Electric Field-II

Phy 108 course
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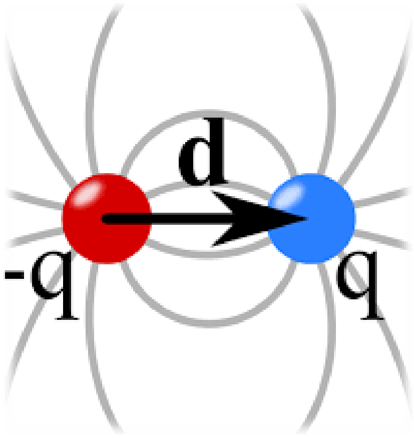
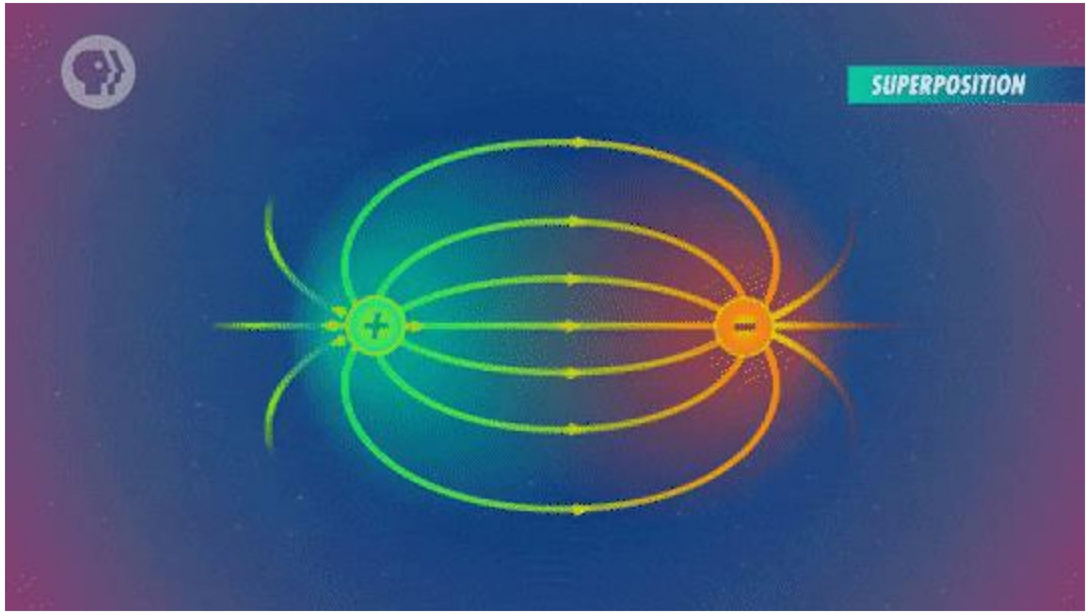
The Electric Field Due to an Electric Dipole

Two particles that have the same charge magnitude q but opposite signs, arrangement known as **Electric dipole**.

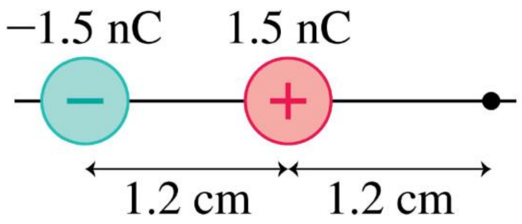
The particles are separated by distance d and lie along the *dipole axis*, an axis of symmetry around which you can imagine rotating the pattern

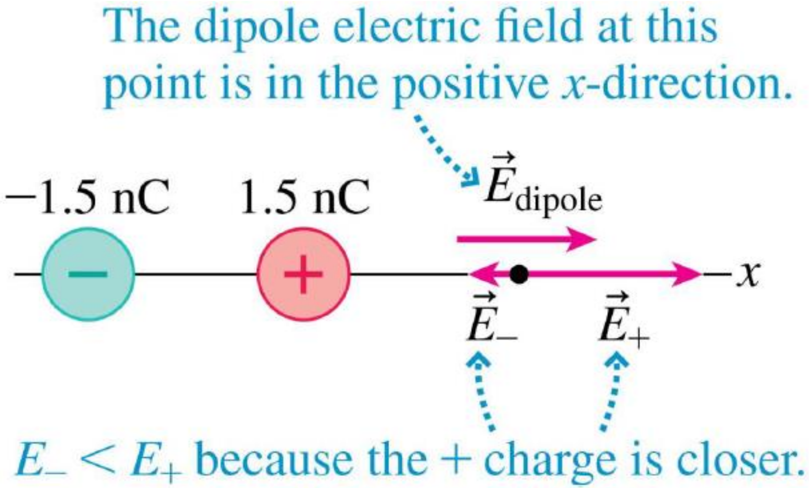




https://phet.colorado.edu/sims/html/charges-and-fields/latest/charges-and-fields_en.html

A dipole consists of a positive and negative charge separated by 1.2 cm, as shown in FIGURE. What is the electric field strength along the line connecting the charges at a point 1.2 cm to the right of the positive charge?





$$E_{\text{dipole}} = E_+ - E_-$$

$$= \frac{\left(9.0 \times 10^9 \dfrac{\text{N}\cdot\text{m}^2}{\text{C}^2}\right)(1.5 \times 10^{-9}\ \text{C})}{(0.012\ \text{m})^2} - \frac{\left(9.0 \times 10^9 \dfrac{\text{N}\cdot\text{m}^2}{\text{C}^2}\right)(1.5 \times 10^{-9}\ \text{C})}{(0.024\ \text{m})^2}$$

$$= 7.0 \times 10^4\ \text{N/C}$$

Electric dipole moment:

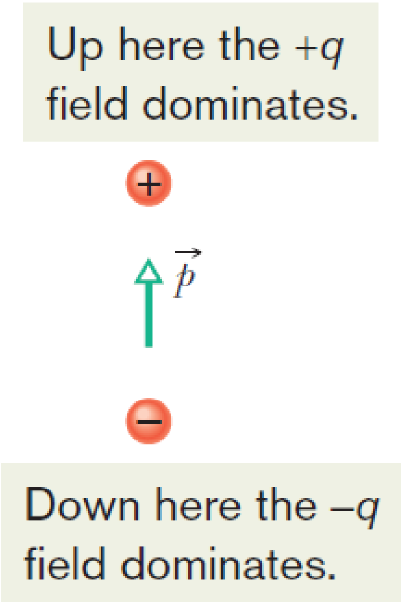
The product of one of the magnitude of one of the charges and the distance between the charges is called the dipole moment, $\vec{p}$, shown in the figure, as the vector:

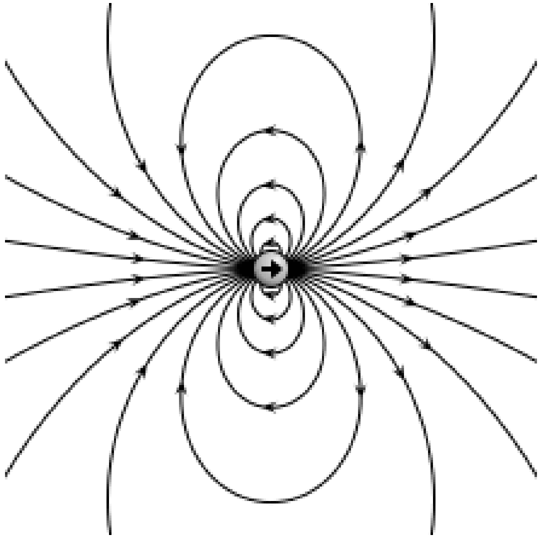
$$\vec{\boldsymbol{p}} = \boldsymbol{q}\vec{\boldsymbol{d}}$$

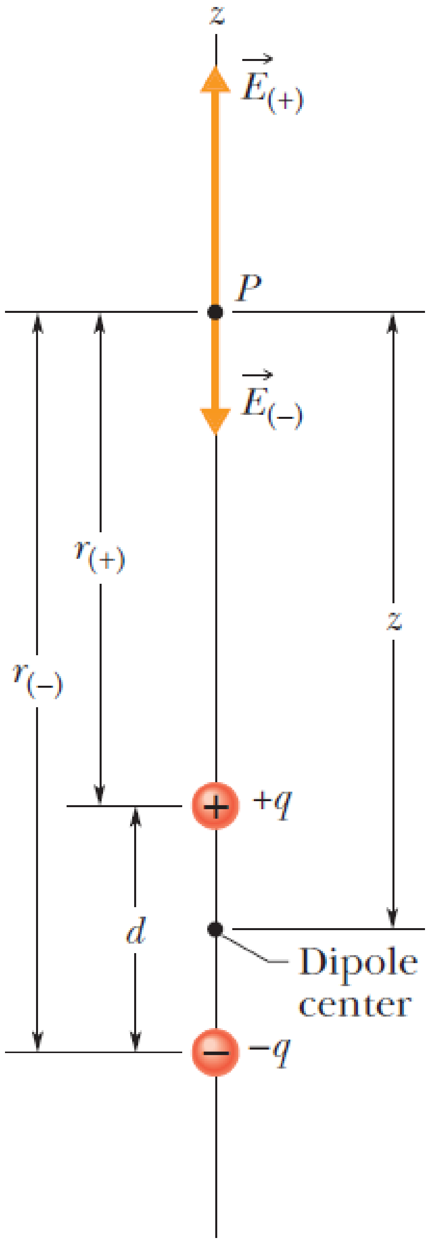
The SI units of the dipole moment are $C - m$

The direction of $\vec{E}$ for distant points on the dipole axis is always the direction of the dipole moment vector $\vec{p}$.

If we measure the electric field of a dipole only at distant points, we can never find q and d separately; instead, we can find only their product. The field at distant points would be unchanged if, for example, q were doubled and d simultaneously halved.







The electric field at a point on the axis of a dipole is:

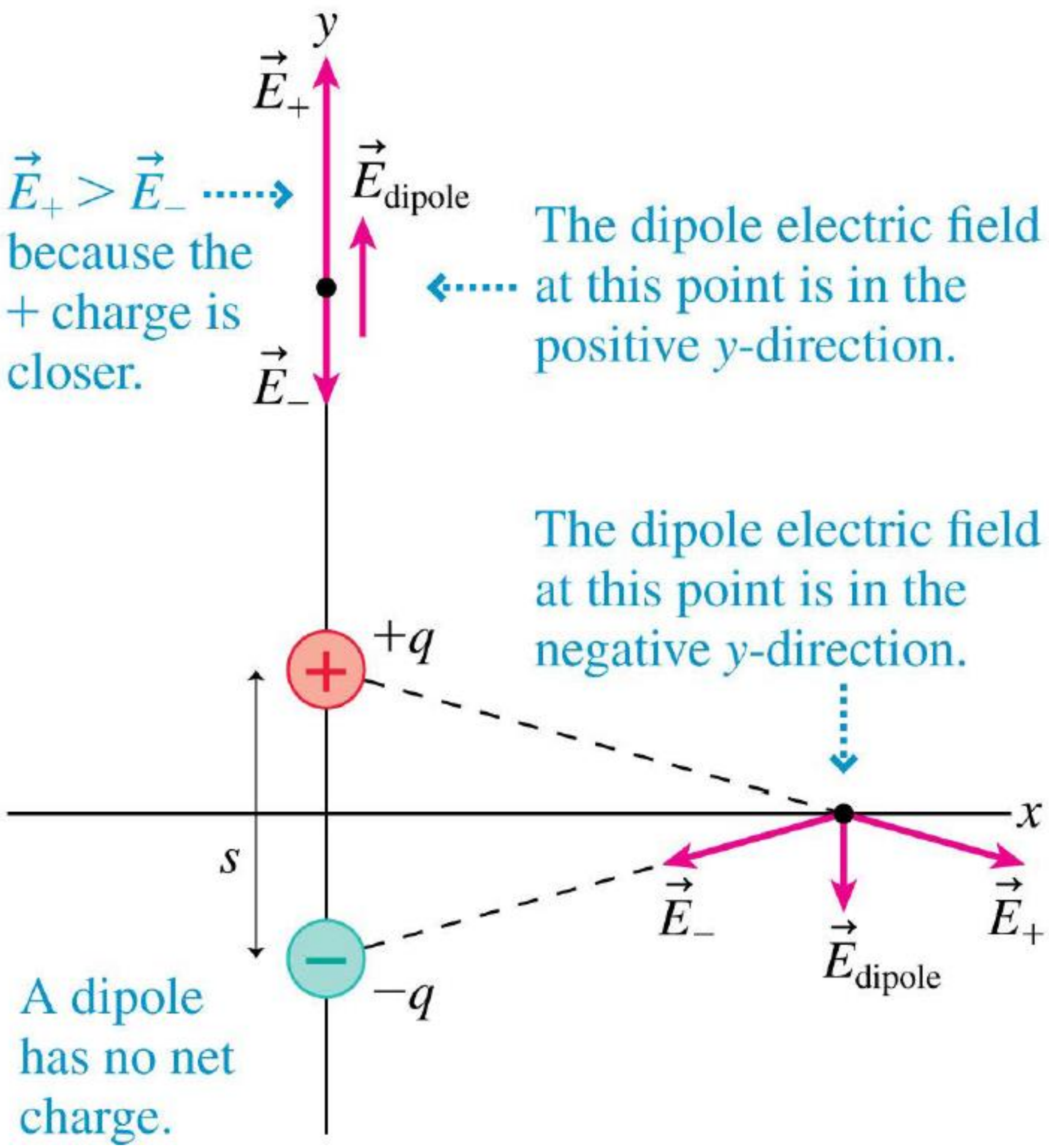
$$\vec{E}_{dipole} \approx \frac{1}{4\pi\epsilon_0}\frac{2\vec{p}}{r^3}$$

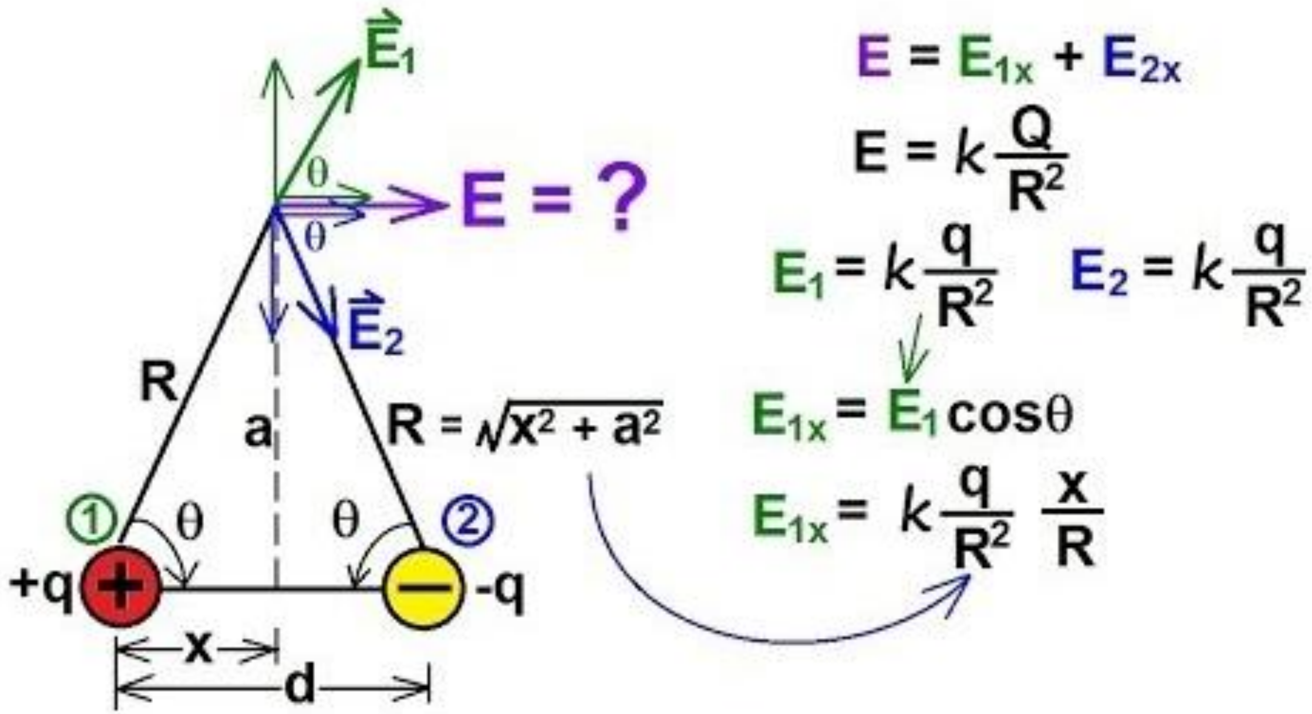
where *r* is the distance measured from the *center* of the dipole.

The electric field in the plane that bisects and is perpendicular to the dipole is

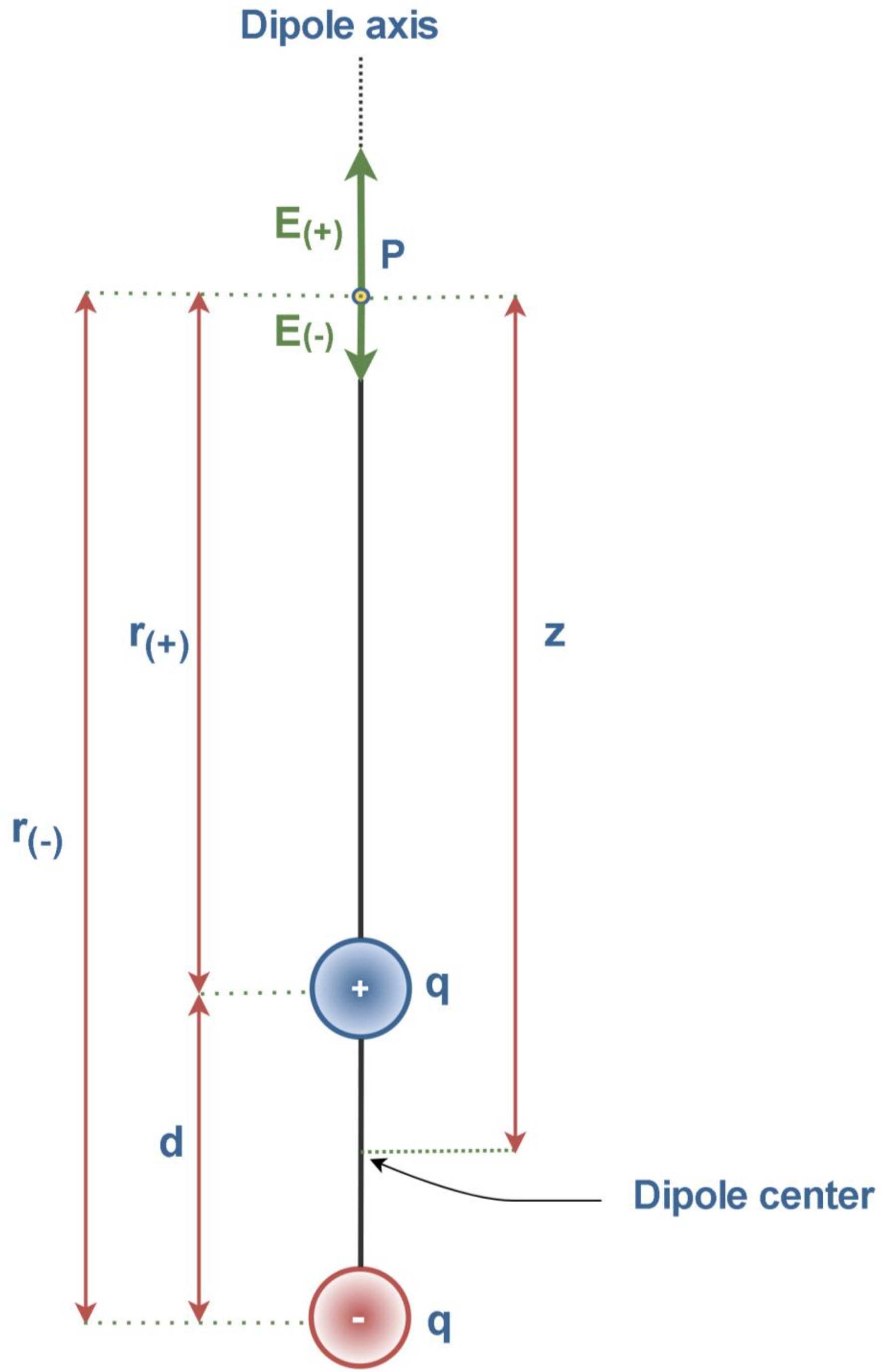
$$\vec{E}_{dipole} \approx -\frac{1}{4\pi\epsilon_0}\frac{\vec{p}}{r^3}$$

This field is opposite to the dipole direction, and it is only half the strength of the on-axis field at the same distance.

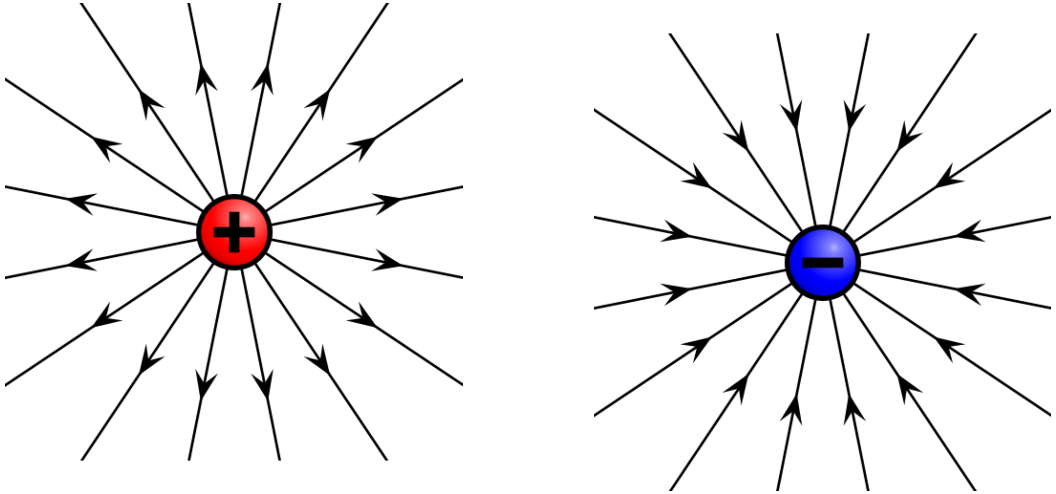


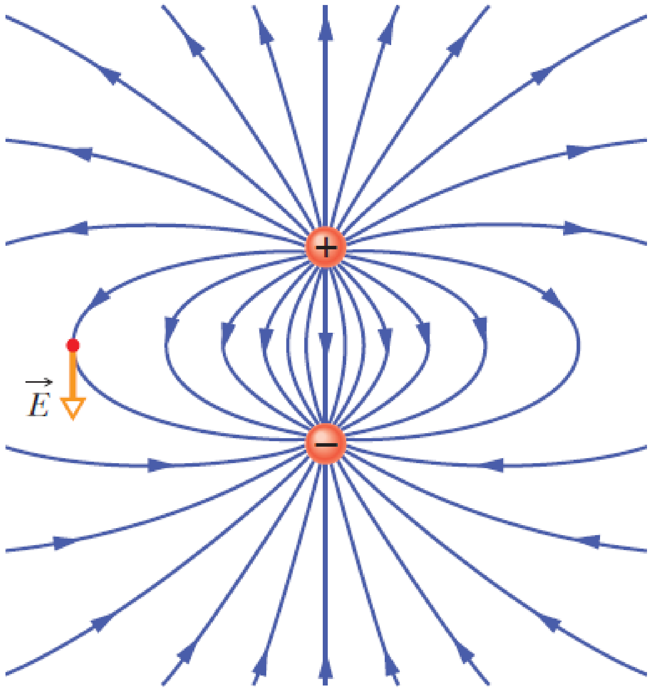


$\vec{E}_1$
$E = ?$
θ
θ
$\vec{E}_2$
R
a
$R = \sqrt{x^2 + a^2}$
①
②
θ
θ
+q
-q
x
d
$E = E_{1x} + E_{2x}$
$E = k\frac{Q}{R^2}$
$E_1 = k\frac{q}{R^2}$
$E_2 = k\frac{q}{R^2}$
$E_{1x} = E_1 \cos\theta$
$E_{1x} = k\frac{q}{R^2}\,\frac{x}{R}$



Dipole axis
$E_{(+)}$
P
$E_{(-)}$
$r_{(+)}$
z
$r_{(-)}$
+
q
d
Dipole center
-
q





$$\vec{E} = \frac{1}{4\pi\epsilon_0}\frac{q}{r^2}\hat{r}$$

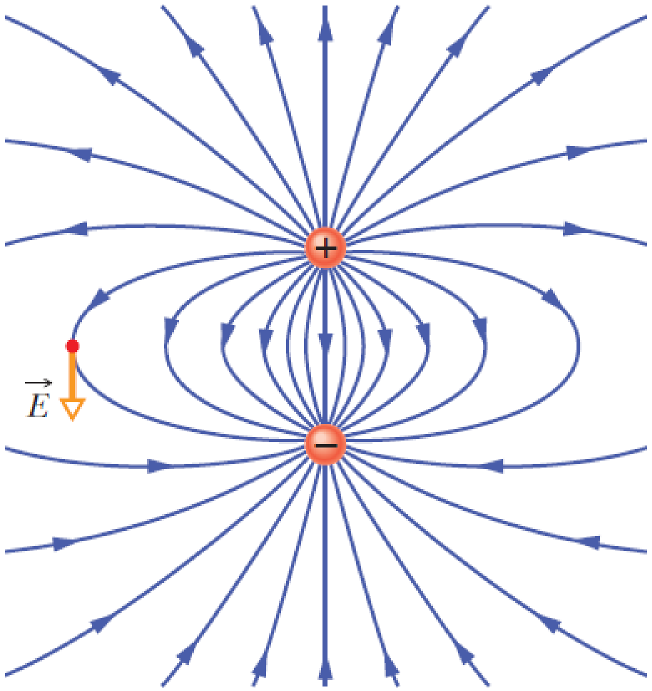
(electric field of a point charge)

$$\vec{E}_{dipole} \approx \frac{1}{4\pi\epsilon_0}\frac{2\vec{p}}{r^3}$$

It turns out that E for a dipole varies as $1/r^3$ for all distant points, regardless of whether they lie on the dipole axis; here r is the distance between the point in question and the dipole center.

The electric field of a dipole decreases more rapidly with distance than does the electric field of a single charge.

If you double the distance of a point from a dipole, the electric field at the point drops by a factor of 8.



The physical reason for this rapid decrease in electric field for a dipole is that from distant points a dipole looks like two particles that almost—but not quite—coincide. Thus, because they have charges of equal magnitude but opposite signs, their electric fields at distant points almost—but not quite—cancel each other.

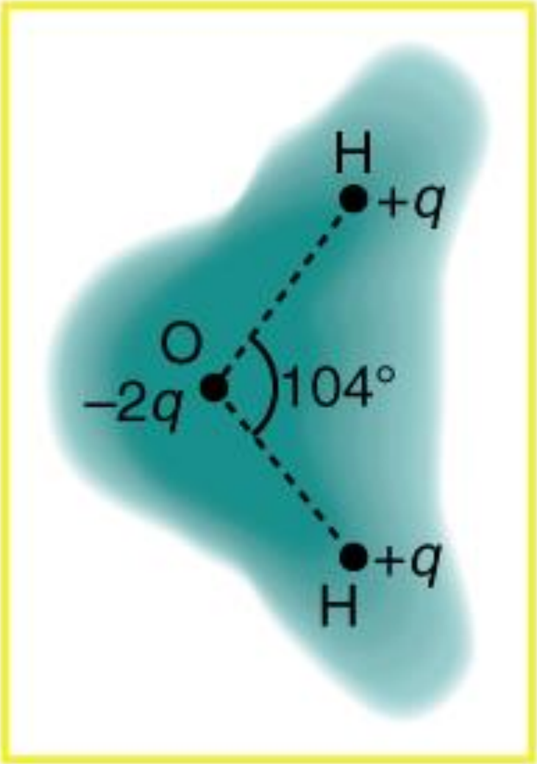
$$\vec{E}_{dipole} \approx \frac{1}{4\pi\epsilon_0}\frac{2\vec{p}}{r^3}$$

It is water's chemical composition and physical attributes that make it such an excellent solvent.

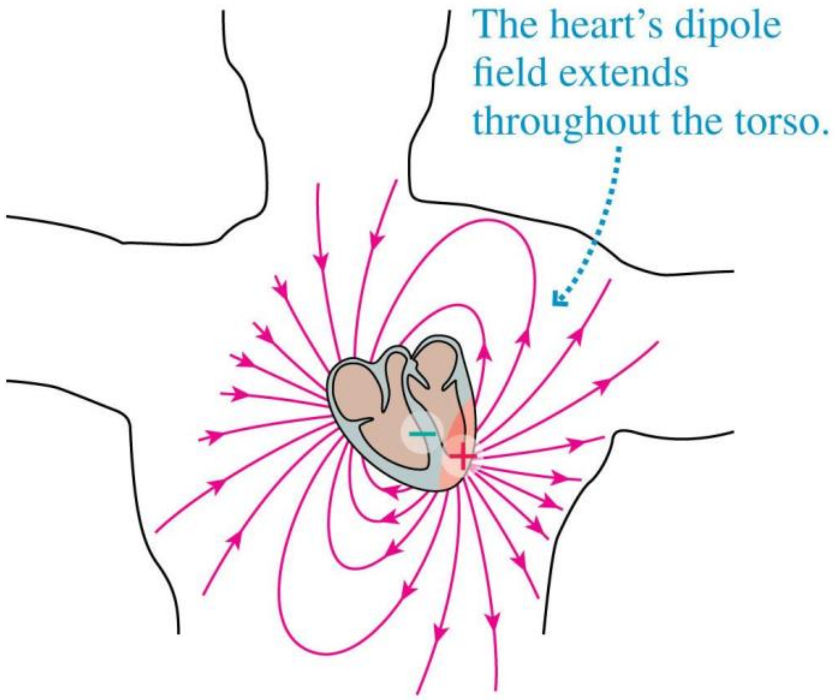
Water molecules have a polar arrangement of the **oxygen** and hydrogen atoms—one side (hydrogen) has a positive **electrical charge** and the other side (**oxygen**) had a negative **charge**.

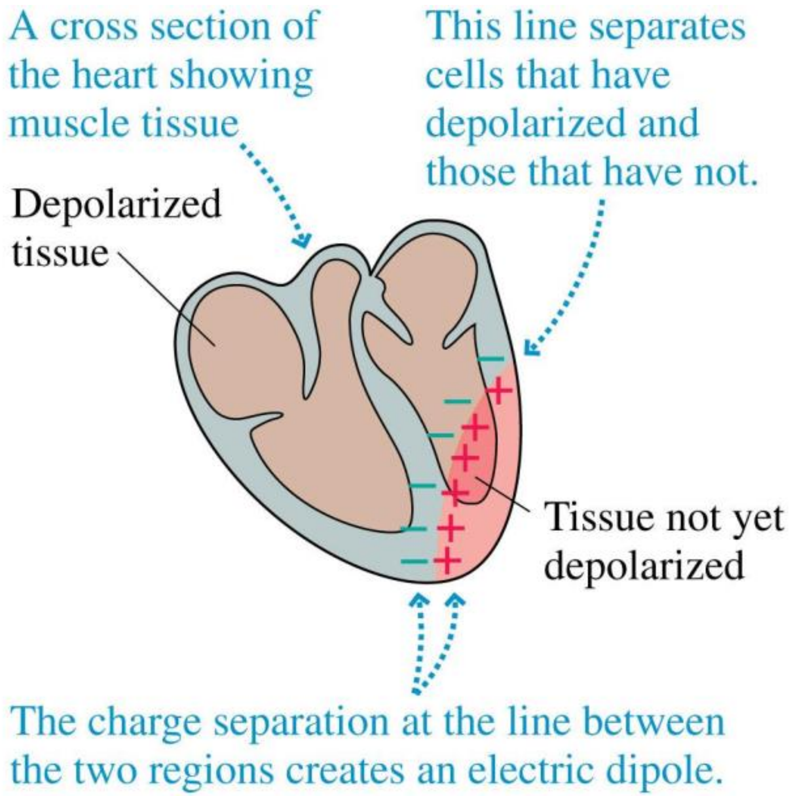
Charge distribution in a water molecule Schematic representation of the outer electron cloud of a neutral water molecule.

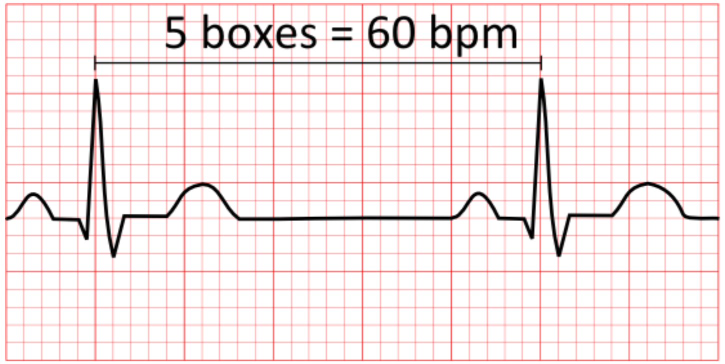
The electrons spend more time near the oxygen than the hydrogens, giving a permanent charge separation as shown.

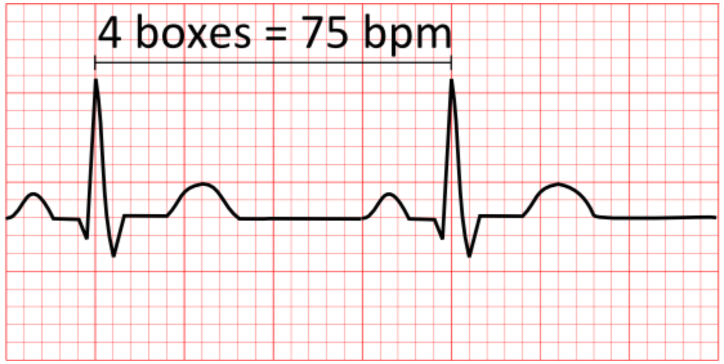


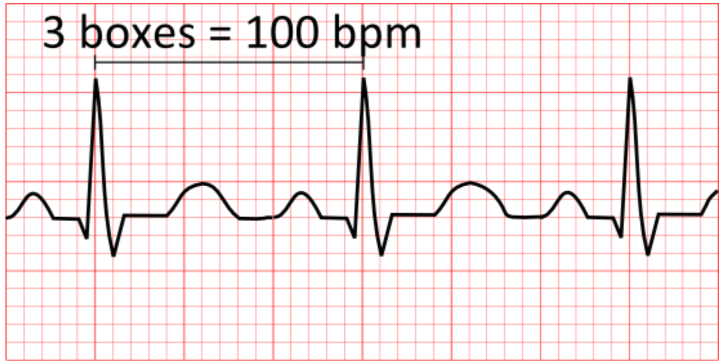
Human heart dipole and ECG



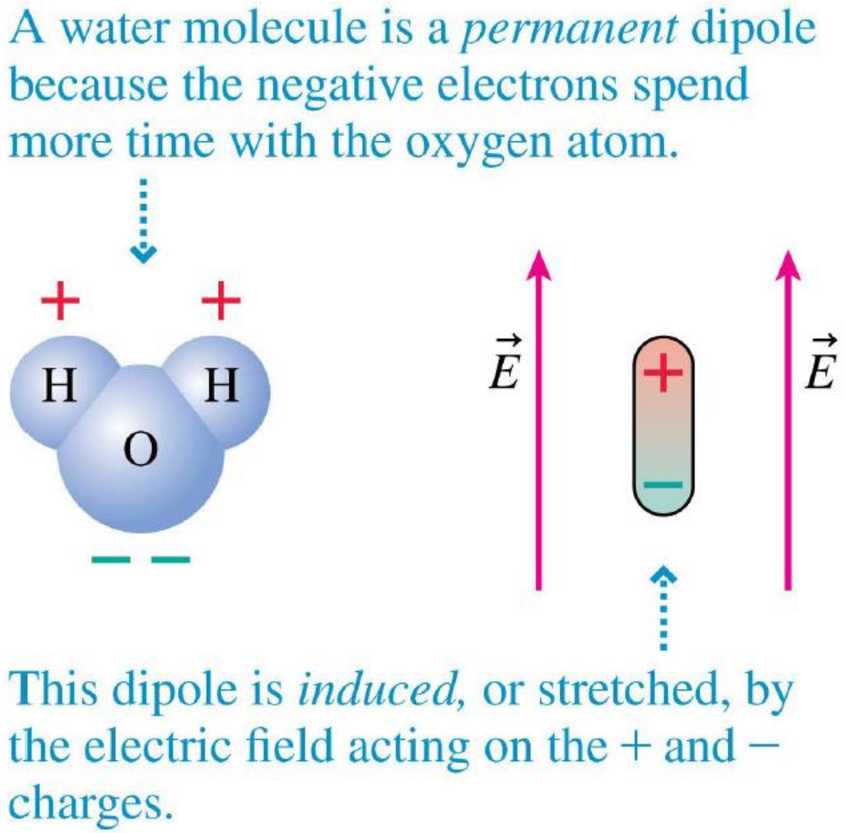




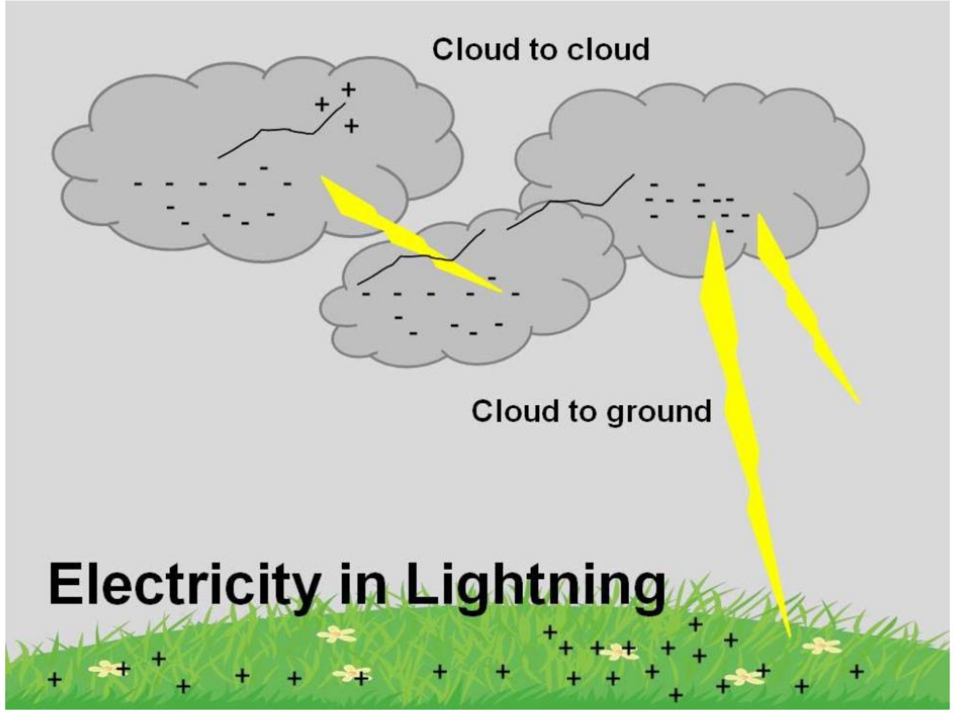




Water is thus a polar molecule. It is more easily affected by electrostatic forces than molecules with uniform charge distributions.

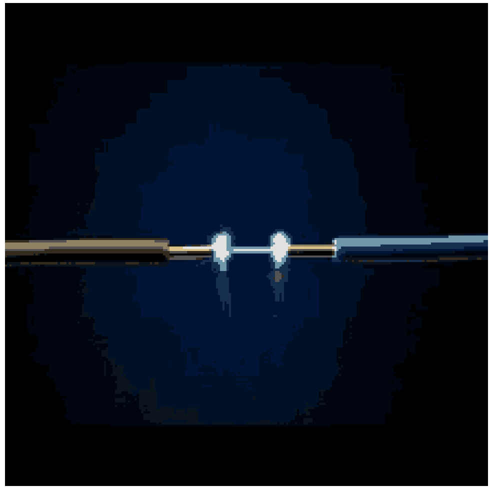


Lightning is a dramatic natural example of static discharge.

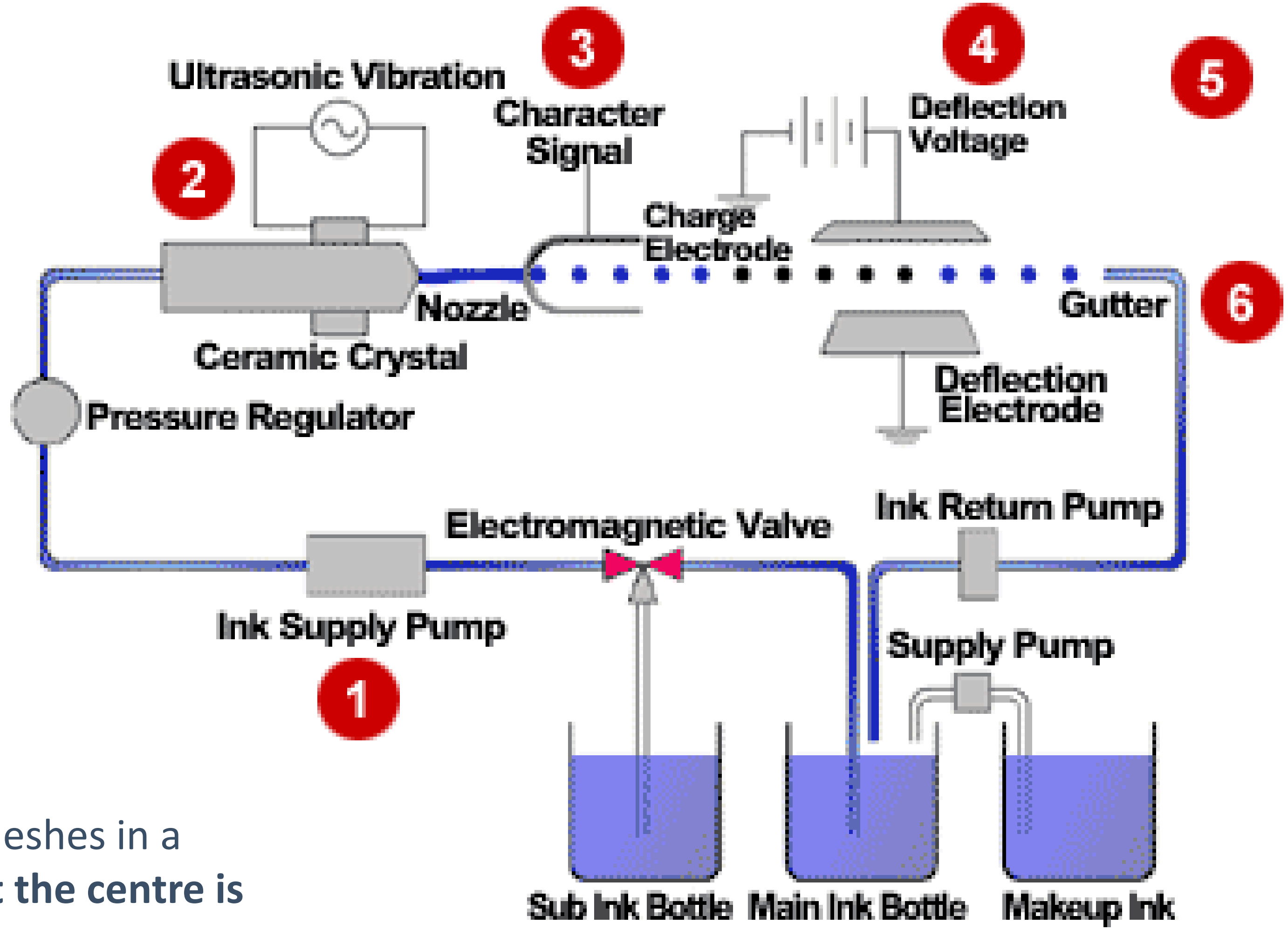


https://www.youtube.com/watch?v=Xp-8xp_3hqU

Electrical Breakdown and Sparking



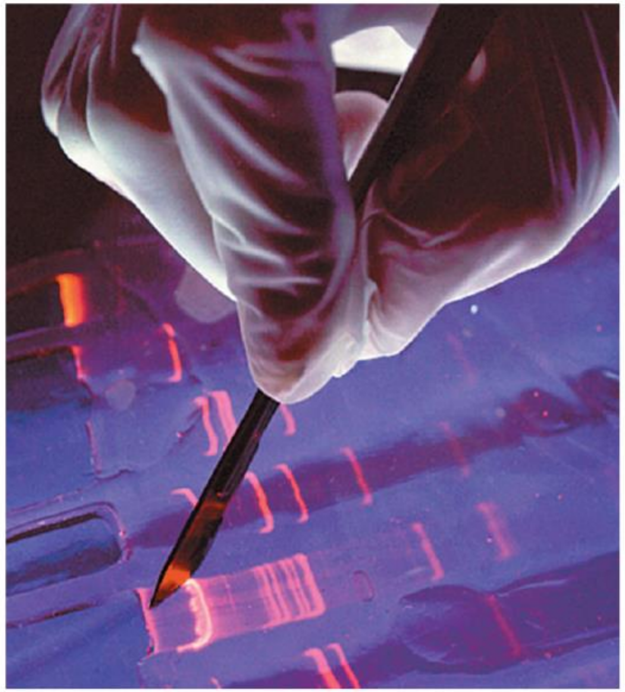
Ink-Jet Printing



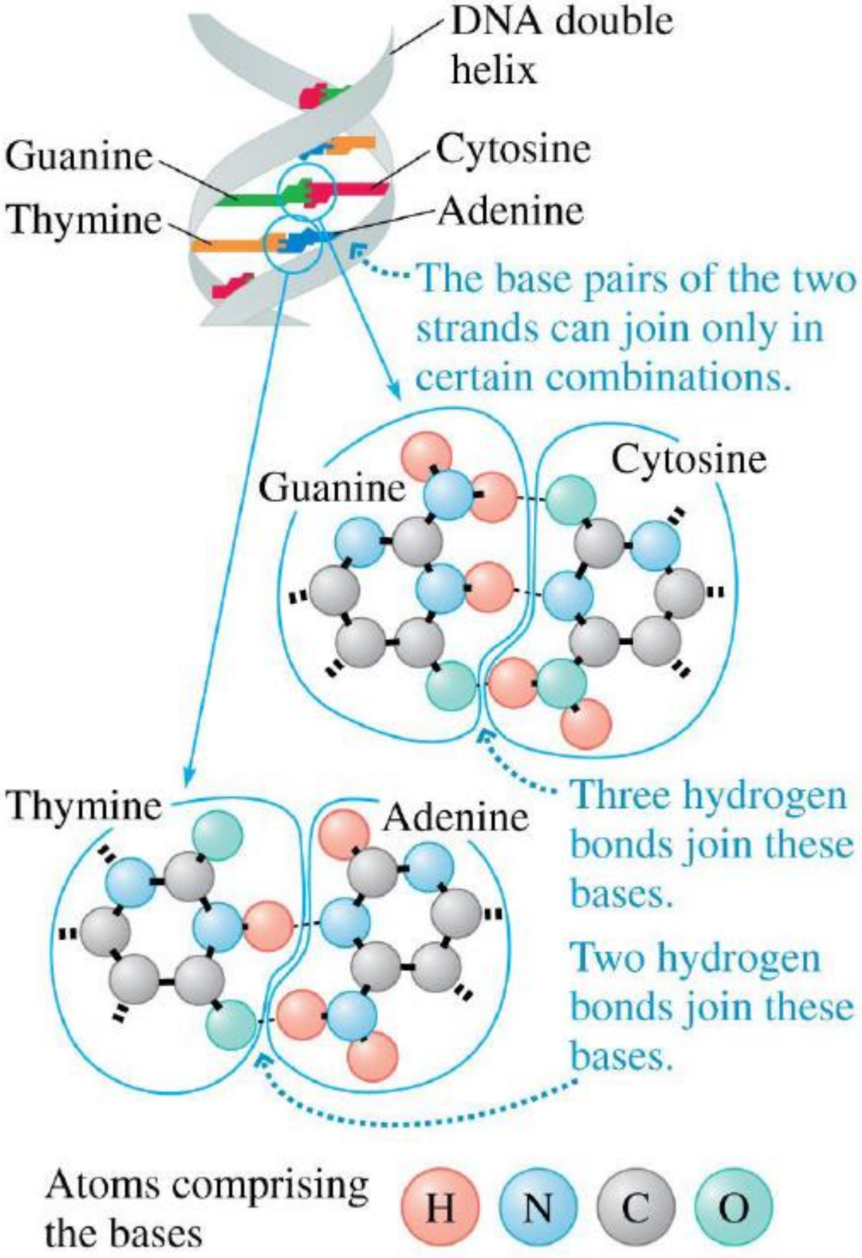
Mosquito Bat

- There are three metal meshes in a mosquito bat. The **one at the centre is positively charged** and the outer ones are **negatively charged**.
- When the layers don’t touch each other, current cannot flow.
- But when a **mosquito connects them,** a current passes through and kills the insect.

Base Pairing in DNA



- "DNA fingerprints" are measured with the technique of *gel electrophoresis*.
- A solution of negatively charged DNA fragments migrate through the gel when placed in a uniform electric field.
- Because the gel exerts a drag force, the fragments move at a terminal speed inversely proportional to their size.

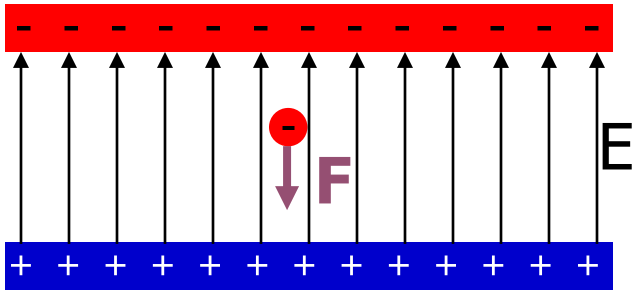


A Point Charge in an Electric Field

A charged particle in an electric field experiences a force, and if it is free to move, an acceleration.

If the only force is due to the electric field, then

$$\sum \vec{F} = m\vec{a} = q\vec{E}.$$

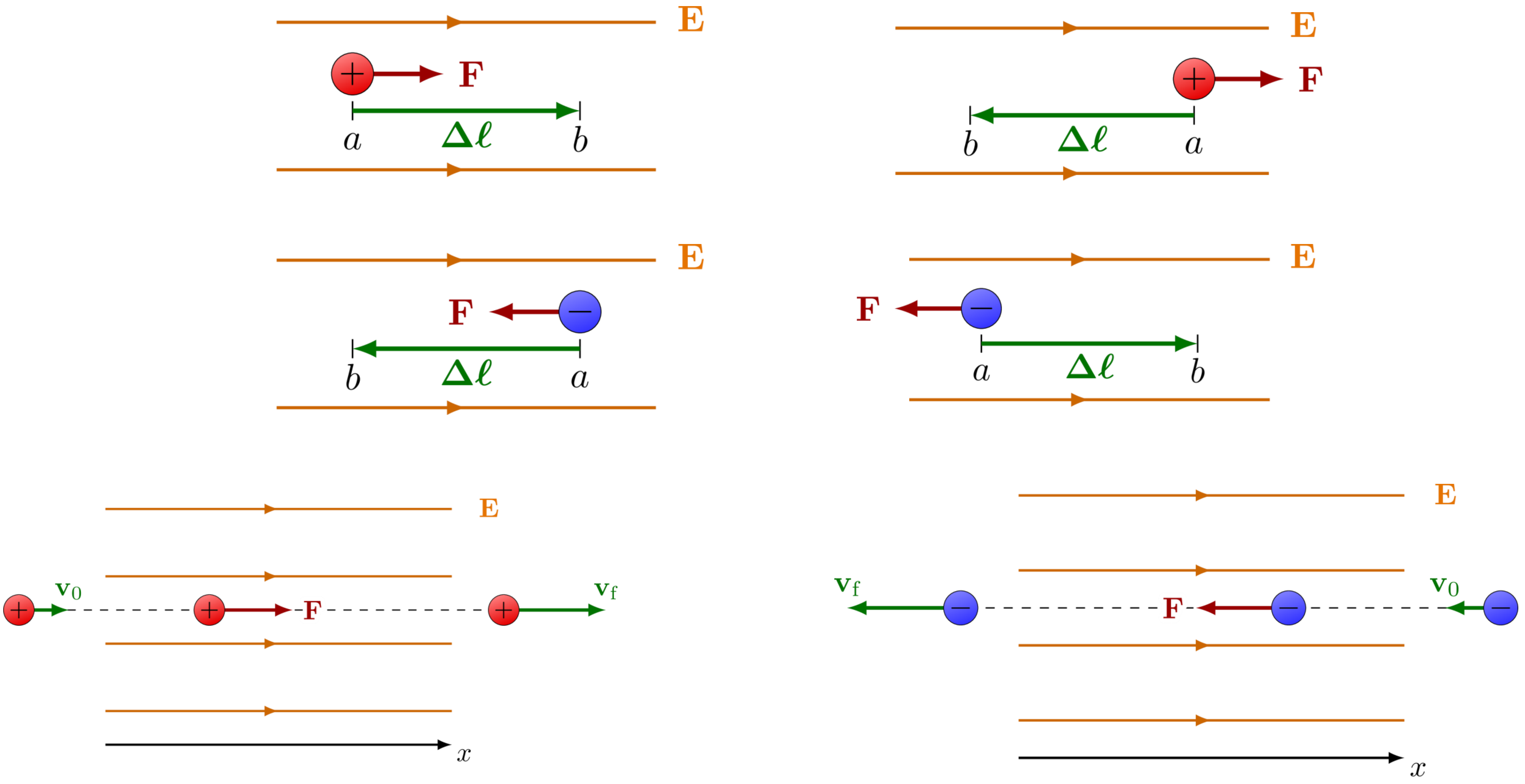


If $\vec{E}$ is constant, then $\vec{a}$ is constant, and you can use the equations of kinematics* (remember way back to the beginning of Physics 107?).

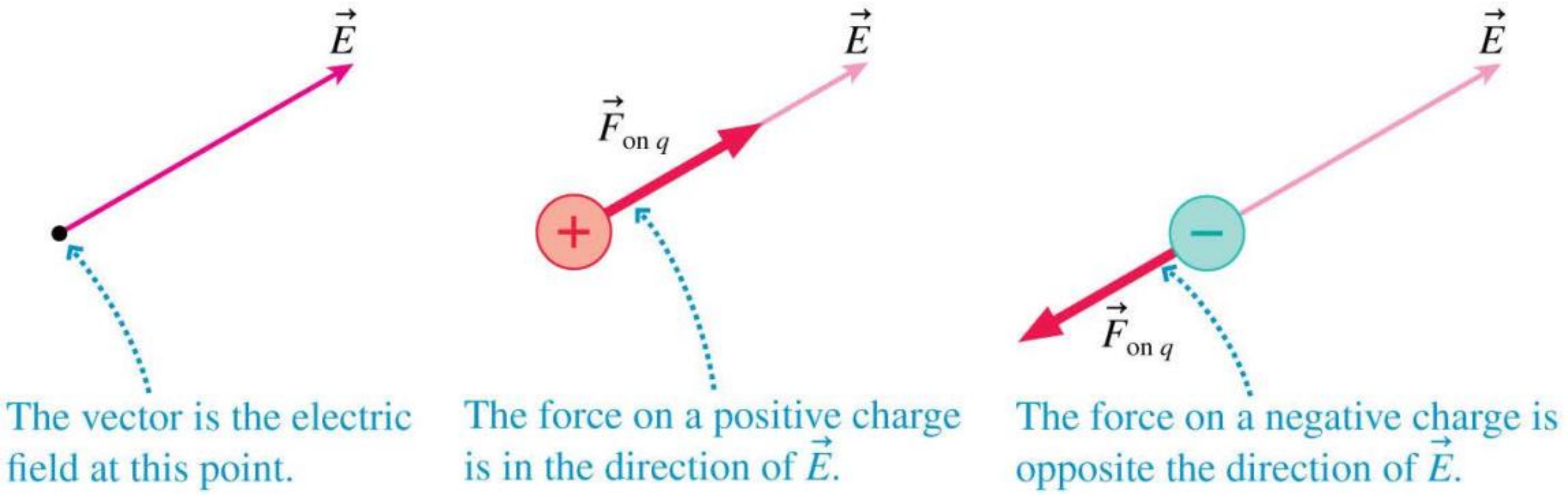
https://ophysics.com/em6.html

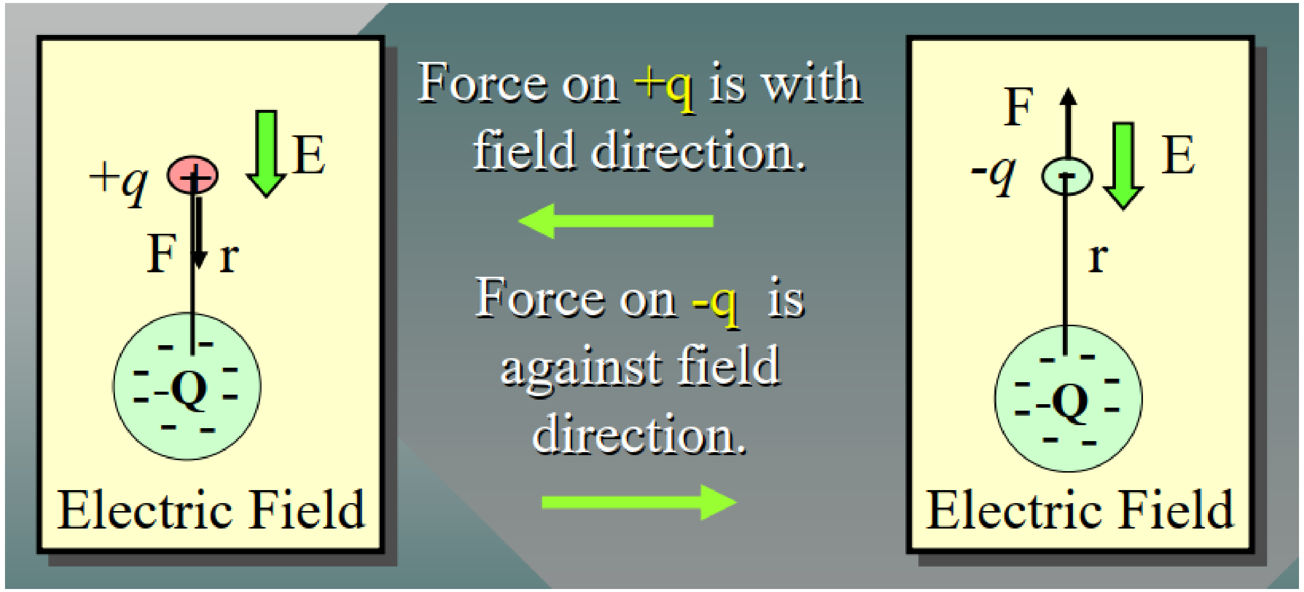
http://physics.bu.edu/~duffy/HTML5/charge_in_EField.html

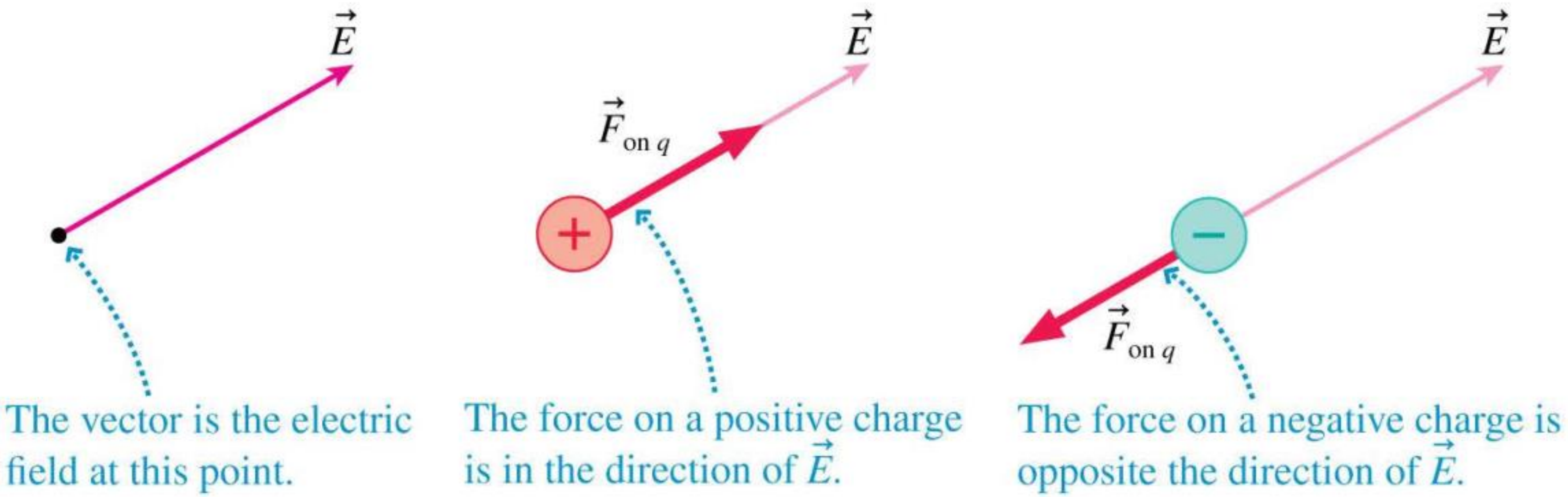
A Point Charge in an Electric Field



Motion of a Charged Particle in an Electric Field







The electrostatic force $\vec{F}$ acting on a charged particle located in an external electric field $\vec{E}$ has the direction of $\vec{E}$ if the charge q of the particle is positive and has the opposite direction if q is negative.

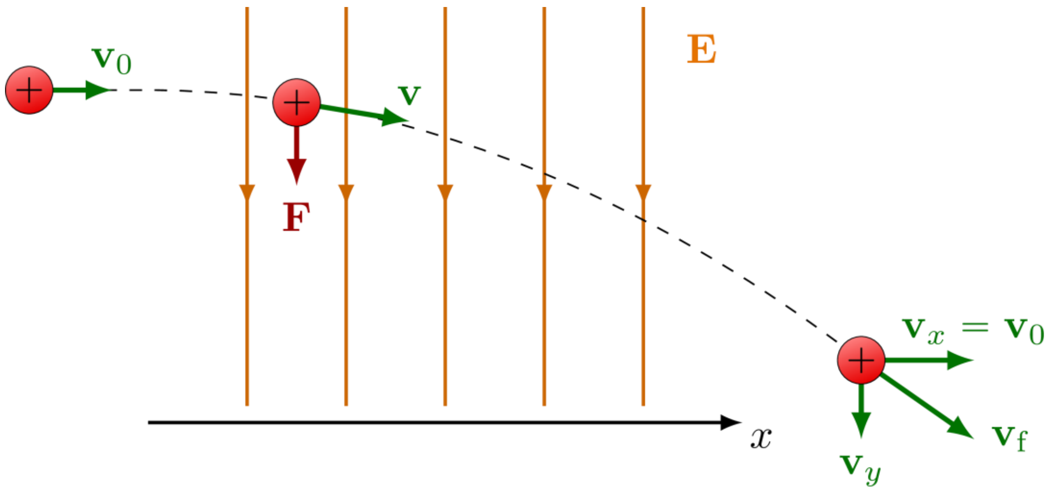
- The electric field exerts a force $\vec{F}_{\text{on } q} = q\vec{E}$ on a charged particle.
- If this is the only force acting on q, it causes the charged particle to accelerate with

$$\vec{a} = \frac{\vec{F}_{\text{on } q}}{m} = \frac{q}{m}\vec{E}$$

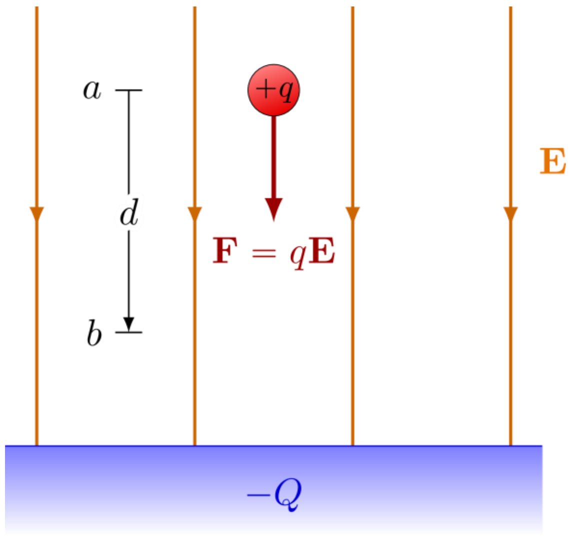
- In a uniform field, the acceleration is constant:

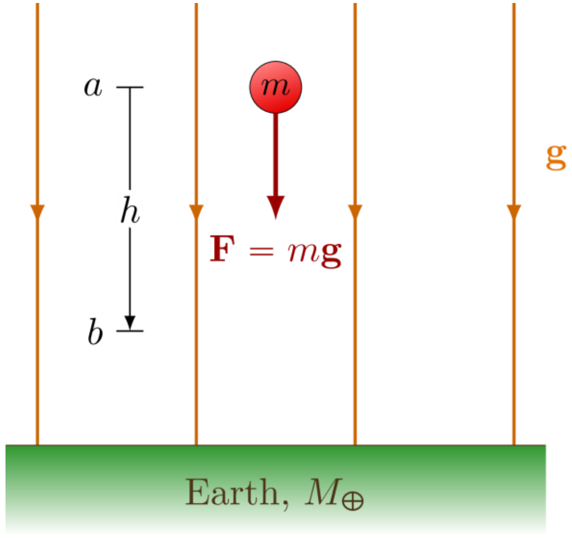
$$a = \frac{qE}{m} = \text{constant}$$

A Point Charge in an Electric Field



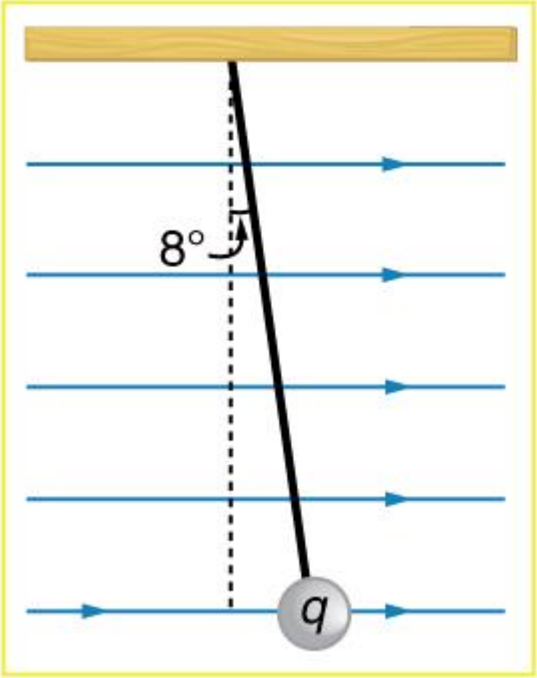
Comparing Electric Field and Gravitational Field





https://seilias.gr/go-lab/html5/motionInUniformElectricField.plain.html

A 5.00 g charged insulating ball hangs on a 30.0 cm long string in a uniform horizontal electric field as shown in Figure. Given the charge on the ball is 1.00 μC, find the strength of the field.



An electron has an initial velocity of 5.00×10^6 m/s in a uniform 2.00×10^5 N/C strength electric field. The field accelerates the electron in the direction opposite to its initial velocity. (a) What is the direction of the electric field? (b) How far does the electron travel before coming to rest? (c) How long does it take the electron to come to rest? (d) What is the electron's velocity when it returns to its starting point?

A Point Charge in an Electric Field

Measuring the Elementary Charge: Millikan oil-drop Experiment

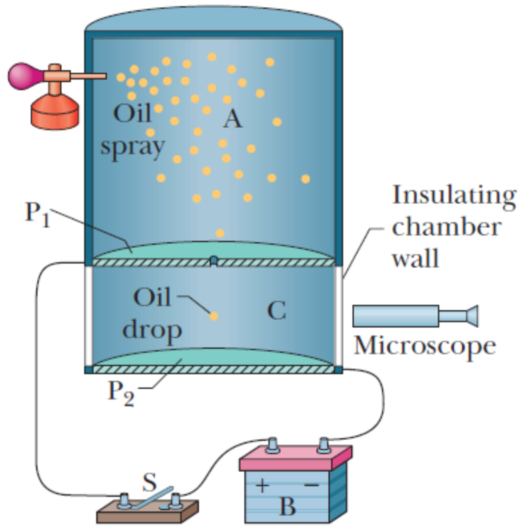
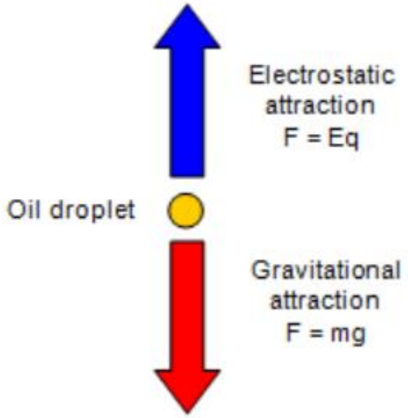


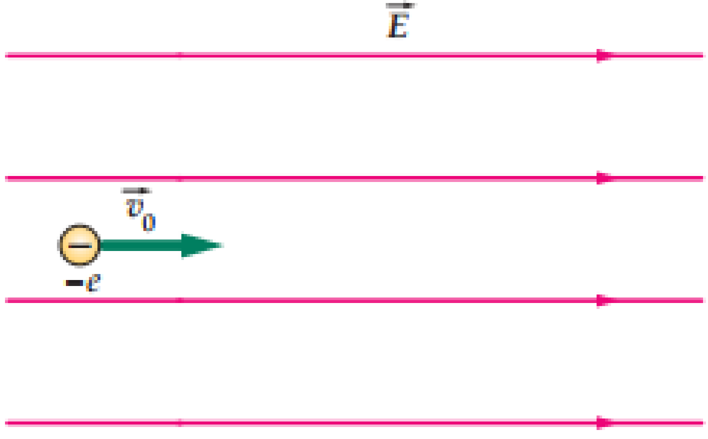
Figure 22-16 The Millikan oil-drop apparatus for measuring the elementary charge *e*. When a charged oil drop drifted into chamber C through the hole in plate P_1, its motion could be controlled by closing and opening switch S and thereby setting up or eliminating an electric field in chamber C. The microscope was used to view the drop, to permit timing of its motion.



https://ophysics.com/em2.html

http://physics.bu.edu/~duffy/HTML5/field_and_string.html

An electron is projected into a uniform electric field $\vec{E} = 1000\frac{N}{C}\hat{\imath}$ with an initial velocity $\vec{v_0} = 2.0 \times 10^6\frac{m}{s}\hat{\imath}$ in the direction of the field (Figure). How far does the electron travel before it is brought momentarily to rest?



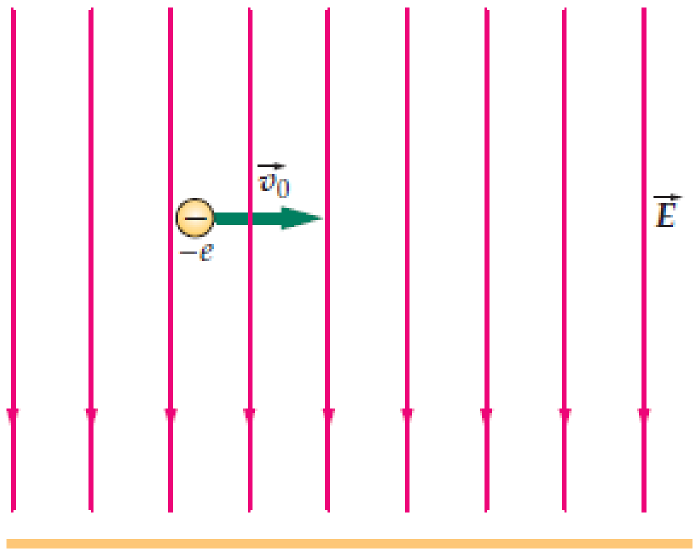
SOLVE

1. The displacement Δx is related to the initial and final velocities: $v_x^2 = v_{0x}^2 + 2a_x\Delta x$
2. The acceleration is obtained from Newton's second law: $a_x = \frac{F_x}{m} = \frac{-eE_x}{m}$
3. When $v_x = 0$, the displacement is:

$$\Delta x = \frac{v_x^2 - v_{0x}^2}{2a_x} = \frac{0 - v_{0x}^2}{2(-eE_x/m)} = \frac{mv_0^2}{2eE} = \frac{(9.11 \times 10^{-31}\text{ kg})(2.00 \times 10^6\text{ m/s})^2}{2(1.60 \times 10^{-19}\text{ C})(1000\text{ N/C})}$$

$$= 1.14 \times 10^{-2}\text{ m} = \boxed{1.14\text{ cm}}$$

An electron enters a uniform electric field $\vec{E} = -2.0\frac{N}{C}\hat{\jmath}$ with an initial velocity $\vec{v_0} = 1.0 \times 10^6\frac{m}{s}\hat{\imath}$ perpendicular to the field (Figure). (*a*) Compare the gravitational force acting on the electron to the electric force acting on it. (*b*) By how much has the electron been deflected after it has traveled 1.0 cm in the direction?



(*a*) 1. Calculate the ratio of the magnitude of the electric force, F_e, to the magnitude of the gravitational force, F_g:

$$\frac{F_e}{F_g} = \frac{eE}{mg} = \frac{(1.60 \times 10^{-19}\text{ C})(2000\text{ N/C})}{(9.11 \times 10^{-31}\text{ kg})(9.81\text{ N/kg})} = \boxed{3.6 \times 10^{13}}$$

(*b*) 1. Express the vertical deflection in terms of the acceleration a and time t:

$$\Delta y = \frac{1}{2}a_y t^2$$

2. Express the time required for the electron to travel a horizontal distance Δx with constant horizontal velocity v_0:

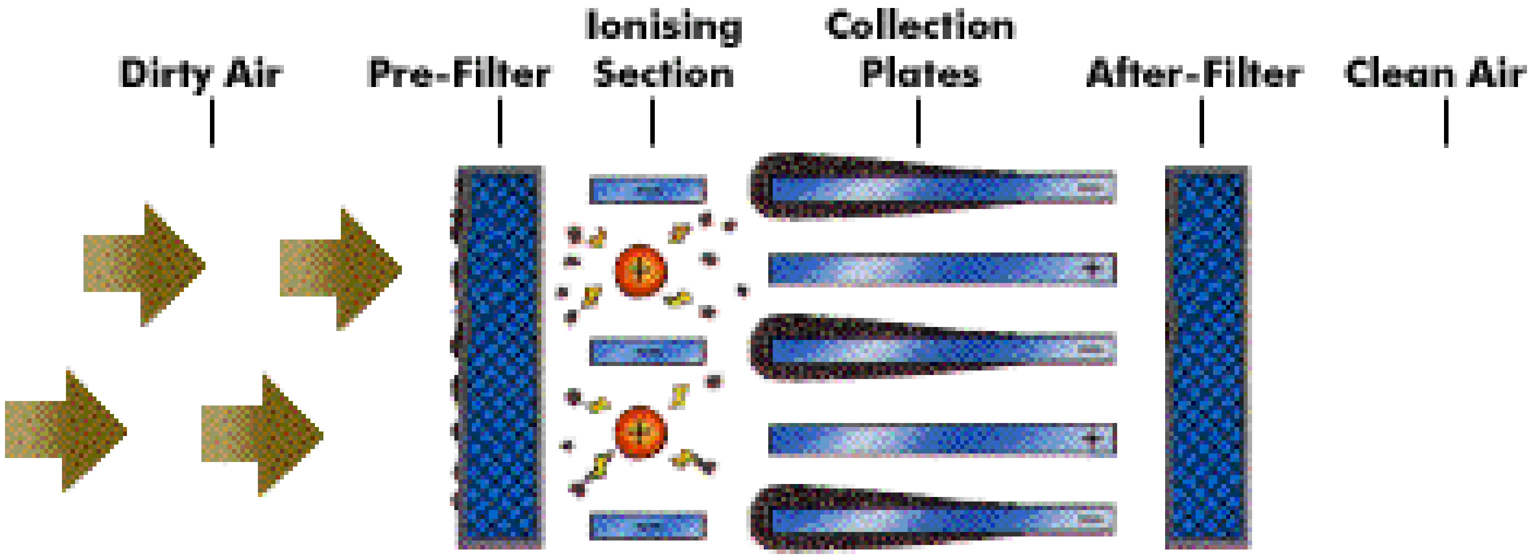
$$t = \frac{\Delta x}{v_0}$$

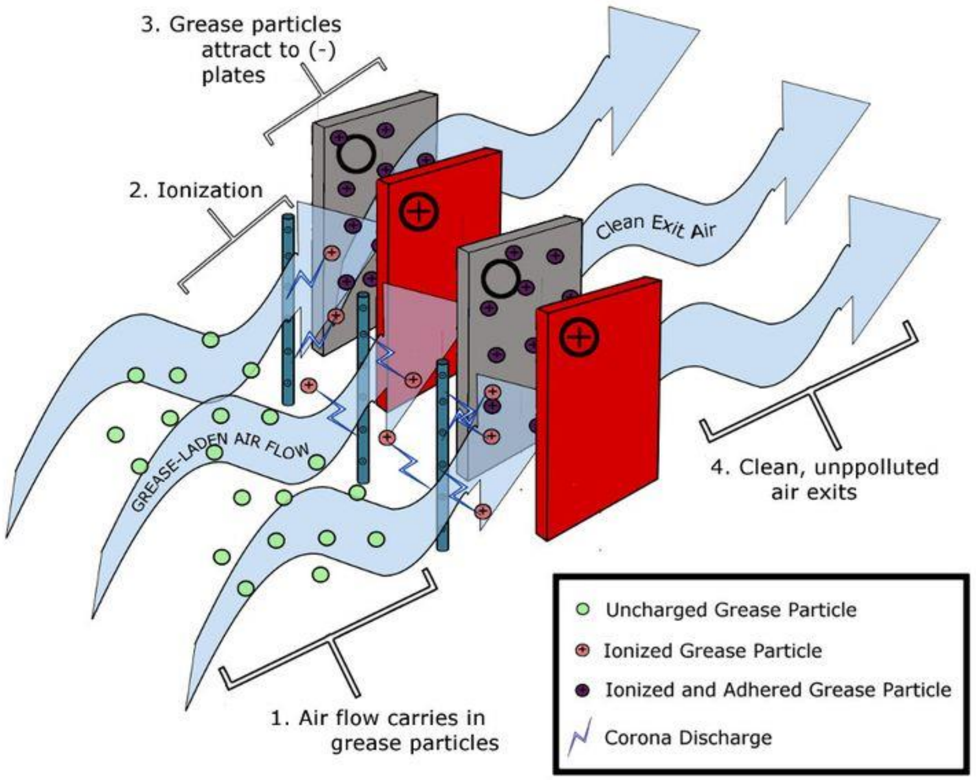
3. Use this result for t and eE/m for a_y to calculate Δy:

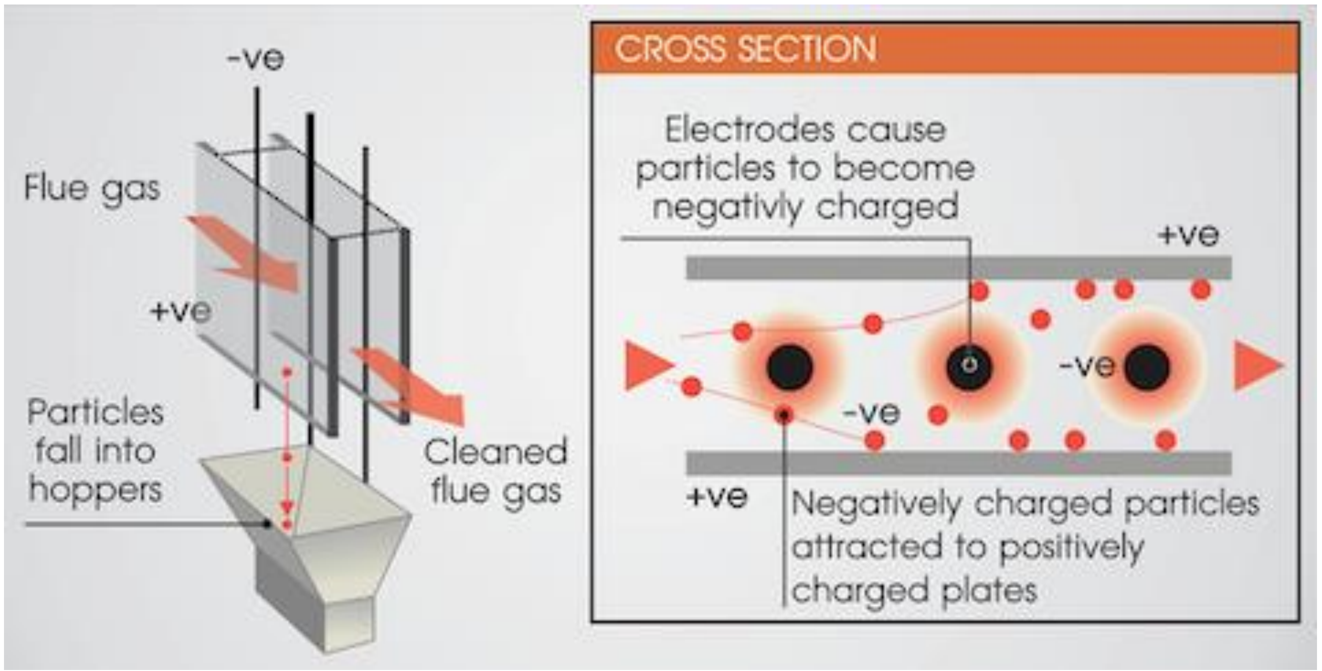
$$\Delta y = \frac{1}{2}\frac{eE}{m}\left(\frac{\Delta x}{v_0}\right)^2 = \frac{1}{2}\frac{(1.6 \times 10^{-19}\text{ C})(2000\text{ N/C})}{9.11 \times 10^{-31}\text{ kg}}\left(\frac{0.010\text{ m}}{10^6\text{ m/s}}\right)^2$$

$$= \boxed{1.8\text{ cm}}$$

Electrostatic Precipitator

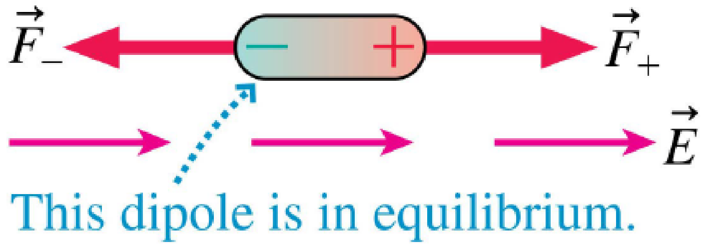




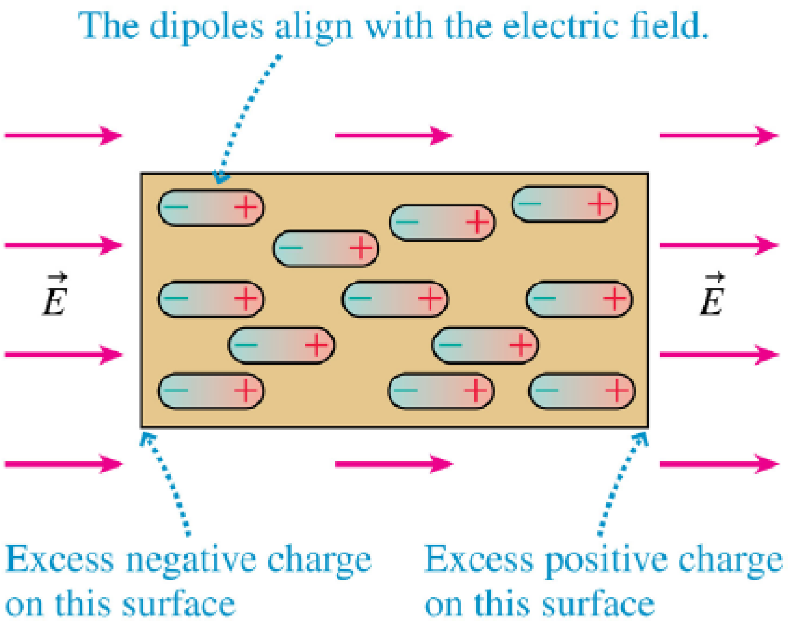


Dipole in an Electric Field

- The figure shows an electric dipole placed in a *uniform* external electric field.
- The torque causes the dipole to rotate until it is aligned with the electric field, as shown.
- Notice that the positive end of the dipole is in the direction in which $\vec{E}$ points.



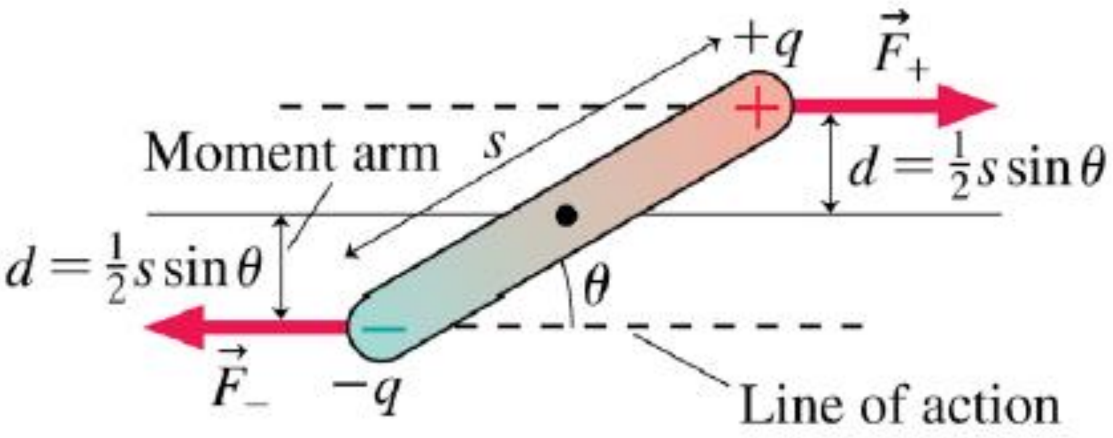
Dipole in an Electric Field



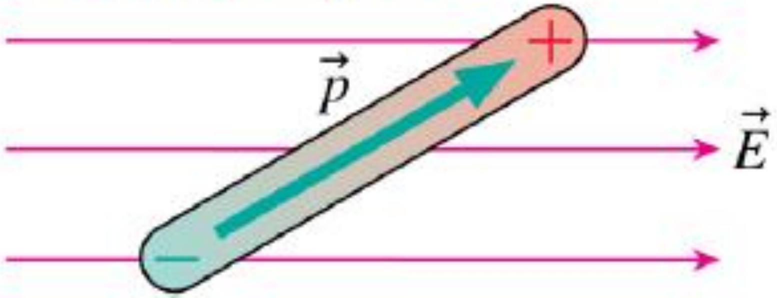
- The figure shows a sample of permanent dipoles, such as water molecules, in an external electric field.
- All the dipoles rotate until they are aligned with the electric field.
- This is the mechanism by which the sample becomes *polarized*.

Dipole in an Electric Field

$$\tau = 2 \times dF_+ = 2(\tfrac{1}{2}s\sin\theta)(qE) = pE\sin\theta$$

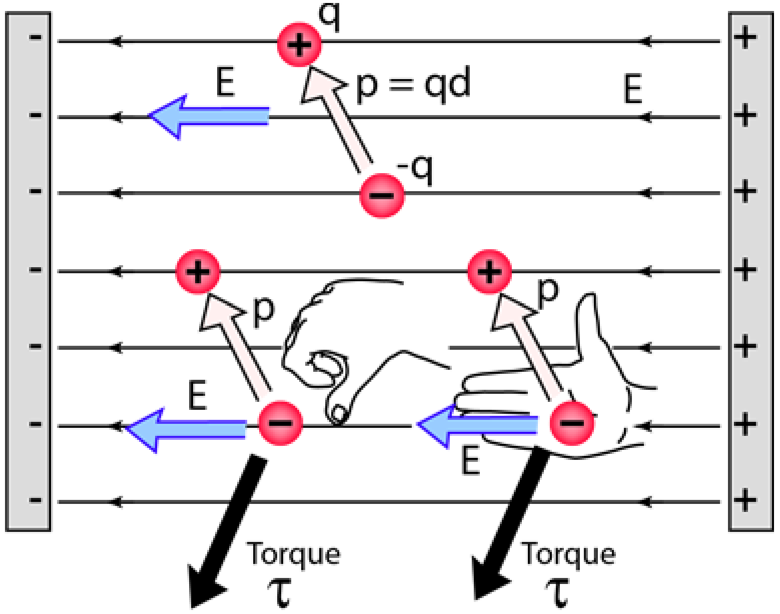


In terms of vectors, $\vec{\tau} = \vec{p} \times \vec{E}$.



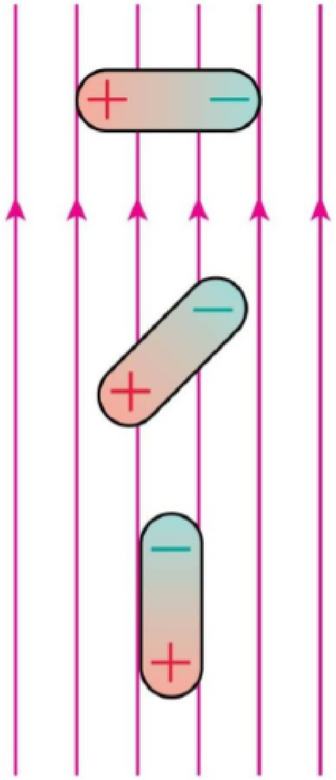
Torque on a electric dipole

$$\vec{\boldsymbol{\tau}} = \vec{\boldsymbol{p}} \times \vec{\boldsymbol{E}}$$



Which dipole experiences no net torque in the electric field?

A. Dipole A
B. Dipole B
C. Dipole C
D. Both dipoles A and C
E. All three dipoles



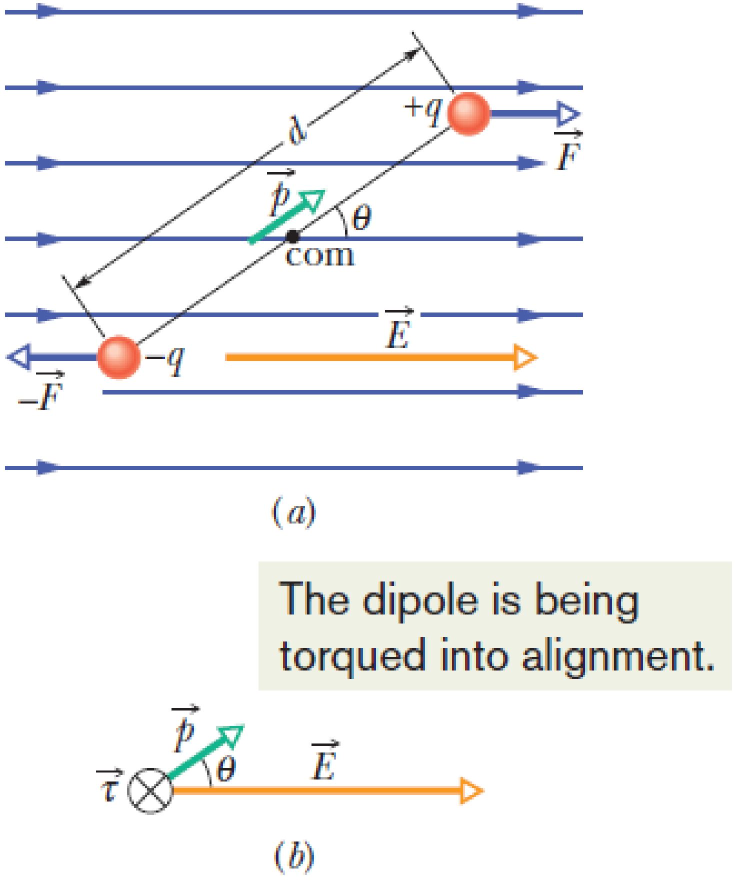
Variations of Torque

The dipole has its least potential energy when it is in its equilibrium orientation, which is when its moment $\vec{p}$ is lined up or parallel with the field $\vec{E}$.

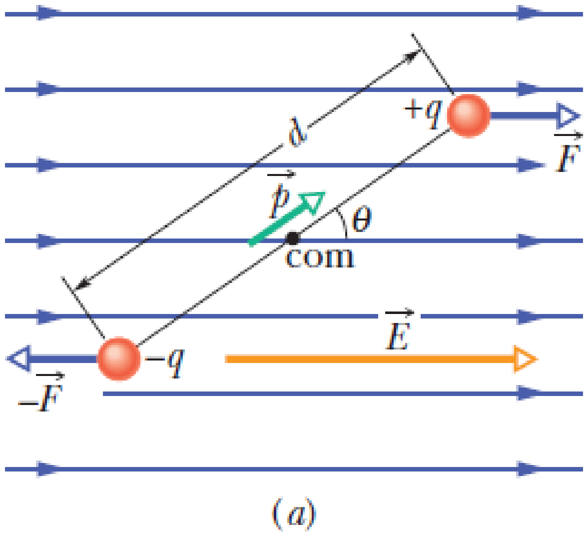
That means, $\vec{\tau} = \vec{p} \times \vec{E} = 0$

It has greater potential energy in all other orientations.

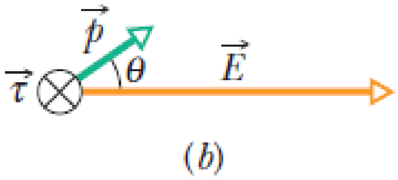
Thus the dipole is like a pendulum, which has its least gravitational potential energy in its equilibrium orientation—at its lowest point. To rotate the dipole or the pendulum to any other orientation requires work by some external agent.



Potential Energy of the configuration



The dipole is being torqued into alignment.



The work dW done by a torque $\vec{\tau}$ during an infinitesimal displacement $d\vec{\theta}$ is given by:

$$dW = \vec{\tau}.d\vec{\theta}$$

Because the torque is in the direction of decreasing angle we must write the torque as,

$$|\vec{\tau}| = -pE\ sin\theta$$

$$dW = -pE\ sin\theta\ d\theta$$

Then total work done,

$$W = -\int pE\ sin\theta\ d\theta$$

$$= pE\ \cos\theta$$

The work is the negative of the change of potential energy,

$$W = -\Delta U$$

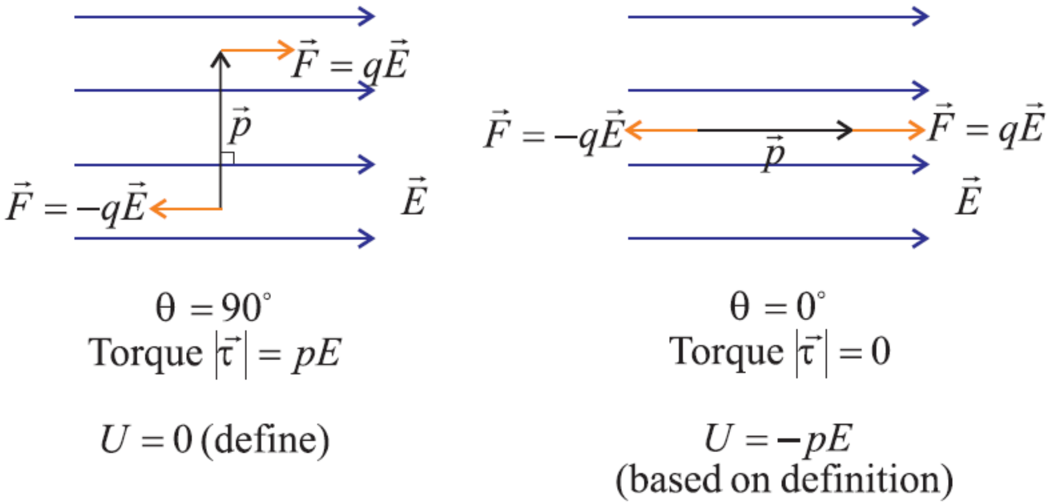
So the expression can be written as, $U(\theta) = -pE\ \cos\theta$

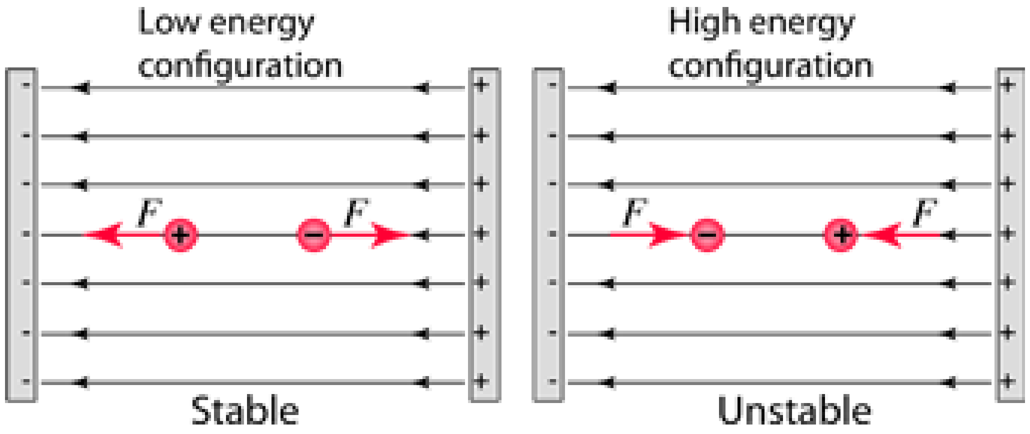
In scalar product form, $U(\theta) = -\vec{p}.\vec{E}$

The potential energy has its minimum (most negative) value, $U = -pE$ at the stable equilibrium position, where $\vec{p}$ is parallel with the field $\vec{E}$, $\theta = 0^0$

The potential energy is maximum $U = +pE$, when $\vec{p}$ is antiparallel with the field $\vec{E}$, $\theta = 180^0$, unstable position

At $\theta = 90^0$ where $\vec{p}$ is perpendicular with $\vec{E}$, $U = 0$.





Thus the dipole is like a pendulum, which has its least gravitational potential energy in its equilibrium orientation—at its lowest point. To rotate the dipole or the pendulum to any other orientation requires work by some external agent.

A neutral water molecule (H_2O) in its vapor state has an electric dipole moment of magnitude 6.2×10^{-30} C·m.

(a) How far apart are the molecule's centers of positive and negative charge?

Calculations: There are 10 electrons and 10 protons in a neutral water molecule; so the magnitude of its dipole moment is

$$p = qd = (10e)(d),$$

in which d is the separation we are seeking and e is the elementary charge. Thus,

$$d = \frac{p}{10e} = \frac{6.2 \times 10^{-30}\ \text{C}\cdot\text{m}}{(10)(1.60 \times 10^{-19}\ \text{C})}$$
$$= 3.9 \times 10^{-12}\ \text{m} = 3.9\ \text{pm}. \qquad \text{(Answer)}$$

(b) If the molecule is placed in an electric field of 1.5×10^4 N/C, what maximum torque can the field exert on it? (Such a field can easily be set up in the laboratory.)

$$\begin{aligned}\tau &= pE \sin \theta \\ &= (6.2 \times 10^{-30}\ \mathrm{C \cdot m})(1.5 \times 10^{4}\ \mathrm{N/C})(\sin 90^\circ) \\ &= 9.3 \times 10^{-26}\ \mathrm{N \cdot m}. \qquad \text{(Answer)}\end{aligned}$$

(c) How much work must an *external agent* do to rotate this molecule by 180° in this field, starting from its fully aligned position, for which $\theta = 0$?

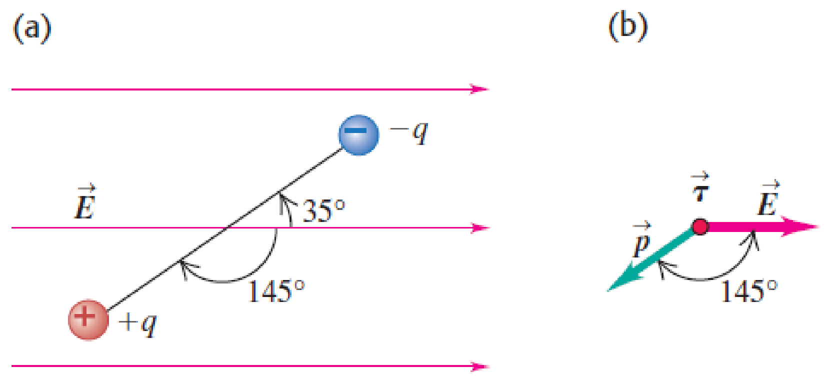
$$
\begin{aligned}
W_a &= U_{180°} - U_0 \\
&= (-pE\cos 180°) - (-pE\cos 0) \\
&= 2pE = (2)(6.2 \times 10^{-30}\ \mathrm{C \cdot m})(1.5 \times 10^{4}\ \mathrm{N/C}) \\
&= 1.9 \times 10^{-25}\ \mathrm{J}. \qquad \text{(Answer)}
\end{aligned}
$$

Example 21.13 Force and torque on an electric dipole

Figure 21.32a shows an electric dipole in a uniform electric field of magnitude 5.0×10^5 N/C that is directed parallel to the plane of the figure. The charges are $\pm 1.6 \times 10^{-19}$ C; both lie in the plane and are separated by 0.125 nm $= 0.125 \times 10^{-9}$ m. Find (a) the net force exerted by the field on the dipole; (b) the magnitude and direction of the electric dipole moment; (c) the magnitude and direction of the torque; (d) the potential energy of the system in the position shown.

Continued

21.32 (a) An electric dipole. (b) Directions of the electric dipole moment, electric field, and torque ($\vec{\tau}$ points out of the page).



SOLUTION

IDENTIFY and SET UP: This problem uses the ideas of this section about an electric dipole placed in an electric field. We use the relationship $\vec{F} = q\vec{E}$ for each point charge to find the force on the dipole as a whole. Equation (21.14) gives the dipole moment, Eq. (21.16) gives the torque on the dipole, and Eq. (21.18) gives the potential energy of the system.

EXECUTE: (a) The field is uniform, so the forces on the two charges are equal and opposite. Hence the total force on the dipole is zero.

(b) The magnitude p of the electric dipole moment $\vec{p}$ is

$$\begin{aligned} p &= qd = (1.6 \times 10^{-19}\ \text{C})(0.125 \times 10^{-9}\ \text{m}) \\ &= 2.0 \times 10^{-29}\ \text{C} \cdot \text{m} \end{aligned}$$

The direction of $\vec{p}$ is from the negative to the positive charge, 145° clockwise from the electric-field direction (Fig. 21.32b).

(c) The magnitude of the torque is

$$\begin{aligned} \tau &= pE\sin\phi = (2.0 \times 10^{-29}\ \text{C})(5.0 \times 10^5\ \text{N/C})(\sin 145°) \\ &= 5.7 \times 10^{-24}\ \text{N} \cdot \text{m} \end{aligned}$$

From the right-hand rule for vector products (see Section 1.10), the direction of the torque $\vec{\tau} = \vec{p} \times \vec{E}$ is out of the page. This corresponds to a counterclockwise torque that tends to align $\vec{p}$ with $\vec{E}$.

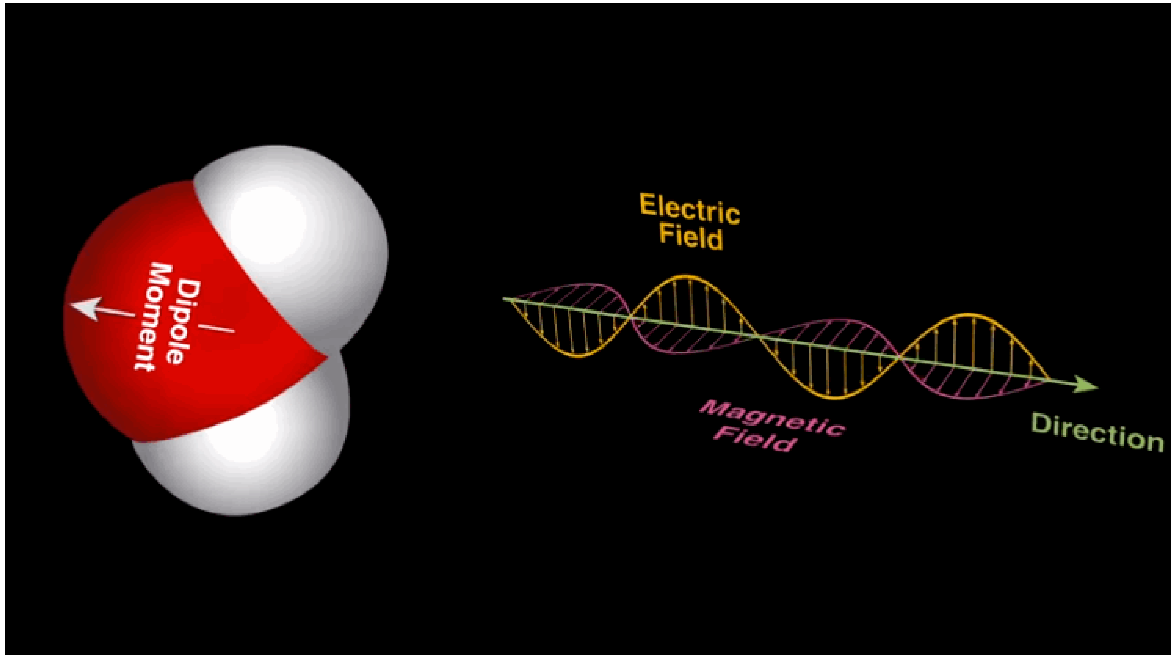
(d) The potential energy

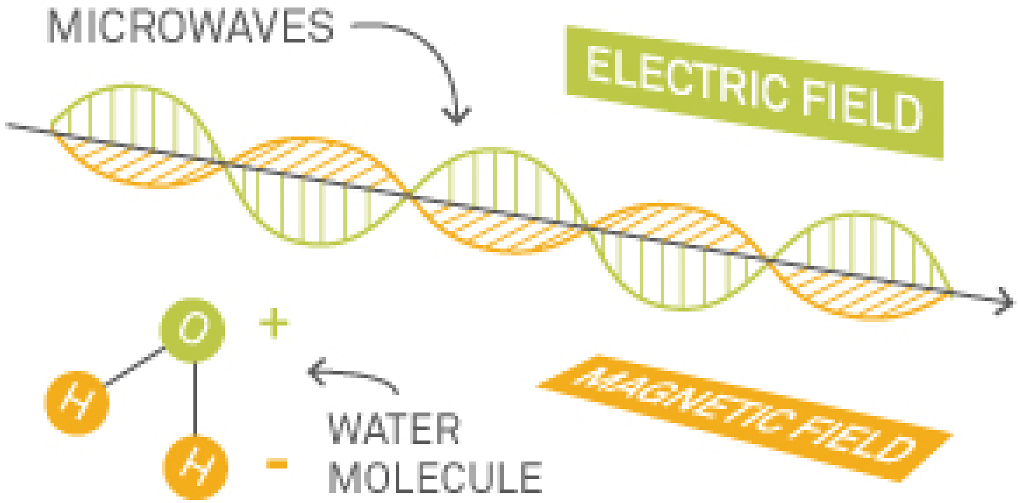
$$\begin{aligned} U &= -pE\cos\phi \\ &= -(2.0 \times 10^{-29}\ \text{C} \cdot \text{m})(5.0 \times 10^5\ \text{N/C})(\cos 145°) \\ &= 8.2 \times 10^{-24}\ \text{J} \end{aligned}$$

EVALUATE: The charge magnitude, the distance between the charges, the dipole moment, and the potential energy are all very small, but are all typical of molecules.

Microwave ovens take advantage of the dipole moment of water molecules to cook food.

Like other electromagnetic waves, microwaves have oscillating electric fields that exert torques on dipoles, torques that cause the water molecules to rotate with significant rotational kinetic energy.
In this manner, energy is transferred from the microwave radiation to the water molecules at a high rate, accounting for the rapid cooking times that make microwave ovens so convenient.





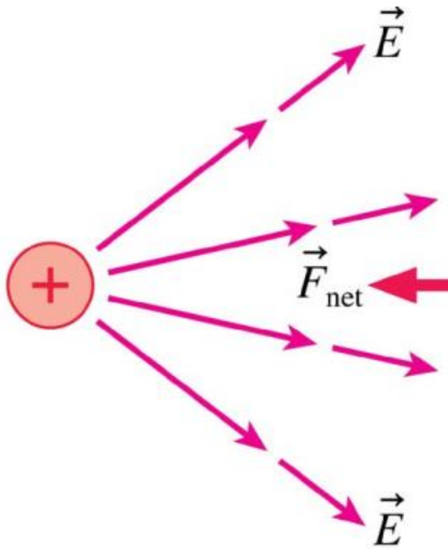
Microwave Cooking

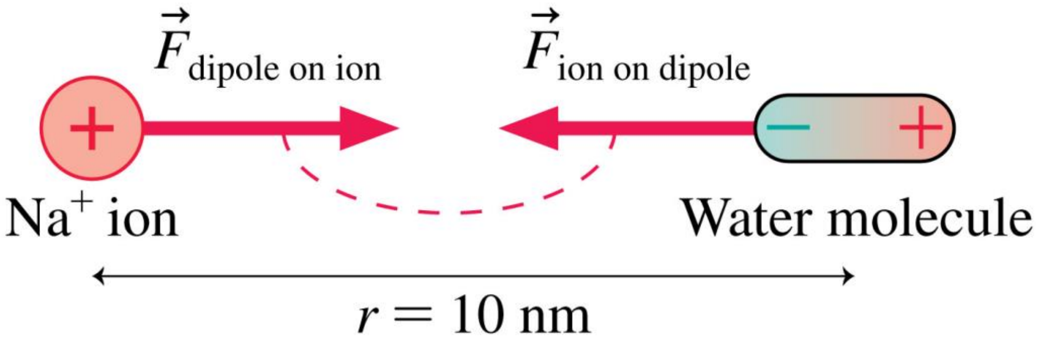
Food can be warmed and cooked in a microwave oven if the food contains water because water molecules are electric dipoles. When you turn on the oven, the microwave source sets up a rapidly oscillating electric field within the oven and thus also within the food.

https://www.youtube.com/watch?v=xDM_Gkpplck

Typically, consumer ovens work around a nominal 2.45 GHz – a wavelength of 12.2 cm, and the corresponding Quantum energy of a microwave photon is about 1×10^{-5} eV.

Dipoles in a Nonuniform Electric Field



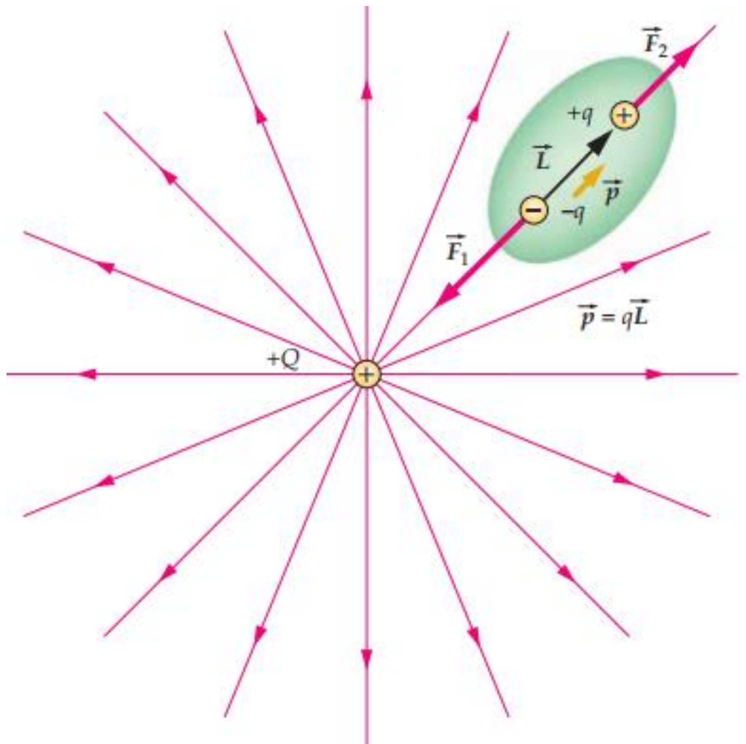


The net force on a dipole is toward the direction of the strongest field.

Nonpolar molecules have no permanent dipole moment. However, all neutral molecules have equal amounts of positive and negative charge.

In the presence of an external electric field the positive and negative charge centers become separated in space.

The molecule thus acquires an induced dipole moment parallel to the external electric field and is said to be **polarized**



Calculating Electric Fields of Charge Distributions

The electric field due to a small "chunk" Δq of charge is

$$\Delta\vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{\Delta q}{r^2}\hat{r}$$

unit vector from Δq to wherever you want to calculate $\Delta\vec{E}$

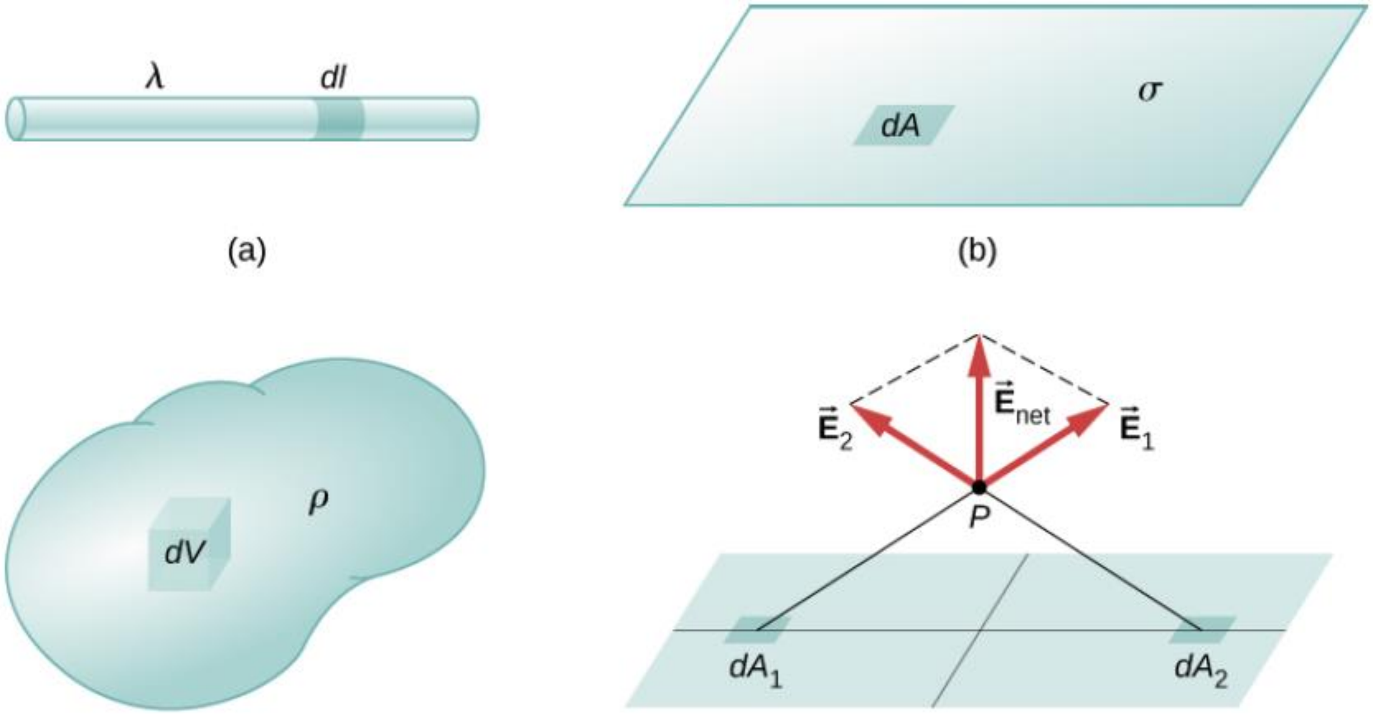
The electric field due to collection of "chunks" of charge is

$$\vec{E} = \sum_i \Delta\vec{E}_i = \frac{1}{4\pi\varepsilon_0}\sum_i \frac{\Delta q_i}{r_i^2}\hat{r}_i$$

As $\Delta q \rightarrow dq \rightarrow 0$, the sum becomes an integral.

unit vector from Δq_i to wherever you want to calculate $\vec{E}$

Calculating Electric Fields of Charge Distributions

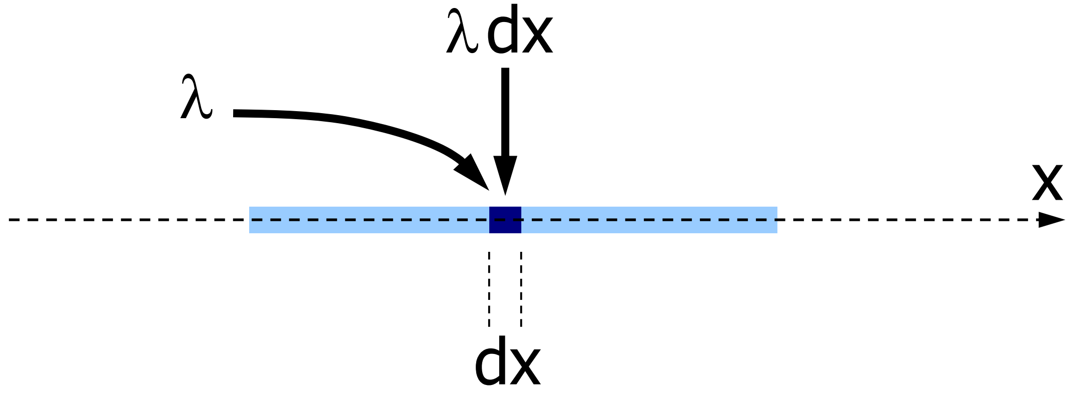


Name	Symbol	Point charge	SI Unit
Charge	q	q	C
Linear charge density	$\lambda = \dfrac{dq}{dx}$	$dq = \lambda\, dx$	C/m
Surface charge density	$\sigma = \dfrac{dq}{dA}$	$dq = \sigma\, dA$	C/m^2
Volume charge density	$\rho = \dfrac{dq}{dV}$	$dq = \rho\, dV$	C/m^3

Consider charge uniformly distributed along a line (e.g., electrons on a thread).

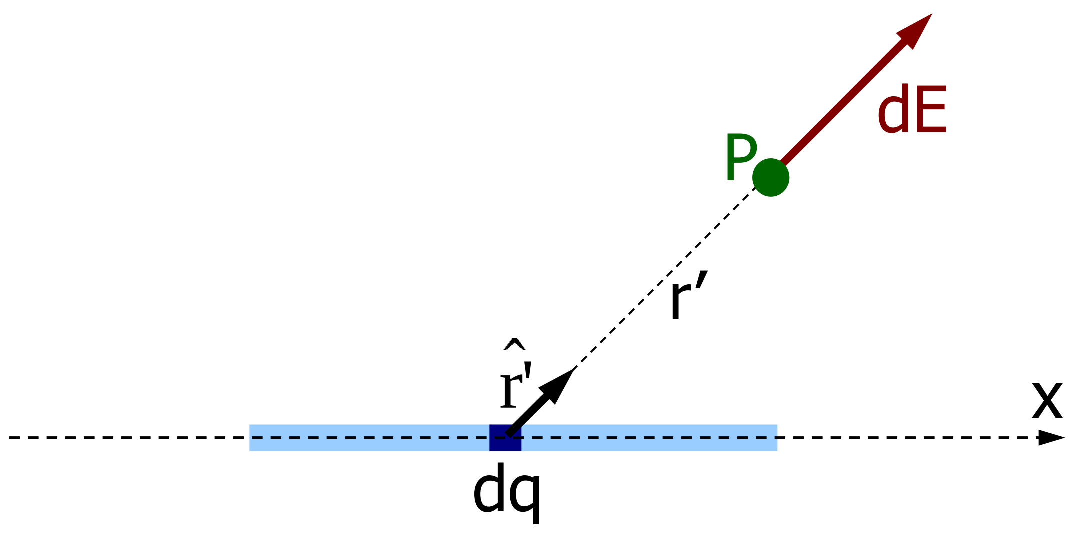
λ is the linear density of charge (amount of charge per unit length). λ may be a function of position.

λ times the length l of line segment is the total charge on the line segment.



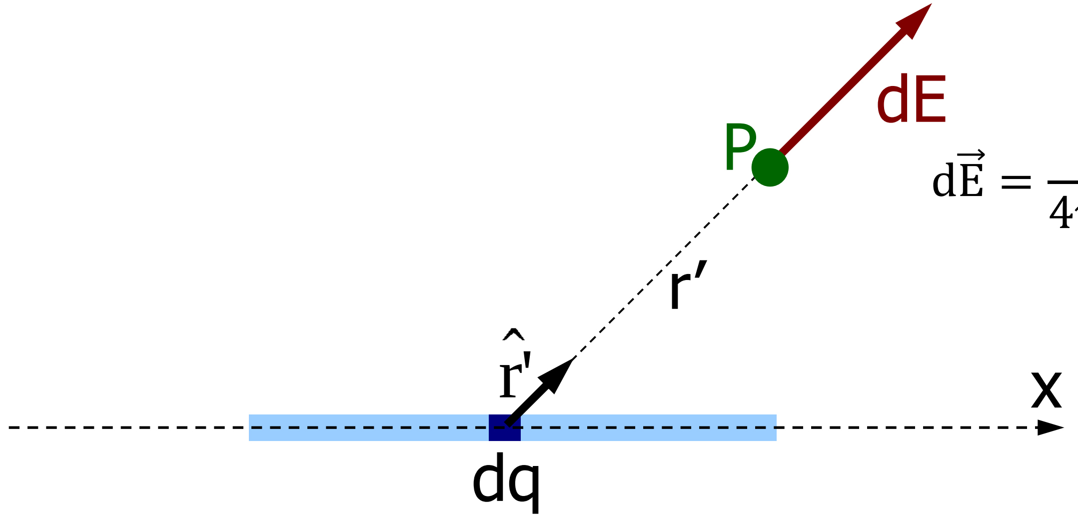
If charge is distributed along a straight line segment parallel to the x-axis, the amount of charge dq on a segment of length dx is $\lambda\, dx$.

Assuming positively charged objects in these "distribution of charges" slides.



The electric field at point P due to the charge dq is

$$d\vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{dq}{r'^2}\hat{r}' = \frac{1}{4\pi\varepsilon_0}\frac{\lambda dx}{r'^2}\hat{r}'$$



$$d\vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{dq}{r'^2}\widehat{r'} = \frac{1}{4\pi\varepsilon_0}\frac{\lambda dx}{r'^2}\widehat{r'}$$

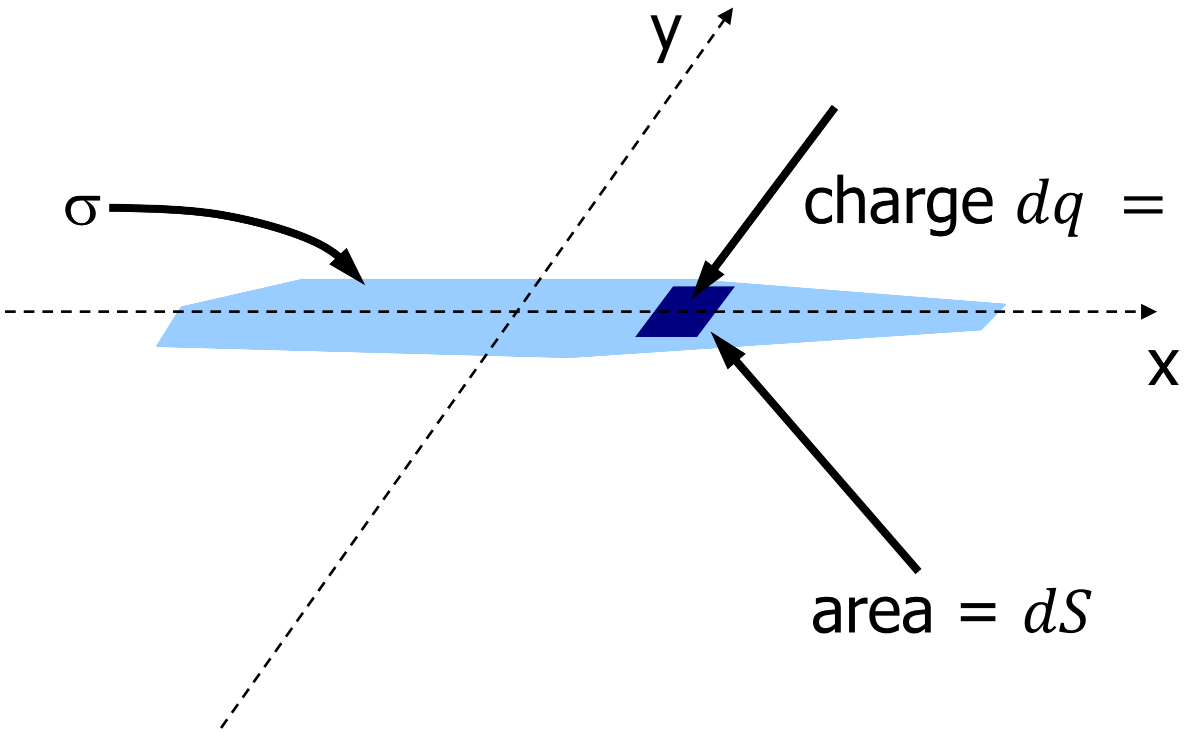
The electric field at P due to the entire line of charge is

$$\vec{E} = \frac{1}{4\pi\varepsilon_0}\int \hat{r}'\frac{\lambda(x)\,dx}{r'^2}.$$

The integration is carried out over the entire length of the line, which need not be straight.

Also, λ could be a function of position, ***and can be taken outside the integral only if the charge distribution is uniform***.

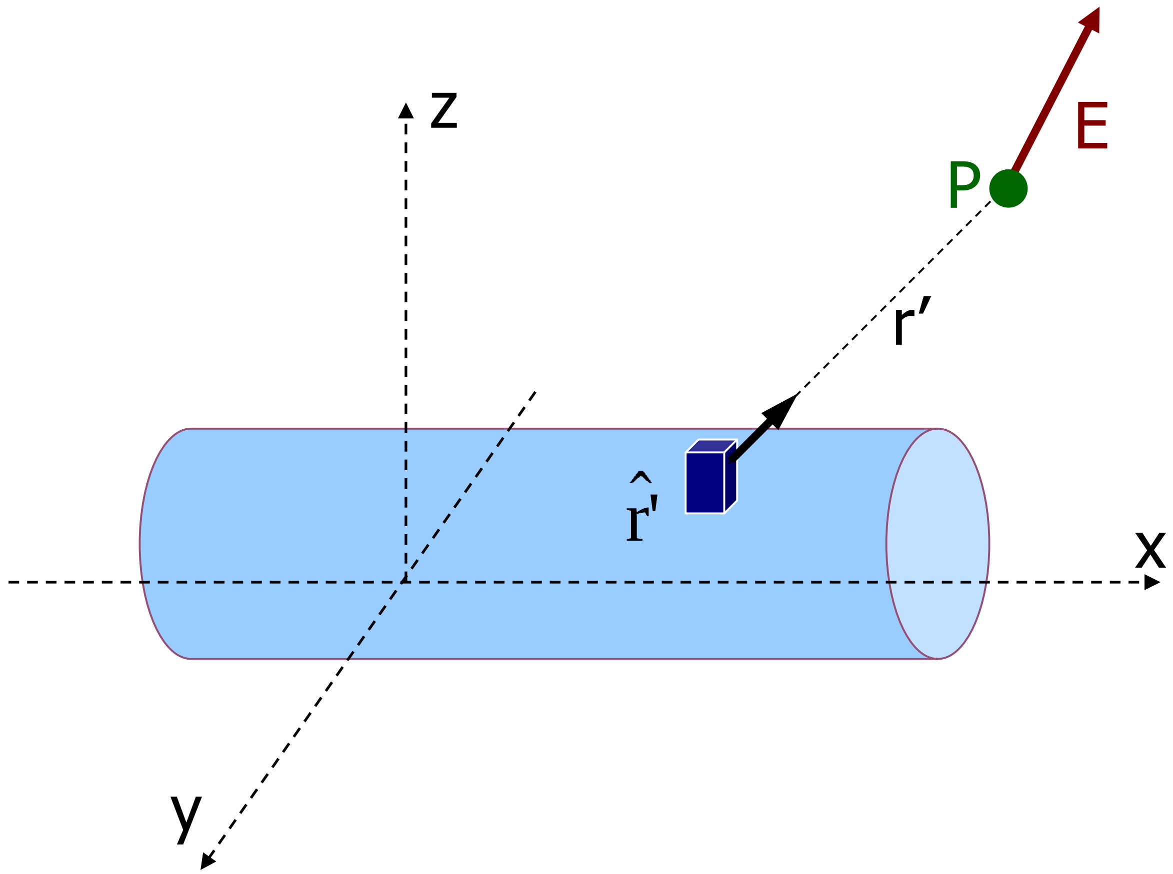
If charge is distributed over a two-dimensional surface, the amount of charge dq on an infinitesimal piece of the surface is $\sigma\, dS$, where σ is the surface density of charge (amount of charge per unit area).



$$d\vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{dq}{r'^2}\hat{r}' = \frac{1}{4\pi\varepsilon_0}\frac{\sigma\ dS}{r'^2}\hat{r}'$$

$$\vec{E} = \frac{1}{4\pi\varepsilon_0}\int_S \hat{r}'\frac{\sigma(x, y)\, dS}{r'^2}$$

Similar to above observations the net electric field at P due to a three-dimensional distribution of charge can be derived as:



$$\vec{E} = \frac{1}{4\pi\varepsilon_0} \int_V \hat{r}' \frac{\rho(x, y, z)\, dV}{r'^2}.$$

Summarizing:

Charge distributed along a line:

$$\vec{E} = \frac{1}{4\pi\varepsilon_0}\int \hat{r}' \frac{\lambda\, dx}{r'^2}.$$

Charge distributed over a surface:

$$\vec{E} = \frac{1}{4\pi\varepsilon_0}\int_S \hat{r}' \frac{\sigma\, dS}{r'^2}.$$

Charge distributed inside a volume:

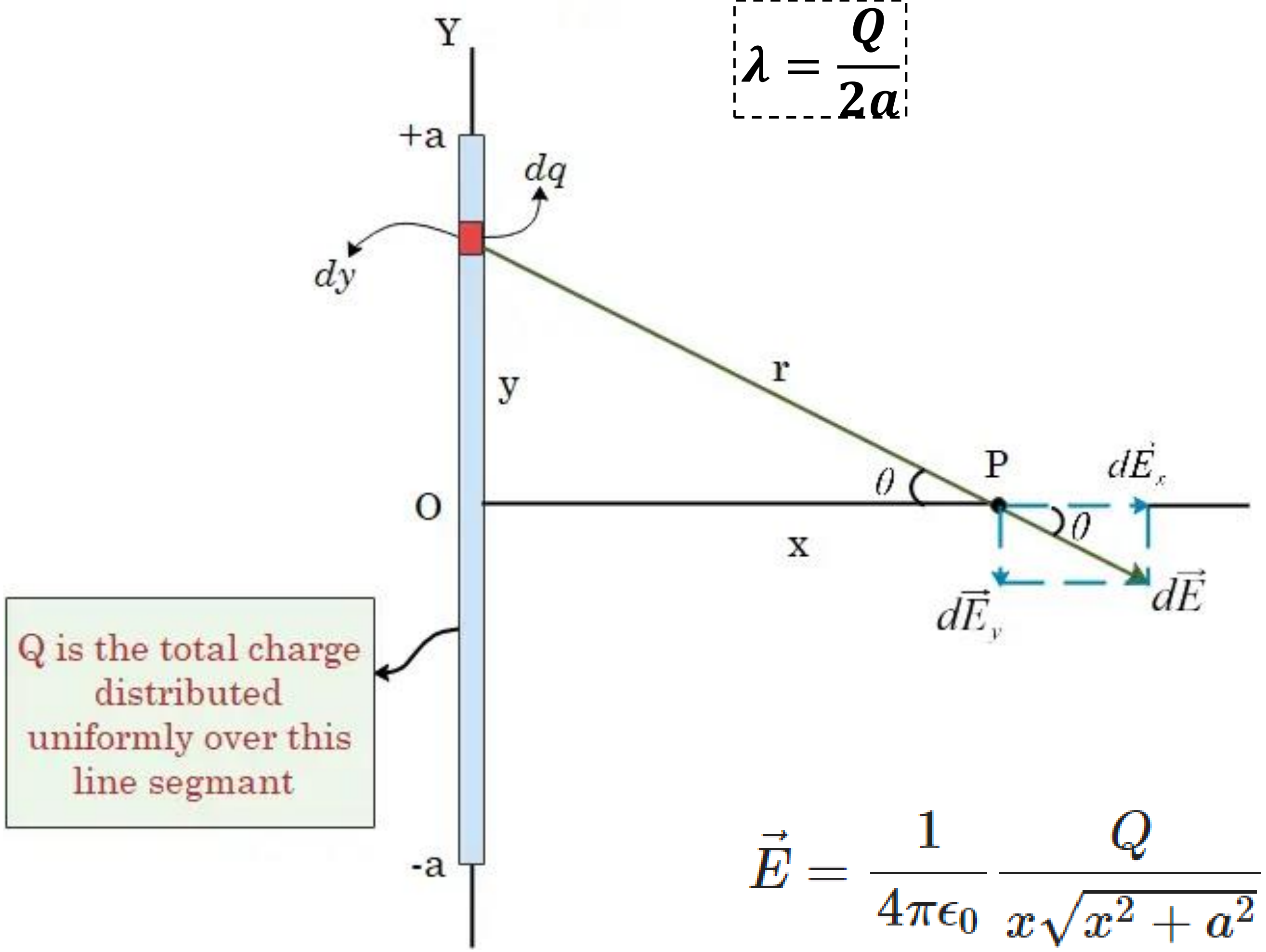
$$\vec{E} = \frac{1}{4\pi\varepsilon_0}\int_V \hat{r}' \frac{\rho\, dV}{r'^2}.$$

If the charge distribution is uniform, then λ, σ, and ρ can be taken outside the integrals.

Field of a charged line

$$dE = \frac{1}{4\pi\epsilon_0}\frac{dQ}{r^2} = \frac{1}{4\pi\epsilon_0}\frac{Q}{2a}\frac{dy}{(x^2+y^2)}$$

$$\boxed{\lambda = \frac{Q}{2a}}$$



$$d\vec{E}_x = \frac{1}{4\pi\epsilon_0}\frac{Q}{2a}\frac{xdy}{(x^2+y^2)^{3/2}}$$

$$d\vec{E}_y = \frac{1}{4\pi\epsilon_0}\frac{Q}{2a}\frac{ydy}{(x^2+y^2)^{3/2}}$$

$$E_x = \frac{1}{4\pi\epsilon_0}\frac{Q}{2a}\int_{-a}^{+a}\frac{xdy}{(x^2+y^2)^{3/2}}$$

$$E_x = \frac{1}{4\pi\epsilon_0}\frac{Q}{2a}\int_{-a}^{+a}\frac{xdy}{(x^2+y^2)^{3/2}}$$

$$\vec{E} = \frac{1}{4\pi\epsilon_0}\frac{Q}{x\sqrt{x^2+a^2}}\hat{i}$$

Field of a charged line

If the size of the line is very large $a \gg x$, the electric field surrounding it is independent of size of the line. It will only depend on the location where we measure the electric field.

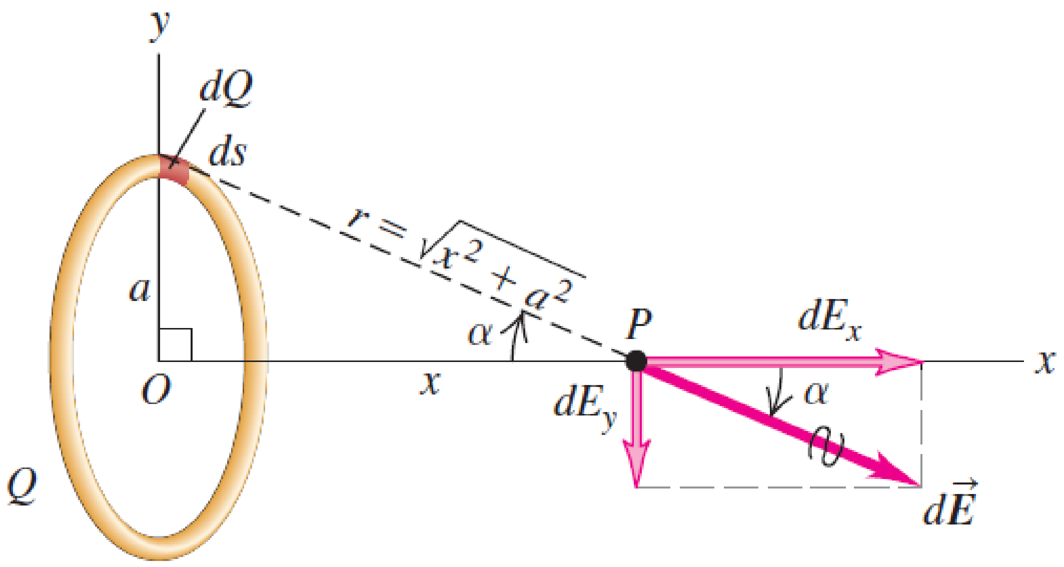
$$\vec{E} = \frac{\lambda}{2\pi\epsilon_0 x}\hat{i}$$

If the observation point is far away from the line charge such that a is negligible in comparison to x, $x \gg a$ then field at this point would be same as that of a point charge.

$$\vec{E} = \frac{1}{4\pi\epsilon_0}\frac{Q}{x^2}\hat{i}$$

Situation	What it Mimics	Key Takeaway
Standing 1 ft from a 1000-ft wall (Line much longer than distance)	Field near an **infinite line**	Wall appears infinitely tall: same as how long charged line looks from very close
Flying 10 km above a 1-km city (Observation point very far away)	Field from **finite line at far point**	City becomes a dot; so does the line charge behave like a point charge
Viewing a streetlight row from the center (Observation is neither very close nor very far)	**Finite line case**	All parts contribute; symmetry matters

Electric Field on the Axis of a Ring of Charge



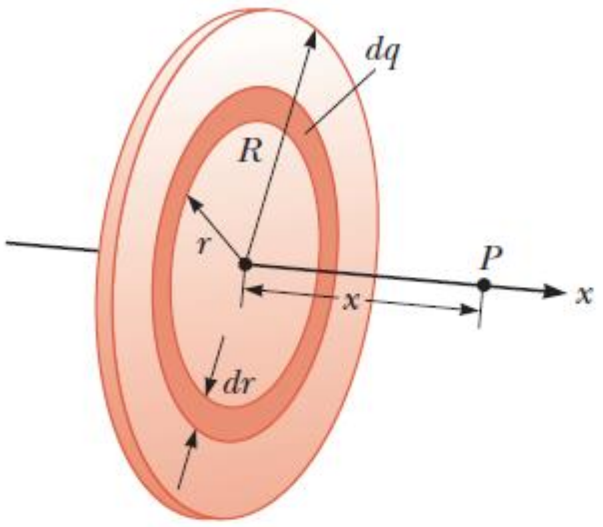
$$dE_x = \frac{kdq}{r^2}\cos\theta = \frac{kdq}{r^2}\frac{x}{r} = \frac{kxdq}{\left(x^2+a^2\right)^{3/2}} \quad where$$

$$r^2 = x^2 + a^2 \quad and \quad \cos\theta = \frac{x}{r} = \frac{x}{\sqrt{x^2+a^2}}$$

We now integrate, noting that r and x are constant for all points on the ring:

$$E_x = \int \frac{kxdq}{\left(x^2+a^2\right)^{3/2}} = \frac{kx}{\left(x^2+a^2\right)^{3/2}}\int dq = \frac{kxQ}{\left(x^2+a^2\right)^{3/2}}$$

Electric Field on the Axis of a Uniformly Charged Disk



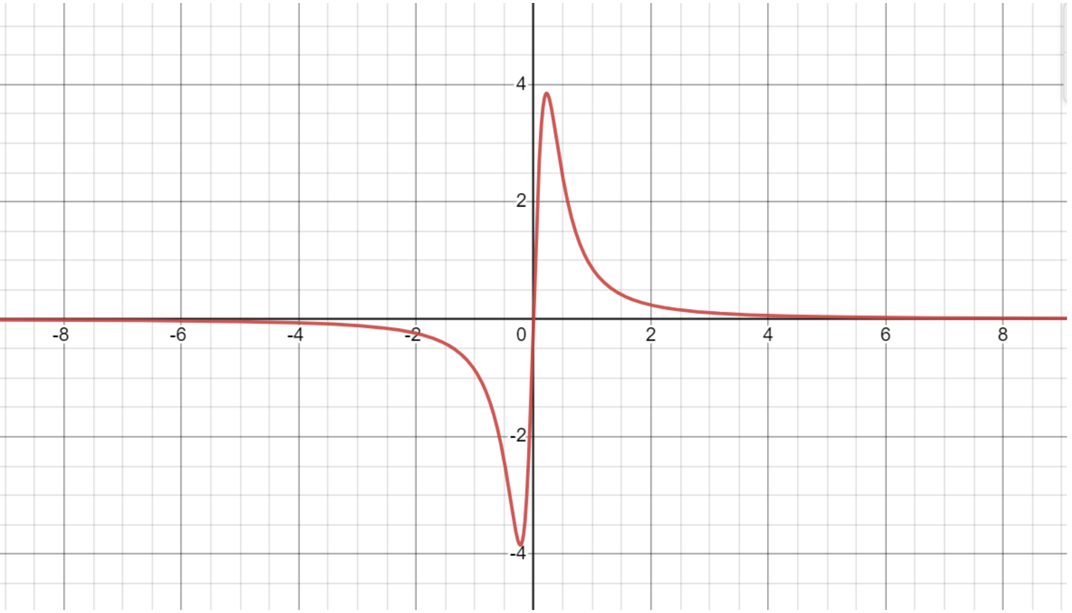
$$dq = \sigma\ dA = \sigma(2\pi r\,dr) = 2\pi\sigma r\,dr$$

$$dE_x = \frac{k_e x}{(r^2 + x^2)^{3/2}}(2\pi\sigma r\,dr)$$

$$E_x = k_e x\pi\sigma\int_0^R \frac{2r\,dr}{(r^2 + x^2)^{3/2}}$$

$$= k_e x\pi\sigma\int_0^R (r^2 + x^2)^{-3/2} d(r^2)$$

$$= k_e x\pi\sigma\left[\frac{(r^2 + x^2)^{-1/2}}{-1/2}\right]_0^R = \boxed{2\pi k_e\sigma\left[1 - \frac{x}{(R^2 + x^2)^{1/2}}\right]}$$



Field of a charged disk

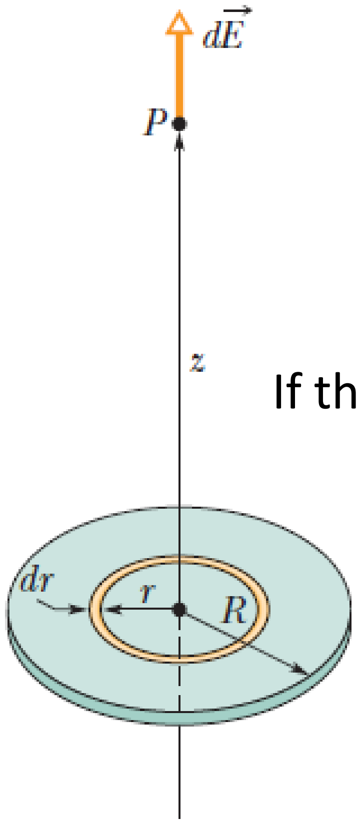


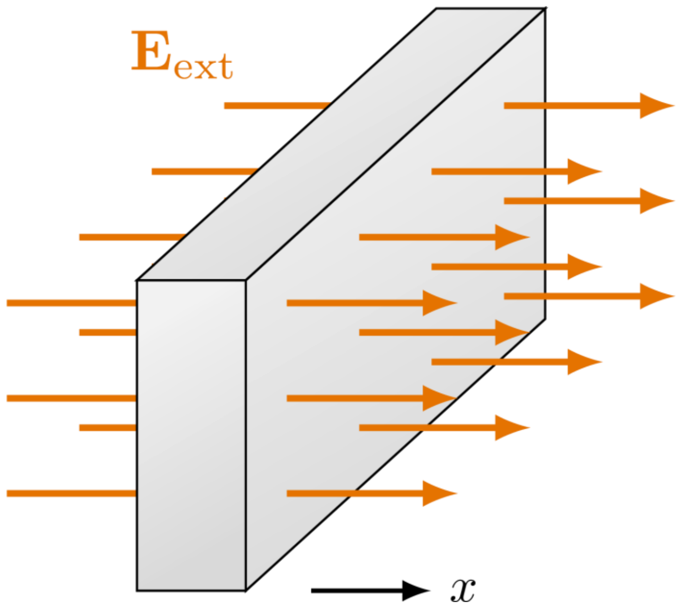
Figure 22-15 A disk of radius R and uniform positive charge. The ring shown has radius r and radial width dr. It sets up a differential electric field $d\vec{E}$ at point P on its central axis.

$$E = \frac{\sigma}{2\epsilon_0}\left[1 - \frac{z}{\sqrt{z^2 + R^2}}\right]$$

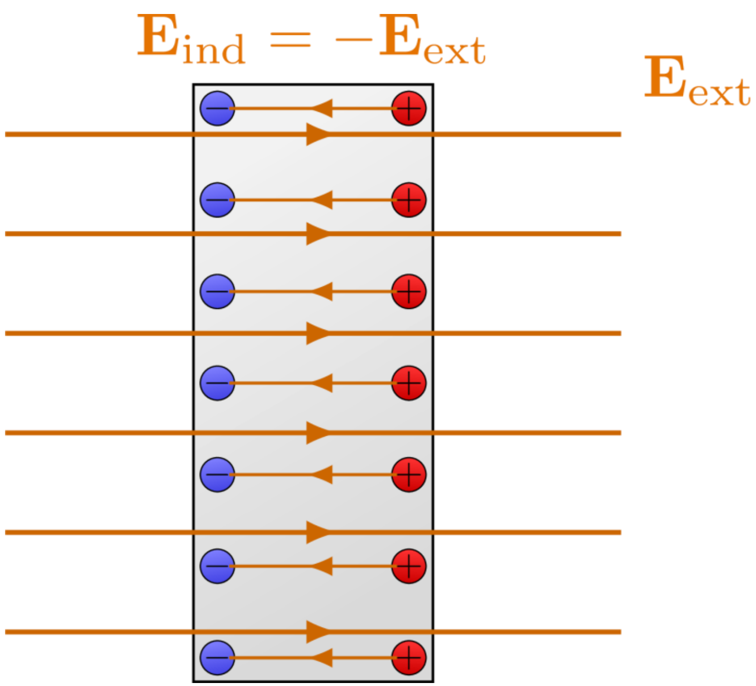
If the disk is very large (or if we are very close to it), $R \gg z$

$$E = \frac{\sigma}{2\varepsilon_0} \quad \text{(infinite sheet).}$$

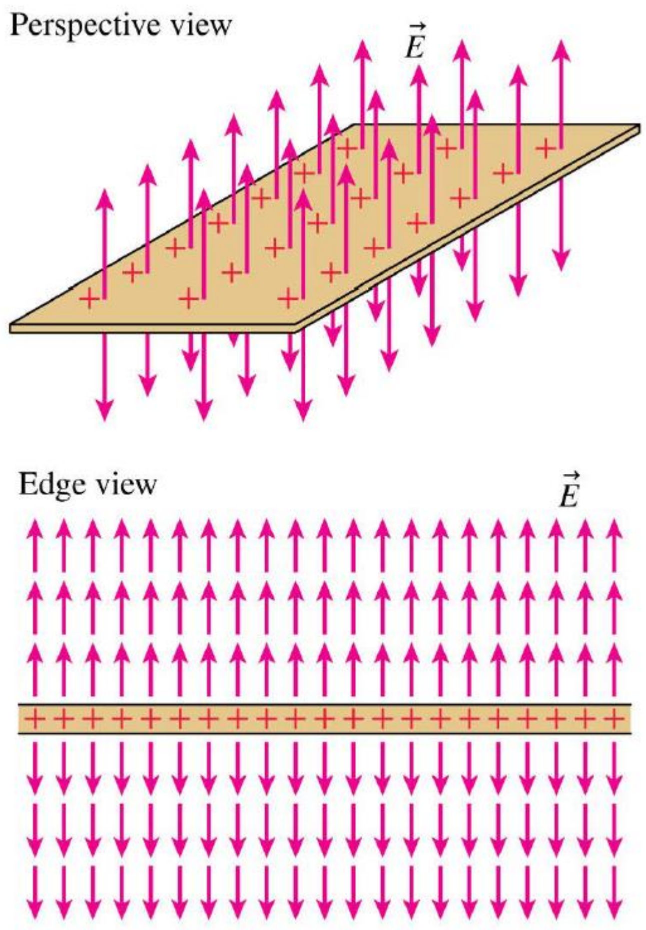
Here we can see that, the electric field is independent of size and dimension of the charged surface; moreover, E is independent of distance where we measure it.



$\mathbf{E}_{\text{ext}}$
x

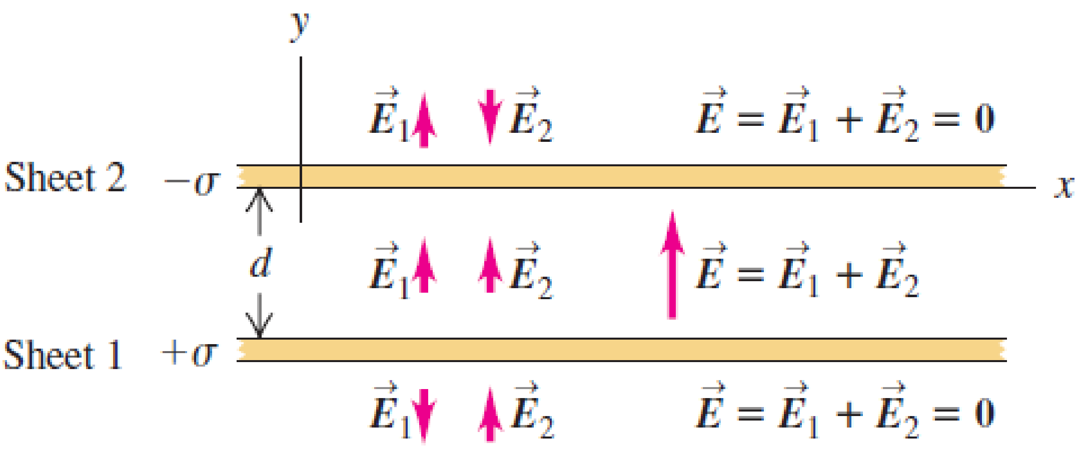


$\mathbf{E}_{\text{ind}} = -\mathbf{E}_{\text{ext}}$
$\mathbf{E}_{\text{ext}}$

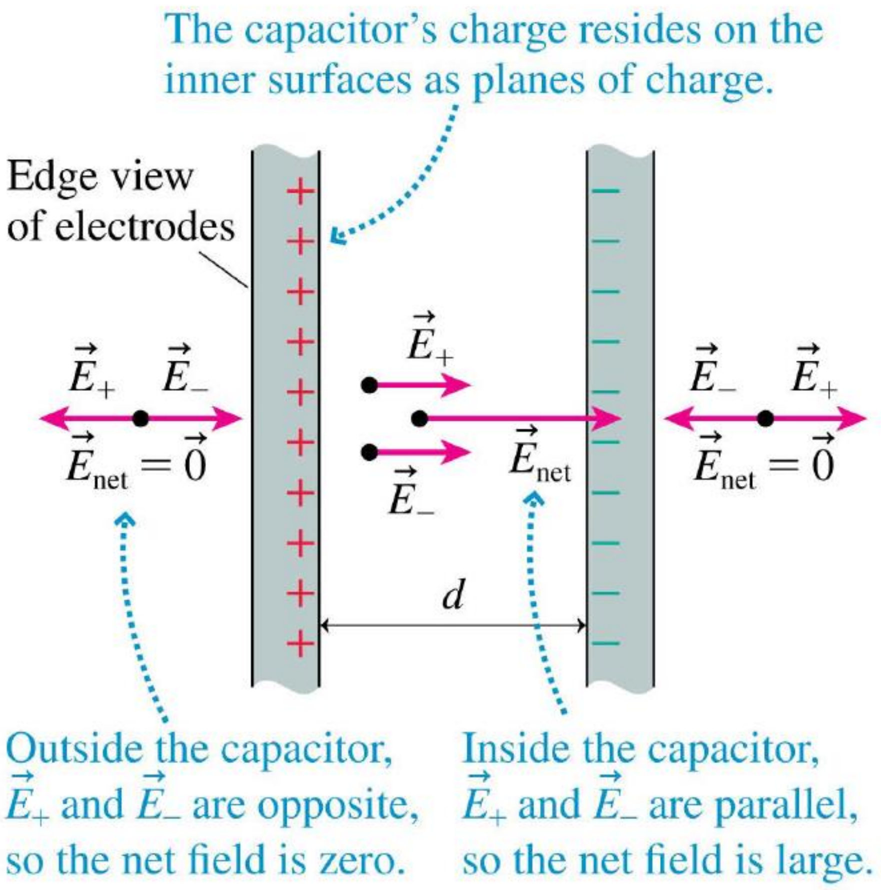


Perspective view
$\vec{E}$
Edge view
$\vec{E}$

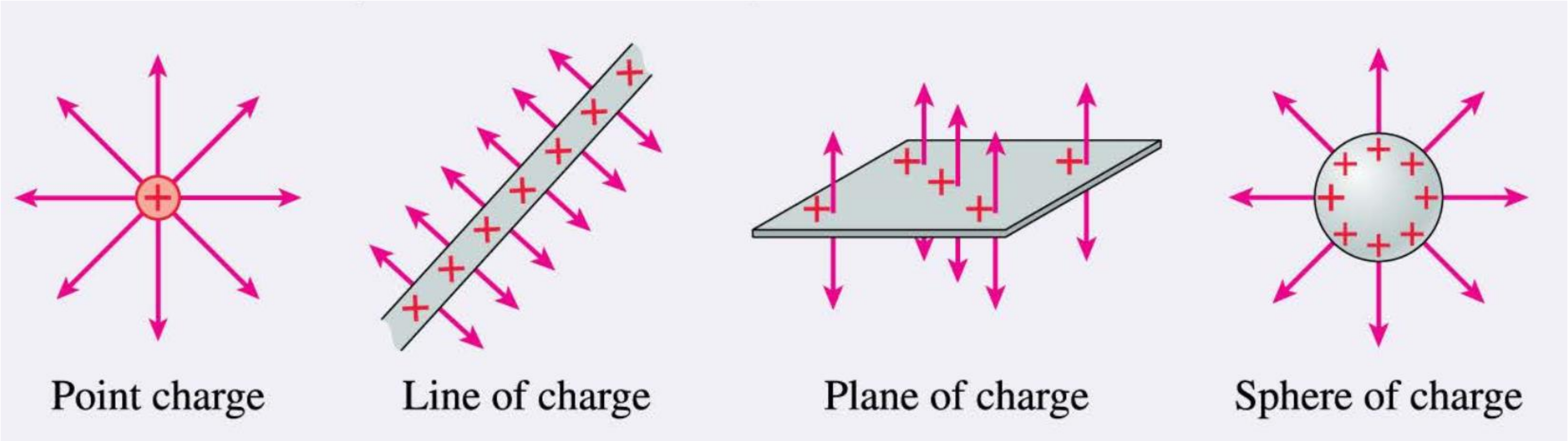
Field of two oppositely charged infinite sheets



$$\vec{E} = \vec{E}_1 + \vec{E}_2 = \begin{cases} 0 & \text{above the upper sheet} \\ \dfrac{\sigma}{\epsilon_0}\hat{j} & \text{between the sheets} \\ 0 & \text{below the lower sheet} \end{cases}$$



Electric Field due to different charge configurations



In Figure 22-13, an infinite plane of surface charge density $\sigma = +4.5\ \text{nC/m}^2$ lies in the $x = 0.00$ m plane, and a second infinite plane of surface charge density $\sigma = -4.50\ \text{nC/m}^2$ lies in the $x = 2.00$ m plane. Find the electric field at (*a*) $x = 1.80$ m and (*b*) $x = 5.00$ m.

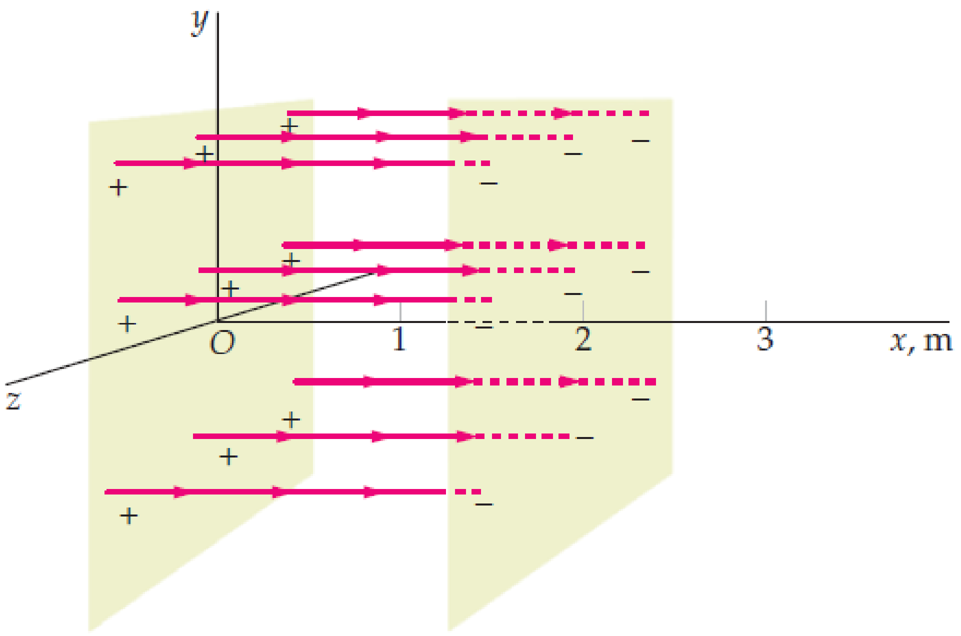


FIGURE 22-13

PICTURE Each charged plane produces a uniform electric field of magnitude $E = \sigma/(2\epsilon_0)$. We use superposition to find the resultant field. Between the planes the fields add, producing a net field of magnitude σ/ϵ_0 in the $+x$ direction. For $x > 2.00$ m and for $x < 0$, the two fields point in opposite directions and thus sum to zero.

SOLVE

(*a*) 1. Calculate the magnitude of the field E produced by each plane:

$$E = |\sigma|/(2\epsilon_0)$$
$$= (4.50 \times 10^{-9}\ \text{N/C})/(2 \cdot 8.85 \times 10^{-12})$$
$$= 254\ \text{N/C}$$

2. At $x = 1.80$ m, between the planes, the field due to each plane points in the $+x$ direction:

$$E_{x\text{net}} = E_1 + E_2 = 254\ \text{N/C} + 254\ \text{N/C}$$
$$= \boxed{508\ \text{N/C}}$$

(*b*) At $x = 5.00$ m, the fields due to the two planes are oppositely directed:

$$E_{x\text{net}} = E_1 - E_2 = \boxed{0.00\ \text{N/C}}$$

Summary Topic

- ❑ Draw an electric dipole, identifying the charges (sizes and signs), dipole axis, and direction of the electric dipole moment.

- ❑ Identify the direction of the electric field at any given point along the dipole axis, including between the charges.

- ❑ Outline how the equation for the electric field due to an electric dipole is derived from the equations for the electric field due to the individual charged particles that form the dipole.

- ❑ The equation for the electric field set up by a particle does not apply to an extended object with charge (said to have a continuous charge distribution).

- ❑ For a charged particle placed in an external electric field (a field due to other charged objects), apply the relationship between the electric field at that point, the particle's charge q, and the electrostatic force that acts on the particle, and identify the relative directions of the force and the field when the particle is positively charged and negatively charged.