ELEC2040 PRACTICAL WORK
Week 1

In working with complex numbers the following is needed and useful.

- The Matlab variables i and j are used to represent $\sqrt{-1}$.
    - For that reason you should avoid using i and j as variables
- The Matlab variable "pi" is used to represent the constant $\pi = 3.14159\ldots$,

Task 1. What is a complex number?

A complex number has two components: a real component, and an "imaginary" component that is indicated by being multiplied by the complex variable "i" (or "j").
For example.

```
z1 = 3+5i;
z2 = -0.5+1.5j;
```

Complex numbers can be plotted on the "complex plane", with the real component on the horizontal axis, and the imaginary component on the vertical axis.

Create a new .m file, (that you may like to call ComplexPlots.m), and enter the following code:

```
z0 = 0+0j;                        % Origin
z1 = 3+5i;
z2 = -0.5+1.5j;

figure(1)
hold off
plot(z1,'*');
hold on
plot(z2,'+');
plot(complex(z0),'gO');
axis([-10 10 -10 10])
axis('square')
grid
```

Check that the points are plotted correctly.

Note: There is Matlab homework to do in week 1 that will help you to become familiar with Matlab commands and plotting in general.

In the above example, the "complex(.)" command tells Matlab that 0 is to be treated as a complex number (rather than just the real number 0). Also, the 'gO' command tells Matlab that the point should be plotted in a green colour, and shown with a circle.

Now use the code below to also plot the additional complex numbers z3, z4, z5, and z6, and confirm that they appear at the locations you expect:

```
z0 = 0+0j;              % Origin
z1 = 3+5i;
z2 = -1+2j;
z3 = z1+4;
```

```
z4 = z1+3i;
z5 = z2+1;
z6 = z2-3j;

figure(1)
hold off
plot(z1,'b*');
hold on
plot(z2,'b+');
plot(complex(z0),'gO');
plot(z3,'g*');
plot(z4,'r*');
plot(z5,'g+');
plot(z6,'r+');
axis([-10 10 -10 10])
axis('square')
grid
```

Task 2: Operations on complex numbers

(i) Complex conjugate

The "complex conjugate" of a complex number is a number with the same "real" part, and the negative "imaginary" part.

The complex conjugate of z = x+jy is $z^* = x - jy$. It is obtained using the conj command.

```
conj(z1);
```

Let z1, z2 be as before, and define new z3, z4 as follows:

```
z0 = 0+0j;              % Origin
z1 = 3+5i;
z2 = -1+2j;
z3 = 8- 3i;
z4 = -4-7i;

figure(1)
hold off
plot(z1,'b*');
hold on
plot(conj(z1), 'r*');
plot(z2,'b+');
plot(conj(z2), 'r+');
plot(complex(z0),'gO');
axis([-10 10 -10 10])
axis('square')
grid

figure(2)
hold off
plot(z3,'b*');
hold on
plot(conj(z3), 'r*');
plot(z4,'b+');
plot(conj(z4), 'r+');
plot(complex(z0),'gO');
axis([-10 10 -10 10])
axis('square')
grid
```

(ii) Addition, subtraction, and multiplication

Enter the following commands:

```
z1 = 3+5i;
z2 = -1+2j;
z3 = z1+z2;
z4 = z1-z2;
z5 = z1*z2;
```

Verify by hand that the correct answers are obtained.

Task 3: Magnitude and phase of a complex number

A complex number can also be represented in a "polar form" as follows, where "r" is the magnitude and "θ" is the phase (argument).

Note that the phase is measured relative to the "real" axis, starting with θ=0 along the "real" axis (ie. to the right), and with θ increasing in the anti-clockwise direction (ie. up from the real axis).

Also note that θ can be indicated in either degrees or radians. Matlab uses radians (ie. the full circle is 2pi radians around).

$$z = x + jy = re^{j\theta} = r\exp(j\theta)$$

To go from rectangular form (ie. $x+jy$) to polar form, the following equations are needed:

$$|z| = \sqrt{x^2 + y^2} = r$$

$$\arg(z) = \angle z = \theta = \tan^{-1}\frac{y}{x}$$

To go from polar form to rectangular form the following equations are needed:

$$z = r\cos\theta + jr\sin\theta$$

i.e.

$$x = r\cos\theta$$

$$y = r\sin\theta$$

(i) The Matlab commands for rectangular to polar form are "abs" (to find r) and "angle" (to find θ). E.g. try

```
abs(z1)
angle(z1)
```

(ii) Polar to rectangular form. Enter the following into the command line:

```
theta = pi/6;
r = 4;
r*exp(j*theta)
```

Verify this gives the correct answer by applying the above equations directly.

Verify that abs(exp(j*theta)) = 1

Verify that abs(-4* exp(j*theta)) = 4

(iii) Note that the complex conjugate of a complex number can be viewed in these "rotation" terms. The complex conjugate of a number has the same magnitude as the number, but it has the negative phase.

(iv) Let s = 2+j*theta; Compute z=exp(s)

Comparing abs(z) with exp(2) explain how your answer was obtained.

(v) Magnitude of a product

```
z1 = 3+5i;
z2 = -1+2j;
z3 = z1*z2 ;
```

Relate abs(z3) to abs(z1) and abs(z2).

Task 4: Rotating Complex Numbers

Complex numbers can be rotated around the origin, by keeping the amplitude constant, and increasing the phase. The following code adds multiples of the angle "(2*pi)/PiFraction" to z1.

Initially, "PiFraction" is set to 32.

Examine the code and confirm that the plot matches with your expectation of where the points z2, z3, z4, z5, and z6, should be located.

Change the value of "PiFraction", and rerun the code. Make sure you understand the resulting plots (especially for values of "PiFraction" = 2 and 1).

```
z1=3+5i;

absz1 = abs(z1);
angz1 = angle(z1);

PiFraction = 32;  % try other values, eg. 16, 8, 4, 2, 1

angz2 = angz1 + (2*pi)/PiFraction;
angz3 = angz1 + 2*(2*pi)/PiFraction;
angz4 = angz1 + 3*(2*pi)/PiFraction;
angz5 = angz1 + 4*(2*pi)/PiFraction;
angz6 = angz1 + 5*(2*pi)/PiFraction;

z2 = absz1 * exp(i*angz2);
z3 = absz1 * exp(i*angz3);
z4 = absz1 * exp(i*angz4);
z5 = absz1 * exp(i*angz5);
z6 = absz1 * exp(i*angz6);

figure(1)
hold off
plot(complex(z0),'gO')
hold on
plot(z1,'gx')
```

```
plot(z2,'r+')
plot(z3,'m+')
plot(z4,'b+')
plot(z5,'b+')
plot(z6,'b+')

axis([-10 10 -10 10])
axis('square')
```

Task 5: Inverses and division

(i) Finding z^(-1) in Matlab:

```
theta = pi/6;
r = 4;
z1 = r*exp(j*theta);
```

Find value of z1 in form x + jy from command line

Now type `z1^(-1)` in the command line to find the value of `z1^(-1)`. What is it?

How to find `z1^(-1)` from the magnitude and phase?

Type `z2=r^(-1)*exp(-j*theta)` in command line. What is the value of `z2`? Relate it to the value of `z1^(-1)`.

Use Matlab to plot both `z1` and `z1^(-1)`.

(ii) Division of complex numbers:

```
z1 = 3+5i;
z2 = -1+2j;
z3 = z1/z2;

z4 = z1*z2^(-1);
```

What is the value of z4? Relate it to the value of z1/z2.

Relate abs(z1) and abs(z2) to abs(z3)

Task 6: Attempt the practice quiz