Computational Bounds to the Quality Factor of a Fabry-Pérot Resonator Through Local Energy Conservation Laws

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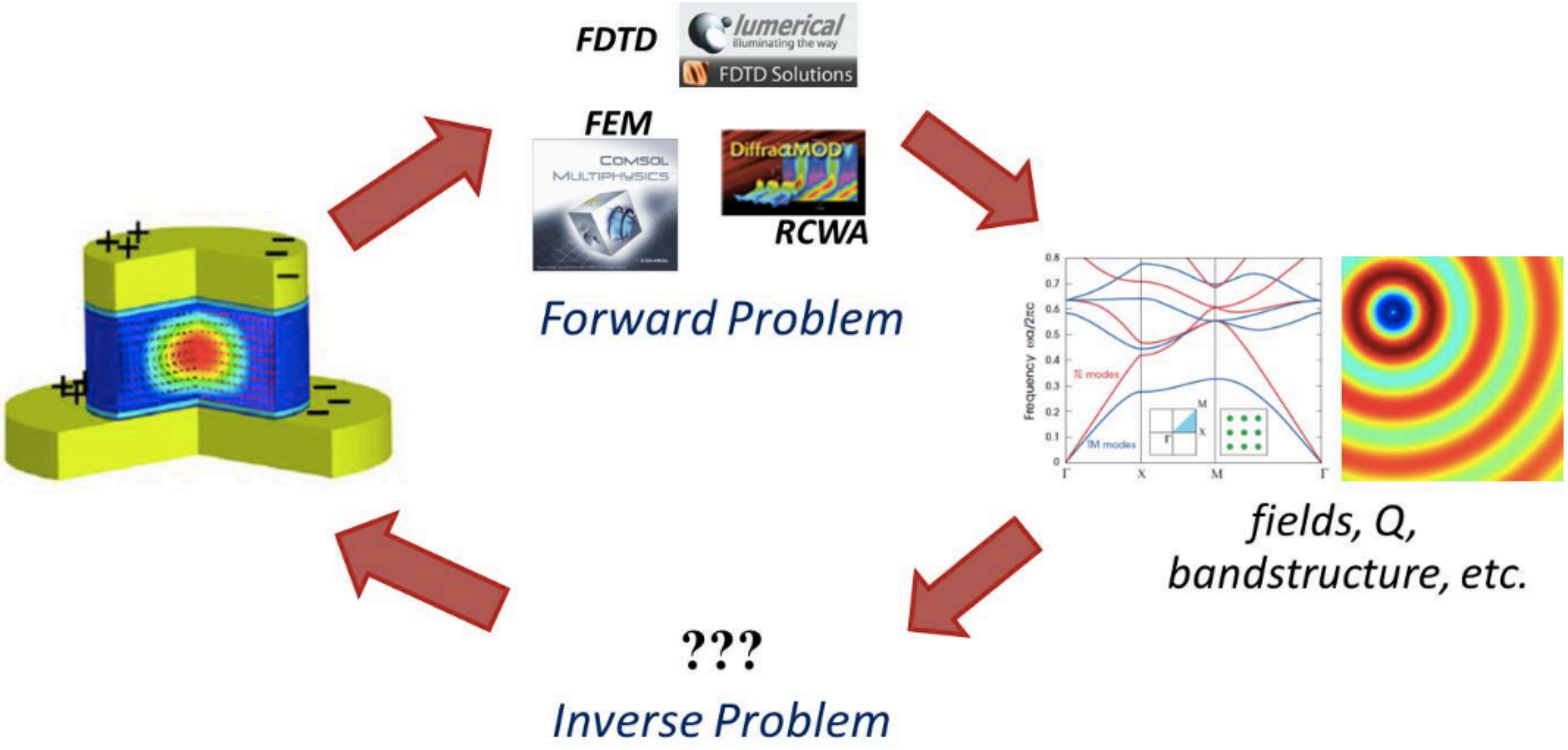
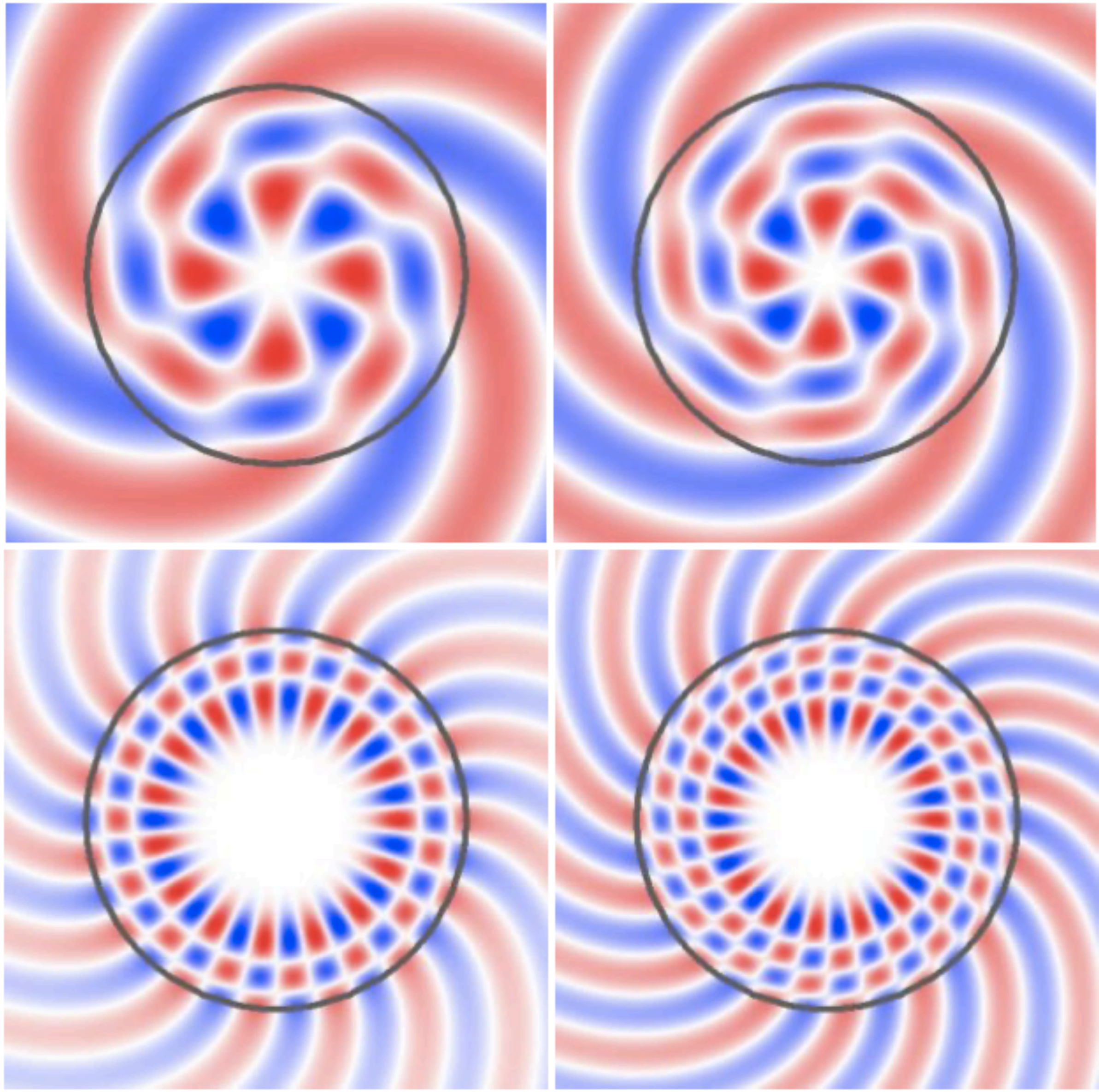
Motivation


FDTD
lumerical
illuminating the way
FDTD Solutions
FEM
COMSOL MULTIPHYSICS
DiffractMOD
RCWA
Forward Problem
TE modes
TM modes
fields, Q,
bandstructure, etc.
???
Inverse Problem

Q Factor

- Lifetime of resonant Energy
- High Q → low damping
- Low Q → high damping

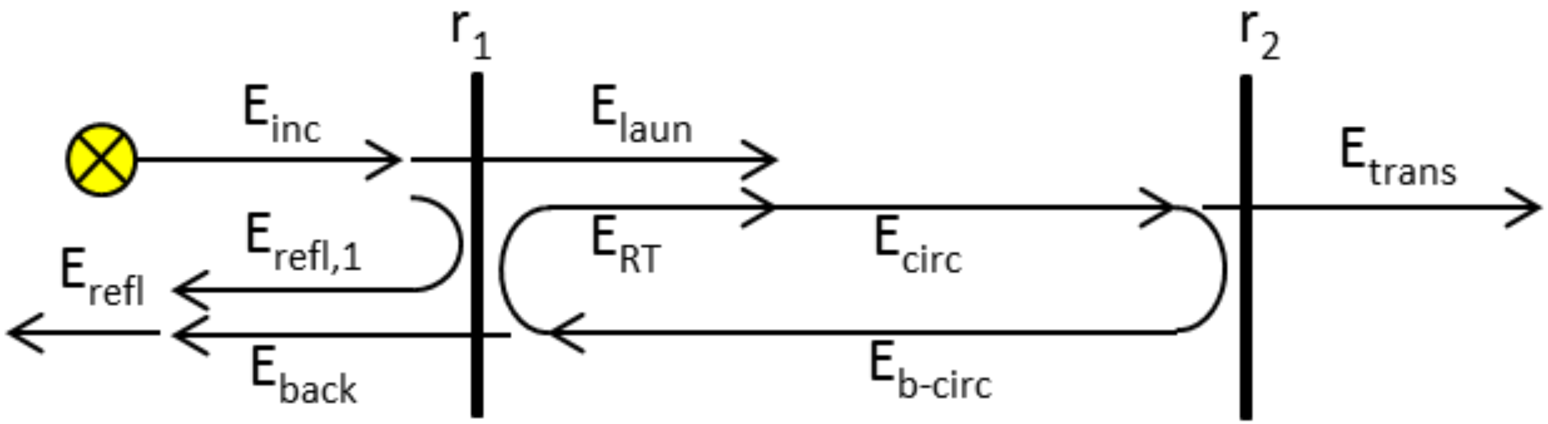


Eigenmode Profiles of Whispering Gallery Resonator

Q factor of Fabry-Pérot Interferometer

- Optical Q factor: $\omega_0 \dfrac{stored\ energy}{power\ loss}$

$$Q = -\frac{Re\{\omega\}}{2Im\{\omega\}}$$



Resonant Modes of FP Resonator

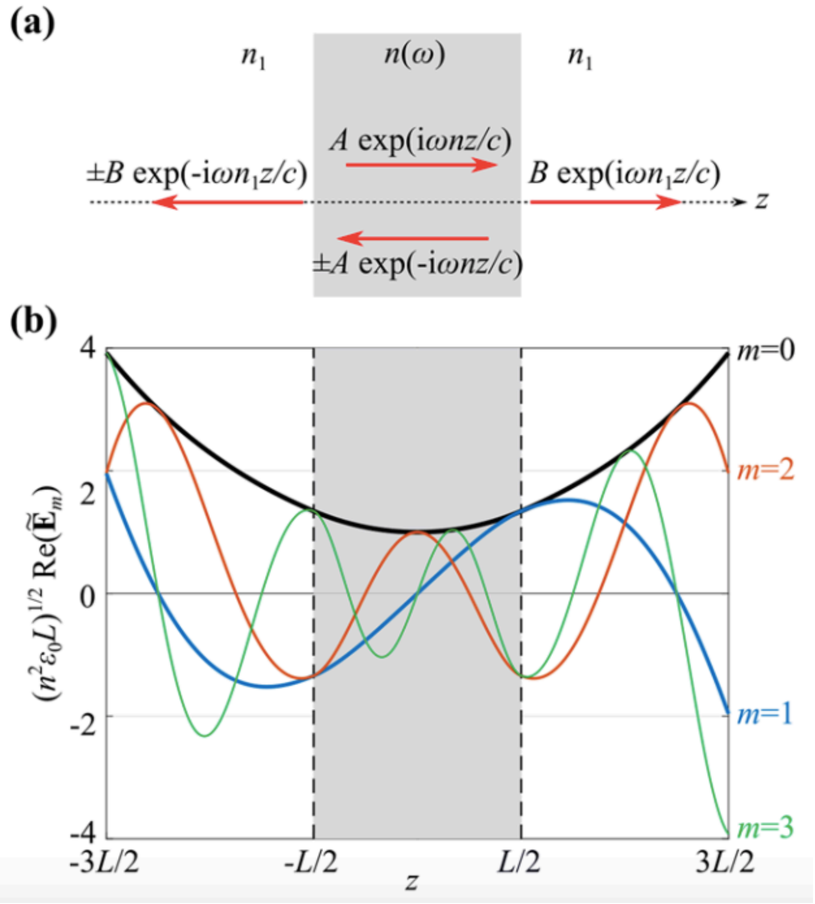
Time-harmonic, source free Maxwell's:

$$\nabla\times\nabla\times\tilde{E}(z) = \omega^2\varepsilon(z)\tilde{E}(z)$$

$$\tilde{A}x = \lambda Bx$$

$$\tilde{E}(z) = Be^{i\omega nz} \pm Be^{-i\omega nz}$$

$$\omega = \frac{\pi m}{nL} + i\frac{\log(r)}{nL}$$

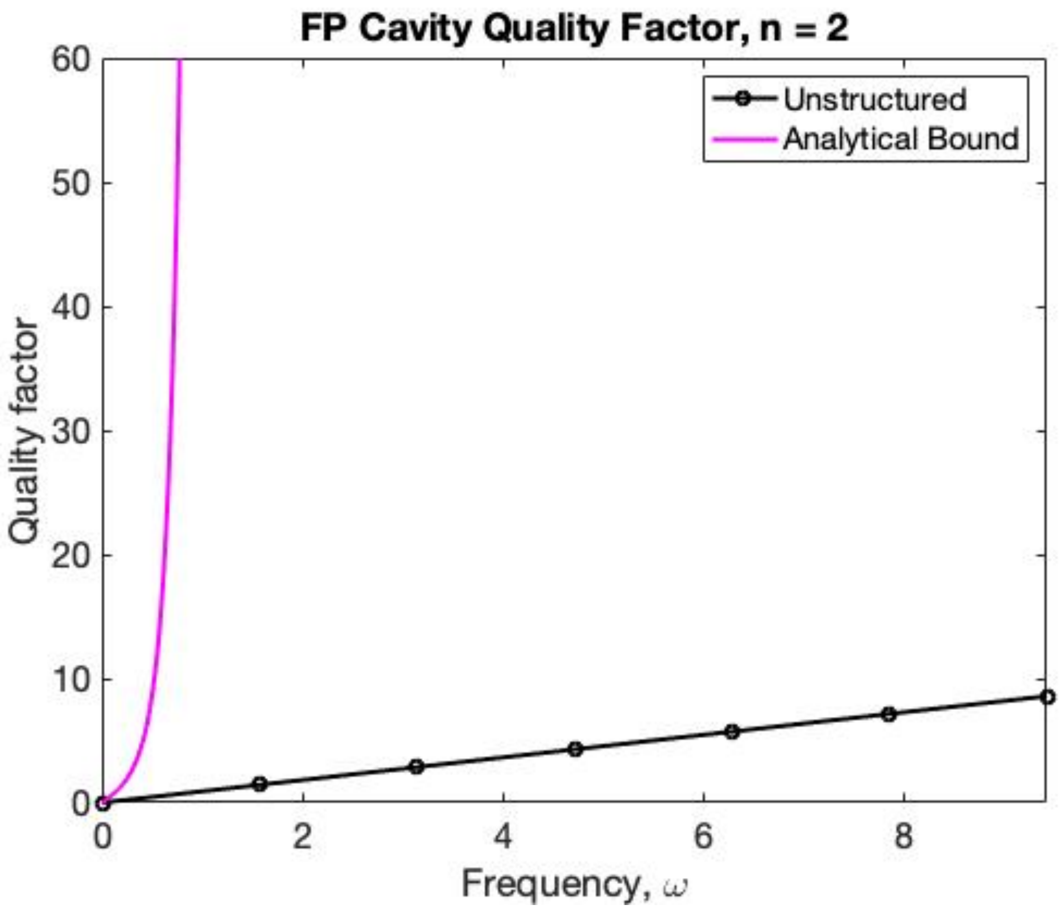


QNM field distribution of 1D FP resonator

Known Analytical Bounds

Previous attempts at Q bound have been loose

Osting and Weinstein (2013)



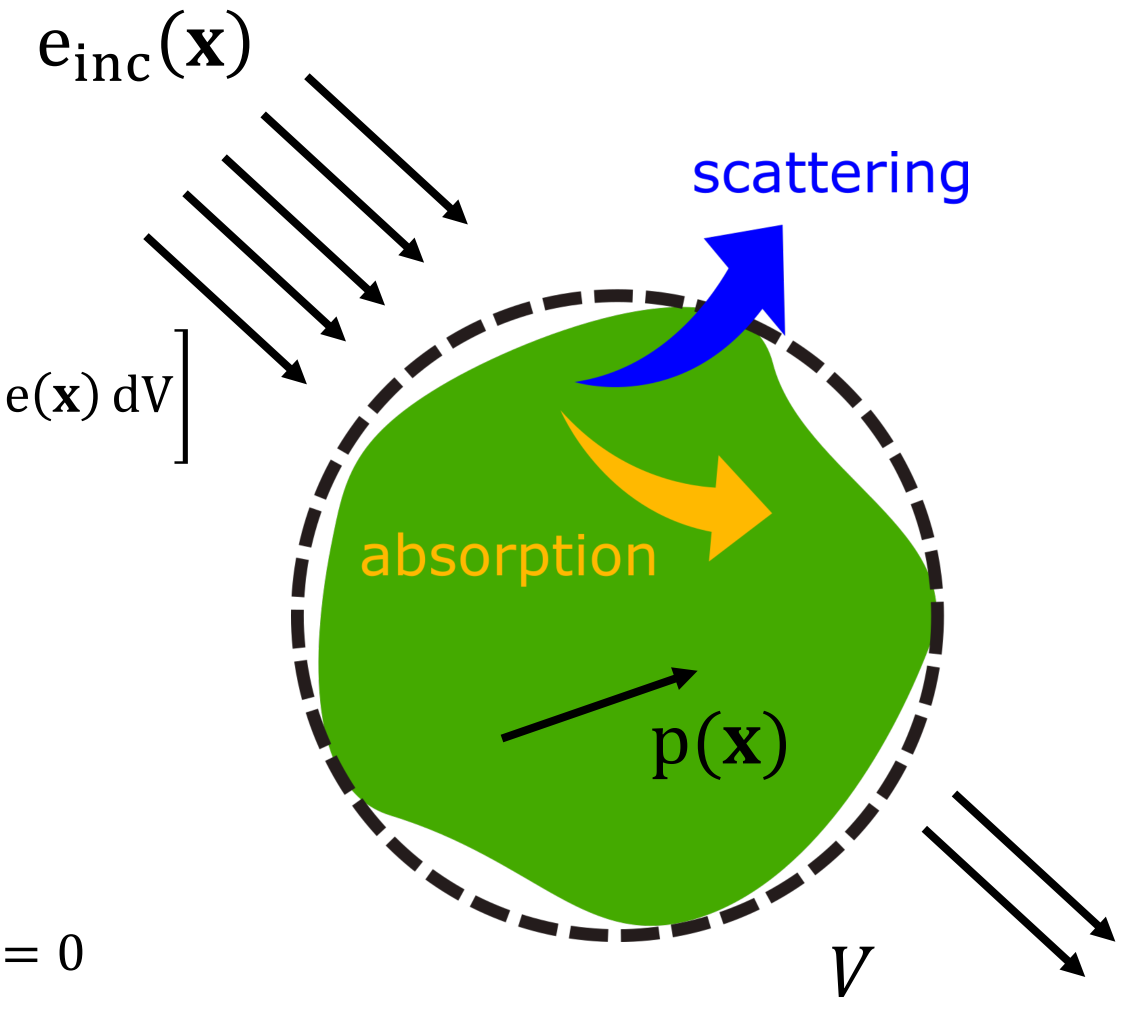
Energy Conservation via Maxwell's Equations

"incident" = "scattered" + "absorbed"

$$-\int_V p^\dagger(\mathbf{x}) e_{inc}(\mathbf{x})\,dV = \int_V p^\dagger(\mathbf{x}) e_{scat}(\mathbf{x})\,dV + \left[-\int_V p^\dagger(\mathbf{x}) e(\mathbf{x})\,dV\right]$$

...but $e_{inc}(\mathbf{x}) = 0$

$$\int_V p^\dagger(\mathbf{x}) e_{scat}(\mathbf{x})\,dV + \left[-\int_V p^\dagger(\mathbf{x}) e(\mathbf{x})\,dV\right] = 0$$



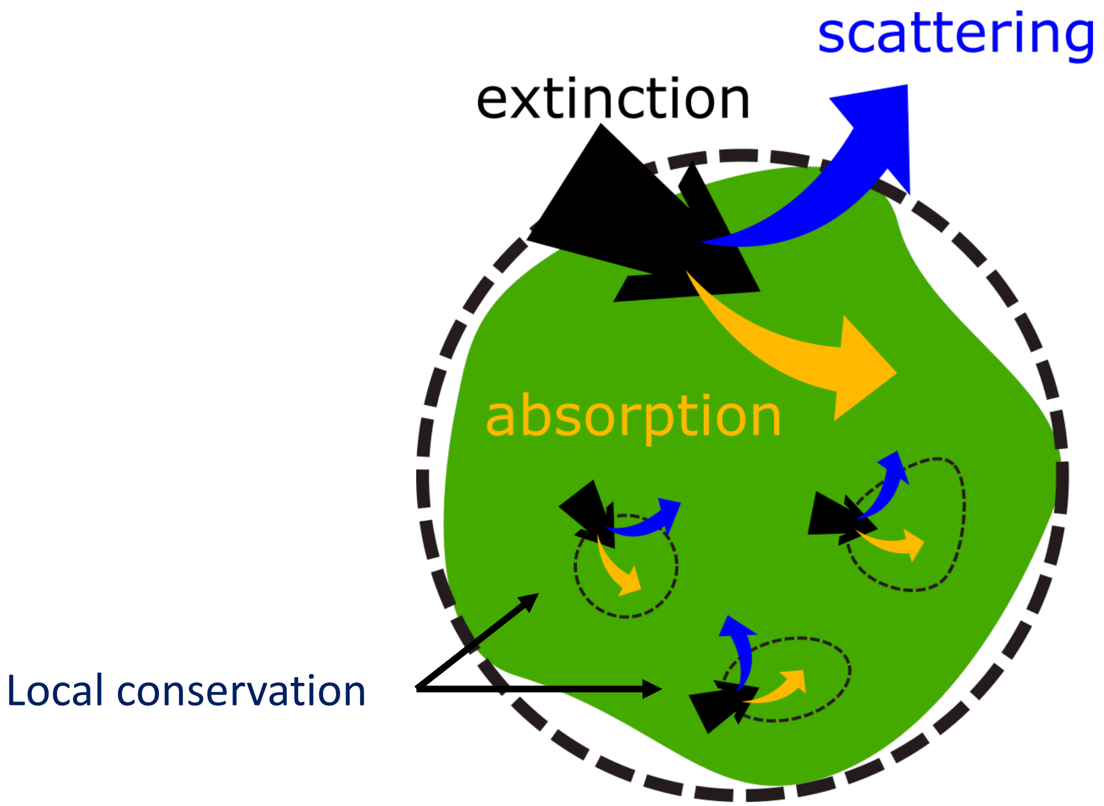
Formulation as Maximization Problem

- Goal: determine upper bound to Q factor
- Analytical definition: $Q = -\frac{Re\{\omega\}}{2Im\{\omega\}}$

Maximize $\qquad \mathrm{Im}\{\omega\}$

s.t. $\qquad$ global conservation
$\qquad\qquad$ + local conservation

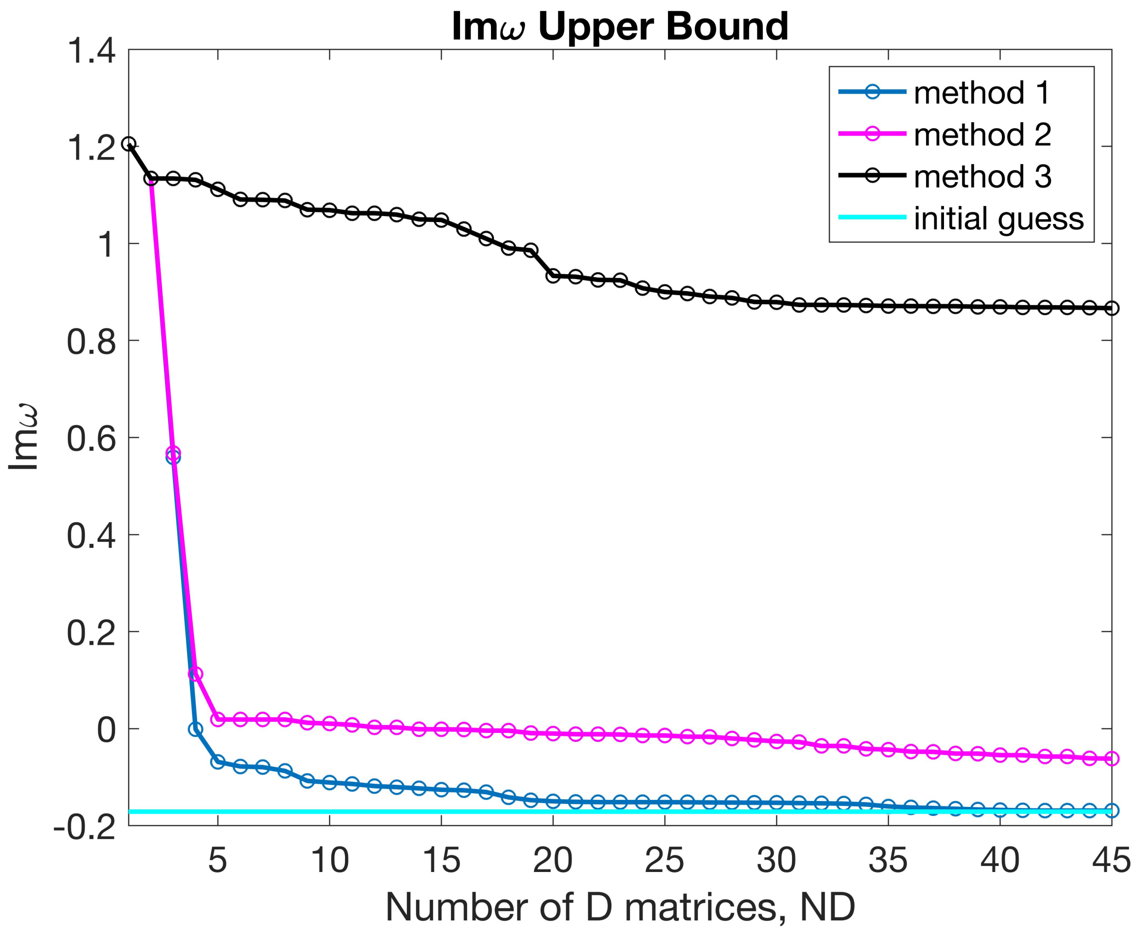
Kuang & Miller (2020)



Global conservation

Choosing Power Constraints

- How do we choose constraints?
- First two constraints: global
- The rest: choose “smart” selection of constraints



MATLAB Implementation

- Nonconvex Quadratically Constrained Quadratic Program (QCQP)
- Solved via Semidefinite Relaxation (SDR)
- Optimal Polarization current is largest eigenvector of SDR variable X

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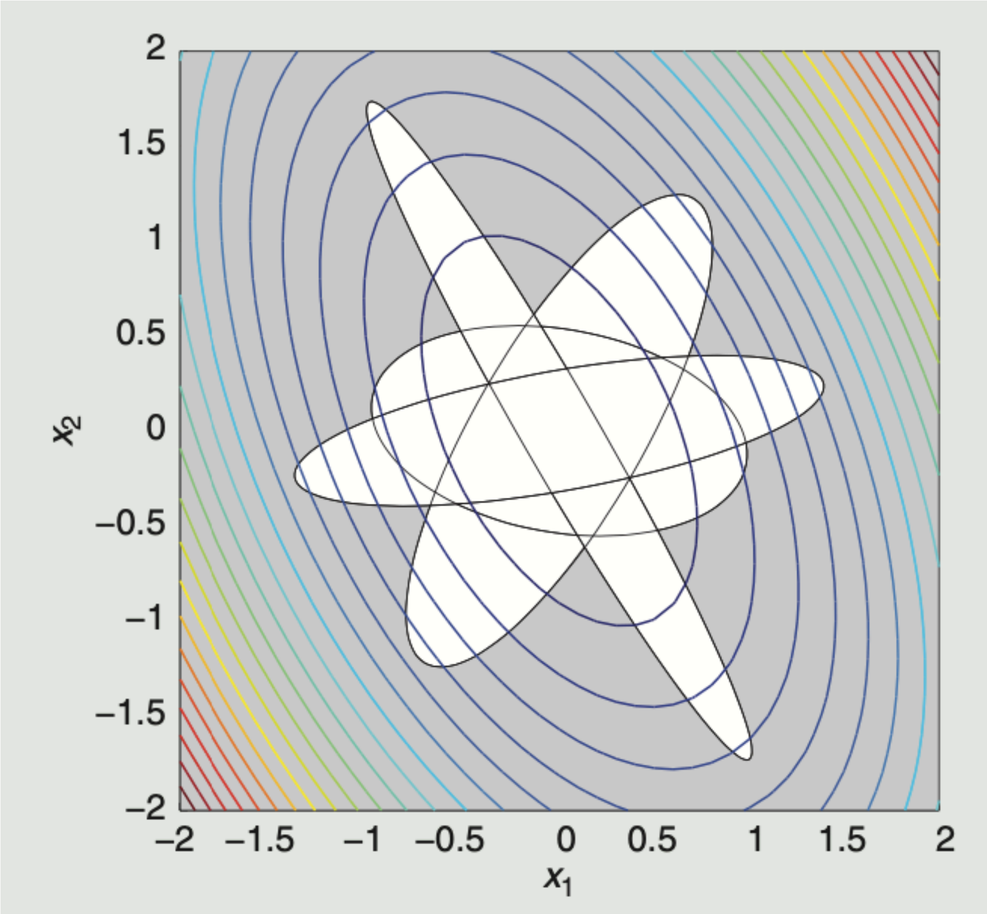
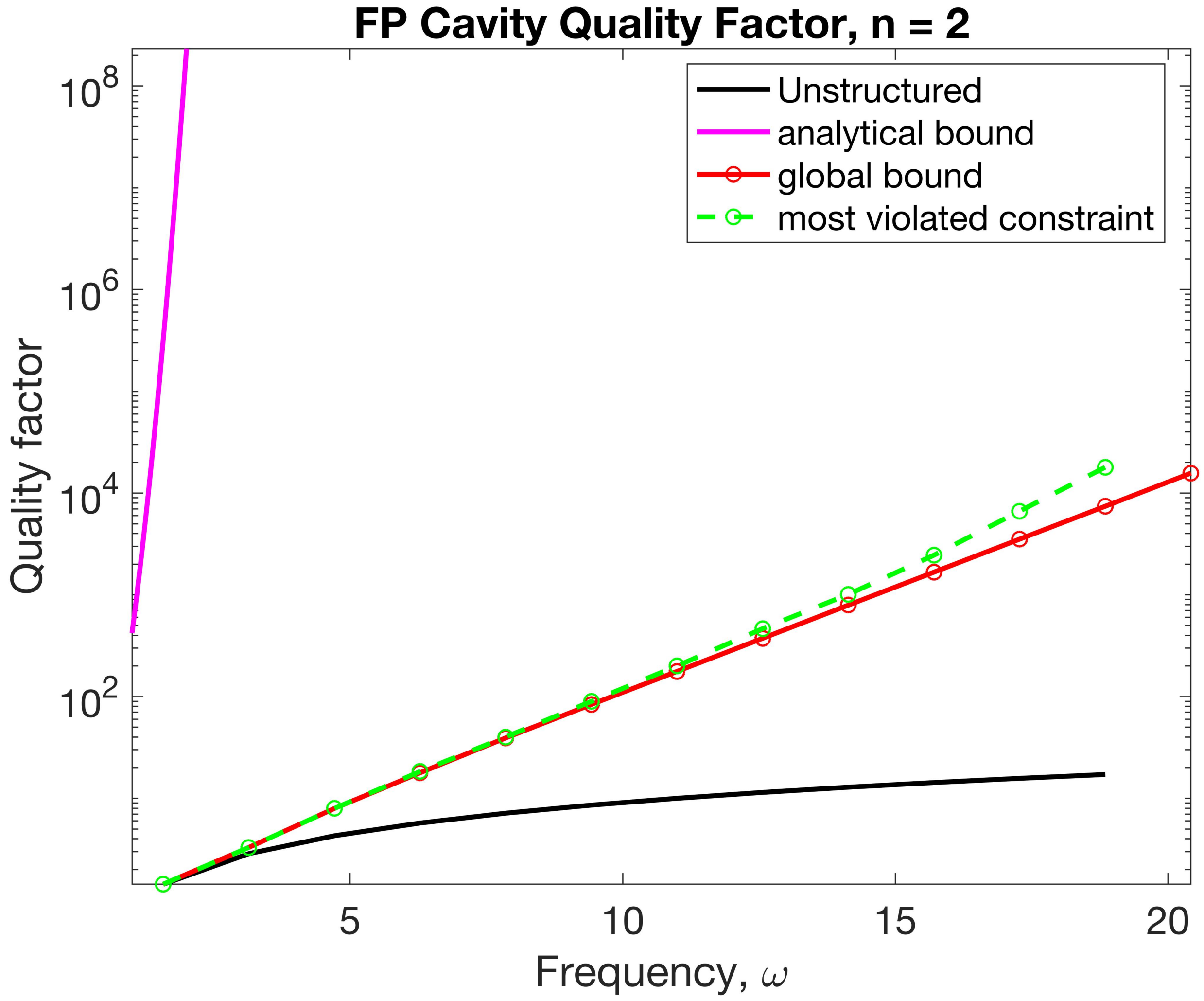
[p_opt,~] = eigs(X,1);
```



Nonconvex QCQP

Results-Q Bound

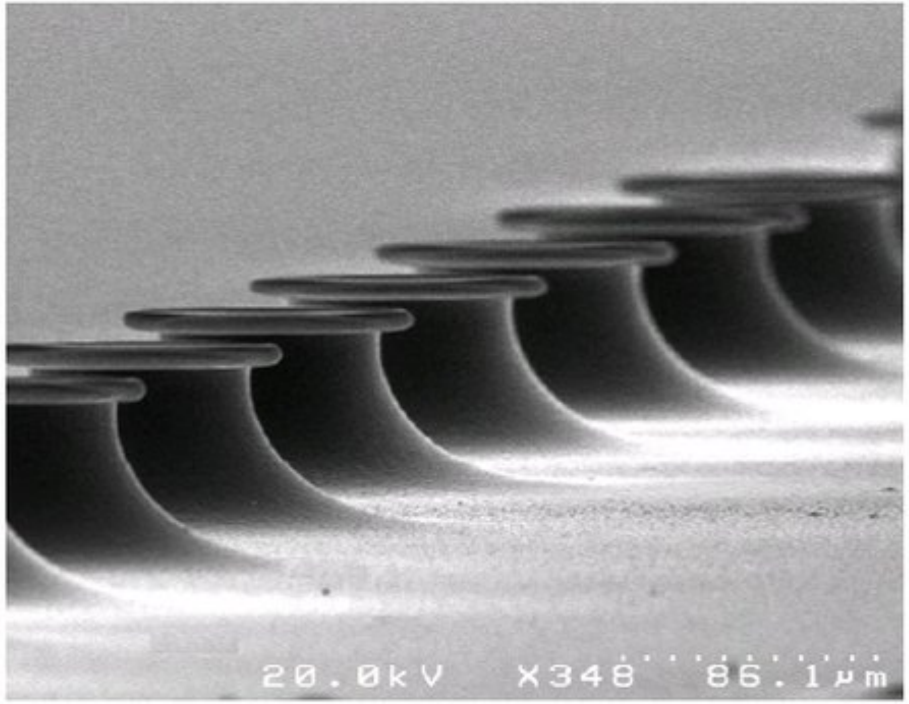
- Calculated an upper bound on quality factor up to $\omega \approx 20$
- Much tighter than previous analytical results



Discussion & Applications

- Local conservation laws derived from Maxwell's equations enable computational bounds to Q factor
- Minimize scattering & absorption
- Optimize transmission spectrum

Array of Ultrahigh Q Microtoids



References

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[3] P. Lalanne, W. Yan, K. Vynck, C. Sauvan, and J.-P. Hugonin, “Light Interaction with Photonic and Plasmonic Resonances,” *Laser & Photonics Reviews*, vol. 12, no. 5, p. 1700113, 2018.
[4] Z. Kuang and O. D. Miller, “Computational Bounds to Light–Matter Interactions via Local Conservation Laws,” *Physical Review Letters*, vol. 125, no. 26, 2020.
[5] Z.-quan Luo, W.-kin Ma, A. So, Y. Ye, and S. Zhang, “Semidefinite Relaxation of Quadratic Optimization Problems,” *IEEE Signal Processing Magazine*, vol. 27, no. 3, pp. 20–34, 2010.