Домашнее задание №1

Табличное интегрирование

sltKaguya

25 февраля 2022 г. — 27 февраля 2022 г.

1. $\int \frac{x-1}{x^2-x-1}dx$:

$$\int \frac{x-1}{x^2-x-1}dx = \int \frac{d(\frac{x^2}{2}-x)}{x^2-x-1} = \frac{1}{2}\int \frac{d(x^2-2x)}{x^2-x-1} =$$

$$= \frac{1}{2}\left(\int \frac{d(x^2-x-1)}{x^2-x-1} - \int \frac{dx}{x^2-x-1}\right) = \left[\begin{array}{ccc} t & = & x^2-x-1 \end{array}\right] =$$

$$= \frac{1}{2}\left(\int \frac{dt}{t} - \int \frac{dx}{\left(x-\frac{1}{2}\right)^2-\frac{5}{4}}\right) = \left[\begin{array}{ccc} u & = & x-\frac{1}{2} \\ du & = & dx \end{array}\right] = \frac{1}{2}\left(\ln|t| - \int \frac{du}{u^2-\frac{5}{4}}\right) =$$

$$\frac{1}{2}\left(\ln|t| + \int \frac{du}{\frac{5}{4}-u^2}\right) = \frac{\ln|x^2-x-1|}{2} + \frac{1}{4\sqrt{\frac{5}{4}}}\cdot\ln\left|\frac{\sqrt{\frac{5}{4}}+x-\frac{1}{2}}{\sqrt{\frac{5}{4}}-x+\frac{1}{2}}\right| + C$$

Ответ: $\frac{\ln|x^2-x-1|}{2} + \frac{1}{4\sqrt{\frac{5}{4}}}\cdot\ln\left|\frac{\sqrt{\frac{5}{4}}+x-\frac{1}{2}}{\sqrt{\frac{5}{4}}-x+\frac{1}{2}}\right| + C$

2. $\int \frac{3x-6}{\sqrt{x^2-4x+5}}dx$:

$$\int \frac{3x-6}{\sqrt{x^2-4x+5}}dx = 3\int \frac{x-2}{\sqrt{\left(x-2\right)^2+1}}dx = \left[\begin{array}{ccc} t & = & x-2 \\ dt & = & dx \end{array}\right] = 3\int \frac{t}{\sqrt{t^2+1}}dt =$$

$$= \left[\begin{array}{ccc} u & = & t^2+1 \\ du & = & 2tdt \\ dt & = & \frac{du}{2t} \end{array}\right] = \frac{3}{2}\int \frac{du}{\sqrt{u}} = \frac{3}{2}\int u^{-\frac{1}{2}}du = \frac{3u^{\frac{1}{2}}}{2\cdot\frac{1}{2}} + C = 3\sqrt{(x-2)^2+1} + C$$

Ответ: $3\sqrt{x^2-4x+5}+C$

3. $\int \frac{x^2-1}{x^4+1}dx$:

$$\int \frac{x^2-1}{x^4+1}dx = \int \frac{x^2(1-\frac{1}{x^2})}{x^2(x^2+\frac{1}{x^2})}dx = \int \frac{1-\frac{1}{x^2}}{(x+\frac{1}{x})^2-2}dx = \left[\begin{array}{ccc} t & = & x+\dfrac{1}{x} \\ dt & = & (1+\dfrac{1}{x^2})dx \\ dx & = & \dfrac{dt}{1+\frac{1}{x^2}} \end{array}\right] =$$

$$= \int \frac{(1-\frac{1}{x^2})dt}{(t^2-2)(1-\frac{1}{x^2})} = -\int \frac{dt}{2-t^2} = -\frac{1}{2\sqrt{2}}\ln\left|\frac{\sqrt{2}+x+\frac{1}{x}}{\sqrt{2}-x-\frac{1}{x}}\right| + C$$

Ответ: $-\frac{1}{2\sqrt{2}}\ln\left|\frac{\sqrt{2}+x+\frac{1}{x}}{\sqrt{2}-x-\frac{1}{x}}\right| + C$

4. $\int \frac{e^x dx}{\sqrt{4-e^{2x}}}$:

$$\int \frac{e^x dx}{\sqrt{4-e^{2x}}} = \begin{bmatrix} t & = & e^x \\ dt & = & e^x dx \\ dx & = & \frac{dt}{e^x} \\ t^2 & = & e^{2x} \end{bmatrix} = \int \frac{tdt}{t\sqrt{2^2-t^2}} = \arcsin\frac{t}{a} + C = \arcsin\frac{e^x}{2} + C$$

Ответ: $\arcsin\frac{e^x}{2} + C$

5. $\int \frac{dx}{x\ln x \ln\ln x}$:

$$\int \frac{dx}{x\ln x\ln\ln x} = \begin{bmatrix} t & = & \ln x \\ dt & = & \frac{dx}{x} \\ dx & = & xdt \end{bmatrix} = \int \frac{xdt}{xt\ln t} = \int \frac{d(\ln t)}{\ln t} = \ln|\ln t| + C =$$

$$= \ln|\ln\ln x| + C$$

Ответ: $\ln|\ln\ln x| + C$

6. $\int \frac{\sin x dx}{\sqrt{1+2\cos x}}$:

$$\int \frac{\sin x dx}{\sqrt{1+2\cos x}} = \begin{bmatrix} t & = & 1+2\cos x \\ dt & = & -2\sin x dx \\ dx & = & \frac{dt}{-2\sin x} \end{bmatrix} = \int \frac{\sin x dt}{-2\sqrt{t}\sin x} = -\frac{1}{2}\int \frac{dt}{\sqrt{t}} =$$

$$= -\frac{1}{2}\int t^{-\frac{1}{2}} dt = -\frac{1}{2}\cdot\frac{t^{\frac{1}{2}}}{\frac{1}{2}} + C = -\sqrt{1+2\cos x} + C$$

Ответ: $-\sqrt{1+2\cos x} + C$

7. $\int \frac{\ln\tan x}{\sin(2x)} dx$:

$$\int \frac{\ln\tan x}{\sin(2x)} dx = \begin{bmatrix} t & = & \tan x \\ dt & = & \frac{dx}{\cos^2 x} \\ dx & = & dt\cos^2 x \\ \sin x & = & t\cos x \end{bmatrix} = \int \frac{\ln t\cos^2 x dt}{2\sin x\cos x} = \int \frac{\ln t\cos x dt}{2t\cos x} =$$

$$= \frac{1}{2}\int \frac{\ln t dt}{t} = \begin{bmatrix} u & = & \ln t \\ du & = & \frac{dt}{t} \\ dt & = & tdu \end{bmatrix} = \frac{1}{2}\int \frac{utdu}{t} = \frac{1}{2}\int udu = \frac{u^2}{4} + C = \frac{\ln^2 t}{4} + C =$$

$$= \frac{\ln^2\tan x}{4} + C$$

Ответ: $\frac{\ln^2\tan x}{4} + C$

8. $\int \frac{\arctan\sqrt{x}}{(1+x)\sqrt{x}} dx$:

$$\int \frac{\arctan\sqrt{x}}{(1+x)\sqrt{x}} dx = \begin{bmatrix} t & = & \arctan\sqrt{x} \\ dt & = & \frac{dx}{(1+x)2\sqrt{x}} \\ dx & = & 2\sqrt{x}(1+x)dt \end{bmatrix} = \int \frac{2t\sqrt{x}(1+x)dt}{(1+x)\sqrt{x}} = 2\int tdt =$$

$$= 2\frac{t^2}{2} + C = \arctan^2\sqrt{x} + C$$

9. $\int \frac{\arccos^2 2x}{\sqrt{1-4x^2}}dx$:

$$\int \frac{\arccos^2 2x}{\sqrt{1-4x^2}}dx = \left[\begin{array}{rcl} t & = & \arccos^2 2x \\ dt & = & -\frac{dx}{\sqrt{1-4x^2}} \\ dx & = & -dt\sqrt{1-4x^2} \end{array}\right] = -\int \frac{tdt\sqrt{1-4x^2}}{\sqrt{1-4x^2}} = -\int tdt =$$

$$= -\frac{t^2}{2} + C = -\frac{\arccos^2 2x}{2} + C$$

Ответ: $-\frac{\arccos^2 2x}{2} + C$

10. $\int \arccos(5x-2)dx$:

$$\int \arccos(5x-2)dx = \frac{1}{5}\int \arccos(5x-2)d(5x-2) = \left[\begin{array}{rcl} t & = & 5x-2 \\ dt & = & d(5x-2) \end{array}\right] =$$

$$= \frac{1}{5}\int \arccos(t)dt = \left[\begin{array}{rcl} u & = & \arccos t \\ dv & = & dt \end{array} \implies \begin{array}{rcl} du & = & -\frac{dt}{\sqrt{1-t^2}} \\ v & = & t \end{array}\right] =$$

$$= \frac{1}{5}\left(\arccos t \cdot t + \int \frac{tdt}{\sqrt{1-t^2}}\right) = \frac{t\arccos t}{5} + \frac{1}{10}\int (1-t^2)^{-\frac{1}{2}}d(t^2) =$$

$$= \frac{t\arccos t}{5} - \frac{1}{10}\int (1-t^2)^{-\frac{1}{2}}d(1-t^2) = \frac{t\arccos t}{5} - \frac{(1-t^2)^{\frac{1}{2}}}{5} + C =$$

$$= \frac{(5x-2)\arccos(5x-2) - \sqrt{1-(5x-2)^2}}{5} + C$$

Ответ: $\frac{(5x-2)\arccos(5x-2)-\sqrt{1-(5x-2)^2}}{5} + C$

11. $\int \frac{\arcsin x}{x^2}dx$:

$$\int \frac{\arcsin x}{x^2}dx = \left[\begin{array}{rcl} u & = & \arcsin x \\ dv & = & \frac{dx}{x^2} \end{array} \implies \begin{array}{rcl} du & = & \frac{dx}{\sqrt{1-x^2}} \\ v & = & -\frac{1}{x} \end{array}\right] =$$

$$= -\frac{\arcsin x}{x} + \int \frac{dx}{x\sqrt{1-x^2}} = \left[\begin{array}{rcl} t & = & \sqrt{1-x^2} \\ dt & = & -\frac{xdx}{\sqrt{1-x^2}} \\ dx & = & -\frac{\sqrt{1-x^2}dt}{x} \\ x^2 & = & 1-t^2 \end{array}\right] = -\frac{\arcsin x}{x} - \int \frac{\sqrt{1-x^2}dt}{x^2\sqrt{1-x^2}} =$$

$$= -\frac{\arcsin x}{x} - \int \frac{dt}{1-t^2} = -\frac{\arcsin x}{x} - \frac{1}{2a}\ln\left|\frac{1+t}{1-t}\right| + C =$$

$$= -\left(\frac{\arcsin x}{x} + \frac{1}{2a}\ln\left|\frac{1+\sqrt{1-x^2}}{1-\sqrt{1-x^2}}\right|\right) + C$$

Ответ: $-\left(\frac{\arcsin x}{x} + \frac{1}{2a}\ln\left|\frac{1+\sqrt{1-x^2}}{1-\sqrt{1-x^2}}\right|\right) + C$

12. $\int \left(\frac{\ln x}{x}\right)^3 dx$:

$$\int \left(\frac{\ln x}{x}\right)^3 dx = \left[\begin{array}{rcl} u & = & \ln^3 x \\ dv & = & \frac{dx}{x^3} \end{array} \implies \begin{array}{rcl} du & = & \frac{3\ln^2 x dx}{x} \\ v & = & -\frac{1}{2x^2} \end{array}\right] =$$

$$= -\frac{\ln^3 x}{2x^2} + \frac{3}{2}\int \frac{\ln^2 x dx}{x^3} = \left[\begin{array}{rcl} u & = & \ln^2 x \\ dv & = & \frac{dx}{x^3} \end{array} \implies \begin{array}{rcl} du & = & \frac{2\ln x dx}{x} \\ v & = & -\frac{1}{2x^2} \end{array}\right] =$$

$$= -\frac{\ln^3 x}{2x^2} - \frac{3\ln^2 x}{4x^2} + \frac{3}{2}\int \frac{\ln x dx}{x^3} = \left[\begin{array}{ccc} u & = & \ln x \\ dv & = & \frac{dx}{x^3} \end{array} \implies \begin{array}{ccc} du & = & \frac{dx}{x} \\ v & = & -\frac{1}{2x^2} \end{array} \right] =$$

$$= -\frac{\ln^3 x}{2x^2} - \frac{3\ln^2 x}{4x^2} - \frac{3\ln x}{4x^2} + \frac{3}{4}\int \frac{dx}{x^3} = -\frac{\ln^3 x}{2x^2} - \frac{3\ln^2 x}{4x^2} - \frac{3\ln x}{4x^2} - \frac{3}{8x^2} + C$$

Ответ: $-\frac{\ln^3 x}{2x^2} - \frac{3\ln^2 x}{4x^2} - \frac{3\ln x}{4x^2} - \frac{3}{8x^2} + C$

13. $\int x^2\sqrt{x^2+1}dx$:

14. $\int xe^x \sin x dx$:

15. $\int \cos^5 x dx$:

$$\int \cos^5 x dx = \frac{\sin x \cos^4 x}{5} + \frac{4}{5}\int \cos^3 x dx = \frac{\sin x \cos^4 x}{5} + \frac{4\sin x \cos^2 x}{25} + \frac{8}{15}\int \cos x dx =$$

$$= \frac{\sin x \cos^4 x}{5} + \frac{4\sin x \cos^2 x}{25} + \frac{8\sin x}{15} + C$$

Ответ: $\frac{\sin x \cos^4 x}{5} + \frac{4\sin x \cos^2 x}{25} + \frac{8\sin x}{15} + C$

16. $\int \frac{x^2e^x}{(x+2)^2}dx$:

$$\int \frac{x^2e^x}{(x+2)^2}dx = \left[\begin{array}{ccc} u & = & x^2e^x \\ dv & = & \frac{dx}{(x+2)^2} \end{array} \implies \begin{array}{ccc} du & = & (2+x)xe^x dx \\ v & = & -\frac{1}{x+2} \end{array} \right] =$$

$$= -\frac{x^2e^x}{x+2} + \int \frac{(2+x)xe^x dx}{(2+x)} = -\frac{x^2e^x}{x+2} + \int xe^x dx =$$

$$= \left[\begin{array}{ccc} u & = & x \\ dv & = & e^x dx \end{array} \implies \begin{array}{ccc} du & = & dx \\ v & = & e^x \end{array} \right] = \frac{x^2e^x}{(x+2)} + xe^x - \int e^x dx = \frac{x^2e^x}{(x+2)} + xe^x - e^x + C$$

Ответ: $e^x\left(\frac{x^2}{(x+2)} + x - 1\right) + C$