Electromagnetic Induction

Phy 108 course
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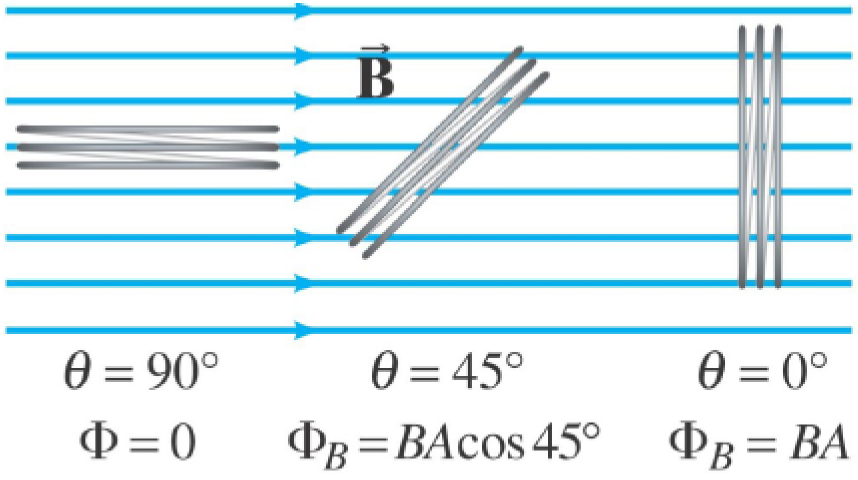
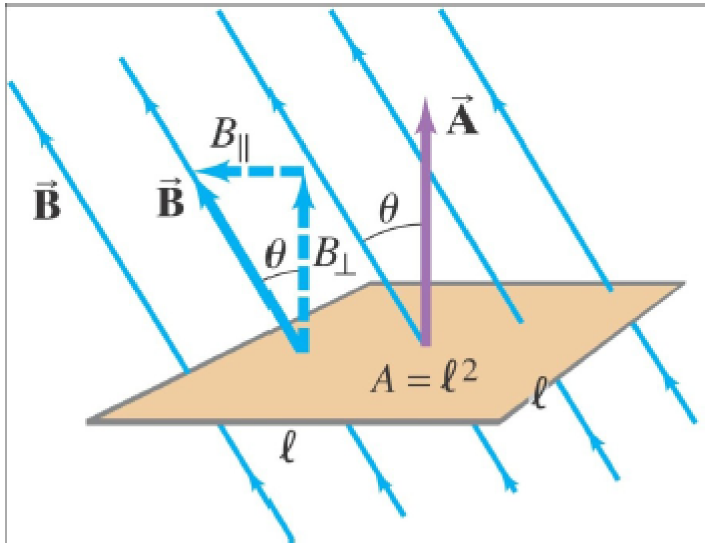
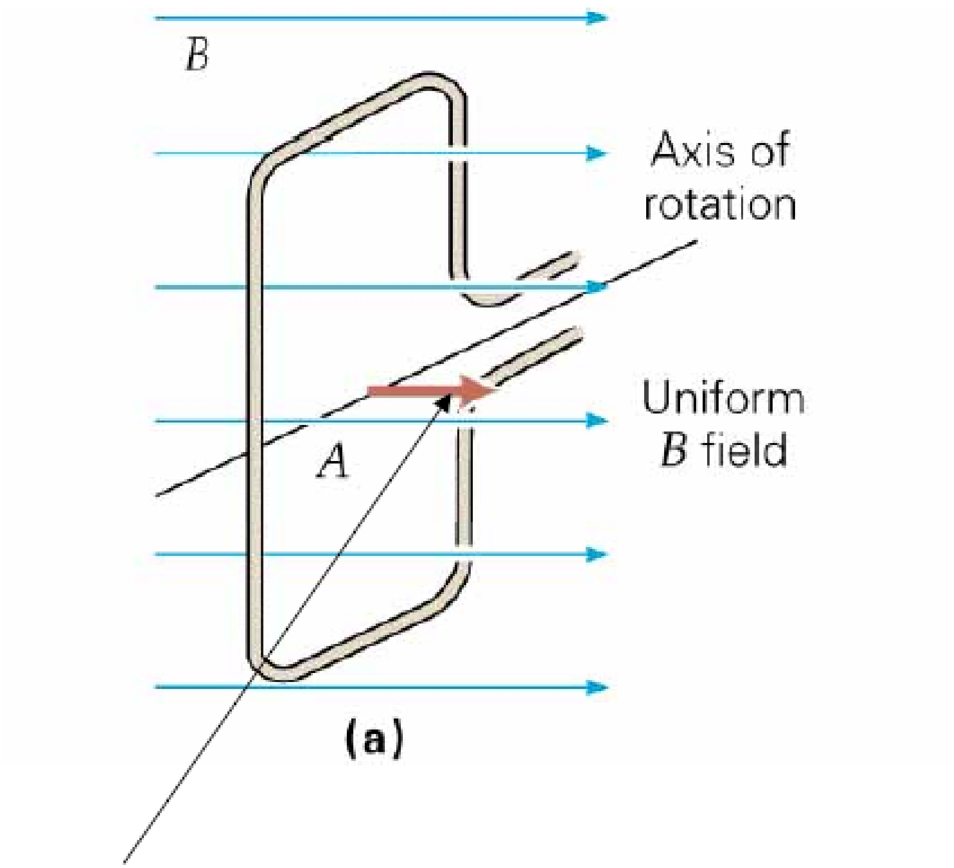
The Magnetic Flux

The magnetic flux is analogous to the electric flux – it is proportional to the total number of magnetic field lines passing through the loop.

$$\Phi_B = \vec{B} \cdot \vec{A} = BA\cos\theta$$







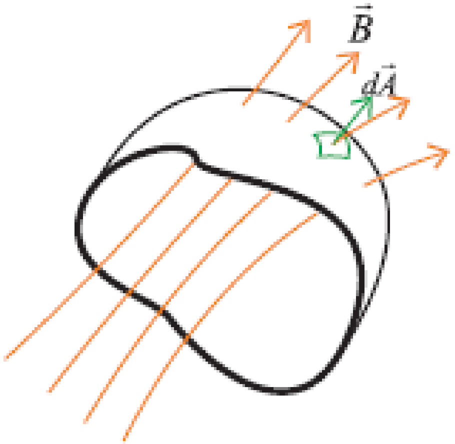
MAGNETIC FLUX,

$\Phi = BA\cos\theta$

Here $\theta=0$ so $\cos\theta = 1$

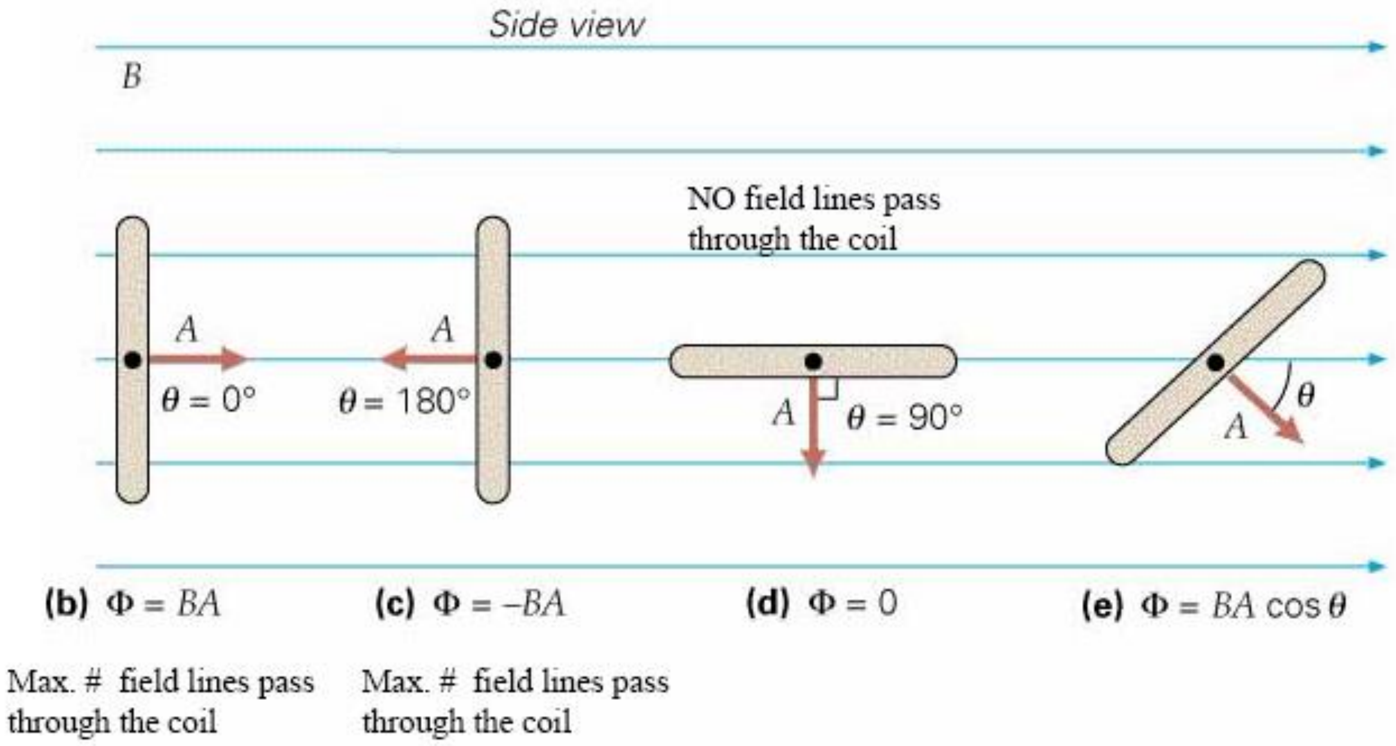
Units: $T.m^2 = Wb$

Area Vector: Direction is perpendicular to plane.
Magnitude is equal to the area of the loop



Magnetic flux through surface S:

$$\boxed{\Phi_m = \int_S \vec{B} \cdot d\vec{A}}$$



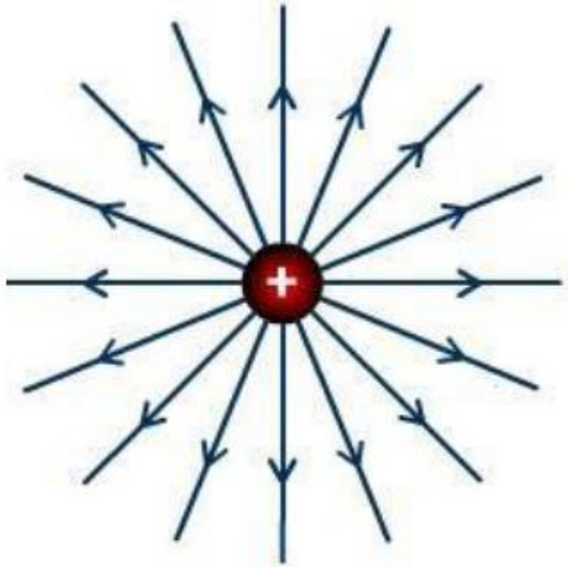
Magnetic Flux is continually changing as coil rotates.

$$\boxed{\oint \vec{B} \cdot d\vec{A} = 0}$$

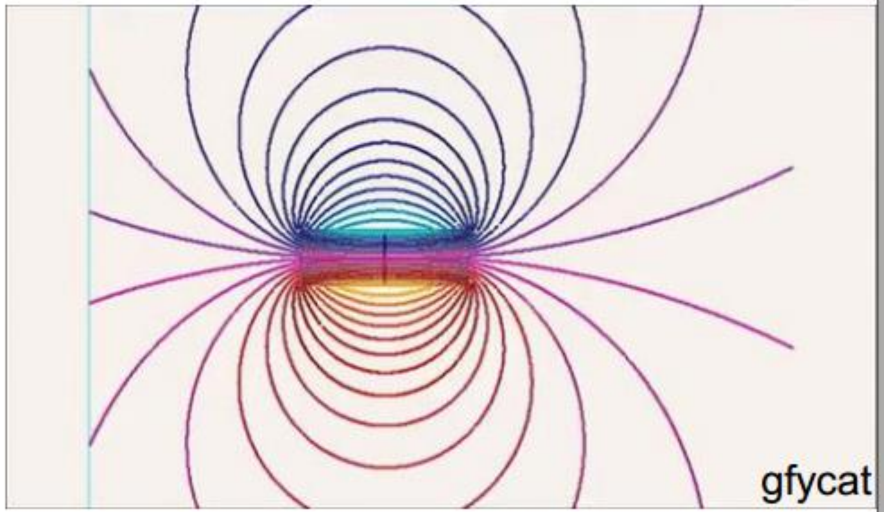
Gauss Law for $\vec{B}$ **(there are no magnetic monopoles)**

Gauss' Law for Magnetic Fields

	Electric field	Magnetic field
	Electric charge	No magnetic "charge" (monopole)
Gauss's law	$$\oint \vec{E} \cdot d\vec{\mathbf{A}} = \frac{q_{encl}}{\varepsilon_0} \quad (1)$$	$$\oint \vec{B} \cdot d\vec{\mathbf{A}} = 0 \quad (2)$$



Electric field lines



Magnetic field lines: always closed

Gauss's law of magnetism

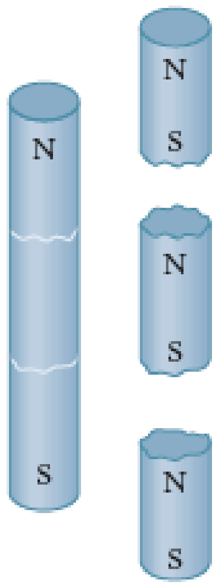
Gauss' Law for Magnetic Fields

One end of the magnet is a *source* of the field (the field lines diverge from it) and the other end is a *sink* of the field (the field lines converge toward it).

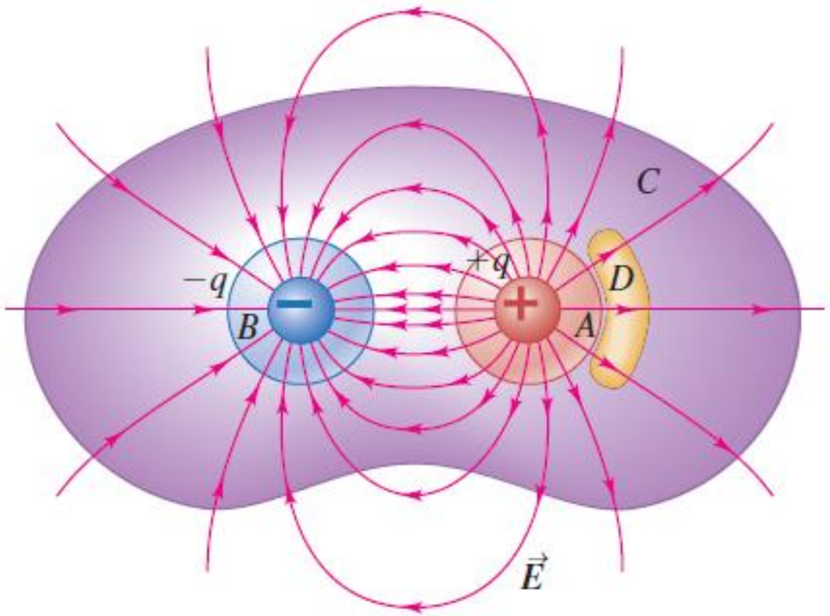
By convention, we call the source the *north pole* of the magnet and the sink the *south pole,* and we say that the magnet, with its two poles, is an example of a **magnetic dipole.**

Suppose we break a bar magnet into pieces the way we can break a piece of chalk. We should, it seems, be able to isolate a single magnetic pole, called a *magnetic monopole.*

However, we cannot—not even if we break the magnet down to its individual atoms and then to its electrons and nuclei. Each fragment has a north pole and a south pole. Thus: The simplest magnetic structure that can exist is a magnetic dipole. Magnetic monopoles do not exist (as far as we know).



Gauss' "Law"



The net number of field lines leaving a closed surface is proportional to the total charge enclosed by that surface.

Two charges, equal in magnitude but opposite in sign, and the field lines that represent their net electric field.

Four Gaussian surfaces are shown in cross section.

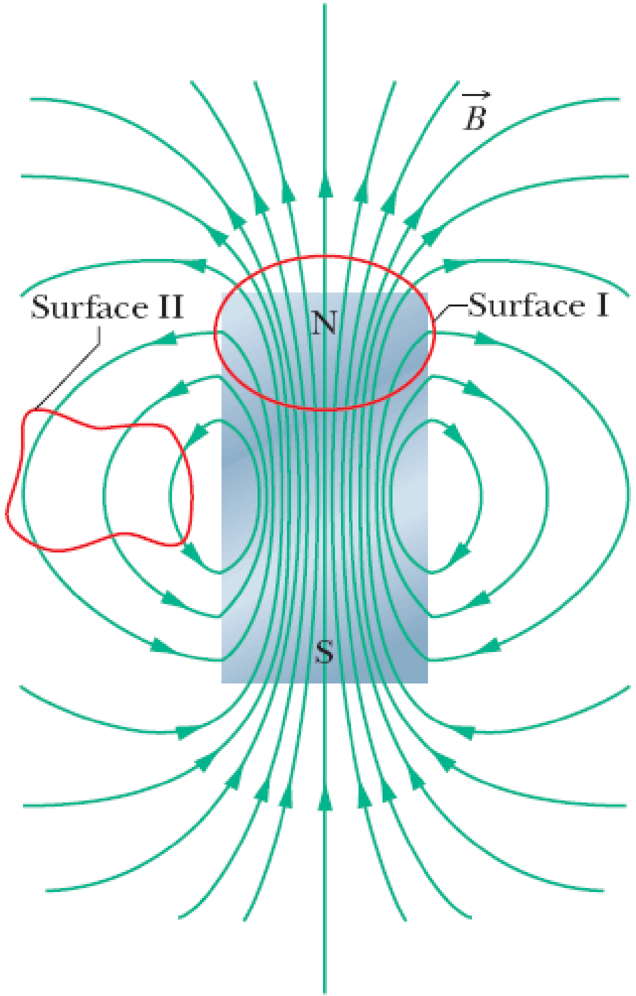
Surface *A* encloses the positive charge.
Surface *B* encloses the negative charge.
Surface *D* encloses no charge.
Surface *C* encloses both charges and thus no net charge.

Gauss’ Law for Magnetic Fields

Gauss’ law for magnetic fields is a formal way of saying that magnetic monopoles do not exist. The law asserts that the net magnetic flux through any closed Gaussian surface is zero:

$$\Phi_B = \oint \vec{B}.d\vec{S} = 0$$

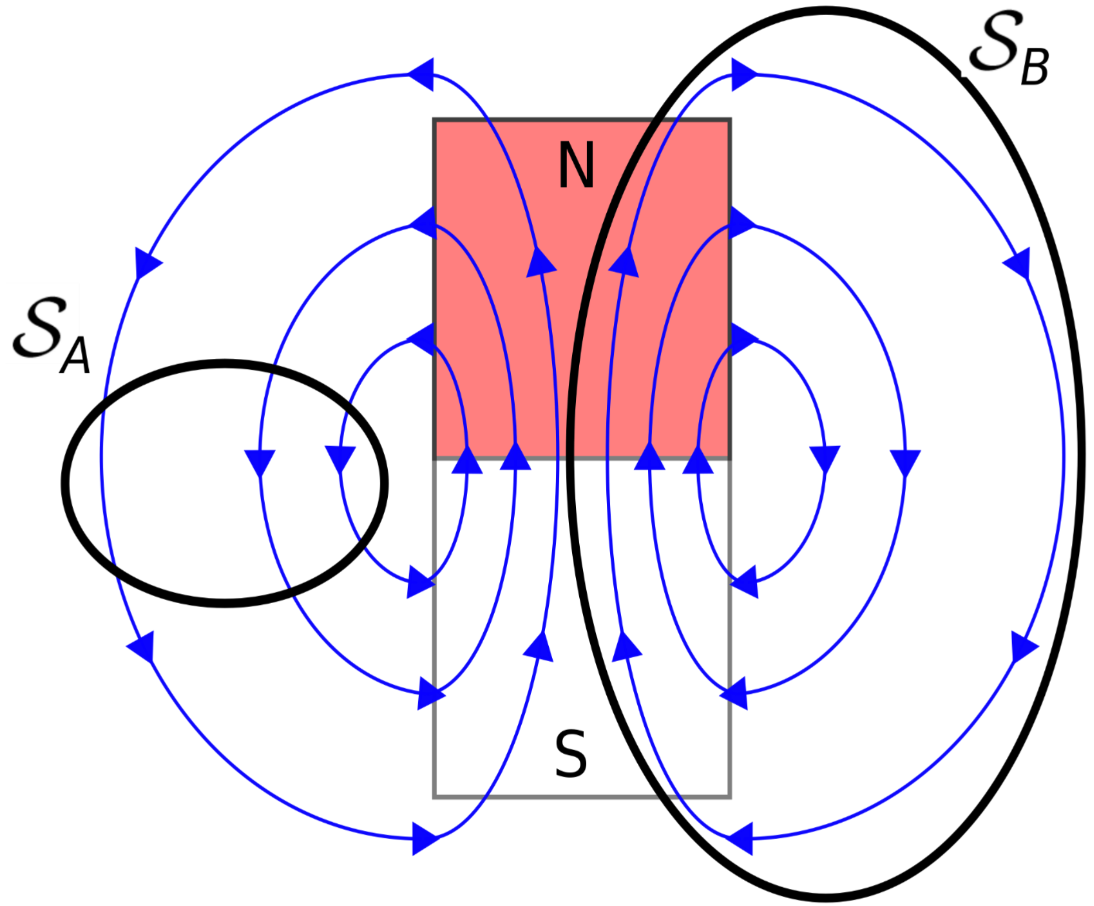
Gauss law for magnetic field

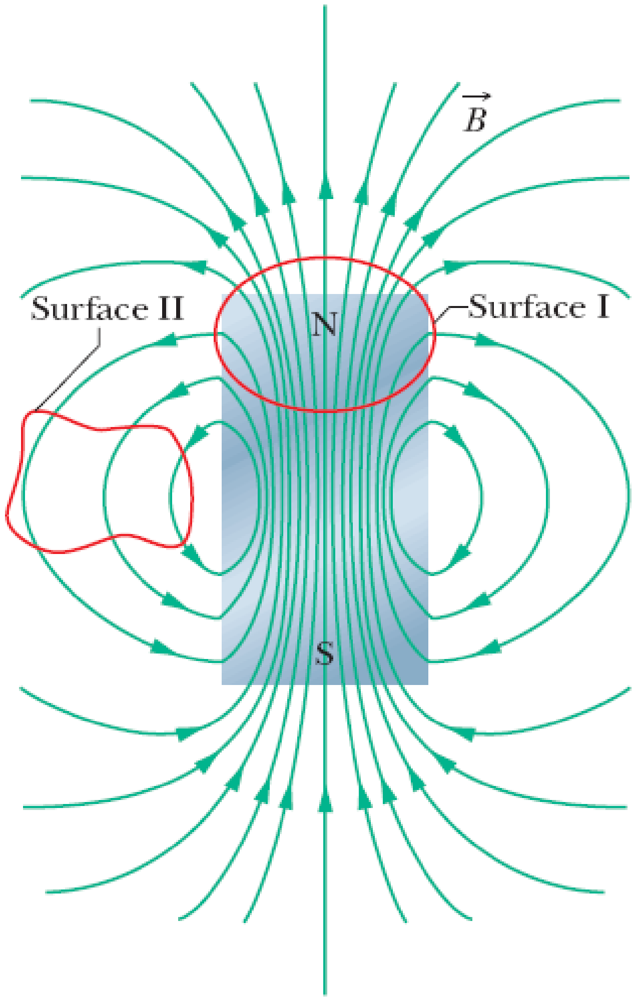


Gauss’ law for magnetic fields holds for structures more complicated than a magnetic dipole, and it holds even if the Gaussian surface does not enclose the entire structure.

Gaussian surface II near the bar magnet of Fig. encloses no poles, and we can easily conclude that the net magnetic flux through it is zero.

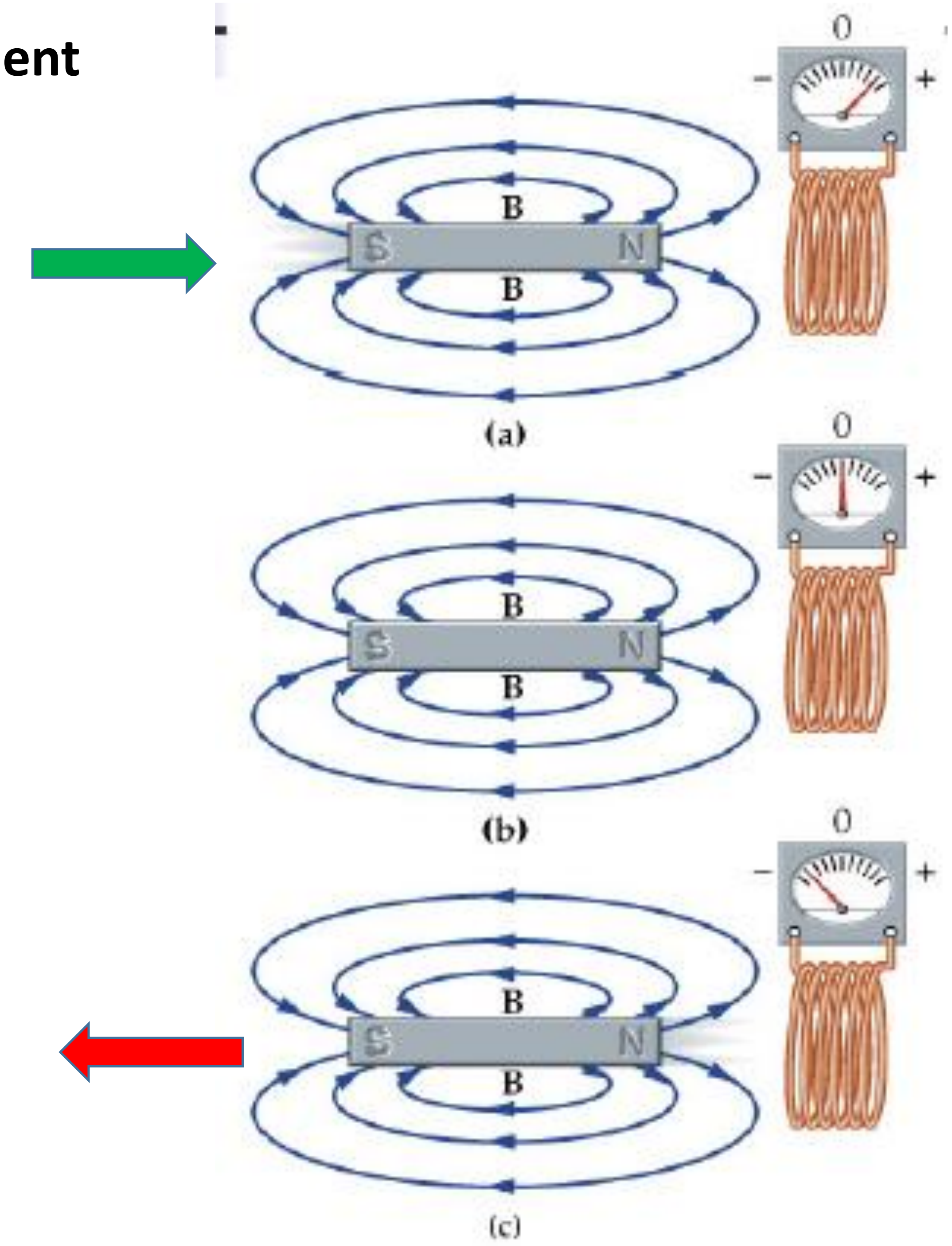
Gaussian surface I is more difficult. It may seem to enclose only the north pole of the magnet because it encloses the label N and not the label S. However, a south pole must be associated with the lower boundary of the surface because magnetic field lines enter the surface there. (The enclosed section is like one piece of the broken bar magnet in Fig.) Thus, Gaussian surface I encloses a magnetic dipole, and the net flux through the surface is zero.

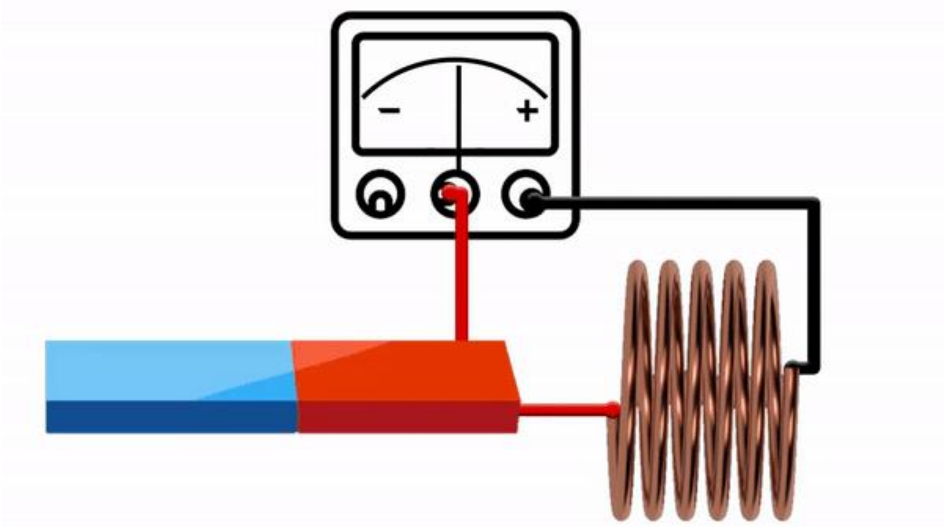

N
S
$\mathcal{S}_A$
$\mathcal{S}_B$


$\vec{B}$
N
S
Surface I
Surface II

Faraday's Experiment

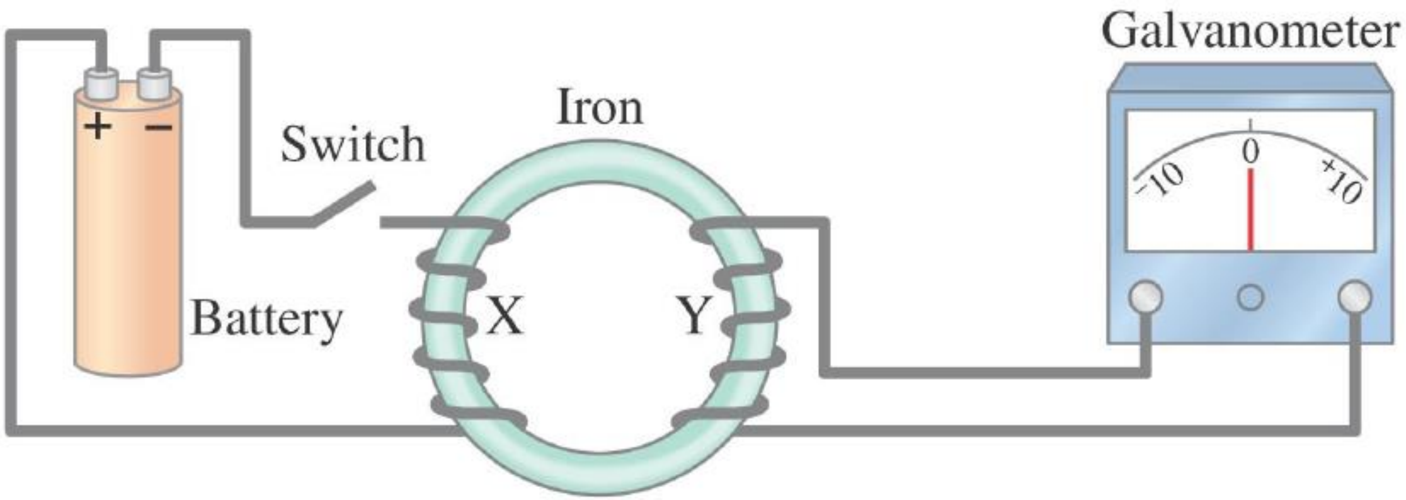
A coil experiences an induced current when the magnetic field passing through it varies. (a) When the magnet moves toward the coil the current is in one direction. (b) No current is induced while the magnet is held still. (c) When the magnet is pulled away from the coil the current is in the other direction.



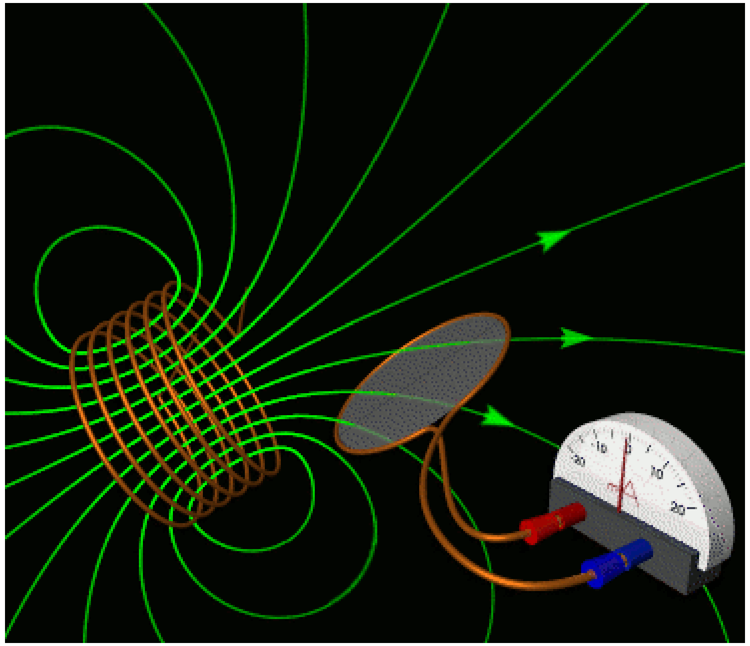


Faraday's Experiment

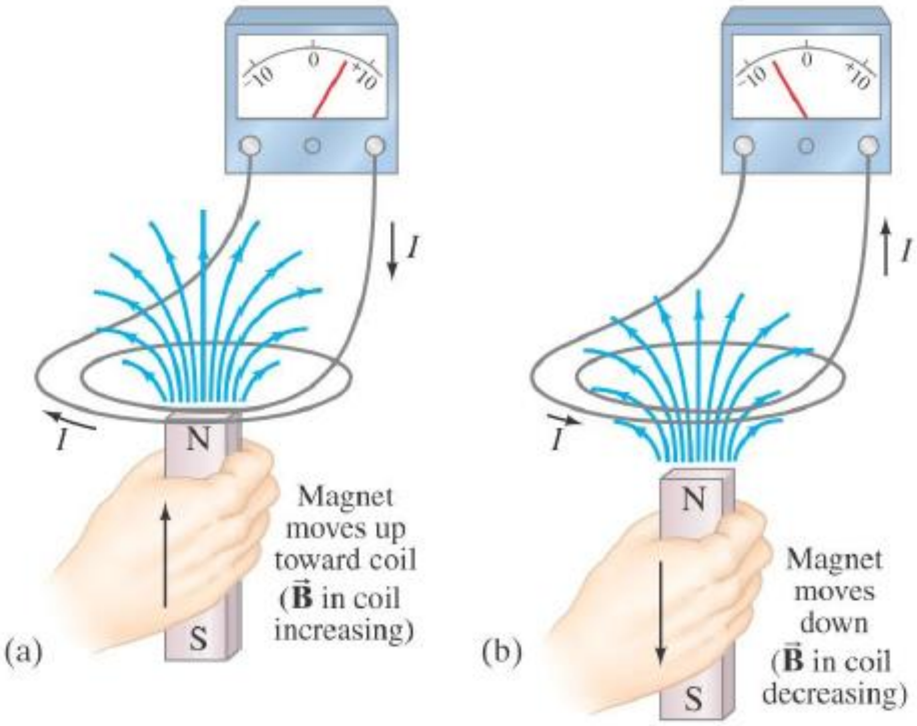
Basic setup of Faraday's experiment on magnetic induction. When the position of the switch on the primary circuit is changed from open to closed or from closed to open, an emf is induced in the secondary circuit.

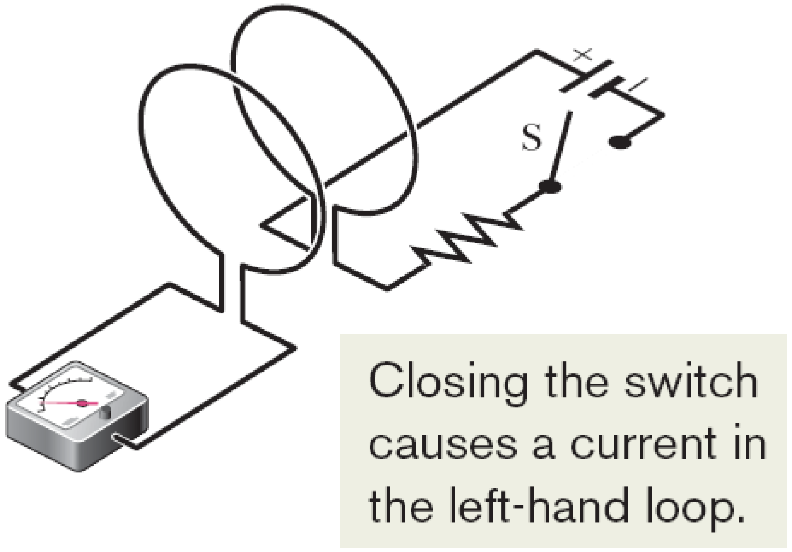


The induced emf causes a current in the secondary circuit, and the current is detected by the ammeter. If the current in the primary circuit does not change, no matter how large it may be, there is no induced current in the secondary circuit.



Faraday's Law **the emf induced in a circuit is equal to the rate of change of magnetic flux through the circuit:**





https://phet.colorado.edu/sims/html/faradays-law/latest/faradays-law_en.html

If the magnetic flux through a loop of wire changes for any reason
either by changing the area, A, of the loop or the field, B, through the loop

Then an EMF (voltage) will be induced in the wire. This voltage will
cause a current to flow (the induced current in the loop).

Faraday quantified the size of the induced voltage:

Faraday's Law

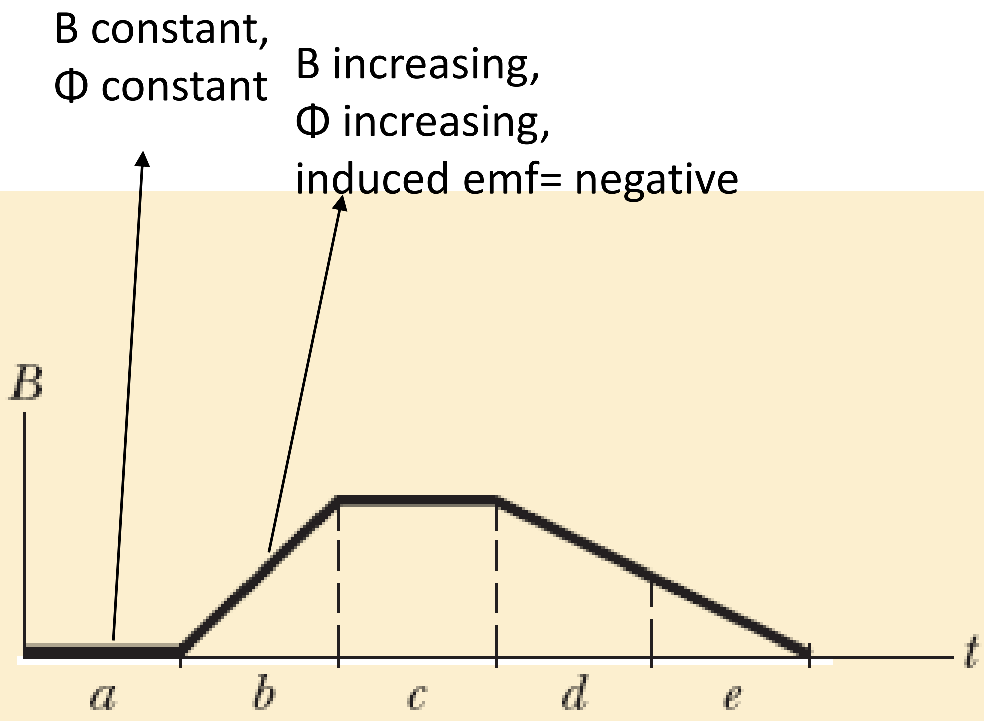
Induced EMF

$$\boxed{\varepsilon = -N\frac{\Delta\Phi}{\Delta t} = -N\frac{\Phi_f - \Phi_i}{t_f - t_i}}$$

The size of the induced EMF depends on how quickly the flux through the coil is changing. There is only an induced EMF if there is a changing flux change through the coil.

Checkpoint 1

The graph gives the magnitude $B(t)$ of a uniform magnetic field that exists throughout a conducting loop, with the direction of the field perpendicular to the plane of the loop. Rank the five regions of the graph according to the magnitude of the emf induced in the loop, greatest first.



Emf is negative proportional to $d\phi/dt$

Faraday’s law and Flux changing

Rotating Loop

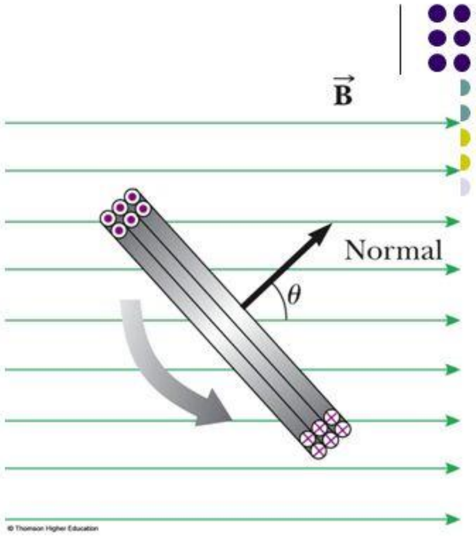
- Assume a loop with N turns, all of the same area rotating in a magnetic field
- The flux through the loop at any time t is

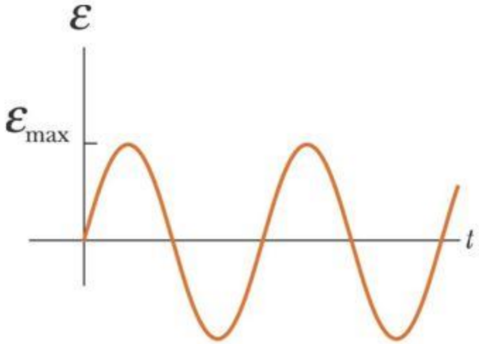
$$\Phi_B = BA\cos\theta$$
$$= BA\cos\omega t$$

So

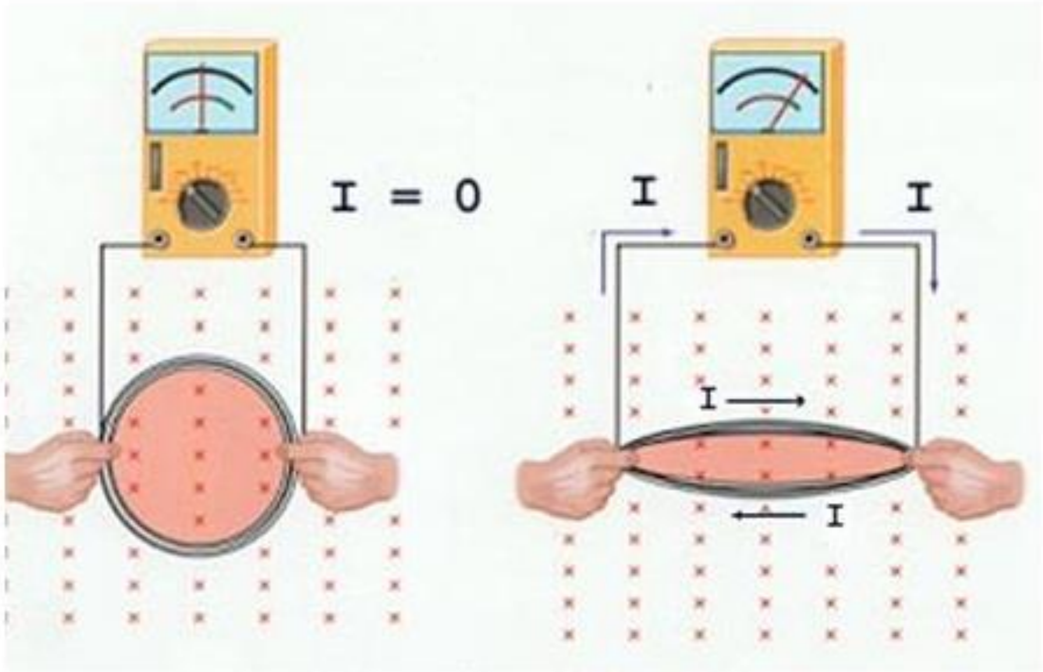
$$emf = -N\frac{d\Phi_B}{dt} = NBA\omega \sin\omega t$$

The *emf* is a sin wave: AC.





Faraday’s law and Flux changing



Induced emf = $N\ \Delta\Phi/\Delta t$
(Omitting negative sign)

Φ = BA
$\Delta\Phi$ = B (ΔA)

Magnetic field doesn't change; area changes.

The more quickly the loop is stretched, the smaller will be Δt and the larger will be the transient emf.

http://physics.bu.edu/~duffy/HTML5/magnet_loop.html

A coil consists of 200 turns of wire. Each turn is a square of side d = 18 cm, and a uniform magnetic field directed perpendicular to the plane of the coil is turned on. If the field changes linearly from 0 to 0.50 T in 0.80 s, what is the magnitude of the induced emf in the coil while the field is changing? Let's assume the coil is connected to a circuit and the total resistance of the coil and the circuit is 2.0 Ω. Then, the current in the coil = ?

Lenz's Law

The minus sign in Faraday's law gives the direction of the induced emf:

A current produced by an induced emf moves in a direction so that the magnetic field it produces tends to restore the changed field.

$$\mathcal{E} = -N\frac{d\varphi_B}{dt}$$

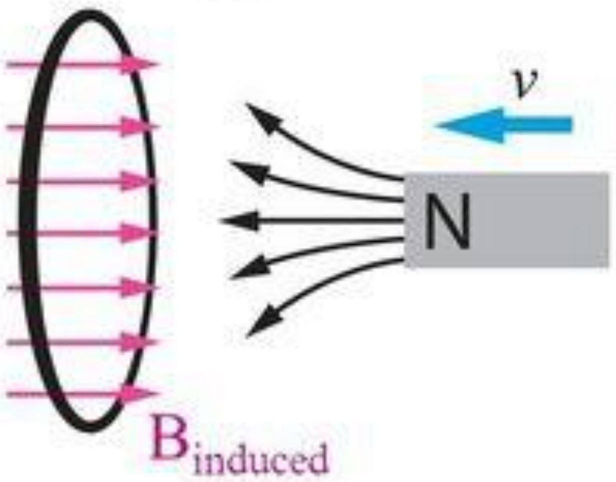
or:

An induced emf is always in a direction that <u>opposes the original change in flux</u> that caused it.

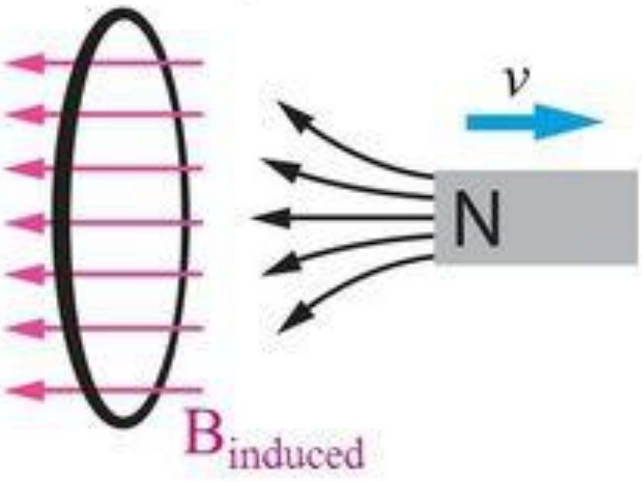
Lenz's Law

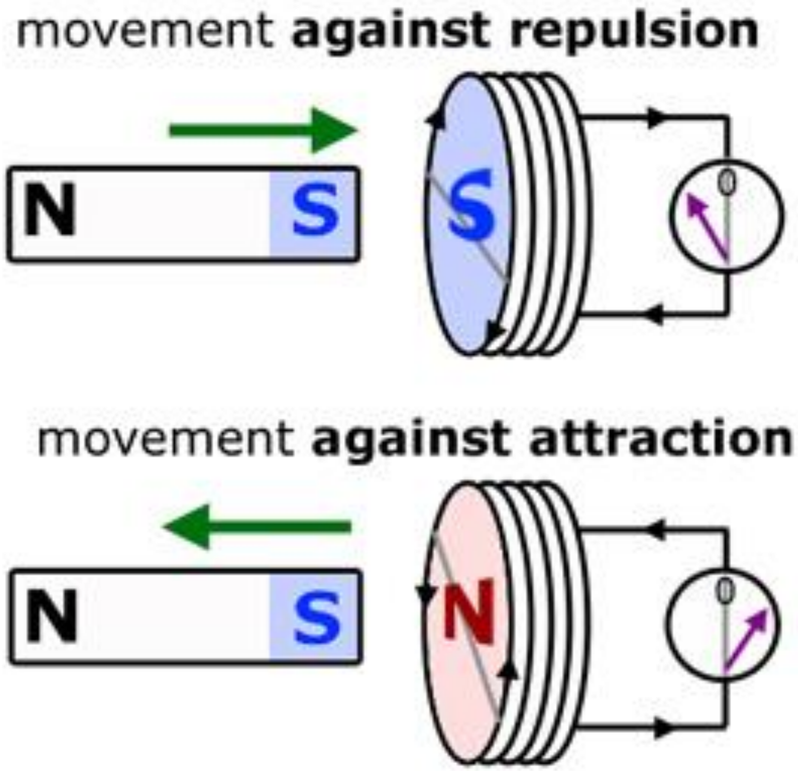
The *induced B field* in a loop of wire will **oppose the change in magnetic flux** through the loop.

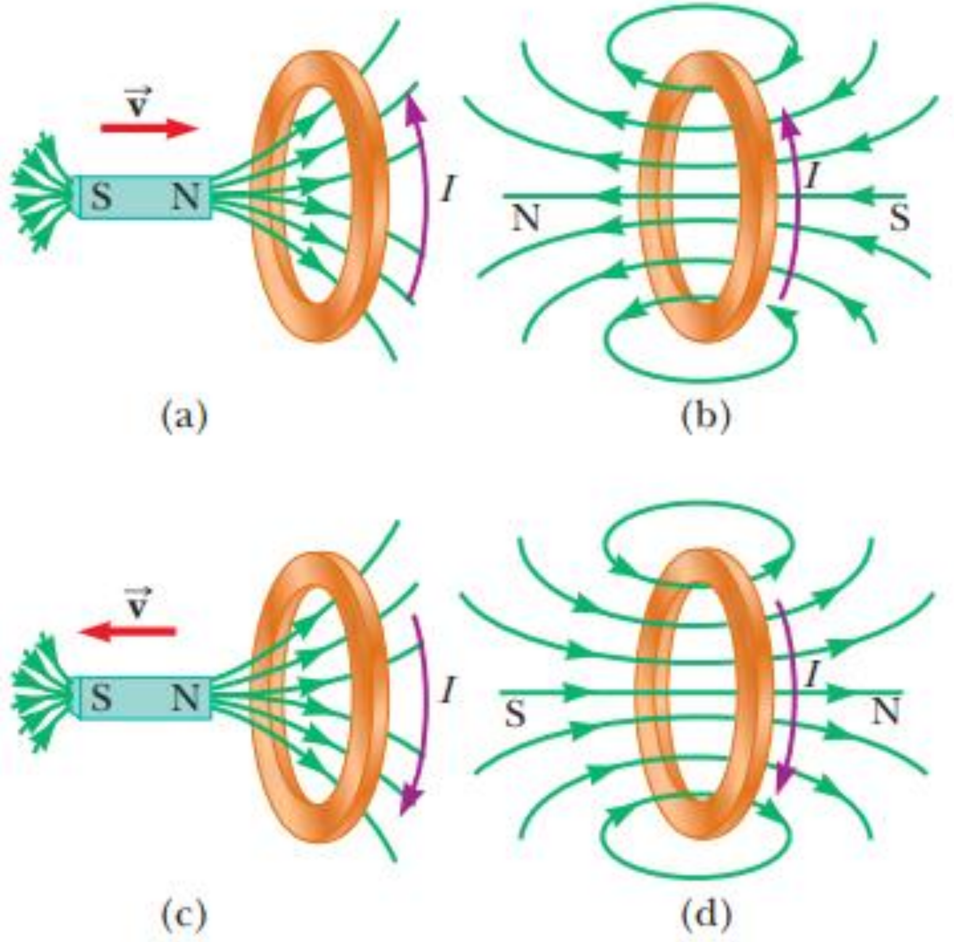
If you try to **increase** the flux through a loop, the induced field will <u>oppose that increase</u>!



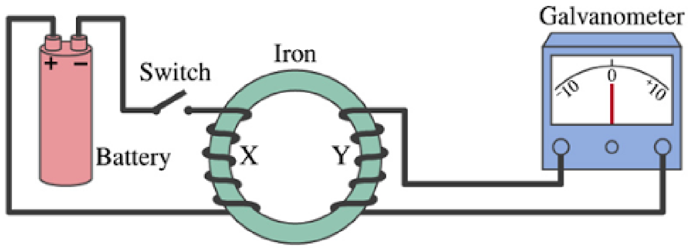
If you try to **decrease** the flux through a loop, the induced field will <u>replace that decrease</u>!







(a) When the magnet is moved toward the stationary conducting loop, a current is induced in the direction shown. The magnetic field lines shown are those due to the bar magnet.
(b) This induced current produces its own magnetic field directed to the left that counteracts the increasing external flux. The magnetic field lines shown are those due to the induced current in the ring.
(c) When the magnet is moved away from the stationary conducting loop, a current is induced in the direction shown. The magnetic field lines shown are those due to the bar magnet.
(d) This induced current produces a magnetic field directed to the right and so counteracts the decreasing external flux. The magnetic field lines shown are those due to the induced current in the ring



$$\varepsilon = -\frac{d\Phi}{dt}$$

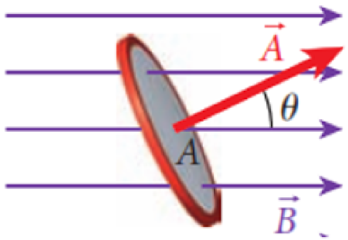
Magnetic flux, ϕ

Faraday's Experiment

Faraday's Law (1831)

Lenz's Law (1834)

$$\Phi = \vec{B} \bullet \vec{A} = BA\cos\theta$$



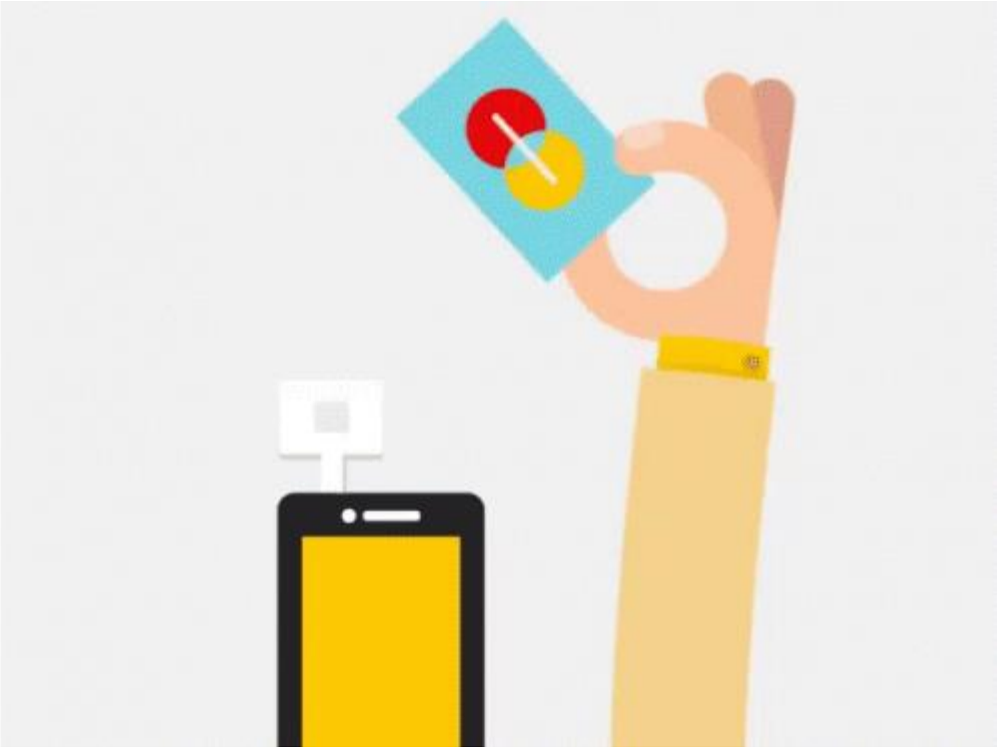
- a *steady magnetic flux/ field* produces *no current*
- a *changing magnetic* flux/ field can *produce an electric current* $\rightarrow$ *induce current*

- the magnitude of the induced emf is **proportional** to the rate of change of the magnetic flux

- an induced electric current always flows in such a direction that it **opposes** the change producing it

Another example is credit card reader. A faster swipe = a BIGGER signal (more induction).

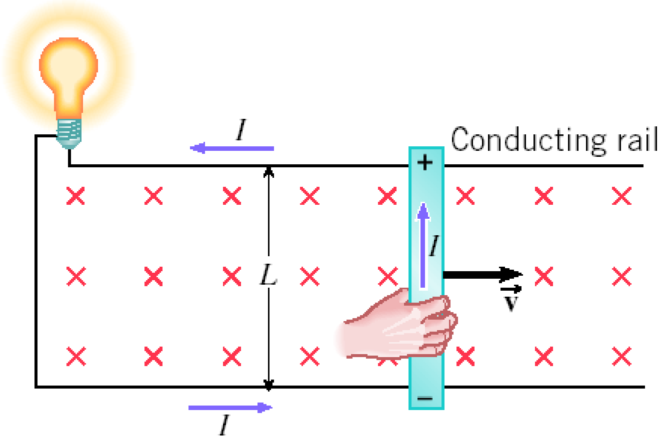
When a credit card is “swiped” through a card reader, the information coded in a magnetic pattern on the back of the card is transmitted to the card-holder’s bank. Why is it necessary to swipe the card rather than holding it motionless in the card reader’s slot?



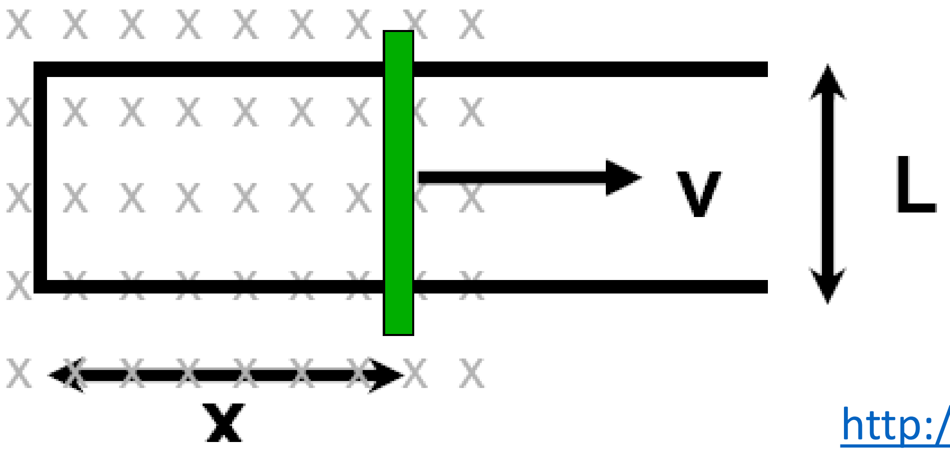
Motional EMF

➔Consider a conducting rod moving on metal rails in a uniform magnetic field:

$$\left|\mathcal{E}\right| = \frac{d\Phi_B}{dt} = \frac{d(BA)}{dt} = \frac{d(BLx)}{dt} = BL\frac{dx}{dt}$$



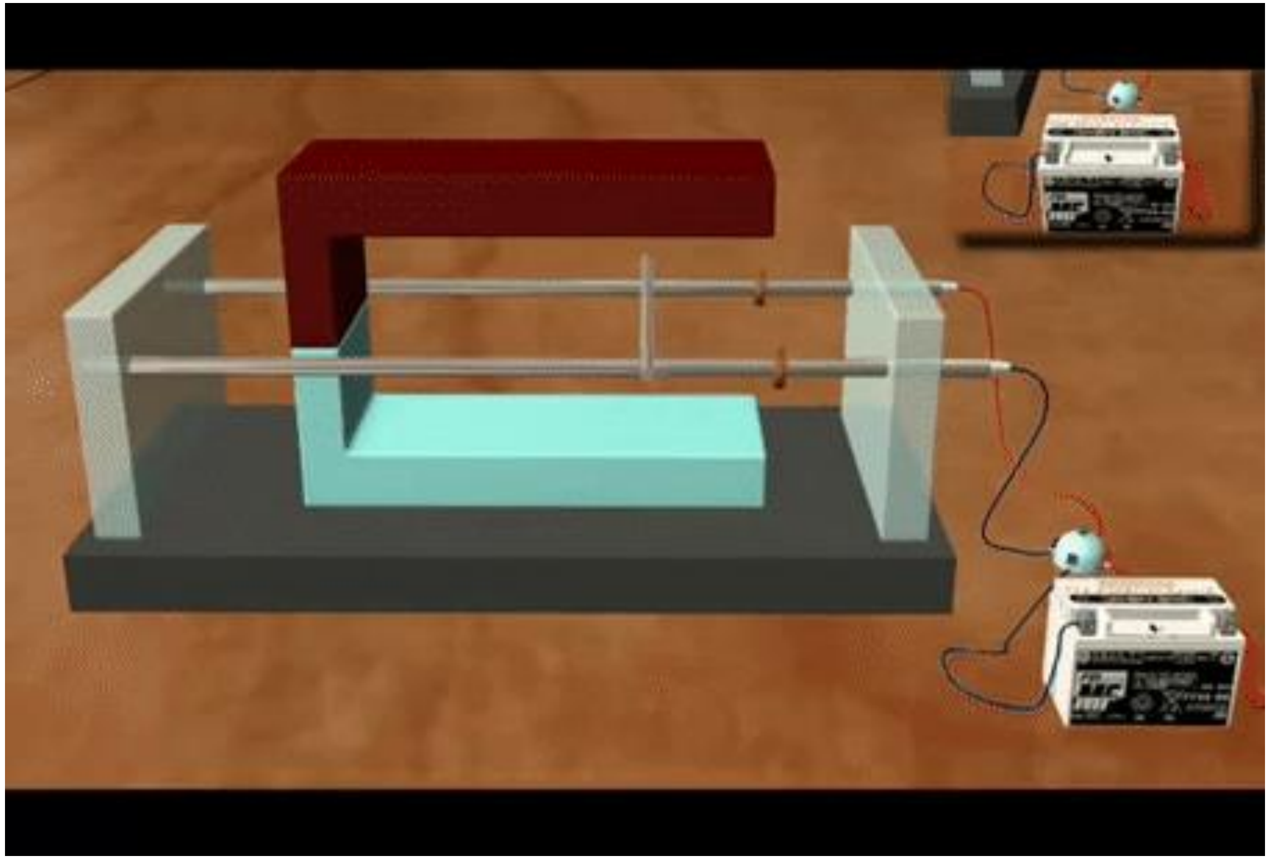
Current will flow counter-clockwise in this "circuit". Why?

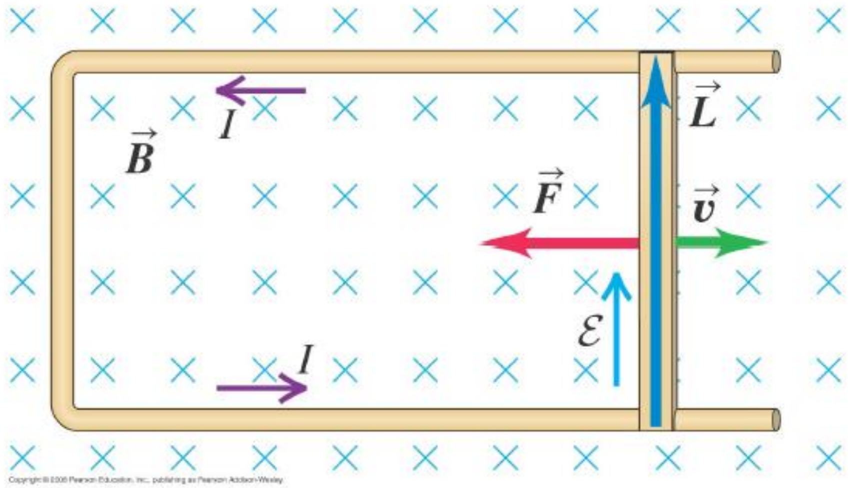


$$\boxed{\left|\mathcal{E}\right| = BLv}$$

http://physics.bu.edu/~duffy/HTML5/motional_emf.html

Motional EMF





http://physics.bu.edu/~duffy/HTML5/EM_railgun.html

Pull a 30cm x 30cm conducting loop of aluminum through a 2T B field at 30cm/sec. Assume it is 1cm thick. Circumference = 120cm = 1.2m, cross sectional area = 10^{-4} m^2, R = 3.3 x $10^{-4}\Omega$

Find the emf, current, force and power while it is moving.

➔EMF

$$\mathcal{E} = BLv = 2 \times 0.3 \times 0.3 = 0.18\,\mathrm{V}$$

➔Current

$$i = \mathcal{E} / R = 0.18 / 3.3 \times 10^{-4} = 545\,\mathrm{A}$$

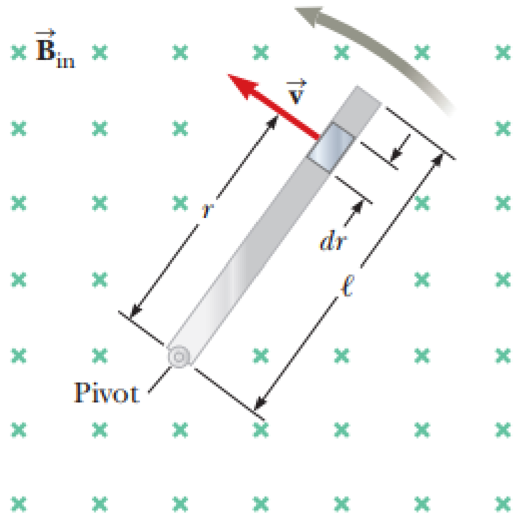
➔Force

$$F = iLB = 545 \times 0.3 \times 2 = 327\,\mathrm{N}$$

➔Power

$$P = i^2 R = 98\,\mathrm{W}$$

A conducting bar of length l, rotates with a constant angular speed ω about a pivot at one end. A uniform magnetic field **B**s is directed perpendicular to the plane of rotation as shown in Figure. Find the motional emf induced between the ends of the bar.



$$d\mathcal{E} = Bv\,dr$$

the magnitude of the emf induced in a segment of the bar of length dr having a velocity **v**

$$\mathcal{E} = \int Bv\,dr$$

Find the total emf between the ends of the bar by adding the emfs induced across all segments

$$\mathcal{E} = B\int v\,dr = B\omega\int_0^{\ell} r\,dr = \boxed{\tfrac{1}{2}B\omega\ell^2}$$

$$v = \omega r$$

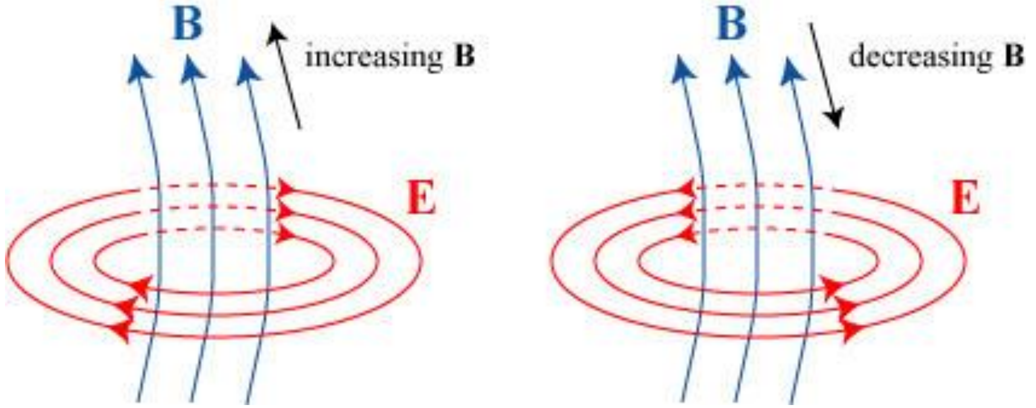
Induced Electric Fields

An induced emf occurs when there is a changing magnetic flux through a stationary conductor. This emf is induced by a changing magnetic flux even if the loop through which the flux is changing is not a physical conductor but an imaginary line.

The changing magnetic field induces an electric field $\vec{E}$ at every point of such a loop; the induced emf is related to $\vec{E}$ by:

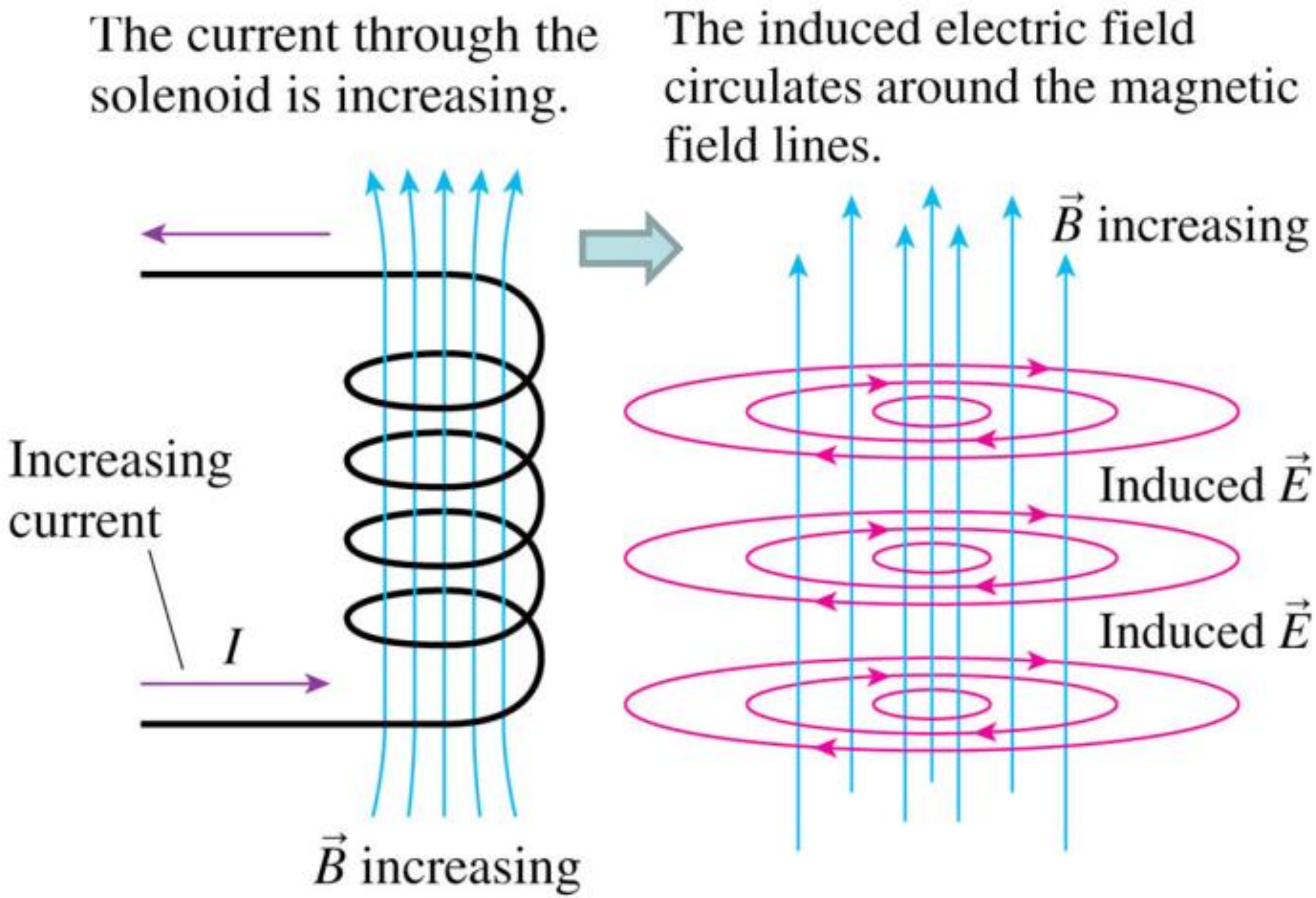
$$\mathcal{E} = \oint \vec{E}.d\vec{S}$$

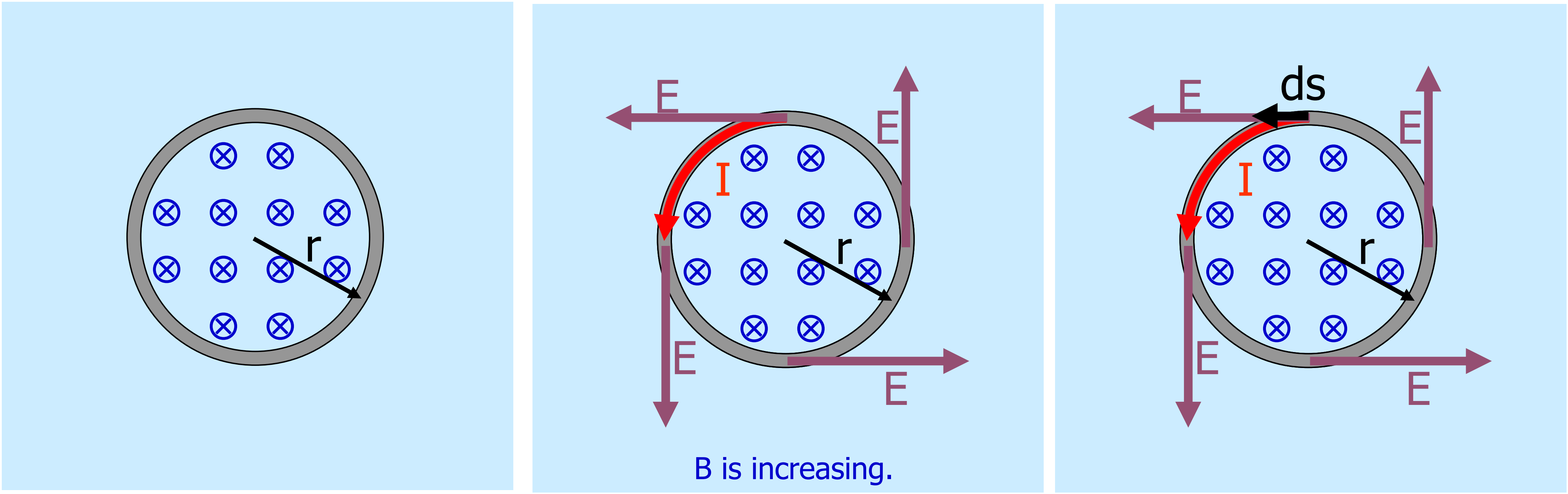
the electric field would appear even if the changing magnetic field were in a vacuum.



By this line of reasoning, we are led to a useful and informative restatement of Faraday's law of induction:

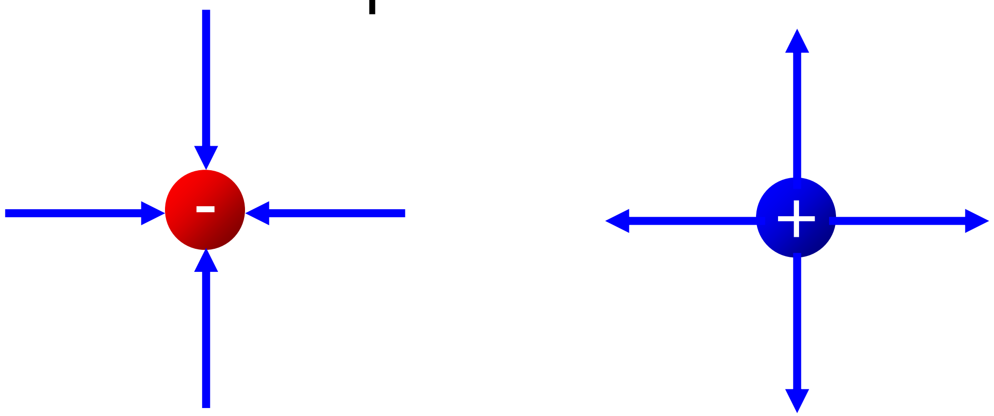
A changing magnetic field produces an electric field.

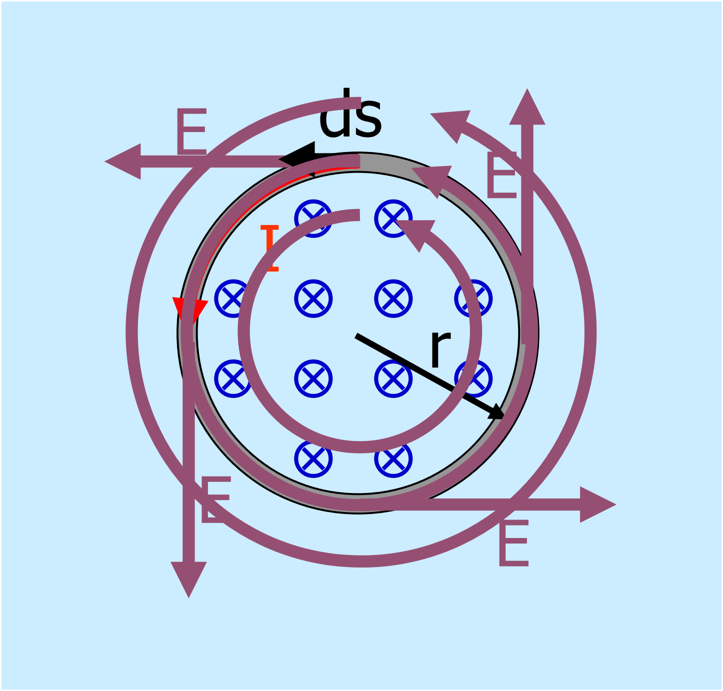

The current through the solenoid is increasing.
The induced electric field circulates around the magnetic field lines.
Increasing current
I
$\vec{B}$ increasing
$\vec{B}$ increasing
Induced $\vec{E}$
Induced $\vec{E}$

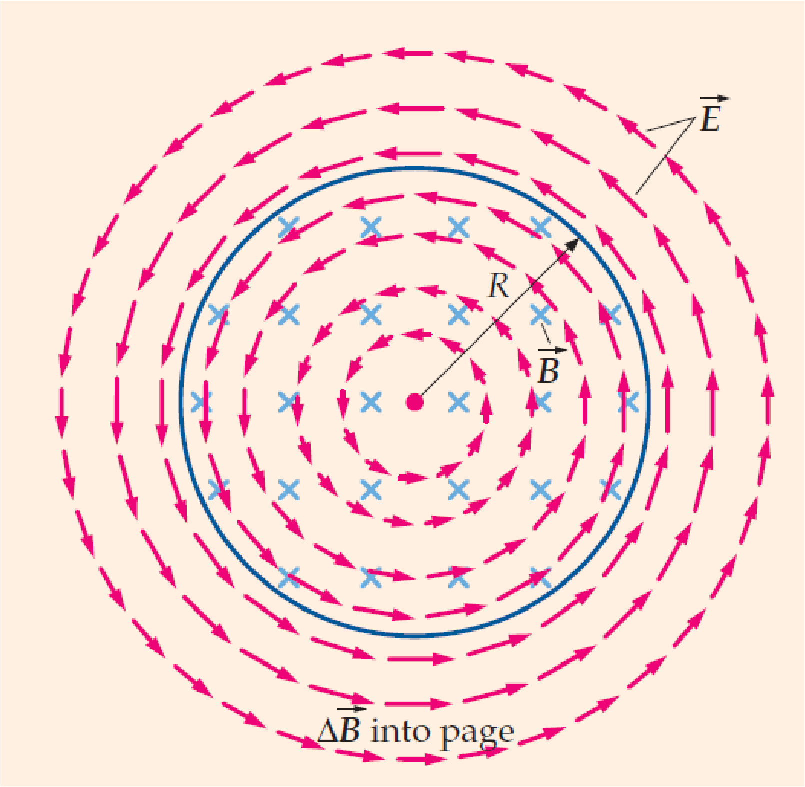


But the magnetic field did not accelerate the charged particles (they aren’t in it). Therefore, there must be a tangential electric field around the loop.

$$E = k\frac{|q|}{r^2}, \text{ away from } +$$



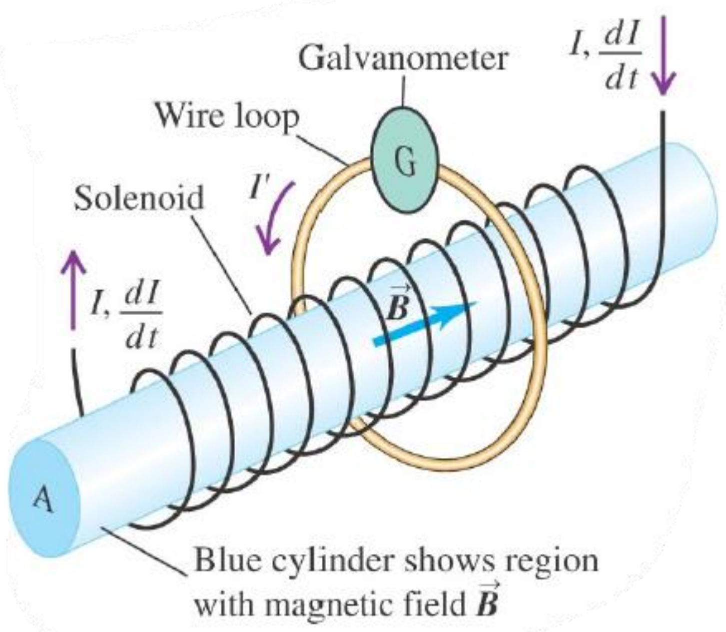




there are no + and – charges. Instead, there are electric field lines that form continuous, closed loops.

Conservative $\vec{E}$ → $\oint \vec{E}\cdot d\vec{l} = 0$ Closed path integration zero, path independent work

Non-conservative $\vec{E}$ → $\oint \vec{E}\cdot d\vec{l} = \varepsilon = -\dfrac{d\Phi_B}{dt}$ (stationary integration path)



The total work done on q by the induced E when it goes once around the loop: $W = qE$, here E is not conservative.

Remember, the magnetic force does no work when it accelerates a charged particle. If the loop has no resistance, the work done by the electric field goes into increasing the charged particle's speed (and therefore kinetic energy). If the loop has resistance, the work done by the electric field is dissipated in the resistance (energy leaves the system).

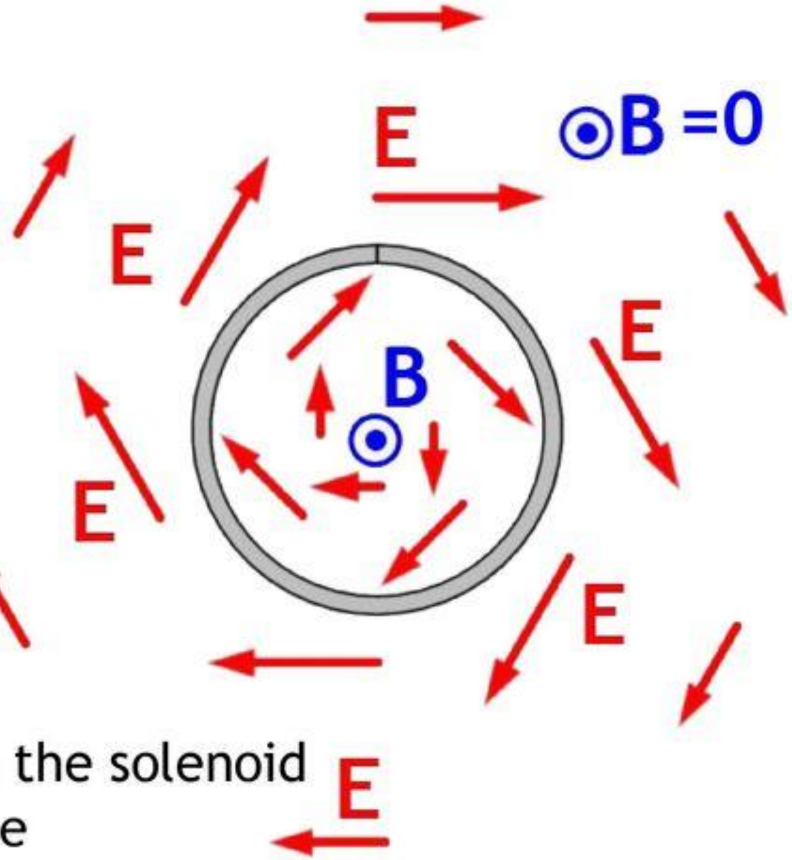
THEREFORE, THERE ARE TWO WAYS TO PRODUCE ELECTRIC FIELD

(1) Coulomb electric field is produced by electric charges according to Coulomb's law:

$$\vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}\hat{r}$$

(2) Non-Coulomb electric field E_{NC} is associated with time-varying magnetic flux density dB/dt

For a solenoid, E_{NC}
- curls around a solenoid
- is <u>proportional to *-dB/dt*</u> through the solenoid
- <u>decreases with *1/r*</u>, where *r* is the radial distance from the solenoid axis



- Faraday's law: 1) an emf is induced by magnetic forces on charges when a conductor moves through $\vec{B}$.

2) a time-varying $\vec{B}$ induces $\vec{E}$ in stationary conductor and emf. $\vec{E}$ is induced even when there is no conductor. Induced $\vec{E}$ is non-conservative, "non-electrostatic". No potential energy associated, but $\vec{F}_E = q\,\vec{E}$.

A New Look at Electric Potential

Induced electric fields are produced not by static charges but by a changing magnetic flux. Although electric fields produced in either way exert forces on charged particles, there is an important difference between them. The simplest evidence of this difference is that the field lines of induced electric fields form closed loops.

Whereas, the field lines produced by static charges never do so but must start on positive charges and end on negative charges. Thus, a field line from a charge can never loop around and back onto itself.

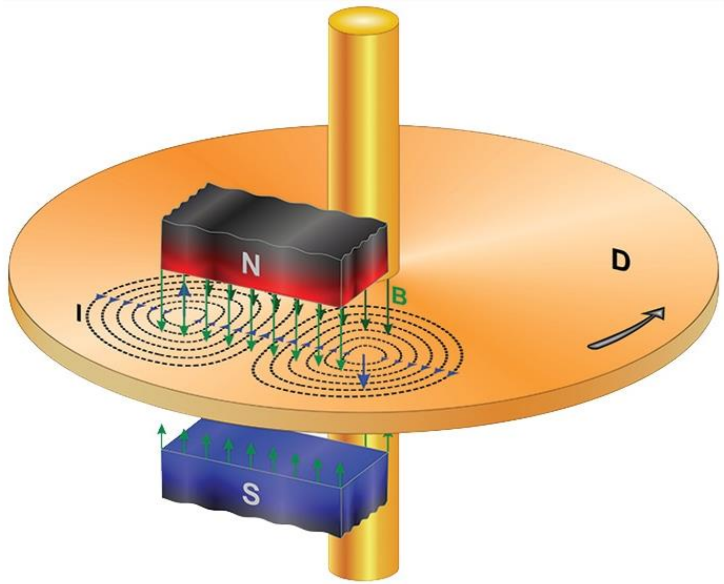
Electric potential has meaning only for electric fields that are produced by static charges; it has no meaning for electric fields that are produced by induction.

Conservative $\vec{E}$ $\rightarrow \oint \vec{E} \cdot d\vec{l} = 0$

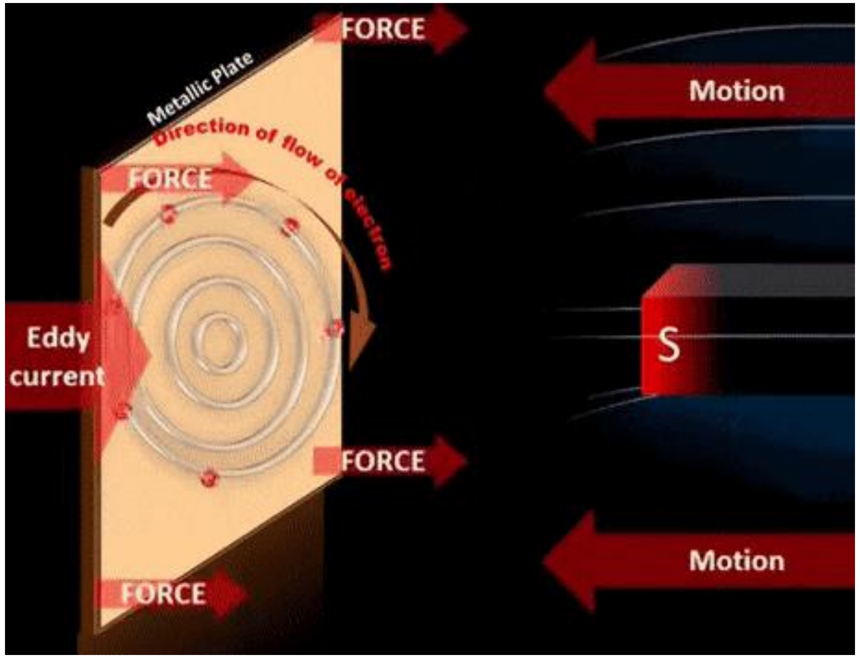
Non-conservative $\vec{E}$ $\rightarrow \oint \vec{E} \cdot d\vec{l} = \varepsilon = -\dfrac{d\Phi_B}{dt}$

Eddy Currents

Suppose we place a solid conducting plate within a magnetic field.



If we then move the plate out of the magnetic field as we did the loop, the relative motion of the field and the conductor again induces a current in the conductor.



The electrons swirl about within the plate as if they were caught in an eddy (whirlpool) of water. Such a current is called an *eddy current, as if* it followed a single path.

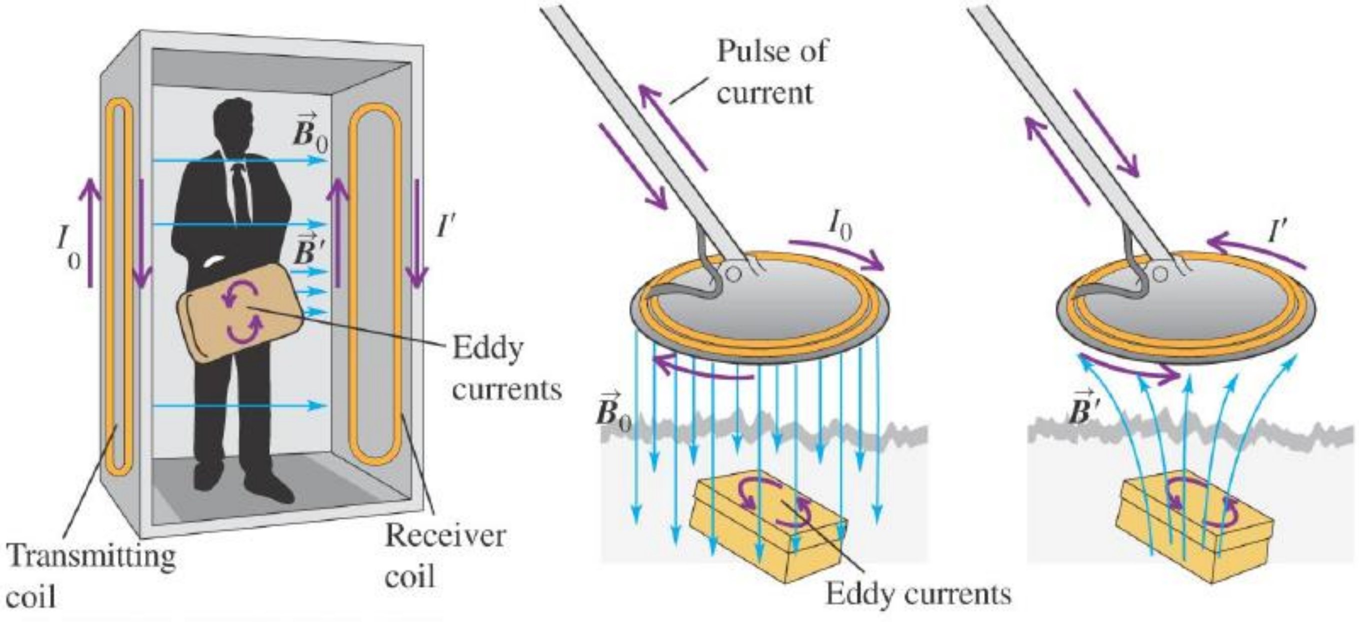


Figure 14.9 The familiar security gate at an airport not only detects metals, but can also indicate their approximate height above the floor. (credit: "Alexbuirds"/Wikimedia Commons)

Metal detector (airport security checkpoint) generates an alternating B0 that induces eddy currents in conducting object (suitcase). These currents produce alternating B' that induces current in detector's receiver (I').

Some Applications

Tape Recording

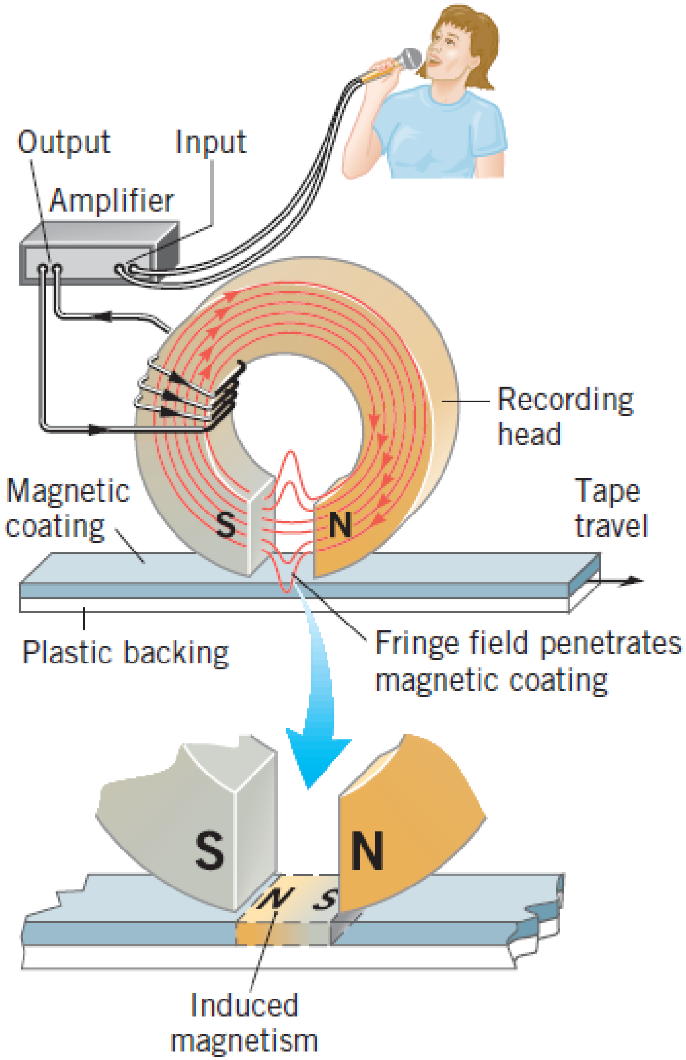
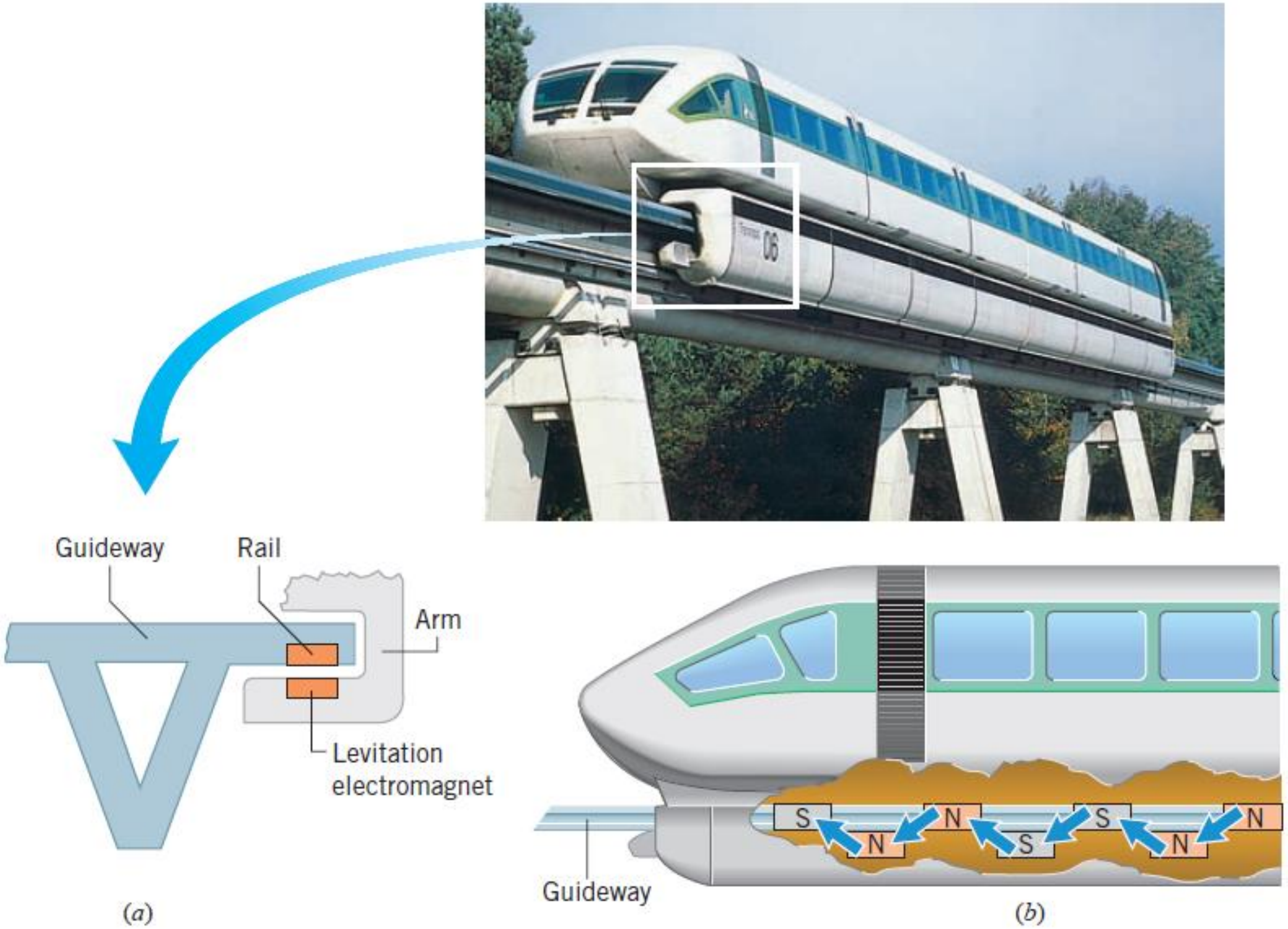


Figure 21.40 The magnetic fringe field of the recording head penetrates the magnetic coating on the tape and magnetizes it.

Maglev Train



Faraday's Law: Summary