

# ELEC2070 Circuits and Devices

Week 3: Ideal Operational Amplifiers

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# The “Big Ideas” in Chap. 6

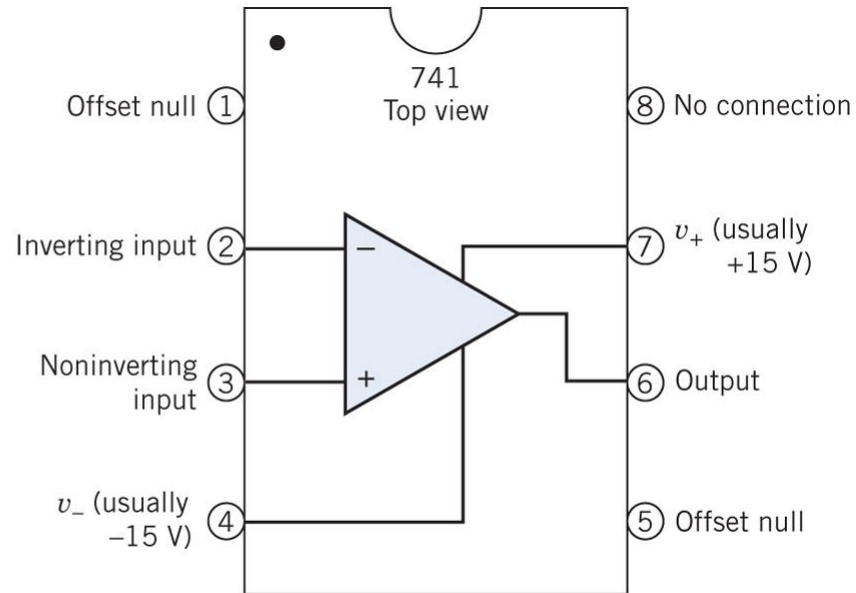
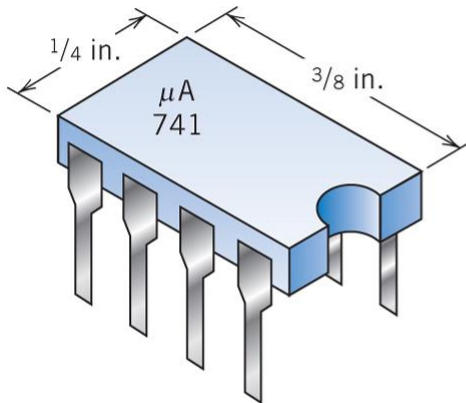
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- Operational amplifiers (or op amps) allow circuits to do “operations” (mathematical functions)
- In this lecture, we cover the basics of **ideal amplifiers**.
- Operational amplifiers are studied using Node Analysis
- Basic circuits for mathematical operations will be introduced

# The operational amplifier

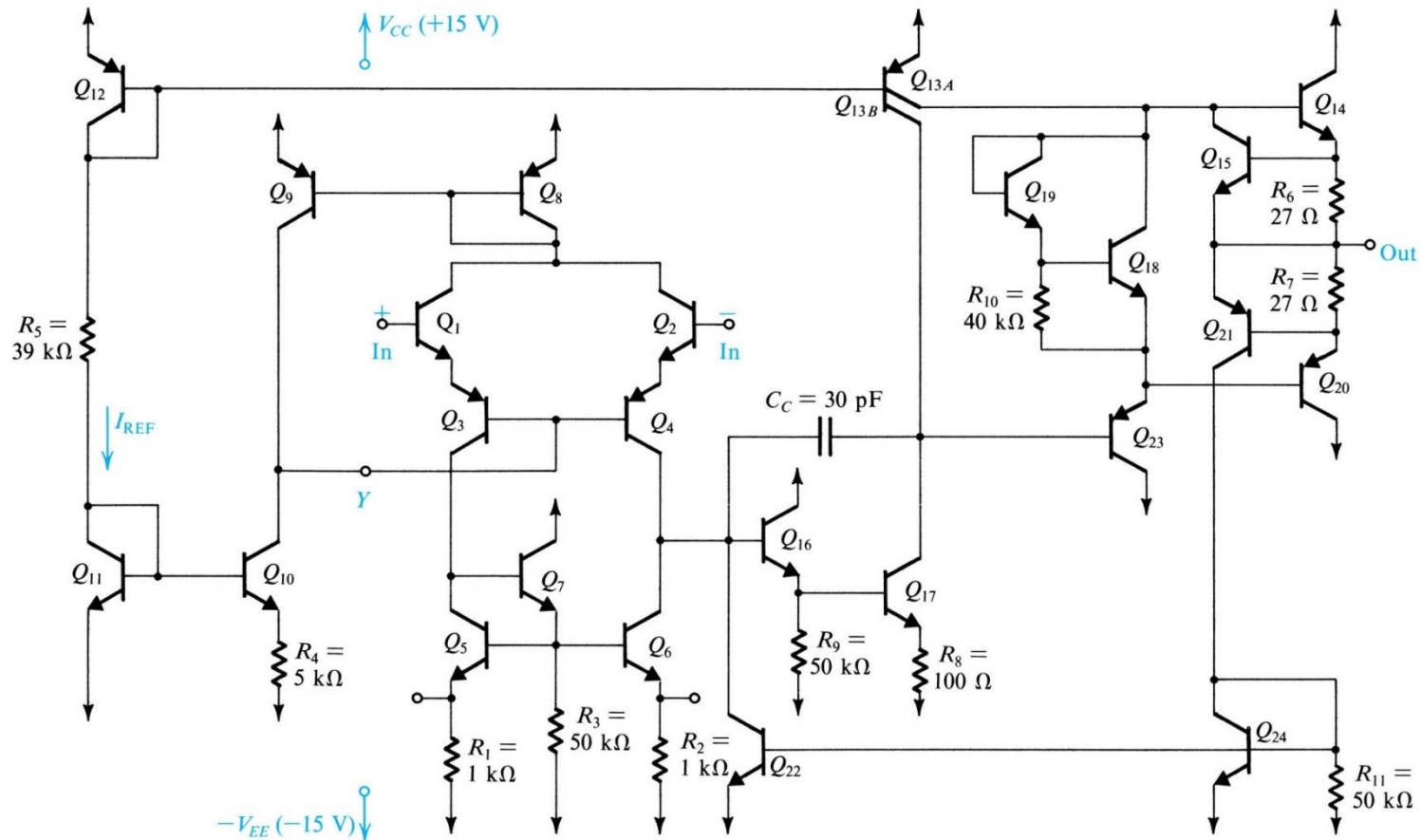


# The most common op amp



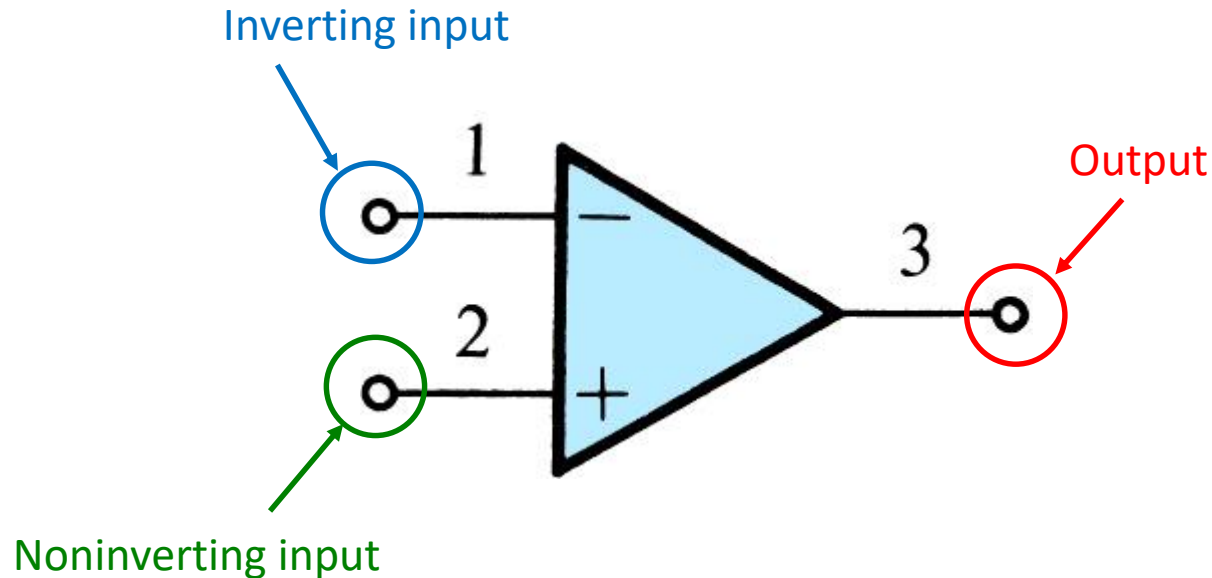
- Operational amplifiers are widely used in signal processing and control circuits.
- $\mu A$ 741 is the most common op amp.
- Op amps are **differential amplifiers**

# The $\mu A741$ Circuit



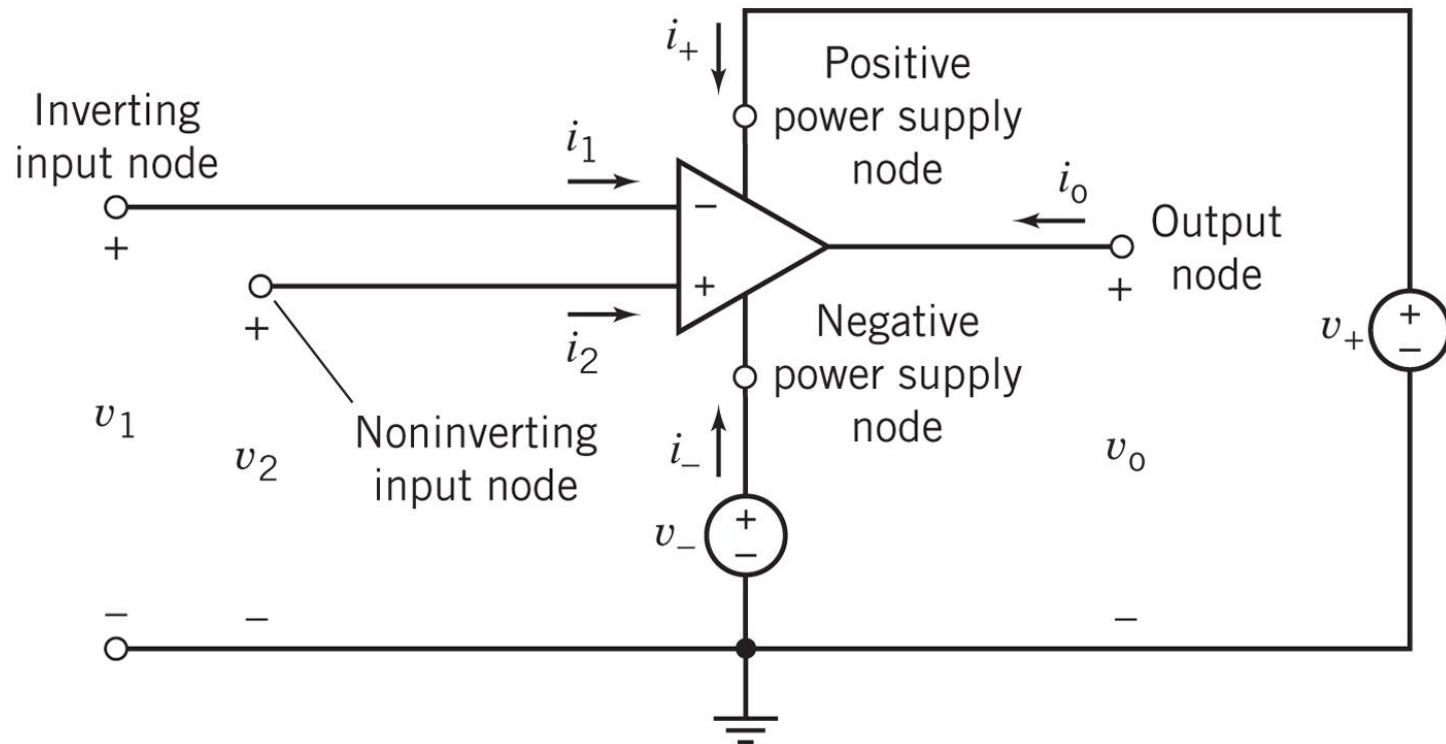
# The op amp - definitions

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In some circuit diagrams, the top terminal is the non-inverting input – be careful!

# Op amp schematic



- The DC power supplies “bias” the op amp
- These are usually **not** included in the schematic
- The power supplies add energy (power) to a circuit

# Use Node Analysis for op amps

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1. Since we generally omit op amp power supplies - using KCL could create some problems
2. At the ground node: we have 2 currents involving the power supplies
  - cannot use KCL at ground node if power supplies are omitted!
3. At the op amp: applying KCL means

$$i_1 + i_2 + i_0 + i_+ + i_- = 0$$

Hence KCL won't work if power supplies omitted

**USE Node Analysis instead for op amps**



# The ideal operational amplifier



# Keeping the op amp in the “linear regime”

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We need:

Data for  
 $\mu\text{A}741$ :

The output voltage:

$$|v_o| \leq v_{\text{sat}} \quad 14 \text{ V}$$

The output current:

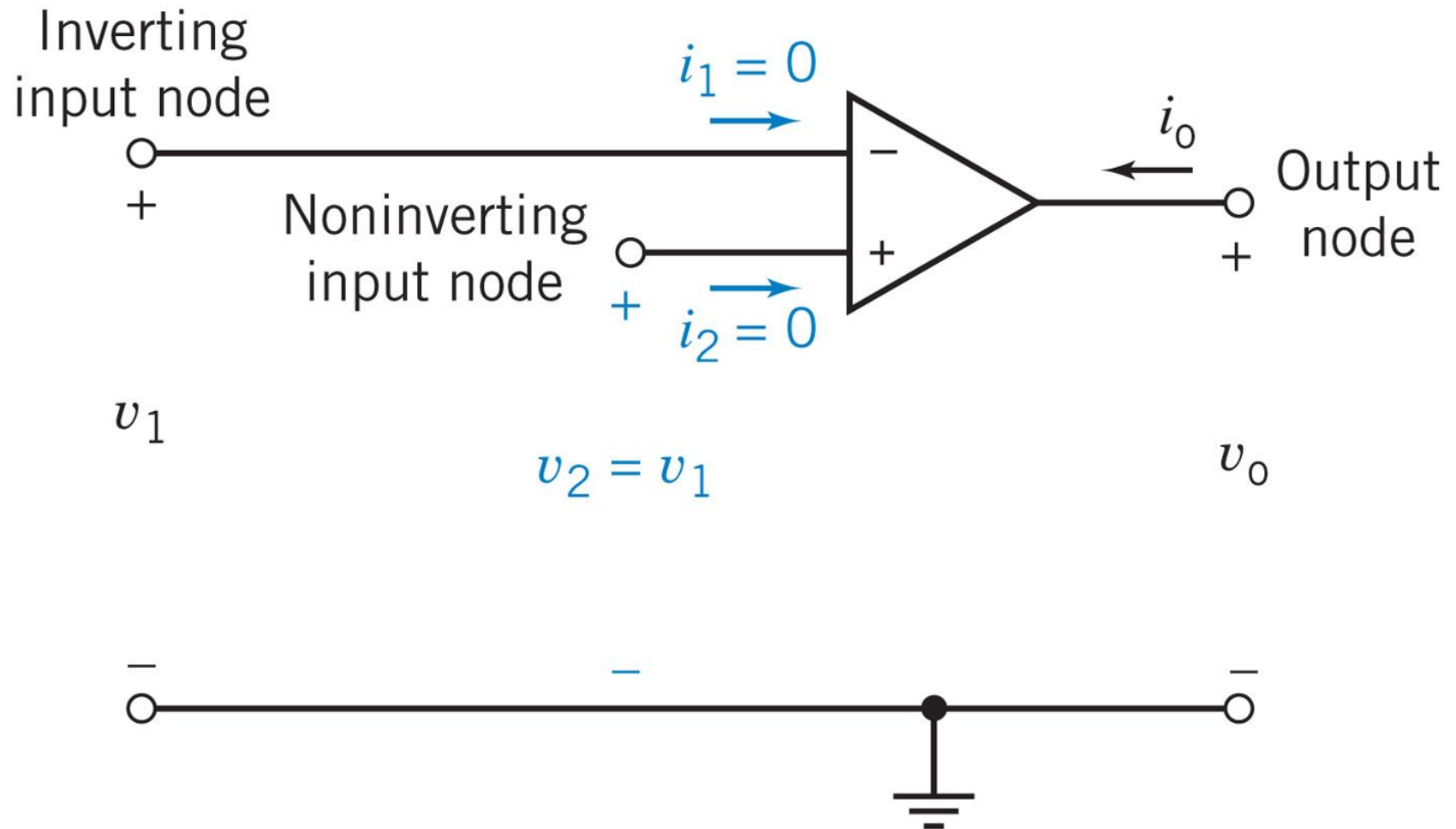
$$|i_o| \leq i_{\text{sat}} \quad 2 \text{ mA}$$

The change in the output voltage

$$\left| \frac{dv_o(t)}{dt} \right| \leq SR \quad \begin{array}{l} \text{The “slew rate”} \\ 500,000 \text{ V/s} \end{array}$$

We can use “ideal operational amplifier” conditions for all our calculations if we stay within these limits

# The ideal operational amplifier



# Operating conditions for an ideal op amp

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VARIABLE	IDEAL CONDITION
Inverting node input current	$i_1 = 0$
Noninverting node input current	$i_2 = 0$
Voltage difference between inverting node voltage $v_1$ and noninverting node voltage $v_2$	$v_2 - v_1 = 0$

These conditions make Node Analysis of circuits with op amps easy!

Remember: the ideal operational amplifier is a **model** for a linear operational amplifier

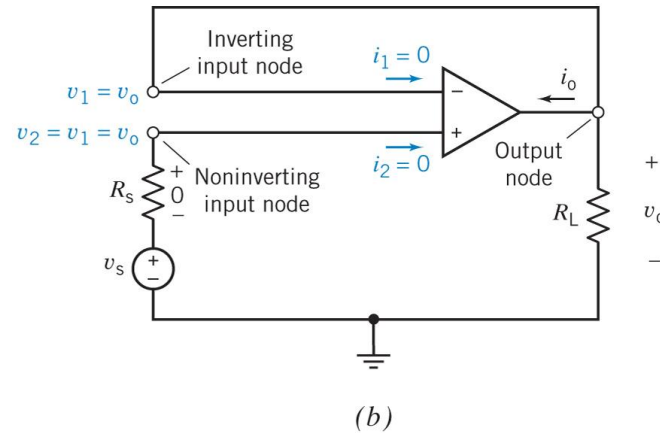
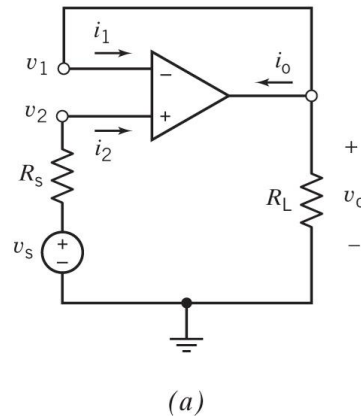
# Input and output

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As opposed to the circuits that we have studied so far, an op amp circuit always has an *output* that depends on some type of *input*. Therefore, we will analyze op amp circuits with the goal of obtaining an expression for the output in terms of the input quantities. *We will find that it is usually a good idea to begin the analysis of an op amp circuit at the input, and proceed from there.*

# Example 6.3.1

Consider the circuit (a):



How does the output voltage change with input voltage?

Figure (b) shows what we know from an ideal op amp. Note that  $v_1 = v_o$  since they are connected via a short.

We can now express the voltage across  $R_s$  as:

$$v_s - v_2 = v_s - v_1 = v_s - v_o \text{ therefore:}$$

$$v_s = v_o \text{ (because no current is flowing through } R_s, \text{ since } i_2 = 0)$$

# Node Analysis of circuits

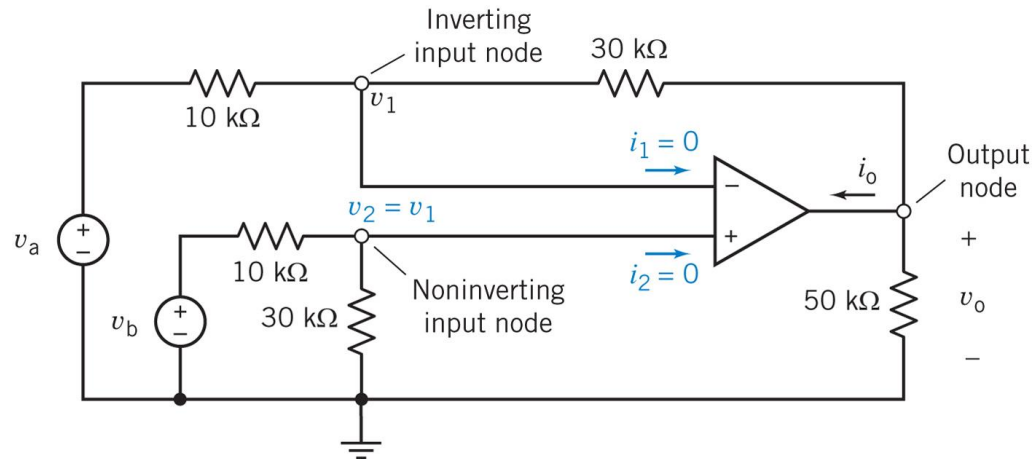
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For ideal op amps we can always say:

1.  $v_1 = v_2$  hence we can eliminate  $v_1$  or  $v_2$  from the node equations
2. Inputs currents are always zero. This helps for node analysis of inputs
3. The output current is not zero and is always involved in a KCL equation at the output node.

# Example 6.4.1

Consider the following difference amplifier:



Find  $v_o$  in terms of the two input voltages.

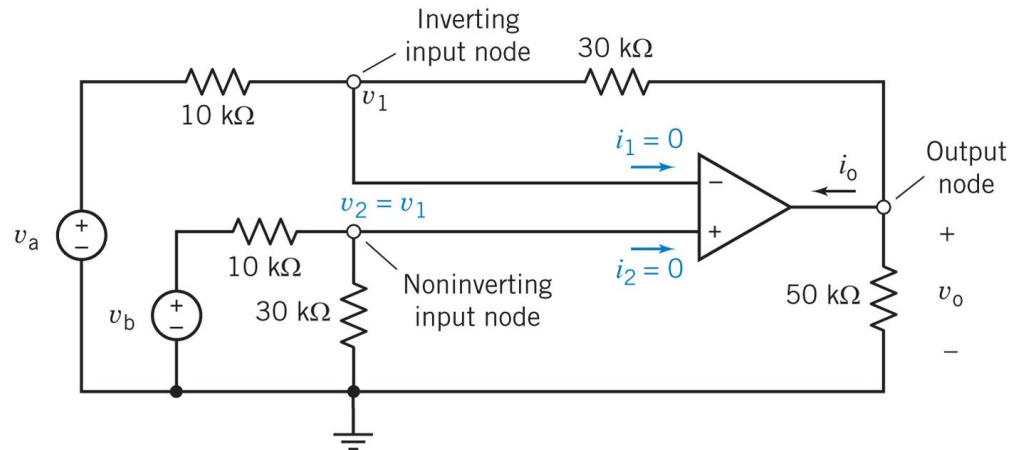
At the noninverting input node (and since  $v_2 = v_1$ ) :

$$\frac{v_1}{30,000} + \frac{v_1 - v_b}{10,000} + i_2 = 0$$

Since  $i_2 = 0$  we get  $v_1 = \frac{3}{4} v_b$



# Example 6.4.1



At the inverting input node:

$$\frac{v_1 - v_a}{10,000} + \frac{v_1 - v_o}{30,000} + i_1 = 0,$$

Of course  $i_1 = 0$  so

$$v_o = 4v_1 - 3v_a$$

and since  $v_1 = \frac{3}{4} v_b$  we get

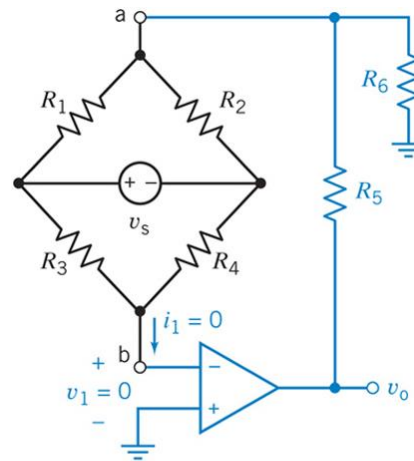
$$v_o = 3(v_b - v_a)$$

# Example 6.4.2

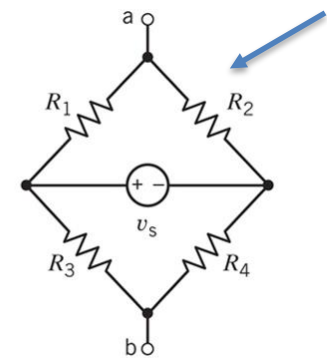
Consider a bridge amplifier:

The resistors  $R_5$  and  $R_6$  are used to amplify the output of the bridge.

Determine the output voltage versus the input voltage.



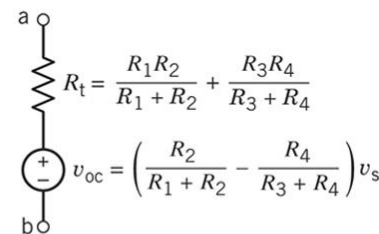
(a)



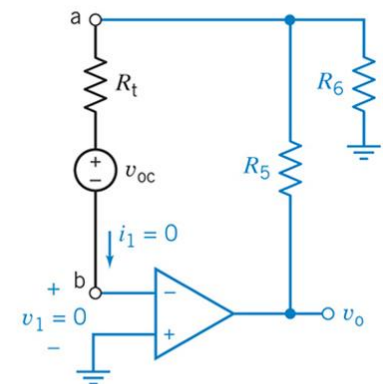
(b)

Solution:

Find Thevenin circuit for the bridge circuit.



(c)



(d)

Hint: Find  $v_{oc}$  using mesh analysis.

Find  $R_{th}$  by switching off  $v_s$

## Example 6.4.2

Now we have a simple circuit which we can apply Node Analysis

Now  $v_b = v_1 = 0$

And  $v_a = v_1 + v_{oc} + i_1 R_t$   
but  $i_1 = 0$

Hence  $v_a = v_{oc}$

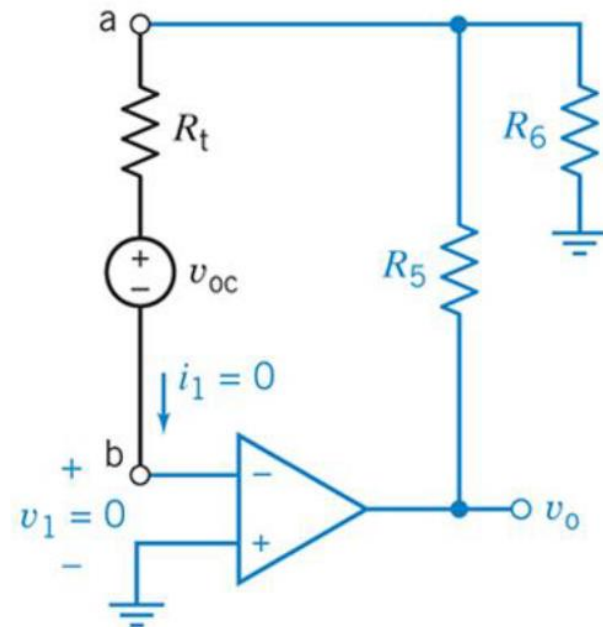
Node equation at node a:

$$i_1 + (v_a - v_0)/R_5 + v_a/R_6 = 0$$

Substituting from above:  $(v_{oc} - v_0)/R_5 + v_{oc}/R_6 = 0$

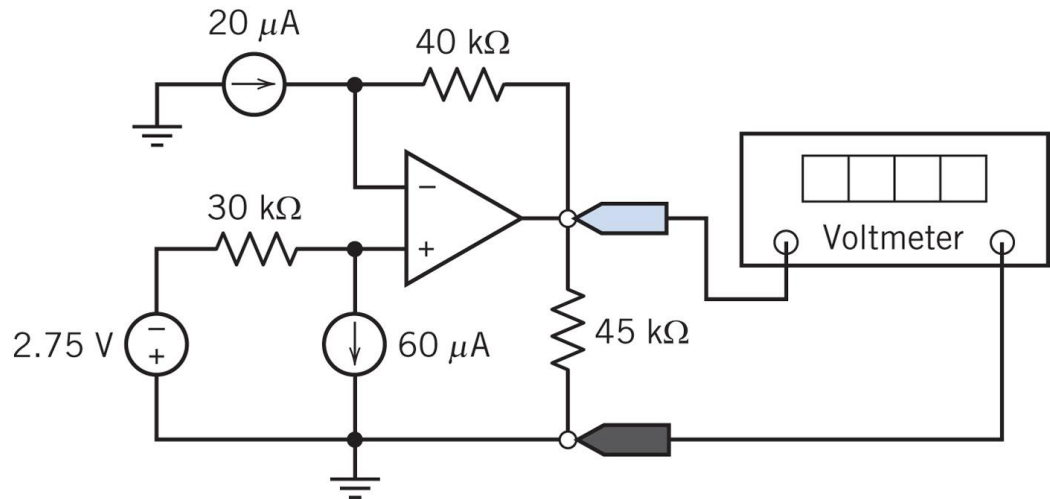
Hence  $v_0 = v_{oc} (1 + R_5/R_6)$

You can now substitute the expression for  $v_{oc}$  to find  $v_0$  in terms of  $v_s$



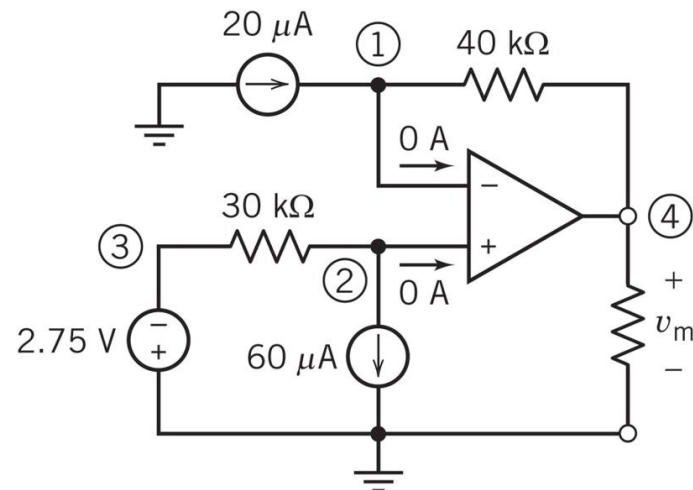
## Example 6.4.3

We need to determine the voltage measured by the voltmeter for this circuit



This is easy since we are dealing with an ideal op amp

The circuit we are dealing with looks like:



## Example 6.4.3

In this circuit we have labelled 4 nodes with the voltages:  $v_1, v_2, v_3, v_4$

Of course  $v_m$  is the output measured by the voltmeter and  $v_m = v_4$

We can also see that  $v_3 = -2.75 \text{ V}$

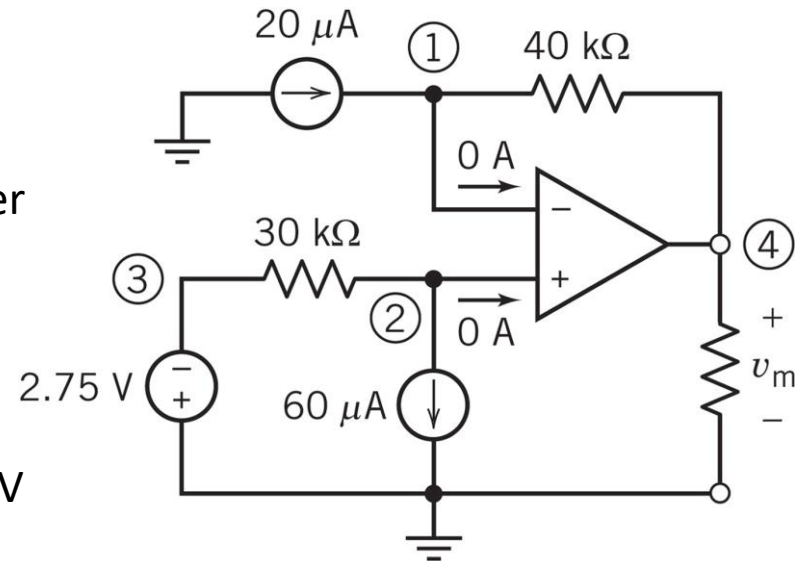
KCL at node 2:  $(v_3 - v_2)/30,000 = 60 \times 10^{-6}$  or  $v_2 = -4.55 \text{ V}$

Since the 2 input voltages are equal,  $v_1 = -4.55 \text{ V}$

Doing KCL at node 1:  $20 \times 10^{-6} = (v_1 - v_4)/40,000$

Or  $v_4 = -5.35 \text{ V}$  (since  $v_1 = -4.55 \text{ V}$ )

This is what the voltmeter will read.

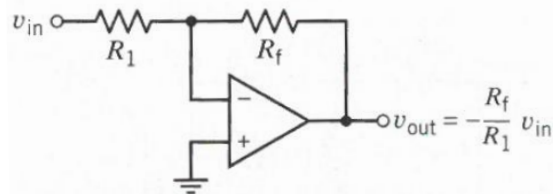


# Designing circuits using op amps

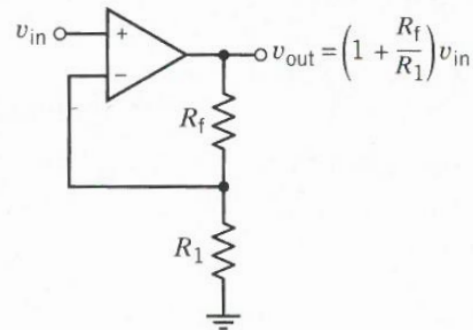




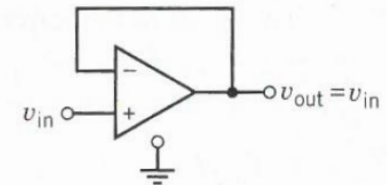
# Functional op amp circuits



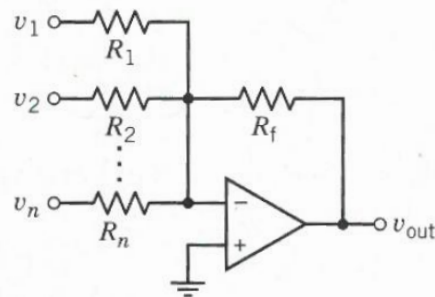
(a) Inverting amplifier



(b) Noninverting amplifier

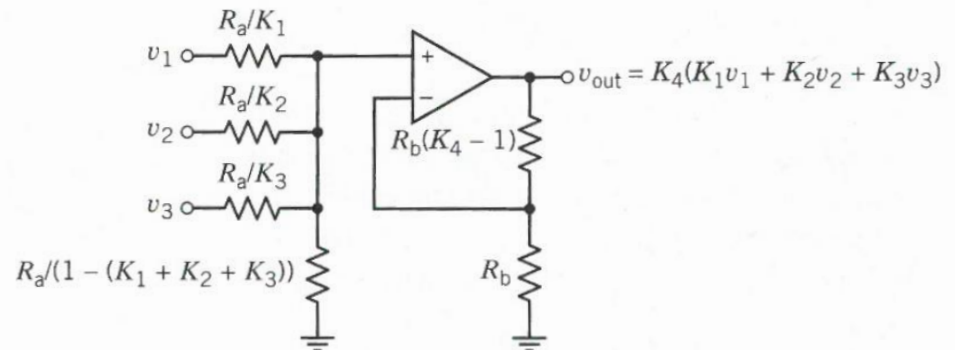


(c) Voltage follower (buffer amplifier)



$$v_{out} = -\left(\frac{R_f}{R_1} v_1 + \frac{R_f}{R_2} v_2 + \dots + \frac{R_f}{R_n} v_n\right)$$

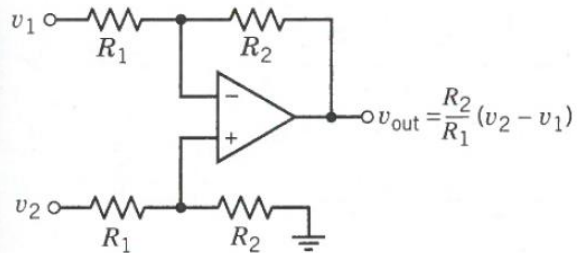
(d) Summing amplifier



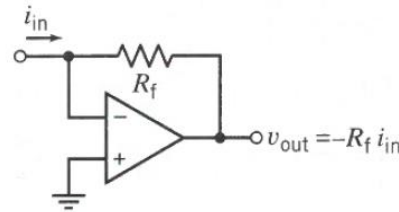
(e) Noninverting summing amplifier



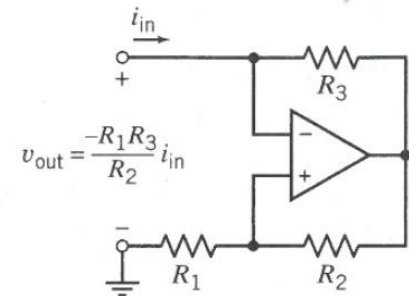
# Functional op amp circuits



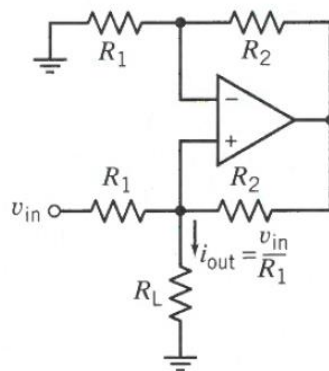
(f) Difference amplifier



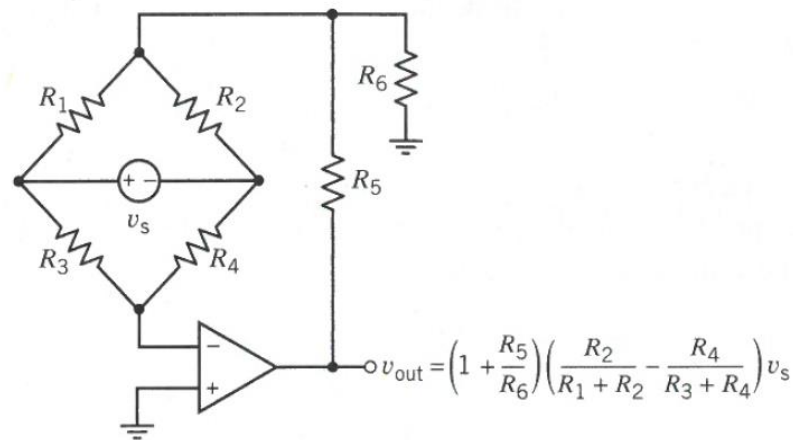
(g) Current-to-voltage converter



(h) Negative resistance convertor



(i) Voltage-controlled current source (VCCS)

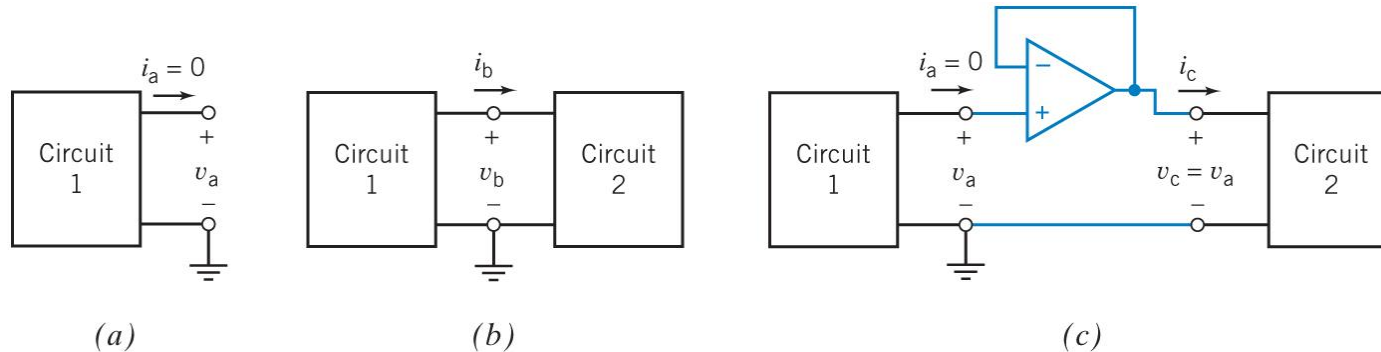


(j) Bridge amplifier

Let's look  
at some  
examples



# Voltage follower

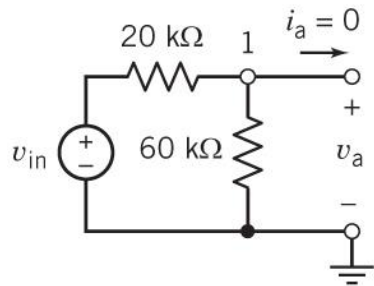


One of the problems when circuit 1 is connected to circuit 2 is **loading**.

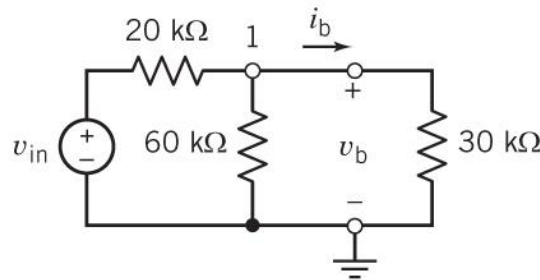
Or we have  $v_a \neq v_b$  (circuit 2 **loads** circuit 1 and  $i_b$  is the **load current**)

Therefore we need a design a solution: use a **voltage follower** – copies the voltage

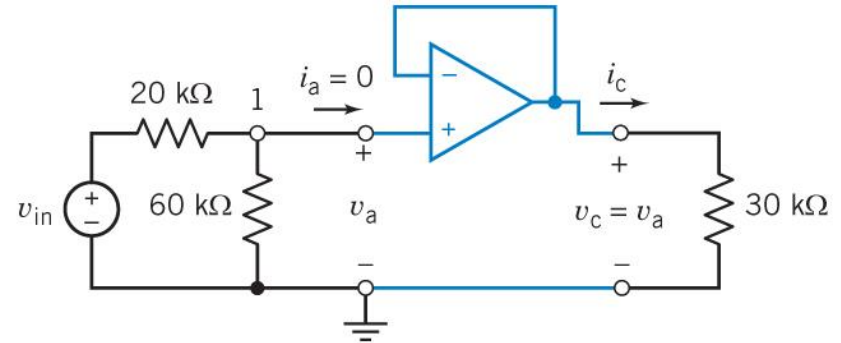
# Example 6.5.1



(a)



(b)



(c)

In this circuit (a) we can apply KCL at node 1:

$$(v_a - v_{in})/20,000 + v_a / 60,000 + i_a = 0$$

$$\text{Or } v_a = 0.75 v_{in}$$

If we connect 30k resistor, i.e., circuit (b) we now have:

$$(v_b - v_{in})/20,000 + v_b / 60,000 + v_b / 30,000 = 0 \text{ or } v_b = 0.5 v_{in}$$

- the loading comes from the new current in the resistor

# Example 6.5.1

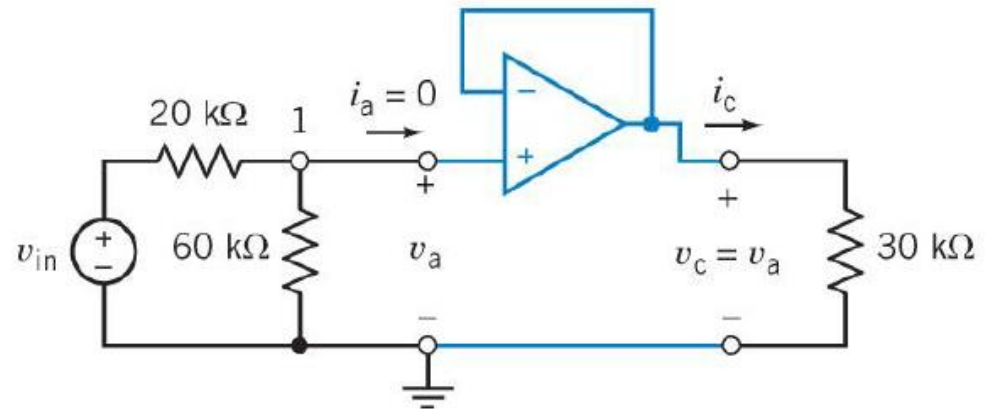
Now we can add a voltage follower:

At node 1:

$$(v_a - v_{in})/20,000 + v_a/60,000 = 0$$

Hence  $v_a = 0.75 v_{in}$  but  $v_c = v_a$

Hence  $v_c = 0.75 v_{in}$



The voltage follower provides the current for the 30k resistor not the independent power supply  $v_{in}$  therefore keeping the voltage the SAME.

# Power supplied to op amps

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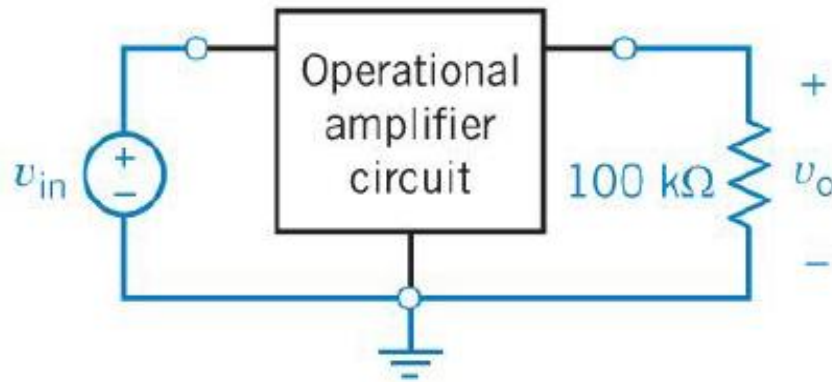
Q: If no current flows into or out of either input terminal of an op amp, why does current flow from the output?

A: In ELEC2070 (and for many other applications) we are treating op amps as an independent element (like a resistor) but in reality they are supplied with an independent power source!

# Scaling a voltage

Want a circuit to create the following:

$$v_o = K v_{in}$$



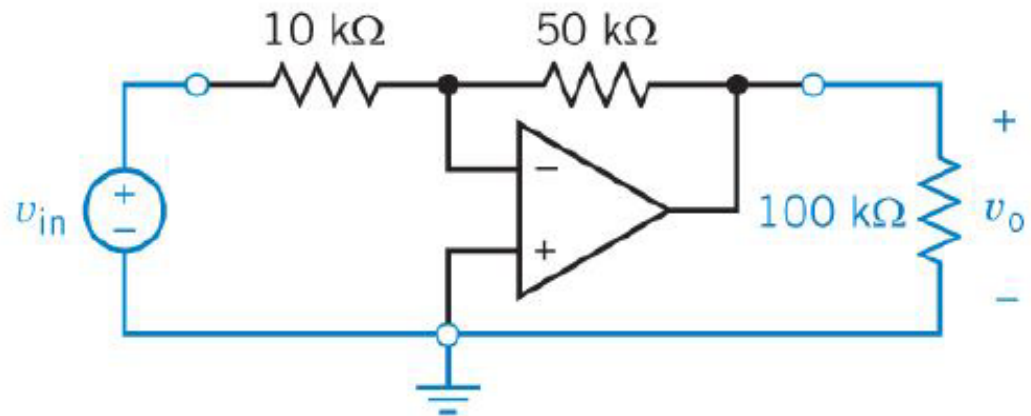
We want an amplifier circuit and  $K$  is called the **gain**. The gain can essentially be any number.

In general: Choose resistors in real op amp circuits between  $5\text{ k}\Omega$  and  $500\text{ k}\Omega$  (this is because the currents in IC's must be small)

# Amplifiers

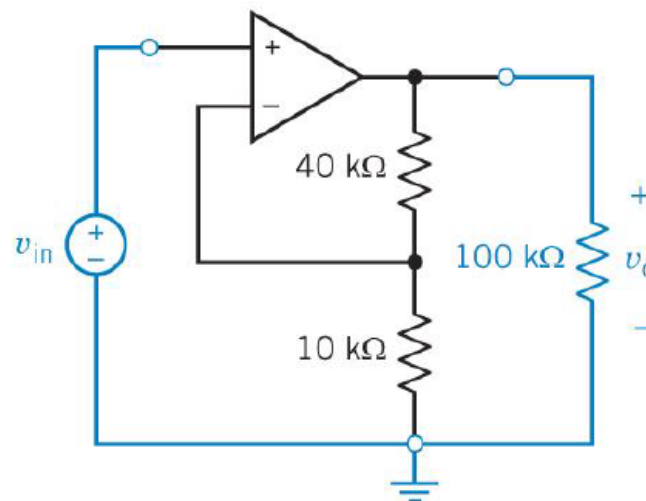
Inverting amplifier, where the gain is  $-R_f / R_1$

$$K = -5$$



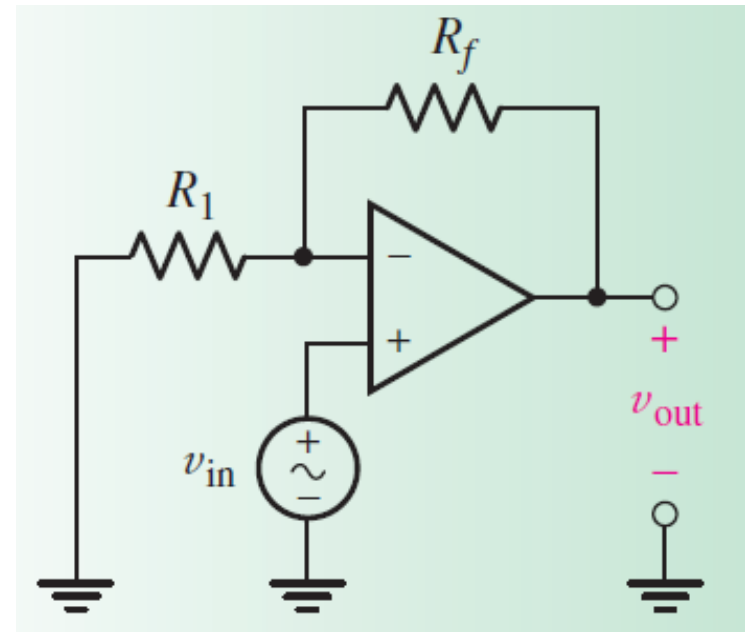
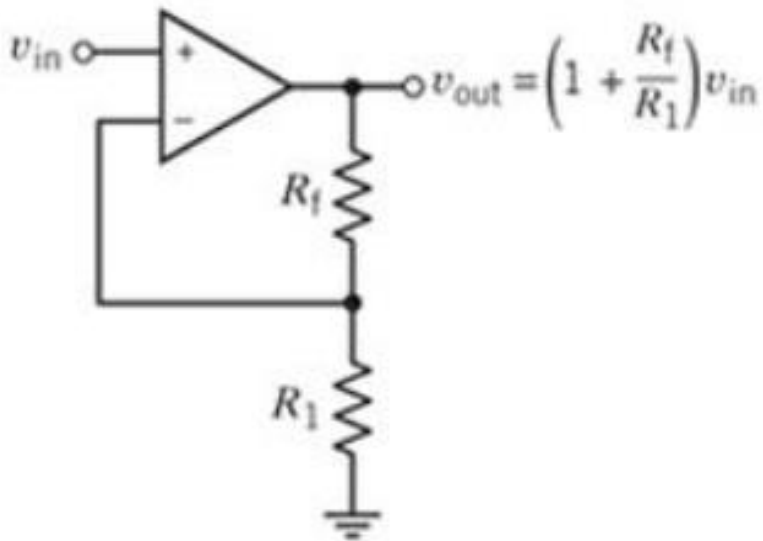
Noninverting amplifier, where the gain is  $(1 + R_f/R_1)$

$$K = 5$$

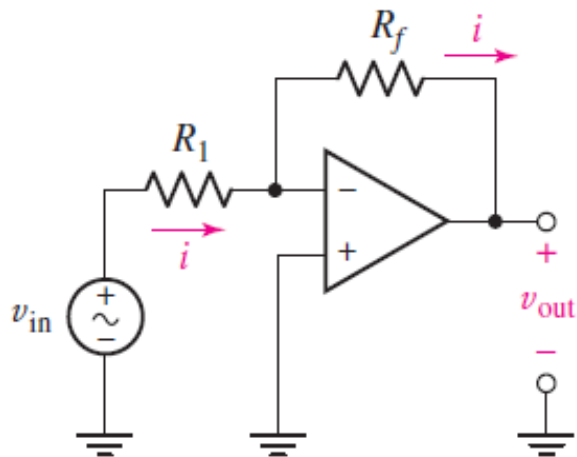


# Top terminal (from each textbook)

For example, a noninverting amplifier.



# Virtual ground



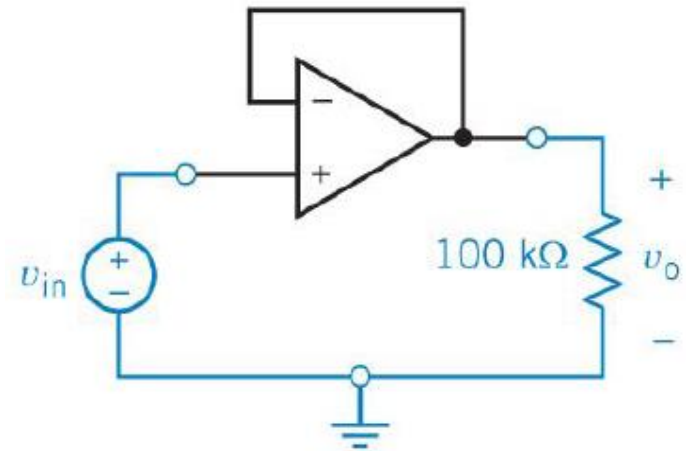
The fact that the inverting input terminal finds itself at zero volts in this type of circuit configuration leads to what is often referred to as a “virtual ground.” This does not mean that the pin is actually grounded, which is sometimes a source of confusion for students. The op amp makes whatever internal adjustments are necessary to prevent a voltage difference between the input terminals. The input terminals are not shorted together.



# Amplifiers

Voltage follower (buffer amplifier)  
(same as  $R_f = 0$ , or  $R_1 = 0$  in noninverting amp )

$$K = 1$$

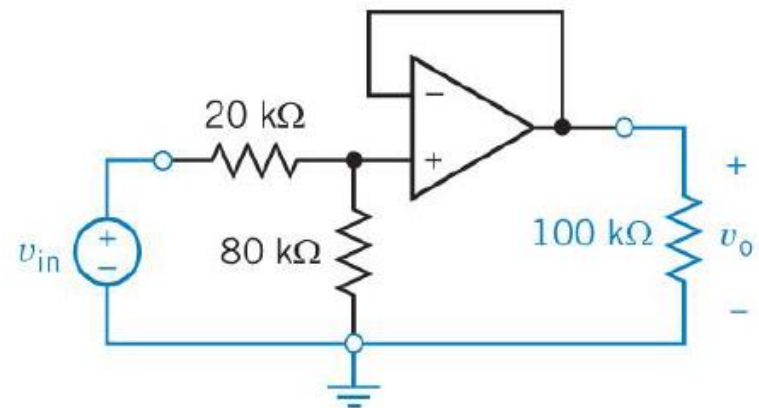


For gains  $0 < K < 1$  no amplifier exists but we can use:

Voltage divider + Follower

Here,  $K = 0.8$

This arrangement could be called an attenuator



# Using the noninverting summing amplifier

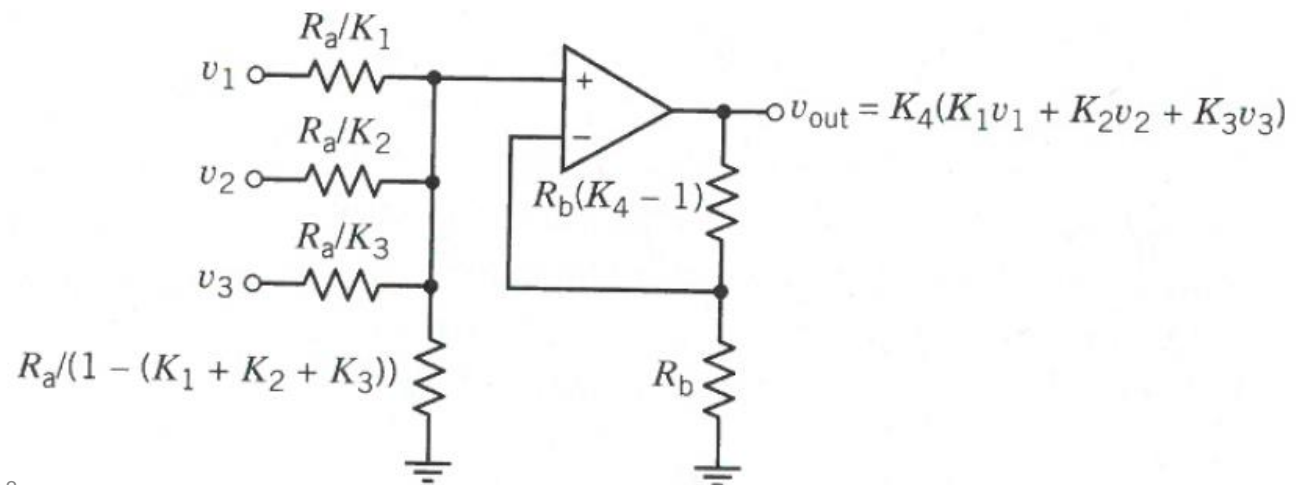
Want to achieve the following summation of voltages:

$$v_0 = 2v_1 + 3v_2 + 4v_3$$

Want inputs between -1 V and 1 V

Consider an op amp that has  $i_{\text{sat}} = 2 \text{ mA}$  and  $v_{\text{sat}} = 15 \text{ V}$

Solution: The noninverting summing amplifier



# Example 6.5.3

Want to achieve the following:  $v_0 = 2v_1 + 3v_2 + 4v_3$

Need to choose the values for  $K_i$  ( $i=1,2,3$ ) and  $R_a$  and  $R_b$

We must have  $K_1 + K_2 + K_3 < 1$  (so that the LH bottom resistor is not negative or infinite)

Choose  $K_1 = 0.2$ ,  $K_2 = 0.3$   $K_3 = 0.4$  (this keeps  $K_1 + K_2 + K_3 < 1$  )

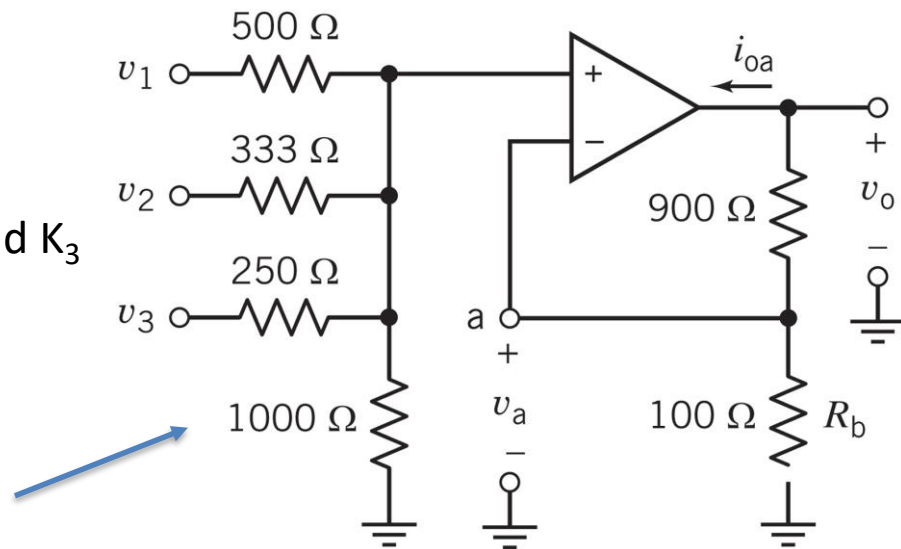
To keep  $v_0 = 2v_1 + 3v_2 + 4v_3$ , the output is  $v_0 = K_4 (K_1 v_1 + K_2 v_2 + K_3 v_3)$  so we choose  $K_4 = 10$

We now have what we :

$$v_0 = 10(0.2v_1 + 0.3v_2 + 0.4v_3)$$

Which means we now have  $K_1$ ,  $K_2$  and  $K_3$

Since we have some flexibility in the values of  $R_a$  and  $R_b$  we choose  $R_a = R_b = 100 \Omega$ , now we have



# Checking the circuit

## Node equations

Noninverting terminal:

$$(v_a - v_1)/500 + (v_a - v_2)/333 + (v_a - v_3)/250 + v_a/1000 = 0$$

Inverting terminal:

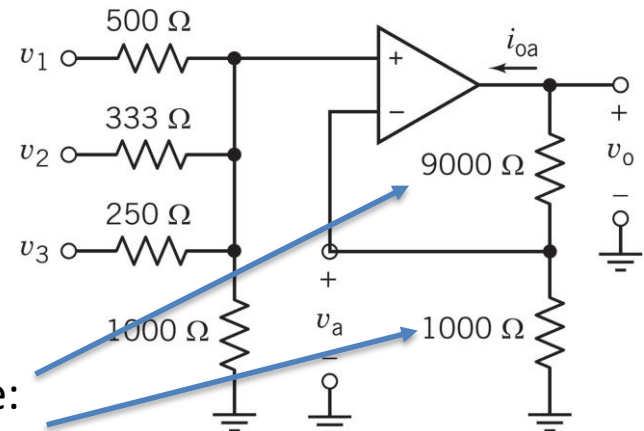
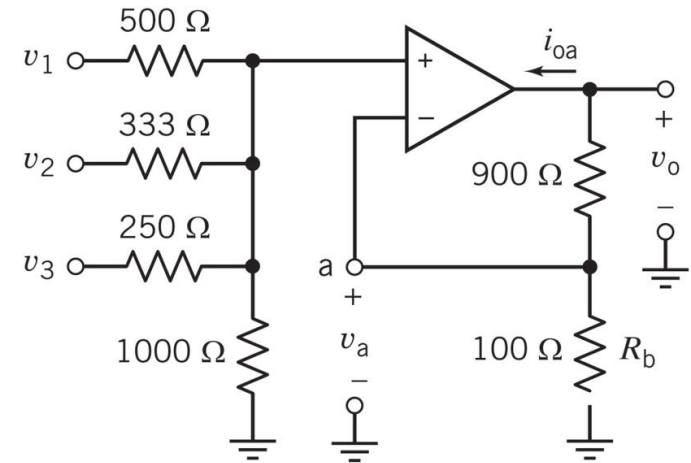
$$(v_a - v_o)/900 + v_a/100 = 0. \text{ Solving gives: } v_a = v_o/10.$$

Putting this into top equation:  $v_o = 2v_1 + 3v_2 + 4v_3$

Let's check to see if it is operating in "ideal" conditions

Max. value:  $|v_o| = 2|v_1| + 3|v_2| + 4|v_3|$  since  $|v_1| < 1 \text{ V}$  then

$|v_o| < 9 \text{ V}$ , which is  $< v_{\text{sat}}$  but  $i_o = |9\text{V}/1000| = 9 \text{ mA} > i_{\text{sat}}$  so use:



# Solving linear algebraic equations



# Algebraic equations

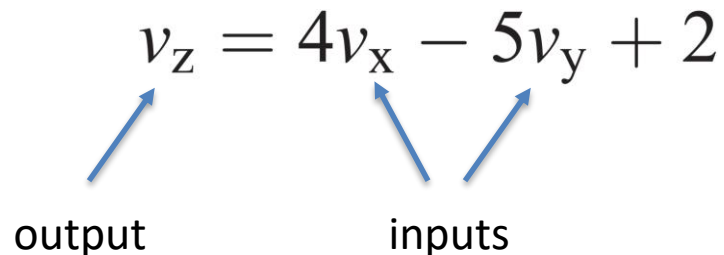
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Assume we want to solve this  $z = 4x - 5y + 2$

This could be solved using an op amp circuit to give:  $v_z = 4v_x - 5v_y + 2$

A **signal** is a voltage or current used to represent something.

Here  $v_x$ ,  $v_y$ ,  $v_z$ , are signals representing  $x$ ,  $y$ ,  $z$

$$v_z = 4v_x - 5v_y + 2$$


output

inputs

# Block diagrams

Our equation can be re-written as:

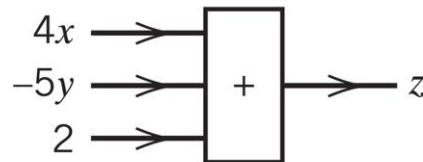
$$z = 4x + (-5)y + 2$$

This can be visualised as:



(a)

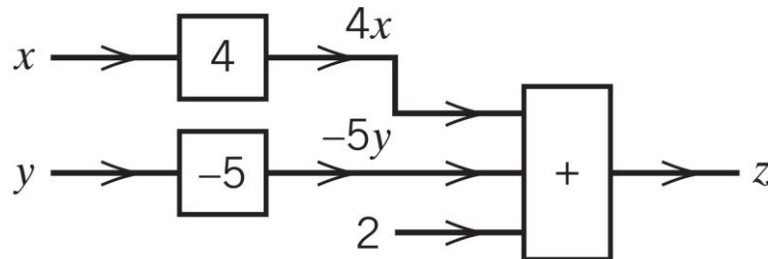
Multiplication block



(b)

Addition block

Block diagram for our equation:

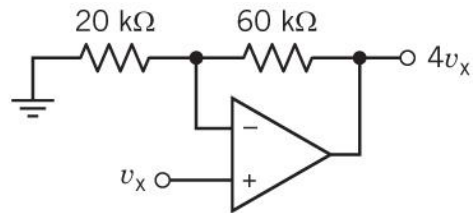


# Designing the circuit

Op amp amplifiers are an obvious choice



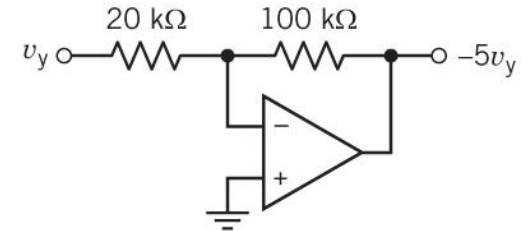
(a)



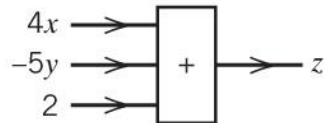
(b)



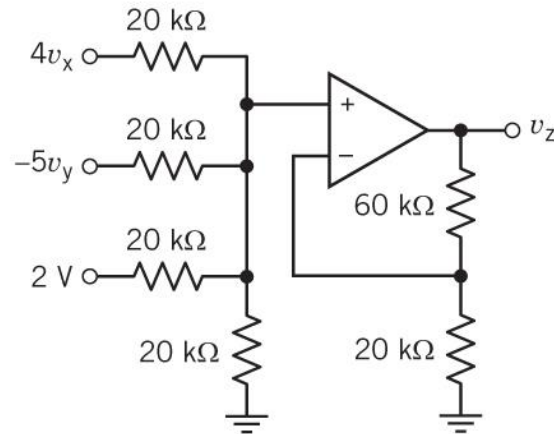
(c)



(d)



(e)



(f)

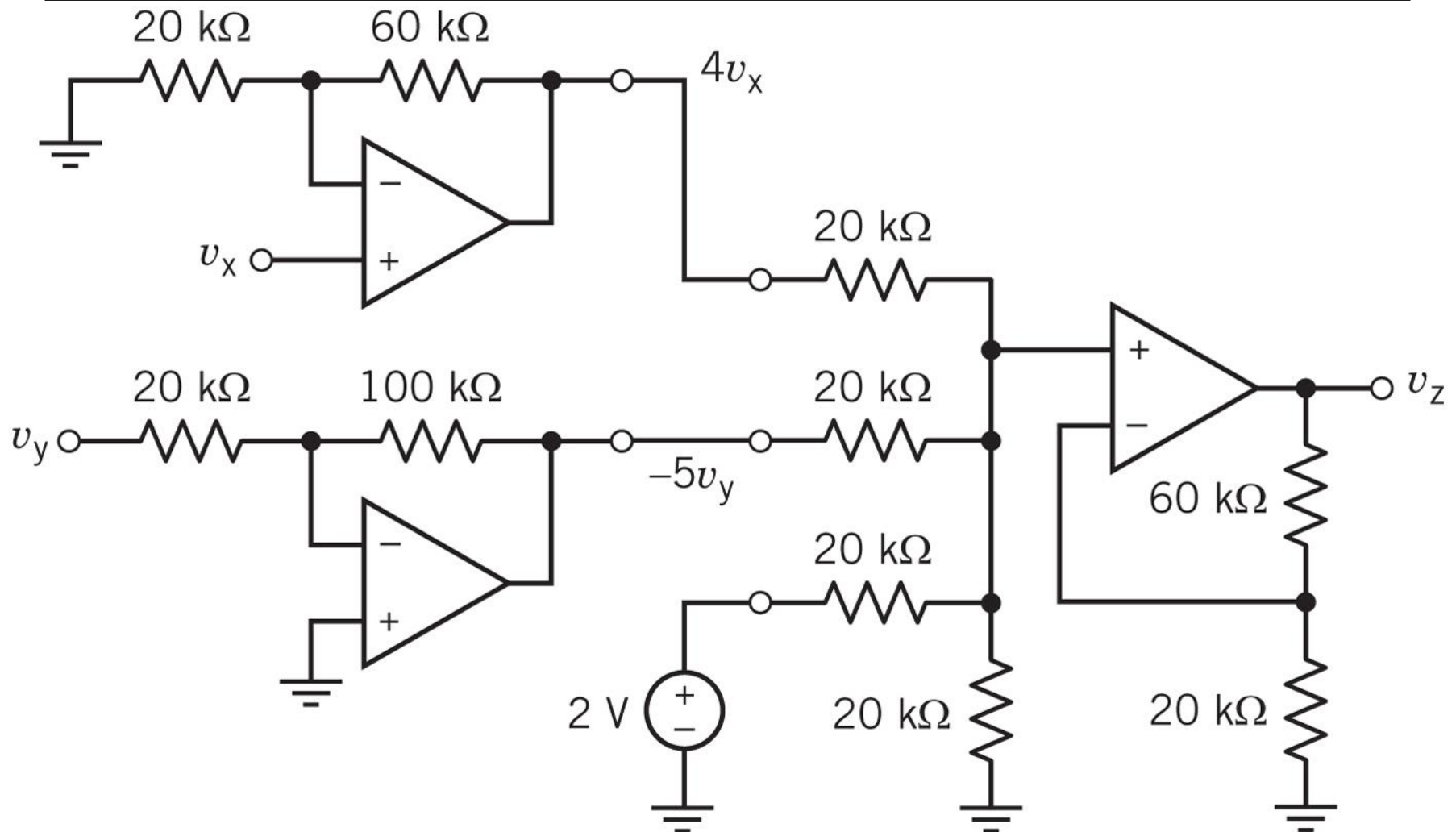
$$K_4 = 4$$

$$K_1 = K_2 = K_3 = 0.25$$



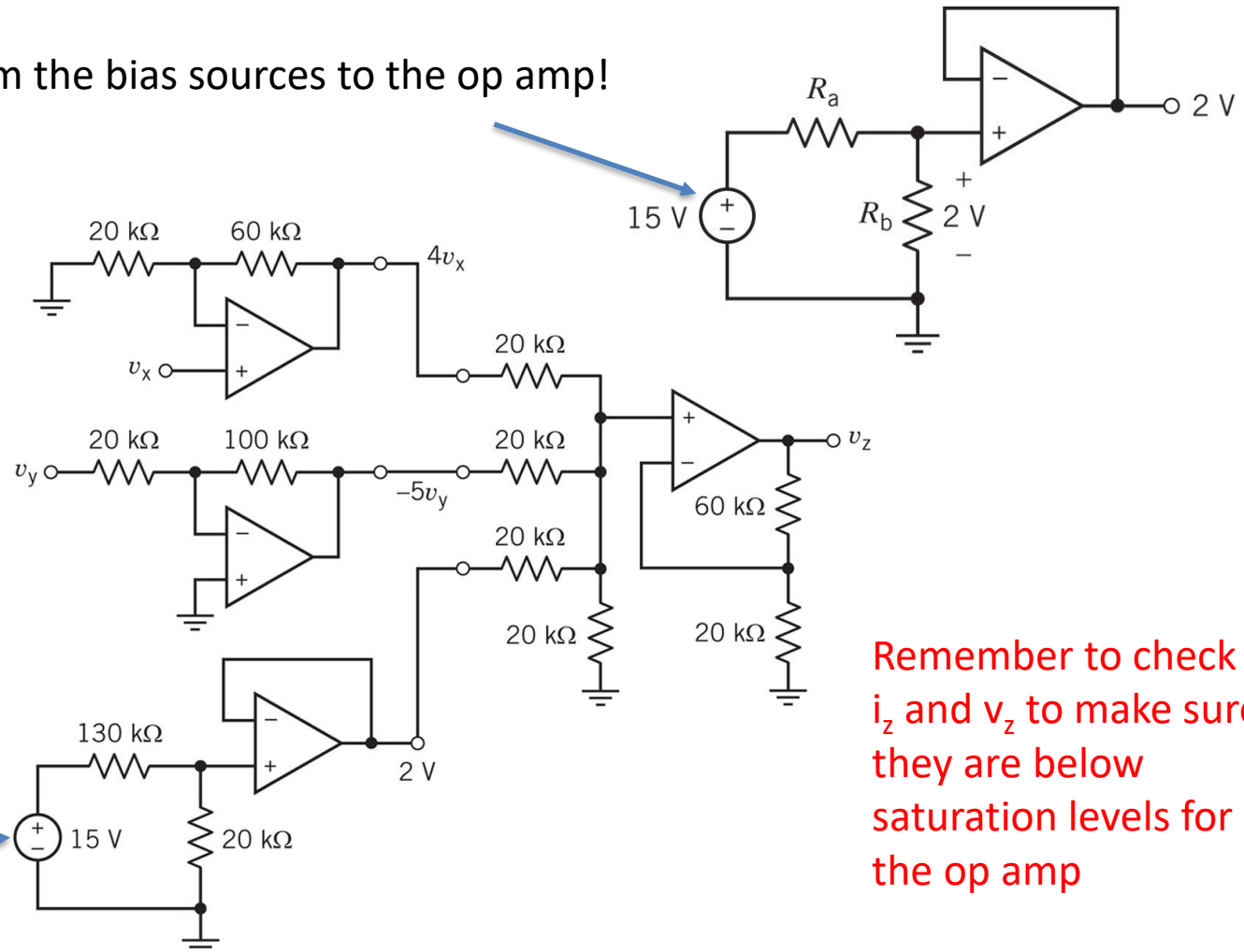


# Overall circuit



# How can we improve this?

Take the 2 V input from the bias sources to the op amp!



Which produces:

bias source to  
the op amp

Remember to check  
 $i_z$  and  $v_z$  to make sure  
they are below  
saturation levels for  
the op amp

# Characteristics of practical operational amplifiers



# Real op amps

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We know that ideal op amps have:  $i_1 = 0$ ,  $i_2 = 0$ , and  $v_1 - v_2 = 0$

Real op amps have:

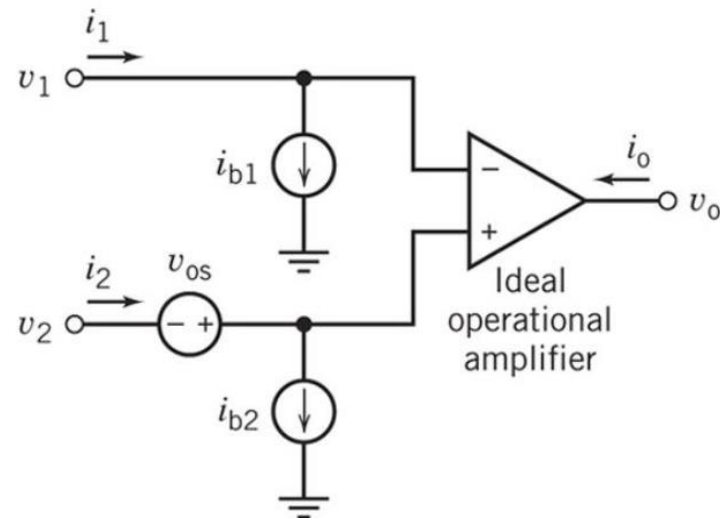
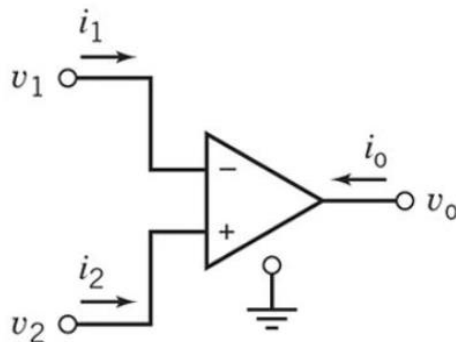
- Nonzero bias currents
- Nonzero input offset voltage
- Finite input resistance
- Nonzero output resistance
- Finite voltage gain

For small signals, the actual currents and voltages in real op amps become very important

# Models for realistic op amps

The **offset model** accounts for:

- Nonzero bias currents
- Nonzero input offset voltage





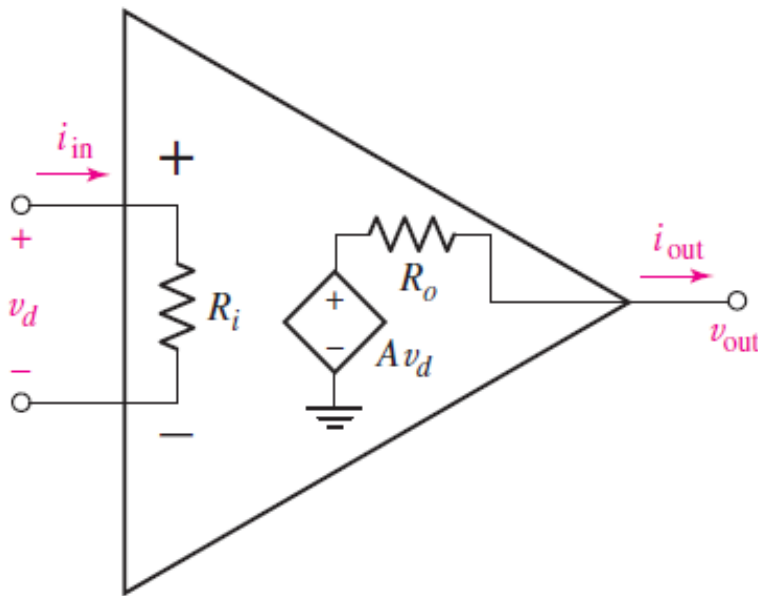
# Actual op amps

TABLE 6.3 Typical Parameter Values for Several Types of Op Amps

Part Number	$\mu$ A741	LM324	LF411	AD549K	OPA690
Description	General purpose	Low-power quad	Low-offset, low-drift JFET input	Ultralow input bias current	Wideband video frequency op amp
Open loop gain $A$	$2 \times 10^5$ V/V	$10^5$ V/V	$2 \times 10^5$ V/V	$10^6$ V/V	2800 V/V
Input resistance	2 M $\Omega$	*	1 T $\Omega$	10 T $\Omega$	190 k $\Omega$
Output resistance	75 $\Omega$	*	$\sim 1$ $\Omega$	$\sim 15$ $\Omega$	*
Input bias current	80 nA	45 nA	50 pA	75 fA	3 $\mu$ A
Input offset voltage	1.0 mV	2.0 mV	0.8 mV	0.150 mV	$\pm 1.0$ mV
CMRR	90 dB	85 dB	100 dB	100 dB	65 dB
Slew rate	0.5 V/ $\mu$ s	*	15 V/ $\mu$ s	3 V/ $\mu$ s	1800 V/ $\mu$ s
PSpice Model	✓	✓	✓		

\* Not provided by manufacturer.  
 ✓ Indicates that a PSpice model is included in Orcad Capture CIS Lite Edition 16.3.

# Open loop gain



Note the actual **open-loop voltage gain** is very large compared to what we encountered in our previous examples.

There is a distinction between this parameter and the closed-loop voltage gain that characterises a particular op amp circuit. The “loop” refers to an external path between the output pin and the inverting input pin; it can be a wire, a resistor etc. depending on the application.