

# Homework Assignment 3 – MA 637-OV

## Graph Theory and Combinatorics

**Due:** Round 1, June 26, **Final due date:** Round 2, July 6

1. Suppose we have a number  $q \neq 1$ . Let us make the ansatz (guess) that for each  $n = 1, 2, \dots$  we have

$$\sum_{j=1}^n jq^{j-1} = aq^n + bnq^n + c, \quad (1)$$

for constants  $a$ ,  $b$  and  $c$ . Explain why we must then have

$$aq + bq + c = 1, \quad (2)$$

and also

$$aq^n + bnq^n + c + nq^n + q^n = a \cdot q \cdot q^n + b(n+1)q \cdot q^n + c. \quad (3)$$

Explain why this implies

$$a + 1 = (a + b)q \text{ and } b + 1 = bq. \quad (4)$$

2. Show your work to solve the linear equations from problem 1 for  $a$ ,  $b$  and  $c$  to obtain the only formula consistent with our ansatz is

$$\sum_{j=1}^n jq^{j-1} = \frac{nq^n}{q-1} - \frac{q^n - 1}{(q-1)^2}. \quad (5)$$

3. Use proof-by-induction to prove that equation (5) is true. (*Show your work.*)

4. Change the order of summation on the sum

$$\sum_{j=1}^n \left( \sum_{k=1}^j q^{j-1} \right) = \sum_{k=1}^n \left( \sum_{j=k}^n q^{j-1} \right). \quad (6)$$

Do the sum on the right-hand-side.