

1. The graph¹ of Europe² $\mathcal{G}^* = \langle V, E \rangle$ is defined as follows: each vertex $v \in V$ is a Europe country; two vertices are adjacent ($\{u, v\} \in E$) if the corresponding countries share a land border. Let \mathcal{G} be the largest connected component of \mathcal{G}^* .
 - (a) Draw \mathcal{G}^* with the minimum number of intersecting edges.
 - (b) Find $|V|$, $|E|$, $\delta(\mathcal{G})$, $\Delta(\mathcal{G})$, $\text{rad}(\mathcal{G})$, $\text{diam}(\mathcal{G})$, $\text{girth}(\mathcal{G})$, $\text{center}(\mathcal{G})$, $\kappa(\mathcal{G})$, $\lambda(\mathcal{G})$.
 - (c) Find the minimum vertex coloring $Z : V \rightarrow \mathbb{N}$ of \mathcal{G} .
 - (d) Find the minimum edge coloring $X : E \rightarrow \mathbb{N}$ of \mathcal{G} .
 - (e) Find the maximum clique $Q \subseteq V$ of \mathcal{G} .
 - (f) Find the maximum stable set $S \subseteq V$ of \mathcal{G} .
 - (g) Find the maximum matching $M \subseteq E$ of \mathcal{G} .
 - (h) Find the minimum vertex cover $R \subseteq V$ of \mathcal{G} .
 - (i) Find the minimum edge cover $F \subseteq E$ of \mathcal{G} .
 - (j) Find the shortest closed walk W that visits every vertex of \mathcal{G} .
 - (k) Find the shortest closed walk U that visits every edge of \mathcal{G} .
 - (l) Find all biconnected components (blocks) and draw the block-cut tree of \mathcal{G}^* .
 - (m) Find all 2-edge-connected components of \mathcal{G}^* .
 - (n) Construct an SPQR tree of the largest biconnected component of \mathcal{G}^* .
 - (o) Add the weight function $w : E \rightarrow \mathbb{R}$ denoting the distance between capitals. Find the minimum (w.r.t. the total weight of edges) spanning tree T for the largest connected component of the weighted Europe graph $\mathcal{G}_w^* = (V, E, w)$.
 - (p) Find $\text{centroid}(T)$ (w.r.t. the edge weight function w).
 - (q) Construct the Prüfer code for T .

¹ Hereinafter, “graphs” are “simple, finite, undirected and unweighted”, unless stated otherwise.

² https://simple.wikipedia.org/wiki/List_of_European_countries used as reference.