Assignment 3 - SOLUTIONS [redacted] (1)

ELEC270

(1)(a) angular frequency $\omega = 10 \text{ rads}^{-1}$

$\therefore$ impedances $Z_L = j\omega L = j10 \times 0.1 = j\,\Omega$

$Z_C = \frac{1}{j\omega C} = \frac{-j}{10 \times 0.22} = -j0.45\,\Omega$

$\therefore$ circuit in frequency domain

(b) $I_1 = 2.5\angle 0^\circ$ mA

$I_2 = 1.5\angle -42^\circ$ mA

Mesh 3: $(I_3 - I_1) \times -j0.45 + (I_3 - I_2)j + 2I_3 = 0$

$I_3 \times -j0.45 + I_1 j0.45 + I_3 j - I_2 j + 2I_3 = 0$

$$\therefore I_3 = \frac{I_2 j - I_1 j0.45}{-j0.45 + j + 2}$$

$$= \frac{1.5\angle -42^\circ \times \angle 90^\circ - 2.5\angle 0^\circ \times 0.45\angle 90^\circ}{2 + j0.55}$$

$$= \frac{1.5\angle 48^\circ - 1.14\angle 90^\circ}{2.05\angle 15.37^\circ}$$

$$= \frac{1.004 + j1.11 - j1.14}{2.05\angle 15.37^\circ}$$

$$= \frac{1.004 - j0.03}{2.05\angle 15.4^\circ} = \frac{1.004\angle -1.7^\circ}{2.05\angle 15.4^\circ}$$

$$= 0.48\angle -17.1^\circ$$

$\therefore\ i_1 = 2.5\cos 10t$ mA

$i_2 = 1.5\cos(10t - 42^\circ)$ mA

$i_3 = 0.48\cos(10t - 17^\circ)$ mA

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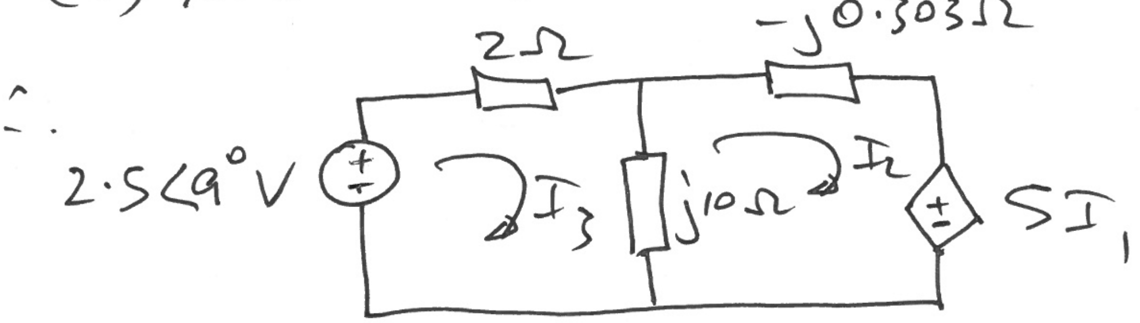
2 (a) $\omega = 10\ \text{rad s}^{-1}$

$\therefore$



Mesh 1:

$$2I_1 + j10(I_1 - I_2) = 2.5\angle 9^\circ$$

$$I_1(2 + j10) - I_2 j10 = 2.47 + j0.391 \quad -①$$

Mesh 2:

$$(I_2 - I_1)j10 + I_2(-j0.303) + 5I_1 = 0$$

$$I_1(5 - j10) + I_2(j9.7) = 0 \quad -②$$

From 2: $I_2 = I_1\left(\dfrac{5-j10}{-j9.7}\right)$

$= I_1\left(\dfrac{11.18\angle -63.43^\circ}{9.7\angle -90^\circ}\right)$

$\therefore I_2 = I_1\left(1.153\angle 26.57^\circ\right)$

Substitute into eqn 1

$I_1(10.2\angle 78.7^\circ) - (1.153\angle 26.57^\circ \times 10\angle 90^\circ)$
$= 2.5\angle 9^\circ$

$I_1\,(10.2\angle 78.7^\circ) - 11.53\angle 116.57^\circ = 2.5\angle 9^\circ$

$I_1\left(2+j10 - [-5.16+j10.3]\right) = 2.5\angle 9^\circ$

$I_1\,(7.16 - j0.3) = 2.5\angle 9^\circ$

$I_1 = \dfrac{2.5\angle 9^\circ}{7.16\angle -2.4^\circ} = 0.349\angle 11.48^\circ$

$\therefore I_1 = 0.349\angle 11.48^\circ$ A

$I_2 = 0.349\angle 11.48 \times 1.153\angle 26.57^\circ$

$= 0.4\angle 38.05^\circ$ A

(b) $i_1 = 0.349\cos(10t + 11.48^\circ)$ A

$i_2 = 0.4\cos(10t + 38.05^\circ)$ A

(c) angular frequency $= 10$ rad s^{-1}

frequency $= f = \dfrac{\omega}{2\pi} = \dfrac{10}{2\times\pi} = 1.59$ Hz

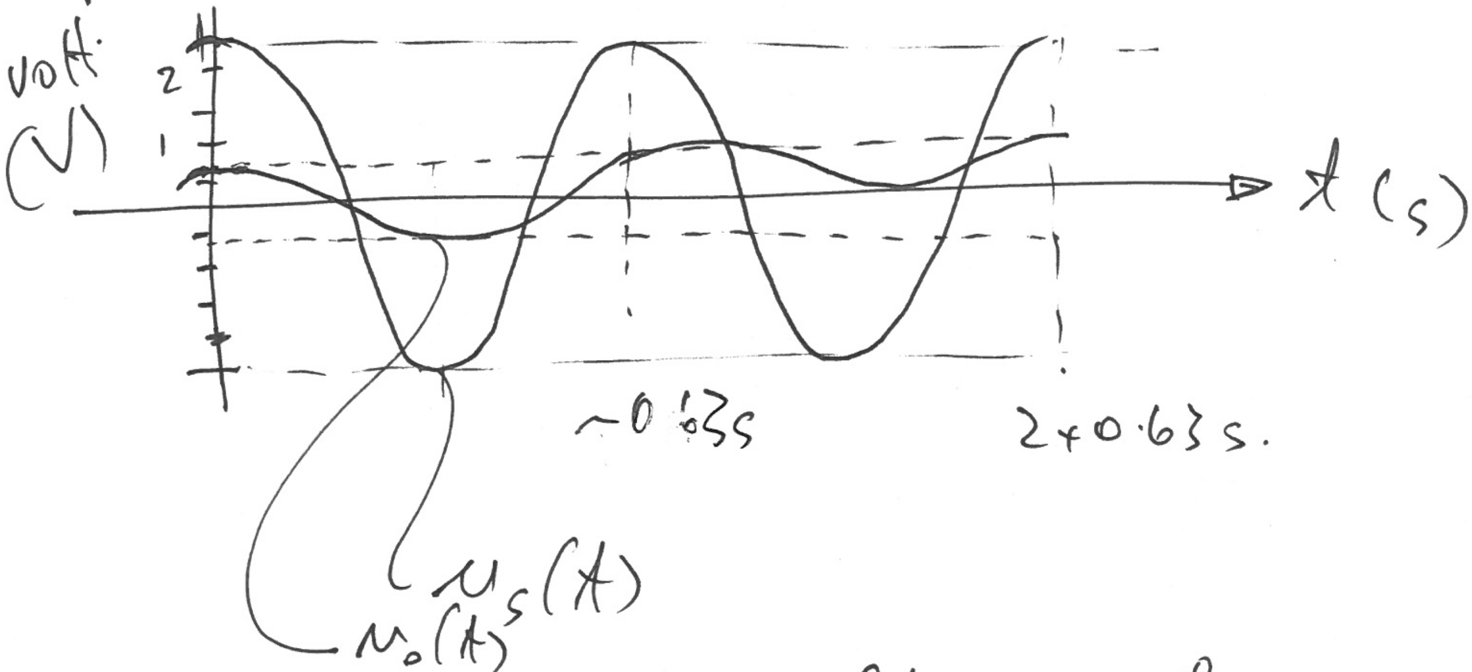
$\therefore$ period $= T = 0.628$ s.

(d) $v = iR$

$= 0.349 \cos(10t + 11.48^\circ) \times 2$

$= 0.7 \cos(10t + 11.48^\circ)\,V$

peak-to-peak voltage $= 1.4\,V$



(e) phase angle of $v_o(t) = 11.48^\circ$

phase angle of $v_s(t) = 9^\circ$

phase difference $= 2.48^\circ$

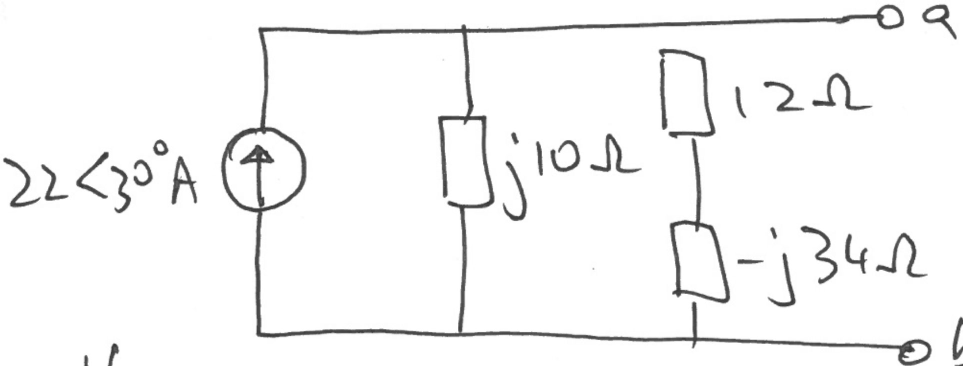
$\therefore$ since $T = 0.628\,s$

$\therefore t = \dfrac{2.48}{360} \times 0.628$

$= 0.0043$

$= 4.3\,ms.$

(3) (a) In frequency domain:



$j10 \parallel 12 - j34$

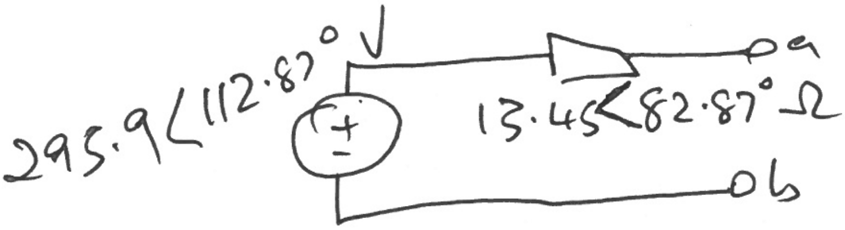
$$\therefore Z_{th} = \frac{(j10)(12-j34)}{12-j24}\ \Omega$$

$$= \frac{10\angle 90^\circ \times 36.1\angle -70.56^\circ}{26.83\angle -63.43^\circ}$$

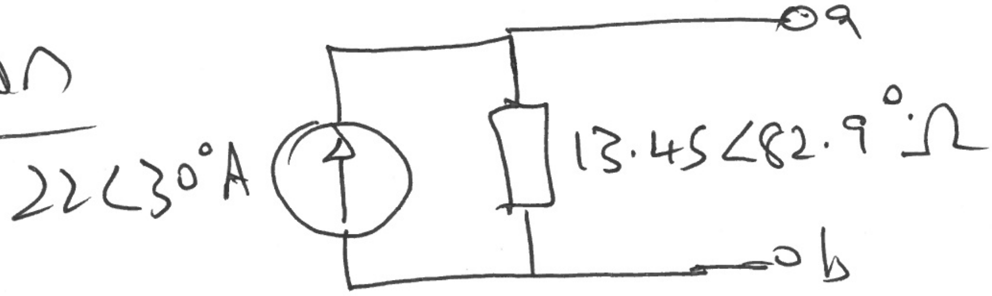
$$= \frac{361\angle 19.44^\circ}{26.83\angle -63.48^\circ} = 13.45\angle 82.87^\circ\ \Omega$$

$$\therefore V_{oc} = 22\angle 30^\circ \times 13.45\angle 82.87^\circ$$

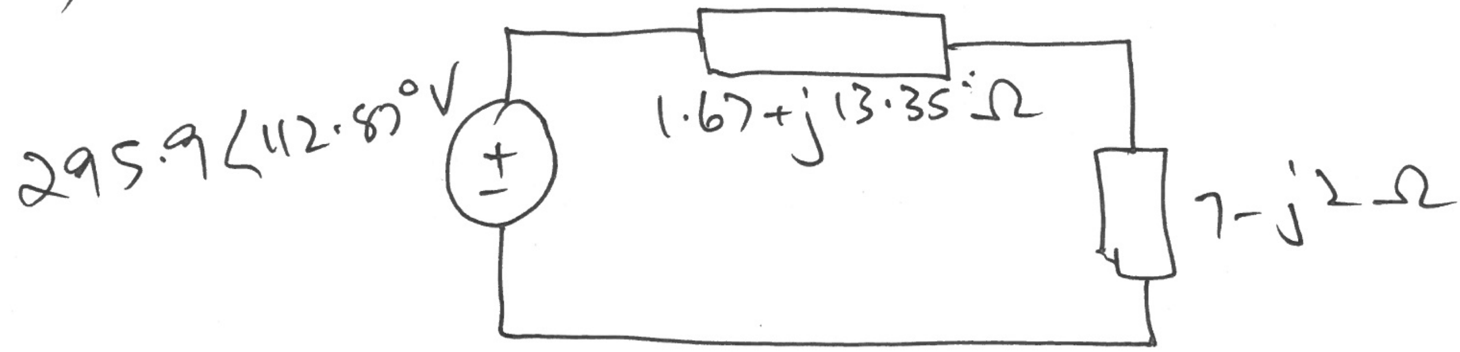
$$= 295.9\angle 112.87^\circ\ V$$



(b) Norton



(c) New circuit



Total impedance $= 8.67 + j11.35\ \Omega$

$= 14.28\angle 52.6°\ \Omega$

$\therefore$ current $I = \dfrac{295.9\angle 112.87°}{14.28\angle 52.6°}$

$= 20.72\angle 60.3°$ A

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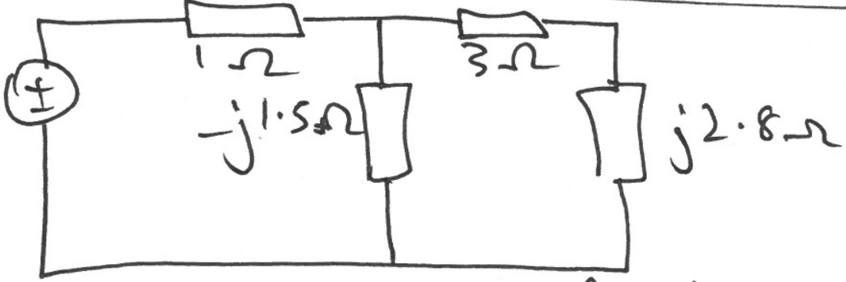
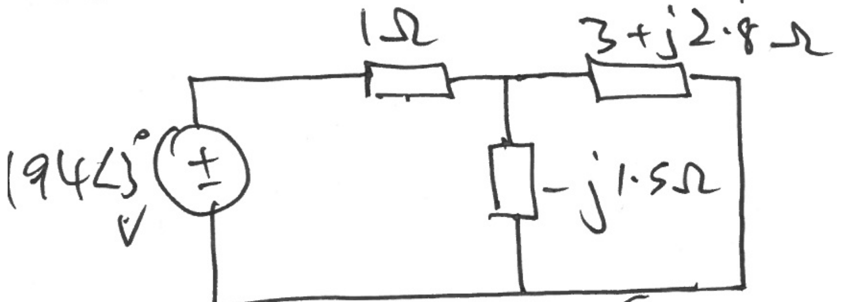
(4) 194∠3° V



Could do mesh analysis or we first simplify



$3+j2.8 \parallel -j1.5$

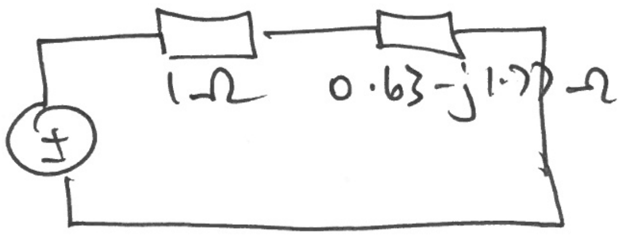
$\Rightarrow Z_{eff} = \dfrac{(3+j2.8)(-j1.5)}{3+j1.3}$

$= \dfrac{4.1\angle 43° \times 1.5\angle -90°}{3.27\angle 23.43°}$

$= 1.88\angle -70.45°\ \Omega$

$= 0.63 - j1.77\ \Omega$

∴ circuit:



current, I from source $= \dfrac{194\angle 3^\circ}{1.63-j1.77}$

$= \dfrac{194\angle 3^\circ}{2.4\angle -47.36^\circ}$

$= 80.8\angle 50.4^\circ$ A

Average power $P = \frac{1}{2} V_m I_m \cos(\theta_V - \theta_I)$

For the source

$P_S = -\frac{1}{2}\, 194 \times 80.8 \cos(3^\circ - 50.4^\circ)$

$= -5.31$ kW

$P_{1\Omega} = \frac{1}{2} I_m^2 \, Re(Z) = \frac{1}{2}(80.83)^2 \times 1$

$= 3.26$ kW

Current through 3Ω resistor:

$I_{3\Omega} = 80.83\angle 50.4 \left[\dfrac{-j1.5}{3+j1.3}\right]$

$= 80.83\angle 50.4 \left(\dfrac{1.5\angle -90^\circ}{3.27\angle 23.43^\circ}\right)$

$= 80.83\angle 50.4^\circ \times 0.459\angle -113.43^\circ$

$= 37\angle -63^\circ$ A

$\therefore P_{3\Omega} = \frac{1}{2}(37)^2 3 = 2.05\ kW$

$\left.\begin{array}{l} P_L = 0\ W \\ P_C = 0\ W \end{array}\right\}$ since reactive elements

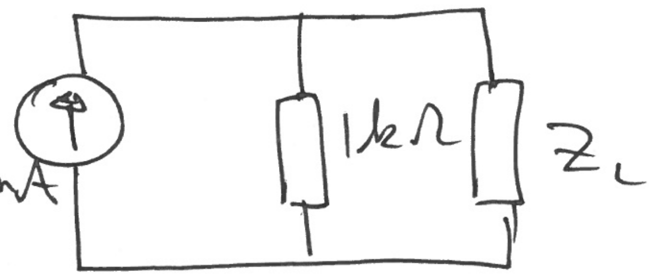
$\sum$ Source $= 5.31\ kW$

$\sum$ Resistances $= 3.26 + 2.06$

$= 5.32\ kW$

(Note: rounding errors here)

5 (a)



$1k\Omega \parallel Z_L \Rightarrow Z_{eff} = \dfrac{Z_L \times 1000}{Z_L + 1000}$

but Z_L is purely resistive $\Rightarrow Z_L = R_L$

$\therefore$ For a purely resistive load

Power factor $= 1$.

(b) $Z_L = 1000 + j900$

(9)

$$\therefore Z_{eff} = \frac{1000\,(1000 + j900)}{2000 + j900}$$

$$= \frac{1000 + j900}{2 + j0.9} = \frac{1345 \angle 42^\circ}{2.19 \angle 24.2^\circ}$$

$$= 613 \angle 17.8^\circ\ \Omega$$

$\therefore$ Voltage, $V = I_S \times Z$

$$= 0.275 \angle 20^\circ \times 613 \angle 17.8^\circ$$

$$= 169 \angle 37.8^\circ\ V$$

$$\therefore PF = \cos(37.8^\circ - 20^\circ)$$

$= 0.952$ lagging (since current lags voltage

(c)

$$Z_{eff} = \frac{1000 \times 500 \angle -5^\circ}{1000 + 498.1 - j43.6}$$

$$= 334 \angle -3.34^\circ\ \Omega$$

$$\therefore V = 0.275 \angle 20^\circ \times 334 \angle -3.34^\circ$$

$$= 92 \angle 16.66^\circ\ V$$

$$\therefore PF = \cos(16.66 - 20^\circ)$$

$= 0.998$ leading

since current leads voltage

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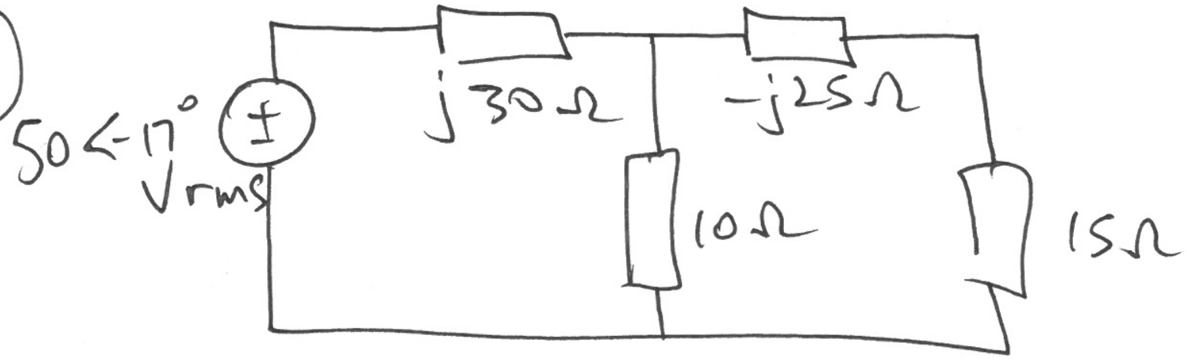

Complex power, $S = \frac{VI^*}{2} = V_{rms} I^*_{rms}$

Need to find currents

$10 \,||\, 15 - j25 \Rightarrow Z_{eff} = \frac{10(15 - j25)}{25 - j25}$

$= 8 - j2\,\Omega$

Current from source, $I_s = \frac{50\angle -17^\circ}{8 + j28}$

$= 1.72\angle -91^\circ$ Arms

Voltage across $j30\,\Omega$ inductance

$V_{j30} = j30 \times 1.72 \angle -91^\circ$

$= 51.6\angle -1^\circ$ Vrms

$S_{j30} = 51.6\angle -1^\circ \times 1.72\angle 91^\circ$

$= 88.75\angle 90^\circ$ VA

Voltage across $10\,\Omega$ resistor

$V_{10\Omega} = 50\angle -17^\circ \times \frac{8.25\angle -14^\circ}{29.1\angle 74.05^\circ}$

$V = 14.18\angle -105^\circ$ Vrms

(11)

current through 10Ω resistor

$$I_{10\Omega} = \frac{14.18\angle -105}{10} = 1.42\angle -105 \text{ Arms}$$

$$\therefore S_{10\Omega} = 14.18\angle -105 \times 1.42\angle 105$$

$$= 20.1\angle 0^\circ \text{ VA}$$

Current through $15 - j25\ \Omega$ impedance

$$= \frac{14.18\angle -105}{29.15\angle -59} = 0.49\angle -46^\circ \text{ A rms}$$

Voltage across $-j25$ capacitance

$$V_{-j25} = 0.49\angle -46^\circ \times -j25$$

$$= 12.25\angle 44^\circ \text{ Vrms}$$

$$\therefore S_{-j25} = 12.25\angle 44^\circ \times 0.49\angle 46^\circ$$

$$= 6\angle 90^\circ \text{ VA}$$

$$V_{15\Omega} = 0.49\angle -46^\circ \times 15$$

$$= 7.35\angle -46^\circ \text{ Vrms}$$

$$\therefore S_{15\Omega} = 7.35\angle -46^\circ \times 0.49\angle 46^\circ$$

$$= 3.68 \text{ VA}$$