Control Systems

Chapter 7: Steady-State Errors

Highlights

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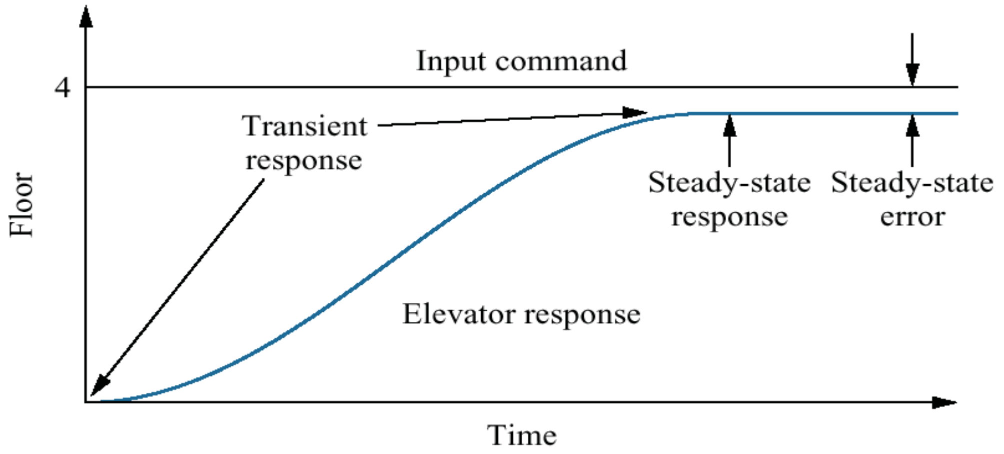
Original slides courtesy to Dr Branislav Hredzak

Response Characteristics and System Configurations



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Steady-State Errors

- How to find the steady-state error for a unity feedback system
- How to specify a system's steady-state error performance
- How to find the steady-state error for disturbance inputs
- How to find the steady-state error for non-unity feedback systems
- How to design system parameters to meet steady-state error performance specifications

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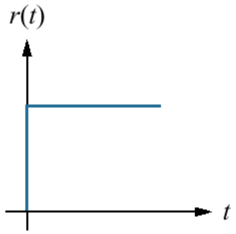
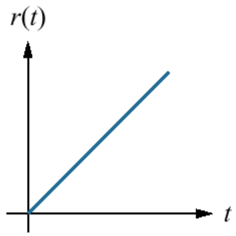
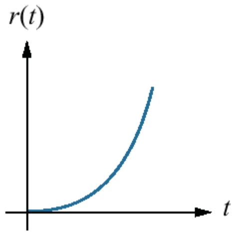
Steady-State Errors

Steady-state error is the difference between the input and the output for a prescribed test input as $t \to \infty$

Test waveforms for evaluating steady-state errors of control systems

Waveform	Name	Physical interpretation	Time function	Laplace transform
	Step	Constant position	1	$\frac{1}{s}$
	Ramp	Constant velocity	t	$\frac{1}{s^2}$
	Parabola	Constant acceleration	$\frac{1}{2}t^2$	$\frac{1}{s^3}$

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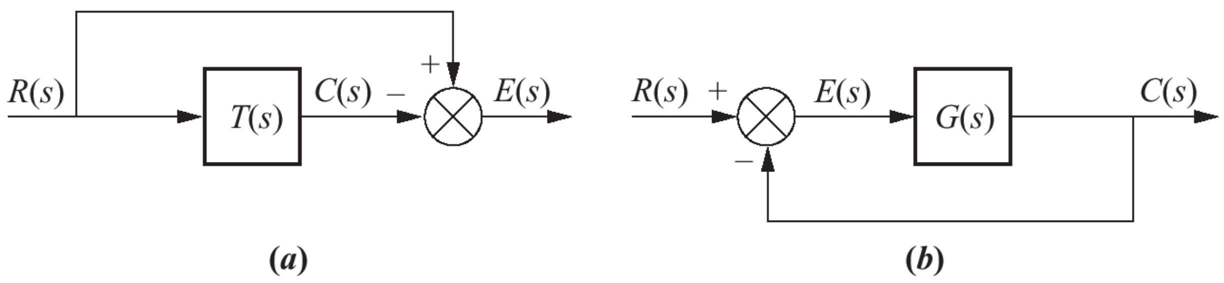
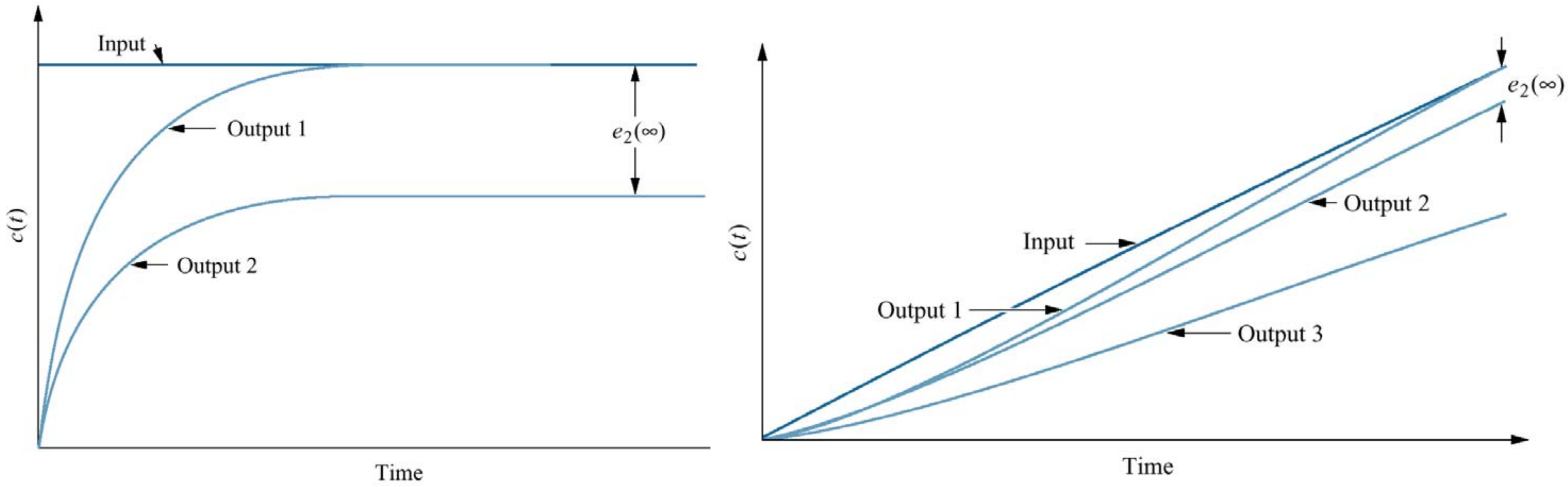
Evaluating Steady-State Errors





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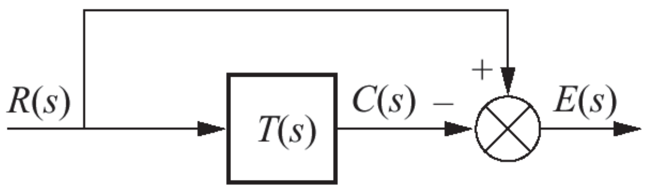
Steady-State Error for Unity Feedback Systems

Steady-State Error in Terms of *T*(*s*) – *closed loop transfer function*



$$E(s) = R(s) - C(s)$$

$$E(s) = R(s)[1 - T(s)]$$

$$e(\infty) = \lim_{t \to \infty} e(t) = \lim_{s \to 0} sE(s)$$ … final value theorem

$$e(\infty) = \lim_{s \to 0} sR(s)[1 - T(s)]$$

Keep in mind the conditions under which this formula is valid!

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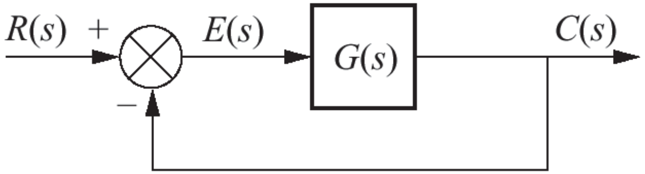
Steady-State Error in Terms of *G*(*s*) – *open loop transfer function*



(*b*)

$$E(s) = R(s) - C(s)$$

$$C(s) = E(s)G(s)$$

$$E(s) = \frac{R(s)}{1 + G(s)}$$

$$e(\infty) = \lim_{s \to 0} \frac{sR(s)}{1 + G(s)}$$

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Static Error Constants and System Type

Static Error Constants

position constant, K_p, where

$$K_p = \lim_{s \to 0} G(s)$$

velocity constant, K_v, where

$$K_v = \lim_{s \to 0} sG(s)$$

acceleration constant, K_a, where

$$K_a = \lim_{s \to 0} s^2 G(s)$$

Thus, for a step input:

$$e_{\text{step}}(\infty) = \frac{1}{1 + \lim_{s \to 0} G(s)} \qquad e(\infty) = \frac{1}{1 + K_p}$$

for a ramp input:

$$e_{\text{ramp}}(\infty) = \frac{1}{\lim_{s \to 0} sG(s)} \qquad e(\infty) = \frac{1}{K_v}$$

for a parabolic input:

$$e_{\text{parabolic}}(\infty) = \frac{1}{\lim_{s \to 0} s^2 G(s)} \qquad e(\infty) = \frac{1}{K_a}$$

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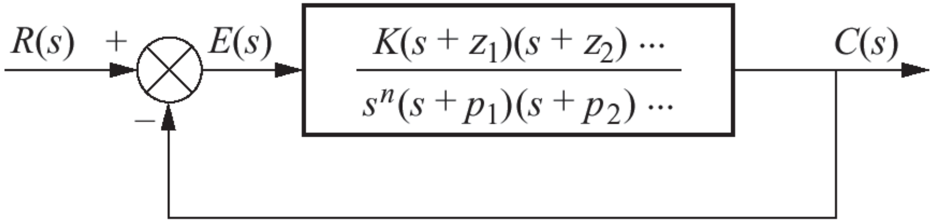
System Type



$n = 0$ … **Type 0** system
$n = 1$ … **Type 1** system
$n = 2$ … **Type 2** system

		Type 0		Type 1		Type 2	
Input	**Steady-state error formula**	**Static error constant**	**Error**	**Static error constant**	**Error**	**Static error constant**	**Error**
Step, $u(t)$	$\dfrac{1}{1+K_p}$	$K_p =$ Constant	$\dfrac{1}{1+K_p}$	$K_p = \infty$	0	$K_p = \infty$	0
Ramp, $tu(t)$	$\dfrac{1}{K_v}$	$K_v = 0$	∞	$K_v =$ Constant	$\dfrac{1}{K_v}$	$K_v = \infty$	0
Parabola, $\frac{1}{2}t^2u(t)$	$\dfrac{1}{K_a}$	$K_a = 0$	∞	$K_a = 0$	∞	$K_a =$ Constant	$\dfrac{1}{K_a}$

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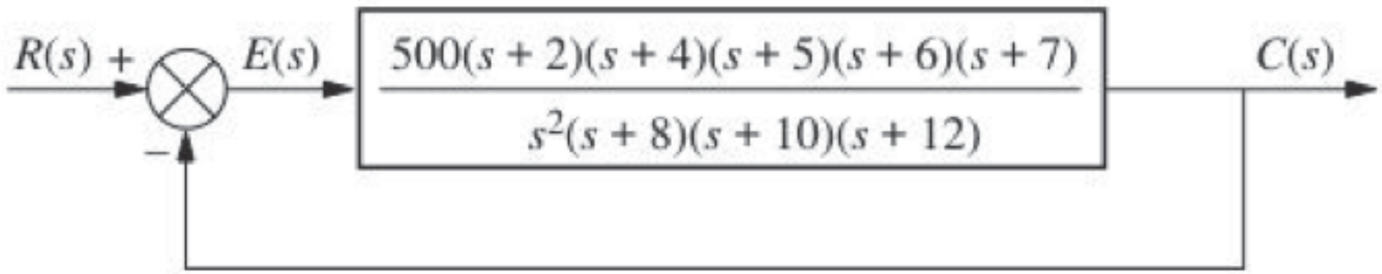
Example 7.4 For the closed-loop system below, find the static error constants and the expected error for the standard step, ramp, and parabolic inputs.



(c)

Solution

$$K_p = \lim_{s\to 0} G(s) = \infty$$

$$K_v = \lim_{s\to 0} sG(s) = \infty$$

$$K_a = \lim_{s\to 0} s^2 G(s) = \frac{500\times 2\times 4\times 5\times 6\times 7}{8\times 10\times 12} = 875$$

$$e_{\text{step}}(\infty) = \frac{1}{1+K_p} = 0$$

$$e_{\text{ramp}}(\infty) = \frac{1}{K_v} = 0$$

$$e_{\text{parabolic}}(\infty) = \frac{1}{K_a} = 1.14\times 10^{-3}$$

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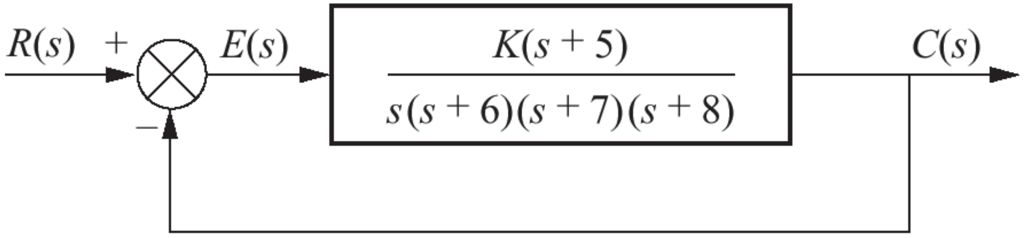
Example 7.6 Given the control system, find the value of K so that there is 10% error in the steady state.



Solution

$$e(\infty) = \frac{1}{K_v} = 0.1$$

$$K_v = 10 = \lim_{s \to 0} sG(s) = \frac{K \times 5}{6 \times 7 \times 8}$$

$$K = 672$$

Applying the Routh-Hurwitz criterion we see that the system is stable at this gain. This is necessary for validity of the calculations above.

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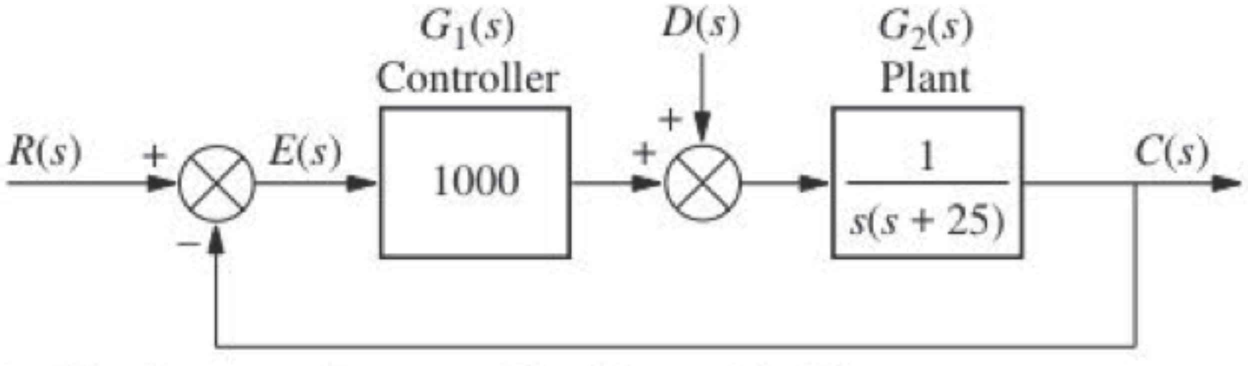
Example 7.7 Find the steady-state error component due to a step disturbance for the system below



Solution

$$e_D(\infty) = -\frac{1}{\lim\limits_{s\to 0}\dfrac{1}{G_2(s)} + \lim\limits_{s\to 0} G_1(s)} = -\frac{1}{0 + 1000} = -\frac{1}{1000}$$

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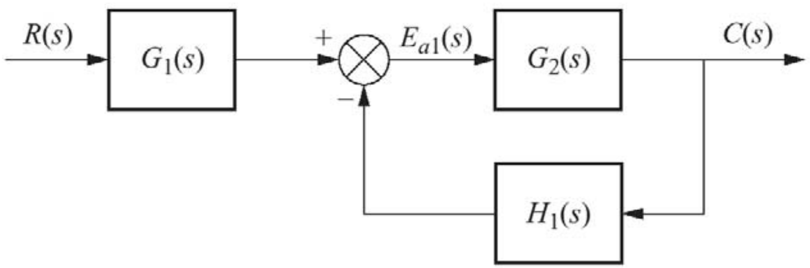
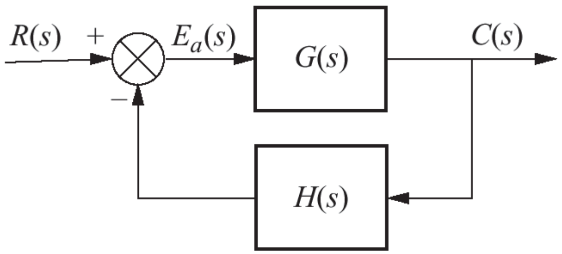
Steady-State Error for Non-unity Feedback Systems

- Control systems often **do not have unity feedback** because of the compensation used to improve performance or because of the physical model for the system.

- When **nonunity feedback** is present, the plant's **actuating signal is not the actual error** or difference between the input and the output.



$G_1(s)$ … input transducer
$G_2(s)$ … plant
$H_1(s)$ … feedback



$$G(s) = G_1(s)G_2(s)$$
$$H(s) = H_1(s)/G_1(s)$$

$E_a(s)$ … actuating signal (not the actual error)

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