MECH2003,
Mechanical Engineering Design
Springs and Spring Selection

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Springs and Spring Selection - Introduction

Where will you find springs being used?

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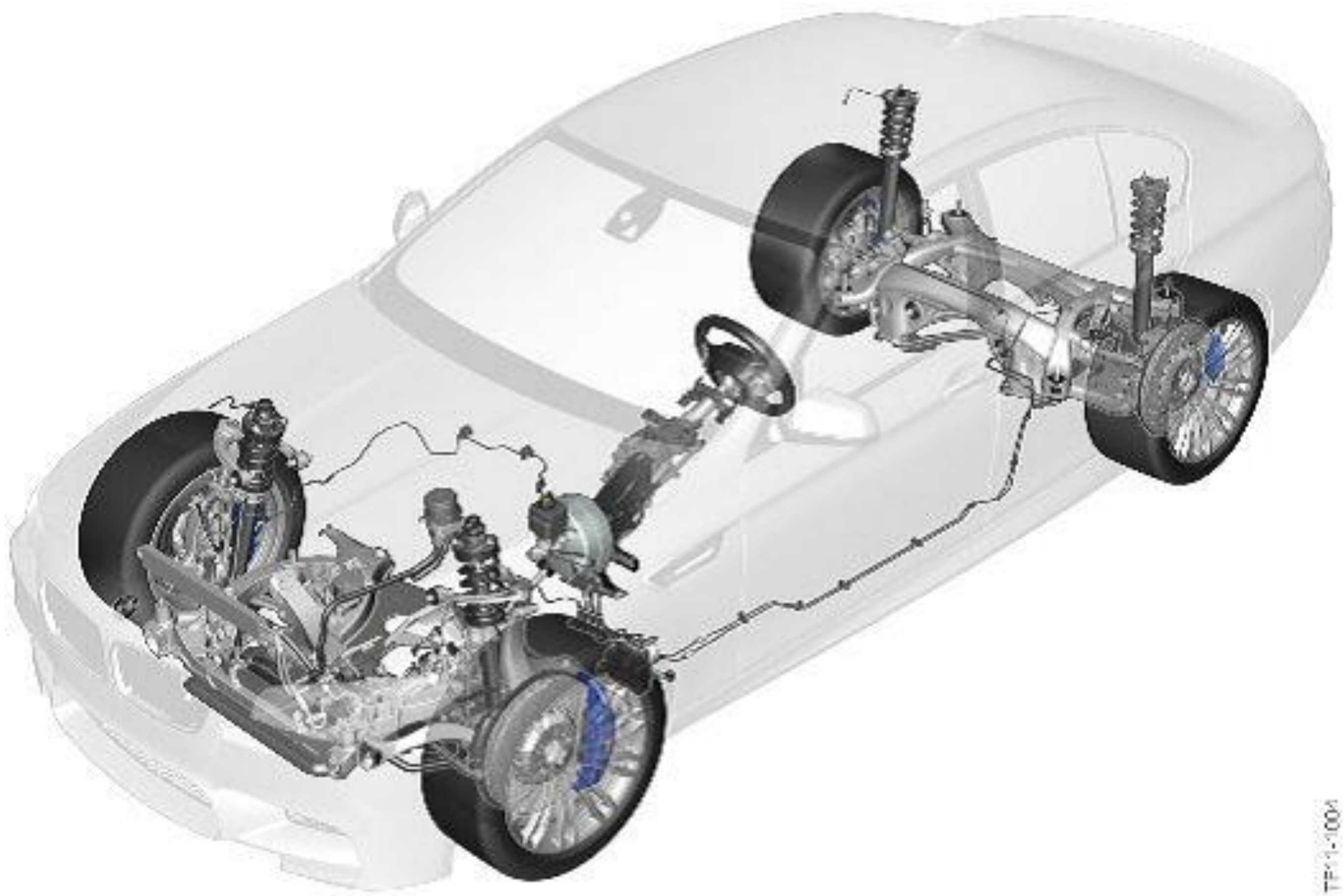
Springs and Spring Selection - Introduction

Where will you find springs being used? *– Suspension for vehicles*



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Springs and Spring Selection - Introduction

Where will you find springs being used? – *Suspension for bicycles*

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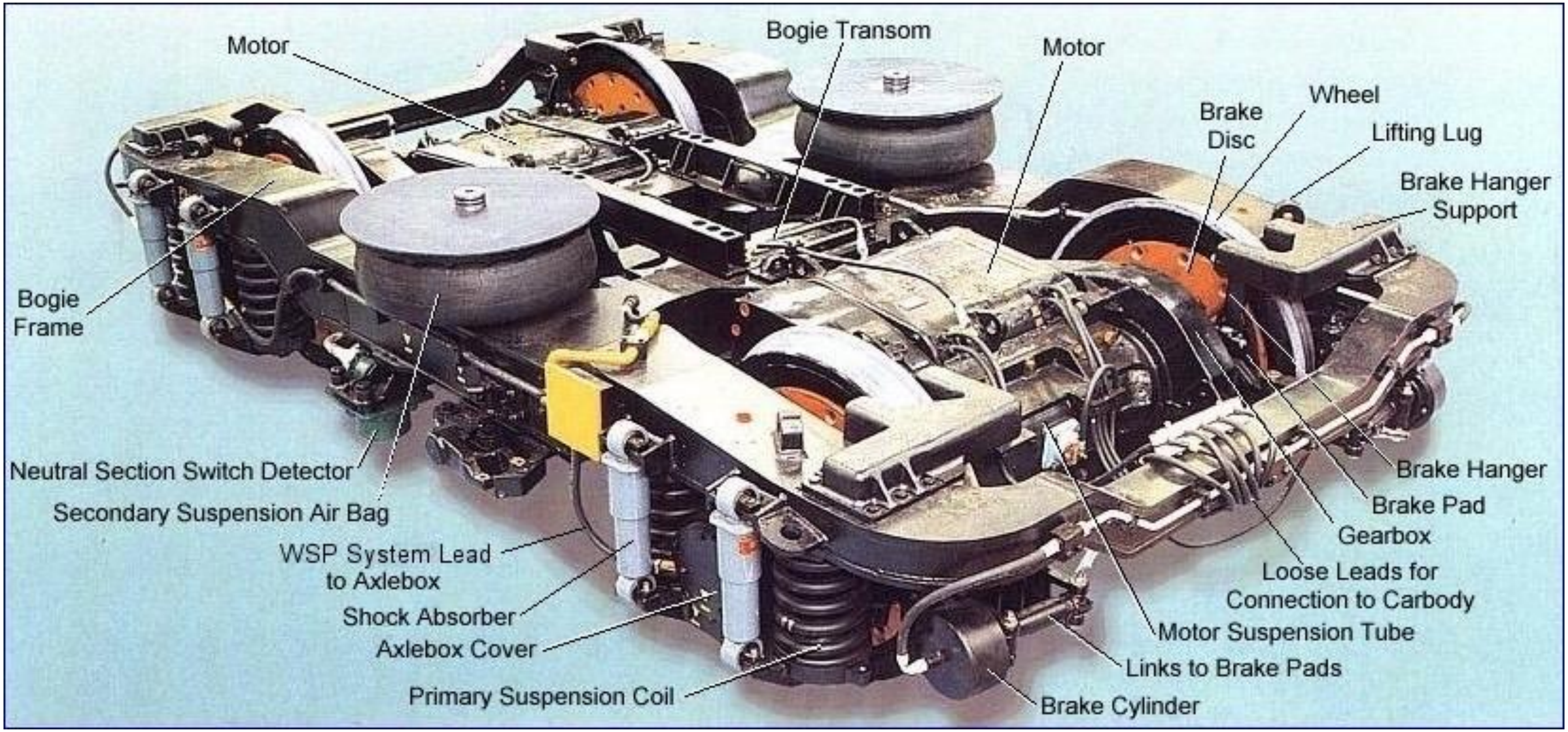
Springs and Spring Selection - Introduction

Where will you find springs being used? *– Suspension for trains*



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Springs and Spring Selection - Introduction

Where will you find springs being used? *– Suspension for trucks*

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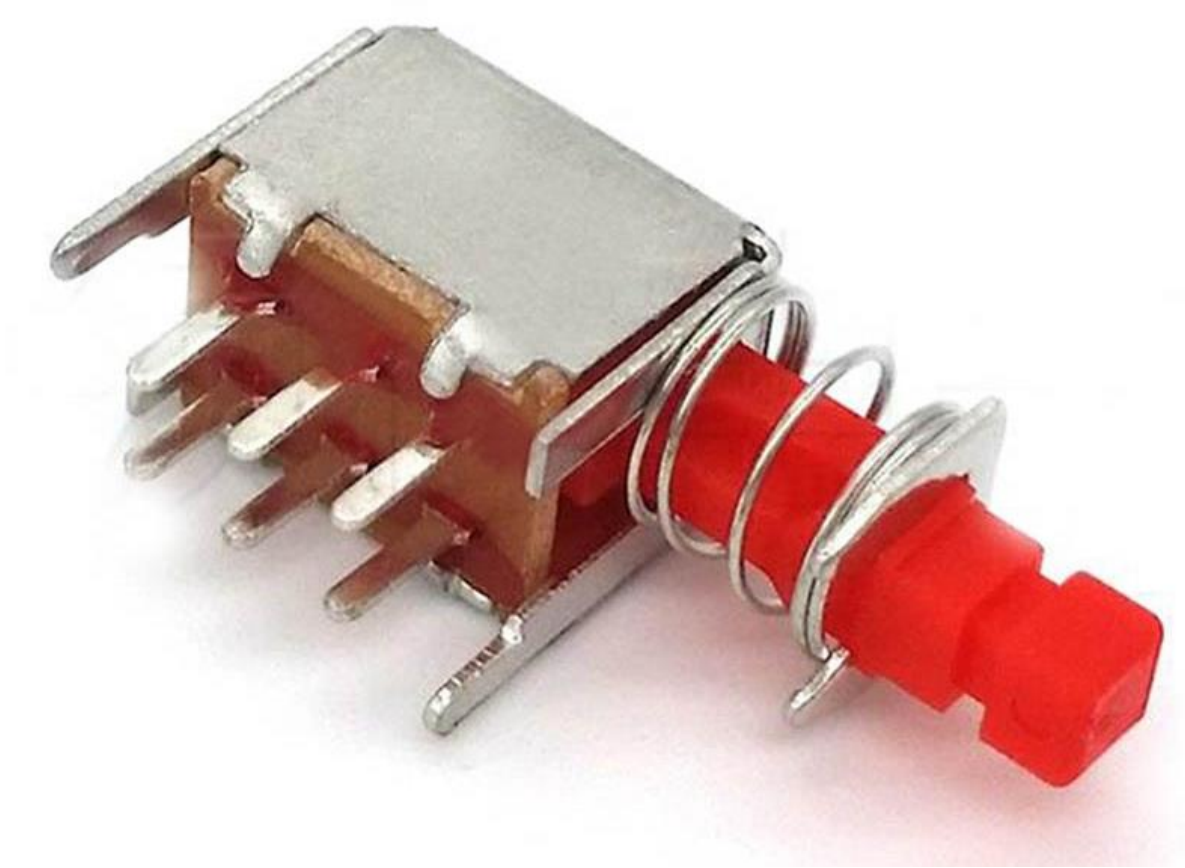
Where will you find springs being used? *– Switches*



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Where will you find springs being used? – *Keyboards*

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Springs and Spring Selection - Introduction

Where will you find springs being used? *– Mouse traps*

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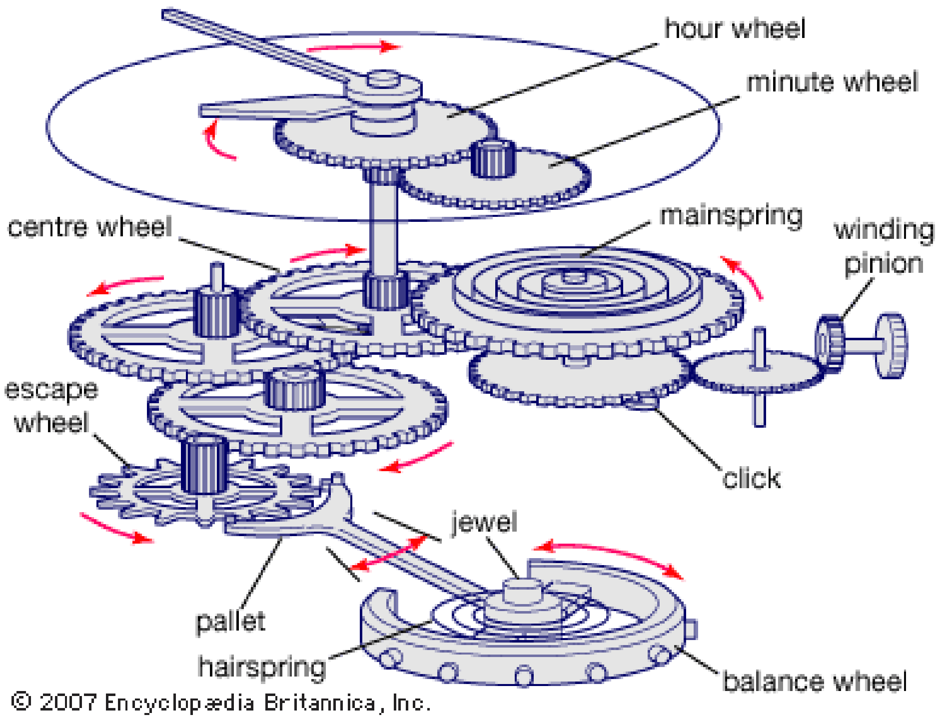
Springs and Spring Selection - Introduction

Where will you find springs being used? – *Watches*



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Springs and Spring Selection - Introduction

Where will you find springs being used? *– Toasters*

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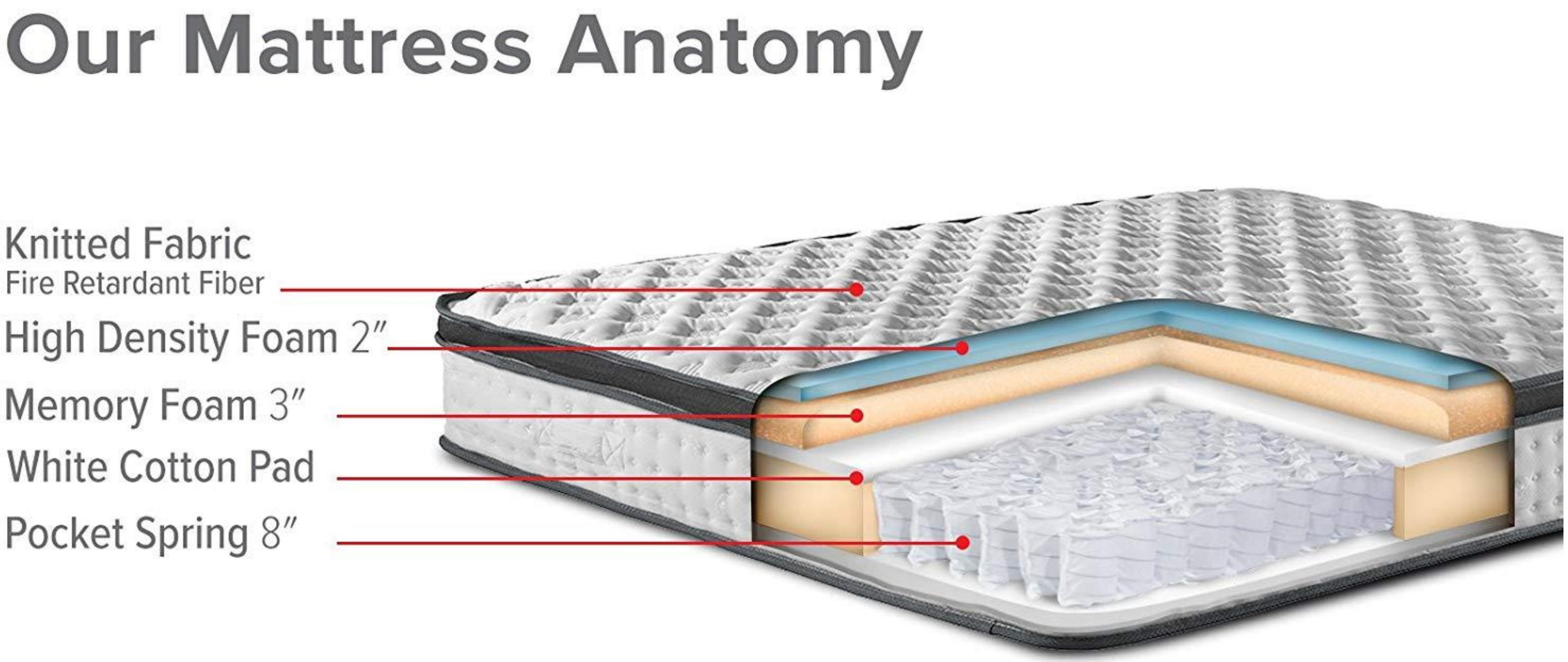
Springs and Spring Selection - Introduction

Where will you find springs being used? *– Mattresses*



Springs and Spring Selection - Introduction

Where will you find springs being used? – *Trampoline*

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Springs and Spring Selection

Where will you find springs being used? *– Toys*

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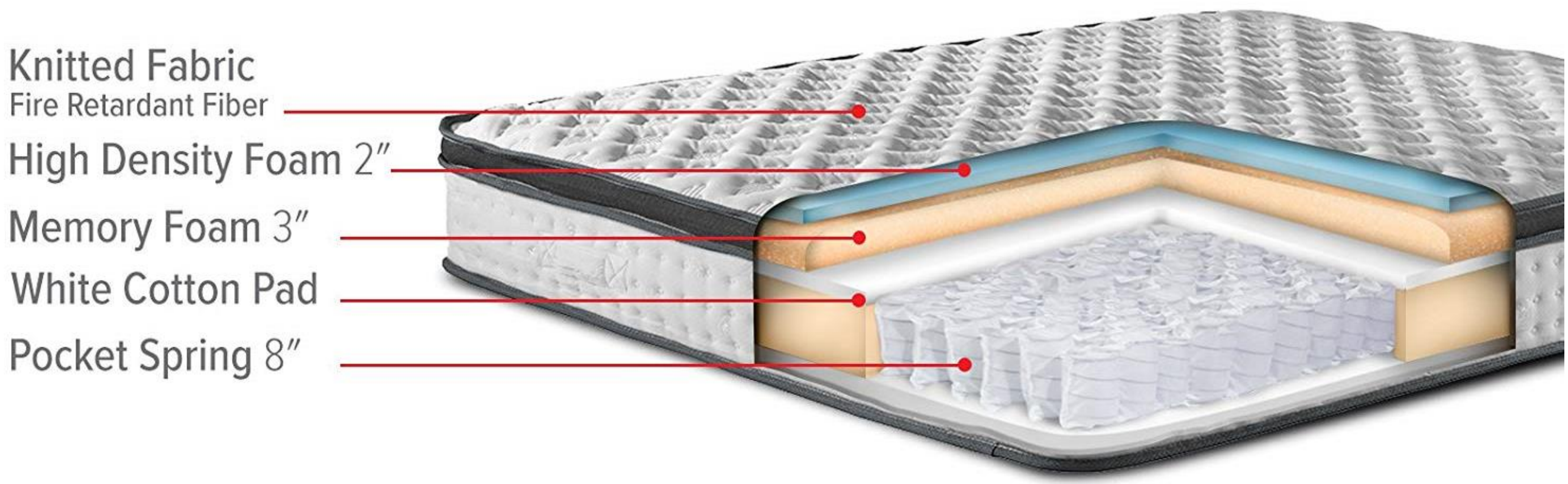
Springs and Spring Selection - Introduction

What is the most exciting thing you can do with a spring…

Springs and Spring Selection – Spring role

What is the role of a spring?

- **Provide flexibility** - for this reason they are used in mattresses. By adopting springs, the mattress can conform to a person's shape and provide support while they sleep.



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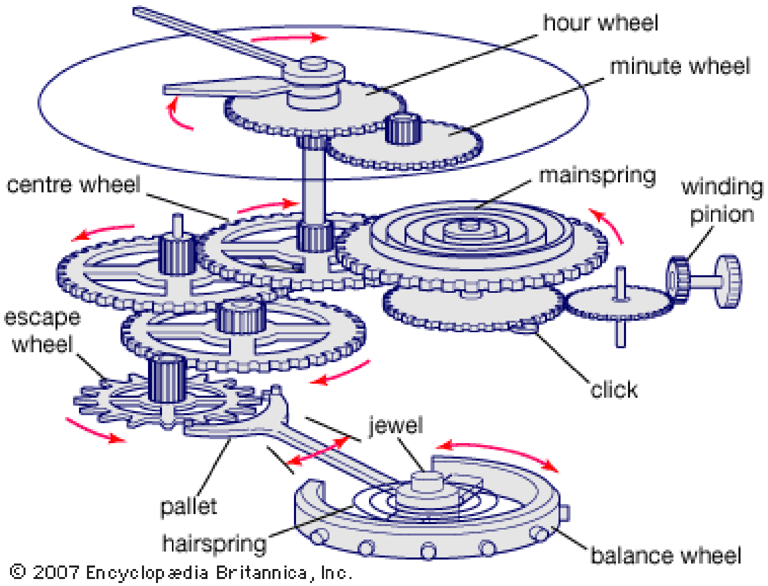
Springs and Spring Selection – Spring role

What is the role of a spring?

- **Store mechanical energy** – for this reason mechanical watches have been using torsional springs. By winding the watch, you are storing energy in the torsional spring to be released over time.



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Springs and Spring Selection – Spring role

What is the role of a spring?

- **Provide force** – helical extension springs are used to provide a force which will eject toast from a toaster once cooked.

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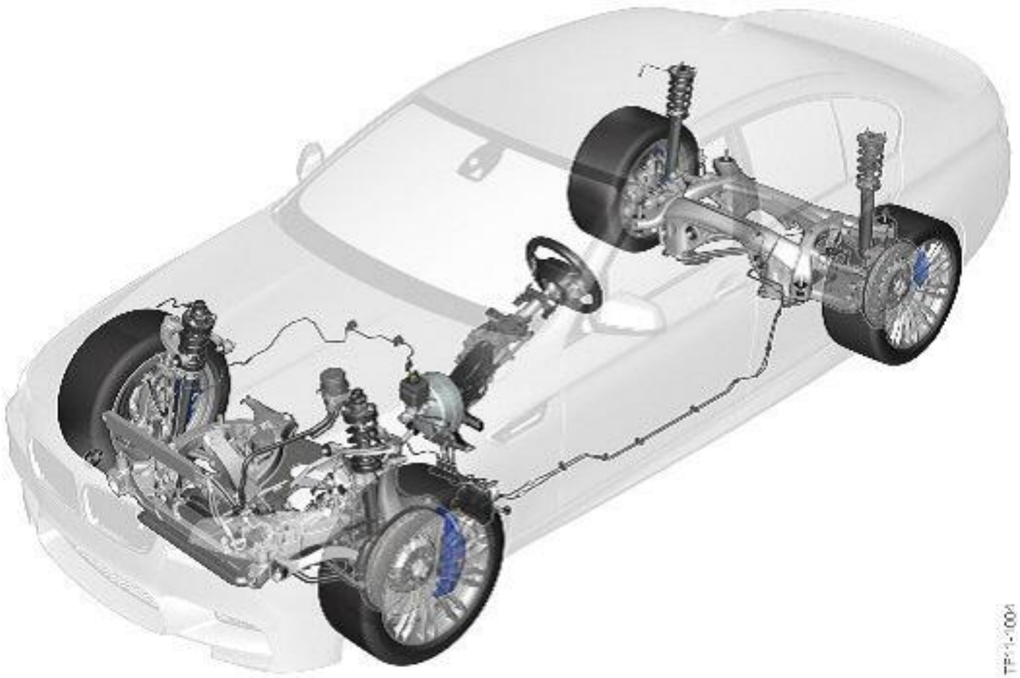
Springs and Spring Selection – Spring role

What is the role of a spring?

- **Reduce impulse** – we see this in the use of suspension members. A spring allows the impulse that is transferred into the vehicle to be reduced makes bumps feel less harsh.



Springs and Spring Selection – Types of springs

Springs can be categorised into three groups. They all work in using the same principles, but their shape and how they store mechanical energy varies.

Flat spring

Helical spring

Torsional spring

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Springs and Spring Selection – Types of springs

The most common of these are **helical** springs, also known as **coil** springs. They are predominantly cylindrical in shape and consist of a constant diameter wire.

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Springs and Spring Selection – Types of springs

These come in two different forms depending on the direction with which they are loaded. If placed in tension, **helical extension** springs are more suitable. If the most frequent load will cause compression, then **helical compression** springs would be required.

Helical Compression spring

Helical extension spring

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Springs and Spring Selection – Types of springs

Space constraints and/or the desire to produce a torque make torsional springs more suitable in some applications. **Torsional** springs store energy by being twisted about their central axis.

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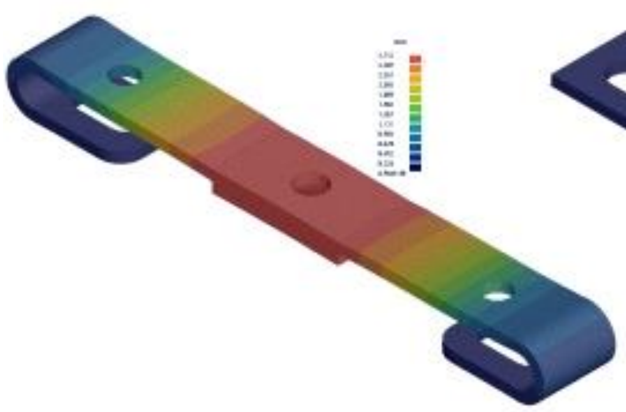
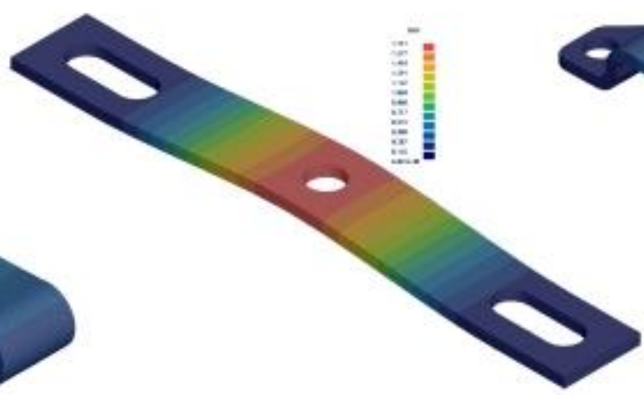
Springs and Spring Selection – Types of springs

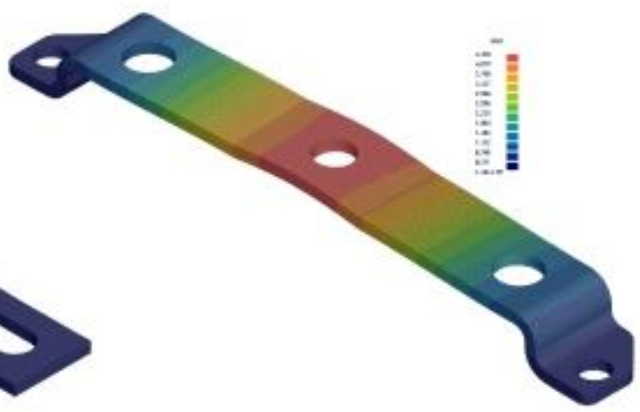
Torsional springs can utilise either rectangular or circular cross sections depending on the desired stiffness and anticipated deflection.

Springs and Spring Selection – Types of springs

Like other springs, **Flat springs** store and release energy when an external load is applied to them causing them to deflect. They are typically manufactured by flat pieces of metal that have been stamped.







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Springs and Spring Selection – Types of springs

Leaf springs are a type of flat spring. These springs are unique as when they are used in a multi leaf configuration, the friction from the flat springs sliding over each other provides damping.

Single leaf spring

Multi leaf spring

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Springs and Spring Selection – Types of springs

Spring washers are also a type of flat spring. These store energy when a nut and bolt is tightened adding to the friction in the assembly and preventing the nut and bolt from coming loose.

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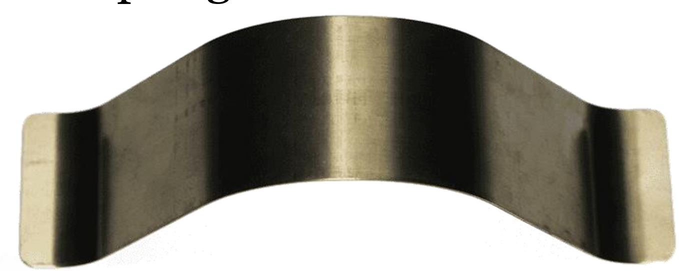
Springs and Spring Selection – Specifications

Because of the different roles that springs can undertake, they are very useful in mechanical engineering applications. Therefore we need to understand how to specify them.

Flat spring



Helical spring

Torsional spring

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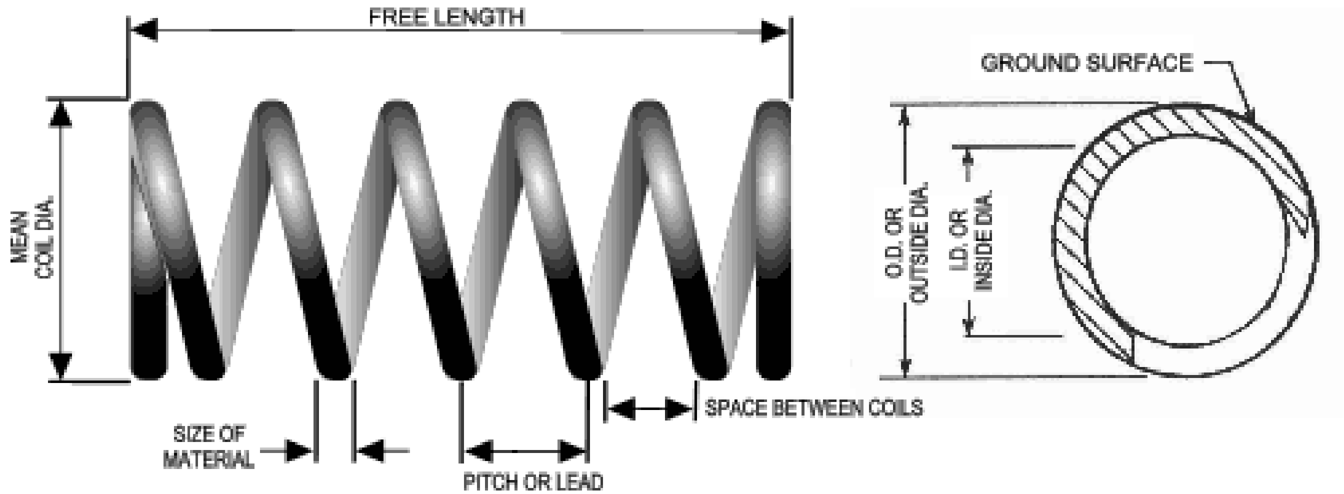
Springs and Spring Selection – Suppliers

To specify a compression helical spring, you must indicate:

- The spring stiffness required. This is defined by; the spring nominal diameter, the wire material and wire diameter
- The length of the spring desired.



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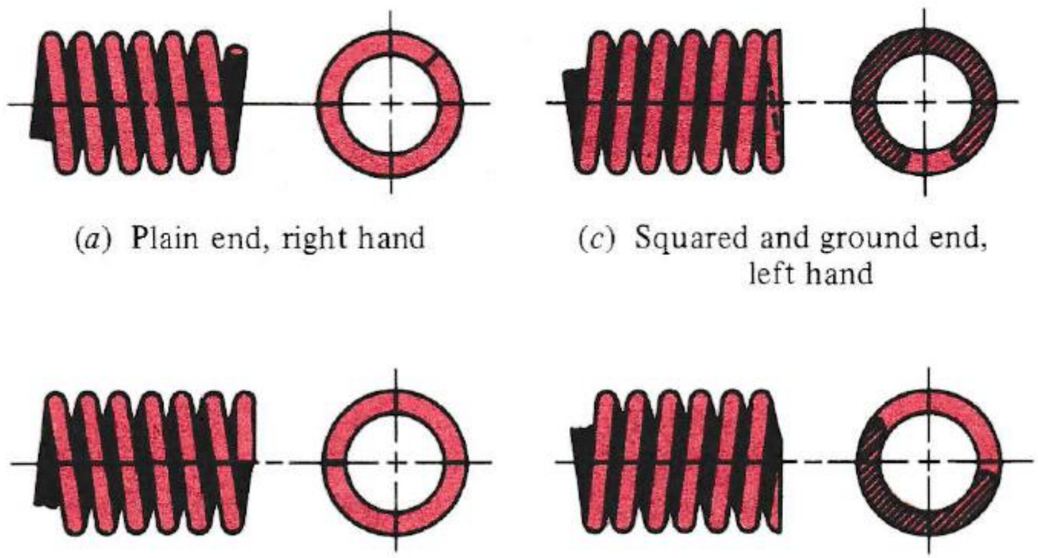
Springs and Spring Selection – Suppliers

To specify a compression helical spring, you must also indicate:

- The type of end that is required; Plain end, squared end and closed end.
- Direction that the spring is wound, left or right handed.



(*a*) Plain end, right hand

(*c*) Squared and ground end, left hand

(*b*) Squared or closed end, right hand

(*d*) Plain end, ground, left hand

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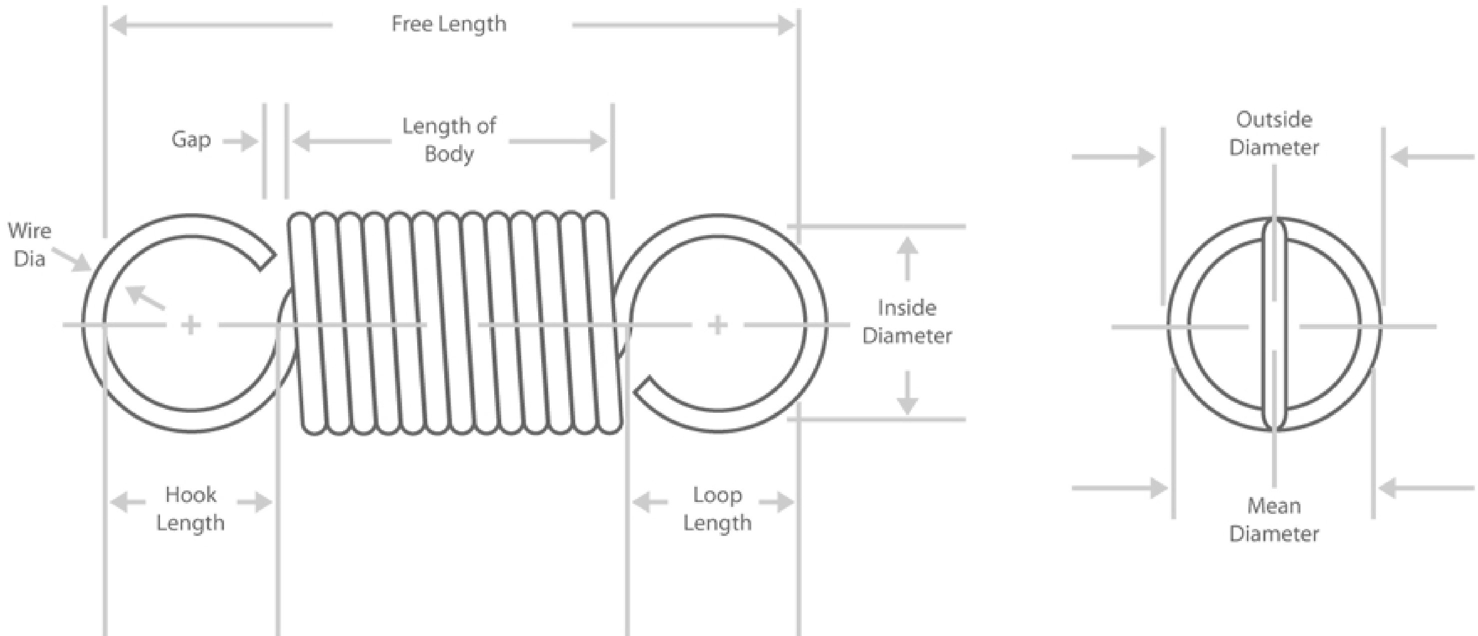
Springs and Spring Selection – Suppliers

To specify an extension helical spring, you must indicate:

- The spring stiffness required. This is defined by; the spring nominal diameter, the wire material and wire diameter
- The length of the spring desired.

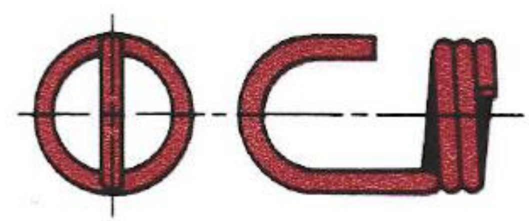


Springs and Spring Selection – Suppliers

To specify an extension helical spring, you must also indicate:

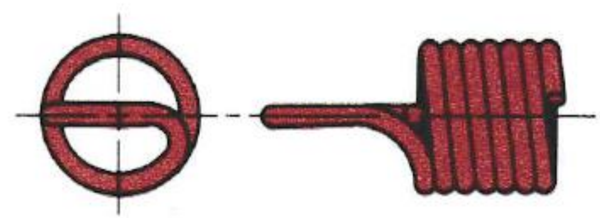
- The type of end that is required; Machine half loop, Raised Hook, Short twisted loop, or, Full twisted loop.
- Direction that the spring is wound, left or right handed.

Machine half loop–open



Raised hook

Short twisted loop



Full twisted loop

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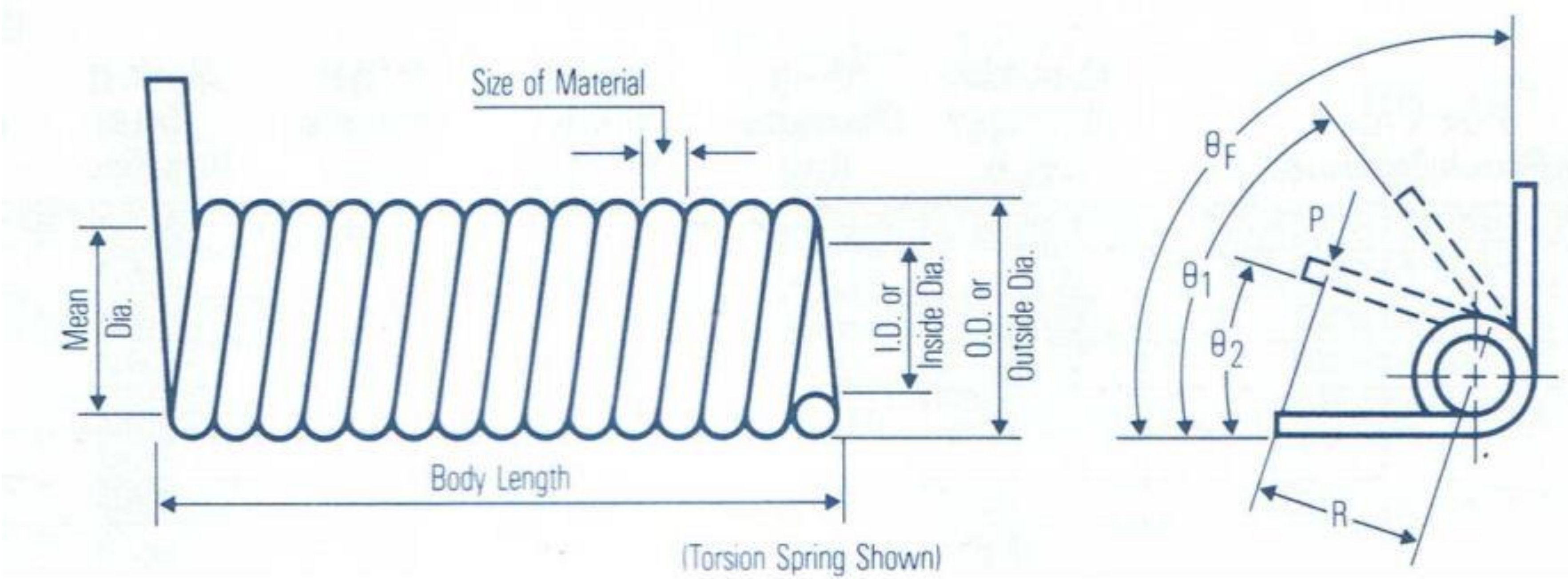
Springs and Spring Selection – Types of springs

To specify a torsional spring, you must indicate:

- The spring stiffness required. This is defined by; the spring nominal diameter, the wire material and wire diameter
- The body length of the spring desired.



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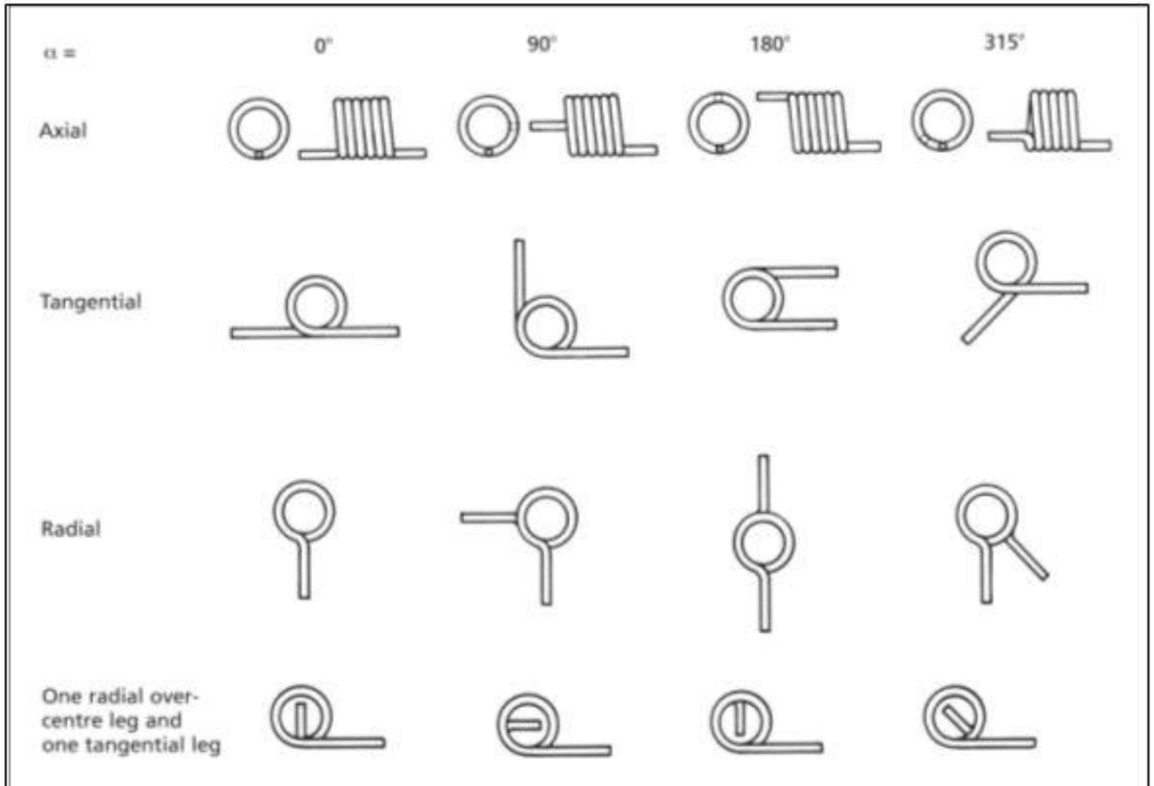
Springs and Spring Selection – Suppliers

To specify a torsional spring, you must also indicate:

- The type of end that is required; Axial, Tangential, Radial and the angle between the ends.
- Direction that the spring is wound, left or right handed.



Conventions for describing relative leg orientation

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Springs and Spring Selection – Manufacturing

Springs are made from a wire that is turned over a mandrel with some tension. This can be done hot…

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DEPARTMENT OF ENGINEERING

DR SAMMY DIASINOS

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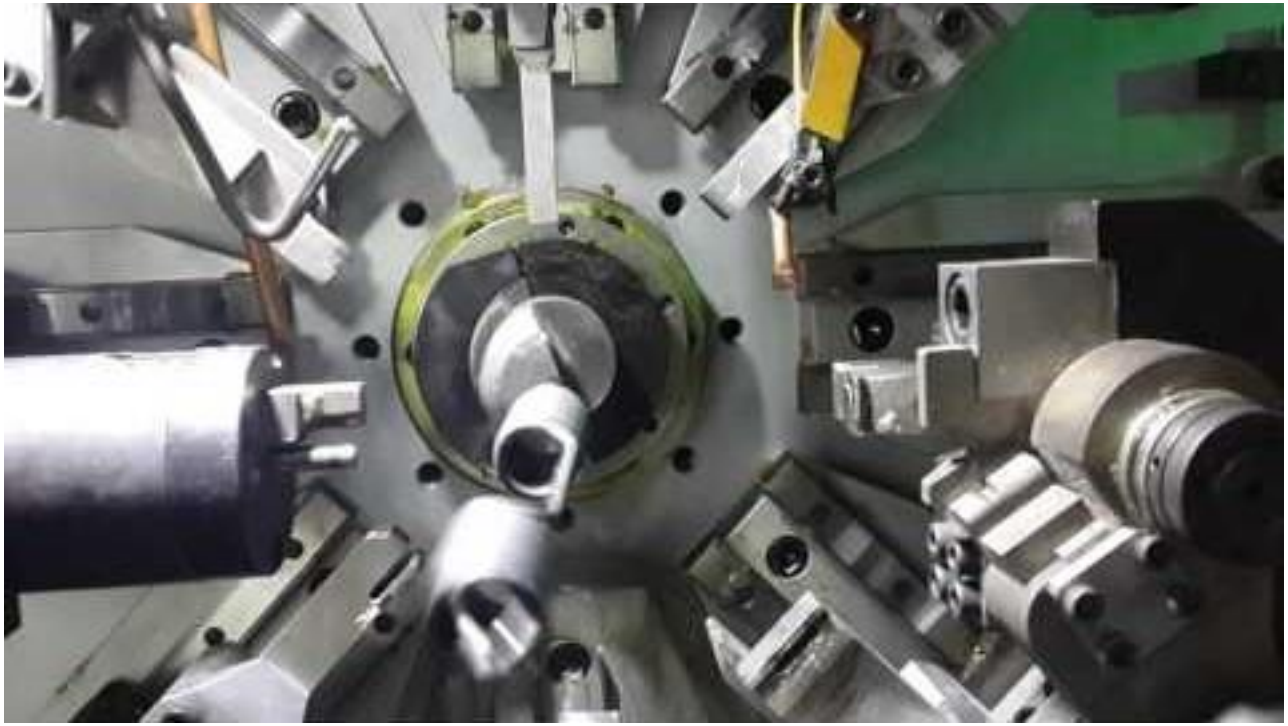
Springs and Spring Selection – Manufacturing

Springs are made from a wire that is turned over a mandrel with some tension. This can be done cold…



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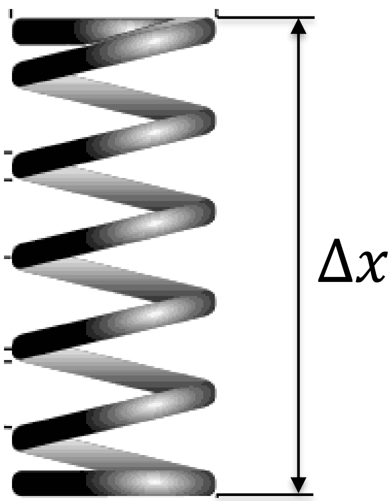
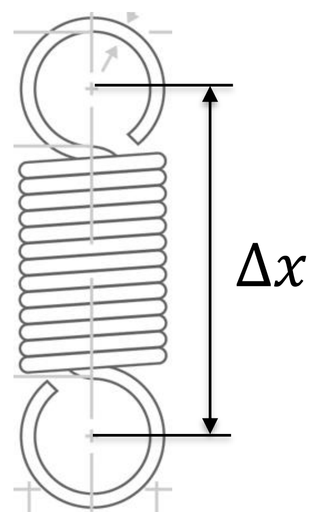
Springs and Spring Selection - Calculations

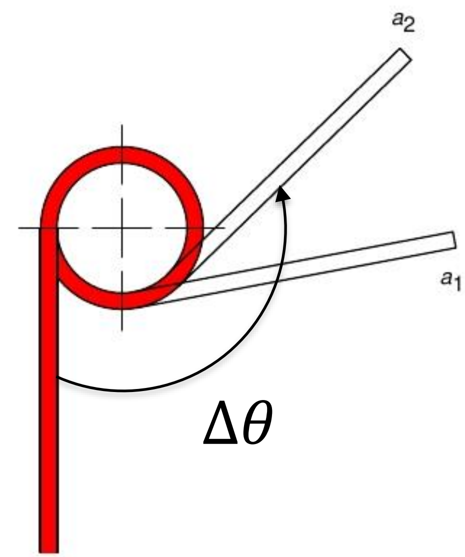
Force that a spring provides is proportional and relative to it's deflection. With no deflection, the spring provides no force, but as it is compressed/stretched/rotated, the force that it provides increases.



$$F_s = k(\Delta x)$$



$$F_s = k(\Delta x)$$



$$T_s = k(\Delta\theta)$$

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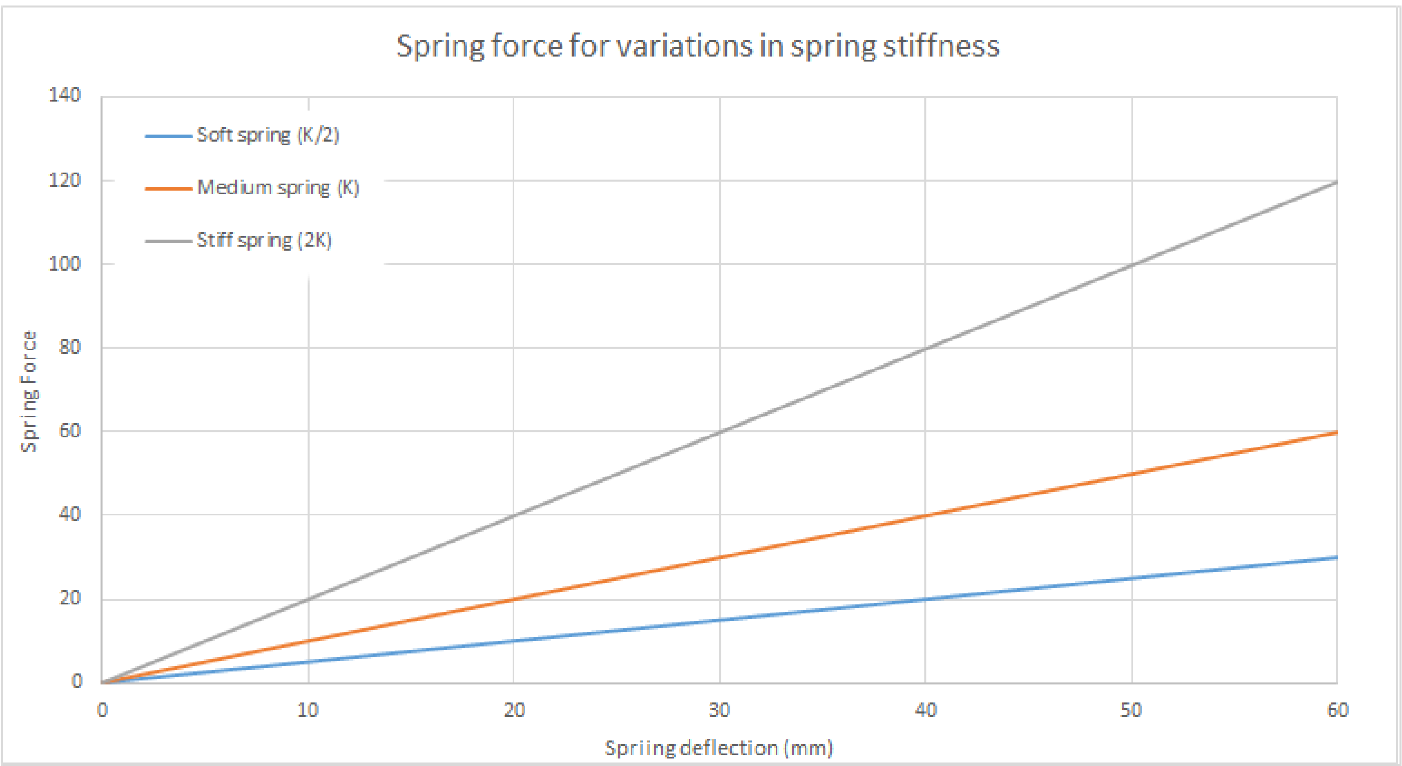
Springs and Spring Selection - Calculations

The rate with which the spring force increases with deflection is known as the **spring coefficient**. The stiffer the spring, the higher the spring coefficient.



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Springs and Spring Selection - Calculations

While keeping all other parameters equal, a spring can be made stiffer by:

- Increasing the wire diameter
- Reducing the mean spring diameter
- Increasing the pitch (for a compression helical spring)

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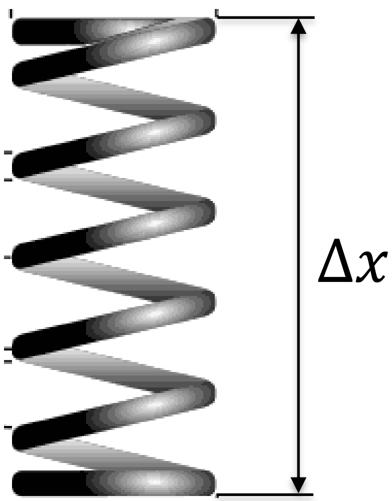
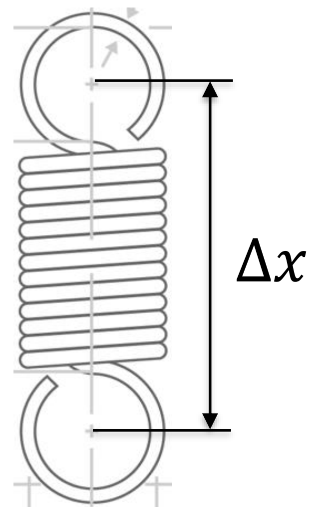
Springs and Spring Selection - Calculations

A deflected spring stores energy. The amount of energy that is stored is proportional to the spring coefficient and the displacement squared. Therefore, every time you double the spring deflection, you quadruple the potential energy it has stored.



$$E_s = \frac{1}{2}k(\Delta x)^2$$



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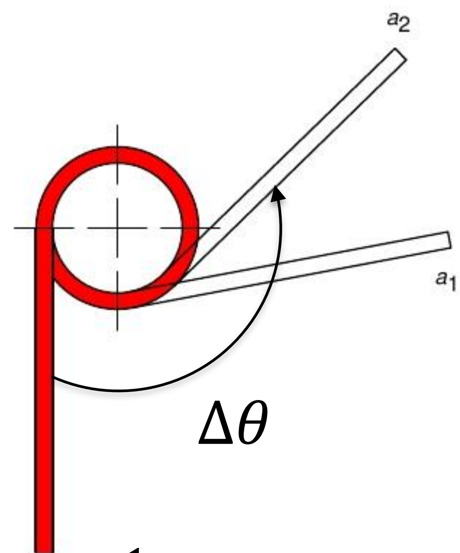

$$E_s = \frac{1}{2}k(\Delta\theta)^2$$

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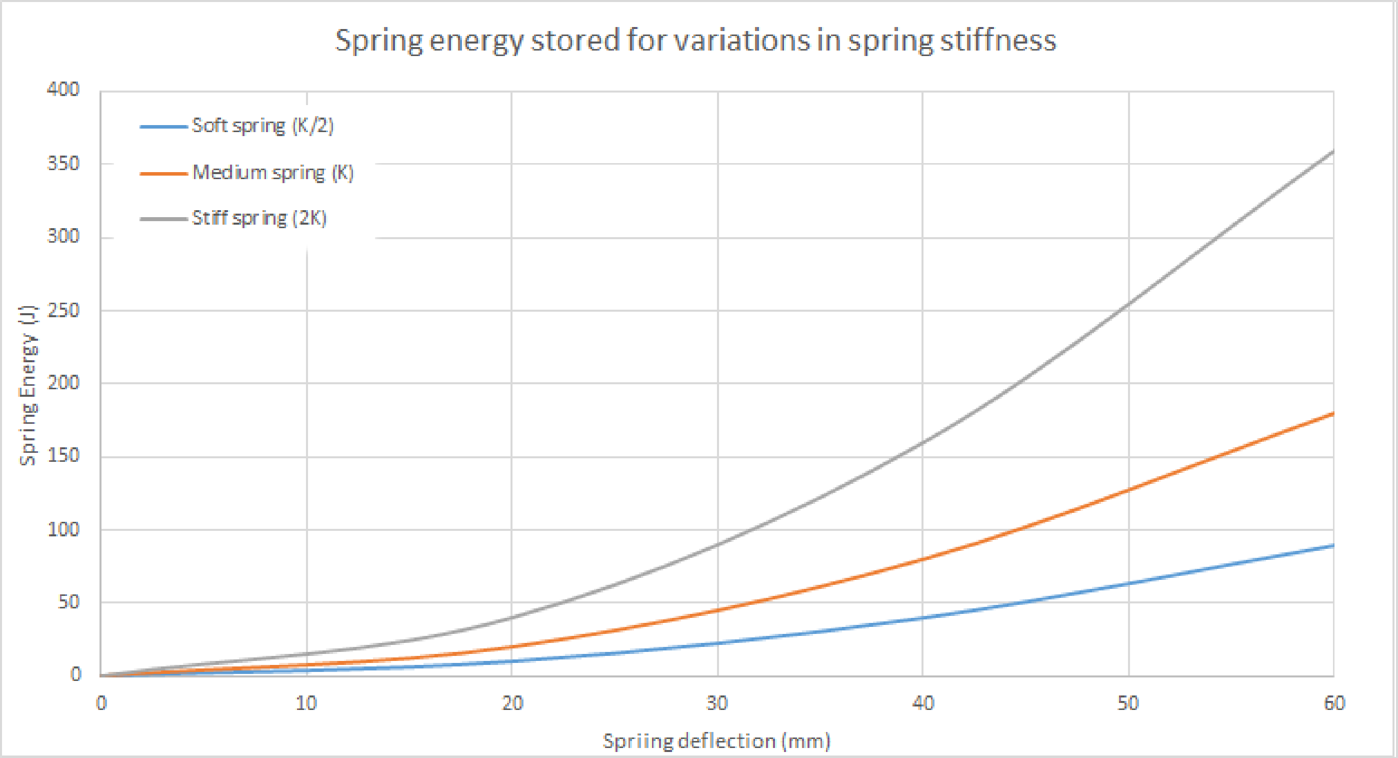
Springs and Spring Selection - Calculations

The energy that can be stored by a spring also increases with the **spring coefficient**. The stiffer the spring, the more energy that can be stored at a given deflection.



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Springs and Spring Selection - Calculations

Example 1

For a spring with a stiffness of 2000N/m, determine the force required to deflect the spring 20mm, 40mm and 60mm. What would be the energy stored in this spring for each of these deflections.

Example 1

For a spring with a stiffness of 2000N/m, determine the force required to deflect the spring 20mm, 40mm and 60mm. What would be the energy stored in this spring for each of these deflections.

Deflection (mm)	Force (N)	Energy (J)
20	40	0.4
40	80	1.6
60	120	3.6

Example 2

a) Determine the spring stiffness that would be required to store twice the energy for the same displacement as that in Example 1.

b) If this spring was being used to shoot an object with mass 0.5kg from a cannon, estimate the speed the object would leave the cannon with if it was deflected 100mm prior to being fired.

Example 2

a) Determine the spring stiffness that would be required to store twice the energy for the same displacement as that in Example 1.

$$E = 2 \times 3.6 = 7.2\,J$$

$$\delta = 60mm$$

$$k = E \times \frac{2}{\delta^2} = 7.2 \times \frac{2}{0.06^2} = 4000\,N/m$$

Springs and Spring Selection - Calculations

Example 2

b) If this spring was being used to shoot an object with mass 0.5kg from a cannon, estimate the speed the object would leave the cannon with if it was deflected 100mm prior to being fired.

$$E = \frac{1}{2}k\delta^2 = \frac{1}{2}mv^2$$

$$v = \sqrt{\frac{k\delta^2}{m}} = \sqrt{\frac{4000 \times 0.1^2}{0.5}} = 8.944\frac{m}{s}$$

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Springs and Spring Selection - Calculations

A spring, which has the ability to store and release energy makes it a useful flexible member.

This characteristic can reduce the impulse that is imparted onto an object making it very useful in suspension systems.

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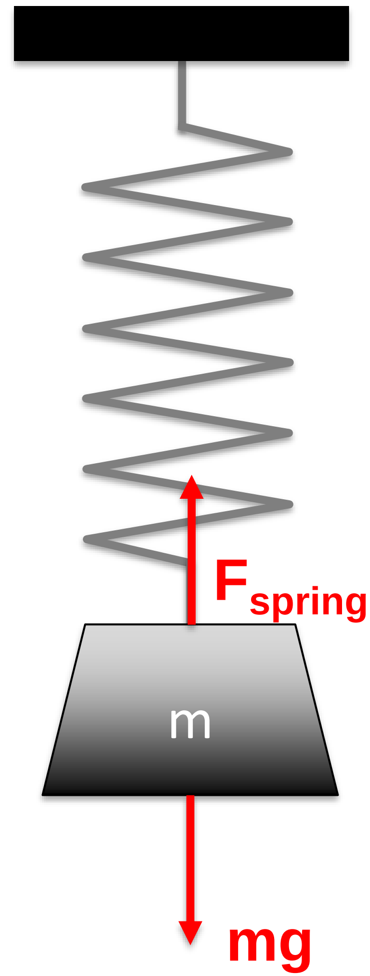
Springs and Spring Selection - Calculations

Since a deflected spring provides a force and stores potential energy that wants to be released, this leads to a challenge associated with their use.

Lets consider a mass (m) hung from a spring. When at rest, the spring force and the weight force are equal and opposite.



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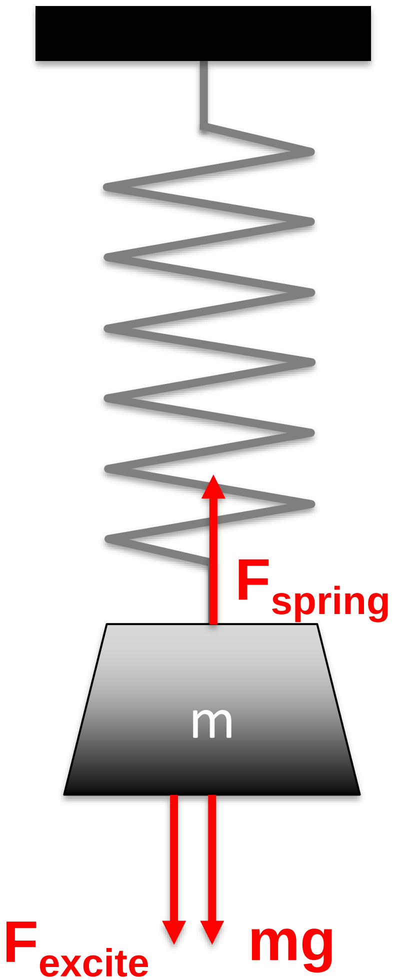
Springs and Spring Selection - Calculations

Now what would happen if the mass was excited by an additional temporary force acting down (F_{excite})…



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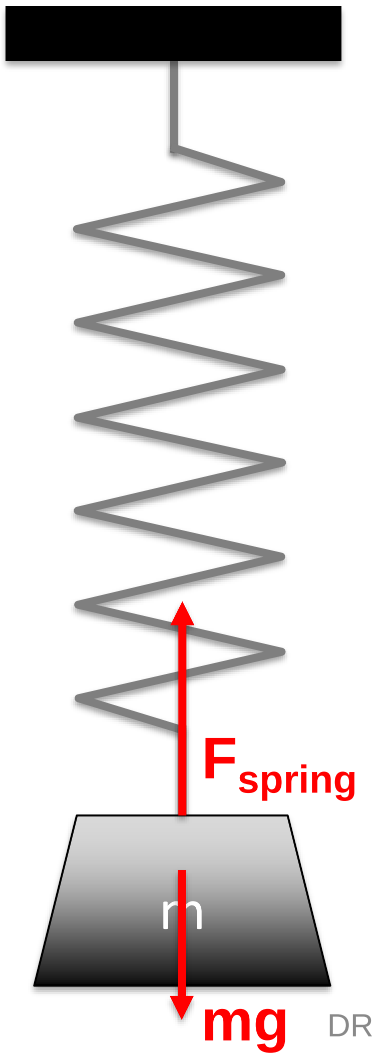
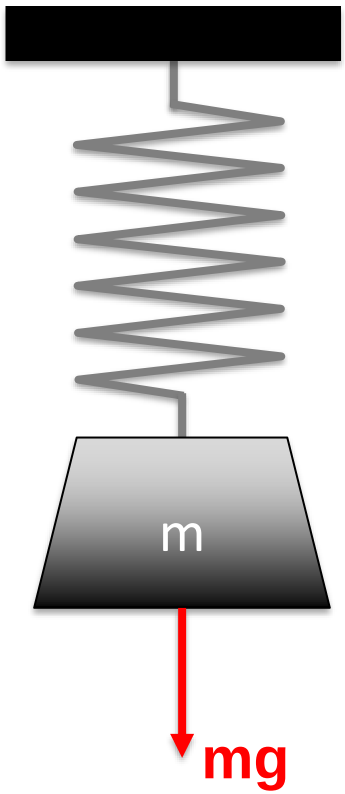
Springs and Spring Selection - Calculations

The excitation will result in the spring extending. Once fully extended, the potential energy in the spring will return a force. Because of the additional spring extension, the force is now greater than the weight resulting in the object being accelerate into the opposite direction until…



all the potential energy in the spring is consumed and instead the mass now has all the potential energy.

At this point there is no spring force and the weight of the mass will cause it to begin falling again…



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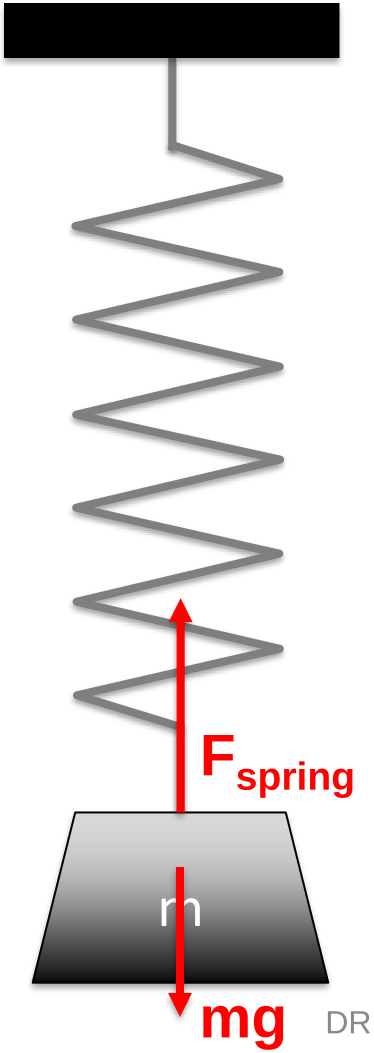
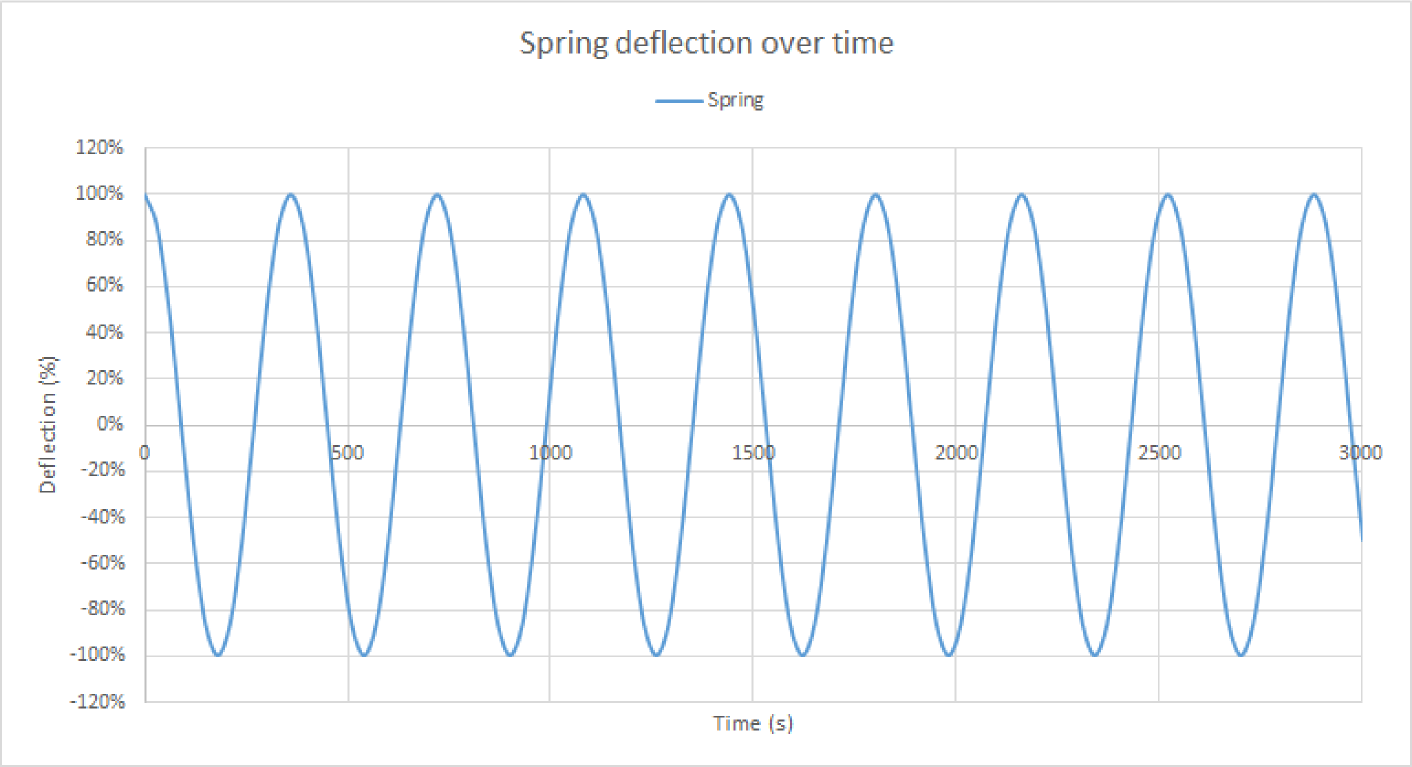
Springs and Spring Selection - Calculations

This initiates the process again causing the spring to deflect, and once again, store energy into the spring!

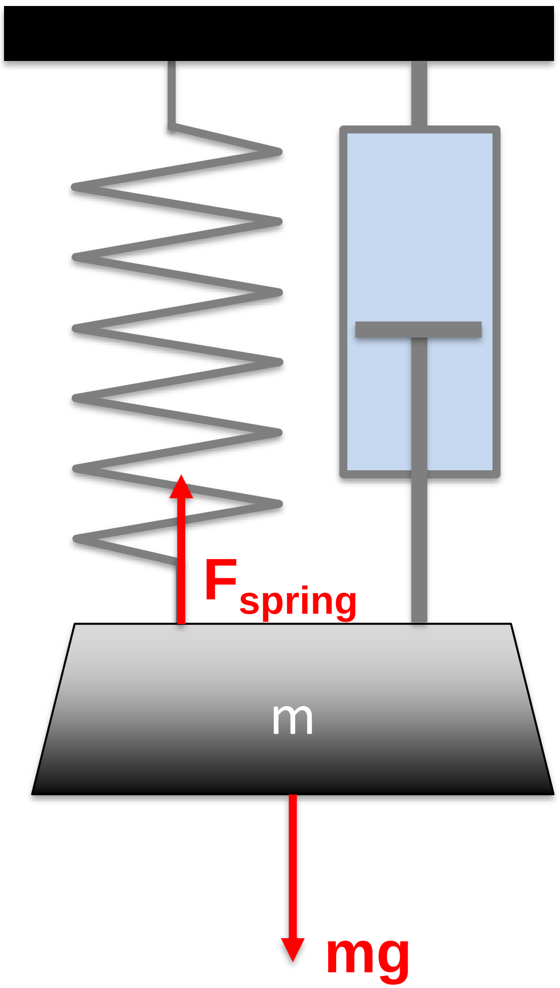


This results in an oscillating motion which, when neglecting friction or gravity, will never stop! But this is undesirable for most mechanical systems.



Springs and Spring Selection - Calculations

For this reason springs are usually coupled with dampers as they provide a force that is proportional to the speed of a piston through a fluid.



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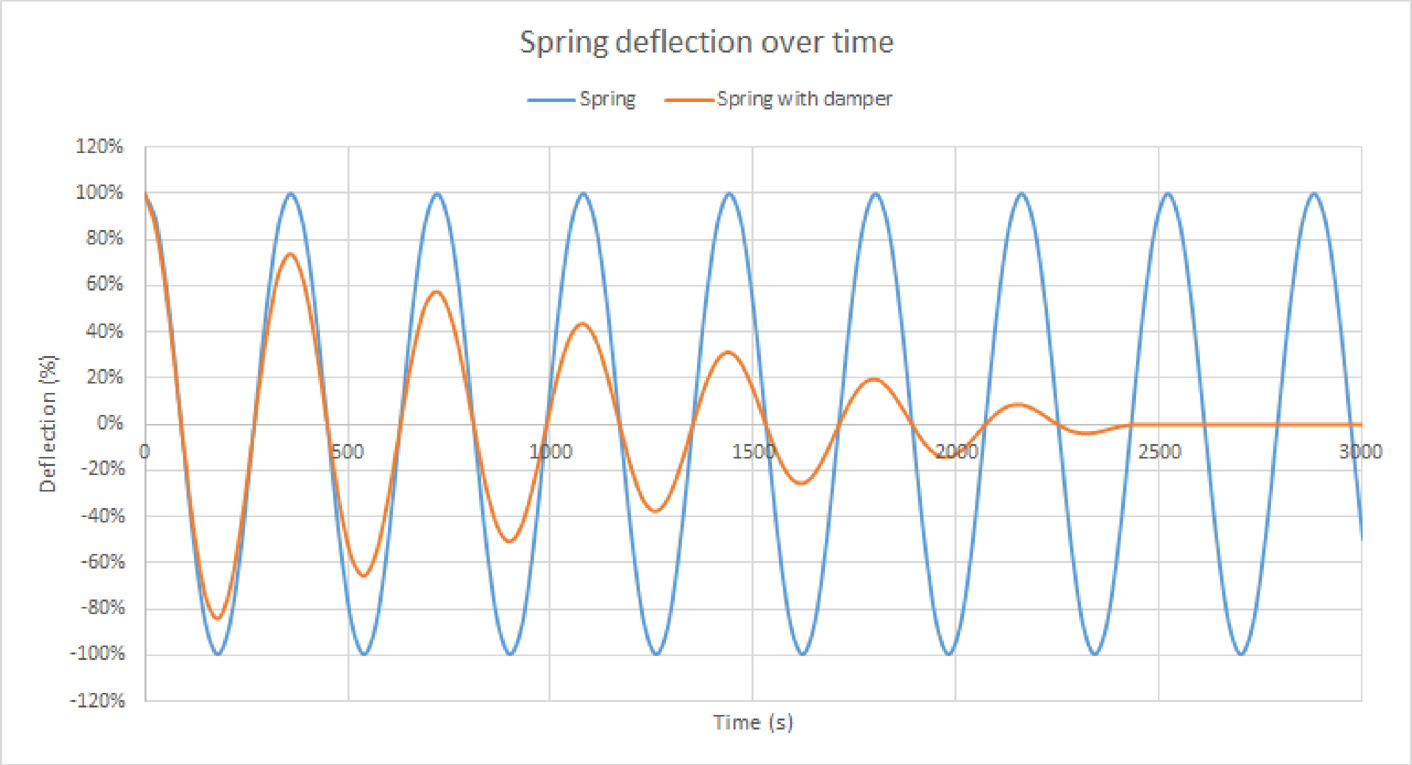
Springs and Spring Selection - Calculations

This dampens the motion and eventually allows the excited mass to come to rest again. Without any damping, the spring and mass would oscillate indefinitely.



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Springs and Spring Selection - Calculations

This combination allows vehicle suspension systems to provide a comfortable ride. The spring is used to offer compliance and minimise the forces transferred to the occupants while the damper minimises the oscillation in the spring.

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Springs and Spring Selection - Calculations

Springs reduce the force experienced at any moment in time by increasing the duration that an impulse is transferred with. Remember that an impulse is the multiple of force and time and also the change in momentum.

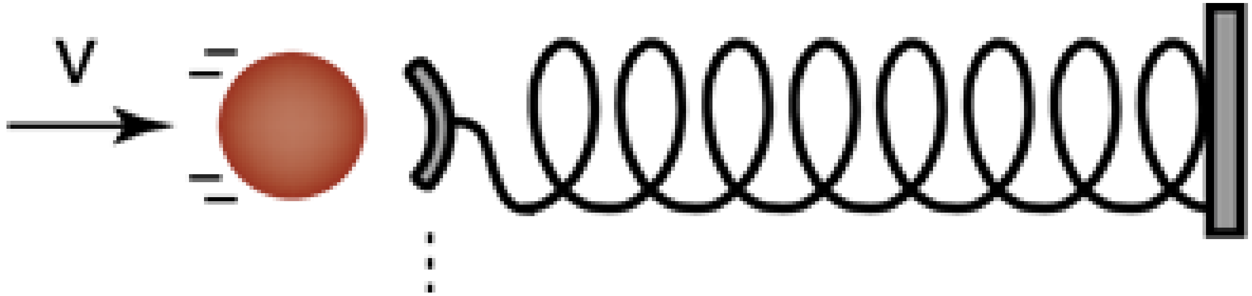
$$\boldsymbol{I = \Delta mv = Ft}$$

Where:
I = Impulse (Ns)
m = mass (kg)
v = velocity (m/s)
F = Force (N)
t = Time (s)

Springs and Spring Selection - Calculations

Example 3

A 0.5 kg object travelling with a speed of 12m/s impacts a spring one side of a spring which is supported by a wall at the opposite end. Assuming an infinitely long spring that will not buckle, determine the maximum reaction force at the wall and the displacement in the spring to bring the ball to rest if the spring has a stiffness of 1000 N/m, 5000 N/m and 10000 N/m.



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Springs and Spring Selection - Calculations

Example 3

A 0.5 kg object travelling with a speed of 12m/s impacts a spring one side of a spring which is supported by a wall at the opposite end. Assuming an infinitely long spring that will not buckle, determine the maximum reaction force at the wall and the displacement in the spring to bring the ball to rest if the spring has a stiffness of 1000 N/m, 5000 N/m and 10000 N/m.

$$E = \frac{1}{2}mv^2 = \frac{1}{2}kx^2$$

$$x = \sqrt{\frac{mv^2}{k}}$$

Example 3

A 0.5 kg object travelling with a speed of 12m/s impacts a spring one side of a spring which is supported by a wall at the opposite end. Assuming an infinitely long spring that will not buckle, determine the maximum reaction force at the wall and the displacement in the spring to bring the ball to rest if the spring has a stiffness of 1000 N/m, 5000 N/m and 10000 N/m.

Spring Stiffness (N/m)	Displacement (m)	Force (N)
1000	0.2683	268.3
5000	0.12	600
10000	0.08485	848.5

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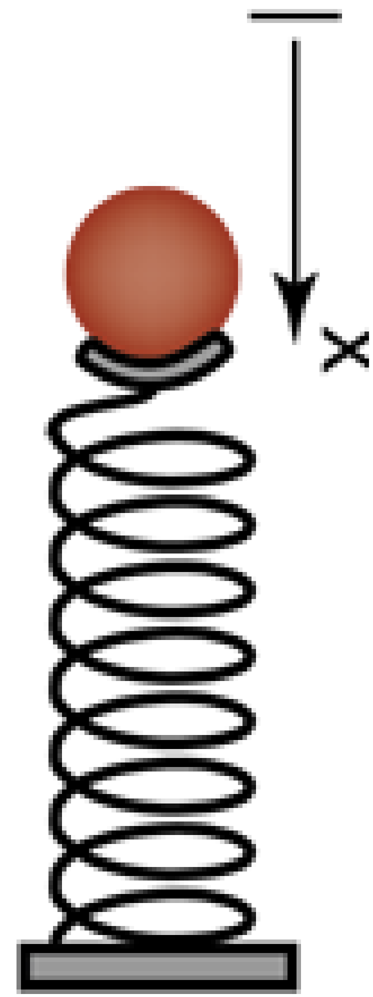
Springs and Spring Selection - Calculations

So what are we trying to achieve with crash structures?

Springs and Spring Selection - Calculations

Example 4

A 0.5 kg is resting on the top of a spring which is supported by a wall at the opposite end. If the system is at rest, determine the maximum reaction force at the wall and the displacement in the spring if the spring has a stiffness of 1000 Nm, 5000 Nm and 10000 Nm.



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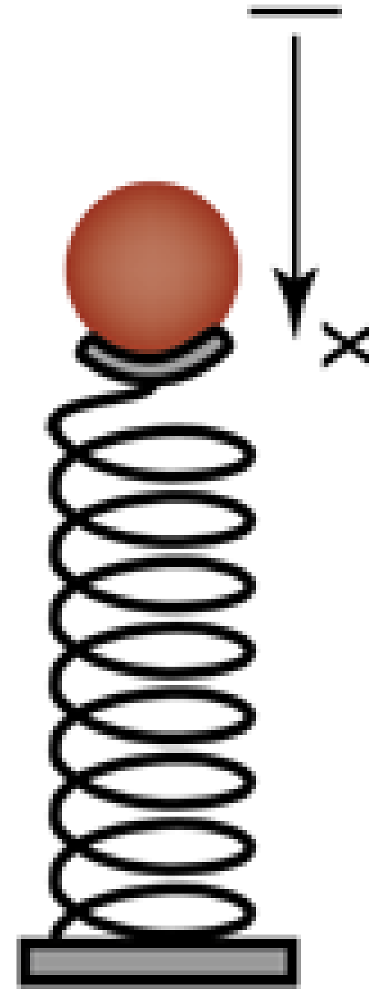
Springs and Spring Selection - Calculations

Example 4

A 0.5 kg is resting on the top of a spring which is supported by a wall at the opposite end. If the system is at rest, determine the maximum reaction force at the wall and the displacement in the spring if the spring has a stiffness of 1000 Nm, 5000 Nm and 10000 Nm.



$$F = mg = 4.905\ N$$

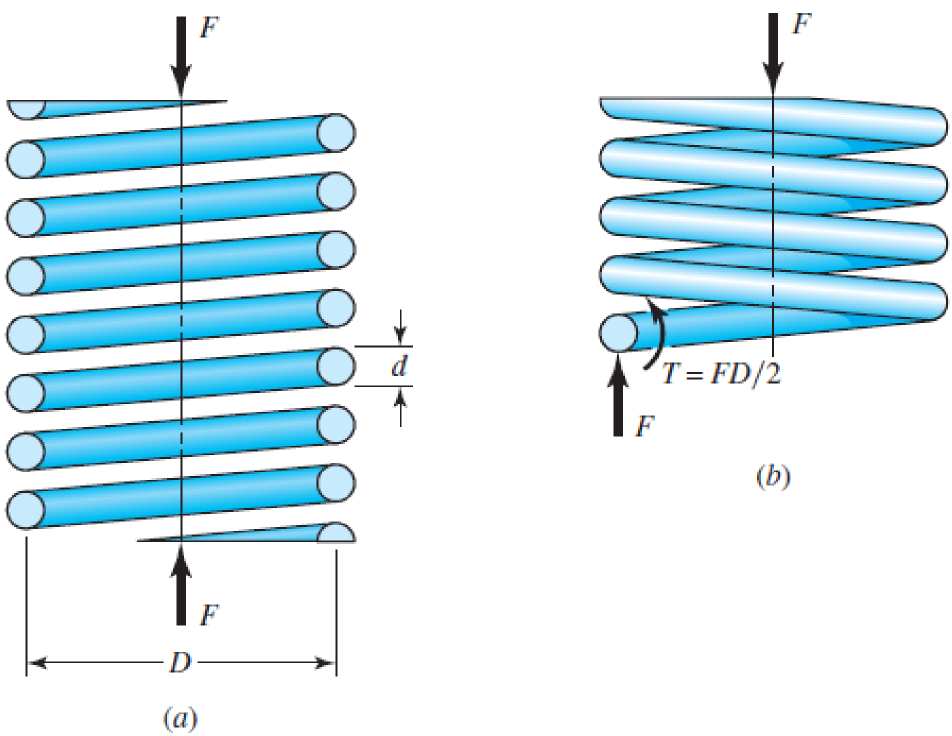
$$F = kx$$

$$x_{1000} = 4.905mm \qquad x_{5000} = 0.981mm$$

$$x_{10000} = 0.4905mm$$

Springs and Spring Selection - Calculations

The maximum stress in a wire of a helical spring can be determined by cutting a spring, and at the cut cross section considering the internal forces/moments to determine the shear force and torsional moment. This reduces to be:



$$\tau_{max} = \frac{8FD}{\pi d^3} + \frac{4F}{\pi d^2}$$

Where:
F = Spring Force (N)
D = Spring mean diameter (m)
d = Wire diameter (m)

The spring index (C) is the ratio of the spring mean diameter to the wire diameter. Where K_s is shear stress correction factor and defined as:

$$C = \frac{D}{d}$$

$$K_s = \frac{2C + 1}{2C}$$

Where:
C = Spring index
K_s = Shear stress correction factor

The stress in the spring reduces to be:

$$\tau = K_s \frac{8FD}{\pi d^3}$$

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Springs and Spring Selection - Calculations

The deflection-force relationship can be obtained by considering the total strain energy. For a helical spring, this is composed of a torsional and shear component. This allows the spring rate, also known as the spring scale, to be determined as:

$$\boldsymbol{k = \frac{d^4 G}{8 D^3 N_a}}$$

Where:
G = Shear Modulus (Pa)
N_a = Number of Active Coils

Term	Type of Spring Ends: Plain	Plain and Ground	Squared or Closed	Squared and Ground
End coils, N_e	0	1	2	2
Total coils, N_t	N_a	$N_a + 1$	$N_a + 2$	$N_a + 2$
Free length, L_0	$pN_a + d$	$p(N_a + 1)$	$pN_a + 3d$	$pN_a + 2d$
Solid length, L_s	$d(N_t + 1)$	dN_t	$d(N_t + 1)$	dN_t
Pitch, p	$(L_0 - d)/N_a$	$L_0/(N_a + 1)$	$(L_0 - 3d)/N_a$	$(L_0 - 2d)/N_a$

Springs and Spring Selection - Calculations

When the load on a compression spring becomes to large, it will buckle. This compromises the spring's ability to continue carrying load and therefore it is necessary to determine at what value this will occur. This is the same as determining if the spring is stable. For steel springs, this occurs when:

$$L_0 < 2.63\frac{D}{\alpha}$$

End Condition	Constant α
Spring supported between flat parallel surfaces (fixed ends)	0.5
One end supported by flat surface perpendicular to spring axis (fixed); other end pivoted (hinged)	0.707
Both ends pivoted (hinged)	1
One end clamped; other end free	2

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Springs and Spring Selection - Calculations

Spring materials tensile strength varies with wire size, the process it undergoes and the material used. The tensile strength of spring materials relative to wire diameter can be represented by:

$$S_{ut} = \frac{A}{d^m}$$

Material	ASTM No.	Exponent m	Diameter, in	A, kpsi · in^m	Diameter, mm	A, MPa · mm^m	Relative Cost of Wire
Music wire*	A228	0.145	0.004–0.256	201	0.10–6.5	2211	2.6
OQ&T wire†	A229	0.187	0.020–0.500	147	0.5–12.7	1855	1.3
Hard-drawn wire‡	A227	0.190	0.028–0.500	140	0.7–12.7	1783	1.0
Chrome-vanadium wire§	A232	0.168	0.032–0.437	169	0.8–11.1	2005	3.1
Chrome-silicon wire‖	A401	0.108	0.063–0.375	202	1.6–9.5	1974	4.0
302 Stainless wire#	A313	0.146	0.013–0.10	169	0.3–2.5	1867	7.6–11
		0.263	0.10–0.20	128	2.5–5	2065	
		0.478	0.20–0.40	90	5–10	2911	
Phosphor-bronze wire**	B159	0	0.004–0.022	145	0.1–0.6	1000	8.0
		0.028	0.022–0.075	121	0.6–2	913	
		0.064	0.075–0.30	110	2–7.5	932	

Example 5:

A helical compression spring is made of 1mm diameter music wire. The outside coil diameter of the spring is 10mm. The ends are squared and there are 12.5 total turns.

a) Estimate the torsional yield strength of the wire.

b) Find the static load corresponding to the yield strength.

c) What is the scale of the spring?

d) Compute the deflection that would result from part b).

e) Compute the solid length of the spring.

Example 5 *(continued)***:**

f) What length should the spring be so that when it is compressed solid and then released, there will be no permanent change in the length of the spring?

g) Given the length found in part f, is buckling a possibility?

h) What is the pitch of the spring?

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Springs and Spring Selection - Calculations

Example 5 *(solution)***:**

a) Estimate the torsional yield strength of the wire.

$Where$:

$A = 2211 \times 10^3$ -> from table on slide 62 for piano wire

$d = 0.001m$ -> this is the wire diameter which is provided in the example

$m = 0.145$ -> from table on slide 62 for piano wire

$$s_{ut} = \frac{A}{d^m} = \frac{2210 \times 10^3}{0.001^{0.145}} = 6.01717 \times 10^6\ Pa$$

$$s_{sy} = 0.45 s_{ut} = 0.45 \times 6.01717 \times 10^6 = 2.70773 \times 10^6 = \mathbf{2.708 \times 10^6 \boldsymbol{Pa}}$$

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Springs and Spring Selection - Calculations

Example 5 *(solution)***:**

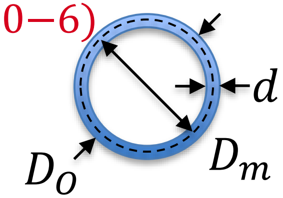
b) Find the static load corresponding to the yield strength.

$Where$:

$D_O = 0.01m$ -> this is the spring outer diameter which is provided in the example

$D_m = ?$ −> this is the spring mean diameter which needs to be determined

$K_B = ?$ −> this is a stress correction factor provided by the text as Equation (10−6)



$$D_m = D_O - d = 0.01 - 0.001 = 0.009m$$

$$C = \frac{D_m}{d} = \frac{0.009}{0.001} = 9$$

$$K_B = \frac{4C + 2}{4C - 3} = \frac{4 \times 9 + 2}{4 \times 9 - 3} = 1.151515$$

$$T_s = \frac{s_{sy} \pi d^3}{8 K_B D_m} = \frac{2.708 \times 10^6 \times \pi \times 0.001^3}{8 \times 1.151515 \times 0.009} = 0.102612 = \mathbf{0.1026\ N}$$

Example 5 *(solution)***:**

c) What is the scale of the spring?

$Where$:

$G = 8.27 \times 10^7 Pa$ -> material stiffness found in Table 10-5 of text and in equation sheet

$N_t = 12.5$ −> number of turns which is provided in the example

$N_a = ?$ −> number of effective turns calculated from appropriate equation from table on slide 60

$$N_a = N_t - 2 = 12.5 - 2 = 10.5\ turns$$

$$k = \frac{d^4 G}{8{D_m}^3 N_a} = \frac{0.001^4 \times 8.27 \times 10^7}{8 \times 0.009^3 \times 10.5} = 1.35051277 = \mathbf{1.351\ N/m}$$

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Springs and Spring Selection - Calculations

Example 5 *(solution)***:**

d) Compute the deflection that would result from part b).

$$y_s = \frac{F_s}{k} = \frac{0.102612}{1.350513} = 75.980001 = \mathbf{75.98mm}$$

e) Compute the solid length of the spring.

$Where$:

$L_s = ?$ -> compressed length calculated from appropriate equation from table on slide 60

$$L_s = d(N_t + 1) = 0.001(12.5 + 1) = 13.5mm$$

Example 5 *(solution)***:**

f) What length should the spring be so that when it is compressed solid and then released, there will be no permanent change in the length of the spring?

To prevent yield from occurring, the spring can not be longer than the solid length (e) plus the deflection to achieve yield (d).

$L_o = L_s + y_s = 13.5 + 75.98 = \mathbf{89.48}\boldsymbol{mm}$

Example 5 *(solution)***:**

g) Given the length found in part f, is buckling a possibility?

$Where:$

$$L_o < 2.63\frac{D_m}{\alpha}$$

$\alpha = 0.5$ -> end condition constant from table on slide 61

$$L_o < 2.63\frac{D_m}{\alpha} < 2.63\frac{0.009}{0.5} < 0.04734 = 47.34mm$$

$84.98 < 47.34$ *is false therefore spring is expected to buckle*

h) What is the pitch of the spring?

$Where:$

$$P = \frac{L_o - 3d}{N_a} = \frac{0.08498 - 3 \times 0.001}{10.5} = 0.00809305 = 8.093mm$$