1. Find the number of different 5-digit numbers using digits 1–9 under the given constraints. For each case, provide representative examples of (non-)complying numbers (*e.g.*, 12345 and 52814 are suitable for (b), but 44521 and 935 are not) and derive a generic[1] formula. Try to express the formula using standard combinatorial terms, *e.g.*, k-combs C_n^k and k-perms $P(n, k)$.

   (a) Digits *can* be repeated.

   - On the first place there could be one of the n digits, so is for the second place *etc.* until k.
   - So, there is n options for the first digit, for each option there is also n options for the second digit *etc.* until k:

   $$\underbrace{n \cdot n \cdot \ldots \cdot n}_{k \text{ times}}$$

   - $\therefore$ the formula to count the k-perms will be

   $$\overline{A}_n^k = n^k$$

   - *Example:* $\overline{A}_9^5 = 9^5 = 59049$

   (b) Digits *cannot* be repeated.

   - On the first place there could be one of the n digits, on the second – one of $n - 1$ remaining *etc.* until k.
   - So, there is n options for the first digit, for each option there is $n - 1$ options (all possible except the first) *etc.* until k:

   $$\underbrace{(n-0) \cdot (n-1) \cdot \ldots \cdot (n-(k-1))}_{\text{from } 0 \text{ to } k-1 \text{ as from } 1^{\text{st}} \text{ to } k^{\text{th}} \text{ digit}}$$

   To make this formula look more prettier and easier we can multiply and divide this by $(n-k)!$ then we'll get:

   $$\frac{(n-0) \cdot (n-1) \cdot \ldots \cdot (n-(k-1)) \cdot (n-k) \cdot \ldots \cdot 1}{(n-k)!} = \frac{n!}{(n-k)!}$$

   - $\therefore$ the formula to count the k-perms will be

   $$A_n^k = \frac{n!}{(n-k)!}$$

   - *Example:* $A_9^5 = \dfrac{9!}{(9-5)!} = 15120$

   (c) Digits *can* be repeated and must be written in *non-increasing*[2] order.

   - So, there is n options for the first digit, then, for each option i there is i options

   (d) Digits *cannot* be repeated and must be written in *increasing* order.

   (e) Digits *can* be repeated, must be written in *non-decreasing* order, and the 4th digit must be 6.

[1] Here, "generic formula" means "depending on the input data". In this particular example, $n = 9$ and $k = 5$, but the sought formula must also be valid for all other (adequate) values of n and k.
Consequently, in this homework abstract n and k will be used in proofs of formulas instead of given numbers.

[2] A sequence (x_n) is said to be *strictly monotonically increasing* if each term is *strictly greater* than the previous one, *i.e.* $x_i < x_{i+1}$.
A sequence is called *non-increasing* if each term is *less than or equal* to the previous one, *i.e.* $x_i \geqslant x_{i+1}$.