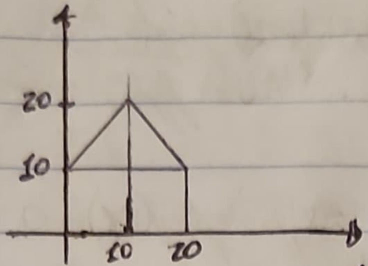
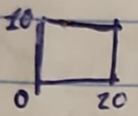
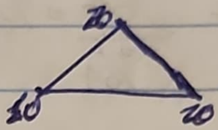
Avaliação 3 - Questão 7
Luan de Barros.

7



a) Determine o valor da constante K





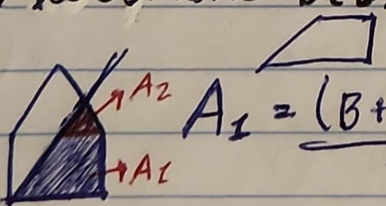
$A_1 = b.h = 20.10 = 200$

$A_2 = \frac{b.h}{2} = \frac{20.10}{2} = 100$

$A_t K = 1$

$K = \frac{1}{A_1 + A_2} = \frac{1}{200+100} = \frac{1}{300}$

b) Determine $Pr[X \geq Y]$



$A_1 = \frac{(B+b)h}{2} = \frac{(20+10).10}{2} = 150$

$A_2 = \frac{b.h}{2} = \frac{10.5}{2} = 25$

$Pr[X \geq Y] = \frac{K.A_1 + A_2}{A_1 + A_2} = \frac{K.150 + 25}{200+100} = 7/12$

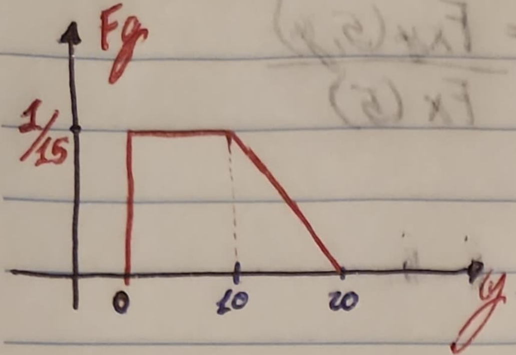
c) Determine e esboce a PDF marginal de Y

$$\begin{cases} 0 < y < 10 \Rightarrow 0 < x < 20 \\ 10 < y < 20 \Rightarrow \triangle\, y = x + 10 \text{ I} \\ y < 0 = 0 \qquad \triangle\, y = -x + 30 \text{ II} \\ y > 20 = 0 \end{cases}$$

$0 < y < 10$

$\int_0^{20} K\,dx \Rightarrow x.K\Big|_0^{20} = \frac{20}{300} = 1/15$

$$\int_{y-10}^{-y+30} \frac{1}{300}\, dx = \frac{(-y+3) - (y-10)}{300} = \frac{-y+20}{150} \qquad 10 < y < 20$$



d) Determine e esboce a CDF marginal de Y

Caso $y < 0$

$= 0$

Caso $0 < y < 10$

$$F_Y(y) = 0 + K \int_0^y dy \Rightarrow \frac{y}{15}$$

Caso $10 < y < 20$

$$F_Y(y) = \frac{10}{15} + \int_{10}^{y} \frac{-y+20}{150}\, du \Rightarrow \boxed{\text{Wolphram Alpha}} \Rightarrow \frac{-y^2}{300} + \frac{2y}{15} - \frac{1}{3}$$

Caso $y > 20$

$$F_Y(y) = \frac{-(20)^2}{300} + \frac{2.20}{15} - \frac{1}{3} + \int_{20}^{y} \cancel{0}^{\,0}\, dy \Rightarrow 1/1$$

e) Determine e esboce a PDF condicional de Y dado $X=5$

$$F_X(5) = \int_0^{X+10} K\, dy = \frac{1}{300}\cdot(X+10) = \frac{X+10}{300} = \frac{5+10}{300} = \frac{1}{20}$$

A PDF conjunta é a constante $K = \frac{1}{300}$, para o intervalo $0 \le y \le X+10$. Ou seja:

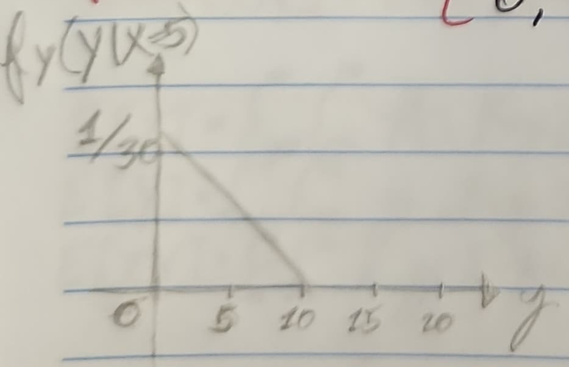
$$f_{X,Y}(5,y) := \begin{cases} 1/300, \text{ se } (0 \le y \le X+10) \\ 0, \text{ c.c} \end{cases}$$

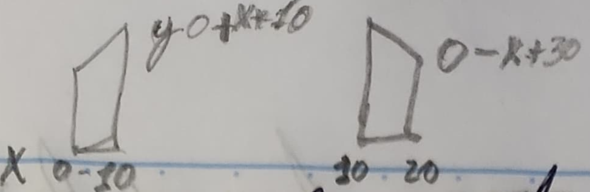
Usando colchete de Iverson:

$$f_{X,Y}(5,y) = \frac{1}{300}\left[(0 \le y \le X+10)\right]$$

$$\frac{f_{X,Y}(5,y)}{f_X(5)} = \frac{\frac{1}{600}}{\frac{1}{20}} = \frac{1}{30}$$

$$f_Y(y \mid X=5) = \begin{cases} 1/30, \text{ se } (0 \le y \le X+10) \\ 0, \text{ c.c} \end{cases}$$





f) Determine a covariância entre X e Y (Corrigida)

$$E[X] = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} x \cdot F_{xy}(x,y)\,dy\,dx$$

$$\int_{0}^{20}\left(\int_{0}^{x+10} \frac{x}{300}\,dy + \int_{0}^{-x+30} \frac{x}{300}\,dy\right)dx \Rightarrow \text{Wolfram} \Rightarrow 26{,}67$$

$$E[Y] = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} y \cdot F_{xy}(x,y)\,dy\,dx$$

$$\int_{0}^{20}\left(\int_{0}^{x+10} \frac{y}{300}\,dy + \int_{0}^{-x+30} \frac{y}{300}\,dy\right)dx \Rightarrow \text{Wolfram} \Rightarrow 28{,}89$$

$$E[XY] = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} xy \cdot F_{xy}(x,y)\,dy\,dx$$

$$\int_{0}^{20}\left(\int_{0}^{x+10} \frac{xy}{300}\,dy + \int_{0}^{-x+30} \frac{xy}{300}\,dy\right)dx \Rightarrow \text{Wolfram} \Rightarrow 288{,}89$$

$$\mathrm{Cov}[X,Y] = E[XY] - E[X]\cdot E[Y]$$

$$= 288{,}89 - 26{,}67 \cdot 28{,}89$$

$$= -481{,}6063$$