ELEC2040 PRACTICAL
Week 3

Linear Time-invariant Systems

Task 1 (Matlab)

(i) Start with:

```
tmax = 5;
dt = 0.01;
t = -tmax:dt:tmax;
```

Plot the following sums of complex exponentials. Identify the functions created.

(a)
```
x = (exp(j*2*pi*0.5*t)+exp(-j*2*pi*0.5*t))/2;
```

(b)
```
x = (exp(j*2*pi*0.5*t)-exp(-j*2*pi*0.5*t))/(2j);
```

(ii) A signal is created by complex exponentials as follows. Plot the resulting signal. Identify the function that is being approximated.

```
tmax = 5;
dt = 0.01;
t = -tmax:dt:tmax;
T = 2;
w0 = 2*pi/T;
x = 0.5 + (-j/(-pi))*exp(-j*w0*t)+(-j/pi)*exp(j*w0*t);
```

What is the effect of varying T?

iii) The signal x is created as follows:

```
T = 2;
w0 = 2*pi/T;
x = 0.5 + (2/pi)*cos(w0*t);
```

x is the input to a two-path channel. The first path attenuates the input by 0.5 and delays it by 1 second. The second path attenuates the signal by 0.125 and delays it by 2 seconds. The result is the signal y, the sum of the two paths.

Write Matlab code to create the output signal y.

What is the frequency of y in Hz?

What happens as we vary T? Does the amplitude of y change? Does the frequency of y change?

How does the frequency of y relate to the frequency of x?

Task 2

Consider the linear system given by the input-output relations:

$$y(t) = x(t-3.5) + x(t-5.5) + 0.5x(t-7)$$

(a) Write down the impulse response $h(t)$ and draw it

(b) Write down the output of the system, $y(t)$, when the input signal is $x(t) = \delta(t-2)$ and draw $y(t)$

(c) Is the system time invariant? Explain.

Task 3

Consider the system

$$y(t) = \left(1 - \exp(-t)\right)x(t-1) + 2x(t-2)$$

(a) Write down the output of the system, $h(t)$, when the input signal is $x(t) = \delta(t)$ and draw $h(t)$

(b) Write down the output of the system, $y(t)$, when the input signal is $x(t) = \delta(t-1)$ and draw $y(t)$

(c) Is the system time invariant? Explain.

Task 4

Consider the system

$$y(t) = x(t+1) + 2x(t-2) + 3u(t-4)$$

Where $u(t)$ is the unit step function.

a) Write down the output of the system, $h(t)$, when the input signal is $x(t) = \delta(t)$ and draw $h(t)$

b) Is the system time invariant? Explain.

c) Is the system linear? Explain.

d) Is the system causal? Explain.

Task 5

For each of the systems below determine if the system is
(i) causal
(ii) time invariant
(iii) linear

a) $x(t) \to y(t) : y(t) = x^3(t)$

b) $x(t) \to y(t) : y(t) = t^2 x(t)$

c) $x(t) \to y(t) : y(t) = \sin(x(t-1))$

d) $x(t) \to y(t) : y(t) = 5x(t) + 6$

e) $x(t) \to y(t) : y(t) = 2x(t+4) - 3$

Integration involving delta functions

Task 6

Evaluate the following integrals. Find the answer in the simplest form.

a) $\int_{-\infty}^{\infty} \cos t \, \delta(t - \pi/4) \, dt$

b) $\int_{-\infty}^{\infty} u(t-2) \, \delta(t-1) \, dt$

c) $\int_{-\infty}^{\infty} u(t-2) \, \delta(t-3) \, dt$

d) $\int_{-\infty}^{\infty} (t+4)^2 \left[\delta(t) + 2\delta(t-3) \right] dt$

e) $\int_{-\infty}^{\infty} \exp(-jt) \, \delta(t + \pi/2) \, dt$

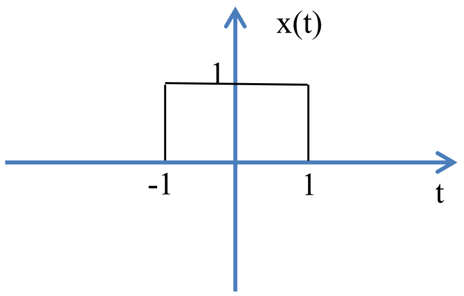
f) $\int_0^{\infty} \left(\frac{1}{t}\right)^2 \sin(2\pi t) \delta\left(t - \frac{1}{12}\right) dt$

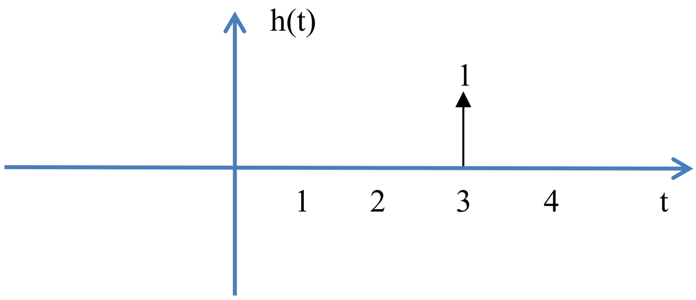
g) $\int_0^{\infty} t^2 \cos(2\pi t) \delta\left(t + \frac{1}{12}\right) dt$

Convolutions involving delta functions

Task 7

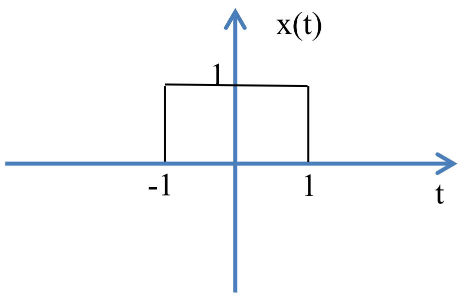
The following binary signal, x(t), is input to a channel with impulse response h(t), both as depicted below. Write down the output, y(t), in terms of x(t), and draw it as well.

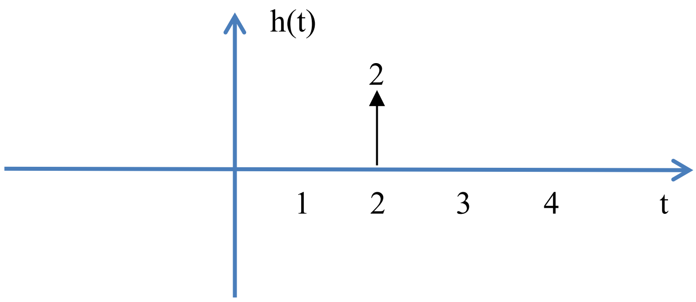




Task 8

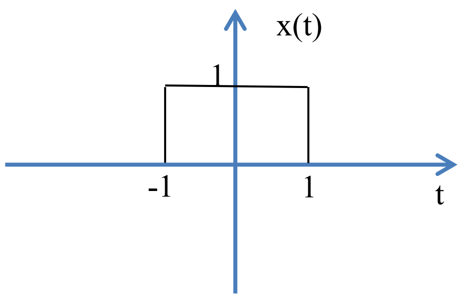
The following binary signal, x(t), is input to a channel with impulse response h(t), both as depicted below. Write down the output, y(t), in terms of x(t), and draw it as well.

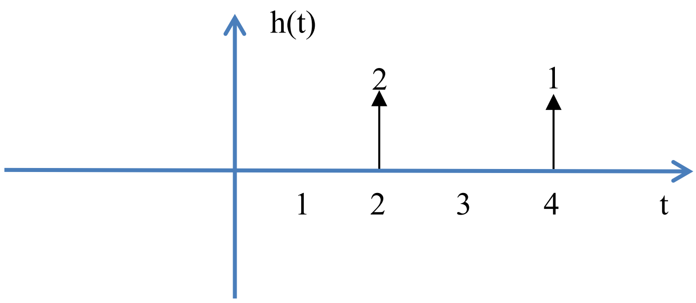




Task 9

The following binary signal, x(t), is input to a channel with impulse response h(t), both as depicted below. Write down the output, y(t), in terms of x(t), and draw it as well.





Task 10

The following binary signal, x(t), is input to a channel with impulse response h(t), both as depicted below. Write down the output, y(t), in terms of x(t), and draw it as well.