Effectiveness of Coupled Mode Theory: Variants, Accuracy, & Examples

Simon Stone

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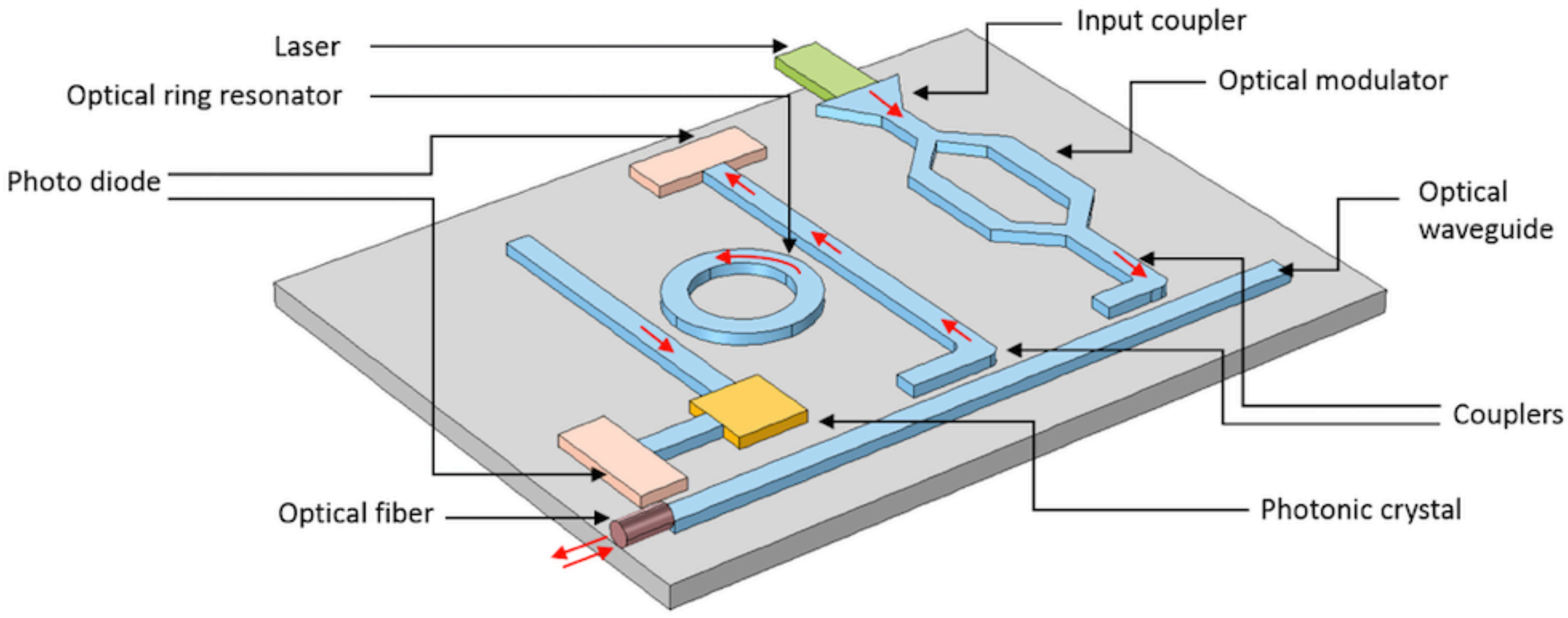
APHY 607

Overview

1. Background & History
2. Conventional CMT
3. CMT in Scattering Problems
4. QCMT: examples & accuracy

Motivation

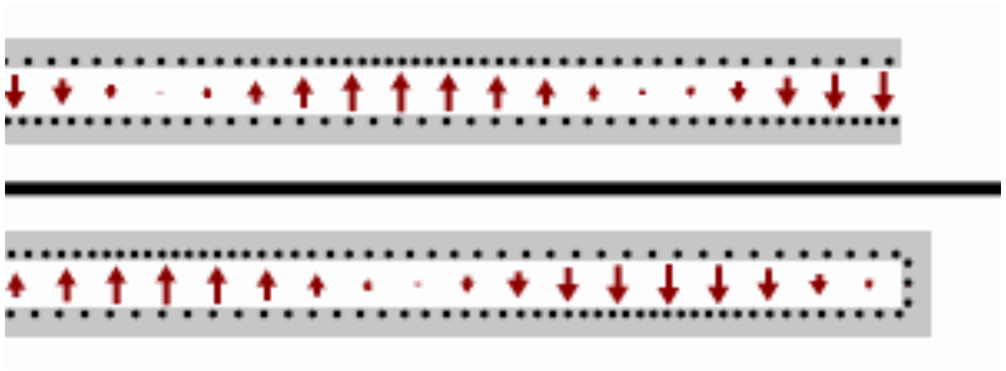
Integrated Photonic Devices: Confine Light in Small Dimensions



History & Background

- Early work: microwave transmission lines
- Linear Superposition of uncoupled system

$$P^{\omega}(z) = \sum_{m}^{n} P_{m}^{\omega}(z) \backsim \sum_{m}^{n} |a_{m}^{\omega}(z)|^{2}$$

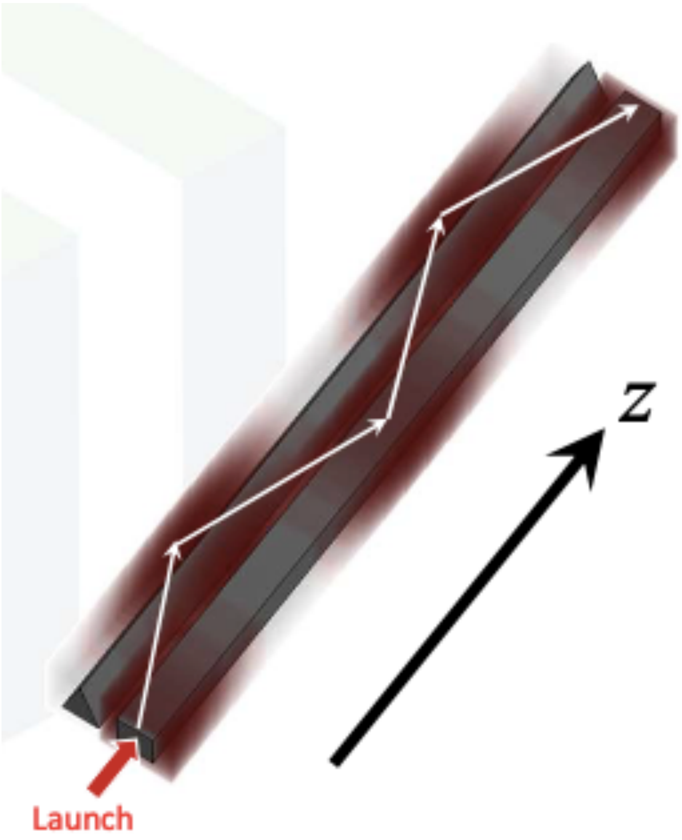


Modes and Coupling

- Example: two waveguides in close proximity → periodic exchange of power

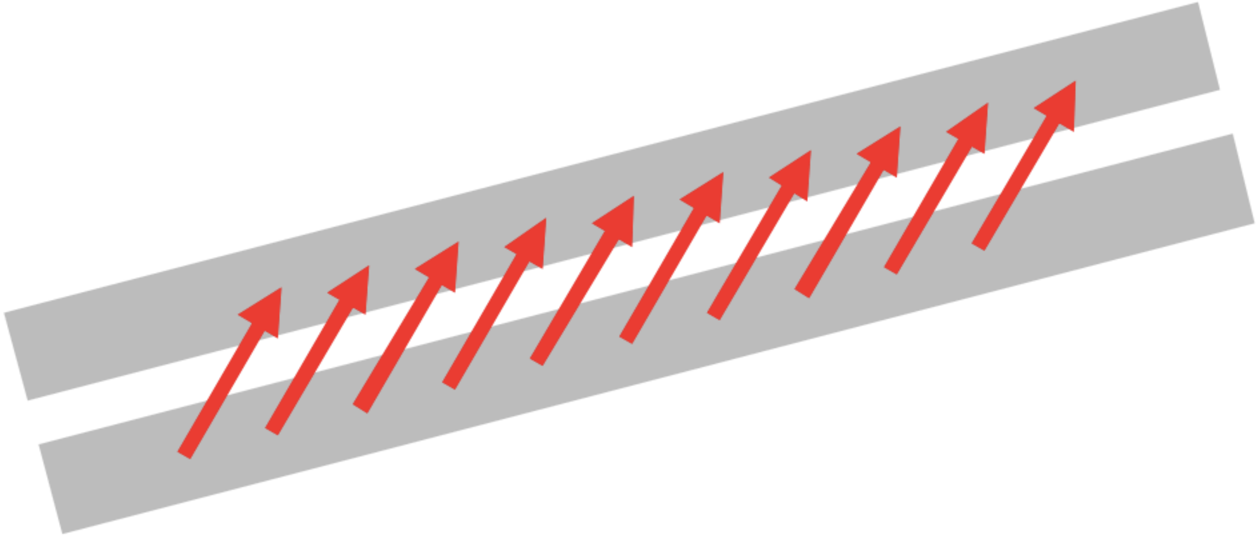
$$\mathbf{E} = A(z)\mathbf{E}_1 + B(z)\mathbf{E}_2$$

$$\mathbf{H} = A(z)\mathbf{H}_1 + B(z)\mathbf{H}_2$$



Coupling of Modes in Space

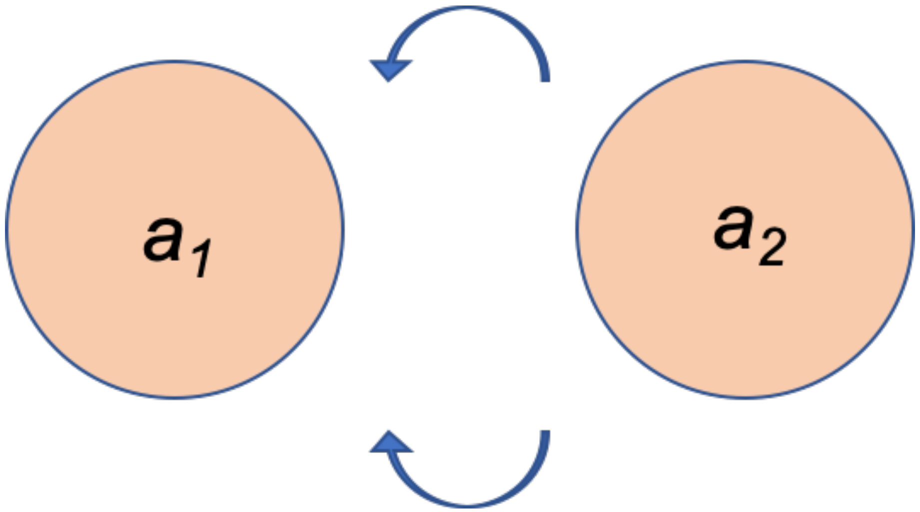
$$\kappa_{pq} = \frac{\omega\epsilon_0 \int_{-inf}^{inf} \int_{-inf}^{inf} (\epsilon_r - \epsilon r, q) \mathbf{E}_p^* \mathbf{E}_q dxdy}{\int_{-inf}^{inf} \int_{-inf}^{inf} \hat{z}(\mathbf{E}_p^* \times \mathbf{H}_p + \mathbf{E}_p \times \mathbf{H}_p^*) dxdy}$$



Coupling of Modes in Time

$$\frac{da_1}{dt} = j\omega_1 a_1 + j\kappa_1 2 a_2$$

$$\frac{da_2}{dt} = j\omega_1 a_2 + j\kappa_1 2 a_1$$



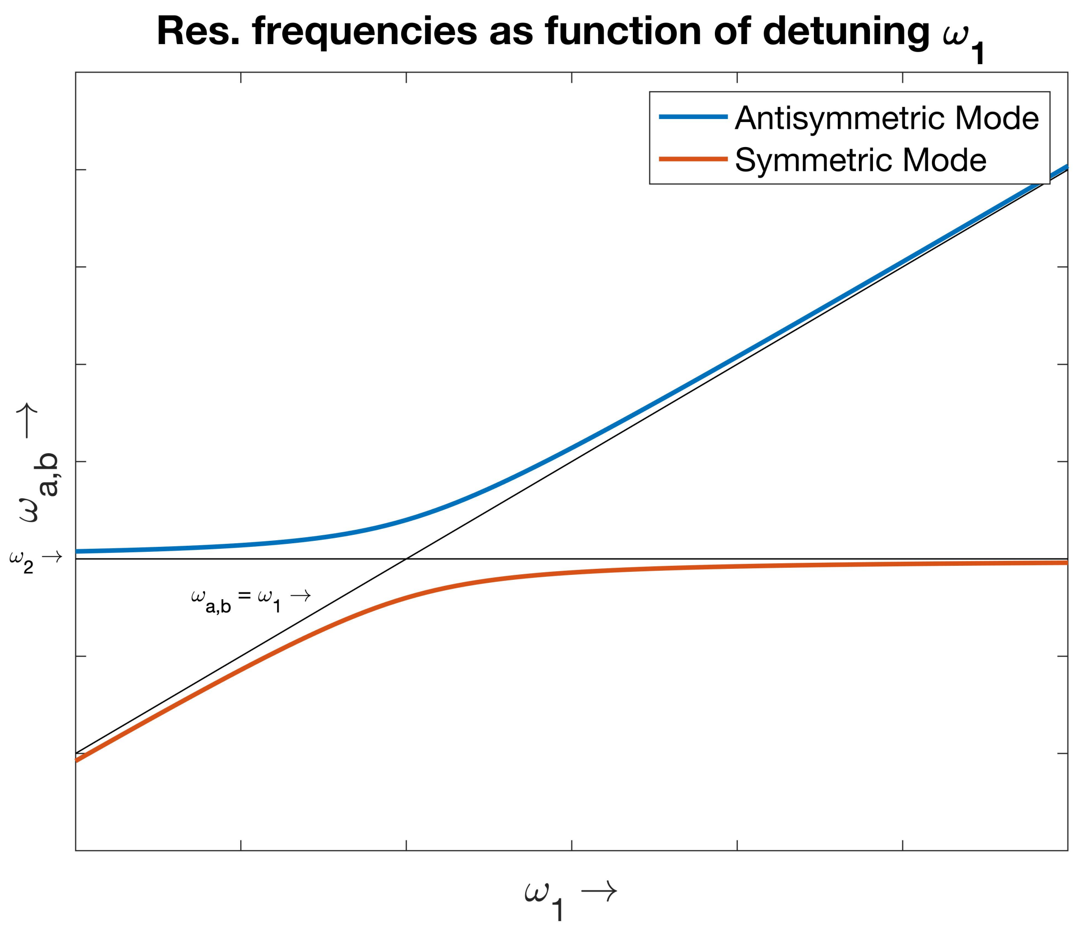
Lossless Energy Orthogonal Modes

Normalize power:

$$|a_1|^2 + |a_2|^2 = E_{tot}$$

Frequency Splitting:

$$\omega = \frac{\omega_1 + \omega_2}{2} \pm \sqrt{(\frac{\omega_1 - \omega_2}{2})^2 + |\kappa|^2}$$

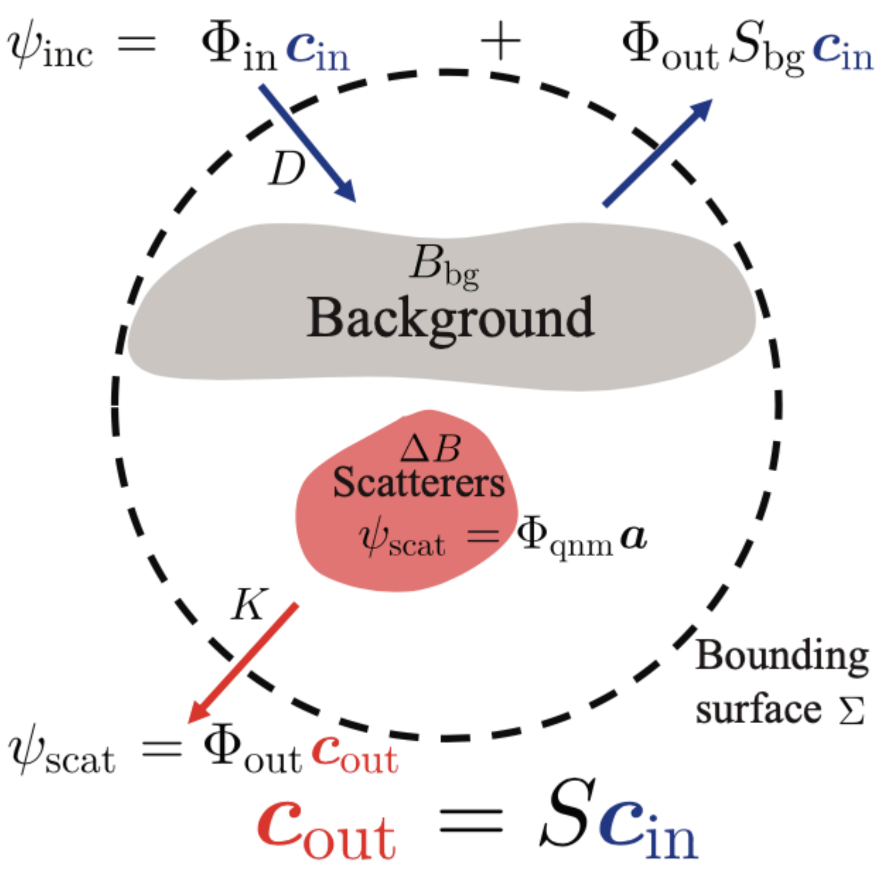


CMT Applied to Scattering Problem

$$i(\Omega - \omega)\mathbf{a} = D^T \mathbf{c}_{in}$$
$$\mathbf{c}_{out} = S_{bg}\mathbf{c}_{in} + K\mathbf{a}$$

Scattering Response:

$$S = S_{bg} - iK(\Omega - \omega)^{-1} D^T$$

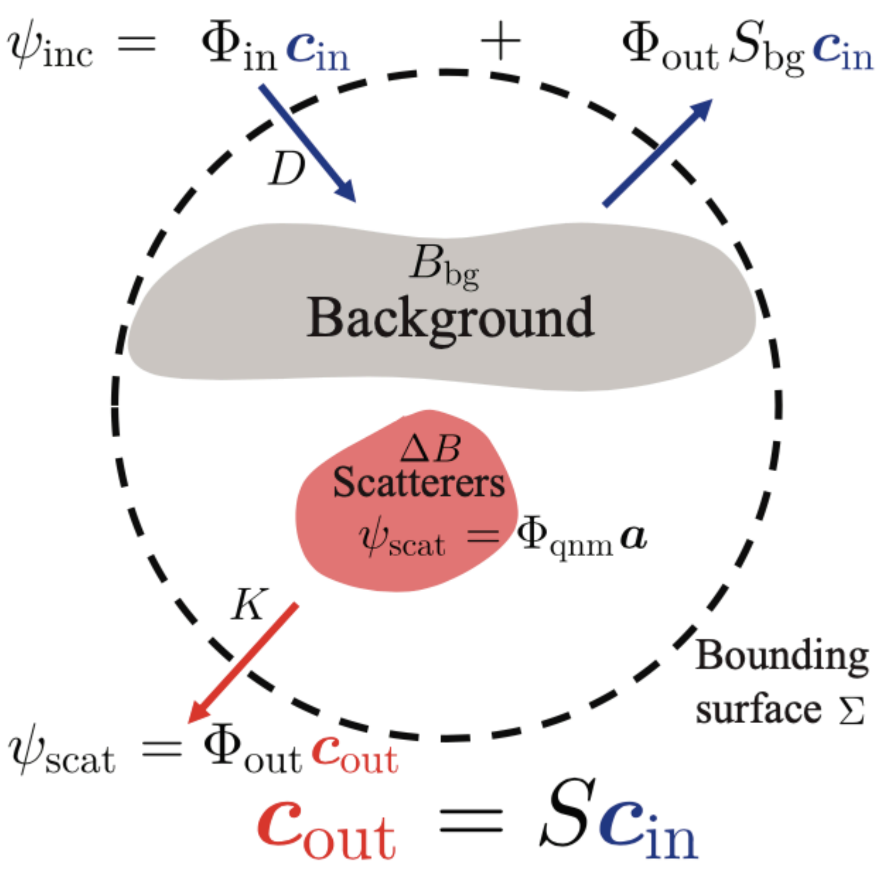


Zhang and Miller (2020)

CMT Applied to Scattering Problem

$$\cancel{\begin{aligned} i(\Omega-\omega)\mathbf{a} &= D^T\mathbf{c}_{in} \\ \mathbf{c}_{out} &= S_{bg}\mathbf{c}_{in} + K\mathbf{a} \end{aligned}}$$

Fails at complex resonant response or high-symmetry scatterer



Zhang and Miller (2020)

CMT Applied to Scattering Problem

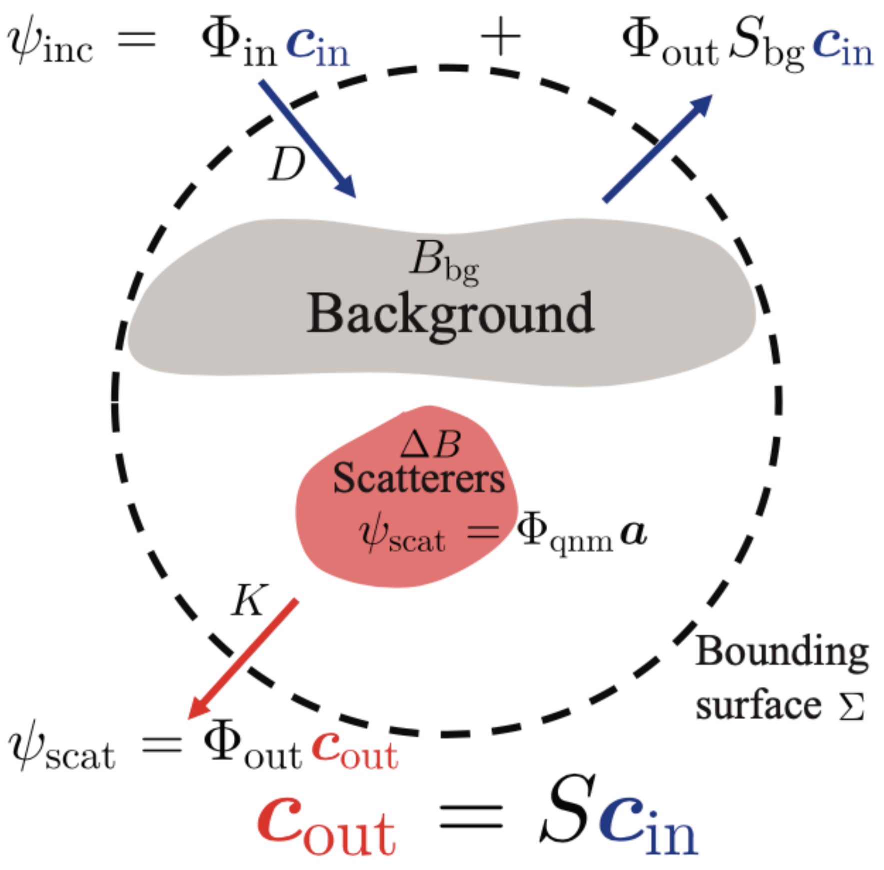
Separate into incident and scattered fields:

$$\psi = \psi_{inc} + \psi_{scat}$$

Solve for Quasinormal Modes (QNMs)

$$\Theta \psi_{R,m} = j\tilde{\omega}_m B(\tilde{\omega}_m)\psi_{R,m}$$

$$\Theta \psi_{L,n} = j\tilde{\omega}_n B^T(\tilde{\omega}_n)\psi_{L,n}$$



Zhang and Miller (2020)

CMT Applied to Scattering Problem

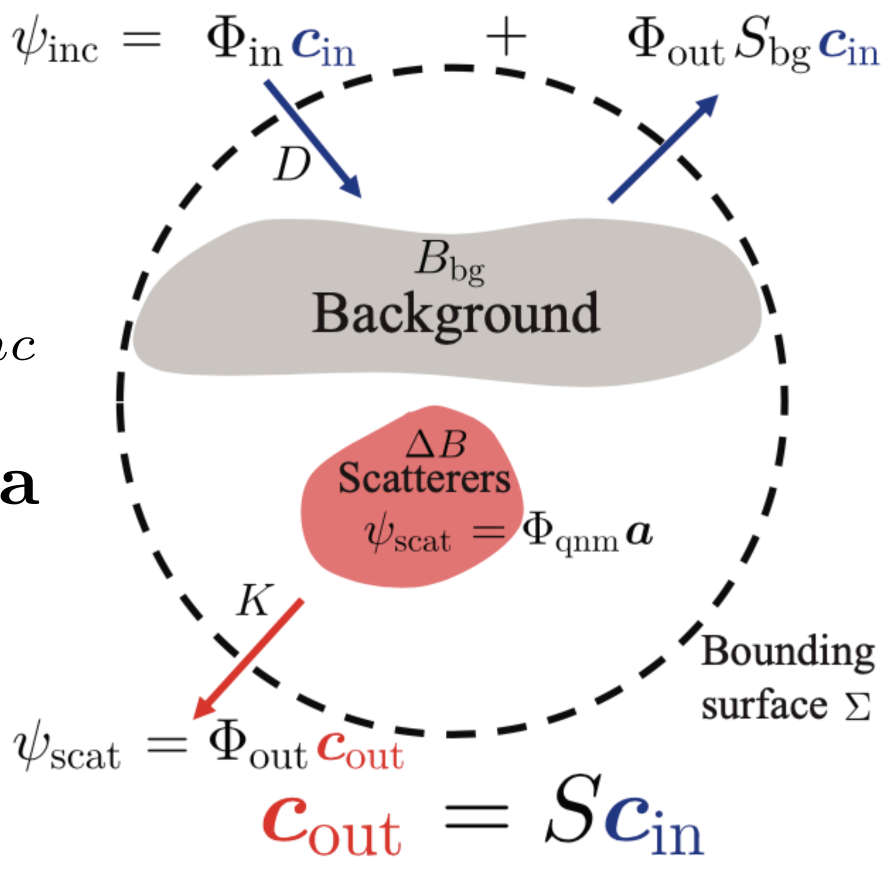
QCMT Equations:

$$i(\Omega - \omega)\mathbf{a} = D^T(\omega)\mathbf{c}_{in}$$

$$\mathbf{c}_{out} = [S_{bg} + \frac{1}{4\alpha\beta^*} i\omega(\Phi_{inc}^{TR}, \Delta B \Phi_{inc})]\mathbf{c}_{inc} + K(\omega)\mathbf{a}$$

Scattering Response:

$$S = \overbrace{S_{bg} + H(\omega) + i\tilde{K}\Omega^{-1}\tilde{D}^T}^{\text{background}} \underbrace{- i\tilde{K}(\Omega - \omega)^{-1}\tilde{D}^T}_{\text{resonance}}$$



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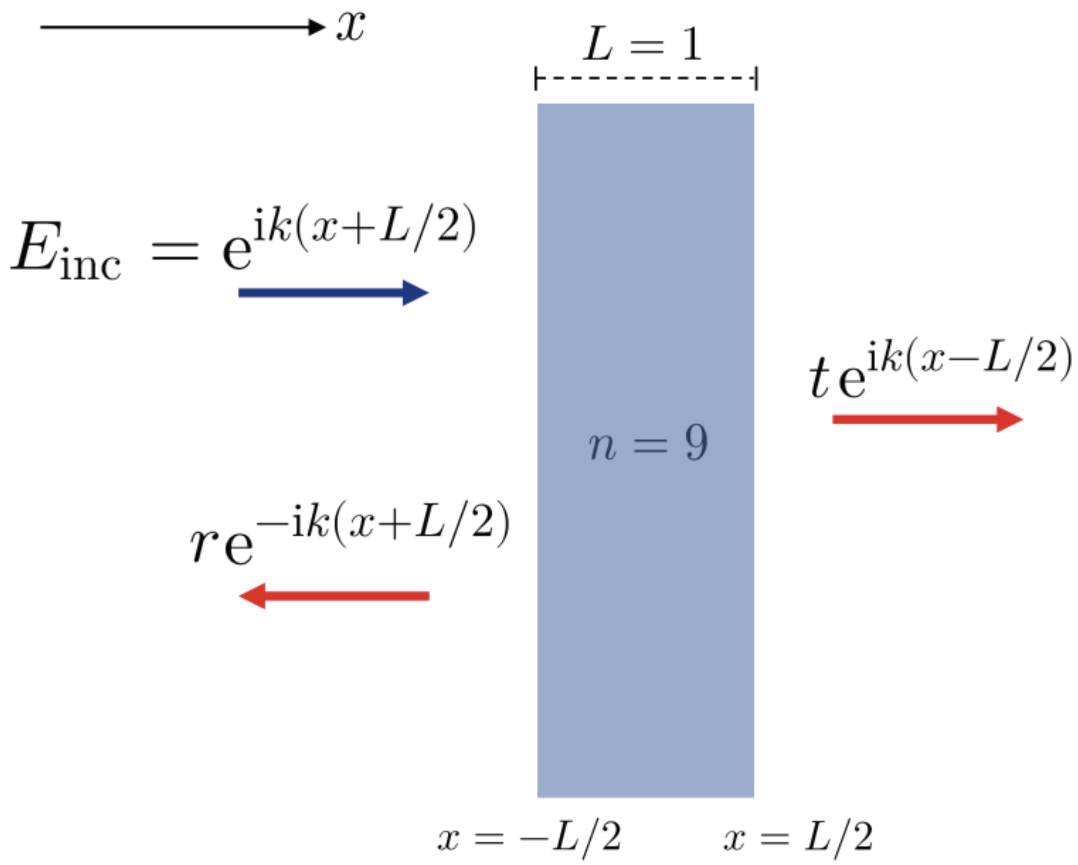
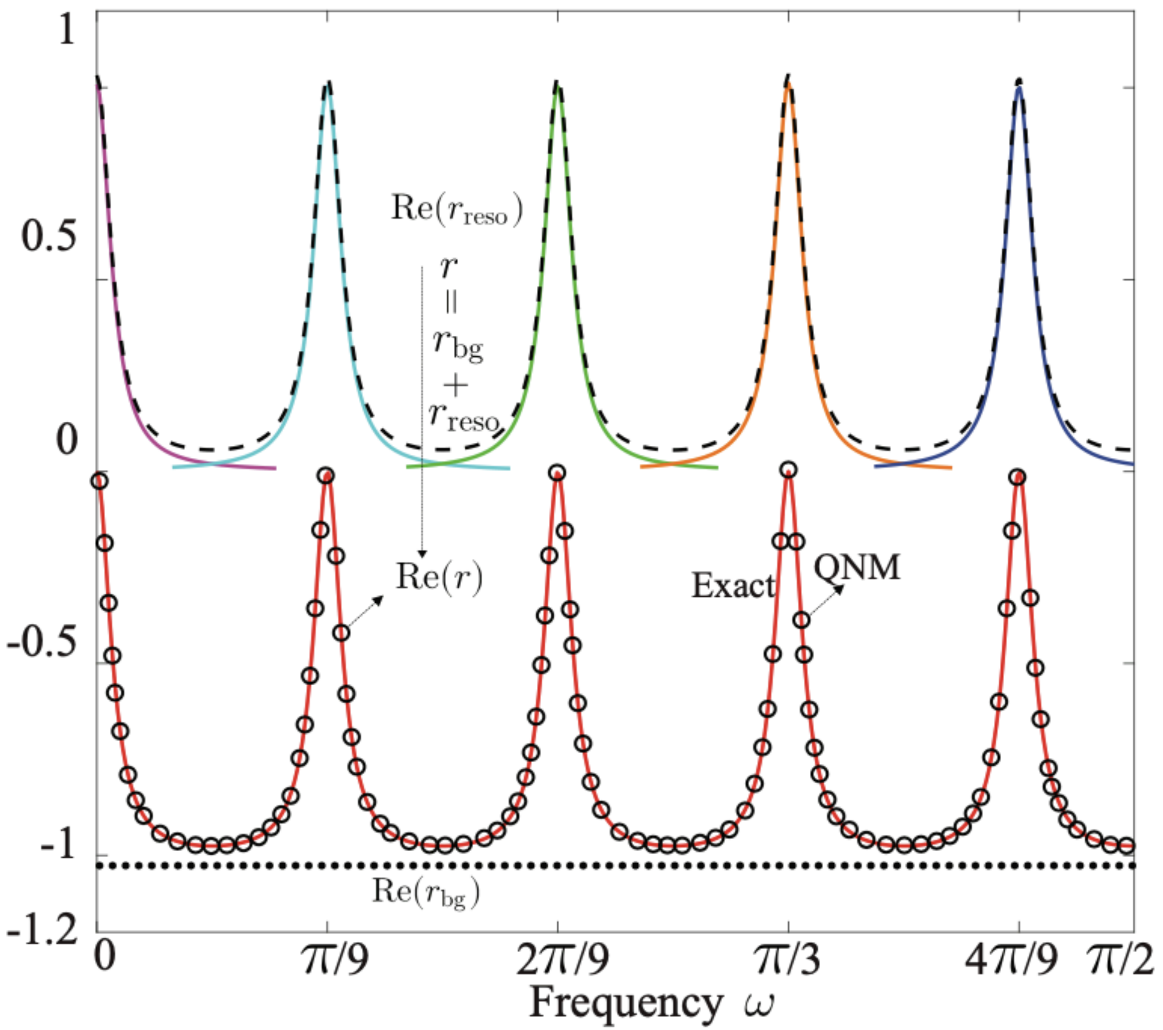
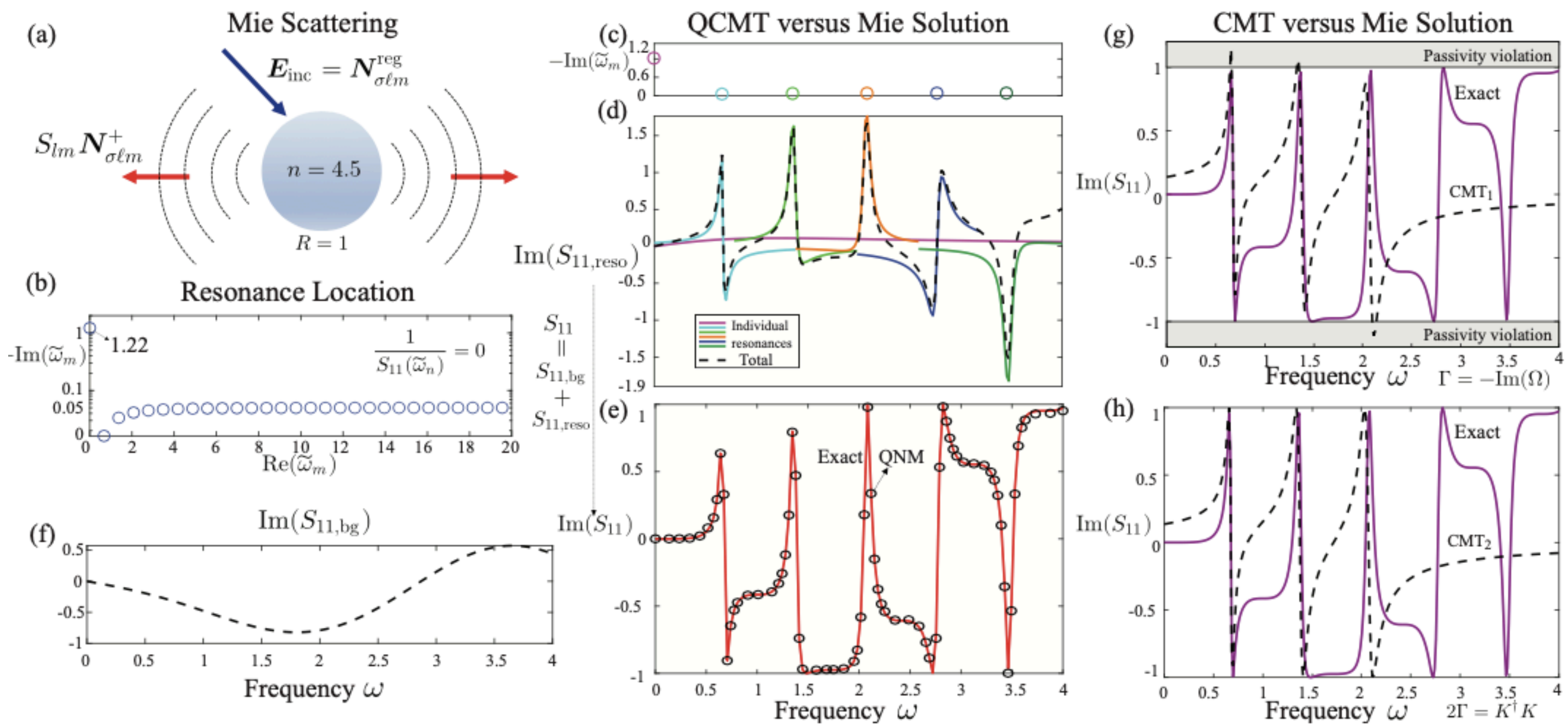
Example: FP Scattering





Zhang and Miller (2020)

Example: Mie Scattering



Zhang and Miller (2020)

Conventional CMT vs QCMT

- QCMT → CMT when:
  - Small background term
  - Coupling is nearly frequency-independent

Conventional CMT vs QCMT

$$\frac{d}{dt}\mathbf{a}(t) = -i\Omega\mathbf{a}(t) + D^T\mathbf{c}_{in}(t)$$

$$\mathbf{c}_{out}(t) = S_{bg}\mathbf{c}_{in}(t) + K\mathbf{a}(t)$$

Conventional, closed system

Quasinormal, open system

$$\frac{d}{dt}\mathbf{a}(t) = -i\Omega\mathbf{a}(t) + \int D^T(t-t^{'})\mathbf{c}_{in}(t-t^{'})dt^{'}$$

$$\mathbf{c}_{out}(t) = \int [S_{bg}(t-t^{'}) + E(t-t^{'}]\mathbf{c}_{in}(t^{'})dt^{'}$$

$$+\int K(t-t^{'}\mathbf{a}(t^{'})dt^{'}$$

References

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[4] Hanwen Zhang and Owen D. Miller. “Quasinormal Coupled Mode Theory”. In: arXiv:2010.08650 [physics] (Oct. 20, 2020). arXiv: 2010.08650. url: http://arxiv.org/abs/2010.08650 (vis- ited on 05/10/2021).

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