

# Домашнее задание №1

## Табличное интегрирование

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1.  $\int \frac{x-1}{x^2-x-1} dx$ :

$$\begin{aligned}\int \frac{x-1}{x^2-x-1} dx &= \int \frac{d(\frac{x^2}{2}-x)}{x^2-x-1} = \frac{1}{2} \int \frac{d(x^2-2x)}{x^2-x-1} = \\&= \frac{1}{2} \left( \int \frac{d(x^2-x-1)}{x^2-x-1} - \int \frac{dx}{x^2-x-1} \right) = \left[ t = x^2-x-1 \right] = \\&= \frac{1}{2} \left( \int \frac{dt}{t} - \int \frac{dx}{(x-\frac{1}{2})^2 - \frac{5}{4}} \right) = \left[ \frac{u}{du} = \frac{x-\frac{1}{2}}{dx} \right] = \frac{1}{2} \left( \ln|t| - \int \frac{du}{u^2 - \frac{5}{4}} \right) = \\&\frac{1}{2} \left( \ln|t| + \int \frac{du}{\frac{5}{4} - u^2} \right) = \frac{\ln|x^2-x-1|}{2} + \frac{1}{4\sqrt{\frac{5}{4}}} \cdot \ln \left| \frac{\sqrt{\frac{5}{4}} + x - \frac{1}{2}}{\sqrt{\frac{5}{4}} - x + \frac{1}{2}} \right| + C\end{aligned}$$

Ответ:  $\frac{\ln|x^2-x-1|}{2} + \frac{1}{4\sqrt{\frac{5}{4}}} \cdot \ln \left| \frac{\sqrt{\frac{5}{4}} + x - \frac{1}{2}}{\sqrt{\frac{5}{4}} - x + \frac{1}{2}} \right| + C$

2.  $\int \frac{3x-6}{\sqrt{x^2-4x+5}} dx$ :

$$\begin{aligned}\int \frac{3x-6}{\sqrt{x^2-4x+5}} dx &= 3 \int \frac{x-2}{\sqrt{(x-2)^2+1}} dx = \left[ \frac{t}{dt} = \frac{x-2}{dx} \right] = 3 \int \frac{t}{\sqrt{t^2+1}} dt = \\&= \left[ \begin{array}{l} u = t^2+1 \\ du = 2t dt \\ dt = \frac{du}{2t} \end{array} \right] = \frac{3}{2} \int \frac{du}{\sqrt{u}} = \frac{3}{2} \int u^{-\frac{1}{2}} du = \frac{3u^{\frac{1}{2}}}{2 \cdot \frac{1}{2}} + C = 3\sqrt{(x-2)^2+1} + C\end{aligned}$$

Ответ:  $3\sqrt{x^2-4x+5} + C$

3.  $\int \frac{x^2-1}{x^4+1} dx$ :

$$\begin{aligned}\int \frac{x^2-1}{x^4+1} dx &= \int \frac{x^2(1-\frac{1}{x^2})}{x^2(x^2+\frac{1}{x^2})} dx = \int \frac{1-\frac{1}{x^2}}{(x+\frac{1}{x})^2-2} dx = \left[ \begin{array}{l} t = x + \frac{1}{x} \\ dt = (1 + \frac{1}{x^2}) dx \\ dx = \frac{dt}{1 + \frac{1}{x^2}} \end{array} \right] = \\&= \int \frac{(1-\frac{1}{x^2}) dt}{(t^2-2)(1+\frac{1}{x^2})} = - \int \frac{dt}{2-t^2} = -\frac{1}{2\sqrt{2}} \ln \left| \frac{\sqrt{2}+x+\frac{1}{x}}{\sqrt{2}-x-\frac{1}{x}} \right| + C\end{aligned}$$

Ответ:  $-\frac{1}{2\sqrt{2}} \ln \left| \frac{\sqrt{2}+x+\frac{1}{x}}{\sqrt{2}-x-\frac{1}{x}} \right| + C$