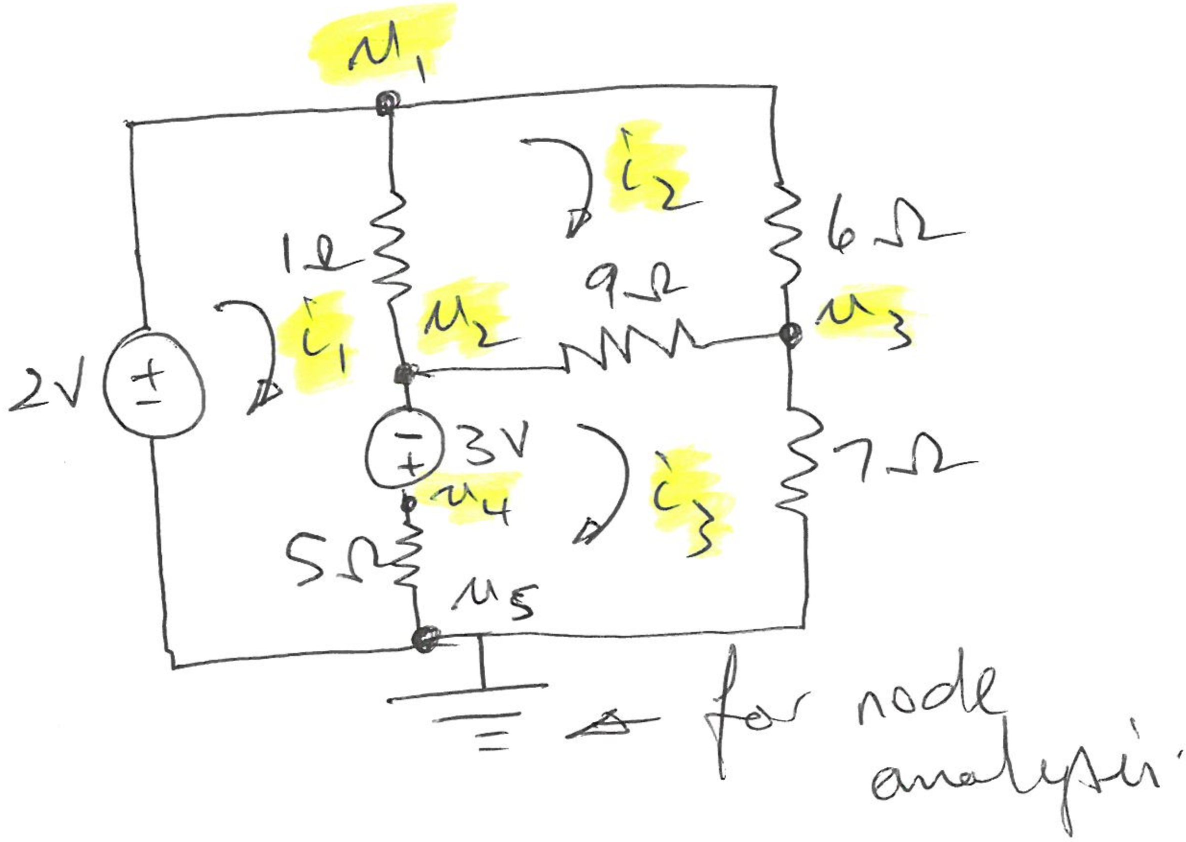
Solutions to Assignment 1
2018

Q 1

(a)



(b) Mesh 1

$$-2+1(i_1-i_2)-3+5(i_1-i_3)=0$$

$$6i_1-i_2-5i_3=5. \quad -1$$

Mesh 2

$$1(i_2-i_1)+6i_2+9(i_2-i_3)=0$$

$$-i_1+16i_2-9i_3=0 \quad -2$$

Mesh 3

$$5(i_3-i_1)+3+9(i_3-i_2)+7i_3=0$$

$$-5i_1-9i_2+21i_3=-3 \quad -3$$

Putting into matrix form: ②

$$\begin{bmatrix} 6 & -1 & -5 \\ -1 & 16 & -9 \\ -5 & -9 & 21 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix} = \begin{bmatrix} 5 \\ 0 \\ -3 \end{bmatrix}$$

$$\therefore \begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix} = \begin{bmatrix} 6 & -1 & -5 \\ -1 & 16 & -9 \\ -5 & -9 & 21 \end{bmatrix}^{-1} \begin{bmatrix} 5 \\ 0 \\ -3 \end{bmatrix}$$

$$= \begin{bmatrix} 0.25 & 0.0647 & 0.0873 \\ 0.0647 & 0.099 & 0.0579 \\ 0.0873 & 0.059 & 0.0932 \end{bmatrix} \begin{bmatrix} 5 \\ 0 \\ -3 \end{bmatrix}$$

$$i_1 = 988 \text{ mA}$$
$$i_2 = 150 \text{ mA}$$
$$i_3 = 157 \text{ mA}$$

__Node Voltages__ Can set $V_5 = 0$ (ground

$$v_1 = +2 \text{ V}$$
$$v_2 = 2 - (0.988 - 0.15)$$
$$= 1.16 \text{ V}$$
$$v_3 = 2 - 6(0.15)$$
$$= 1.1 \text{ V.}$$

OR $v_3 = 7 \times 0.157$

$= 1.1\ V.$ (Check)

$v_4 = 5 \times (0.988 - 0.157)$

$= 4.16\ V.$

(C) Voltage sources.

$2V$: $P = vi$

$= 2 \times 0.988 = 1.976\ W$

$3V$: $P = vi = 3 \times (0.988 - 0.157)$

$= 2.493\ W$

Total source $= 4.469\ W$

$1\ \Omega$ resistor : $P = i^2 R = (0.988 - 0.15)^2$

$= 0.702\ W$

$6\ \Omega$ resistor : $P = (0.15)^2 \times 6 = 0.135\ W$

$9\ \Omega$ resistor : $P = (0.15 - 0.157)^2 \times 9 = 0.00044$

$7\ \Omega$ resistor : $P = (0.157)^2 \times 7 = 0.173\ W$

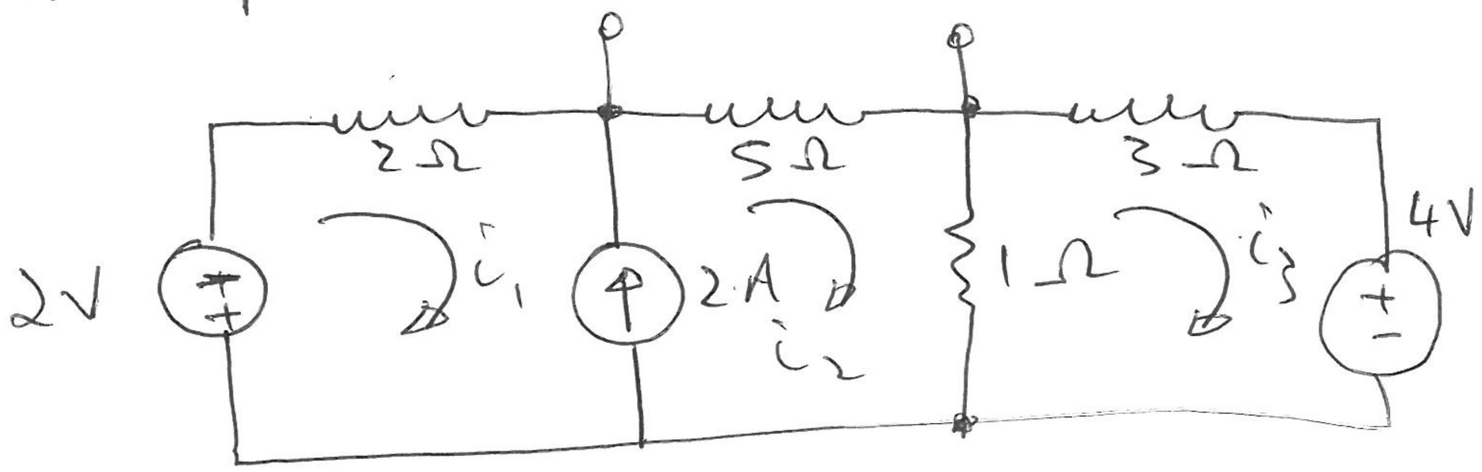
$5\ \Omega$ resistor : $P = (0.988 - 0.157)^2 \times 5$

$= 3.45\ W$

Resistor total $= 4.469\ W$

$\therefore$ Power conserved.

2 (a) Since we have a source between 2 meshes, we can use the supermesh method.



1 $2 = i_1 - i_2$ (current between mesh 1 & mesh 2)

Supermesh:

2 $2 + 2i_1 + 5i_2 + (i_2 - i_3) = 0$

$$2i_1 + 6i_2 - i_3 = -2.$$

Mesh 3:

$$(i_3 - i_2) + 3i_3 + 4 = 0$$

$$-i_2 + 4i_3 = -4$$

Using matrix method:

$$\begin{bmatrix} 1 & -1 & 0 \\ 2 & 6 & -1 \\ 0 & -1 & 4 \end{bmatrix} \begin{bmatrix} \hat{i}_1 \\ \hat{i}_2 \\ \hat{i}_3 \end{bmatrix} = \begin{bmatrix} 2 \\ -2 \\ -4 \end{bmatrix}$$

$$\begin{bmatrix} \hat{i}_1 \\ \hat{i}_2 \\ \hat{i}_3 \end{bmatrix} = \begin{bmatrix} 1 & -1 & 0 \\ 2 & 6 & -1 \\ 0 & -1 & 4 \end{bmatrix}^{-1} \begin{bmatrix} 2 \\ -2 \\ -4 \end{bmatrix}$$

$$= \begin{bmatrix} -0.742 & 0.129 & 0.0323 \\ 0.258 & 0.129 & 0.0323 \\ 0.0645 & 0.0323 & 0.258 \end{bmatrix} \begin{bmatrix} 2 \\ -2 \\ -4 \end{bmatrix}$$

$\therefore \hat{i}_1 = -1.87\ A$

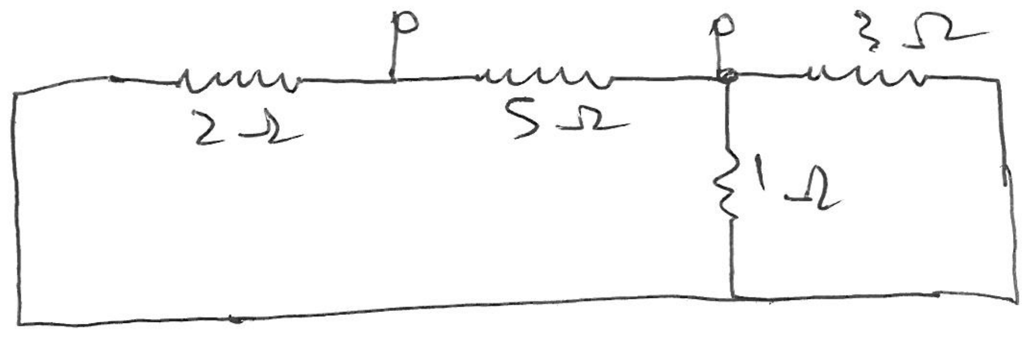
$\hat{i}_2 = 0.129\ A$

$\hat{i}_3 = -0.968\ A$

(iv) $V_{oc} = 5 \times \hat{i}_2 = V_x = 0.645V$

Need to find R_{th}.
voltage sources become short circuit
current sources become open circuit

$\therefore$

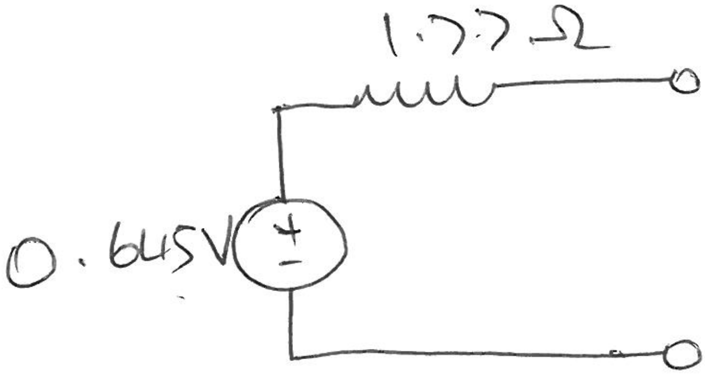


$R_{th} = 5 \| (2 + 3 \| 1)$

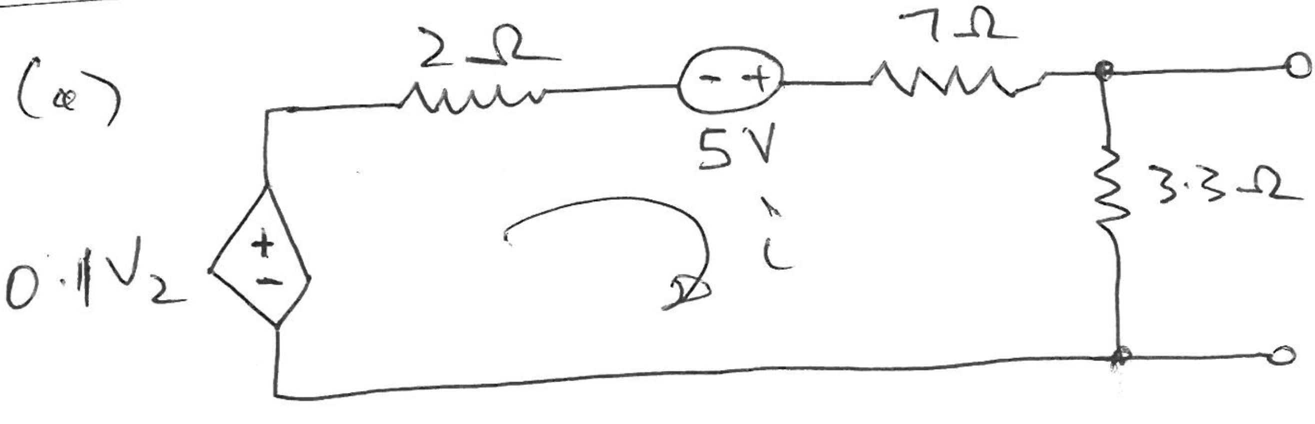
$= 5 \| 2 + 3/4 = 5 \| 2.75$

$= 1.77 \, \Omega.$

$\therefore$ Thevenin circuit :



③ (a)



KVL :

$-0.1V_2 + 2i - 5 + 7i + V_2 = 0$

$9i + 0.9V_2 = 5. \quad -①$

But $V_2 = 3.3i \;\therefore$

$\therefore ① \Rightarrow \quad 9i + 3.3i \times 0.9 = 5$

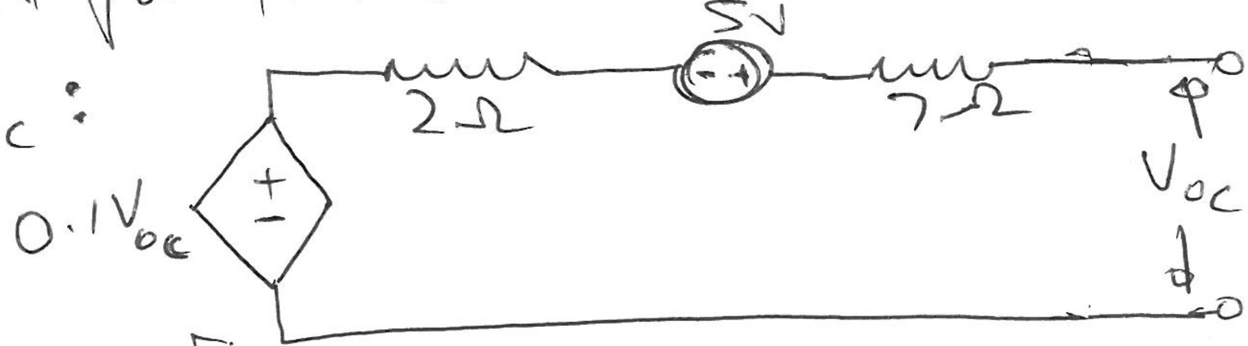
$i = 0.418 \text{ A}$

$V_2 = 3.3 \times 0.418$
$= 1.378\ V$

$P_{3.3} = 1.378 \times 0.418 = 0.576\ W$
$= 576\ mW$

(h) Need to find Thevenin equivalent circuit for this circuit

First V_{oc}:



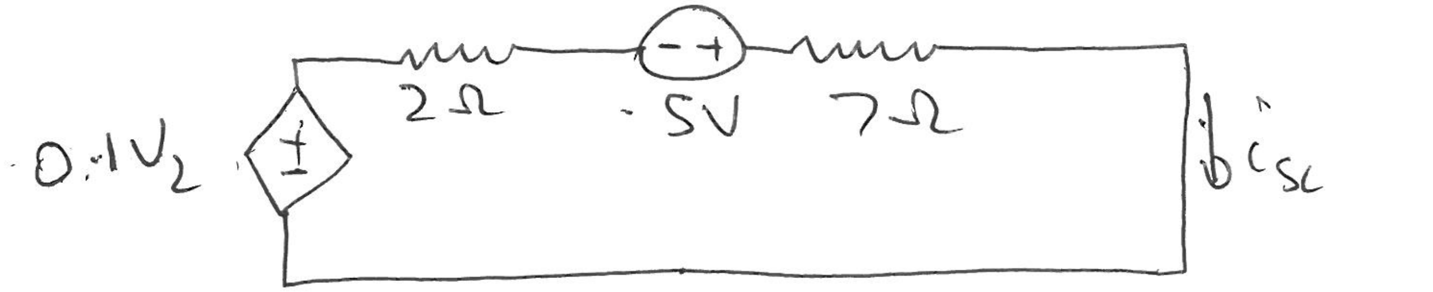
No current Flows
KVL

$-0.1V_{oc} - 5 + V_{oc} = 0$

$-5 = -0.9V_{oc}$

$V_{oc} = 5/0.9 = 5.56\ V$

Since we have a dependent source, need to find i_{sc} (shorts out 3.3 Ω resistor)



$V_2 = 0$

(8)

$$2i_{sc} - 5 + 7i_{sc} = 0$$

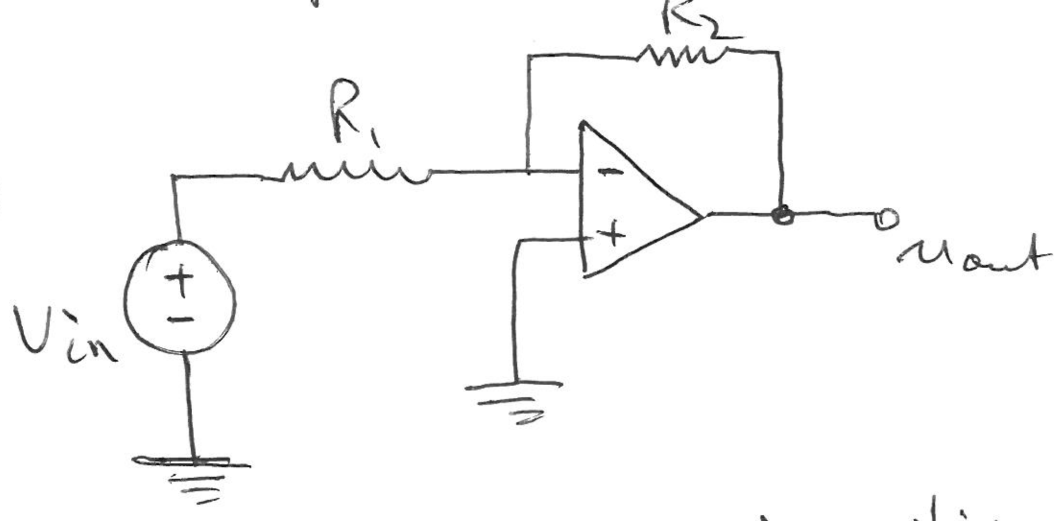
$$9i_{sc} = 5$$

$$i_{sc} = 5/9 = 0.556\ A$$

$$\therefore R_{th} = \frac{5.56}{0.556} = 10\ \Omega.$$

$\therefore$ For maximum power transfer, $3.3\ \Omega$ should be replaced with $10\ \Omega$ resistor.

(4) (a)



This configuration is a inverting amplifier

i.e. $$\frac{v_{out}}{v_{in}} = -\frac{R_2}{R_1}$$

OR $$v_{out} = v_{in} \times -\frac{R_2}{R_1}$$

$\therefore R_1 = R_2 = 100\ \Omega \Rightarrow v_{out} = -v_{in}$

$= -5\ V$

(b) $$v_{out} = \frac{-200 R_1}{R_1} \times 1$$

$$= -200\,V$$

(c) $$v_{out} = -\frac{47\,k\Omega}{4.7\,k\Omega} \times 20 \sin 5t$$

$$= -200 \sin 5t.$$

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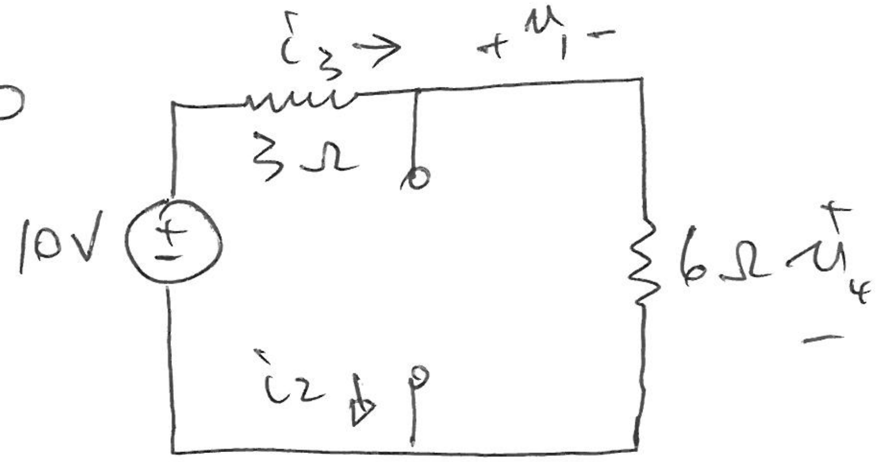
5 for $t < 0$



capacitor is an open circuit
inductor is a short circuit.

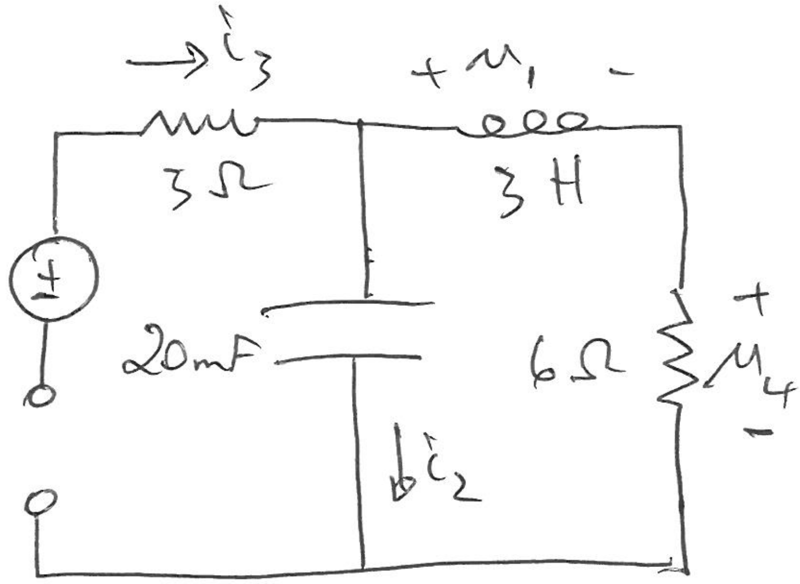
$$i_3(0^-) = \frac{10}{3+6} = 1.11A$$

$$v_1(0^-) = 0$$

$$v_4(0^-) = 6.66A$$

$$i_2(0^-) = 0$$

for $t > 0$



1) Capacitor voltage does not change instaneously
2) ~~Inductor~~ Inductor current does not change instaneously

1) $\Rightarrow$ $v_{capacitor}(0^+) = 6.66\,V$

$i_{inductor}(0^+) = 1.11\,A$

$i_3(0^+) = 0$ (since the LHS mesh is open circuit)

Applying KCL at top node

$$i_3 = i_2 + i_{inductor}(0^+)$$

$$0 = i_2 + 1.1(0^+)$$

$$\therefore i_2(0^+) = -1.11\,A.$$

$v_1(0^+) = 0 \rightarrow$ since $v_1(0^+) = L \dfrac{d i_{inductor}(0^+)}{dt}$

$$= L \frac{d}{dt}(1.1)$$

$$= 0$$

since $v_1(0^+) = 0$

$\therefore \quad v_4(0^+) = v_{capacitor}(0^+)$
$= 6.66\ V.$

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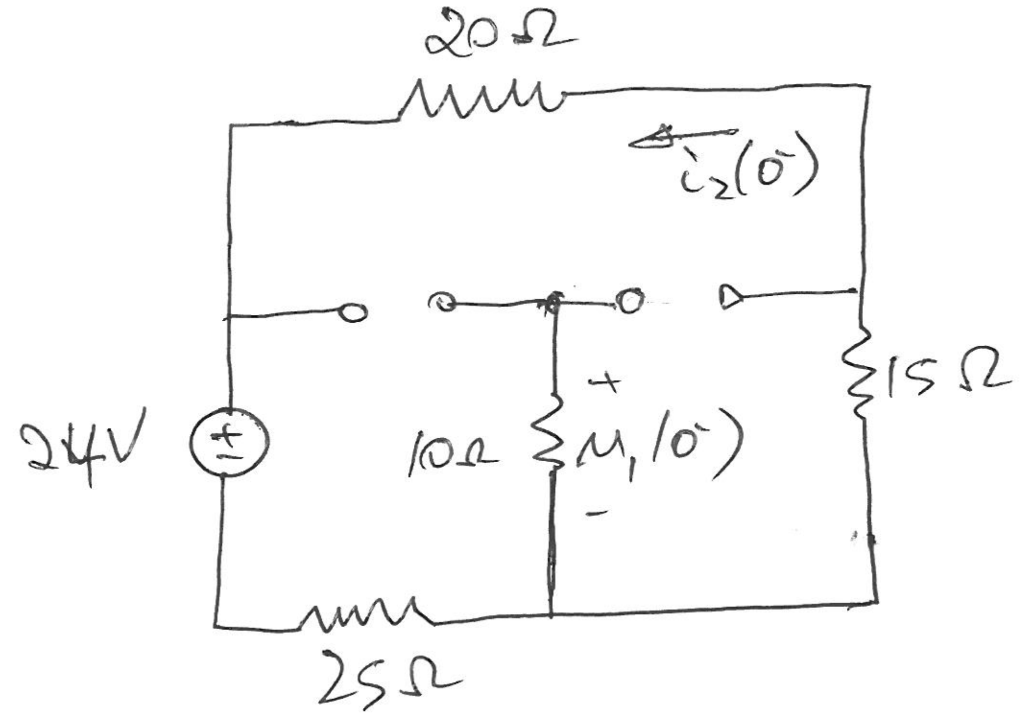
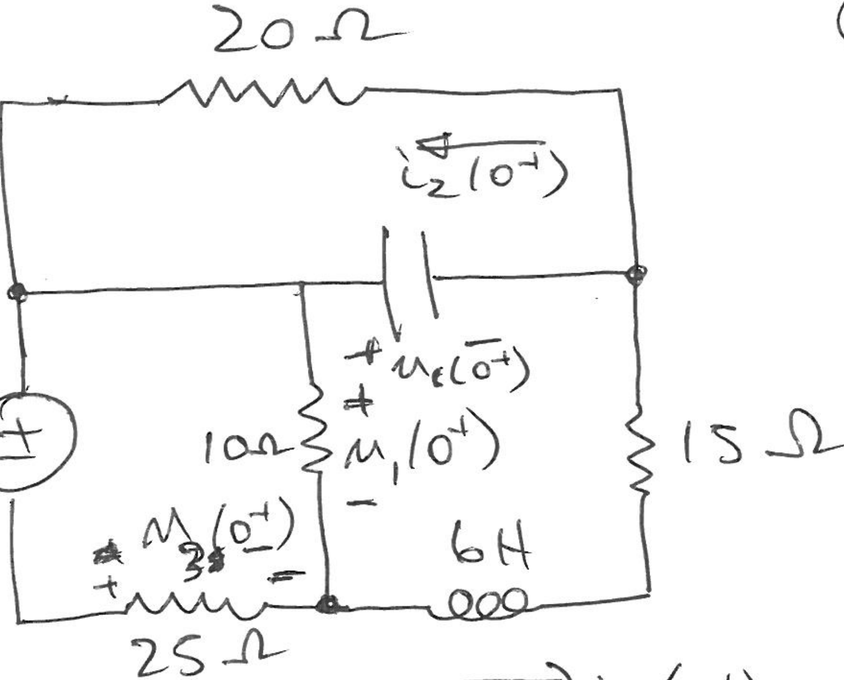
(6) for $t < 0$ (Note DC independent source)



$$i_2(0^-) = \frac{-24V}{20+15+25} = \frac{24}{60} = -0.4A$$

Since NO current passes through 10 Ω resistor $\therefore v_1(0^-) = 0$

for $t > 0$



inductor current does not change instantly

$\Rightarrow i_L(0^+) = -0.4\,A$ (as defined above)

Capacitor voltage does not change instantly

$\Rightarrow v_c(0^+) =$ voltage across 15 Ω resistor for $t < 0$

$= 15 \times i_2(0^-)$

$= 15 \times -0.4 = -6\,V.$

KVL at bottom LHS mesh

$-24 + v_1(0^+) - v_3(0^+) = 0$

$\therefore v_1(0^+) - v_3(0^+) = 24$ — ①

KCL at bottom node:

(note all currents are leaving node as defined in figure

$$\therefore \frac{u_3(0^+)}{25} + \frac{u_1(0^+)}{10} + i_L(0^+) = 0$$

$$\frac{u_3(0^+)}{25} + \frac{u_1(0^+)}{10} = 0.4 \quad - ②$$

From ①

$$u_1(0^+) = 24 + u_3(0^+)$$

Putting this into ②

$$\frac{u_3(0^+)}{25} + \frac{24 + u_3(0^+)}{10} = 0.4$$

$$\frac{10u_3 + \quad + 25u_3(0^+)}{250} = 0.4 - 2.4$$

$$35u_3 = -500$$

$$u_3 = -500/35 = -14.28$$

$$\therefore u_1 = 24 - 14.28$$

$$u_1(0^+) = 9.72\ V.$$

KVL in top mesh: $20i_2(0^+) + u_c(0^+) = 0$

$$20i_2(0^+) - 6 = 0$$

$$i_2(0^+) = 6/20$$

$$= 0.3\ A.$$

$$\left.\begin{aligned} i_2(0^+) &= 0.3A \\ u_1(0^+) &= 9.72V \end{aligned}\right\}$$