Current and Resistance-II

Phy 108 course
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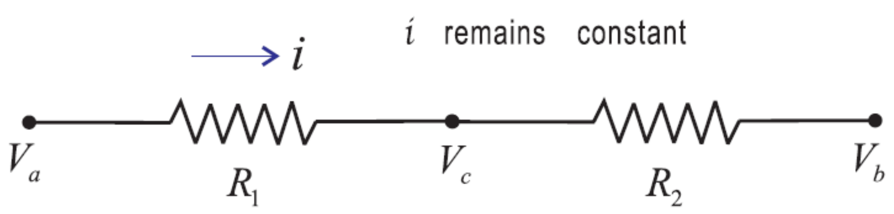
DMP, SEPS, NSU

Resistance in combination

Resistances in series



Potential differece (P.D.)

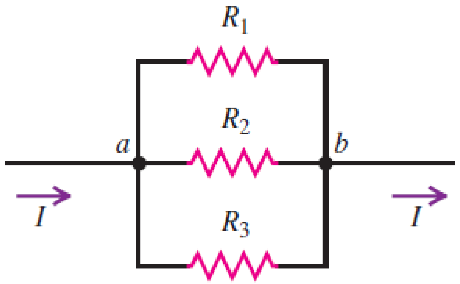
$$\begin{aligned} V_a - V_b &= (V_a - V_c) + (V_c - V_b) \\ &= iR_1 + iR_2 \end{aligned}$$

$\therefore$ Equivalent Resistance

$$R = R_1 + R_2 \qquad \text{for resistors in series}$$

The equivalent resistance of *any number* of resistors in series equals the sum of their individual resistances.

Resistances in parallel



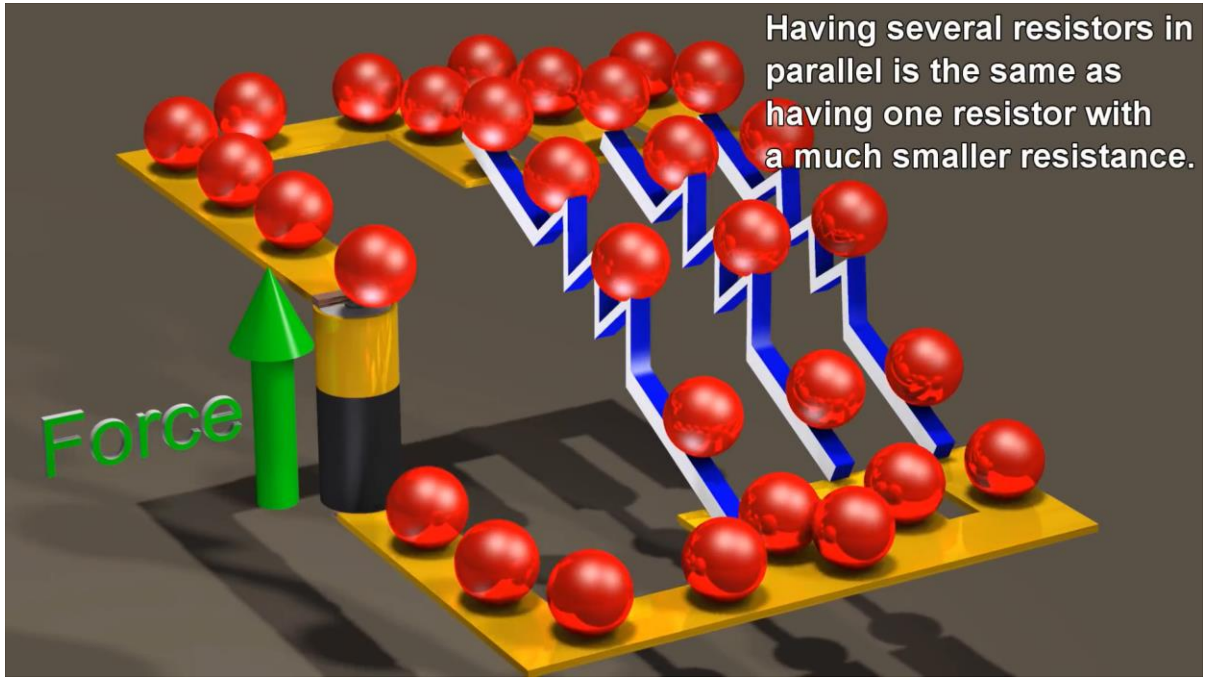
$$I_1 = \frac{V_{ab}}{R_1} \qquad I_2 = \frac{V_{ab}}{R_2} \qquad I_3 = \frac{V_{ab}}{R_3}$$

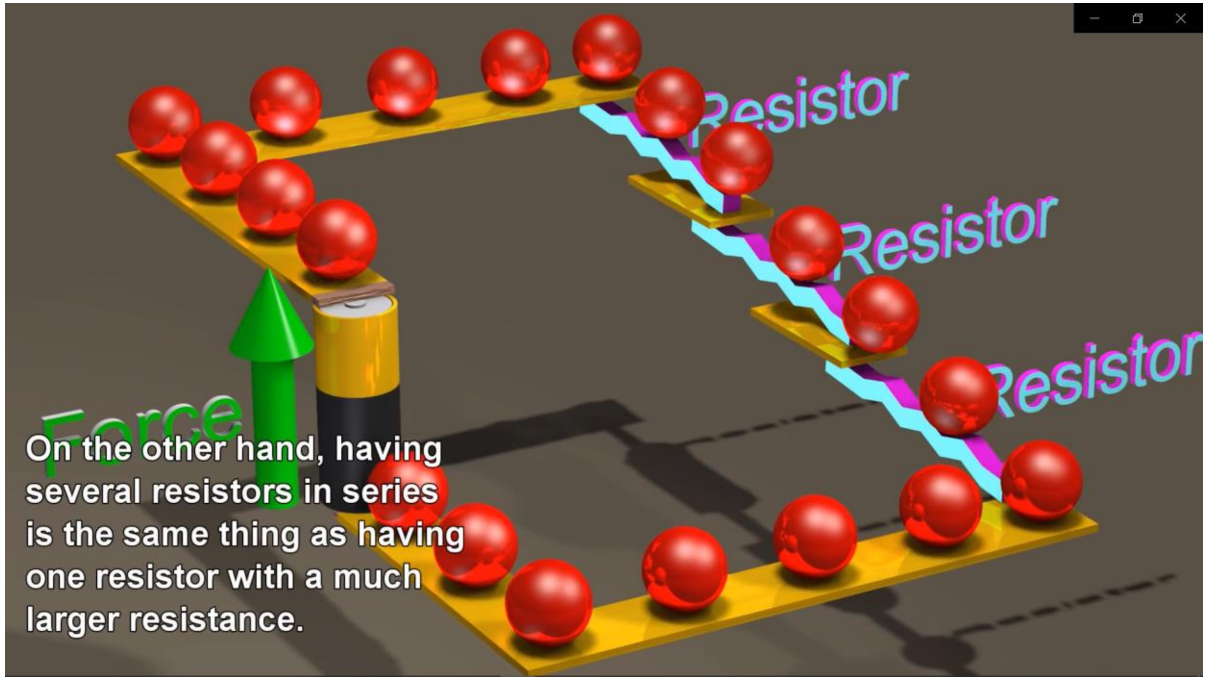
$$I = I_1 + I_2 + I_3 = V_{ab}\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right) \quad \text{or}$$

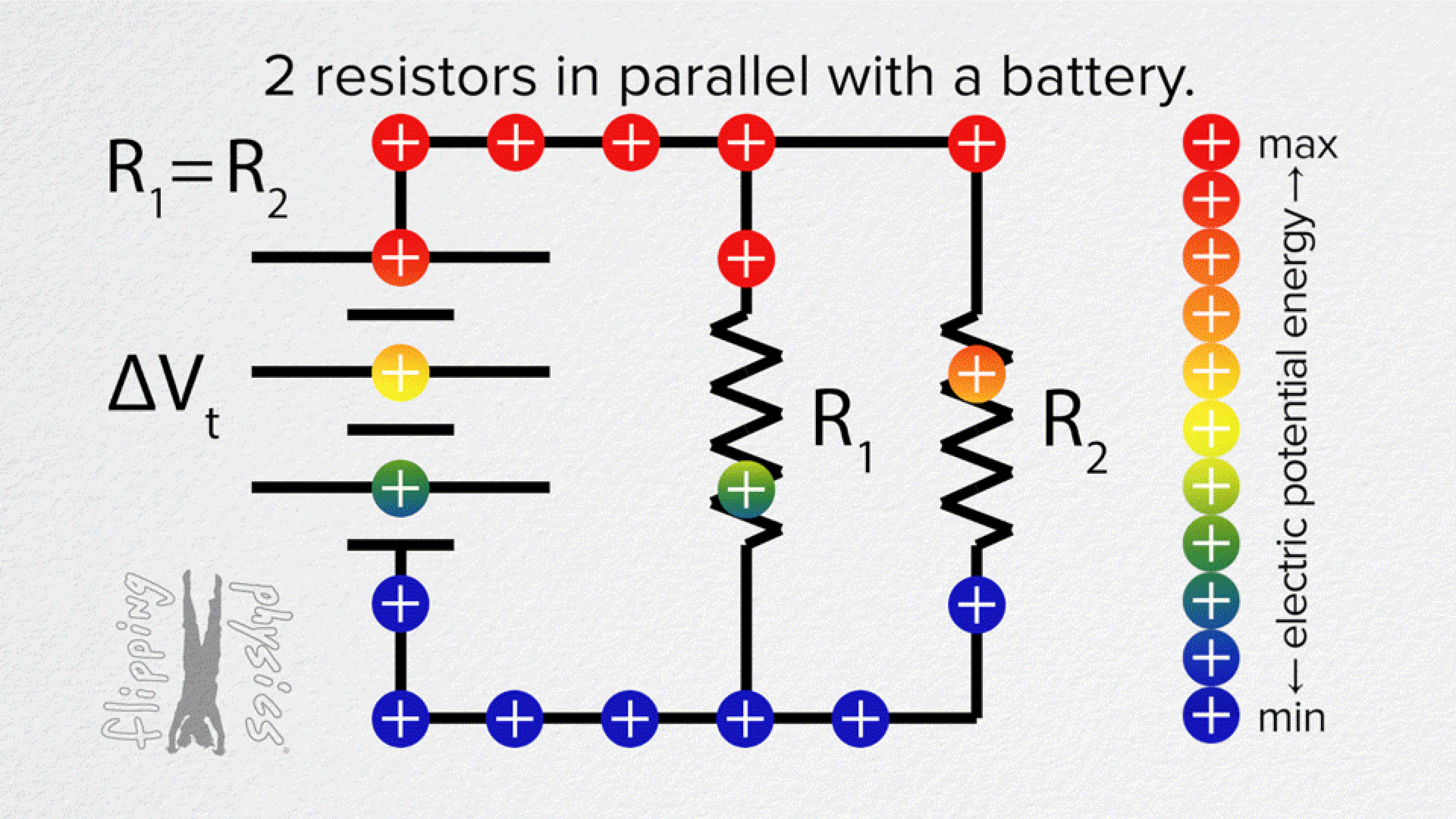
$$\frac{I}{V_{ab}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} \qquad I/V_{ab} = 1/R_{eq}$$

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

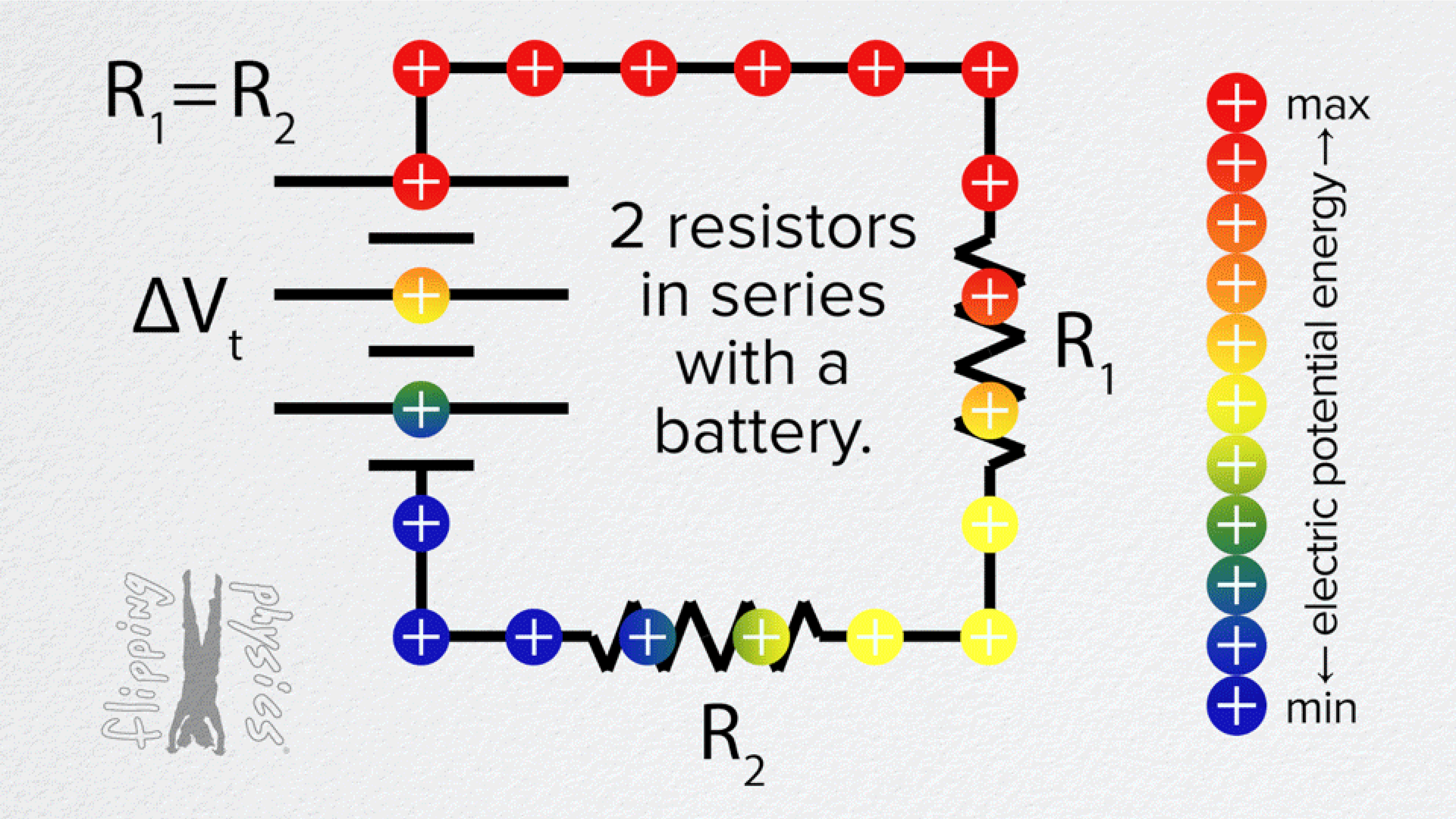
For *any number* of resistors in parallel, the *reciprocal* of the equivalent resistance equals the *sum of the reciprocals* of their individual resistances.


Having several resistors in parallel is the same as having one resistor with a much smaller resistance.
Force


Resistor
Resistor
Resistor
Force
On the other hand, having several resistors in series is the same thing as having one resistor with a much larger resistance.



2 resistors in parallel with a battery.
R1=R2
ΔVt
R1
R2
max
← electric potential energy →
min
flipping physics

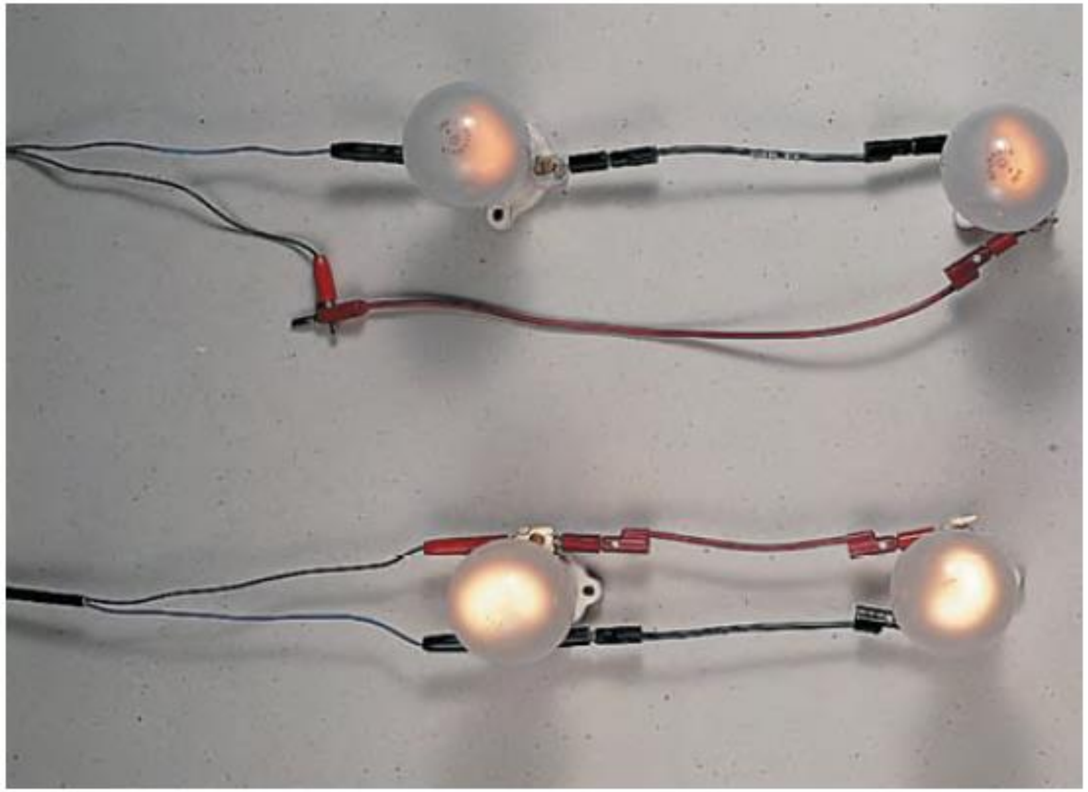


$R_1 = R_2$
ΔV_t
2 resistors in series with a battery.
R_1
R_2
max
min
← electric potential energy →
flipping physics

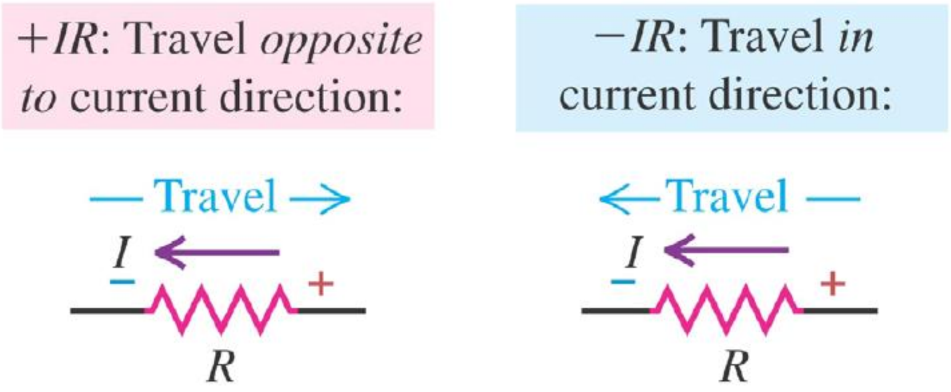
Resistance in combination

26.2 A car's headlights and taillights are connected in parallel. Hence each light is exposed to the full potential difference supplied by the car's electrical system, giving maximum brightness. Another advantage is that if one headlight or tail-light burns out, the other one keeps shining (see Example 26.2).

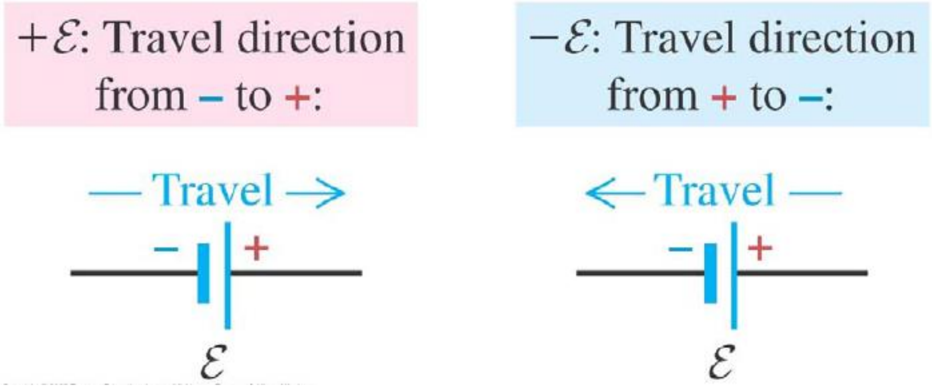
26.5 When connected to the same source, two light bulbs in series (shown at top) draw less power and glow less brightly than when they are in parallel (shown at bottom).



RESISTANCE RULE:
For a move through a resistance in the direction of the current, the change in potential is $-iR$; in the opposite direction it is $+iR$.



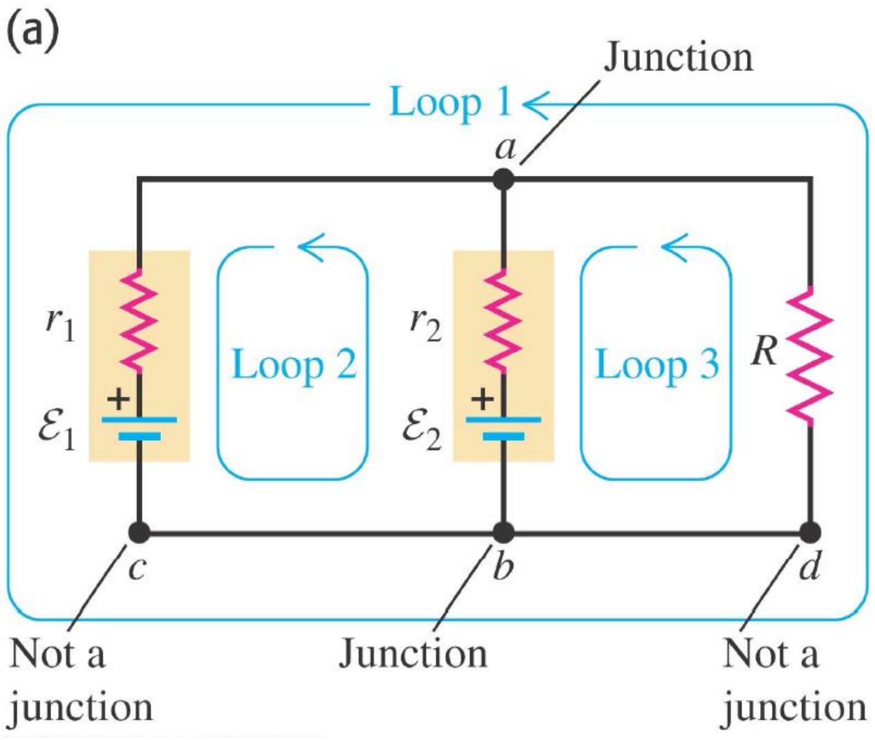
EMF RULE:
For a move through an ideal emf device in the direction of the emf arrow, the change in potential is $+\mathcal{E}$; in the opposite direction it is $-\mathcal{E}$.

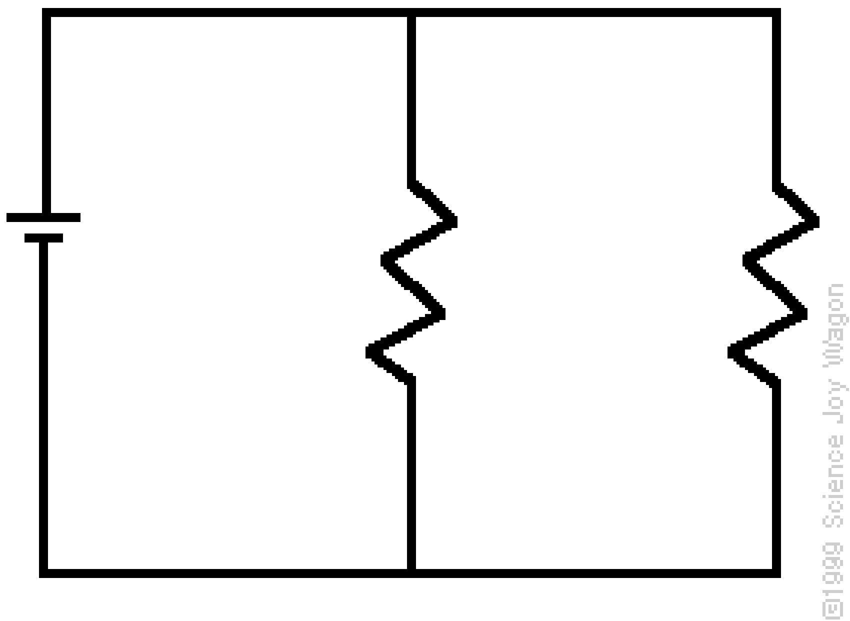


KIRCHOFF'S LAWS:

LOOP RULE:

The algebraic sum of the changes in potential encountered in a complete traversal of any loop of a circuit must be zero.
This is often referred to as *Kirchhoff's loop rule* (or *Kirchhoff's voltage law*), after German physicist Gustav Robert Kirchhoff. Energy conservation law





LOOP RULE:

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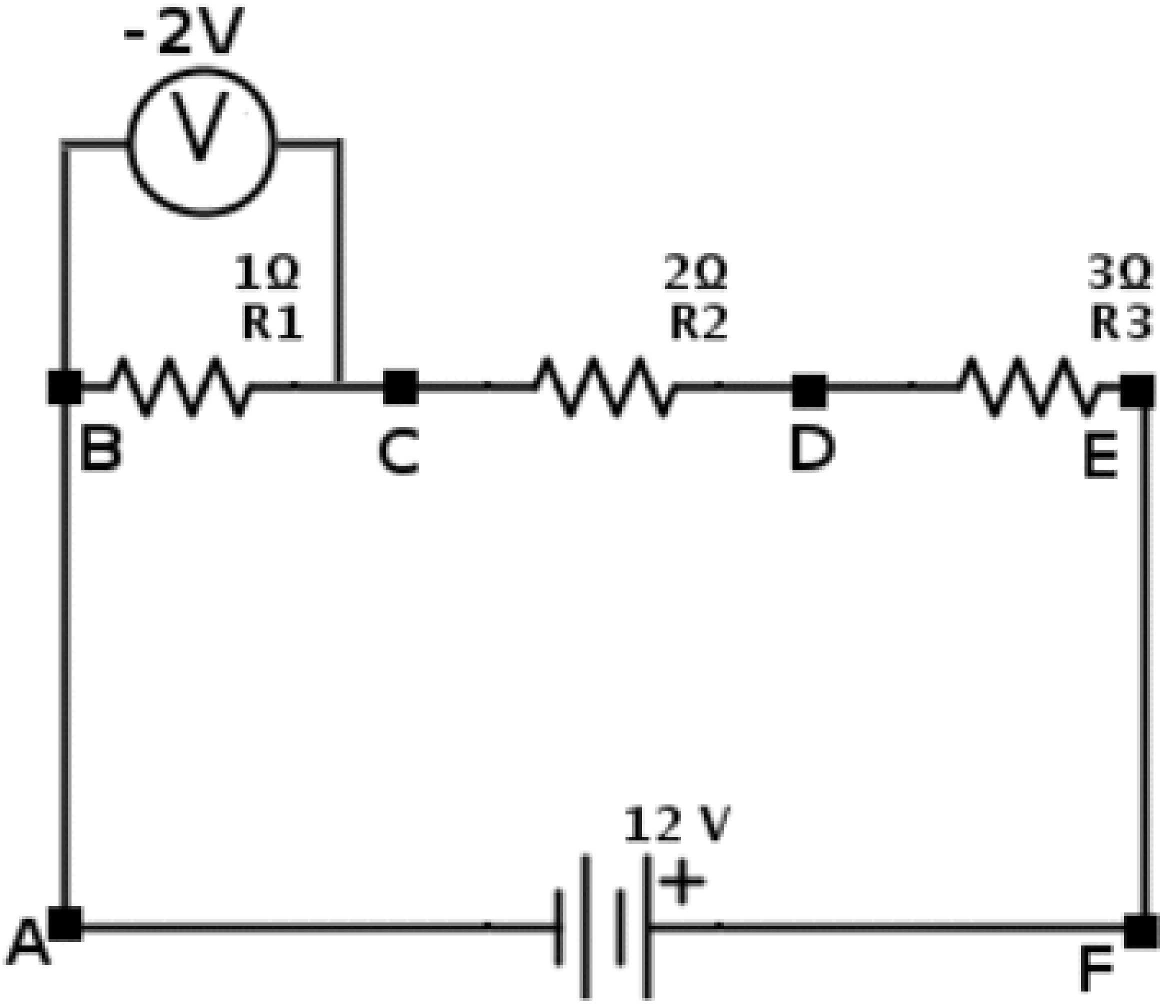


Energy Conservation

$$\sum V_i = 0$$

Kirchhoff's Law

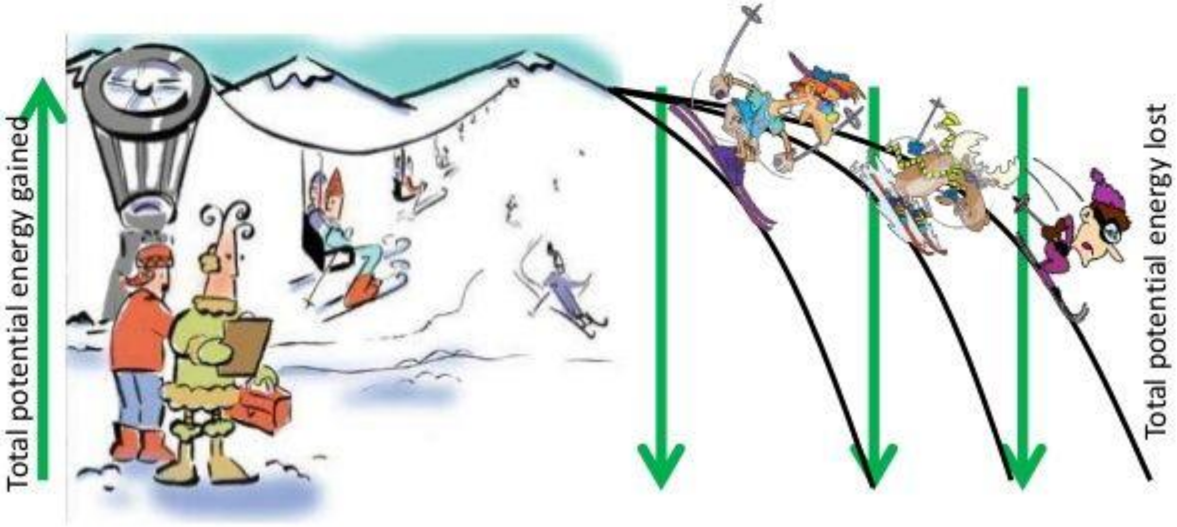
Kirchhoff's Voltage Law: In a series circuit, the total voltage supplied by the source must equal the total voltage used by all of the loads in that path.

Analogy:

Kirchhoff's Law

Kirchhoff's Voltage Law: In a parallel circuit, the total voltage supplied by the source is equal to the voltage across each parallel branch.

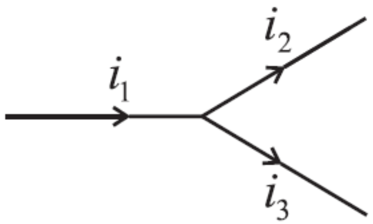
Analogy:



JUNCTION RULE:

The sum of the currents entering any junction must be equal to the sum of the currents leaving that junction.
This rule is often called Kirchhoff's junction rule (or Kirchhoff's current law). It is simply a statement of the conservation of charge for a steady flow of charge—there is neither a buildup nor a depletion of charge at a junction.

Conservation of charge



$$\Rightarrow i_1 = i_2 + i_3$$

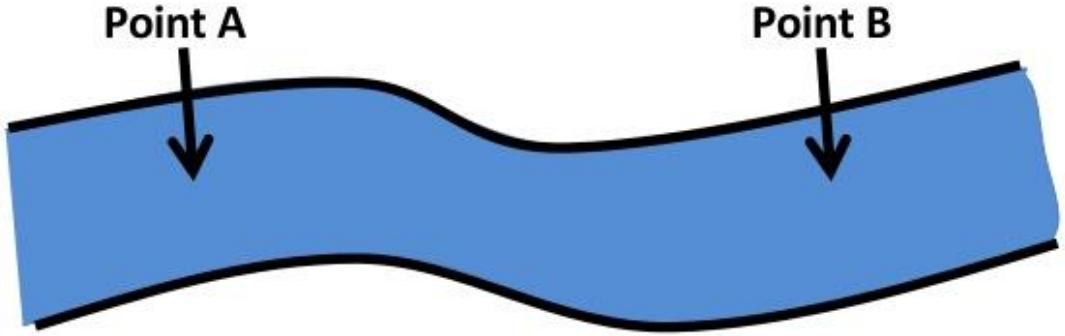
Charge Conservation

$$\sum I_j = 0$$

Kirchhoff's Law

Kirchhoff's Current Law: In a series circuit, the total current flowing through the circuit is identical at any point in the circuit.

Analogy:

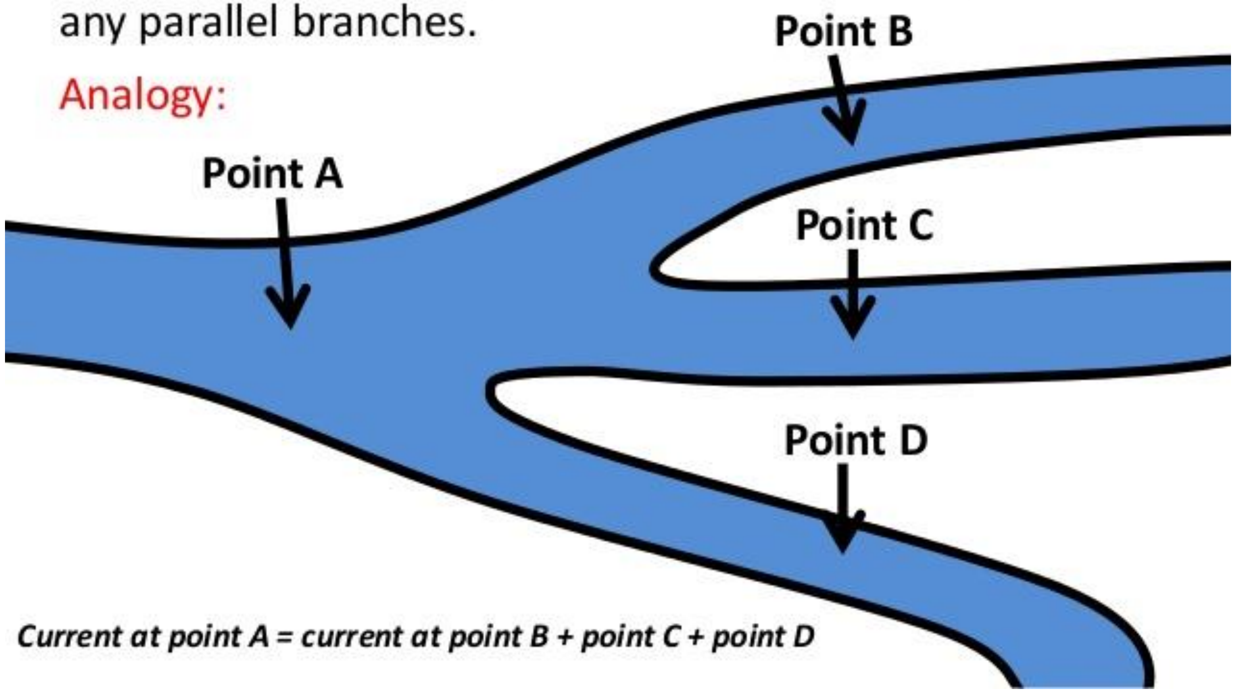


Same current

Kirchhoff's Law

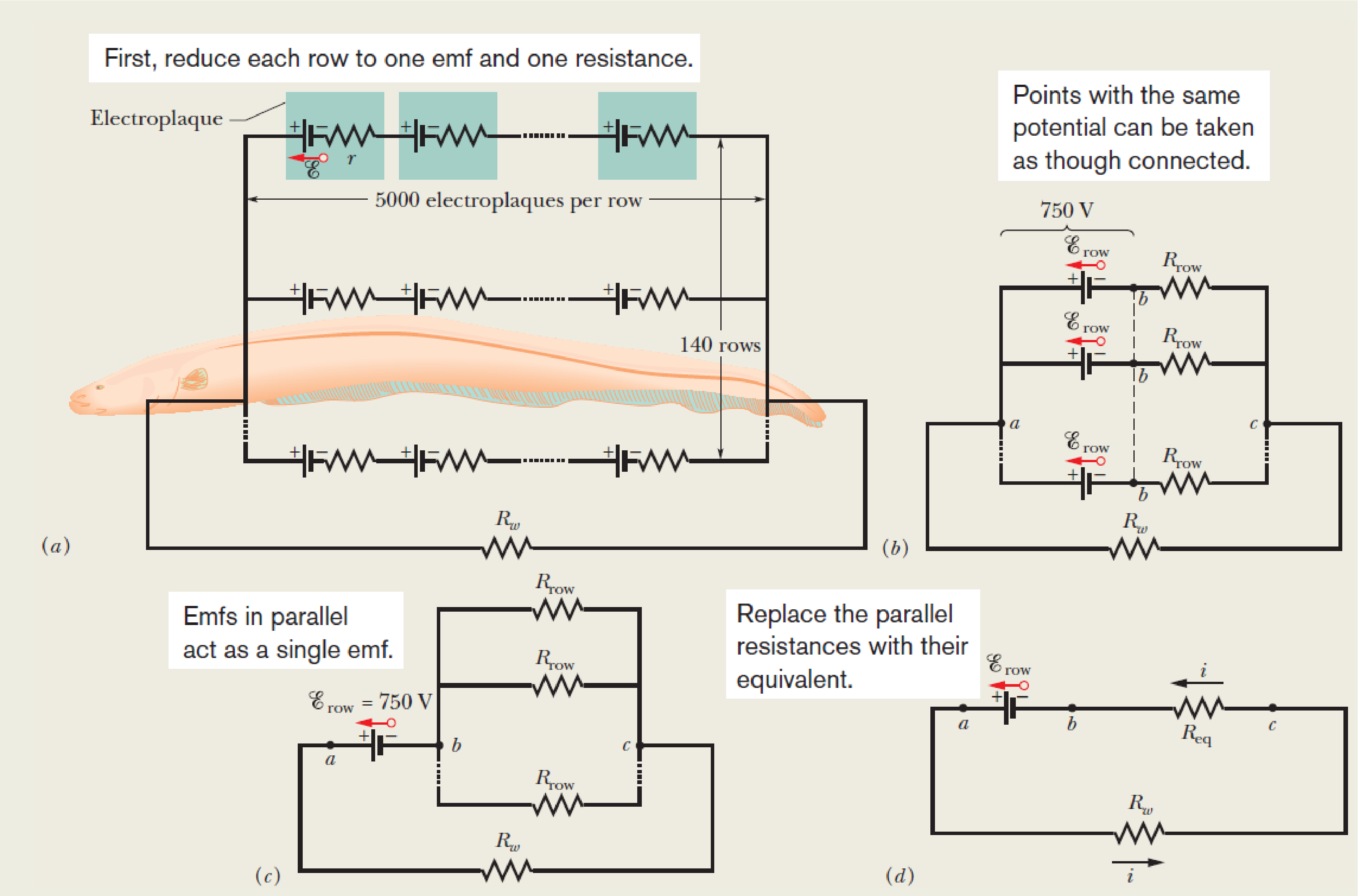
Kirchhoff's Current Law: In a parallel circuit, the total current flowing through the circuit is divided up among any parallel branches.

Analogy:



Current at point A = current at point B + point C + point D

Electric fish can generate current with biological emf cells called *electroplaques*. In the South American eel they are arranged in 140 rows, each row stretching horizontally along the body and each containing 5000 cells. Each electroplaque has an emf of 0.15 V and an internal resistance *r* of 0.25Ω.The water surrounding the eel completes a circuit between the two ends of the electroplaque array, one end at the head of the animal and the other near the tail.



Application **Danger: Electric Ray!**

Electric rays deliver electric shocks to stun their prey and to discourage predators. (In ancient Rome, physicians practiced a primitive form of electroconvulsive therapy by placing electric rays on their patients to cure headaches and gout.) The shocks are produced by specialized flattened cells called electroplaques. Such a cell moves ions across membranes to produce an emf of about 0.05 V. Thousands of electroplaques are stacked on top of each other, so their emfs add to a total of as much as 200 V. These stacks make up more than half of an electric ray's body mass. A ray can use these to deliver an impressive current of up to 30 A for a few milliseconds.

https://www.youtube.com/watch?v=FS-tmBD9Cjk

https://www.youtube.com/watch?v=PHqdT0B2B68&t=21s

Calculating the Current

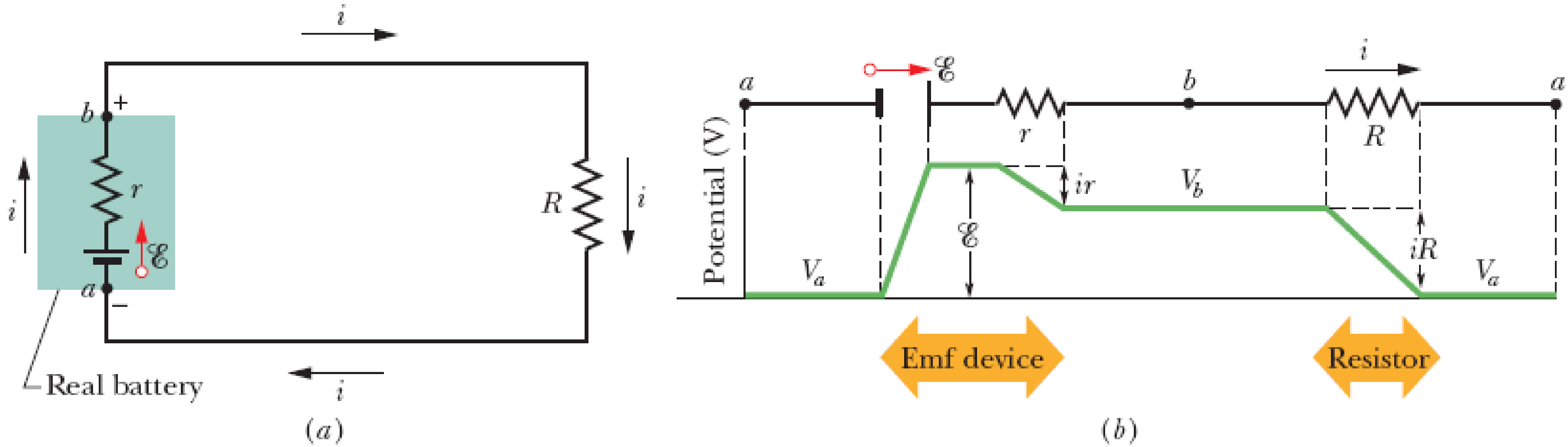
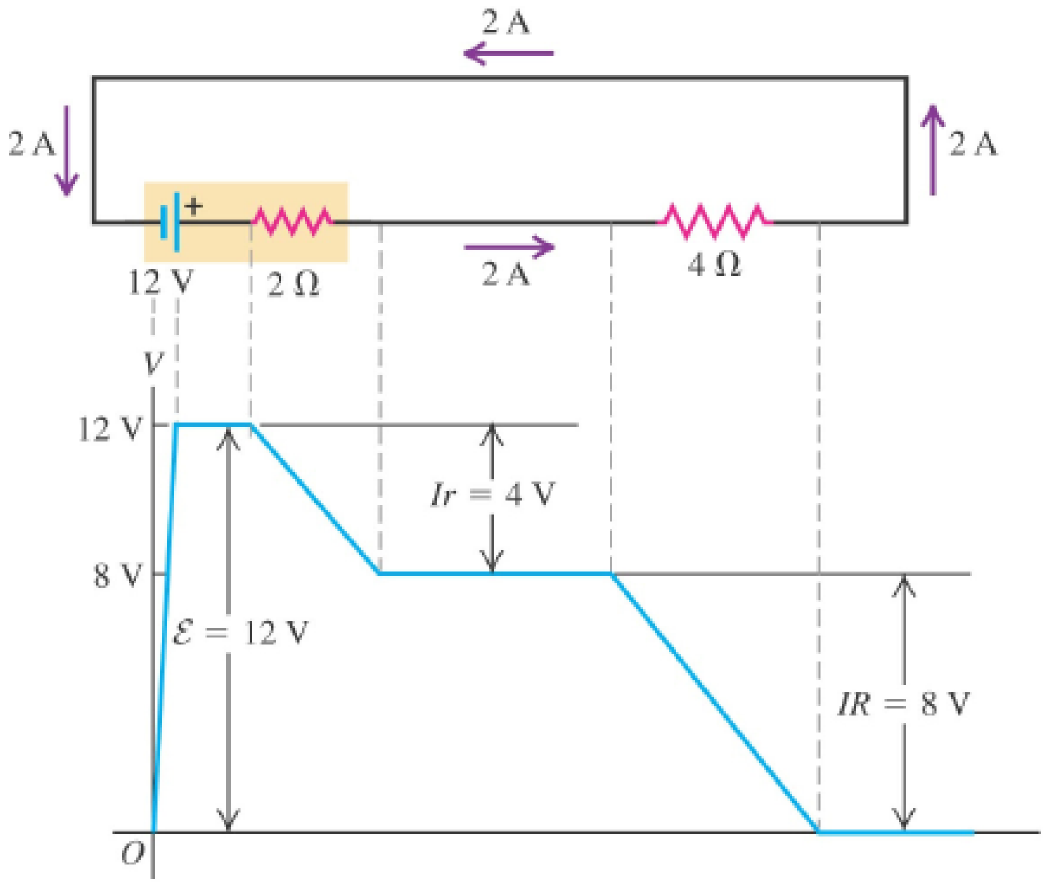


Figure 27-4 (*a*) A single-loop circuit containing a real battery having internal resistance r and emf $\mathscr{E}$. (*b*) The same circuit, now spread out in a line. The potentials encountered in traversing the circuit clockwise from a are also shown. The potential V_a is arbitrarily assigned a value of zero, and other potentials in the circuit are graphed relative to V_a.

Potential changes around a circuit

- The net change in potential energy for a charge *q* making a round trip around a complete circuit must be zero.

- Local differences in potential occur.



https://phet.colorado.edu/en/simulations/capacitor-lab-basics

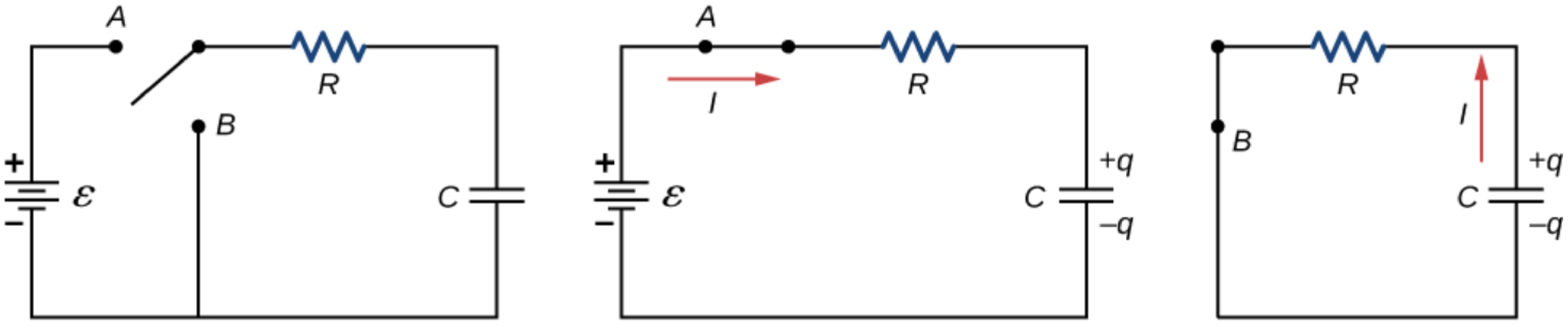
https://phet.colorado.edu/sims/html/ohms-law/latest/ohms-law_en.html

https://phet.colorado.edu/sims/html/resistance-in-a-wire/latest/resistance-in-a-wire_en.html

https://phet.colorado.edu/sims/html/circuit-construction-kit-dc/latest/circuit-construction-kit-dc_en.html

https://phet.colorado.edu/sims/html/circuit-construction-kit-dc-virtual-lab/latest/circuit-construction-kit-dc-virtual-lab_en.html

RC circuits:

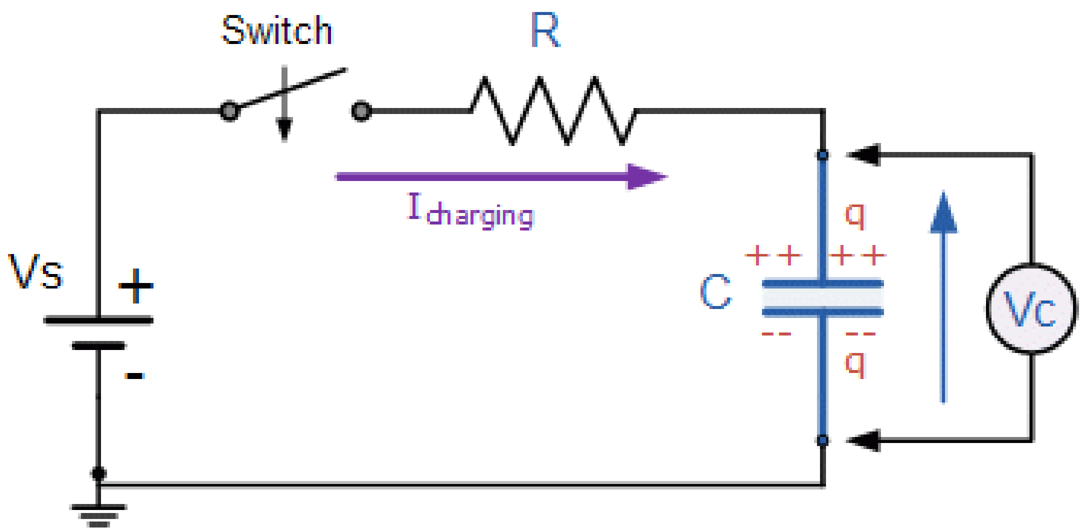


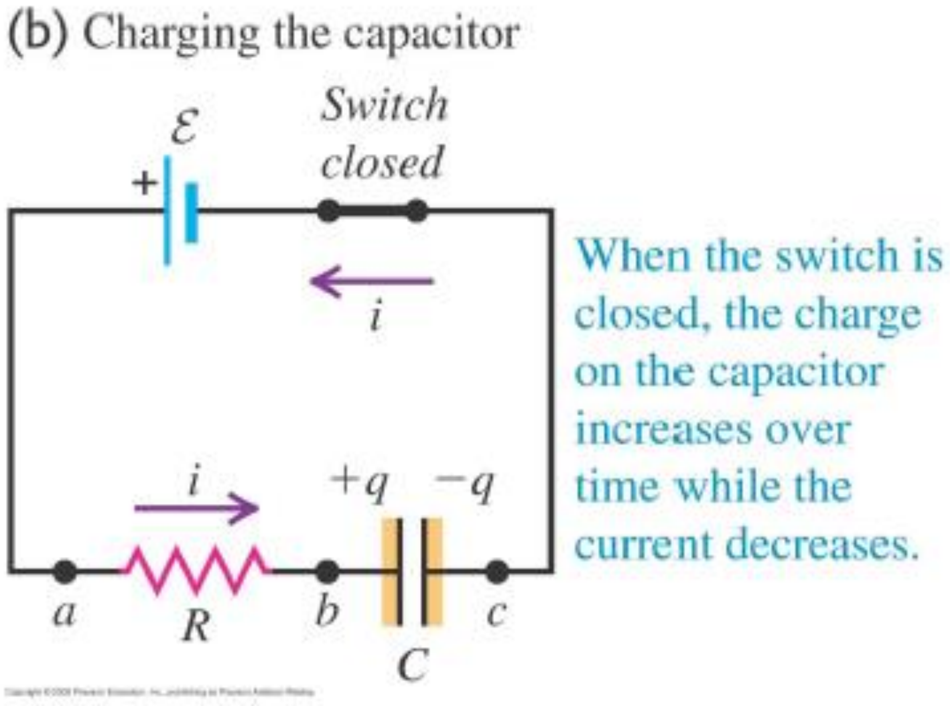
(a) Original circuit (b) Charging capacitor (b) Discharging capacitor

RC circuits:

Then close switch.

Charging Capacitor:





https://www.geogebra.org/m/rFEV4HJx

Apply Kirchhoff's Loop Rule.

Across resistor potential change IR

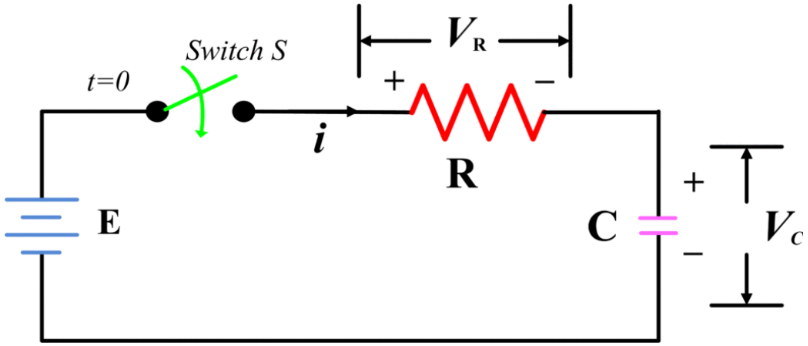
Across battery potential change ε

Across capacitor potential change $\frac{Q}{C}$

Charging Capacitor:

Start at a.

$$-\varepsilon + \frac{Q}{C} + iR = 0$$



The current is $i = \frac{dQ}{dt}$

Put in equation.

$$-\varepsilon + \frac{Q}{C} + \frac{dQ}{dt}R = 0$$

$$\varepsilon = \frac{Q}{C} + R\frac{dQ}{dt}$$

$$\frac{dQ}{dt} = \frac{\varepsilon - \frac{Q}{C}}{R} = \frac{C\varepsilon - Q}{RC}$$

$$\frac{dQ}{C\varepsilon - Q} = \frac{dt}{RC}$$

The charge on the capacitor goes from 0 to Q. Start the stopwatch when the switch is closed and time goes from 0 to t.

Integrate

$$\int_0^Q \frac{dQ}{C\varepsilon - Q} = \frac{1}{RC}\int_0^t dt$$

Charging Capacitor:

$$ln\frac{C\varepsilon - Q}{C\varepsilon} = -\frac{t}{RC}$$

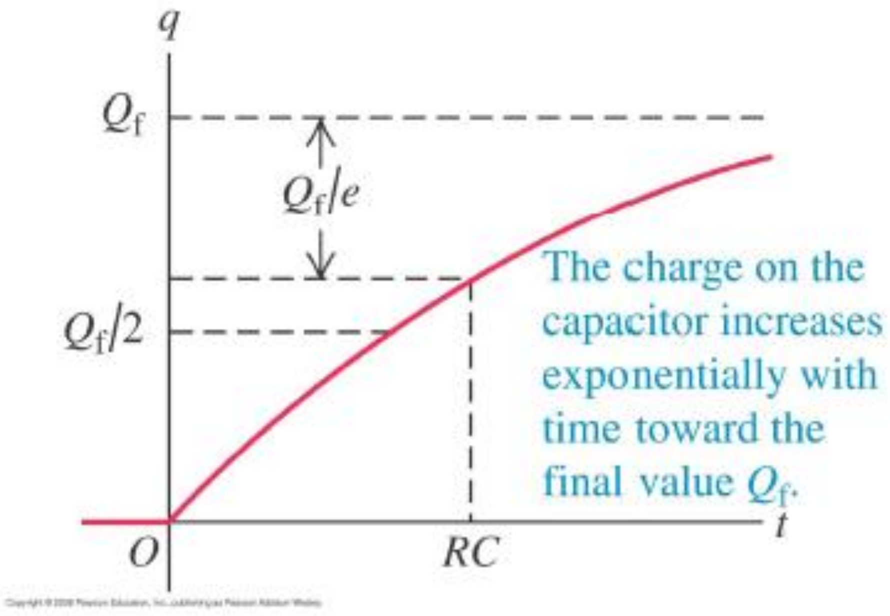
$$1 - \frac{Q}{C\varepsilon} = e^{-\frac{t}{RC}}$$

$$\frac{Q}{C\varepsilon} = 1 - e^{-\frac{t}{RC}}$$

$$Q = C\varepsilon\left(1 - e^{-\frac{t}{RC}}\right)$$

The charge on the capacitor increases exponentially with time.

(b) Graph of capacitor charge versus time for a charging capacitor



$$Q_f = C\varepsilon$$

Charging Capacitor:

Charge on capacitor:

$$Q = Q_f\left(1 - e^{-\frac{t}{RC}}\right)$$

Current in circuit:

$$i = \frac{dQ}{dt} = \frac{d}{dt}\left[Q_f\left(1 - e^{-\frac{t}{RC}}\right)\right]$$

$$i = Q_f\left[0 - \left(-\frac{1}{RC}\right)e^{-\frac{t}{RC}}\right]$$

$$i = \frac{Q_f}{RC}e^{-\frac{t}{RC}}$$

$$Q_f = C\varepsilon$$

Charging Capacitor:

$$i = \frac{\varepsilon}{R} e^{-\frac{t}{RC}}$$

And $\frac{\varepsilon}{R} = i_0$

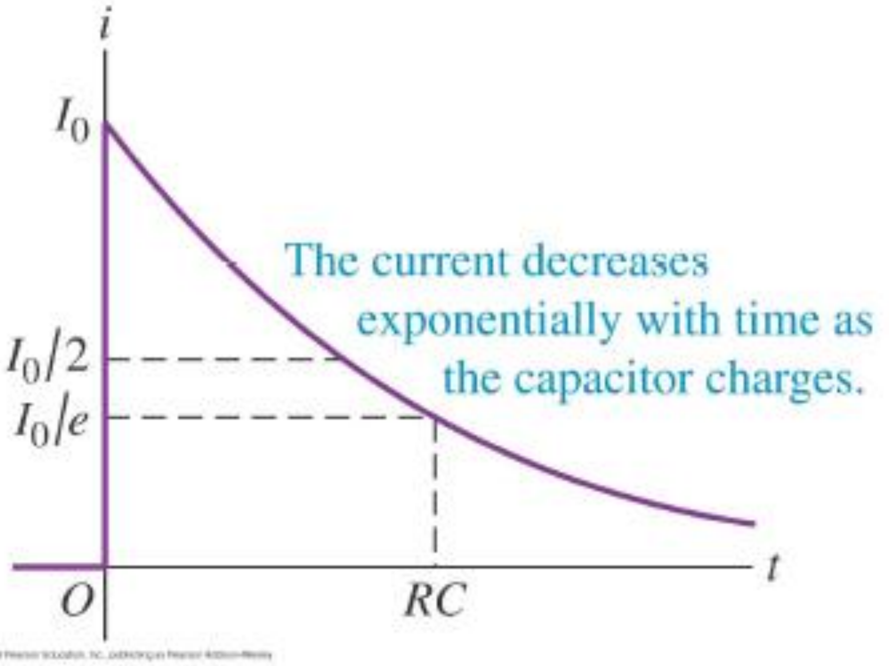
$$i = i_0 e^{-\frac{t}{RC}}$$

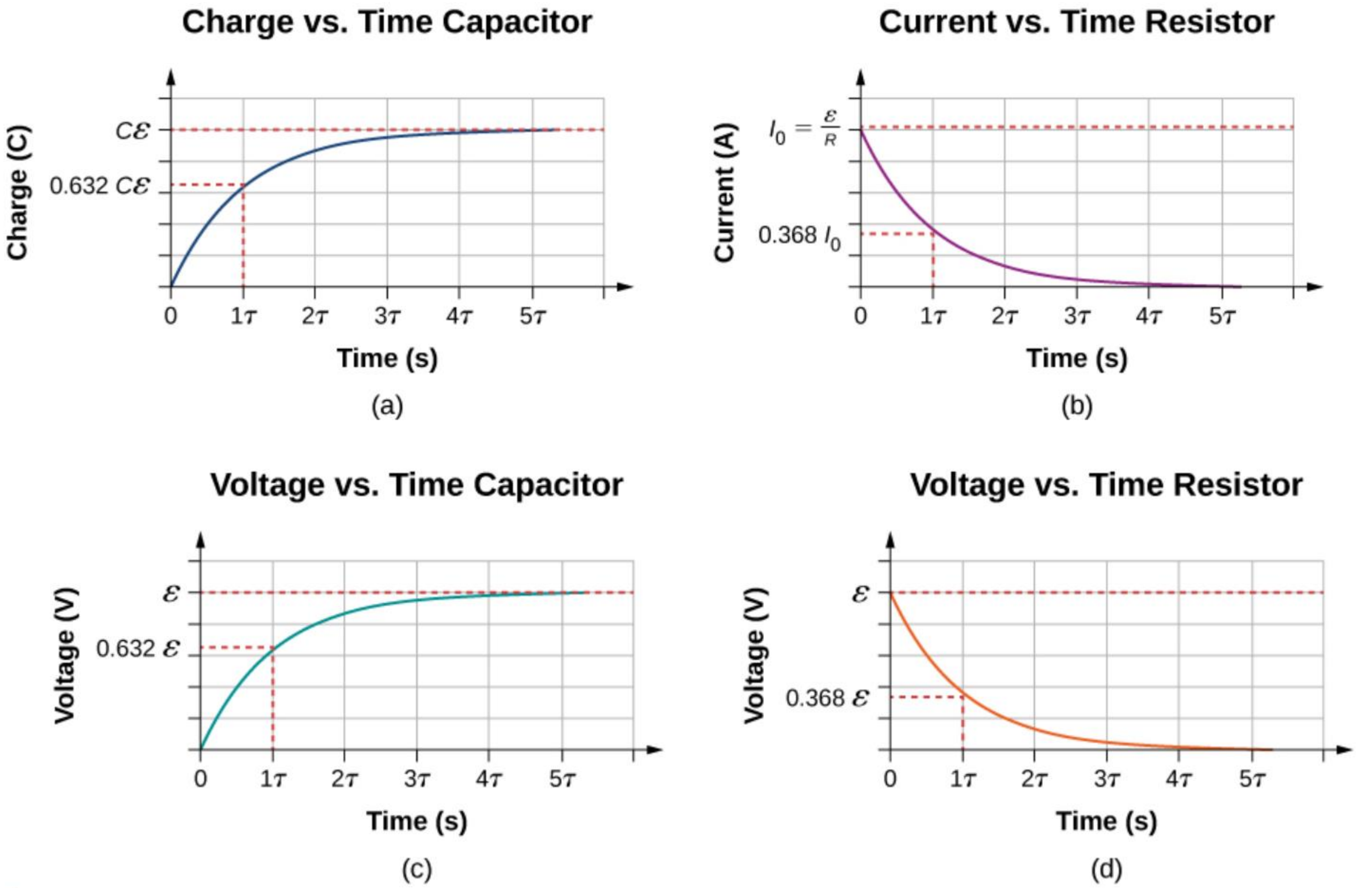
Define $\tau = RC = Circuit\ Time\ Constant$

$$i = i_0 e^{-t/\tau}$$

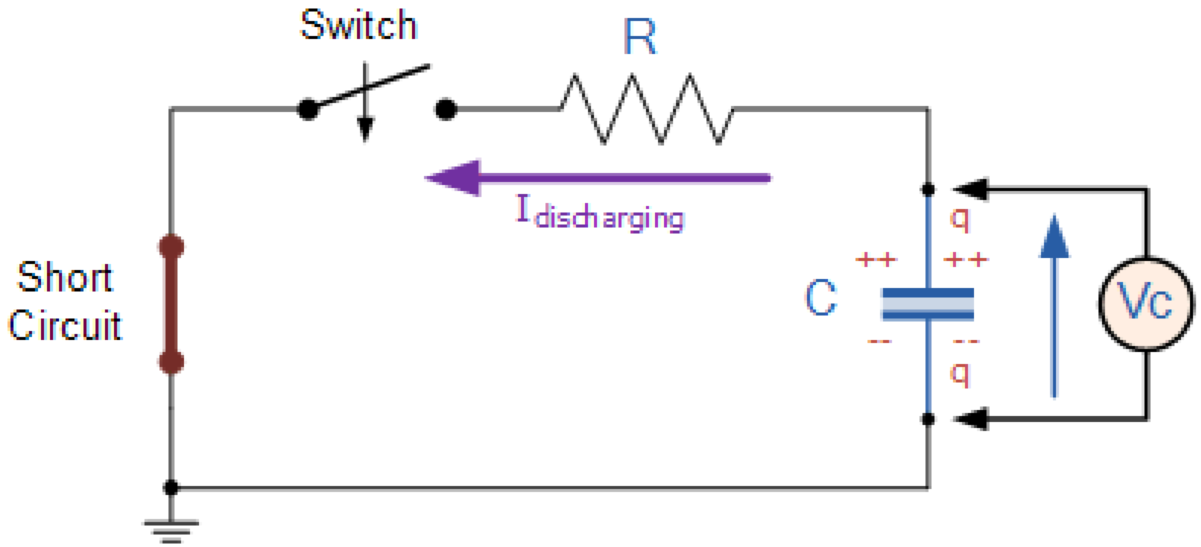
Current starts $i = i_0$ and decreases

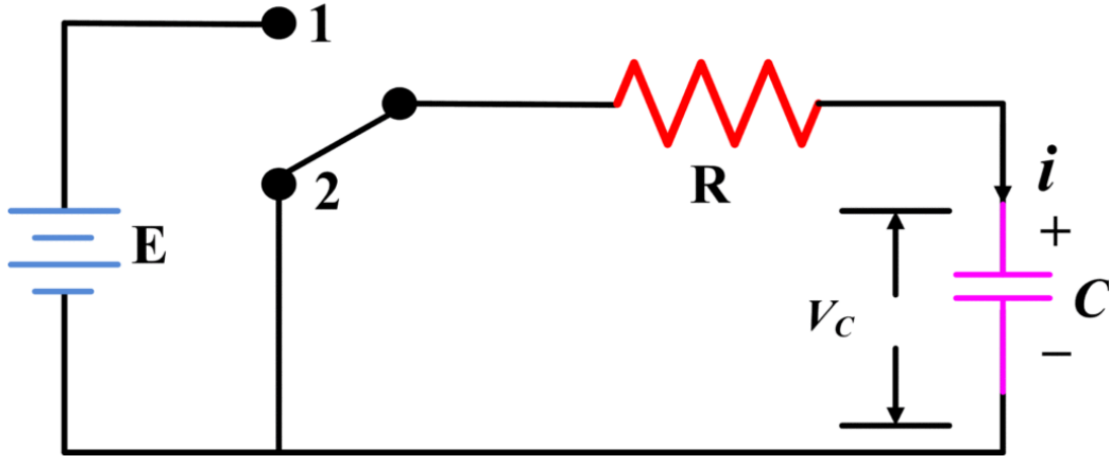
(a) Graph of current versus time for a charging capacitor




Charge vs. Time Capacitor
Charge (C)
$C\mathcal{E}$
$0.632\ C\mathcal{E}$
0
1τ
2τ
3τ
4τ
5τ
Time (s)
(a)
Current vs. Time Resistor
Current (A)
$I_0 = \frac{\mathcal{E}}{R}$
$0.368\ I_0$
Time (s)
(b)
Voltage vs. Time Capacitor
Voltage (V)
$\mathcal{E}$
$0.632\ \mathcal{E}$
Time (s)
(c)
Voltage vs. Time Resistor
Voltage (V)
$\mathcal{E}$
$0.368\ \mathcal{E}$
Time (s)
(d)

Discharging Capacitor:





Now capacitor discharges through the resistor and:

$$IR = V_C = \frac{Q}{C}$$

$$I = -\frac{dQ}{dt}$$

$$-\frac{dQ}{dt}R = \frac{Q}{C}$$

$$\frac{dQ}{Q} = -\frac{1}{RC}dt$$

Integrate from $t = 0\ to\ t = t$ while $O_0\ goes\ to\ Q$.

$$\int_{Q_0}^{Q} \frac{dQ}{Q} = -\frac{1}{RC}\int_0^t dt$$

$$ln\frac{Q}{Q_0} = -\frac{t}{RC}$$

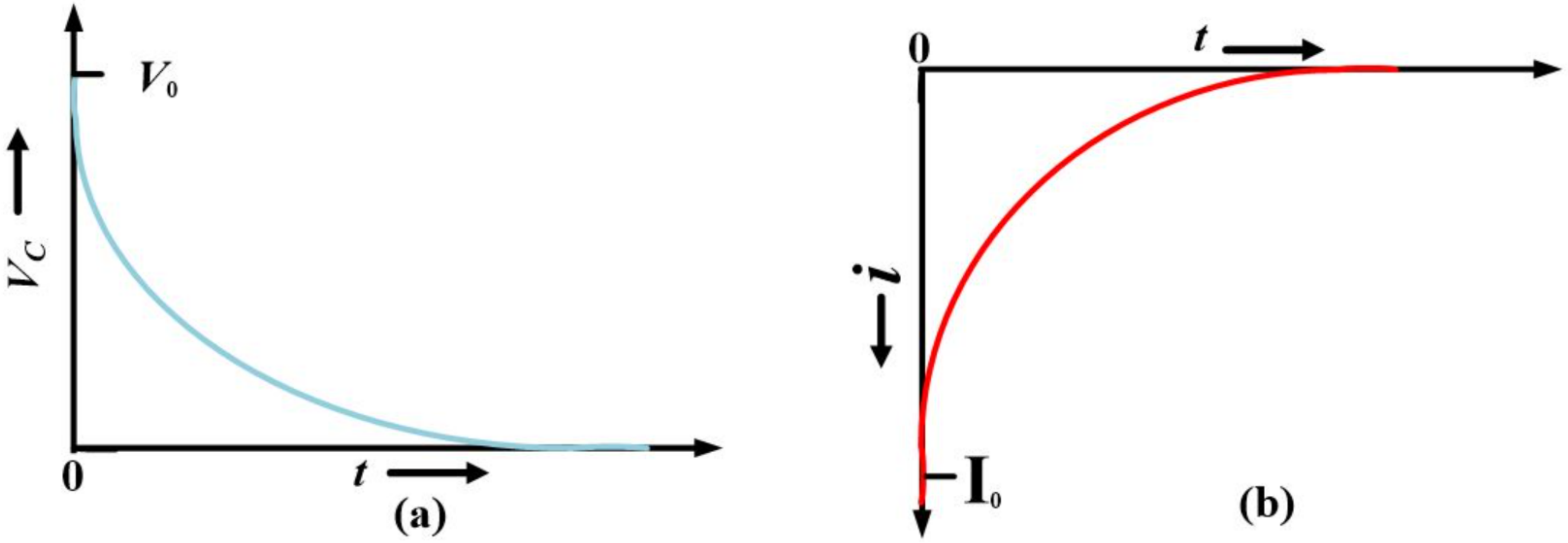
$$\frac{Q}{Q_0} = e^{-\frac{t}{RC}}$$

$$Q = Q_0 e^{-\frac{t}{RC}}$$

Then

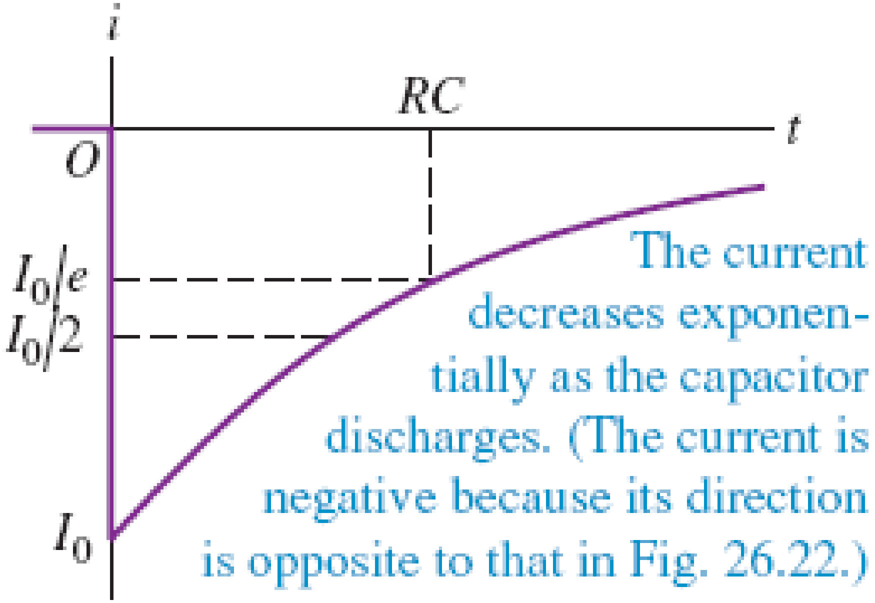
$$I = -\frac{dQ}{dt} = \frac{Q_0}{RC}e^{-\frac{t}{RC}}$$

$$I = \frac{V_0}{R}e^{-\frac{t}{RC}} = I_0 e^{-\frac{t}{RC}}$$

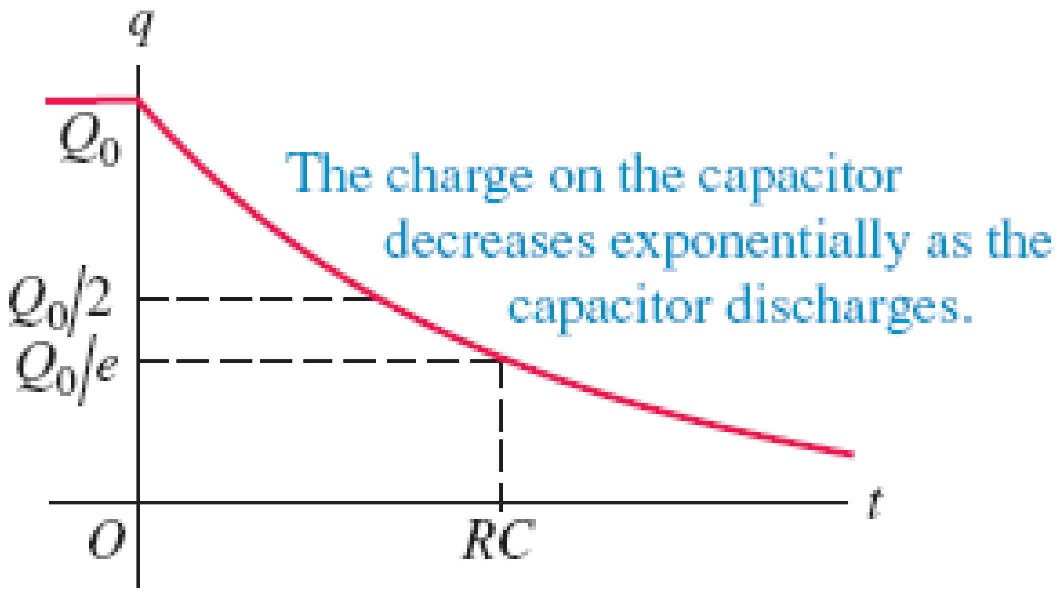


http://physics.bu.edu/~duffy/HTML5/RC_circuit.html

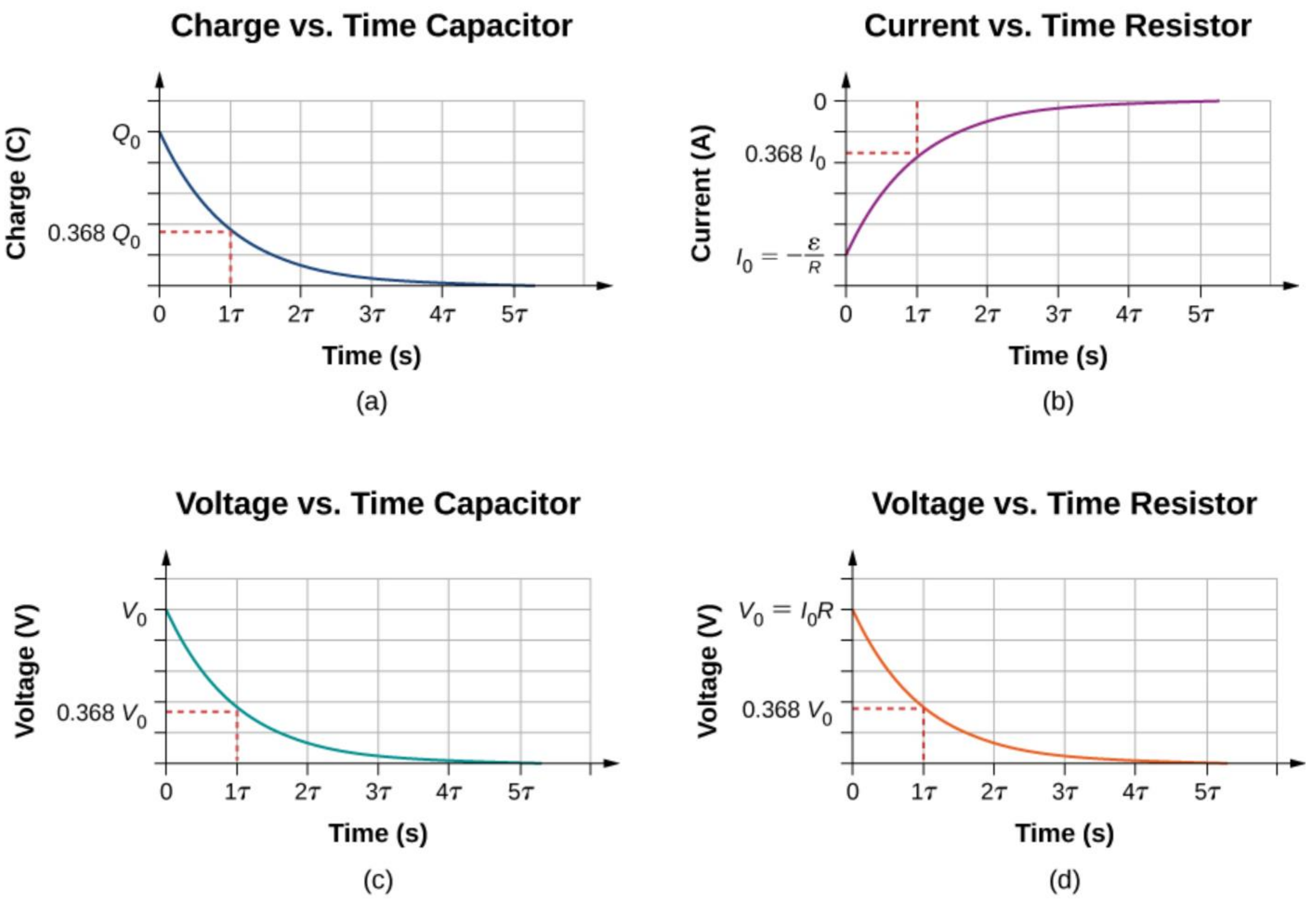
(a) Graph of current versus time for a discharging capacitor

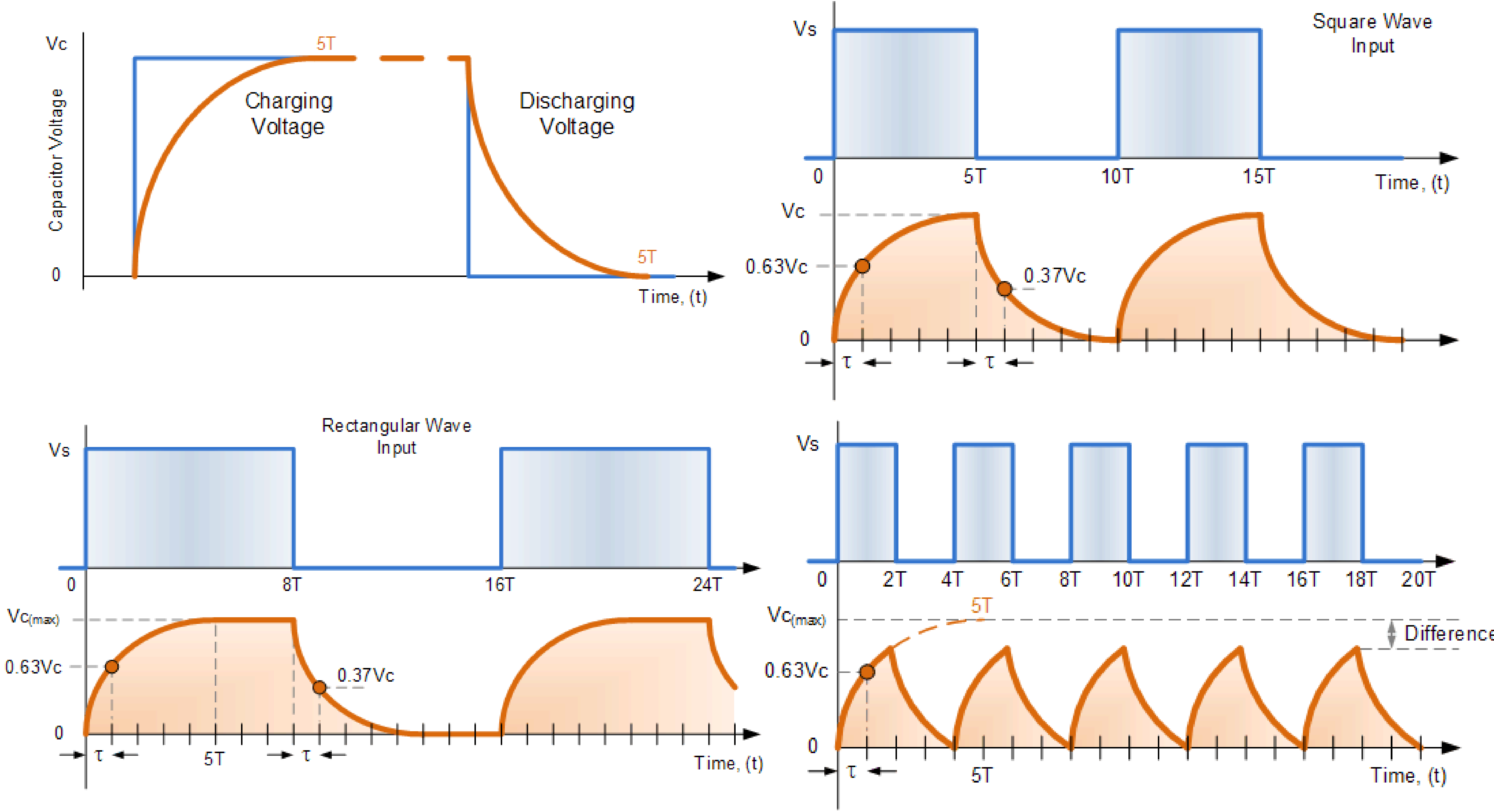


(b) Graph of capacitor charge versus time for a discharging capacitor



https://ophysics.com/em5.html


Charge vs. Time Capacitor
Charge (C)
Q_0
0.368 Q_0
0 1τ 2τ 3τ 4τ 5τ
Time (s)
(a)
Current vs. Time Resistor
Current (A)
0
0.368 I_0
$I_0 = -\frac{\varepsilon}{R}$
0 1τ 2τ 3τ 4τ 5τ
Time (s)
(b)
Voltage vs. Time Capacitor
Voltage (V)
V_0
0.368 V_0
0 1τ 2τ 3τ 4τ 5τ
Time (s)
(c)
Voltage vs. Time Resistor
Voltage (V)
$V_0 = I_0R$
0.368 V_0
0 1τ 2τ 3τ 4τ 5τ
Time (s)
(d)



Vc
Capacitor Voltage
0
5T
Charging Voltage
Discharging Voltage
5T
Time, (t)
Square Wave Input
Vs
0
5T
10T
15T
Time, (t)
Vc
0.63Vc
0.37Vc
0
τ
τ
Rectangular Wave Input
Vs
0
8T
16T
24T
Vc(max)
0.63Vc
0.37Vc
0
τ
5T
τ
Time, (t)
Vs
0
2T
4T
6T
8T
10T
12T
14T
16T
18T
20T
Vc(max)
5T
Differenc
0.63Vc
0
τ
5T
Time, (t)

$\tau = RC$ is the characteristic time of any RC circuit, called time constant

- Only t / τ is meaningful

→ $t = \tau$

- Current falls to $1/e$ = 37% of maximum value
- Current rises to 63% of maximum value

→ $t = 2\tau$

- Current falls to $1/e^2$ = 13.5% of maximum value
- Current rises to 86.5% of maximum value

→ $t = 3\tau$

- Current falls to $1/e^3$ = 5% of maximum value
- Current rises to 95% of maximum value

→ $t = 5\tau$

- Current falls to $1/e^5$ = 0.7% of maximum value
- Current rises to 99.3% of maximum value

During the charging process, a total charge $Q_f = \mathcal{E}C$ flows through the battery. The battery therefore does work

$$W = Q_f\mathcal{E} = C\mathcal{E}^2$$

The rate at which energy is dissipated by the resistance is $\dfrac{dW_R}{dt} = I^2R$

$$\frac{dW_R}{dt} = \left(\frac{\mathcal{E}}{R}e^{-t/(RC)}\right)^2 R = \frac{\mathcal{E}^2}{R}e^{-2t/(RC)}$$

We find the total energy dissipated by integrating from $t = 0$ to $t = \infty$: $W_R = \displaystyle\int_0^\infty \frac{\mathcal{E}^2}{R}e^{-2t/(RC)}\,dt = \frac{\mathcal{E}^2}{R}\frac{RC}{2}$

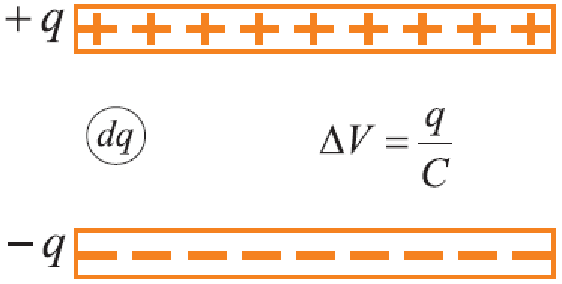
The total amount of Joule heating is thus

$$W_R = \frac{1}{2}\mathcal{E}^2C = \frac{1}{2}Q_f\mathcal{E}$$

Half of this work is accounted for by the energy stored in the capacitor $U = \frac{1}{2}Q_f\mathcal{E}$

Thus, when a capacitor is charged through a resistor by a constant source of emf, half the energy provided by the source of emf is stored in the capacitor and half goes into thermal energy. This thermal energy includes the energy that is dissipated by the internal resistance of the source of emf.

Energy Stored in an Electric Field



In charging a capacitor, *positive charge* is being moved from the *negative plate* to the *positive plate*.
$\Rightarrow$ NEEDS WORK DONE!

Suppose we move charge dq from $-ve$ to $+ve$ plate, *change in potential energy*

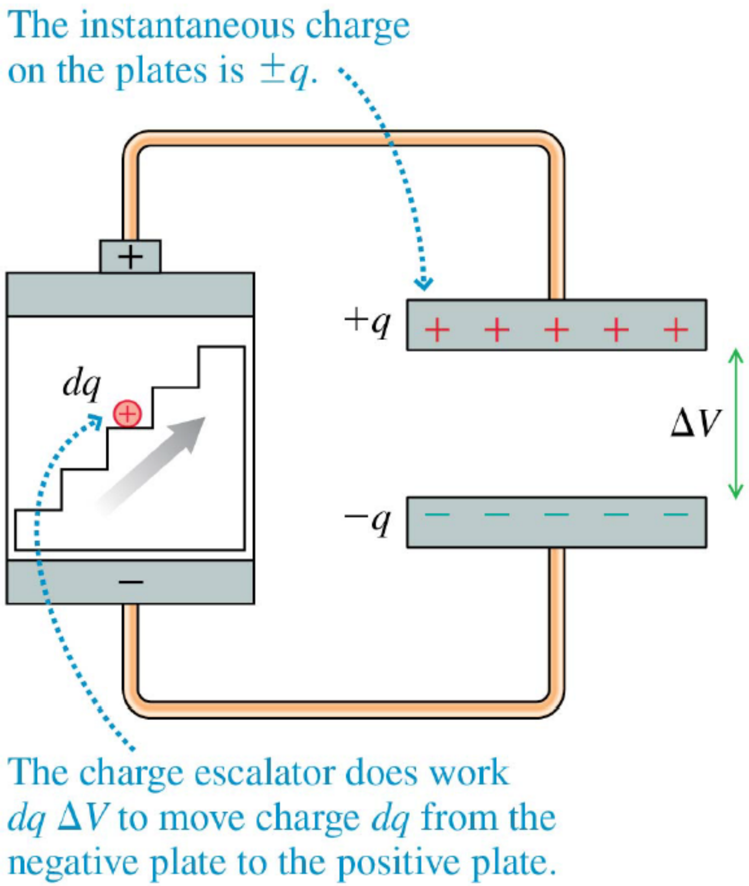
$$dU = \Delta V \cdot dq = \frac{q}{C}\, dq$$

Suppose we keep putting in a total charge Q to the capacitor, the *total potential energy*

$$U = \int dU = \int_0^Q \frac{q}{C}\, dq$$

$$\therefore \quad U = \frac{Q^2}{2C} = \tfrac{1}{2}CV^2 = \tfrac{1}{2}QV \quad (\because Q = C\Delta V)$$

The energy stored in the capacitor is stored in the **electric field** between the plates.

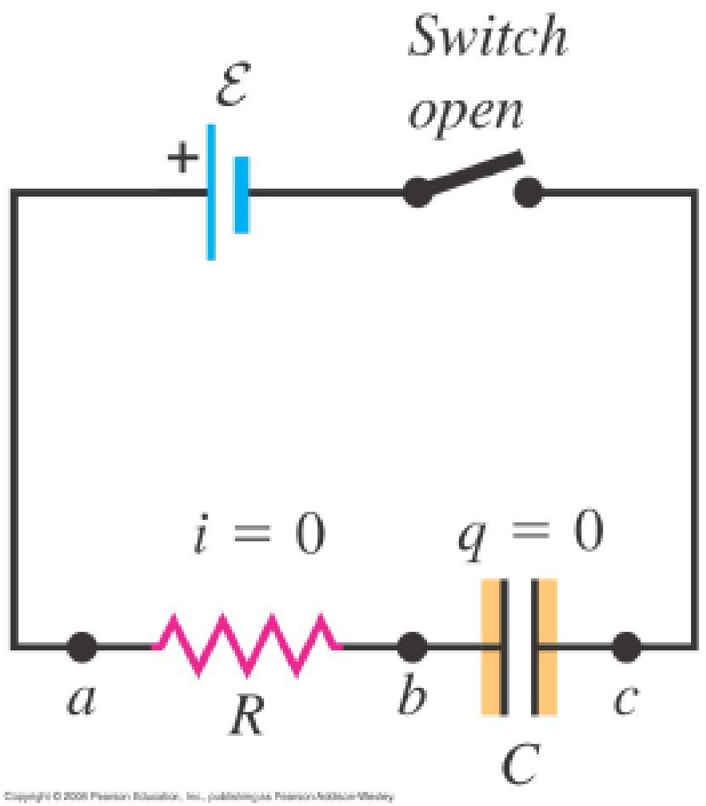


The potential energy of a charged capacitor may be viewed as being stored in the electric field between its plates.

Charging Capacitor:

EXALMPLE: Consider this circuit:

(a) Capacitor initially uncharged



$\varepsilon = 12V;\ C = 0.3x10^{-6}F;\ R = 20x10^{3}\Omega$

Find:

a. The time constant

b. The maximum charge on C

c. The time for the charge to reach 99% of C_{max}.

d. The current when $Q = \frac{1}{2}Q_{max}$

e. I_{max}

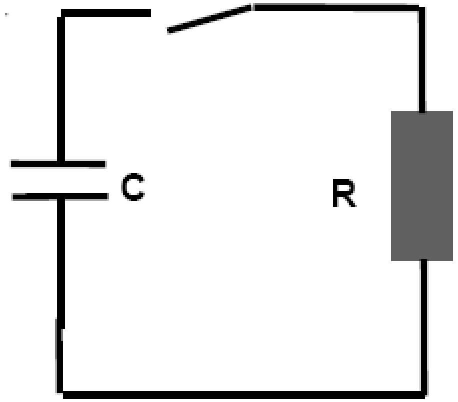
f. Q when $I = 0.2I_{max}$

A heart pacemaker fires 72 times a minute, each time a 25.0-nF capacitor is charged (by a battery in series with a resistor) to 0.632 of its full voltage. What is the value of the resistance?

A capacitor **C** with an initial charge **Q** discharges through a resistor **R**. How many time constants $\boldsymbol{\tau} = \mathbf{RC}$ must elapse in order for the capacitor to lose **2/3** of its charge?

Problem 5:

A charged capacitor **C** has a resistance **R** connected across its terminals to form the **RC circuit** shown in the Figure. If it takes **2 seconds** for the capacitor to loose one-half of its stored energy, how long does it take (**in s**) for it to loose **90%** of its initial charge?



Answer: 13.3

Solution: The stored energy in the **RC** circuit is given by

$$U(t) = \frac{Q(t)^2}{2C} = U_0 e^{-2t/\tau},$$

and solving for **τ** gives

$$\tau = RC = \frac{-2t_1}{\ln(U(t)/U_0)} = \frac{-2t_1}{\ln(0.5)},$$

where $\mathbf{t_1 = 2\ s}$. The charge as a function of time is

$$Q(t) = Q_0 e^{-t/\tau} \quad \text{and} \quad t = -\tau \ln(Q(t)/Q_0).$$

Thus,

$$t = \frac{2t_1 \ln(0.1)}{\ln(0.5)} = \frac{2(2s)\ln(0.1)}{\ln(0.5)} = 13.3s.$$