Homework Assignment 1 – MA 637-OV

Graph Theory and Combinatorics

Due: Round 1, June 12, **Final due date:** Round 2, June 22

1. In class we proved that

$$\sum_{i=1}^{n} i \;=\; \frac{n(n+1)}{2} \quad \text{and also} \quad \sum_{i=1}^{n} 1 \;=\; n\,. \tag{1}$$

What function $f(i)$ must we take to get

$$\sum_{i=1}^{n} f(i) \;=\; n^2\,? \tag{2}$$

(Show your work.)

2. Use proof-by-induction or some other proof technique to prove that

$$\sum_{i=1}^{n} \frac{i(i+1)(i+2)}{6} \;=\; \frac{n(n+1)(n+2)(n+3)}{24}\,. \tag{3}$$

3. What is the value of the sum

$$\sum_{i=1}^{n} \left(\sum_{j=1}^{i} \frac{j(j+1)(j+2)}{6} \right)\,? \tag{4}$$

(Show your work.)

4. Prove that

$$\sum_{j=1}^{n} (n+1-j)\cdot j \;=\; \frac{n(n+1)(n+2)}{6}\,. \tag{5}$$