Electric Potential-II

Phy 108 course
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If a charged particle moves through an electric field with no force acting on it other than the electric force due to the field, then the mechanical energy is conserved.

$$U_i + K_i = U_f + K_f$$

or, $\Delta K = -\Delta U$

$$\Delta K = -q\Delta V = -q(V_f - V_i)$$

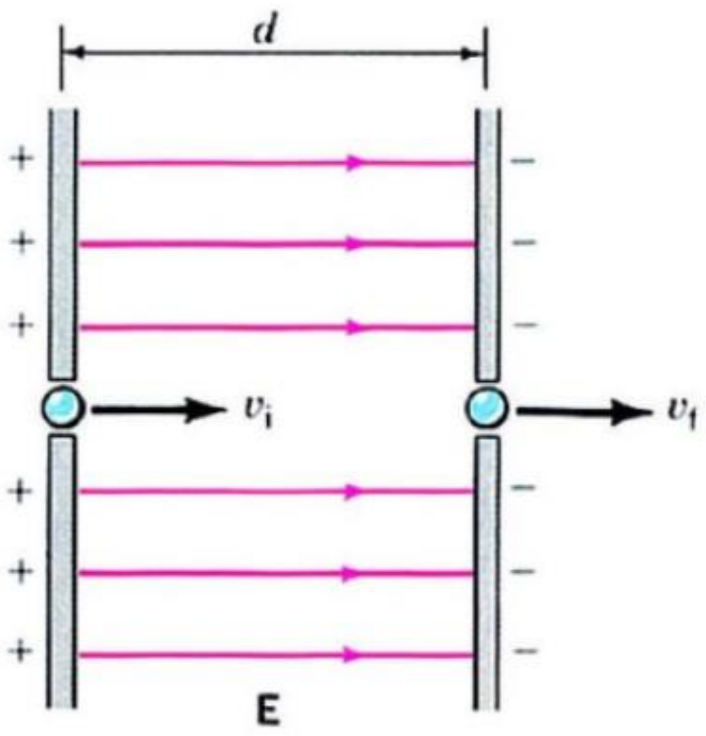
(initial energy) + (work by applied force) = (final energy)

$$U_i + K_i + W_{app} = U_f + K_f$$

$$\Delta K = -\Delta U + W_{app}$$

A proton, of mass 1.67×10^{-27} kg, enters the region between parallel plates a distance 20 cm apart. There is a uniform electric field of 3×10^{5} V/m between the plates, as shown in the Figure. If the initial speed of the proton is 5×10^{6} m/s, what is its final speed?

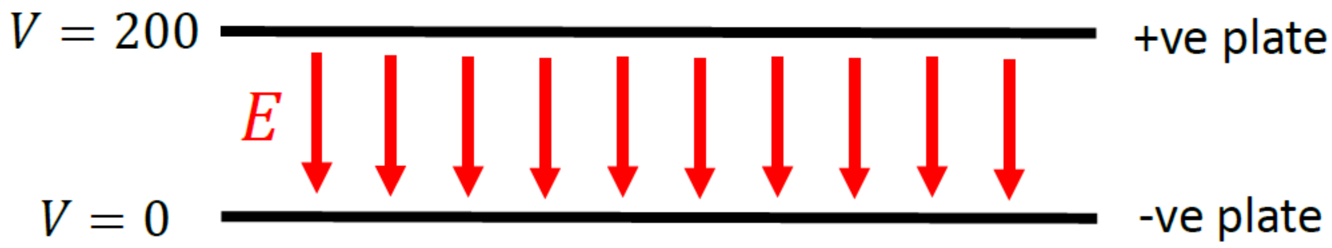
$$\Delta K = -q\Delta V = -q(V_f - V_i) \qquad \Delta V = -\int \vec{E}.d\vec{s} = -Ed$$



<u>Exercise:</u> a potential difference of 200 V is applied across a pair of parallel plates 0.012 m apart. (a) calculate E and draw its direction between the plates.

The electric field is the gradient in potential

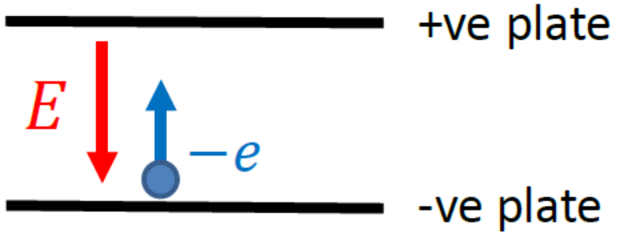
$$E = \frac{\Delta V}{\Delta x} = \frac{200}{0.012} = 1.7 \times 10^4 \; V \, m^{-1} \; [or \; N \; C^{-1}]$$



Exercise: a potential difference of 200 V is applied across a pair of parallel plates 0.012 m apart. (b) an electron is placed between the plates, next to the negative plate. Calculate the force on the electron, the acceleration of the electron, and the time it takes to reach the other plate.

Force $F = qE = (-1.6 \times 10^{-19}) \times (1.7 \times 10^{4}) = -2.7 \times 10^{-15}\ N$

$F = ma$ Acceleration $a = \frac{F}{m} = \frac{2.7 \times 10^{-15}}{9.1 \times 10^{-31}} = 3.0 \times 10^{15}\ m\, s^{-2}$



$d = \frac{1}{2}at^2$ Time $t = \sqrt{\frac{2d}{a}}$

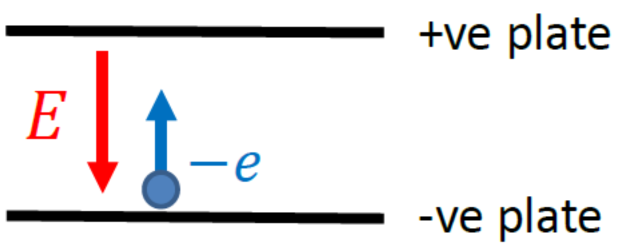
$t = \sqrt{\frac{2 \times 0.012}{3.0 \times 10^{15}}} = 2.8 \times 10^{-9}\ s$

$e = 1.6 \times 10^{-19}$ C; $m_e = 9.1 \times 10^{-31}$ kg

Exercise: a potential difference of 200 V is applied across a pair of parallel plates 0.012 m apart. (c) calculate the work done on the electron as it travels between the plates.

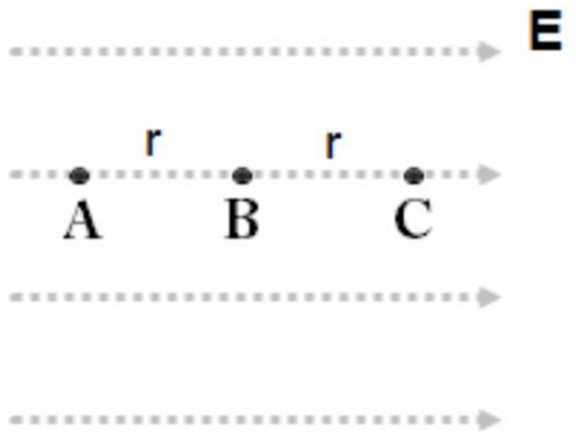
The potential difference is the work done on 1C charge

$$\text{Work } W = qV \quad = (1.6 \times 10^{-19}) \times 200 \quad = 3.2 \times 10^{-17}\, J$$



$e = 1.6 \text{ x } 10^{-19}$ C; $m_e = 9.1 \text{ x } 10^{-31}$ kg

A point charge q is released from rest at point A and accelerates in a uniform electric field E. What is the ratio between the work done by the field on the charge: $W_{A\to B}/W_{B\to C}$? What is the ratio between velocities of the charge V_B/V_C?



$$W_{A\to B} = qEd$$
$$W_{B\to C} = qEd$$

$$\frac{W_{A\to B}}{W_{B\to C}} = \frac{qEd}{qEd} = 1$$

$$W = \Delta KE = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2$$

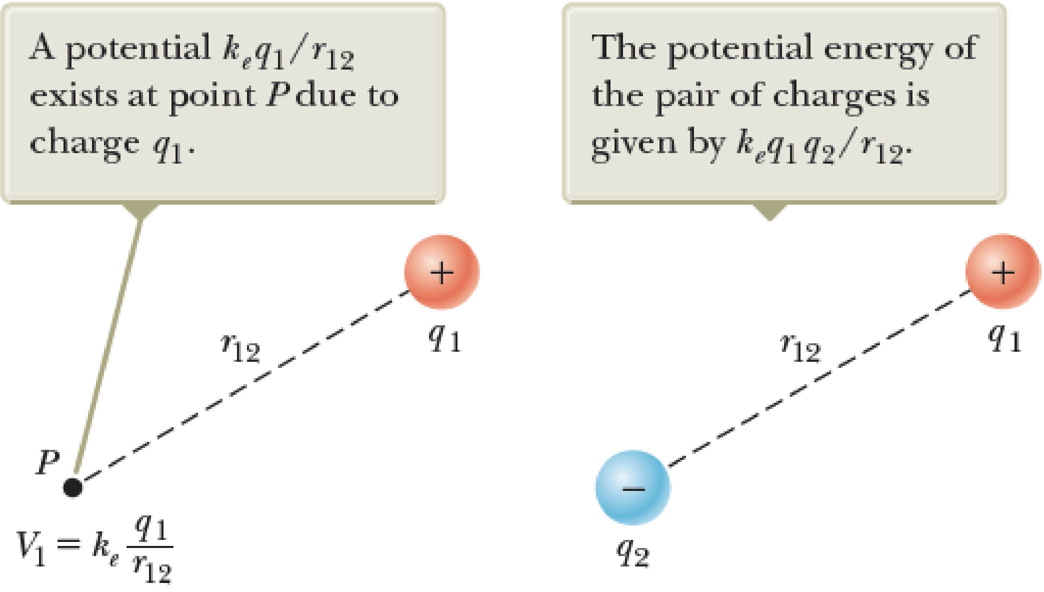
$$W_{A\to B} = \frac{1}{2}mv_B^2$$
$$W_{B\to C} = \frac{1}{2}mv_C^2 - \frac{1}{2}mv_B^2$$

Since $W_{A\to B} = W_{B\to C}$, we substitute:

$$\frac{1}{2}mv_B^2 = \frac{1}{2}m(v_C^2 - v_B^2)$$

$$\frac{v_B}{v_C} = \frac{1}{\sqrt{2}}$$

Potential Due to a Group of Charged Particles



- Use superposition

$$\boxed{V = -\int_{\infty}^{r} \vec{E}\cdot d\vec{s} = -\sum_{i=1}^{n}\int_{\infty}^{r} \vec{E}_i \cdot d\vec{s} = \sum_{i=1}^{n} V_i}$$

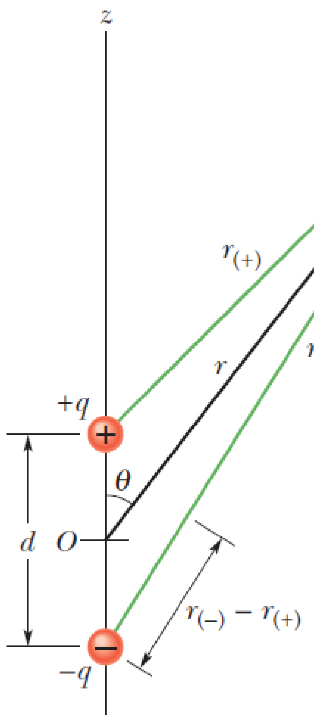
- For point charges

$$\boxed{V = \sum_{i=1}^{n} V_i = \frac{1}{4\pi\varepsilon_0}\sum_{i=1}^{n}\frac{q_i}{r_i}}$$

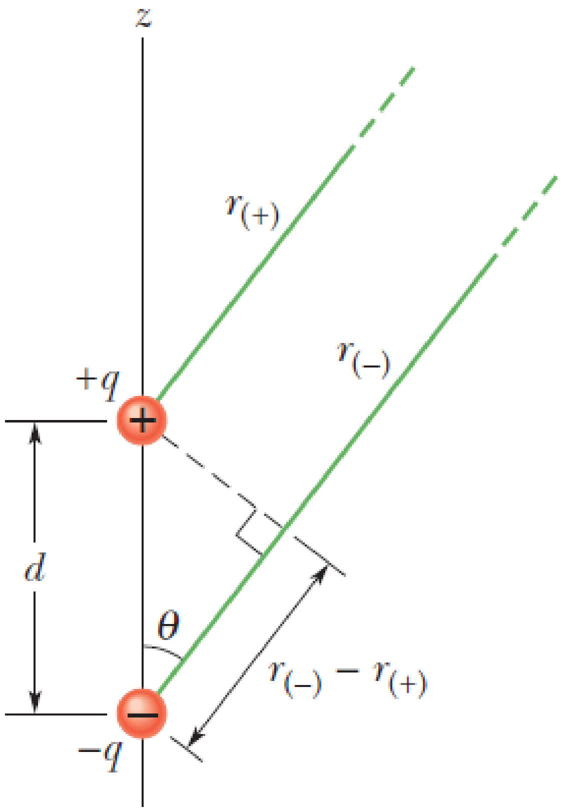
- The sum is an algebraic sum, not a vector sum.
- E may be zero where V does not equal to zero.
- V may be zero where E does not equal to zero.

$$V = \sum_{i=1}^{n} V_i = \frac{1}{4\pi\varepsilon_0}\sum_{i=1}^{n}\frac{q_i}{r_i} \quad (n \text{ charged particles}).$$

Potential Due to an Electric Dipole



Naturally occurring dipoles—such as those possessed by many molecules—are quite small; so we are usually interested only in points that are relatively far from the dipole, such that $r \gg d$, where *d* is the distance between the charges and *r* is the distance from the dipole's midpoint to *point P*.



$$r_{(-)} - r_{(+)} \approx d\cos\theta \quad \text{and} \quad r_{(-)}r_{(+)} \approx r^2.$$

$$V = \frac{q}{4\pi\varepsilon_0}\frac{d\cos\theta}{r^2}$$

$$V = \sum_{i=1}^{2} V_i = V_{(+)} + V_{(-)} = \frac{1}{4\pi\varepsilon_0}\left(\frac{q}{r_{(+)}} + \frac{-q}{r_{(-)}}\right)$$

$$= \frac{q}{4\pi\varepsilon_0}\frac{r_{(-)} - r_{(+)}}{r_{(-)}r_{(+)}}.$$

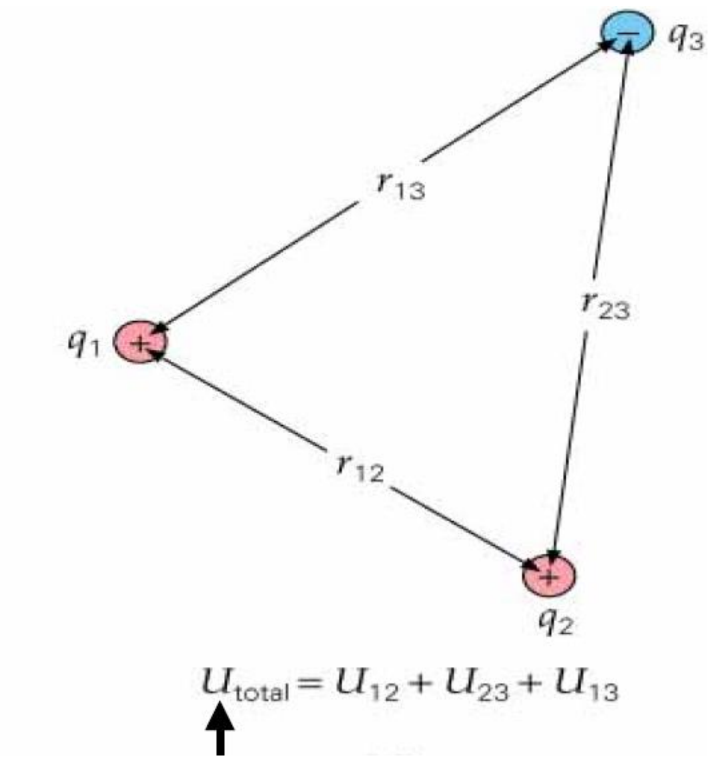
$$V = \frac{1}{4\pi\varepsilon_0}\frac{p\cos\theta}{r^2} \quad \text{(electric dipole)}$$

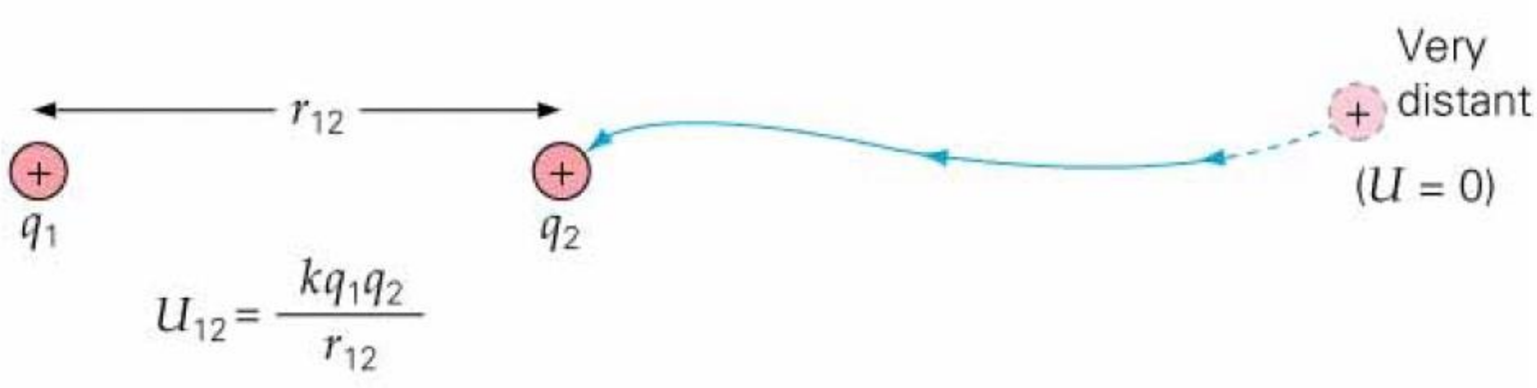
Electric dipole moment, $\vec{p} = q\vec{d}$

Electric Potential Energy of a System of Point Charges

The idea of electric potential energy isn't restricted to the special case of a uniform electric field. Indeed, we can apply this concept to a point charge in *any* electric field caused by a static charge distribution.

The total potential energy of a system of particles is the sum of the potential energies for every pair of particles in the system.



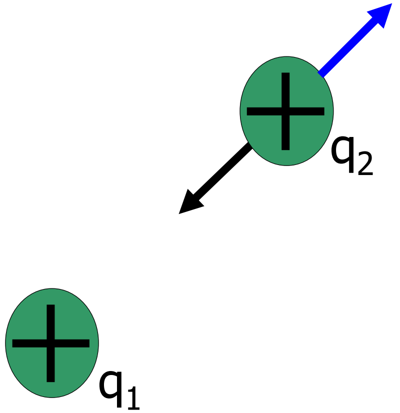


$$U = k_e\left(\frac{q_1q_2}{r_{12}} + \frac{q_1q_3}{r_{13}} + \frac{q_2q_3}{r_{23}}\right)$$

Electric Potential Energy of a System of Point Charges

$$\boxed{\Delta U = U_f - U_i = -W} \quad \boxed{W = \vec{F}\cdot\Delta\vec{r} = q\vec{E}\cdot\Delta\vec{r}}$$

$$\boxed{W_{app} = -W} \quad \boxed{\Delta U = U_f - U_i = W_{app}}$$



- Start with (set U_i=0 at ∞ and U_f=U at r) $\boxed{V = \frac{1}{4\pi\varepsilon_0}\frac{q_1}{r}}$

- We have $\boxed{U = q_2 V = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r}}$

- If the system consists of more than two charged particles, calculate U for each pair of charges and sum the terms algebraically. $\boxed{U = U_{12} + U_{13} + U_{23} = \frac{1}{4\pi\varepsilon_0}\left(\frac{q_1q_2}{r_{12}} + \frac{q_1q_3}{r_{13}} + \frac{q_2q_3}{r_{23}}\right)}$

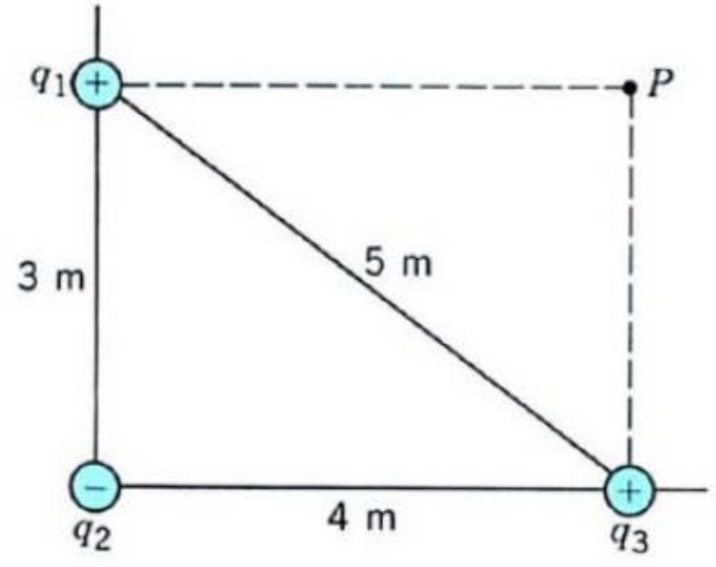
Electric Potential Energy for point charges

Three point charge, $q1 = 1\mu C, q2 = -2\mu C, and\ q3 = 3\mu C$ are fixed at the positions shown in Figure.

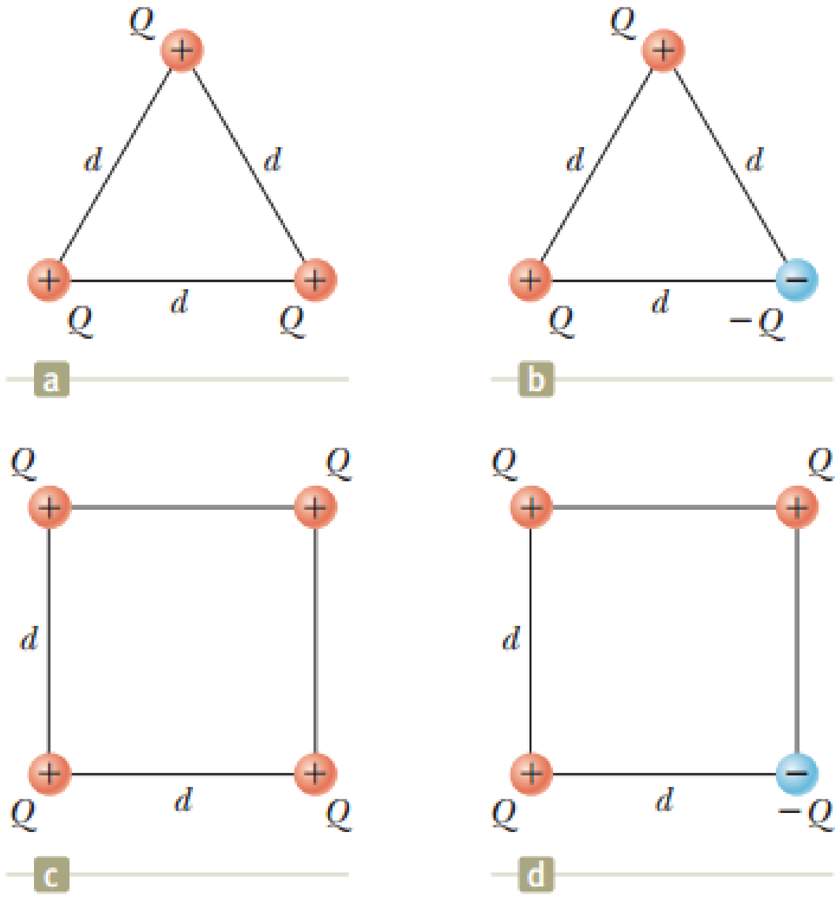
(a) What is the potential at point P at the corner of the rectangle?

(b) What is the total potential energy of q1, q2, and q3?

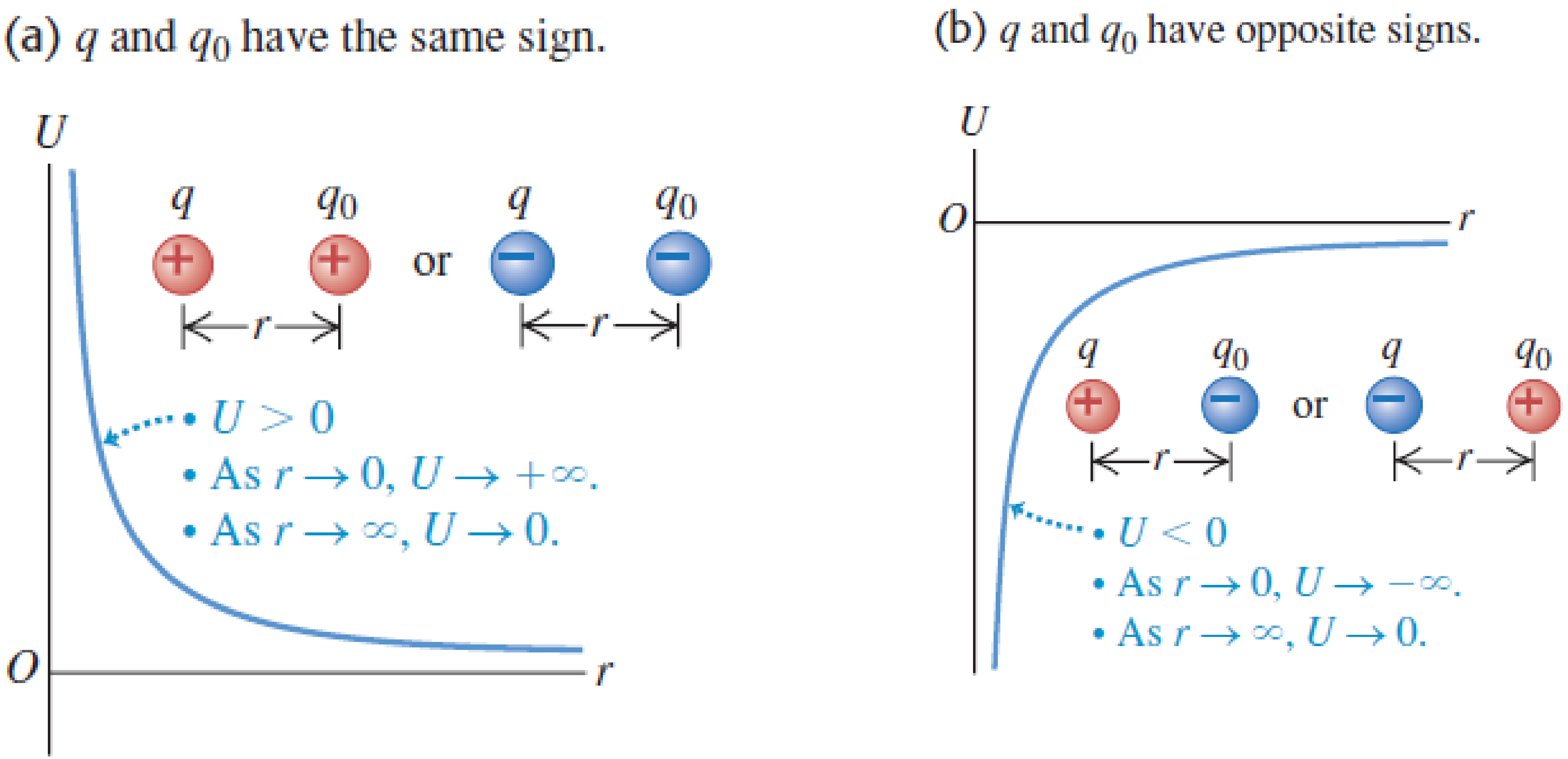
(c) How much work would be needed to bring a charge $q4 = 2.5\mu C$ from infinity and to place it at P?



Rank the electric potential energies of the systems of charges shown in Figure from largest to smallest. Indicate equalities if appropriate.



Potential and Potential Energy of Point Charges



http://physics.bu.edu/~duffy/HTML5/potential2.html

http://physics.bu.edu/~duffy/HTML5/force_PE2.html

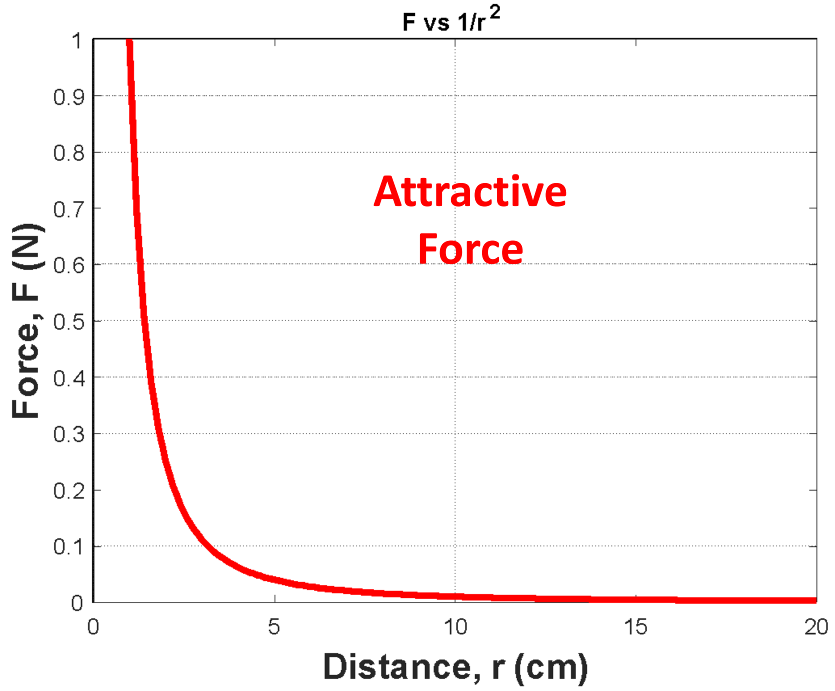
Coulomb’s Law

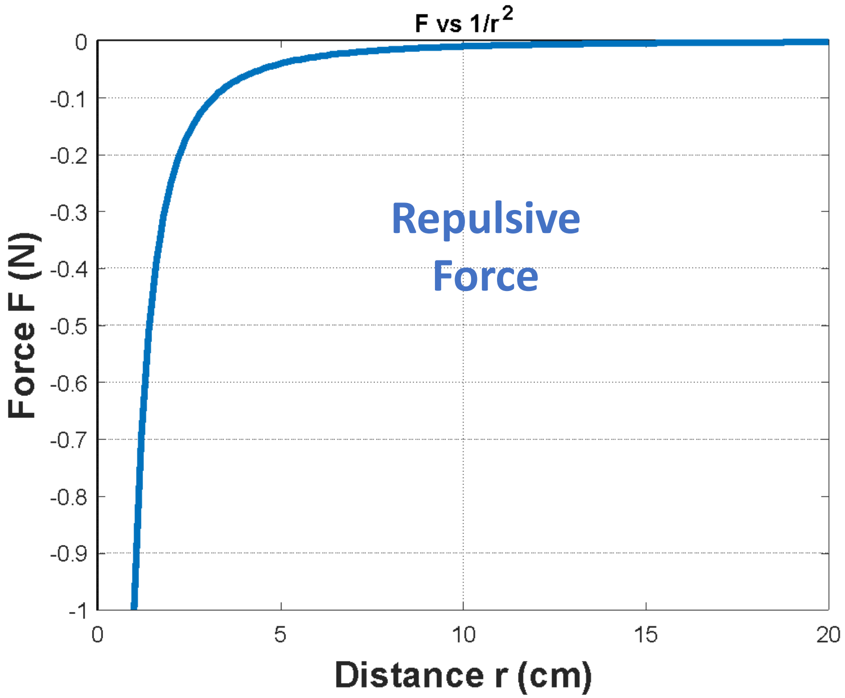
Comparison with Gravitational law:

However, the laws differ in that gravitational forces are always attractive but electrostatic forces may be either attractive or repulsive, depending on the signs of the charges.

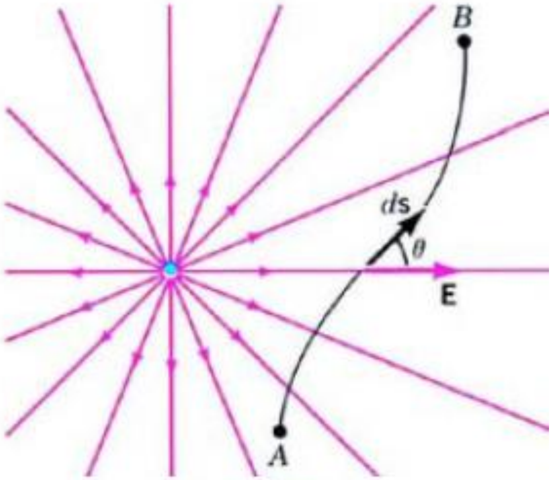
This difference arises from the fact that there is only one type of mass but two types of charge.

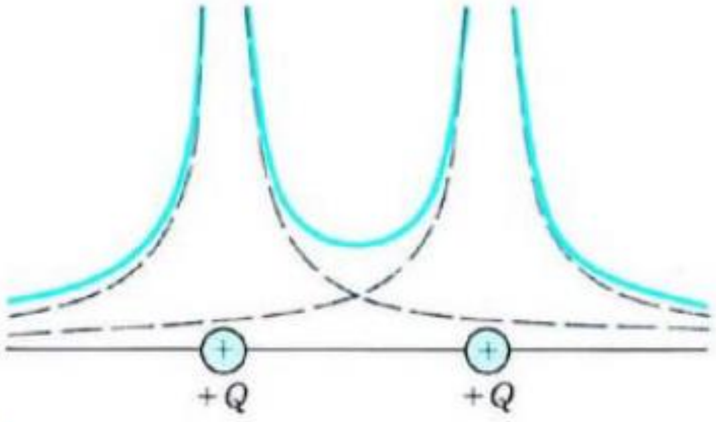
We will see the difference in strength of these two forces.

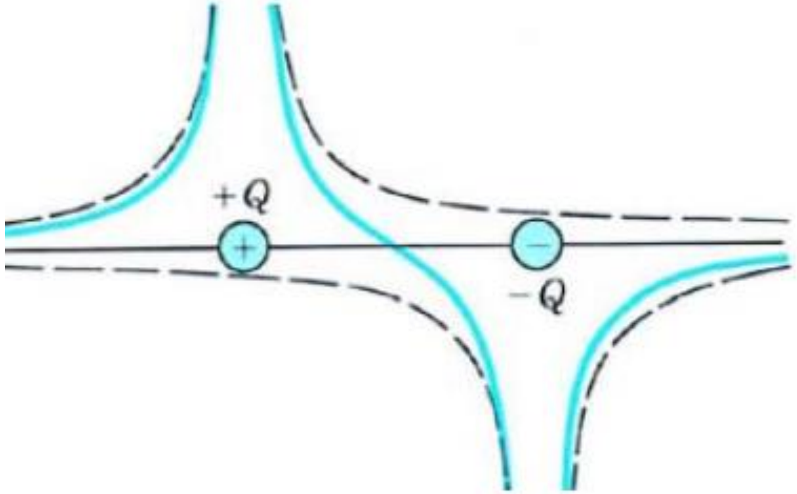


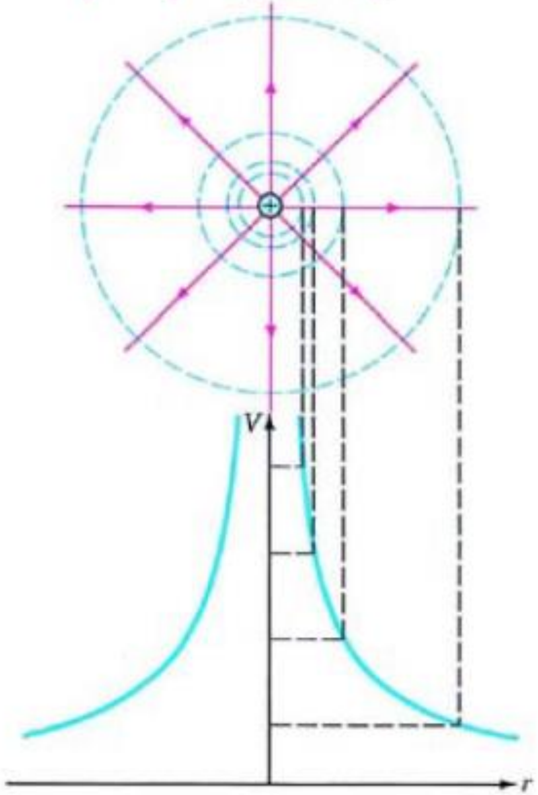


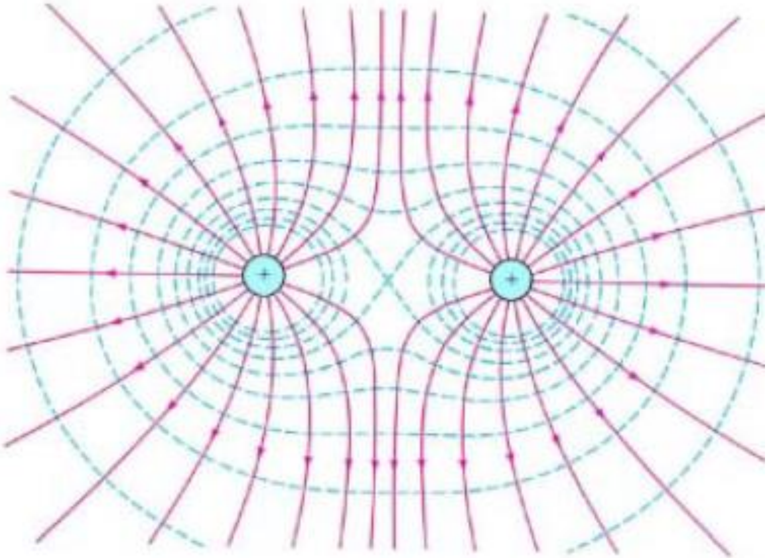
Potential and Potential Energy of Point Charges

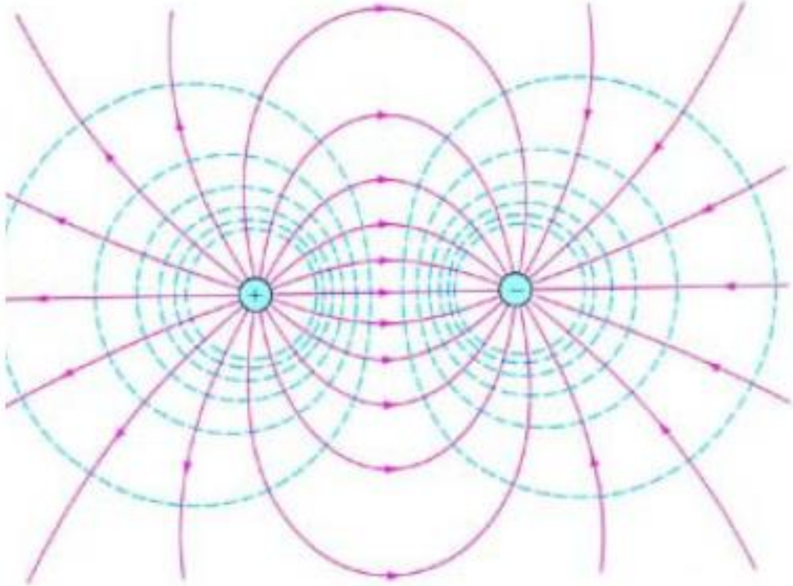




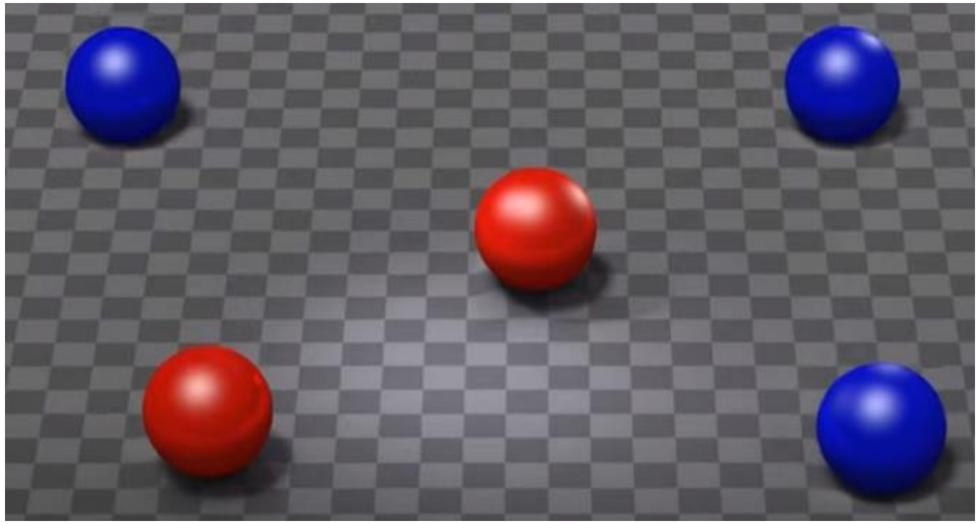


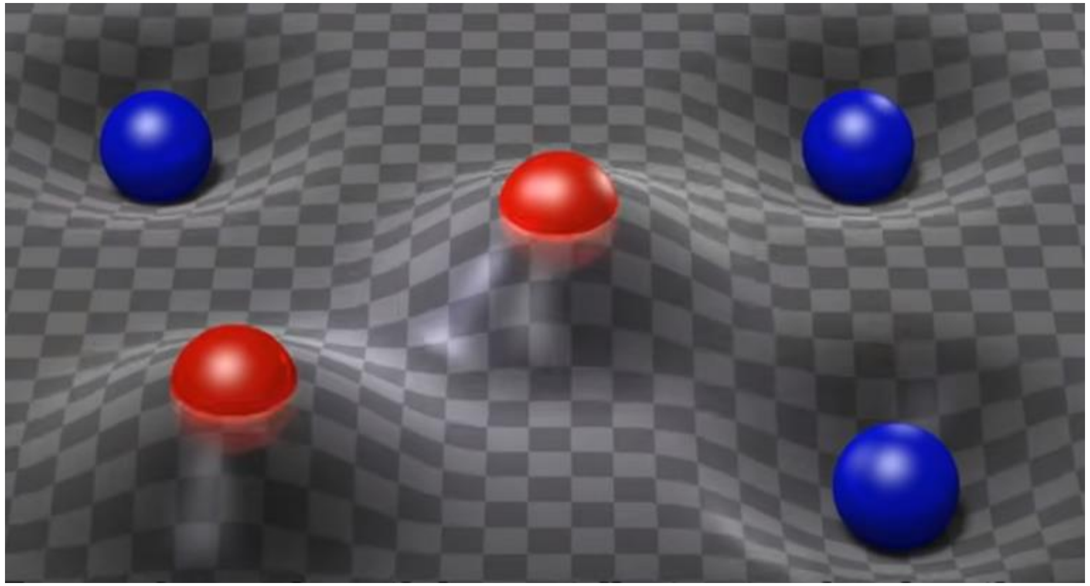






http://physics.bu.edu/~duffy/HTML5/field_potential3.html



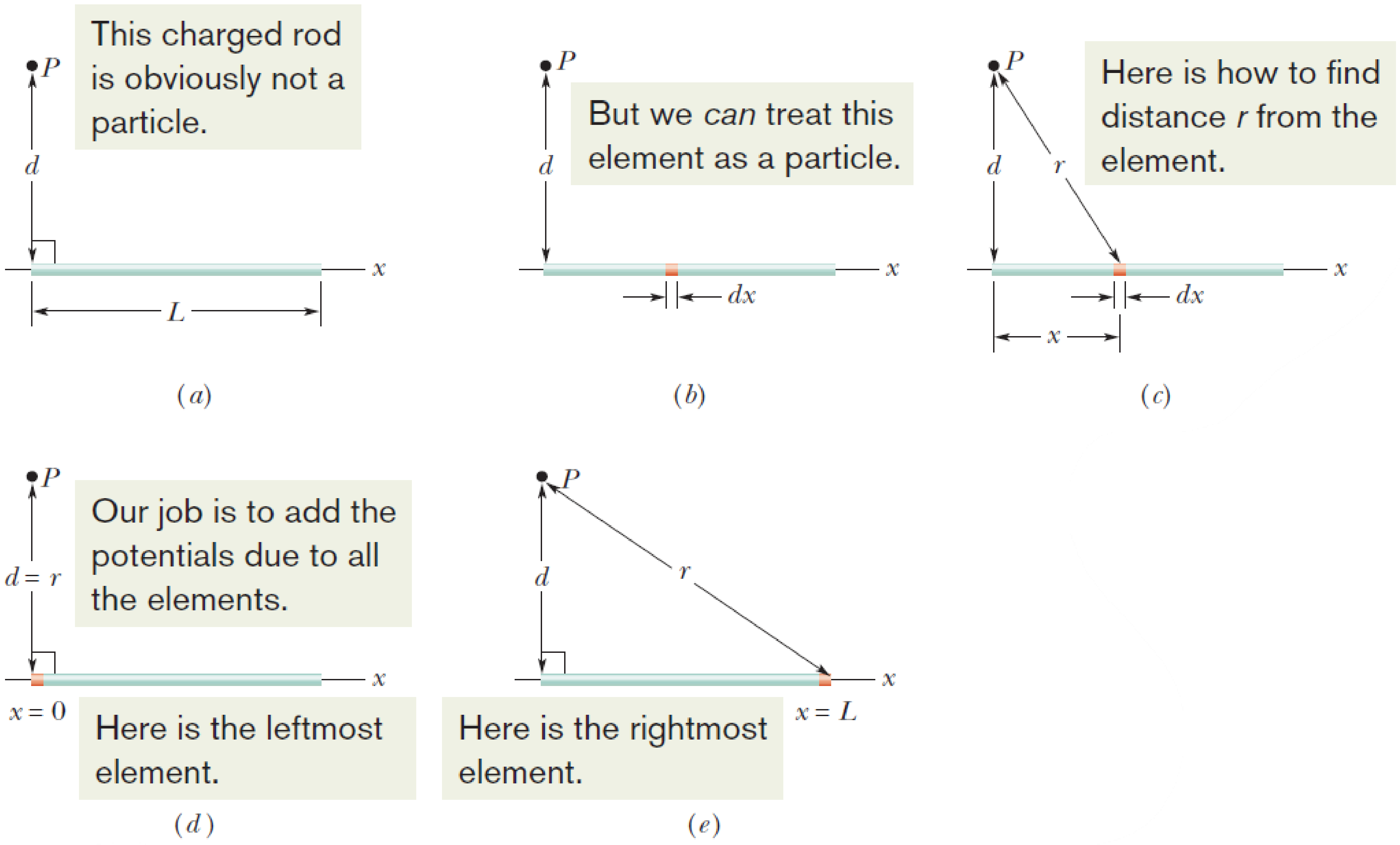


Potential Due to a Continuous Charge Distribution

When a charge distribution q is continuous (as on a uniformly charged thin rod or disk), we must choose a differential element of charge dq, determine the potential dV at P due to dq, and then integrate over the entire charge distribution.

$$dV = \frac{1}{4\pi\varepsilon_0}\frac{dq}{r} \quad \text{(positive or negative } dq\text{).} \quad \Rightarrow \quad V = \int \frac{k}{r}dq$$

Electric Potential Due to a Finite Line of Charge



Electric Potential Due to a Finite Line of Charge

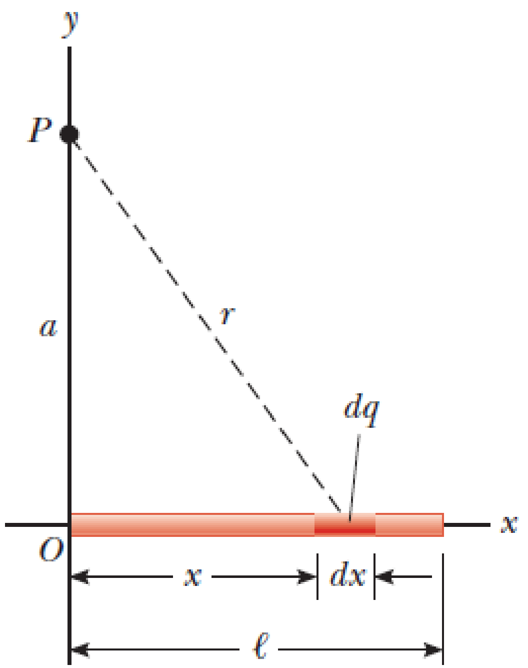


Figure 25.16 (Example 25.7) A uniform line charge of length ℓ located along the x axis. To calculate the electric potential at P, the line charge is divided into segments each of length dx and each carrying a charge $dq = \lambda\, dx$.

$$dV = k_e \frac{dq}{r} = k_e \frac{\lambda\, dx}{\sqrt{a^2 + x^2}}$$

$$V = \int_0^\ell k_e \frac{\lambda\, dx}{\sqrt{a^2 + x^2}}$$

$$V = k_e \lambda \int_0^\ell \frac{dx}{\sqrt{a^2 + x^2}} = k_e \frac{Q}{\ell} \ln\left(x + \sqrt{a^2 + x^2}\right)\Big|_0^\ell$$

$$V = k_e \frac{Q}{\ell}\left[\ln\left(\ell + \sqrt{a^2 + \ell^2}\right) - \ln a\right] = \boxed{k_e \frac{Q}{\ell} \ln\left(\frac{\ell + \sqrt{a^2 + \ell^2}}{a}\right)}$$

Electric Potential Due to a Uniformly Charged Ring

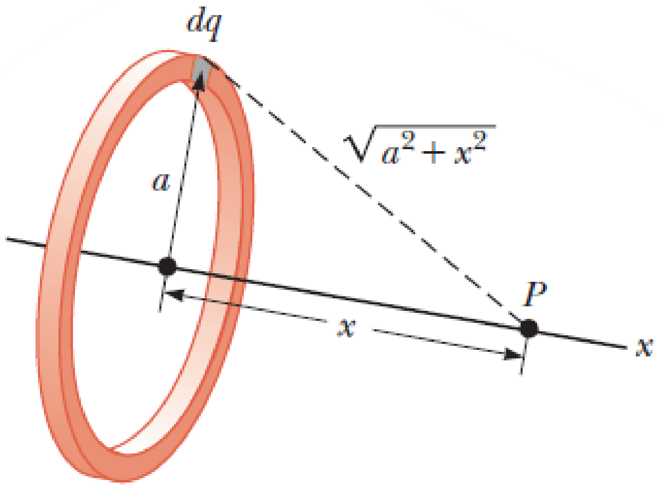


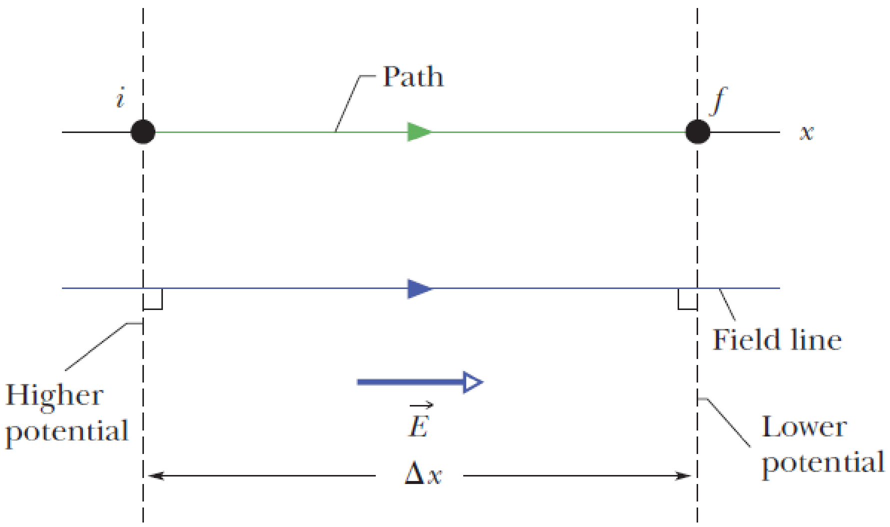
Figure 25.14 (Example 25.5) A uniformly charged ring of radius a lies in a plane perpendicular to the x axis. All elements dq of the ring are the same distance from a point P lying on the x axis.

$$V = k_e \int \frac{dq}{r} = k_e \int \frac{dq}{\sqrt{a^2 + x^2}}$$

$$V = \frac{k_e}{\sqrt{a^2 + x^2}} \int dq = \boxed{\frac{k_e Q}{\sqrt{a^2 + x^2}}}$$

Potential of a Charged Isolated Conductor

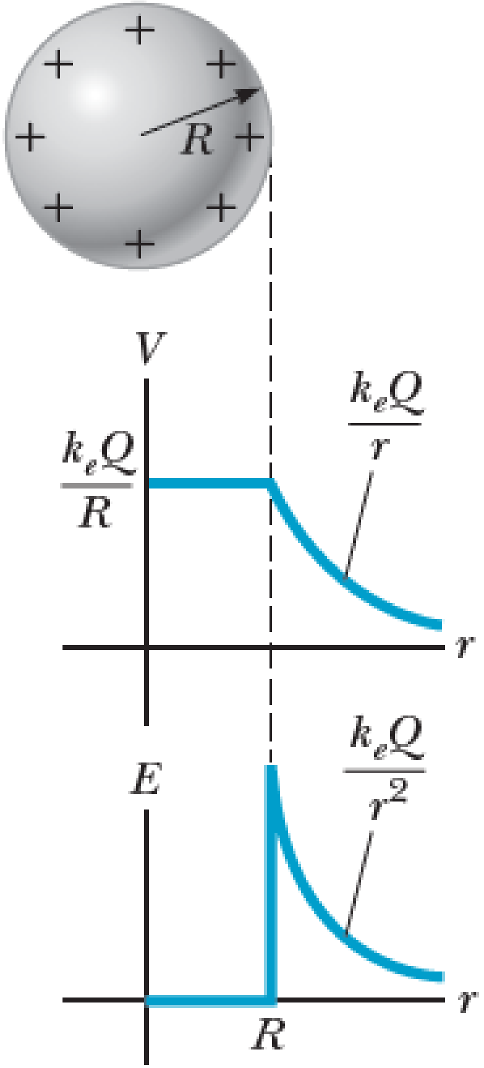
An excess charge placed on an isolated conductor will distribute itself on the surface of that conductor so that all points of the conductor—whether on the surface or inside—come to the same potential. This is true even if the conductor has an internal cavity and even if that cavity contains a net charge.



$$V_f - V_i = -\int_i^f \vec{E} \cdot d\vec{s}. \qquad E = -\frac{dV}{dx}$$

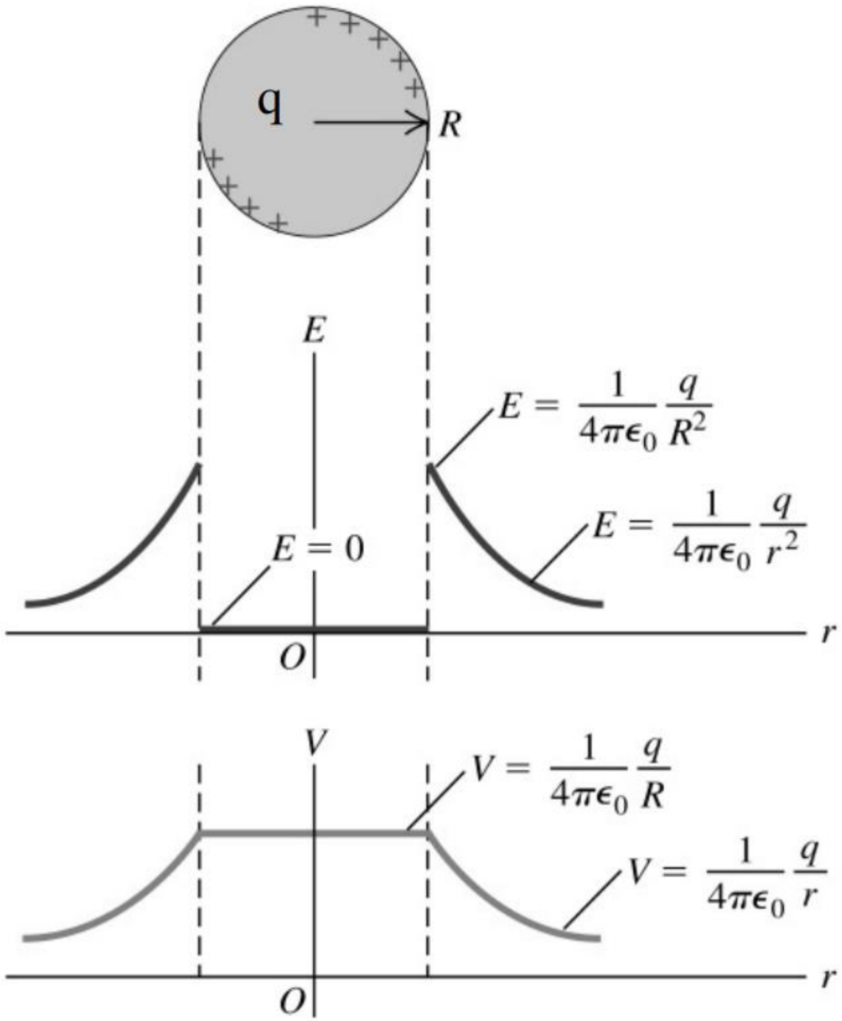
This result applies to any two points on the surface. Therefore, V is constant everywhere on the surface of a charged conductor in equilibrium.

Potential of a Charged Isolated Conductor

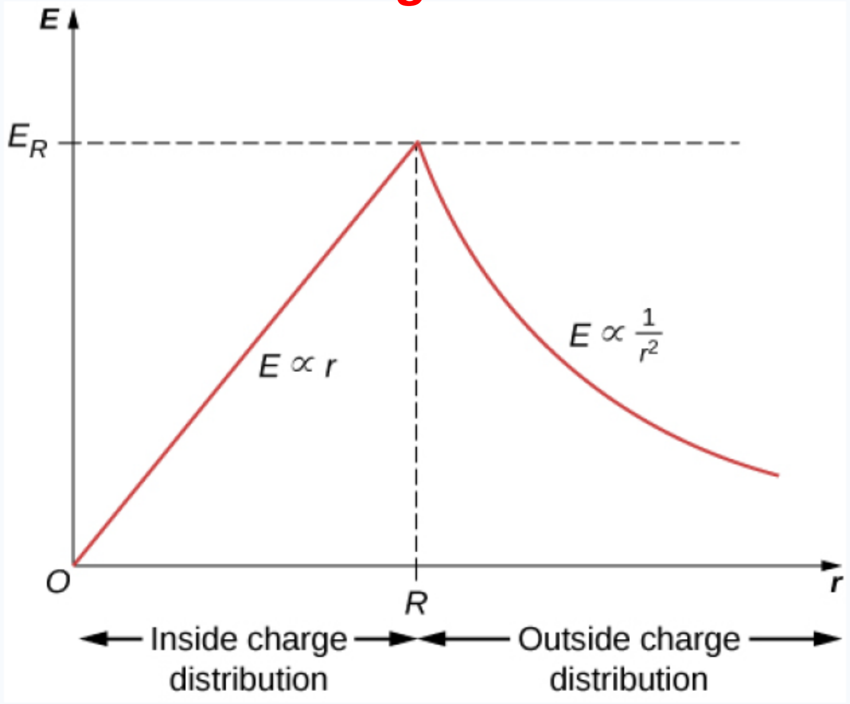


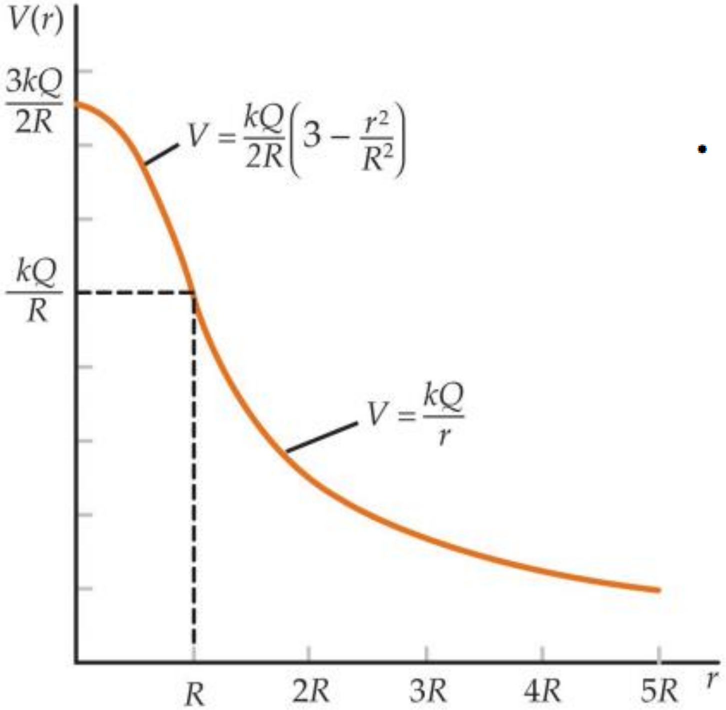
The surface of any charged conductor in electrostatic equilibrium is an equipotential surface: every point on the surface of a charged conductor in equilibrium is at the same electric potential.

Furthermore, because the electric field is zero inside the conductor, the electric potential is constant everywhere inside the conductor and equal to its value at the surface.



A Charged Non Conductor





- Electric charge on sphere: $Q = \rho V = \frac{4\pi}{3}\rho R^3$
- Electric field at $r > R$: $E = \frac{kQ}{r^2}$
- Electric field at $r < R$: $E = \frac{kQ}{R^3}r$
- Electric potential at $r > R$:

$$V = -\int_{\infty}^{r} \frac{kQ}{r^2}\,dr = \frac{kQ}{r}$$

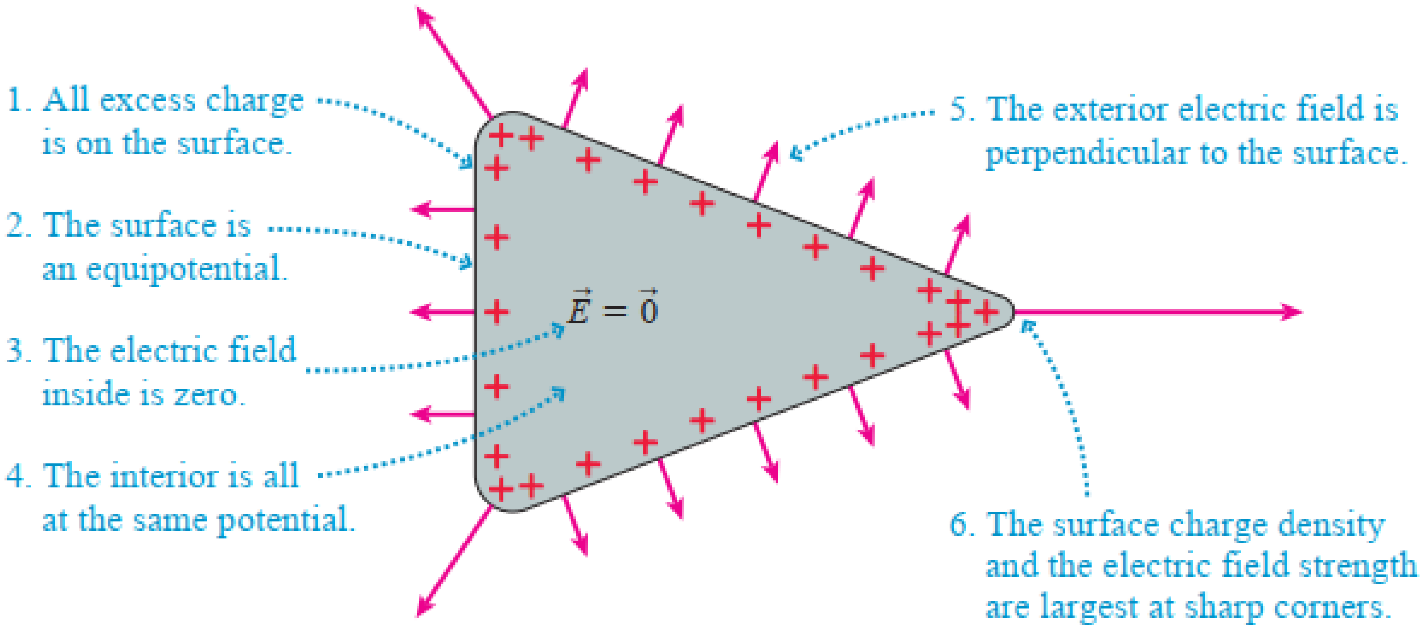
- Electric potential at $r < R$:

$$V = -\int_{\infty}^{R} \frac{kQ}{r^2}\,dr - \int_{R}^{r} \frac{kQ}{R^3}\,r\,dr$$

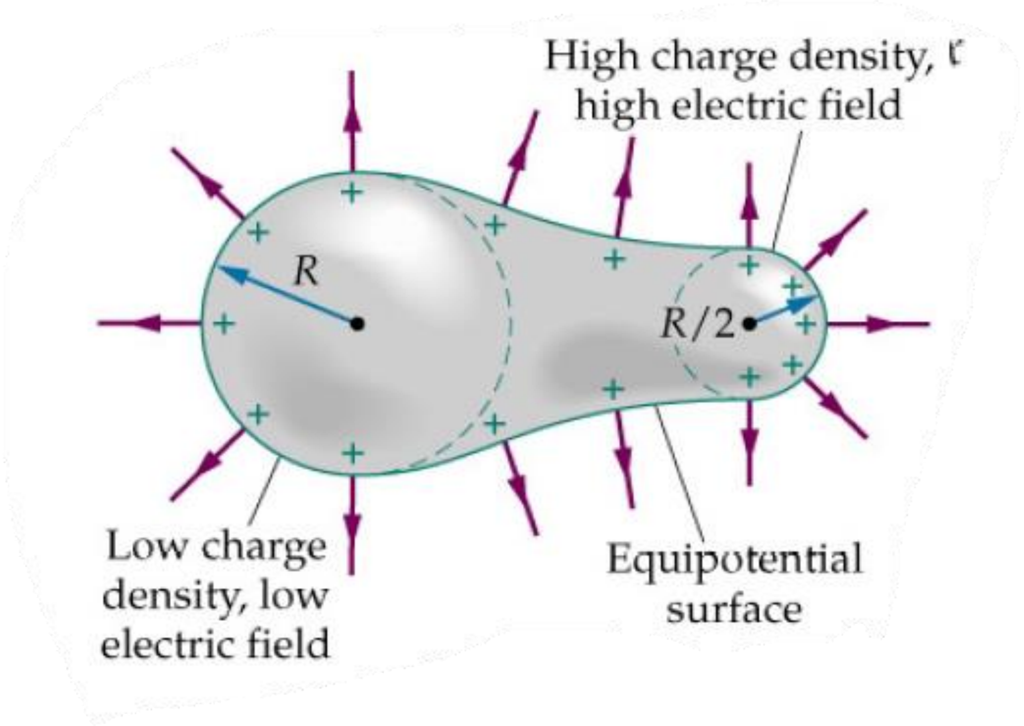
$$\Rightarrow V = \frac{kQ}{R} - \frac{kQ}{2R^3}\left(r^2 - R^2\right) = \frac{kQ}{2R}\left(3 - \frac{r^2}{R^2}\right)$$

A Conductor in electrostatic equilibrium

In other words, any two points inside a conductor in electrostatic equilibrium are at the same potential.



An arbitrarily shaped conductor can be approximated by spheres with the same potential at the surface and varying radii of curvature.
Thus more sharply curved end of a conductor has a greater charge density and a more intense field.



Smaller spheres have higher charge densities to be at the same potential. Also electric field is larger for the small sphere and at right angles to the surface.

$\therefore$ Potential everywhere is identical.

Potential of radius R_1 sphere $V_1 = \dfrac{q_1}{4\pi\epsilon_0 R_1}$

Potential of radius R_2 sphere $V_2 = \dfrac{q_2}{4\pi\epsilon_0 R_2}$

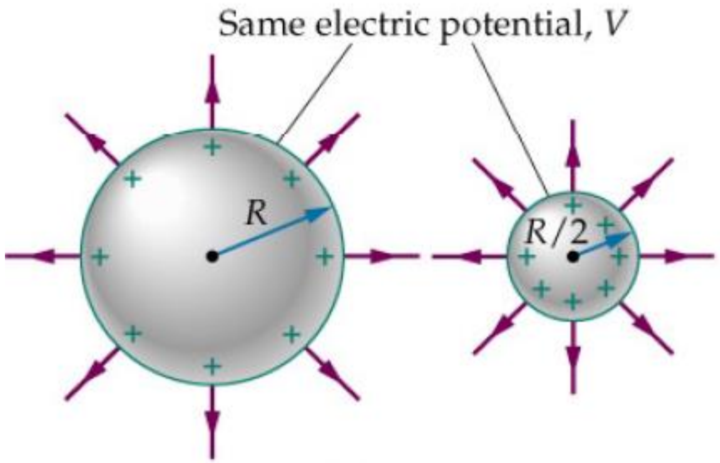
$$\boldsymbol{V_1 = V_2}$$

$$\Rightarrow \frac{q_1}{R_1} = \frac{q_2}{R_2} \quad \Rightarrow \quad \frac{q_1}{q_2} = \frac{R_1}{R_2}$$

$$\sigma_1 = \underbrace{\frac{q_1}{4\pi R_1^2}}_{\text{Surface area of radius } R_1 \text{ sphere}}$$

$$\therefore \quad \frac{\sigma_1}{\sigma_2} = \frac{q_1}{q_2} \cdot \frac{R_2^2}{R_1^2} = \frac{R_2}{R_1}$$

$$\therefore \quad \text{If } R_1 < R_2, \text{ then } \sigma_1 > \sigma_2$$



We infer that $\sigma \propto 1/R$: The surface charge density on each sphere is inversely proportional to the radius.

The surface charge density on each sphere is inversely proportional to the radius...thus the electric field strength

If the field strength is great enough (about 3×10^6 V/m for dry air) it can cause an electrical discharge in air..

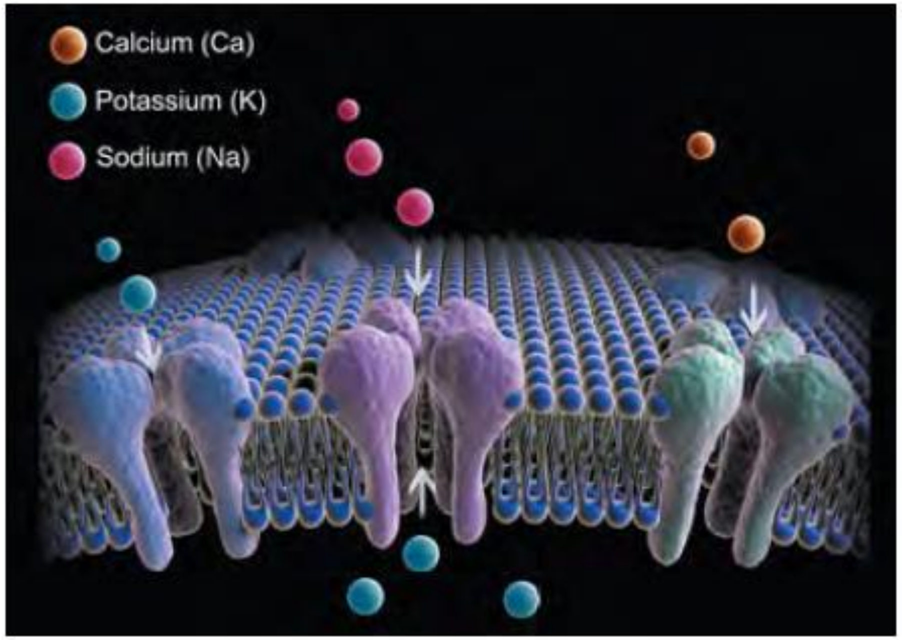
The potential of a sphere of radius 10 cm may be raised to 3×10^5 V before breakdown.

On the other hand, a 0.05 mm dust particle can initiate a discharge at 150 V.

A high voltage system must keep at very clean condition!!!

Application Potential Gradient Across a Cell Membrane

The interior of a human cell is at a lower electric potential V than the exterior. (The potential difference when the cell is inactive is about –70 mV in neurons and about –95 mV in skeletal muscle cells.) Hence there is a potential gradient $\vec{\nabla} V$ that points from the *interior* to the *exterior* of the cell membrane, and an electric field $\vec{E} = -\vec{\nabla} V$ that points from the *exterior* to the *interior*. This field affects how ions flow into or out of the cell through special channels in the membrane.

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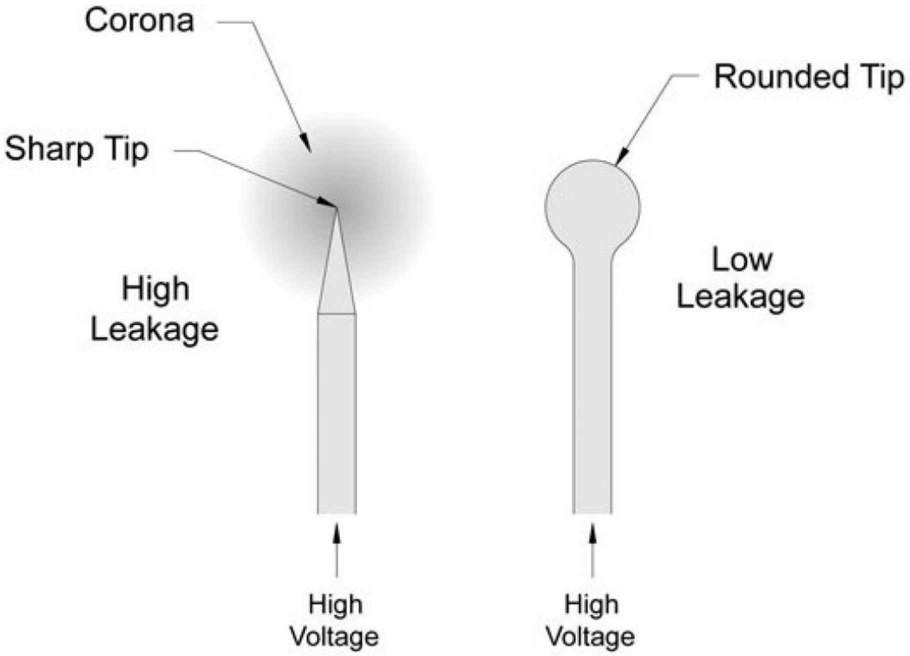
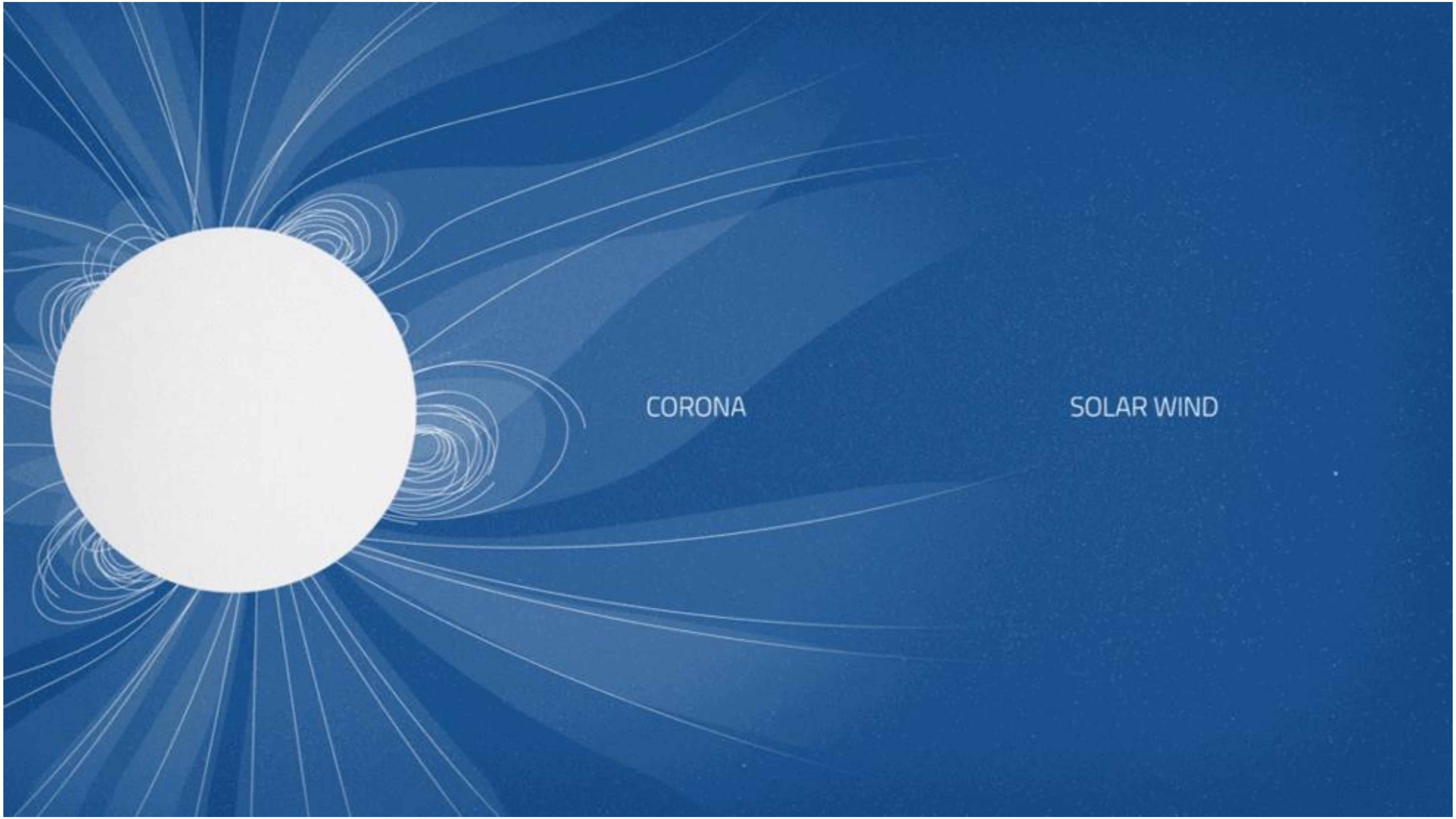

Corona
Sharp Tip
High
Leakage
High
Voltage
Rounded Tip
Low
Leakage
High
Voltage

The **corona** is the outermost part of the Sun's atmosphere.

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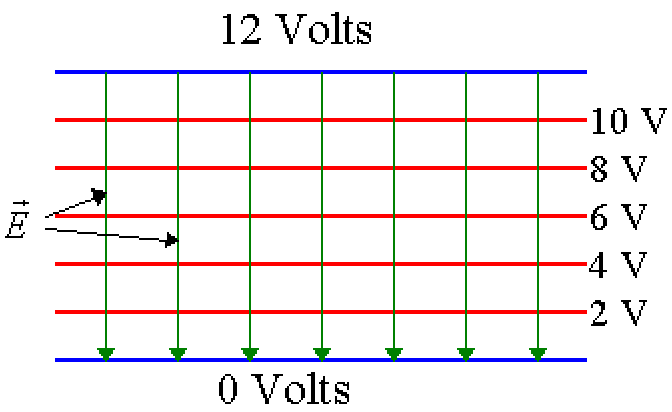
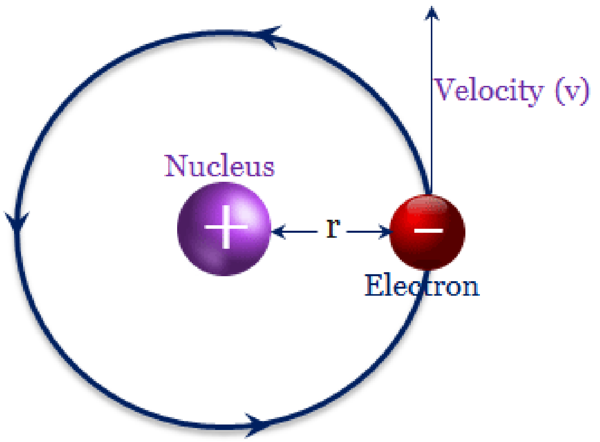
The potential difference between the two plates of the capacitor shown below is 12 V. Equipotential surfaces are shown. If the separation between the plates is 1 mm, what is the strength of the electric field between the plates?



In 1913, Bohr proposed a model of the hydrogen atom in which an electron orbits a stationary proton in a circular path.
Find the total mechanical energy of the electron given that the radius of the orbit is $0.53 \times 10^{-10}\ m$.

The mechanical energy is the sum of the kinetic and potential energies, E=K+U. The centripetal force is provided by the coulomb attraction.



$$U = -\frac{ke^2}{r}$$

$$F = \frac{ke^2}{r^2} = \frac{mv^2}{r} \Rightarrow K = \frac{1}{2}mv^2 = \frac{ke^2}{2r}$$

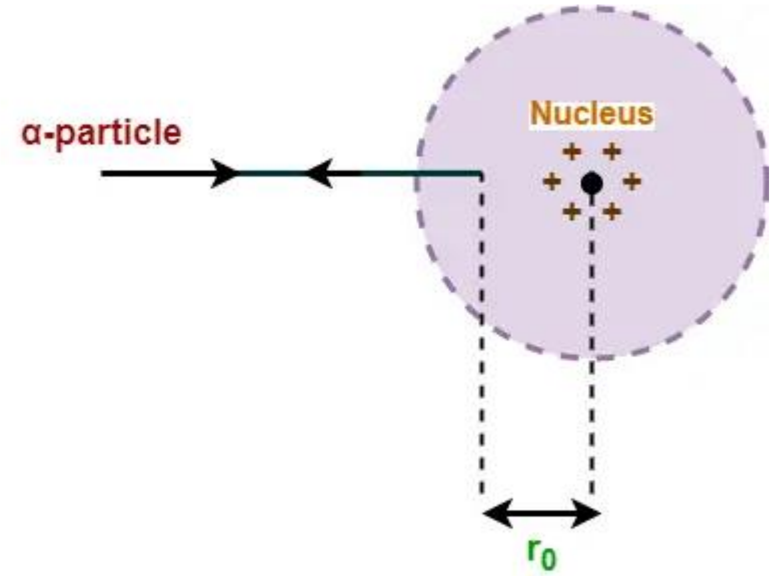
$$E = U + K = -\frac{1}{2}U = -\frac{9\times10^9\times(1.6\times10^{-19})^2}{2\times0.53\times10^{-10}} = -2.18\times10^{-18}\ \mathrm{J} = -13.6\ \mathrm{eV}$$

1

How much work must be done to completely separate the electron and the proton..

To remove the electron from the atom, i.e. to move it very far away and give it zero kinetic energy, 13.6 eV of work must be done by an external force. 13.6 eV is the ionization energy of hydrogen.

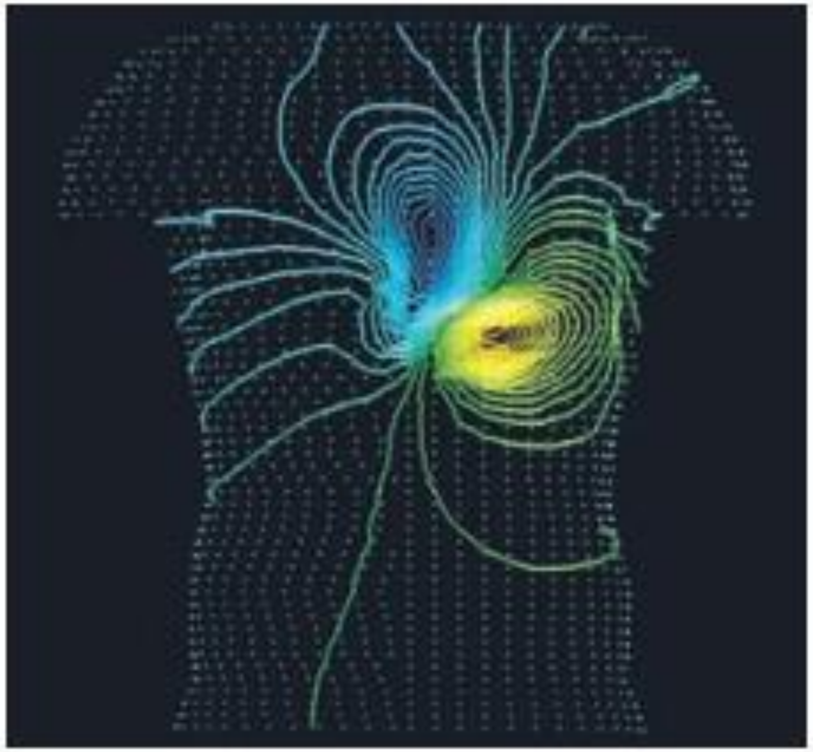
An alpha particle containing two protons is shot directly towards a platinum nucleus containing 78 protons from a very large distance with a kinetic energy of 1.7×10^{-12} J. What will be the distance of closest approach?



Hint: The alpha particle and the nucleus repel each other. As the alpha particle moves towards the nucleus, some of its kinetic energy will be converted into electrostatic potential energy. At the distance of closest approach, the alpha particle's velocity is zero, and all its initial kinetic energy has been converted into electrostatic potential energy.

Application **Electrocardiography**

The electrodes used in an electrocardiogram—EKG or ECG for short—measure the potential differences (typically no greater than $1\ mV = 10^{-3}\ V$) between different parts of the patient's skin. These are indicative of the potential differences between regions of the heart, and so provide a sensitive way to detect any abnormalities in the electrical activity that drives cardiac function.



Equipotentials on the chest of a human are a slightly distorted electric dipole.