surface.

2.Two equipotential surfaces can never intersect.

3.The values of the equipotential surface is from high potential to low potential.

4. No work is required to move a charge from different points of an equipotential surface.

5.For an isolated point charge, the equipotential surface is a sphere, i.e. concentric spheres around the point charge are different equipotential surfaces.

6.In a uniform electric field, any plane normal to the field direction is an equipotential surface.

A charged particle (q =1.4 μC) moves a distance of 0.4 m along an equipotential surface of 10 V. Calculate the work done by the field during this motion.

A positive particle of charge 1.0 C accelerates in a uniform electric field of 100 V/m. The particle started from rest on an equipotential plane of 50 V. After *t* = 0.0002 seconds, the particle is on an equipotential plane of V = 10 volts. Determine the distance travelled by the particle.

Calculating the Potential from the Field

In mechanics, the definition of potential energy in terms of the work done by the conservative force is,

$$\Delta U = -W_c$$

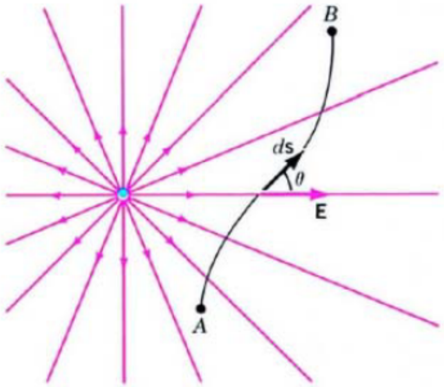
The negative sign tells us that positive work by the conservative force leads to a decrease in potential energy.

Therefore, the change in potential energy, associated with an infinitesimal displacement d**s**, is

$$dU = -\mathbf{F}_c \cdot d\mathbf{s} = -q\mathbf{E} \cdot d\mathbf{s}$$

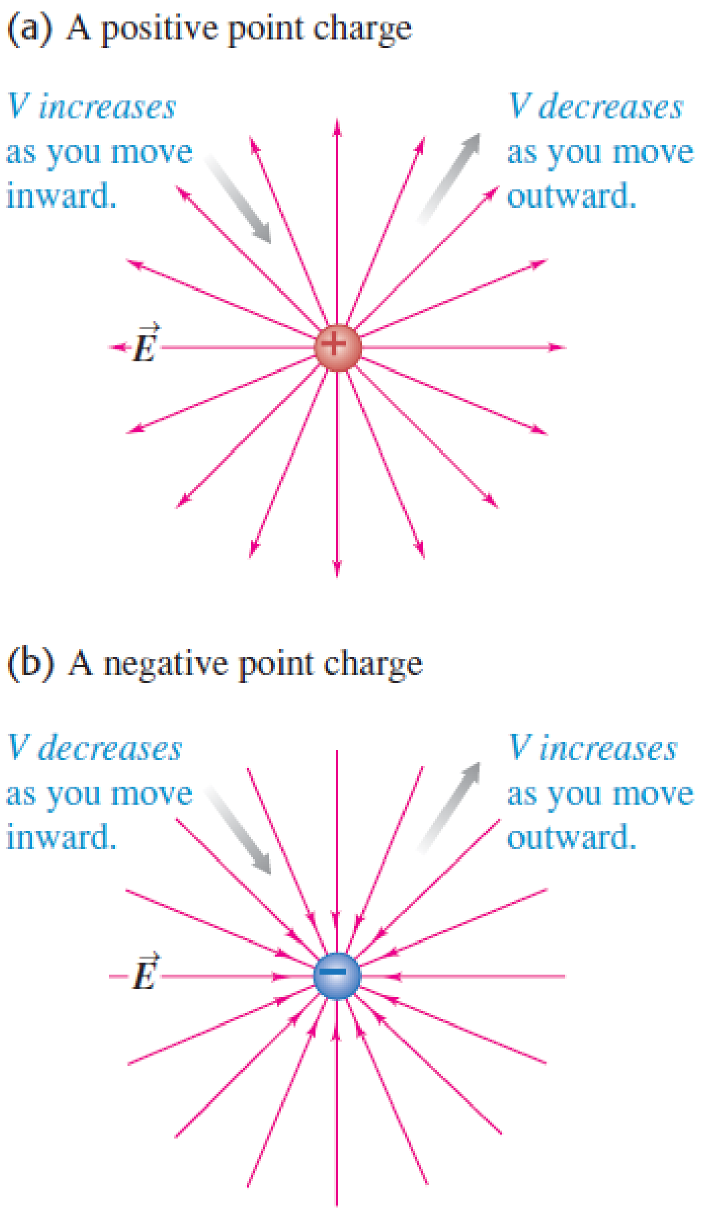
$$dV = \frac{dU}{q} = -\mathbf{E} \cdot d\mathbf{s}$$

$$V_B - V_A = -\int_A^B \mathbf{E} \cdot d\mathbf{s}$$



Since the electrostatic field is conservative, the value of this line integral depends only on the end points A and B, not on the path taken.

Electric field unit: 1 V/m = 1 N/C



Potential Due to a Point Charge

- Start with (set V_f=0 at ∞ and V_i=V at R)

$$\boxed{\Delta V = V_f - V_i = -\int_i^f \vec{E}\cdot d\vec{s} = -\int_i^f (E\cos 0°)ds = -\int_R^\infty E dr}$$

- We have

$$\boxed{E = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}}$$

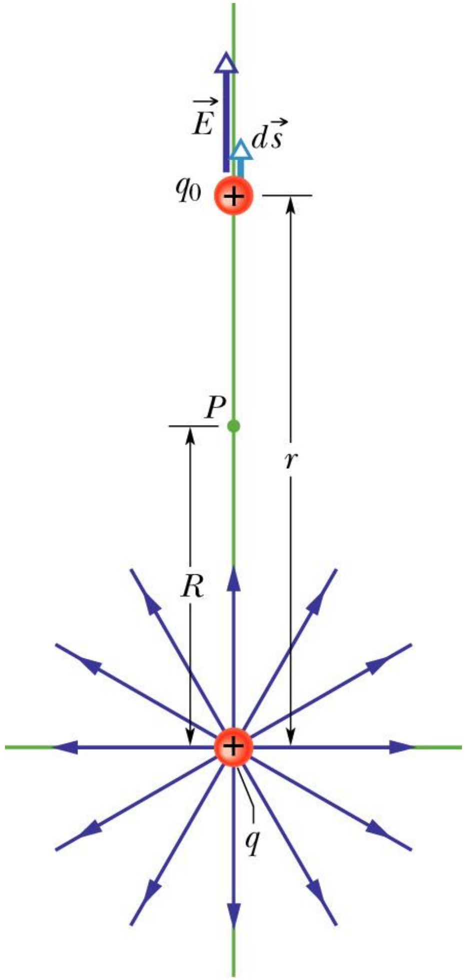
- Then

$$\boxed{0 - V = -\frac{q}{4\pi\varepsilon_0}\int_R^\infty \frac{1}{r^2}dr = \frac{q}{4\pi\varepsilon_0}\left[\frac{1}{r}\right]_R^\infty = -\frac{1}{4\pi\varepsilon_0}\frac{q}{R}}$$

- So

$$\boxed{V(r) = \frac{1}{4\pi\varepsilon_0}\frac{q}{r}}$$

- A positively charged particle produces a positive electric potential.
- A negatively charged particle produces a negative electric potential



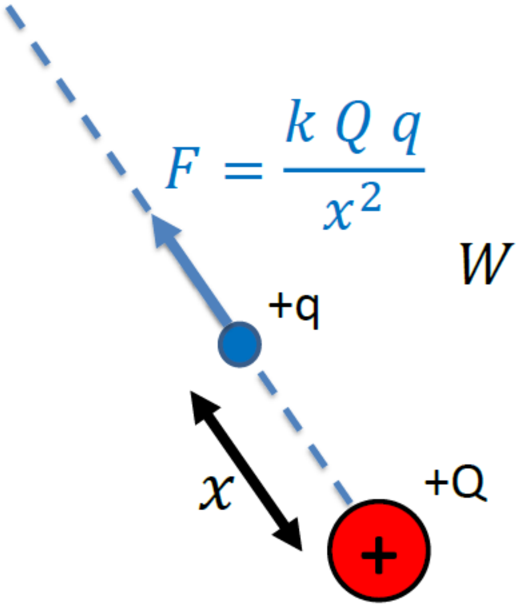
Potential Due to a Charged Particle

- What is the electric potential near a charge +Q?

Consider a point P at distance x from a fixed particle of positive charge $+Q$

Work = Force x Distance

$$F = \frac{k\,Q\,q}{x^2}$$



Force is varying with distance, need integral!

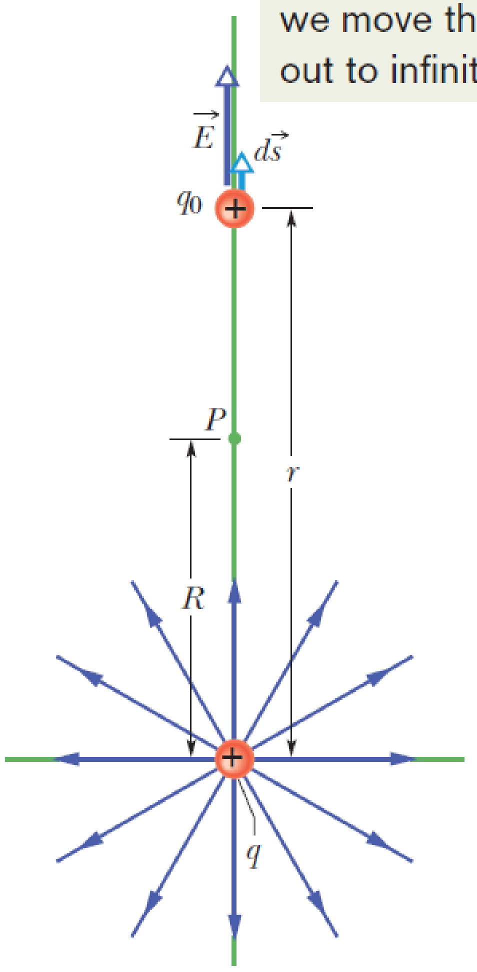
$$W = \int_{\infty}^{r} F\,dx = \int_{\infty}^{r} -\frac{k\,Q\,q}{x^2}\,dx = \frac{k\,Q\,q}{r}$$

Potential energy $U = \frac{k\,Q\,q}{r}$

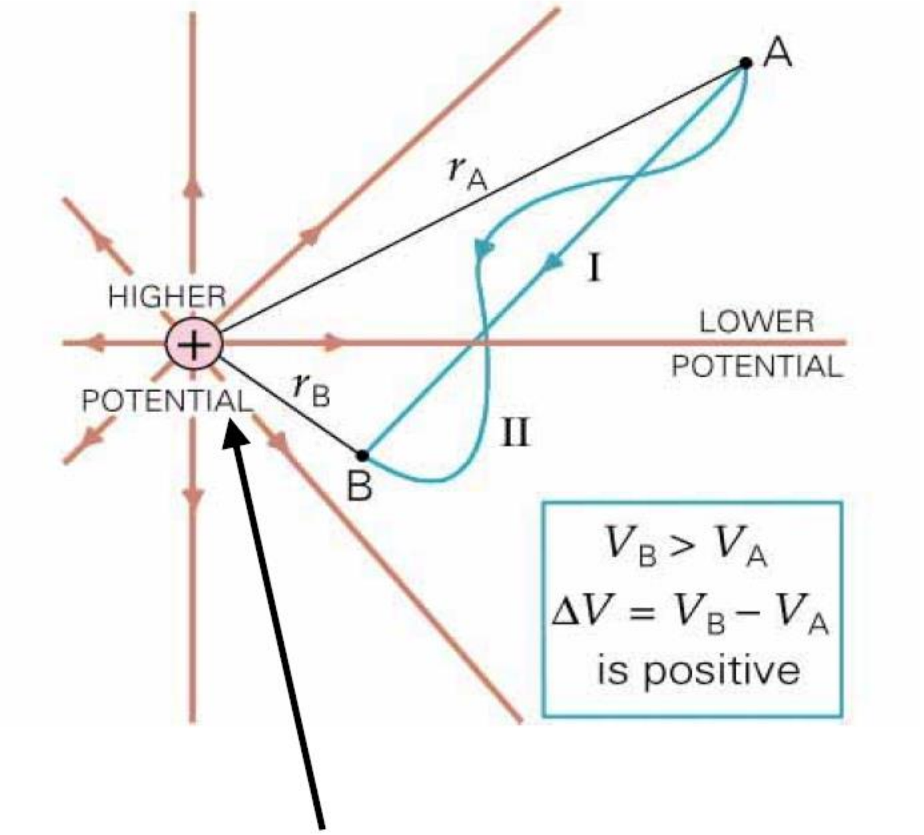
Electric potential $V = \frac{U}{q} = \frac{k\,Q}{r}$

The electric potential V due to a particle of charge q at any radial distance r from the particle.

To find the potential of the charged particle, we move this test charge out to infinity.



Potential Due to a Charged Particle



charge Q

$$\Delta V = V_B - V_A = \frac{kQ}{r_B} - \frac{kQ}{r_A}$$

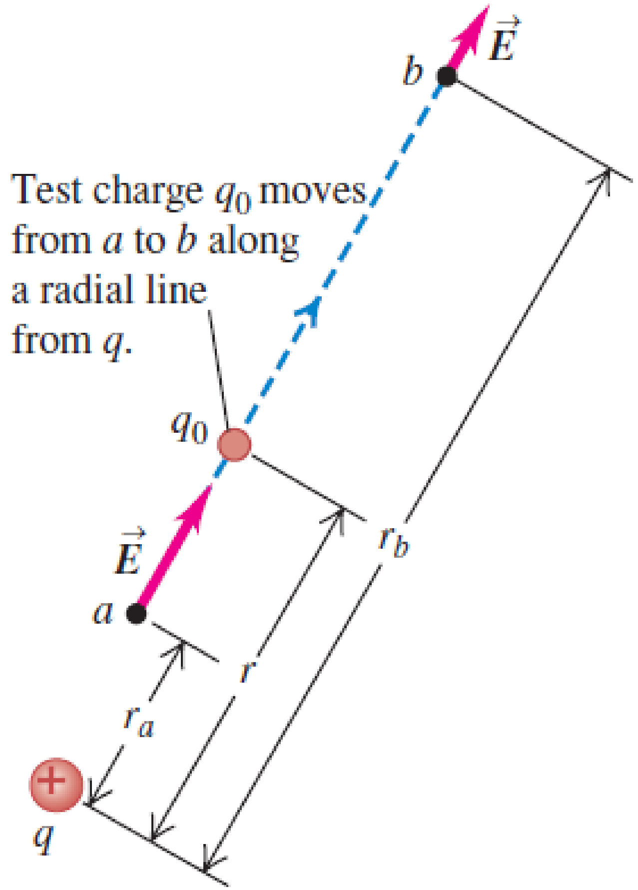
At r_A= infinity, the potential is defined to be ZERO

$$\Delta V = V_B = \frac{kQ}{r_B}$$

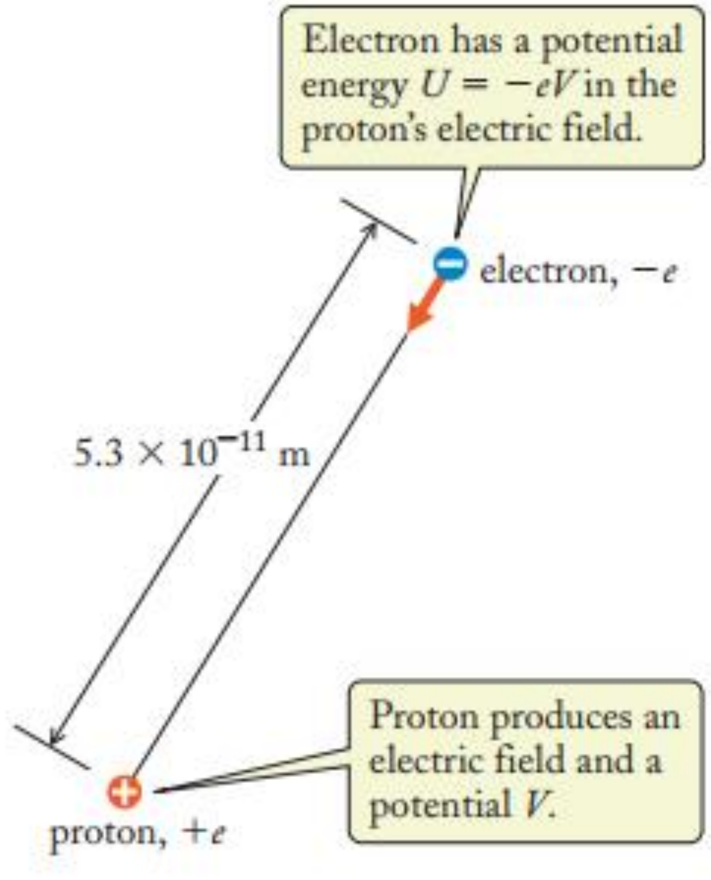
or

$$\boxed{V = \frac{kQ}{r}}$$

Equation is valid ONLY for a point charge



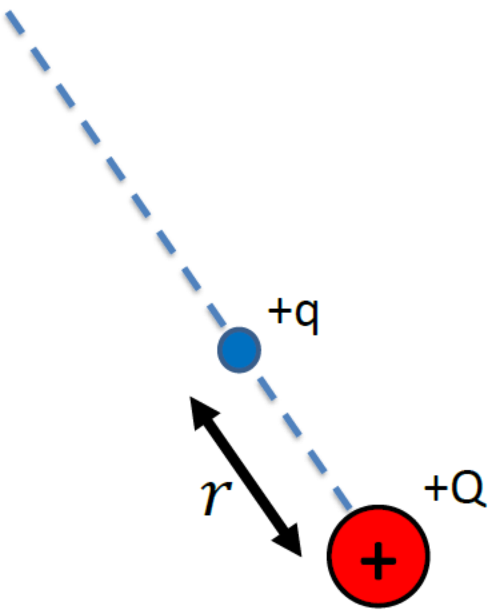
The electron in a hydrogen atom is at a distance of 5.3×10^{-11} m from the proton. The proton is a small ball of charge with 1.60×10^{-19} C. What is the electrostatic potential generated by the proton at this distance? What is the potential energy of the electron?

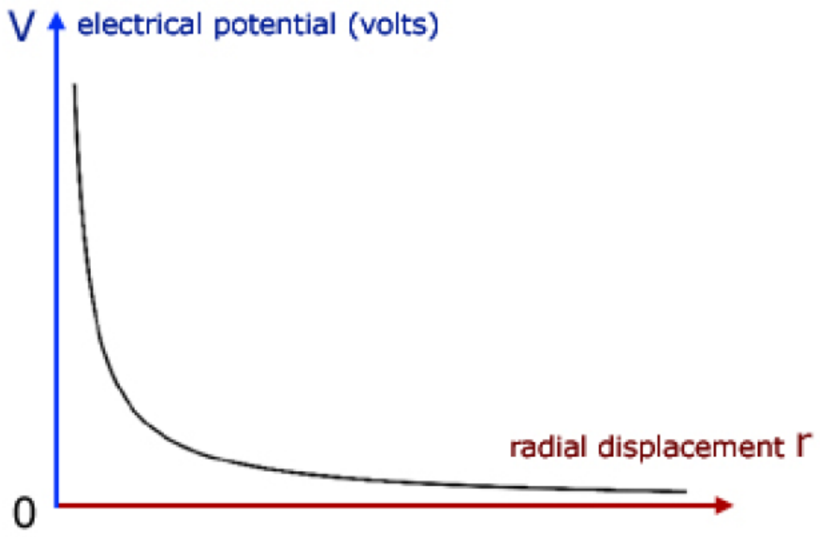


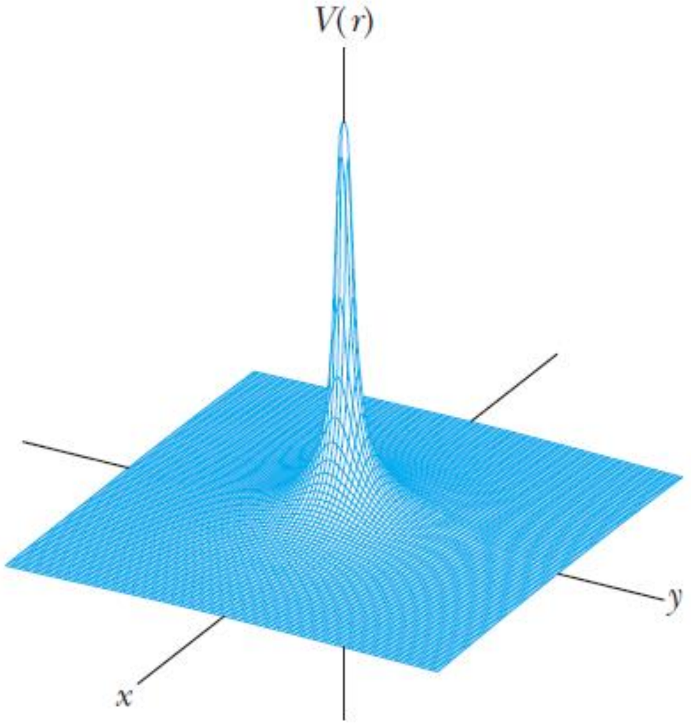
Potential Due to a Charged Particle

Note that the magnitude increases as $r = 0$.
V is infinite at $r = 0$

- What is the electric potential near a charge +Q?







Electric potential $V = \frac{k\,Q}{r}$

Figure 24-10 A computer-generated plot of the electric potential $V(r)$ due to a positively charged particle located at the origin of an xy plane. The potentials at points in the xy plane are plotted vertically. (Curved lines have been added to help you visualize the plot.) The infinite value of V predicted by Eq. 24-26 for $r = 0$ is not plotted.

A positively charged particle produces a positive electric potential.

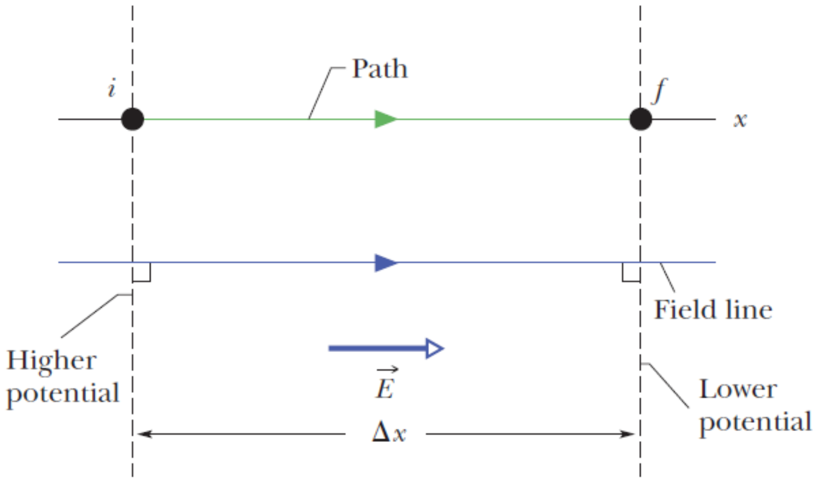
A negatively charged particle produces a negative electric potential.

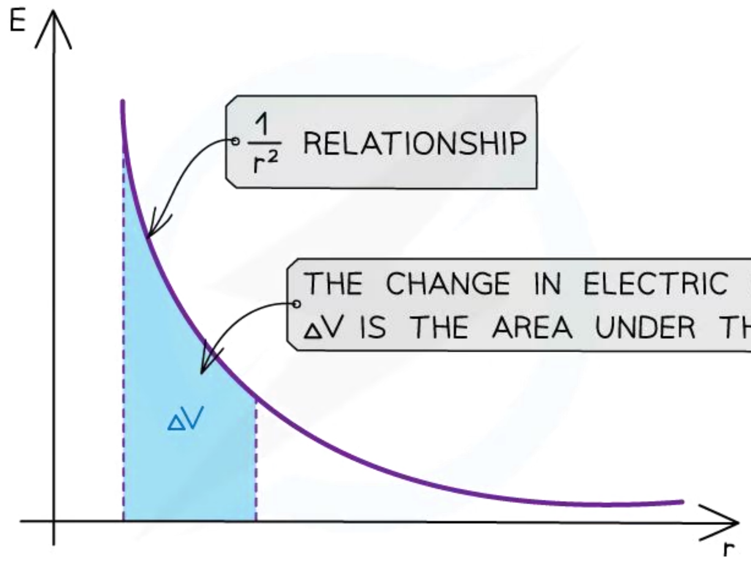
Calculating the Field from the Potential

The electric field is a measure of the rate of change of the electric potential with respect to position. For uniform electric field:

$$\Delta V = -\int \vec{E}.d\vec{s} = -Ed \text{ thus, } E = -\frac{dV}{dx} \text{ (one dimension)}$$

The electric field vector points from higher potential toward lower potential.





Calculating the Field from the Potential

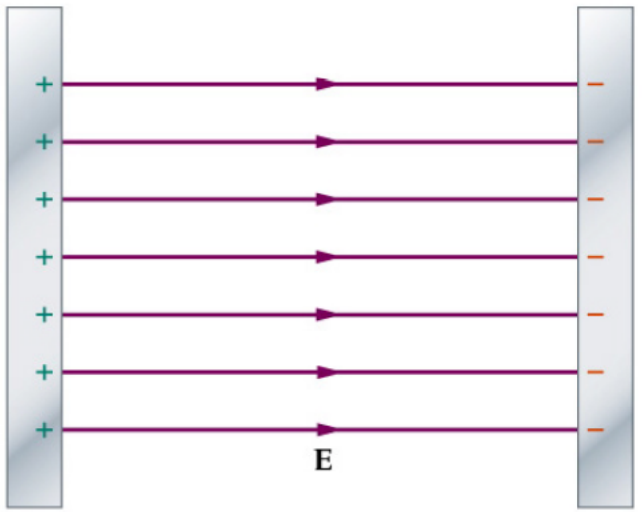
The electric potential, V, decreases as one moves in the direction of the electric field. In the case shown here, the electric field is constant; as a result, the electric potential decreases uniformly with distance.

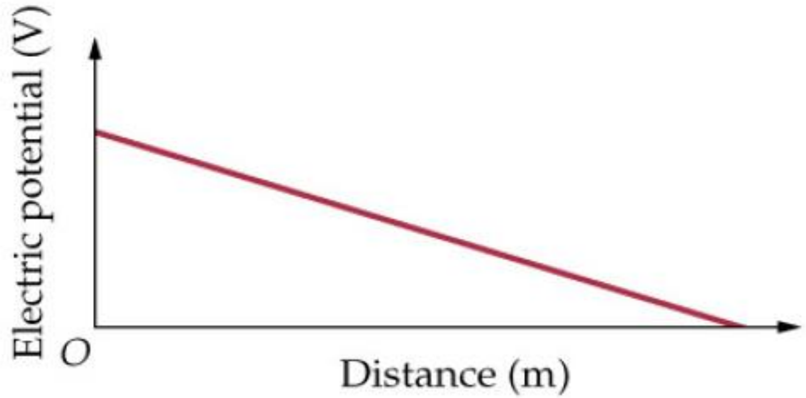
$$E = -\frac{dV}{dx} \text{ (one dimension)}$$

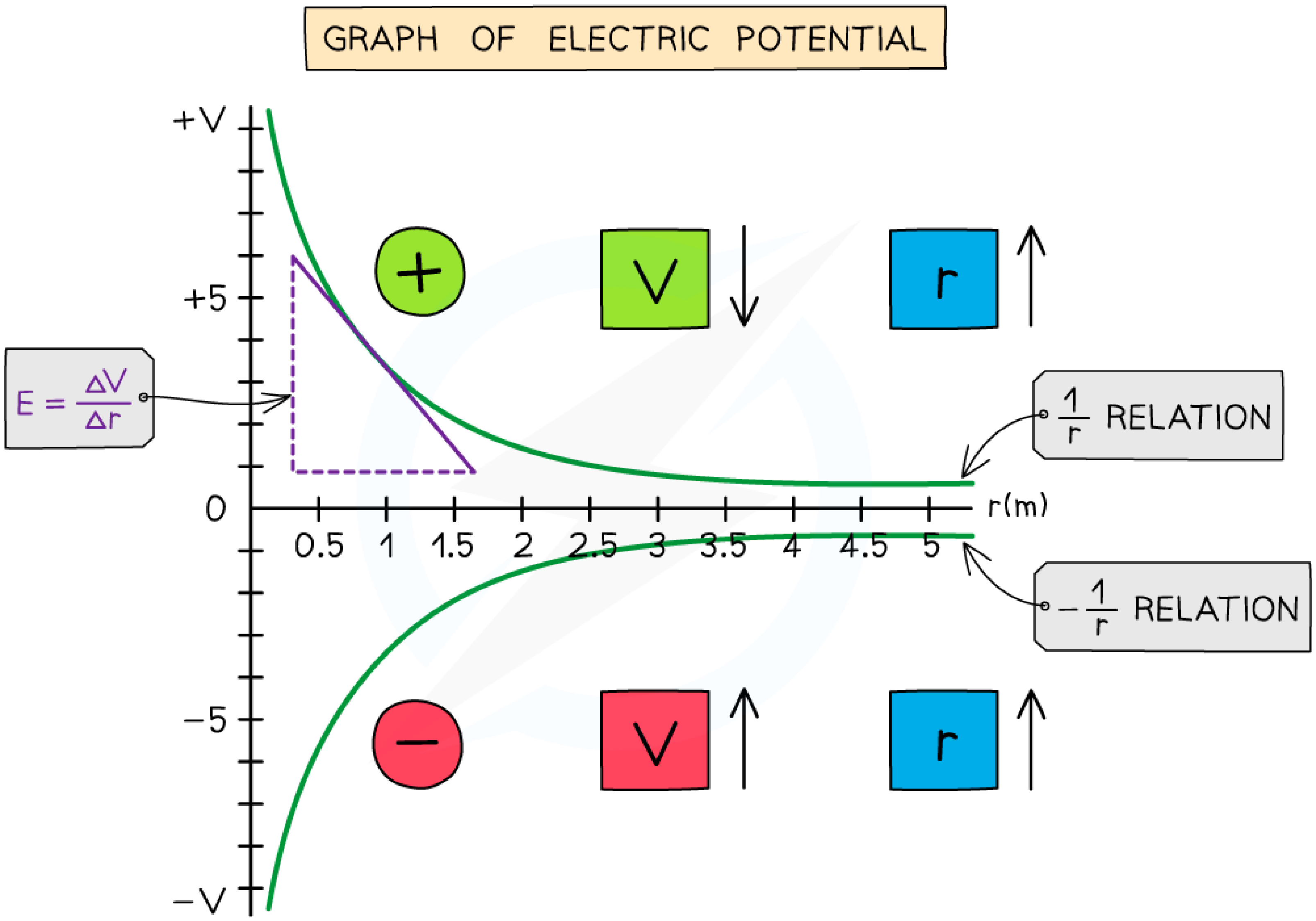
In mechanics we observed, the relations,

$$F = -\frac{dU}{dx} \text{ (force and potential energy)}$$

$$W = -U = -\int F dx \text{ (one dimension)}$$

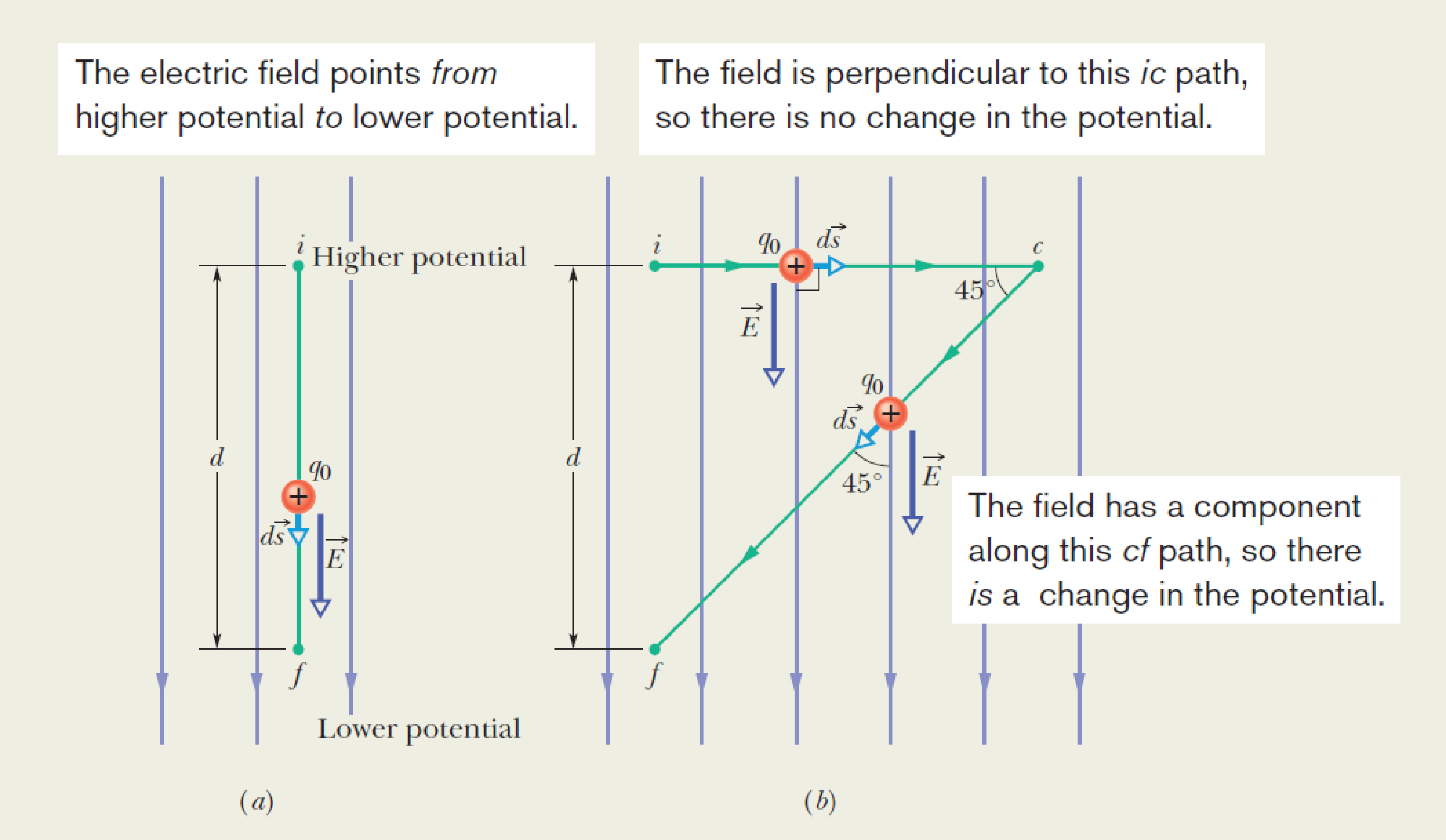






GRAPH OF ELECTRIC POTENTIAL
+V
+5
0
−5
−V
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5
r(m)
$E = \frac{\Delta V}{\Delta r}$
+
V
r
−
V
r
$\frac{1}{r}$ RELATION
$-\frac{1}{r}$ RELATION

Finding the potential change from the electric field



$$\Delta V = V_f - V_i = -\int_i^f \vec{E}\cdot d\vec{s} = -\int_i^f (E\cos 0°)ds = -\int_i^f E ds$$

$$\Delta V = V_f - V_i = -E\int_i^f ds = -Ed$$

$$\Delta U = q_0 \Delta V = -q_0 Ed$$

$$V_c - V_i = -\int_i^c \vec{E}\cdot d\vec{s} = -\int_i^c (E\cos 90°)ds = 0$$

$$V_f - V_c = -\int_c^f \vec{E}\cdot d\vec{s}$$
$$= -\int_c^f (E\cos 45°)ds = -E\cos 45°\int_c^f ds$$

$$V_f - V_i = -E\cos 45° \frac{d}{\sin 45°} = -Ed$$

Finding the potential change from the electric field

A uniform electric field of 1200 N/C points to the left as shown.
A. What is the difference in potential (V_B-V_A) between points B and A?
B. What is the difference in potential (V_B-V_C) between points B and C?
C. What is the difference in potential (V_C-V_A) between points C and A?