Capacitor & Capacitance

Phy 108 course
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DMP, SEPS, NSU


nichicon
4700µF 35v
VITAMIN Q
CAPACITOR
22 - 400 D.C.
196P22404T16
189-505-5
193-10 8216
103K
500V
151
15

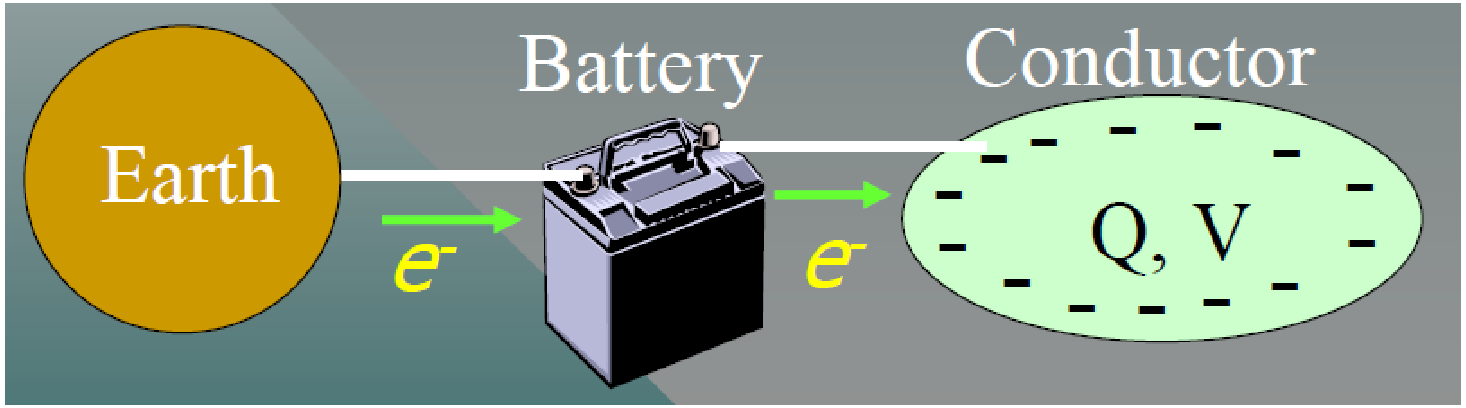

2200µF 50v

The **capacitance** *C* of a capacitor is defined as the ratio of the magnitude of the charge on either conductor to the magnitude of the potential difference between the conductors

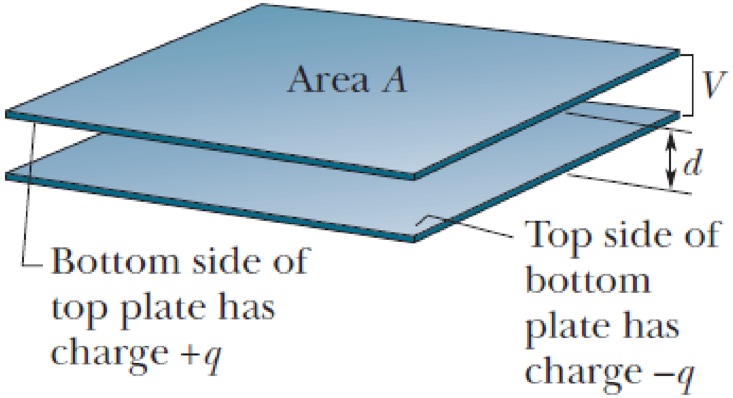
$$Q \propto V$$

$$C = Q/V$$

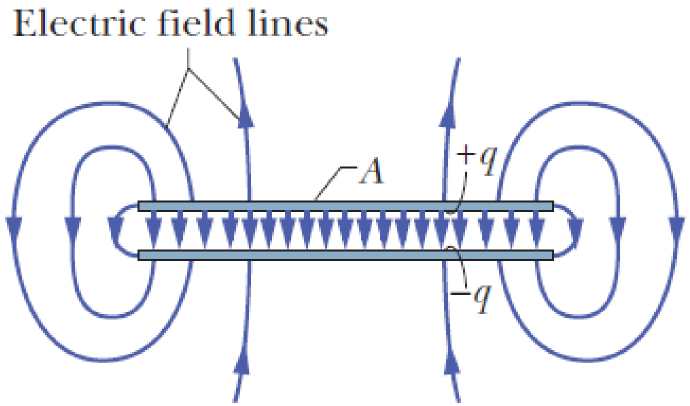
A battery establishes a difference of potential that can pump electrons from a ground (earth) to a conductor



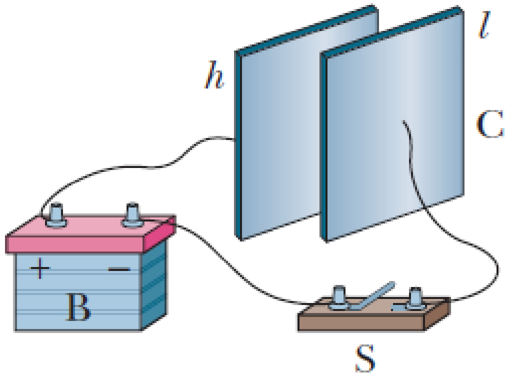
A capacitor is a system of *two conductors* that carries *equal and opposite charges*. A capacitor *stores charge and energy* in the form of electro-static field.



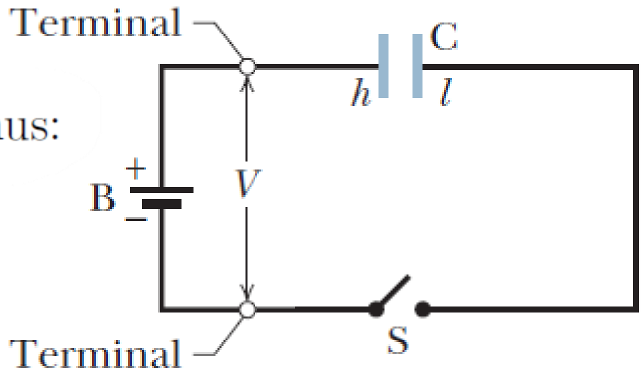
(a)



(b)



(a)



(b)

The SI unit of capacitance is then $1\,\frac{\text{C}}{\text{V}}$, a combination which is called the **farad**[1]. Thus:

$$1\,\text{farad} = 1\,\text{F} = 1\,\frac{\text{C}}{\text{V}}$$

The permittivity constant can be expressed in terms of this new unit as:

$$\epsilon_0 = 8.85 \times 10^{-12}\,\frac{\text{C}^2}{\text{N}{\cdot}\text{m}^2} = 8.85 \times 10^{-12}\,\frac{\text{F}}{\text{m}}$$

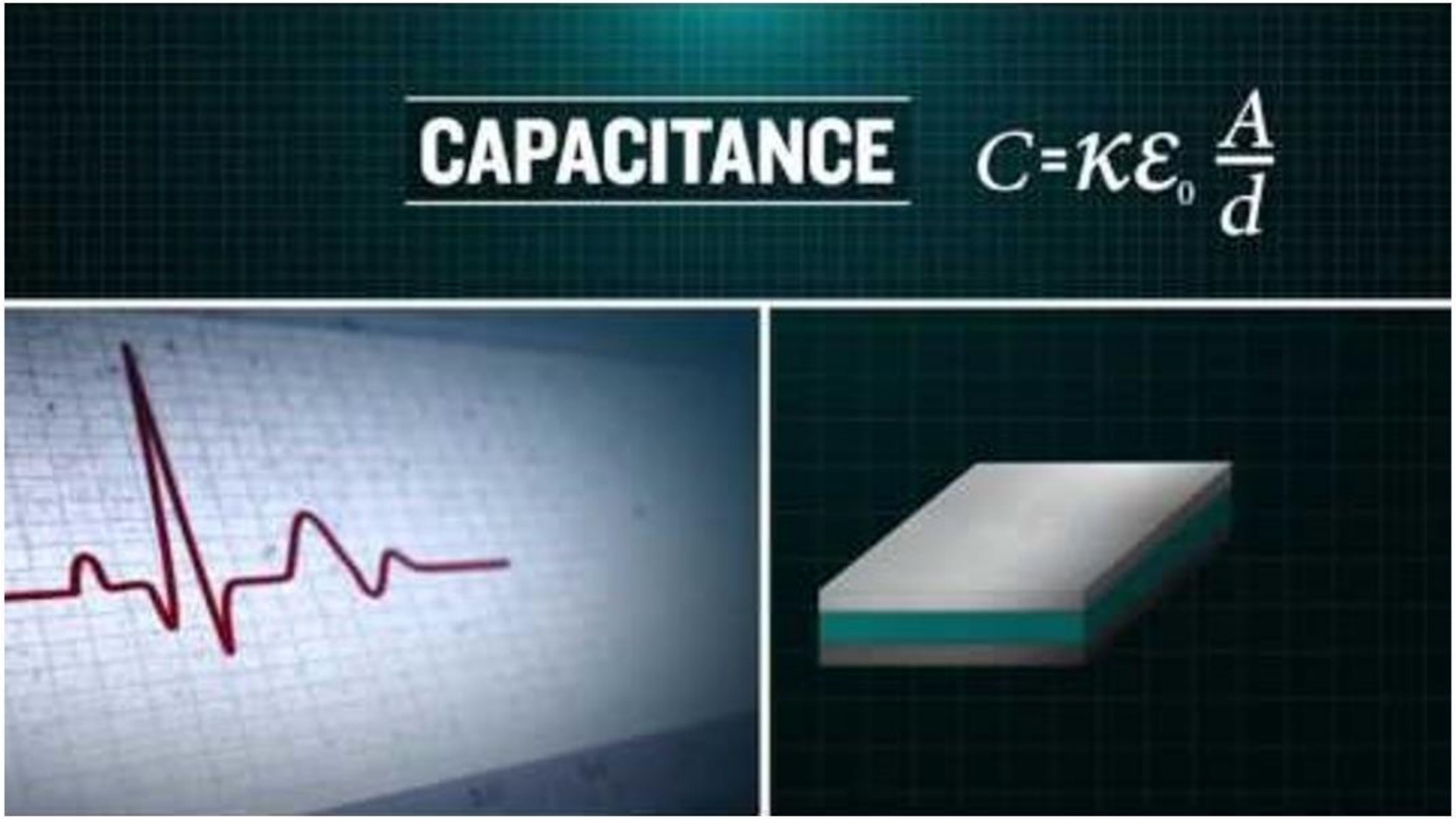
One farad is a very large capacitance. In many applications the most convenient units of capacitance are the *microfarad (μF) and picofarad (pF)*

Permittivity constant, ϵ_0

It represents the capability of a vacuum to permit electric fields. It is also connected to the energy stored within an electric field and capacitance. Perhaps more surprisingly, it's fundamentally related to the speed of light.

The ability of a material to permit or transmit electric field actually relates to permittivity . Permittivity of free space is a constant of proportionality and it specifies the strength of electric force between electric charges in vacuum.

It relates the electric field in a material to the electric displacement in that material. It characterizes the tendency of the atomic charge in an insulating material to distort in the presence of an electric field. The larger the tendency for charge distortion (also called electric polarization), the larger the value of the permittivity.



https://www.youtube.com/watch?v=COKBImkkJKw

Calculating the Capacitance

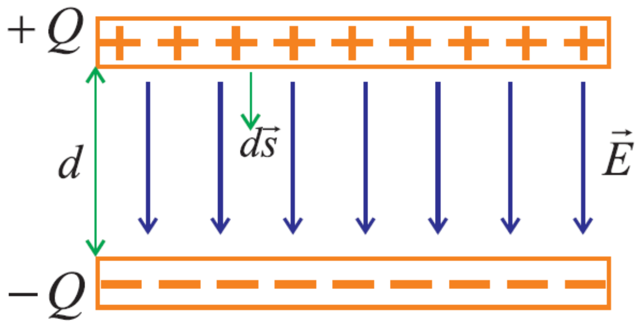
(1) Assume a charge q on the plates;

(2) Calculate the electric field $\vec{E}$ between the plates in terms of this charge, using Gauss' law;

(3) Knowing $\vec{E}$, calculate the potential difference V between the plates;

(4) Calculate C.

Parallel-Plate Capacitor

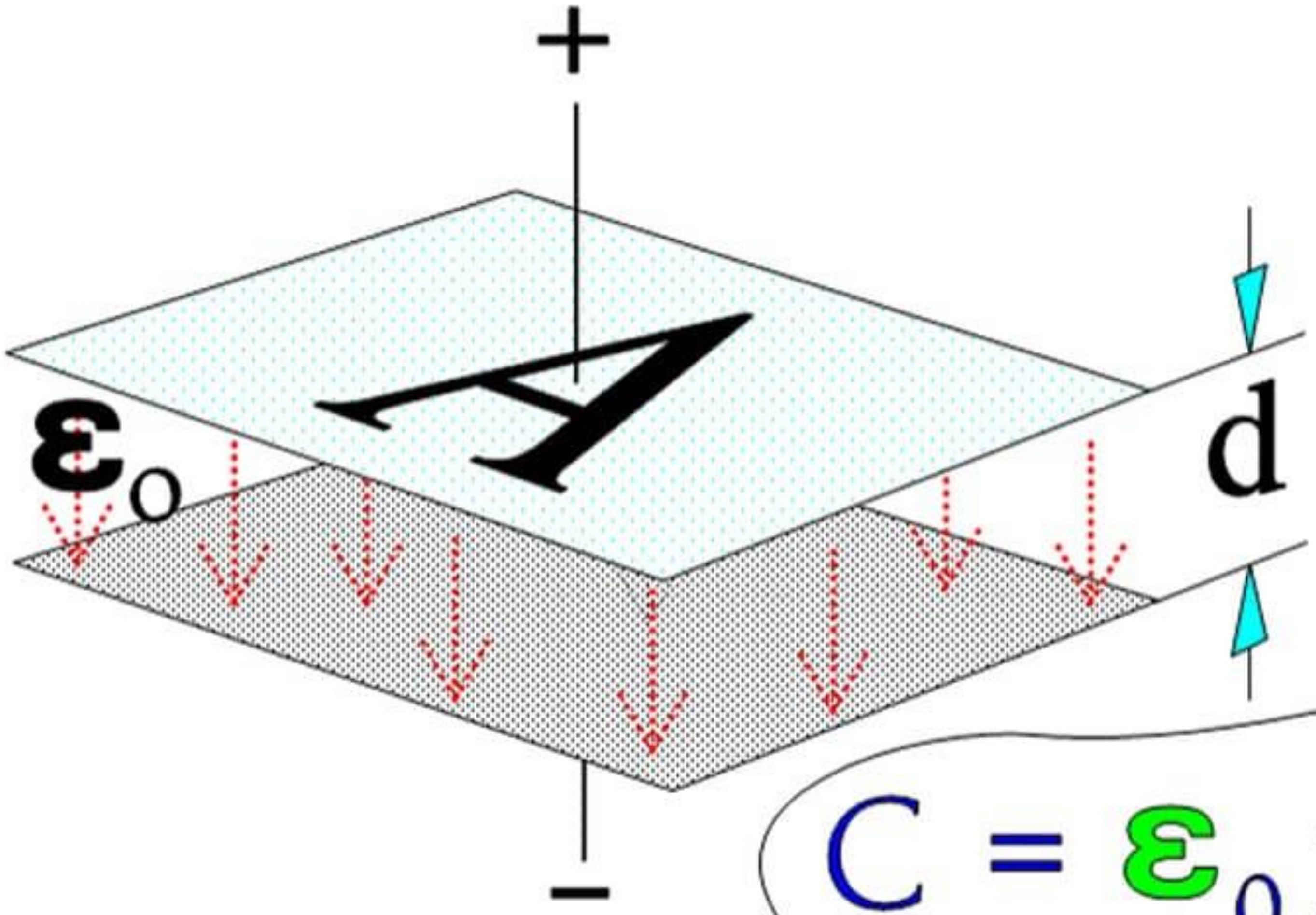


$$\varepsilon_0 \oint \vec{E} \cdot d\vec{A} = q. \quad |\vec{E}| = \frac{\sigma}{\epsilon_0} \qquad q = \varepsilon_0 E A$$

$$\Delta V = V_+ - V_- = -\int_-^+ \vec{E} \cdot d\vec{s}$$

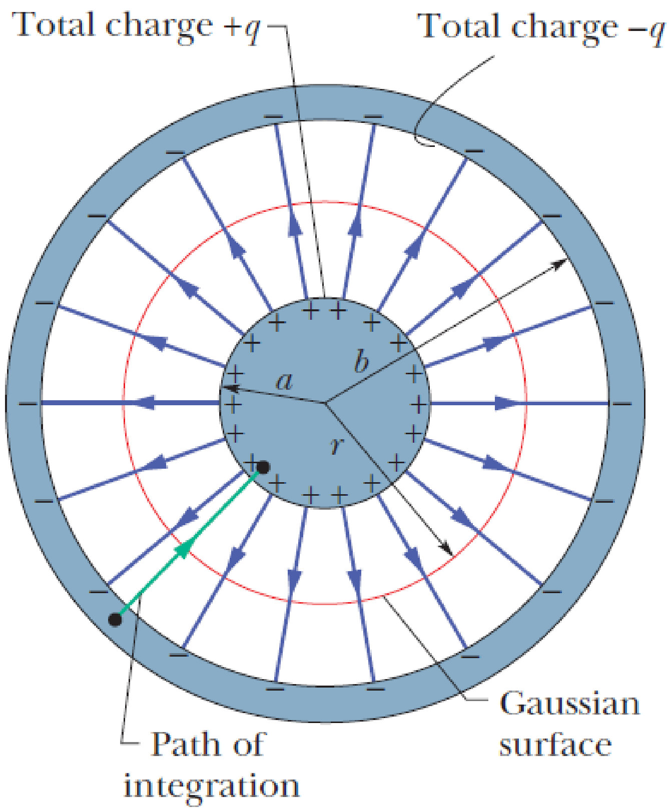
$$\boxed{C = \frac{q}{\Delta V} = \frac{\varepsilon_0 A}{d}}$$

$$\begin{aligned} \Delta V &= \int_+^- \vec{E} \cdot d\vec{s} \\ &= \int_+^- E \cdot ds \quad = \frac{q}{\varepsilon_0 A} \int_-^+ ds \end{aligned}$$



$$C = \varepsilon_0 \times \frac{A}{d} \ (F)$$

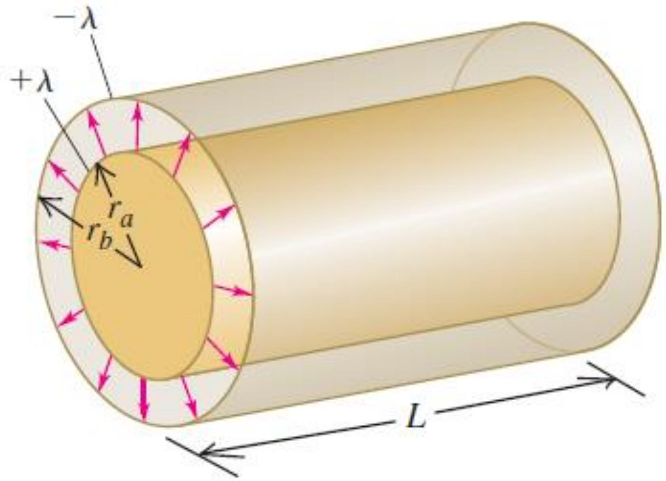
Cylindrical Capacitor

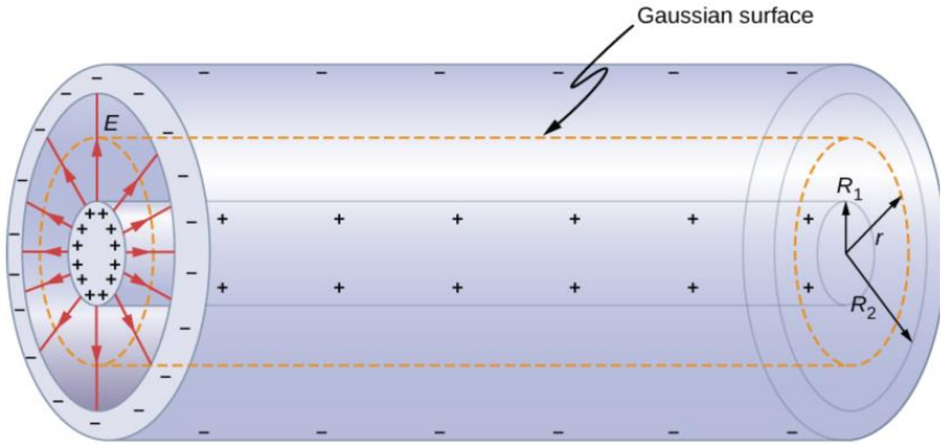


$$q = \varepsilon_0 EA = \varepsilon_0 E(2\pi r L) \qquad E = \frac{q}{2\pi\varepsilon_0 L r}$$

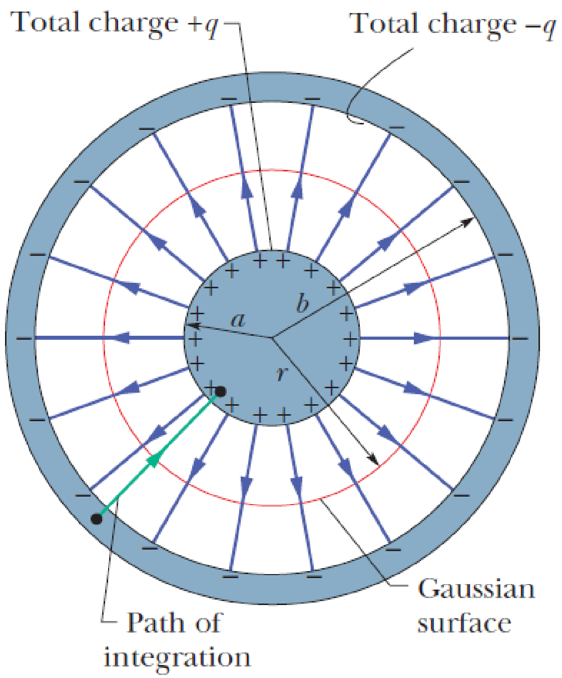
$$V = \int_{-}^{+} E\,ds = -\frac{q}{2\pi\varepsilon_0 L}\int_{b}^{a}\frac{dr}{r} = \frac{q}{2\pi\varepsilon_0 L}\ln\left(\frac{b}{a}\right)$$

$$\boxed{C = \frac{q}{\Delta V} = \frac{2\pi\varepsilon_0 L}{\ln(b/a)}}$$





Spherical Capacitor

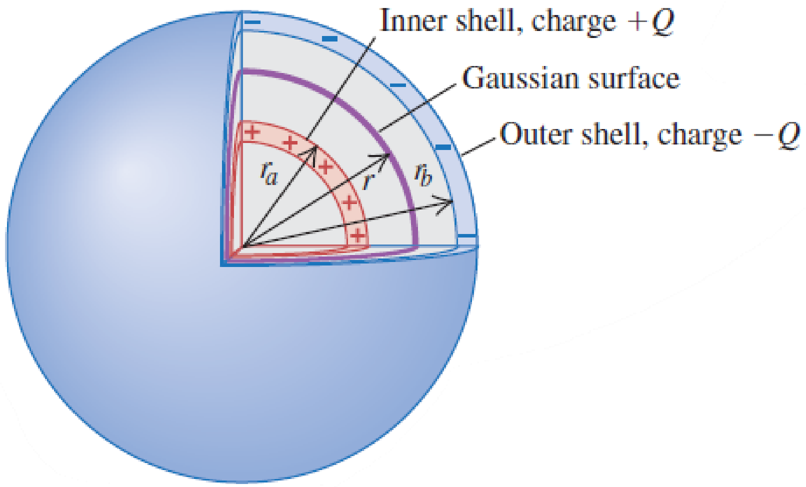


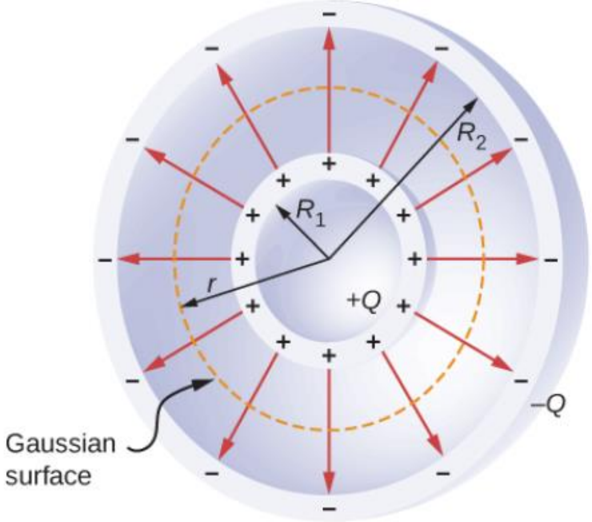
$$q = \varepsilon_0 E A = \varepsilon_0 E(4\pi r^2), \qquad E = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}$$

$$V = \int_{-}^{+} E\,ds = -\frac{q}{4\pi\varepsilon_0}\int_b^a \frac{dr}{r^2} = \frac{q}{4\pi\varepsilon_0}\left(\frac{1}{a} - \frac{1}{b}\right) = \frac{q}{4\pi\varepsilon_0}\frac{b-a}{ab}$$

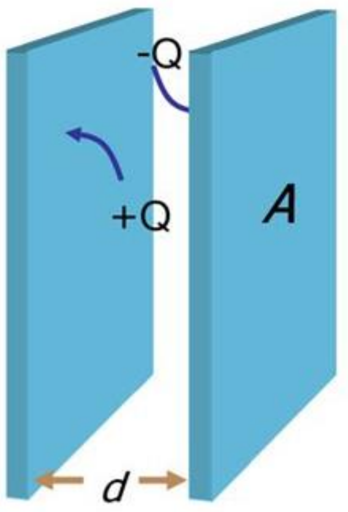
$$\boxed{C = \frac{q}{\Delta V} = \frac{4\pi\varepsilon_0 ab}{b-a}}$$

A spherical capacitor.





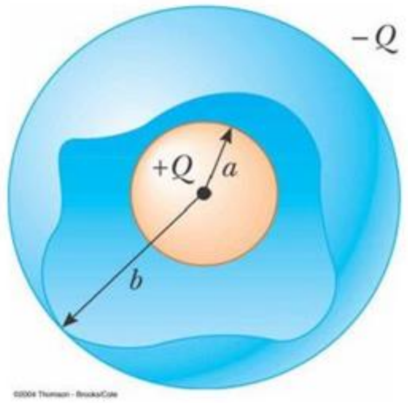
Different geometries of capacitors



Parallel plate capacitor

$$C = \frac{Q}{\Delta V} = \frac{\varepsilon_o A}{d}$$

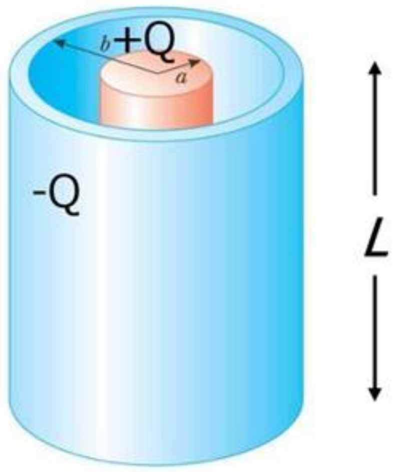
plate area *A*
plate separation *d*.



Spherical capacitor

$$C = \frac{Q}{\Delta V} = 4\pi\varepsilon_o\left(\frac{1}{a} - \frac{1}{b}\right)^{-1}$$

two radii *b* and *a*.



Cylindrical capacitor

$$C = \frac{Q}{\Delta V} = \frac{2\pi\varepsilon_o L}{\ln(b/a)}$$

length *L* and the two radii *b* and *a*.

For *any* capacitor in vacuum, the capacitance depends only on the shapes, dimensions, and separation of the conductors that make up the capacitor.

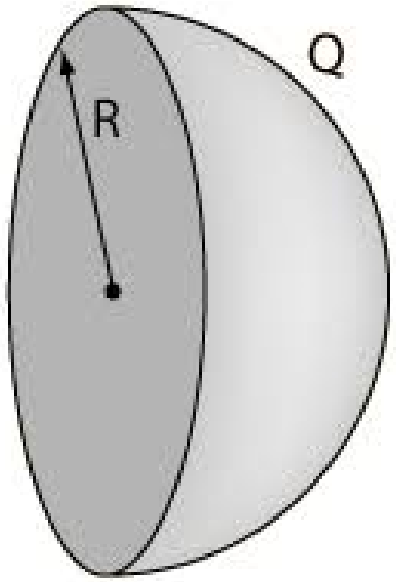
An Isolated Sphere

We can assign a capacitance to a *single* isolated spherical conductor of radius R by assuming that the "missing plate" is a conducting sphere of infinite radius. After all, the field lines that leave the surface of a positively charged isolated conductor must end somewhere; the walls of the room in which the conductor is housed can serve effectively as our sphere of infinite radius.

$$C = 4\pi\varepsilon_0 \frac{a}{1 - a/b}.$$

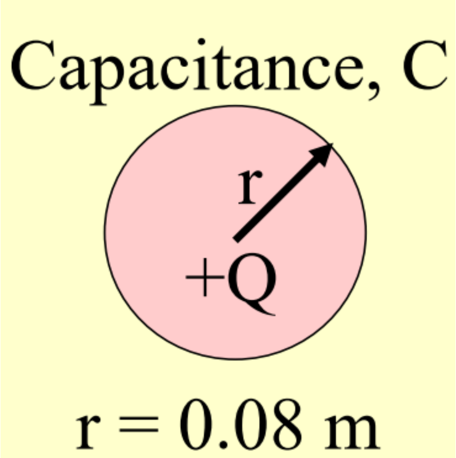
If we then let $b \to \infty$ and substitute R for a, we find

$$C = 4\pi\varepsilon_0 R \quad \text{(isolated sphere).}$$



https://www.compadre.org/Physlets/electromagnetism/illustration26_1.cfm

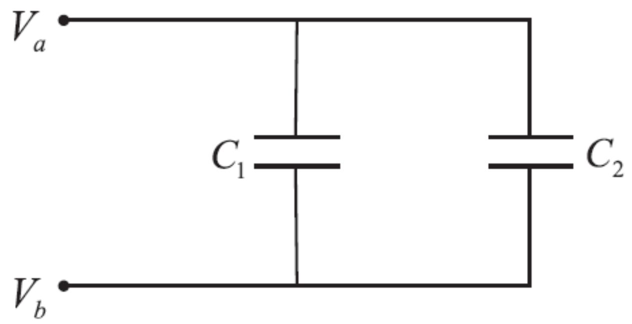
What is the capacitance of a metal sphere of radius 8 cm?



What charge Q is needed to give a potential of 400 V?

Combinations of Capacitors

(a) Capacitors in Parallel

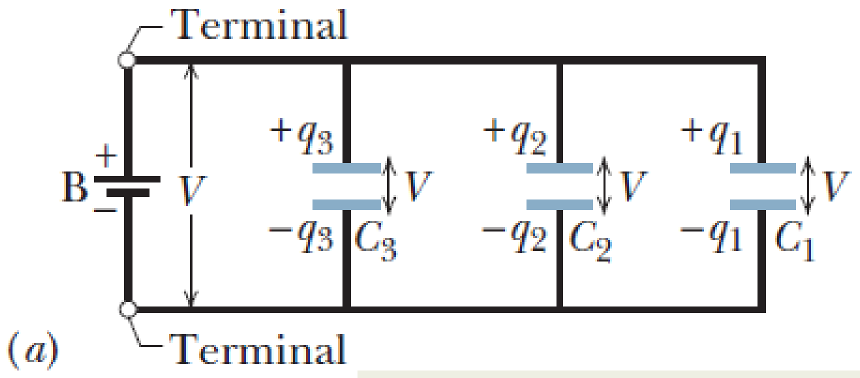


In this case, it's the *potential difference* $V = V_a - V_b$ that is the same across the capacitor.

<u>BUT</u>: *Charge* on each capacitor different

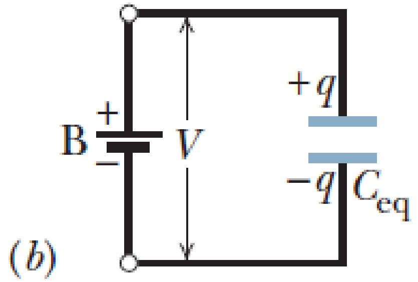
$$\begin{aligned} \text{Total charge} \quad Q &= Q_1 + Q_2 \\ &= C_1 V + C_2 V \\ Q &= \underbrace{(C_1 + C_2)}_{\text{Equivalent capacitance}} V \end{aligned}$$

$\therefore$ For capacitors in parallel: $\boxed{C = C_1 + C_2}$



$$C_{eq} = \sum_{j=1}^{n} C_j \qquad (n \text{ capacitors in parallel}).$$

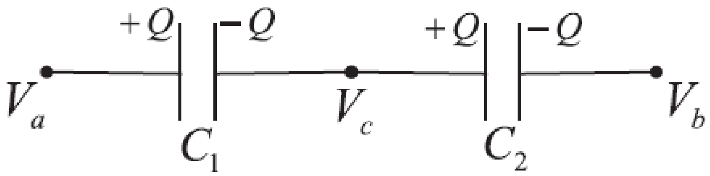
Parallel capacitors and their equivalent have the same V ("par-V").



Capacitors connected in parallel can be replaced with an equivalent capacitor that has the same *total* charge q and the same potential difference V as the actual capacitors.

Combinations of Capacitors

(b) Capacitors in Series



The *charge across capacitors* are the same.

<u>BUT</u>: *Potential difference* (P.D.) across capacitors different

$$\Delta V_1 = V_a - V_c = \frac{Q}{C_1} \qquad \text{P.D. across } C_1$$

$$\Delta V_2 = V_c - V_b = \frac{Q}{C_2} \qquad \text{P.D. across } C_2$$

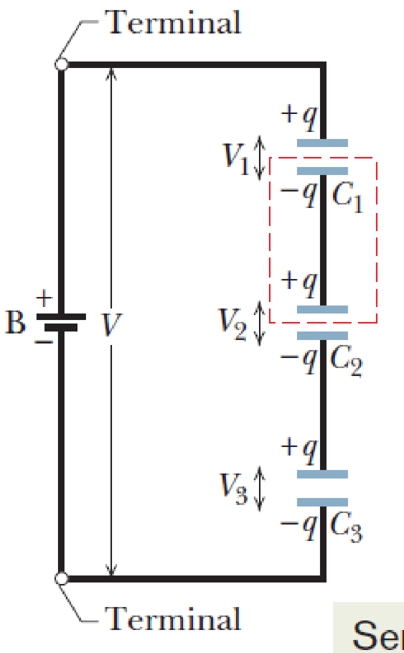
∴ Potential difference

$$\begin{aligned} \Delta V &= V_a - V_b \\ &= \Delta V_1 + \Delta V_2 \\ \Delta V &= Q\left(\frac{1}{C_1} + \frac{1}{C_2}\right) = \frac{Q}{C} \end{aligned}$$

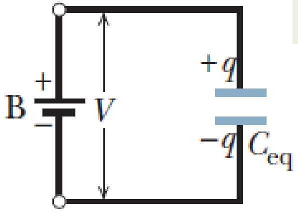
where C is the **Equivalent Capacitance**

$$\therefore \quad \boxed{\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2}}$$

$$\frac{1}{C_{eq}} = \sum_{j=1}^{n} \frac{1}{C_j} \qquad (n \text{ capacitors in series}).$$



(*a*)

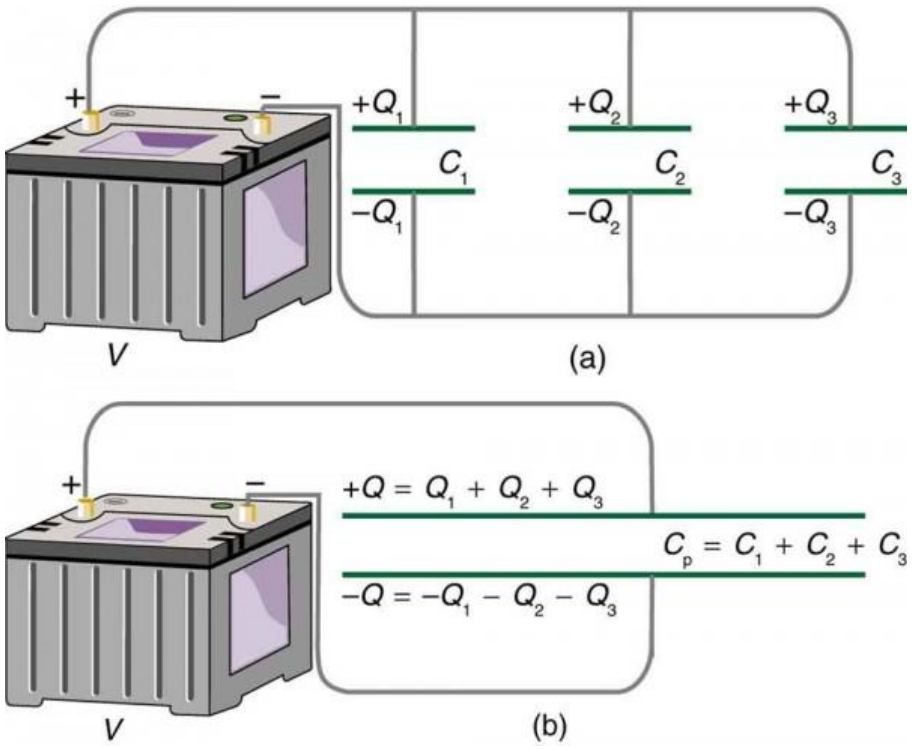


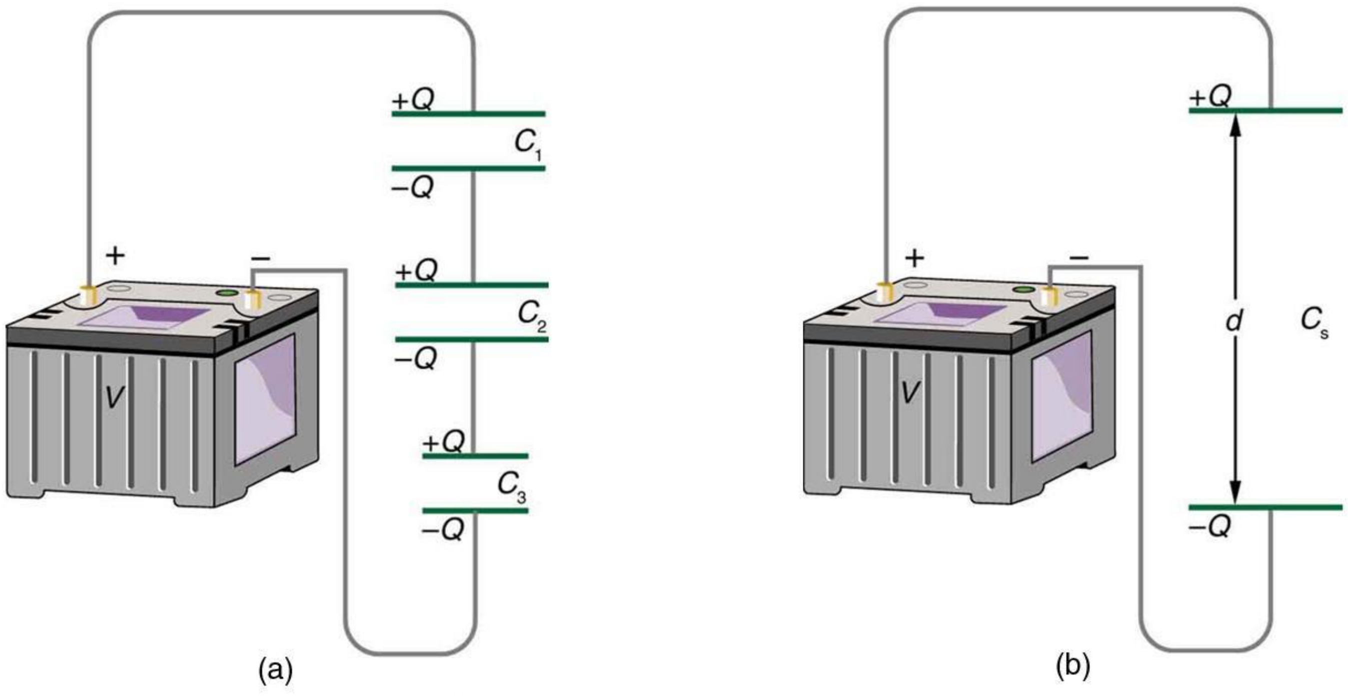
(*b*)

Series capacitors and their equivalent have the same q ("seri-q").

Capacitors that are connected in series can be replaced with an equivalent capacitor that has the same charge q and the same *total* potential difference V as the actual series capacitors.

Combinations of Capacitors

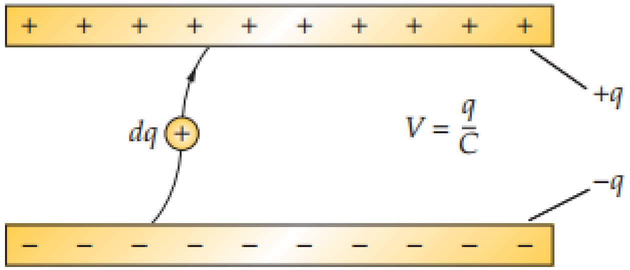




https://www.compadre.org/Physlets/electromagnetism/illustration26_4.cfm

Energy Stored in an Electric Field

In charging a capacitor, *positive charge* is being moved from the *negative plate* to the *positive plate*.
$\Rightarrow$ NEEDS WORK DONE!



Suppose we move charge dq from $-ve$ to $+ve$ plate, *change in potential energy*

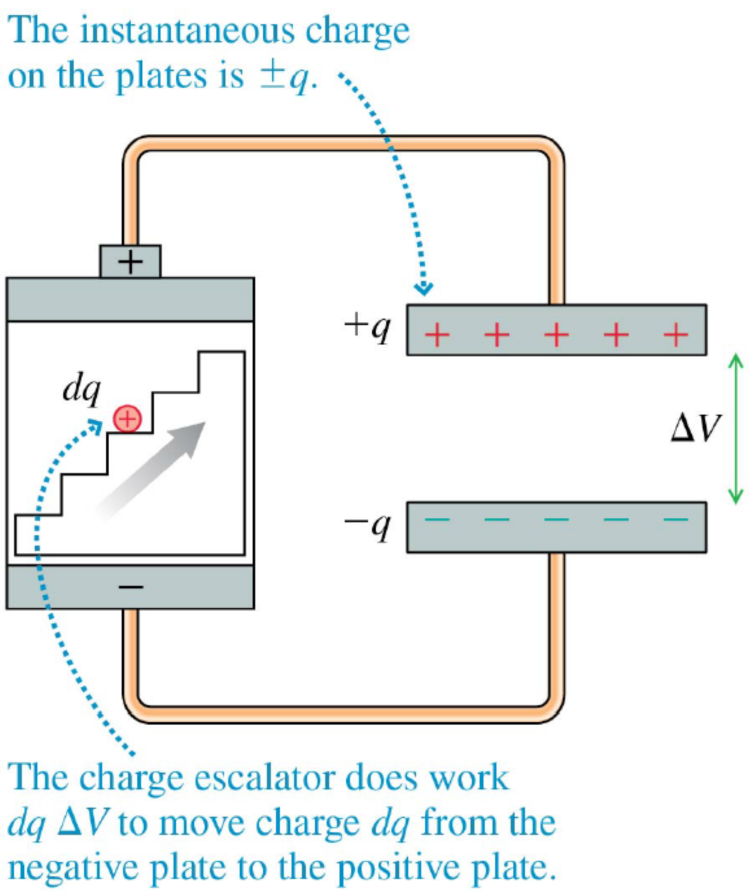
$$dU = \Delta V \cdot dq = \frac{q}{C}\, dq$$

Suppose we keep putting in a total charge Q to the capacitor, the *total potential energy*

$$U = \int dU = \int_0^Q \frac{q}{C}\, dq$$

$$\therefore \quad \boxed{U = \frac{Q^2}{2C} = \tfrac{1}{2}CV^2 = \tfrac{1}{2}QV} \quad (\because Q = C\Delta V)$$

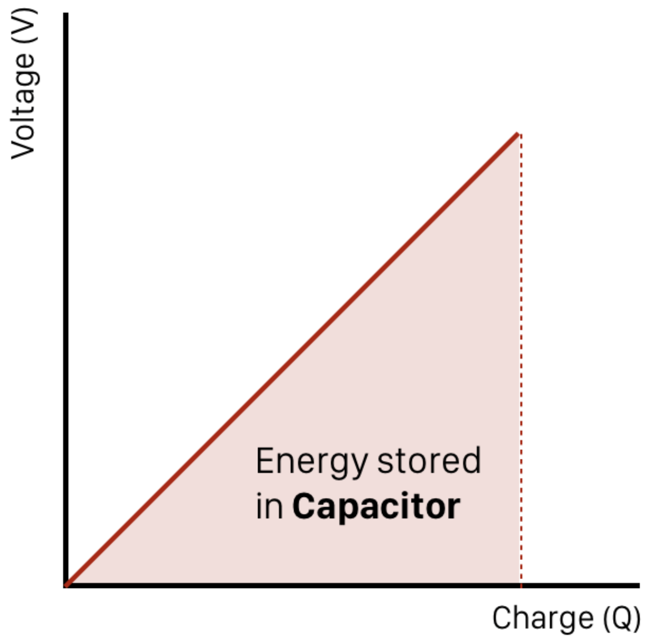
The energy stored in the capacitor is stored in the **electric field** between the plates.



The potential energy of a charged capacitor may be viewed as being stored in the electric field between its plates.

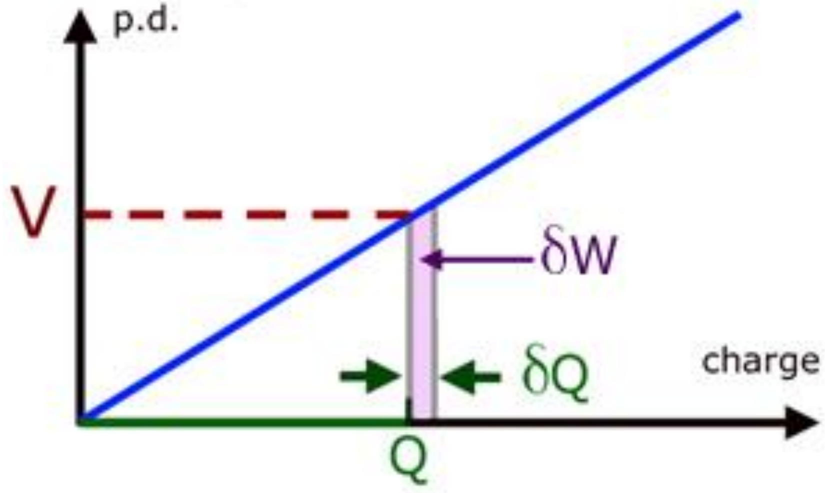
Energy Stored in a Charged Capacitor

$$dW = \Delta V dq = \frac{q}{C} dq$$

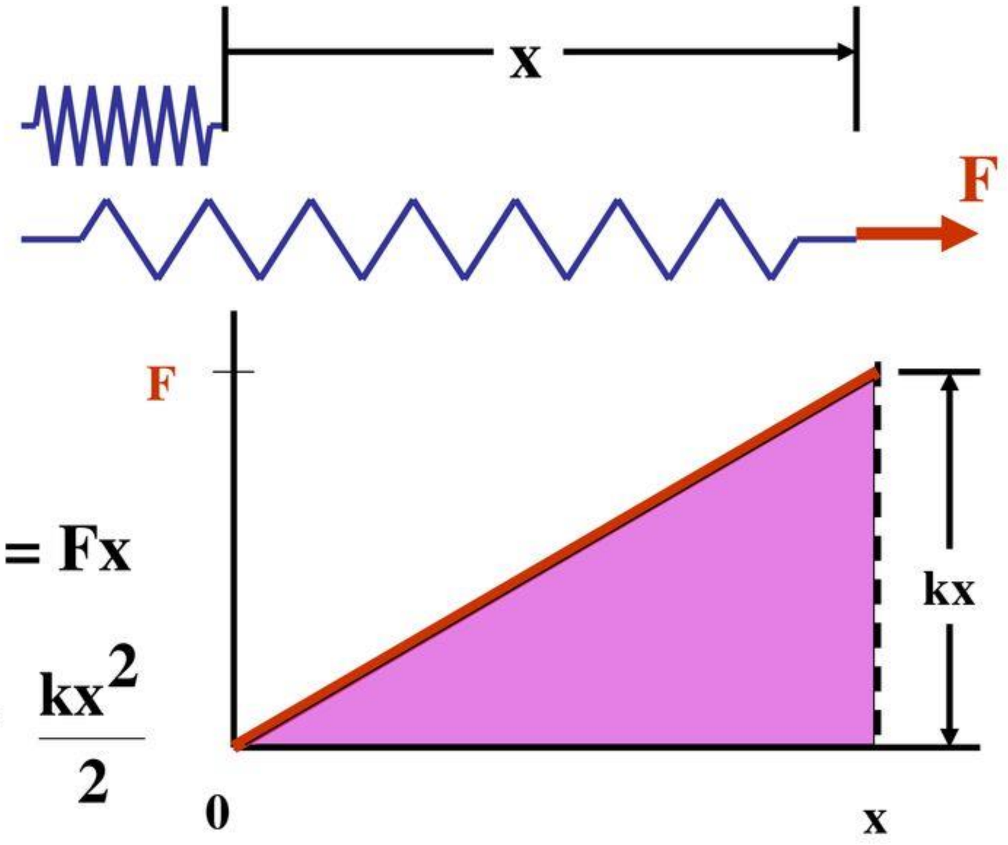


$$W = \int_0^Q \frac{q}{C} dq = \frac{1}{C} \int_0^Q q\,dq = \frac{Q^2}{2C}$$

$$U = \frac{Q^2}{2C} = \frac{1}{2} Q\Delta V = \frac{1}{2} C(\Delta V)^2$$

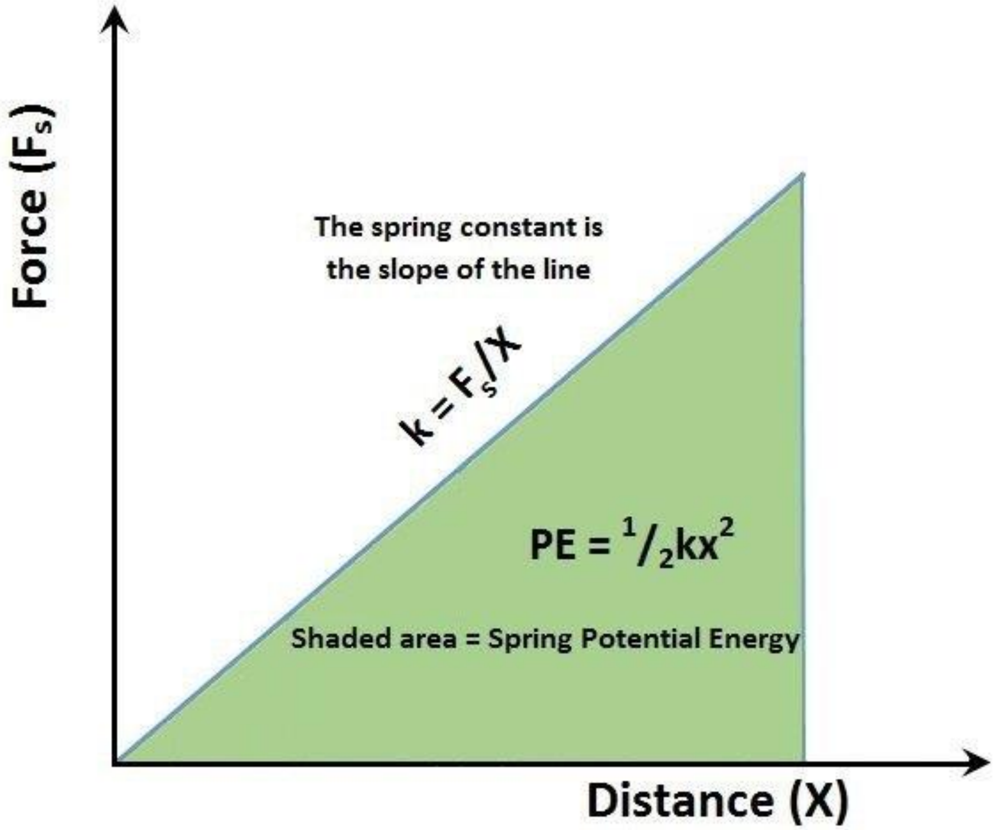


Stretching an Ideal Spring



Work = Fx

$$W_s = \frac{kx^2}{2}$$



Energy Stored in an Electric Field

The expression $U = \frac{1}{2}\frac{Q^2}{C}$ shows that a charged capacitor is the electrical analog of a stretched spring with elastic potential energy $U = \frac{1}{2}kx^2$ analogous to $U = \frac{1}{2}(1/C)Q^2$

The charge Q is analogous to the elongation x and *reciprocal* of the capacitance $1/C$, is analogous to the force constant k.

The energy supplied to a capacitor in the charging process is analogous to the work we do on a spring when we stretch it.

The spring constant is constant for a particular spring and different for different springs. Moreover the spring constant is purely geometric, same is the capacitance.

Energy Density

In a parallel-plate capacitor, neglecting fringing, the electric field has the same value at all points between the plates. Thus, the **energy density** *u*—that is, the potential energy per unit volume between the plates—should also be uniform.

$$\text{density u} = \frac{\text{Total energy stored}}{\text{Total volume with E-field}}$$

$$\therefore \quad u = \frac{U}{\underbrace{Ad}_{\text{Rectangular volume}}}$$

$$u = \frac{U}{Ad} = \frac{CV^2}{2Ad}.$$

$$u = \tfrac{1}{2}\varepsilon_0 \left(\frac{V}{d}\right)^2$$

$$\begin{cases} C = \dfrac{\epsilon_0 A}{d} \\[2ex] E = \dfrac{\Delta V}{d} \quad \Rightarrow \quad \Delta V = Ed \end{cases}$$

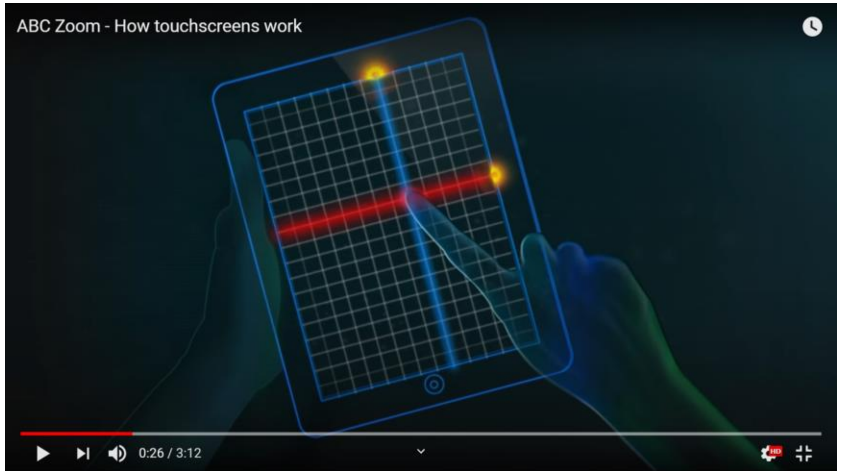
$$\boxed{u = \tfrac{1}{2}\varepsilon_0 E^2 \qquad \text{(energy density).}}$$

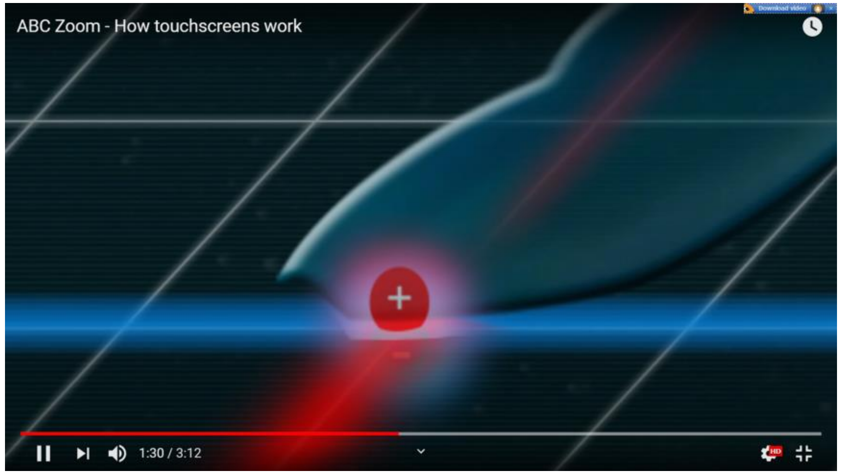
$$\therefore \quad u = \frac{1}{2}\,(\overbrace{\frac{\epsilon_0 A}{d}}^{C}) \cdot (\overbrace{Ed}^{(\Delta V)^2})^2 \cdot \overbrace{\frac{1}{Ad}}^{\frac{1}{Volume}}$$

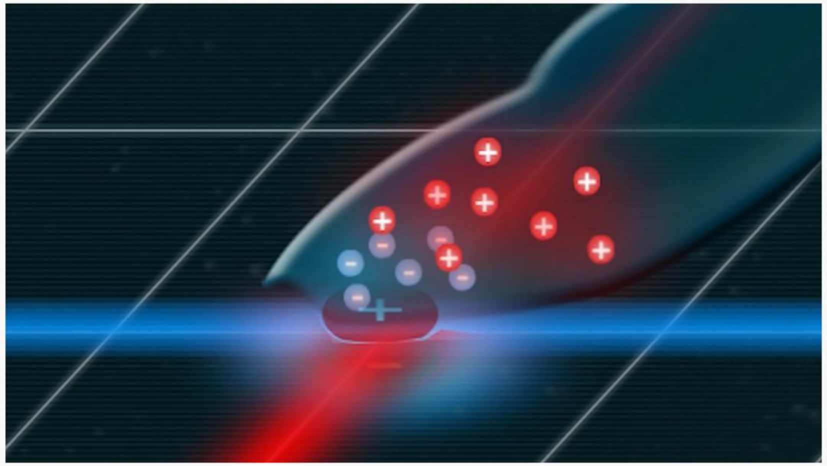
Although we have derived this relationship only for a parallel-plate capacitor, it turns out to be valid for any capacitor in vacuum and indeed *for any electric field configuration in vacuum.* This result has an interesting implication.
We think of vacuum as space with no matter in it, but vacuum can nevertheless have electric fields and therefore energy. Thus “empty” space need not be truly empty after all.

Energy Stored in an Electric Field

Touch Screen

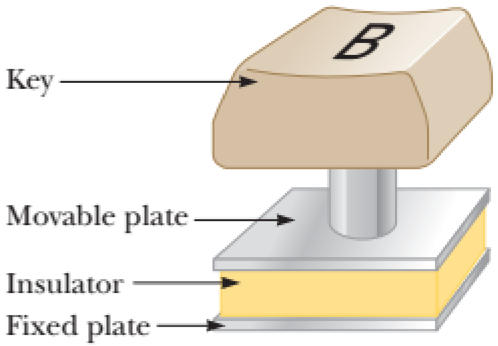






What Is a Capacitive Keyboard?

A capacitive keyboard is a type of keyboard that leverages capacitance to detect key presses. They use a similar method of operation as capacitive touchscreens.



The keys on most computer keyboards are capacitor switches. Pressing the key pushes two capacitor plates closer together, increasing their capacitance.

What is the energy stored in the 10.0 μF capacitor of a heart defibrillator charged to 9.00×10^3 V? (b) Find the amount of stored charge.

In open heart surgery, a much smaller amount of energy will defibrillate the heart. (a) What voltage is applied to the 8.00 μF capacitor of a heart defibrillator that stores 40.0 J of energy? (b) Find the amount of stored charge.

Capacitors and Dielectrics

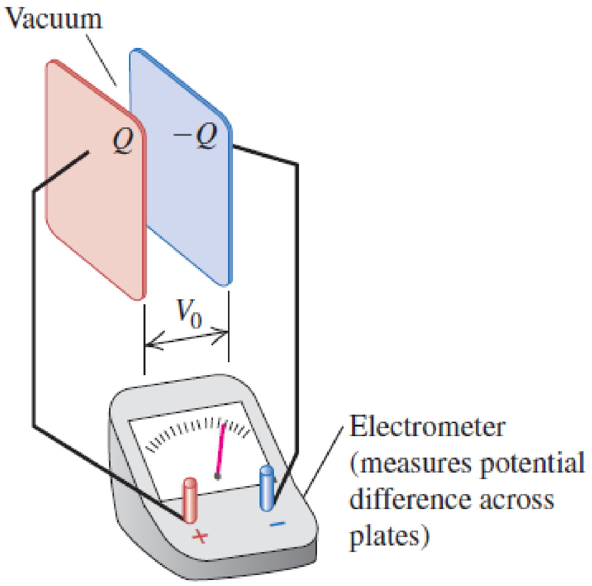
If we fill the region between the plates of a capacitor with an insulating material the capacitance will be increased by some numerical factor :

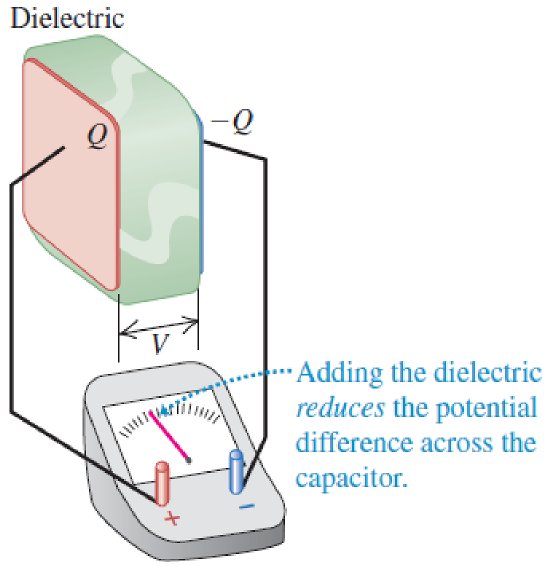
$$C = \kappa C_{air}$$

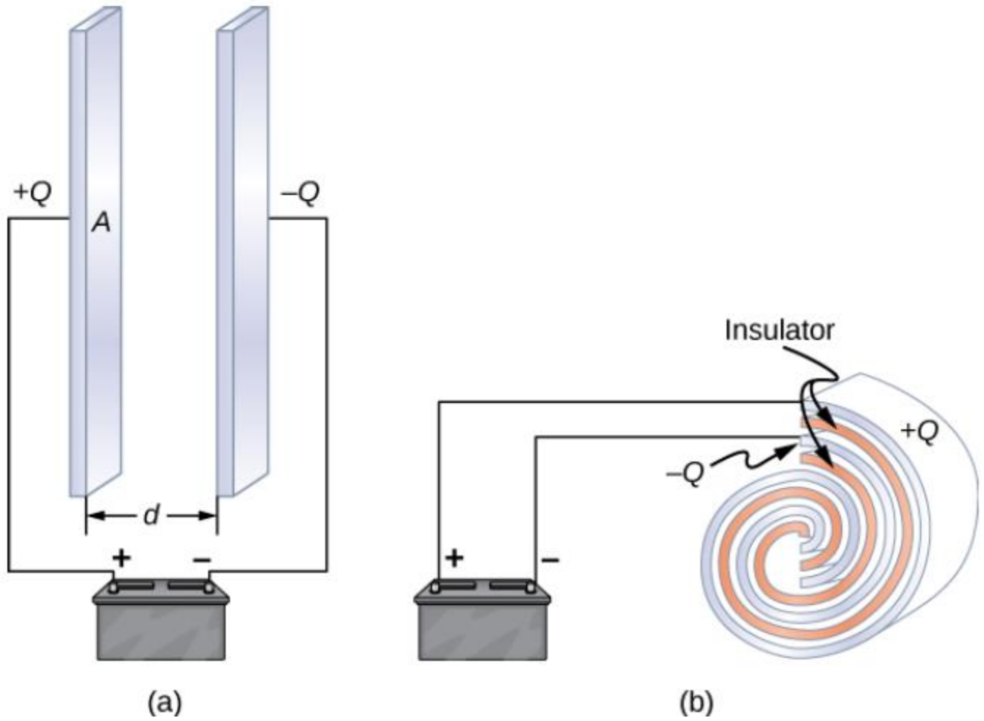
The number κ (which is unitless) is called the **dielectric constant** of the insulating material

$$\boldsymbol{\kappa = \frac{C}{C_{air}}}$$

In a region completely filled by a dielectric material of dielectric constant κ, all electrostatic equations containing the permittivity constant ε_0 are to be modified by $\kappa\varepsilon_0$



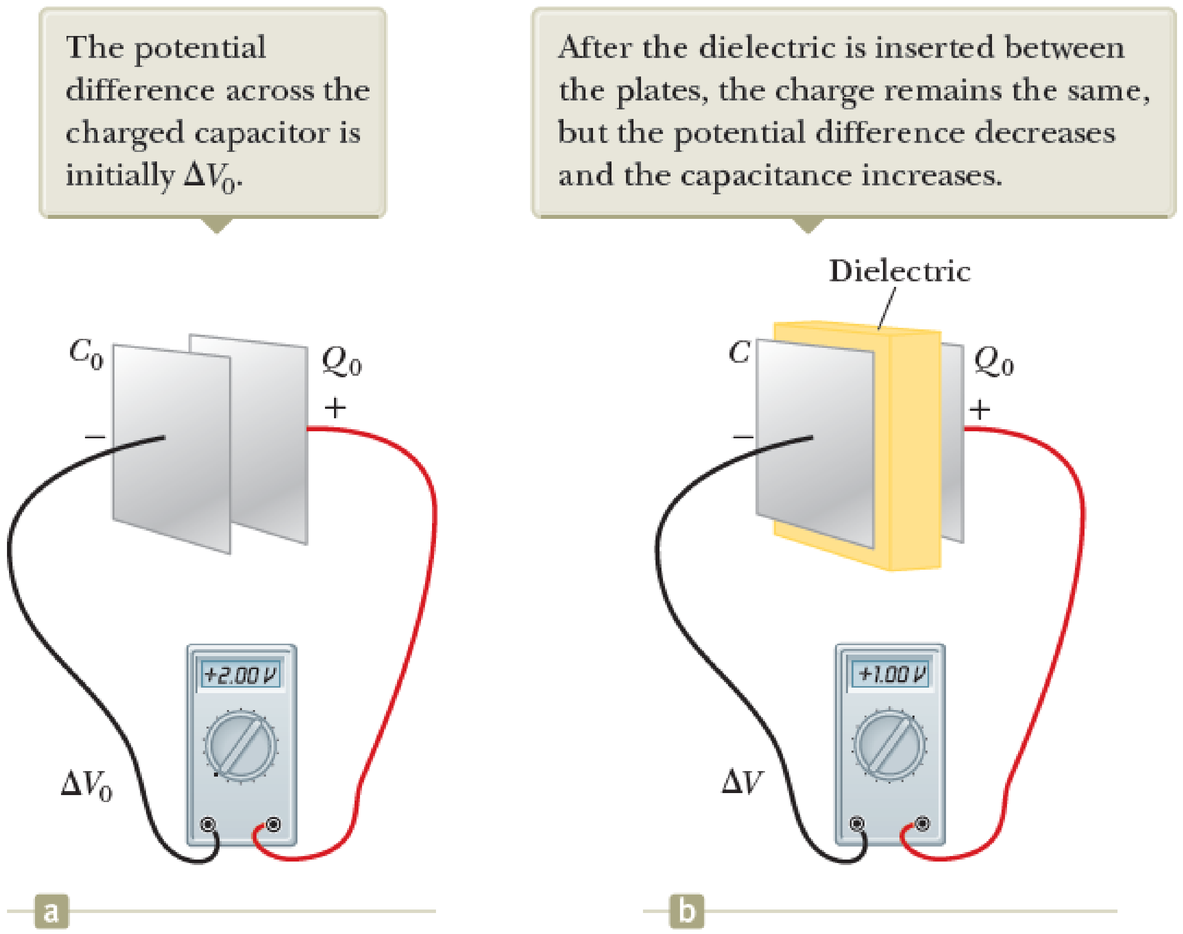


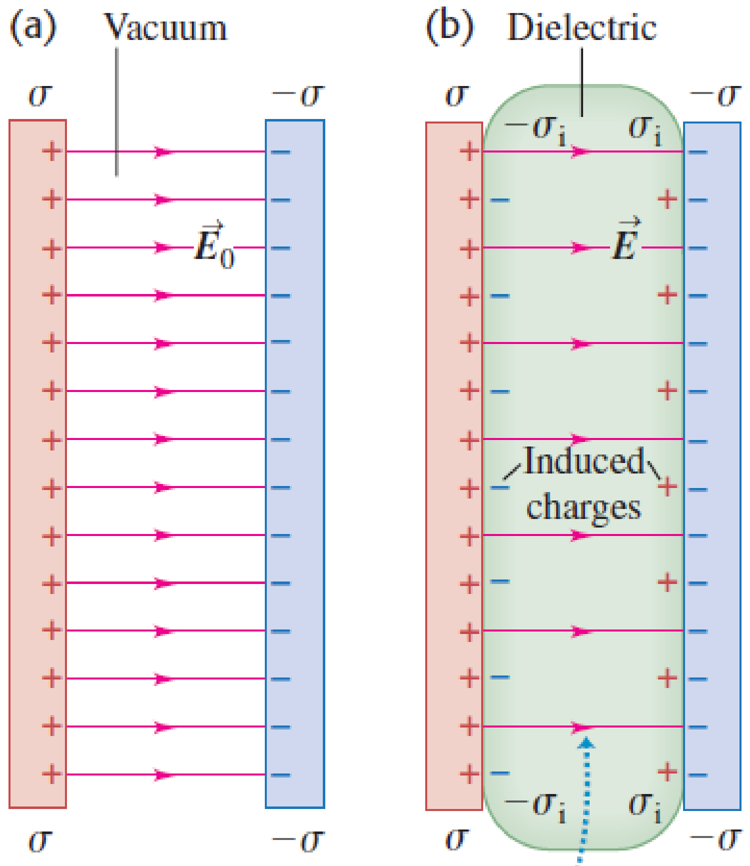


Both capacitors shown here were initially uncharged before being connected to a battery. They now have charges of $+Q$ and $-Q$ (respectively) on their plates.

(a) A parallel-plate capacitor consists of two plates of opposite charge with area A separated by distance d.

(b) A rolled capacitor has a dielectric material between its two conducting sheets (plates).





For a given charge density σ, the induced charges on the dielectric's surfaces reduce the electric field between the plates.

When a dielectric material is inserted between the plates while the charge is kept constant, **the potential difference between the plates decreases by a factor κ.**

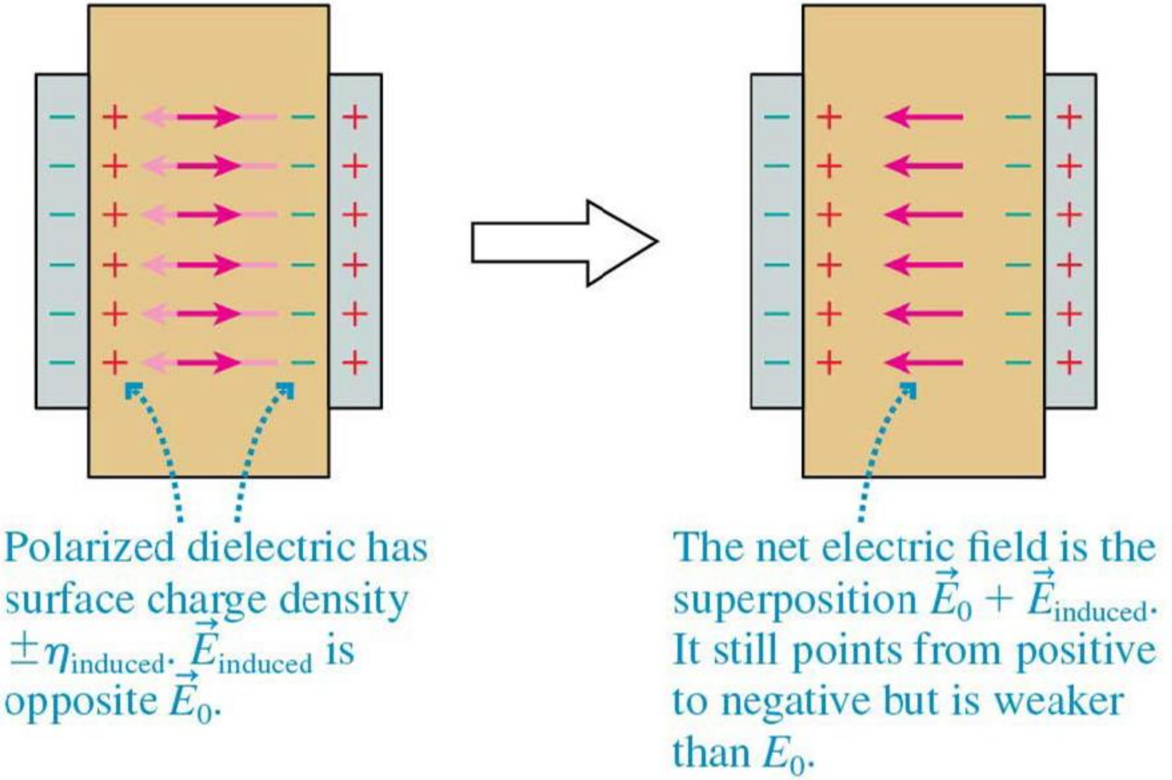
Therefore, when Q is constant **the electric field between the plates must decrease by the same factor κ**,

$$E = \frac{E_{air}}{\kappa}$$

We will assume that the induced surface charge is *directly proportional* to the electric-field magnitude E in the material; this is indeed the case for many common dielectrics. (This direct proportionality is analogous to Hooke's law for a spring.) In that case, κ is a constant for any particular material.

Because κ is always greater than unity, both these equations show that *for a fixed distribution of charges, the effect of a dielectric is to weaken the electric field* that would otherwise be present.

https://javalab.org/en/dielectric_in_capacitor_en/


Polarized dielectric has surface charge density $\pm\eta_{\text{induced}}$. $\vec{E}_{\text{induced}}$ is opposite $\vec{E}_0$.
The net electric field is the superposition $\vec{E}_0 + \vec{E}_{\text{induced}}$. It still points from positive to negative but is weaker than E_0.

Weaker Field, More Charge

Due to the opposing field from the polarized dielectric, the overall electric field between the plates gets weaker compared to a vacuum gap. However, the voltage difference between the plates remains the same (assuming a constant charge source).

Capacitance (C) is defined as the ratio of stored charge (Q) to the voltage difference (V) across the plates: $C = Q/V$.
Even with a weaker electric field, the capacitor can store the same amount of charge (Q) at a lower voltage (V) due to the presence of the dielectric.
This essentially means the capacitor becomes more efficient at storing charge for a given voltage, increasing its capacitance.

Net Field

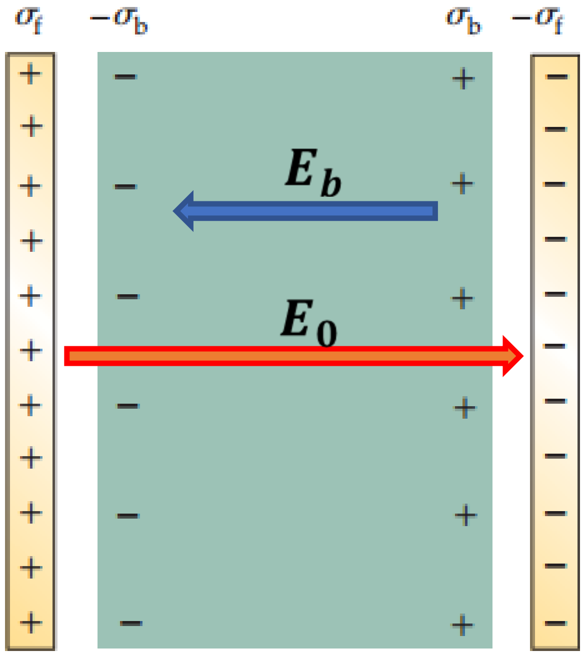
$$E = \frac{E_0}{\kappa}$$

Field due to free charges

$$E_0 = \frac{\sigma_f}{\epsilon_0}$$

Field due to bound charges

$$E_b = \frac{\sigma_b}{\epsilon_0}$$



$$\boxed{E = E_0 - E_b}$$

$$E = E_0 - E_b = \frac{E_0}{\kappa}$$

$$E_b = E_0 - \frac{E_0}{\kappa} = (1 - \frac{1}{\kappa})E_0$$

Relation between charge densities within capacitor dielectric

$$\sigma_b = (1 - \frac{1}{\kappa})\sigma_f$$

DIELECTRICS and Permittivity

The dielectric constant coincides with the relative Dielectric permittivity in a homogeneous medium and is defined as the ratio of the Dielectric permittivity of the medium (ε) to the Dielectric permittivity in a vacuum (ε_0).

Their ratio $\kappa = \varepsilon/\varepsilon_0$ called the dielectric constant. $\varepsilon = \varepsilon_0\kappa$

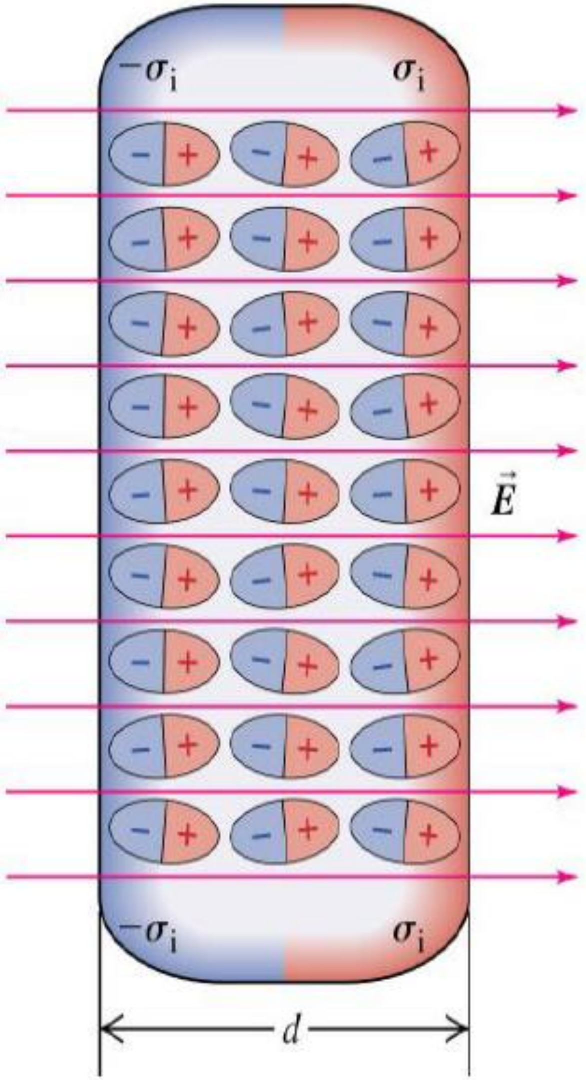
Permittivity is the ability of a material to store an electric field in the polarization of the medium.

Air approximates to a perfect vacuum and so the dielectric constant for air is approximately zero.

Polarization

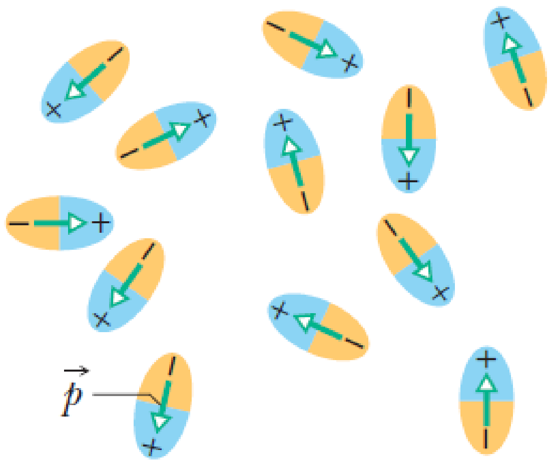
Without the dielectric in the capacitor, we have the electric field points undiminished from the positive to the negative plate.

With the dielectric in place we have t**he electric field between the plates of the capacitor is reduced because some of the material within the dielectric rearranges so that their negative charges are oriented towards the positive plate**

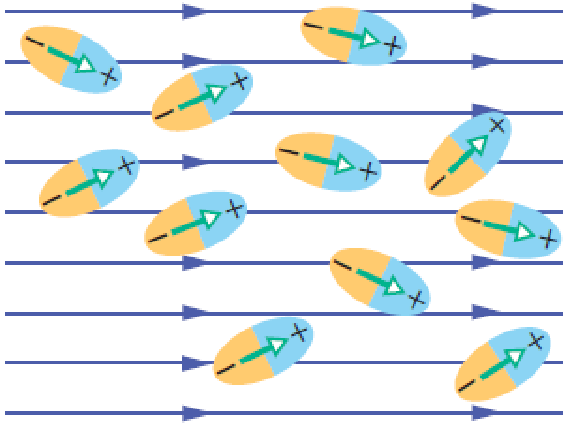


Polar dielectrics.

The molecules of some dielectrics, like water, have permanent electric dipole moments. In such materials (called *polar dielectrics*), the electric dipoles tend to line up with an external electric field.
The alignment of the electric dipoles produces an electric field that is directed opposite the applied field and is smaller in magnitude.



(a)



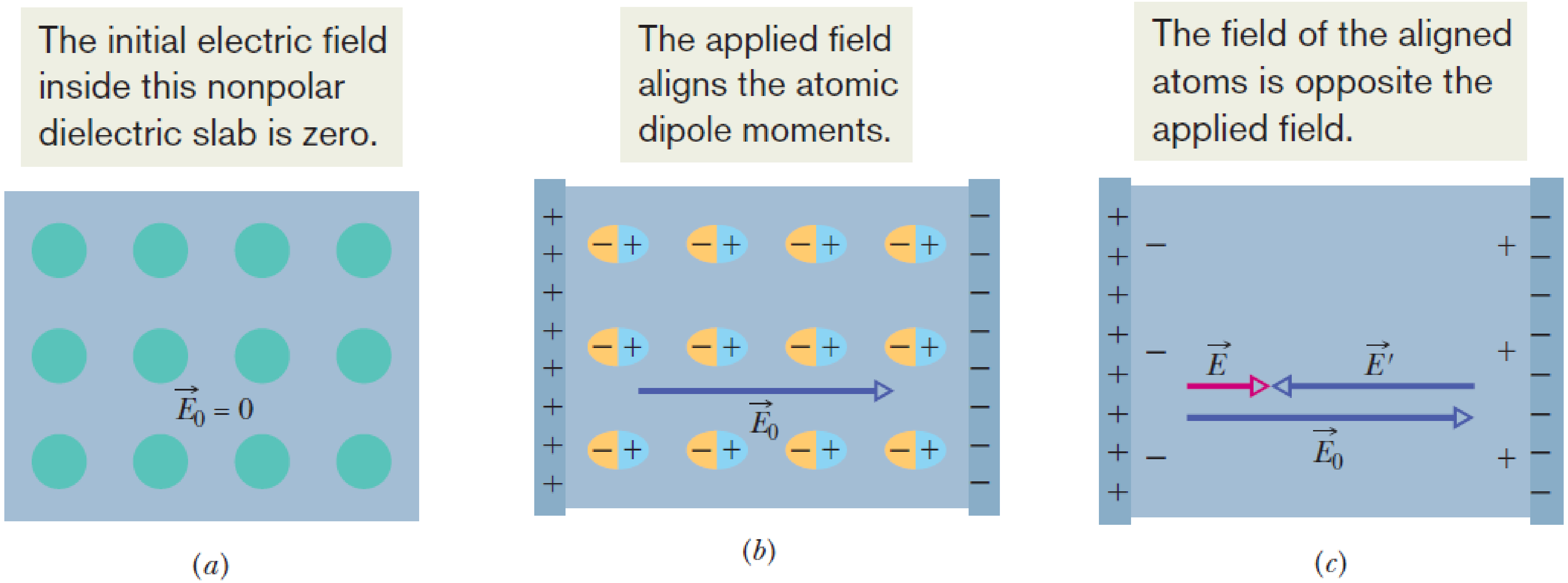
(b)

Polar capacitors have a higher dielectric constant than nonpolar capacitors. This means that they can store more charge for a given voltage.

However, polar capacitors are also more susceptible to dielectric breakdown, which is when the dielectric material suddenly loses its insulating properties.

***Nonpolar dielectrics*.**

Regardless of whether they have permanent electric dipole moments, molecules acquire dipole moments by induction when placed in an external electric field.



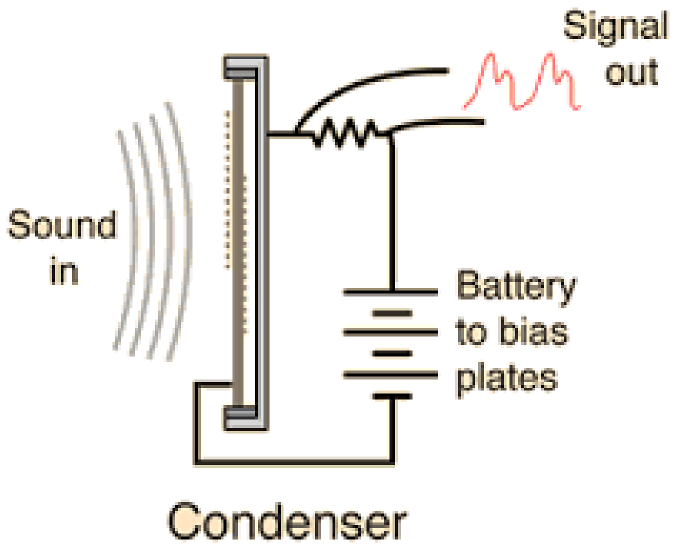
Nonpolar capacitors are less susceptible to dielectric breakdown than polar capacitors. However, they also have a lower dielectric constant, so they cannot store as much charge for a given voltage.

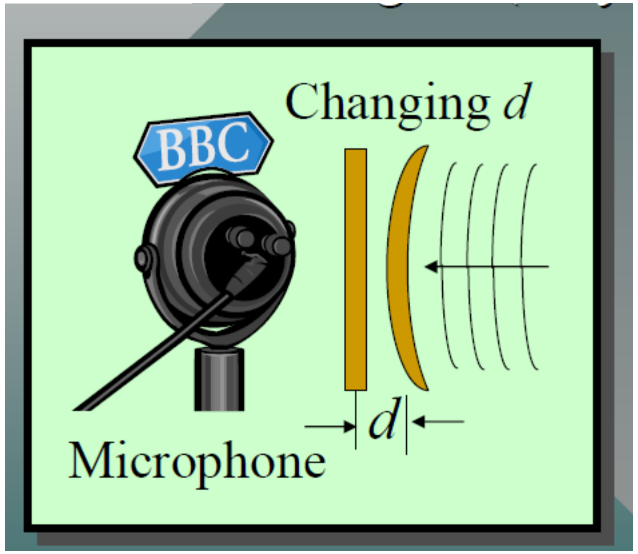
NONPOLARIZED **POLARIZED**

Feature	Polar Capacitor	Nonpolar Capacitor
Dielectric constant	High	Low
Susceptibility to dielectric breakdown	High	Low
Typical applications	High-voltage	Low-voltage

The tree-like branch patterns in this clear glass block are known as a Lichtenberg figure, named for the German physicist Georg Christof Lichtenberg (1742–1799), who was the first to study these patterns. The “branches” are created by the dielectric breakdown produced by a strong electric field.

https://www.youtube.com/watch?v=kJEAtH_BsSo

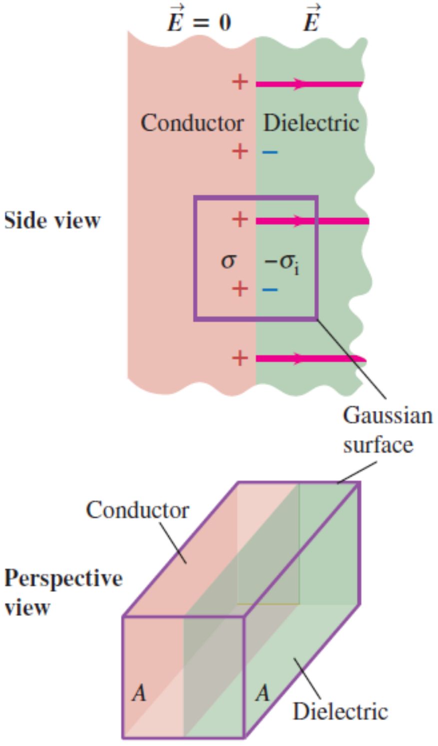


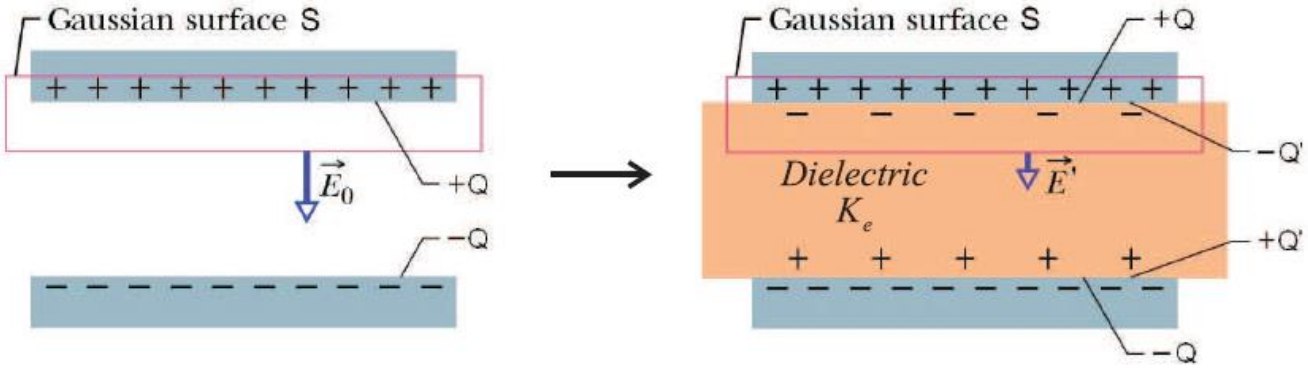


The tuner in a radio is a variable capacitor. The changing area alters capacitance until desired signal is obtained.

A microphone converts sound waves into an electrical signal (varying voltage) by changing

Dielectrics and Gauss' Law





Free charge on plates	$\pm Q$	$\pm Q$
Induced charge on dielectric	0	$\mp Q'$

Gauss' Law

$$\oint_S \vec{E} \cdot d\vec{A} = \frac{Q}{\epsilon_0}$$

$$\Rightarrow \quad E_0 = \frac{Q}{\epsilon_0 A} \qquad (1)$$

Gauss' Law:

$$\oint_S \vec{E}' \cdot d\vec{A} = \frac{Q - Q'}{\epsilon_0}$$

$$\therefore \quad E' = \frac{Q - Q'}{\epsilon_0 A} \qquad (2)$$

However, we define $\quad E' = \dfrac{E_0}{K_e} \qquad (3)$

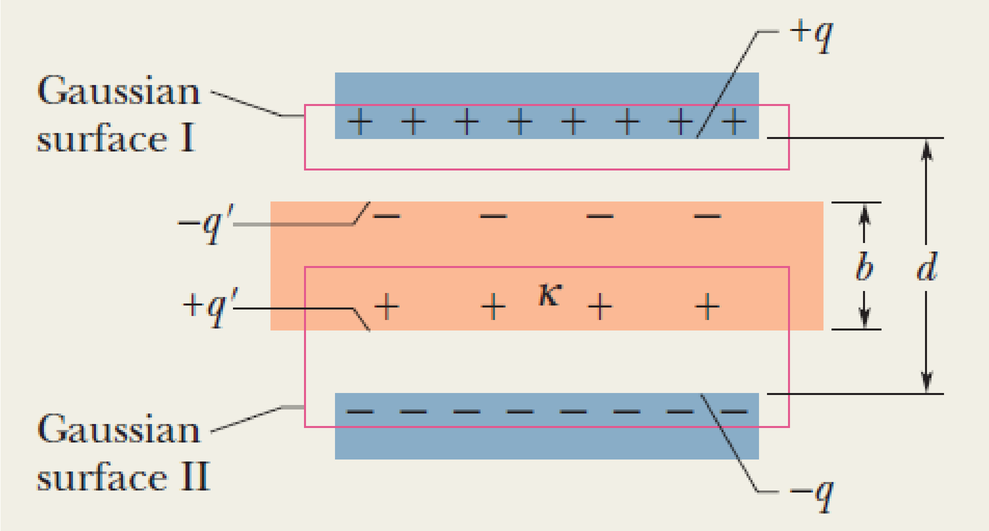
From (1), (2), (3) $\quad \therefore \quad \dfrac{Q}{K_e \epsilon_0 A} = \dfrac{Q}{\epsilon_0 A} - \dfrac{Q'}{\epsilon_0 A}$

$$\boxed{\text{Induced charge density } \sigma' = \frac{Q'}{A} = \sigma\left(1 - \frac{1}{K_e}\right) < \sigma}$$

where σ is free charge density.

$$\epsilon_0 \oint_S \vec{E}' \cdot d\vec{A} = \underbrace{Q}_{\text{free charge}} - \underbrace{Q'}_{\text{induced charge}}$$

E-field in dielectric — free charge — induced charge



$$\varepsilon_0 \oint \kappa \vec{E} \cdot d\vec{A} = q \quad \text{(Gauss' law with dielectric).}$$

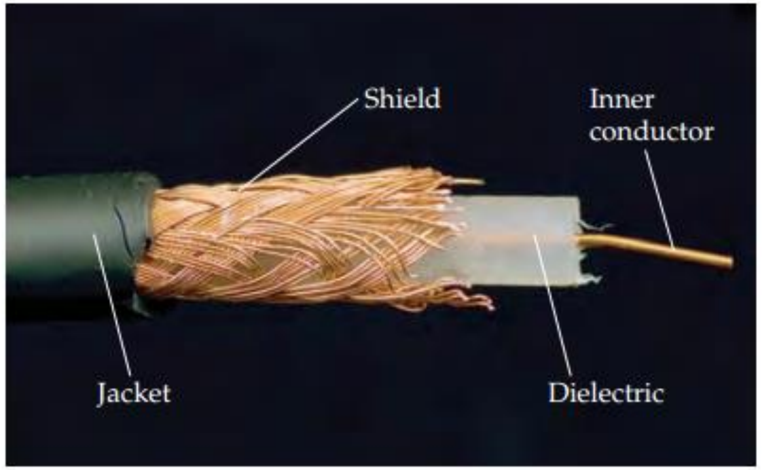
The flux integral now involves $\kappa\vec{E}$ not just $\vec{E}$

The vector $\varepsilon_0 \kappa \vec{E}$ is sometimes called the ***electric displacement*** $\boldsymbol{\vec{D} = \varepsilon_0 \kappa \vec{E}}$

$$\oint \vec{D} . d\vec{A} = q$$

(Gauss law of electricity)

$$\oint \vec{E} . d\vec{A} = q/\epsilon_0$$



Coaxial cable, is a type of electrical cable consisting of an inner conductor surrounded by a concentric conducting shield, with the two separated by a dielectric (insulating material); many coaxial cables also have a protective outer sheath or jacket. The term "coaxial" refers to the inner conductor and the outer shield sharing a geometric axis.

Coaxial cable is a type of transmission line, used to carry high-frequency electrical signals with low losses.

It is used in such applications as telephone lines, broadband internet networking cables, high-speed computer data busses, cable television signals, and connecting radio transmitters and receivers to their antennas. It differs from other shielded cables because the dimensions of the cable and connectors are controlled to give a precise, constant conductor spacing, which is needed for it to function efficiently as a transmission line.

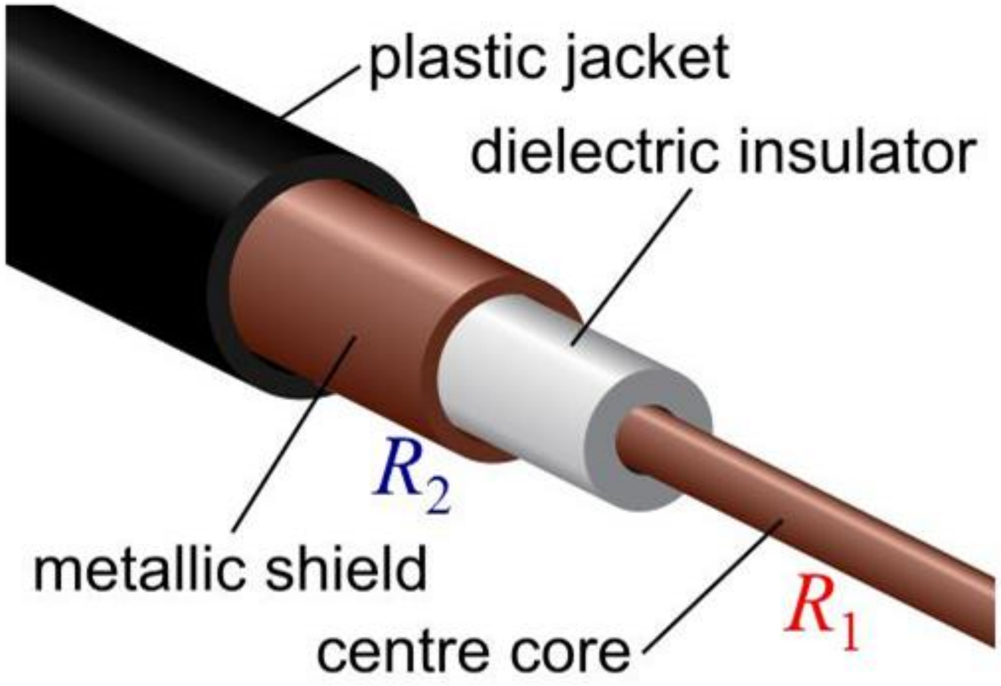
https://phet.colorado.edu/en/simulations/capacitor-lab

https://www.knowatom.com/interactive-simulations/capacitor-lab

https://ophysics.com/em5.html

A coaxial cable of length ***L*** is an example of a cylindrical capacitor

$$\boxed{C = \frac{2\pi\varepsilon_0 L}{\ln(R_2 / R_1)}}$$

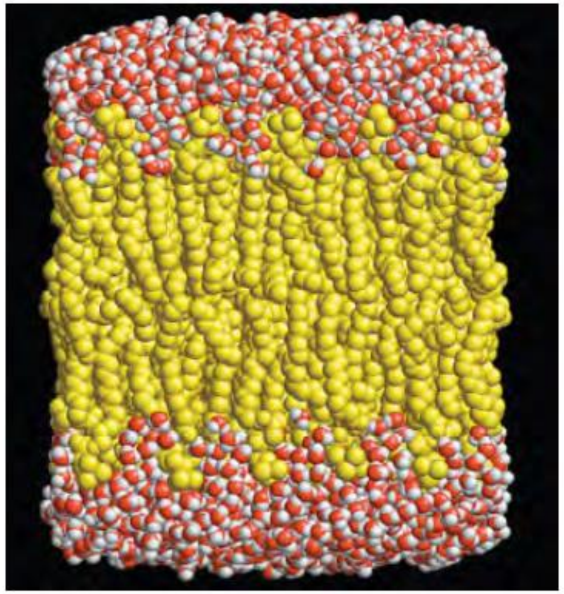


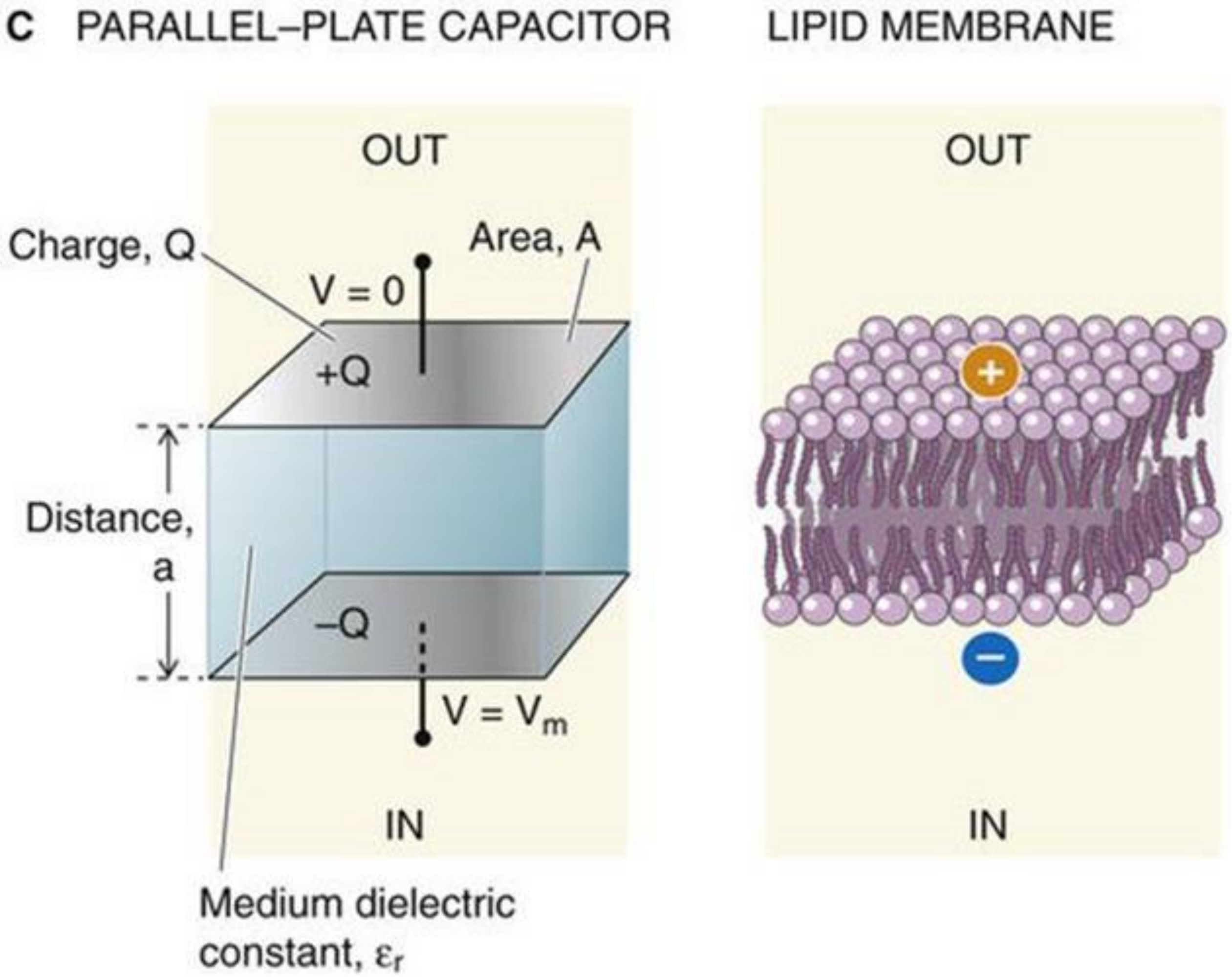
What is the maximum charge that can be placed on a spherical surface one meter in diameter? (R = 0.50 m)

The dielectric strength of a material is that electric intensity for which the material becomes a conductor.
For air $E_m = 3 \times 10^6\ N/C$

Application **Dielectric Cell Membrane**

The membrane of a living cell behaves like a dielectric between the plates of a capacitor. The membrane is made of two sheets of lipid molecules, with their water-insoluble ends in the middle and their water-soluble ends (shown in red) on the surfaces of the membrane. The conductive fluids on either side of the membrane (water with negative ions inside the cell, water with positive ions outside) act as charged capacitor plates, and the nonconducting membrane acts as a dielectric with K of about 10. The potential difference V across the membrane is about 0.07 V and the membrane thickness d is about 7×10^{-9} m, so the electric field $E = V/d$ in the membrane is about 10^7 V/m—close to the dielectric strength of the membrane. If the membrane were made of air, V and E would be larger by a factor of $K \approx 10$ and dielectric breakdown would occur.

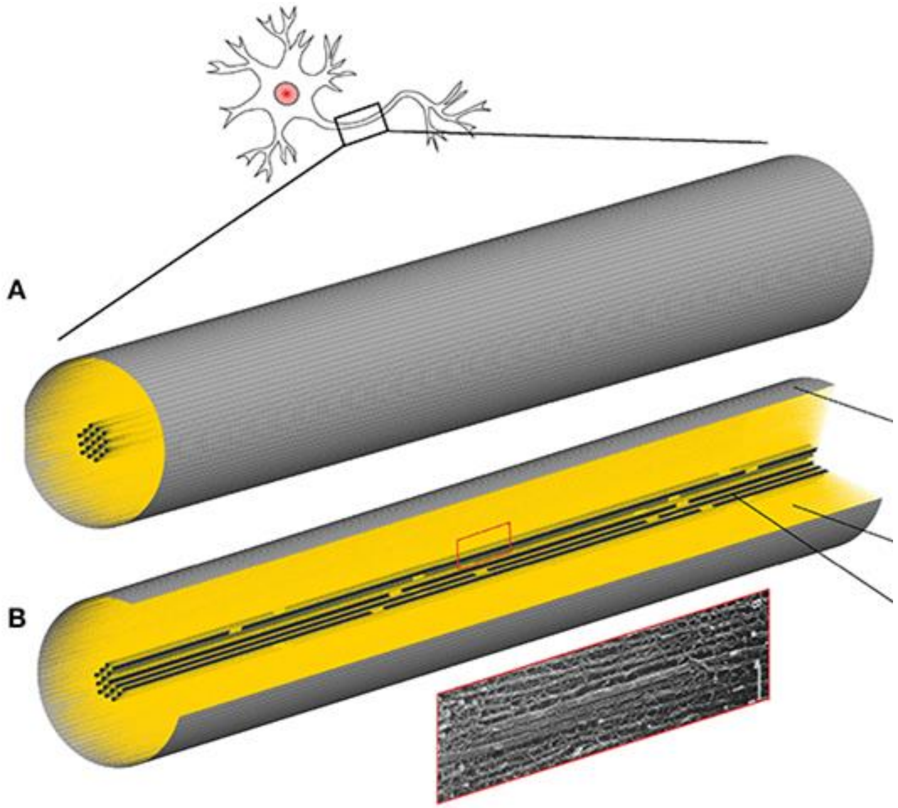




https://aberinstruments.com/biotech/capacitance-measurement/#gref

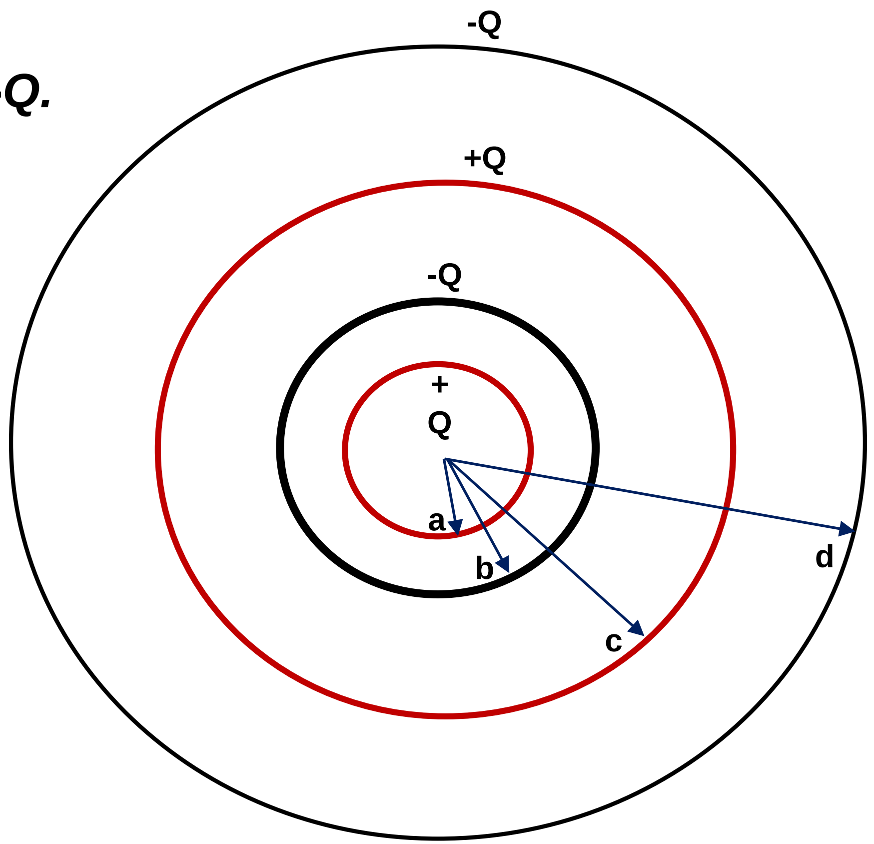
The membrane of the axon of a nerve cell can be modeled as a thin cylindrical shell of radius $1.0 \times 10^{-5} m$ having a length of 10.0 cm and a thickness of 10.0 nm. The membrane has a positive charge on one side and a negative charge on the other, and the membrane acts as a parallel plate capacitor of area $2\pi rL$ and separation *d*. Assume the membrane is filled with a material whose dielectric constant is 3.00.

(a) Find the capacitance of the membrane. If the potential difference across the membrane is 70.0 mV find (b)



Given 4 concentric cylinders of radii *a*,*b*,*c*,*d* and charges +*Q*, -*Q*, +*Q*, -*Q*.

Question: What is the capacitance between *a* and *d*?



- **A cylinder of radius r_1: $b < r_1 < c$ encloses zero charge!**

$$V_{ad} = \int_a^b \frac{Q}{2\pi\varepsilon_0 rL} dr + 0 + \int_c^d \frac{Q}{2\pi\varepsilon_0 rL} dr = \frac{Q}{2\pi\varepsilon_0 L}\left(\ln\left(\frac{b}{a}\right) + \ln\left(\frac{d}{c}\right)\right)$$

$$C = \frac{Q}{V_{ad}} = \frac{2\pi\varepsilon_0 L}{\left(\ln\left(\frac{b}{a}\right) + \ln\left(\frac{d}{c}\right)\right)}$$

https://phet.colorado.edu/sims/html/capacitor-lab-basics/latest/capacitor-lab-basics_en.html