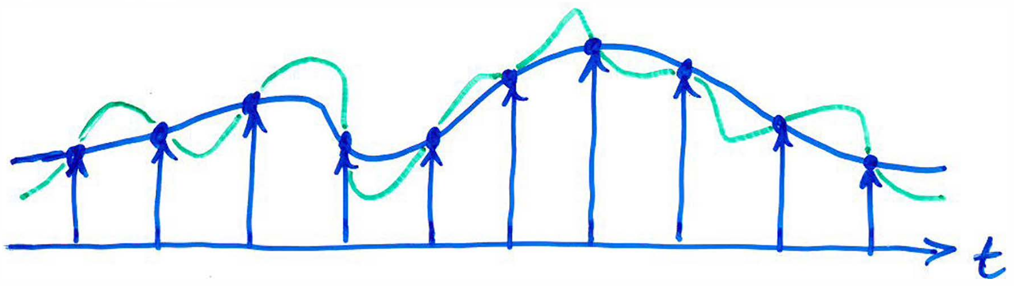
SAMPLING (p 514)

CT SIGNAL AND ITS SAMPLES

Q. WHEN CAN YOU SAMPLE A C.T. SIGNAL AND THEN <u>EXACTLY</u> RECONSTRUCT IT FROM THE SAMPLES?



IMPULSE TRAIN SAMPLING

IMPULSE TRAIN $\quad p(t) = \sum_{n=-\infty}^{\infty} \delta(t - nT)$

SAMPLING OF $\quad x(t)$

GIVES $\quad x_p(t) = x(t)\, p(t)$

$$= \sum_{n=-\infty}^{\infty} x(nT)\, \delta(t - nT)$$

Now,

$$X_p(j\omega) = \frac{1}{2\pi} X(j\omega) * P(j\omega)$$

$$\& \quad P(j\omega) = \frac{2\pi}{T} \sum_{k=-\infty}^{\infty} \delta(\omega - k\omega_s)$$

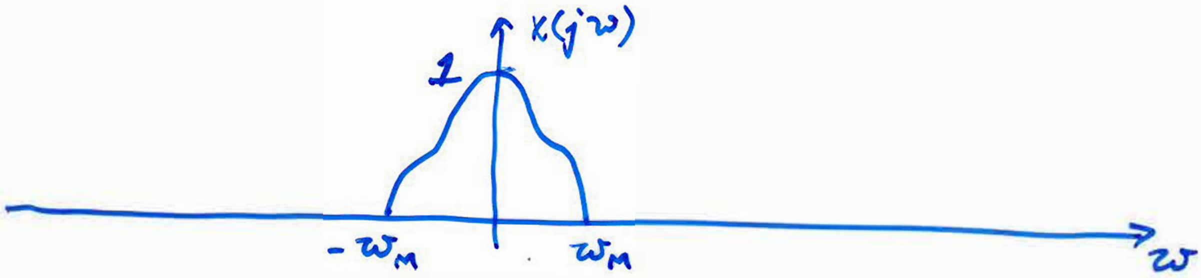
$$\omega_s = \frac{2\pi}{T}$$

from $p(t) \Rightarrow a_k = \frac{1}{T} \quad \left(a_k = \frac{1}{T} \int_{-T/2}^{T/2} p(t) e^{-jk\omega_0 t}\, dt \right)$

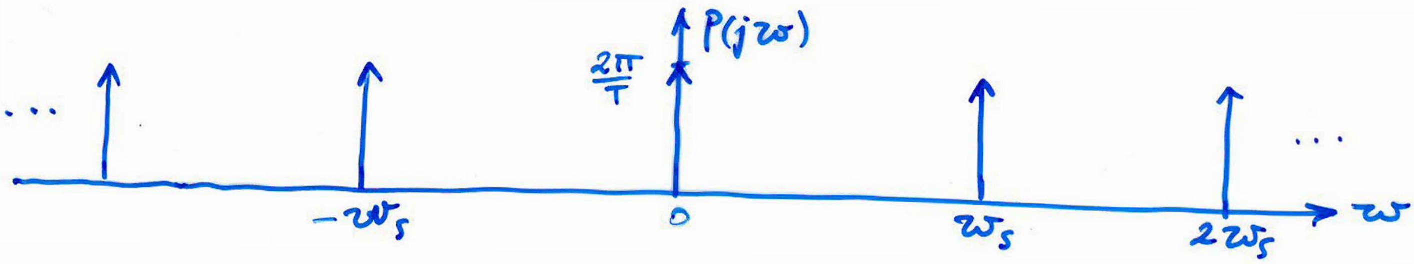
$P(j\omega) = \sum_{k=-\infty}^{\infty} 2\pi a_k \delta(\omega - k\omega_0)$

$$\Rightarrow X_p(j\omega) = \frac{1}{T} X(j\omega) * \sum_{k=-\infty}^{\infty} \delta(\omega - k\omega_s)$$

$$= \frac{1}{T} \sum_{k=-\infty}^{\infty} X(j(\omega - k\omega_s))$$



$*$



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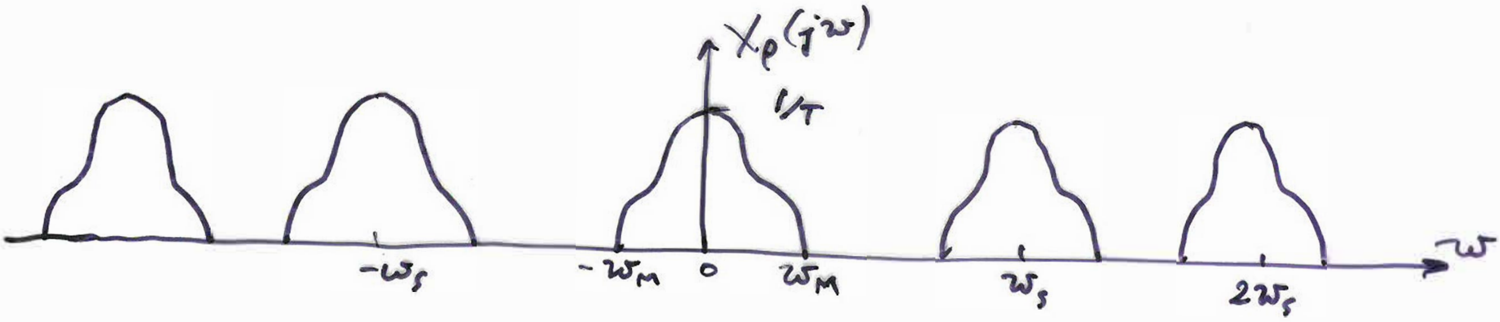
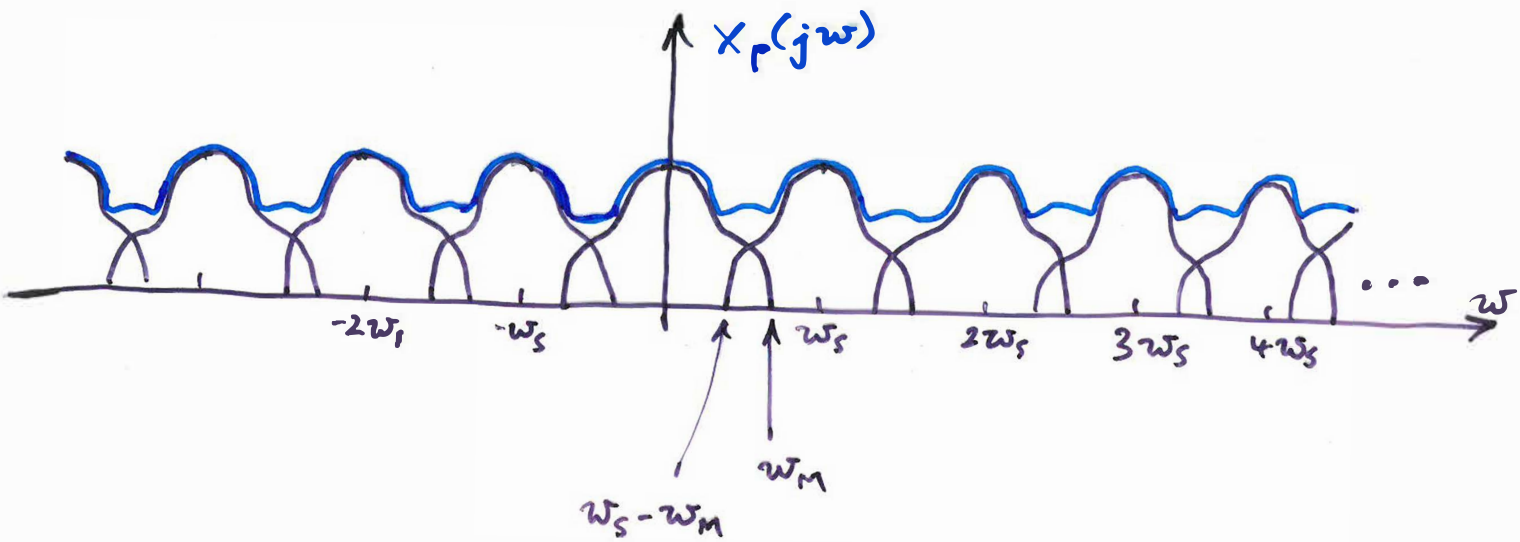
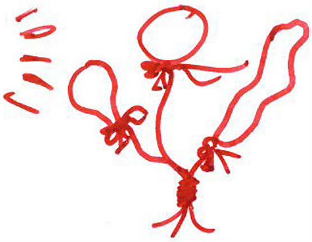


$\omega_s = 2\pi/T$

OR, IF SAMPLING RATE IS SLOW ie ω_s IS SMALLER



SAMPLING THEOREM



FOR $x(t)$ BANDLIMITED $\quad X(j\omega) = 0$ FOR $|\omega| > \omega_M$

IF SAMPLING IS DONE AT PERIOD T

THEN $x(t)$ IS UNIQUELY DETERMINED BY SAMPLES $x(nT)$

IF

$$\boxed{\omega_s = \frac{2\pi}{T} > 2\,\omega_M}$$

$2\omega_M$ is CALLED NYQUIST RATE

ω_M is CALLED NYQUIST FREQUENCY

RECOVER $x(t)$ FROM $x_p(t)$ USING IDEAL L.P.F.

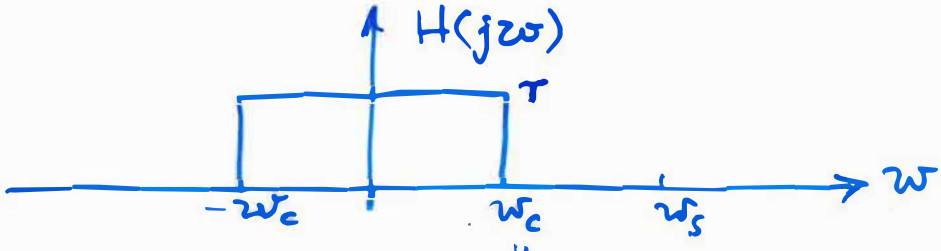
& $x_r(t) = x(t)$ IF $\omega_s > 2\omega_M$

& $h(t)$ IDEAL LPF

RECONSTRUCTION FROM SAMPLES

INTERPOLATION (p 522)

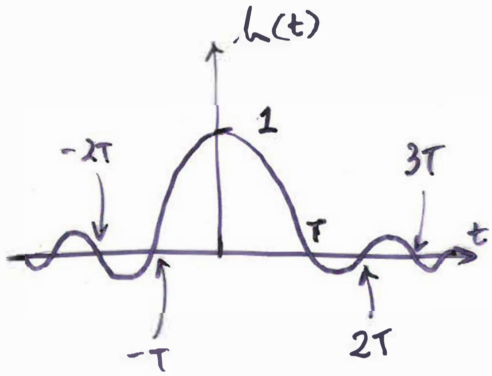
IDEAL LOW PASS FILTER (WITH GAIN T)



$\omega_c = \frac{\omega_s}{2}$

$\omega_s = \frac{2\pi}{T}$

$$\Rightarrow \quad h(t) = T \quad \frac{\sin(\omega_c t)}{\pi t}$$



$h(t) = 0$ when

$\omega_c t = k\pi$

i.e. $t = \frac{2k\pi}{\omega_s} = kT$

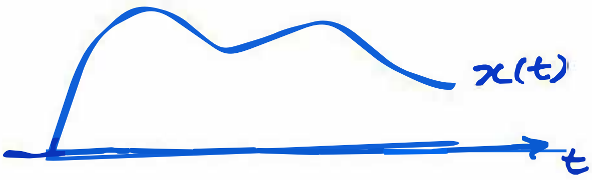
RECALL,

$$x_p(t) = \sum_{n=-\infty}^{\infty} x(nT)\,\delta(t - nT)$$

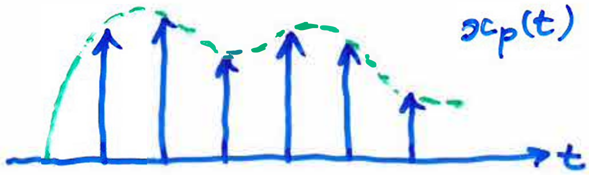
$$\Rightarrow \quad x_r(t) = \sum_{n=-\infty}^{\infty} x(nT)\,\underline{h(t - nT)}$$

shifted sinc $f^{\underline{n}}$

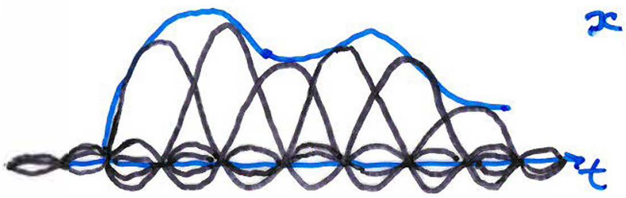
ORIGINAL



SAMPLED



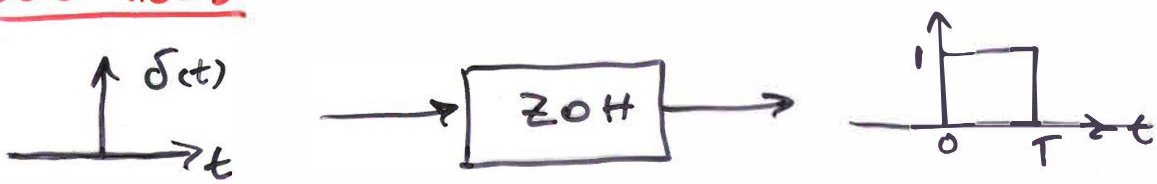
RECONSTRUCTED

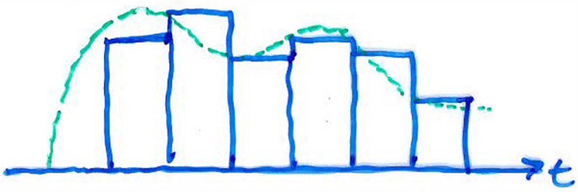


$$x_r(t) = x_p(t) * h(t)$$

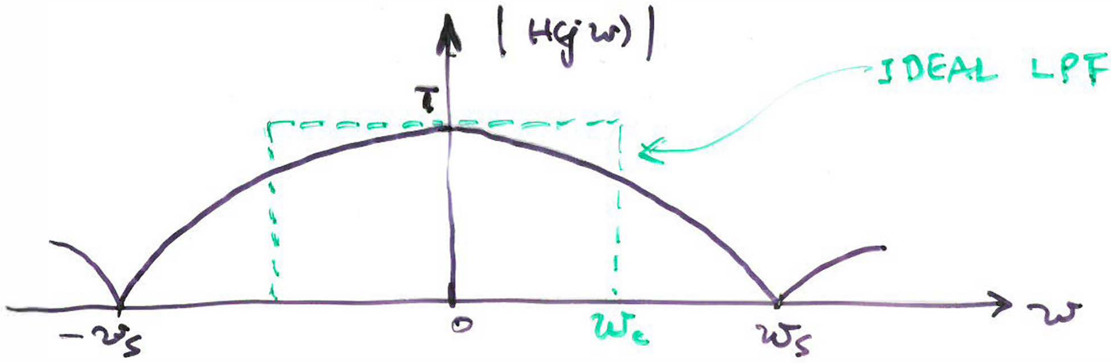
RECONSTRUCTED <u>EXACTLY</u> IF NYQUIST SATISFIED

<u>ZERO ORDER HOLD</u>

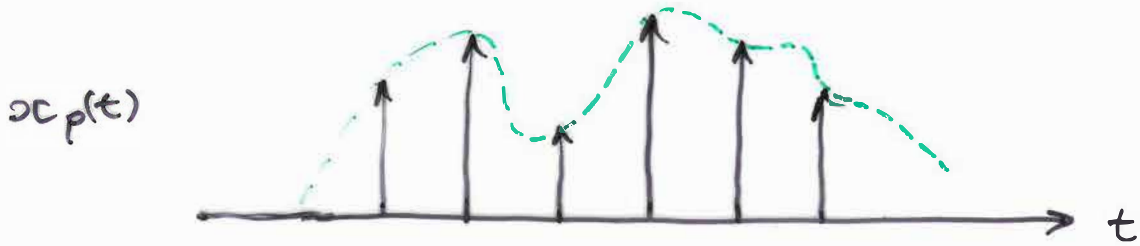


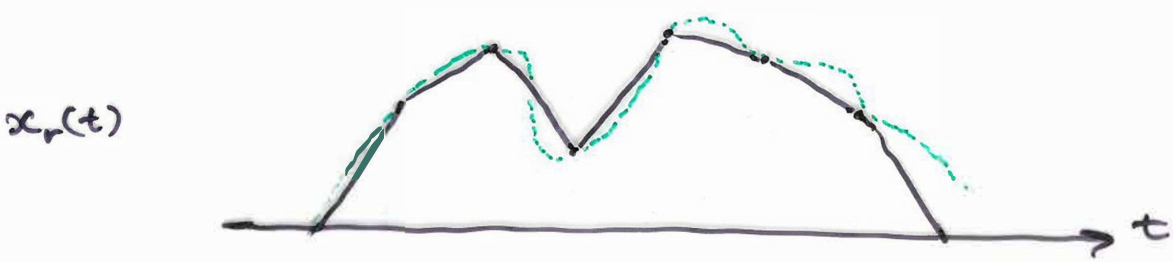


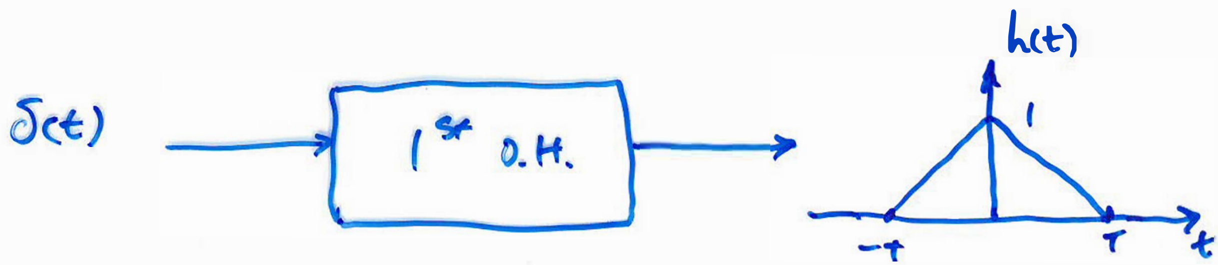
$$H(j\omega) = e^{-j\omega T/2} \frac{2 \sin(\omega T/2)}{\omega}$$

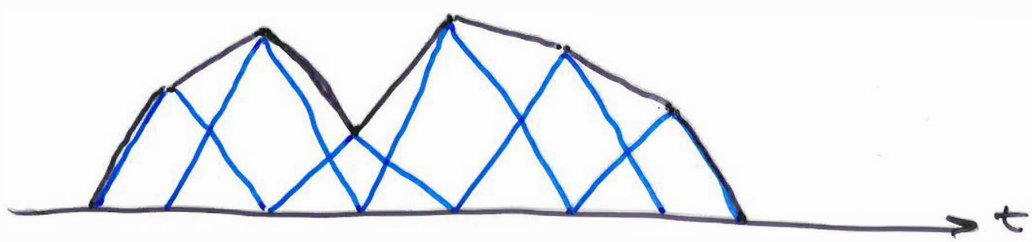


FIRST ORDER HOLD (LINEAR INTERPOLATION)

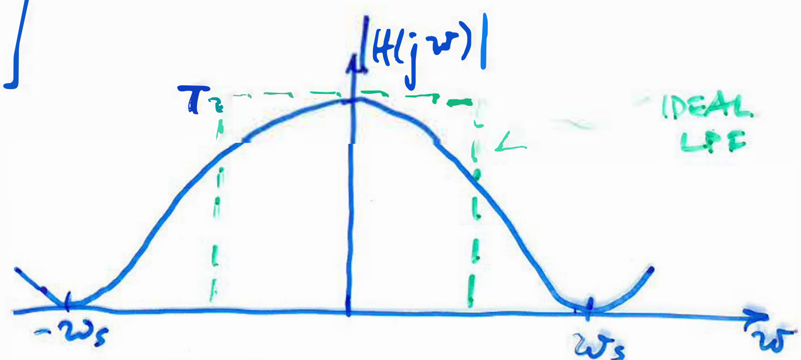








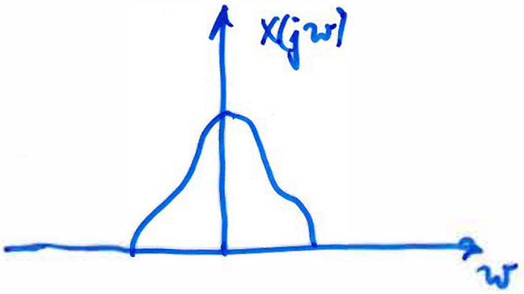
$$H(j\omega) = \frac{1}{T}\left[\frac{\sin\left(\frac{\omega T}{2}\right)}{\omega/2}\right]^2$$



THE EFFECT OF UNDERSAMPLING

ALIASING (p 527)

SUPPOSE $\omega_s < 2\omega_M$



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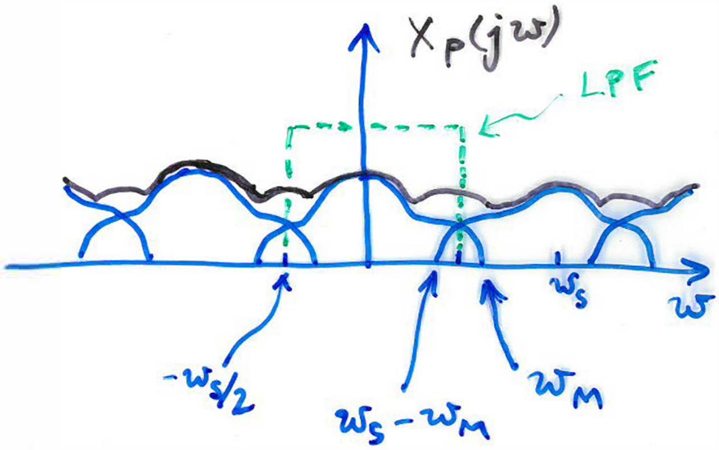
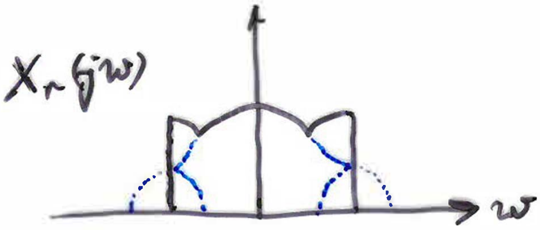




[HERE, $\omega_s - \omega_M < \omega_M$

$\Rightarrow \omega_s < 2\omega_M$]

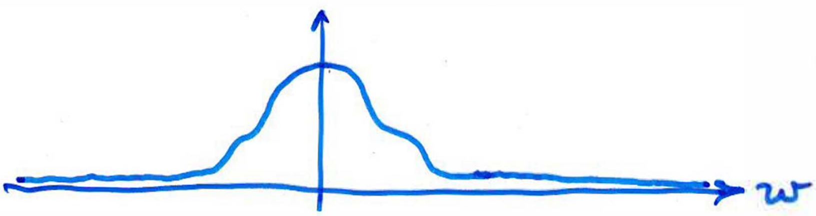
USING IDEAL LPF DOES NOT RECOVER SIGNAL

* HIGH FREQUENCIES ARE MAPPED TO LOW FREQS!!

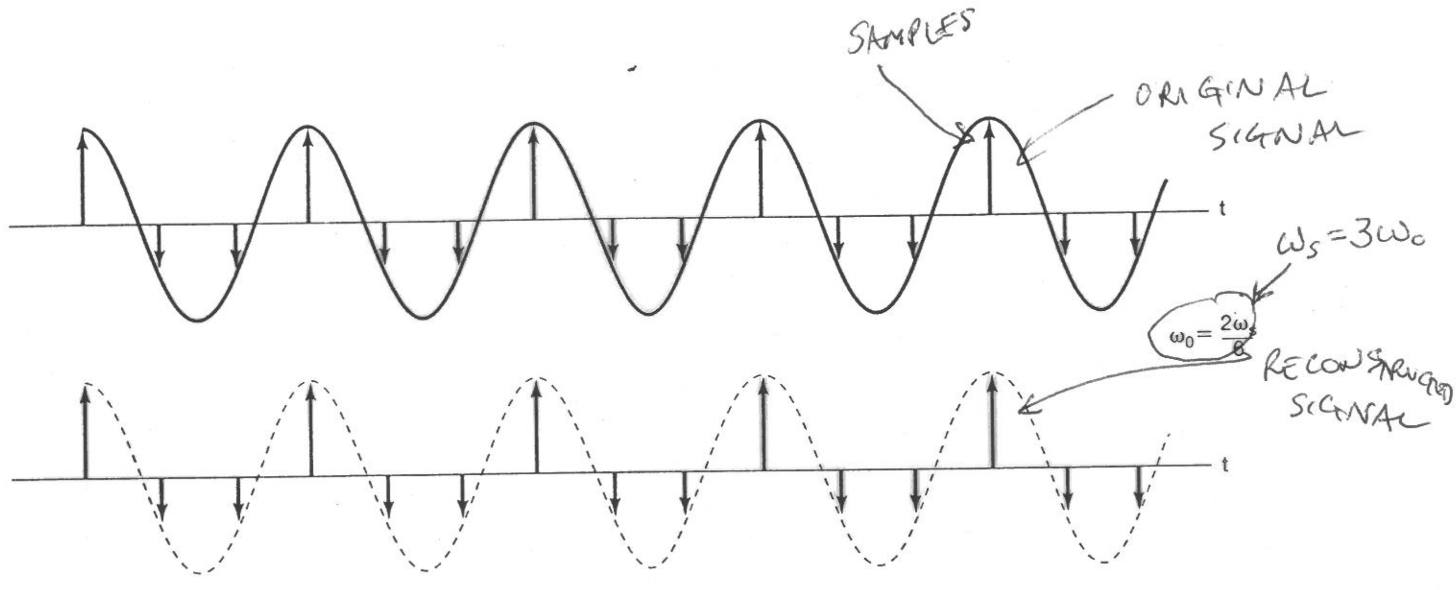
(FOLDED BACK)

SINCE "NOISE" IN A SYSTEM HAS BROAD SPECTRUM

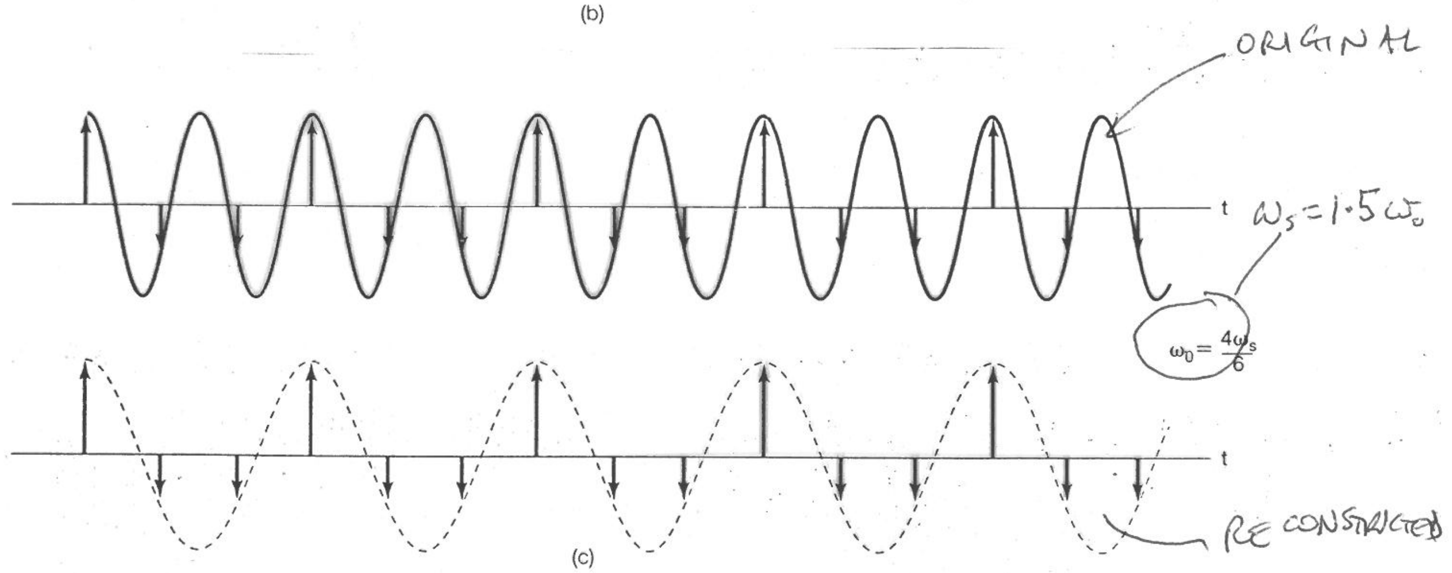
A TYPICAL SIGNAL REALLY LOOKS LIKE :



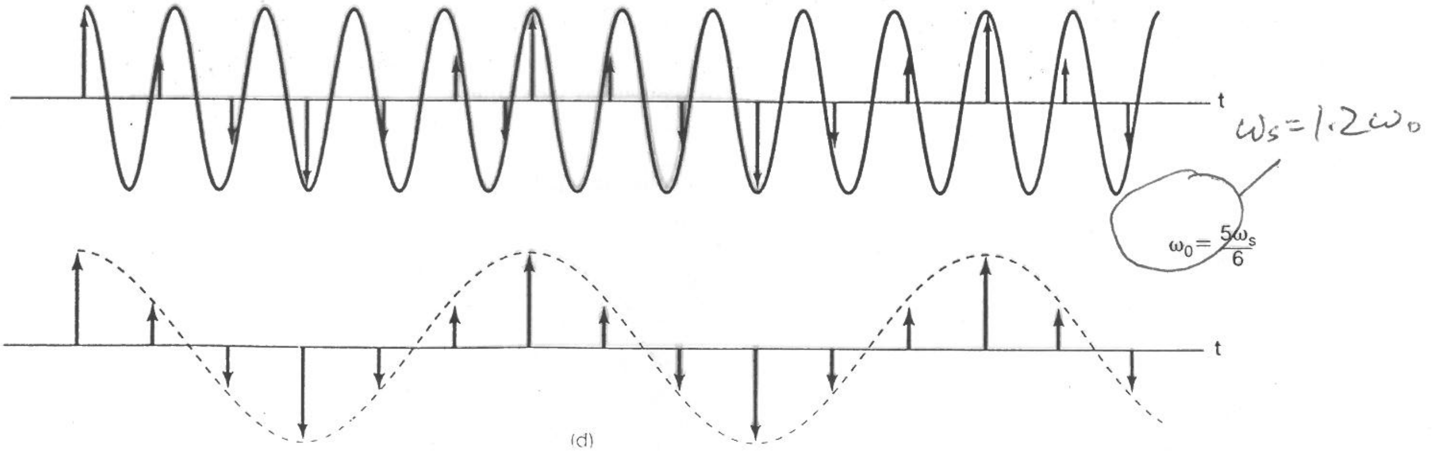
∴ NEED TO L.P.F. BEFORE SAMPLING SO THAT THE HIGH FREQ NOISE DOESN'T GET FOLDED BACK AFTER RECONSTRUCTION!


SAMPLES
ORIGINAL SIGNAL
t
$\omega_S = 3\omega_0$
$\omega_0 = \frac{2\omega_s}{6}$
RECONSTRUCTED SIGNAL
t

(b)


ORIGINAL
t
$\omega_S = 1.5\omega_0$
$\omega_0 = \frac{4\omega_s}{6}$
t
RECONSTRUCTED

(c)


t
$\omega_S = 1.2\omega_0$
$\omega_0 = \frac{5\omega_s}{6}$
t

(d)