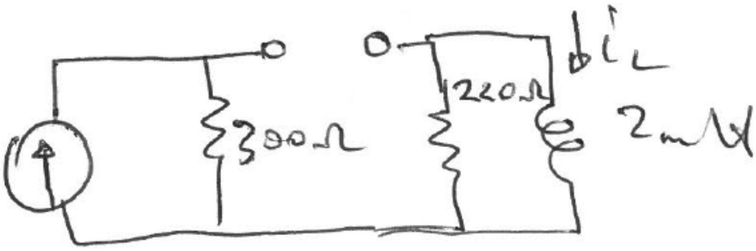
Assignment 2 [redacted]

SOLUTIONS

1 (a) for $t < 0$

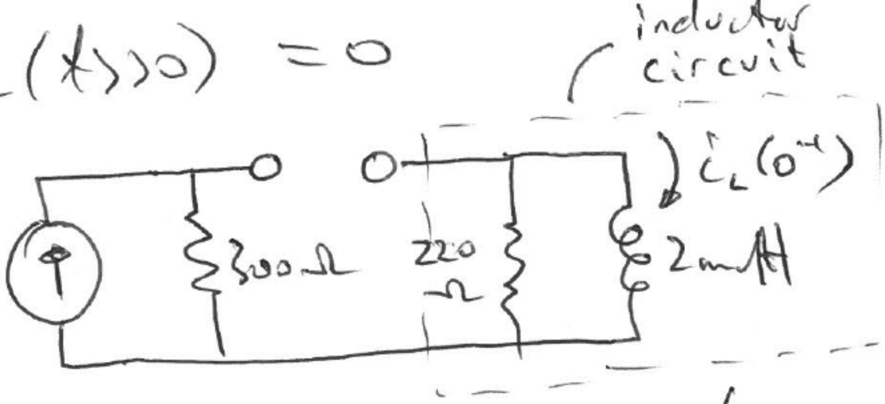
$$i_L(0^-) = 4mA = i_L(0^+)$$

for $t > 0$



for $t \gg 0$, $i_L(t \gg 0) = 0$

(b) for $t > 0$



$i_L(0^+) = 4mA$

$$\tau = \frac{L}{R} = \frac{2\times10^{-3}}{220} = 9\times10^{-6}\ s$$

$$i_L(t) = 4\times10^{-3}\, e^{-1.1\times10^{5} t}\ A$$

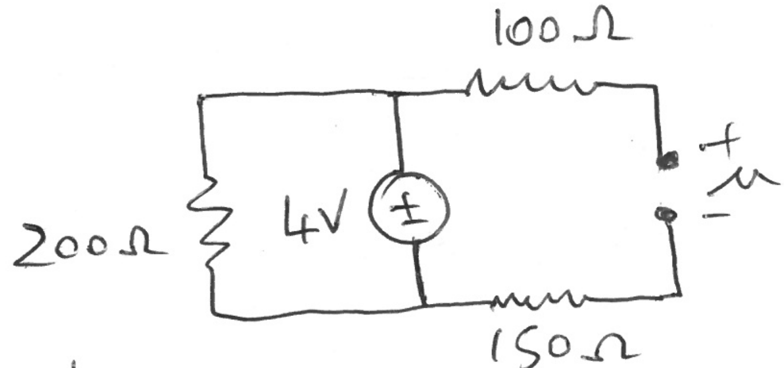
$$= 4\, e^{-1.1\times10^{5} t}\ mA$$

(c) $t = 15.8\ \mu s \rightarrow i_L = 0.703\ mA$ ②

$t = 31.5\ \mu s \rightarrow i_L = 0.125\ mA$

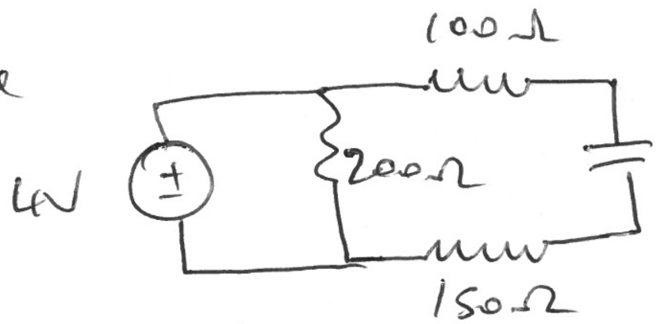
$t = 78.8\ \mu s \rightarrow i_L = 6.88\times10^{-4}\ mA$

② (a) for $t<0$



voltage across capacitor = source
$= 4V$

(b) for $t<0$ we have



$R_t = 450\,\Omega$

$\tau = R_t C = 9\times10^{-7}\ s$

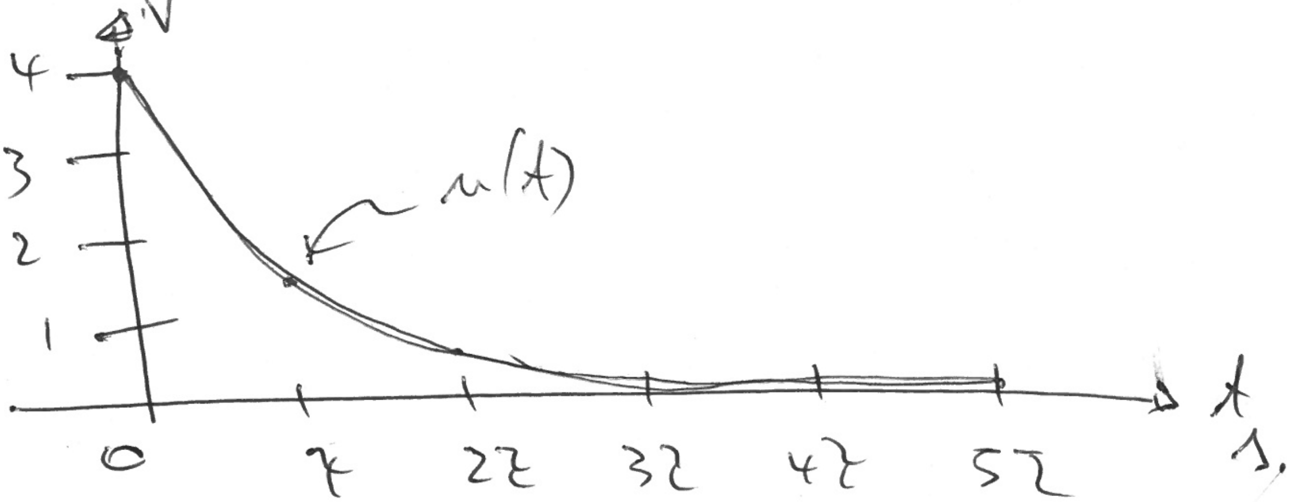
$$\therefore v(t) = 4e^{-t/9\times10^{-7}s}$$
$$= 4e^{-1.1\times10^{6}t}\ V$$

(c) $u(\tau) = 1.47\,V = 4e^{-1}\,V$

$u(2\tau) = 4e^{-2} = 0.54\,V$

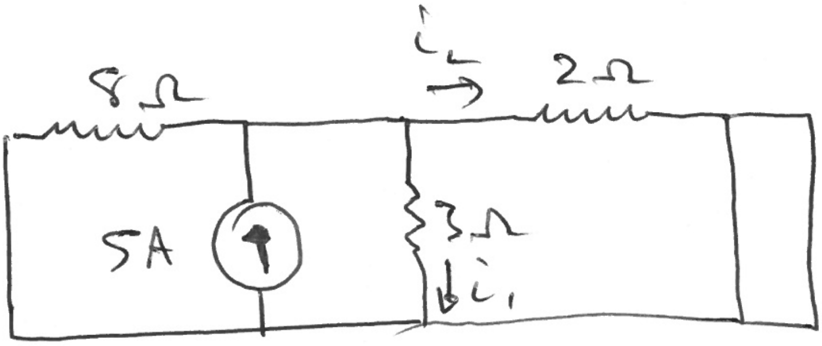
$u(5\tau) = 4e^{-3} = 0.027\,V$

(d)



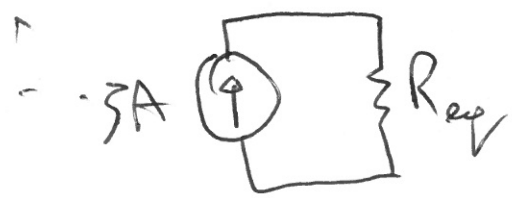
3 For $t<0$

3 resistors in parallel



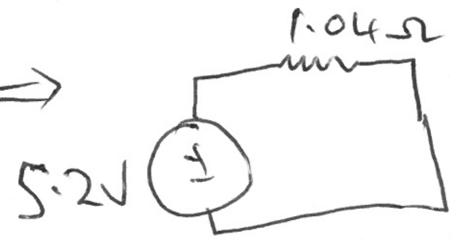
$$\frac{1}{R_{eq}} = \frac{1}{8} + \frac{1}{3} + \frac{1}{2}$$

$$\Rightarrow R_{eq} = 1.04\,\Omega$$



Source transformation

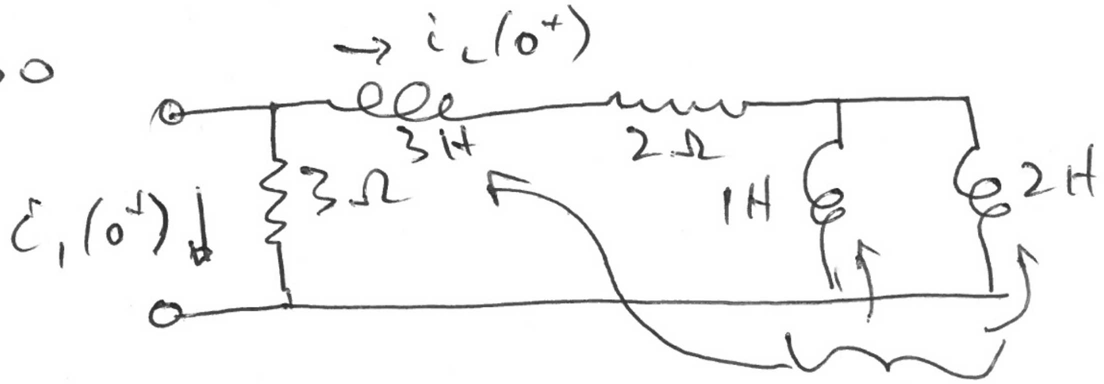
$\Rightarrow$



$\therefore\ i_L(0^-) = \dfrac{5.2}{2} = 2.6\ A$

$i_1(0^-) = \dfrac{5.2}{3} = 1.73\,A$

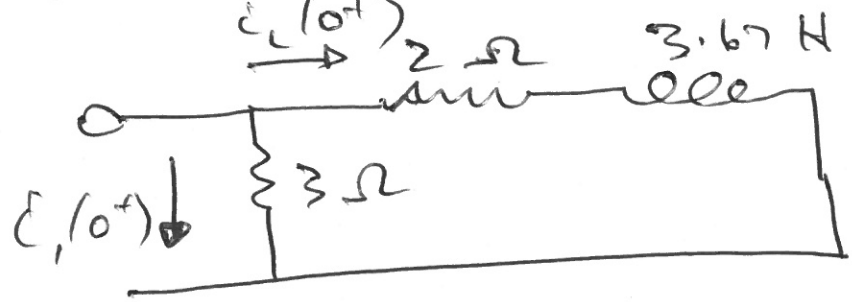
for $t > 0$



find equivalent inductance.

$1H \,\|\, 2H + 3H = 3.67\,H.$

$\therefore$ circuit



$R_t = 5\,\Omega \quad \therefore\ L/R_t = \dfrac{3.67}{5} = 0.734\,s$

$\therefore\ i_L(t) = i_L(0^-)e^{-t/\tau}$

$= 2.6e^{-t/0.734}$

$= 2.6e^{-1.36t}\ A$

$i_1(t) = 1.73e^{-1.36t}\,A$

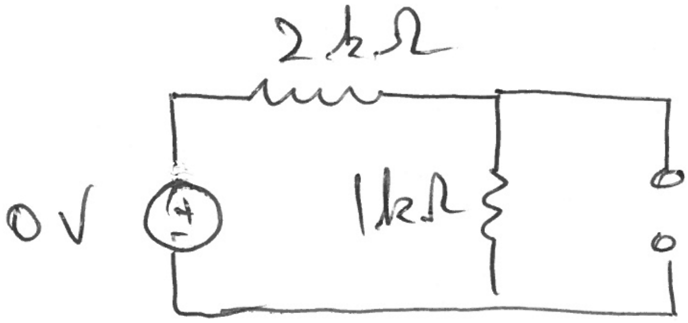
$i_{total}(t) = i_L(t) - i_1(t)$ (since going in opposite directions

5

$= (2.6 - 1.73)e^{-1.36t}$

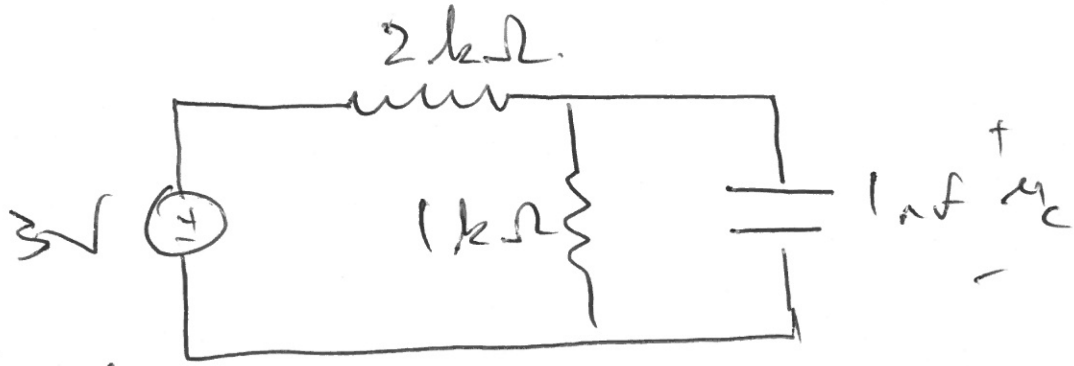
$i_7(t) = 0.87e^{-1.36t}$ A.

4 (a) for $t<0$



$v_c(0^-) = v_c(0^+) = 0V$

for $t>0$



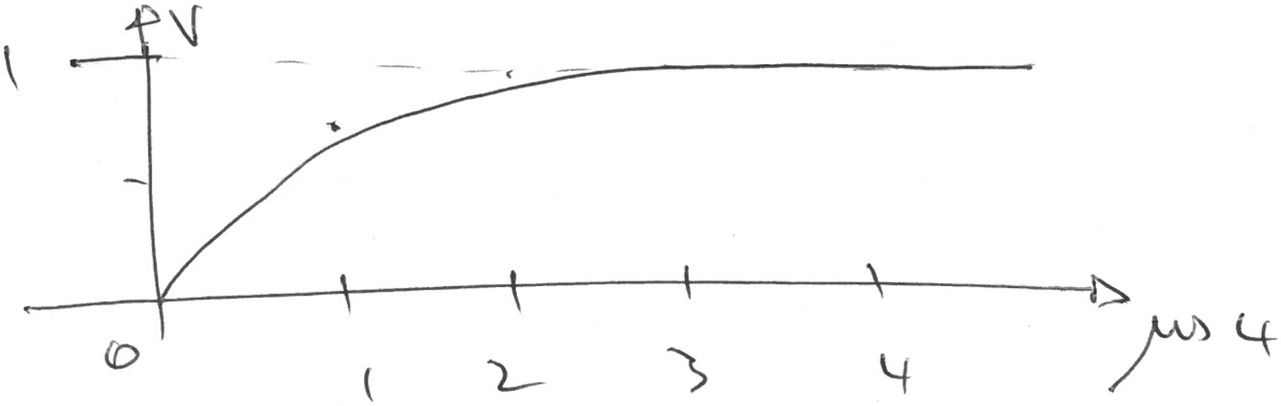
$R_{eq} = 0.667\ k\Omega$

$\therefore \tau = RC = 0.667\times10^3\times10^{-9}$
$= 0.667\times10^{-6}$ s

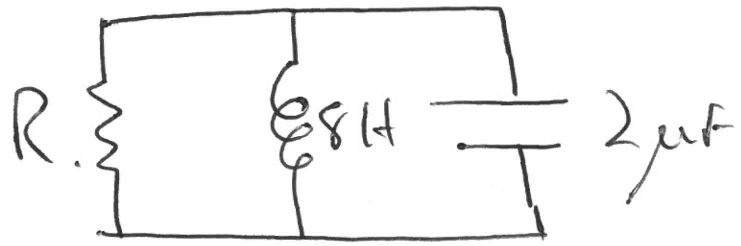
$V_c(\infty) = 1V$

$\therefore v_c(t) = 1 + (0-1)e^{-1.5\times10^6 t}$
$= 1 - e^{-1.5\times10^6 t}$ V.

$t = 1\mu s$ $v_c = 0.777$ V
2 μs = 0.93 V
3 μs = 0.989 V
4 μs = 0.998 V



(5) (a)



for critically damped $\alpha^2 = \omega_0^2$

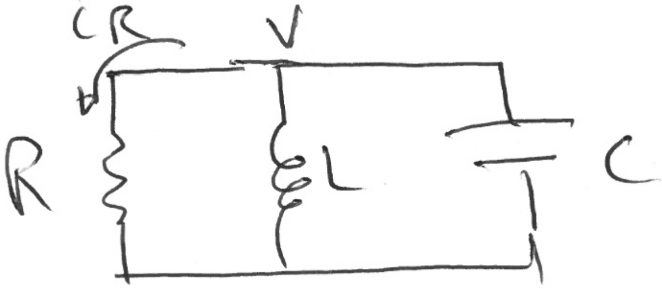
or $\alpha = \omega_0$

i.e. $\dfrac{1}{2RC} = \dfrac{1}{\sqrt{LC}}$

$$R = \sqrt{L/4C} = 1\,k\Omega.$$

(b) $\alpha = \dfrac{1}{2RC} = 250\,s^{-1}$

(c)



the voltage is common to all elements in circuit.

critically damped $\Rightarrow$ $v = e^{-\alpha t}(A_1 + A_2 t)$

$\alpha = 250$ $\therefore$ $V = e^{-250t}(A_1 + A_2 t)$ V

$$\therefore i_R = \frac{V}{R} = \frac{1}{R} e^{-250t} \left[A_1 + A_2 t\right] V$$

(d) Now keeping α

we have

$$i_R = \frac{1}{R} e^{-\alpha t} \left[A_1 + A_2 t\right] \quad V$$

$$\therefore \frac{di_R}{dt} = \frac{1}{R}\left[-\alpha e^{-\alpha t}(A_1 + A_2 t) + e^{-\alpha t} A_2\right]$$

$$= \frac{e^{-\alpha t}}{R}\left(-\alpha A_1 - \alpha A_2 t + A_2\right)$$

$$= \frac{e^{-\alpha t}}{R}\left(A_2 - \alpha A_1 - \alpha A_2 t\right)$$

$$\frac{d^2 i_R}{dt^2} = -\frac{\alpha e^{-\alpha t}}{R}\left(A_2 - \alpha A_1 - \alpha A_2 t\right) + \frac{e^{-\alpha t}}{R}\left(-\alpha A_2\right)$$

$$= \frac{-\alpha e^{-\alpha t}}{R}\left[A_2 + A_2 - \alpha A_1 - \alpha A_2 t\right]$$

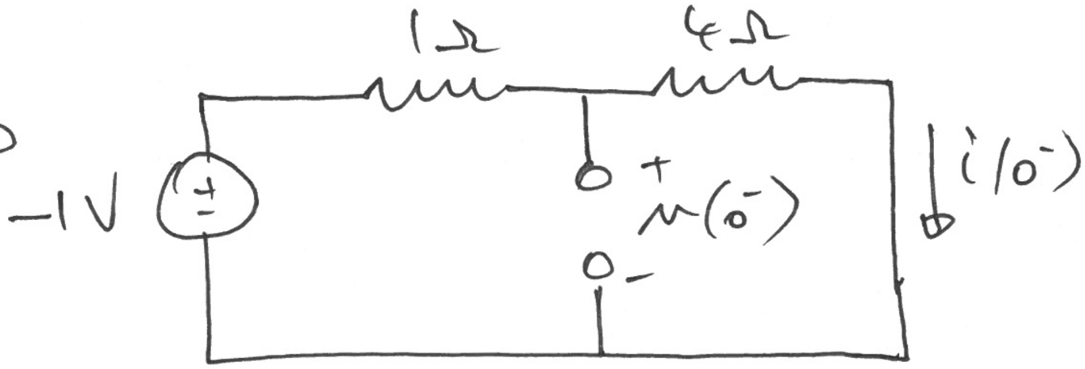
$$= -\frac{\alpha e^{-\alpha t}}{R}\left[2A_2 - \alpha A_1 - \alpha A_2 t\right]$$

$$\therefore \frac{d^2 i_R}{dt^2} + 2\alpha \frac{d i_R}{dt} + \alpha^2 i_R$$

$$= \frac{e^{-\alpha t}}{R}\Big[-2\alpha A_2 + \alpha^2 A_1 + \alpha^2 A_2 t + 2\alpha A_2 - 2\alpha^2 A_1 - 2\alpha^2 A_2 t + \alpha^2 A_1 + \alpha^2 A_2 t\Big]$$

$$= 0$$

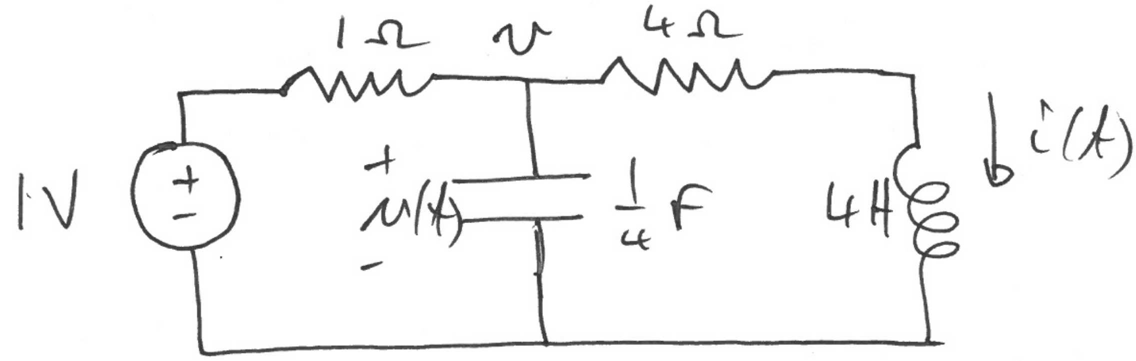
(6) for $t<0$



$$i(0^-) = \frac{-1}{5} = -0.2\,A = i(0^+)$$

$$v(0^-) = -1 + \frac{4}{5} = -0.8\,V = v(0^+)$$

for $t>0$



KCL at top node:

$$\frac{v(t)-1}{1} + \frac{1}{4}\frac{dv}{dt} + i(t) = 0$$

$$v(t) - 1 + \frac{1}{4}\frac{dv(t)}{dt} + i(t) = 0 \qquad (1)$$

since we want $i(t)$
need to remove $u(t)$ from KCL eqn.
$\therefore$ Carry out KVL on right mesh.

$$-u(t) + 4i(t) + 4\frac{di}{dt} = 0$$

$$\therefore u(t) = 4\frac{di}{dt} + 4i(t) \quad - (2)$$

Substitute into eqn. (1)

$$4\frac{di}{dt} + 4i - 1 + \frac{1}{4}\frac{d}{dt}\left(4\frac{di}{dt} + 4i\right) + i = 0$$

$$4\frac{di}{dt} + 4i - 1 + \frac{d^2i}{dt^2} + \frac{di}{dt} + i = 0$$

$$\therefore \boxed{\frac{d^2i}{dt^2} + 5\frac{di}{dt} + 5i = 1.}$$

<u>Find forced response.</u>

Since source $= 1V =$ constant

$$\therefore i_f = A$$

substituting into DE.

$$\Rightarrow \quad 0 + 5 \times 0 + 5A = 1$$

$$A = 0.2$$

Find natural response

Characteristic equation: $s^2 + 5s + 5 = 0$

$$\alpha = 2.5, \quad \omega_0 = \sqrt{5} = 2.24$$

$$\Rightarrow \alpha > \omega_0 = \text{overdamped}$$

$$s_1 = -2.5 + \sqrt{6.25 - 1} = -1.38$$

$$s_2 = -2.5 - 1.118 = -3.618$$

$$\therefore i_n = A_1 e^{-3.62t} + A_2 e^{-1.38t}$$

Complete response

$$i = 0.2 + A_1 e^{-3.62t} + A_2 e^{-1.38t}.$$

Apply initial conditions

$$i(0^+) = -0.2 = 0.2 + A_1 + A_2$$

$$\therefore A_1 + A_2 = -0.4 \quad - (1)$$

$$v(0^+) = -0.8\text{V}.$$

KVL at RH mesh

$$\Rightarrow v(t) = 4i(t) + 4\frac{di}{dt}$$

$$\frac{di}{dt} = -3.62 A_1 e^{-3.62t} - 1.38 A_2 e^{-1.38t}. \quad (1)$$

∴ KVL equation ⇒

$$v(t) = 4\left(0.2 + A_1 e^{-3.62t} + A_2 e^{-1.38t}\right) + 4\left(-3.62 A_1 e^{-3.62t} - 1.38 A_2 e^{-1.38t}\right)$$

apply at $t = 0^+$

$$v(0^+) = -0.8 = 0.8 + 4A_1 + 4A_2 - 14.48 A_1 - 5.52 A_2$$

$$\therefore\ 10.48 A_1 + 1.5 A_2 = 1.6 \quad -(2)$$

Solve (1) & (2)

$$\Rightarrow A_1 = 0.247$$
$$A_2 = 0.647$$

∴ Complete Response

$$i(t) = 0.2 + 0.247 e^{-3.62t} - 0.647 e^{-1.38t}\ \text{A}.$$