Problem Set 2

Name
MATH-UA 120 Discrete Mathematics

due February 10, 2023

These are to be written up and turned in to Gradescope.

<u>LaTeXInstructions:</u> You can view the source (`.tex`) file to get some more examples of LaTeX code. I have commented the source file in places where new LaTeX constructions are used.

Remember to change `\showsolutionsfalse` to `\showsolutionstrue` in the document's preamble (between `\documentclass{article}` and `\begin{document}`)

Assigned Problems

1.

(a) Disprove: Every triangle has at least one obtuse angle.

(b) Disprove: For all real numbers x, $x^2 \geq x$.

(c) Disprove: For every positive nonprime integers n, if some prime p divides n, then some other prime q ($q \neq p$) also divides n.

2. (Scheinerman, Exercise 7.12:) Another method to prove that certain Boolean formulas are tautologies is to use the properties in Theorem 7.2 together with the fact that $x \to y$ is equivalent to $(\neg x) \vee y$ (Proposition 7.3) For example, Exercise 7.11, part (b) asks you to establish that the formula $(x \wedge (x \to y)) \to y$

is a tautology. Here is a derivation of that fact:

$$\begin{aligned}
(x \wedge (x \to y)) \to y &= [x \wedge (\neg x \vee y)] \to y && \text{translate } \to \\
&= [(x \wedge \neg x) \vee (x \wedge y)] \to y && \text{distributive} \\
&= [\text{FALSE} \vee (x \wedge y)] \to y && \text{inverse elements} \\
&= (x \wedge y) \to y && \text{identity element} \\
&= \neg(x \wedge y) \vee y && \text{translate } \to \\
&= (\neg x \vee \neg y) \vee y && \text{De Morgan's laws} \\
&= \neg x \vee (\neg y \vee y) && \text{associativity} \\
&= \neg x \vee \text{TRUE} && \text{inverse elements} \\
&= \text{TRUE} && \text{identity element}
\end{aligned}$$

Use this technique [not truth tables] to prove that these formulas are tautologies:

(a) $(x \to \text{FALSE}) \to \neg x$

(b) $(x \to y) \wedge (x \to \neg y) \to \neg x$

3. (Scheinerman, Exercise 7.16:) Here is another Boolean operation called *exclusive or*; it is denoted by the symbol $\veebar$. It is defined in the following table.

x	y	$x \veebar y$
T	T	F
T	F	T
F	T	T
F	F	F

(a) Prove that $x \veebar y$ is logically equivalent to $(x \wedge \neg y) \vee ((\neg x) \wedge y)$.

(b) Prove that $x \veebar y$ is logically equivalent to $(x \vee y) \wedge (\neg(x \wedge y))$.

(c) Explain why the operation $\veebar$ is called *exclusive or*.

4. Suppose each single character stored in a computer uses eight bits. Then each character is represented by a different sequence of eight 0's and 1's called a bit pattern, such as 10011101 or 00100010. Provide a brief reasoning for the following questions.

(a) How many different bit patterns are there?

(b) How many different bit patterns are palindromes (the same backwards as forwards)?

(c) How many different bit patterns have an even number of 1's?

(d) How many different bit patterns have the property that their second and fourth digits are 1's?

(e) How many different bit patterns have the property that their second or fourth digits are 1's?

5. Suppose we want to make a list of length 5 from the letters $A, B, C, D, E, F, G, H, I, J$. Provide a brief reasoning for the following questions.

(a) How many such lists can be made if repetition is not allowed and the list must begin with a vowel?

(b) How many such lists can be made if repetition is not allowed and the list must end with a vowel?

(c) How many such lists can be made if repetition is not allowed and the list must contain exactly one A?