Magnetic Fields-II

Phy 108 course

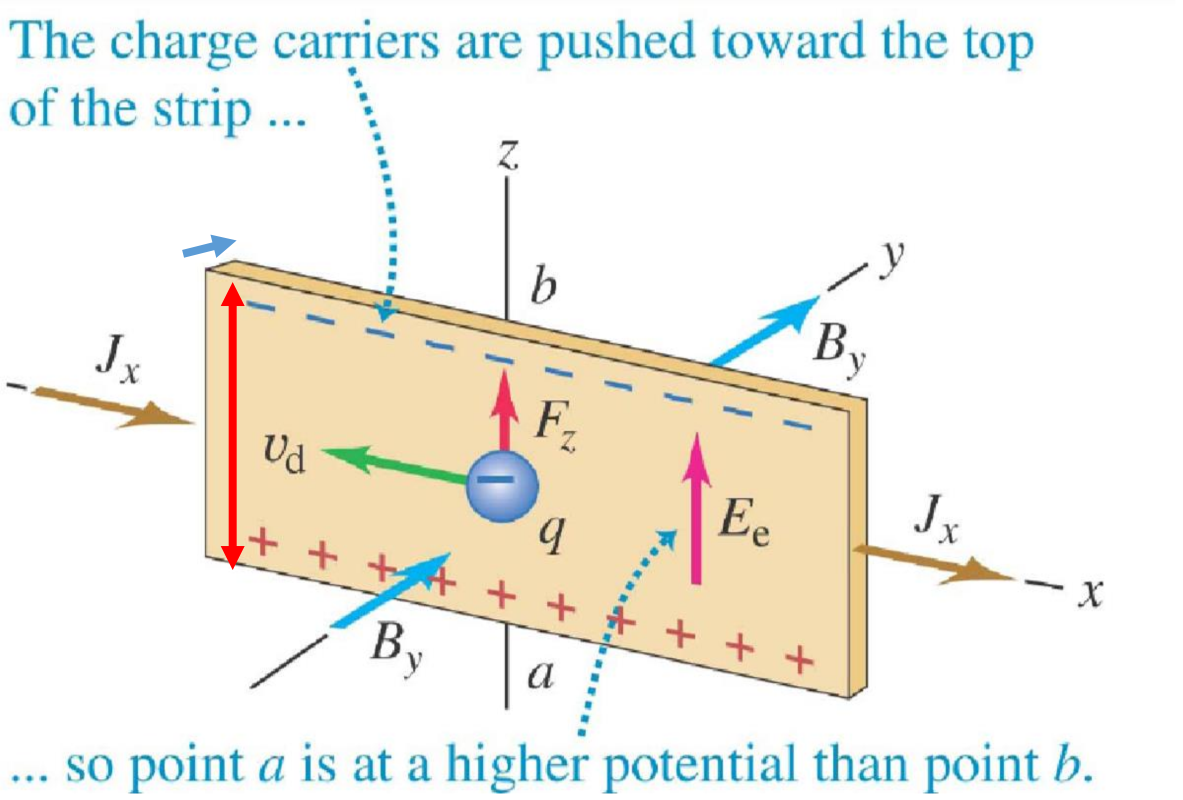
Zaid Bin Mahbub (ZBM)

DMP, SEPS, NSU

Magnetic Force on a Current-Carrying Wire

We have already seen (in connection with the Hall effect) that a magnetic field exerts a sideways force on electrons moving in a wire.

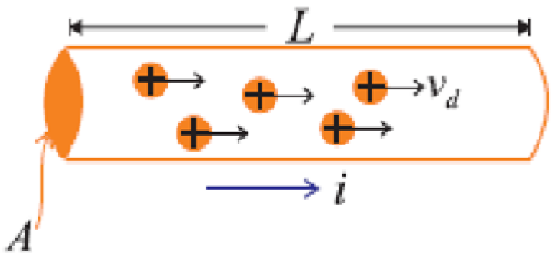
This force must then be transmitted to the wire itself, because the conduction electrons cannot escape sideways out of the wire.



Magnetic Force on a Current-Carrying Wire

Current = many charges moving together

Consider a wire segment, length L, carrying current i in a magnetic field.



$$\text{Total magnetic force} = (\underbrace{q\vec{v}_d \times \vec{B}}_{\text{force on one charge carrier}}) \cdot \underbrace{n\,A\,L}_{\text{Total number of charge carrier}}$$

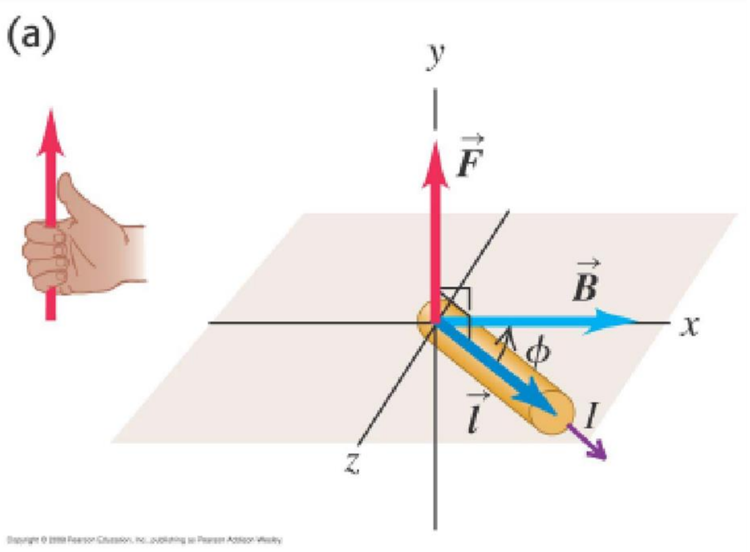
Recall $i = nqv_dA$

$$\therefore \quad \boxed{\text{Magnetic force on current } \vec{F} = i\vec{L} \times \vec{B}}$$

where $\vec{L}$ = Vector of which: $|\vec{L}|$ = length of current segment; direction = direction of current

For an infinitesimal wire segment $d\vec{l}$

$$\boxed{d\vec{F} = i\,d\vec{l} \times \vec{B}}$$



Fingers in direction of first vector, I.

Magnetic Force on a Current-Carrying Wire

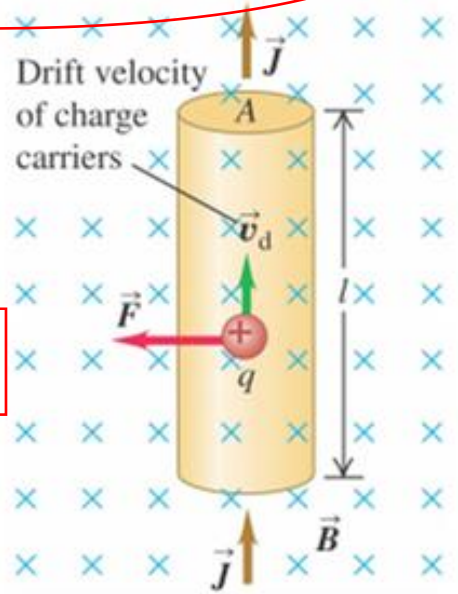
$$\vec{F}_m = q\vec{v}_d \times \vec{B}$$

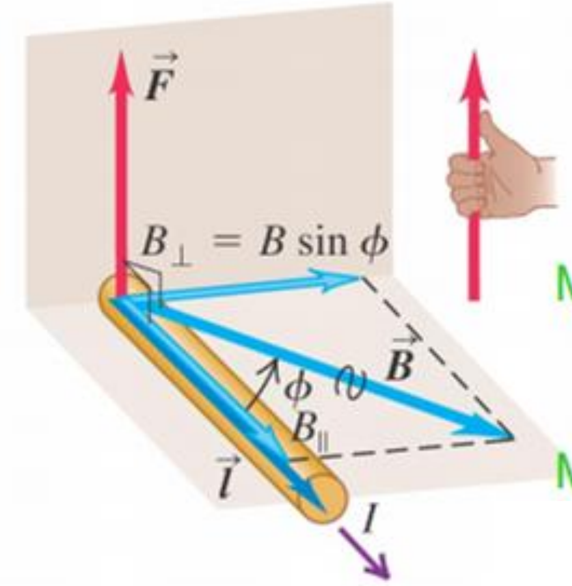
$$F_m = qv_d B$$ Force on one charge

- Total force: $$\boxed{F_m = (nAl)(qv_d B)}$$

n = number of charges per unit volume
A l = volume

$$\boxed{F_m = (nqv_d)(A)(lB) = (JA)(lB) = IlB \qquad (B \perp \text{wire})}$$





In general:

$$F = IlB_\perp = IlB\sin\varphi$$

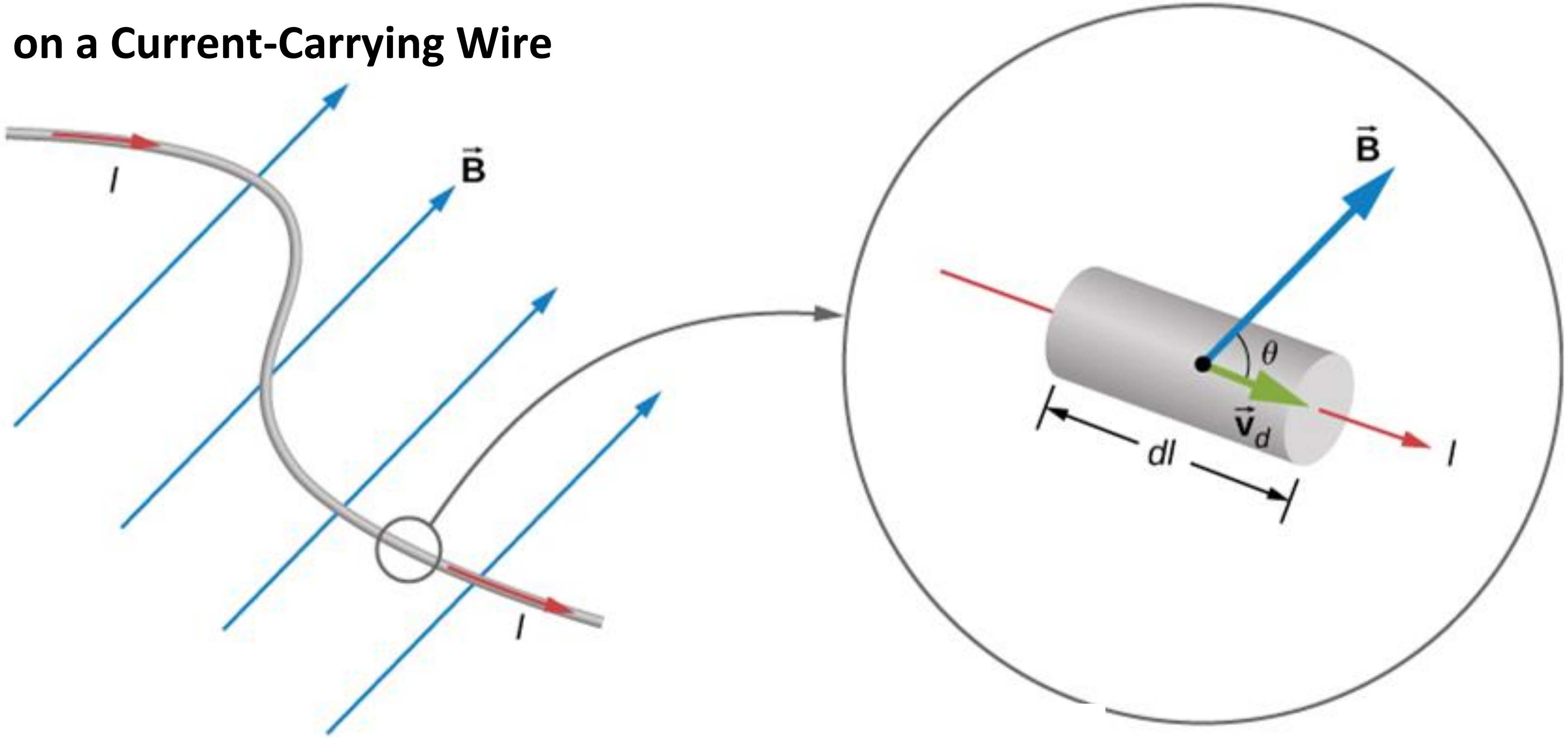
Magnetic force on a straight wire segment:

$$\vec{F} = i\vec{l} \times \vec{B}$$

Magnetic force on an infinitesimal wire section:

$$d\vec{F} = id\vec{l} \times \vec{B}$$

Magnetic Force on a Current-Carrying Wire



$$d\,\vec{\mathbf{F}} = neAv_{\mathrm{d}}\,\vec{\mathbf{dl}} \times \vec{\mathbf{B}},$$

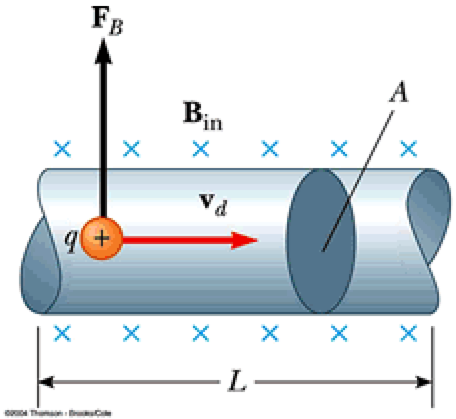
$$\vec{F} = i\vec{l} \times \vec{B}$$

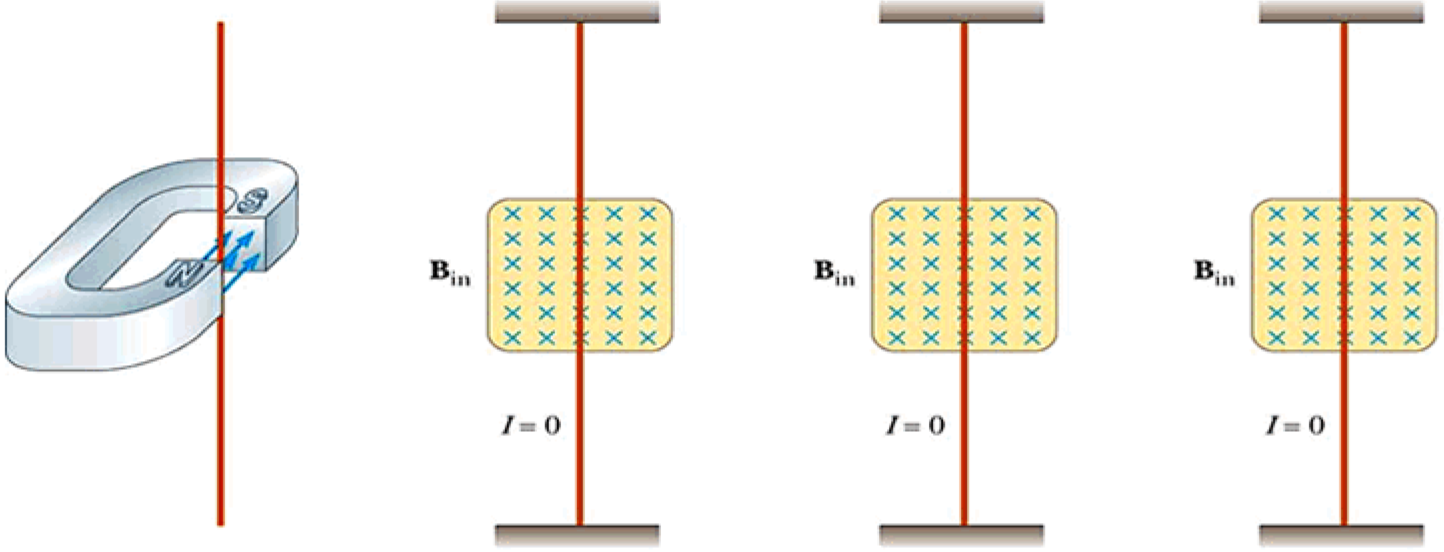
Magnetic Force **Magnetic Field**

$$\vec{F}_B = I\vec{L} \times \vec{B}$$

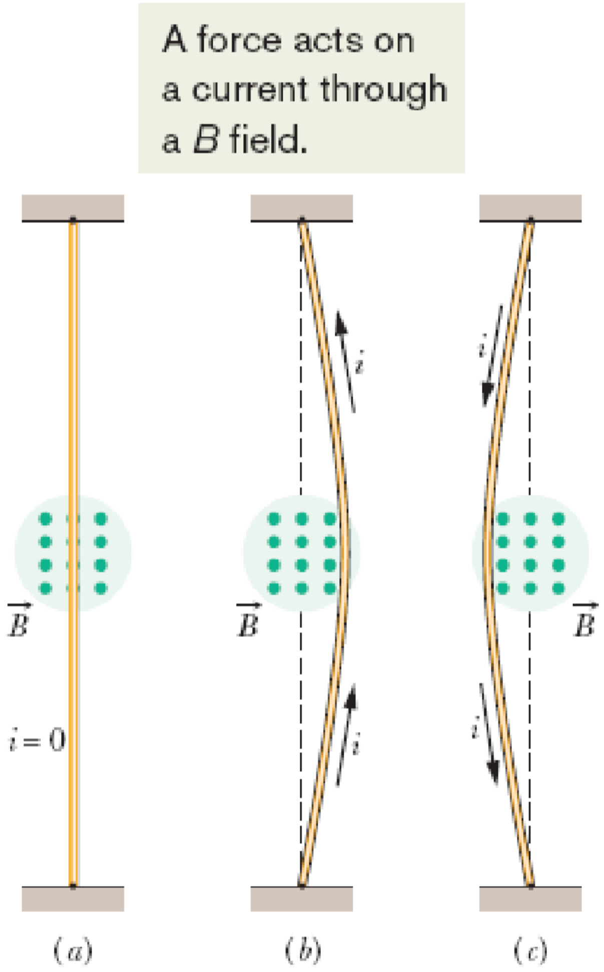
current in wire

The moving electrons in the wire is immersed in an external B-Field and feels a magnetic force given by the right hand rule as shown.

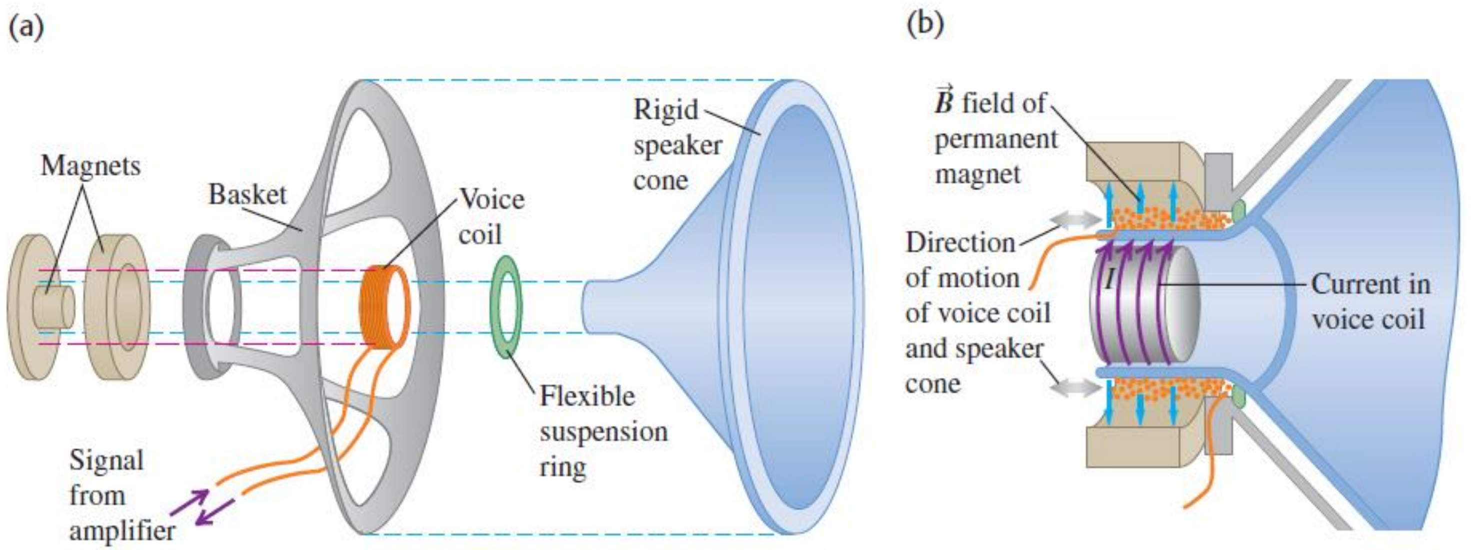




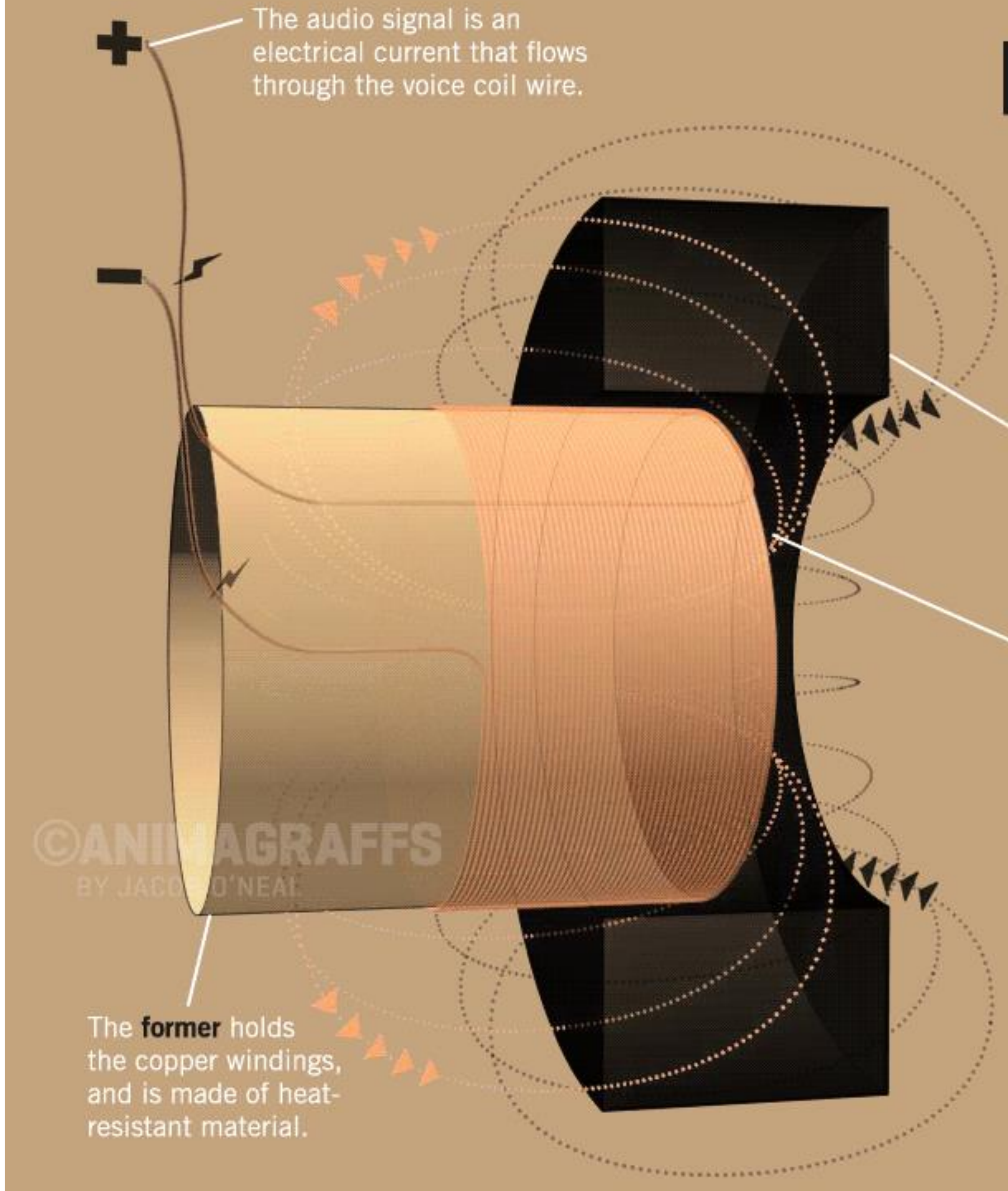
Magnetic Force on a Current-Carrying Wire



27.28 (a) Components of a loudspeaker. (b) The permanent magnet creates a magnetic field that exerts forces on the current in the voice coil; for a current I in the direction shown, the force is to the right. If the electric current in the voice coil oscillates, the speaker cone attached to the voice coil oscillates at the same frequency.



MAGNETS



Magnetic field flow direction and shape.

Electric current flow direction.

Permanent magnet

The voice coil is suspended inside a ring-shaped permanent magnet.

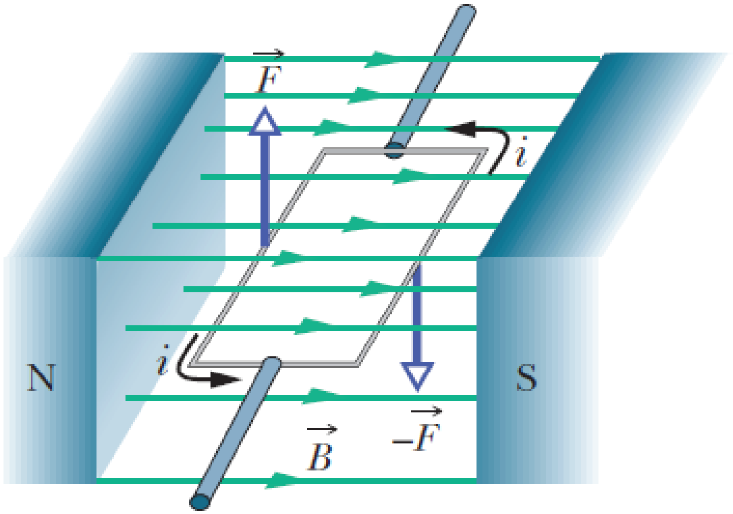
Voice coil = electromagnet

As electricity flows through the voice coil winding, it creates a magnetic field around the wire. A magnet made by electric current is called an **electromagnet**.

Moving the voice coil

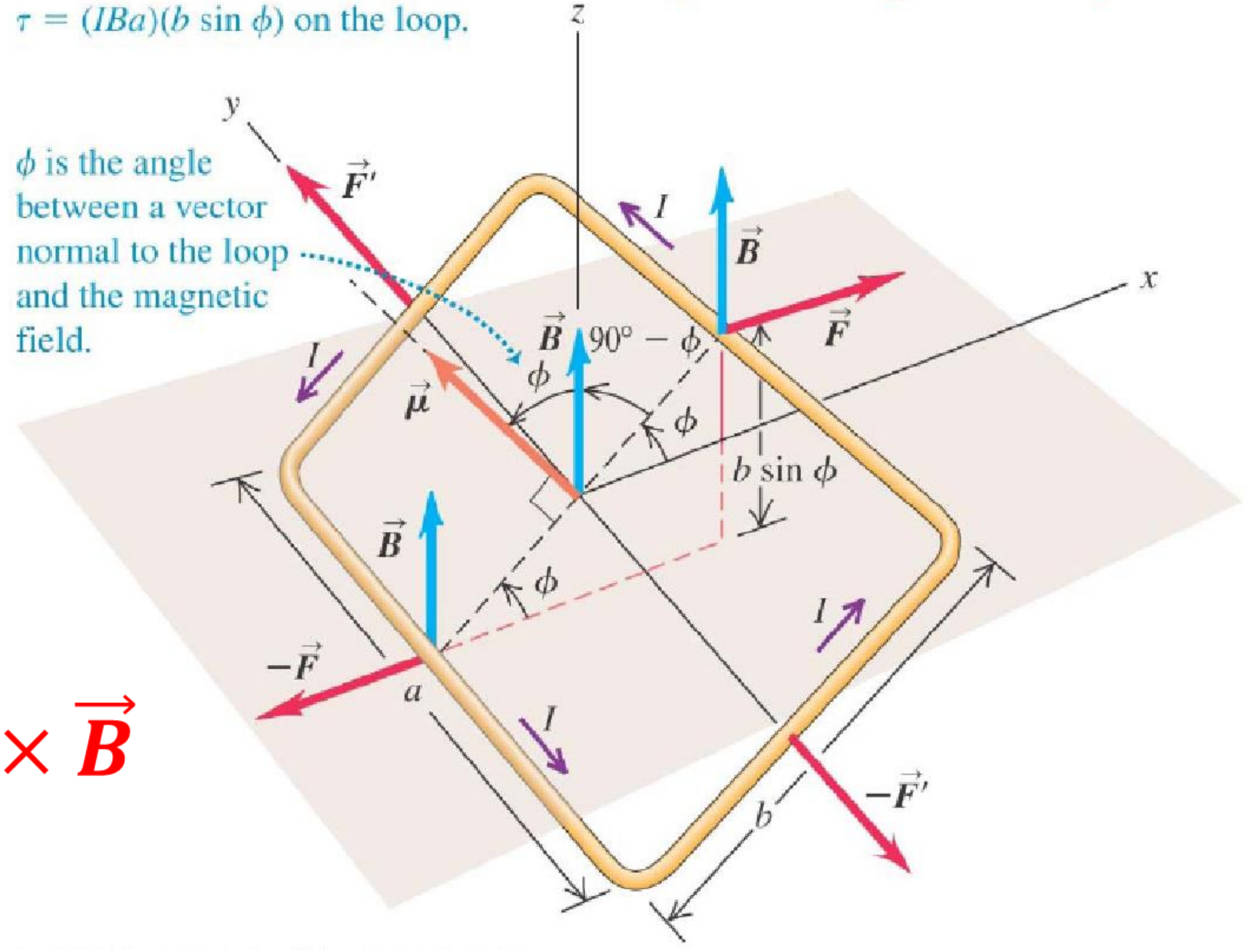
The magnetic field direction and intensity is controlled by altering electric current as it flows through the copper windings. The suspended voice coil is either repulsed or attracted to the permanent magnet in varying degrees.

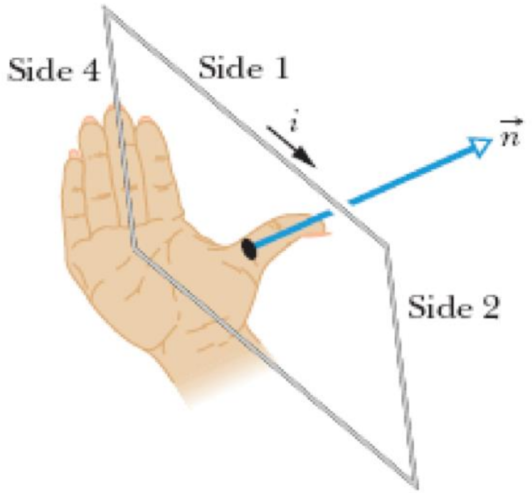
TORQUE ON A CURRENT LOOP



The two pairs of forces acting on the loop cancel, so no net force acts on the loop.

However, the forces on the a sides of the loop ($\vec{F}$ and $-\vec{F}$) produce a torque $\tau = (IBa)(b \sin \phi)$ on the loop.





$$\vec{\tau} = i\vec{A} \times \vec{B}$$

TORQUE ON A CURRENT LOOP

Let us examine the force on each segment of the loop in Figure (motor) to find the torques produced about the axis of the vertical shaft.

We take the magnetic field to be uniform over the rectangular loop, which has width *w* and height *l*.

$$\boldsymbol{\tau} = \frac{w}{2}\, \boldsymbol{F} \sin \boldsymbol{\theta}\, \frac{w}{2}\, \boldsymbol{F} \sin \boldsymbol{\theta} = \boldsymbol{wF} \sin \boldsymbol{\theta}$$

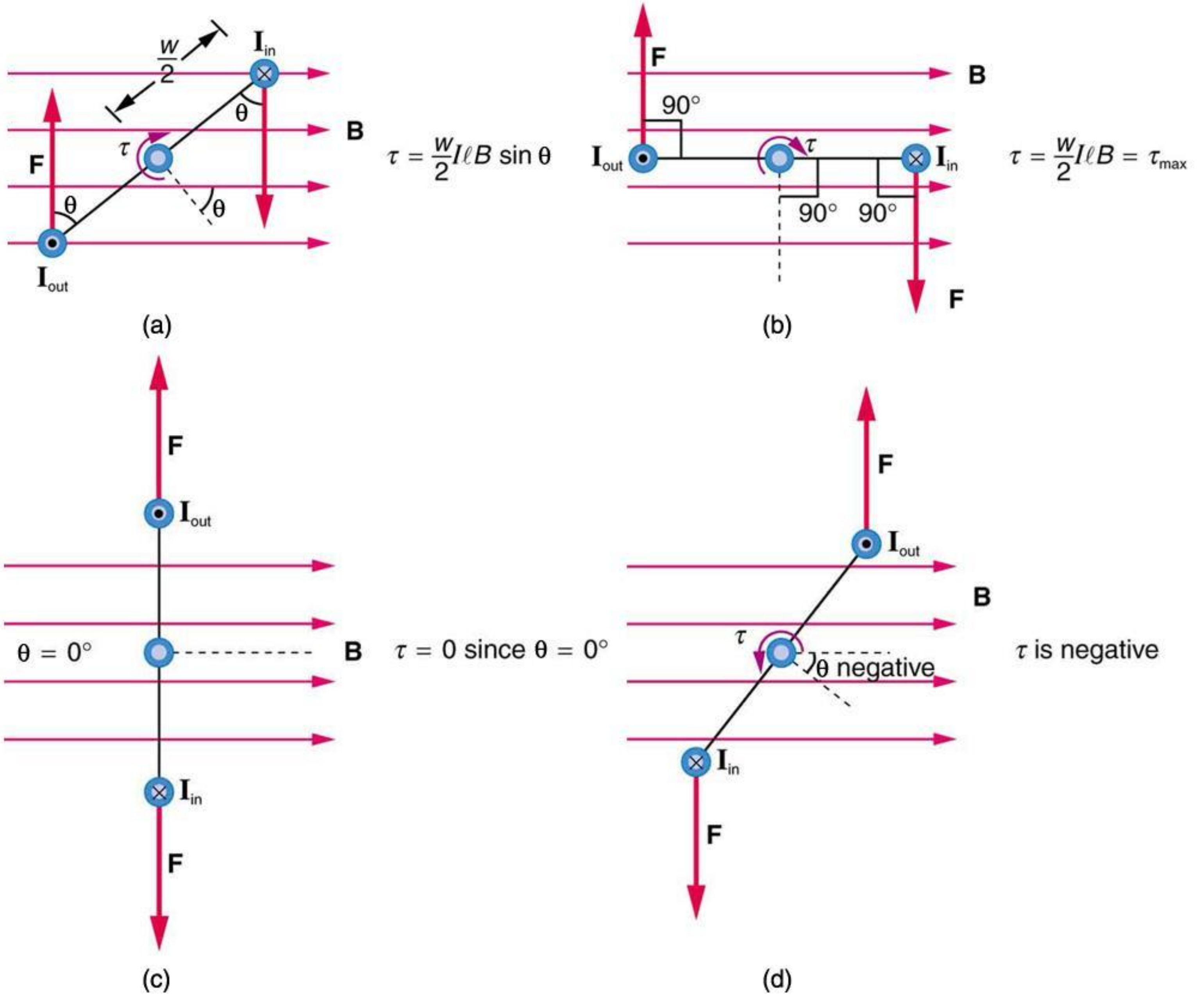
Now, each vertical segment has a length *l* that is perpendicular to *B*, so that the force on each is $F = ilB$. Entering F into the expression for torque yields

$$\boldsymbol{\tau} = \boldsymbol{wIlB} \sin \boldsymbol{\theta}.$$

Area, $A = wl$ and with N turns

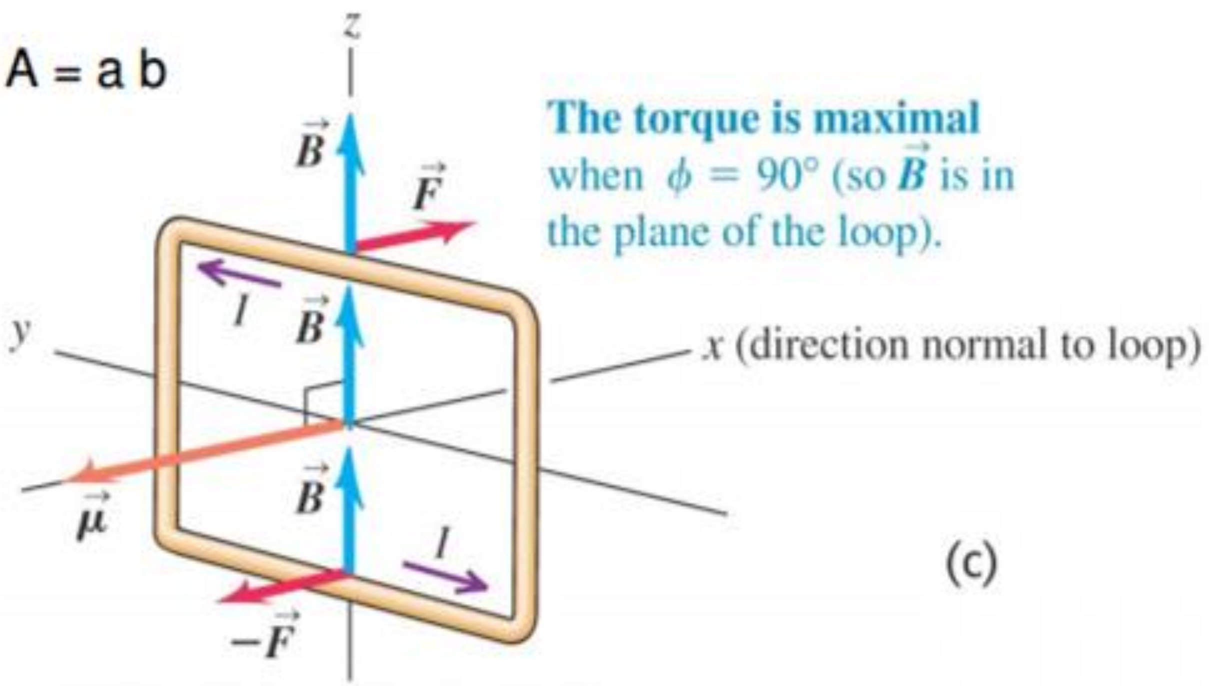
$$\boldsymbol{\tau} = \boldsymbol{NIAB} \sin \boldsymbol{\theta}.$$

$$\vec{\tau} = i\vec{A} \times \vec{B}$$

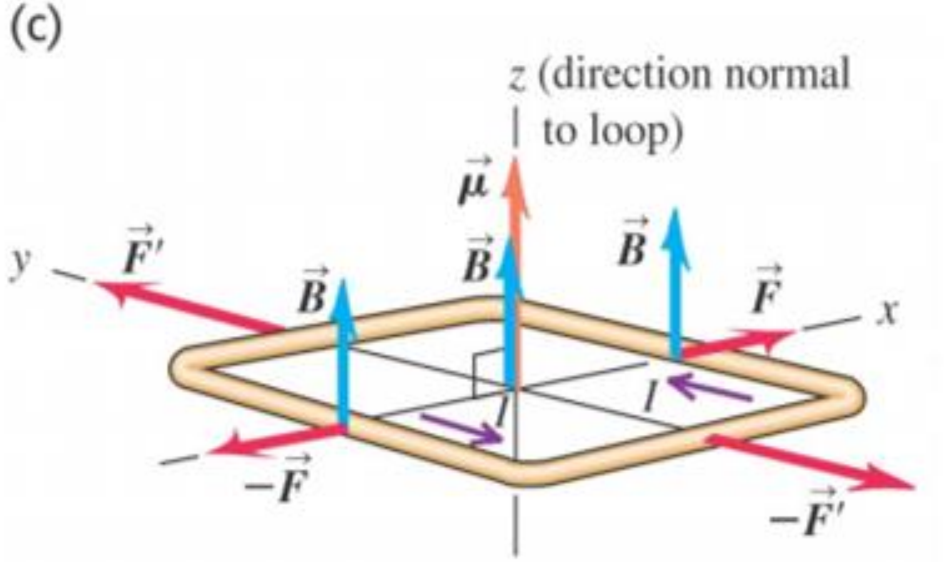


TORQUE ON A CURRENT LOOP

A = a b



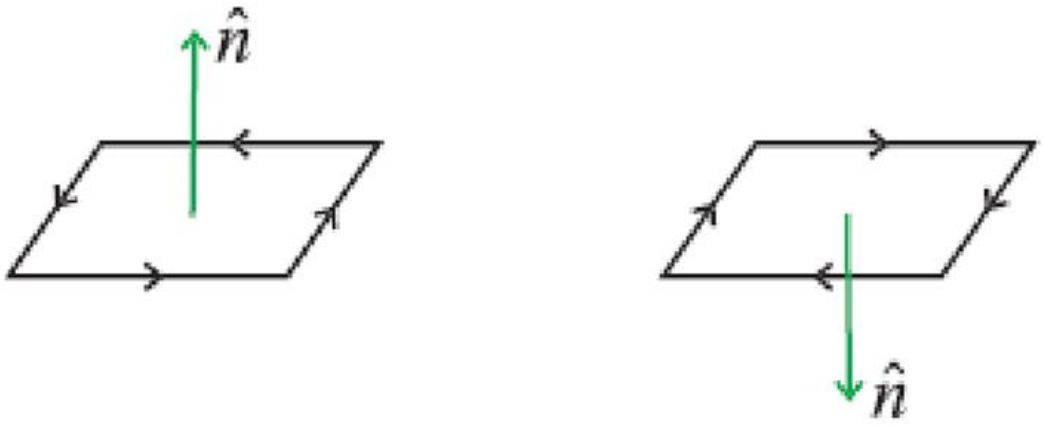
Torque is zero, φ = 0º

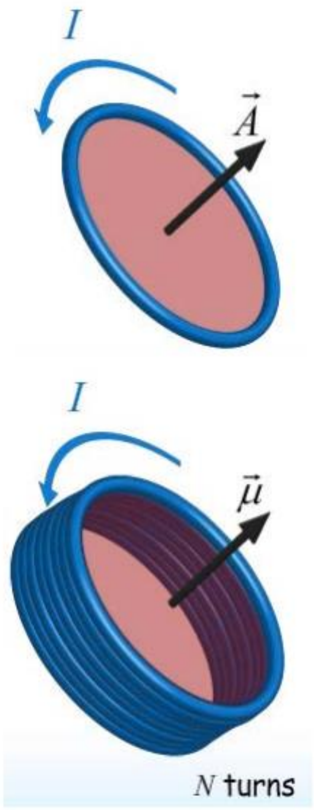


φ is angle between a vector perpendicular to loop and $\vec{B}$

TORQUE ON A CURRENT LOOP

Unit vector $\hat{n}$ represents the area vector direction use right hand rule





We can define an area vector as having its magnitude equal to the loop area, and a direction perpendicular to the loop (and in a sense given by the right-hand rule and the current direction). Thus, a vector form of the torque is:

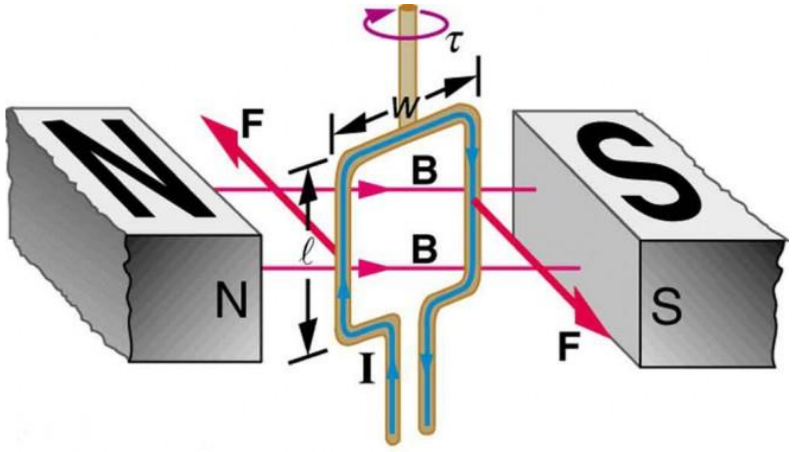
$$\vec{\tau} = i\vec{A} \times \vec{B}$$

If there are N windings of the wire around the loop, the formula becomes

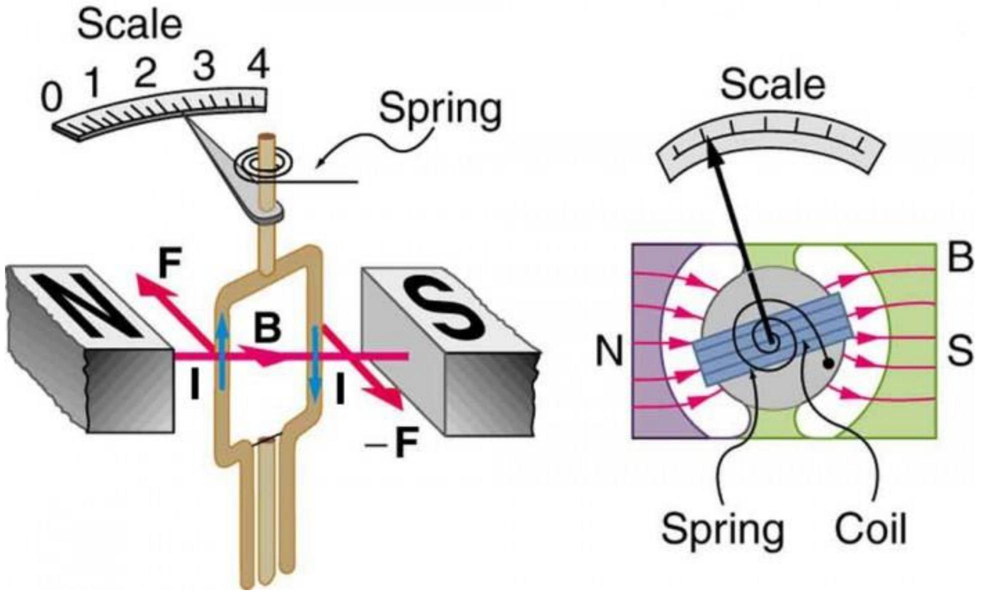
$$\vec{\tau} = iN\vec{A} \times \vec{B}$$

Find the maximum torque on a 100-turn square loop of a wire of 10.0 cm on a side that carries 15.0 A of current in a 2.00-T field.

TORQUE ON A CURRENT LOOP



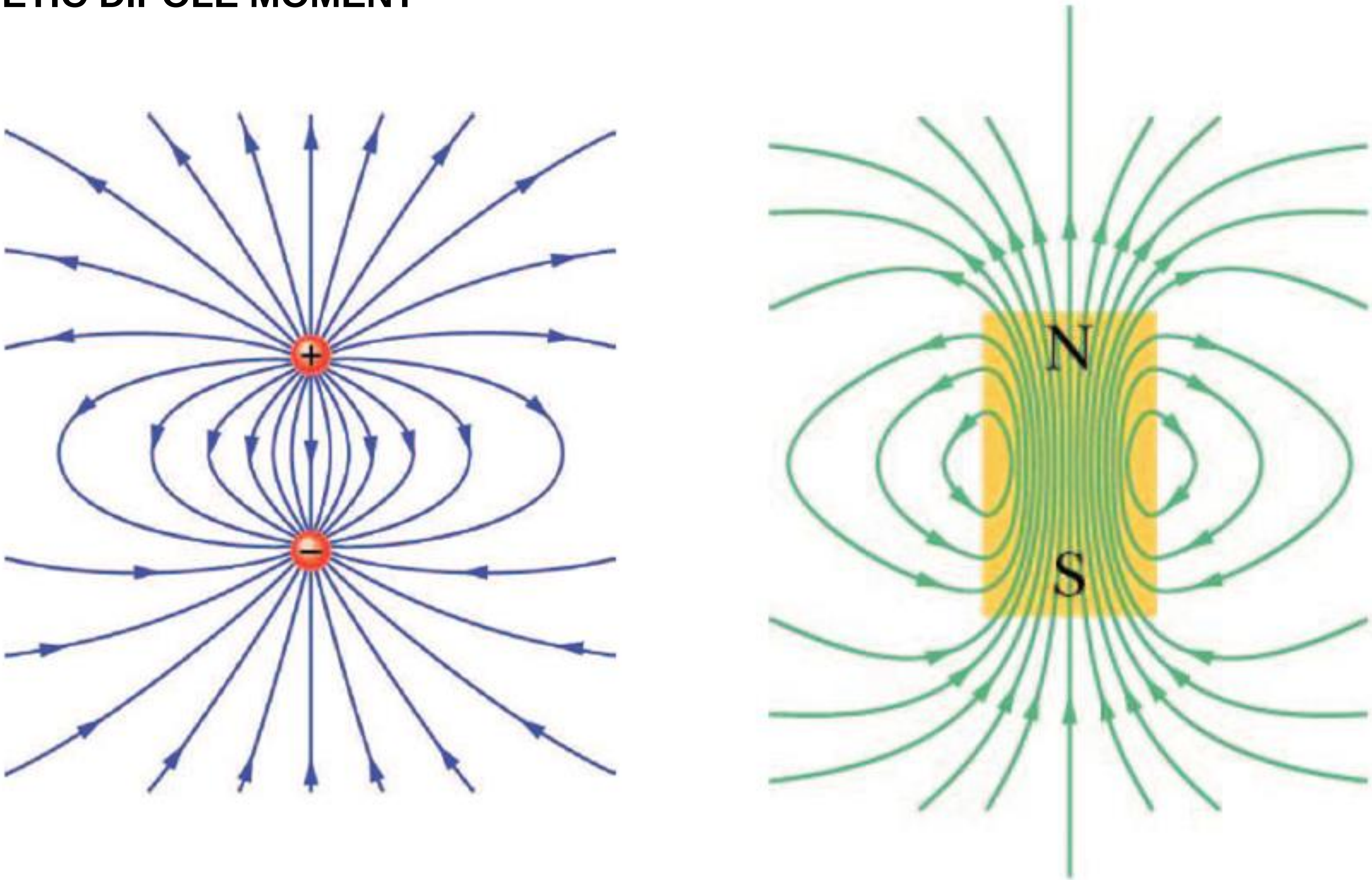
Motors are the most common application of magnetic force on current-carrying wires. Motors have loops of wire in a magnetic field. When current is passed through the loops, the magnetic field exerts torque on the loops, which rotates a shaft. Electrical energy is converted to mechanical work in the process.



Meters, such as those in analog fuel gauges on a car, are another common application of magnetic torque on a current-carrying loop.

https://physics.bu.edu/~duffy/semester2/c13_motor.html

THE MAGNETIC DIPOLE MOMENT



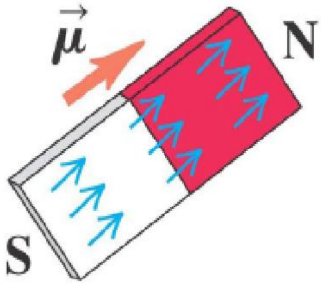
Note: Isolated magnetic monopoles do not exist.

THE MAGNETIC DIPOLE MOMENT

The magnetic dipole moment of an object is a measure of its tendency to align with a magnetic field. It is a vector quantity. The direction of the magnetic dipole moment points from the south pole to the north pole of the object.

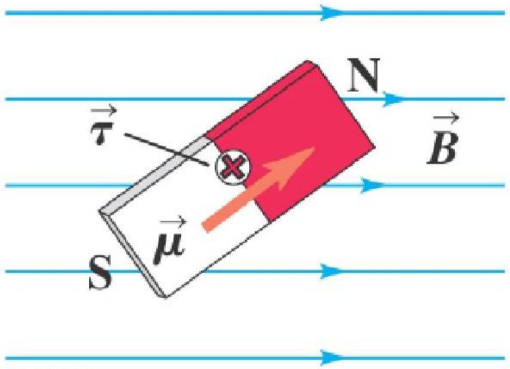
Magnets have magnetic moment:

(b) In a bar magnet, the magnetic moments are aligned.



Magnetic moment wants to be aligned with B.

(c) A magnetic field creates a torque on the bar magnet that tends to align its dipole moment with the $\vec{B}$ field.

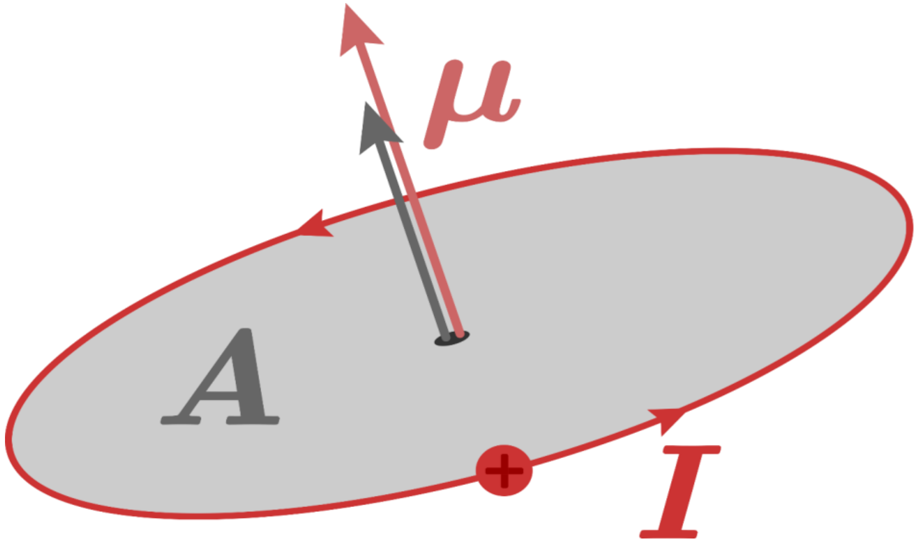


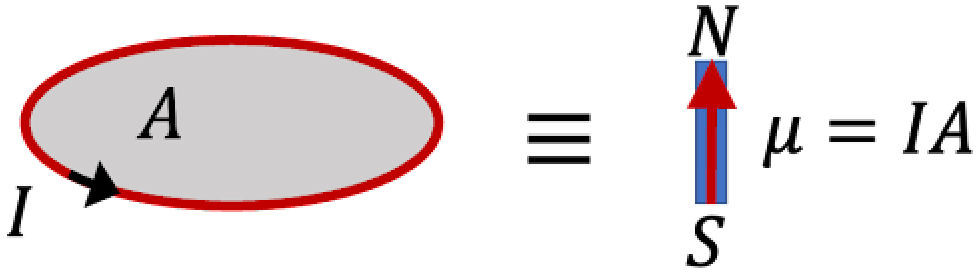
Torque

$$\vec{\tau} = \vec{\mu} \times \vec{B}$$

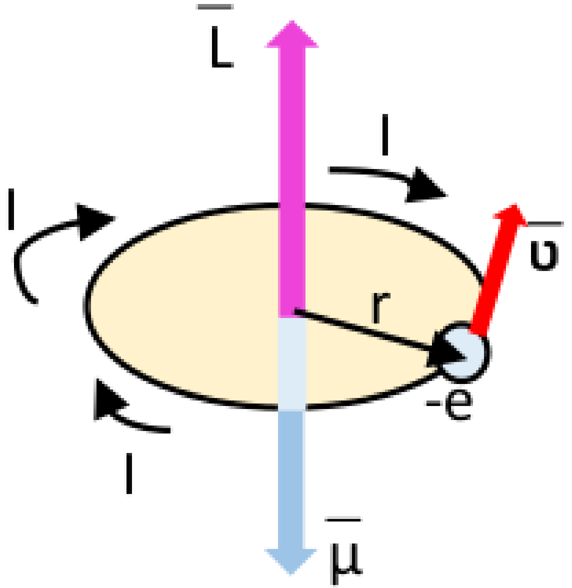
THE MAGNETIC DIPOLE MOMENT

Magnetic dipole moments can be created by either electric currents or by the spin of elementary particles. For example, a current loop acts as a magnetic dipole, with the magnetic dipole moment proportional to the current and the area of the loop.



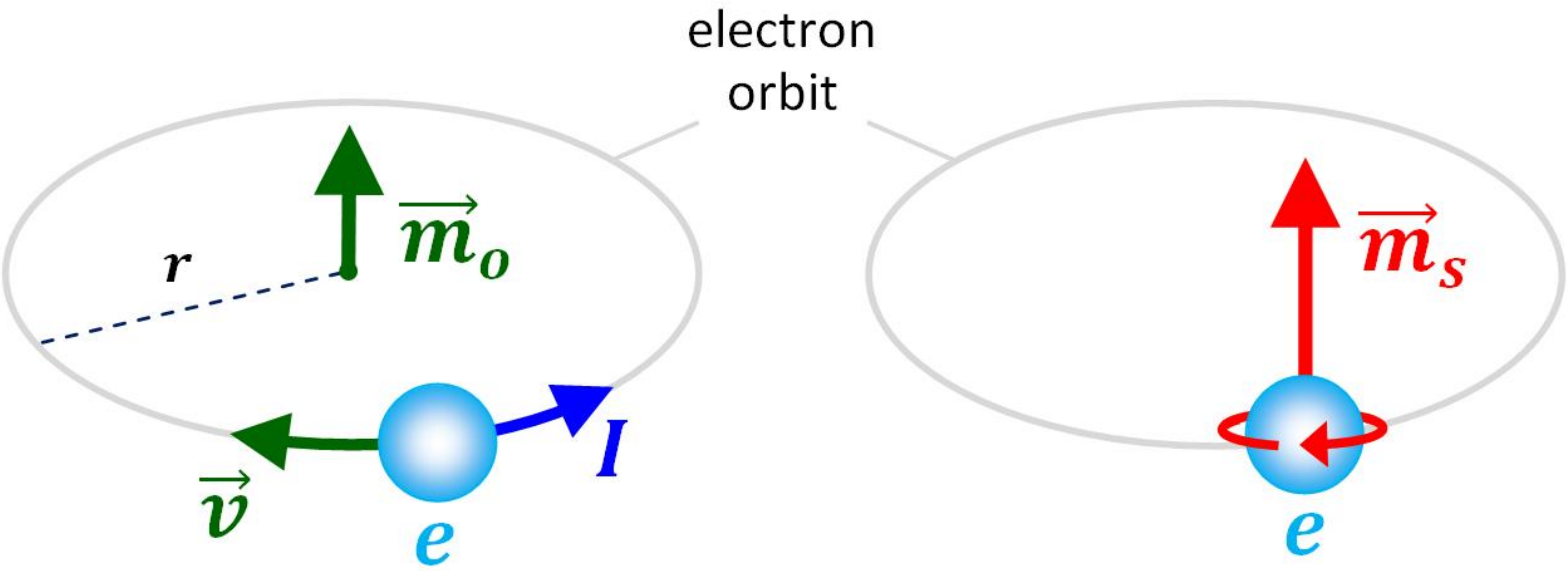


Elementary particles such as electrons and protons also have magnetic dipole moments due to their spin.



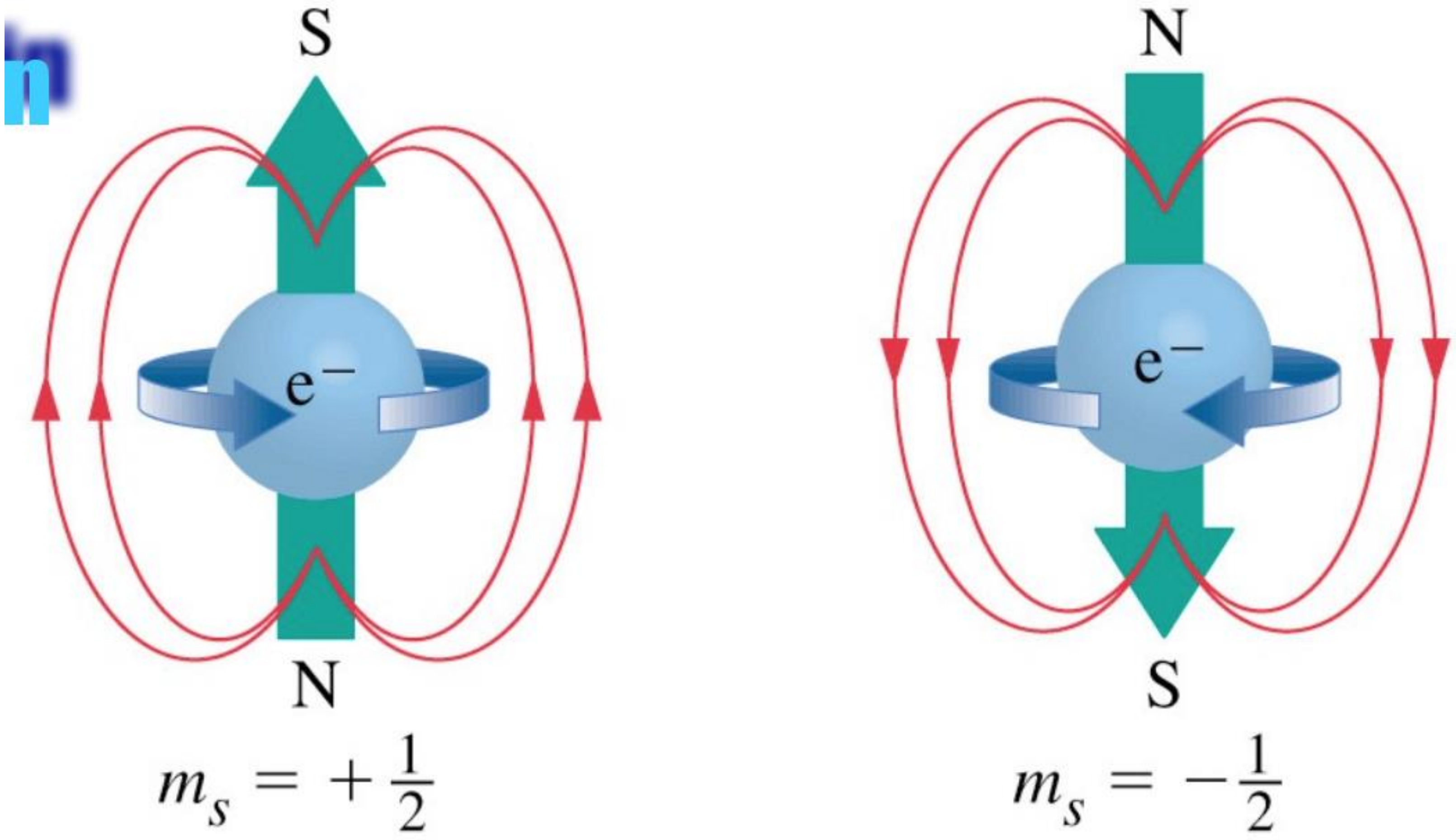
THE MAGNETIC DIPOLE MOMENT

Magnetic dipole moments can be created by either electric currents or by the spin of elementary particles. For example, a current loop acts as a magnetic dipole, with the magnetic dipole moment proportional to the current and the area of the loop.



Elementary particles such as electrons and protons also have magnetic dipole moments due to their spin.

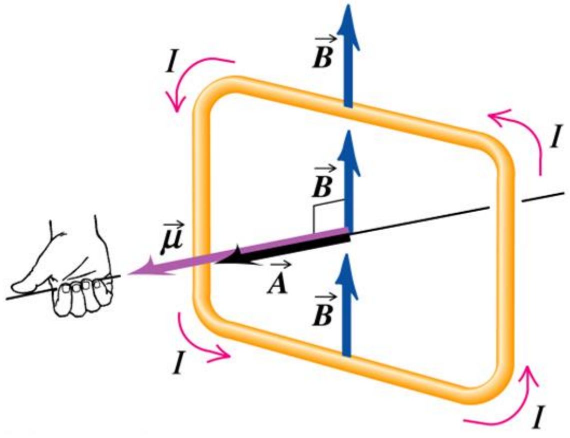
THE MAGNETIC DIPOLE MOMENT

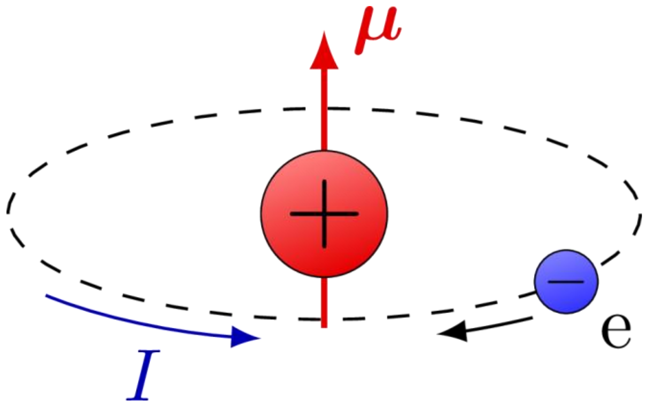


Elementary particles such as electrons and protons also have magnetic dipole moments due to their spin.

THE MAGNETIC DIPOLE MOMENT

The right-hand rule determines the direction of the magnetic moment of a current-carrying loop. This is also the direction of the loop's area vector $\vec{A}$; $\vec{\mu} = I\vec{A}$ is a vector equation.



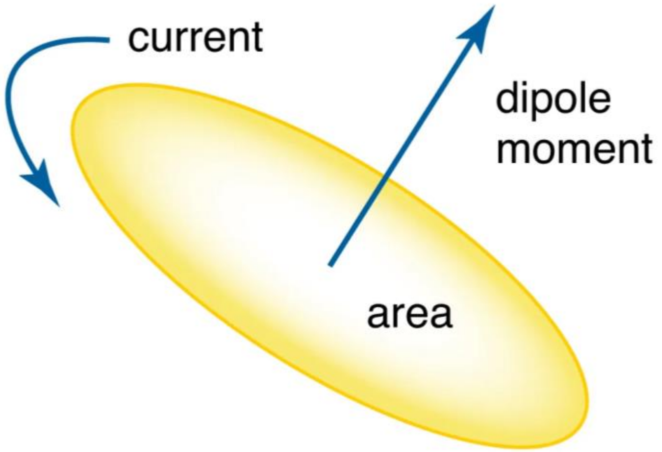


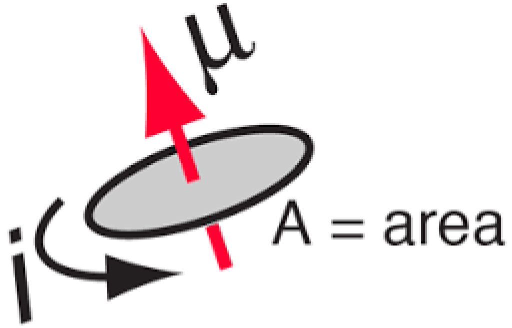
Dipole Moment

$$\vec{\mu} = i\vec{A}$$

$$\boxed{\boldsymbol{\tau} = \boldsymbol{\mu} \times \mathbf{B}}$$

This formula holds for any current loop shape.

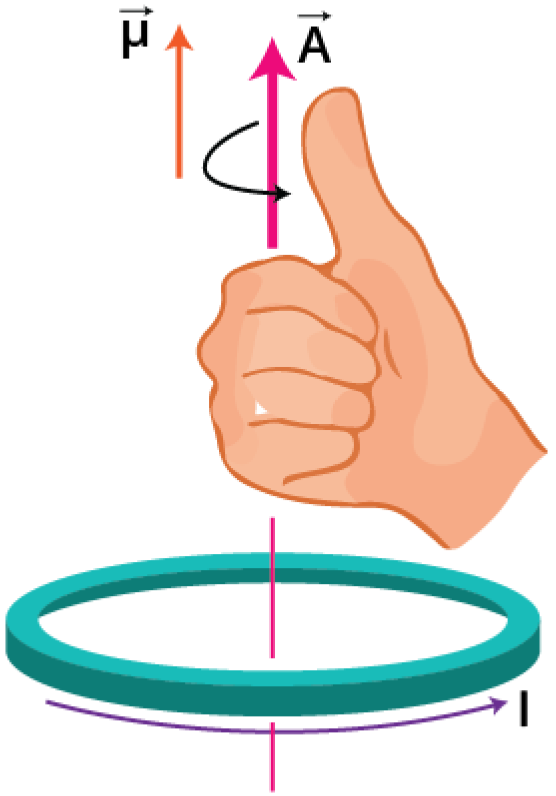


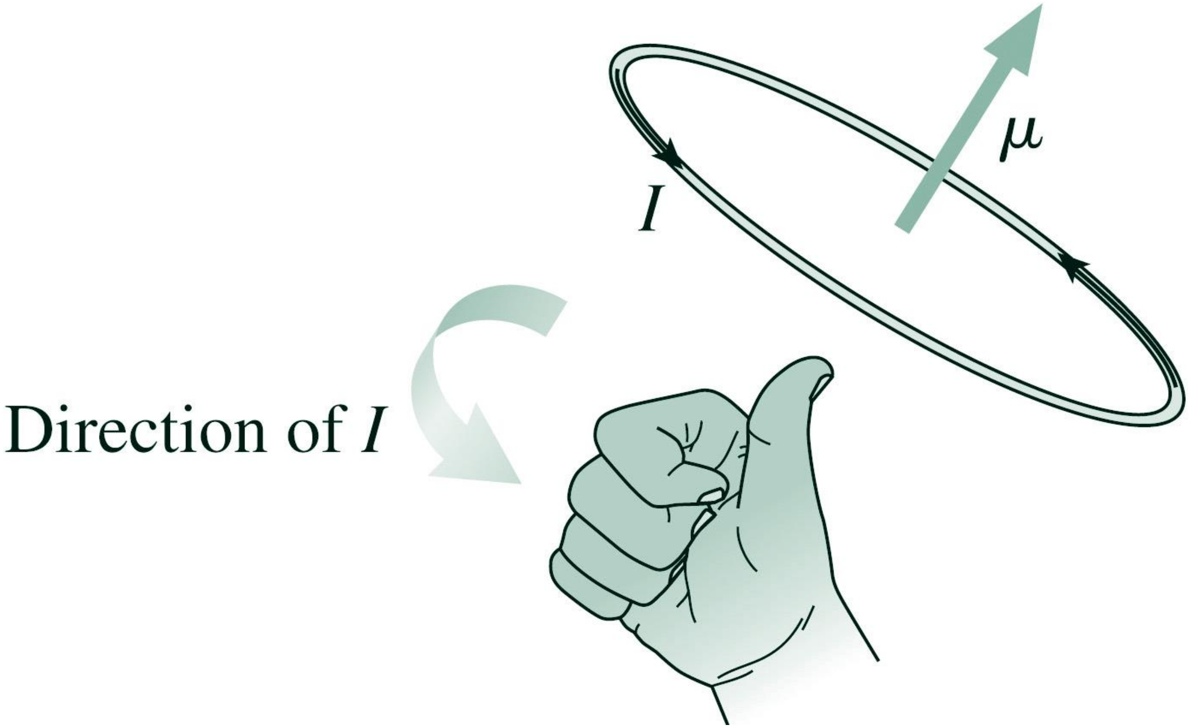


THE MAGNETIC DIPOLE MOMENT

Dipole Moment

$$\vec{\mu} = i\vec{A}$$



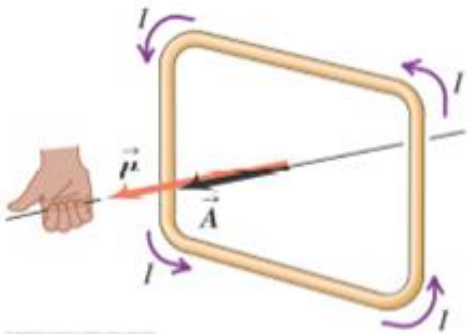


THE MAGNETIC DIPOLE MOMENT

$$\tau_{total} = IBA\sin\varphi$$

Magnetic dipole moment: $\boxed{\vec{\mu} = I\vec{A}}$

Direction: perpendicular to plane of loop (direction of loop's vector area → right hand rule)



$$\tau_{total} = \mu B\sin\varphi$$

Magnetic torque: $\boxed{\vec{\tau} = \vec{\mu}\times\vec{B}}$

Potential Energy for a Magnetic Dipole:

$$\boxed{U = -\vec{\mu}\cdot\vec{B} = -\mu B\cos\varphi}$$

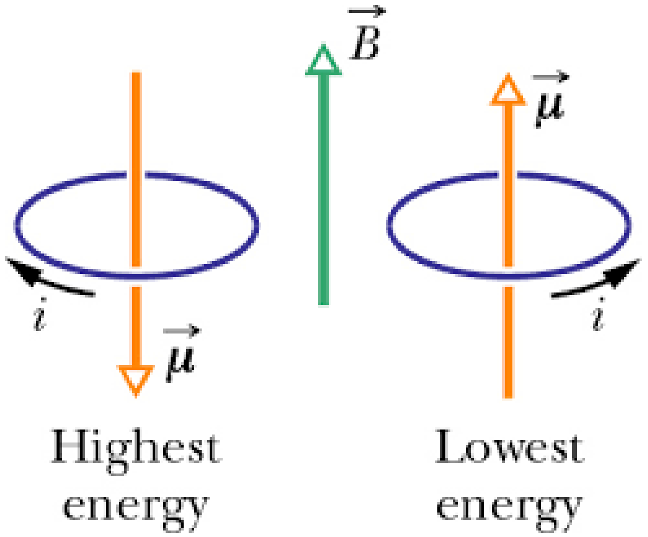
Electric dipole moment: $\boxed{\vec{p} = q\vec{d}}$

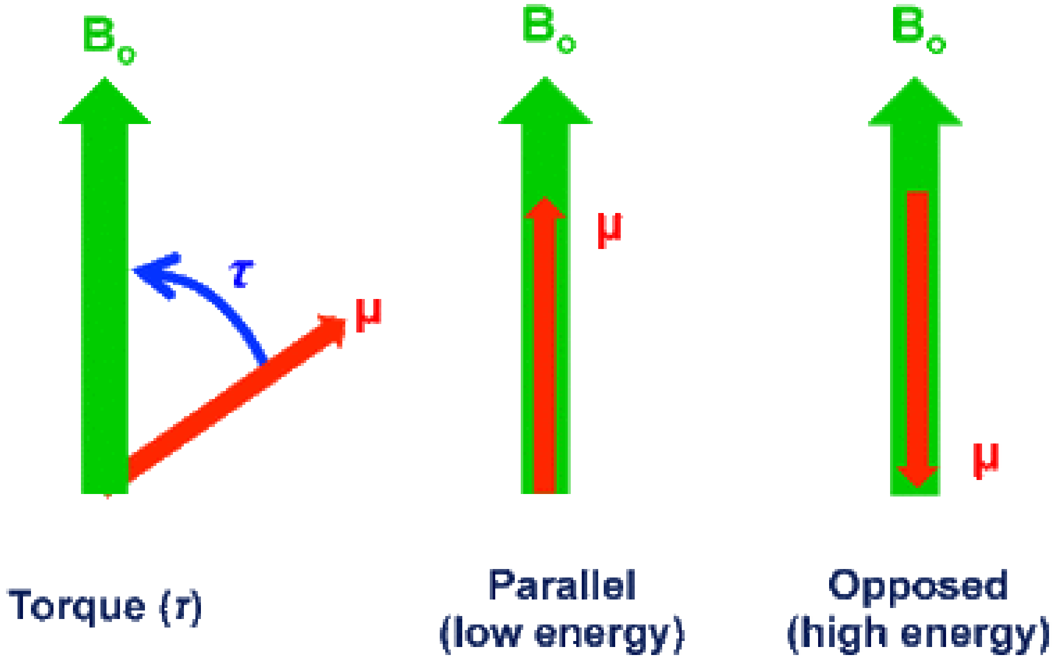
Electric torque: $\boxed{\vec{\tau} = \vec{p}\times\vec{E}}$

Potential Energy for an Electric Dipole:

$$\boxed{U = -\vec{p}\cdot\vec{E}}$$

THE MAGNETIC DIPOLE MOMENT





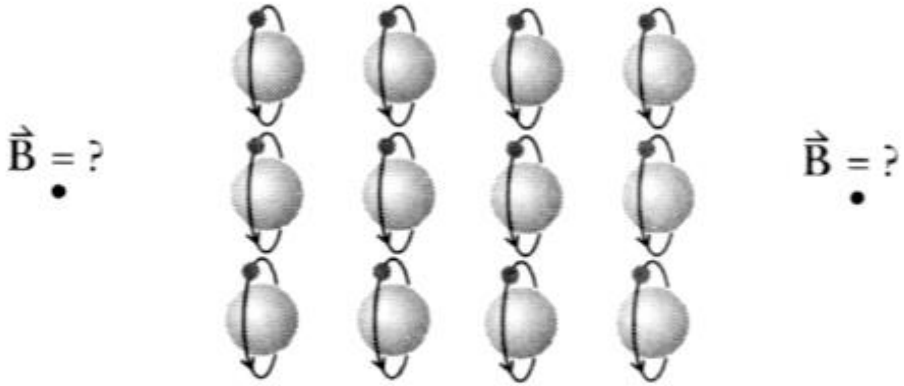
Magnetic dipole has lowest energy when $\vec{\mu}$ and $\vec{B}$ are in the same direction

Magnetic dipole has highest energy when $\vec{\mu}$ and $\vec{B}$ are in the opposite direction

Work done due to change in energy, $W_a = U_f - U_i$

THE MAGNETIC DIPOLE MOMENT of a magnet

Magnetic dipole moment of 1 atom: $\mu \approx 10^{-23} \quad \mathrm{A \cdot m^2/atom}$



Mass of a magnet: m~5g

$6{\cdot}10^{23}$ atoms

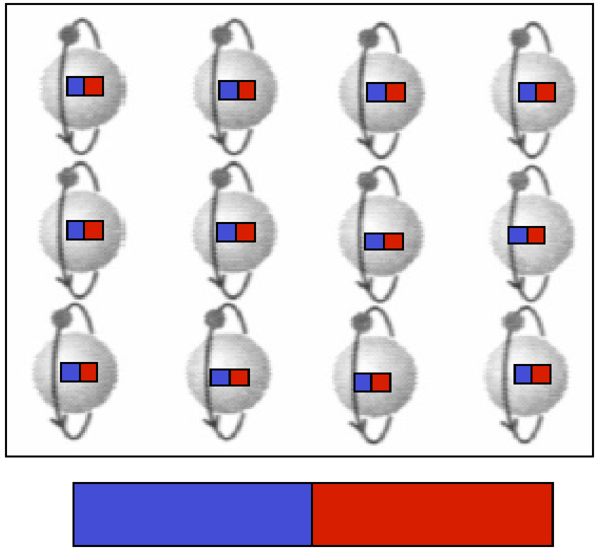
Assume magnet is made of iron: 1 mole – 56 g

number of atoms = $5g/56g \cdot 6{\cdot}10^{23} \sim 6{\cdot}10^{22}$

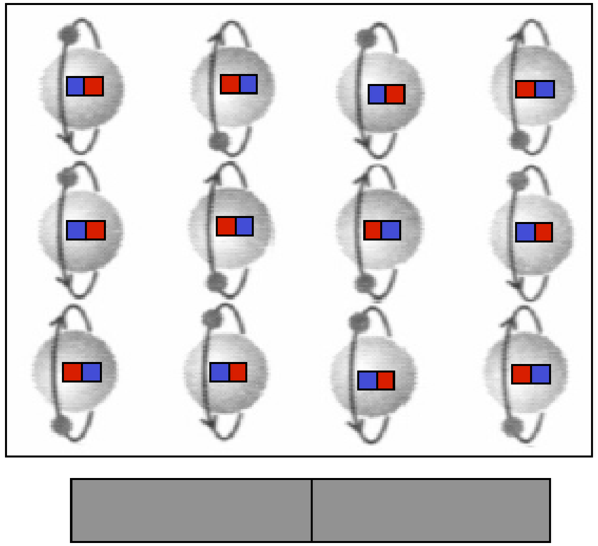
$$\mu_{magnet} \approx 6 \times 10^{22} \cdot 10^{-23} = 0.6 \quad \mathrm{A \cdot m^2}$$

THE MAGNETIC DIPOLE MOMENT of a magnet

Alignment of atomic dipole moments:

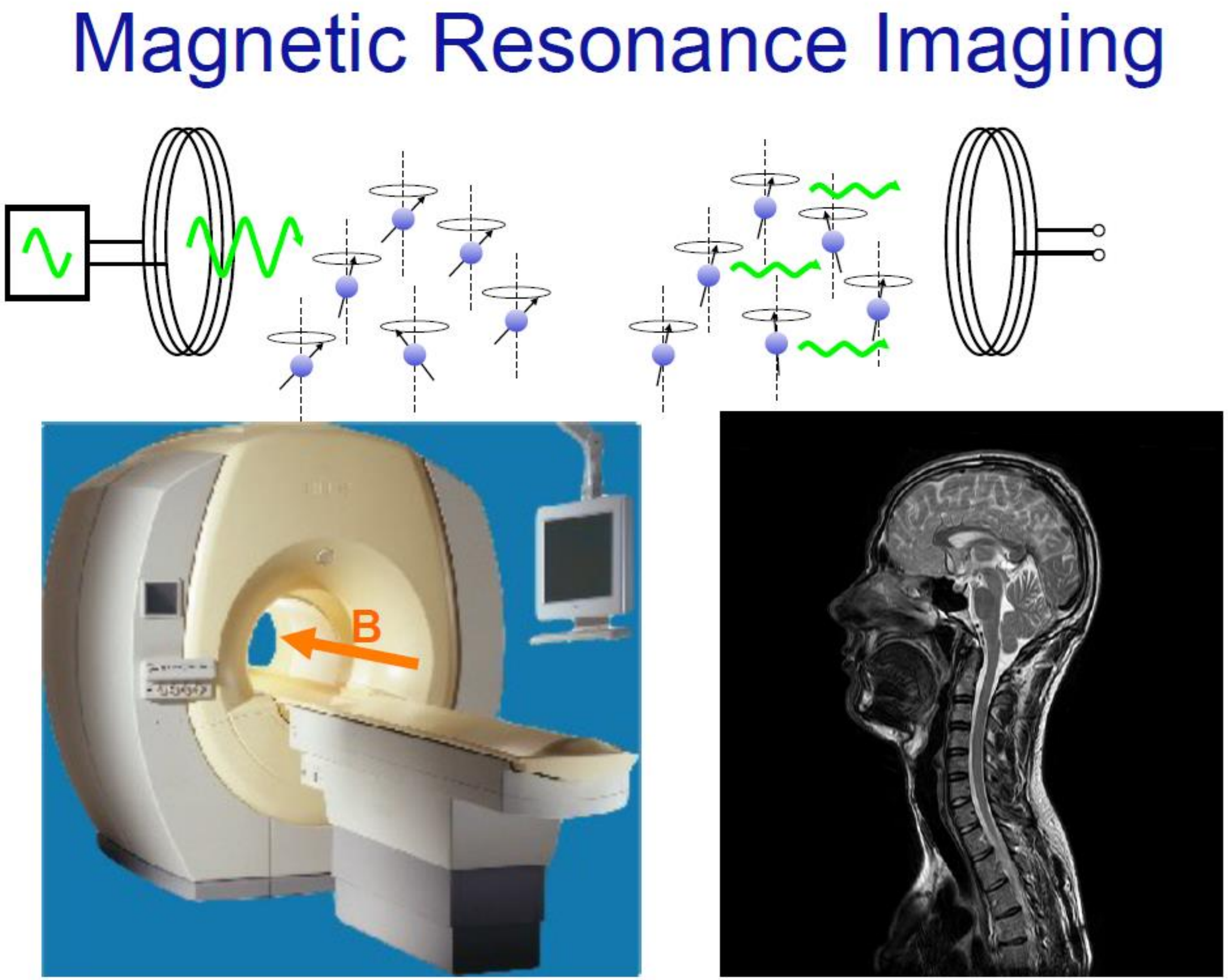


ferromagnetic materials:
iron, cobalt, nickel



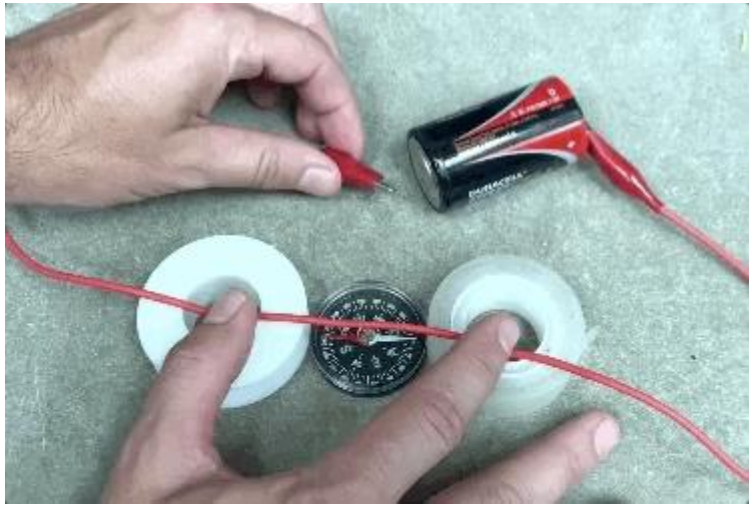
most materials

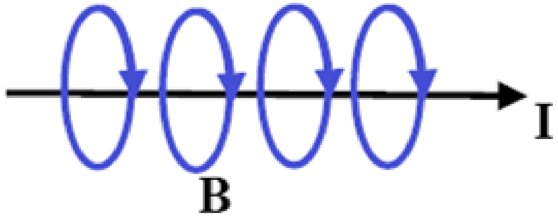
THE MAGNETIC DIPOLE MOMENT

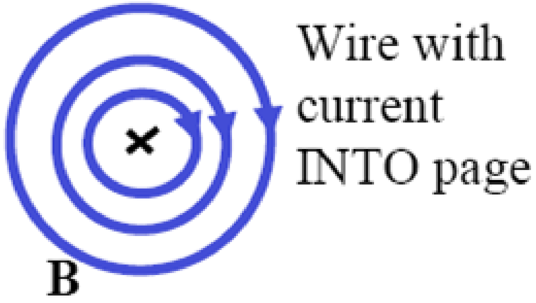


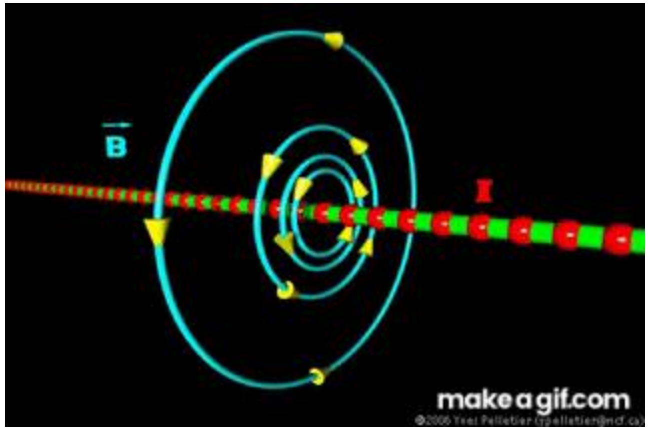
Electric Current: A Source of Magnetic Field

- **Observation: an electric current creates a magnetic field**
- **Simple experiment: hold a current-carrying wire near a compass needle!**

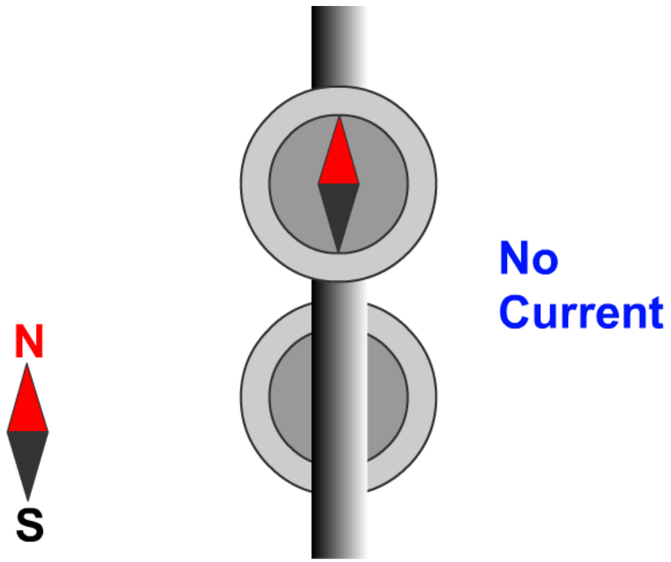


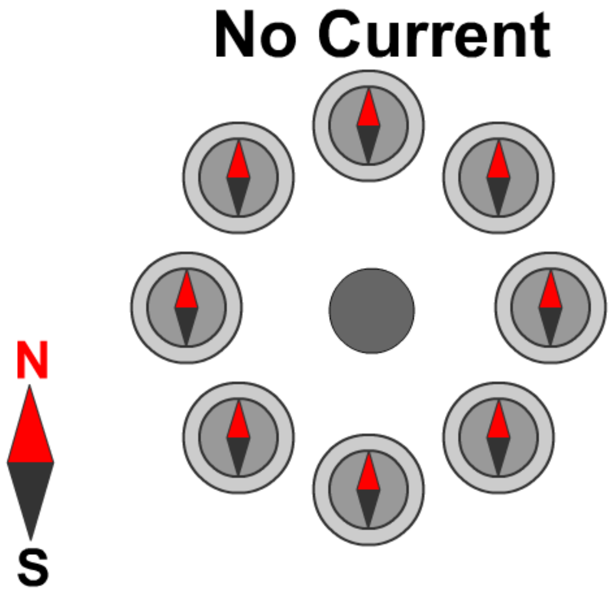




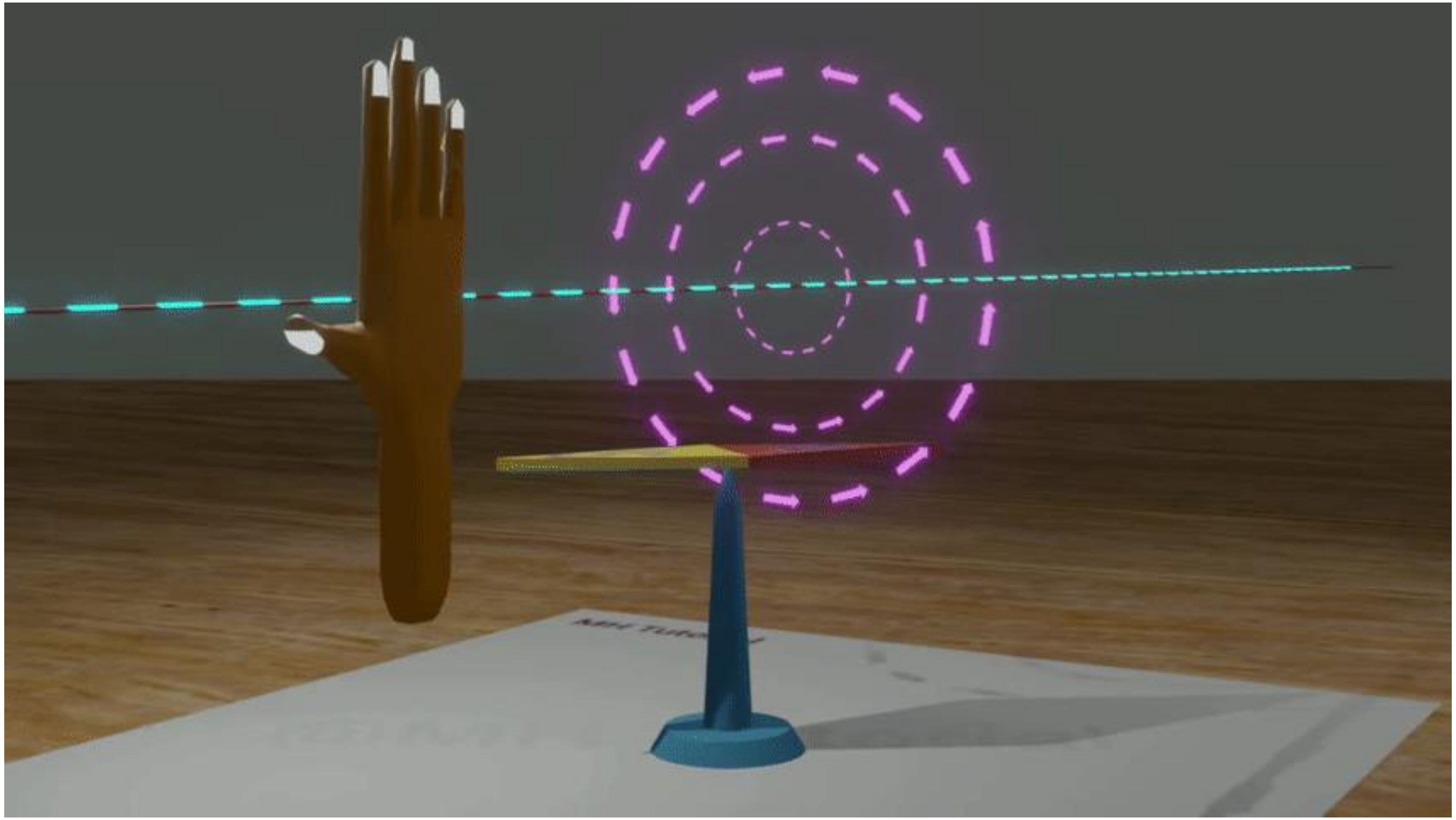


MAGNETIC FIELD DUE TO A CURRENT



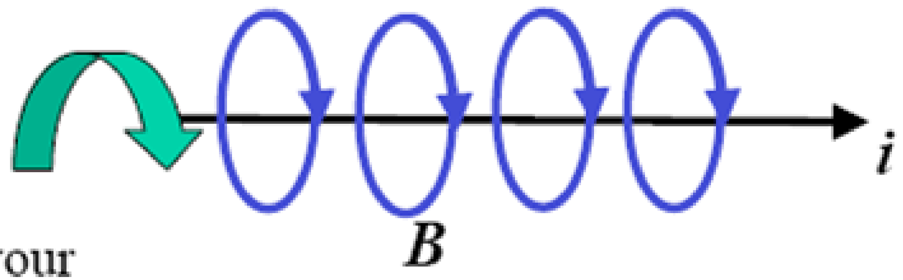


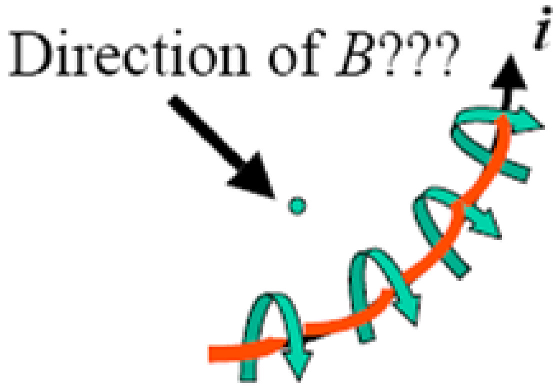
MAGNETIC FIELD DUE TO A CURRENT



Yet Another Right Hand Rule!

- Point your thumb along the direction of the current in a straight wire
- The magnetic field created by the current consists of circular loops directed along your curled fingers.
- The magnetic field gets weaker with distance.
- You can apply this to ANY straight wire (even a small differential element!)
- What if you have a curved wire? Break into small elements.





MAGNETIC FIELD DUE TO A CURRENT

A moving charge → Experiences magnetic force in magnetic field

Can generate magnetic field

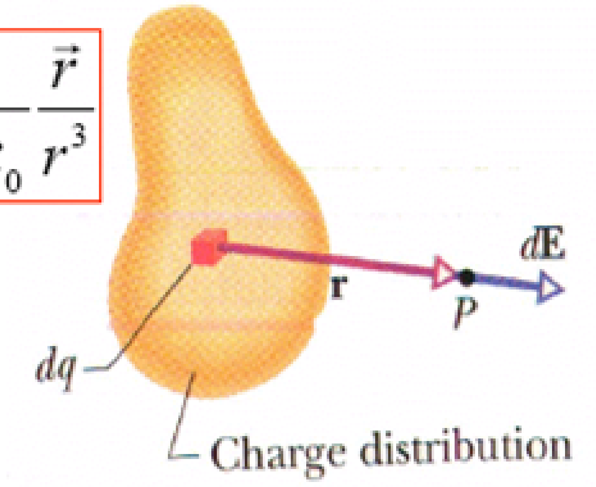
- A single moving charge will NOT generate a steady magnetic field
- Stationary charges generate steady electric field, E
- Steady currents generate steady magnetic field, B

When we computed the electric field due to charges we used Coulomb's law. If one had a large irregular object, one broke it into infinitesimal pieces and computed,

$$d\vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{dq}{r^2}\hat{r}$$

Which we write as,

$$\boxed{d\vec{E} = \frac{dq}{4\pi\varepsilon_0}\frac{\vec{r}}{r^3}}$$



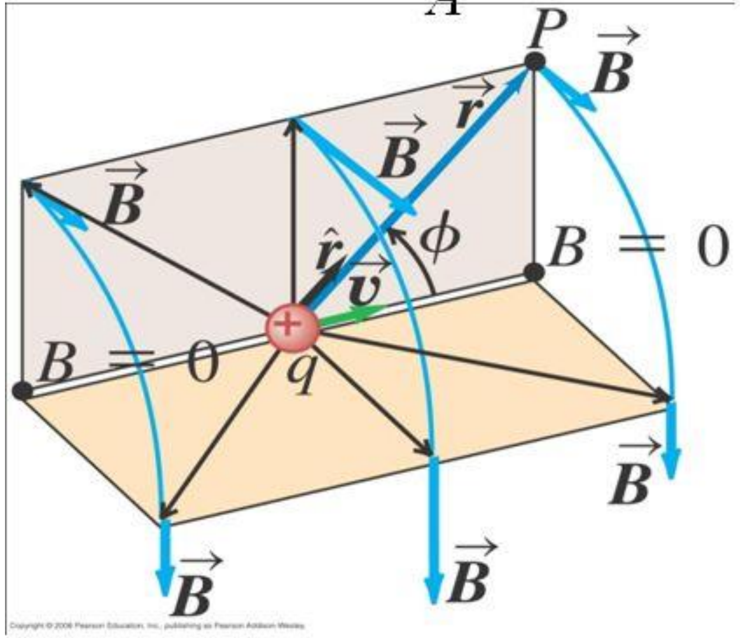
If you wish to compute the magnetic field due to a current in a wire, you use the law of Biot and Savart.

B of a moving charge

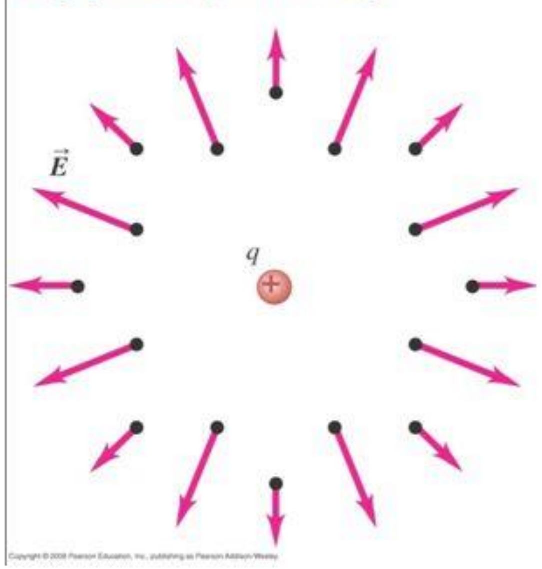
$$\vec{B} = \frac{\mu_0}{4\pi} \frac{q\vec{v} \times \vec{r}_0}{r^2}$$

$$\vec{E} = \frac{1}{4\pi\varepsilon_0} \frac{q}{r^2} \vec{r}_0$$

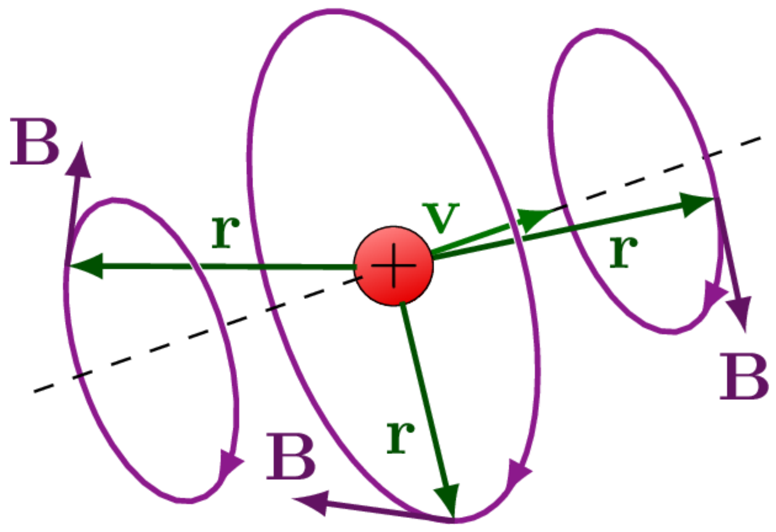
$$\mu_0 = 4\pi \times 10^{-7} \frac{Tm}{A}$$



(a) The field produced by a positive point charge points *away from* the charge.



Magnetic field of a moving point charge



Magnetic field $\vec{B}$ due to moving point charge:

$$\vec{B} = \frac{\mu_0}{4\pi} \cdot \frac{q\vec{v} \times \hat{r}}{r^2} = \frac{\mu_0}{4\pi} \cdot \frac{q\vec{v} \times \vec{r}}{r^3}$$

where $\quad \mu_0 = 4\pi \times 10^{-7}\ \mathrm{Tm/A}\ (\mathrm{N/A^2})$

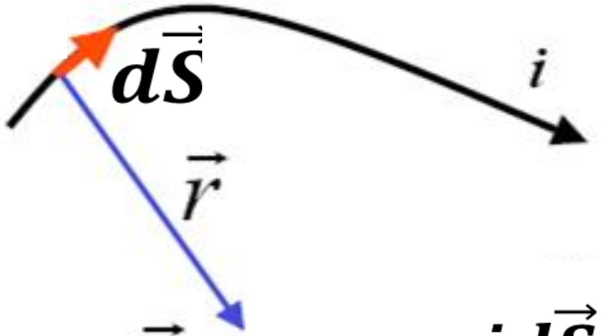
$$|\vec{B}| = \frac{\mu_0}{4\pi} \cdot \frac{qv\sin\theta}{r^2} \quad \begin{cases} \mathit{maximum} & \text{when } \theta = 90^\circ \\ \mathit{minimum} & \text{when } \theta = 0^\circ/180^\circ \end{cases}$$

Jean-Baptiste
Biot (1774-1862)

The Biot-Savart Law

Felix Savart
(1791-1841)

- Quantitative rule for computing the magnetic field from any electric current
- Choose a differential element of wire of length ***dS*** and carrying a current *i*
- The field ***dB*** from this element at a point located by the vector *r* is given by the Biot-Savart Law



$$d\vec{B} = \frac{\mu_0}{4\pi}\frac{i d\vec{S} \times \vec{r}}{r^3}$$

$\mu_0 = 4\pi x 10^{-7}$ Tm/A
(permeability constant)

Compare with $d\vec{E} = \dfrac{dq}{4\pi\varepsilon_0}\dfrac{\vec{r}}{r^3}$

Magnetic field at point P can be obtained by integrating the contribution from individual current segments (principle of superposition)

$$\vec{dB} = \frac{\mu_0}{4\pi}\frac{dq\vec{v} \times \hat{r}}{r^2}$$

Now, $dq\vec{v} = dq.\frac{d\vec{s}}{dt} = i\,d\vec{s}$

$$\vec{dB} = \frac{\mu_0}{4\pi}\frac{i\vec{dS} \times \hat{r}}{r^2}$$

$$\vec{dB} = \frac{\mu_0}{4\pi}\frac{i\vec{dS} \times \vec{r}}{r^3}$$

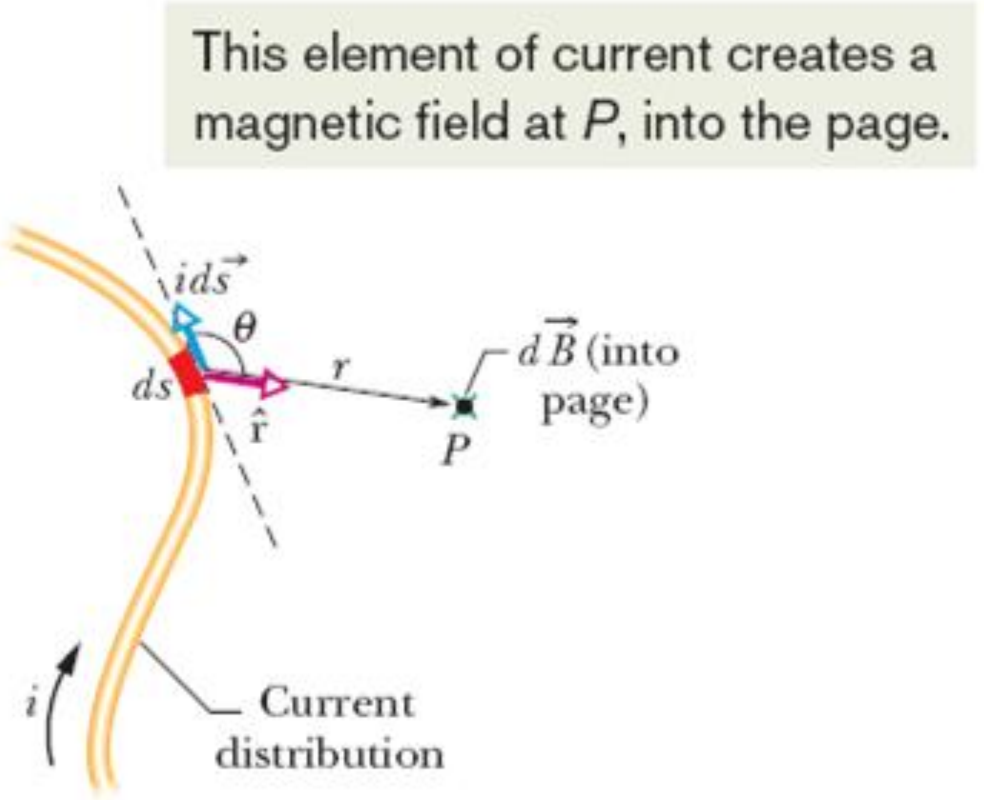
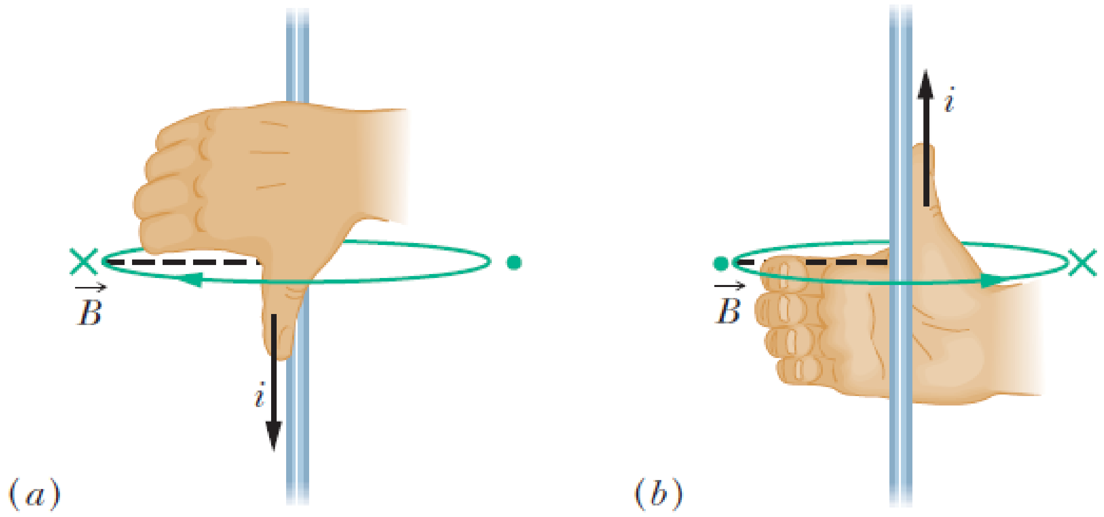
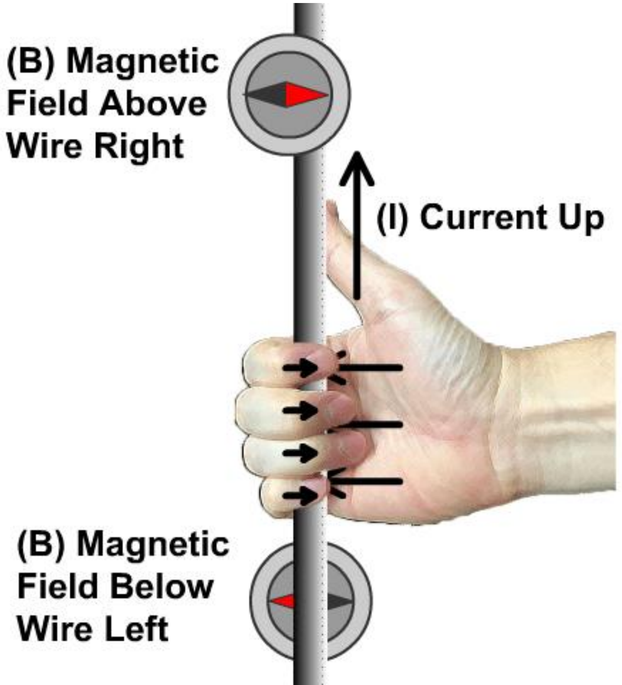


Figure 29-1 A current-length element $i\,d\vec{s}$ produces a differential magnetic field $d\vec{B}$ at point P. The green × (the tail of an arrow) at the dot for point P indicates that $d\vec{B}$ is directed *into* the page there.

Magnetic field due to straight current carrying wire



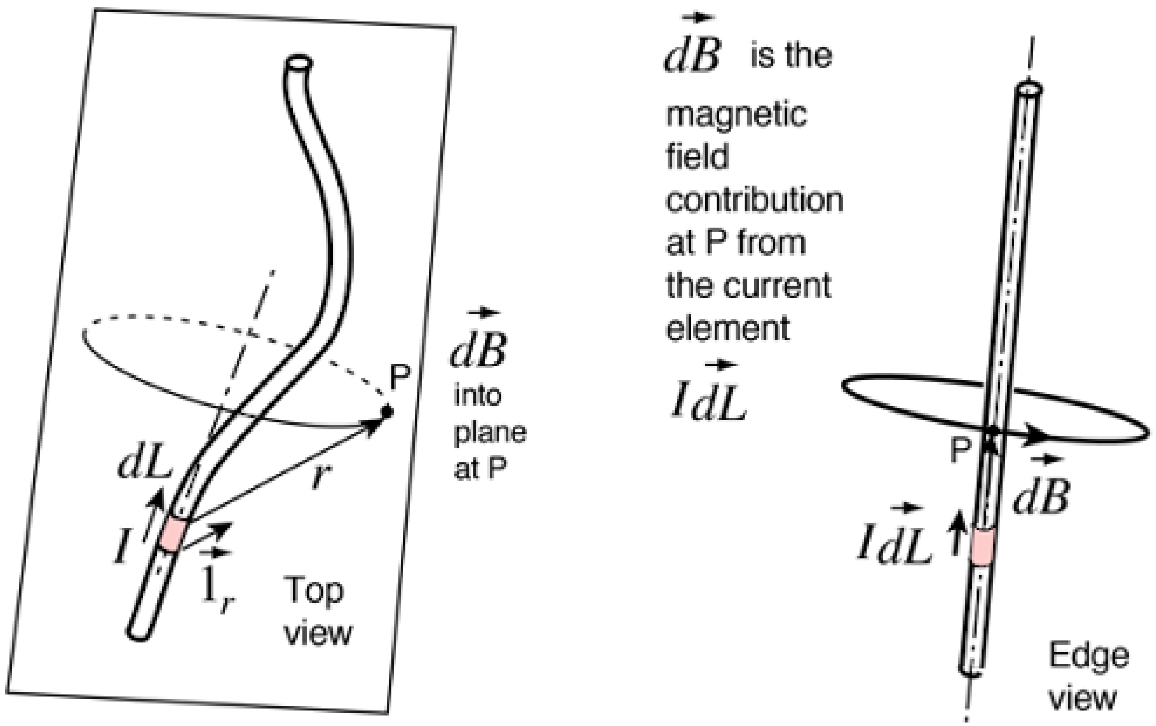
The thumb is in the current's direction. The fingers reveal the field vector's direction, which is tangent to a circle.



The Biot Savart Law

For current around the whole circuit

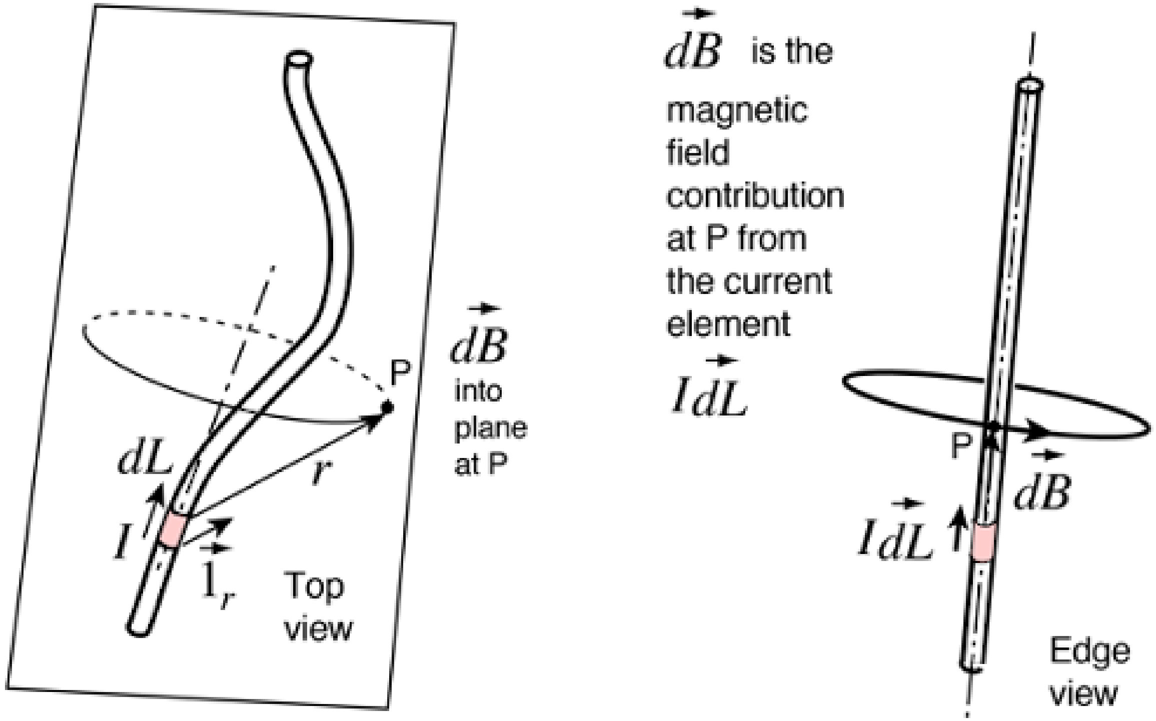
$$\vec{B} = \int d\vec{B} = \frac{\mu_0}{4\pi}\int \frac{id\vec{S}\times\vec{r}}{r^3} = \frac{\mu_0}{4\pi}\int \frac{id\vec{S}\times\hat{r}}{r^2}$$

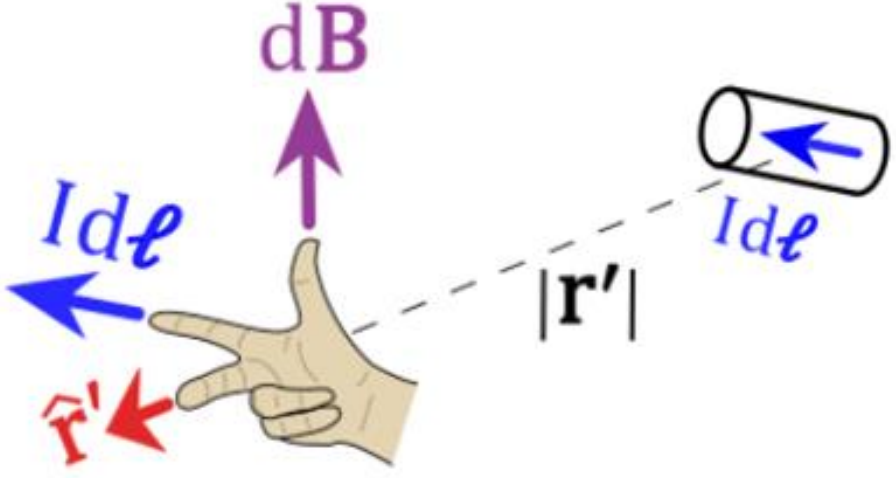


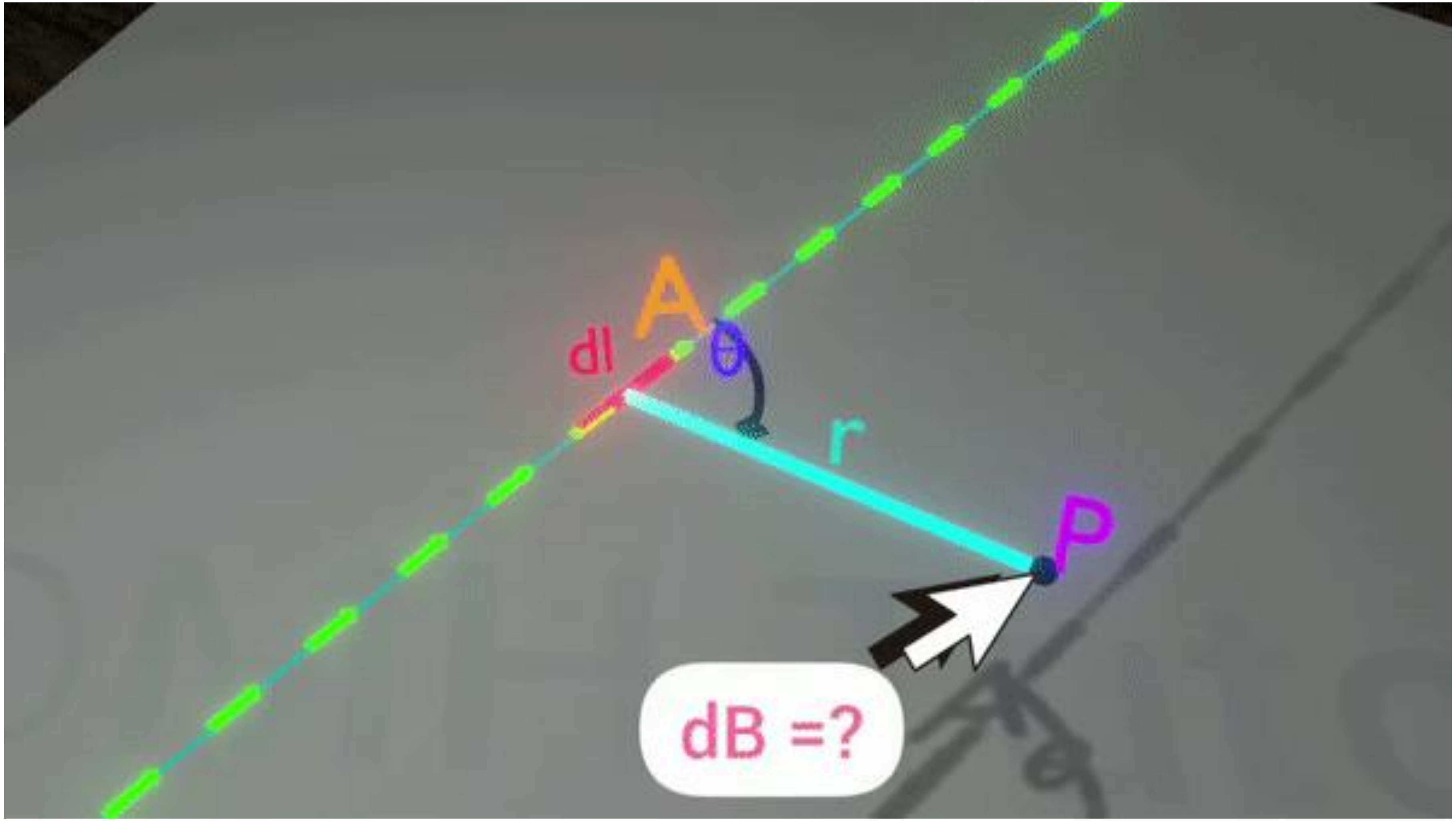
Biot-Savart law in magnetic field (current element $id\vec{s}$)

Coulomb's law in electric field (electric charge element dq)

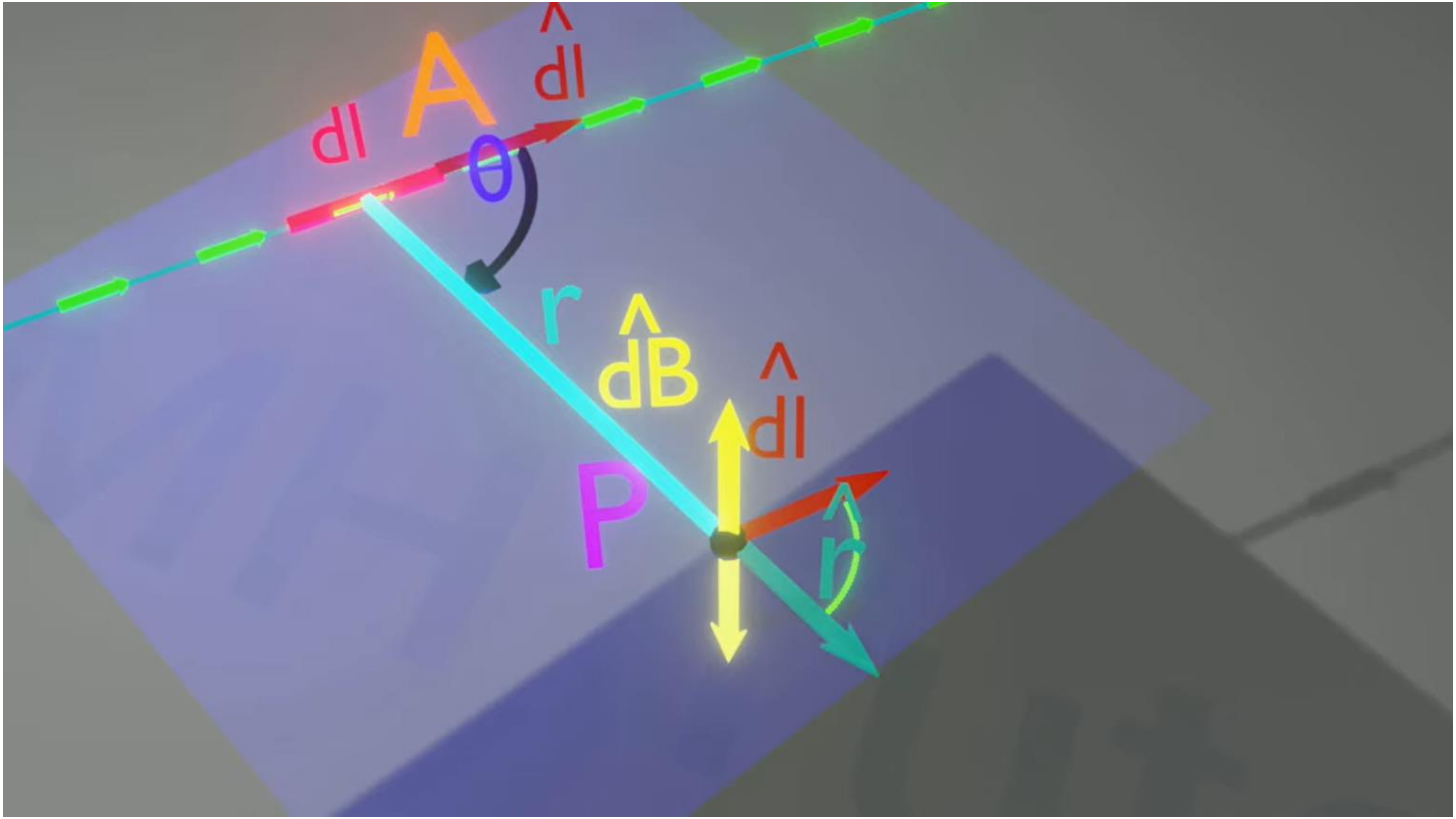
The Biot Savart Law





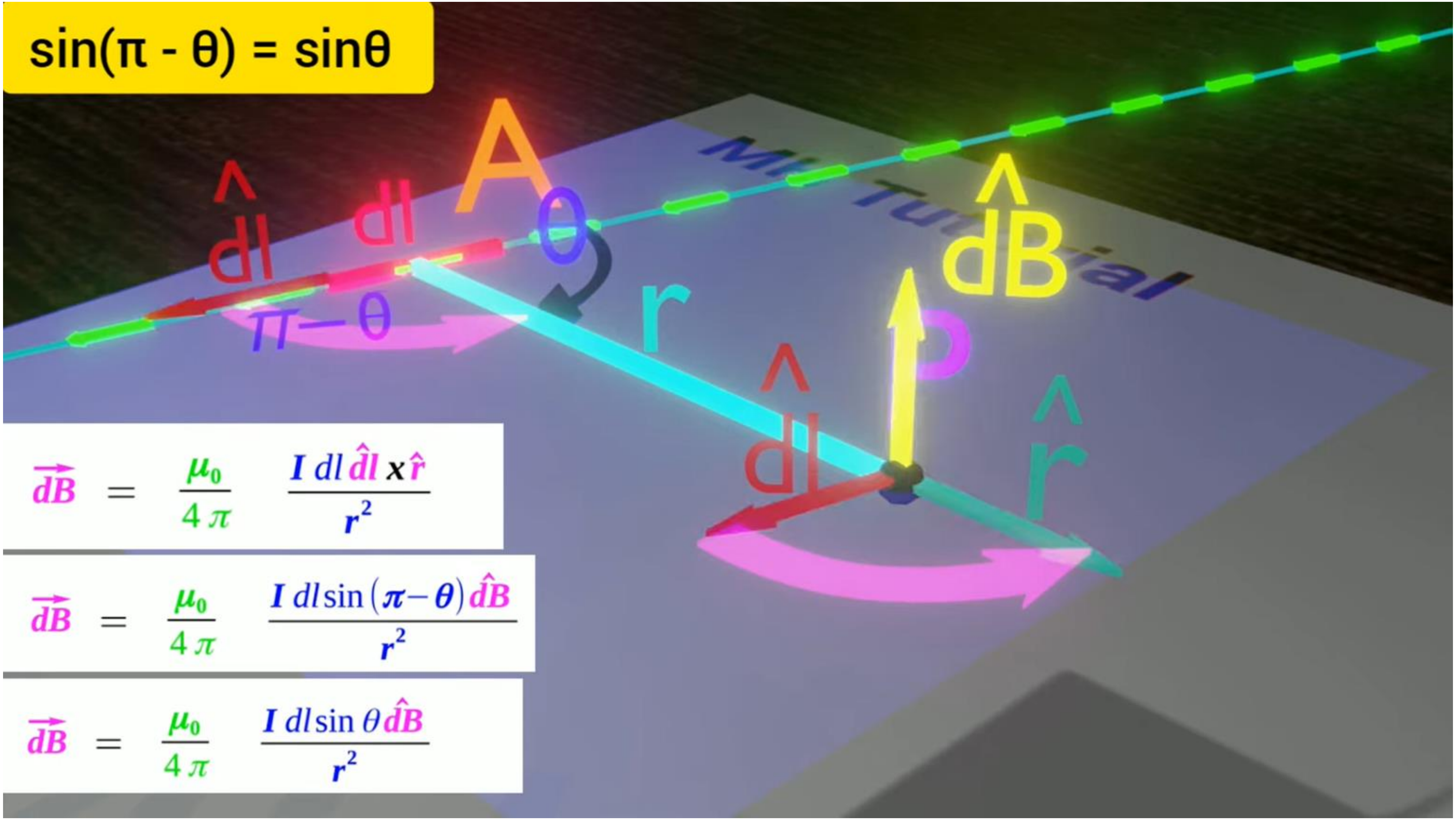


A
dl
θ
r
P
dB =?



A
dl
$\hat{dl}$
θ
r
$\hat{dB}$
$\hat{dl}$
P
$\hat{r}$

$\sin(\pi - \theta) = \sin\theta$



$$\vec{dB} = \frac{\mu_0}{4\pi} \frac{I\, dl\, \hat{dl} \times \hat{r}}{r^2}$$

$$\vec{dB} = \frac{\mu_0}{4\pi} \frac{I\, dl \sin(\pi - \theta)\, \hat{dB}}{r^2}$$

$$\vec{dB} = \frac{\mu_0}{4\pi} \frac{I\, dl \sin\theta\, \hat{dB}}{r^2}$$