Magnetic Fields-III

Phy 108 course
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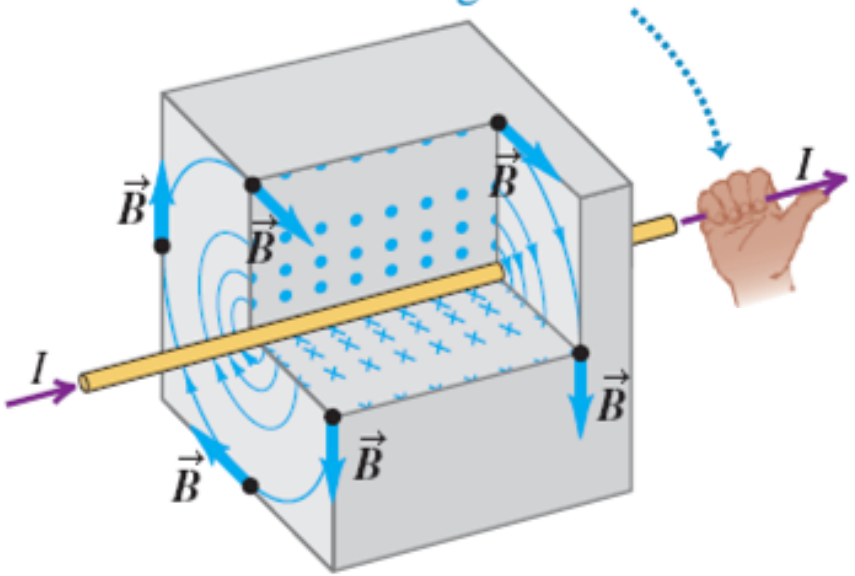
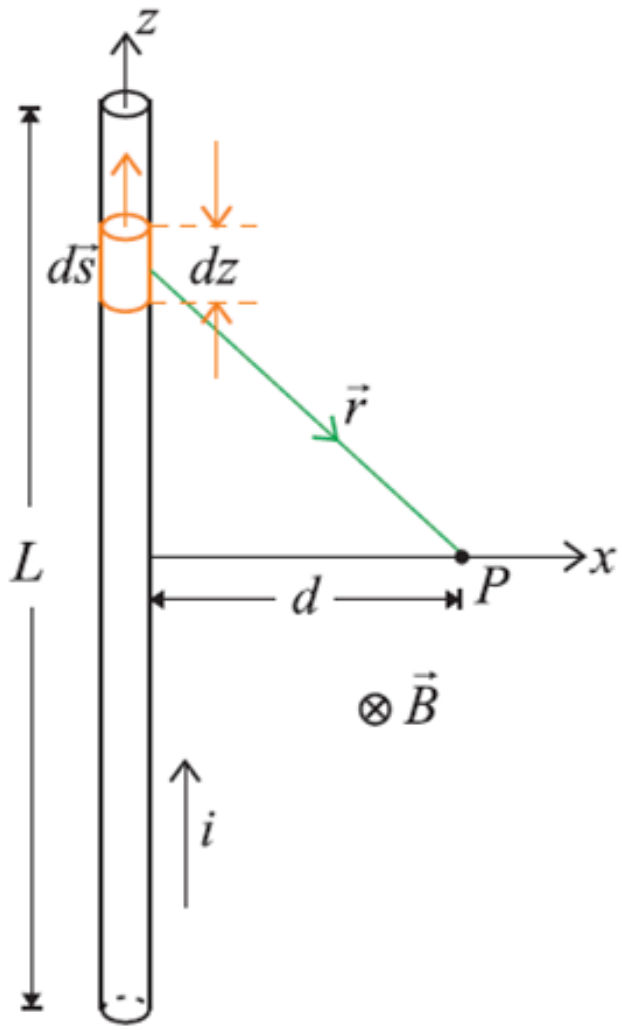
Magnetic field due to straight current carrying wire

Right-hand rule for the magnetic field around a current-carrying wire: Point the thumb of your right hand in the direction of the current. Your fingers now curl around the wire in the direction of the magnetic field lines.





$$d\vec{B} = \frac{\mu_0}{4\pi}\frac{i d\vec{S}\times\hat{r}}{r^2}$$

considering the magnitude

$$dB = \frac{\mu_0}{4\pi}\frac{i dS \sin\theta}{r^2}$$

$$\vec{B} = \int d\vec{B} = \frac{\mu_0}{4\pi}\int \frac{i d\vec{S}\times\vec{r}}{r^3}$$

$$B = \frac{\mu_0 i}{2\pi d}$$

direction of field determined from *right-hand screw rule*

Magnetic field due to straight current carrying wire

$$d\vec{\mathbf{s}} \times \hat{\mathbf{r}} = |d\vec{\mathbf{s}} \times \hat{\mathbf{r}}|\hat{\mathbf{k}} = \left[dx \sin\left(\frac{\pi}{2} - \theta\right)\right]\hat{\mathbf{k}} = (dx \cos\theta)\hat{\mathbf{k}}$$

$$(1) \quad d\vec{\mathbf{B}} = (dB)\hat{\mathbf{k}} = \frac{\mu_0 I}{4\pi} \frac{dx \cos\theta}{r^2} \hat{\mathbf{k}}$$

$$(2) \quad r = \frac{a}{\cos\theta}$$

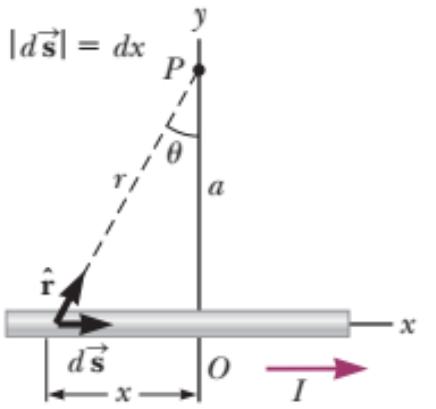
$$x = -a \tan\theta$$

$$(3) \quad dx = -a \sec^2\theta \, d\theta = -\frac{a \, d\theta}{\cos^2\theta}$$

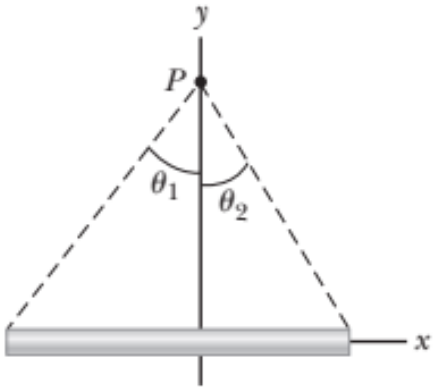
$$(4) \quad dB = -\frac{\mu_0 I}{4\pi}\left(\frac{a \, d\theta}{\cos^2\theta}\right)\left(\frac{\cos^2\theta}{a^2}\right)\cos\theta = -\frac{\mu_0 I}{4\pi a}\cos\theta \, d\theta$$

$$B = -\frac{\mu_0 I}{4\pi a}\int_{\theta_1}^{\theta_2} \cos\theta \, d\theta = \frac{\mu_0 I}{4\pi a}(\sin\theta_1 - \sin\theta_2)$$

$$B = \frac{\mu_0 i}{2\pi d}$$

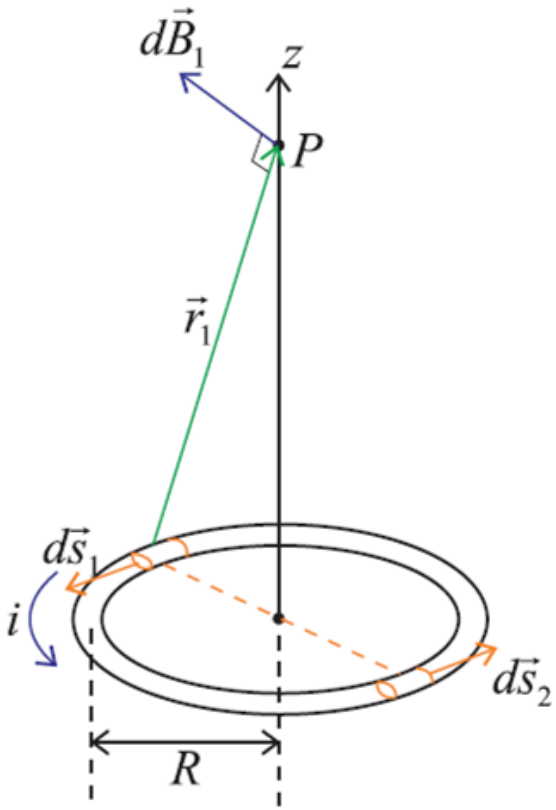


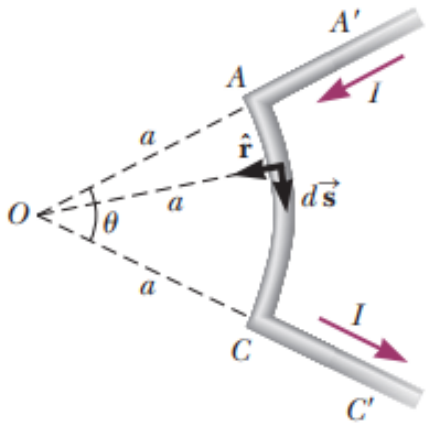
a



b

Magnetic Field Due to a Current in a Circular Arc of Wire





Only vertical component of B field needs to be considered at point *P*

$$dB = \frac{\mu_0}{4\pi}\frac{I\,ds}{a^2}$$

$$B = \frac{\mu_0 I}{4\pi a^2}\int ds = \frac{\mu_0 I}{4\pi a^2}\,s$$

$$B = \frac{\mu_0 I}{4\pi a^2}(a\theta) = \boxed{\frac{\mu_0 I}{4\pi a}\theta}$$

$$\boldsymbol{B = \frac{\mu_0 i \varphi}{4\pi R}}$$

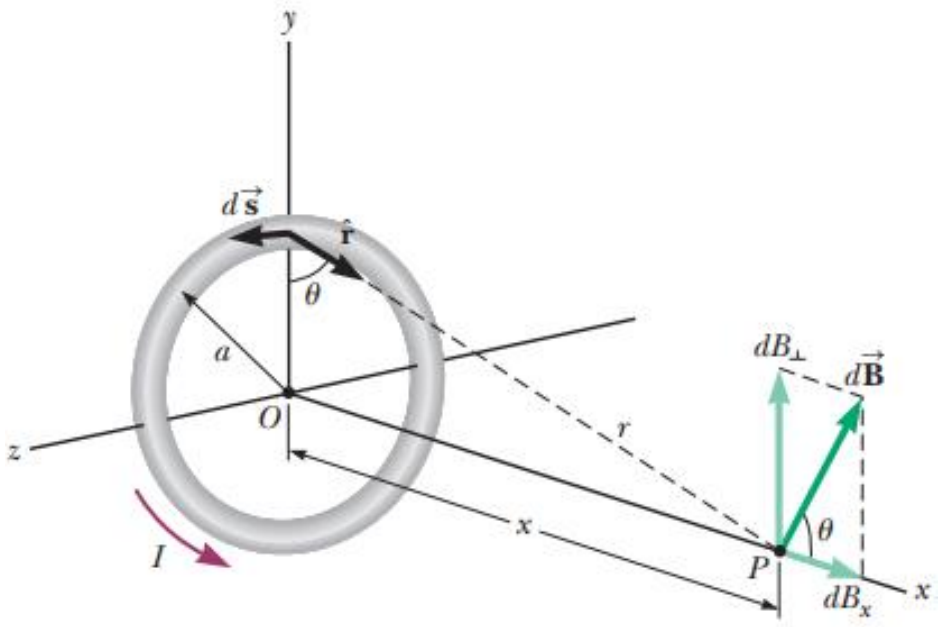
(at the centre of circular arc)
φ is in radians,

For a whole circle φ = 2π

$$\boldsymbol{B = \frac{\mu_0 i}{2R}}$$

https://javalab.org/en/magnetic_field_around_a_circular_wire_en/ https://javalab.org/en/magnetic_field_around_a_wire_en/

Magnetic Field on the Axis of a Circular Current Loop



$$dB = \frac{\mu_0 I}{4\pi}\frac{|d\vec{s} \times \hat{r}|}{r^2} = \frac{\mu_0 I}{4\pi}\frac{ds}{(a^2 + x^2)}$$

$$dB_x = \frac{\mu_0 I}{4\pi}\frac{ds}{(a^2 + x^2)}\cos\theta$$

$$B_x = \oint dB_x = \frac{\mu_0 I}{4\pi}\oint\frac{ds\cos\theta}{a^2 + x^2}$$

$$\cos\theta = \frac{a}{(a^2 + x^2)^{1/2}}$$

$$B_x = \frac{\mu_0 I}{4\pi}\oint\frac{ds}{a^2 + x^2}\left[\frac{a}{(a^2 + x^2)^{1/2}}\right] = \frac{\mu_0 I}{4\pi}\frac{a}{(a^2 + x^2)^{3/2}}\oint ds$$

$$B_x = \frac{\mu_0 I}{4\pi}\frac{a}{(a^2 + x^2)^{3/2}}(2\pi a) = \boxed{\frac{\mu_0 I a^2}{2(a^2 + x^2)^{3/2}}} \qquad \textbf{(30.7)}$$

$$B = \frac{\mu_0 I}{2a} \quad (\text{at } x = 0)$$

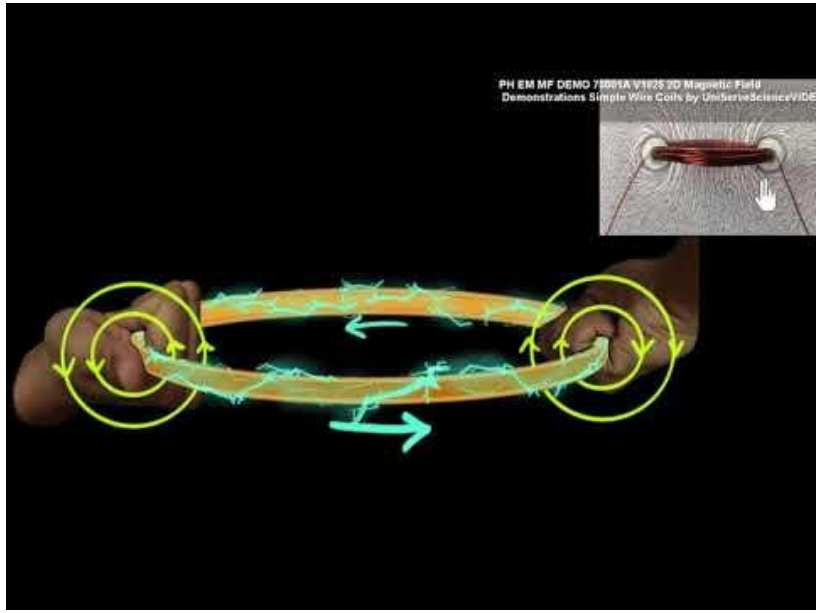
Magnetic Field Due to a Current in a Circular Arc of Wire

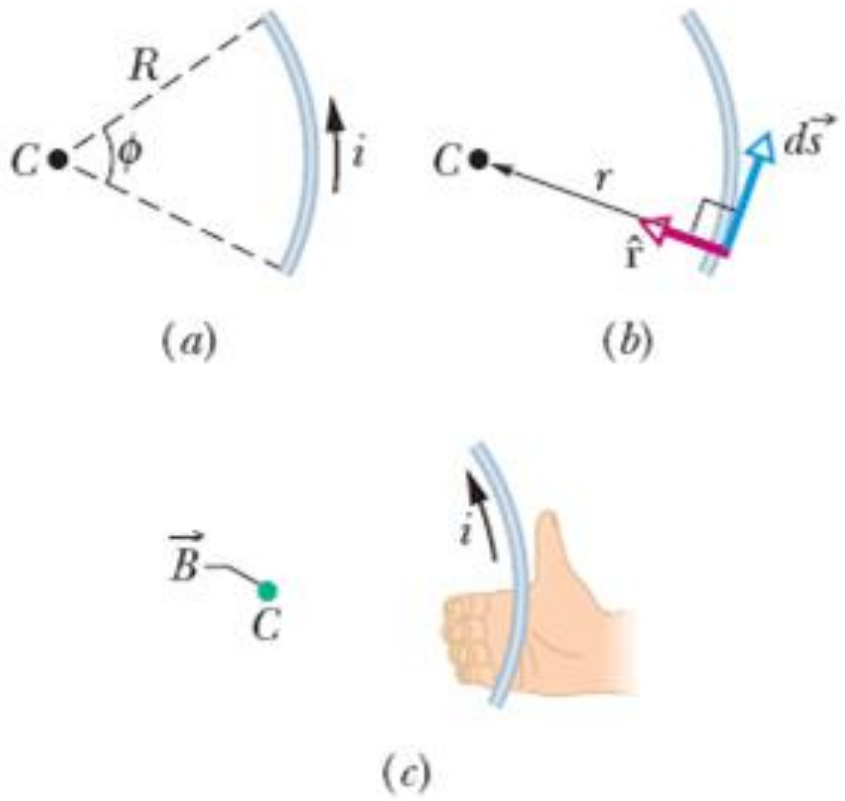
Direction of B??

TWO right hand rules!:

If your thumb points along the CURRENT, your fingers will point in the same direction as the FIELD.

If you curl our fingers around direction of CURRENT, your thumb points along FIELD!





The right-hand rule reveals the field's direction at the center.

http://physics.bu.edu/~duffy/HTML5/magnetic_field_1D.html

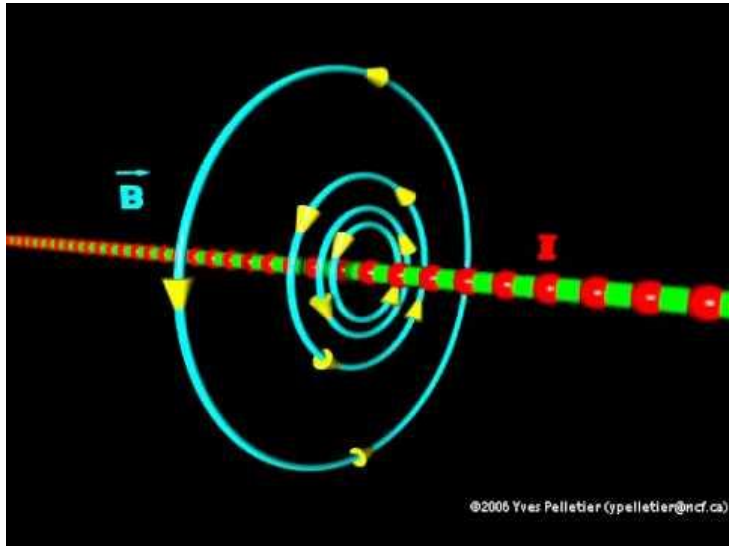
Example : A Practical Matter

A power line carries a current of 500 A. What is the magnetic field in a house located 100 m away from the power line?

$$B = \frac{\mu_0 i}{2\pi R}$$

$$= \frac{(4\pi x 10^{-7} T.m/A)(500A)}{2\pi(100m)}$$

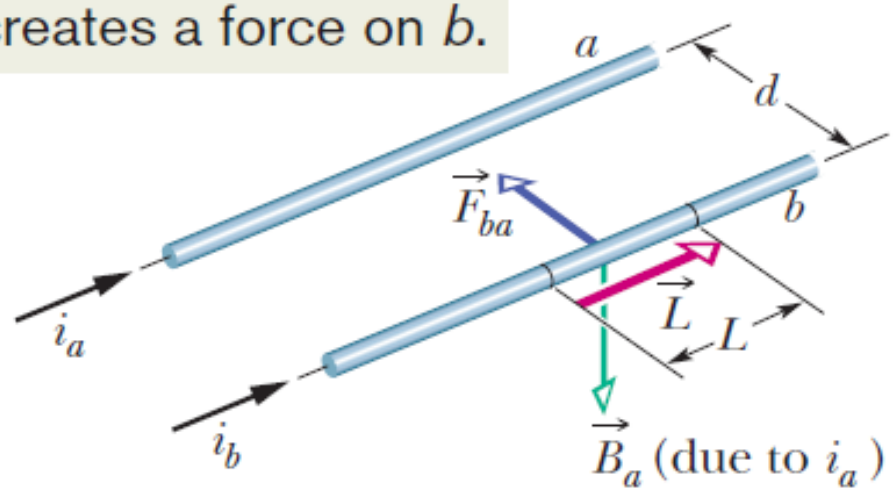
$$\boxed{= 1\ \mu T!!}$$



Recall that the earth's magnetic field is ~10^{-4}T = 100 μT

Force Between Two Parallel Currents

The field due to *a* at the position of *b* creates a force on *b*.

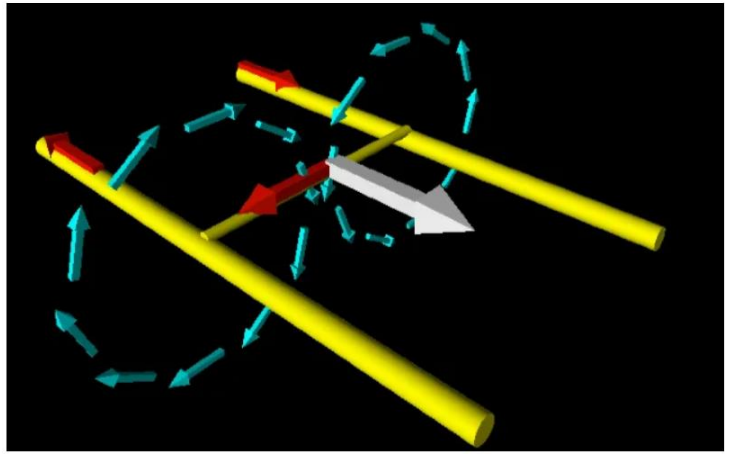


Two long parallel wires carrying currents exert forces on each other. Figure shows two such wires, separated by a distance d and carrying currents i_a and i_b. Let us analyze the forces on these wires due to each other.

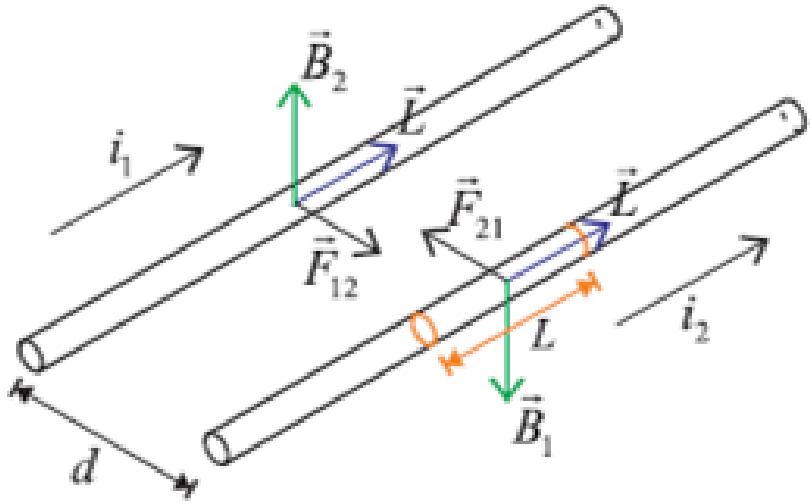
$$B_a = \frac{\mu_0 i_a}{2\pi d}$$

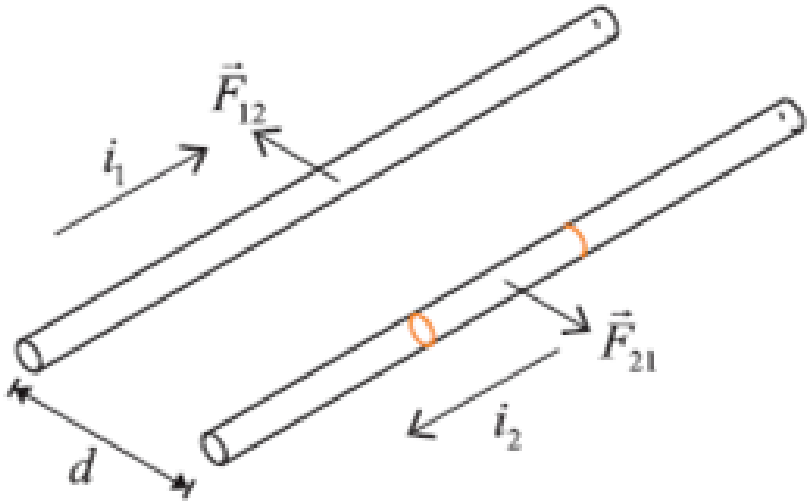
$$\vec{F}_{ba} = i_b \vec{L} \times \vec{B}_a$$

$$F_{ba} = i_b L B_a \sin 90° = \frac{\mu_0 L i_a i_b}{2\pi d}$$



Force Between Parallel Currents :





Consider a segment of length L on i_2 :

$$\vec{B}_1 = \frac{\mu_0 i_1}{2\pi d} \quad \text{(pointing down)} \qquad\qquad \vec{B}_2 = \frac{\mu_0 i_1}{2\pi d} \quad \text{(pointing up)}$$

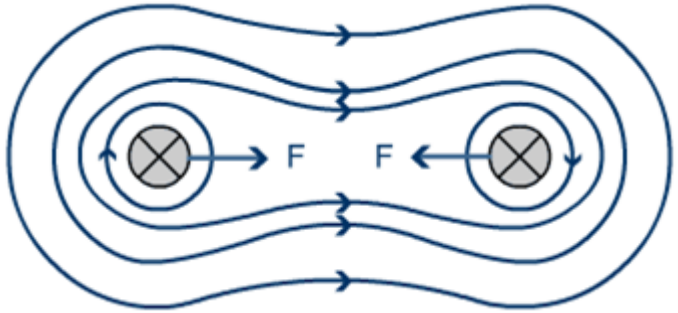
Force on i_2 coming from i_1:

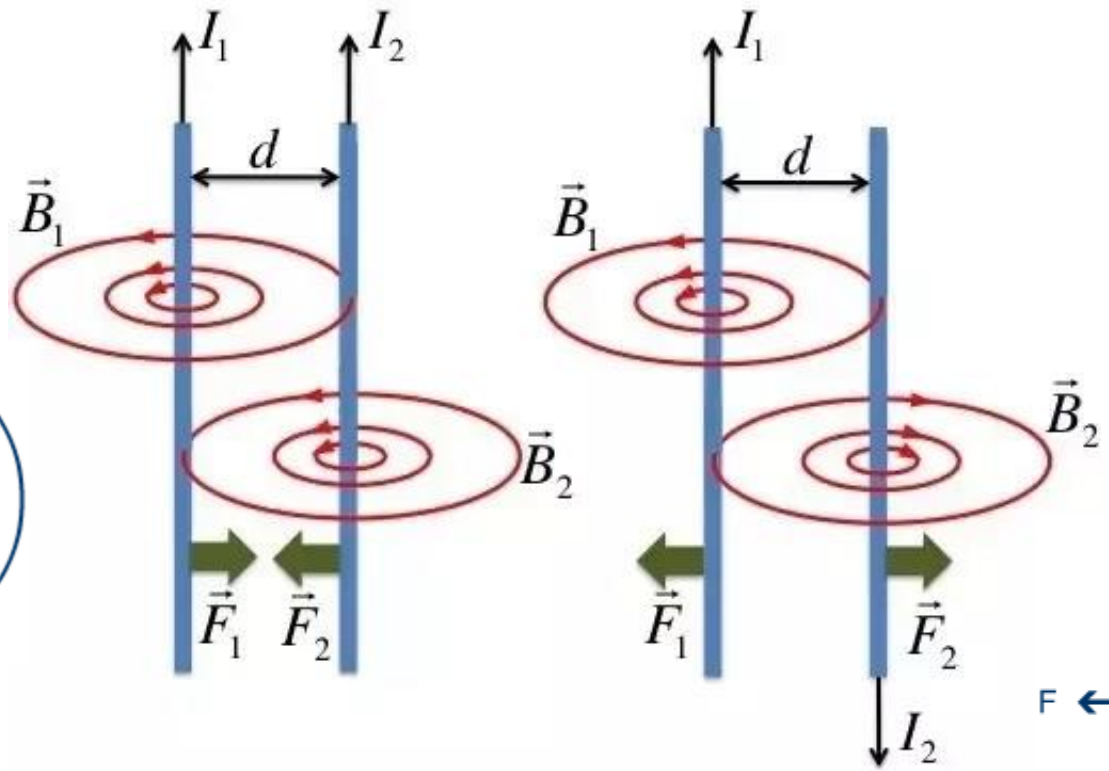
$$|\vec{F}_{21}| = i_2 \vec{L} \times \vec{B}_1 = \frac{\mu_0 L i_1 i_2}{2\pi d} = |\vec{F}_{12}| \qquad \textit{(Def'n of ampere, A)}$$

$\therefore$ *Parallel currents attract, anti-parallel currents repel.*

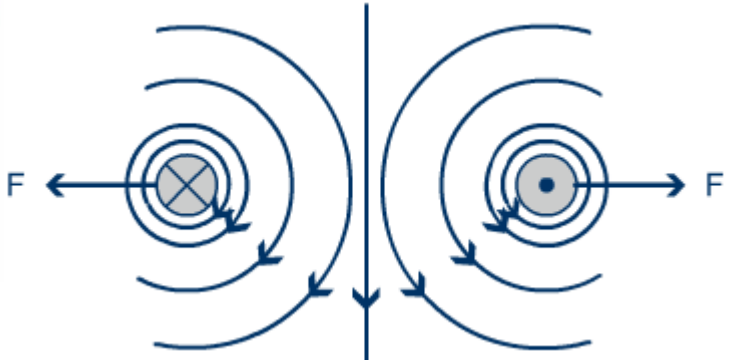
http://physics.bu.edu/~duffy/HTML5/BField.html

Same direction parallel currents



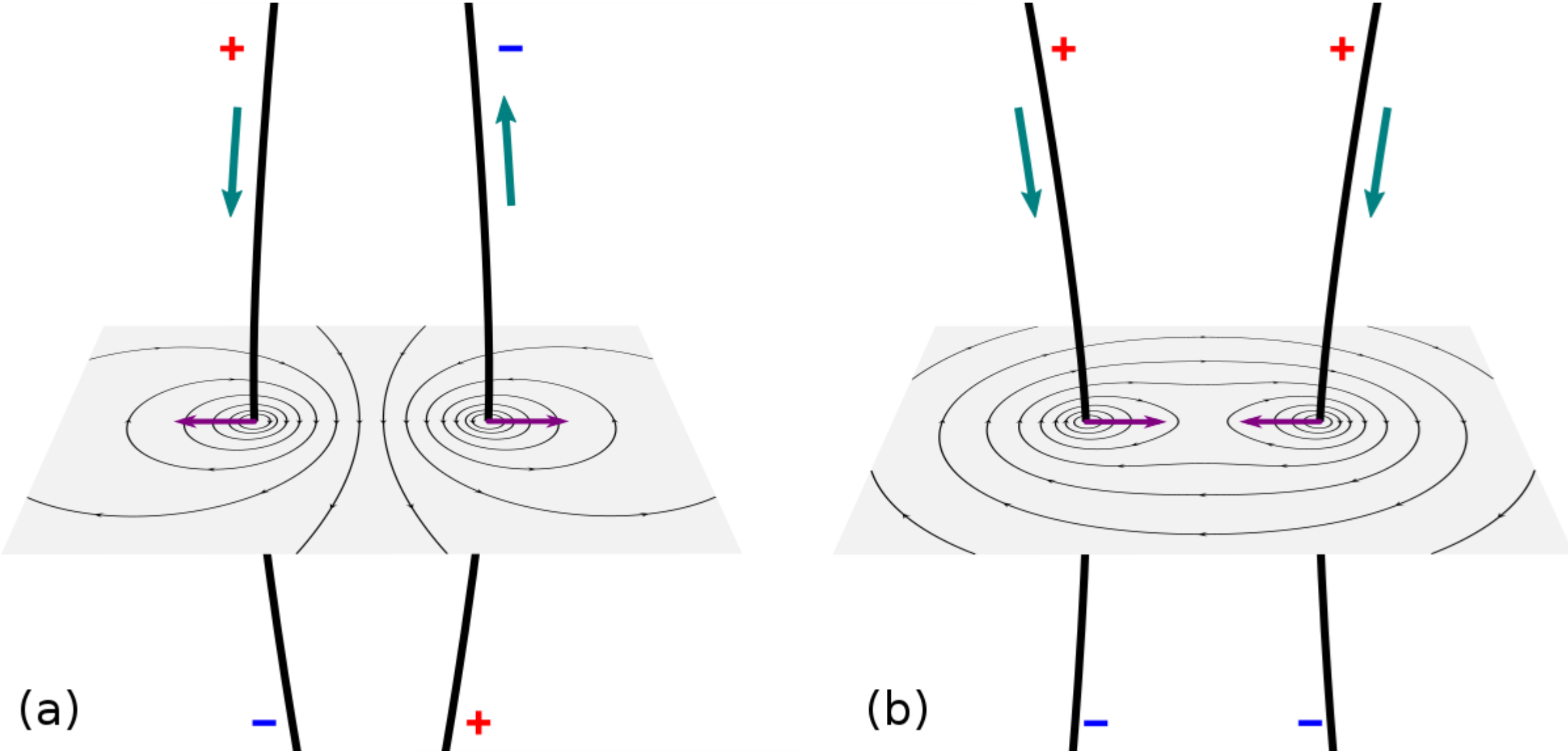


Opposite direction parallel currents



Parallel currents attract each other, and antiparallel currents repel each other.

http://physics.bu.edu/~duffy/HTML5/current_interaction.html



The force acting between currents in parallel wires is the basis for the definition of the ampere, which is one of the seven SI base units.

The definition, adopted in 1946, is this:

The ampere is that constant current which, if maintained in two straight, parallel conductors of infinite length, of negligible circular cross section, and placed 1 m apart in vacuum, would produce on each of these conductors a force of magnitude 2×10^{-7} newton per meter of wire length.

For $i_1 = i_2 = 1$ A and $d = 1$ m force between the wires, $F = 2\times10^{-7}$ N

$$\frac{F}{l} = \frac{(4\pi\times10^{-7}\ \mathrm{T\cdot m/A})(1\ \mathrm{A})^2}{(2\pi)(1\ \mathrm{m})} = 2\times10^{-7}\ \mathrm{N/m}.$$

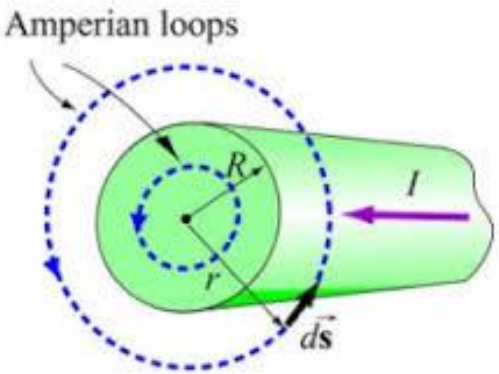
Two straight rods have central axes that are 1.50 mm apart in a current balance, and carry currents of 15.0 A each in opposite directions. What mass must be placed on the upper rod to balance the magnetic force of repulsion? The wire has a mass of 0.12 g per meter of length.

Ampere's Law

In our study of electricity, we notice that the inverse square force law leads to Gauss' Law, which is useful for *finding E-field for systems with high level of symmetry*.

Similarly, we can find the net magnetic field due to *any* distribution of currents by first writing the differential magnetic field due to a current-length element

$$d\vec{B} = \frac{\mu_0}{4\pi}\frac{i d\vec{S} \times \hat{r}}{r^2}$$



then summing the contributions of from all the elements.

$$\oint \vec{B}.d\vec{S} = \mu_0 i_{enc}$$

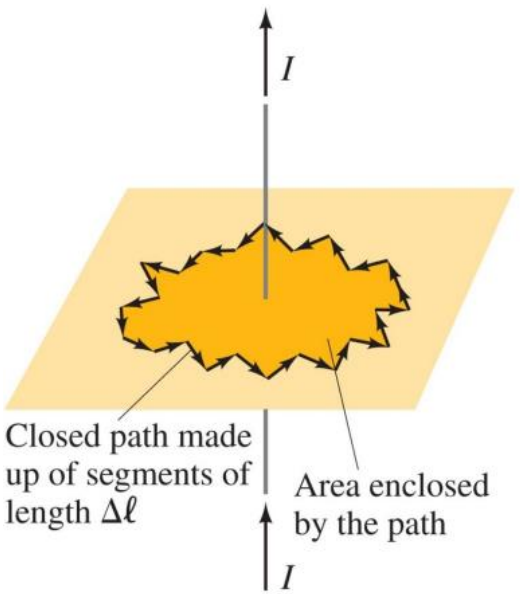
The loop on the integral sign means that the scalar (dot) product $\vec{B}.d\vec{S}$ is to be integrated around a *closed* loop, called an *Amperian loop*. The current i_{enc} is the *net* current encircled by that closed loop.

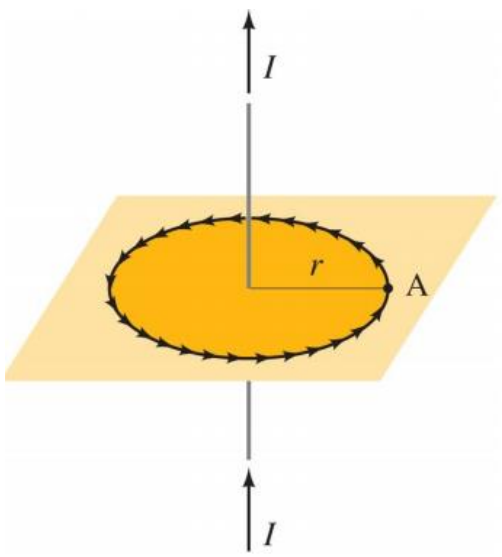
Again we may have to use a computer for a complicated distribution. However, if the distribution has some symmetry, we may be able to apply **Ampere's law** to find the magnetic field with considerably less effort.

Ampère's law relates the magnetic field around a closed loop to the total current flowing through the loop

The integral around a closed path of the component of the magnetic field tangent to the direction of the path equals μ_0 times the current intercepted by the area within the path:

$$\oint \vec{B}.d\vec{s} = \mu_0 I_{enc}$$





Ampère's Law is one of the general laws of magnetism.

The only restriction on this law is that the currents must be steady; that is, they must be constant in time.

Is independent of the shape of the curve around the current.

Is independent of where the current passes through the curve.

Depends only on the total amount of current through the area enclosed by the integration path.

Ampere's Law

Amperian loop is an imaginary closed loop around a current carrying conductor through which ampere's law is applied to that conductor.

$$\oint \vec{B}.d\vec{S} = \mu_0 i_{enc}$$

where the integral on the left is a “path integral”, similar to how we calculate the work done by a force over a particular path. The circle sign on the integral means that this is an integral over a “closed” path; a path where the starting and ending points are the same

This is different from Gauss’ Law, where the integral is over a closed surface (not a closed path, as it is here).
In the context of Gauss’ Law, we refer to “calculating the **flux** of the electric field **through** a closed surface”

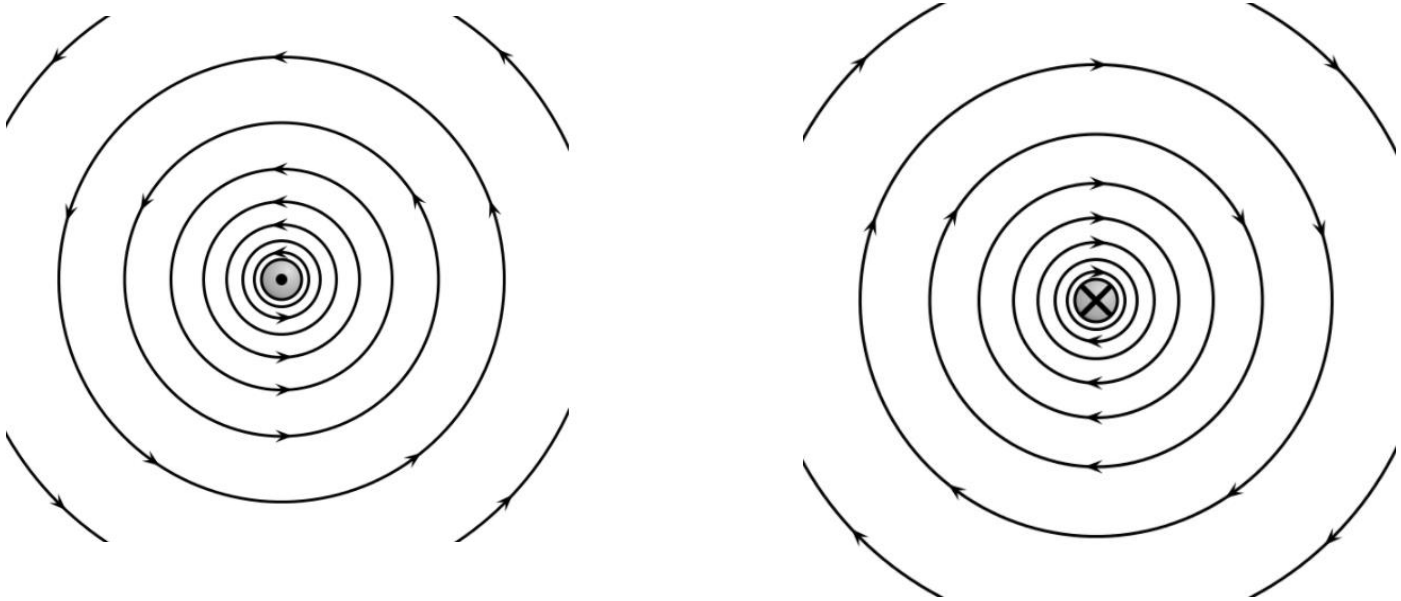
In the context of Ampere’s Law, we refer to “calculating the **circulation** of the magnetic field **along** a closed path”

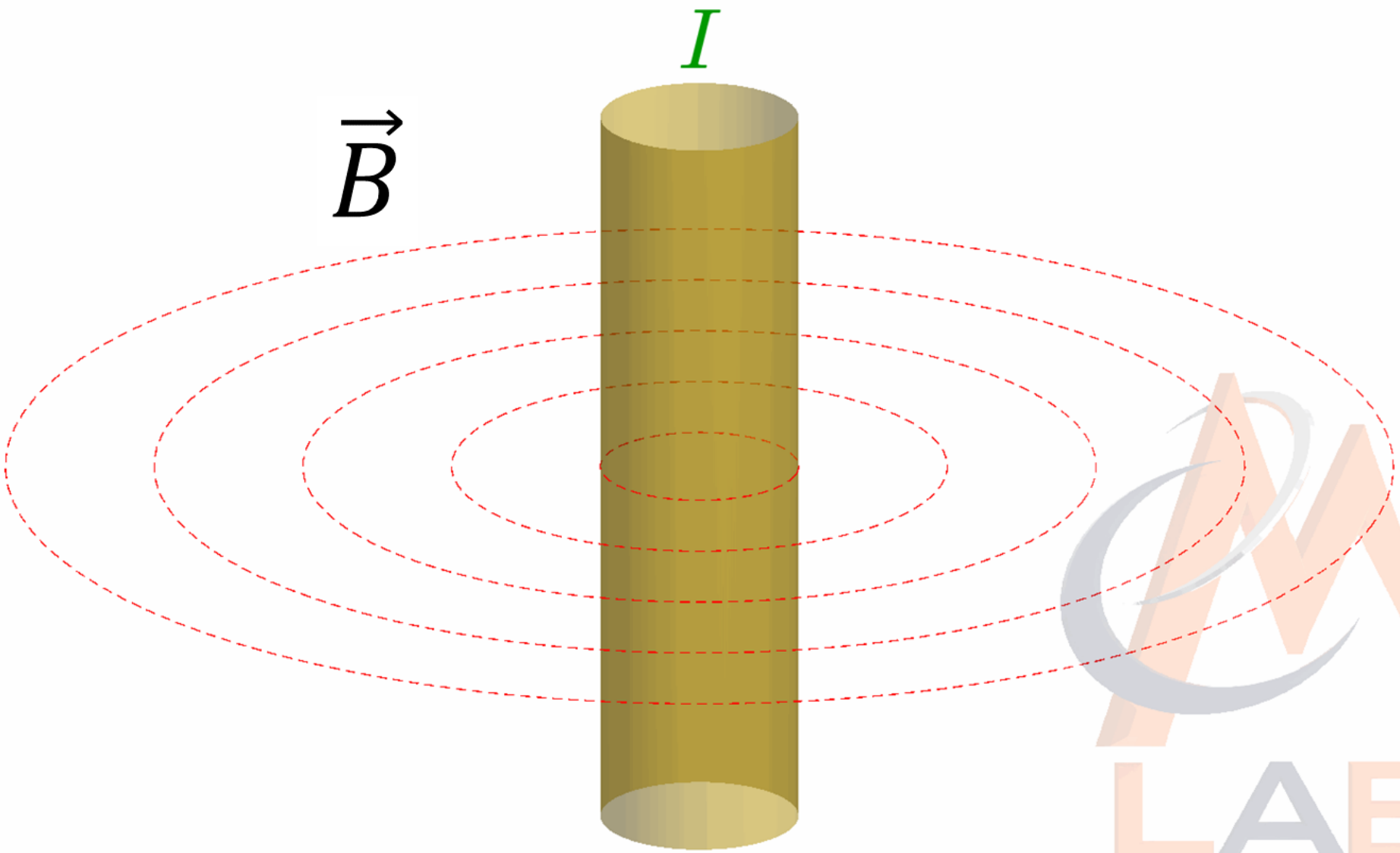
Ampere's Law

1. The angle between $\vec{B}$ and $d\vec{s}$ is constant along the path, so that: $\oint \vec{B} \cdot d\vec{s} = \oint B ds cos\theta = \cos\theta \oint B ds$ where θ is the angle between $\vec{B}$ and $d\vec{s}$.

2. The magnitude of $\vec{B}$ is constant along the path, so that: $\cos\theta \oint B ds = B cos\theta \oint ds$

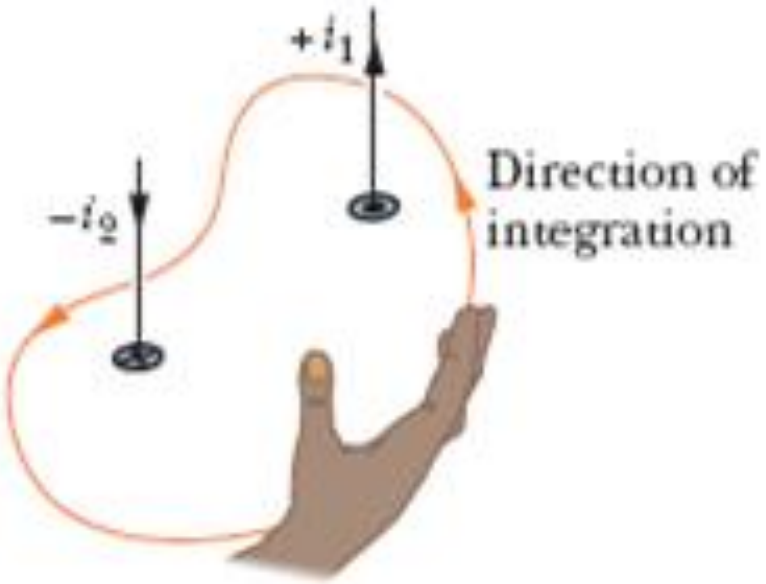
Choosing a path that meets these two conditions is only possible if there is a high degree of symmetry.




I
$\vec{B}$

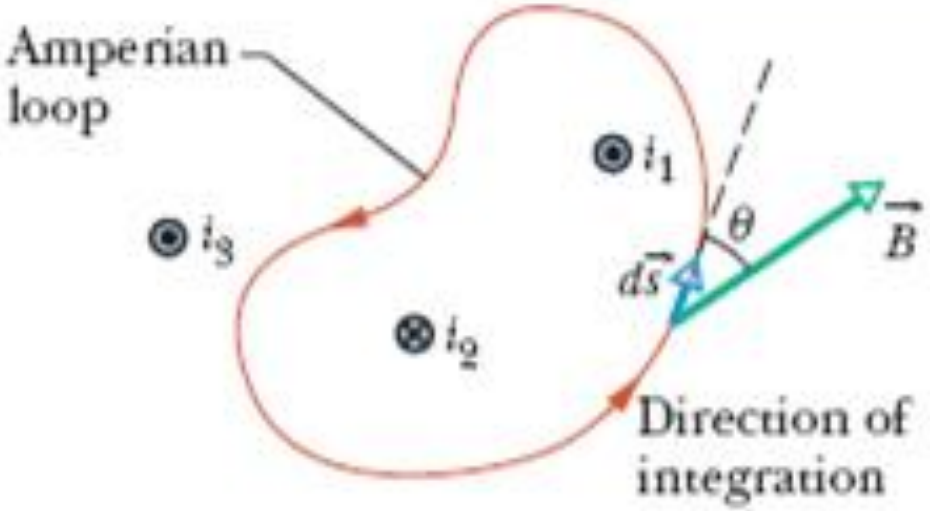
Ampere's Law

This is how to assign a sign to a current used in Ampere's law.



$$i_{enc} = i_1 - i_2$$

Only the currents encircled by the loop are used in Ampere's law.

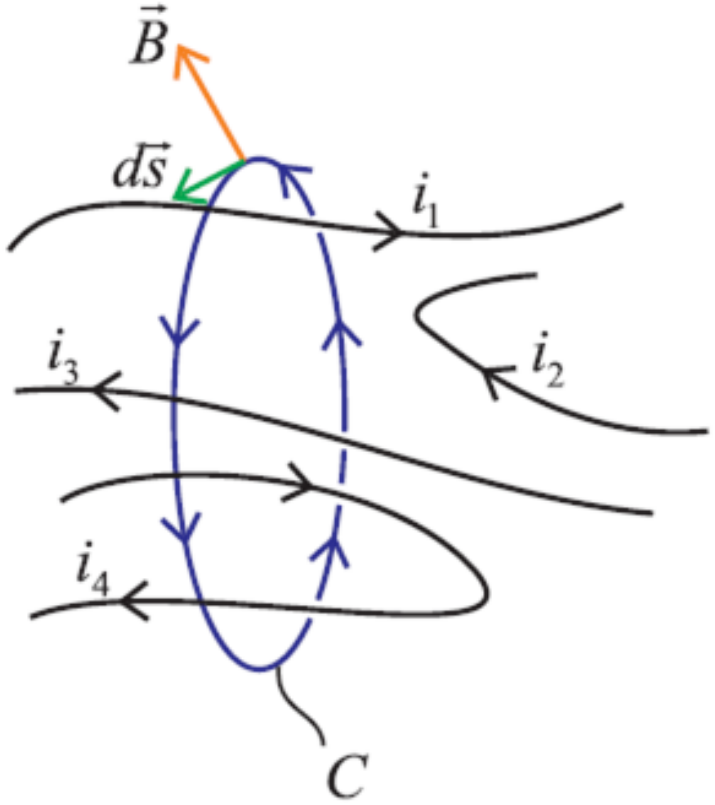


$$i_{enc} = i_1 - i_2$$

$$\oint B \cos\theta \, ds = \mu_0(i_1 - i_2).$$

Ampere's Law

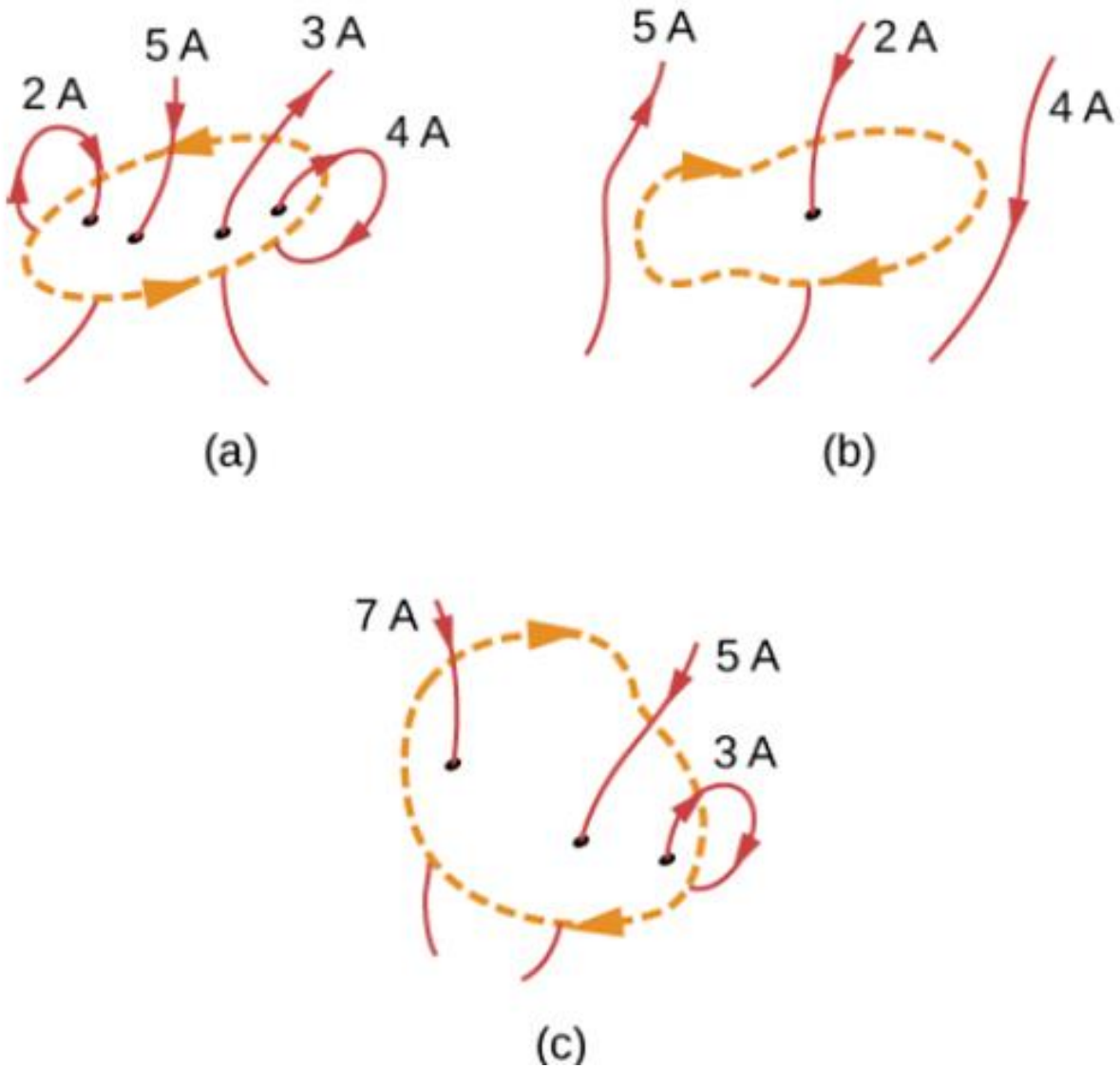
Convention : Use the right-hand screw rule to determine the *sign* of current.



$$\oint_C \vec{B} \cdot d\vec{s} = \mu_0(i_1 - i_3 + i_4 - i_4)$$

$$= \mu_0(i_1 - i_3)$$

Using Ampère's Law with Arbitrary Paths



Ampere's Law

Ampere's Law is thus a statement that an electric current will result in a field with a magnitude proportional to the current, that has some degree of rotation to it. The direction of rotation of that field corresponds to the right-hand rule for axial vectors as applied to the current (your thumb points in the direction of the current so that your fingers curl in the direction of the rotation of the associated field).

Ampere's law integral form: $\oint \vec{B}.d\vec{S} = \mu_0 i_{enc}$

By considering the current density, $i_{enc} = \iint \vec{J}.d\vec{S}$

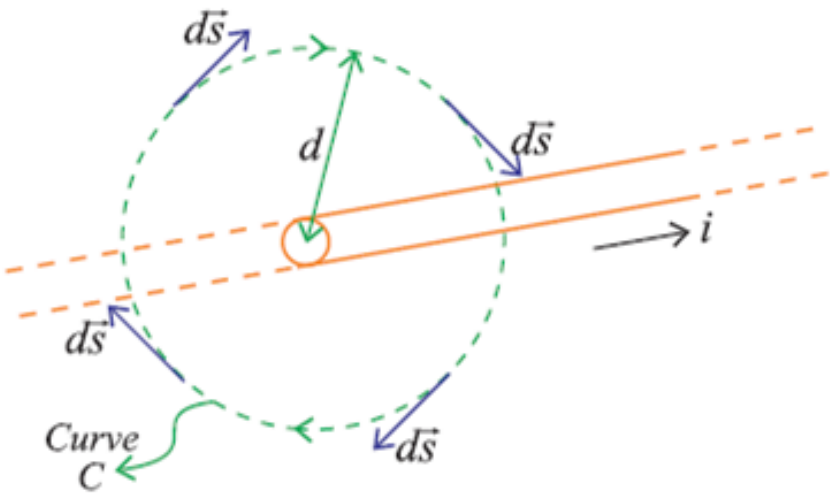
Ampere's law differential form: $\vec{\nabla} \times \vec{B} = \mu_0 \vec{J}$

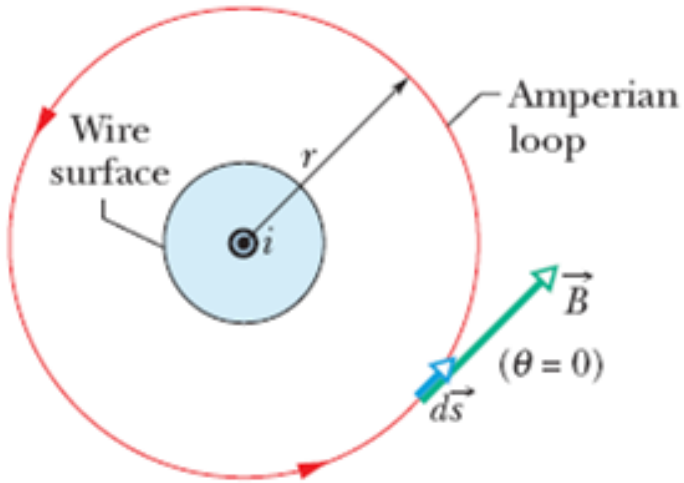
a current density will create a (magnetic) field that has non-zero curl

Applications of the Ampere's Law :

(1) Long-straight wire

All of the current is encircled and thus all is used in Ampere's law.





By symmetry argument, we know *B field* only has *tangential component* $\oint \vec{\boldsymbol{B}}.\boldsymbol{d}\vec{\boldsymbol{S}} = \boldsymbol{\mu_0 i}$

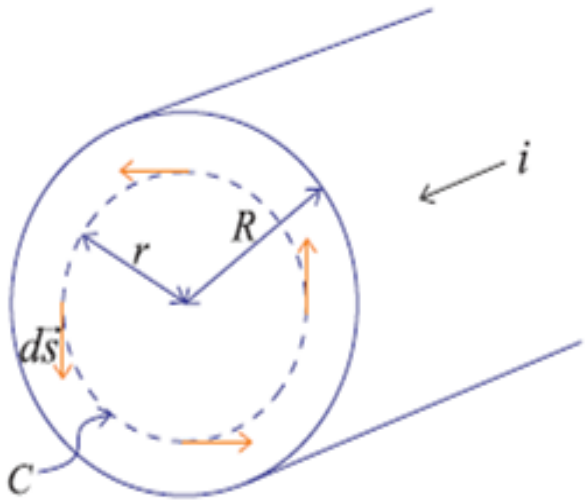
Taking $d\vec{S}$ to be the tangential vector around the circular path: $\vec{\boldsymbol{B}}.\boldsymbol{d}\vec{\boldsymbol{S}} = B\ dS$

Then considering the circular path: $B\ \oint dS = B(2\pi r) = \boldsymbol{\mu_0 i}$

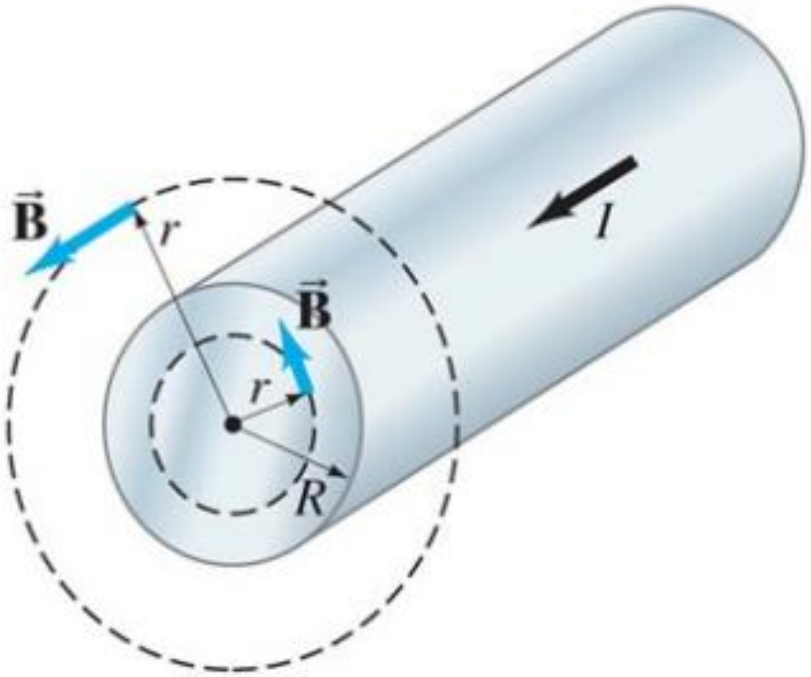
Thus the field, $\boldsymbol{B} = \frac{\boldsymbol{\mu_0 i}}{(\boldsymbol{2\pi r})}$ (observed from Biot Savart law)

Magnetic Field Inside the current carrying wire

Inside a current-carrying wire



Again, symmetry argument implies that $\vec{B}$ is *tangential* to the Amperian curve and $\vec{B} \rightarrow B(r)\hat{\theta}$

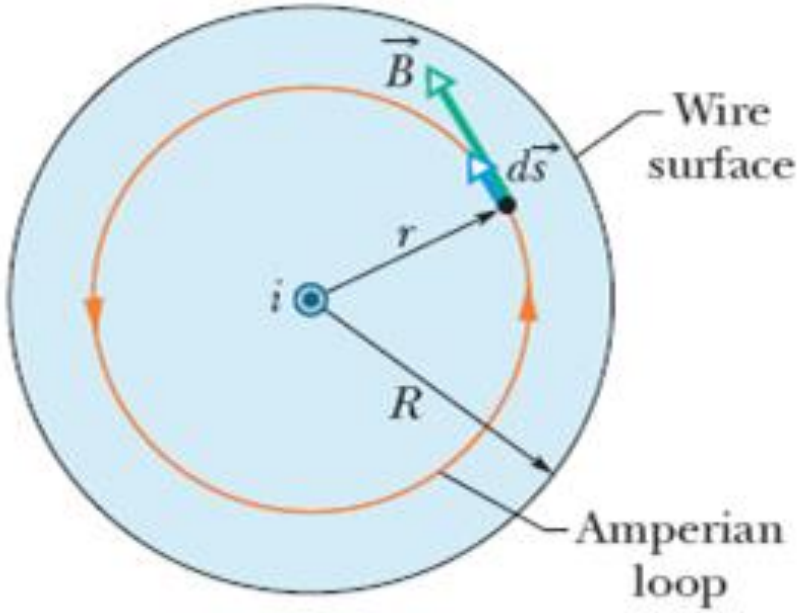


Consider an Amperian curve of radius $r(< R)$

$$\oint_C \vec{B} \cdot d\vec{s} = B \oint ds = B(2\pi r) = \mu_0 i_{included}$$

Magnetic Field Inside the current carrying wire

Only the current encircled by the loop is used in Ampere's law.

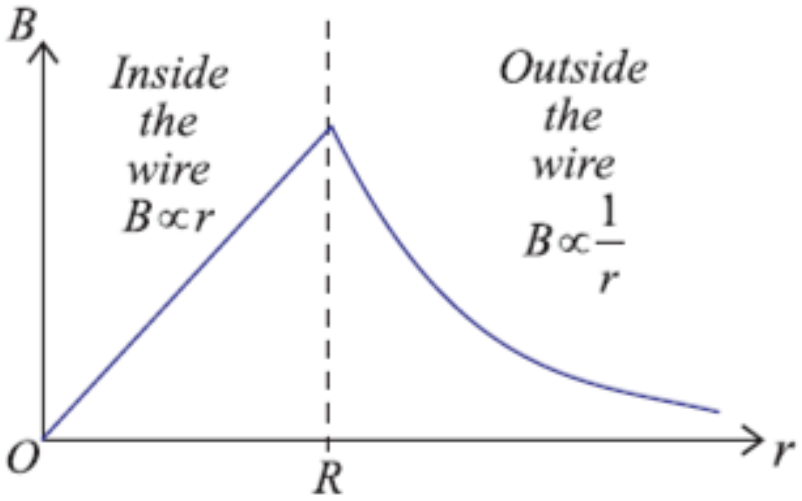


$$\frac{i_{included}}{i} = \frac{\pi r^2}{\pi R^2}$$

$$i_{included} = \frac{r^2}{R^2} i$$

$$B = \frac{\mu_0 i}{2\pi R^2} \cdot r \propto r$$

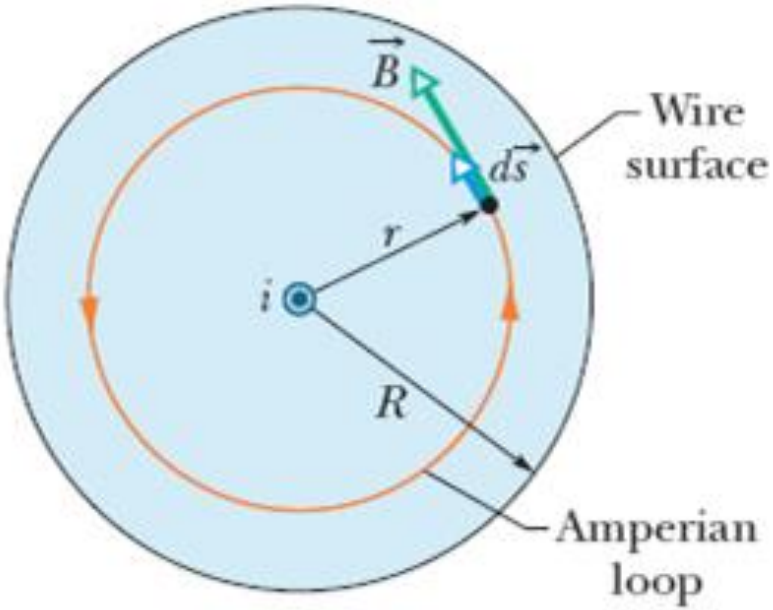
But $i_{included} \propto$ cross-sectional area of C

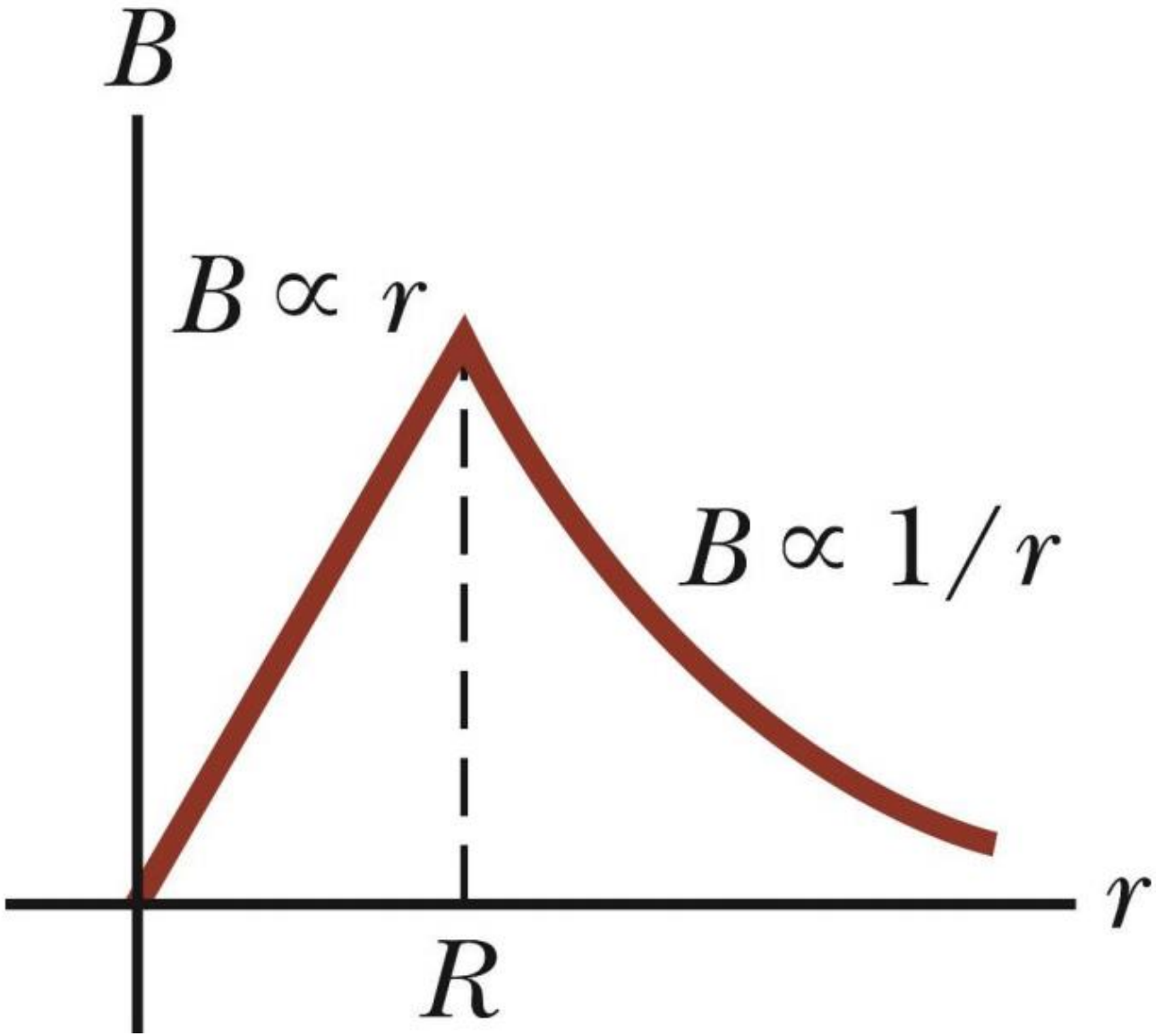


Recall: Uniformly charged infinite long rod

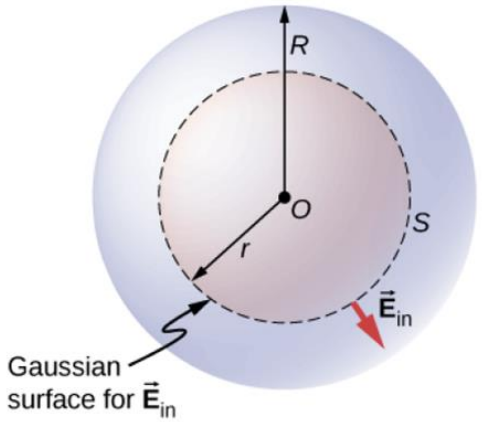
Magnetic Field Inside the current carrying wire

Only the current encircled by the loop is used in Ampere's law.

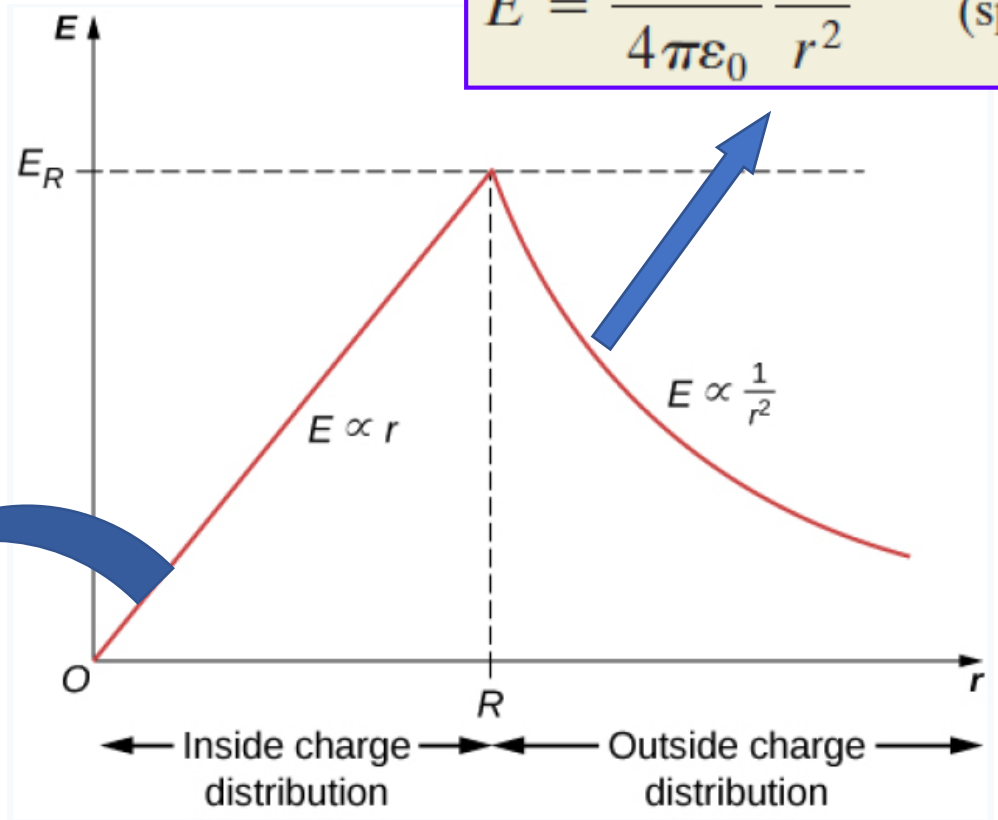


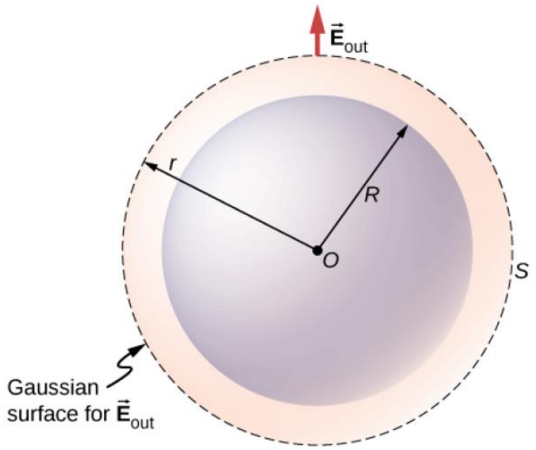


Recall the case for Electric field



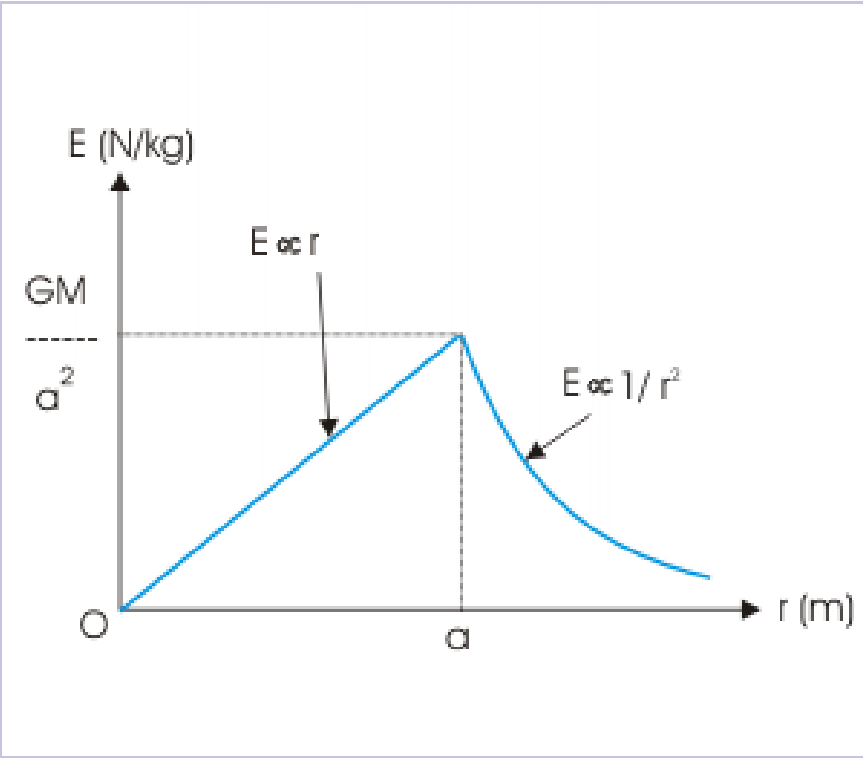
$$E = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2} \qquad \text{(spherical shell, field at } r \geq R\text{).}$$



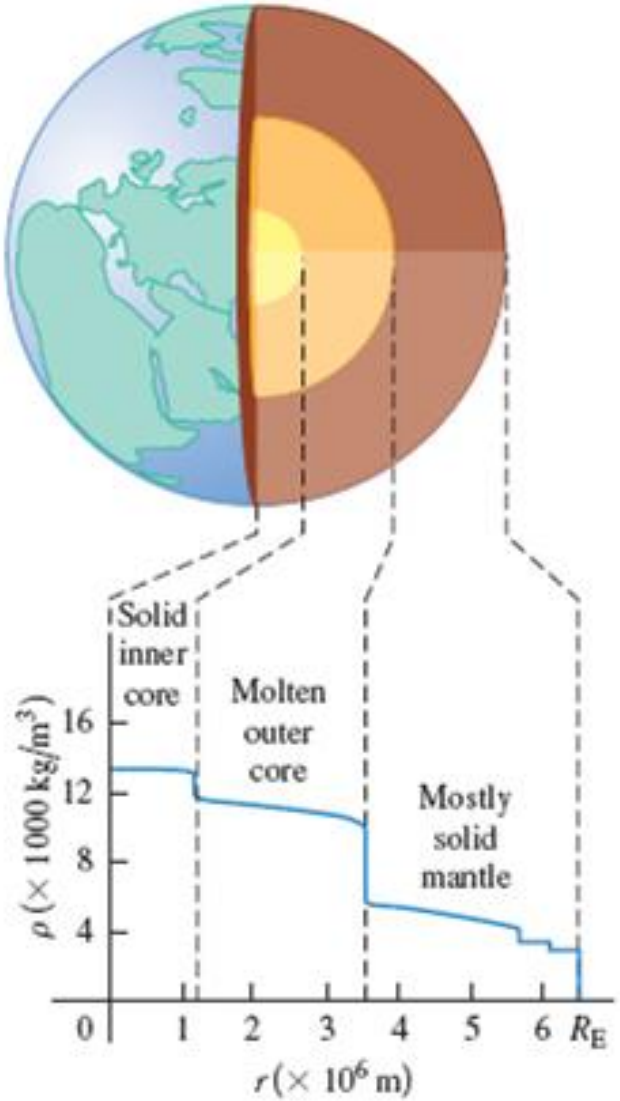


$$E = \left(\frac{q}{4\pi\varepsilon_0 R^3}\right)r \qquad \text{(uniform charge, field at } r \leq R\text{).}$$

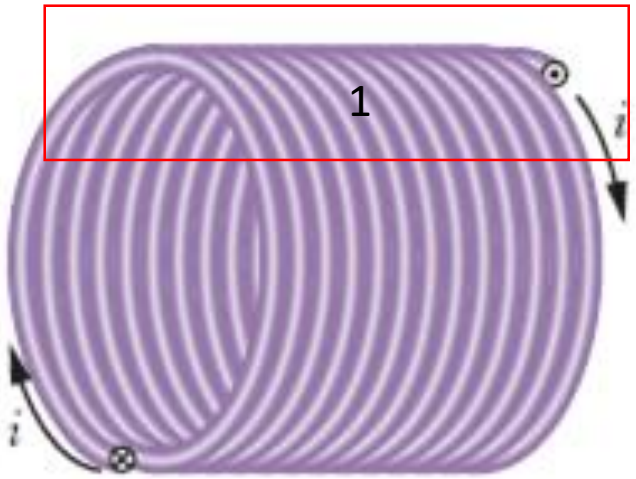
Recall the case for Gravitational field



13.9 The density of the earth decreases with increasing distance from its center.



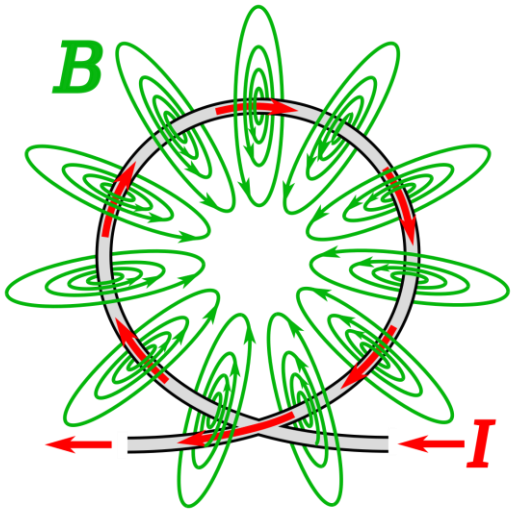
Magnetic Field of a Solenoid

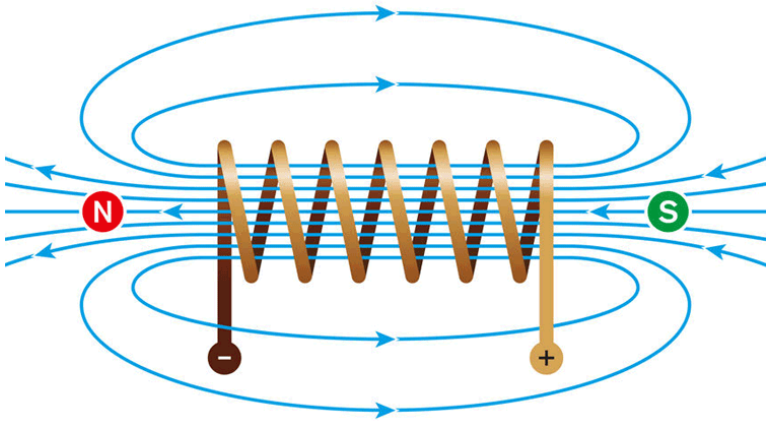


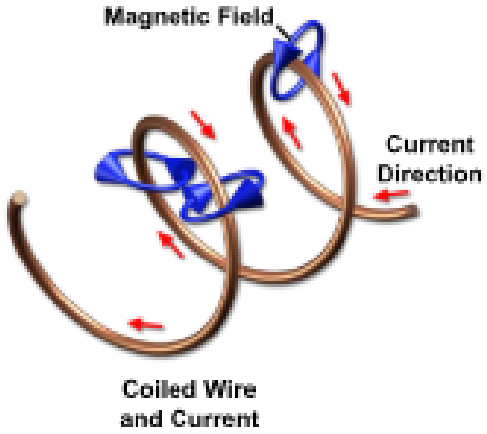
A solenoid carrying current *i*.

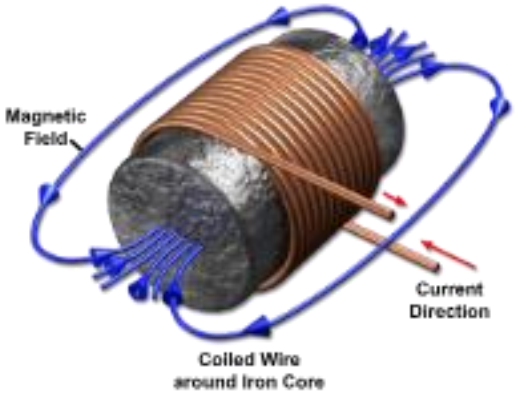
≈

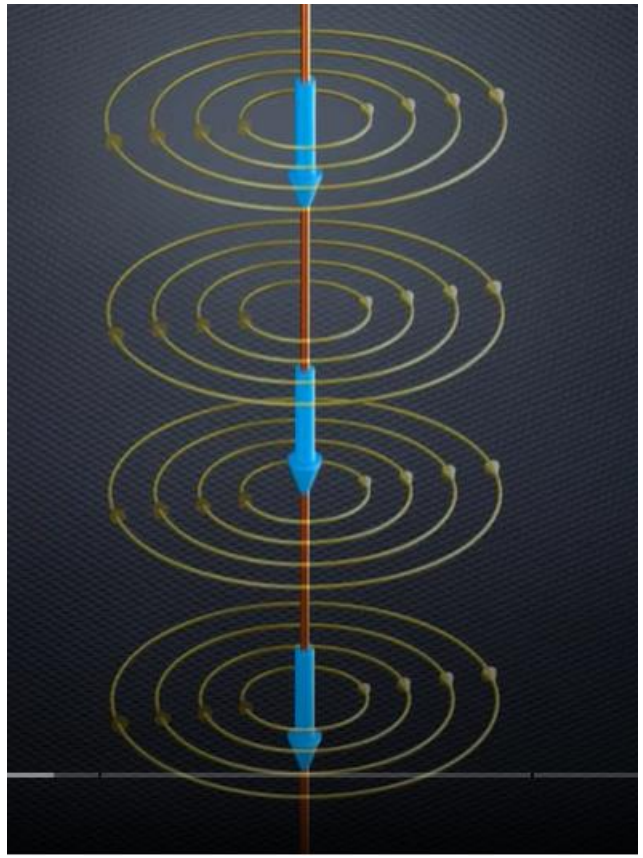
Tightly-packed coils of wire

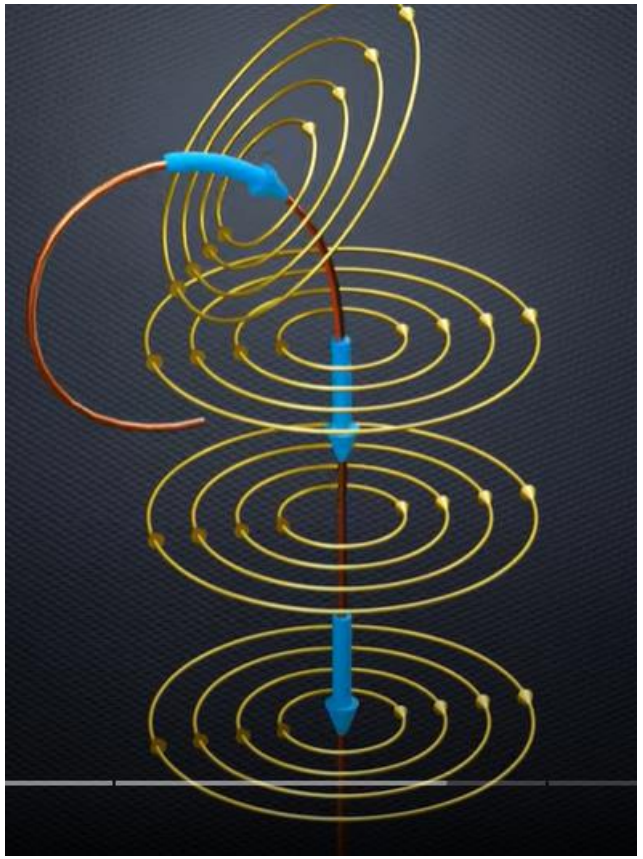


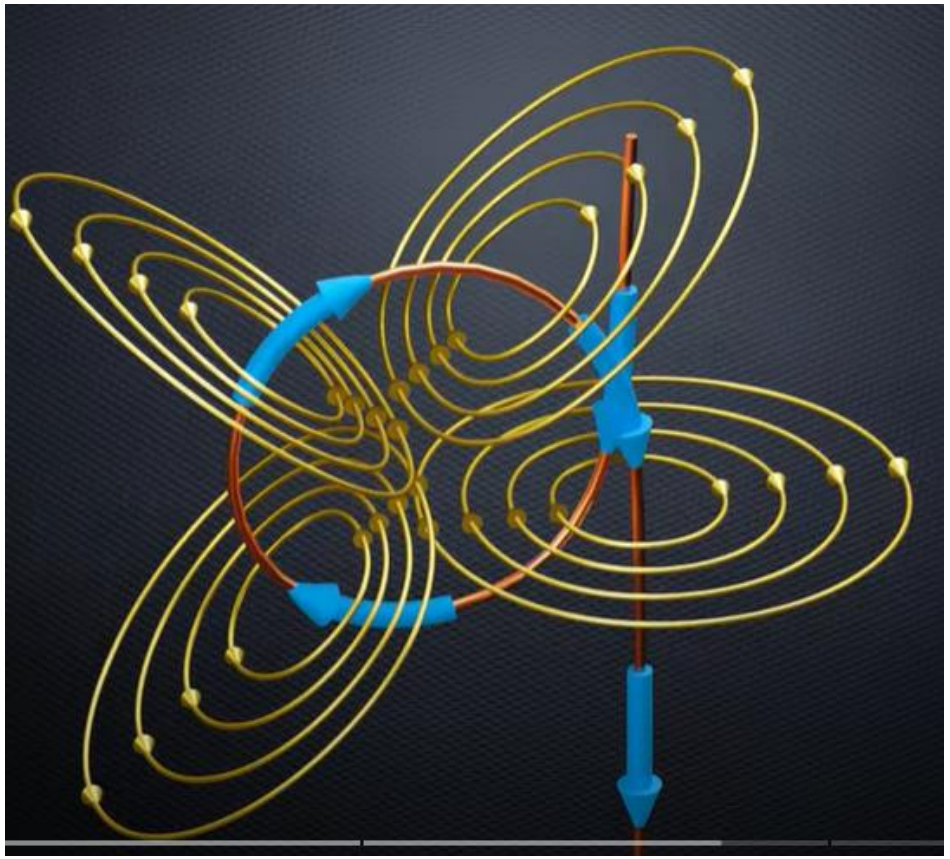


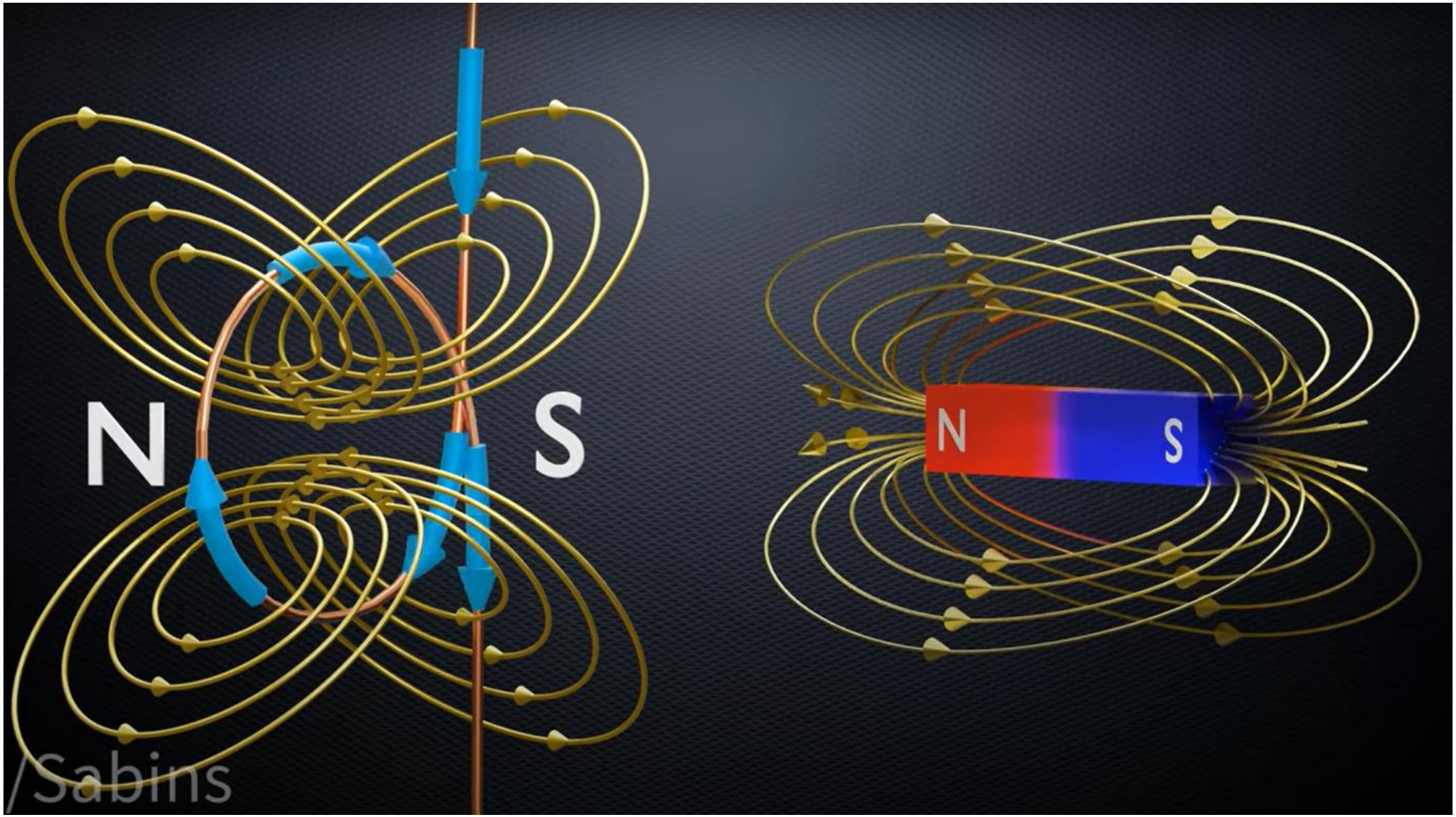








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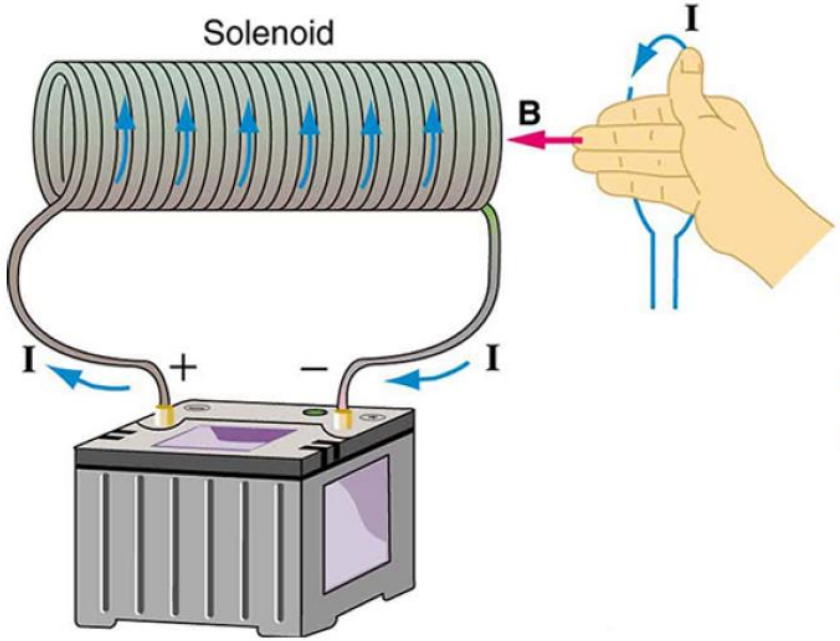
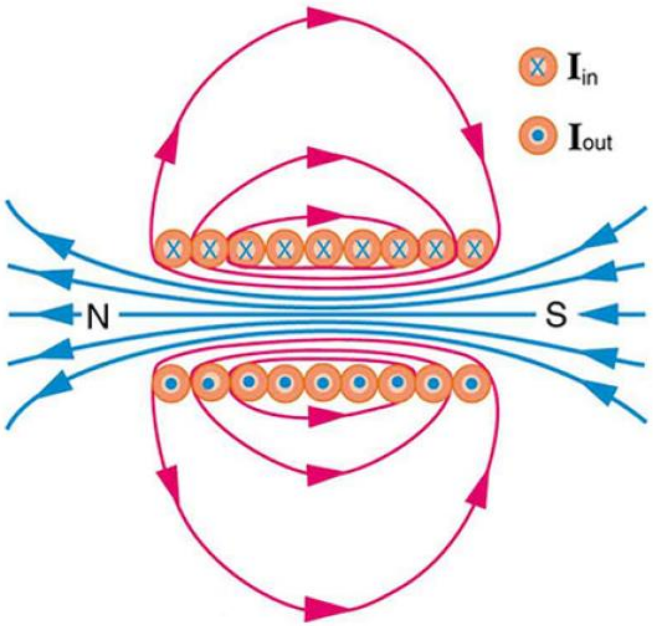
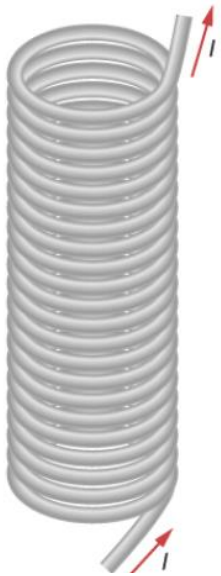
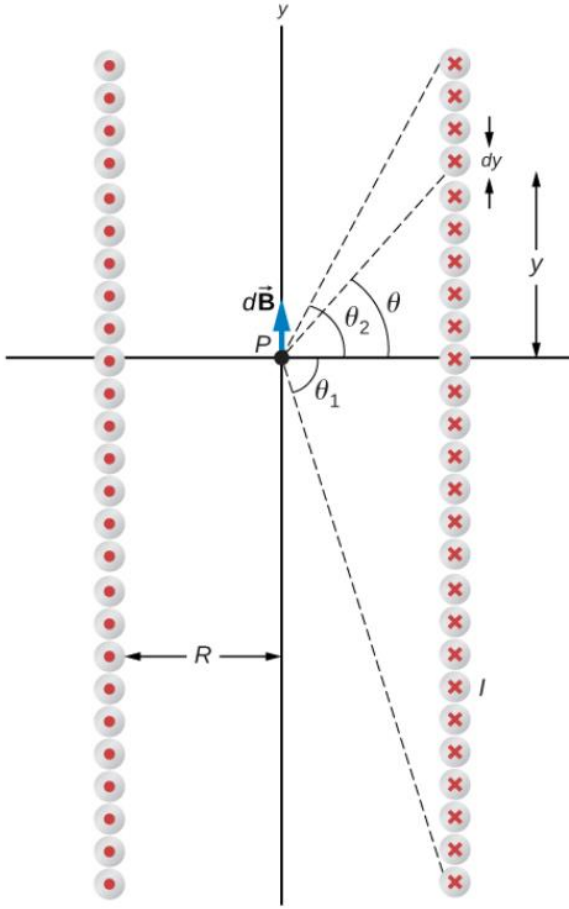


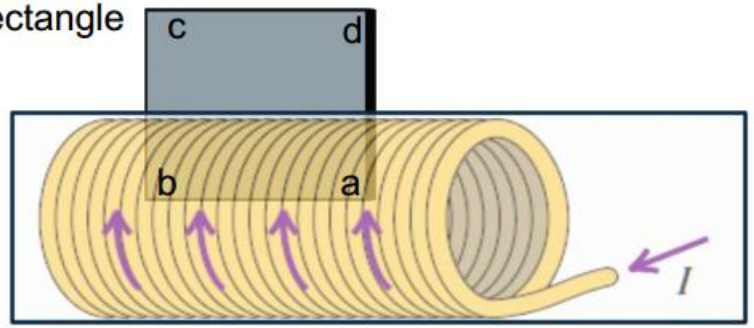


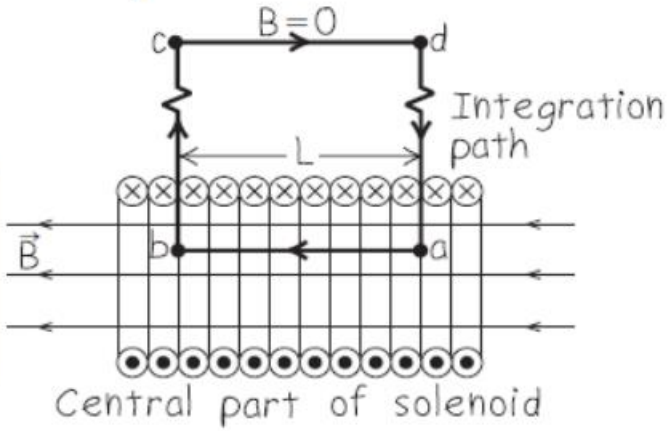




Applying Ampere's law to a solenoid: $\oint \vec{\mathbf{B}} \cdot d\vec{s} = \mu_o I_{encl}$

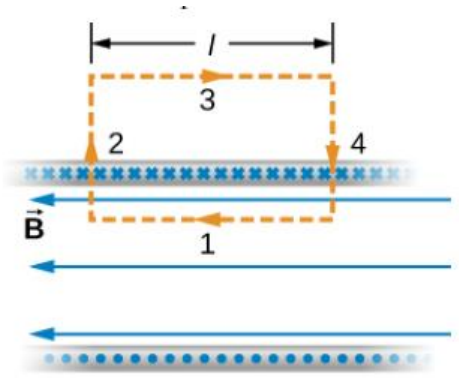
Chose an integration rectangle





https://www.geogebra.org/m/NC7aXeRS

Ampere's Law. Let us now apply Ampere's law,



Consider the rectangular Amperian curve 1234.

$$\oint_C \vec{B}\cdot d\vec{s} = \int_1 \vec{B}\cdot d\vec{s} + \cancel{\int_2 \vec{B}\cdot d\vec{s}} + \cancel{\int_3 \vec{B}\cdot d\vec{s}} + \cancel{\int_4 \vec{B}\cdot d\vec{s}}$$

$$\int_2 = \int_4 = 0 \quad \because \quad \begin{cases} \vec{B}\cdot d\vec{s} & = 0 \quad \text{inside solenoid} \\ \vec{B} & = 0 \quad \text{outside solenoid} \end{cases}$$

$$\int_3 = 0 \quad \because \quad \vec{B} = 0 \quad \text{outside solenoid}$$

$$\therefore \quad \oint_C \vec{B}\cdot d\vec{s} = \int_1 \vec{B}\cdot d\vec{s} = Bl = \mu_0 i_{tot}$$

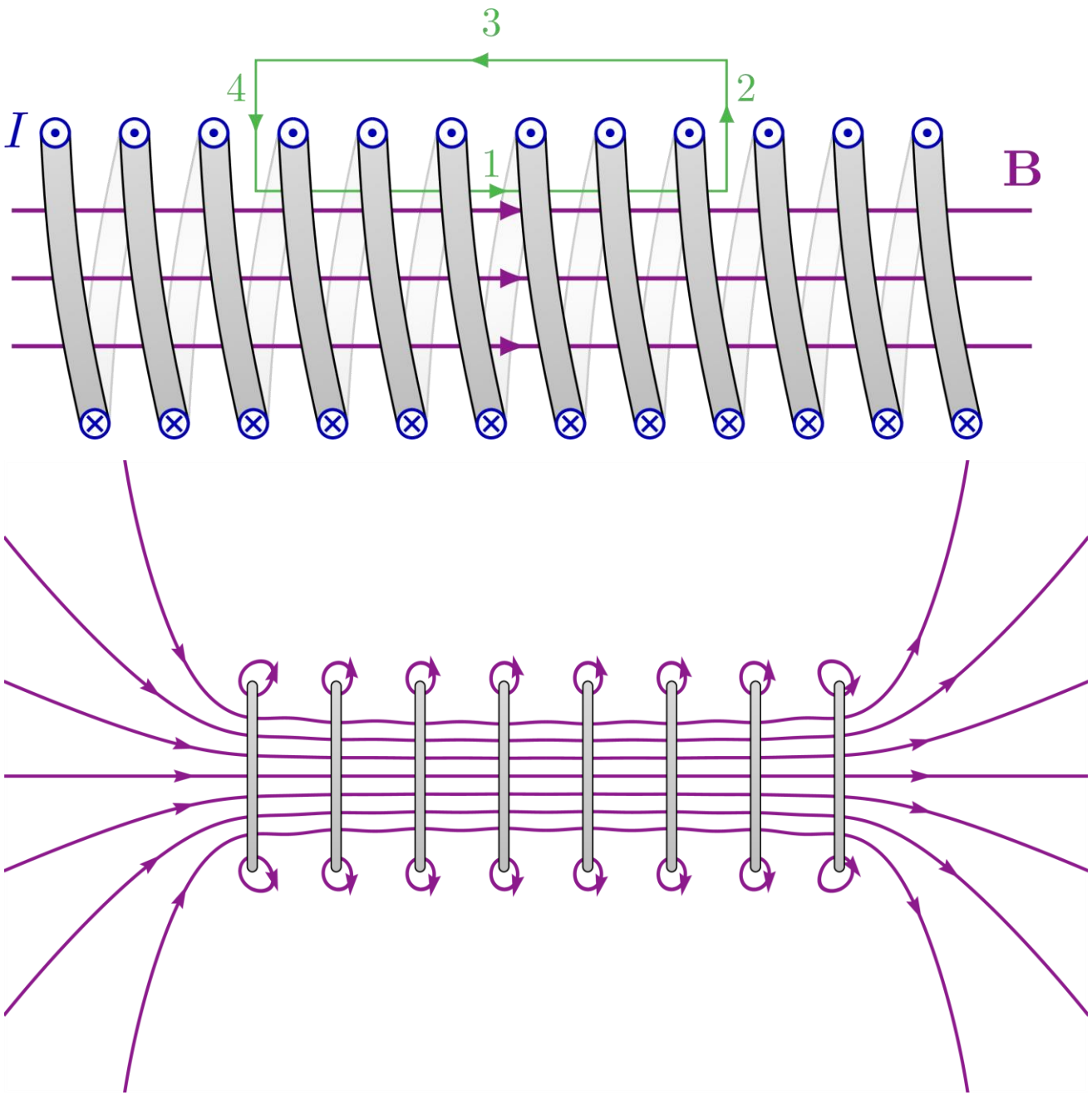
But $i_{tot} = \underbrace{nl}_{\text{Number of coils included}} \cdot i$

$$\therefore \boxed{B = \mu_0 n i}$$

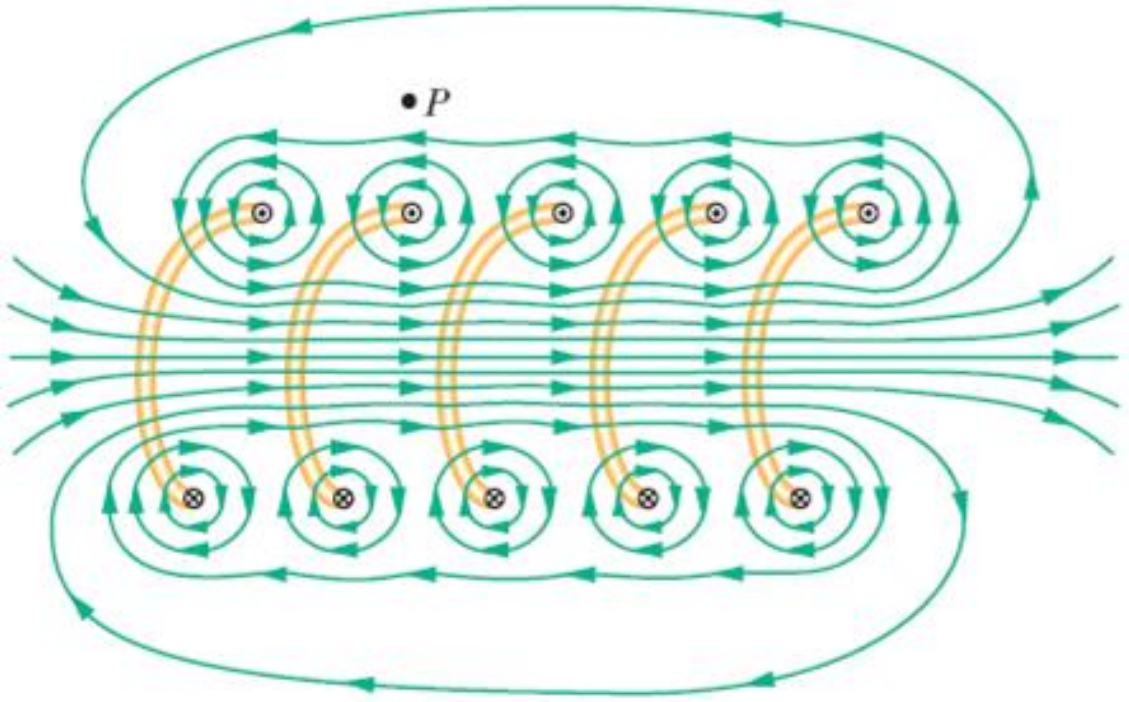
Note :

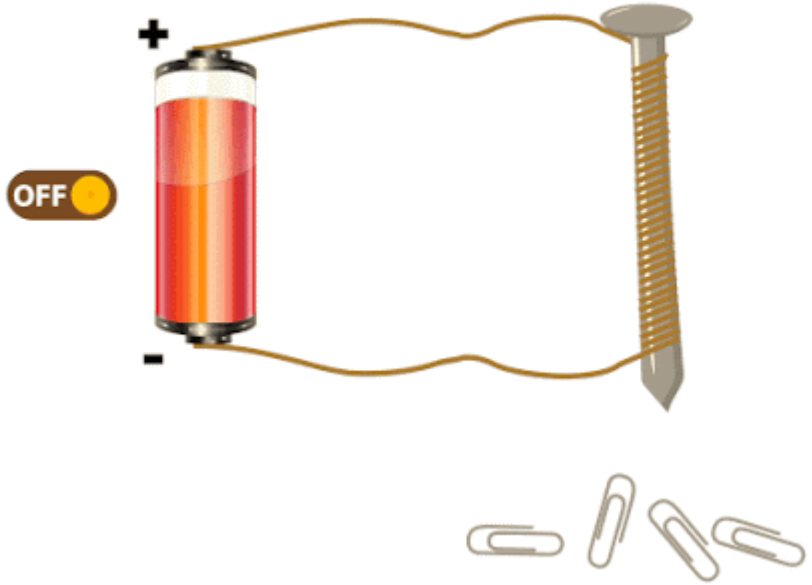
(i) The assumption that $\vec{B} = 0$ outside the ideal solenoid is only *approximate*. (Halliday, Pg.763)

(ii) B-field everywhere inside the solenoid is a *constant*. (for ideal solenoid)



https://javalab.org/en/magnetic_field_around_a_coil_en/



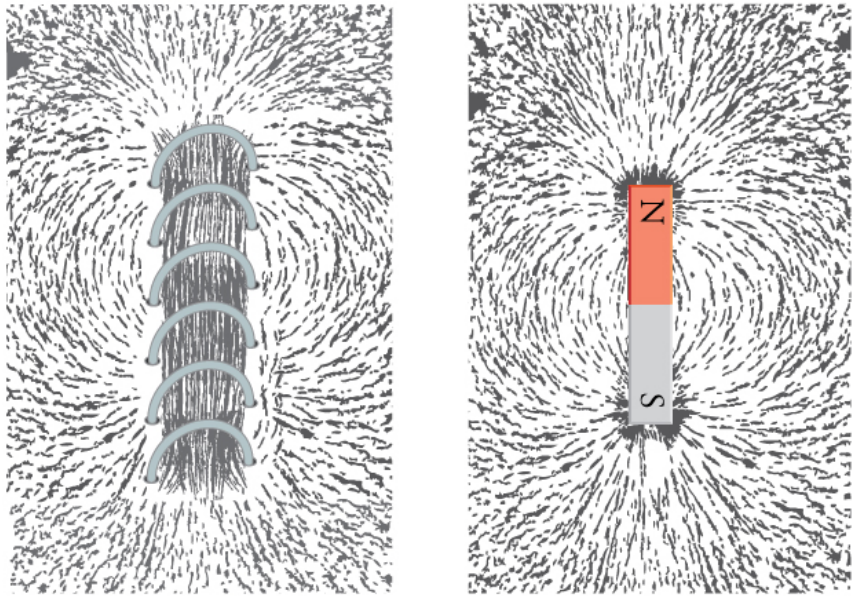


A vertical cross section through the central axis of a “stretched-out” solenoid. The back portions of five turns are shown, as are the magnetic field lines due to a current through the solenoid.

Each turn produces circular magnetic field lines near itself.

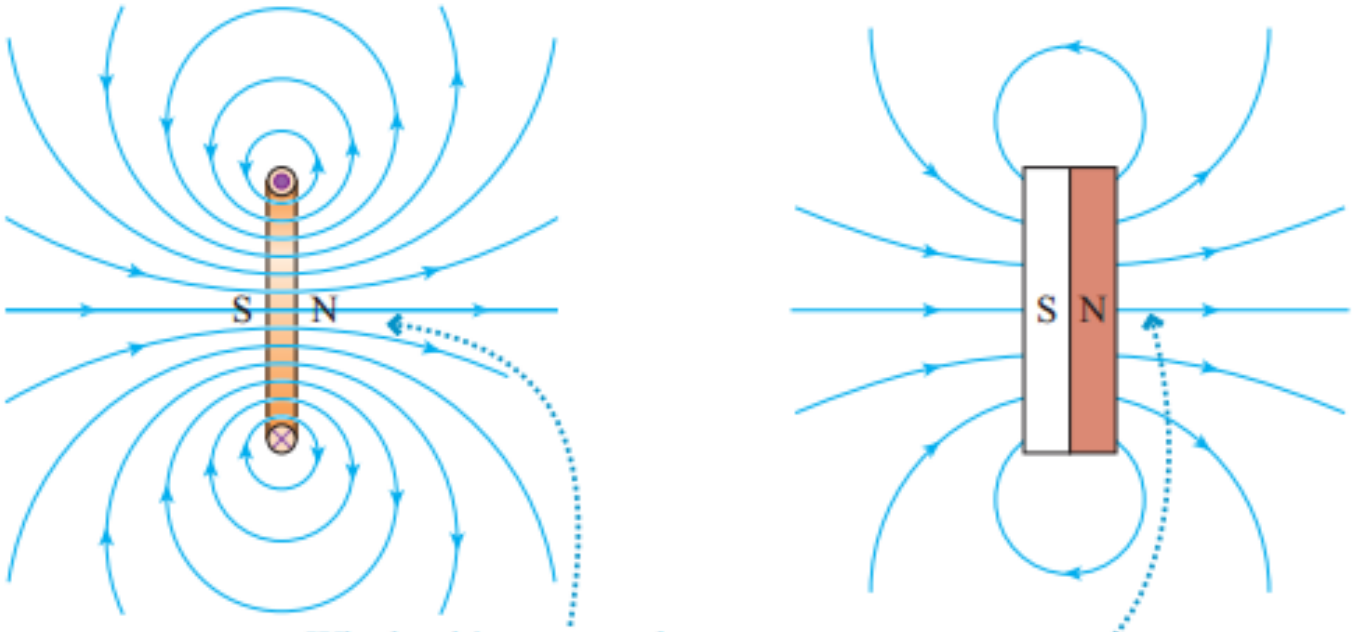
Near the solenoid’s axis, the field lines combine into a net magnetic field that is directed along the axis. The closely spaced field lines there indicate a strong magnetic field.

Outside the solenoid the field lines are widely spaced; the field there is very weak



(a) Current loop

(b) Permanent magnet

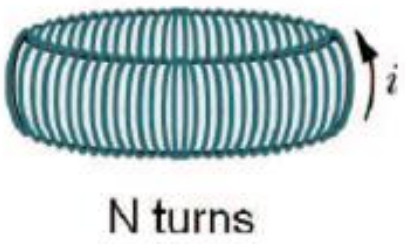


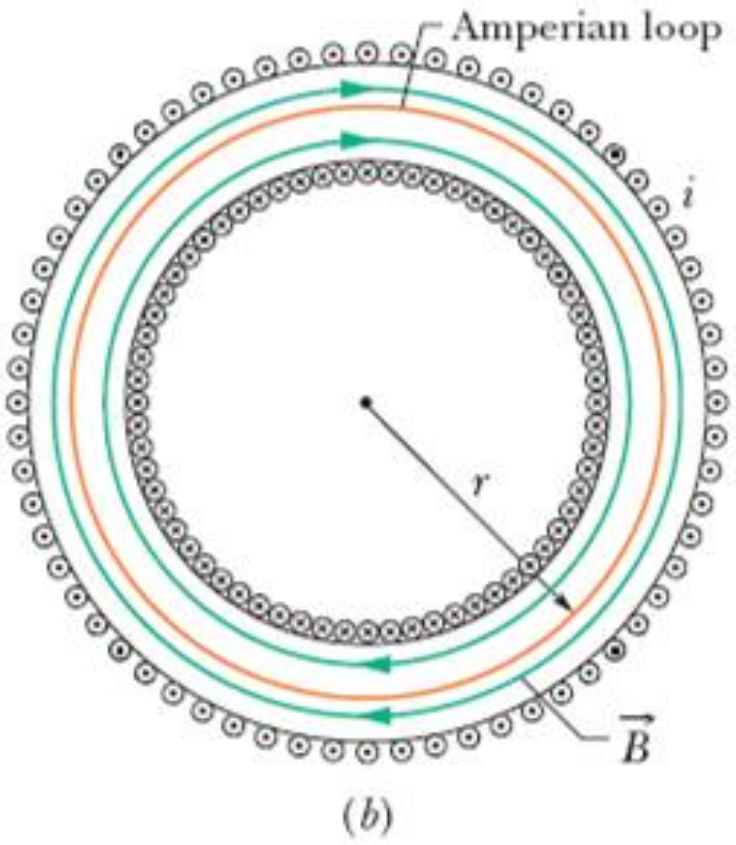
Whether it's a current loop or a permanent magnet, the magnetic field emerges from the north pole.

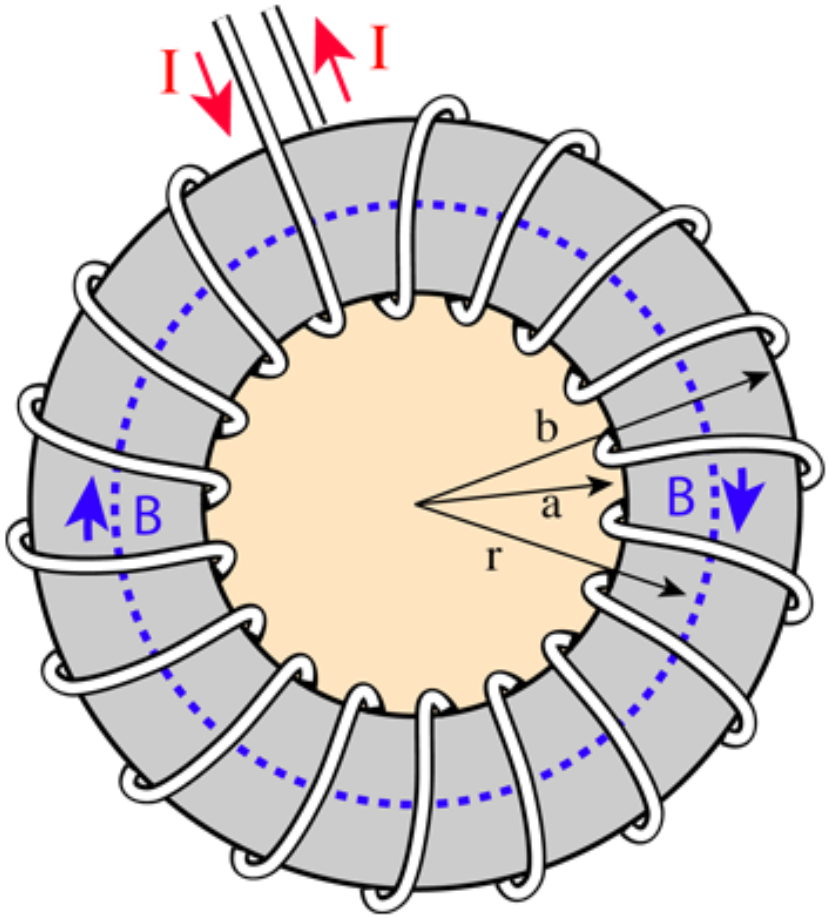
Figure 12.21 In an MRI machine, a large magnetic field is generated by the cylindrical solenoid surrounding the patient. (credit: Liz West)

Magnetic Field of a Toroid

A **toroid** may be described as a (hollow) solenoid that has been curved until its two ends meet, forming a sort of hollow bracelet.

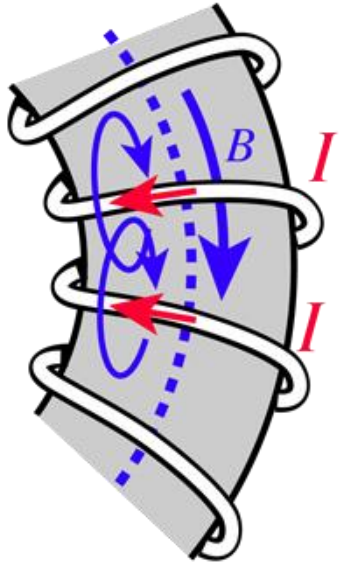






What magnetic field is set up inside the toroid (inside the hollow of the bracelet)? We can find out from Ampere's law and the symmetry of the bracelet.

By symmetry argument, the B-field lines form *concentric circles inside the toroid*.



Take Amperian curve C to be a circle of radius *r* inside the toroid.

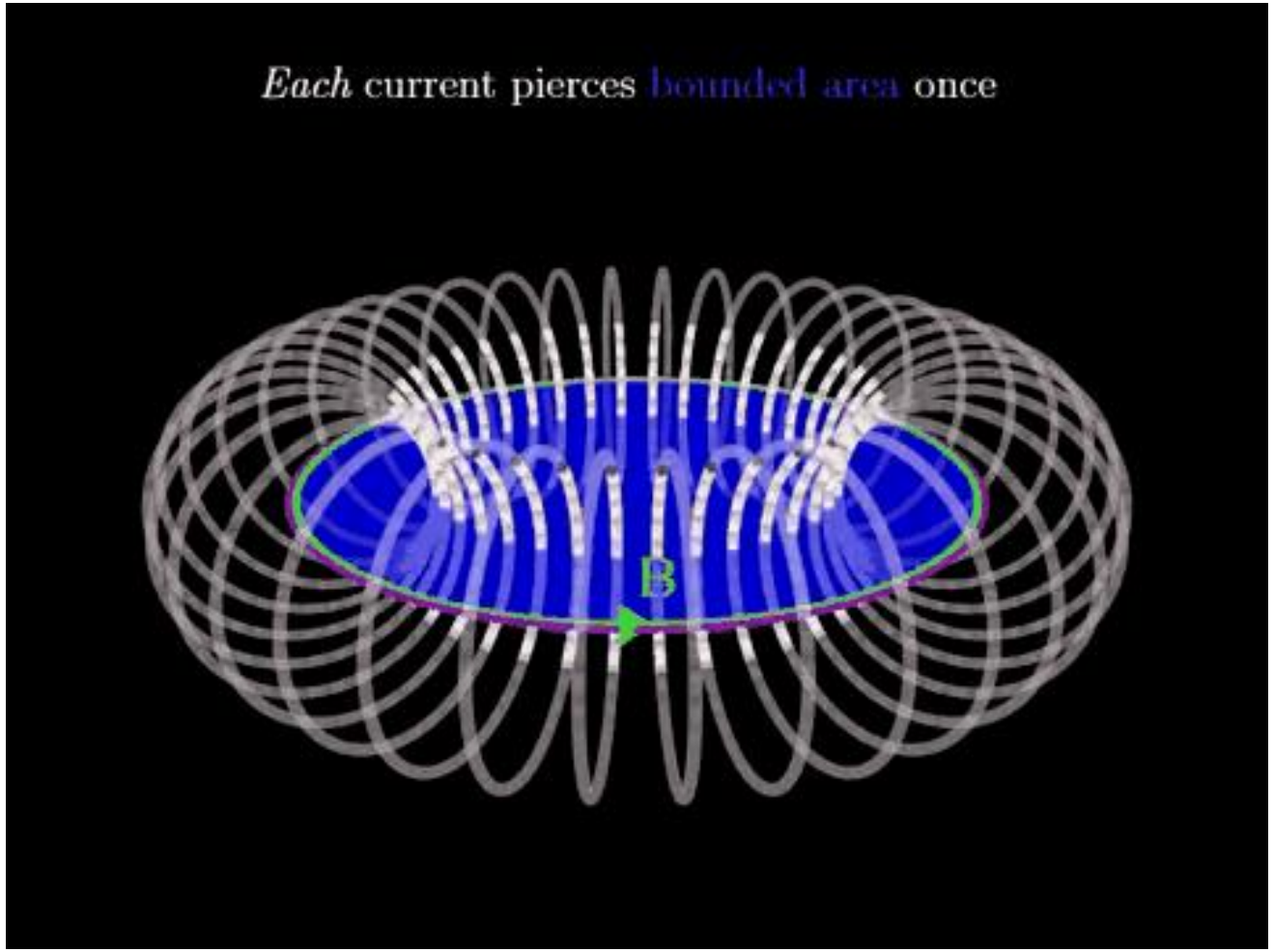
$$\oint \vec{\boldsymbol{B}}.\boldsymbol{d}\vec{\boldsymbol{S}} = \boldsymbol{B}\oint \boldsymbol{dS} = B(2\pi r)$$

$$B(2\pi r) = \boldsymbol{N\mu_0 i}$$

where *i* is the current in the toroid windings (and is positive for those windings enclosed by the Amperian loop) and *N* is the total number of turns.

$$B = \frac{\boldsymbol{N\mu_0 i}}{(2\pi r)}$$

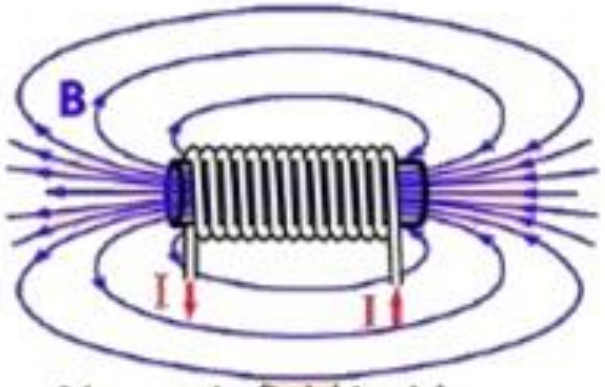
In contrast to the situation for a solenoid, *B* is not constant over the cross section of a toroid.



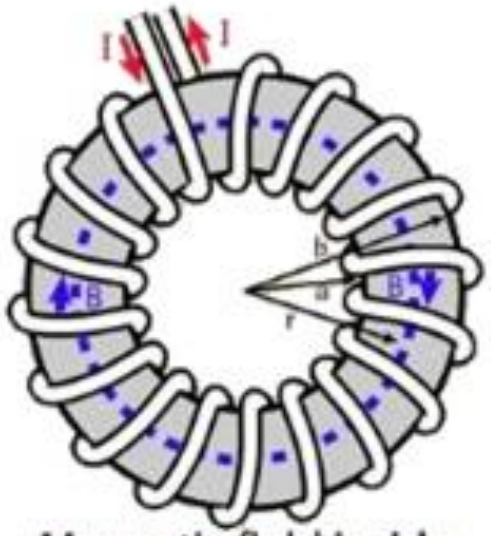
It is easy to show, with Ampere's law, that $B = 0$ for points outside an ideal toroid (as if the toroid were made from an ideal solenoid).

The direction of the magnetic field within a toroid follows from our curled–straight right-hand rule:
Grasp the toroid with the fingers of your right hand curled in the direction of the current in the windings; your extended right thumb points in the direction of the magnetic field.

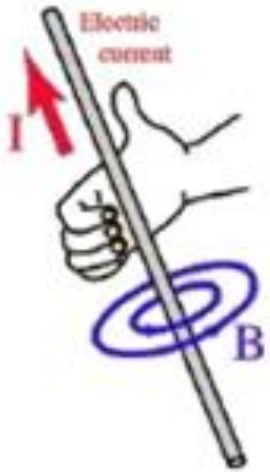
Ampere's Law Applications



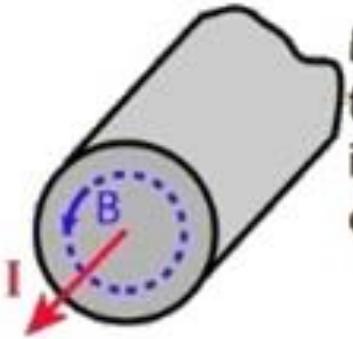
Magnetic field inside a long solenoic.



Magnetic field inside a toroidal coil.



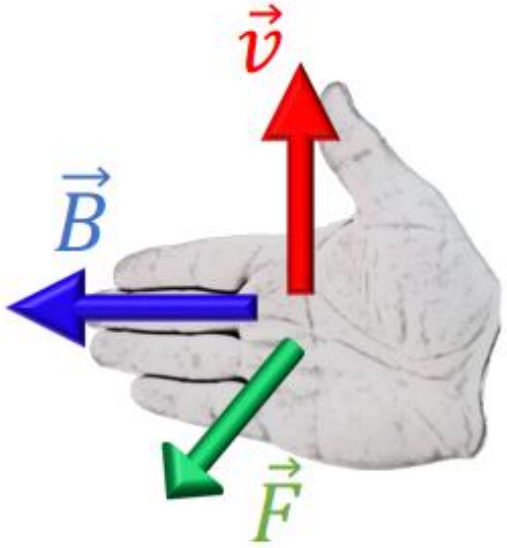
Magnetic field from a long straight wire.



Magnetic field inside a conductor.

(1) Magnetic force:

$$q\,\vec{v} \times \vec{B} = \vec{F}$$



(2) Linear current:

(3) circular current: (solenoid)

- In the context of introductory Electromagnetic theory, you can use Ampere's law when the symmetry of the problem permits i.e. when the magnetic field around an 'Amperian loop' is constant.

- Ampere's Law is analogous to Gauss's Law, referring to closed loops and enclosed current instead of Gaussian surfaces and enclosed charge.

- Any shape of loop can be chosen, however just like with Gaussian surfaces, there are easier and harder choices when it comes to calculations.

- Biot-Savart law is the more brute force approach, you evaluate this integral when there is not enough symmetry to use Ampere's law.

- Biot-Savart law is analogous to Coulomb’s law for electrostatics.

Electric and Magnetic Fields: Differences

Work

- The electric force does work in displacing a charged particle.

- The **magnetic force associated with a steady magnetic field does no work when a particle is displaced.**

- This is because the force is perpendicular to the displacement of its point of application

$$\Delta W = \underline{F} \cdot d\underline{r} = q\left(\underline{v} \times \underline{B}\right) \cdot d\underline{r} = q\left(\underline{v} \times \underline{B}\right) \cdot \frac{d\underline{r}}{dt} dt$$

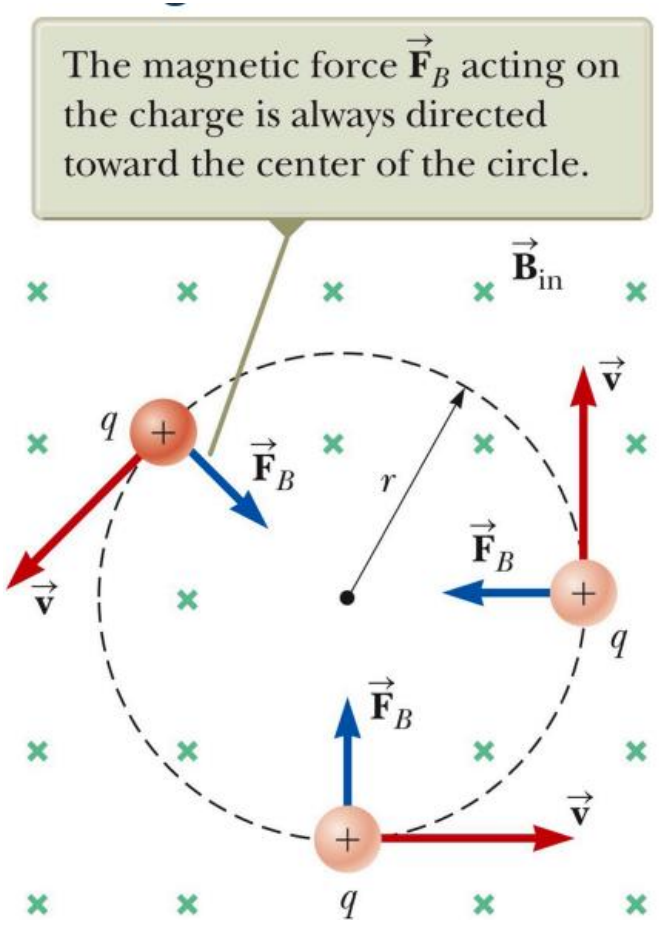
$$\Rightarrow \quad \Delta W = q\underbrace{\left(\underline{v} \times \underline{B}\right) \cdot \underline{v}}_{\text{zero}} dt = 0$$

Electric and Magnetic Fields: Differences

Kinetic Energy

- The electric field alone can alter the kinetic energy of a charged particle.

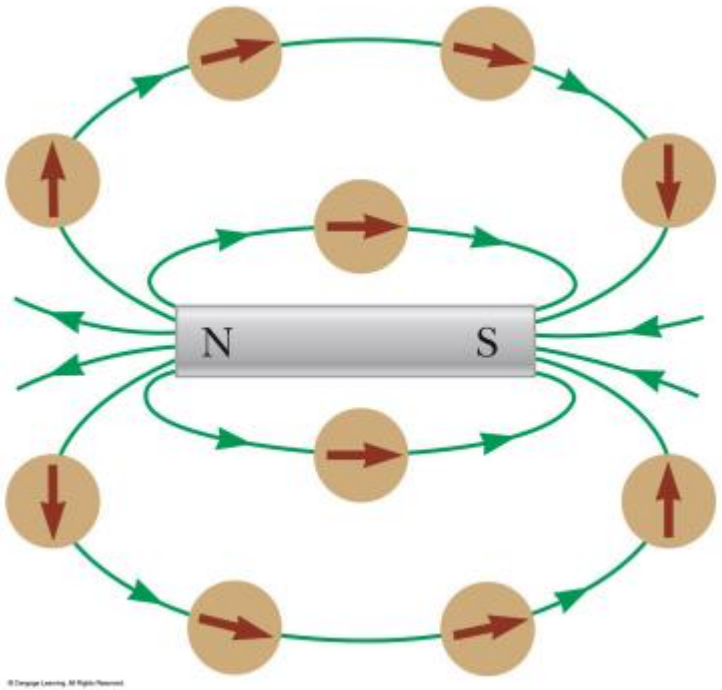
- The magnetic field cannot alter the kinetic energy of a charged particle.



Magnetic Field Lines

Unlike electric fields lines, magnetic field lines:

- can never be infinite, but always form closed loops.
- do not represent the path of the charged particle
- do not originate on charges, but originate on poles



A fundamental property of a static magnetic field is that, unlike an electrostatic field, it is not conservative.

A conservative field is one that does the same amount of work on a particle moving between two different points regardless of the path chosen.

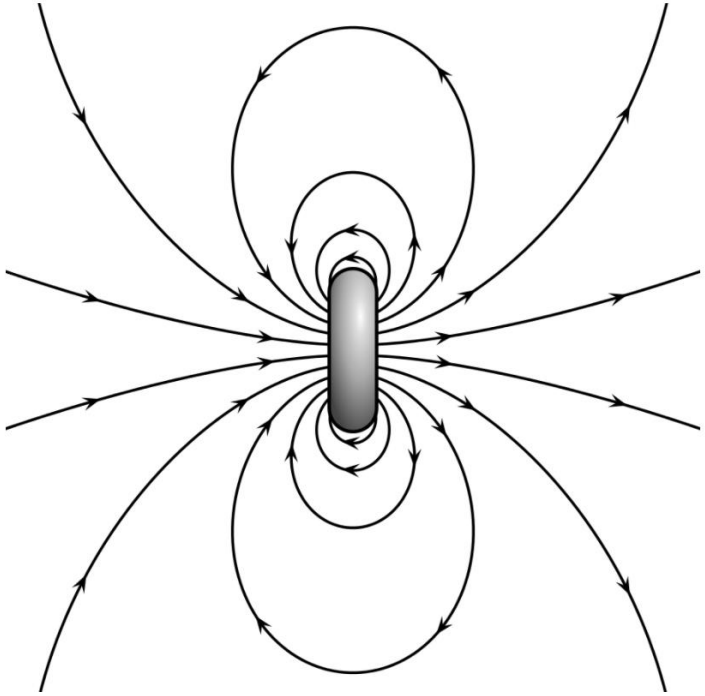
- Magnetic fields do not have such a property. Instead, there is a relationship between the magnetic field and its source, electric current.

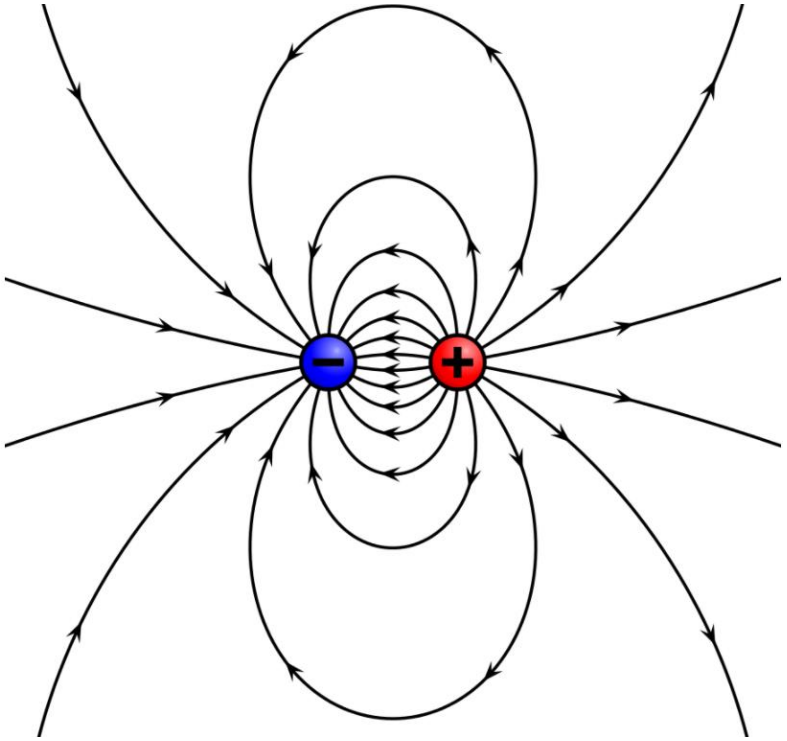
- Unlike $\vec{\boldsymbol{E}}$, the magnetic field $\vec{\boldsymbol{B}}$ is *sourceless*; there are no sources or sinks; the magnetic field lines are closed loops.

- The force on a charge q in an electric field is $q\vec{\boldsymbol{E}}$, and it depends only on where the charge is in the electric field – i.e. on its position. Thus the force is *conservative*, and we understand from any study of classical mechanics that only conservative forces can be expressed as the derivative of a potential function.

- The force on a charge q in a *magnetic* field is $q\vec{\boldsymbol{v}} \times \vec{\boldsymbol{B}}$. This force does not depend only on the position of the particle, but also on its velocity (speed and direction). Thus the force is not conservative.

Magnetic dipole and Electric dipole





Permeability and Permitivity constants

In electromagnetism, **permeability** is the measure of magnetization that a material obtains in response to an applied magnetic field. Permeability is typically represented by the (italicized) Greek letter μ.

Magnetic field strength, also called **magnetic intensity** or **magnetic field intensity**, the part of the magnetic field in a material that arises from an external current and is not intrinsic to the material itself. It is expressed as the vector H and is measured in units of amperes per metre.

$$\vec{B} = \mu\vec{H}$$

Recall

Electric displacement, denoted by $\vec{D}$, is the charge per unit area that would be displaced across a layer of conductor placed across an electric field $\vec{D}$. It is also known as electric flux density.

In electromagnetism, the **absolute permitivity**, often simply called **permitivity** and denoted by the Greek letter ε (epsilon), is a measure of the electric polarizability of a dielectric.

$$\vec{D} = \varepsilon\vec{E}$$

Permeability and Permitivity constants

Magnetic constant Permeability of free space, $\mu_0 = 4\pi \times 10^{-7}$T.m/A

We discussed about Permittivity constant, $\varepsilon_0 = 8.854 \times 10^{-12}$ F/m

When we study electromagnetic waves, we will find that their speed of propagation in vacuum, which is equal to the speed of light is given by,

$$c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}}$$