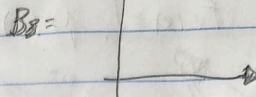
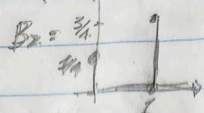


## Avaliação 2 - Questão 4

Luan de Barros

Data: 15/04/2024 - Avaliação 2 - questão 4  
Luan de Barros.



a) PMF Conjunta de  $X$  e  $Y$

$B_1$	$B_2$	$B_3$	$X$	$Y$	$P_{X,Y}$	$p_{X,Y}(X,Y)$
0	0	0	0	0	$\frac{1}{64}$	$y=0 \quad y=1$
0	0	1	1	0	$\frac{3}{64}$	$X=0 \quad \frac{1}{64} \quad 0$
0	1	0	1	0	$\frac{3}{64}$	$X=1 \quad \frac{9}{64} \quad 0$
0	1	1	2	0	$\frac{9}{64}$	$X=2 \quad \frac{27}{64} \quad 0$
1	0	0	1	0	$\frac{3}{64}$	$X=3 \quad 0 \quad \frac{27}{64}$
1	0	1	2	0	$\frac{9}{64}$	
1	1	0	2	0	$\frac{9}{64}$	
1	1	1	3	1	$\frac{27}{64}$	

b) PMFs marginais de  $X$  e  $Y$

$$p_X(X) = \sum_{y=0}^1 p_{X,Y}(X,Y)$$

$$p_X(X=0) = p_{X,Y}(X=0, Y=0) + p_{X,Y}(X=0, Y=1) = \frac{1}{64}$$

$$p_X(X=1) = p_{X,Y}(X=1, Y=0) + p_{X,Y}(X=1, Y=1) = \frac{3}{64}$$

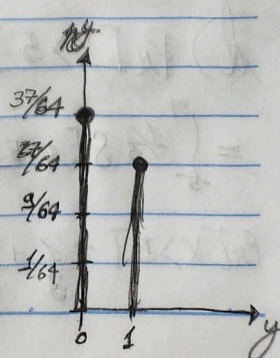
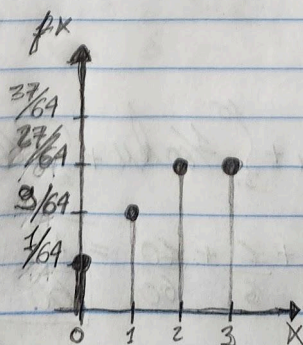
$$p_X(X=2) = p_{X,Y}(X=2, Y=0) + p_{X,Y}(X=2, Y=1) = \frac{27}{64}$$

$$p_X(X=3) = p_{X,Y}(X=3, Y=0) + p_{X,Y}(X=3, Y=1) = \frac{27}{64}$$

$p_{X,Y}(X,Y)$	$y=0$	$y=1$	$p_X(X)$
$X=0$	$\frac{1}{64}$	0	$\frac{1}{64}$
$X=1$	$\frac{9}{64}$	0	$\frac{9}{64}$
$X=2$	$\frac{27}{64}$	0	$\frac{27}{64}$
$X=3$	0	$\frac{27}{64}$	$\frac{27}{64}$
$p_Y(Y)$	$\frac{37}{64}$	$\frac{27}{64}$	1

$$p_Y(Y=0) = p_{X,Y}(X=0, Y=0) + p_{X,Y}(X=1, Y=0) + p_{X,Y}(X=2, Y=0) + p_{X,Y}(X=3, Y=0) = \frac{37}{64}$$

$$p_Y(Y=1) = p_{X,Y}(X=0, Y=1) + p_{X,Y}(X=1, Y=1) + p_{X,Y}(X=2, Y=1) + p_{X,Y}(X=3, Y=1) = \frac{27}{64}$$





c) PMFs condicionais de  $X$  dado que  $Y=y$

$$p_X(X|Y=y) = \frac{p_{X,Y}(X,y)}{p_Y(y)}$$

$$p_X(X|Y=1) = \frac{p_{X,Y}(0,1)}{p_Y(1)} = 0$$

$$p_X(X|Y=0) = \frac{p_{X,Y}(X=0,0)}{p_Y(0)} = \frac{1/64}{37/64} = 1/37 \quad p_X(X|Y=1) = \frac{p_{X,Y}(3,1)}{p_Y(1)} = 0$$

$$p_X(X|Y=0) = \frac{p_{X,Y}(1,0)}{p_Y(0)} = \frac{9/64}{37/64} = 9/37$$

$$p_X(X|Y=1) = \frac{p_{X,Y}(2,1)}{p_Y(1)} = 0$$

$$p_X(X|Y=1) = \frac{p_{X,Y}(3,1)}{p_Y(1)} = \frac{27/64}{27/64} = 1$$

$$p_X(X|Y=0) = \frac{p_{X,Y}(2,0)}{p_Y(0)} = \frac{27/64}{37/64} = 27/37$$

$$p_X(X|Y=0) = \frac{p_{X,Y}(3,0)}{p_Y(0)} = 0$$

	X=0	X=1	X=2	X=3
$p_Y(X Y=0)$	1/37	9/37	27/37	0
$p_Y(X Y=1)$	0	0	0	1

