Inductor and Inductance-I

Phy 108 course
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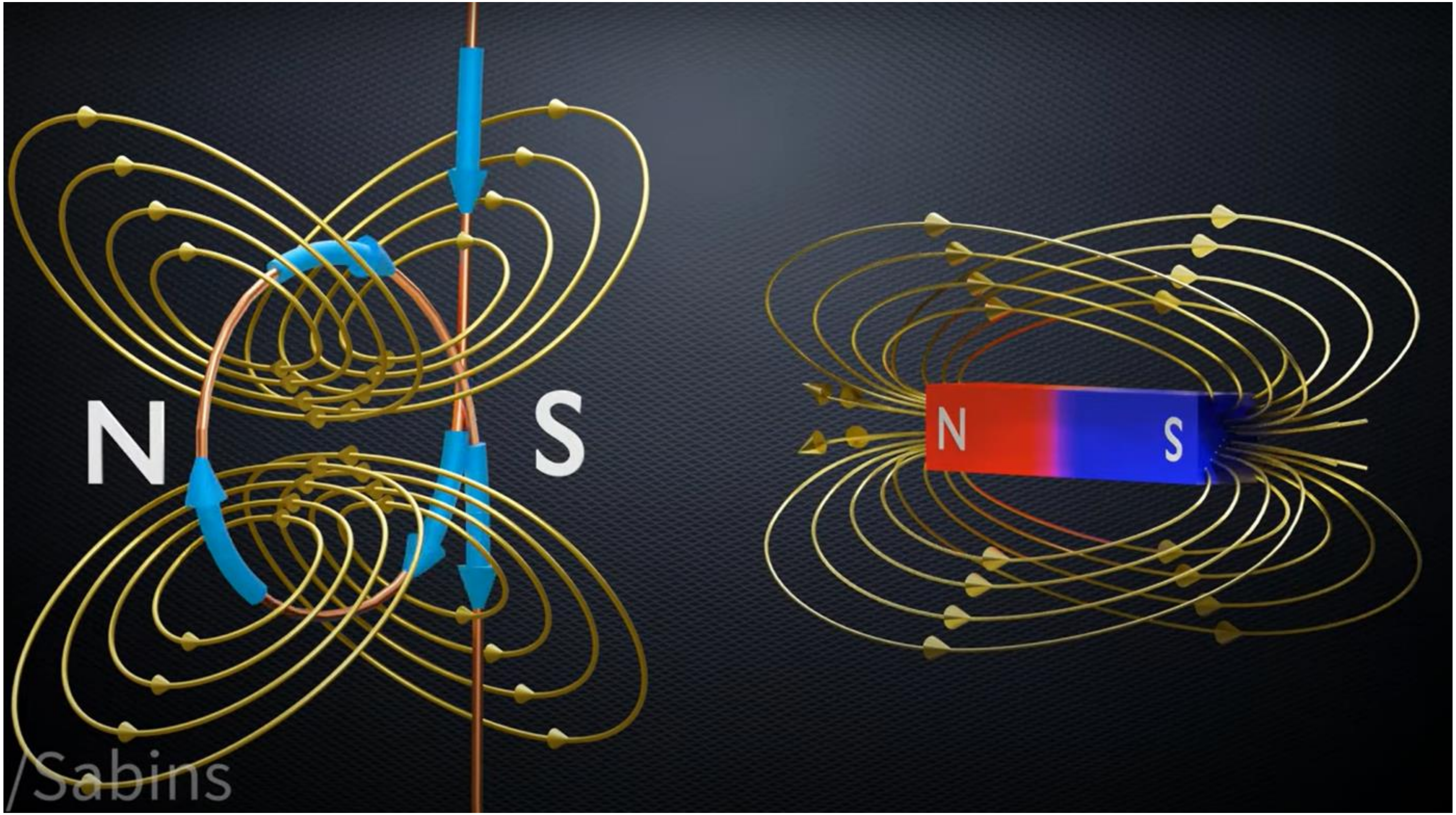
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The **inductor** is a passive component which stores the energy in the magnetic field when the electric current passes through it.
Inductance is the property of a device that tells us how effectively it induces an emf in another device. In other words, it is a physical quantity that expresses the effectiveness of a given device.

Like resistors and capacitors, inductors are among the indispensable circuit elements of modern electronics. One of their purpose is to oppose any variations in the current through the circuit.

An inductor in a direct-current circuit helps to maintain a steady current despite any fluctuations in the applied emf; in an alternating current circuit, an inductor tends to suppress variations of the current that are more rapid than desired.

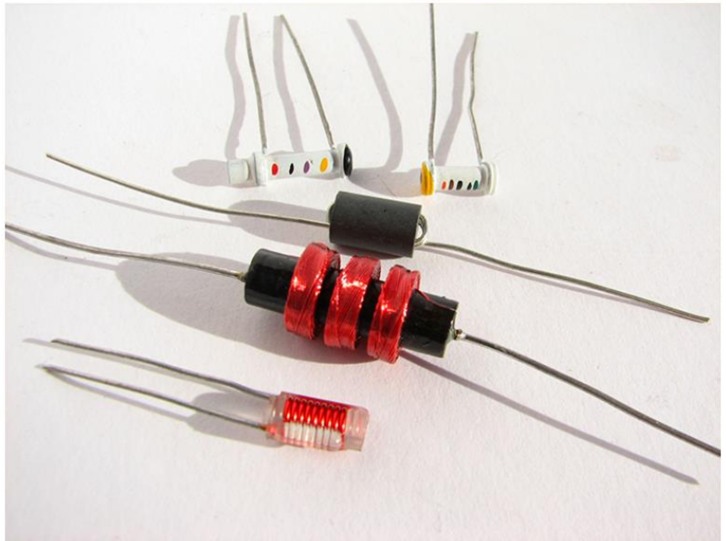
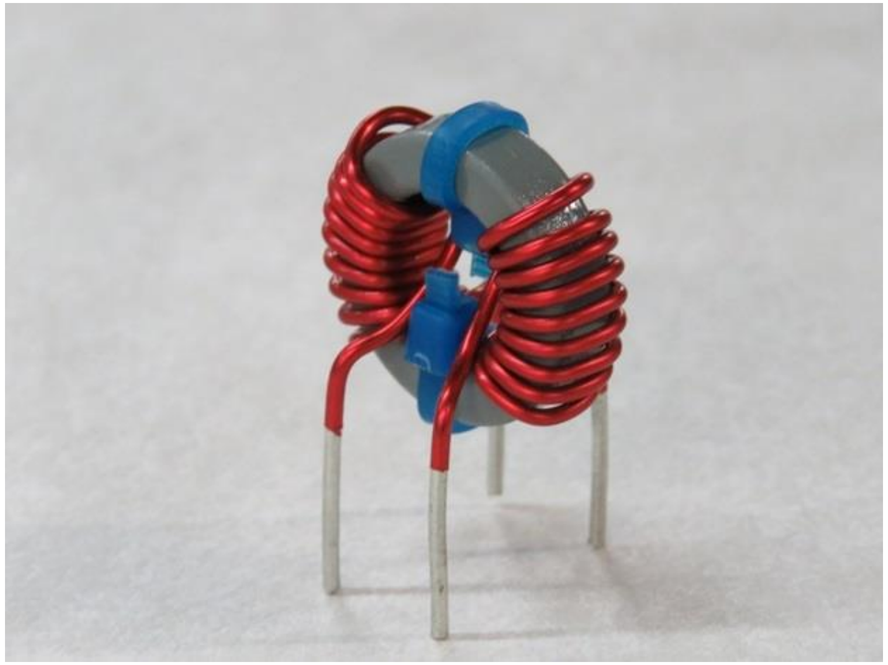


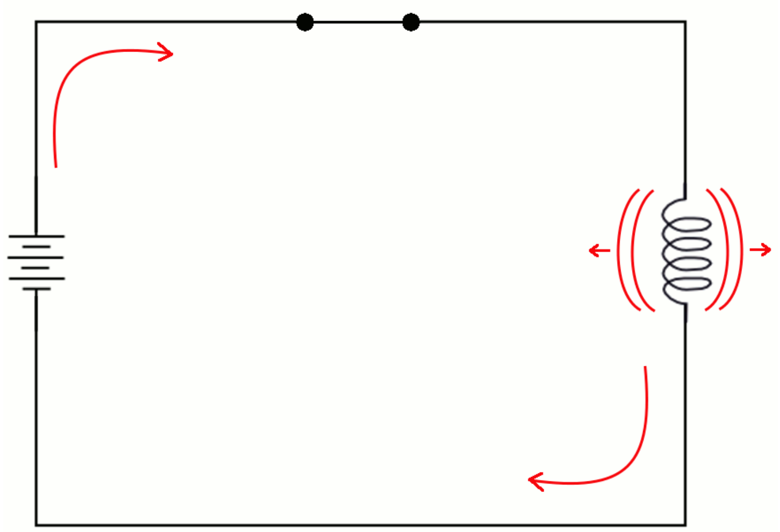
Figure 14.7 A variety of inductors. Whether they are encapsulated like the top three shown or wound around in a coil like the bottom-most one, each is simply a relatively long coil of wire. (credit: Windell Oskay)

An inductor is an electromagnetic component used to resist the changes in current in a circuit. It consists of a wire wound into a coil, which temporarily stores energy in a magnetic field — they are like an electrical version of a flywheel.



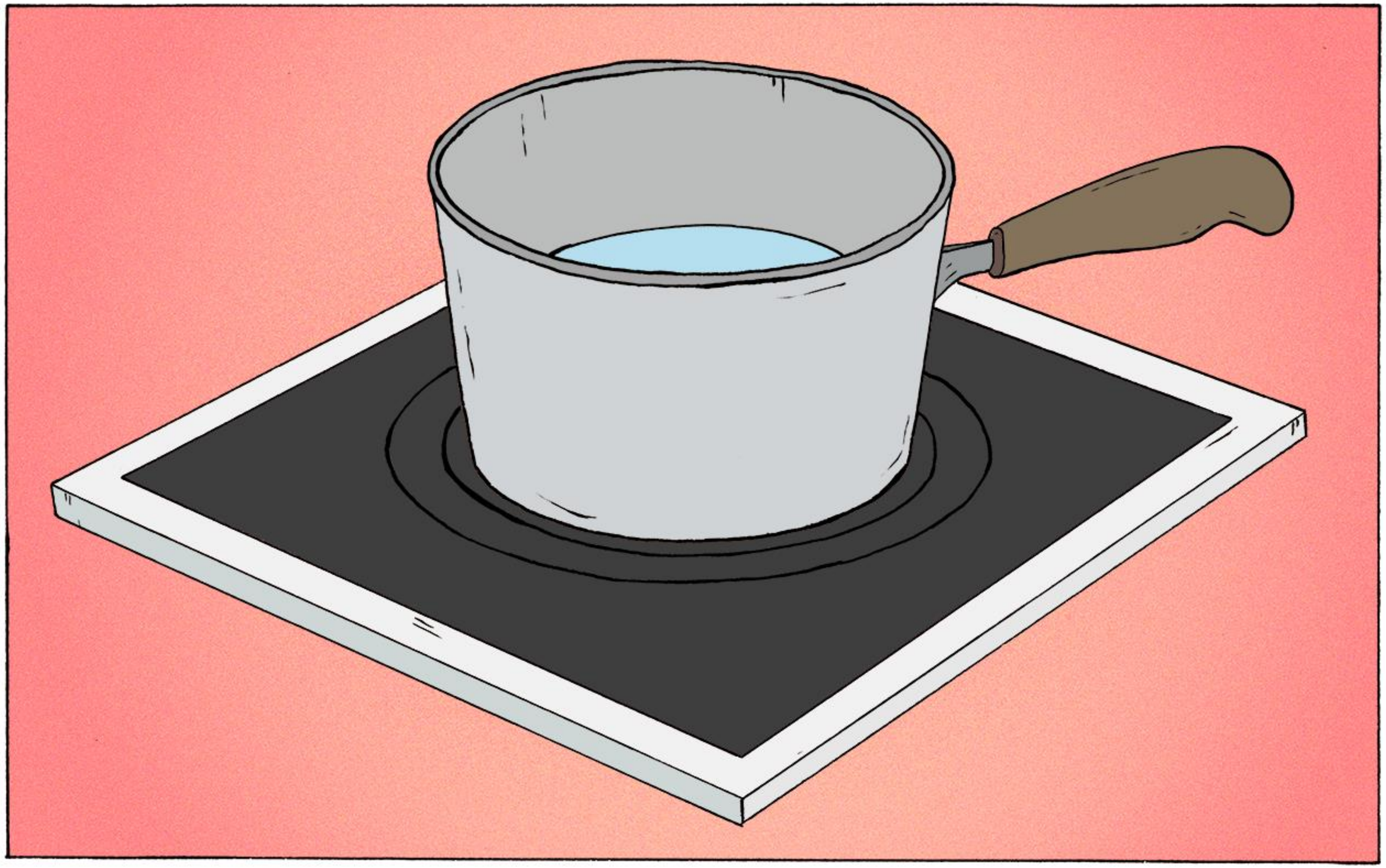
Inductors are the complementary component to the capacitor. They are bulky and expensive, and practical inductors are far from ideal.

Inductors are used as antennae for sending and receiving radio signals, and form part of transformers used in wireless charging.



To minimize lightning effects, large inductors are incorporated into the transmission system. These use the principle that an inductor opposes and suppresses any rapid changes in the current.

A smartphone charging mat contains a coil that receives alternating current, or current that is constantly increasing and decreasing. The varying current induces an emf in the smartphone, which charges its battery.



HOW AN INDUCTION COOKTOP WORKS

$$\Phi_B \propto i \qquad \Phi_B = Li \qquad \text{Magnetic Flux}$$

$$q \propto V \qquad q = CV \qquad \text{Electric Charge}$$

The windings of the inductor are said to be *linked* by the shared flux, and the product $N\Phi_B$ is called the *magnetic flux linkage*.

The inductance L is thus a measure of the flux linkage produced by the inductor per unit of current $\frac{N\Phi_B}{i}$

Because the SI unit of magnetic flux is the tesla–square meter, the SI unit of inductance is the tesla–square meter per ampere (Tm^2/A).

Inductance SI Unit: Henry (H), $1\ \boldsymbol{henry},\ 1\ \boldsymbol{H} = 1\ \boldsymbol{Tm^2/A}$

We shall consider a long solenoid (a short length near the middle of a long solenoid, no fringing effects) as our basic type of inductor.

If we establish a current i in the windings (turns) of the solenoid we are taking as our inductor, the current produces a magnetic flux Φ_B through the central region of the inductor.

- ➔Inductance in a coil of wire defined by $L = \dfrac{N\Phi_B}{i}$
- ➔Can also be written $Li = N\Phi_B$
- ➔From Faraday's law $\mathcal{E} = -N\dfrac{d\Phi_B}{dt} = -L\dfrac{di}{dt}$
  - ◆ This is a more useful way to understand inductance
- ➔Inductors play an important role in circuits when current is changing!

Self - Inductance

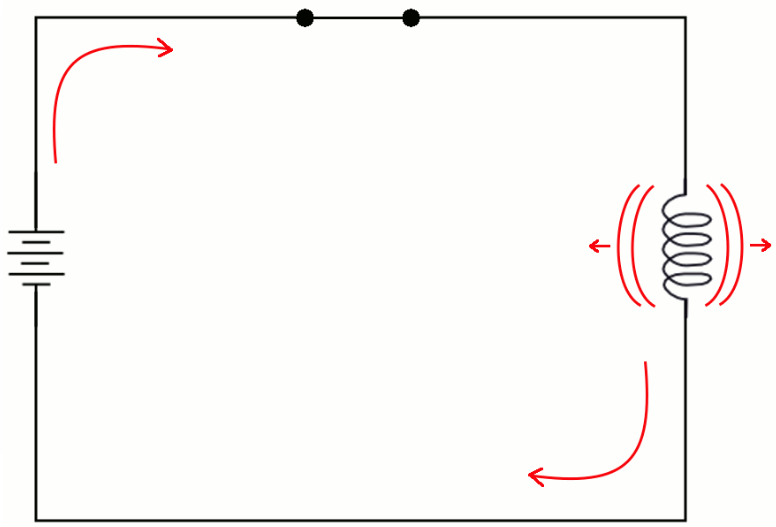
➔Consider a single isolated coil:

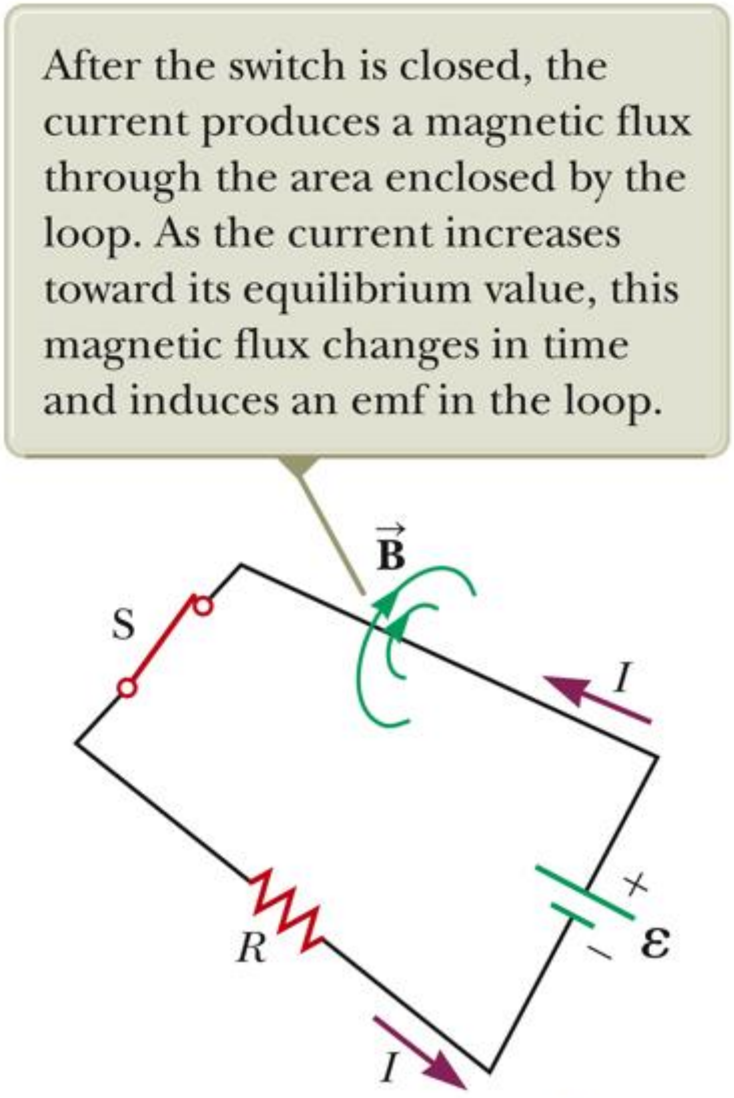
- Current (red) starts to flow clockwise due to the battery
- But the buildup of current leads to changing flux in loop
- Induced emf (green) opposes the change

This is a self-induced emf (also called "back" emf)

$$\mathcal{E} = -N\frac{d\Phi}{dt} = -L\frac{di}{dt}$$

L is the self-inductance
units = "Henry (H)"





Consider a uniformly wound solenoid having *N* turns and length ,L. Assume , L is much longer than the radius of the windings and the core of the solenoid is air. Find the inductance of the solenoid.

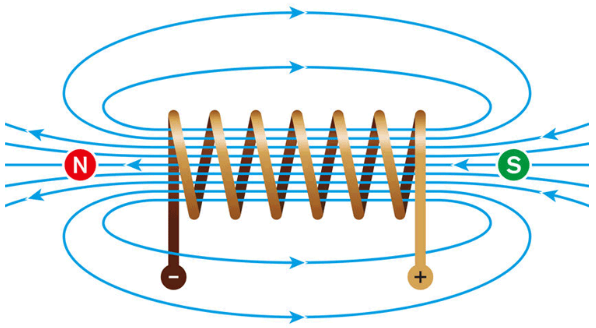
Inductance of Solenoid

➔Total flux (length *l*)

$B = \mu_0 i n$ Solenoid magnetic field

Consider a length *l* near the middle of this solenoid. The flux linkage there is

$$N\Phi_B = (nl)(BA) = \mu_0 n^2 A l i$$



$$\mathcal{E} = -N\frac{d\Phi_B}{dt} = -\mu_0 n^2 A l \frac{di}{dt} = -L\frac{di}{dt}$$

$$L = \mu_0 n^2 A l$$

To make large inductance:
- Lots of windings
- Big area
- Long

Inductance like capacitance depends only on the geometry of the device.

$$\boldsymbol{L = \mu_0 n^2 A l}$$

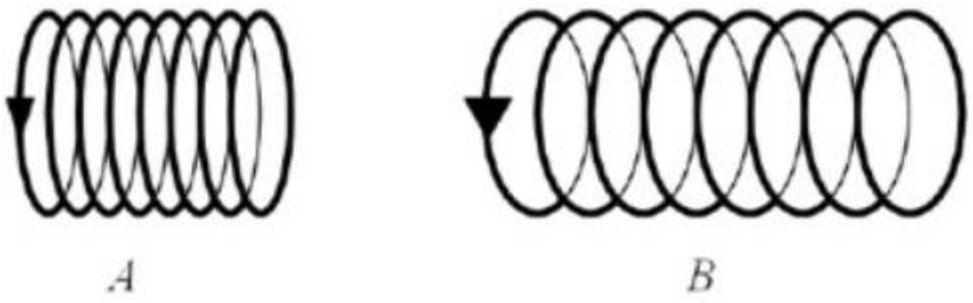
Here *n* is a number per unit length, and the permeability constant unit can be expressed as,

$$\mu_0 = 4\pi \times 10^{-7} T.\frac{m}{A} = 4\pi \times 10^{-7} H/m$$

$$\boldsymbol{L = \frac{\mu_0 N^2 A}{l}}$$

The inductance of a solenoid depends only on its geometry, not at all on the current.

Two solenoids are made with the same cross sectional area and total number of turns. Inductor *B* is twice as long as inductor *A. Compare the inductance of the two solenoids*

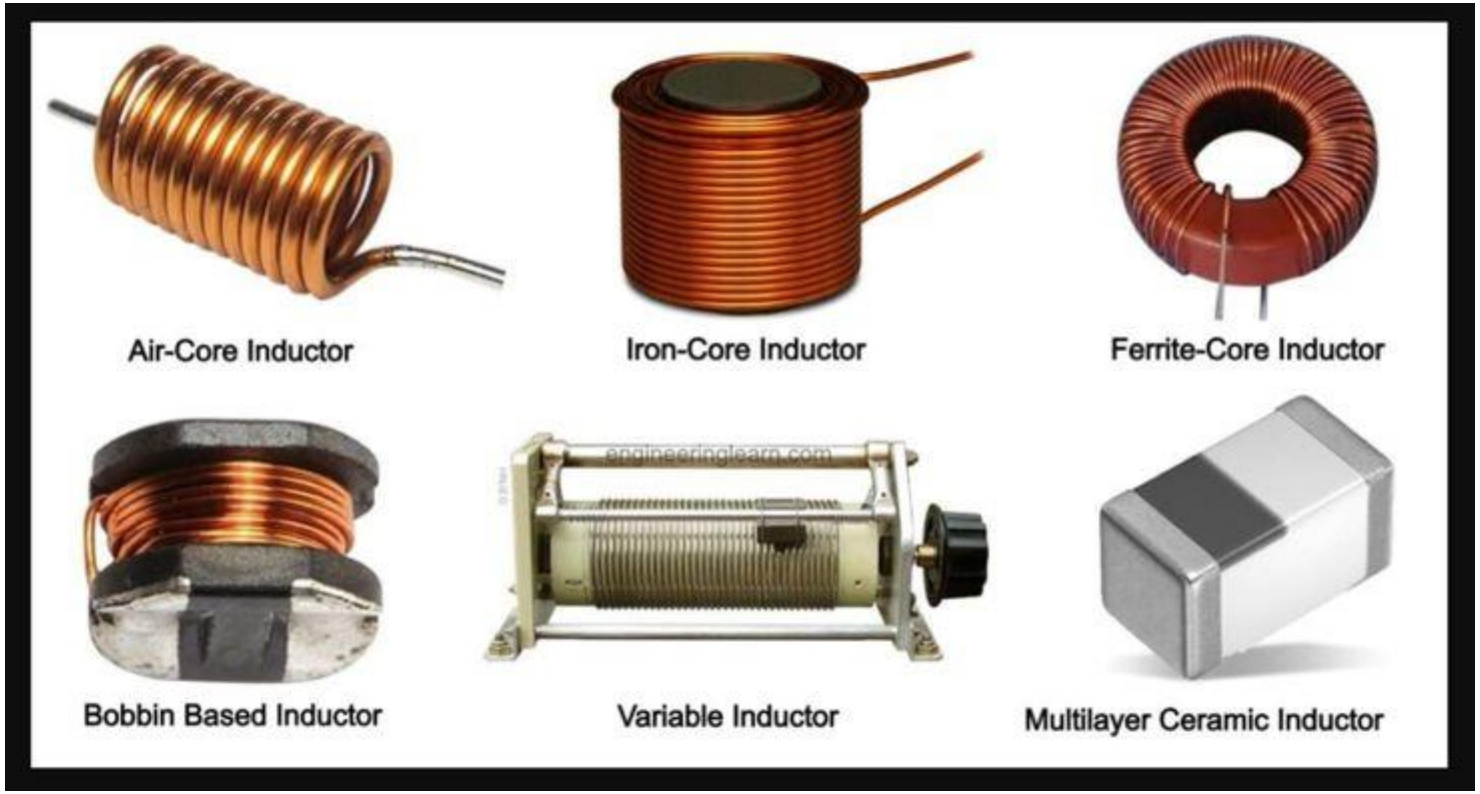


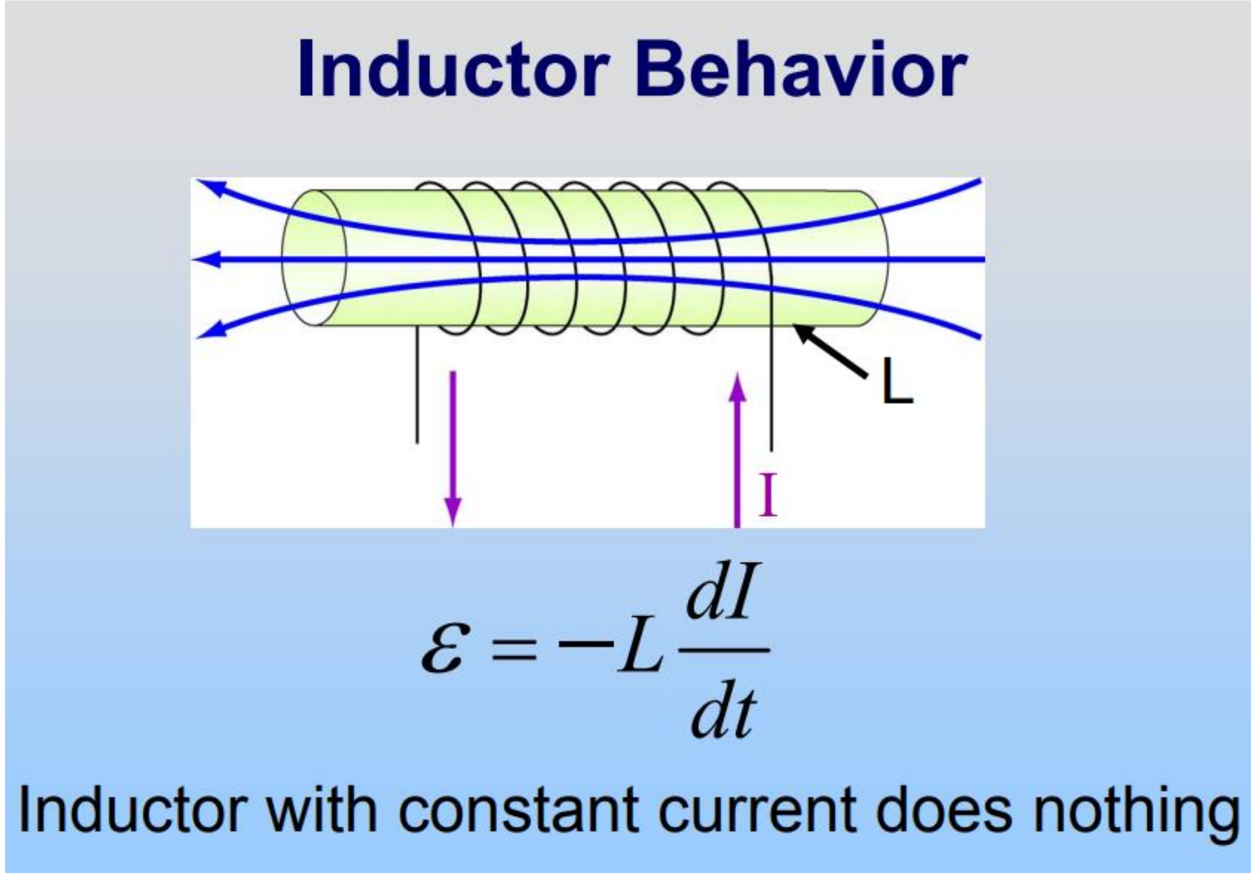
$$L = \frac{\mu_0 N^2 A}{l}$$

Inductors consist of a magnetic iron or ferrite core inside an insulated copper coil.

For high frequency applications, ferrite or powdered (distributed gap) cores are used. The high permeability of the ferromagnetic core creates a large magnetic field and, therefore, a greater level of inductance.

Low frequency inductors can be built more in the style of transformers, featuring cores of electrical steel.





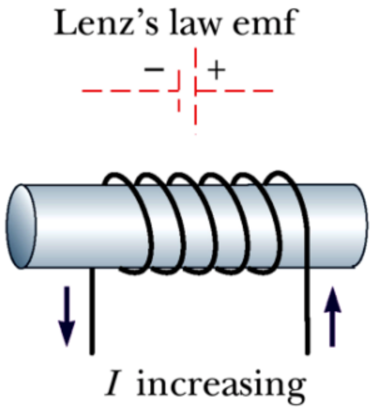
emf induced across L tries to keep I constant.

Inductors prevent discontinuous current changes!

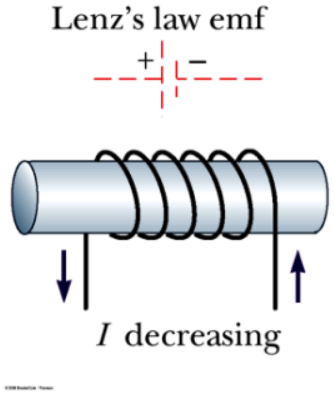
Self-Inductance: Analogous to inertia

ANY magnetic flux change is resisted. Changing current in a single coil induces a “back EMF” ε_{ind} in the same coil opposing the current change, an induced current i_{ind}, and a consistent induced field B_{ind}.

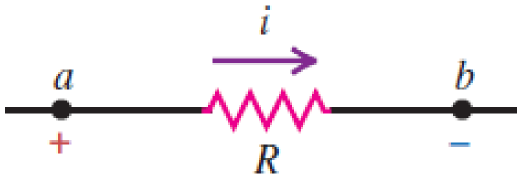
- For increasing current, back EMF limits the rate of increase



- For decreasing current, back EMF sustains the current

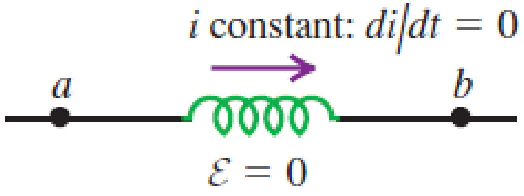


(a) Resistor with current i flowing from a to b: potential drops from a to b.



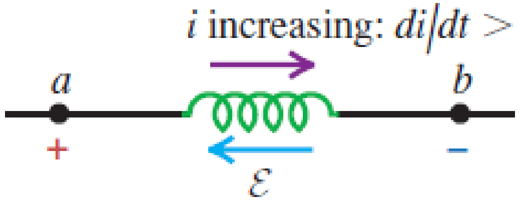
$V_{ab} = iR > 0$

(b) Inductor with *constant* current i flowing from a to b: no potential difference.



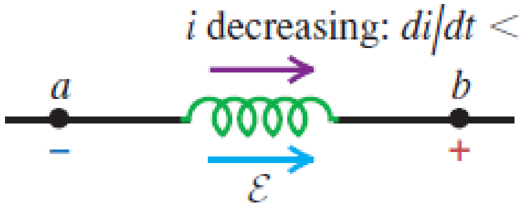
$V_{ab} = L\dfrac{di}{dt} = 0$

(c) Inductor with *increasing* current i flowing from a to b: potential drops from a to b.

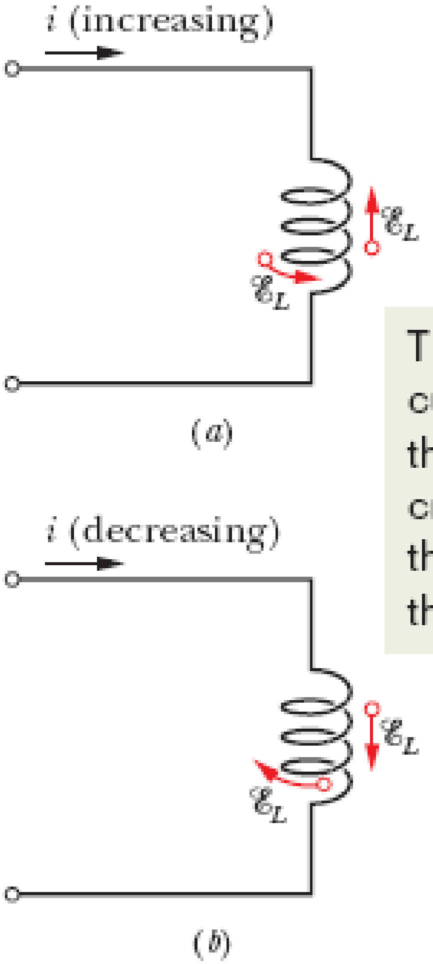


$V_{ab} = L\dfrac{di}{dt} > 0$

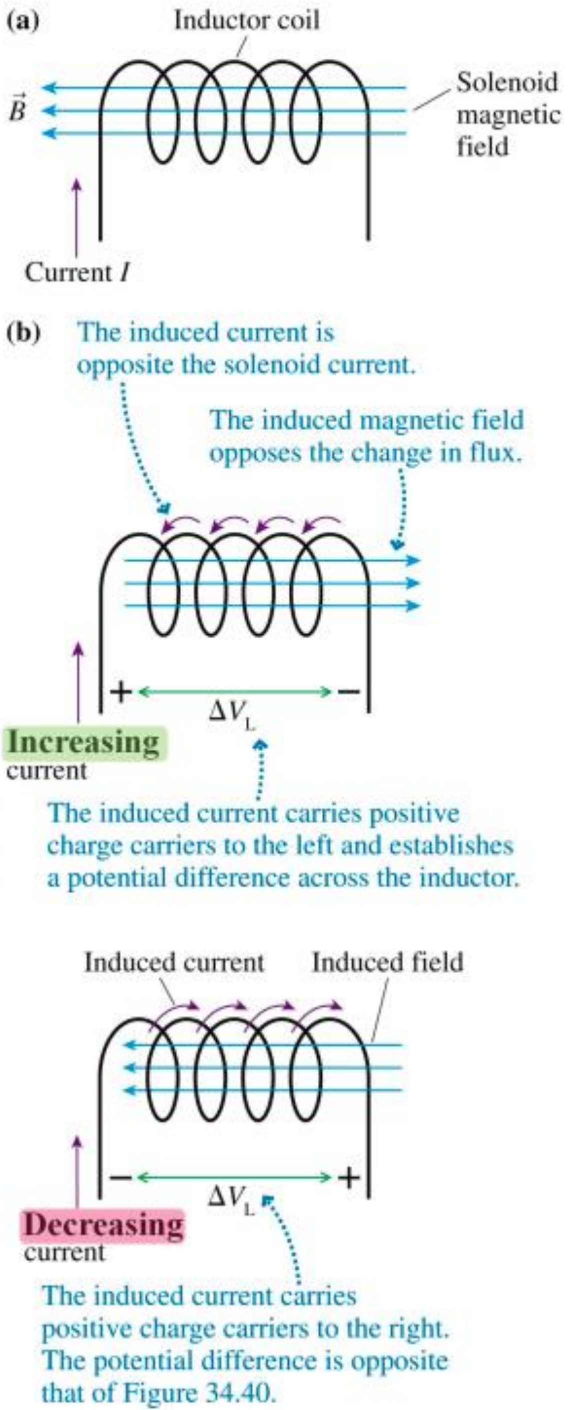
(d) Inductor with *decreasing* current i flowing from a to b: potential increases from a to b.



$V_{ab} = L\dfrac{di}{dt} < 0$



The changing current changes the flux, which creates an emf that opposes the change.



If i is <u>increasing</u>: $\dfrac{d\Phi_B}{dt} > 0$

$\therefore$ $\mathcal{E}_L$ <u>opposes increase</u> in i

Power is being stored in B field of inductor

If i is <u>decreasing</u>: $\dfrac{d\Phi_B}{dt} < 0$

$\therefore$ $\mathcal{E}_L$ <u>opposes decrease</u> in i

Power is being tapped from B field of inductor

Calculate the inductance of the solenoid if it contains 300 turns, its length is 25.0 cm, and its cross-sectional area is 4.0 cm^2. Calculate the self-induced emf in the solenoid if the current it carries decreases at the rate of 50.0 A/s.

Current I increases uniformly from 0 to 1 A. in 0.1 seconds. Find the induced voltage (back EMF) across a 50 mH (milli-Henry) inductance.

Series inductors:

$$v = v_1 + v_2 = L_1\frac{di}{dt} + L_2\frac{di}{dt}$$
$$= (L_1 + L_2)\frac{di}{dt}$$

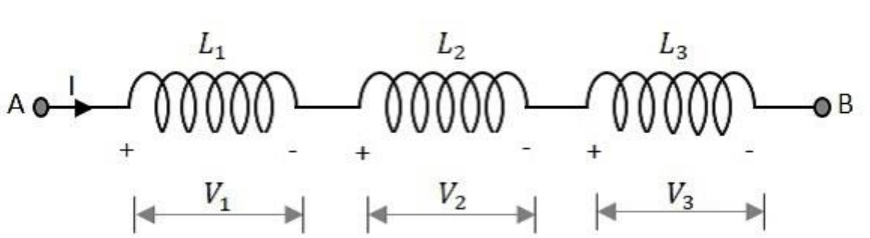
- Same equation as a single inductor of value $L_1 + L_2$

Parallel inductors:

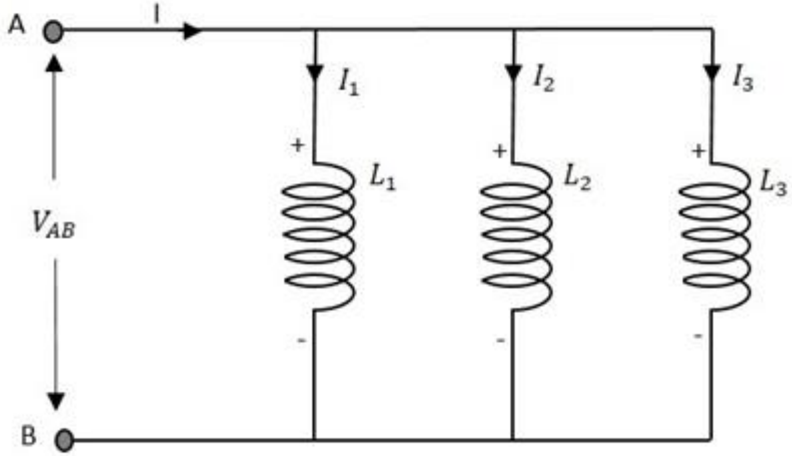
$$\frac{di}{dt} = \frac{d(i_1+i_2)}{dt} = \frac{di_1}{dt} + \frac{di_2}{dt}$$
$$= \frac{v}{L_1} + \frac{v}{L_2} = v\left(\frac{1}{L_1} + \frac{1}{L_2}\right)$$
$$v = \frac{1}{\frac{1}{L_1}+\frac{1}{L_2}}\frac{di}{dt}$$

- Same as a single inductor of value $\frac{1}{\frac{1}{L_1}+\frac{1}{L_2}} = \frac{L_1 L_2}{L_1+L_2}$

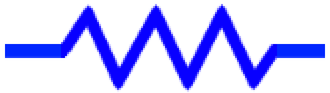
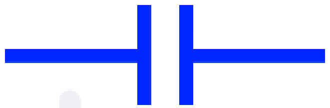
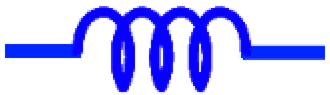
Inductors in Series



Inductors in Parallel



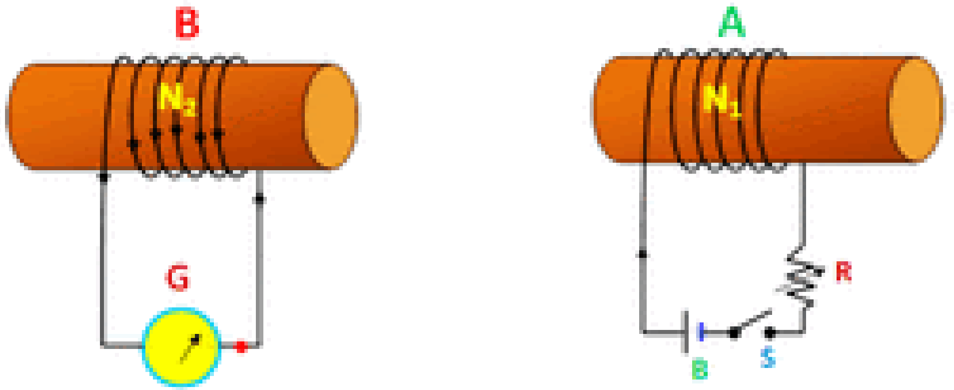
Series and parallel inductors combine just like resistors do.

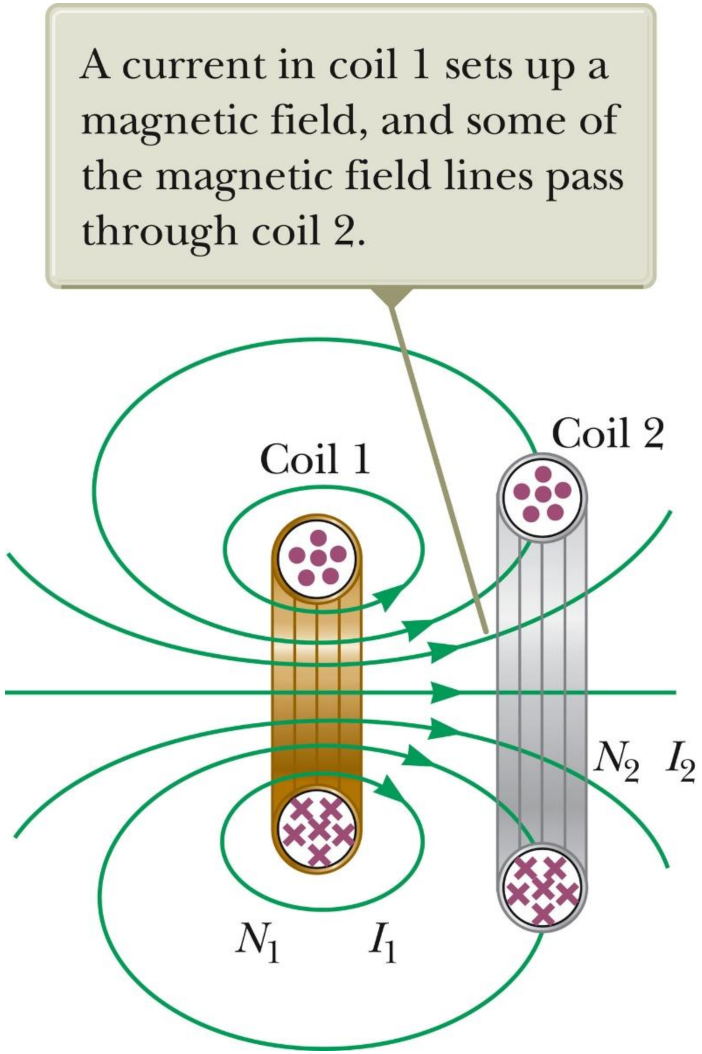
Elements Symbol	**RESISTOR** 	**CAPACITOR** 	**INDUCTOR** 
Denoted by	**R**	**C**	**L**
Equation	$R = \frac{V}{I}$	$C = \frac{Q}{V}$	$L = \frac{V_L}{(di/dt)}$
Series	$R_T = R_1 + R_2$	$\frac{1}{C_T} = \frac{1}{C_1} + \frac{1}{C_2}$	$L_T = L_1 + L_2$
Parallel	$\frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2}$	$C_T = C_1 + C_2$	$\frac{1}{L_T} = \frac{1}{L_1} + \frac{1}{L_2}$

Mutual Induction

As we have seen previously, changes in the magnetic flux due to one circuit can effect what goes on in other circuits. The changing magnetic flux induces an emf in the second circuit

If coils 1 and 2 are near each other, a changing current in either coil can induce an emf in the other.

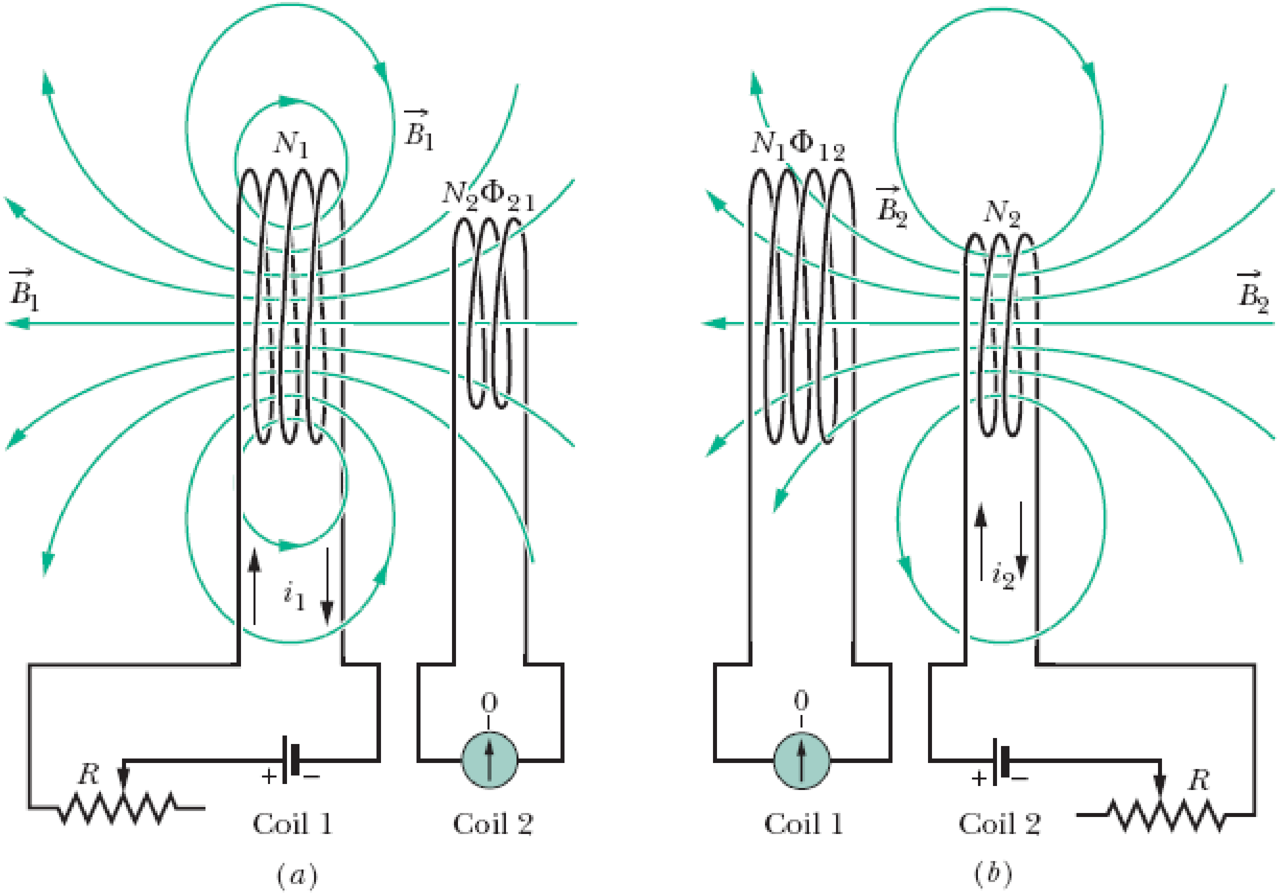




Suppose that we have two coils.

Coil 1 with N_1 turns and Coil 2 with N_2 turns, Coil 1 has a current i_1 which produces a magnetic flux, Φ_{B1}, going through one turn of Coil 2 if i_1 changes, then the flux changes and an emf is induced in Coil 2 which is given by,

$$\mathcal{E}_2 = -N_2 \frac{d\varphi_{B1}}{dt}$$

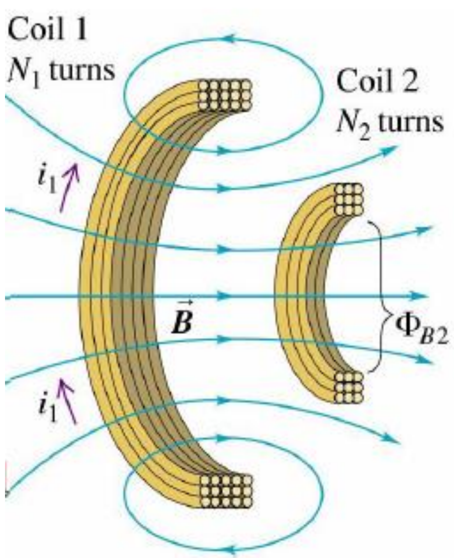


The flux through the second coil is proportional to the current in the first coil

$$N_2\Phi_{B2} = M_{21}i_1 \qquad M = \frac{N\Phi_B}{i} = \frac{(nl)BA}{i}$$

where M_{21} is called the mutual inductance

Taking the time derivative of this we get $N_2\frac{d\Phi_{B2}}{dt} = M_{21}\frac{di_1}{dt}$ or $\varepsilon_2 = -M_{21}\frac{di_1}{dt}$



If we were to start with the second coil having a varying current, we would end up with a similar equation with an M_{12}

We would find that, $M_{12}=M_{21}=M$

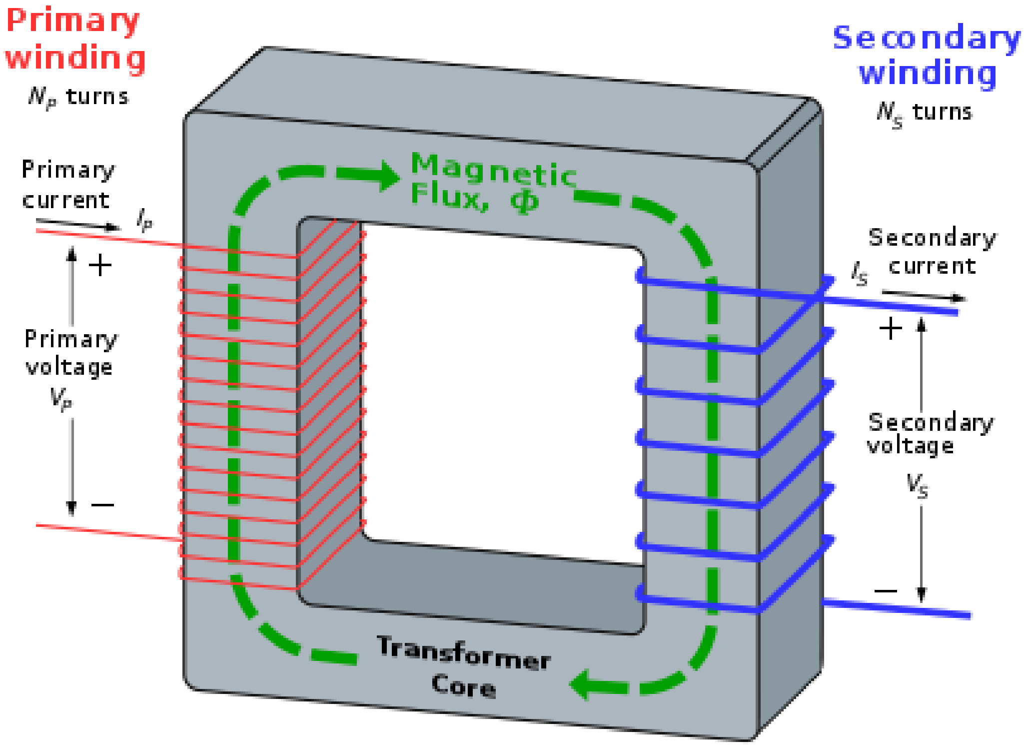
The two mutual inductances are the same because the mutual inductance is a geometrical property of the arrangement of the two coils.

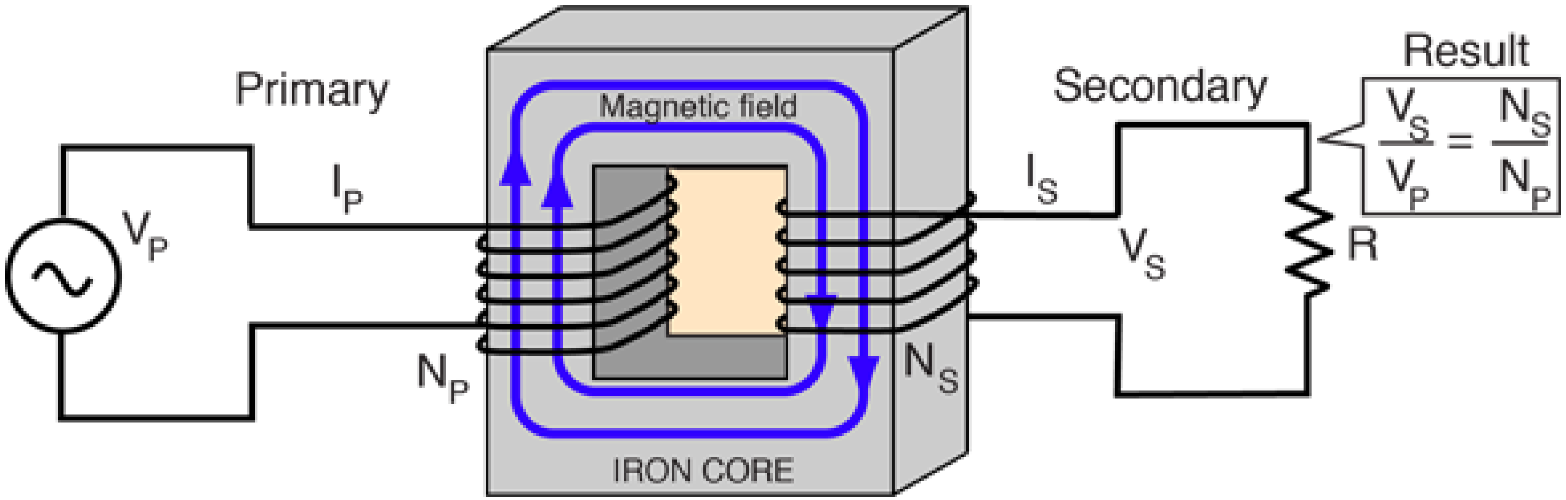
To measure the value of the mutual inductance you can use either

$$\mathcal{E}_2 = -M\frac{di_1}{dt} \text{ or } \mathcal{E}_1 = -M\frac{di_2}{dt}$$

This persistent generation of voltages which oppose the change in magnetic field is the operating principle of a transformer. The fact that a change in the current of one coil affects the current and voltage in the second coil is quantified in the property called mutual inductance.

The iron core acts to strengthen the flux and to bring it through the secondary winding





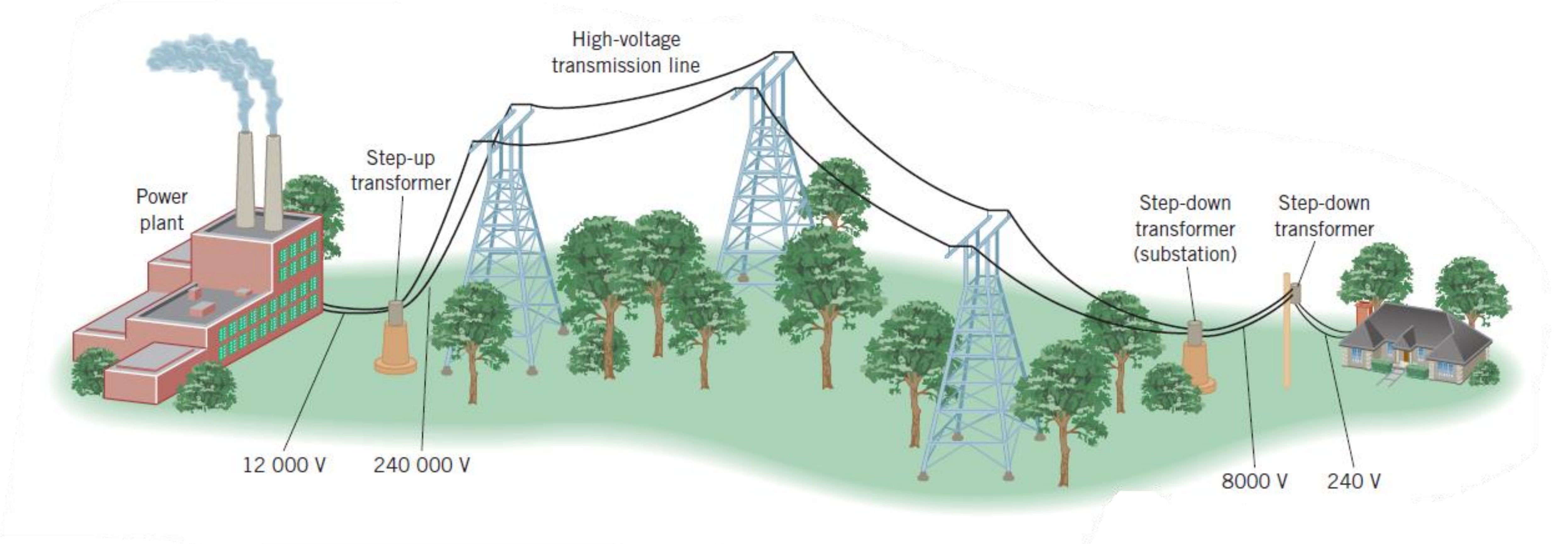
Across the primary, the voltage Vp is the product of $\mathcal{E}turn$ and the number of turns in the primary, Np, i.e.,

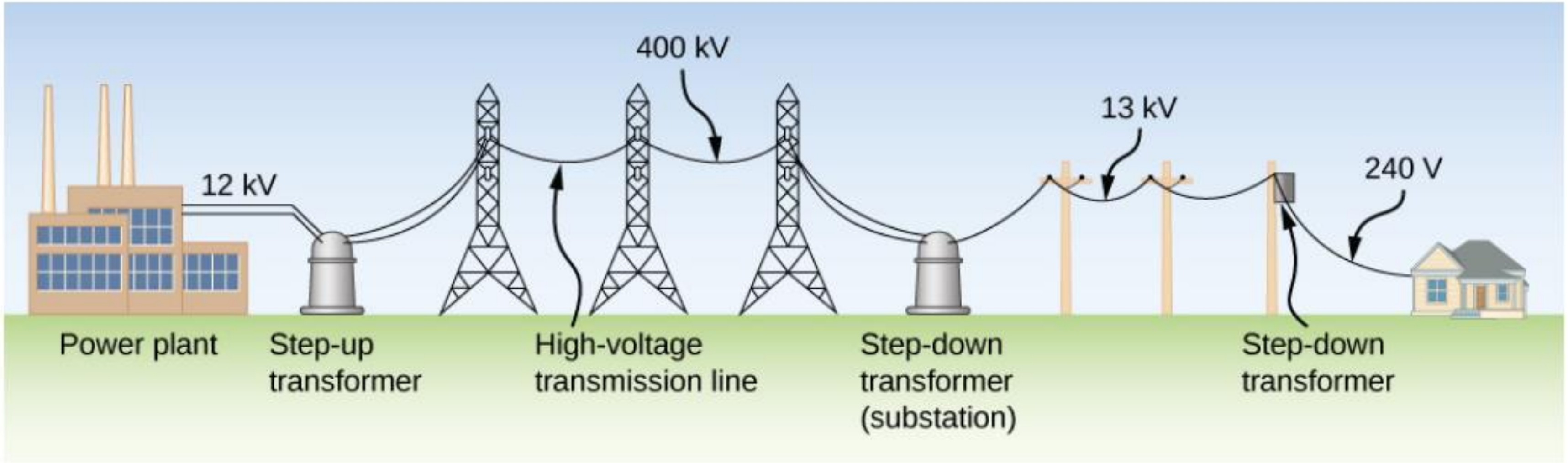
Vp = $\mathcal{E}_{ind} * N_p$ and for the secondary Vs = $\mathcal{E}_{ind} * N_s$

$$\varepsilon_{ind} = \frac{V_p}{N_p} = \frac{V_S}{N_S}$$

If $Ns > Np$, the device is called a step-up transformer because the secondary voltage is greater than the primary voltage.

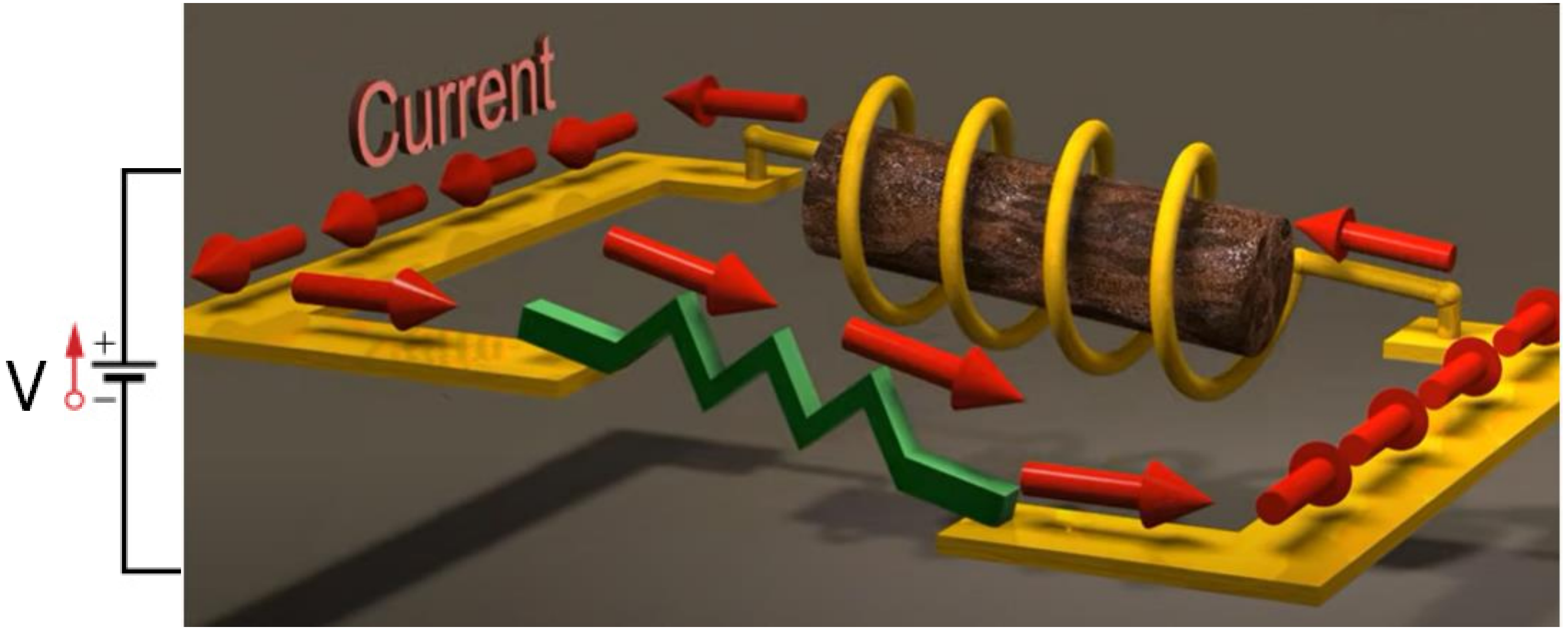
If $Np > Ns$, the device is called a step-down transformer because the secondary voltage is smaller than the primary voltage.


High-voltage
transmission line
Step-up
transformer
Power
plant
Step-down
transformer
(substation)
Step-down
transformer
12 000 V
240 000 V
8000 V
240 V


400 kV
13 kV
12 kV
240 V
Power plant
Step-up
transformer
High-voltage
transmission line
Step-down
transformer
(substation)
Step-down
transformer

LR Circuits

- ➔Inductance and resistor in series with battery of EMF V
- ➔Start with no initial current in circuit
  - ◆ Close switch at t = 0
  - ◆ Current is initially 0 (initial increase causes voltage drop across inductor)
- ➔Find $i(t)$
  - ◆ Resistor: $\Delta V = Ri$
  - ◆ Inductor: $\Delta V = L\, di/dt$



$$V - Ri - L di / dt = 0$$

Please follow the steps we used in RC circuit

Analysis of LR Circuit

➔Differential equation is $di/dt + i(R/L) = V/R$

➔General solution: $i = \frac{V}{R}\left(1 - e^{-(R/L)t}\right)$

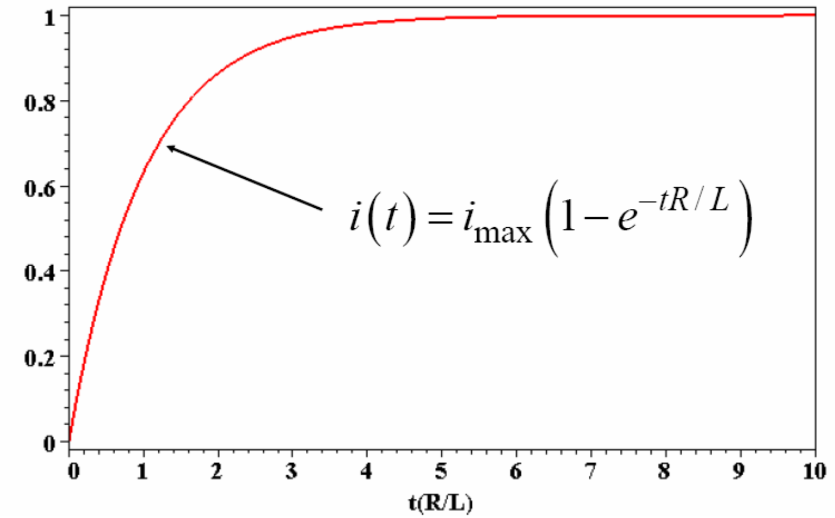
- (Check and see!)
- K = –V/R (necessary to make i = 0 at t = 0)

$$i = \frac{V}{R}\left(1 - e^{-tR/L}\right)$$

Rise from 0 with time constant $\tau = L / R$

Final current (maximum)

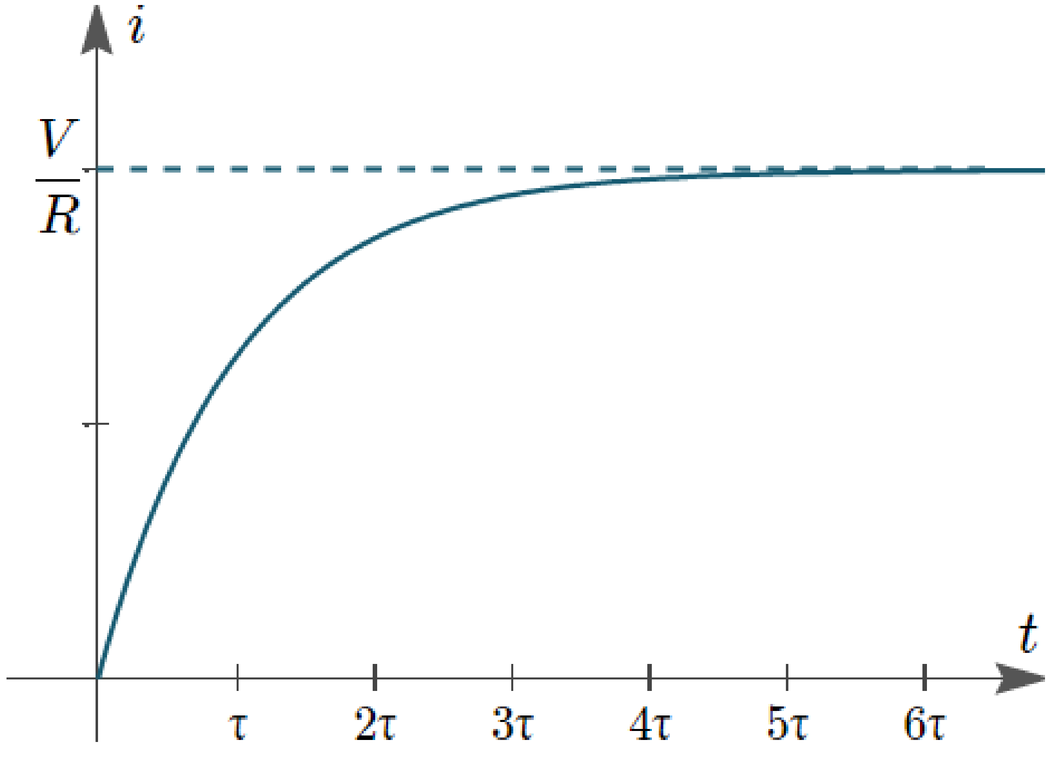
Current vs Time in RL Circuit
(Initially Zero Current in Inductor)



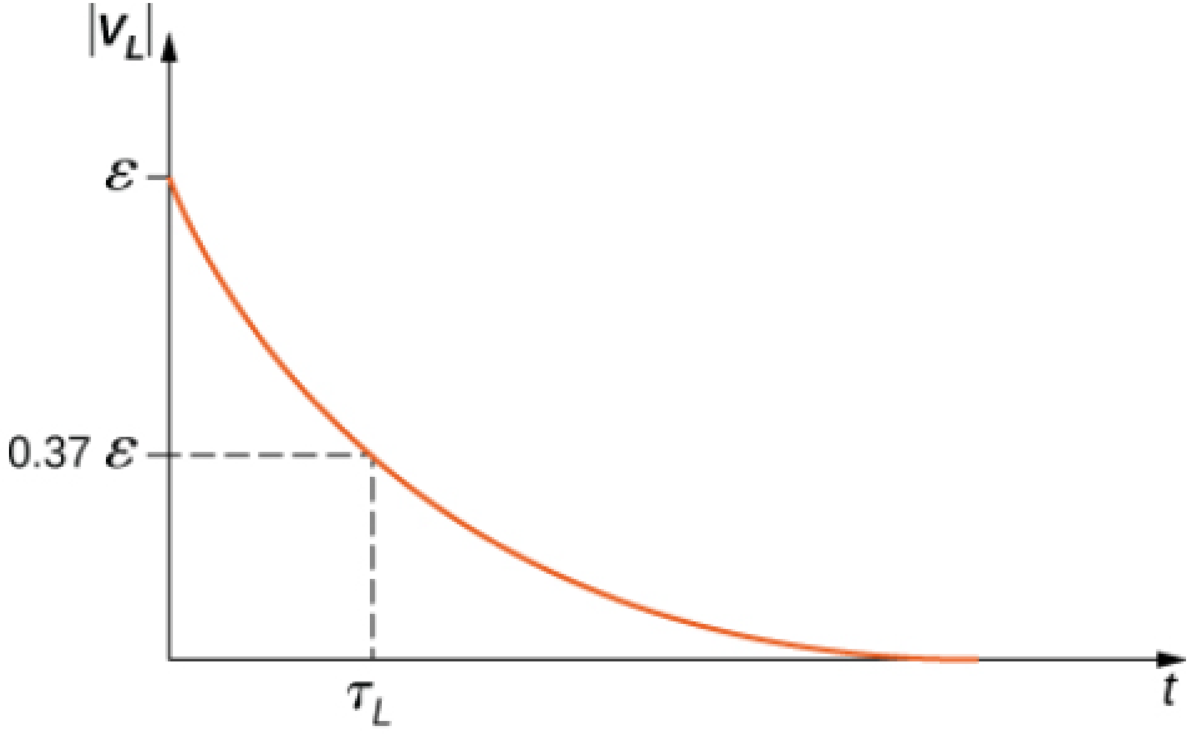
$$i = i_{max}\left(1 - e^{-t/\tau_L}\right)$$

$\tau_L = \frac{L}{R}$ time constant for LR circuit

Time variation of (a) the electric current and (b) the magnitude of the induced voltage across the coil in the circuit



Graph of $i = \frac{V}{R}\left(1 - e^{-(R/L)t}\right)$.



L-R Circuits (2)

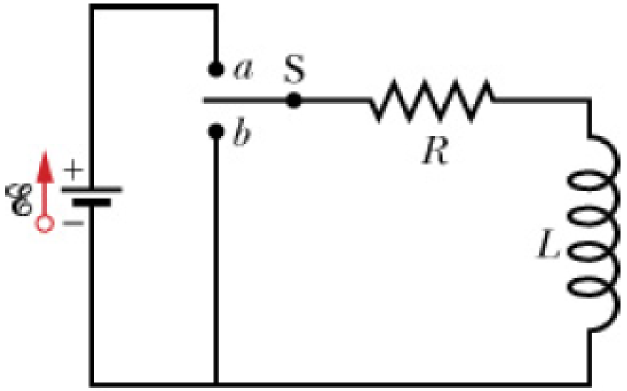
➔Switch off battery: Find $i(t)$ if current starts at i_0

$$0 = L\,di/dt + Ri$$

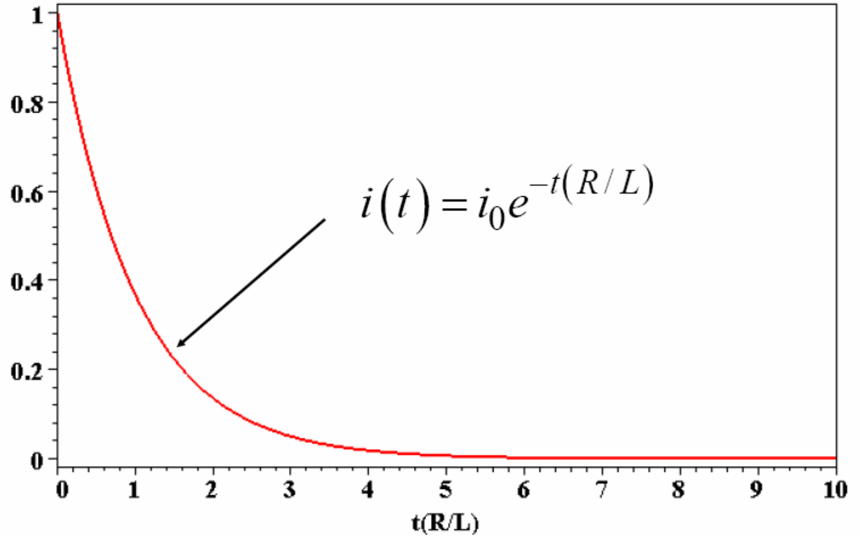
$$i = i_0 e^{-tR/L}$$

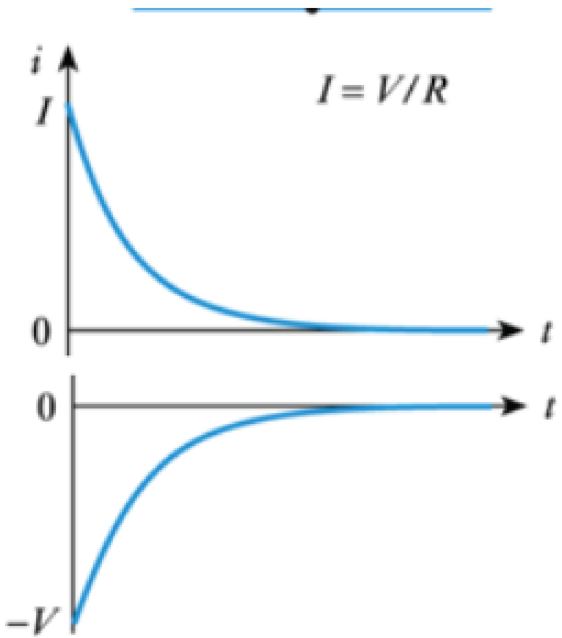
Exponential fall to 0 with time constant $\tau = L / R$

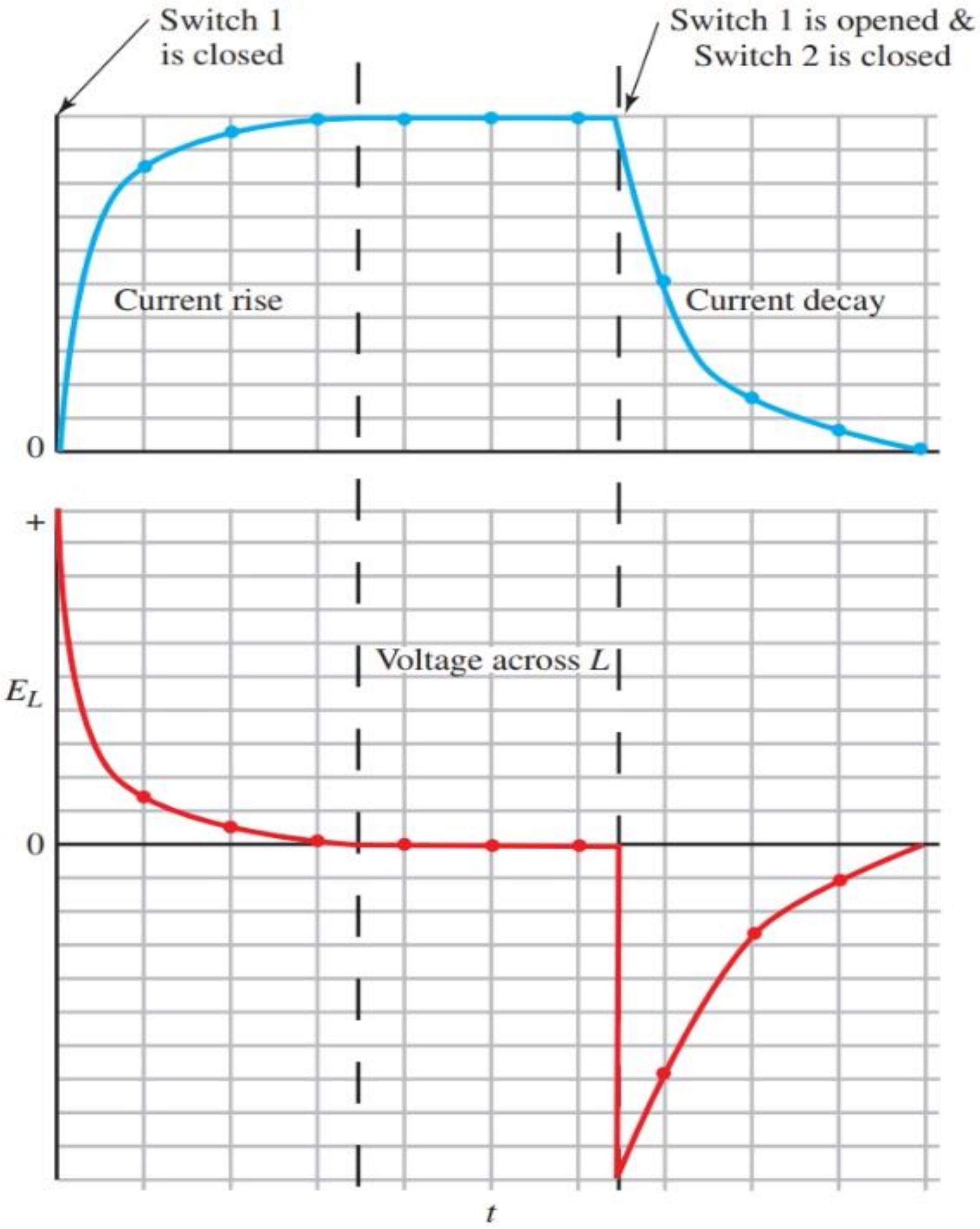
Initial current (maximum)

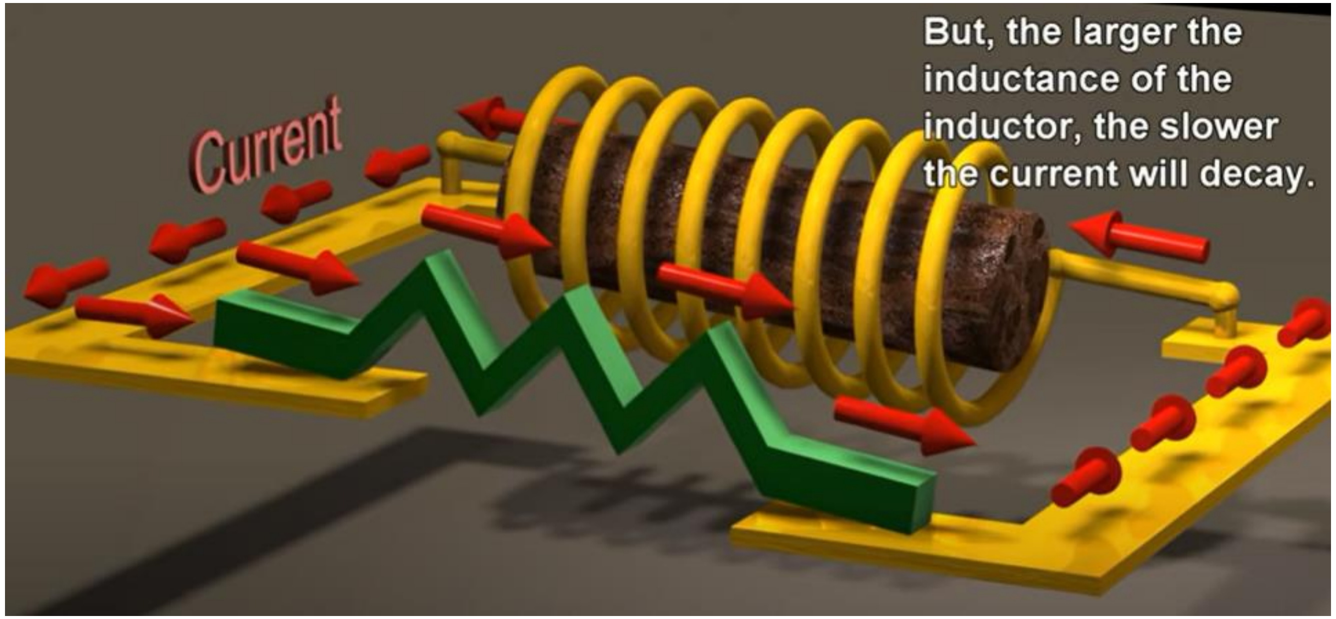


Current vs Time in RL Circuit (For Initial Current i_{max} in Inductor)



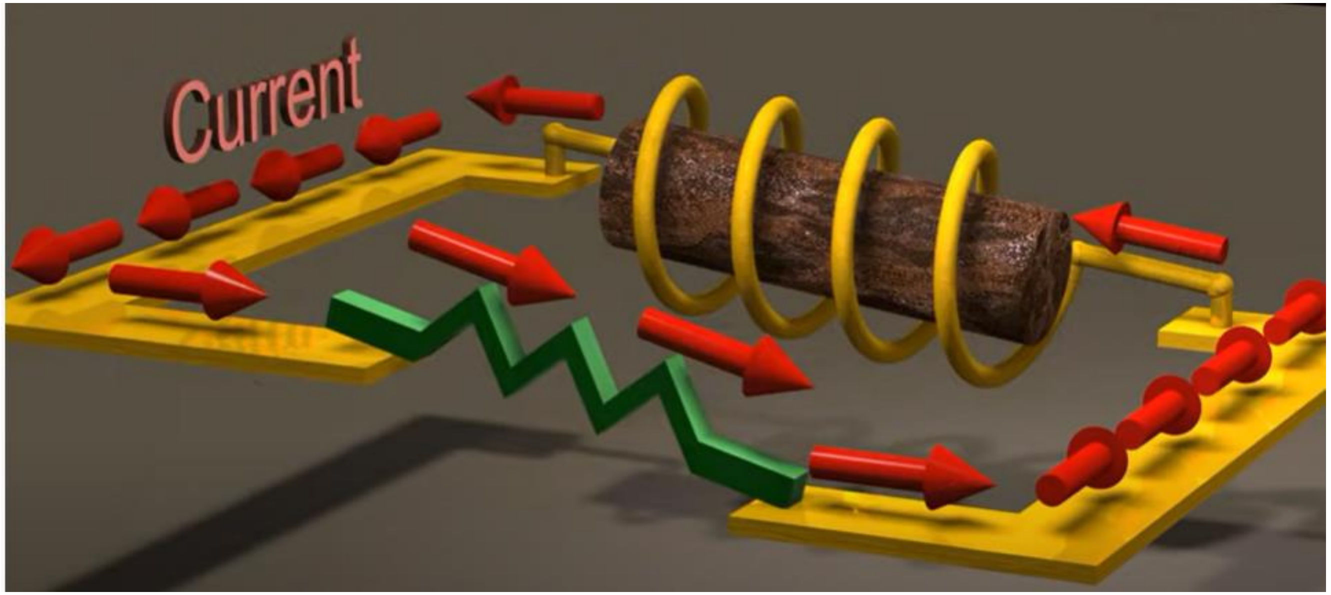



Switch 1
is closed
Switch 1 is opened &
Switch 2 is closed
Current rise
Current decay
0
+
Voltage across L
E_L
0
t



$$\tau_L = \frac{L}{R}$$

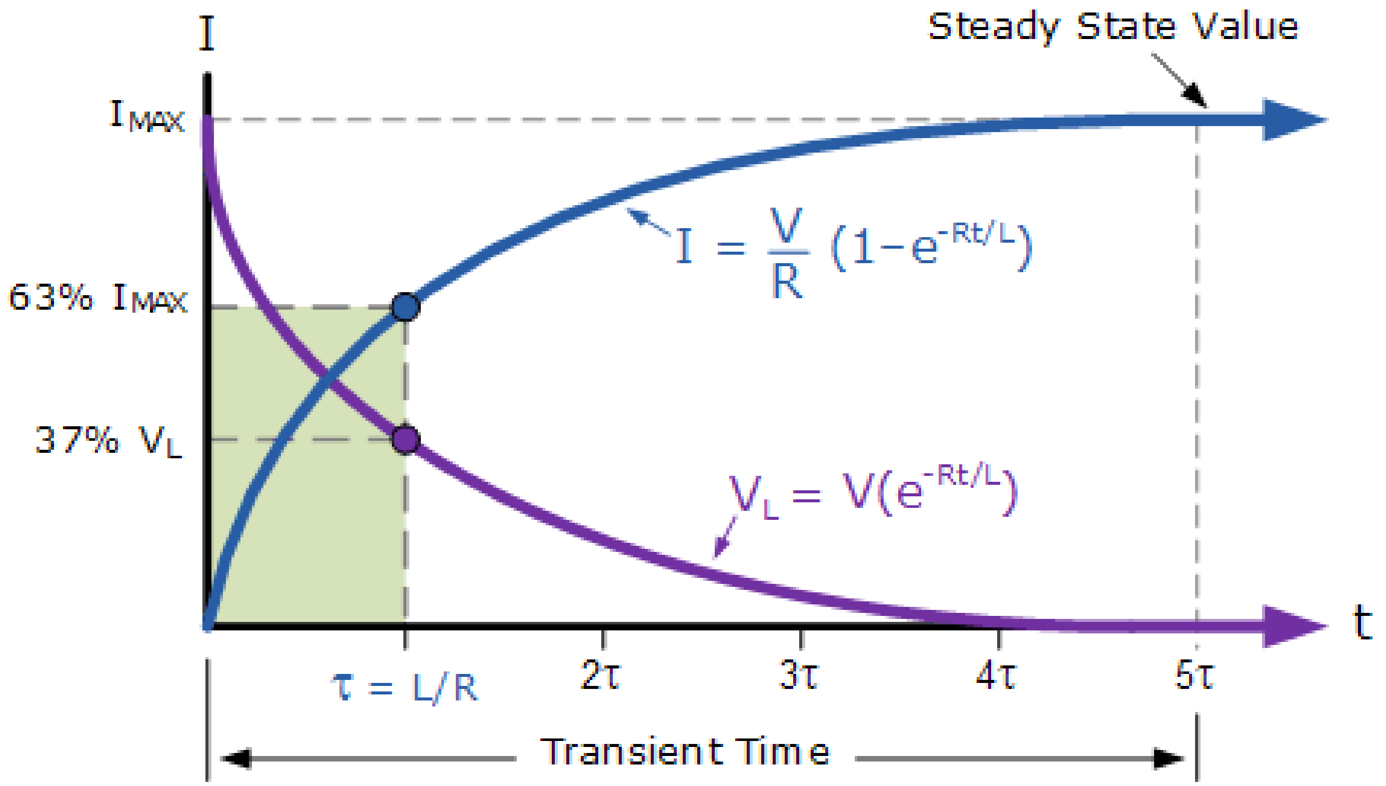
time constant for LR circuit



The induced emf ε is directly proportional to $-\frac{di}{dt}$, or the slope of the curve.

Hence, while at its greatest immediately after the switches are thrown, the induced emf decreases to zero with time as the current approaches its final value of ε/R.

The circuit then becomes equivalent to a resistor connected across a source of emf.



➔ τ = L/R is the "characteristic time" of any RL circuit

- Only t / τ is meaningful

➔ $t = \tau$

- Current falls to 1/e = 37% of maximum value
- Current rises to 63% of maximum value

➔ $t = 2\tau$

- Current falls to $1/e^2$ = 13.5% of maximum value
- Current rises to 86.5% of maximum value

➔ $t = 3\tau$

- Current falls to $1/e^3$ = 5% of maximum value
- Current rises to 95% of maximum value

➔ $t = 5\tau$

- Current falls to $1/e^5$ = 0.7% of maximum value
- Current rises to 99.3% of maximum value

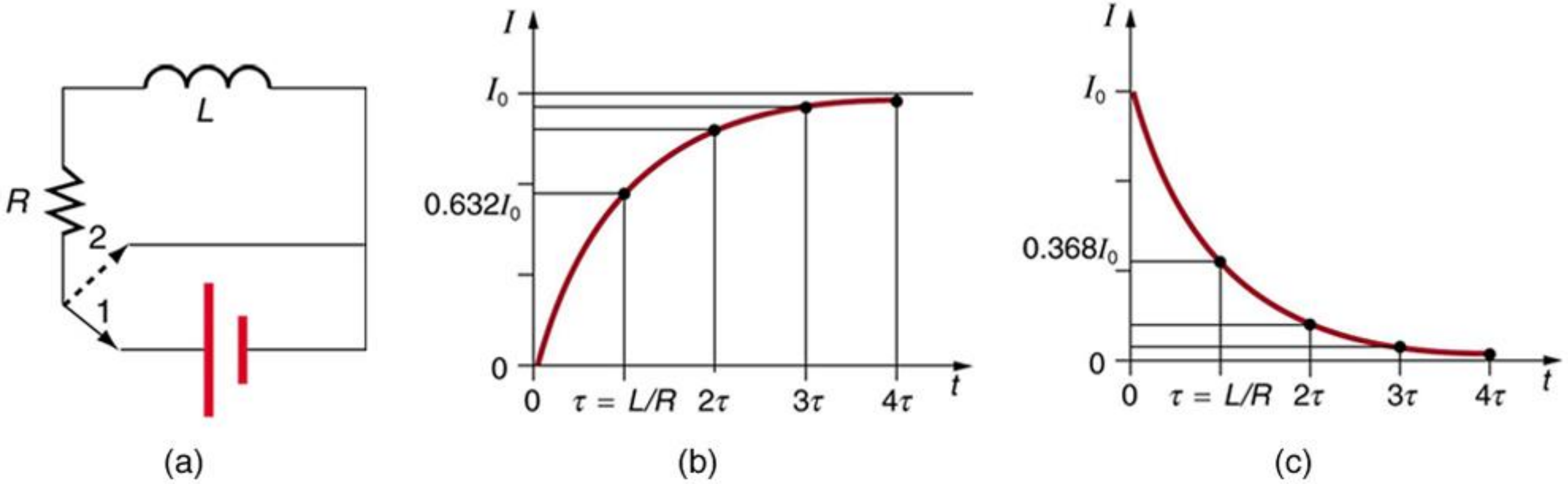
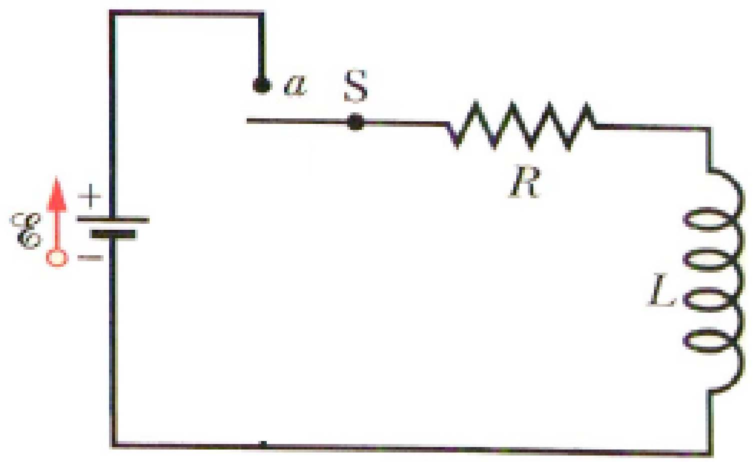
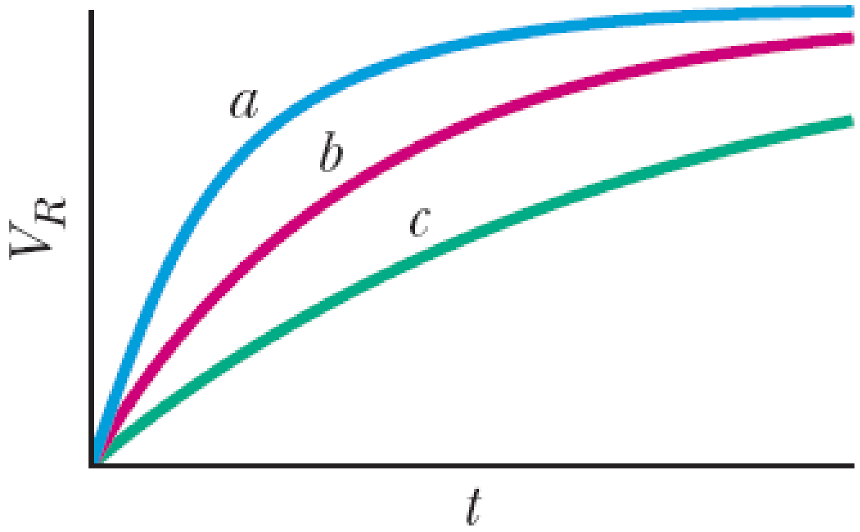


Figure gives the variation with time of the potential difference V_R across a resistor in three circuits wired.

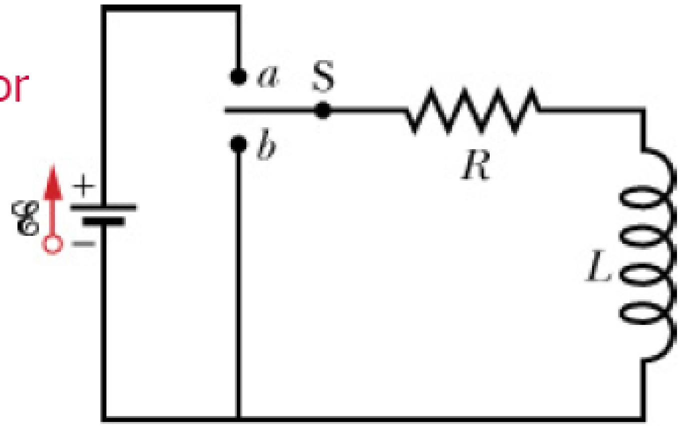
The circuits contain the same resistance R and emf but differ in the inductance L.

Rank the circuits according to the value of L, greatest first.





Energy of an Inductor



- How much energy is stored in an inductor when a current is flowing through it?
- Start with loop rule

$$\mathcal{E} = iR + L\frac{di}{dt}$$

- Multiply by I to get power equation

$$\mathcal{E}i = i^2 R + Li\frac{di}{dt} \qquad P_L = Li\frac{di}{dt} = \frac{d}{dt}\left(\tfrac{1}{2}Li^2\right)$$

$$P_{\text{in}} = P_R\,(\text{heat}) + P_L\,(\text{store})$$

- P_L = rate at which energy is being stored in inductor
  - Energy stored in inductor $\boxed{U_L = \frac{1}{2}Li^2}$
  - Similar to capacitor: $U_C = \dfrac{q^2}{2C}$

The energy in an inductor is actually stored in the magnetic field of the coil, just as the energy of a capacitor is stored in the electric field between its plates.

The work done is equal to the potential energy stored in the inductor.

- Current through inductor: I (increasing)
- Voltage induced across inductor: $|\mathcal{E}| = L\dfrac{dI}{dt}$
- Power absorbed by inductor: $P = |\mathcal{E}|I = \dfrac{dU}{dt}$
- Increment of potential energy: $dU = Pdt = LIdI$
- Potential energy of inductor with current I established:
  $U = L\int_0^I IdI = \dfrac{1}{2}LI^2$

$\therefore$ Energy stored in inductor:

$$U_B = \frac{1}{2} L i^2 = \frac{1}{2} \mu_0 n^2 i^2 \underbrace{lA}_{\text{Volume of solenoid}}$$

$\therefore$ **Energy density** (= Energy stored per unit volume) inside inductor

$$u_B = \frac{U_B}{lA} = \frac{1}{2} \mu_0 n^2 i^2$$

Recall magnetic field inside solenoid

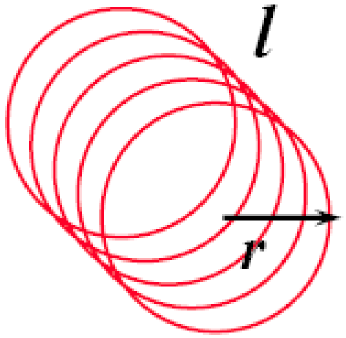
$$B = \mu_0 n i$$

$$\therefore \quad u_B = \frac{B^2}{2\mu_0}$$

This is a **general result** of **energy stored in a magnetic field**

Energy in Magnetic Field (2)

➔Apply to solenoid (constant B field)



$$U_L = \tfrac{1}{2}Li^2 = \tfrac{1}{2}\left(\mu_0 n^2 l A\right)i^2$$

➔Use formula for B field: $B = \mu_0 n i$

$$U_L = \frac{B^2}{2\mu_0} l A$$

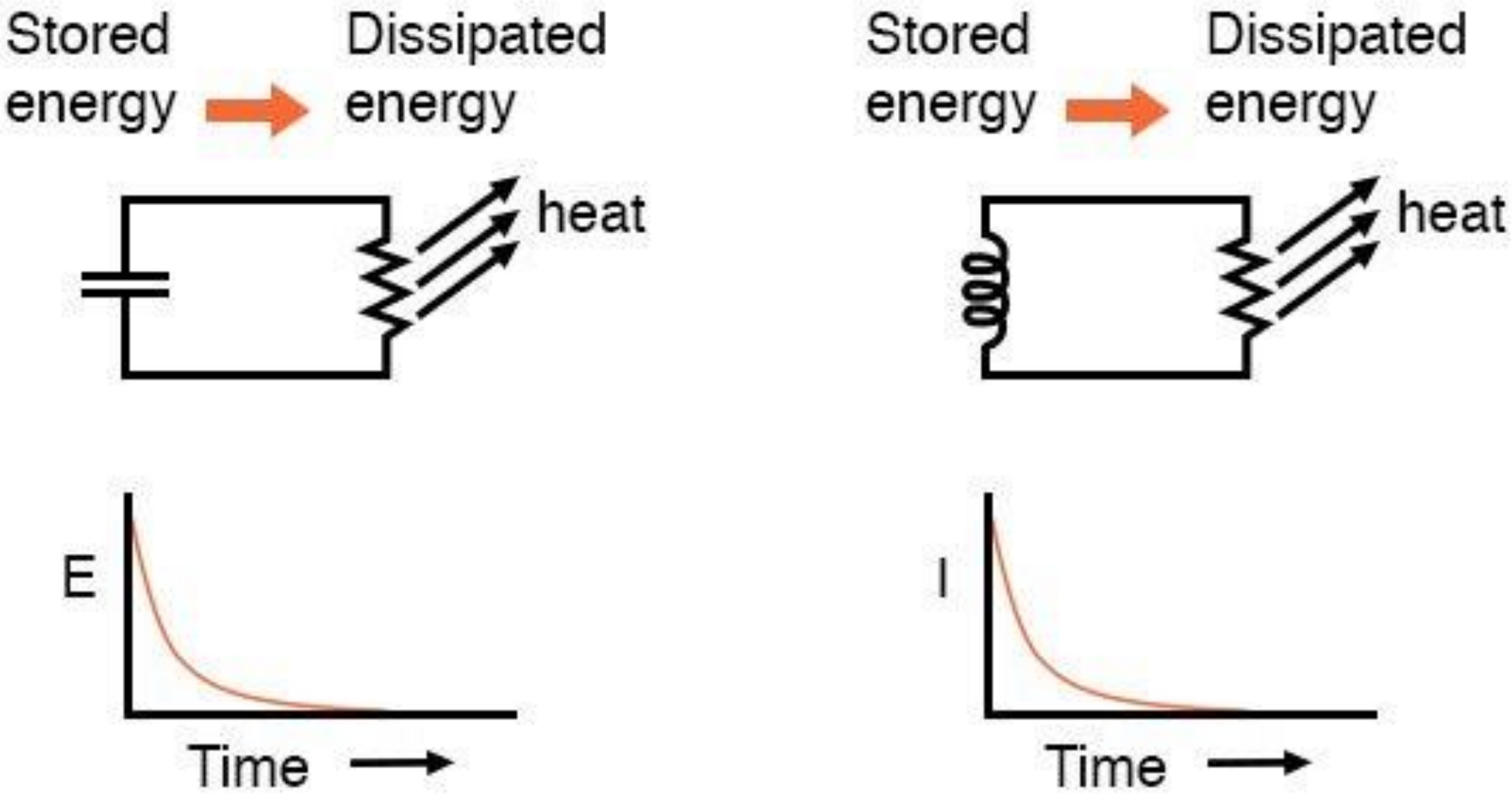
➔Calculate energy density: $u_B = \frac{U_L}{V}$ $V = Al$ volume

$$\boxed{u_B = \frac{B^2}{2\mu_0}}$$ B field

$$\boxed{u_E = \tfrac{1}{2}\varepsilon_0 E^2}$$ E field

➔This is generally true even if B is not constant

Capacitor and inductor discharge



Energy Calculation Examples

→Calculate u_B for earth field, $B = 5 \times 10^{-5}$ T

$$u_B = \frac{B^2}{2\mu_0} = \frac{\left(5\times 10^{-5}\right)^2}{2\times 4\pi \times 10^{-7}} \simeq 10^{-3} \text{ J/m}^3$$

→Calculate u_B for neutron star, $B = 10^8$ T

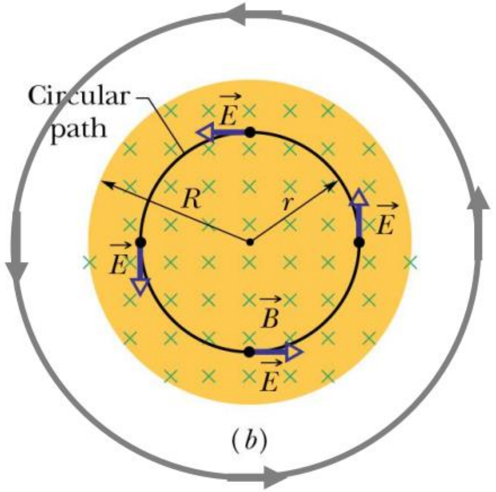
$$u_B = \frac{B^2}{2\mu_0} = \frac{\left(10^{8}\right)^2}{2\times 4\pi \times 10^{-7}} \simeq 4\times 10^{21} \text{ J/m}^3$$

Energizing an inductor is similar to that of charging a capacitor.
Except that the inductor CURRENT (as suppose to the capacitor voltage) is rising exponentially.

The inductor voltage is decreasing exponentially. The time constant is L/R as suppose to RC.

Similarly when we de-energize the inductor, we get the exponential characteristics as we did for discharging the capacitor.

Find the induced electric field: find the expression for the magnitude E of the induced electric field at points within and outside the magnetic field.



$$\mathcal{E}_{ind} = \oint_{loop} \mathbf{E}_{ind} \circ d\vec{s} = -\frac{d\Phi_B}{dt}$$

$$\oint \vec{B} \circ d\vec{s} = \mu_0 i_{enc}$$

Due to symmetry: $\oint \vec{E} \cdot \vec{ds} = \oint E ds = E\oint ds = E(2\pi r)$

For r < R: $\Phi_B = BA = B(\pi r^2)$

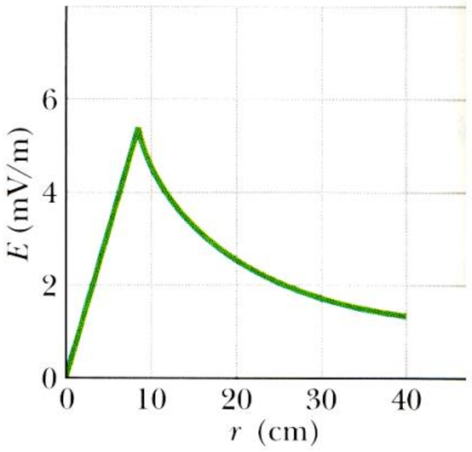
So $E(2\pi r) = \pi r^2 \dfrac{dB}{dt}$

$$\boxed{E = \frac{r}{2}\frac{dB}{dt}}$$

For r > R: $\Phi_B = BA = B(\pi R^2)$

So $E(2\pi r) = \pi R^2 \dfrac{dB}{dt}$

$$\boxed{E = \frac{R^2}{2r}\frac{dB}{dt}}$$



The magnitude of induced electric field grows linearly with r, then falls off as 1/r for r>R