Design via Frequency Response

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Modified from original slides courtesy of Dr Branislav Hredzak

Overview

1. Introduction
2. Transient response via gain adjustment
3. Lead compensation
4. Lag Compensation (optional)
5. Summary

1. Introduction

1. Introduction

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How to use the frequency response to:

- adjust the gain to **meet a transient response specification**

1. Introduction

Review of root locus design:

- the **transient response** of a control system can be designed by **adjusting the gain** along the root locus
  - finding the transient specification on the root locus
  - setting the gain accordingly
  - settling for the resulting steady-state error
- **disadvantage**
  - only the transient response and steady-state error represented by **points along the root locus** are available
- **cascade compensators** have been introduced to meet transient response specifications represented by **points not on the root locus** and, independently, steady-state error requirements

1. Introduction

Bode plots vs. root locus

- **Stability and transient response design via gain adjustment**
  - unlike root locus techniques, Bode plots can be **implemented conveniently without a computer** or other tool except for testing the design
  - we can easily draw Bode plots using asymptotic approximations and **read the gain from the plots**
  - root locus **requires repeated trials** to find the desired design point from which the gain can be obtained
  - the **computational disadvantage** of root locus **vanishes** when using tools like MATLAB

1. Introduction

Bode plots vs. root locus

- **Transient response design via cascade compensation**
  - Bode plots are **not as intuitive** as the root locus
  - it is **something of an art** to design cascade compensation with Bode plots
  - in Bode plots, phase margin is related to percent overshoot and bandwidth is related to both damping ratio and settling time (or peak time), and the **equations are rather complicated**
  - the reshaping of the open-loop transfer function’s frequency response **can lead to several trials** until all transient response requirements are met

1. Introduction

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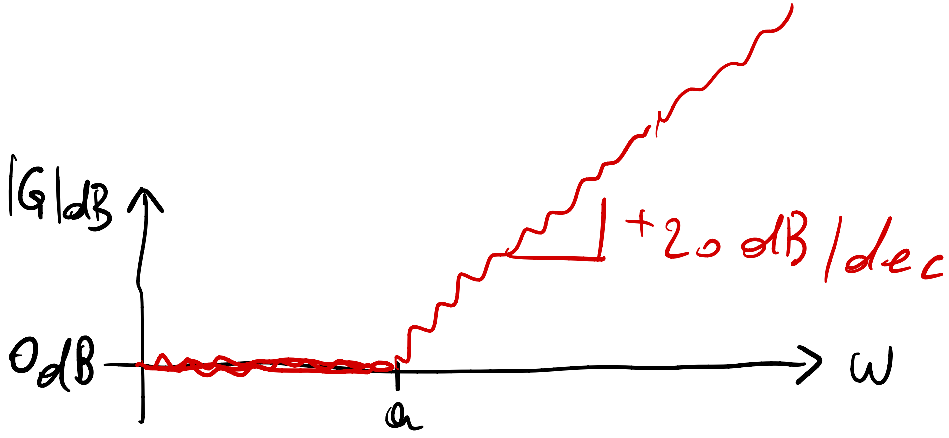
Bode plots vs. root locus

- **Steady-state error design via cascade compensation**
  - an advantage of using Bode plots is the **ability to design derivative compensation**, such as lead compensation, to improve both transient response and steady-state error
  - using root locus there are an **infinite number of possible solutions** to the design of a lead compensator
  - root locus **requires numerous tries** to arrive at the solution that yields the required steady-state error performance

Ideal Derivative (Root Locus) controller:

$G(s) = (s + a) \longrightarrow$



(ideal derivative compensator magnitude)

1. Steady-State Error Insights from Bode Plots

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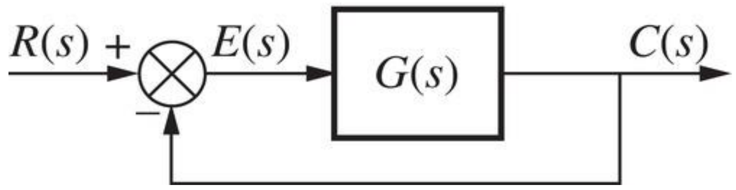
Step input, assuming G(s) type 0:



Type 0 system:

$$G(s) = K\frac{\prod_{i=1}^{n}(s+z_i)}{\prod_{i=1}^{m}(s+p_i)}$$

Position constant K_p:

$$K_p = K\frac{\prod_{i=1}^{n} z_i}{\prod_{i=1}^{m} p_i}$$

At dc, or close to it (ω_0), plot magnitude proportional to K_p:

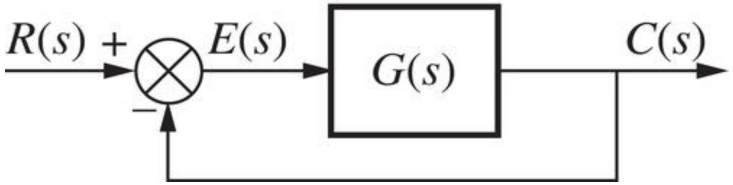
$$20\log M = 20\log K\frac{\prod_{i=1}^{n} z_i}{\prod_{i=1}^{m} p_i} = 20\log K_p$$

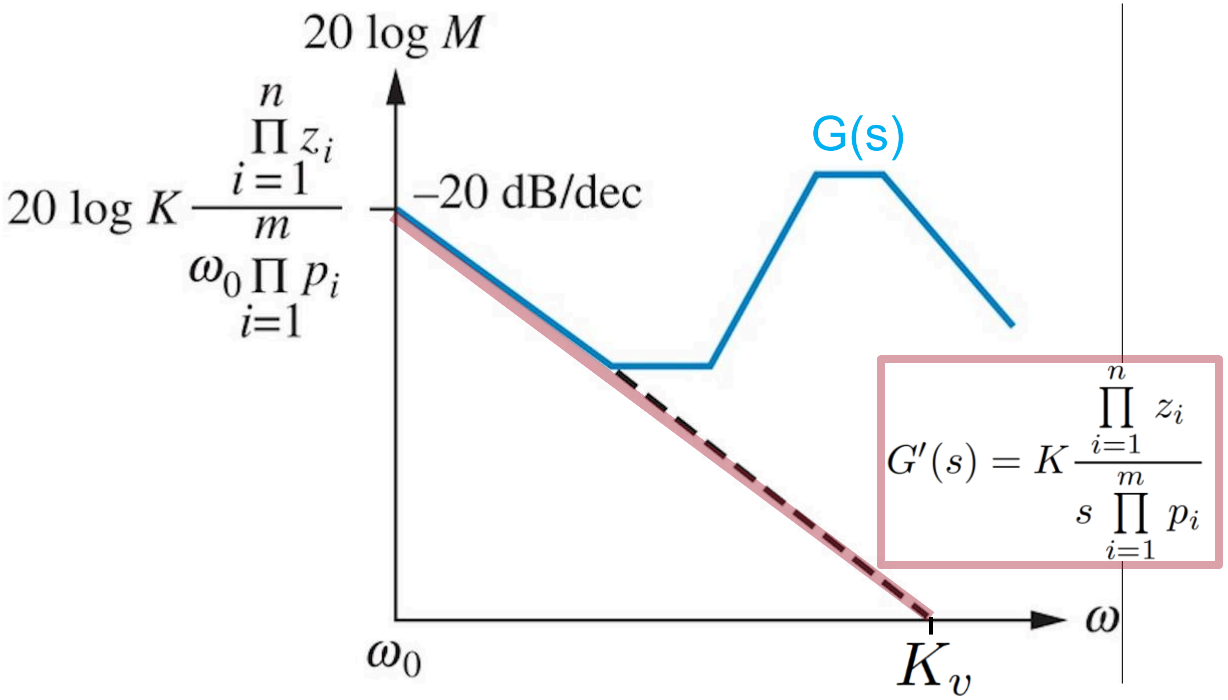
Error to unit step decreases if magnitude at ω_0 increases:

$$e_{ss-step} = \frac{1}{1+K_p}$$

1. Steady-State Error Insights from Bode Plots

Ramp input, assuming G(s) type 1:





Type 1 system:

$$G(s) = K \frac{\prod_{i=1}^{n} (s + z_i)}{s \prod_{i=1}^{m} (s + p_i)}$$

Velocity constant K_v:

$$K_v = \lim_{s \to 0} sG(s) = K \frac{\prod_{i=1}^{n} z_i}{\prod_{i=1}^{m} p_i}$$

G'(s) = 1, i.e., it intersects the frequency axis (0 dB) at:

$$\omega = K \frac{\prod_{i=1}^{n} z_i}{\prod_{i=1}^{m} p_i} = K_v$$

Error to unit ramp decreases if G'(s) intercept ω increases:

$$e_{ss-ramp} = \frac{1}{K_v}$$

1. Steady-State Error Insights from Bode Plots

Visual inspection of Bode Magnitude plot at dc (low frequency) where s → 0

- **Type 0 systems [no common "s" the denominator of G(s)]**
  - Magnitude is horizontal line when s→0
  - Shifting the Magnitude plot upwards where s → 0, reduces steady state error to step input
- **Type 1 systems [one common "s" the denominator of G(s)]**
  - Magnitude decrease by -20db/decade when s → 0, because of the 1/s term in G(s)
  - Intercept of initial magnitude slope line with frequency axis inversely proportional to ramp error
- **Type 2 systems [one common "s^2" the denominator of G(s)]**
  - Similar considerations to type 1 systems apply

- Conclusion:
  - Bode plot magnitude for s→ 0 relates to steady-state error of time response
  - higher magnitude (or higher –ve slope) where s→0 means lower, or none, steady state error!

2. Transient response via gain adjustment

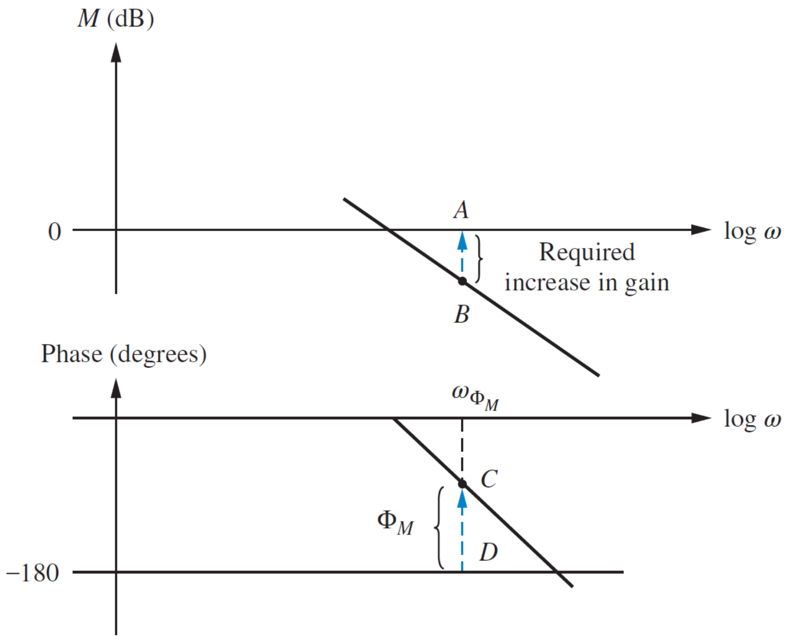
2. Transient response via gain adjustment

Design procedure

- **Draw the Bode** magnitude and phase plots for a convenient value of gain.
- **Determine the required phase margin** from the percent overshoot.

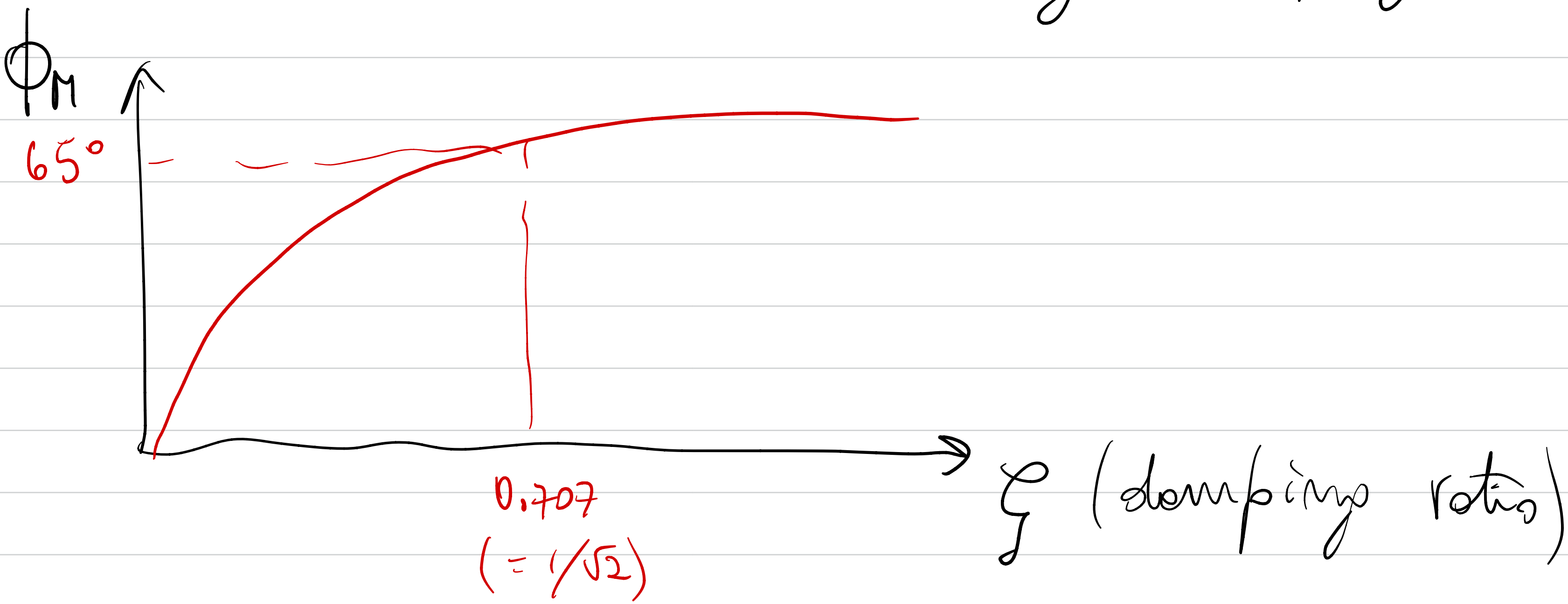
$$\zeta = \frac{-\ln(\%OS/100)}{\sqrt{\pi^2 + \ln^2(\%OS/100)}} \qquad \Phi_M = \tan^{-1}\frac{2\zeta}{\sqrt{-2\zeta^2 + \sqrt{1+4\zeta^4}}}$$

- **Find the frequency, $\omega_{\Phi M}$**, on the Bode phase diagram that yields the desired phase margin, CD.
- **Change the gain** by an amount AB to force the magnitude curve to go through 0 dB at $\omega_{\Phi M}$.



Bode plots showing gain adjustment for a desired phase margin

OS%
0
1
ζ (damping ratio)

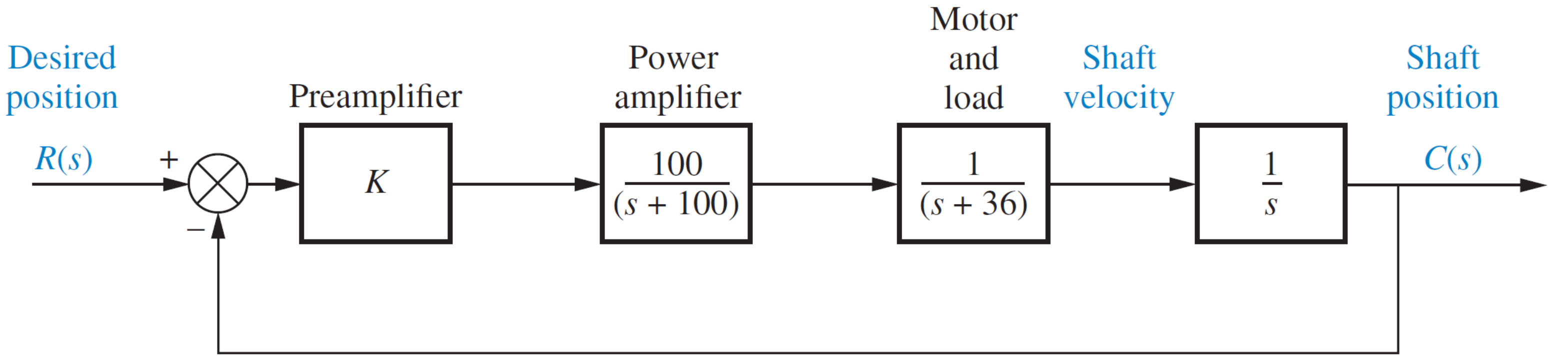


ΦM
65°
0.707
(= 1/√2)
ζ (damping ratio)

2. Transient response via gain adjustment

Problem

- For the position control system below, **find the value of the preamplifier gain *K* to yield a 9.5% overshoot** in the transient response for a step input.



Solution

- %OS = 9.5% → ζ = 0.6 → PM = 59.2°. Draw the Bode plot for *K* = 3.6 so that the plot starts at 0 dB at ω = 0.1 rad/s.

$$OS\% = 9.5\% \Rightarrow \zeta = \frac{-\ln(OS\%/100)}{\sqrt{\pi^2 + \ln^2(OS\%/100)}} = 0.6 \Rightarrow \phi_M = \tan^{-1}(\cdots) = 59.2^\circ$$

2. Transient response via gain adjustment

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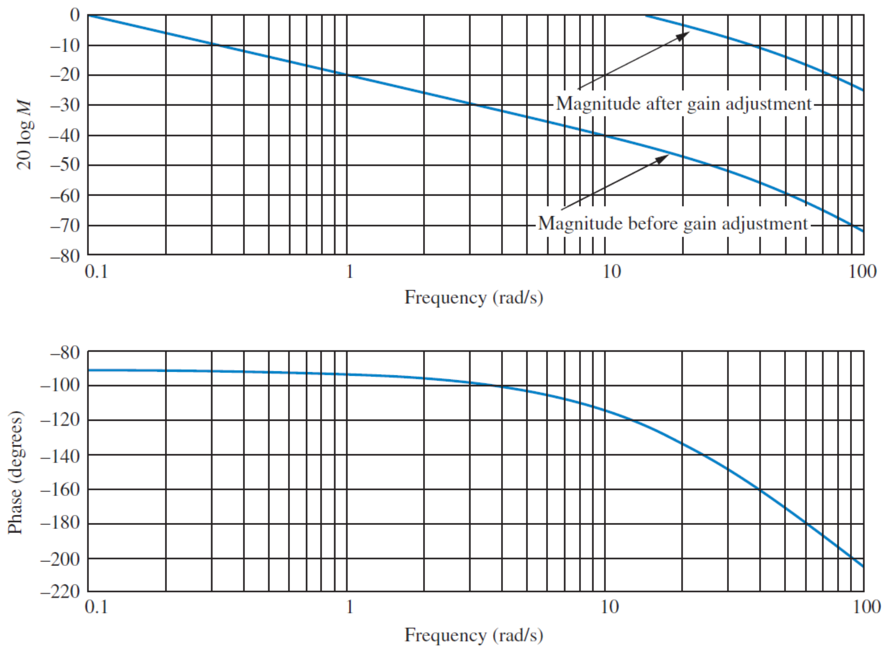
Problem

- In the plot, PM = 59.2° is achieved at ω = 14.8 rad/s.
- At ω = 14.8 rad/s the magnitude response is -44.2 dB.
- Thus K must be increased so that the magnitude response at this frequency is 0 dB.
- To do this K must be increased from 3.6 by a factor of $10^{44.2/20}$ = 162.18.
- Thus the desired value of K is 3.6 × 162.18 = **583.9**.



$$20 \log_{10} K = +44.2$$

$$K = 10^{44.2/20}$$

2. Transient response via gain adjustment

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Problem

- The value K = 583.9 yields a phase margin for **%OS = 9.48%.**
- Open-loop transfer function becomes $$G(s) = \frac{58{,}390}{s(s+36)(s+100)}$$
- Stability of the closed-loop system and validity of second order assumptions need to be verified.

Parameter	Proposed specification	Actual value
K_v	—	16.22
Phase margin	59.2°	59.2°
Phase-margin frequency	—	14.8 rad/s
Percent overshoot	9.5	10
Peak time	—	0.18 second

3. Lead compensation

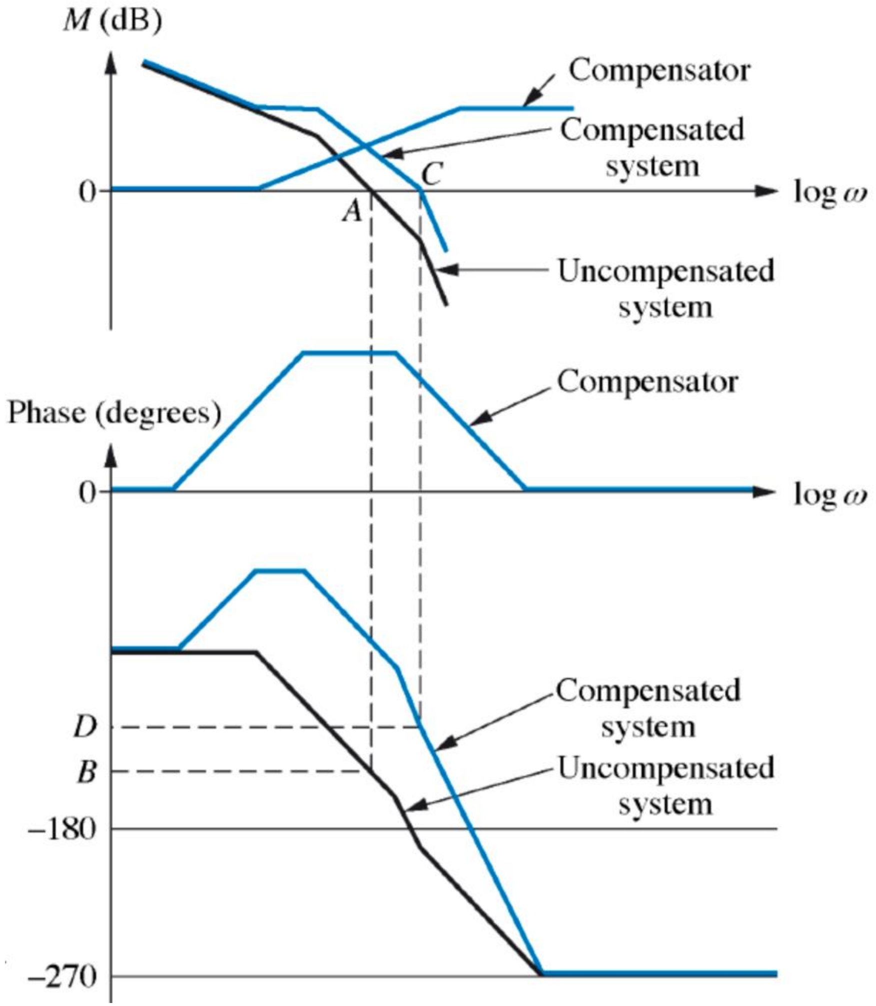
3. Lead compensation

Lead compensation improves:

- **the phase margin** to reduce the percent overshoot.
- **the gain crossover** to realise a faster transient response.

Note that

- the low frequency portion of the Bode plot is unchanged (DC gain is unchanged)



Note: notice that the initial slope, which determines the steady-state error, is not affected by the design

3. Lead compensation

$$G_c(s) = \frac{1}{\beta}\frac{s+\frac{1}{T}}{s+\frac{1}{\beta T}}$$

Lead compensator frequency response

- This form has DC gain equal to 1, where $\beta < 1$.
- frequency, ω_{max}, at which the maximum phase angle, φ_{max}, occurs can be found using

$$\omega_{\max} = \frac{1}{T\sqrt{\beta}}$$

(d) ω_{max}

(e) Find T

- the maximum phase angle φ_{max}:

(a) ϕ MAX

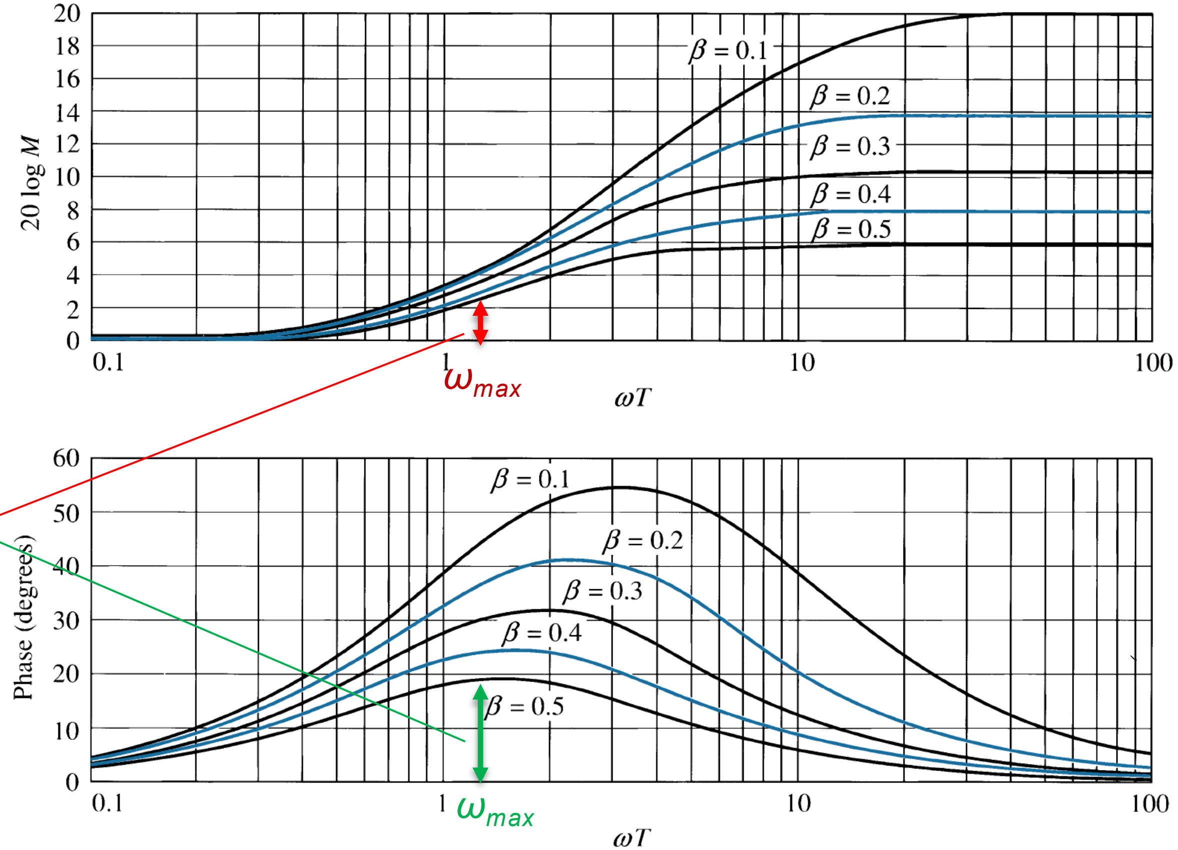
$$\phi_{\max} = \tan^{-1}\frac{1-\beta}{2\sqrt{\beta}} = \sin^{-1}\frac{1-\beta}{1+\beta}$$

(b) β

- compensator's magnitude at ω_{max} is

$$|G_c(j\omega_{\max})| = \frac{1}{\sqrt{\beta}}$$

(c) $|G_c|$



$$G_c = \frac{1}{\beta}\,\frac{s+z_c}{s+p_c} = \frac{1}{\beta}\,\frac{s+\frac{1}{T}}{s+\frac{1}{\beta T}} = \frac{1}{\cancel{\beta}}\,\frac{\frac{1}{\cancel{T}}\left(\frac{s}{\frac{1}{T}}+1\right)}{\frac{1}{\cancel{\beta}\cancel{T}}\left(\frac{s}{\frac{1}{\beta T}}+1\right)} = \frac{sT+1}{\frac{sT}{\frac{1}{\beta}}+1}$$

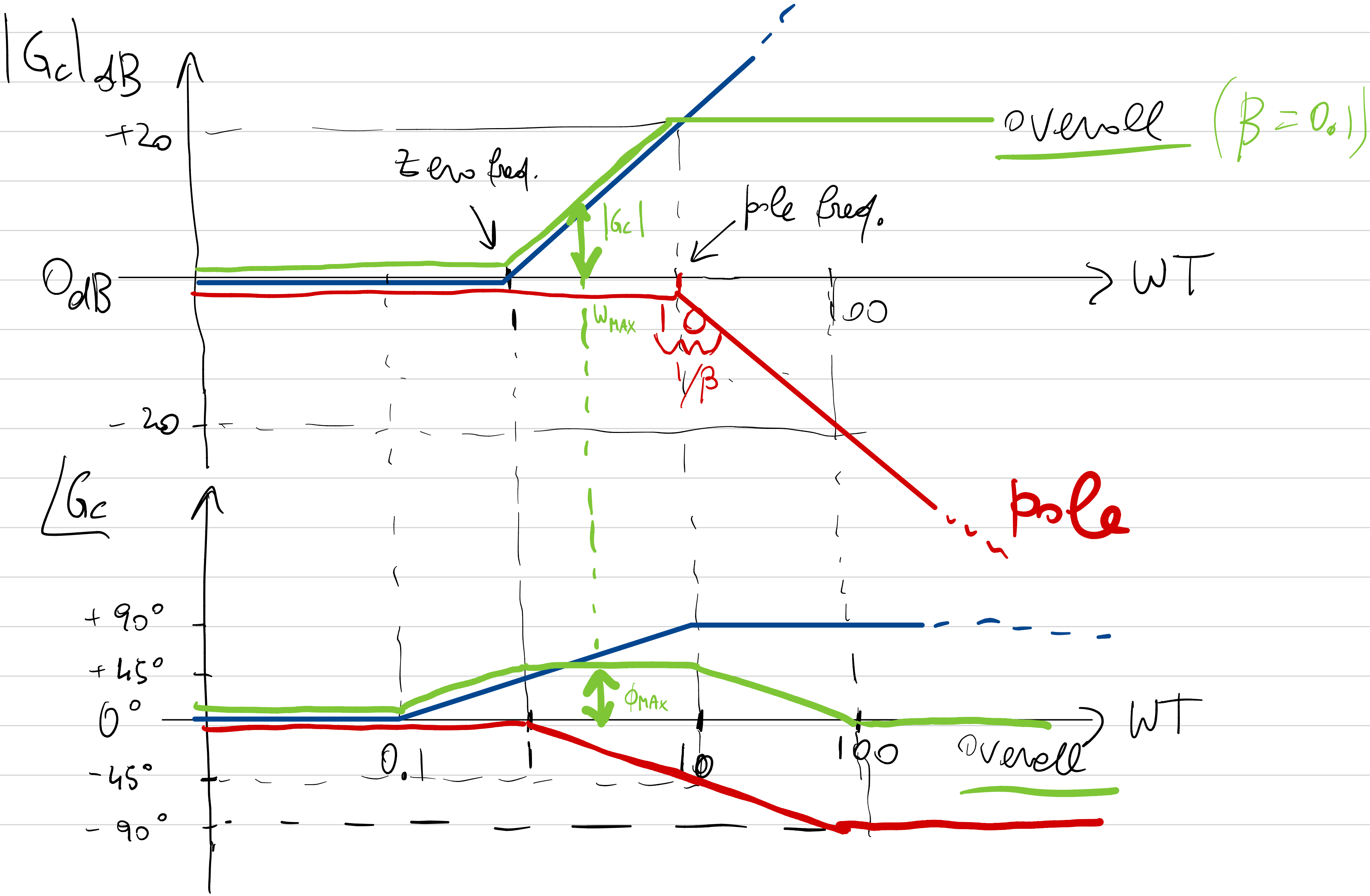
Example for $\beta = 0.1$, plot frequency response

$$G_c(j\omega) = \frac{j\omega T+1}{\frac{j\omega T}{\frac{1}{\beta}}+1}$$

$$\boxed{G_c(j\omega)} = \frac{j\omega T + 1}{\frac{j\omega T}{\frac{1}{\beta}} + 1}$$

with $\beta = 0.1$

Lead compensator



3. Lead compensation

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Design procedure

1. **Find the closed-loop bandwidth** required to meet the settling time, peak time, or rise time requirement
2. **Set the gain**, K, of the uncompensated system to the value that satisfies the steady-state error requirement.
3. **Plot the Bode** magnitude and phase diagrams for this value of gain and determine the uncompensated system's phase margin.
4. **Find the phase margin** to meet the damping ratio or percent overshoot requirement. Evaluate the additional phase contribution required from the compensator.
5. **Determine the value of β** from the lead compensator's required phase contribution.
6. **Determine the compensator's magnitude** at the peak of the phase curve
7. **Determine the new phase-margin frequency** by finding where the uncompensated system's magnitude curve is the negative of the lead compensator's magnitude at the peak of the compensator's phase curve.
8. **Design** the lead compensator's **break frequencies**
9. **Perform simulation** to check that all specs are met by the design, **if not redesign**.

3. Lead compensation

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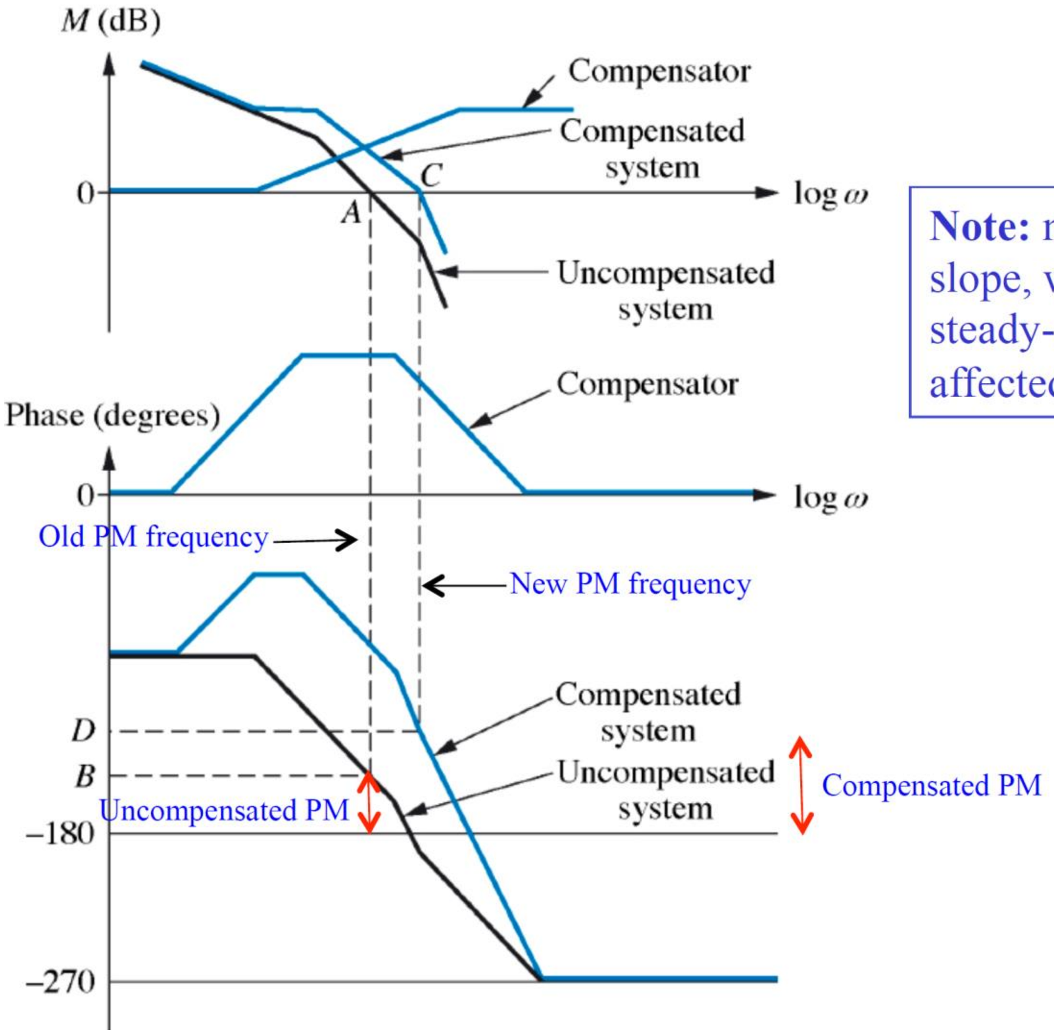
Design procedure

$w_{BW} = \frac{1}{T_s} \cdot f(\xi)$



Note: notice that the initial slope, which determines the steady-state error, is not affected by the design

4. Lag compensation (optional)

Lag compensation improves:

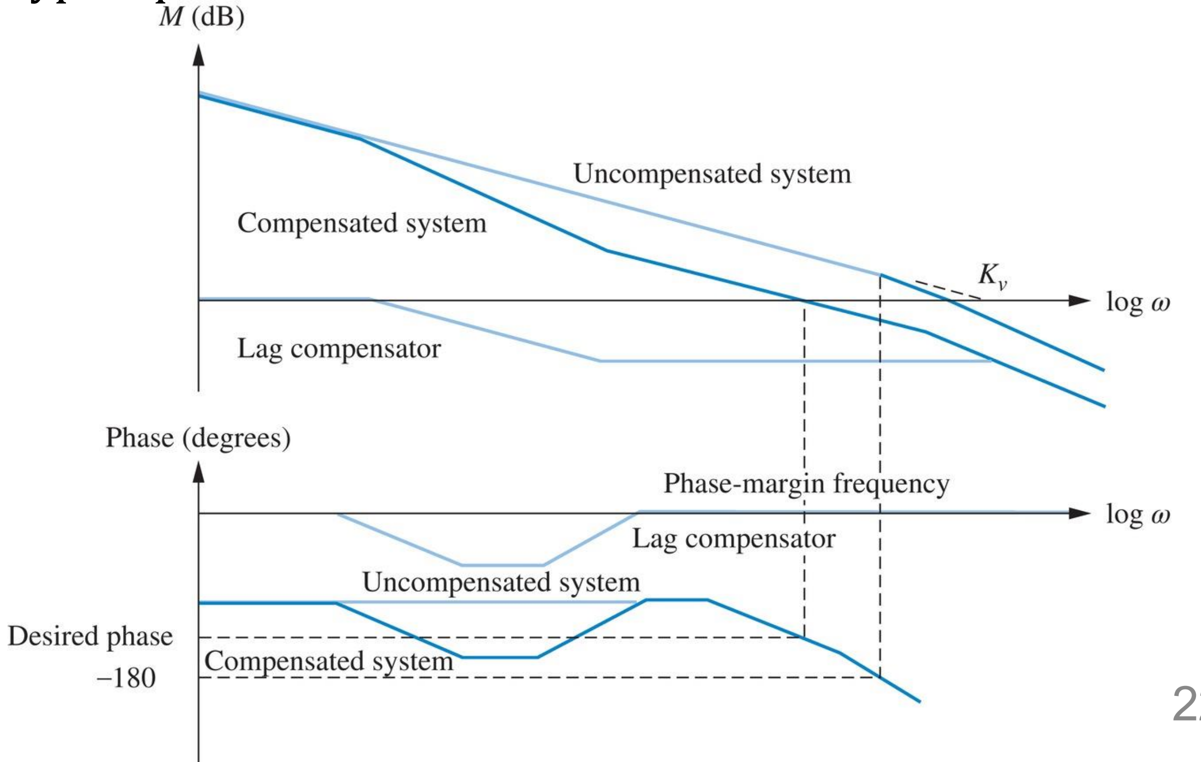
- **stability**, by increasing **the gain margin** without changing crossover frequency or phase margin
  - phase margin determined during gain-adjustment design stage: unchanged by lag compensator
  - ideally, no change in transient response between uncompensated and compensated system
- **the steady-state error**
  - Kv is high, while keeping the system stable (from the fig., the Uncomp. system with desired Kv would be unstable due to M > 0dB at crossover frequency)

Type equation here.

$$G_c(s) = \frac{1}{\alpha}\frac{s+\frac{1}{T}}{s+\frac{1}{\propto T}}$$

  - note: this expression provides

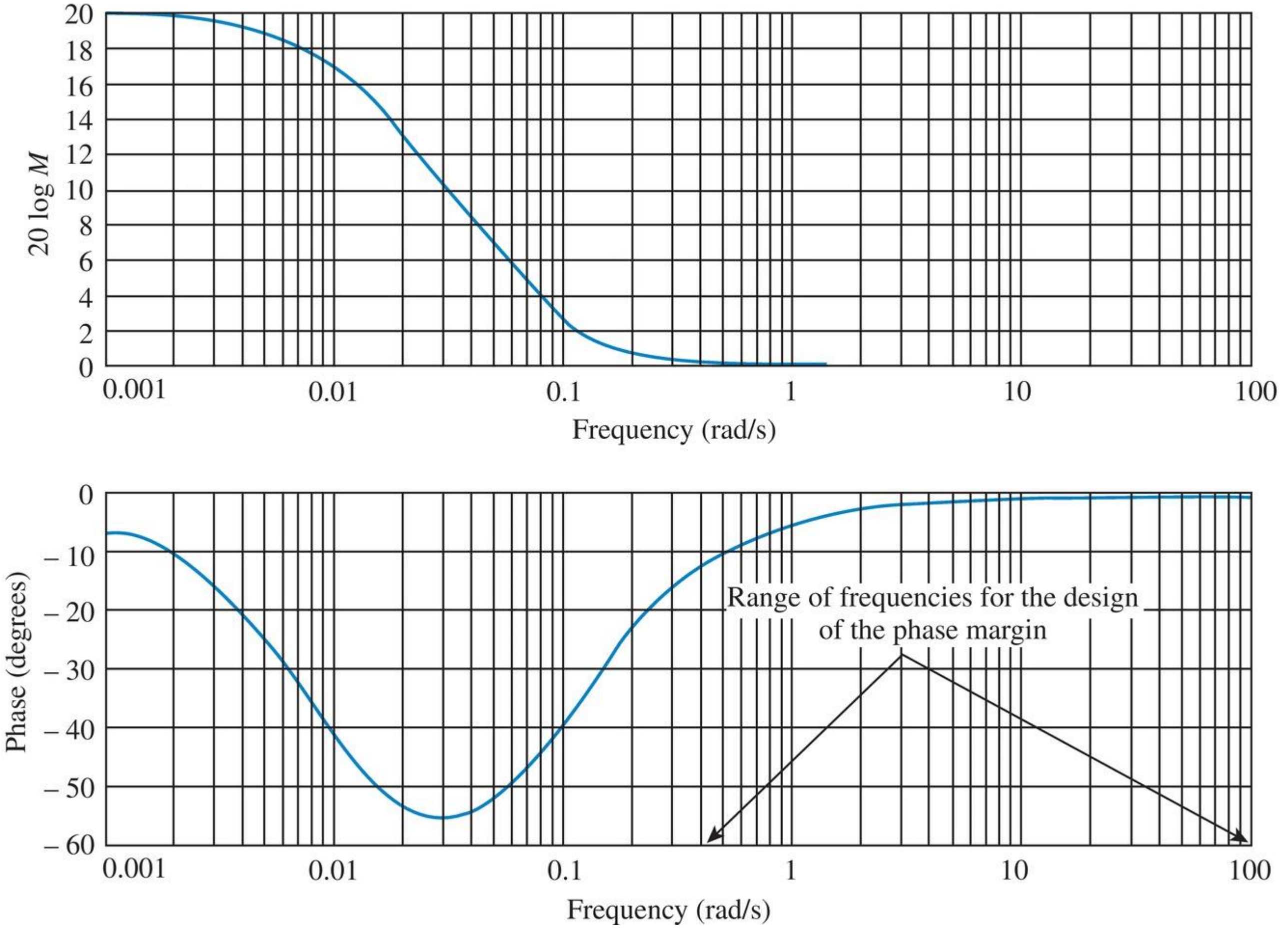
$$G_c(0) = 1$$



4. Lag compensation (optional)

Bode Plot of a Lag compensator

- Overall phase is negative, i.e., lagging



4. Lag compensation (optional)

Design procedure

1. Set the gain, K, to the value that satisfies the steady-state error specification and plot the Bode magnitude and phase diagrams for this value of gain.
   i. Assumes initial gain adjustment done to operate at desired phase margin (%OS)
2. Find frequency where phase margin is 10° higher than desired value, to compensate for compensator phase lag
3. Select a lag compensator whose magnitude response yields a compensated Bode magnitude diagram that goes through 0 dB at the frequency found in Step 2 as follows:
   i. Draw the compensator's high-frequency asymptote to yield 0 dB for the compensated system at the frequency found in Step 2. Thus, if the gain at the frequency found in Step 2 is 20 log K_{PM}, then the compensator's high-frequency asymptote will be set at -20 log K_{PM}
   ii. select the upper break frequency (compensator zero) to be 1 decade below the frequency found in Step 2;
   iii. select the low-frequency asymptote to be at 0 dB (compensator has a dc gain = 1, i.e., 0 dB);
   iv. connect the compensator's high- and low-frequency asymptotes with a -20 dB/decade line to locate the lower break frequency
4. Reset the system gain, K, to compensate for any attenuation in the lag network in order to keep the static error constant the same as that found in Step 1.

5. Summary

5. Summary

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Lectorial learning outcomes

- Compensators design using Bode plots vs. root locus: **advantages/disadvantages**
- Designed transient response via **gain adjustment**
- Designed cascade **lead compensator** to improve transient response
- Provided principles around **lag compensator**
  - Remember that lag compensators reduce, but do not eliminate the steady-state error
  - In practice, proportional integral (PI) compensators are used more freqeuntly, rather than lag compensators