CHRISTOPHER PRENER, PH.D.
FALL, 2016

WEEK 06
LECTURE 08–EXTRA

QUANTITATIVE ANALYSIS

RANDOM VARIABLES

A NEGATIVE BINOMIAL DISTRIBUTION

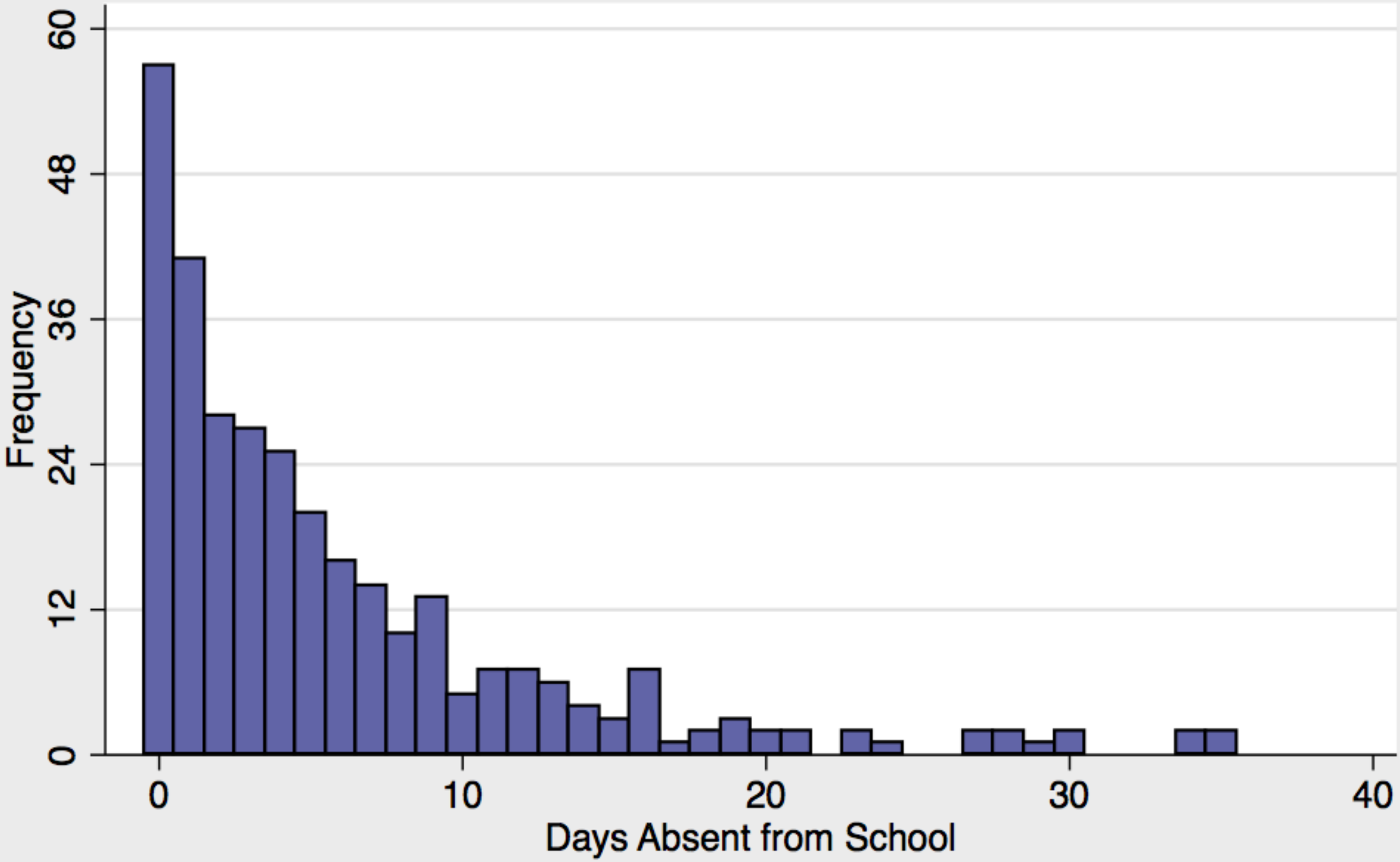
PROBLEM

- With the Poisson distribution, μ and σ^2 should be approximately equal.
- However, with many types of count data, σ^2 can be larger than μ. This is a condition known as *overdispersion*.

```
. use http://www.ats.ucla.edu/stat/stata/dae/nb_data, clear

. summarize daysabs

    Variable |        Obs        Mean    Std. Dev.       Min        Max
-------------+---------------------------------------------------------
     daysabs |        314    5.955414    7.036958           0         35


. display 7.036958^2
49.518778
```

Example of Overdispersed Count Data

Days Absent from School



Data via UCLA ATS

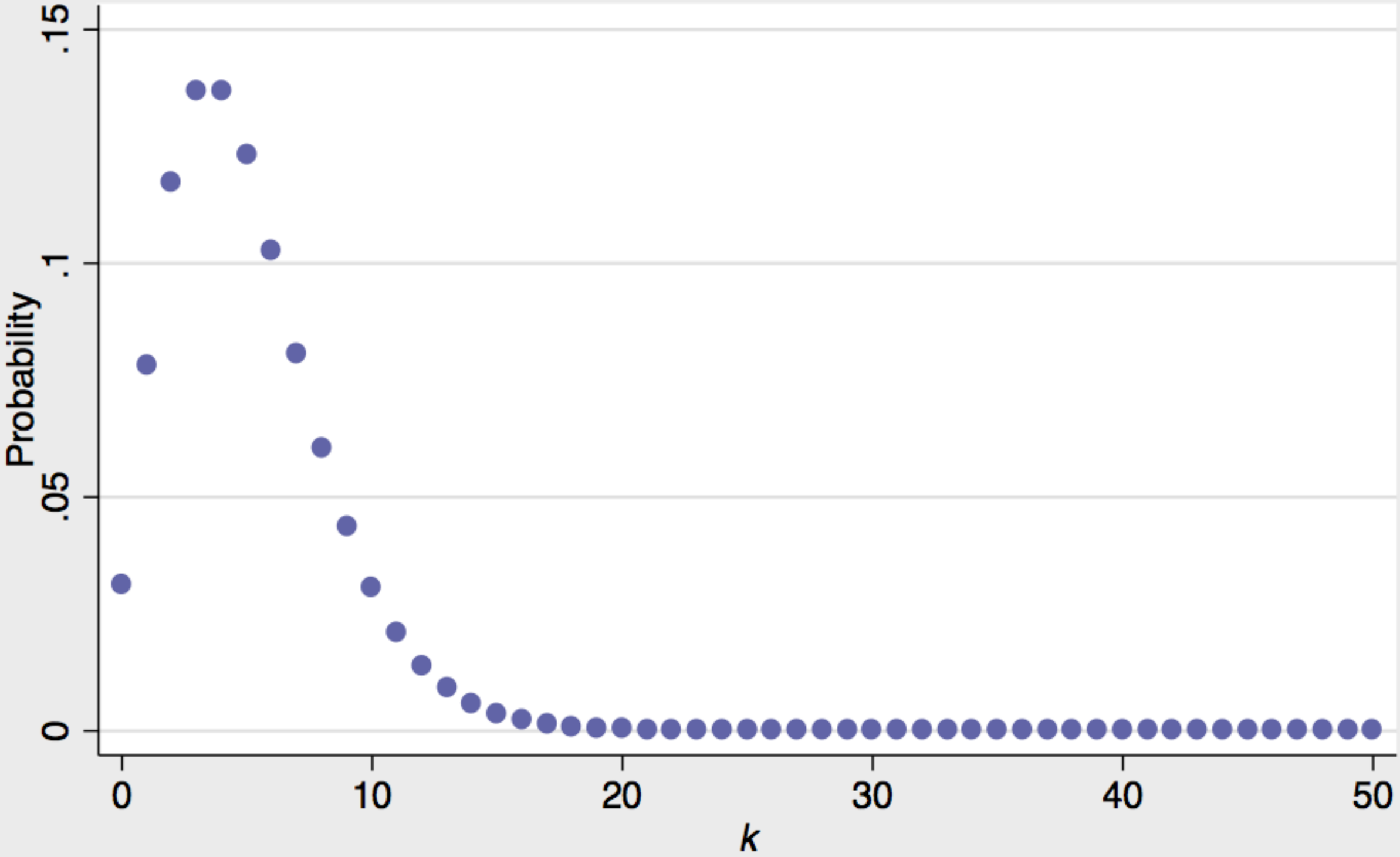
DEFINITION

- A sequence of independent trials with constant probability of success at each trial (p) where we are interested in the number of failures (k) required to produce a set number of successes (n).
- r is often used instead of n.
- Like the Poisson distribution, it is characterized by count data.
- Unlike the Poisson distribution, it is able to accommodate variances that differ from the mean.

$$P(X = x) = \binom{x-1}{r-1}(1-p)^{x-r}p^r$$

Negative Binomial Distribution Probability Mass Function

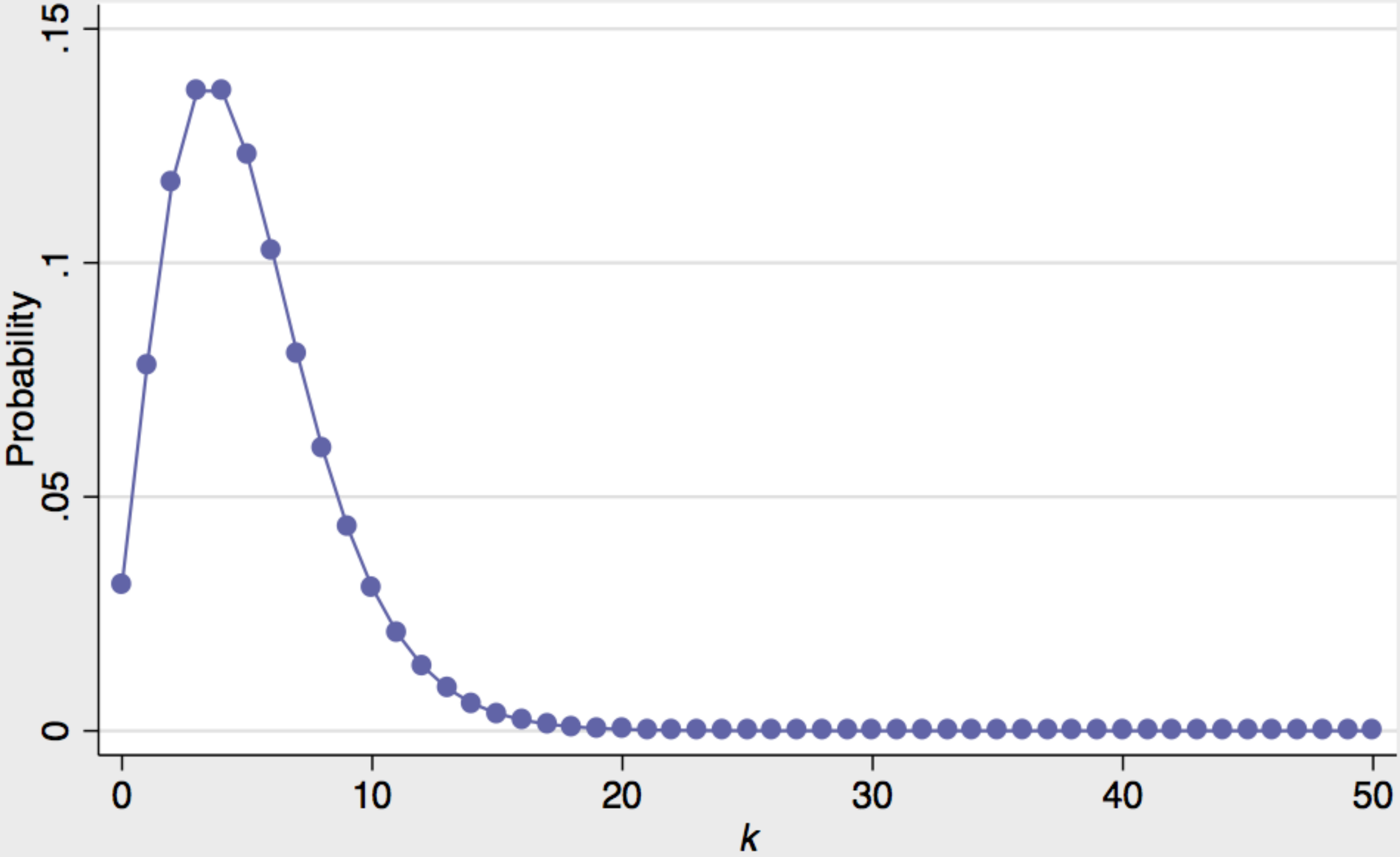
p=0.5 and n=5



Line between calculated probabilities included for visualization purposes only.

Negative Binomial Distribution Probability Mass Function

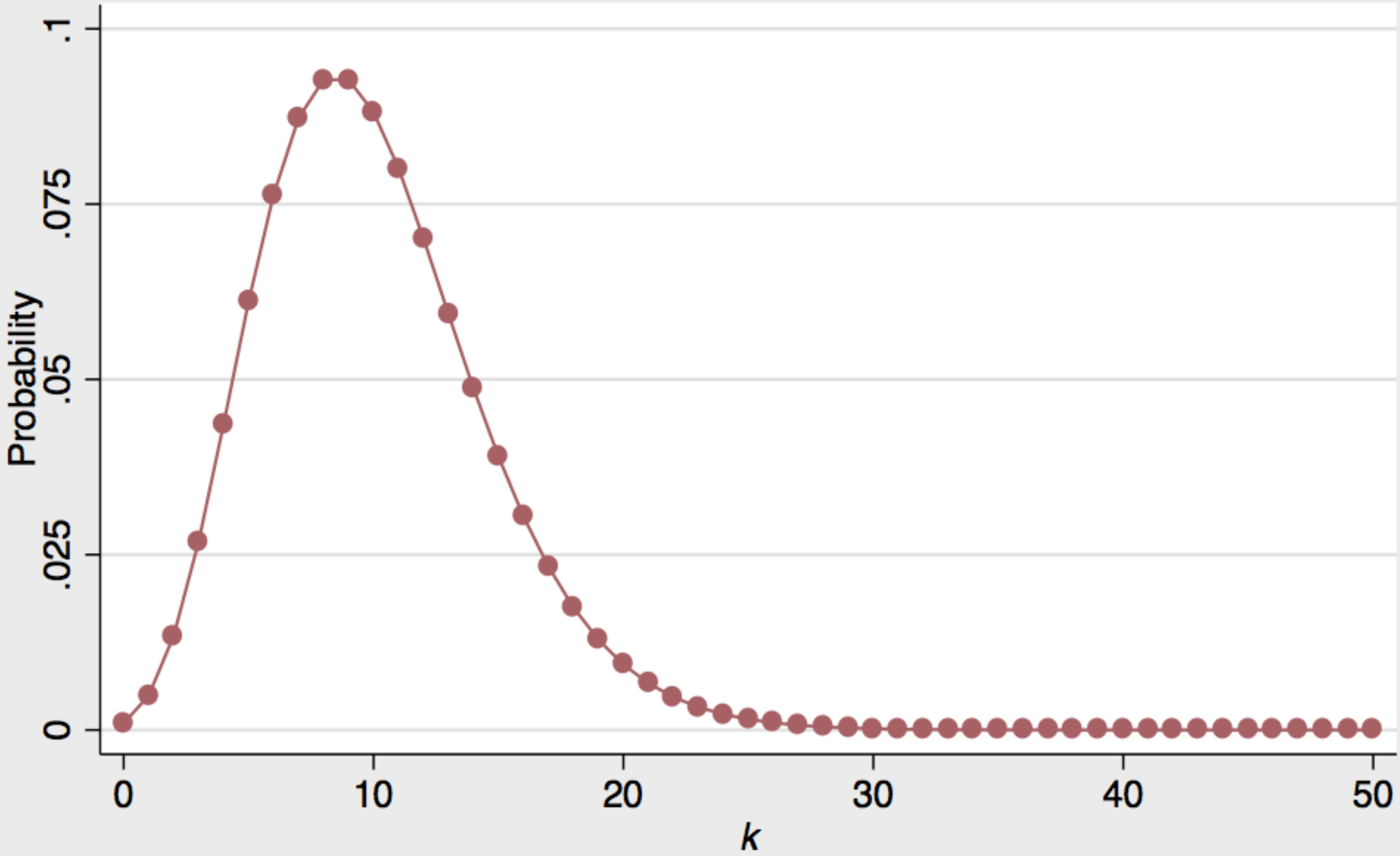
p=0.5 and n=5



Line between calculated probabilities included for visualization purposes only.

Negative Binomial Distribution Probability Mass Function

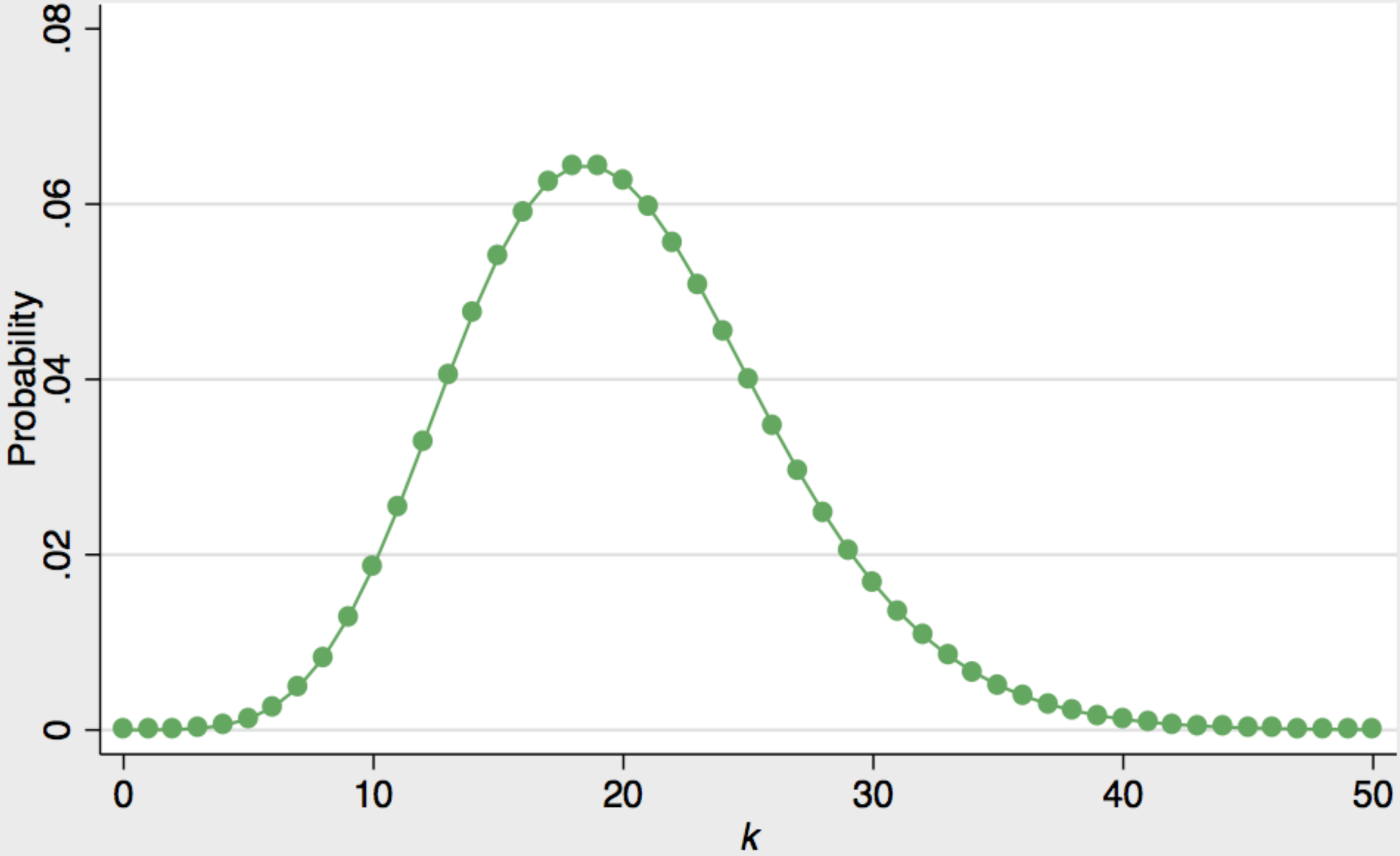
p=0.5 and n=10



Line between calculated probabilities included for visualization purposes only.

Negative Binomial Distribution Probability Mass Function

p=0.5 and n=20



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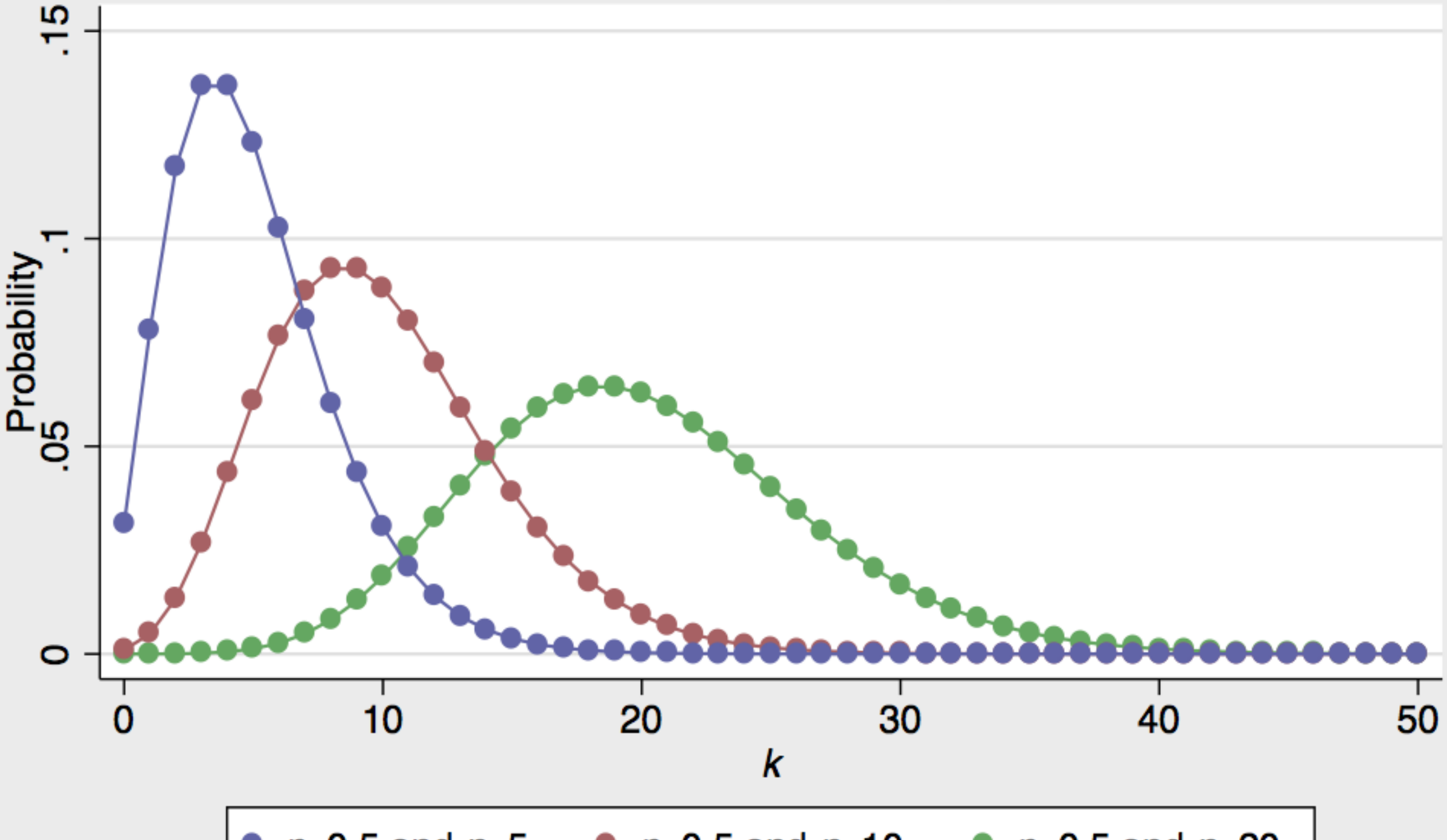
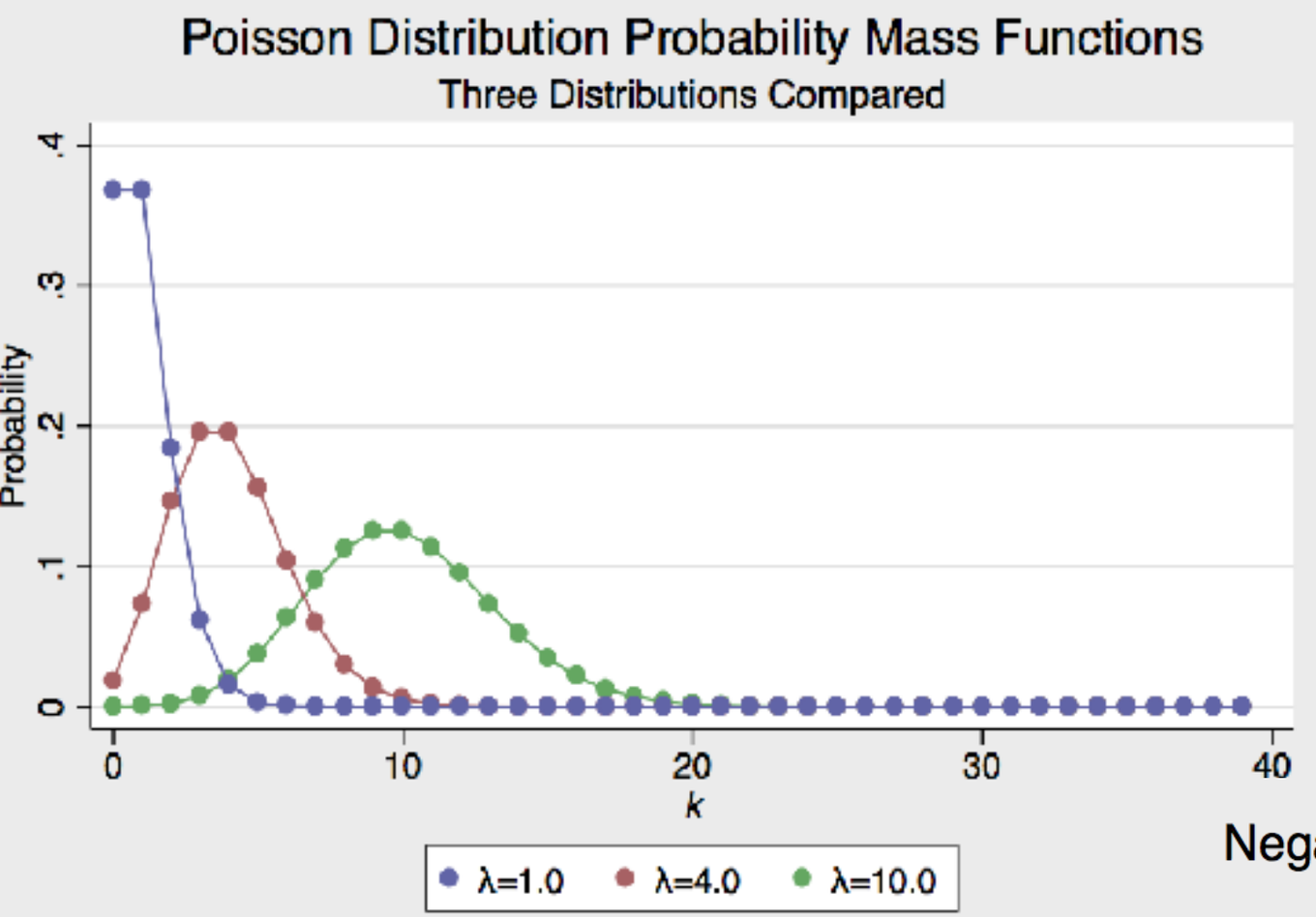
Negative Binomial Distribution Probability Mass Functions

Three Distributions Compared



Line between calculated probabilities included for visualization purposes only.



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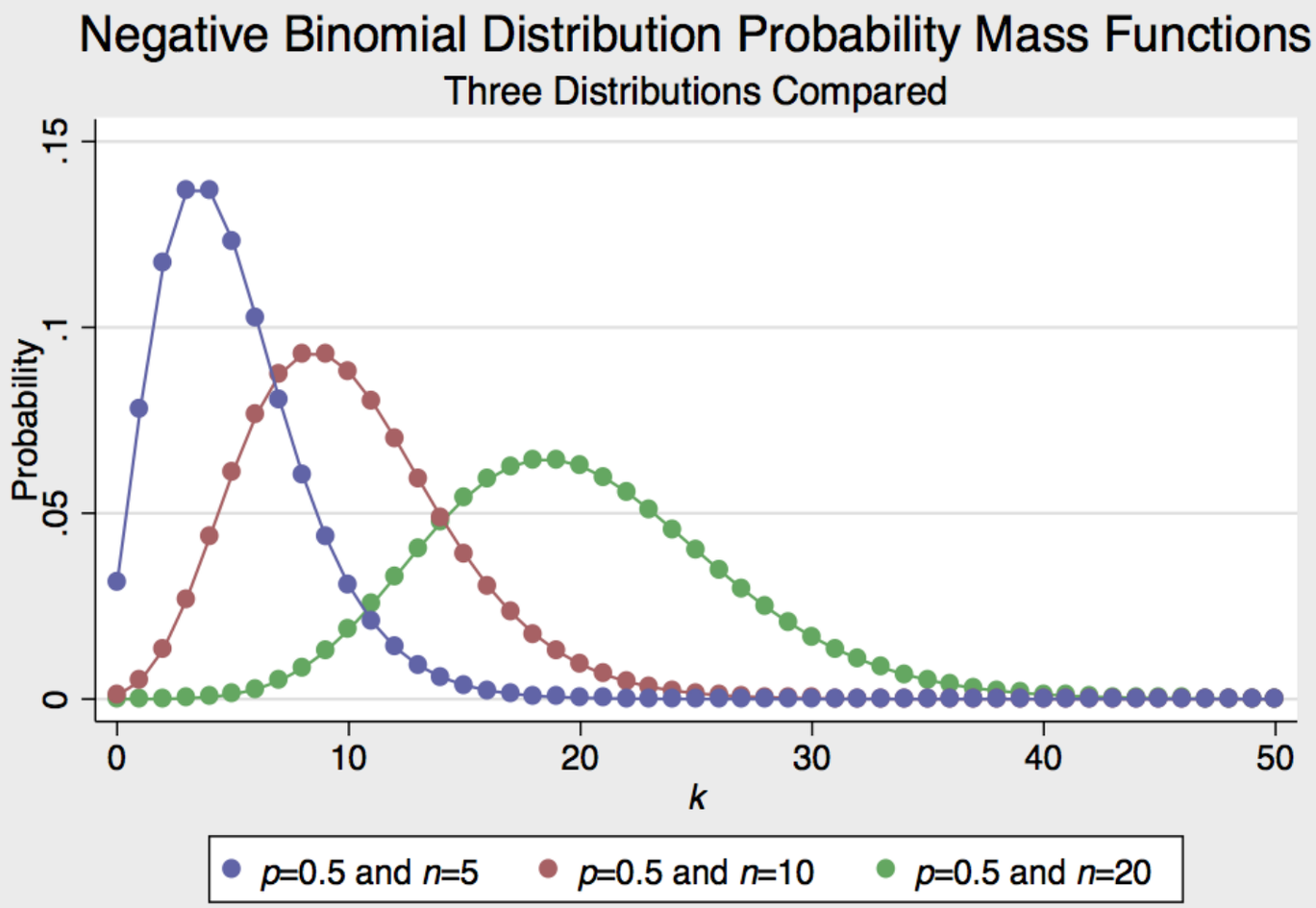

Line between calculated probabilities included for visualization purposes only.

STATA FUNCTIONS

`nbinomialp(n,k,p)`	returns probability of observing ***k*** failures before the n^{th} success	P(x = k)
`nbinomial(n,k,p)`	returns probability of observing ***k* or fewer** failures before the n^{th} success	P(x ≤ k)
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NEGATIVE BINOMIAL WORKFLOW

- A social service agency determines that 20% of clients approached on a given day will enroll in an intervention program. What is the probability that the first enrollment comes on the fifth client after four clients refuse (fail) to enroll?
- Is the negative binomial distribution appropriate?

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```
. display nbinomialp(1,4,.20)
.08192
```

DOCUMENT DETAILS

Document produced by Christopher Prener, Ph.D for the Saint Louis University course SOC 5050: QUANTITATIVE ANALYSIS - APPLIED INFERENTIAL STATISTICS. See the course wiki and the repository README.md file for additional details.